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Forward-looking Models and Monetary Policy in Japan

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1. Introduction

The role of expectations has long been recognized in economic theory. For instance, stock prices depend not only on present performance of firms and interest rates, but also on the expectations about future performance and interest rates. Private fixed investment is affected not only by present profitability of a firm, but also by expectations for future profitability. Despite this recognition, there is little consensus on how expectations should be incorporated in macro-econometric models. In practice, some econometric models incorporate very simple mechanisms of expectation formation, such as adaptive expectations. However, since adaptive expectations are based on present and past information, they cannot take into account events that are expected to occur in the future. Nonetheless, it is easy to imagine that future events affect present activity. Suppose for instance that a government announces that it will have a tight fiscal policy several years in the future. If private economic agents expect this policy to be adopted, and change their economic behavior accordingly, there may be changes in economic variables, such as long-term interest rates, even though there is no actual reduction in fiscal expenditure and even though the policy is only announced. This mechanism of expectation formation is called rational expectations in general, and "model-consistent expectations" in macro-econometric models. The models that incorporate such expectations are called "forward-looking models" and are distinguished from "backward-looking models," in which expectations are based on current and past information, as are adaptive expectations. In this paper, we build a forward-looking model by incorporating rational expectations into a backward-looking model and then investigate the properties of the forward-looking model.

Forward-looking models have many applications. Among them, the analysis of monetary policy rules is the most relevant for central bankers.[1] Suppose that a central bank adopts a

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[1] Much of the literature uses small models for policy rule analysis; for example, see Fuhrer and Moore

particular policy rule. In reaction to changing economic situations, the rule requires a specific policy, and the central bank implements that policy. Meanwhile, private economic agents choose their present behavior, based on their expectations of future monetary policy by the central bank. Taking into consideration expectation formation and behavioral change among private economic agents, the central bank should choose an optimal policy rule to achieve its goals. Forward-looking models allow for easy incorporation of monetary policy rules into macro-econometric models and enable us to investigate the implications of rational expectations in the models.[2] In practice, many central banks devote resources to intense analysis of monetary policy rules that uses rational expectations.[3] In contrast, in Japan, there are virtually no analyses based on forward-looking models and there has been little accumulation of empirical analyses for monetary policy rules.

Monetary policy analysis based on forward-looking models emphasizes the impact of a change in a policy rule on the dynamic processes by which output and prices revert to long-run equilibrium values. Since forward-looking models take long-run equilibrium as given, they provide no way to investigate how the long-run equilibrium is affected by a change in a policy rule. Forward-looking models are merely tools with which we analyze how an economy reverts to long-run equilibrium. Analysis that uses a forward-looking model is still under development. Hence, the focus is currently on a qualitative analysis of the model's properties under the assumptions of a particular mechanism of expectation formation and a particular rule of monetary policy. The reason is that the monetary authorities lack quantitative information on how rational private economic agents are and on how helpful a central bank's credibility is when it comes to economic stabilization. Consequently, forward-looking models are too underdeveloped to calculate the short-run effects of monetary tightening and easing and to compare a rule targeting interest rates with a rule targeting something else.

This paper will give an introductory explanation of forward-looking models, build and estimate a small-sized forward-looking model for the Japanese economy, and discuss the

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[1995] and McCallum and Nelson [1999]. The Board of Governors of the Federal Reserve System (FRB) continues to develop large models, such as FRB/US (Brayton and Tinsley [1996]) and FRB/Global (Levin et al. [1997]).

[2] Impetus for using forward-looking models comes from the Lucas critique that argues simulations with fixed parameters are useless, since changes in policy parameters may change behavioral patterns of economic agents.

[3] In the countries whose central banks adopt inflation-targeting policy, such as the Bank of England (the Bank of England [1999] and Batini and Haldane [1999]), the Bank of Canada (Donald et al. [1996] and Black and Rose [1997]), and the Reserve Bank of New Zealand (Black et al. [1997]), analysis of macro-econometric models that explicitly incorporate rational expectations is very popular. The FRB is also developing a large forward-looking model (see Reifschneider et al. [1997] for the relationships between monetary policy and macro-econometric models).

properties of the forward-looking model through various simulations. The remaining part of this paper is constructed as follows: Chapter 2 gives a theoretical explanation of forward-looking models. We assume that in asset markets, participants form rational expectations for long-term interest rates and foreign exchange rates. Chapter 3 investigates the properties of a forward-looking model by comparing it with a backward-looking model. We show that when a future expansion in demand and acceleration in inflation are expected, present long-term interest rates rise, which exerts negative pressure on the present economy before the actual future expansion occurs. Chapter 4 uses real data and estimates a forward-looking model for the Japanese economy. Chapter 5 uses that estimated model and analyzes policy rules by implementing simulations. Chapter 6 uses stochastic simulations, in which models are under random shocks, and presents analysis of policy rules under uncertainty. Appendix A explains how to implement stochastic simulations. Appendix B investigates the role of the zero-percent constraint on nominal interest rates.

2. Basics for Forward-looking Models

2.1. Forward-looking Expectations

There are various mechanisms of expectation formation. In traditional macro-econometric models, adaptive expectations and static expectations are popular. They are called backward-looking expectations, since they make the *ad hoc* assumption that expectations are based solely on present and past information.[4] In contrast, rational expectations assume that economic agents have complete knowledge of an economic system and calculate the future value of economic variables correctly according to that knowledge. Thus, rational expectations are called forward-looking expectations and, in macro-econometric models, model-consistent expectations.[5]

Chart 1 shows how expectations are formed in forward-looking models. The thick arrow denotes the transmission route that is specific to forward-looking models, while the thin arrows denote transmission routes that are common to forward-looking models and backward-looking models, in which expectations are based solely on present and past information. For instance,

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[4] Muth [1960] shows that adaptive expectations are not necessarily irrational when a temporary shock cannot be distinguished from a permanent shock. Nonetheless, adaptive expectations are not rational in that they cannot deal with new information from the future.

[5] When a model is deterministic, rational expectations are the same as perfect foresight. Rational expectations are an excessively strong assumption in analyzing real economies. Models under development mix adaptive expectations and rational expectations or explicitly take into consideration a learning process. (See Tetlow and von zur Muehlen [1999] for instance.)

present fiscal expenditure boosts present output. In addition, current consumption may depend on past consumption due to habit-formation effects. These effects are common to forward-looking models and backward-looking models. Meanwhile, since a future rise in long-term interest rates is expected to push down asset prices, economic agents try to sell their assets in the present, which in turn pushes down current asset prices and pushes up current long-term interest rates. This transmission route is specific to forward-looking models, as present long-term interest rates depend on future long-term interest rates. A central bank can respond to existing inflation rates as well as future inflation rates. In the latter case, the model is forward-looking. Thus, in backward-looking models, all variables depend solely on present and past information, and hence exogenous shocks and changes in monetary policy have no impact on an economy until they happen actually. In contrast, in forward-looking models, present variables are affected by future exogenous shocks and future changes in a policy rule, thus exogenous shocks have an impact on an economy before they happen, as discussed in the example of long-term interest rates. These effects distinguish forward-looking models from backward-looking models.

As we explained, forward-looking models incorporate rational expectations, and economic agents know the correct future values of every variable. Technically, forward-looking models are solved this way: All variables are assumed to have long-run equilibrium values (steady-state values); they are assumed to revert to equilibrium values eventually; the present value of all variables is solved by working backward from the equilibrium values.[6] The existence of long-run equilibrium is a necessary condition for a forward-looking model. (Unless we know our destination, we cannot find a path to reach it.) It is also a necessary condition for a forward-looking model that all variables revert to their steady-state values. Assuming rational expectations does not guarantee that all variables will revert to their steady-state values. A monetary policy rule should be so chosen as to prevent the models from exploding. If a steady state exists and the reversion condition holds, a forward-looking model reverts to long-run equilibrium after variables depart from their steady-state values in the short run as a result of a temporary shock. Thus, a forward-looking model connects the short-run disequilibrium to the long-run equilibrium. In this paper, we assume Keynesian disequilibrium in the short run and neo-classical equilibrium in the long run.

In this paper, the mechanisms of expectation formation are not derived systematically by solving the dynamic optimization problem of a representative agent, but assumed separately

[6] Steady-state values can be calibrated or estimated. As discussed later, we exploit both calibration and estimation in this paper.

sector by sector.[7] This *ad hoc* assumption may be problematic in estimating a model's structural parameters. This paper is free from the simultaneous equation bias by employing the econometric method of full information maximum likelihood. From the viewpoint of a representative agent, however, the model's structural parameters may not be estimated free from restrictions. This paper does not take into consideration possible restrictions among structural parameters.

2.2. Monetary Policy Rules

Another feature of forward-looking models is that a central bank's behavior is formalized as a monetary policy rule. Traditional policy simulations using a macro-econometric model take as given each period's policy. In such simulations, attention is not necessarily paid to interactions between the model's endogenous variables and policies. For instance, policies are kept fixed even though an economic situation changes. Suppose that a central bank implements a tight monetary policy against high inflation. If it continues the same policy even after the high inflation disappears, an economy is driven into a deflationary situation. From the viewpoint of the central bank's goal, deflation is not thought of as preferable. Thus, when monetary policies are exogenous, they are likely to become inconsistent over time with the central bank's goal.

In contrast, in a forward-looking model, a central bank's behavior is incorporated as a policy rule and changes in reaction to economic situations. In the above example, as high inflation disappears, a central bank should weaken its tight monetary policy gradually. In this paper, as the output gap and inflation rate approach their target values, the central bank is assumed to take the call rate or its instrument back to its normal level. Private economic agents rationally expect such monetary policy and make adjustments in their own behavior. In the wake of such adjustments by private agents, the central bank changes monetary policy again according to the rule. Incorporating monetary policy as a rule keeps policies consistent with the central bank's goals over time.[8]

In this paper, we use real data and estimate a policy reaction function. We do not claim that the central bank has actually adopted the particular rule we have imputed to it. Nonetheless, we proceed and show that, whatever the central bank's true intention, past monetary policy could be approximated through use of the Taylor rule (Taylor [1993a]). That is, the central bank adjusts

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[7] Real business-cycle models and computable general equilibrium models are based on micro-economic theory and are derived explicitly from the optimal behavior of economic agents. They form research areas that are separated from that of forward-looking models.

[8] With policy determined endogenously, we can compare the qualitative properties of various rules from the viewpoint of stability of output and inflation rates.

the call rate as the output gap and inflation rate diverge their target values. Furthermore, when private economic agents identify the central bank's behavioral pattern as estimated here, it is reasonable to incorporate the estimated policy reaction function into our forward-looking model as a subjective monetary policy rule used by the private economic agents.

A caveat is in order. This paper investigates the economic implications of various monetary policy rules, but the effects of a change in a monetary policy rule on long-run economic performance are beyond our scope. This paper assumes the long-run neutrality of money. Thus, in the long run, real variables cannot be affected by monetary policy, the instrument of which is the call rate or a nominal variable. The purpose of this paper is to analyze how a change in a monetary policy rule affects the model's dynamic path that reaches long-run equilibrium after a temporary shock.

We must note that a central bank's true policy reaction function may not be the subjective policy reaction function that is perceived by private economic agents. Although this diversion is likely to disappear over time, its adjustment speed depends on the reputation the central has acquired and on the economic environment surrounding the central bank.[9] For instance, suppose that the central bank announces that it will raise its target rate of inflation. If the central bank has poor reputation, private economic agents make little adjustments in their subjective policy reaction function. Even if the central bank has good reputation, the announcement that it is raising its target rate of inflation will have no effect on private economic agents' expectations of the inflation rate, when the central bank has no effective way to increase money supply and when an increase in money supply does not accelerate inflation. In addition, consider a temporary policy: The central bank raises its target rate of inflation and immediately lowers it to the initial level. As long as private economic agents correctly anticipate this policy, the effects are smaller than those of a permanent rise in the target rate of inflation. Thus, when the central bank raises the target rate of inflation, the effects vary widely. Therefore, we note that our simulation results are relevant only if all the assumptions hold.

As mentioned above, we estimate the whole system simultaneously, including a policy reaction function, by the method of full information maximum likelihood. This implies that the effects of a policy change are free from the Lucas critique, as long as the change follows the same political rule. The effects of a change in a policy rule, however, is subject to the Lucas critique, since parameters may change in reduced-form equations along with the change in the rule.[10]

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[9] See Sargent [1993] for instance.

[10] For instance, Rotemberg and Woodford [1997] are free from the Lucas critique.

3. Basic Model

This chapter presents a basic model and explains the properties of forward-looking models. For comparison, we first introduce a backward-looking model, that is, a model with no forward-looking expectations. The model is a standard structural model, including the IS balance, a sticky inflation rate, Fisher's identity on the relationship between a short-term interest rate and an inflation rate, and a Taylor-type policy reaction function.[11] Additionally, in the backward-looking model, static expectations are applied to future short-term interest rates; that is, they are expected to stay at the present level forever. From the term structure of interest rates, this implies that the present long-term interest rate coincides with the present short-term interest rate. Next, we build a forward-looking model on the basis of this backward-looking model. Specifically, it is assumed that future short-term interest rates are subject to rational expectations. As discussed later, the present long-term interest rate depends not only on the present short-term interest rate, but also on the expectations for the next period's long-term interest rate. To investigate the properties of a forward-looking model, this chapter compares simulation results produced by the forward-looking model and the backward-looking model. Here, we use fictitious models, whose structural parameters are so chosen as to magnify the properties of forward-looking models. We discuss the properties of the forward-looking model that are estimated with real data from the next chapter.

3.1. Backward-looking Model

(Model)

$$y_t = y^* - a(rl_t - rl^*) + e_t , \qquad (1)$$

$$p_t = p_{t-1} + b(y_t - y^*) + h_t , \qquad (2)$$

$$i_t = p_t + rs^f + g(y_t - y^f) + d(p_t - p^f) , \qquad (3)$$

$$rs_t = i_t - p_t , \qquad (4)$$

$$rl_t = rs_t , \qquad (5)$$

where

y_t : output gap (%) (negative);
y^* : steady-state output gap (%) (negative);
y^f : target output gap (%) (negative);
i_t : call rate (%);
rs^f : target real short-term interest rate (%);
e_t , h_t : error terms.
rl_t : real long-term interest rate (%);
rl^* : equilibrium real long-term interest rate (%);
p_t : inflation rate (%);
p^f : target rate of inflation (%);
a, b, g, d : structural parameters (positive);

[11] The backward-looking model in this chapter is based on Taylor [1994].

(Theoretical Background)

Equation (1) is the IS balance in a reduced form. The y_t is not the output itself but the output gap, which is determined by the difference between the real long-term interest rate (rl_t) and its equilibrium value (rl^*). When the real long-term interest rate rises, the output gap expands, as consumption is postponed to the future and private fixed investment is reduced. The e_t summarizes the demand movements caused by non-interest rate factors, such as an increase in fiscal expenditure, an increase in net exports accompanied with improvements in overseas economies, and an increase in private fixed investment expanded by expectations for an increase in future revenue.[12]

Equation (2) is the Phillips curve with the Non-Acceleration Inflation Rate of Unemployment (NAIRU). The inflation rate rises above the previous period's level when the output gap closes beyond its equilibrium level (y^*). From the viewpoint of the expectation-augmented Phillips curve, the first term on the right-hand side corresponds to the expected rate of inflation of private economic agents. Hence, we interpret that equation (2) assumes static expectations: Period t's expected rate of inflation p_t^e is given by period $(t-1)$'s realized rate of inflation p_{t-1} (the superscript e denotes expectations). It is not essential, however, to interpret equation (2) from the viewpoint of expectations for the inflation rate. For our purposes, it is enough to understand that the equation describes sticky movements in the inflation rate. The h_t summarizes exogenous inflation shocks, such as a rise in oil prices.

Equation (4) determines the real short-term interest rate and is obtained by combining the two assumptions of Fisher's identity ($i_t = rs_t + p_{t+1}^e$) and the static expectations for the future inflation rate ($p_{t+1}^e = p_t$). These assumptions are combined to say that players are irrational in the short-term money market. Suppose that it is certain that an economy will be boosted ($y_{t+1} > y^*$). From equation (4), the *ex post* inflation rate is surely above the *ex ante* inflation rate ($p_{t+1} > p_{t+1}^e = p_t$). This implies that, since the short-term interest rate is set too low, the *ex post* real short-term interest rate ($i_t - p_{t+1}$) is below the *ex ante* real short-term interest rate (rs_t).

Equation (5) determines the real long-term interest rate. According to the term structure of interest rates, the present real long-term interest rate is given by the weighted average of the present and future real short-term interest rates if a term premium is ignored. If the static expectations are assumed for the real short-term interest rates ($rs_t = rs_{t+1}^e = rs_{t+2}^e = \cdots$), the

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[12] $y^* = 0$ in Taylor [1994], since he defines the potential output not as maximal output, but as average output. Here, we define the potential output as the maximal output that can be produced with the full utilization of capital stocks and labor. As a result, the steady-state output gap is not zero, but takes on a negative value, as in the models.

real long-term interest rate coincides with the present real short-term interest rate, since the former is given by the weighted average of the real short-term interest rates.

Equation (3) is a Taylor-type policy reaction function by a central bank: The central bank adjusts the level of the call rate in response to the divergence of the output gap and inflation rate from their target values. The true purpose of the central bank is to control the real short-term interest rate by adjusting the nominal short-term interest rates, while the inflation rate moves slowly. If the first term on the right-hand side of equation (3) is interpreted as the central bank's static expectations for the inflation rates ($p^e_{t+1} = p_t$), its sum with the second term implies the target nominal short-term interest rate of the central bank.

Notice that the Taylor rule describes the following process in a single equation. A central bank provides monetary base in reaction to the expansion of the output gap and the deceleration of the inflation rate beyond their target values. Money supply expands in the process of credit creation. The call rate falls to clear the money market. As the Japanese economy experienced in the 1990s, this process may break down, when nominal short-term interest rates decline sharply. In such situations, the Taylor rule is no longer a feasible policy rule (chart 2).

The above assumption of static expectations for the real short-term interest rate is inconsistent with the central bank's policy reaction function. Combining the static expectations for the inflation rate and those for the real short-term interest rate gives the static expectations for the nominal short-term interest rate. This implies that private economic agents expect the central bank to keep the nominal short-term interest rate constant, irrespective of the changing economic environment. This contradicts with the assumption in equation (4) that the central bank adjusts the nominal short-term interest rate in reaction to the output gap and inflation rate.

(Steady State)

To solve for the steady state of the model, let $y_t = \bar{y}$, $p_t = \bar{p}$, $i_t = \bar{i}$, $rs_t = \overline{rs}$, and $rl_t = \overline{rl}$ in equations (1) to (5) and set all the error terms to be zero. Then the results follow:

$$\bar{y} = y^*,$$
$$\overline{rs} = \overline{rl} = rl^*,$$
$$\bar{i} = rl^* + \bar{p},$$
$$\bar{p} = p^f + (rl^* - rs^f)/d + (y^f - y^*)g/d .$$

Some remarks are in order. In the model, the classical dichotomy holds in the long run: A change in a policy rule by a central bank has no effect on the long-run equilibrium of the output gap and real long- and short- interest rates. In other words, the neutrality of money holds.

Suppose that the central bank targets a smaller output gap than the long-run equilibrium gap ($y^f > y^*$) by aiming at a real short-term interest rate lower than its long-rum equilibrium level ($rl^* > rs^l$). Then the central bank is penalized with a higher inflation rate than its target rate ($\bar{p} > p^f$), but cannot affect the equilibrium values of real variables, such as the output gap and real interest rates. Furthermore, the central bank's policy of raising its target rate of inflation leads to the same amount of increase in the actual inflation rate, but it has no effect on real variables, such as the output gap, in the long-run equilibrium. As seen in the later chapters' shock simulations, we should be sure that the model reverts to its long-run equilibrium.

(Transmission Mechanism)

We overview the total behavior of the backward-looking model by the flow chart in chart 3 and the simulation results in chart 4-1. Suppose that in this period, the output gap is hit by a positive shock, for instance, by a temporary increase in fiscal expenditure. The output gap closes in this period (①), and the inflation rate accelerates (③). A central bank raises the level of the call rate in reaction to the divergence of the output gap and inflation rate from the policy goals (④). Expectations for the inflation rate are unchanged, since they are formed according to static expectations. Hence, the real interest rate rises (⑤). This pushes down aggregate demand, thereby reducing the closing of the output gap and slowing the acceleration of the inflation rate.

Subsequently, after the inflation rate accelerates in this period, the expected inflation rate rises in the next period, which in turn accelerates the actual inflation rate in the next period according to the Phillips curve (⑧). The central bank raises the level of the call rate (④'). Thus, the real interest rate rises up (⑤'). Since there occurs no additional fiscal expenditure and the real interest rate rises, the output gap widens beyond its baseline value (⑦'). This reduces the acceleration of the inflation rate (③'). This process is repeated in the next period. Eventually, all variables revert to their steady state values. The flow chart in chart 3 and the simulation results in chart 4-2 are used to explain system behavior in response to an inflation shock.

3.2. Forward-looking model

(Model)

$$y_t = y^* - a(rl_t - rl^*) + e_t, \quad (1)$$

$$p_t = p_{t-1} + b(y_t - y^*) + h_t, \quad (2)$$

$$i_t = p_t + rs^f + g(y_t - y^f) + d(p_t - p^f), \quad (3)$$

$$rs_t = i_t - p_t, \quad (4)$$

$$rl_t - (rl_{t+1} - rl_t)/rl^* = rs_t. \quad (5)'$$

(Theoretical Background)

Here, we keep the assumptions for the backward-looking model that were used in the previous section, except that private economic agents have rational expectations for real short-term interest rates, which in turn means that they have similar rational expectations for real long-term interest rates. Shiller et al. [1983] show that the term structure of interest rates is given by $rl_t - (rl^e_{t+1} - rl_t)/rl^* = rs_t$.[13] Equation (5)' obtains from combining the term structure with the rational expectations hypothesis ($rl^e_{t+1} = rl_{t+1}$). Equation (5)' is rewritten as

$$rl_t = l\,rl_{t+1} + (1 - l)rs_t \text{, where } l = 1/(1 + rl^*) .$$

That is, this period's real long-term interest rate is the weighted average of the next period's real long-term interest rate and this period's real short-term interest rate. Note that the equation is rewritten further as

$$rl_t = (1 - l)(rs_t + l\,rs_{t+1} + l^2 rs_{t+2} + l^3 rs_{t+3} + \cdots) .$$

This equation shows that the rational expectations for the real long-term interest rate obtains from the weighted average of the infinite series of future real short-term interest rates.

Here we encounter the difference between the *ex post* and *ex ante* short-term interest rates discussed previously. Based on static expectations, when an output gap is smaller than usual, lenders who roll over short-term lending always expect the inflation rate to be lower than is realized *ex post*. As a result, the *ex post* real short-term interest rate is always less than the *ex ante* rate. Nonetheless, short-term lenders incur only small losses, since they raise their expectations for inflation rates gradually with time lags. In comparison, long-term lenders with static expectations set the nominal long-term interest rates with the assumption that the low inflation rate will continue for a long time. Hence, long-term lenders may incur larger losses *ex post* than do short-term lenders. To analyze the financial market with the completely rational expectations, it is necessary to replace equation (4)'s static expectations for the inflation rate p_t with the rational expectations p_{t+1}. Furthermore, a completely rational policy rule requires the same replacement. Here, we did not aim at incorporating these assumptions of rationality all at once, but introduced rationality only for expectations for real short-term interest rates, given the *ex ante* real short-term interest rates.

(Steady State)

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[13] Equation (5)' assumes that the real long-term and short-term interest rates, coupon rates, and time preferences are all moving in the neighborhood of the equilibrium real long-term interest rate rl^*.

The steady state of the forward-looking model is exactly the same as obtained in the backward-looking model. This follows from the fact that the forward-looking model coincides with the backward-looking model if $rl_{t+1} = rl_t$ in equation (5)'.

(Transmission Mechanism)

The behavior of the forward-looking model is roughly similar to that of the backward-looking model. The forward-looking model, however, shows complicated movements to the extent that the present variables are affected by future variables, as the expectations for the long-term interest rate are formed rationally. We explain this point in the flow chart in chart 5 and the simulation results in chart 6-1. Chart 3 is the same as chart 5 except for the new arrows that direct from the future to the present (thick arrows ⑨ and ⑩). These arrows show the behavior in the forward-looking model that is not shared with the backward-looking model.

Let us discuss a case that is different from the one for the backward-looking model: Suppose that it is certain that there will be a positive demand shock on the output gap: say, a temporary increase in fiscal expenditure in the next period. The system behavior from the next period is the same as discussed above and is not repeated here. But one thing to note is that the next period's long-term interest rate rises. In the forward-looking model, if the real long-term interest rate is expected to rise in the next period, the real long-term interest rate rises in this period (⑩). This widens the output gap (⑦), and deflation occurs (③). Thus, when a future boom is expected, downward pressures are put on the present economy. A central bank lowers the level of the call rate preemptively to reduce those downward pressures (④). It is important to note that, when a future shock is expected in a forward-looking model, the system moves only from the announcement effects, even though no real shocks actually occur at the time. As in the case of a demand shock, chart 5 can be used to interpret the simulation results in chart 6-2 in the case of an inflation rate shock.

4. Extended Model and Estimation

In this chapter, Japan's real data is applied to the forward-looking model introduced in the previous chapter. In doing so, we extend the basic model in several directions: ① a forward-looking exchange rate derived from the interest rate arbitrage between domestic and foreign interest rates, ② fiscal expenditure, and ③ a forecast-based monetary policy. It should be noted that a simple rule like the Taylor rule does not suite well the Japanese economy in the late 1990s. We fit a policy reaction function to the sample through the second quarter in 1996, and compare the fitted values of the call rates with the actual call rates through 1998 in chart 2. The chart shows that the Taylor rule follows the real movements in the call rate roughly up to 1995. After that, however, the Taylor rule diverges from the actual movements in the call rate. For instance, the Taylor rule overestimates the call rates in 1996. The reason is that in 1996, though the Japanese economy recorded a high growth rate, the central bank did not raise the level of the call rate since it expected a rise in the consumption tax from 1997 and a reduction in fiscal deficits.[14] Another reason is that, faced with an unprecedentedly wide output gap recently, the central bank must strengthen monetary easing policy more than the Taylor rule shows has been the average reaction of the central bank in the past.

4.1. Extended Model

(i) IS balance

$$y_t = y^* + a_1(y_{t-1} - y^*) + a_2(rl_t - rl^*) + a_3(\ln s_{(t-7,t-1)} - \ln s^*) + a_4 g_{(t-6,t)} + e_t ,$$

(ii) inflation rate

$$p_t - p^* = b_1(p_{t-1} - p^*) + b_2(y_t - y^*) + h_t ,$$

(iii) policy reaction function

$$i_t = p_t + rs^f + g(y_{(t-4,t-1)} - y^f) + d(p_{(t,t+3)} - p^f) ,$$

(iv) real short-term interest rate

$$rs_t = i_t - p_t ,$$

(v) real long-term interest rate

$$rl_t - (rl_{t+1} - rl_t)/rl^* = i_t - p_t ,$$

(vi) real exchange rate

$$\ln s_t = \ln s^e_{t+1} - (rs_t - rs^{us}_t) .$$

In the above, the s_t is the real exchange rate, and the g_t is the quarter-to-quarter growth rate of real government expenditure. $x_{(t-j,t-k)}$ is the moving average of variable x from j period past to k period past. Similarly, $x_{(t+j,t+k)}$ is the moving average of variable x from j period future to k period future. This model is basically the same as the basic forward-looking model introduced in the previous chapter. We, however, have extended the basic model in the following fashion: The model treats an open economy with the introduction of the real exchange rate. As seen in equation (vi), an expected change in the real exchange rate balances with the difference between the domestic real interest rate and foreign real interest rate through investors' interest rate arbitrage. The equation says that this period's real exchange rate is determined in a forward-looking way, since it depends on the next period's real exchange rate.

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[14] The Taylor rule is different from the actual level of call rates in 1997. The reason is that the consumption tax was raised from 3 percent to 5 percent that year. A similar departure is observed in 1989, when the 3-percent consumption tax was introduced.

Next, as in equation (ii), we make a new assumption that the divergence of the inflation rate from its equilibrium value is determined by the past divergence of the inflation rate and also by the divergence of the output gap from its equilibrium value. Note that equation (ii) is rewritten as follows:

$$p_t = b_1 p_{t-1} + (1 - b_1) p^* + b_2 (y_t - y^*) + h_t .$$

From the viewpoint of the expectation-augmented Phillips curve, the sum of the first and second terms on the right-hand side corresponds to the expectations for the inflation rate. Hence, the equation is interpreted that expectations for the inflation rate of private economic agents revert to its long-run equilibrium value p^* at the speed of $100(1 - b_1)$ % per quarter.[15] That is, the b_1 is a measure of the inertia in expectations for the inflation rate.

In the policy reaction function, we use the moving average of the future inflation rates that the central bank should control. The reason is that the central bank implements monetary policy preemptively against future inflation rates that the central bank forecasts, as well as the present inflation rate.

4.2. Estimation

Among the above six equations, equations (iv) and (vi) are identities and require no estimation. Equation (v) cannot be estimated meaningfully, since the identified equilibrium real long-term interest rate has a negative value. Hence, equations (i) to (iii) are estimated in this paper.[16] For estimation, we collect several constants into one for each equation. That is,

$$y_t = con_1 + a_1 \cdot y_{t-1} + a_2 \cdot r_t + a_3 \cdot \ln s_{(t-7,t-1)} + a_4 \cdot g_{(t-6,t)} , \quad \text{(i)'}$$

$$p_t = con_2 + b_1 \cdot p_{t-1} + b_2 \cdot y_t , \quad \text{(ii)'}$$

$$i_t = con_3 + f\hat{I}_t + c \cdot y_{(t-4,t-1)} + d \cdot p_{(t,t+3)} . \quad \text{(iii)'}$$

The frequency of the sample is a quarter. The sample period is from the third quarter of 1983 to

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[15] When equation (ii) is applied to data in the 1980s and 1990s, the b_1 is significantly smaller than one. See Higo and Kuroda Nakada [1999] for an estimation of the Phillips curve in Japan.

[16] We use the real capital cost as rl_t. See the Bank of Japan [1996] for the procedure to compute the real capital cost. There are various ways to measure the potential output, which is required to compute the output gap. Here we assume the Cobb=Gouglas production function and a linear positive trend in the total productivity factor. The potential output obtains by substituting the potential amount of each production factor. See Watanabe [1997] for details. The real exchange rate is converted to the real value from the nominal yen/dollar rate with the GDP deflator and foreign prices (1990 = 100, the average of foreign country prices weighted by the amounts of exports of foreign countries). The p_t is a quarter-to-quarter change in the GDP deflator (the TC factor). The i_t is a level of the call rate.

the second period of 1996, when the Japanese call rate virtually reached its floor. Chart 2 shows that the Japanese call rate is almost flat from 1996, despite fluctuations in the output gap and inflation rate. It is obviously impossible to fit a linear policy reaction function like equation (iii)'. Around the middle of 1996, the actual call rate did not rise, despite the closing of the output gap. The reason is that monetary policy did not need to react to the superficial closing of the output gap, since the central bank recognized that closing of the output gap during this period was caused by the surge in demand before the consumption tax hike in 1997. For these reasons, our sample ends in the second quarter of 1996.

Monetary policy always reacts to the model's endogenous variables, such as the output gap and inflation rate. Thus, the simultaneous equation bias may cause a serious problem, when the model includes a policy reaction function. For this reason, we avoid an equation-by-equation estimation and estimate the three equations simultaneously by using the method of full information maximum likelihood (FIML). Furthermore, even when policy changes, such changes are free from the Lucas critique, as long as the changes follow a policy rule. It should be noted that the structural parameters in the model may be related to each other and cannot be estimated free from restrictions. Nevertheless, such restrictions among the structural parameters are not taken into account in this paper. The estimation results are as follows:

(i)'	con_1	a_1	a_2	a_3	a_4	R^2
	-16.65 [-3.8]	0.64 [8.6]	-0.90 [-4.8]	3.94 [4.0]	0.10 [2.0]	0.88
(ii)'	con_2	b_1	b_2			R^2
	1.34 [3.2]	0.64 [6.9]	0.27 [3.0]			0.74
(iii)'	con_3	c	d			R^2
	4.04 [6.8]	0.26 [2.1]	0.33 [2.1]			0.73

(Note) sample period: from 1983Q3 to 1996Q2; estimation method: FIML (full information maximum likelihood); t-values in parentheses.

4.3. Identification

The estimation in the previous section is not enough to determine all the equilibrium values and target values (y^*, y^f, p^*, p^f, r^*, r^f, s^*). To analyze monetary policy rules, it is necessary to identify these values explicitly. This section presents a method to identify these parameters in a monetary policy rule. The explicit identification of the equilibrium values and target values are important for the purpose of checking how reliable the estimated parameters are.

We have to identify many variables, i.e., the seven equilibrium values and target values. We assume that the central bank's purpose is to keep the economy from diverging from the long-run equilibrium as far as possible. That is, we let $y^f = y^*$, $p^f = p^*$, and $r^f = r^*$. Matching equations (i) to (iii) with equations (i)’ to (iii)’ gives

$$con_1 = (1 - a_1) \cdot y^* - a_2 \cdot r^* - a_3 \cdot \ln s^* , \quad \text{(i)”}$$

$$con_2 = (1 - b_1) \cdot p^* - b_2 \cdot y^* , \quad \text{(ii)”}$$

$$con_3 = -c \cdot y^* - d \cdot p^* + r^* . \quad \text{(iii)”}$$

There are the four undetermined variables y^*, p^*, r^*, and s^* in the three equations, which implies that we have one degree of freedom. We use the average of the inflation rates in the sample period as the p^* and estimate the remaining three variables. Given the p^*, the y^* obtains from equation (ii)”. Given the p^* and y^*, the r^* obtains from equation (iii)’’. Substituting these three into equation (i)” gives the s^*. Following this method, we calculate the equilibrium values as follows:

$$y^* = y^f = -3.40\% ,\ p^* = p^f = 1.15\% ,\ r^* = r^f = 3.52\% ,\ s^* = 102.35 .$$

This finishes the estimation of the whole system of equations (i) to (vi).

5. Policy Rule Analysis by Shock Simulations

In this chapter, we take the Japanese forward-looking model elaborated on in the previous chapter as the standard case and implement various simulations to investigate how a policy change affects the stability of the economy. Our focus is on how differences between various policy rules affect economic dynamics by reacting to the choice of private economic agents. It should be noted that, since a forward-looking model treats money as a veil in the long run, real variables are determined independently of monetary policy and their long-run equilibrium values cannot be affected by changes in policy. A forward-looking model can envision nothing but how economic variables move toward given long-run equilibrium values and what policy rule is useful for economic stabilization in the short run. Our discussion, however, focuses on qualitative analysis rather than quantitative analysis.

Here we cover the policy rules that can be defined as Taylor-type rules. Thus, we do not compare the Taylor rule with other policy rules. Since we focus on the Taylor rule, a central bank can choose the values of the three parameters in changing a policy rule, as seen in equation (iii): ① the target rate of inflation (p^f), ② the sensitivity to the output gap (g), ③ the sensitivity to the inflation rates (d). By varying these three parameters, we produce various policy rules and investigate how differences in the rules affect dynamic paths of the economy toward the long-run equilibrium. Here we should note the Lucas critique that a revision in a policy rule may change the structural parameters, thereby reducing the accuracy of the simulation for that policy rule revision. Since our model does not necessarily treat deep parameters, it is not free from the Lucas critique. Furthermore, the reaction of private economic agents to a change in a policy rule depends on the reputation a central bank has and the economic conditions under which the central bank implements monetary policy. Thus, we must note that the simulation results presented are meaningful only in the specific conditions identified in this paper. Beside the policy changes discussed in this paper, a central bank can also choose the lag and lead length of the output gap and inflation rate. We, however, keep the lag and lead length that has obtained in the estimation and do not discuss the effects of changing either.

Finally, this chapter's simulations assume away the zero constraint on nominal interest rates and the central bank can lower the level of the call rate as far as possible. The effects of the zero constraint on nominal interest rates are discussed in Appendix B.

5.1. Change in the Target Rate of Inflation (Chart 7)

Consider the following case. A central bank announces that it will raise its target rate of inflation (p^{f}) by 1 percent permanently from the next year and simultaneously lowers the level of the call rate. Private economic agents, however, do not believe immediately that the central bank will raise the target rate of inflation, but they do believe the announcement after the inflation rate actually begins to rise. Here we assume that one year will pass after the central bank raises its target rate of inflation and before private economic agents believe the announcement.

In this paper, since the long-run neutrality of money is assumed, a change in the target rate of inflation has no long-run effects on real variables, such as the output gap. The purpose of this paper is to investigate the transitory effects of the policy change on real variables. We assume a permanent change in the target rate of inflation, not a temporary change in the target, raising the target and then lowering it immediately. If a change in the target is temporary, private economic agents take that into account, which reduces the effect of the policy change. Furthermore, the short-term effects on real variables depend on the sticky movements of the inflation rate and the resulting sticky movements of the expected inflation rate. When the adjustment speed of actual and expected inflation rises, the short-run effect of the policy change on real variables is reduced. Furthermore, our results are affected by the reputation the central bank has acquired in monetary policy. The higher reputation is, the more private economic agents believe the announcement that it is raising the target rate of inflation. However, there is no reason to believe the central bank's announcement if private economic agents think that the central bank has no way to achieve the announced rate of inflation.

a. Effects after a Change in the Target Rate of Inflation

The central bank raises its target rate of inflation and lowers the level of the call rate simultaneously. The inflation rate, however, does not decline by the same amount due to its sticky movement. This induces a decline in the real short-term interest rate, which leads to a decrease in the real long-term interest rate and the depreciation of the real exchange rate. Consequently, the output gap closes and the inflation rate accelerates. When the inflation rate actually accelerates, private economic agents raise their expected rate of inflation, which in turn accelerates the actual rate of inflation. Against the rise in the inflation rate, the central bank raises the level of the call rate. Due to the sticky movements of the inflation rate, the real long-term interest rate and real short-term interest rate increase to their equilibrium values and the output gap expands toward its equilibrium level. It should be noted that, when the central bank raises its target rate of inflation, only nominal variables, such as inflation rates and nominal interest rates, increase in the long run, but real variables, such as the output gap and real interest rates, revert to their equilibrium values.

b. Effects before a Change in the Target Rate of inflation

Before lowering the level of the call rate, the central bank announces that it will raise its target rate of inflation in the next year. As discussed above, the real long-term interest rate declines, as soon as the call rate actually declines after a year. If private economic agents expect a future decline in the real long-term interest rate, the real long-term interest rate declines in the present. Similarly, if the real exchange rate is expected to depreciate in the future, the real exchange rate depreciates in the present. Thus, as soon as the rise in the target rate of inflation is announced, the real long-term interest rate declines and the real exchange rate depreciates. Thus, the output gap closes and the inflation rate accelerates in the present.

5.2. Change in Policy Sensitivity

Here we consider two cases: a demand shock and an inflation shock. We compare the stabilizing effects of ① a policy rule that is relatively sensitive to the output gap (a high g) and of ② a policy rule that is relatively sensitive to an inflation rate (a high d) against these shocks. Note that the following results depend heavily on the lag and lead structure of the output gap and inflation rate in the policy reaction function.

5.2.1. Demand Shock (Chart 8)

Here we give a 1 percent shock on the error term (e_t) of the IS balance equation (i) in each quarter and repeat this for four quarters from period 0. This sequence of shocks is not foreseen before it happens. When the first shock happens in period 0, however, the four consecutive shocks are expected to happen.

A. Output Gap Sensitive Rule ($g = 4$, $d = 1$)

The output gap closes by the positive demand shocks for four quarters from period 0. When the rule is relatively sensitive to the output gap, the call rate rises substantially, as does the real short-term interest rate. Consequently, the real long-term interest rate rises substantially and the real exchange rate appreciates greatly. These effects restrict the initial closing of the output gap and reduce inflationary pressures.

B. Inflation Sensitive Rule ($g = 1$, $d = 4$)

When the rule is relatively sensitive to the inflation rate, the call rate is raised substantially against the expected increase in the inflation rate, following the closing of the output gap. Since the policy is expected to decelerate the inflation rate gradually in the future, the rise in the call rate is halted and reverts to the initial equilibrium value gradually, even though the output gap is still small. Consequently, the output gap closes and the inflation rate rises more than they do with the output gap sensitive rule.

5.2.2. Inflation Shock (Chart 9)

Here we give a 1 percent shock on the error term (h_t) of the inflation determination equation (i) in each quarter and repeat this for four quarters from period 0. This sequence of shocks is not foreseen before it actually happens. When the first shock happens in period 0, however, the four consecutive shocks are expected to happen.

A. Output Gap Sensitive Rule ($g = 4$, $d = 1$)

When the inflation rate accelerates for a year from period 0, the central bank raises the level of the call rate according to the rule. When the rule is relatively sensitive to the output gap, the rise in the call rate is small, and thus the real short-term interest rate does not rise very much. Consequently, the real long-term interest rate does not rise very much and the real exchange rate does not appreciate very much. This results in the relatively small expansion of the output gap and the long-lasting inflation rate.

B. Inflation Sensitive Rule ($g = 1$, $d = 4$)

When the rule is relatively sensitive to the inflation rate, the call rate rises substantially against an increase in the inflation rate, which induces a substantial rise in the real short-term interest rate. Hence, the real long-term interest rate rises significantly and the real exchange rate appreciates very much. Therefore, the output gap expands more, but the inflation rate rises less than they do with the output gap sensitive rule.

The Japanese forward-looking model reveals that, against a demand shock, the output gap sensitive rule lessens the volatility of both the output gap and the inflation rate. Thus, a central bank is not confronted with a tradeoff between the weights on the two policy targets. In contrast, against an inflation shock, the output gap sensitive rule reduces the volatility of the output gap, but increases the volatility of the inflation rate. On the other hand, the inflation sensitive rule reduces the volatility of the inflation rate, but increases the volatility of the output gap. As a result, the central bank faces a tradeoff between the weights on the two policy targets.[17]

6. Policy Analysis by Stochastic Simulations

In this chapter, which uses the Japanese forward-looking model explained in chapter 4, we analyze monetary policy rules under uncertainty by implementing stochastic simulations. So far, we have evaluated policy performance by observing how a change in a policy rule affects the dynamic paths of the output gap and inflation rate after deterministic demand and inflation shocks. In reality, these exogenous shocks happen stochastically and successively. From this point of view, the recent analysis of monetary policy rules emphasizes how effectively various monetary policy rules stabilize the movements of the output gap and inflation rate by giving random shocks on models. Appendix A gives the procedure for stochastic simulations in detail.

6.1. Tradeoff between the Variances of the Output Gap and Inflation Rate

Among the many criteria for evaluating the performance of monetary policy rules is the tradeoff between variances in the output gap and the inflation rate; Taylor [1994] is an especially keen advocate of this benchmark. There is consensus that, between the levels of the output gap (or the unemployment rate) and inflation rate, there is a tradeoff in the short run, but not in the long run. Thus, the level of the output gap is a poor criterion in evaluating policy performance over time. For this reason, Taylor claims that a central bank can aim at reducing the variances in the output gap and the inflation rate as one of its policy objectives. Fix a pair of policy weights

[17] With a more complex model, where the output gap affects the inflation rates with long lags, there may be a tradeoff in the case of demand shocks.

(g, d) in the Taylor rule. Providing various demand and inflation shocks to the model produces different dynamic paths. These paths move around the long-run equilibrium value. The variability of these paths can be measured by the two variances in the output gap and the inflation rate. As noted in the previous chapter, a change in policy weights affects the dynamic paths of the output gap and the inflation rate toward their long-run equilibrium values and thus the variances of the output gap and the inflation rate. If no change reduces both variances at once, the associated policy rule is "an efficient rule." An efficient frontier is the set of pairs of variances of the output gap and the inflation rate produced by various efficient rules. Taylor [1994] claims that an efficient frontier often shows tradeoff relationships between the variances.[18]

These concepts are summarized in chart 10. In the chart, the shaded area is the feasible set of variances of the output gap and the inflation rate that a central bank can achieve by changing the sensitivity parameters in the policy rule. Suppose that the central bank adopts a certain policy rule and achieves a pair of variances, point *X*, which is located in the shaded area. Starting from this point, the central bank can adopt a better policy rule to move to point *Y*, which is located to the lower left of point *X*. At this point, the central bank achieves smaller variances in the output gap and the inflation rate. In this case, the policy rule associated with point *X* is "an inefficient policy rule." In contrast, variances in the output gap and the inflation rate cannot be lowered simultaneously from point *Y*. In this case, the policy rule associated with point *Y* is "an efficient policy rule." The set of pairs of variances associated with efficient policy rules is "an efficient frontier" and is denoted by the downward-sloping curve in the chart. From point *Y*, if the central bank wants to reduce the variance of the output gap, it has to allow for an increase in the variance of the inflation rate instead. If the central bank wants to reduce the variance of the inflation rate, it has to allow for an increase in the variance of the output gap. In this case, the central bank faces a policy tradeoff. Recent analysis of monetary policy rules focuses on how far from the efficient frontier a pair of variances of the output gap and the inflation rate associated with a certain policy rule are located.[19] Note that the variance tradeoff depends on the model's structure and much attention is paid to the causes that bring about changes in the shape of the variance tradeoff.

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[18] In Solow and Taylor [1998], Taylor gives an intuitive explanation of this tradeoff. He claims that differences in sensitivity give rise to the tradeoff. Ball [1994], however, shows that some models may have no tradeoff in the case of demand shocks and claims that the source of the tradeoff is inflation shocks. In our model, there is no tradeoff without inflation shocks.

[19] For the US economy, there have been many studies of the variance tradeoff, e.g., Taylor [1979], Fuhrer [1994], Fuhrer [1997], and Levin et al. [1999].

6.2. Variance Tradeoff in Japan

Chart 11 presents the Japanese tradeoff between variances in the output gap and the inflation rate, which obtains from the stochastic simulations that use the Japanese forward-looking model estimated in chapter 4. The chart shows a tradeoff between the variances of the output gap and the inflation rate.

We also find point *A* associated with the estimated parameters of the policy reaction function ($g = 0.26$, $d = 0.33$) to the upper right of the tradeoff. The Japanese forward-looking model of this paper seems to show that the monetary policy rule was not completely efficient during the sample period (from the third quarter of 1983 to the second quarter of 1996). It also seems that the central bank could reduce both variances in the output gap and the inflation rate by controlling the level of the call rate more aggressively. For instance, point *B*, where the variance of the inflation rate is smaller than at point *A*, can be achieved with larger parameters ($g = 1.30$, $d = 2.85$) than the estimated policy reaction parameters. Similarly, point *C*, where the variance of the output gap is smaller, can be achieved with extremely large parameters ($g = 2.75$, $d = 1.85$).

More investigation is required before we can conclude that these results show the inefficiency of this monetary policy, however. The first reason is the “Brainard Conservatism.”[20] Brainard [1967] showed that when the central bank does not know a model’s true parameters and thus implements monetary policy under uncertainty, its policy reaction should be more conservative than when it knows the true parameters. In other words, the central bank can adopt an efficient policy rule only if the following conditions hold: ① Both private economic agents and the central bank have complete knowledge of the economic structure; ② without any measurement errors *ex post*, they have accurate data on economic variables, such as GDP data, which are subject to possible substantial revision. These conditions, however, are hard to satisfy (Orphanides and van Norden [1999]). If the central bank conducts a cautious monetary policy by taking into account the uncertainty of its model, it is plausible that the narrow policy parameters reflect the optimal choice of a policy rule.

7. Conclusion

[20] For instance, when a central bank thinks that a large fluctuation of the call rate will cause a disturbance in the market and wants to avoid a sharp change in the call rate, the optimal parameters become smaller. Furthermore, when a central bank has policy objectives other than the output gap and inflation rate, the parameters of the output gap and inflation rate may become smaller in order to achieve the additional objectives.

This paper has introduced the basic concepts of forward-looking models that use rational expectations and has shown their properties through various simulations. Forward-looking models have the following merits:

① Forward-looking models treat rational expectations other than static and adaptive expectations. Thus, it becomes possible to analyze the effects of future events that are independent of past information.

② Forward-looking models incorporate monetary policy rules. Thus, it becomes possible to take into consideration the interactions between economic policy and private economic agents' expectations for the future economy.

③ Forward-looking models produce a tradeoff between variances of the output gap and the inflation rate through stochastic simulations. Thus, it becomes possible to evaluate the performance of a certain policy rule from the efficiency point of view.

Next, this paper discussed a procedure to find a central bank's policy rule from real data by estimating and identifying a model. Using this procedure, this paper built a model, estimated its parameters, and implemented various monetary policy simulations. Note that our sample ends at the second quarter of 1996. After that, the policy reaction function fails to fit the data for the following reasons: ① The central bank did not raise the short-term interest rate, despite high growth rate in 1996, since it expected a future restrictive fiscal policy; ② recently, the nominal short-term interest rate has been virtually subject to the zero-percent constraint. Appendix B gives examples of simulations subject to the zero-percent constraint.

Some caveats are in order. Japan's long-run equilibrium may change in the 1990s. Forward-looking models, however, fail to deal with changes in long-run equilibrium. In addition, it is necessary to measure to what extent the expectation-formation mechanism of private economic agents is rational. From the viewpoint of short-horizon forecasting, forward-looking models are still developed and their performance is not necessarily comparable with traditional macro-econometric models.

This paper focuses on the analysis of monetary policy rules and thus introduces forward-looking expectations for financial variables, such as interest rates and exchange rates. Forward-looking models have other applications, however. For instance, we can assume forward-looking expectations for real variables, as the permanent income hypothesis can be introduced in a consumption function. As forward-looking models are applicable more extensively, more research should be conducted.

Appendix A. Procedure for Stochastic Simulations

① Fix the parameter set of the policy reaction function (parameters of the output gap and inflation rate) at certain values.[21]

② Give random shocks in the error terms of equations (i) and (ii) each time and produce the time series of the output gap and inflation rate. The sample size is 52 quarters, which is the same size as that in the model's estimation. In this paper, the sizes of random shocks (standard deviations) are the same as the standard deviations of the estimation of equations (i) and (ii).[22]

③ Calculate the variances of the time series of the output gap and inflation rate obtained in ②.

④ Repeat simulation in ② certain times (for instance, 300 times) with the parameters of the policy reaction function unchanged and calculate the variances of the time series of the output gap and inflation rate each time as in ③.

⑤ Calculate a pair of averages of the 300 variances of the output gap and the inflation rate obtained in ④ and mark the pair of averages on the scatter diagram. The more the repetition in ④, the closer the averages in ⑤ are to the "true" values and the more accurately the dots in the scatter diagram denote the true performance.

⑥ Change the parameter set of the policy reaction function and repeat the process from ②.

Following this procedure, we present the performance of various policy rules on the scatter diagram, whose abscissa measures the variances of the output gap and whose ordinate measures the variances of the inflation rate.[23] In chapter 6, we move each g and d of equation (iii) by 0.05 from 0 to 3 and conduct simulations.

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[21] In the original Taylor Rule, the parameters of the output gap and inflation rate are both 0.5 (Taylor [1993a]).

[22] This is equivalent to assuming that the future volatility of exogenous shocks on the economy is the same as the past volatility during the sample period for the estimation of the model.

[23] It is not strenuous to compute the expected variances algebraically in the case of a linear model, in which stochastic simulation is not needed. When a model is large or non-linear, stochastic simulation is necessary.

Appendix B. Zero-percent Constraint on Nominal Short-term Interest Rates

B.1. Effects of the Zero-percent Constraint

In the late 1990s, the call rate fell virtually to the zero-percent constraint in Japan. As a result, the central bank could not lower the level of the call rate any further, despite the slowdown of the economy. Here we investigate the effects on the economy of the zero-percent constraint for nominal short-term interest rates. Specifically, we replace the reaction function (iii) in chapter 4 as follows:[24]

$$i_t = \max\{0, p_t + rs^f + g(y_{(t-4,t-1)} - y^f) + d(p_{(t,t+3)} - p^f)\},$$

which prevents the call rate from falling below zero percent. In the above equation, $\max\{a, b\}$ means the larger of a and b. According to the new policy reaction function, in a normal situation, a central bank controls an interest rate as the Taylor rule commands; when the Taylor rule commands negative interest rates, the central bank chooses zero percent. If the default risk is taken into consideration, the call rate does not fall completely to zero percent, and thus the "0" in the above equation should be replaced with 0.1 for instance.

To investigate the economic effects of the zero-percent constraint for the nominal short-term interest rates, we simulate and compare the transmission of a large negative shock in the economy with and without the zero percent-constraint. Chart A1 shows the simulation results. When a negative demand shock hits the economy, the output gap expands. Without the zero-percent constraint, a central bank lowers the level of the call rate substantially to counter the downward pressure on the economy. Then the real short-term interest rate and real long-term interest rate fall substantially, and the real exchange rate of the yen depreciates significantly. This reduces the initial expansion of the output gap, which in turn reduces the decline in the inflation rate. With the zero-percent constraint, however, the central bank cannot lower the level of the call rate below zero percent. Hence, the real short-term interest rate and real long-term interest rate do not fall very much, and the real exchange rate of the yen does not depreciate very much. Thus, the output gap expands more, and the inflation rate falls substantially. In this way, the zero-percent constraint on the nominal short-term interest rate restricts the effectiveness of monetary policy.

Here it is worth to note that monetary policy is still effective even when the nominal short-term interest rate reaches the zero-percent constraint. Forward-looking models assume that

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[24] The zero-percent constraint can be modeled in various ways other than that used in this paper. See Fuhrer and Madigan [1997] for instance.

private economic agents expect an economy to revert to equilibrium in the long run and that the nominal short-term interest rate departs from the zero-percent constraint eventually. When the nominal short-term interest rate is above zero percent, a central bank's monetary policy has real effects. In a forward-looking model, expectations of the effectiveness of future monetary policy affect future long-term interest rates and exchange rates, which in turn affect the current economy. Thus, the effectiveness of monetary policy when the nominal short-term interest rate equals zero percent depends on the assumption that the economy returns to the steady state in the long run. Therefore, when the interest rate reaches zero percent, the effectiveness of monetary policy depends on how much private economic agents are forward-looking and how much they believe the effects of the future monetary easing.

B.2. Effects of a Rise in the Target Rate of Inflation (charts A2 and A3)

Here we analyze the effects of raising the target rate of inflation by a central bank when a large negative demand shock occurs. We assume that the central bank anticipates the demand shock and raises the target rate of inflation by 3 percent a year before the shock. The effects of this policy change depend on the expectations of private economic agents. We assume that half a year will pass after the central bank raises its target rate of inflation before private economic agents raise expectations concerning the inflation rate.

The simulation results are presented as the thick curves in chart A2 (the thin curves are for the case when there is no rise in the target rate of inflation). When the central bank raises its target rate of inflation and private economic agents believe it, the expected rate of inflation increases and the actual rate of inflation rises.[25] This shifts upward the level of the call rate, which departs from the zero-percent constraint. Since the central bank can lower the level of the call rate from the higher level, the real short-term interest rate can fall substantially. This makes possible a large decline in the real long-term interest rate and a large depreciation of the real exchange rate, which in turn reduces the expansion of the output gap.

As a reference, we see the effects of raising the target rate of inflation when a demand shock is so small that the nominal short-term interest rate does not reach zero percent. Chart A3 shows the simulation results. According to it, when the nominal short-term interest rate is above zero and the central bank raises its target rate of inflation, the actual rate of inflation rises with the expected rate. However, since the level of the call rate rises immediately, the path of real short-term interest rate is virtually unaffected and consequently the output gap does not

[25] Note that as in $b_1 p_{t-1} + (1-b_1)p^*$ in chapter 4, the current expectation of the future inflation rate by private economic agents includes the equilibrium rate of inflation, which is assumed to be equal to a central bank's target rate of inflation.

close. Thus, when a demand shock is small and the time lag is the same as the past average after the central bank raises its target rate of inflation and before private economic agents raise their expectations regarding inflation, raising the target rate of inflation has almost no effect on real variables.

Note the assumption in the above simulation that the target rate of inflation is raised before a shock actually happens. The simulation is therefore not applicable when the target rate of inflation is raised after a shock happens and the call rate reaches zero percent. Suppose that the target rate of inflation is raised permanently after the call rate reaches zero percent. This policy change has positive effects on the present economy by guaranteeing lower interest rates in the future. In contrast, the policy of raising the target rate of inflation and lowering it when the actual rate of inflation rises has no policy effects, since private economic agents expect that policy. Additionally, in the late 1990s, private economic agents are likely to be pessimistic enough to expect that Japan's long-run equilibrium has fallen substantially. Here, however, we do not attempt simulations for dealing with the problem of a declining long-run equilibrium.

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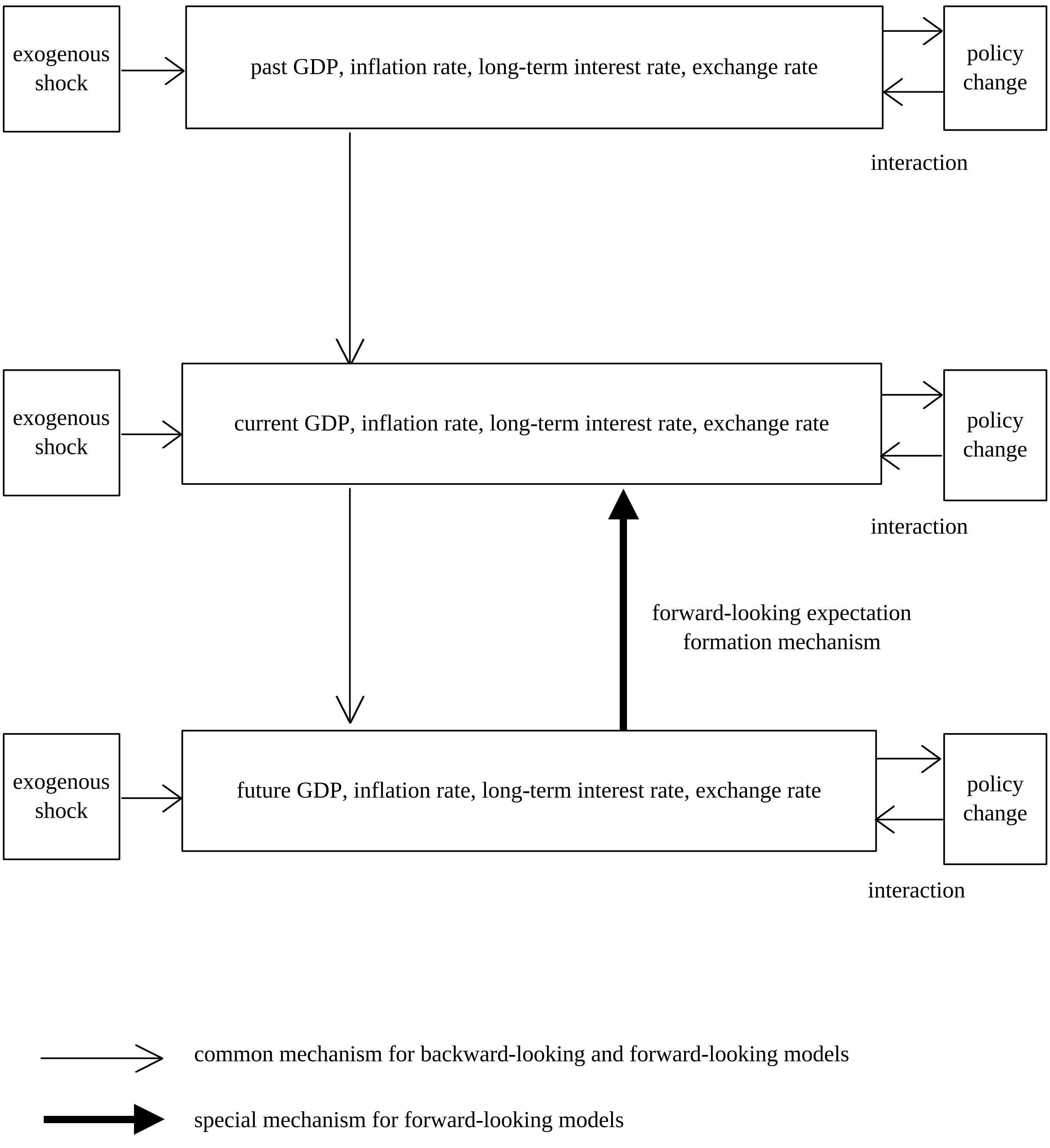
Chart 1

Image of Expectation Formation in Forward-looking Model



Japan's Monetary Policy from the Viewpoint of the Taylor Rule

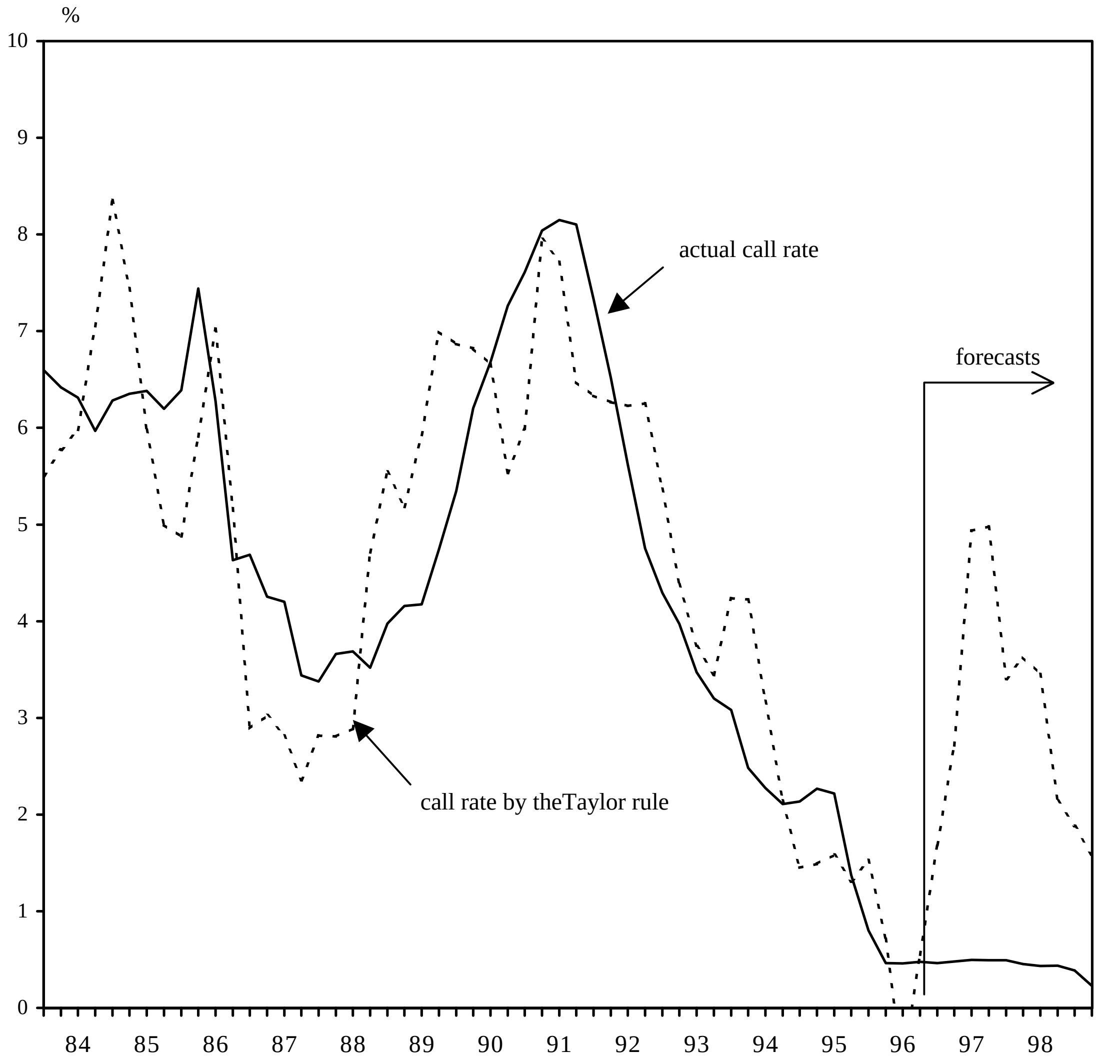


Chart 3

Transmission Mechanism in Backward-looking Models

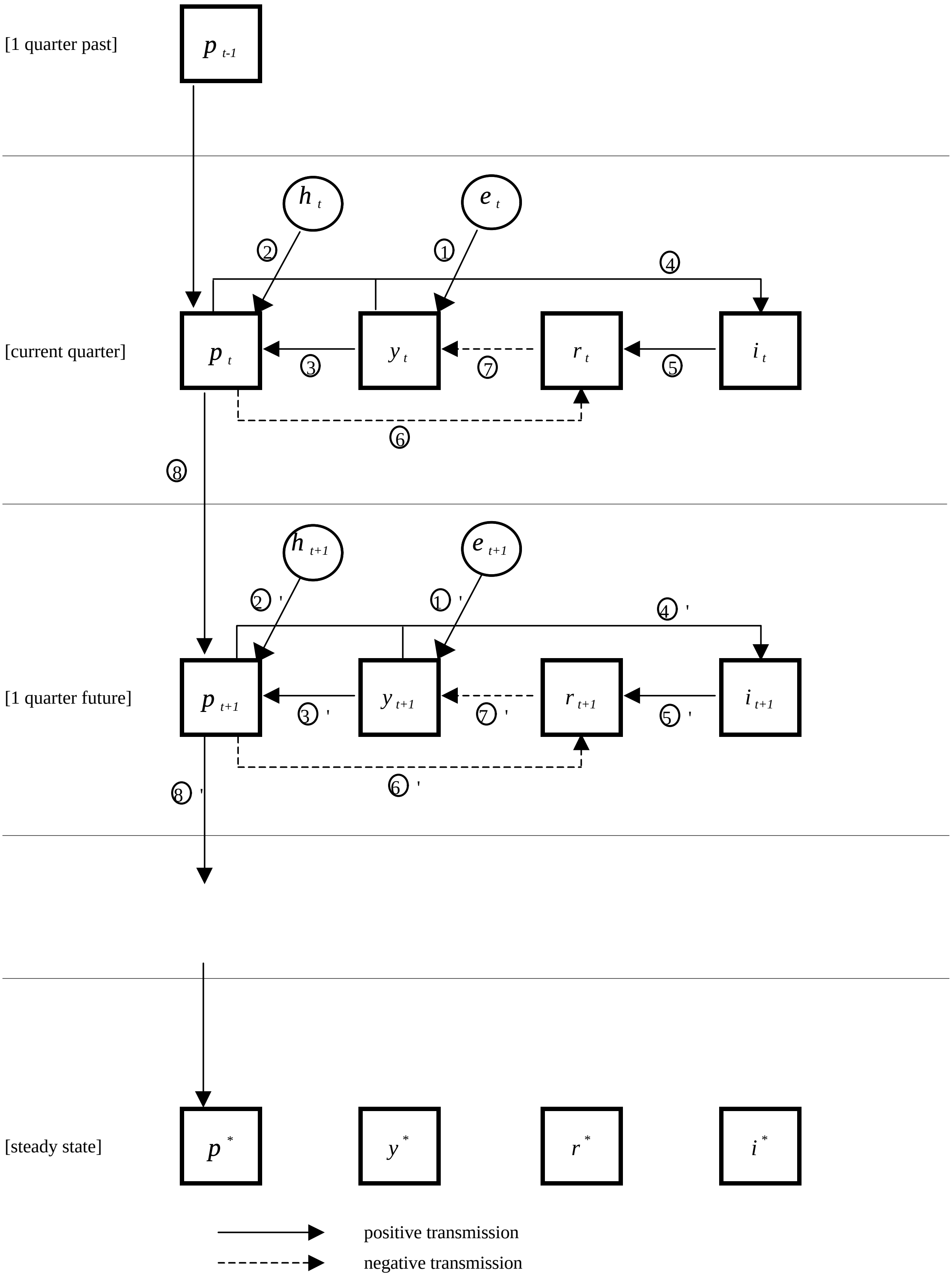
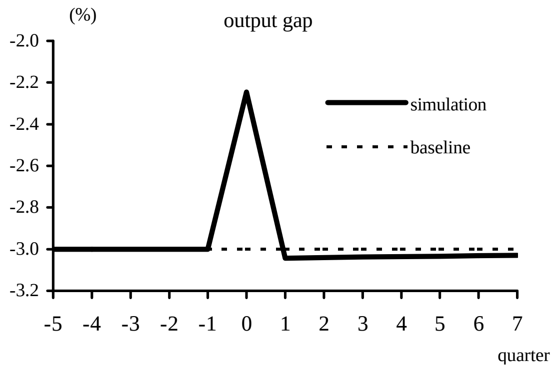


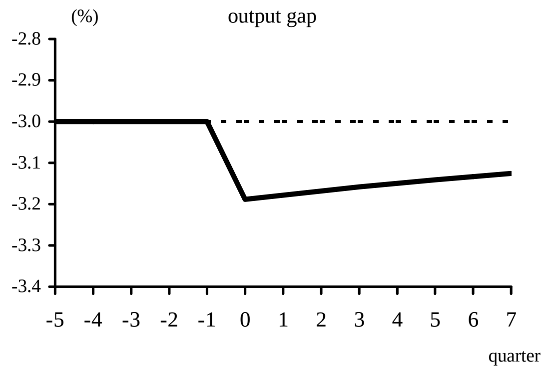
Chart 4

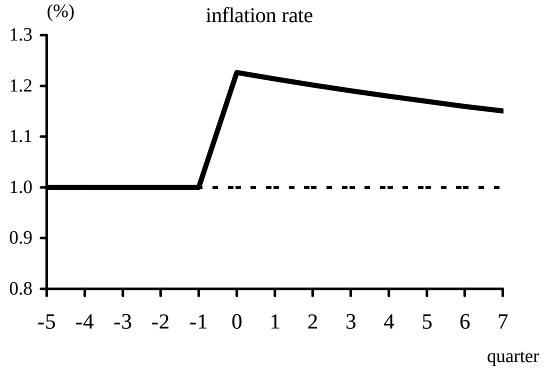
Impulse Response in Backward-looking Models

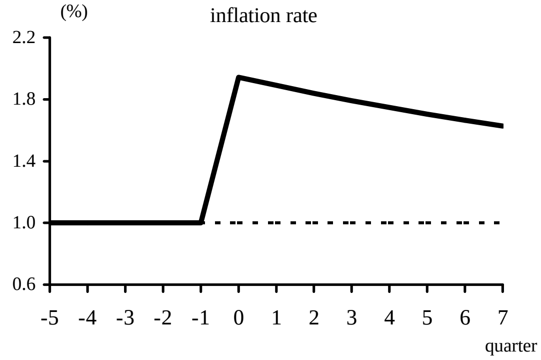
1. Demand Shock

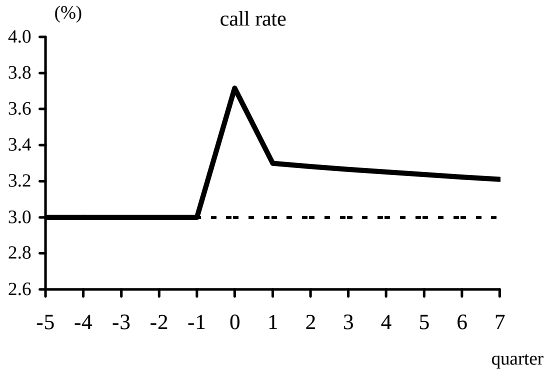
2. Inflation Shock

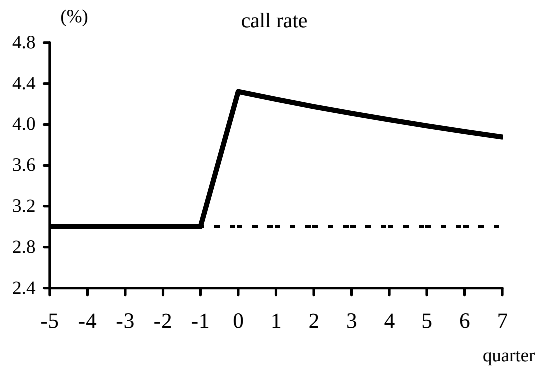


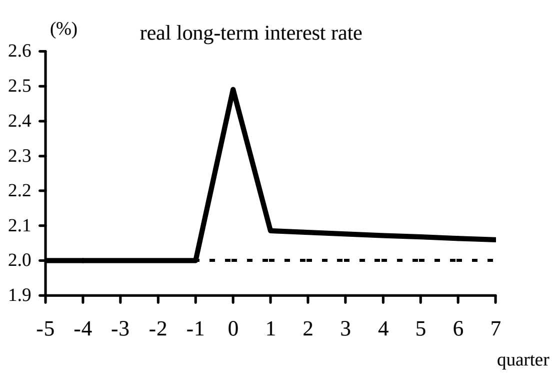


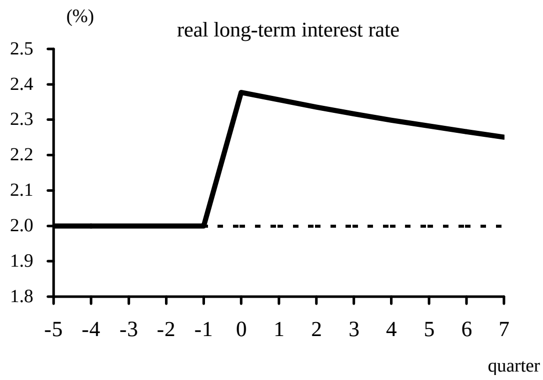








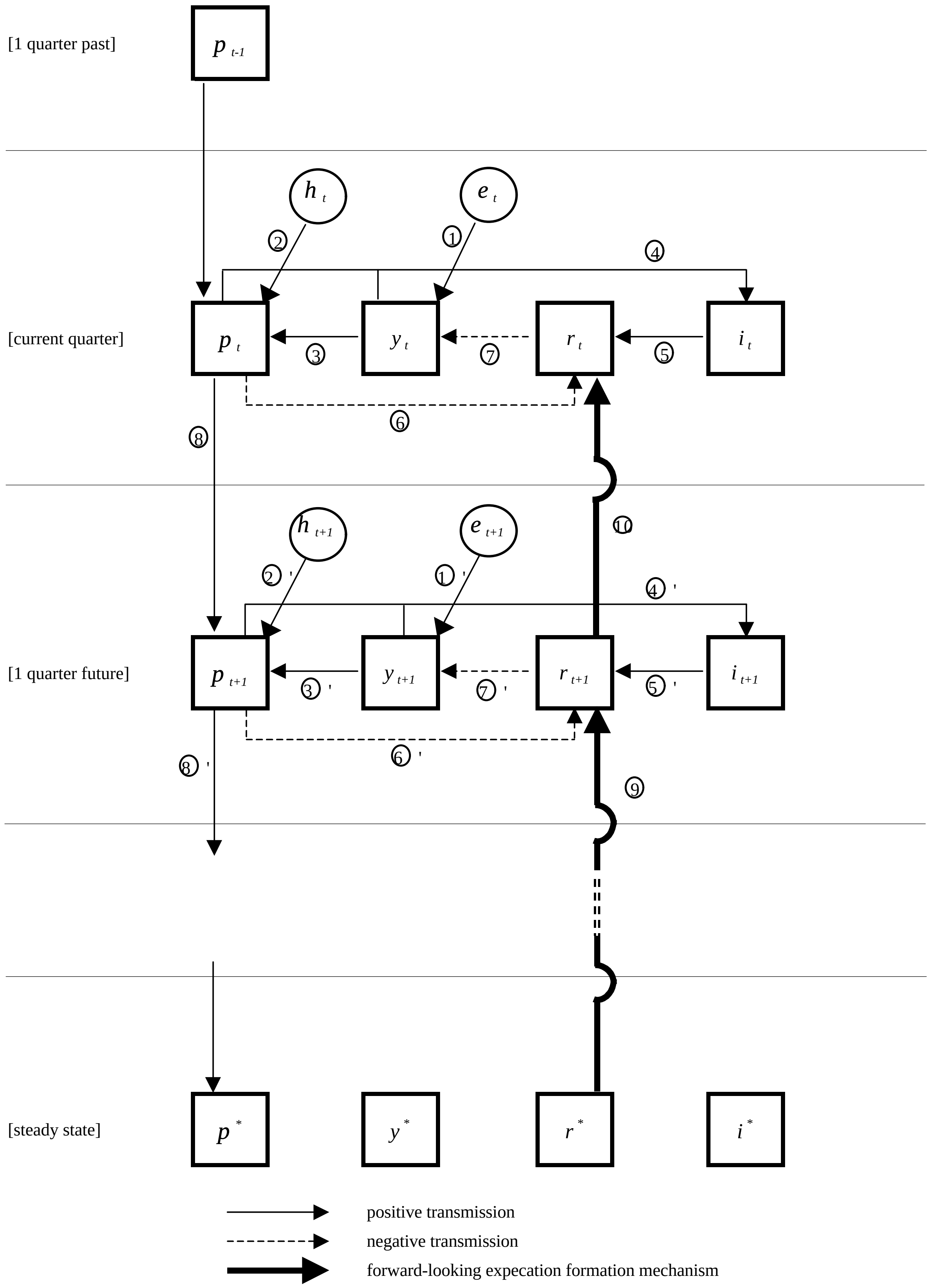




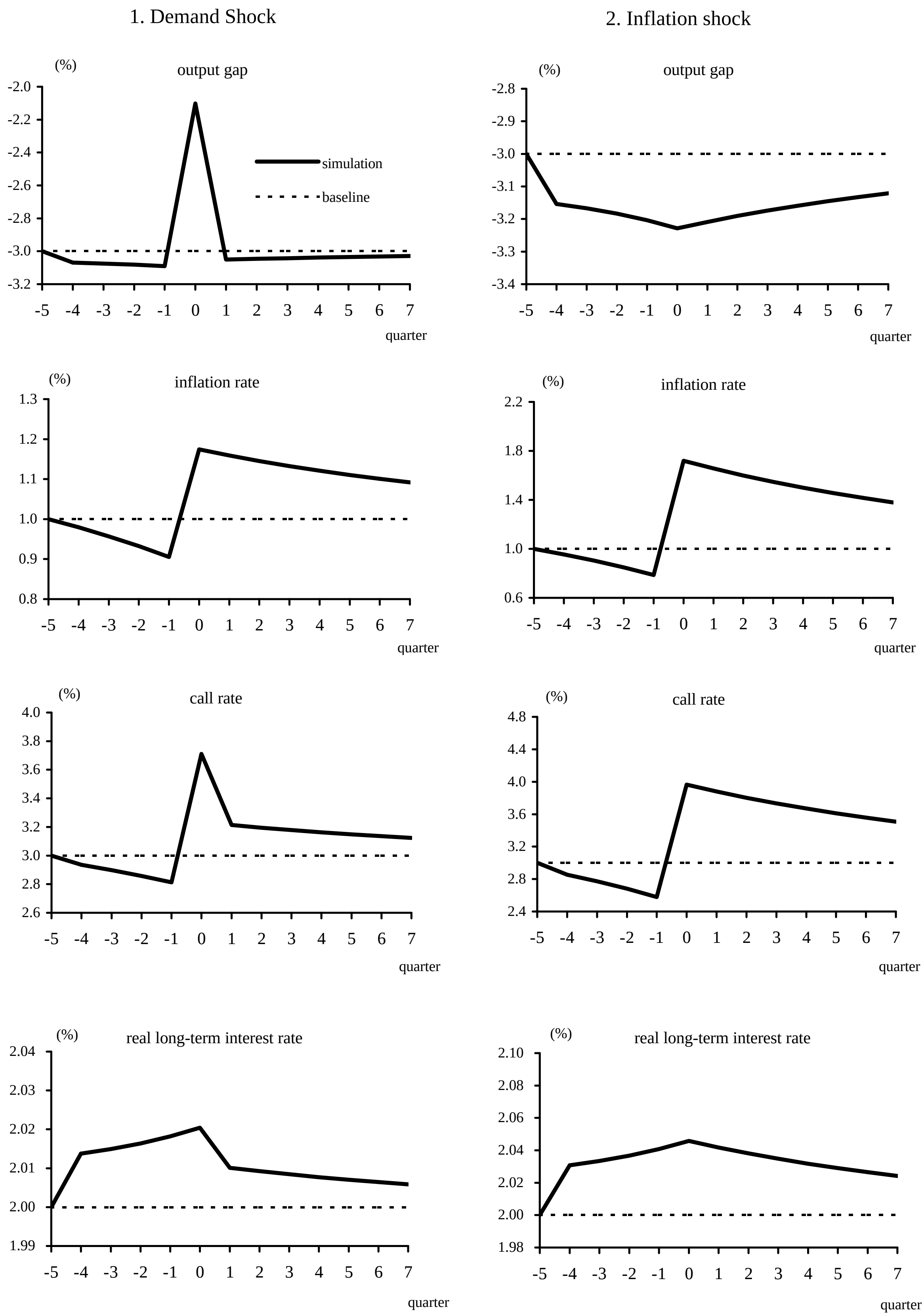
Note: Fictional parameters are used to magnify simulation properties of the model.

Chart 5

Transmission Mechanism in Forward-looking Models



Impulse Responses in Forward-looking Models



Note: Fictional parameters are used to magnify simulation properties of the model.

Chart 7

Increase in Target Rate of Inflation

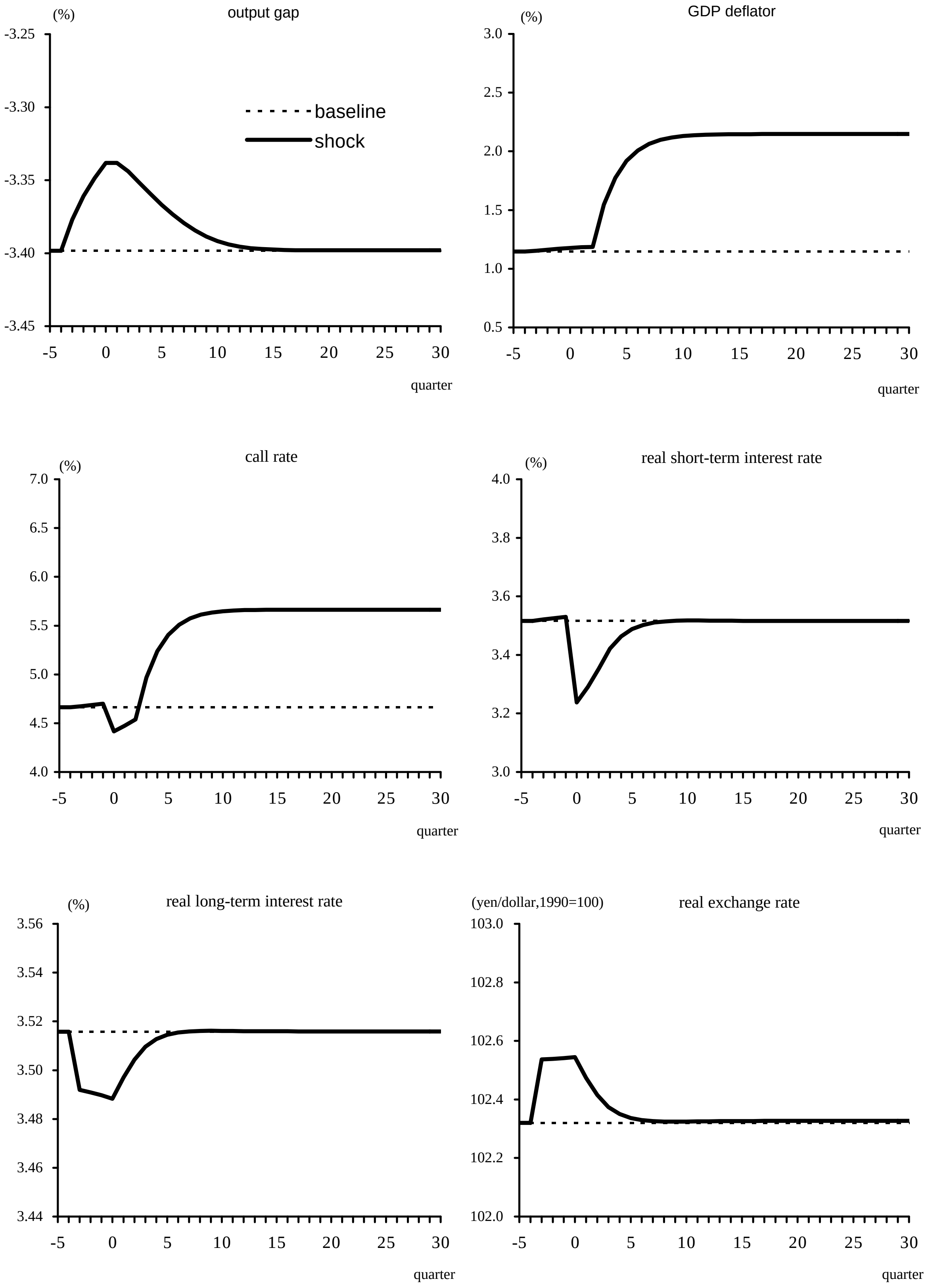


Chart 8

Comparing Policy Rules (Demand Shock)

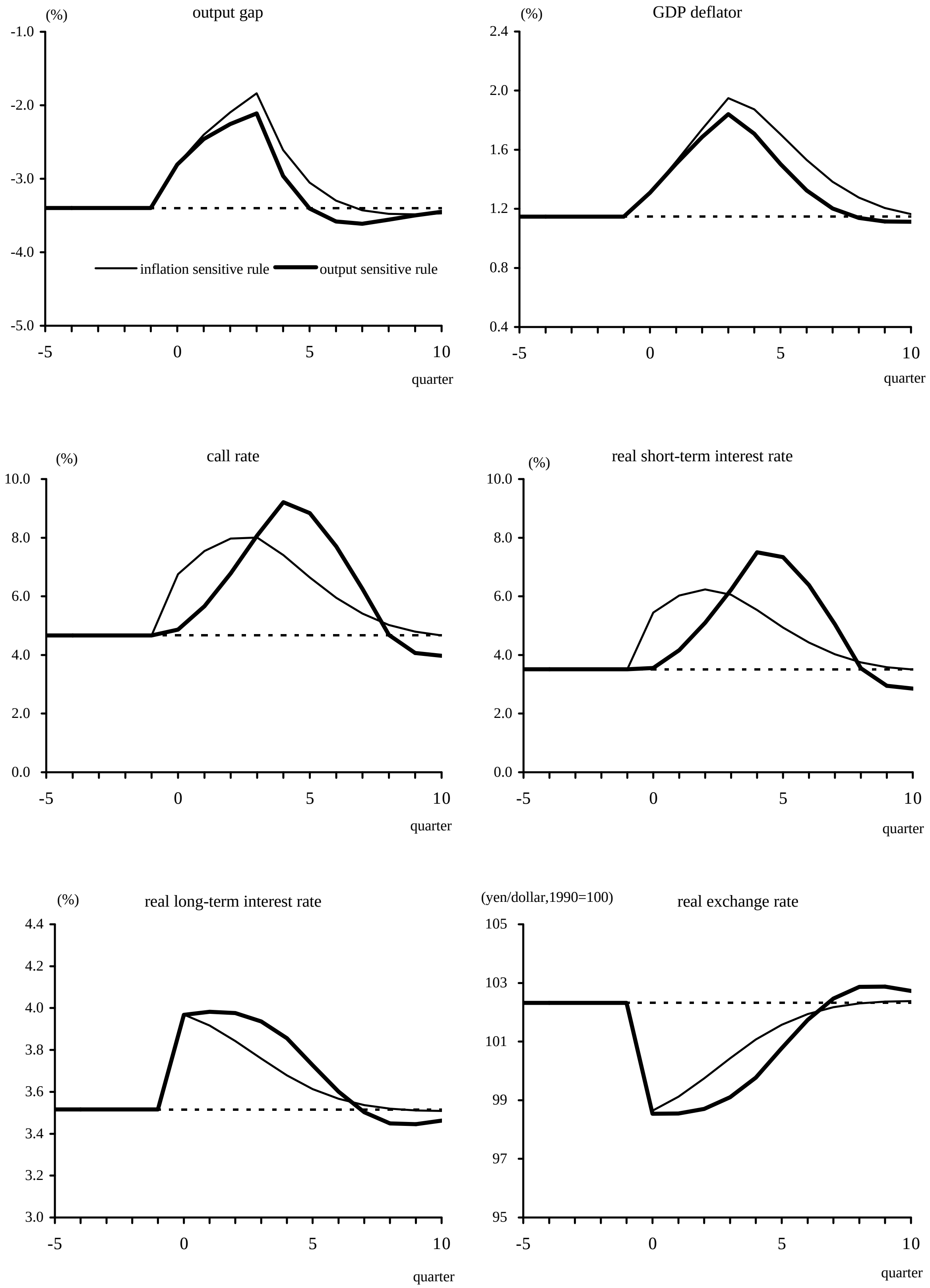


Chart 9

Comparing Policy Rule (Inflation Shock)

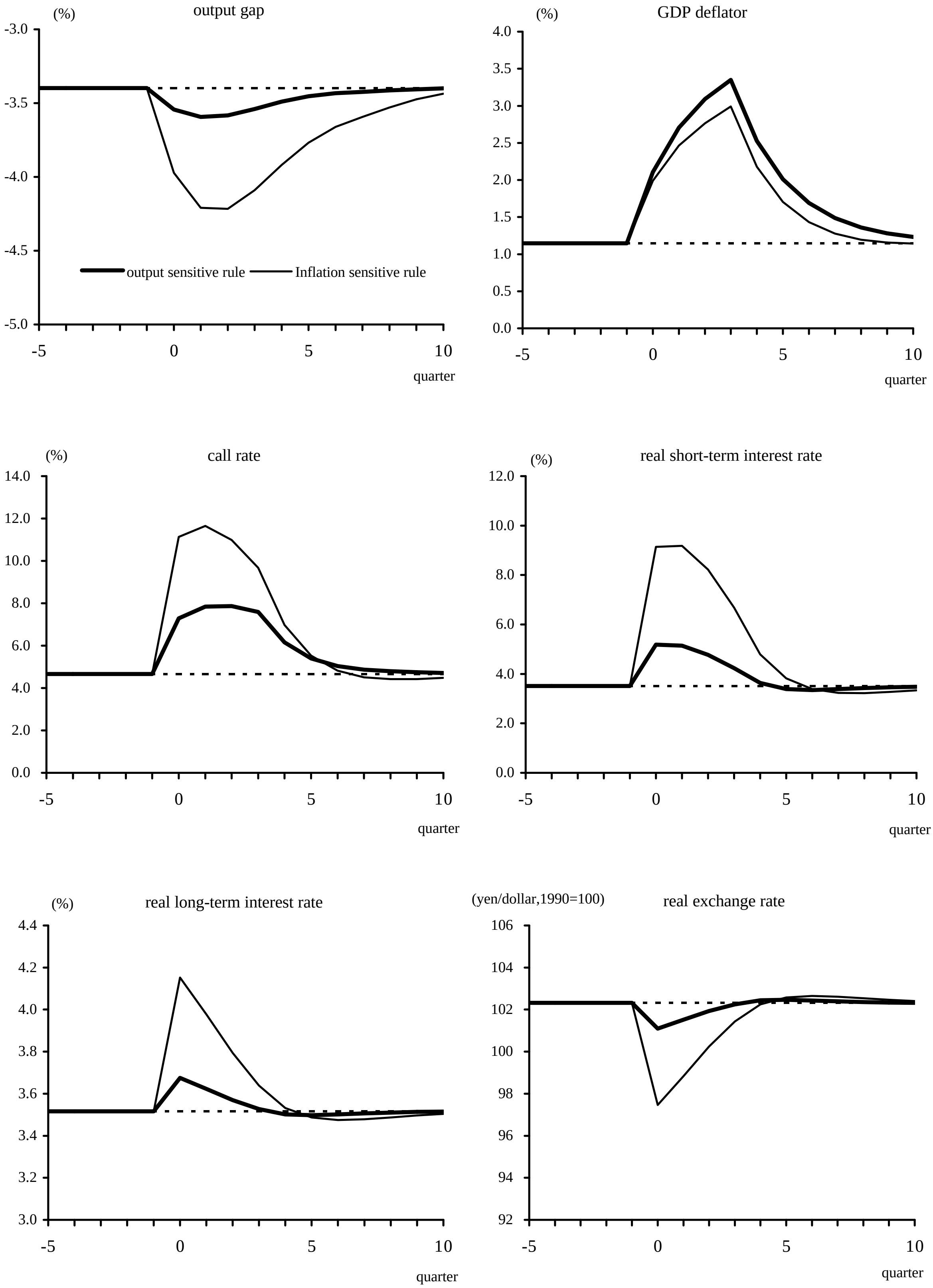
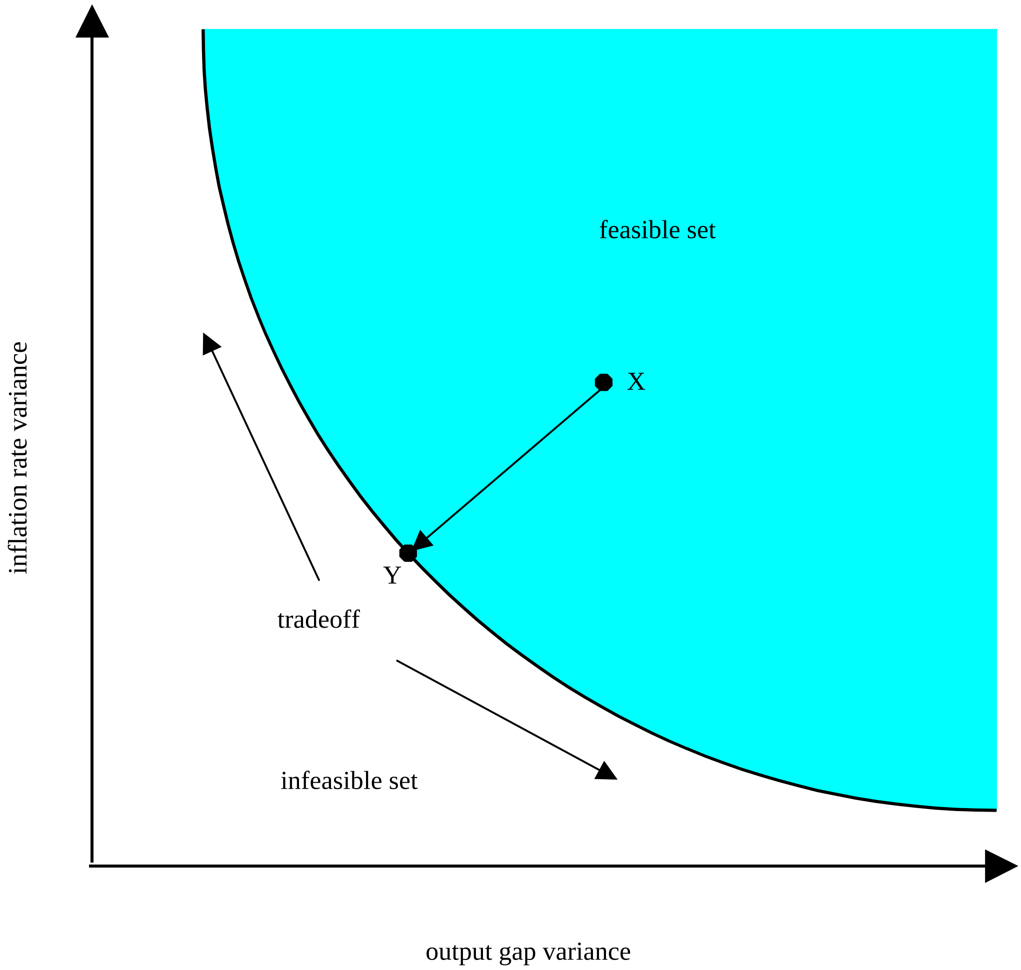


Chart10

Variance Tradeoff



Varinace Tradeoff by the Japanese Forward-looking Model

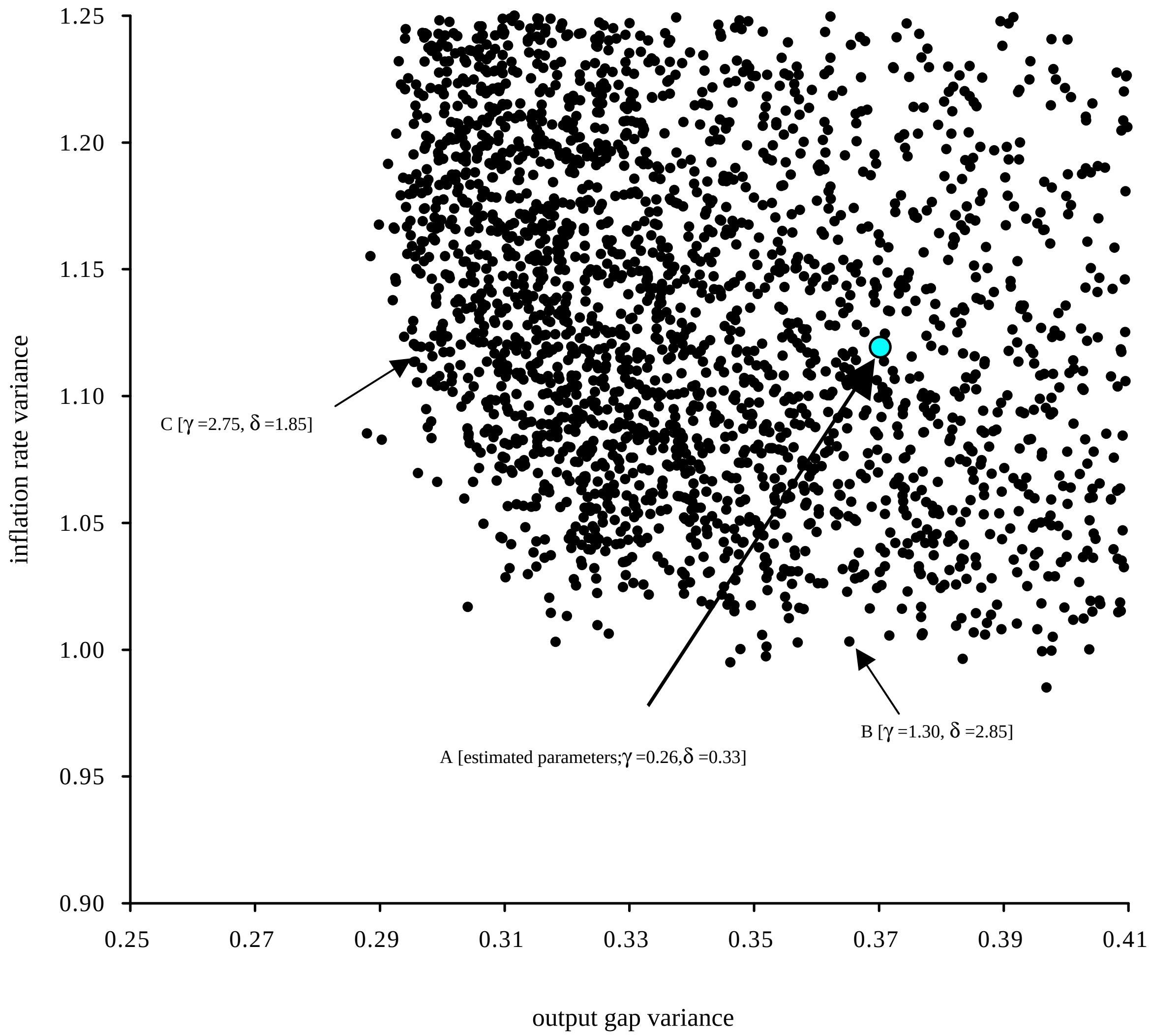


Chart A1

Zero-percent Constraint on Nominal Interest Rates (Demand Shock)

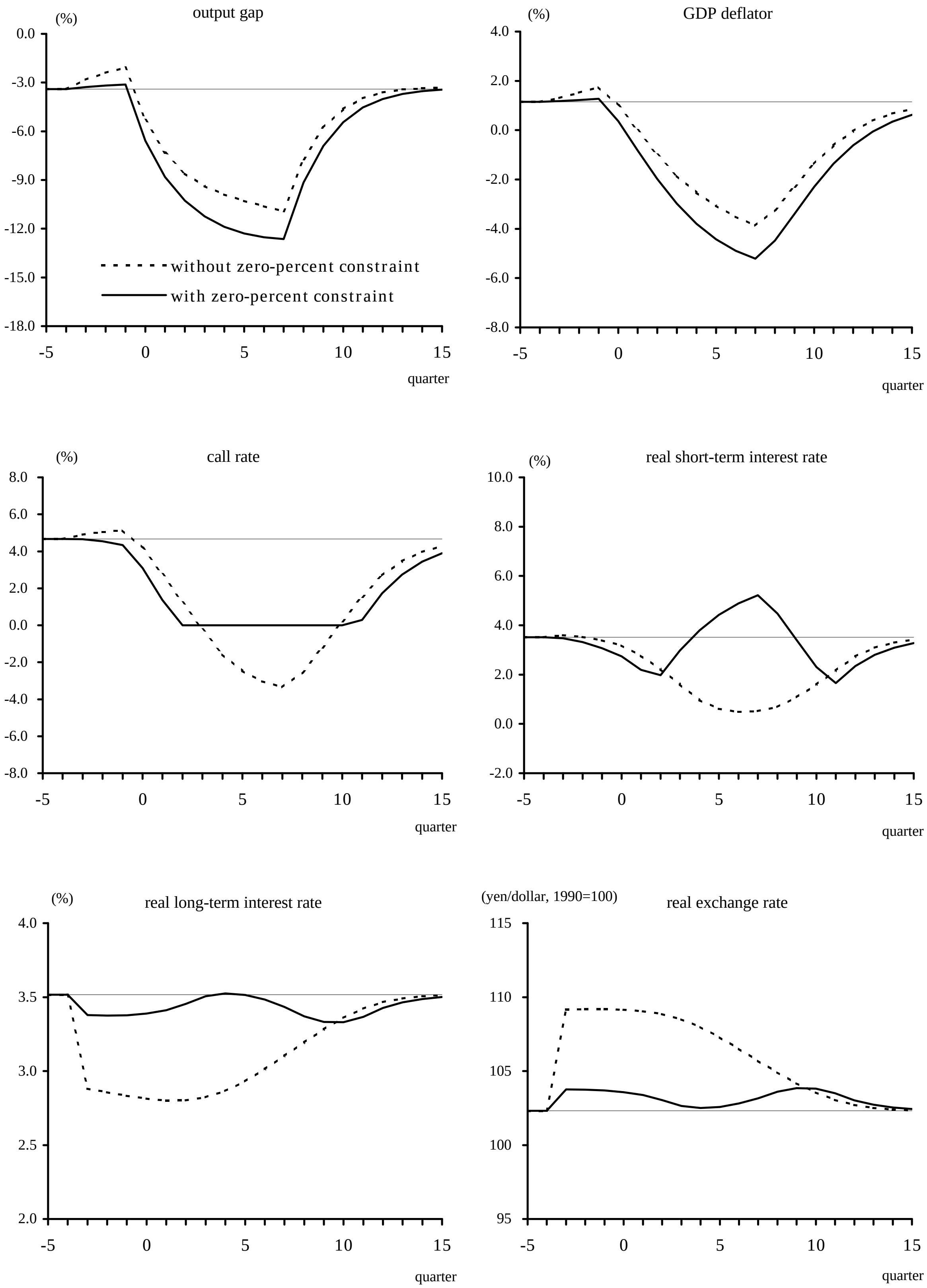
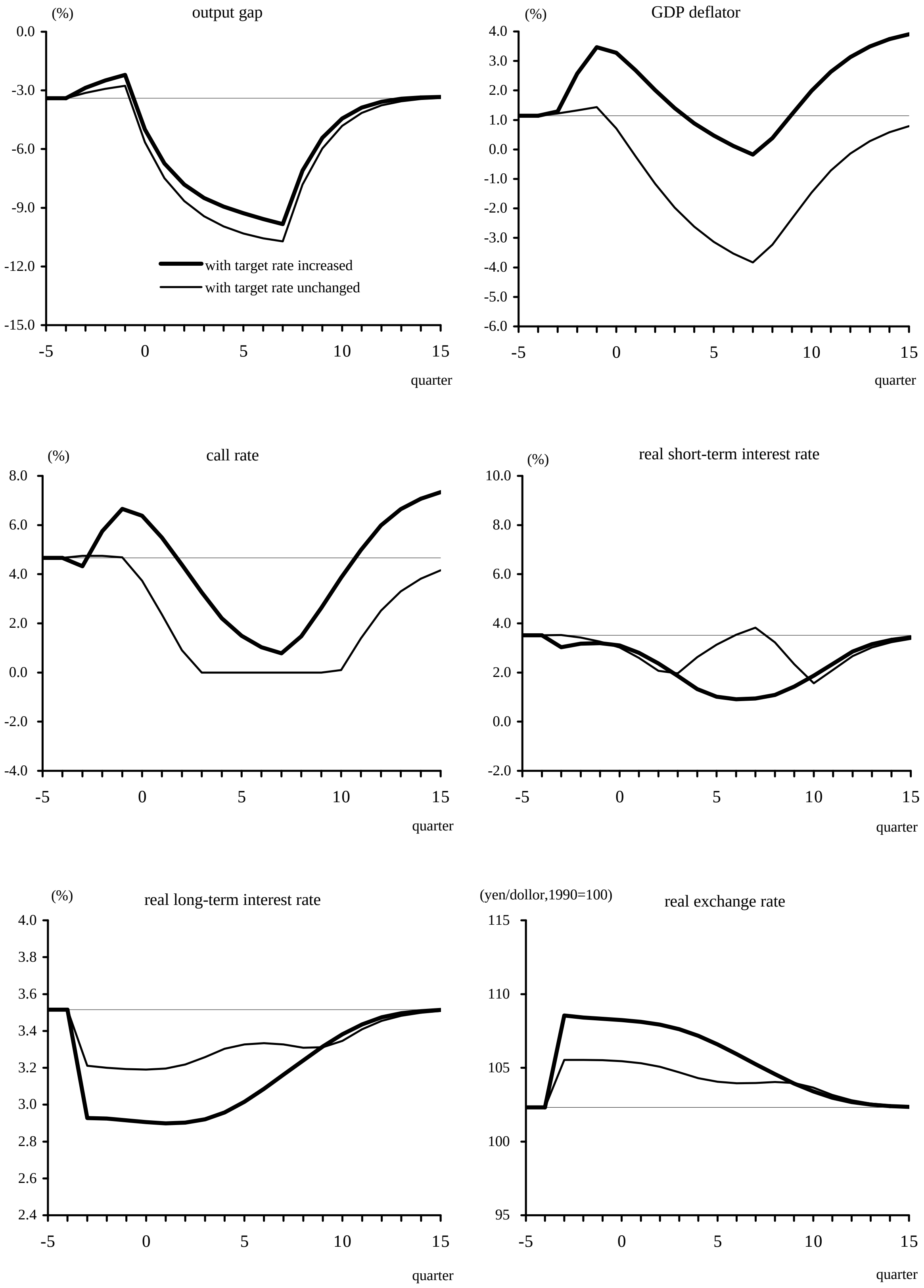


Chart A2

Raising Target Rate of Inflation (Demand Shock) (1)

(The demand shock is so large that nominal interest rates face the zero-percent constraint.)



<u>Chart A3</u>

Raising Target Rate of Inflation (Demand Shock) (2)

(The demand shock is so small that nominal interest rates do not face the zero-percent constraint.)