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Measuring Potential Growth in Japan: Some Practical Caveats

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Economists at central banks and in academia have made various efforts to measure potential growth, something that cannot be observed directly. This review introduces some of these estimation techniques and applies them to the Japanese data. The estimates of the potential growth rate can differ considerably, depending on the methods used; all of these estimates are subject to substantial errors; and the reliability of the estimates is severely hampered whenever turbulence is rife in the economy. Although all approaches introduced in this review suggest that the potential growth rate in Japan has recently declined, significant uncertainty remains regarding the magnitude of its fall. Substantial margins of error must be taken into account with any estimates of the potential growth rate.

1. *Introduction*

Potential growth, which frequently appears in macroeconomic analyses, is notoriously difficult to handle in practice. Much of the difficulty arises from its measurement. Since it is a latent variable that cannot be observed directly, potential growth needs to be estimated in one way or another for its measurement.

Estimating potential growth of the economy is likened to measuring the potential of an athlete such as a marathon runner. Her day-to-day performance depends not only on her potential, but also on her physical and mental conditions at the time. Even if she has not run fast several times in a row, it is difficult to tell whether her potential has declined or her conditions at the time happened to be subnormal. As described below, the core of estimating potential growth lies in the distinction between potential (a long-run trend) and conditions (short-run oscillations mainly incurred by business cycles). As in the case of the marathon runner, it is very difficult to distinguish between the two of them, despite all the sophisticated techniques economists have developed.

Estimating potential growth of the latest—the most relevant for policy organizers—is particularly problematic. With the benefit of hindsight, we know, say, how long a marathon runner has continued to register lackluster records. If she had continued her poor performance over a long period, it would be more likely that her potential had declined. Due to lack of this knowledge—we do not know how she will do *tomorrow*—measuring her potential today becomes much more challenging. Likewise, potential growth of the recent periods is more difficult to estimate than measuring just one past performance. This is especially so at the time when the economy shows exceptionally large volatility, as has been the case for the past couple of years.

In addition, the data revision causes another problem for estimating potential growth. While the records of a marathon runner are seldom revised, economic data such as real GDP, on which any estimates of potential growth are based, *are* subject to substantial revision. The revision sometimes goes back to data points several (or more) years old. As a consequence, the estimates of potential growth, even including those in the deeper past, are also revised substantially.[1]

The rest of this review is structured as followed. Section 2 briefly explains what potential growth is at a conceptual level. Section 3 describes how potential growth is estimated in practice, and shines a light on it to see what it looks like once these techniques are applied to recent Japanese data. Section 4 discusses whether potential growth has recently declined in Japan. Section 5 concludes the review.

2. *What Is Potential Growth?*

Potential growth is defined in various ways. Indeed, no "definitive" definition of potential growth even exists in economics. However, in economic commentaries, it is often used as economic growth that can be sustained over a reasonably long run. It is also called *cycle-adjusted trend growth*, since the effects of business cycles may well be smoothed out in the long run. Such growth rate plays a central role in long-run economic projections and policy simulations. An example can be found in the latest IMF Article IV consultation of Japan, in which the 1.2 percent growth rate is assumed for the long-run simulations of the public sector debt sustainability. This 1.2 percent corresponds to the IMF staff's estimate of the potential growth of the Japanese economy.

Potential growth is closely related with the output gap, which is the difference between potential and actual outputs. It is generally thought that potential output—the change in which is potential growth—reflects the supply capacity of the economy and that actual output mainly depends on effective demand at the time. As a consequence, the output gap reveals the supply-demand conditions of the economy. The larger the positive the output gap becomes, the faster prices tend to increase, and vice versa. This relationship, known as the Phillips curve, makes potential growth and the output gap indispensable for the analyses of price developments.

Potential growth is also closely related with the equilibrium real interest rate or the natural rate of interest.[2] It is reasonable to think that, in the long run, the expected return from investment projects is more or less same as the long-run sustainable growth rate of the economy. In theory, if the actual real interest rate is above (below) the equilibrium real interest rate, it restricts (stimulates) effective demand and hence the output gap deteriorates (improves). The relationship, known as the IS equation, is used for assessing the level of the policy interest rate.

3. *How to Estimate Potential Growth?*

The staff at the Bank of Japan, like researchers at other central banks and in academia, has tried various approaches to estimate potential growth. These approaches can be categorized into those utilizing (1) filtering techniques; (2) a production function; (3) the Phillips curve; and (4) a Dynamic Stochastic General Equilibrium (DSGE) model.[3] Let's take a look at each.

(1) Filtering Approach

The filtering approach tries to estimate potential growth from a trend derived by a time-series analysis. Time-series data such as real GDP is supposed to comprise a trend, cycle components, and others:

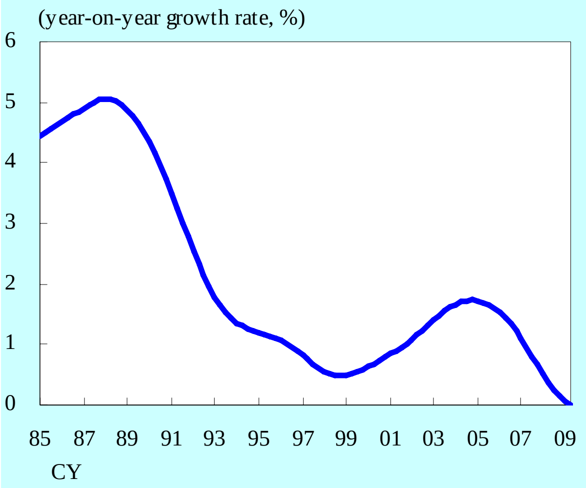
Real GDP = Trend × Cycle × Others (an irregular component, etc.)

"Filtering" in a time-series analysis is a statistical method that enables researchers to obtain a desired component (in this case, a trend component) by filtering the original data. A number of filtering methods have been developed. These include, among others, a simple one such as taking a backward moving average, and a more sophisticated (but also complicated) one such as applications using frequency-domain estimation and a state-space form.[4]

All these filtering techniques presume that a trend component must be something smooth; they try to identify it by making an a priori assumption on its smoothness explicitly or implicitly. For instance, the number of averaged periods (or the size of the window, in a statistical jargon) needs to be specified for taking a backward moving average. The longer the periods are, the smoother the estimated trend becomes, because fluctuations of individual observations are better smoothed out.

In other words, there is a smaller chance of mixing the trend component with the cycle and the irregular components. At the same time, however, there is a larger chance of omitting a turning point in the case of a sudden change in the trend component. Graph 1 shows potential growth obtained by the Hodrick-Prescott filter, which is one of the most frequently used filtering methods, with a commonly assumed degree of smoothness.[5]

【Graph 1】 Filtering Approach



The filtering approach is quite popular among practitioners because of its straightforwardness—it only requires real GDP as an input and a simple filter like the Hodrick-Prescott filter, which can be easily implemented by a standard statistical package. However, it is quite difficult to detect the latest potential growth by the filtering approach. Having reliable estimates requires enough observations not only prior to the data point, but also after it. Furthermore, depending on an assumed smoothness, the approach may falsely detect a change in potential growth in the presence of large cyclical and/or irregular movements. It may also underestimate a change in potential growth in the presence of an abrupt shift in the level of potential output.

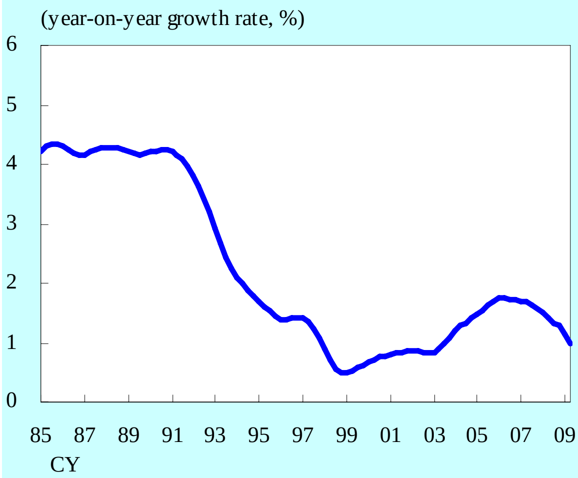
These drawbacks can be seen as a cost associated with its straightforwardness. The straightforwardness even enables a practitioner to use the approach as a statistical black box without considering the economics behind it. However, this would mean that he does not fully exploit available information. The other approaches discussed below try to overcome these difficulties by taking account of a structure of the economy such as a production function, the Phillips curve, and the IS equation.

(2) Production Function Approach

The production function approach tries to gauge the supply capacity of the economy by estimating a production function. According to growth accounting, which is derived from a production function, a change in production of the economy is the weighted average of changes in factor inputs (such as capital and labor) plus a contribution of total factor productivity (TFP). Potential growth as a change in supply capacity can be estimated from this growth accounting using the capital and labor inputs available at the time (instead of actual capital and labor inputs) and the TFP.

Graph 2 presents potential growth estimated by the production function approach. Research and Statistics Department of the Bank of Japan uses the estimates by this approach as a benchmark, which are reported in the Bank of Japan *Outlook for Economic Activity and Prices*.[6]

【Graph 2】 Production Function Approach



The production function approach has several advantages over the filtering approach. First, instead of deriving a trend component of real GDP from a statistical black box, it can rely on richer information. For instance, it can exploit a long-run trend of labor force, which is relatively easy to detect, given stable population growth. Second, it allows economic interpretation of a change in potential growth. Using growth accounting, the change in potential growth is attributable to the capital and labor inputs and the TFP.

The corresponding output gap has a broadly positive correlation with the CPI inflation rate in Japan.[7] However, this may not be necessarily the case, because, in contrast to the other two approaches discussed below, the production function approach does not explicitly take into account the Phillips curve relationship.

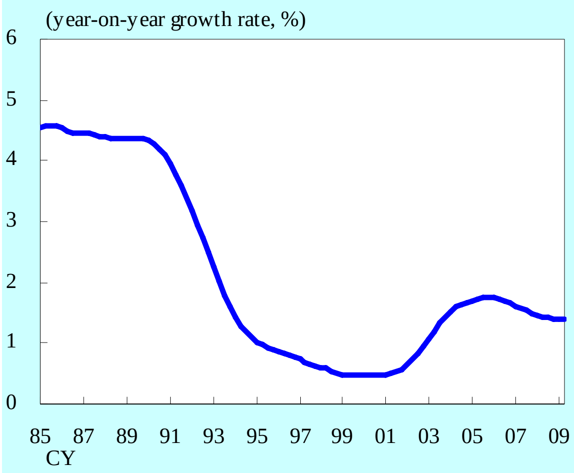
Although it relies on richer information compared with the filtering approach, the production function approach is not immune to the identification problem between a long-run trend and short-run oscillations. To obtain a long-run trend of the TFP, the production function approach needs to use the time-series techniques of filtering. A change in the TFP, commonly called the "Solow residual" after the economist who first calculated this,[8] is a residual that cannot be ascribed to capital and labor inputs. In other words, the TFP is supposed to reflect anything that is not captured by capital and labor inputs. It may include effects of technological progress as well as business cycles, and hence, for estimates of potential growth, the actual TFP needs to be filtered to obtain its trend part. This implies that the production function approach, to a certain degree, is subject to the same problems as the filtering approach discussed above.

(3) Phillips Curve Approach

The Phillips curve approach extends the filtering approach by taking into account not only real GDP, but also inflation. It tries to estimate potential output so as to maximize the fit of its implied output gap with inflation.[9] Technically, it simultaneously estimates an equation that decomposes actual GDP to the trend and the cycle components, and an equation that corresponds to the Phillips curve. Graph 3 exhibits potential growth obtained by this approach.

It is not surprising to see that the output gap estimated by this approach outperforms those obtained by the above two approaches in terms of the fit with the inflation rate. However, this does not necessarily mean that the approach provides better forecasts of inflation. In fact, there is an element of circularity: if an analyst tries to use thus estimated output gap to predict the future course of inflation, he needs to determine future inflation by another method, since it is required to estimate the future output gap.

【Graph 3】Phillips Curve Approach



The approach is not used much by practitioners compared with the above two approaches, presumably because its computation is technicality more demanding. In addition, the estimated potential growth (and the output gap) tends to be sensitive to the specification of the Phillips curve.

Despite these caveats, the approach has attracted some researchers. These researchers have extended the approach so as to take into account the IS equation in addition to the Phillips curve.[10]

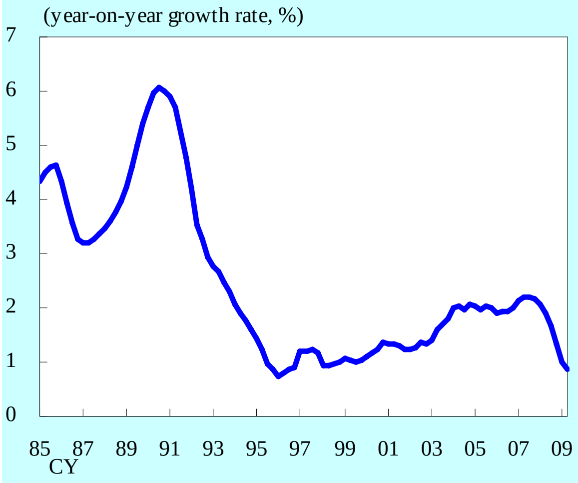
(4) DSGE approach

A Dynamic Stochastic General Equilibrium (DSGE) model is a theory-oriented macroeconomic model, in which developments of the economy are described as a general equilibrium determined by rational forward-looking behaviors of households and firms. Recently, the model has gained popularity among researchers and has been applied to analyze various economic issues.

Many DSGE models define the potential output as the natural level of output, which would be attainable under flexible prices. Accordingly, potential growth and the output gap are calculated based on this natural level of output.[11] The model usually embeds various adjustment costs, including the "menu" cost, which deters firms from adjusting their nominal prices flexibly. Due to these costs, the actual output may be different from one without these rigidities.

The DSGE approach tries to obtain the potential growth as what is implied by the estimated DSGE model. Thanks to advances in computation, a number of researchers have matched the theory-oriented DSGE model with the actual data.[12] However, in most cases, potential growth has been given exogenously by assuming some statistical trends. In recent years, some researchers have attempted to estimate the DSGE model including these trends.[13] Graph 4 presents thus obtained potential growth in Japan.[14]

【Graph 4】DSGE Approach



In the model, changes in the natural level of output are incurred by exogenous population growth as well as various shocks. Among these shocks, the most relevant shocks for the analysis of long-run growth are productivity shocks, of which the model assumes transitory ones and permanent ones (see Box below). In the above graph, changes in the natural level of output generated by permanent productivity shocks and population growth are shown, since those generated by transitory shocks, including transitory productivity shocks, can be ignored in the long run. This makes the potential growth rate estimated by the DSGE approach comparable with those by the other approaches that seek the long-run trend economic growth rate.

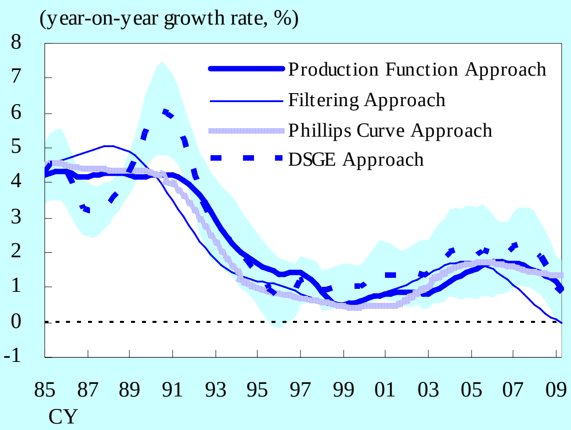
There are several advantages to the DSGE approach. First, given the theory-consistency of the model, researchers can easily trace the theoretical background of the estimate results. Second, as a byproduct of estimation, the confidence intervals of the potential growth rate can be obtained.[15]

Despite its attractiveness, the DSGE approach also has a number of caveats for its practical usage. For instance, the computational burden is much heavier than it is with the other approaches. As discussed in the Box, there is some chance that estimation results, such as the smoothness of potential growth, are sensitive to the specification of the model. This presumably suggests that the approach is not perfectly immune to the identification problem discussed above. Since the approach is still in the early stages of research, further progress in this field should resolve some of these problems in the future.

4. Recent Decline in Potential Growth

Graph 5 redraws the potential growth rates indicated in Graphs 1 to 4. All of them suggest: (i) potential growth declined to about one percent in the latter half of the 1990s from much higher levels in the latter half of the 1980s; and (ii) after the turn of the millennium, potential growth rose somewhat, but it fell again in the last couple of years. According to studies by international institutions and other central banks, a decline in potential growth, albeit to a different degree, has also been observed recently in other industrial countries.[16]

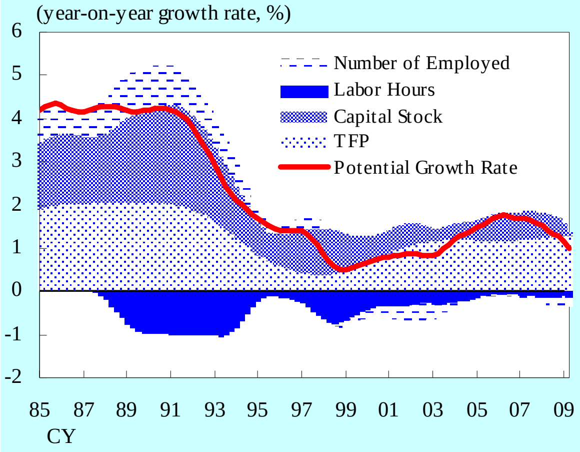
【Graph 5】Various Estimates of Potential Growth Rate



Note: Shaded region is the 90% confidential interval around potential output growth rate estimated by DSGE approach.

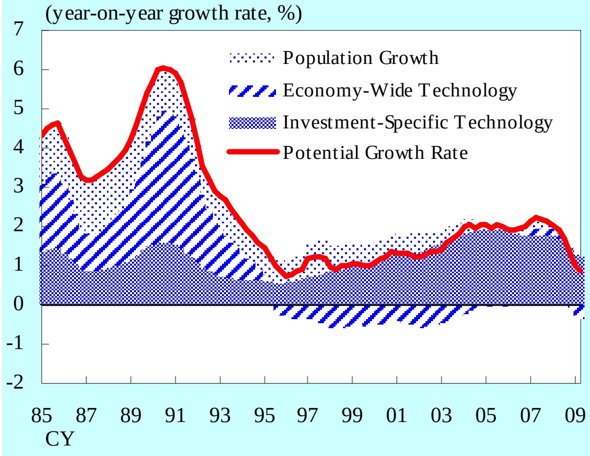
The production function approach indicates that the deceleration in capital accumulation accounts for the recent decline in potential growth (Graph 6). This reflects a plunge in business fixed investment after 2008. Meanwhile, the TFP has continued to raise potential growth by about one percentage point up to the latest period. As discussed above, because the TFP is subject to the filtering, there is increasingly large uncertainty regarding its estimation toward the end point of the data. Given an almost free fall of economic activity from the late 2008 to early 2009, the TFP, including that of past couple of years, is likely to be revised downward unless the economy registers a sharp recovery in the near future to compensate the large fall during those periods.

【Graph 6】 Decomposition of Potential Growth Rate (Production Function Approach)



On the other hand, the DSGE approach suggests that permanent productivity shocks are mainly responsible for the recent decline in potential growth (Graph 7). In the DSGE model, both capital and labor inputs are endogenous variables and their changes are further traced to permanent productivity shocks and exogenous population growth. Following the recent literature, the estimated DSGE model takes into account rapid technological progress in the sector producing investment goods, and hence permanent productivity shocks can be decomposed to those for the economy wide and those specific to the investment goods sector.[17] In Graph 7, both productivity shocks lowered potential growth. However, to the extent that the DSGE model faces the identification problem discussed above, as with the production function approach, considerable uncertainty remains regarding the estimation of these productivity shocks.

【Graph 7】 Decomposition of Potential Growth Rate (DSGE Approach)



5. *Conclusion*

Despite all the efforts made by economists, there is no silver bullet for estimating potential growth accurately. This is because, given the limited information of observed data, it is inherently difficult to identify a long-term trend and short-tem oscillations, both of which are unobservable. This is especially so for the latest observations. Such difficulty further increases when the economy shows exceptionally large volatility, as has been the case for the recent recession. If the level of potential output shifted downward abruptly, any approaches discussed in this paper necessarily lag behind to detect it, to the extent that a change in potential growth is assumed to be smooth.[18]

Substantial uncertainty is associated with the recent estimates of potential growth. The latest reading of the potential growth rate can be from nil to one percent, depending on the approaches used (Graph 5 above). In light of the 90% confidence intervals of the DSGE model alone, the spread of estimates further increases to the range of zero to two percent. On top of that, the spread would become wider, if the effects of data revision and uncertainty with estimation at the end point of the data were fully taken into account in calculation of the confidence intervals. The bottom line for policymakers is that they should be sure enough about how unsure the recent estimates of potential growth and the output gap really are.

BOX: Applying the DSGE approach to Japan's economy

In Graph 4 in the text, a DSGE model is estimated using Japanese data from 1981Q1 to 2009 Q2. In most respects, the estimated DSGE model follows now-conventional New Keynesian DSGE models in the literature: it has the forward-looking Phillips curve and IS equation plus the monetary policy rule; it embeds some rigidity in nominal price and wage setting; and it assumes that price and wage are marked up owing to monopolistic competition. A main difference comes from the fact that the estimated potential growth rate at each point of time is endogenously determined in the model rather than exogenously given as a trend derived by, say, the Hodrick-Prescott filter.

More specifically, the model assumes two types of productivity shocks: one is permanent and the other is transitory. Suppose the production function is expressed as

$$Y_t = (A_t Z_t L_t)^{\alpha} K_t^{1-\alpha},$$

where Y_t is output, L_t is labor input, K_t is capital input, and α is the labor share. A_t and Z_t are transitory and permanent productivity shocks, respectively, that evolve along the following dynamics:

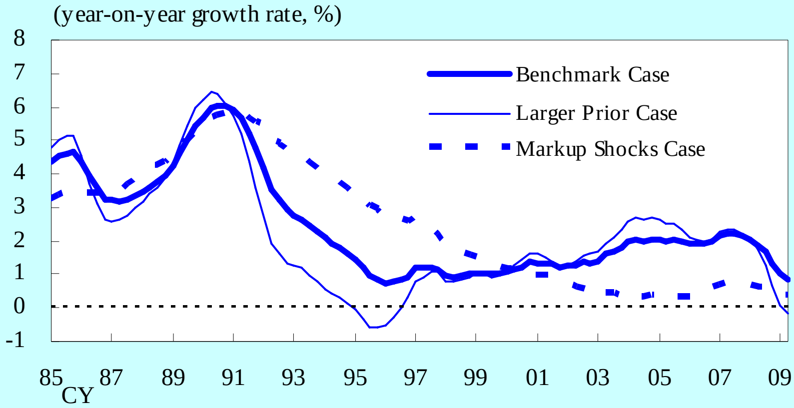
$$\log A_t = (1-\rho_A)\cdot \log A_0 + \rho_A \cdot \log A_{t-1} + \varepsilon_{A,t},$$

$$\log Z_t - \log Z_{t-1} = (1-\rho_Z)\cdot g + \rho_Z \cdot (\log Z_{t-1} - \log Z_{t-2}) + \varepsilon_{Z,t},$$

where ρ_A, ρ_Z, A_0, g are parameters to be estimated and $\varepsilon_{A,t}$, $\varepsilon_{Z,t}$ are error terms. In the first equation, $\log A_t$ depends on its own lag $\log A_{t-1}$. This implies that $\log A_t$ will become larger for a while, if there is a positive one-time shock in $\varepsilon_{A,t}$, but it will return to the original level $\log A_0$ in the end, as long as ρ_A is smaller than unity in the absolute value. Because the positive effects of A_t on Y_t will die out, this is called a "transitory" shock. On the other hand, in the second equation, instead of $\log A_t$, $\log Z_t - \log Z_{t-1}$ depends on its own lag. As $\log A_t$ converges to $\log A_0$, $\log Z_t - \log Z_{t-1}$ converges to g after a positive one-time shock in $\varepsilon_{Z,t}$: i.e., $\log Z_t = \log Z_{t-1} + g$ at the time of convergence. Because of a unit coefficient on $\log Z_{t-1}$, Z_t permanently takes a higher values once its level is lifted—nonstationarity in econometrics. Because the positive effects of Z_t on Y_t will not wane, this is called a "permanent" shock. As explained in the text, the potential growth rate in Graph 4 is the sum of effects due to (i) population growth, which is an exogenous component of a change in labor input, and (ii) a change in permanent productivity shocks, $\log Z_t - \log Z_{t-1}$.

The smoothness of the estimated potential growth rate partly depends on the size of $\varepsilon_{Z,t}$ that is originally assumed by a researcher—a "prior" in the lexicon of Bayesian econometrics, which is used for estimating the DSGE model in this paper. If $\varepsilon_{Z,t}$ is assumed to take relatively small values, $\log Z_t - \log Z_{t-1}$ tends to become smooth, and vice versa. In the graph, following the existing studies, a prior on $\varepsilon_{Z,t}$ is set to be within the plus or minus 0.2 percent range with a probability of 95 percent. If the range is set as wide as plus or minus 1 percent, the estimated potential growth rate becomes more volatile, as indicated by "larger prior case" in the following graph.

【BOX Graph】 Various Estimates of Potential Growth Rate by DSGE Approach



The estimated potential growth rate also depends on how to specify other parts of the DSGE model. If the model assumes that markups are subject to some structural shocks, then the potential growth rate becomes higher in the 1990s and lower in the 2000s ("markup shocks case" in the same graph). This is presumably because the model attributes the weak price and wage developments in the 2000s to a shock that lowered the markup ratio rather than to the larger negative output gap. That is, the potential growth rate should be lower so as to make the output gap less negative. The opposite force seems to have worked in the 1990s.

The DSGE model assumes the two-sector production structure: the sector producing consumption goods and that producing investment goods; and each sector is affected by both transitory and permanent shocks to productivity. Production functions of two sectors are expressed as:

$$Y_{C,t} = (A_t Z_t L_{C,t})^{\alpha} K_{C,t}^{1-\alpha} ,$$
$$Y_{K,t} = (A_t Z_t A_{K,t} Z_{K,t} L_{K,t})^{\alpha} K_{K,t}^{1-\alpha} ,$$

where subscript C represents the consumption goods sector and subscript K represents the investment goods sector. A_t and Z_t are the economy-wide transitory and permanent productivity shocks, and $A_{K,t}$ and $Z_{K,t}$ are the investment goods specific transitory and permanent productivity shocks, respectively.

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[1] See Box 2 of the Bank of Japan *Outlook for Economic Activity and Prices* (April 2009). Uncertainty about the measurement of potential growth is discussed by a number of papers including Orphanides, A., "Monetary Policy Rules Based on Real-Time Data," *American Economic Review* 91(4), 964-985, 2001.

[2] In a standard macroeconomic model, subject to the relative risk aversion and the discount rate, the natural rate of interest is more or less equal to the potential growth rate. See, for instance, a textbook of Galí, J., *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press, 2008.

[3] (1) and (3) are combined as "aggregate approaches" in the terminology of Mishkin, F. S. "Estimating Potential Output," speech at the Conference on Price Measurement for Monetary Policy, Federal Reserve Bank of Dallas, May 24, 2007.

[4] See Higo, M. and S. K. Nakada, "How Can We Extract a Fundamental Trend from an Economic Time-Series?," *Monetary and Economic Studies*, Bank of Japan, 16 (2), 61-111, 1998.

[5] As the degree of smoothness, it is a common practice to assume 100 for annual series, 1,600 for quarterly series, and 14,400 for monthly series.

[6] Technical details of estimation can be found in Hara, N., N. Hirakata, Y. Inomata, S. Ito, T. Kawamoto, T. Kurozumi, M. Minegishi and I. Takagawa, "The New Estimates of Output Gap and Potential Growth Rate," Bank of Japan Review Series, 06-E-3, 2006.

[7] See Chart 42 of the Bank of Japan *Outlook for Economic Activity and Prices* (April 2009).

[8] Solow, R. M., "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics* 39 (3), 312-320, 1957.

[9] Kuttner, K. N., "Estimating Potential Output as a Latent Variable," *Journal of Business and Economic Statistics* 12(3), 361-368, 1994; Hirose, Y. and K. Kamada "A New Technique for Simultaneous Estimation of Potential Output and the Phillips Curve," *Monetary and Economic Studies*, Bank of Japan, 21 (2), 93-112, 2003.

[10] Laubach, T. and J. Williams, "Measuring the Natural Rate of Interest," *Review of Economics and Statistics* 85(4), 1063-1070, 2003.

[11] Some DSGE models define the potential output as the *efficient* level of output, which would be attainable under perfect competition as well as flexible prices.

[12] An example for the Japanese economy is found in Sugo, T. and K. Ueda, "Estimating a Dynamic Stochastic General Equilibrium Model for Japan," *Journal of the Japanese and International Economies* 22, 476-502, 2008.

[13] Edge, R.M., M. T. Kiley, and J. Laforte, "Natural Rate Measures in an Estimated DSGE Model of the U.S. Economy," *Journal of Economic Dynamics and Control* 32(8), 2512-2535, 2008.

[14] Details can be found in Fueki, T., I. Fukunaga, H. Ichiue, and T. Shirota, "Measuring Potential Growth with an Estimated DSGE Model of Japan's Economy," mimeo.

[15] The confidence intervals can also be constructed by the Phillips curve approach and some variants of the filtering approach.

[16] OECD, *Economic Outlook*, No. 85, Chapter 4, June, 2009. Weidner, J. and J. C. Williams, "How Big Is the Output Gap?" FRBSF Economic Letter, 2009.

[17] See, for example, Edge, Kiley, and Laforte (2008), cited above.

[18] A downward shift in potential output tends to be observed when the economy is hit by a huge negative shock such as a financial crisis. See the study of 88 financial crises in the past 40 years by the International Monetary Fund, *World Economic Outlook*, Chapter 4, October, 2009.

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