Bank of Japan Working Paper Series

Confidence Erosion and Herding Behavior in Bond Markets: An Essay on Central Bank Communication Strategy

Koichiro Kamada*
kouichirou.kamada@boj.or.jp

Ko Miura**
kou.miura@boj.or.jp

No.14-E-6
April 2014

Bank of Japan
2-1-1 Nihonbashi-Hongokucho, Chuo-ku, Tokyo 103-0021, Japan

* Monetary Affairs Department
** Monetary Affairs Department (currently Matsue Branch)

CONFIDENCE EROSION AND HERDING BEHAVIOR IN BOND MARKETS:*

AN ESSAY ON CENTRAL BANK COMMUNICATION STRATEGY

Koichiro Kamada† and Ko Miura‡

April 2014

Abstract

This paper examines the distinctive behavior of long-term interest rates observed after the Bank of Japan's introduction of quantitative and qualitative monetary easing, by focusing on changes in traders' confidence and herding behavior. When participants in bond markets lose confidence in their outlook for future interest rates, their investment decision depends heavily on the developments of market prices. This often leads to herding behavior among traders and destabilizes market prices: demand fuels further demand, or supply fuels further supply. This study develops a theoretical model and employs it for stochastic simulations to show that volatility of bond prices and trading volumes is affected by a number of factors, such as investors' confidence in the financial environment, the usefulness or value of information available in the market, and the market liquidity of bonds. In addition, the model is fitted to actual data to specify the driving forces underlying the changes in long-term interest rate volatility observed in 2013. The analysis shows that the key to understanding the developments in long-term interest rates during this period lies in how traders interpreted information flows in the market, especially the announcement by the Bank of Japan regarding its policy change, and in capturing the extent to which their confidence was weakened or strengthened by those information flows. The findings of the analysis highlight the importance of formulating a communication strategy as part of the conduct of monetary policy and the challenges in implementing such a strategy.

---

* We would like to thank Makoto Nirei and the staff of the Bank of Japan for their helpful comments and discussion. The opinions expressed here, as well as any remaining errors, are those of the authors and should not be ascribed to the Bank of Japan or the Monetary Affairs Department.

† Monetary Affairs Department, Bank of Japan (kouichirou.kamada@boj.or.jp).

‡ Monetary Affairs Department, Bank of Japan (currently Matsue Branch, kou.miura@boj.or.jp).

1. INTRODUCTION

On April 4, 2013, the Bank of Japan (BOJ) introduced quantitative and qualitative monetary easing. The aim of this new and bold policy was to raise inflation to the target rate of 2 percent within the space of 2 years, to double the amount of base money within 2 years through the purchase of mainly long-term government bonds, and to double the maturity of government bonds purchased. The policy had a substantial impact on bond markets.[1] 10-year government bond yields rose rapidly from a low of 0.446 percent in April to a peak of 0.933 percent in May and continued to fluctuate throughout the summer (Figure 1). This jump in bond yields and the sudden increase in volatility following a relatively mild downward trend until then show how much the BOJ's policy change came as a surprise to market participants. The bond market, however, soon calmed down again without falling into serious turmoil. Yields on 10-year government bonds fell to a monthly average of 0.635 percent for October, and as yields declined, so did the volatility of yields.

In this study, we focus on confidence erosion and herding behavior as two key concepts to understand the developments in government bond yields in 2013. Under the new policy, the BOJ embarked on monetary easing on an unprecedented scale. However, controversy abounded among market participants as to whether the policy would have the effects desired by the BOJ. When confidence is eroded, markets are driven by herding behavior among investors. Feeling that they lack sufficient information, investors pay more attention to other investors' actions and try to pick up as much information as they can. Herding emerges from these rational actions of individual investors and amplifies price fluctuations in the market. A possible explanation for the increased volatility of long-term interest rates observed around May 2013 is the emergence of herding behavior due to the erosion of confidence among investors. This view is supported by a survey on expectations of long-term interest rates and can also be inferred from actual developments in long-term interest rates

[1] In fact, the shock was sufficiently large for circuit breakers to be triggered twice in the market for government bond futures on April 5, 2013. However, the impact on 10-year government bond yields was not as large as that of the *unyobu shock* in 1998 and the *Var shock* in 2003.

(Figure 2).[2] Conversely, the decrease in the volatility of long-term interest rates observed thereafter can be explained by the diminishing of herding behavior due to a recovery in confidence among investors.

A variety of herding models have been proposed to express herding behavior in financial markets (e.g., Banerjee, 1992). One such model is the one developed by Nirei (2013) to investigate herding behavior in stock markets. Nirei's model has two important ingredients: (i) investors obtain private information on future stock prices before making investment decisions; (ii) they make inferences on other investors' private information based on their observations of market conditions. Investors exploit both private and market information to update their subjective probability regarding future stock prices and decide whether to make an investment or not. When stock prices are high, investors infer that prices will rise further. This, in turn, leads to more investment, pushing up actual stock prices. When stock prices are low, investors infer that prices will fall further. This discourages investment, driving down stock prices. In a market where demand fuels further demand and supply fuels further supply, security prices and trading volumes tend to be volatile, which creates so-called fat-tail distributions. Given these considerations, Nirei's model provides a useful analytical tool to explain the increased uncertainty in the bond market and the increased volatility of long-term interest rates during the first half of fiscal 2013 triggered by the BOJ's policy change.

However, Nirei's model, which deals only with private information, alone

---

[2] Figure 2(a) presents the distributions of 6 months ahead expectations of the yields of newly issued 10-year government bonds. The figure shows that market participants' expectations were more dispersed in May 2013 than in October 2012 and April 2013, which can be interpreted as a sign that market participants' confidence in the financial environment may have deteriorated. Figure 2(a) is base data provided by *QUICK Report*. We are grateful to QUICK Corp. for allowing us to use the data. Figure 2(b) focuses on the day-to-day changes in long-term interest rates since May 2013 and plots the log of the change on the horizontal axis and the log of the relative frequency of changes greater than or equal to that change on the vertical axis. If the distribution has a fat tail, a straight line can be fitted. If the distribution is normal, a rapidly downward sloping arc emerges. In the figure, we find that the interest rate changes since mid-May are on a straight line, indicating that the distribution is far from normal.

cannot explain the rapid increase in long-term interest rates around May 2013. The introduction of quantitative and qualitative easing by the BOJ, which triggered the increase in long-term interest rates, was not private information available exclusively to one investor, but public information to which all market participants had equal access. The existing literature on market microstructure, including the studies by Fleming and Remolona (1997, 1999), shows empirically that public information, such as the release of statistics and particularly central banks' policy announcements, has a strong impact on price formation in government bond markets.[3] Clearly, the public information in April 2013—i.e., the BOJ's policy announcement—had an impact on long-term interest rates and thus should be distinguished from private information. In order to incorporate government bond markets into Nirei's model, we extend his model to deal with a double-layered information structure consisting of both public and private information.

The more drastic a policy change is, the larger its impact is likely to be. Thus, in pursuing their policy goals, policymakers should try to keep market disturbances to a minimum. For this purpose, it is particularly important to know what determines the volatility of long-term interest rates and to find ways to minimize increases in volatility. In the theoretical model developed in this study, the volatility of long-term interest rates depends on a number of factors such as investors' confidence in the financial environment, the usefulness or value of information available in the market, and the market liquidity of bonds.[4] The model is then fitted to actual data on long-term interest rates and the forces underlying changes in the volatility of long-term interest rates observed in 2013 are examined. In recent years, with interest rates first in Japan and then the United States and Europe approaching the zero lower bound, the role of

---

[3] Previous studies such as Cutler et al. (1989) provide little evidence that public information has significant effects on stock markets. Instead, stock prices are largely determined by various types of private information, such as unconfirmed information about the development of new products and changes in management strategy. This suggests that public information is of relatively little importance in stock markets.

[4] In this study, the value of information is a concept influenced by both the degree of certainty and the relevance to interest rates. We will discuss this concept in detail later in Section 2.

central banks' communication strategy, including forward guidance, has attracted considerable attention as a means to avoid financial turmoil (see, e.g., Blinder et al., 2008). This study is not so ambitious as to derive an optimal communication strategy, but at the very least it aims at providing evidence that is useful in discussing communication strategies.

The remainder of the study is organized as follows. Section 2 extends Nirei's (2013) herding model so as to include public information in addition to private information. Section 3 implements various simulations to demonstrate the characteristics of the model. Section 4 fits the model to actual data and provides an interpretation of developments in long-term interest rates in 2013. Section 5 concludes.

2. THE MODEL

2.1. Public and private information

In this section, we develop a simple model to describe bond markets. In doing so, we extend Nirei's (2013) herding model, which deals only with private information, by incorporating a double-layered information structure consisting of public and private information (Figure 3).

Suppose that traders face uncertainty about the financial environment. For simplicity, we assume that there are two states regarding the financial environment, H and L, and denote the corresponding bond prices by p_H and p_L, respectively. We assume $p_L < p_H$. Thus, state H is a low interest rate environment where bond prices are high; and state L is a high interest rate environment where bond prices are low. Traders do not know which state they find themselves in, i.e., H or L, but have an *ex ante* subjective probability distribution regarding the current environment. They believe that they are trading in state H with probability b_0 and in state L with probability $1-b_0$. Denote the odds of state L over H by $\theta_0 \equiv (1-b_0)/b_0$. We call θ_0 traders' *prior confidence* in the financial environment. In the special case where $b_0 = 0.5$, i.e., $\theta_0 = 1$, traders have no confidence and are completely uncertain about the financial environment. As discussed later, we assume that b_0 and θ_0 are common to all traders.

We extend Nirei (2013) by introducing public information into his original model. Public information on the financial environment is released to traders at the end of the period. In this study, by public information we mean all information that is related to interest rates and to which all traders have equal access. Public information includes not only statistics and other data that have a direct impact on interest rates, such as inflation expectations, the potential rate of growth, and overseas interest rates, but also a range of other types of information that affect interest rates indirectly, such as labor statistics and various surveys. Public information of particular importance is changes in central bank policy as well as associated speeches. Note, however, that public information does not always convey correct information about the financial environment. We assume that public information is correct with probability q (>0.5) and wrong with probability $1-q$.[5] For instance, if the true financial environment is state H, public information indicates state H with probability q and state L with probability $1-q$. We assume that q is common and known to all traders.[6]

Below, we call q the value of information. The role of q is discussed in detail in the following sections. Here, we would only like to point out that q is a parameter measuring both the *degree of certainty* of public information and its *relevance* to interest rates. No matter how precise, public information has no information value if it has nothing to do with interest rates. And no matter how relevant to interest rates, public information has no information value if it is completely wrong.

On the other hand, by private information we mean unpublicized information that each trader collects to predict public information. Consider a trader who wants to

---

[5] We exclude the case of $q \le 0.5$. Suppose, for instance, that a provider of public information releases information indicating state H and that a receiver takes it as indicating state H with probability 0.4. Thus, in our setting, the receiver takes the information as indicating state L with probability 0.6 (= 1-0.4).

[6] Whether these assumptions are realistic is a matter of debate and considering them further is beyond the scope of this study. The interested reader is referred to Keynes (1921) and Keynes's reply to the criticism by Frank. P. Ramsey (Keynes, 1933).

predict which state, H or L, public information will indicate. For this purpose, trader i collects private information x_i. The trader knows that private information x_i is generated from distribution F_H when public information indicates state H and distribution F_L when public information indicates state L. Denote the densities of F_H and F_L by f_H and f_L, respectively. We assume that the likelihood ratio, $\delta(x) \equiv f_L(x)/f_H(x)$, is monotonically decreasing in x. Intuitively, this assumption implies that traders make the following conjecture. If x is high, it is likely that state H public information will be released and thus that the true financial environment is state H. For the simulation in the next section, we assume that F_H and F_L are normal distributions with means μ_H and μ_L ($< \mu_H$), respectively, and common standard deviation σ.

2.2. *Informed traders on the buying side*

We call traders who collect private information *informed traders* and distinguish them from *uninformed traders* who do not attempt to obtain private information. Furthermore, informed traders are divided into two groups: those on the buying side who choose between buying a bond or doing nothing and those on the selling side who choose between selling a bond or doing nothing.[7] To simplify the discussion, we assume that buying-side traders are always on the buying side. The same assumption applies to traders on the selling side.

Trader i updates his/her subjective probability using bond prices as well as his/her private information x_i. Denote the number of buying-side informed traders by n^d, of which k traders ($0 \le k \le n^d$) are ready to buy a bond, while the remaining $n^d - k$ do nothing. If traders buy bonds, they do so at the asking (selling) price offered by uninformed traders. Let $p^a(k)$ be the ask price when k buying-side informed traders are ready to buy a bond. Assume further that $p^a(k)$ is an increasing function of k. Informed traders know this function. Thus, when $p^a(k)$ is offered, traders can

[7] Nirei (2013) considers only demand-side informed traders, while we consider both demand-side and supply-side informed traders.

infer that k buying-side informed traders are ready to buy a bond. Combining these pieces of information, traders update their subjective probability distributions. Denote the posterior subjective probability of state H by b_{1i}^d and that of state L by $1-b_{1i}^d$, and further denote the odds of state L over H by $\theta_{1i}^d \equiv (1-b_{1i}^d)/b_{1i}^d$. θ_{1i}^d will be referred to as trader i's *posterior confidence* in the financial environment. In general, when traders receive information set $\{x_i, p^a(k)\}$, they update their confidence according to Bayes' rule as follows:

$$\theta_{1i}^d(x_i, p^a(k)) \equiv \frac{\Pr(L \mid x_i, p^a(k))}{\Pr(H \mid x_i, p^a(k))} = \frac{\Pr(x_i, p^a(k) \mid L)}{\Pr(x_i, p^a(k) \mid H)}\theta_0 . \tag{1}$$

Therefore, when θ_0 is given, θ_{1i}^d is calculated from $\Pr(x_i, p^a(k) \mid H)$ and $\Pr(x_i, p^a(k) \mid L)$.

Suppose that there is a critical value $\bar{x}(k)$ such that when $p^a(k)$ is offered, a trader buys a bond if private information x_i is greater than the critical value, but otherwise does nothing:

$$x_i \begin{cases} \geq \bar{x}(k) & \Rightarrow \textit{Buy a bond.} \\ < \bar{x}(k) & \Rightarrow \textit{Do nothing.} \end{cases} \tag{2}$$

When bond price $p^a(k)$ is offered, each trader infers that there are $k-1$ traders who are ready to buy a bond other than him/herself and the remaining n^d-k traders do nothing. Therefore, under the condition that the true financial environment is state H or L, the probability of $\{x_i, p^a(k)\}$ being generated is given as follows, taking account of the possibility of public information being wrong:

$$\Pr(x_i, p^a(k) \mid L) = qF_L(\bar{x}(k))^{n^d-k}(1-F_L(\bar{x}(k)))^{k-1} f_L(x_i)$$
$$+(1-q)F_H(\bar{x}(k))^{n^d-k}(1-F_H(\bar{x}(k)))^{k-1} f_H(x_i)\,; \tag{3}$$

$$\Pr(x_i, p^a(k) \mid H) = (1-q)F_L(\bar{x}(k))^{n^d-k}(1-F_L(\bar{x}(k)))^{k-1} f_L(x_i)$$
$$+qF_H(\bar{x}(k))^{n^d-k}(1-F_H(\bar{x}(k)))^{k-1} f_H(x_i)\,. \tag{4}$$

Substituting these into equation (1) gives

$$\theta_{1i}^{d}(x_i, p^a(k)) = \frac{qA(\bar{x}(k))^{n^d-k} B(\bar{x}(k))^{k-1}\delta(x_i) + 1 - q}{(1-q)A(\bar{x}(k))^{n^d-k} B(\bar{x}(k))^{k-1}\delta(x_i) + q}\theta_0, \qquad (5)$$

where

$$A(x) \equiv \frac{F_L(x)}{F_H(x)}; \qquad (6)$$

$$B(x) \equiv \frac{1-F_L(x)}{1-F_H(x)}. \qquad (7)$$

Since $\delta(x)$ is decreasing in x, the following inequalities hold:

$$A(x) > \delta(x) > B(x). \qquad (8)$$

In addition, $A(x)$, $\delta(x)$, and $B(x)$ are all decreasing in x (Nirei, 2013).

To close the model, we need to solve for critical value $\bar{x}(k)$. Assume that informed traders are risk neutral. Then, when they expect bond prices to rise, they buy a bond now and sell it at the end of the period. Consider zero-coupon bonds. Then, trader i buys a bond under the following condition:

$$b_{1i}^{d} p_H + (1-b_{1i}^{d}) p_L \geq p^a \qquad (9)$$

$$\Rightarrow \qquad \frac{p_H - p^a}{p^a - p_L} \geq \theta_{1i}^{d}. \qquad (10)$$

If $x_i = \bar{x}(k)$, bond trading gives rise to no profits by definition. Therefore, both equations (5) and (10) hold with equality through $\theta_{1i}^{d}(\bar{x}(k), p^a(k))$. That is,

$$\frac{p_H - p^a(k)}{p^a(k) - p_L} = \frac{qA(\bar{x}(k))^{n^d-k} B(\bar{x}(k))^{k-1}\delta(\bar{x}(k)) + 1 - q}{(1-q)A(\bar{x}(k))^{n^d-k} B(\bar{x}(k))^{k-1}\delta(\bar{x}(k)) + q}\theta_0. \qquad (11)$$

This yields $\bar{x}(k)$.

Traders' investment decision based on $\bar{x}(k)$ is incentive compatible. Define $C_i \equiv A(\bar{x}(k))^{n^d-k} B(\bar{x}(k))^{k-1} \delta(x_i)$. Then C_i is decreasing in x_i, since $\delta(x)$ is decreasing in x. The right-hand side of equation (5) is increasing in C_i when $q > 0.5$. Thus, the right-hand side of equation (5) is decreasing in x_i. By definition, if the trader is ready to buy a bond, inequality $x_i \geq \bar{x}(k)$ must hold. This implies that the right-hand side of equation (5) is smaller than the right-hand side of equation (11). Thus, equation (10) is satisfied. This shows that the above action rule of buying-side informed traders is incentive compatible.

Next, let us consider the bond demand function of buying-side informed traders. As in the model by Nirei (2013), $\bar{x}(k)$ is decreasing in k as n^d goes to infinity (see Lemma 1 in the Appendix). Figure 4(a) provides a graphic representation of the relationship between private information and demand, with the horizontal axis showing demand k, the vertical axis depicting private information x_i, and the upward-sloping curve representing critical value $\bar{x}(k)$. Suppose that trader 1 obtains private information x_1. Trader 1 is ready to buy a bond if the price is equal to or higher than $p^a(k_1)$, but does nothing otherwise. Suppose that trader 2 obtains private information x_2 $(> x_1)$. Trader 2 is ready to buy a bond if the price is equal to or higher than $p^a(k_2)$, but does nothing otherwise. If there are only two informed traders on the buying side, market demand is constructed as shown in Figure 4(b), where the demand function is an upward-sloping curve as a consequence of herding behavior among market participants.

The upward-sloping demand curve shown in Figure 4(b) contrasts with the usual downward-sloping demand curve found in text books. Suppose that each market participant makes investment decisions only based on his/her own private information, without inferring other participants' private information from bond prices. This is the same as assuming $A(\bar{x}(k))^{n^d-k} B(\bar{x}(k))^{k-1} = 1$ in equation (11). When bond prices go up, the left-hand side of equation (11) falls. Since $\delta(x)$ is decreasing in x, $\bar{x}(k)$ must be increasing in k, as shown in Figure 5(a), in order for the right-hand side of the equation to fall in line with the left-hand side. Suppose that traders 1 and 2 obtain the same private information as above, i.e., x_1 and x_2, respectively. Trader 1 buys a bond if the price is lower than $p^a(k_1')$ and trader 2 does so if the price is lower than $p^a(k_2')$. The market demand is given by the sum of their demand, shown as the downward-sloping curve in Figure 5(b).

The equilibrium bond price and trading volume are determined as follows. Suppose that there exists an auctioneer in the bond market and that he/she offers $p^a(k)$. Each buying-side informed trader compares his/her private information, x_i, with critical value $\bar{x}(k)$. If the former is greater than or equal to the latter, he/she buys a unit of bonds. Otherwise, his/her demand is zero. The market demand is given by the sum of all buying-side informed traders' demand and is denoted by $\Gamma^a(k)$. On the other hand, the market supply of bonds from uninformed traders is k by definition. Therefore, equilibrium trading volume k^* satisfies the equality $\Gamma^a(k^*) = k^*$ (see Proposition 1 in the Appendix for the existence of equilibrium). When there are multiple k^*, the minimum k^* is chosen as a unique solution, as is in Nirei (2013).

2.3. *Informed traders on the selling side*

A similar argument applies to informed traders on the selling side. Suppose that there are n^s selling-side informed traders. Let $p^b(h)$ be the bid price offered by uninformed traders when h selling-side informed traders are ready to sell. Assume also that the bid price function is decreasing in h. Define critical value $\underline{x}(h)$ such that each trader sells a bond if his/her private information is equal to or smaller than this critical value, but otherwise does nothing. That is,

$$x_j \begin{cases} \leq \underline{x}(h) & \Rightarrow \textit{ Sell a bond.} \\ > \underline{x}(h) & \Rightarrow \textit{ Do nothing.} \end{cases} \tag{12}$$

When each supply-side trader is offered bond price $p^b(h)$, he/she infers that there are h traders beside him/herself ready to sell bonds and the remaining $n^s - h$ traders do nothing. Therefore, the odds of state L over H, θ^s_{1j}, measure trader j's *posterior confidence* and are calculated as follows:

$$\theta_{1j}^{s}(x_j, p^b(h)) = \frac{qA(\underline{x}(h))^{h-1} B(\underline{x}(h))^{n^s-h} \delta(x_j) + 1 - q}{(1-q)A(\underline{x}(h))^{h-1} B(\underline{x}(h))^{n^s-h} \delta(x_j) + q} \theta_0 . \tag{13}$$

Informed traders are assumed to be risk-neutral. Thus, if they expect bond prices to go down, they will sell a bond and buy it back at the end of the period. Consider a zero-coupon bond market. Trader j sells a bond, if

$$b_{1j}^{s} p_H + (1 - b_{1j}^{s}) p_L \leq p^b \tag{14}$$

$$\Rightarrow \quad \frac{p_H - p^b}{p^b - p_L} \leq \theta_{1j}^{s} . \tag{15}$$

By definition, traders' profits are zero if $x_j = \underline{x}(h)$. Thus, equations (13) and (15) hold with equality through $\theta_{1j}^{s}(\underline{x}(h), p^b(h))$. That is,

$$\frac{p_H - p^b(h)}{p^b(h) - p_L} = \frac{qA(\underline{x}(h))^{h-1} B(\underline{x}(h))^{n^s-h} \delta(\underline{x}(h)) + 1 - q}{(1-q)A(\underline{x}(h))^{h-1} B(\underline{x}(h))^{n^s-h} \delta(\underline{x}(h)) + q} \theta_0 . \tag{16}$$

This solves for $\underline{x}(h)$. It is easy to show that this rule of action is incentive compatible. Note also that $\underline{x}(h)$ is increasing in h as n^s goes to infinity (see Lemma 1 in the Appendix). This is in contrast with $\overline{x}(k)$, which decreases in k. Denote market supply from selling-side informed traders at price $p^b(h)$ by $\Gamma^b(h)$. Since uninformed traders' bond demand is h by definition, equilibrium volume h^* must satisfy the equality $\Gamma^b(h^*) = h^*$ (see Proposition 1 in the Appendix for existence of equilibrium). When there exist multiple h^*, the minimum h^* is chosen as a unique solution.

2.4. Confidence updating when public information is released

When public information is released at the end of the period, all traders update their subjective probability regarding the financial environment. Private information here is defined as information which traders use to predict public information. Therefore, once public information is released, all private information loses its value. This implies that all traders have the same posterior probability distribution. This setting differs somewhat from the model developed by Nirei (2013), in which private information does not become obsolete but accumulates over time. In this case, as time goes by, each trader's information structure becomes more and more complex. However, we think that the definition employed in this study more realistically describes the actual situation in government bond markets and has the added benefit of substantially simplifying the model.

The way traders update their confidence depends on which state, H or L, public information indicates. Denote both traders' prior subjective probability of state H and the odds at the beginning with a time script, i.e., $b_0(t)$ and $\theta_0(t)$. These are updated as follows and used as traders' prior subjective probability and the odds at the beginning of the next period:[8]

$$b_0(t+1) = \begin{cases} \dfrac{qb_0(t)}{qb_0(t)+(1-q)(1-b_0(t))} & \textit{for state } H \textit{ public information.} \\ \dfrac{(1-q)b_0(t)}{(1-q)b_0(t)+q(1-b_0(t))} & \textit{for state } L \textit{ public information.} \end{cases} \tag{17}$$

Define $\eta \equiv q/(1-q)$. Then, the above equation simply becomes

$$\theta_0(t+1) = \begin{cases} \eta^{-1}\theta_0(t) & \textit{for state } H \textit{ public information.} \\ \eta\theta_0(t) & \textit{for state } L \textit{ public information.} \end{cases} \tag{18}$$

Let us start with an initial confidence level of $\theta_0(0)$ at time 0. Suppose that t periods have passed, during which public information indicating state H and L is released $\tau_H(t)$ times and $\tau_L(t)$ $(= t - \tau_H(t))$ times, respectively. Then the confidence at time t is given by

$$\theta_0(t) = \eta^{\tau_L(t)-\tau_H(t)}\theta_0(0)\,. \tag{19}$$

[8] In this study, we assume that q at the beginning of a period and q at end of the period are the same. However, the two can differ, especially when released public information has more value than expected. In this case, the extent of bond price volatility is not necessarily large.

This equation shows that given initial confidence, traders' confidence depends only on how many times more often state H public information is released than state L public information. When state L public information is released more frequently, confidence in state L is stronger than initially. When state H public information is released more frequently, confidence in state L is weaker than initially. If the true financial environment is state L, the frequency of state L public information becomes larger than that of state H public information over time. Therefore, confidence in state L grows infinitely large. Conversely, if the true financial environment is state H, confidence in state L shrinks to zero.

Lastly, we point out two important theoretical consequences here. First, there exists a band in which bond prices can fluctuate when traders obtain private information; and second, the band has a close relationship with bond prices that are available at the end of the period (or the beginning of the next period). To see the first point, note that $\overline{C} \equiv A(\overline{x}(k))^{n^d-k} B(\overline{x}(k))^{k-1} \delta(\overline{x}(k))$ on the right-hand side of equation (11) takes any value from zero to infinity. In addition, $\underline{C} \equiv qA(\underline{x}(h))^{h-1} B(\underline{x}(h))^{n^s-h} \delta(\underline{x}(h))$ on the right-hand side of equation (16) also takes any value from zero to infinity. Therefore, the range of motion of $p^a(k)$ and $p^b(h)$ is given by the following inequality:

$$\frac{(1-q)b_0 p_H + q(1-b_0) p_L}{(1-q)b_0 + q(1-b_0)} \leq p^b(h), p^a(k) \leq \frac{qb_0 p_H + (1-q)(1-b_0) p_L}{qb_0 + (1-q)(1-b_0)}. \quad (20)$$

Note that the rightmost term of equation (20) coincides with the expected bond price in the case of public information indicating state H, while the leftmost term is the same as the expected bond price in the case of public information indicating state L. The reason is simple. The bond price will go up as the probability increases that state H public information will be released. The bond price, however, does not go beyond the price available when traders know that state H public information is released. Similarly, the bond price will fall as the probability decreases that state H public information will be released. The bond price, however, does not fall below the price available when traders know that state L public information is released. This relationship is important when we explore how the volatility of bond prices or interest rates is determined.

2.5. Uninformed traders' ask and bid price functions

Lastly, we define uninformed traders' ask and bid price functions employed below. They are given by

$$p^a(k) = p_0 + \lambda\phi^a\left(\frac{k}{n^d}\right)^\gamma \quad \text{for} \quad 0 \le k \le n^d \tag{21}$$

and

$$p^b(h) = p_0 - \lambda\phi^b\left(\frac{h}{n^s}\right)^\gamma \quad \text{for} \quad 0 \le h \le n^s, \tag{22}$$

where p_0, ϕ^a, and ϕ^b are defined as follows:

$$p_0 = p_L + \frac{1}{1+\theta_0}(p_H - p_L); \tag{23}$$

$$\phi^a = \left(\frac{1}{1+\eta^{-1}\theta_0} - \frac{1}{1+\theta_0}\right)(p_H - p_L); \tag{24}$$

$$\phi^b = \left(\frac{1}{1+\theta_0} - \frac{1}{1+\eta\theta_0}\right)(p_H - p_L). \tag{25}$$

Note also that λ is a parameter measuring market liquidity of bonds and satisfies $0 \le \lambda \le 1$. When λ is close to 1, market liquidity of bonds is low. Conversely, when λ is close to zero, market liquidity is high.[9]

Some comments are in order here:

(a) Equation (23) is obtained by transforming the following equation:

---

[9] This study assumes $0 \le \lambda \le 1$ so that $\overline{x}(k)$ and $\underline{x}(h)$ become interior solutions. However, this is not a necessary condition for the analysis here. Alternatively, we can assume $\lambda > 1$ and define the ask price as the smaller of the following two values: the price indicated by equation (21) and the bond-price ceiling defined by equation (20). Similarly, we can define the bid price as the larger of the following two values: the price indicated by equation (22) and the bond-price floor defined by equation (20). In addition, although we use the same λ in equations (21) and (22), the λ in equation (21) is larger than that in equation (22) when there is a supply shortage of bonds in the market.

$$p_0 = b_0 p_H + (1 - b_0) p_L . \tag{26}$$

That is, p_0 is a fair price at the beginning of the period. Suppose that the ask price is set lower than the fair price. Traders gain from buying a bond at the ask price and then selling at the fair price, with no new information. However, if the ask price is set higher than the fair price, traders have no incentive to make such transactions. Here, we assume that the ask price available when the demand for bonds is zero is set at the fair price ($p^a(0) = p_0$). A similar assumption applies for the bid price ($p^b(0) = p_0$).

(b) Equation (20), which determines bond prices' range of motion after traders collect private information, is transformed into

$$\begin{aligned} p_L + \frac{1}{1+\eta\theta_0}(p_H - p_L) \le p^b(h) \le p_0 \\ \le p^a(k) \le p_L + \frac{1}{1+\eta^{-1}\theta_0}(p_H - p_L) . \end{aligned} \tag{27}$$

Note that the second and third inequalities hold by the definitions of the ask and bid price functions, i.e., equations (21) and (22). Below, the interval between the leftmost and rightmost terms in equation (27) is called the *information-constrained range of motion*. This range is distinguished from the *maximum range of motion*, which is defined as the band between the ceiling and floor of bond prices, i.e., p_H and p_L.

(c) In this study, we define ϕ^a such that the highest ask price ($p^a(1) = p_0 + \phi^a$) is equal to the ceiling of the information-constrained range of motion with $\lambda = 1$. Moreover, ϕ^b is defined such that the lowest bid price ($p^b(1) = p_0 - \phi^b$) is equal to the floor of the information-constrained range of motion.

(d) The *liquidity-constrained range of motion*, which emerges with $\lambda < 1$, is smaller than the information-constrained range of motion. As equations (21) and (22) show, the liquidity-constrained range is large when λ is large (market liquidity is low) and is small when λ is small (market liquidity is high).

(e) Lastly, when conducting our simulation, we assume $\gamma = 0.5$ following previous studies (e.g., Lillo et al., 2003).[10]

3. SIMULATION ANALYSIS

3.1. Fat-tail distributions of bond prices and trading volumes

The distributions of bond prices and trading volumes in the present study follow the power law and thus are characterized by fat tails. This is because the model employed here is an extension of Nirei's (2013) and thus inherits the latter's most important characteristic. This section conducts simulations to examine whether the distributions of bond prices and trading volumes are indeed characterized by fat tails. Below, we use the following parameter set as a benchmark: $n^d = n^s = 10{,}000$, $\mu_H = 1$, $\mu_L = -1$, $\sigma = 200$, $q = 0.8$ (or $\eta = 4$), $\lambda = 0.8$, $p_H = 100$, and $p_L = 86$. Private information is generated from distribution F_H. The simulation is iterated 25,000 times each for the selling and the buying side.

Figure 6(a) shows the distribution of changes in bond prices with the benchmark parameter set. The horizontal axis measures the percent change in bond prices, while the vertical axis depicts the relative frequency. Compared with the theoretical values based on a normal distribution, the simulated distribution clearly shows fat tails.[11]

To examine whether the simulated distribution is a power law distribution, we plot the log of the percent change in bond prices on the horizontal axis and the log of the relative frequency of percent changes greater than or equal to that percent change on the vertical axis. If the distribution is a power law distribution, we would see a

---

[10] Market liquidity of bonds is determined not only by λ but also by γ. Gabaix et al. (2006), for instance, show that the cost of restoring inventories reduced through transactions to their initial level depends on market liquidity and theoretically derive an ask price function that is similar to equation (21) when uninformed traders are risk-averse. However, when considering market liquidity, it is sufficient to examine only the role of λ and take γ as constant.

[11] Note that the distribution in Figure 6(a) is skewed toward the right. This is because private information is generated from F_H.

downward-sloping straight line. On the other hand, if the distribution is a normal distribution, we would see that a curve slopes downward in an arc. Figure 6(b) shows the result of this experiment. We find that the simulated curve clearly is closer to a straight line than a curved line based on a normal distribution. Figure 7 shows the simulated results for trading volumes, i.e., k^* / n^d and h^* / n^s, confirming that they follow a power law, as bond price changes do.

3.2. *Effects of traders' confidence on bond markets*

We start by examining the effects of traders' confidence on bond prices. Remember that bond prices depend on the ask and bid price functions offered by uninformed traders. When combined, these functions form an S-shaped curve as shown in Figure 8. We refer to this curve as the bid-ask price function. The shape and location of this function depend on various parameters. In particular, its average height is determined by p_0, which in turn also depends on various parameters, as shown in equation (26) (or equation (23)). With other parameter values given, p_0 is decreasing in θ_0. Therefore, as confidence in state L grows (i.e., θ_0 increases), average bond prices decline. Furthermore, as seen in equation (19), θ_0 depends on how many times state L public information is released in comparison to state H public information. This means that average bond prices are low when state L public information is released more frequently than state H public information.

Next, we investigate how traders' confidence influences bond price volatility. Bond prices are determined by the bid-ask price function. Remember that in our analysis here the price function moves in accordance with the information-constrained range of motion (see equation (27)). Therefore, the key to finding the determinants of bond price volatility lies in how this range is constructed. The width of this range is obtained from equation (27):

$$\begin{pmatrix} \textit{Width of information} \\ \textit{-constrained range} \end{pmatrix} = \left(\frac{1}{1+\eta^{-1}\theta_0} - \frac{1}{1+\eta\theta_0} \right)(p_H - p_L). \qquad (28)$$

Simple calculation shows that this width is maximized when $\theta_0 = 1$.[12] In Figure 8, the S-shaped bid-ask price function closes as θ_0 diverges from 1. This means that as traders' confidence in the financial environment weakens (i.e., θ_0 approaches 1), the volatility of bond prices increases.

Figure 9(a) shows the simulated distributions of percent changes in bond prices. The horizontal axis represents the percent change in bond prices and the vertical axis shows the log of its relative frequency. In the simulation, we assume three values for θ_0: 0.1, 1, and 10. As can be seen in the figure, the tails of the simulated distribution are fattest when $\theta_0 = 1$. Next, Figure 9(b) shows the simulated distributions of trading volumes, i.e., k^* / n^d and h^* / n^s. However, no clear pattern can be discerned.

3.3. *Effects of the value of public information on bond markets*

To begin with, we examine the effects of the value of public information on the volatility of bond prices. In Figure 10, two bid-ask price functions are depicted assuming two values for q: 0.6 and 0.8. The figure clearly shows that the bid-ask price function extends both upwards and downwards as q increases. This can also be proved mathematically by showing that the width of the information-constrained range of motion (i.e., equation (28)) is increasing in η and thus in q. Thus, the volatility of bond prices increases as the value of public information increases (i.e., q increases).

Next, Figure 11(a) presents the simulated distributions of percent changes in bond prices. In the simulation, we again assume two values for q: 0.6 and 0.8 (i.e., η is 1.5 and 4, respectively). We find that the tails of the simulated distribution are fatter when $q = 0.8$ than when $q = 0.6$, implying that the volatility of bond prices increases as the value of public information increases.

[12] As seen in equations (24) and (25), the ranges of motion of ask and bid prices depend respectively on ϕ^a and ϕ^b. These are maximized when θ_0 equals $\eta^{1/2}$ and $\eta^{-1/2}$, respectively, in the neighborhood of 1.

On the other hand, as seen in Figure 11(b), the simulated distributions of trading volumes are almost identical regardless of the value of q. It can be shown analytically in the model that when $\theta_0 = 1$ a change in q is absorbed in the change in prices and has no impact on the trading volume (this is the case for any λ). It should be noted that this result is due to the way that the model is set up in that the bid-ask price function moves in accordance with the information-constrained range of motion. Yet, we think that the result is quite plausible. As shown above, bond prices move considerably when q is large. This gives informed traders the opportunity to make large profits. However, at the same time, uninformed traders raise their ask prices and lower their bid prices in the hope of gaining from the price change. This reduces the opportunity for informed traders to make a profit and thus lowers transactions by them. Furthermore, there is also the well-known rule of thumb among practitioners that interest rate volatility in the government bond market often increases when major statistics are released, but that it takes time for trading volume volatility to react. Taking into account this fact, we can say that the simulation results obtained here are consistent with the reality of the market.

3.4. Effects of market liquidity on bond markets

In the analysis here, uninformed traders, when selling bonds to informed traders, infer that it becomes costly to restore their initial inventory levels as bonds become scarce in the market. Parameter λ captures these costs, which increase as market liquidity declines. In Figure 12, three bid-ask price functions are shown assuming three values for λ: 0.4, 0.8, and 1. Clearly, as market liquidity decreases (i.e., λ increases), the bid-ask price function extends both upwards and downwards. When $\lambda = 1$, the range within which bond prices can fluctuate reaches its maximum and coincides with the information-constrained range of motion (i.e., equation (28)).

The above discussion means that the volatility of bond prices declines when market liquidity increases. However, another force emerges that amplifies bond price volatility when market liquidity increases. Namely, as market liquidity increases, more favorable prices are offered to informed traders by uninformed traders; and thus more informed traders are attracted into the market to gain from bond transactions. Bond price volatility is determined by the balance of these two forces. Therefore, high market liquidity does not necessarily mean small volatility in bond prices. On the other hand, trading volume volatility increases as market liquidity increases.

Figure 13(a) shows the simulated distributions of bond prices. We again assume three values for λ: 0.4, 0.8, and 1. The simulation results indicate that bond price volatility does not necessarily fall even when market liquidity increases. On the other hand, Figure 13(b) clearly shows that trading volume volatility increases as market liquidity increases.

3.5. Effects of the accuracy of private information on bond markets

Lastly, we examine the effects of the accuracy of private information on bond markets. In the model developed in this study, private information is generated from F_H or F_L, depending on which state, H or L, public information indicates. These probability distributions are characterized by means μ_H and μ_L and by common standard deviation σ. When we say that the accuracy of private information is high, this means that the difference in the two means is large or the common standard deviation is small. When the accuracy of private information is high, its information value is also high. This prompts herding behavior among market participants and thus increases the volatility of bond prices and trading volumes.

Figure 14 shows the simulated distributions for two different assumptions regarding the difference between the means, i.e., when the difference is 1 and when it is 2. The figure shows that the volatility of bond prices and trading volumes is greater when the mean difference is greater, i.e., the accuracy of private information is greater. Figure 15 shows the simulated distributions obtained under two different assumptions regarding the standard deviation, i.e., when it is 200 and when it is 400. The panels clearly show that the volatility of bond prices and trading volumes is greater when the standard deviation is smaller, i.e., the accuracy of private information increases.

4. EMPIRICAL ANALYSIS

4.1. Fitting the model

In this section, we examine how well the model captures reality by fitting it to actual data. In estimating the model, we assume that the trend or the average level of bond prices is driven by public information, while fluctuations around the trend are caused by private information. In this subsection, q and thus η are assumed constant, but this assumption is relaxed in a later subsection.

We start by explaining how we set the values of p_H and p_L. Japan's recent potential rate of growth has been estimated at around 0.5 percent per annum. The inflation rate has been around zero percent or slightly negative; however, the BOJ raised the target rate of inflation to 2 percent on January 2013, and this decision may exert upward pressure on inflation expectations. Thus, it is reasonable to assume that inflation expectations are in the range of -0.5 to 2 percent. A term premium should be added on to long-term interest rates. Note that the size of the term premium varies over time. In fact, the BOJ has been implementing policies to squeeze the term premium. In addition, the size of the term premium is uncertain and includes measurement errors. Thus, we assume that the term premium is between 0 and 0.5 percent. Taking all these various aspects into account, it seems fair to assume that long-term interest rates vary in the range of 0 to 3 percent. We convert this interest-rate based range into a zero-coupon bond price range using the following relationship:

$$Bond\ prices = 100 \times e^{-10 \times Interest\ rates} \tag{29}$$

We thus obtain $p_H = 100$ and $p_L = 74$.[13]

The trend of bond prices is given by equation (23). Combining equation (23) with equation (19), i.e., the difference equation of traders' evolving confidence yields

[13] Both p_H and p_L are not constant from a long-run perspective. They will change if the potential rate of growth or mid- to long-term inflation rates change.

$$\frac{p_H - p_0(t)}{p_0(t) - p_L} = \eta^{\tau_L(t) - \tau_H(t)} \theta_0(0) . \tag{30}$$

Taking the log of both sides of the equation provides the following linear relationship:

$$y(t) = \alpha + \beta z(t) + \varepsilon(t) , \tag{31}$$

where t denotes the time when public information is released. $\varepsilon(t)$ is an error term. The dependent and independent variables respectively are defined as follows:

$$y(t) = \ln \frac{p_H - p_0(t)}{p_0(t) - p_L} ; \tag{32}$$

$$z(t) = \tau_L(t) - \tau_H(t) . \tag{33}$$

Note that coefficients α and β are related to the original parameters as follows:

$$\theta_0(0) = e^{\alpha} \tag{34}$$

$$\eta = e^{\beta} \quad \Rightarrow \quad q = e^{\beta} / (1 + e^{\beta}) \tag{35}$$

In the empirical analysis, the data on p_0 are given as the two-week average prices of actual zero-coupon 10-year government bonds and are used to construct the data for variable y .[14] Variable z is defined as the difference between the number of times state L public information is released and the number of times state H public information is released. No one is sure, however, which public information is released, or more precisely, whether the public information released is perceived as state L or state H information. Thus, we need to estimate time series z in order to estimate the parameters in equation (31). We obtain z as follows. Note that the amount by which

[14] In the analysis here, the unit of time is two weeks. The reason is as follows. The BOJ holds monetary policy meetings every month. Thus, an alternative time unit would be one month. However, in addition, speeches by the Governor and a variety of statistics are published at various times during a month. From this perspective, the unit of time should be shorter than one month. At the same time, the unit of time should be longer than one week in order to capture the trend in the long-term interest rate. Hence, using two weeks as the unit of time seems like a good compromise that takes these various considerations into account.

z rises or falls in each period is the same. Thus, given a sample size of T, we have 2^{T-1} potential paths of $z \equiv \{0, z(1), \cdots, z(T-1)\}$. Using each of these potential series as an explanatory variable, we conduct regression analyses and pick the series that maximizes the coefficient of determination as our estimate of z.[15]

To evaluate the performance of the model, we compare the estimate obtained using the model with the actual trend of bond prices. In Figure 16(a), the solid curve represents the trend of actual long-term interest rates since January 2013, while the dotted curve shows the model estimation. The figure indicates that the model traces, albeit roughly, the actual trend of long-term interest rates. Note that q is estimated to be 0.53. Based on these estimates, we can construct the information-constrained range of motion, the band depicted by the dotted lines in Figure 16(b). We see that the actual daily fluctuations in long-term interest rates (solid line) are more or less within this band.

4.2. Determinants of interest rate volatility in 2013

Although the model describes the overall development in actual long-term interest rates quite well, Figure 16 also shows that especially from June 2013 onward the estimate jumps considerably compared to the smooth movement of actual long-term interest rates. In addition, the estimated model fails to capture the decline in long-term interest rate volatility during the same period. To improve the fit of the model, we reexamine the assumptions made above in order to more accurately replicate the changes in long-term interest rate volatility in 2013.

According to the model, there are four potential sources of the increase in volatility in the bond market: (i) a decline in traders' confidence (i.e., θ_0 moves in the direction of 1), (ii) an increase in the value of public information (i.e., a rise in q or η), (iii) an increase in the accuracy of private information (i.e., μ_H and μ_L diverge or σ increases), and (iv) an increase in the maximum range of bond price motion (i.e., an

[15] It should be noted that the restriction $\eta > 1$ must hold in the regression analysis.

increase in $p_H - p_L$).[16] The potential decline in traders' confidence was taken into account in the analysis in the preceding section. Therefore, Figure 16 tells us that the change in traders' confidence is not sufficient to explain the change in interest rate volatility during the observation period. Regarding the accuracy of private information, while there is no reason to reject this as a potential factor underlying the increase in bond market volatility, it is too difficult to treat empirically due to the lack of adequate data.

An increase in the maximum range of bond price motion is a potential reason for the increase in bond prices, but we skipped this aspect in the previous section since from a theoretical perspective it is trivial. Nevertheless, it is not necessarily a straightforward matter whether this is a potential factor explaining the increase in interest rate volatility observed in 2013 when we trace the BOJ's recent policy changes. In March 2006, the BOJ introduced its "understanding" of medium- to long-term price stability, stating: "[A]n approximate range between zero and two percent was generally consistent with the distribution of each Board members' understanding of medium- to long-term price stability. Most Board members' median figures fell on both sides of one percent" (Bank of Japan, 2006). Then, in February 2012, the BOJ announced the price stability goal in the medium to long term, stating: "[The goal] is in a positive range of 2 percent or lower ... and, more specifically, [the Bank has] set a goal … [of] 1 percent for the time being" (Bank of Japan, 2012). Furthermore, in January 2013, the BOJ set the price stability target at 2 percent as a reference for monetary policy.

As indicated in the above summary, the BOJ has changed the terminology used to refer to price stability from an "understanding" to a "goal" and to a "target," but the ceiling of 2 percent has remained unchanged. Therefore, p_L did not necessarily change in early 2013. However, some may argue that the change from a price stability "goal" of 1 percent to a price stability "target" of 2 percent was in essence an increase in the ceiling of inflation rates. If this were the case, the ceiling of interest rates would have risen, and p_L would have declined, which would have

[16] We do not discuss a decline in market liquidity of government bonds here. As shown by the simulation in Section 3, it has no strong impact on interest rate volatility.

widened $p_H - p_L$ and thus increased interest rate volatility. In this case, however, the floor of inflation rates would also have changed by the same amount, and thus the floor of interest rates and p_H would have changed by the same amount. Taking this into consideration, $p_H - p_L$ and thus interest rate volatility would have not changed, even if the ceiling of inflation rates had changed. Moreover, although this argument is useful to explain the volatility increase in April and May, it fails to explain the volatility decline since June.

Given these considerations, in the following analysis we focus on the value of public information and allow q to vary over time. As mentioned above, we think of q as measuring the aggregate value of all public information released during one period. The amount and importance of financial statistics released varies across periods. Moreover, communications by the BOJ, such as speeches, reports and policy announcements, as well as their frequency, content, and implications differ across periods. Therefore, the aggregate value of public information can also be regarded as time variant. In order to more closely investigate the changes in interest rate volatility in 2013, we take two approaches to estimate time-varying q. The first approach assumes that q changes twice during the observation period (January to September 2013). The second approach assumes that q changes every period.

In the first approach, the two changes in q, at time t_1 and t_2, are incorporated using the following two dummy variables:

$$d_1(t) = \begin{cases} 0 & \text{for} \quad t < t_1. \\ (\tau_L(t) - \tau_H(t)) - (\tau_L(t_1) - \tau_H(t_1)) & \text{for} \quad t_1 \leq t. \end{cases} \tag{36}$$

$$d_2(t) = \begin{cases} 0 & \text{for} \quad t < t_2. \\ (\tau_L(t) - \tau_H(t)) - (\tau_L(t_2) - \tau_H(t_2)) & \text{for} \quad t_2 \leq t. \end{cases} \tag{37}$$

We take changes in q into account by estimating equation (31) including these dummies. In doing so, t_1 and t_2 are estimated simultaneously to maximize the coefficient of determination. Figure 17 presents the estimation result. Figure 17(a) shows that the estimated model traces the interest rate developments since June very well. Furthermore, Figure 17(b) indicates that the estimated model captures both the increase in interest rate volatility in April and May and the decline since June.

Next, Figure 18(a) shows the estimates of q adjusted by the two dummy variables. In the figure, q increased in early February 2013 and decreased in late June.[17] The white bars indicate that state H public information was released, while the black bars indicate that state L public information was released. The high white bars in February and March in 2013 imply that public information led to a decline in long-term interest rates during this period. In contrast, the high black bars in April, May, and early June 2013 imply that public information pushed up long-term interest rates during this period. Since then, the white bars have been consistently low, indicating that long-term interest rates are following a gradual downward trend.

Next, we describe the implementation of the second approach. Note that equation (30) is fully satisfied if η is time variant. We look at the two-week average of actual 10-year government bond prices. If the average goes up, we judge that state H public information was released and otherwise that state L public information was released. Thus, we obtain $\theta_0(0)$ and $\{\eta(1), \eta(2), \cdots, \eta(T-1)\}$ as follows:

$$\theta_0(0) = \frac{p_H - p_0(0)}{p_0(0) - p_L} \tag{38}$$

$$\eta(t) = \begin{cases} \dfrac{p_H - p_0(t)}{p_0(t) - p_L} \Big/ \dfrac{p_H - p_0(t+1)}{p_0(t+1) - p_L} & \textit{for state } H \textit{ public information.} \\ \dfrac{p_H - p_0(t+1)}{p_0(t+1) - p_L} \Big/ \dfrac{p_H - p_0(t)}{p_0(t) - p_L} & \textit{for state } L \textit{ public information.} \end{cases} \tag{39}$$

Figure 18(b) shows the estimated series of q. The interpretation is almost the same as in the preceding paragraph, so we do not repeat it here.

Figure 18 highlights important issues concerning central bank communication strategy. First, central bank communication becomes particularly difficult at the time of

---

[17] Denoting the coefficients on the two dummies by β_1 and β_2, we can calculate q adjusted by the two dummies as $e^{\beta+\beta_1+\beta_2}/(1+e^{\beta+\beta_1+\beta_2})$.

a policy shift, as suggested by the black bars that appear from April to early June 2013. At the January 2013 monetary policy meeting, the BOJ replaced its price stability goal of 1 percent with the price stability target of 2 percent. This policy announcement was received by market participants as plausible information indicating further monetary easing and created the expectation that nominal interest rates would fall further. In contrast, the policy change announced in April was wrongly interpreted by the majority of market participants as indicating that the BOJ would allow interest rates to rise. However, from the BOJ's actions it is clear that it had no such intention.

Second, the central bank cannot arbitrarily control the size of q, as suggested by the sequence of low values of q. While the BOJ continued with large-scale asset purchases to squeeze term premiums, its communication was taken to indicate that nominal interest rates would decline. Nonetheless, by the end of our observation period in September 2013, q had not been restored to its January 2013 level. One of the reasons that the BOJ cannot increase q rapidly is that although the information it provides has a strong impact on the economy, it is only one element of a vast amount of information that market participants have to digest. Therefore, when countervailing information emerges, the BOJ's information loses its relative value. For instance, US long-term interest rates have started to rise since Federal Reserve Chairman Ben S. Bernanke made statements in May and June 2013 concerning the exit from quantitative easing (Bernanke, 2013a, b). The Chairman's statements likely exerted upward pressure on Japanese long-term interest rates, thereby keeping q at a low level.

Note, however, that an increase in q is not always desirable. Figure 19 shows how the information-constrained range of motion of interest rates evolves over time. When q is large (0.6), the width of the range temporarily expands but shrinks rapidly thereafter. On the other hand, when q is small (0.55), the width of the range remains small but shrinks only gradually. Given this, the central bank must make the most appropriate choice: it can allow temporary high volatility in interest rates and try to restore market participants' confidence; or it can use time to increase market participants' confidence and avoid an increase in interest rate volatility. The estimated q at least partially reflects the central bank's optimal actions.

5. CONCLUSION

In this study, we developed a theoretical model to explain long-term interest rate volatility, focusing on two key aspects: confidence erosion and herding behavior. We then fitted the model to actual interest rate data and showed that the model can explain, albeit quite roughly, the developments in long-term interest rates around the time of the BOJ's introduction of quantitative and qualitative easing. Furthermore, examining the fitness of the model, we find that the value of public information, including the central bank's communication, plays an important role in explaining fluctuations of long-term interest rates in 2013.

When the policy interest rate is close to the zero lower bound, the central bank must put high priority on its communication strategy to affect long-term interest rates. From this point of view, the information-flow approach adopted in this study points out one possible direction for analyzing the role of central bank communication, including forward guidance, and of unconventional monetary policy, such as quantitative easing, more generally. In this concluding section, we summarize the main findings obtained in the preceding sections, focusing on central bank communication strategy. In doing so, we come back to Figure 3, which represents the double-layered information structure in bond markets employed in our model.

Our analysis showed that in terms of Figure 3, the nexus between the true financial environment and public information is crucial and that the central bank must choose its language carefully to get the intended message across to market participants. In doing so, the central bank must bear in mind that market participants interpret central bank communications and choose their actions based on their own risk-and-return perspective. The empirical analysis showed that the rise in long-term interest rates and the increase in interest rate volatility in April and May 2013 took place in a tense atmosphere in which market participants had no clear idea of how the BOJ's policy change would affect future interest rates. However, central bank communication always and everywhere poses a challenge. For instance, as mentioned in Section 4, the speeches by Federal Reserve Chairman Ben S. Bernanke in May and June 2013 concerning the exit from quantitative easing unintentionally triggered a rapid rise in long-term interest rates.

Regarding the link between public information and bond price volatility, the analysis provided two important findings. First, the central bank cannot arbitrarily control the value of public information. Information provided by the central bank represents only one element in the vast amount information that bond market participants process and the value of central bank information is determined in relation to other public information. Information coming from the United States and other major economies abroad is particularly important, since it can have a large impact on Japanese long-term interest rates. Thus, with global financial markets closely intertwined, the BOJ has only limited control over the relative value of its announcements. Second, when using public information, the central bank must have a clear idea of how fast market participants' confidence should be increased. In the herding model employed in this study, a rise in the value of public information temporarily amplifies interest rate volatility. On the other hand, as the value of public information increases, market participants' confidence strengthens faster than otherwise would be the case, and thus interest rate volatility is contained faster. The central bank must make the optimal choice subject to this trade-off between short-run and long-run market stability in implementing its communication strategy.

APPENDIX. PROOF OF THE EXISTENCE OF MARKET EQUILIBRIUM

Lemma 1. $\bar{x}(k)$ is monotonically decreasing in k when n^d is sufficiently large. $\underline{x}(h)$ is monotonically increasing in h when n^s is sufficiently large.

Transforming equation (11) yields

$$V^a(k) = A(\bar{x}(k))^{n^d - k} B(\bar{x}(k))^{k-1} \delta(\bar{x}(k)), \tag{A1}$$

where

$$V^a(k) \equiv \frac{(1-q) - q\dfrac{1}{\theta_0}\dfrac{p_H - p^a(k)}{p^a(k) - p_L}}{(1-q)\dfrac{1}{\theta_0}\dfrac{p_H - p^a(k)}{p^a(k) - p_L} - q}. \tag{A2}$$

Taking the log of both sides of equation (A1) yields

$$\ln\frac{A(\bar{x}(k))}{B(\bar{x}(k))} + \ln\frac{V^a(k+1)}{V^a(k)}$$

$$= (n^d - k - 1)\ln\frac{A(\bar{x}(k+1))}{A(\bar{x}(k))} + k\ln\frac{B(\bar{x}(k+1))}{B(\bar{x}(k))} + \ln\frac{\delta(\bar{x}(k+1))}{\delta(\bar{x}(k))}. \tag{A3}$$

The first term on the left-hand side is positive from equation (8). It is clear from equation (21) that as n^d increases, the difference between $(k+1)/n^d$ and k/n^d converges to zero, and so does the difference between $p^a(k+1)$ and $p^a(k)$. Thus, the difference between $V^a(k+1)$ and $V^a(k)$ converges to zero, and so does the second term on the left-hand side. This implies that the left-hand side of equation (A3) is positive when n^d is sufficiently large. Since $A(x)$, $B(x)$, and $\delta(x)$ are all decreasing in x, the right-hand side is positive only if $\bar{x}(k) > \bar{x}(k+1)$. This shows that $\bar{x}(k)$ is decreasing in k. Similarly, we can use equations (8) and (22) to show that $\underline{x}(h)$ is increasing in h when n^s is sufficiently large.

Proposition 1. There exists a k^* that satisfies $\Gamma^a(k^*) = k^*$ when n^d is sufficiently large. Moreover, there exists an h^* that satisfies $\Gamma^b(h^*) = h^*$ when n^s is sufficiently large.

We know from Lemma 1 that $\bar{x}(k)$ is monotonically decreasing in k. Thus, Γ^a is a monotonic mapping. Therefore, following Nirei (2013), we can show the existence of equilibrium k^* using Tarski's fixed point theorem for a discrete monotonic mapping. The existence of equilibrium h^* can be proved in a similar manner.

REFERENCES

Banerjee, Abhijit V. (1992), "A Simple Model of Herd Behavior," *Quarterly Journal of Economics*, Vol. 107, No. 3, pp. 797-817.

Bank of Japan (2006), "The Introduction of a New Framework for the Conduct of Monetary Policy," March 9, 2006, http://www.boj.or.jp/en/announcements/release_2006/k060309b.htm/.

-------------------- (2012), "The Price Stability Goal in the Medium to Long Term," February 14, 2012, http://www.boj.or.jp/en/announcements/release_2012/k120214b.pdf.

Bernanke, Ben S. (2013a), Statement by Ben S. Bernanke, Chairman of the Board of Governors of the Federal Reserve System before the Joint Economic Committee, May 22, 2013, http://www.federalreserve.gov/newsevents/testimony/bernanke20130522.pdf.

------------------ (2013b), Transcript of Chairman Bernanke's Press Conference, June 19, 2013, http://www.federalreserve.gov/mediacenter/files/FOMCpresconf20130619.

Blinder, Alan S.; Michael Ehrmann; Marcel Fratzscher; Jakob De Haan; and David-Jan Jansen (2008), "Central Bank Communication and Monetary Policy: A Survey of Theory and Evidence," *Journal of Economic Literature*, Vol. 46, No. 4, pp. 910-945.

Cutler, David M.; James M. Poterba; and Lawrence H. Summers (1989), "What Moves Stock Prices?" *Journal of Portfolio Management*, Vol. 15, pp. 4-12.

Fleming, Michael J., and Eli M. Remolona (1997), "What Moves the Bond Market?" *FRBNY Economic Policy Review*, December, 1997, pp. 31-50.

--- (1999), "Price Formation and Liquidity in the U.S. Treasury Market: The Response to Public Information," *Journal of Finance*, Vol. LIV, No. 5, pp. 1901-1915.

Gabaix, Xavier; Parameswaran Gopikrishnan; Vasiliki Plerou; and H. Eugene Stanley (2006), "Institutional Investors and Stock Market Volatility," *Quarterly Journal of Economics*, Vol. 121, No. 2, pp. 461-504.

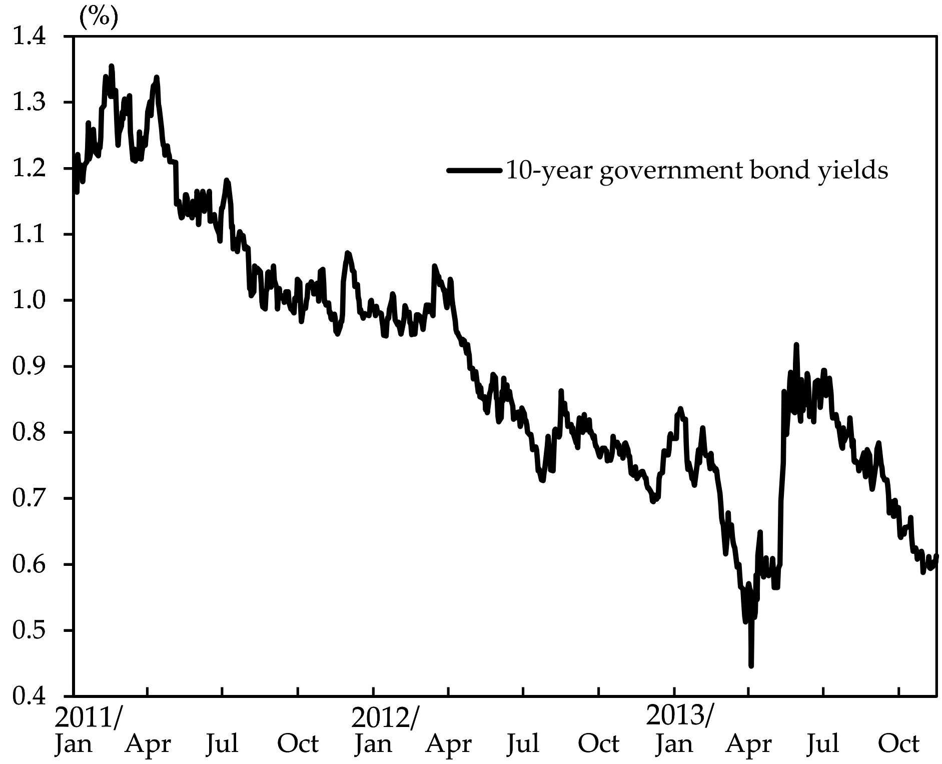
Keynes, John M. (1921), *A Treatise on Probability*, reprinted in *The Collected Writings of John Maynard Keynes*, Vol. VIII, London: Macmillan.

-------------------- (1933), *Essays in Bibliography*, reprinted in *The Collected Writings of John Maynard Keynes*, Vol. X, London: Macmillan.

Lillo, Fabrizio; J. Doyne Farmer; and Rosario N. Mantegna (2003), "Master Curve for Price-Impact Function," *Nature*, Vol. 421, pp. 129-130.

Nirei, Makoto (2013), "Beauty Contests and Fat Tails in Financial Markets," http://ssrn.com/abstract=2362341.

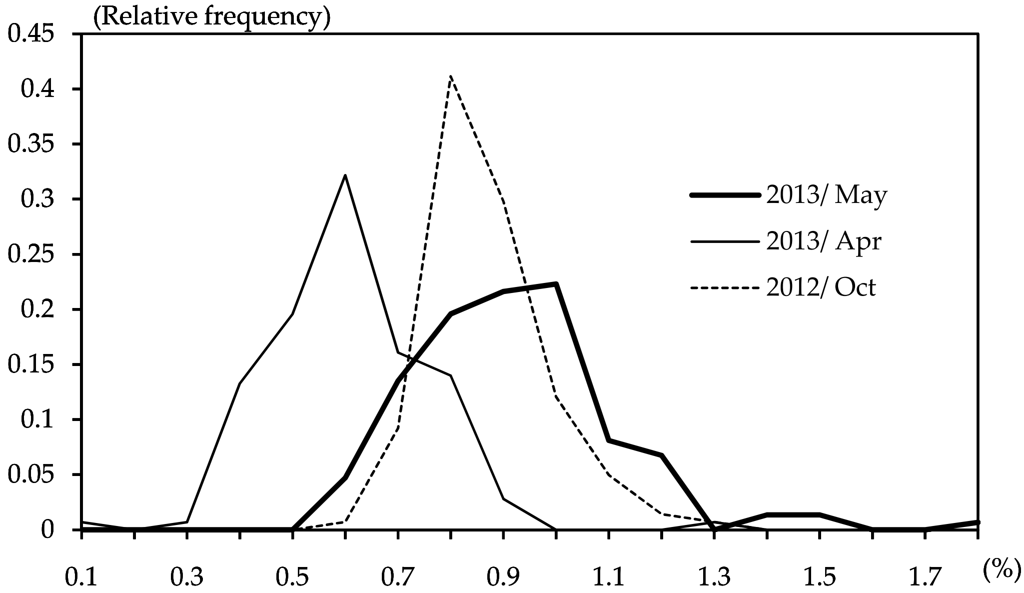
Figure 1. Long-Term Interest Rates



Source: Bloomberg.

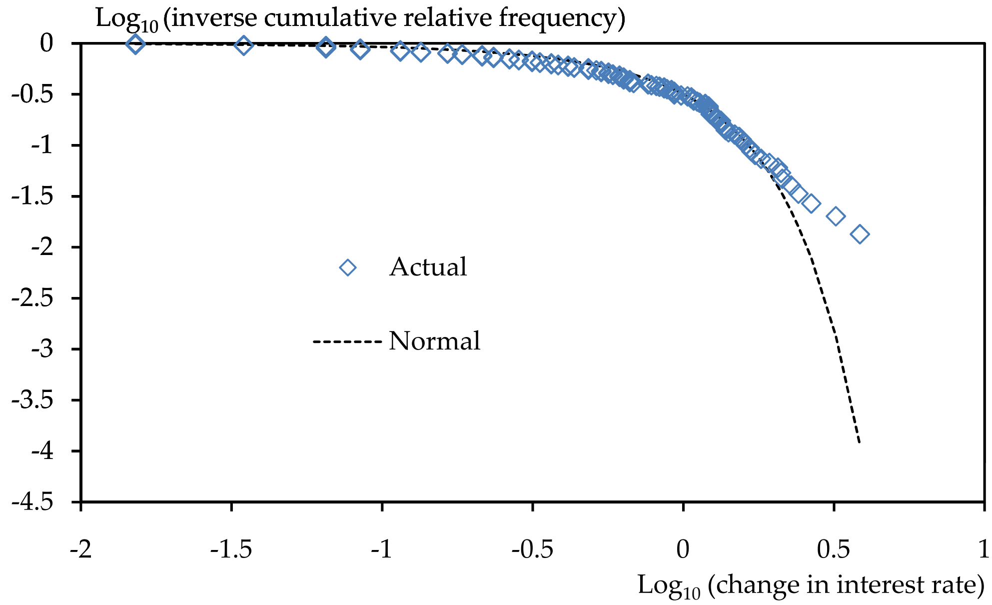
Figure 2. Confidence Erosion and Herding Behavior

(a) Distributions of 6 month ahead expectations of yields on newly issued 10-year government bond



Source: QUICK Report

(b) Day-to-day changes in 10-year government bond yields since May 2013



Notes: 1. The horizontal axis measures the log of the day-to-day change in 10-year government bond yields normalized by the mean and standard deviation; the vertical axis measures the log of the relative frequency of changes greater than or equal to that change.

2. The sample covers the period from May 15, 2013 to December 6, 2013.

Figure 3. Information Structure in the Model

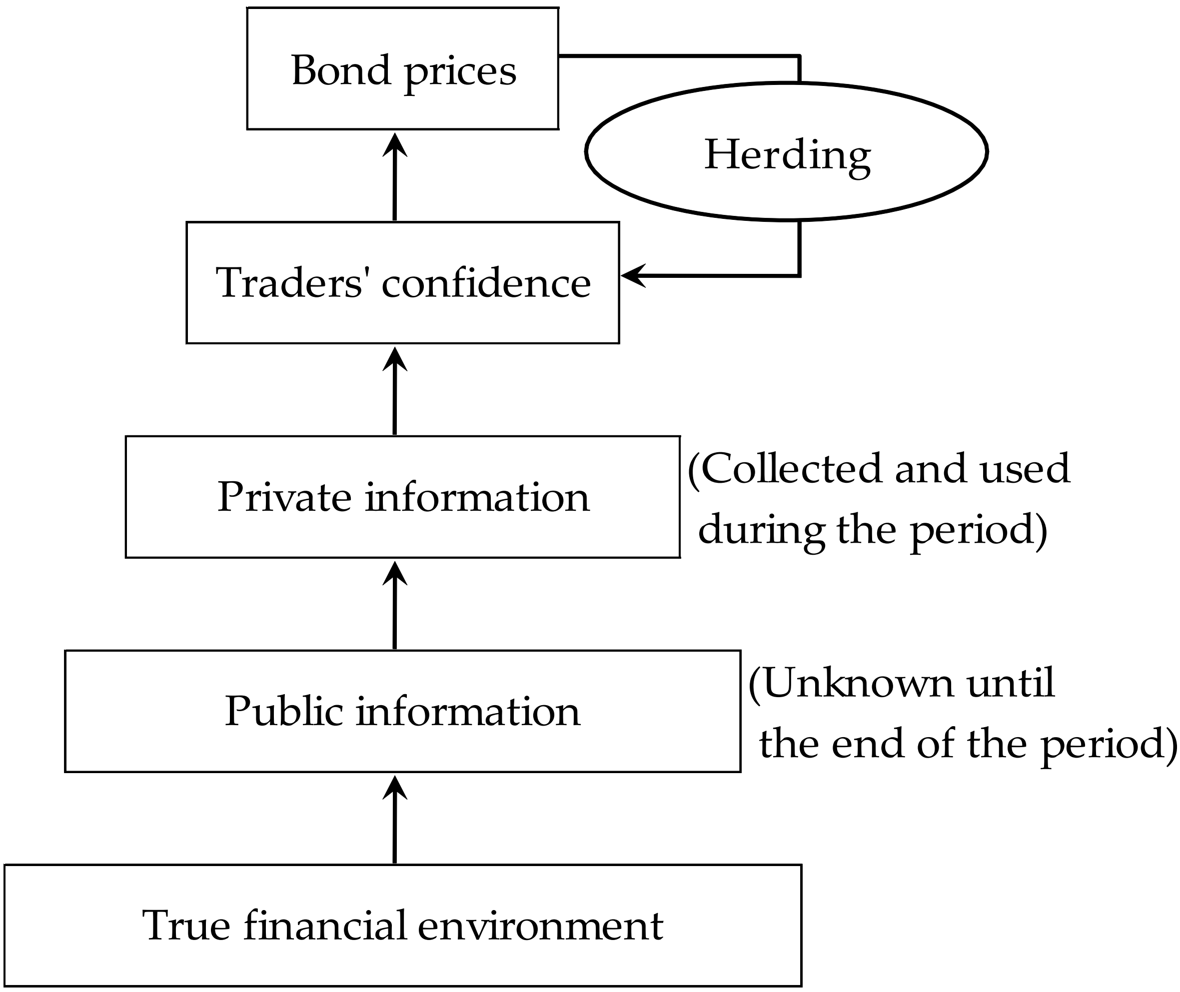
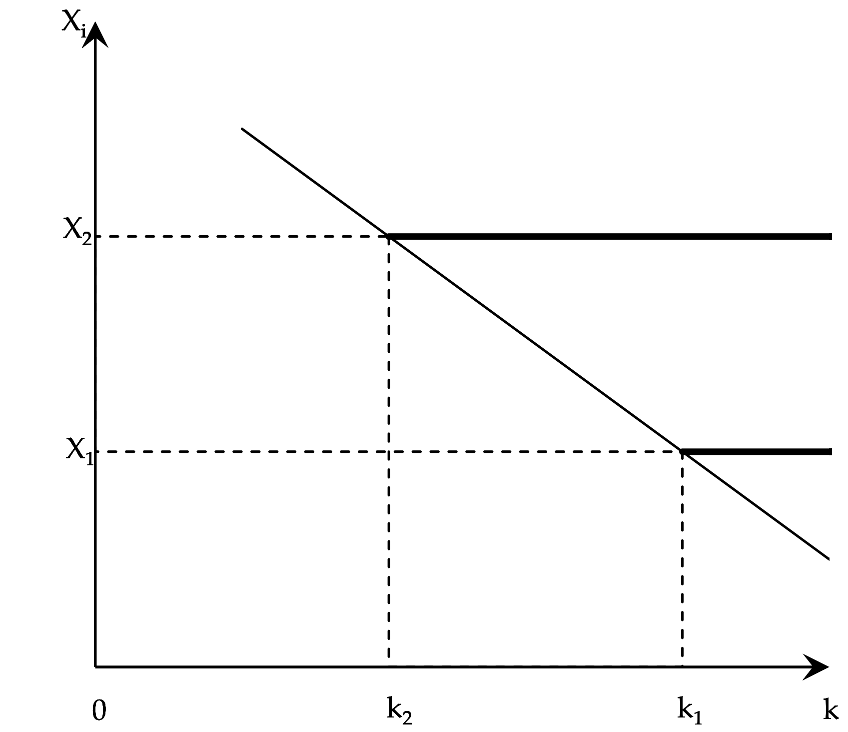


Figure 4. Buying-Side Informed Traders

(a) Investment decision-making based on private information



(b) Informed traders' aggregate demand function

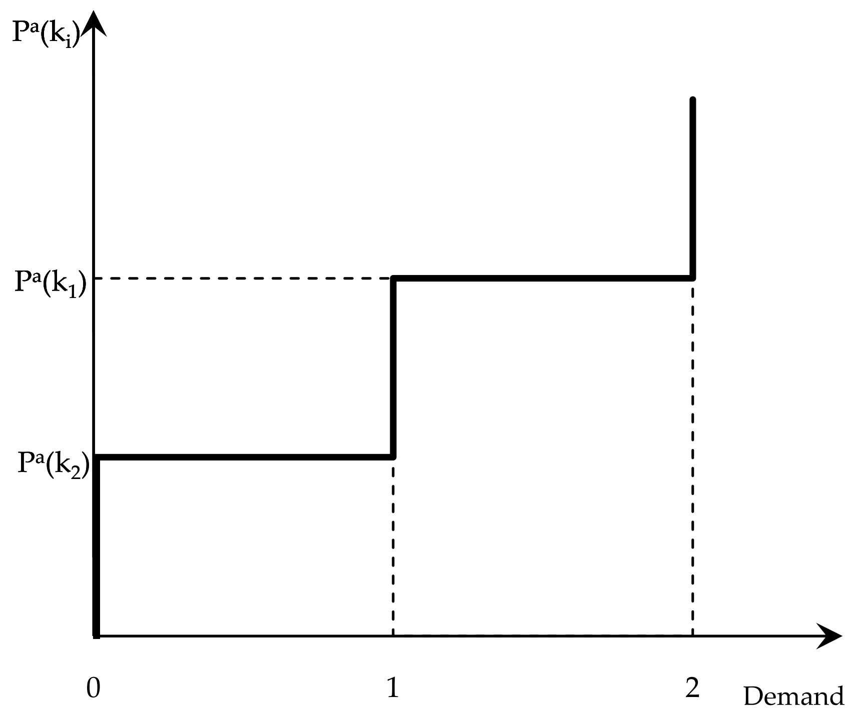
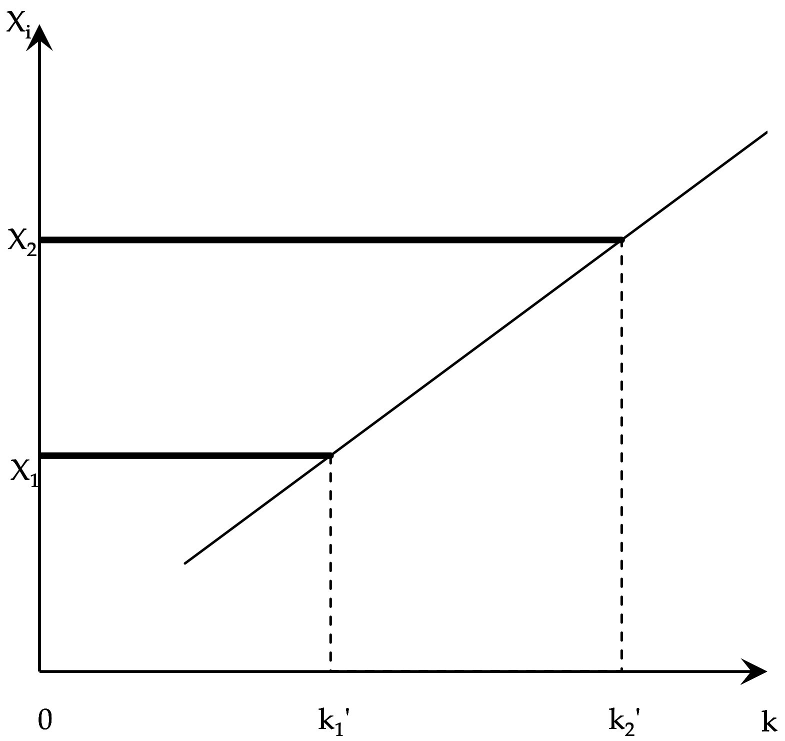


Figure 5. Buying-Side Informed Traders without Herding

(a) Investment decision-making based on private information



(b) Informed traders' aggregate demand function

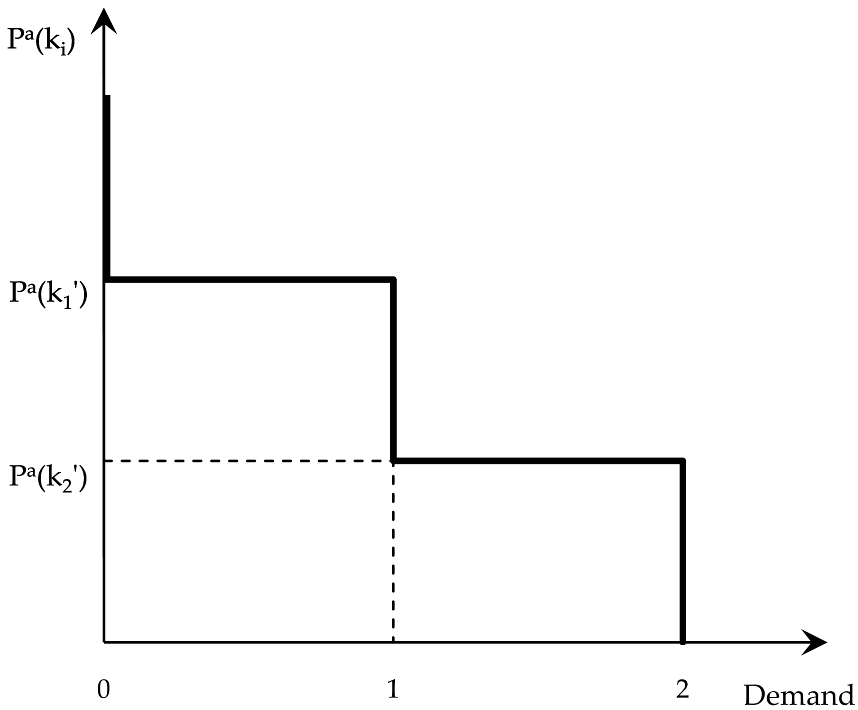
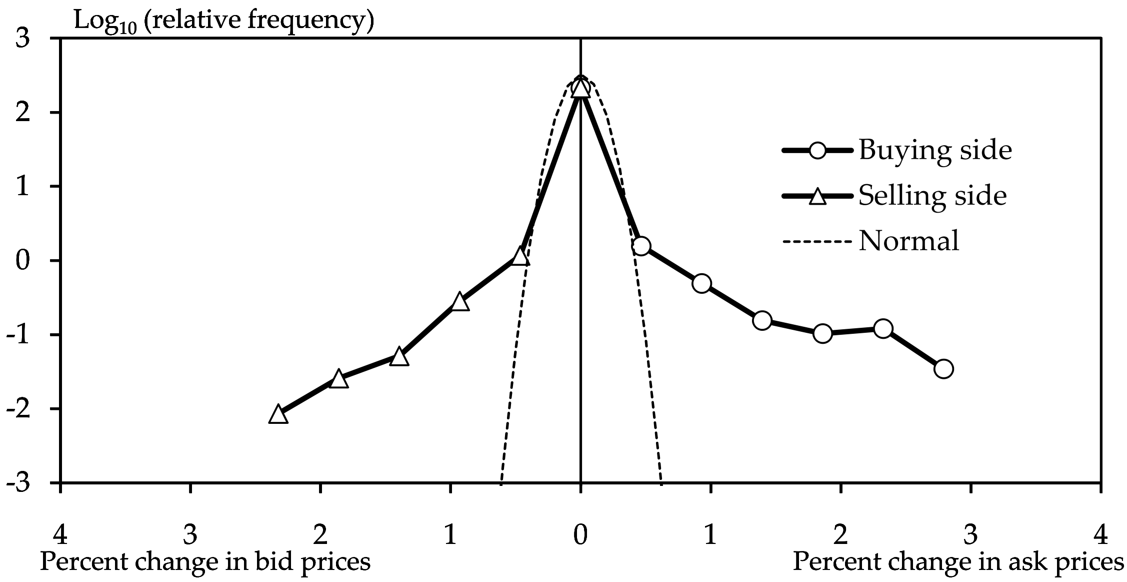
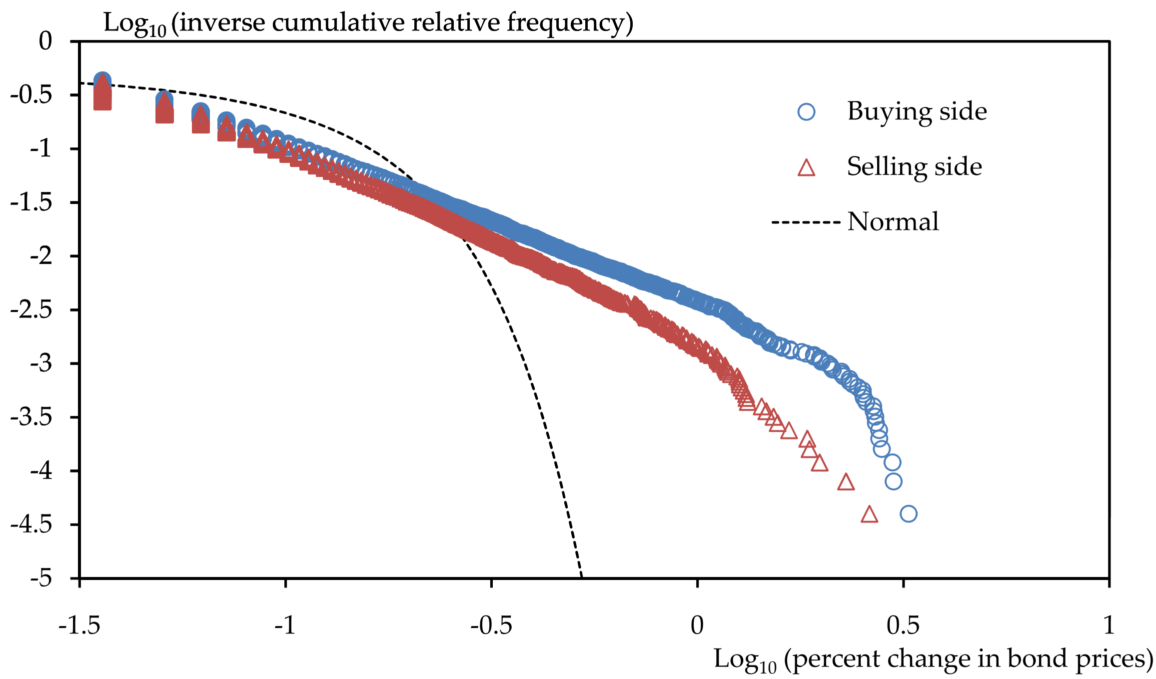


Figure 6. Fat-Tailed Bond Price Distribution

(a) Bond price change distribution



(b) Inverse cumulative distributions of percent change in bond prices

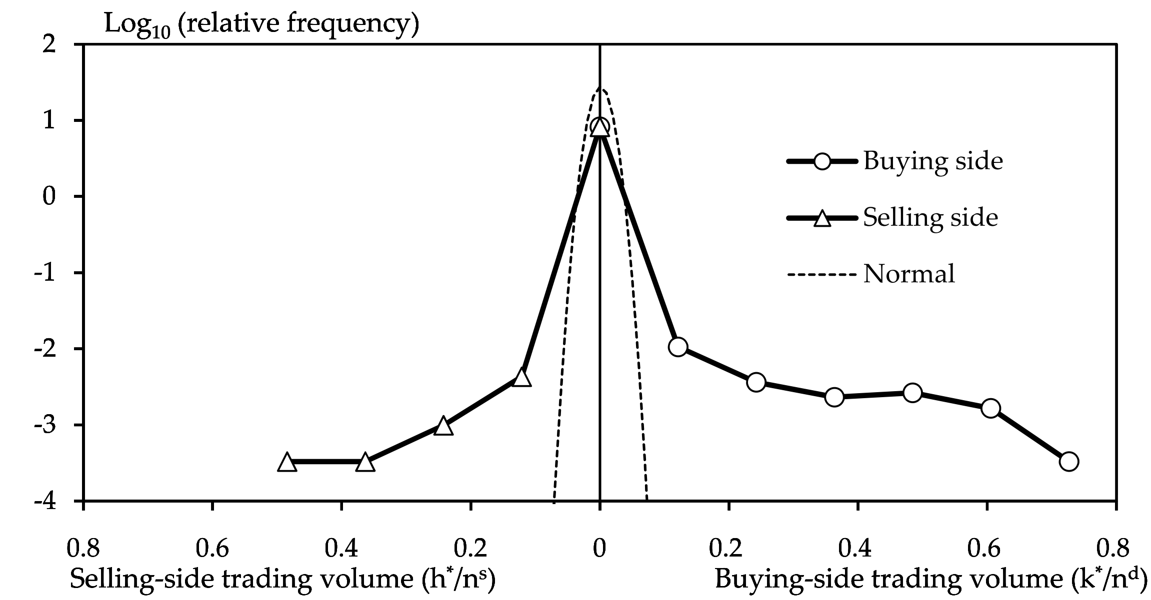


Notes: 1. $n^d = n^s = 10{,}000$, $\mu_H = 1$, $\mu_L = -1$, $\sigma = 200$, $\lambda = 0.8$, $q = 0.8$, $\theta_0 = 1$, $P_H = 100$, and $P_L = 86$.

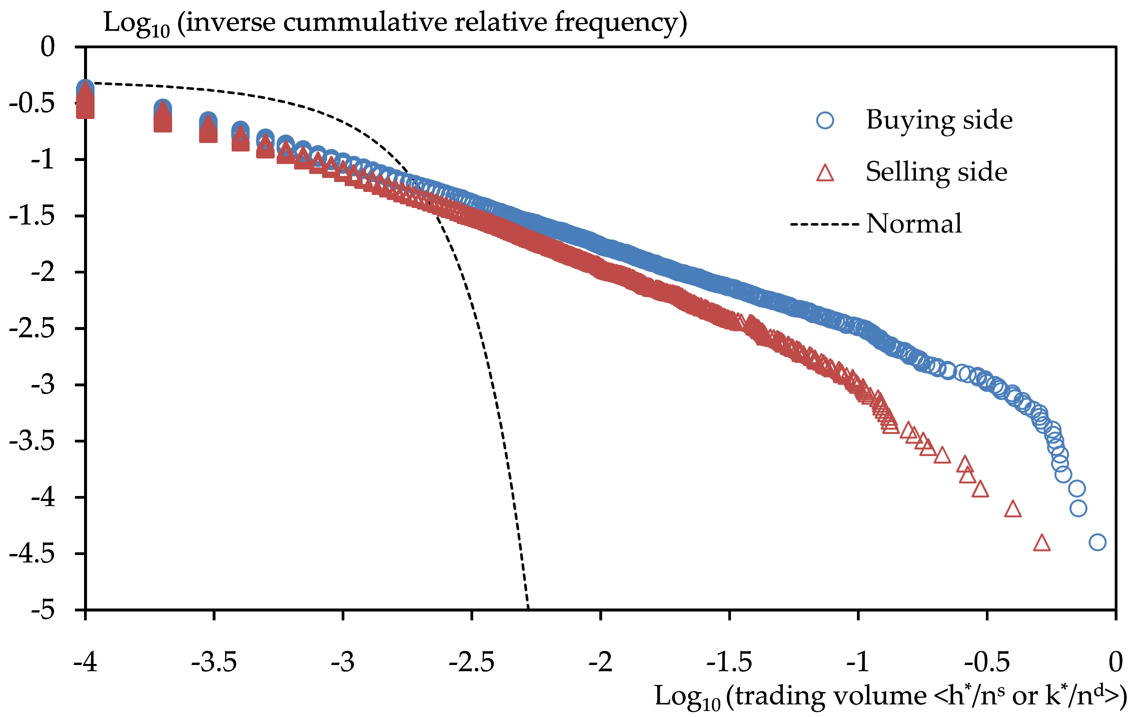
2. Private information is generated from F_H. The simulation is iterated 25,000 times each for the selling and the buying side.

Figure 7. Fat-Tailed Trading Volume Distribution

(a) Trading volume distributions



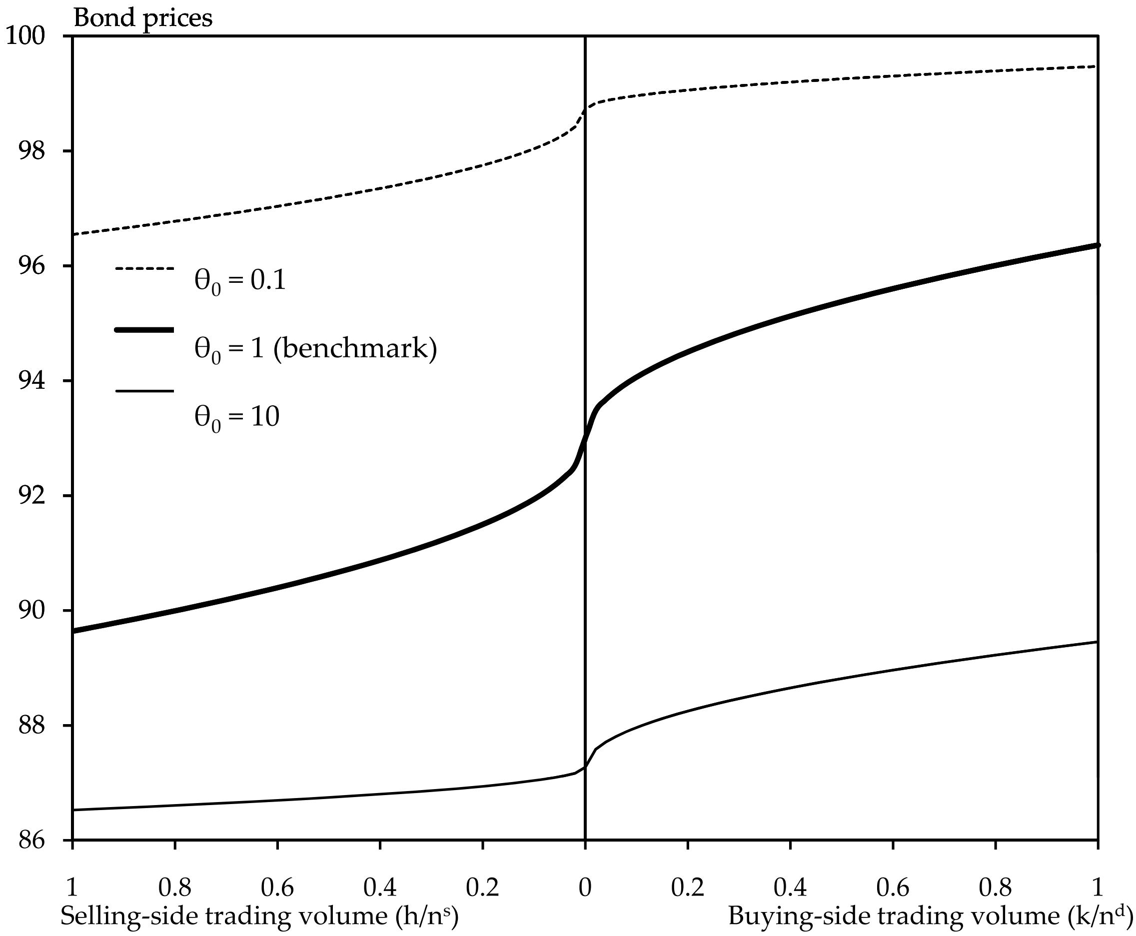
(b) Inverse cumulative distributions of trading volume



Notes: 1. $n^d = n^s = 10{,}000$, $\mu_H = 1$, $\mu_L = -1$, $\sigma = 200$, $\lambda = 0.8$, $q = 0.8$, $\theta_0 = 1$, $P_H = 100$, and $P_L = 86$.

2. Private information is generated from F_H. The simulation is repeated 25,000 times each for selling- and buying-side traders.

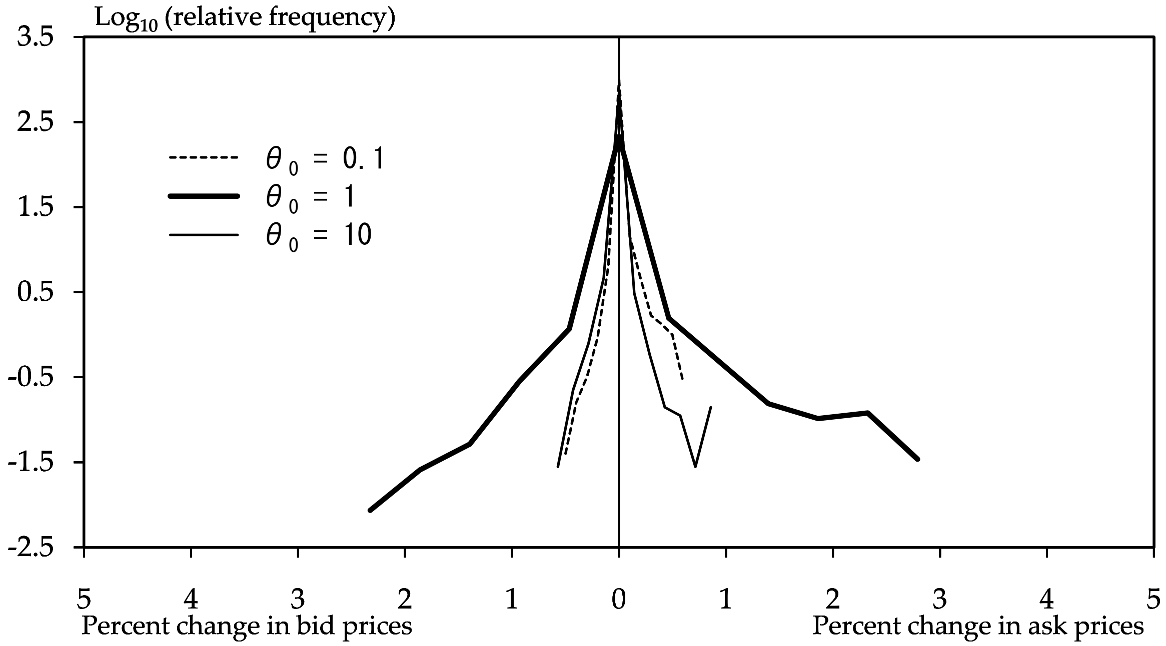
Figure 8. Effects of Traders' Confidence on Bond Price Functions



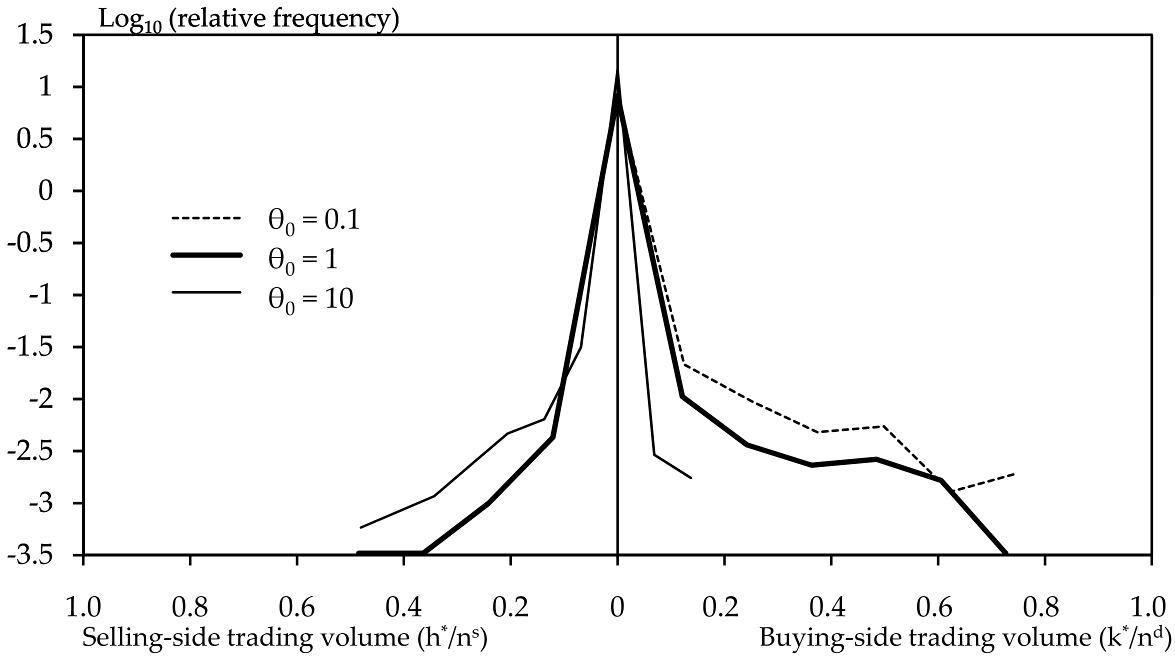
Note: $\lambda = 0.8$, $q = 0.8$, $P_H = 100$, and $P_L = 86$.

Figure 9. Effects of Traders' Confidence on Bond Markets

(a) Bond price distributions



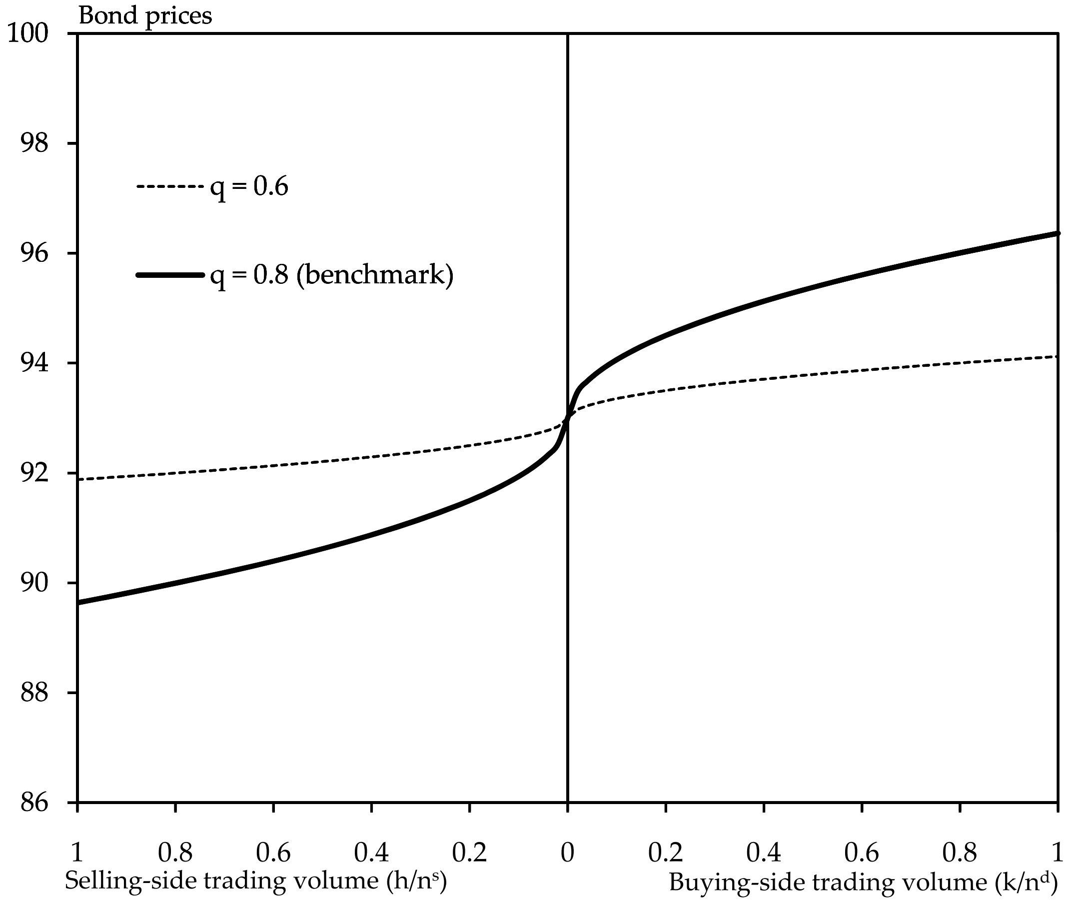
(b) Trading volume distributions



Notes: 1. $n^d = n^s = 10{,}000$, $\mu_H = 1$, $\mu_L = -1$, $\sigma = 200$, $\lambda = 0.8$, $q = 0.8$, $P_H = 100$, and $P_L = 86$.

2. Private information is generated from F_H. The simulation is iterated 25,000 times each for the selling and the buying side.

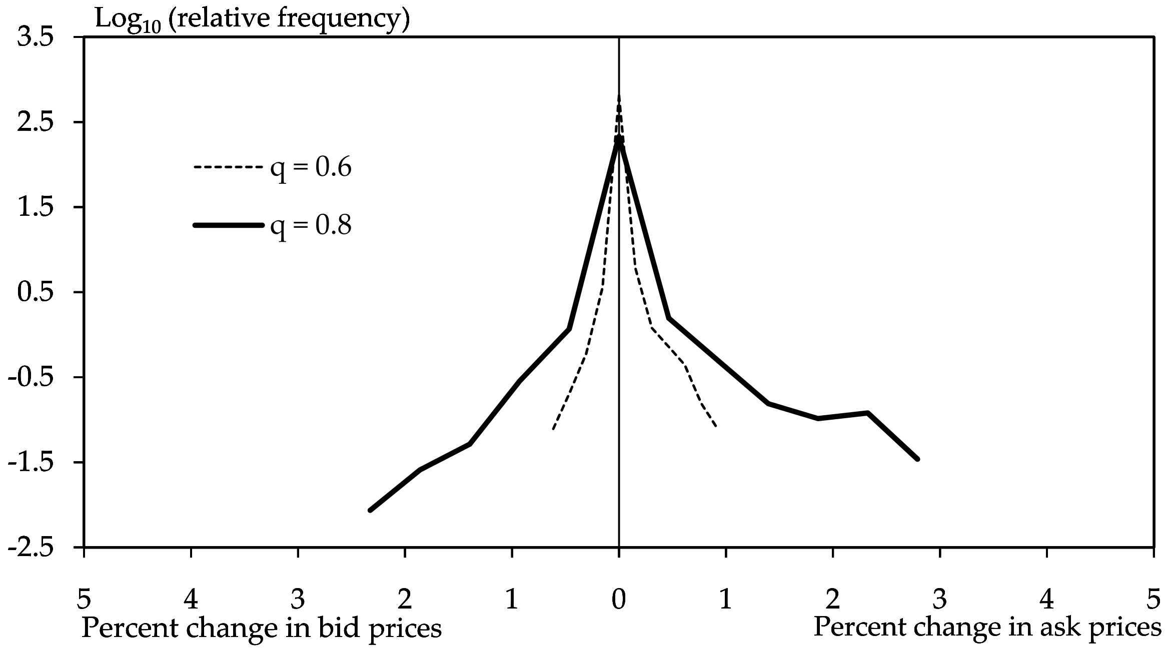
Figure 10. Effects of the Value of Public Information on Bond Price Functions



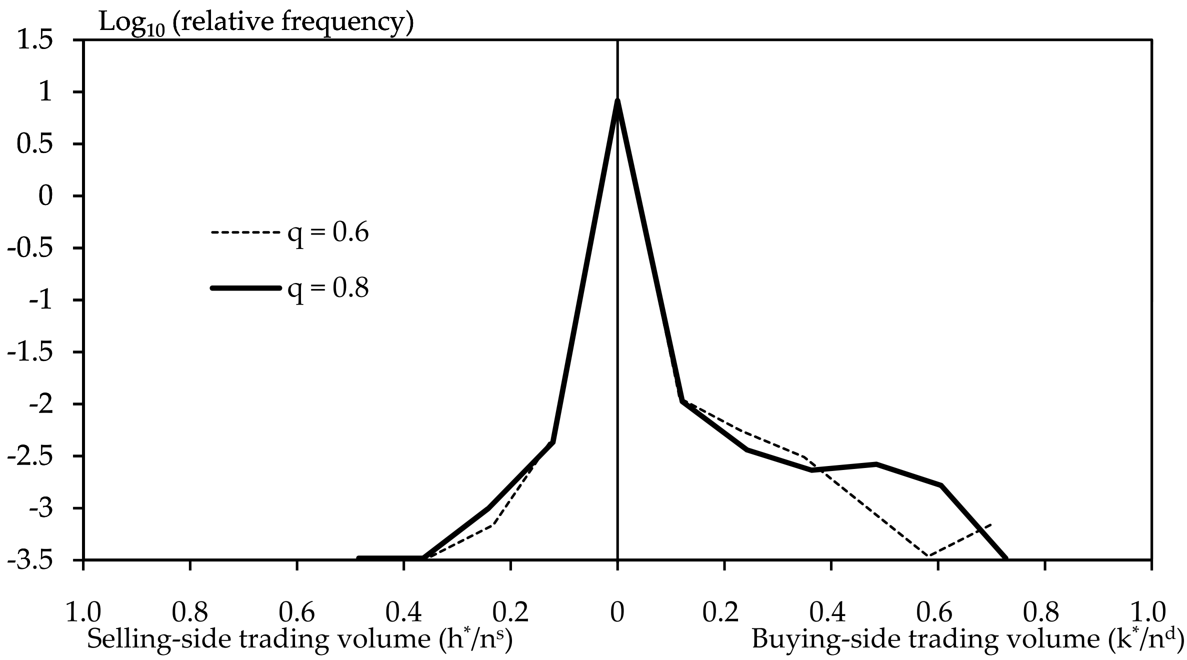
Note: $\theta_0 = 1$, $\lambda = 0.8$, $P_H = 100$, and $P_L = 86$.

Figure 11. Effects of the Value of Public Information on Bond Markets

(a) Bond price distributions



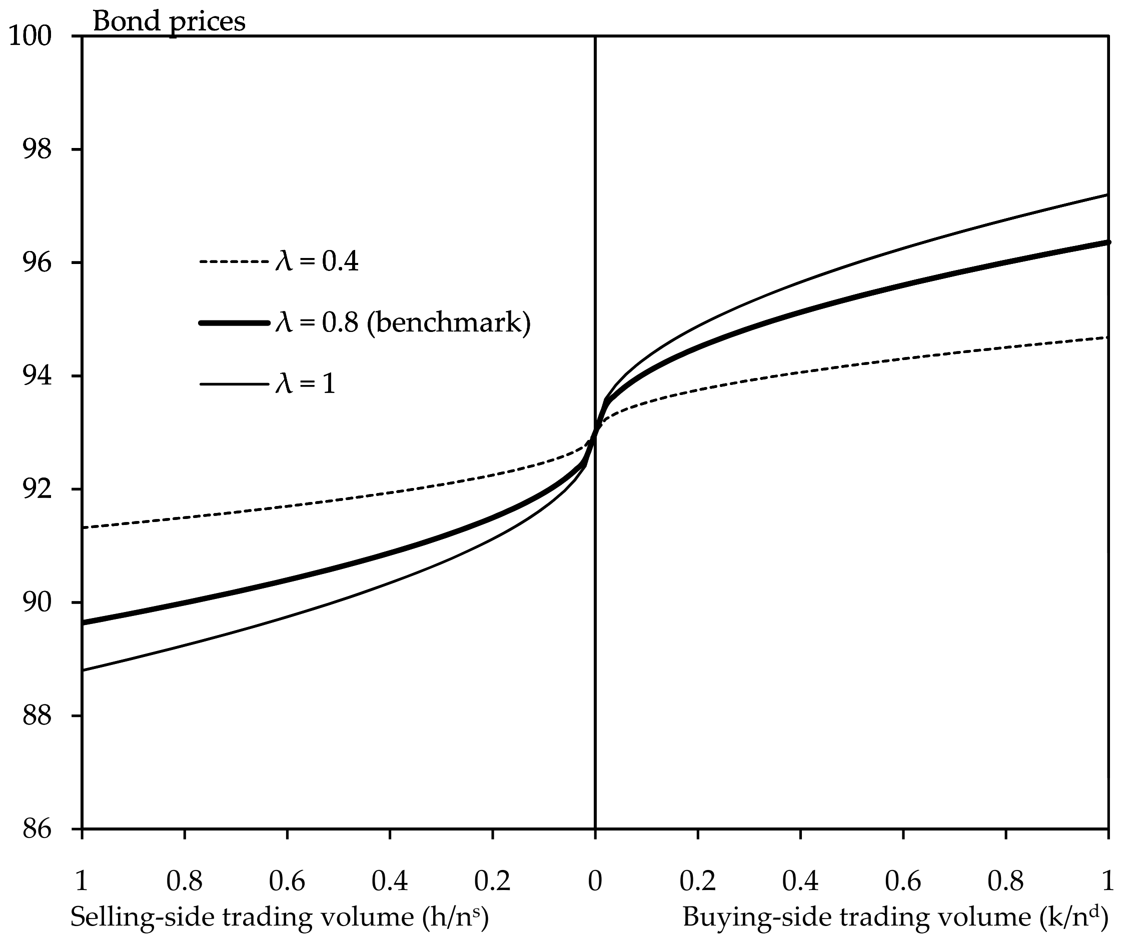
(b) Trading volume distributions



Notes: 1. $n^d = n^s = 10{,}000$, $\mu_H = 1$, $\mu_L = -1$, $\sigma = 200$, $\lambda = 0.8$, $\theta_0 = 1$, $P_H = 100$, and $P_L = 86$.

2. Private information is generated from F_H. The simulation is iterated 25,000 times each for the selling and the buying side.

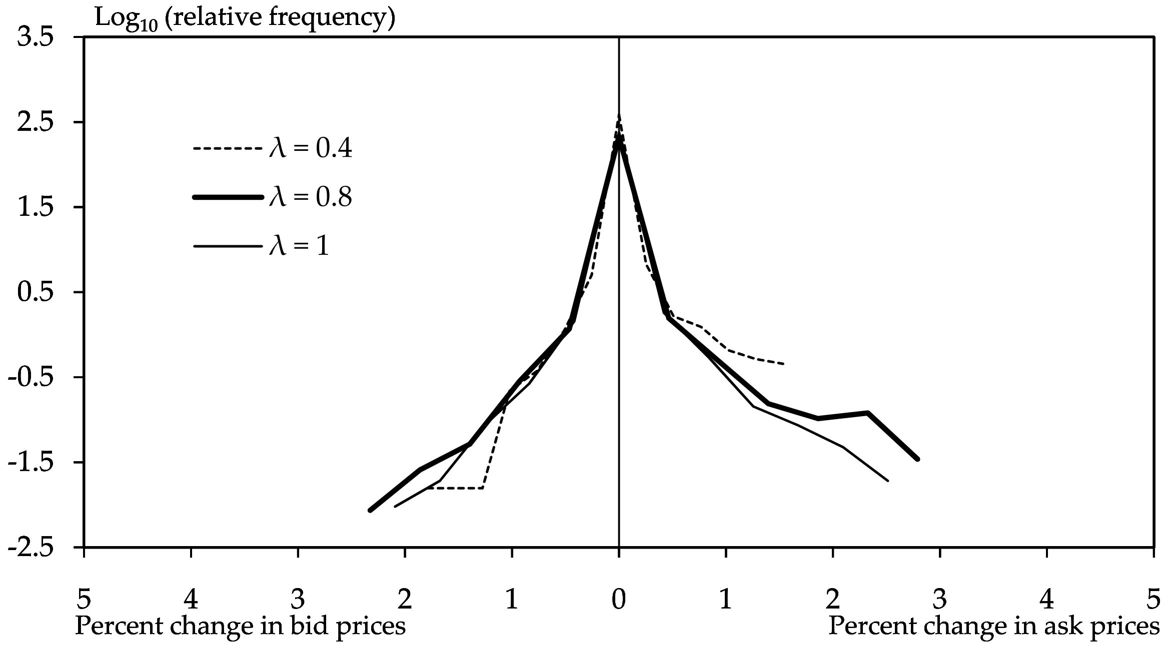
Figure 12. Effects of Market Liquidity on Bond Price Functions



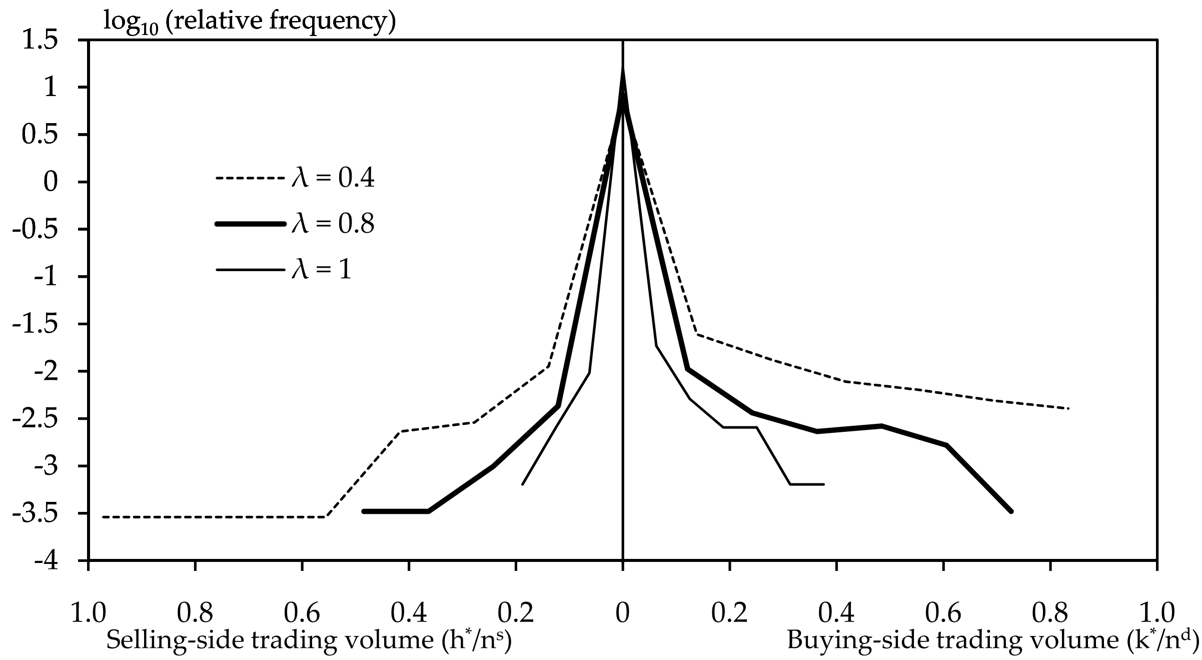
Note: $\theta_0 = 1$, $q = 0.8$, $P_H = 100$, and $P_L = 86$.

Figure 13. Effects of Market Liquidity on Bond Markets

(a) Bond price distributions



(b) Trading volume distributions

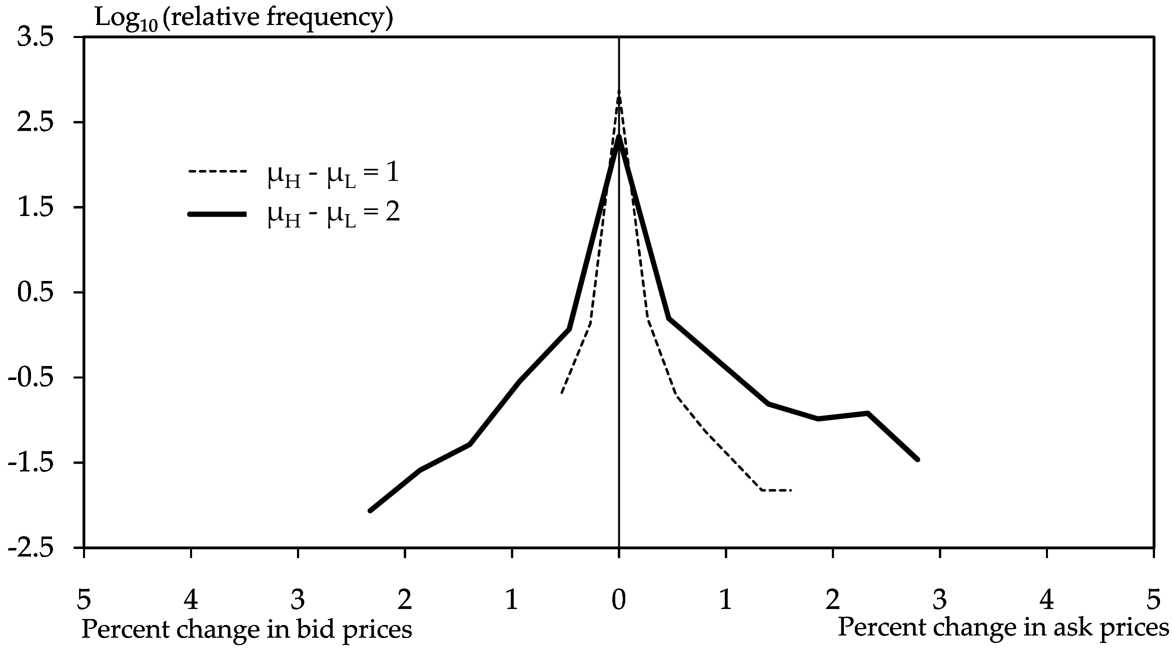


Notes: 1. $n^d = n^s = 10{,}000$, $\mu_H = 1$, $\mu_L = -1$, $\sigma = 200$, $q = 0.8$, $\theta_0 = 1$, $P_H = 100$, and $P_L = 86$.

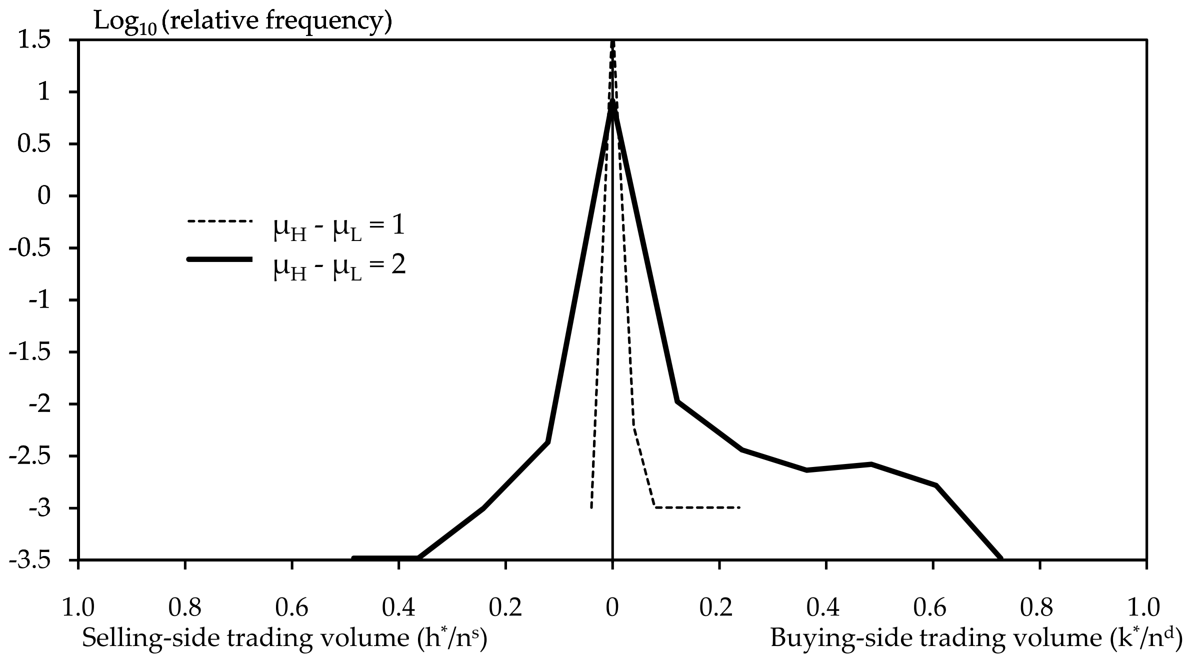
2. Private information is generated from F_H. The simulation is iterated 25,000 times each for the selling and the buying side.

Figure 14. Effects of the Accuracy of Private Information on Bond Markets (Mean Difference)

(a) Bond price distributions



(b) Trading volume distributions

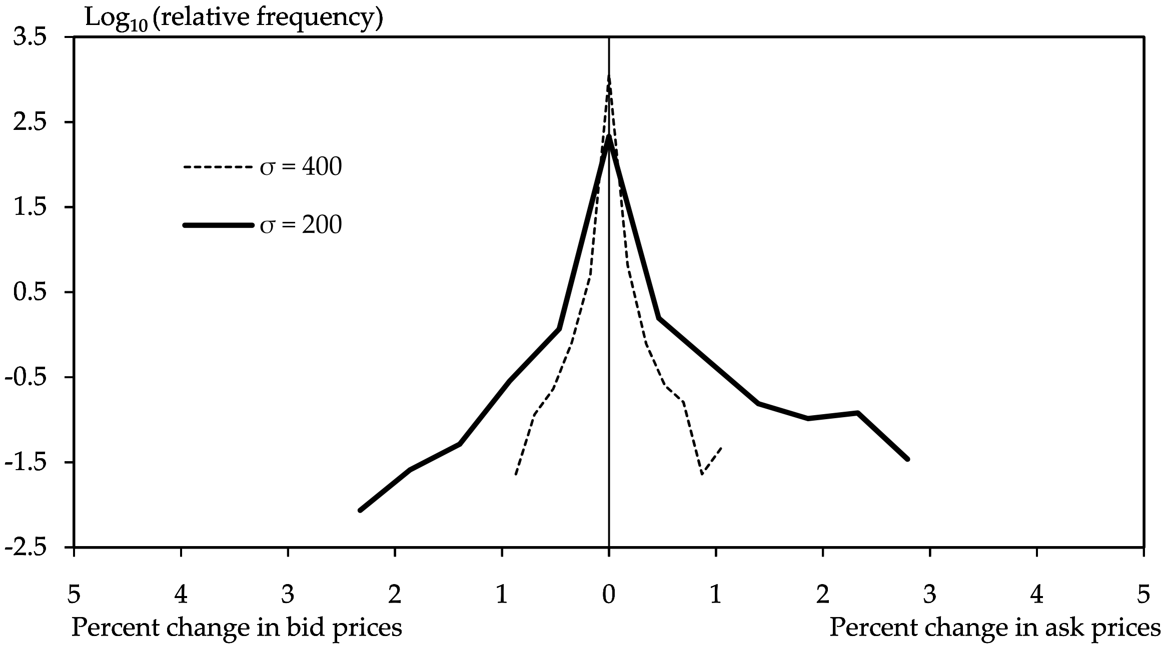


Notes: 1. $n^d = n^s = 10{,}000$, $\sigma = 200$, $\lambda = 0.8$, $q = 0.8$, $\theta_0 = 1$, $P_H = 100$, and $P_L = 86$.

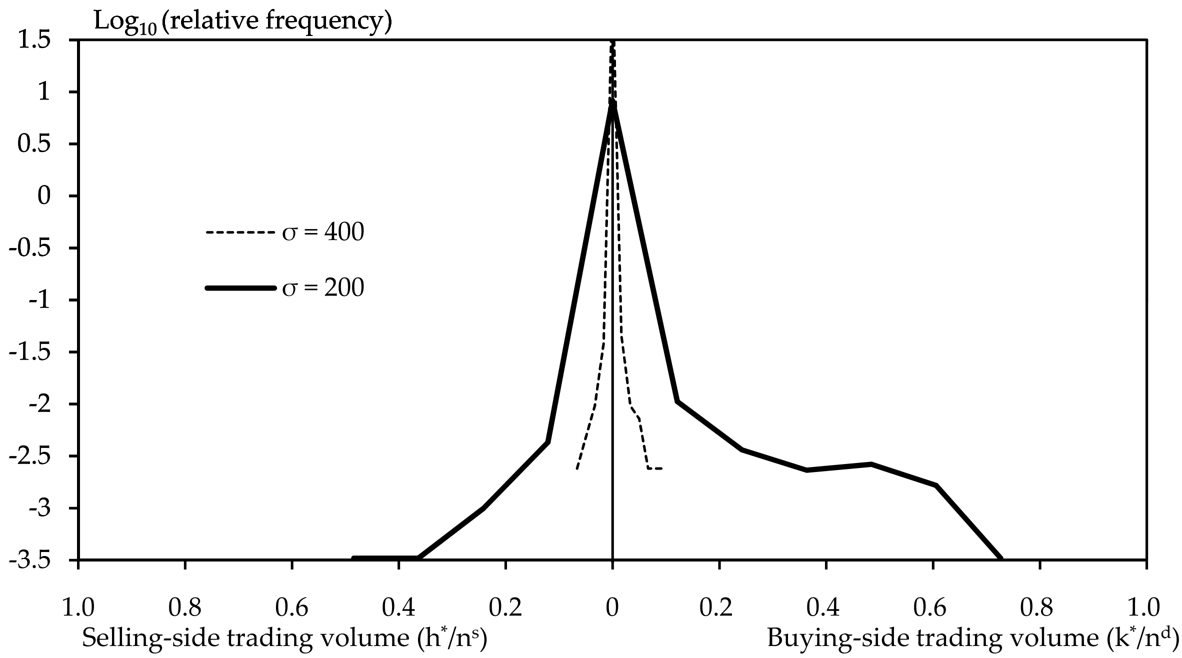
2. Private information is generated from F_H. The simulation is iterated 25,000 times each for the selling and the buying side.

Figure 15. Effects of the Accuracy of Private Information on Bond Markets (Standard Deviation)

(a) Bond price distributions



(b) Trading volume distributions

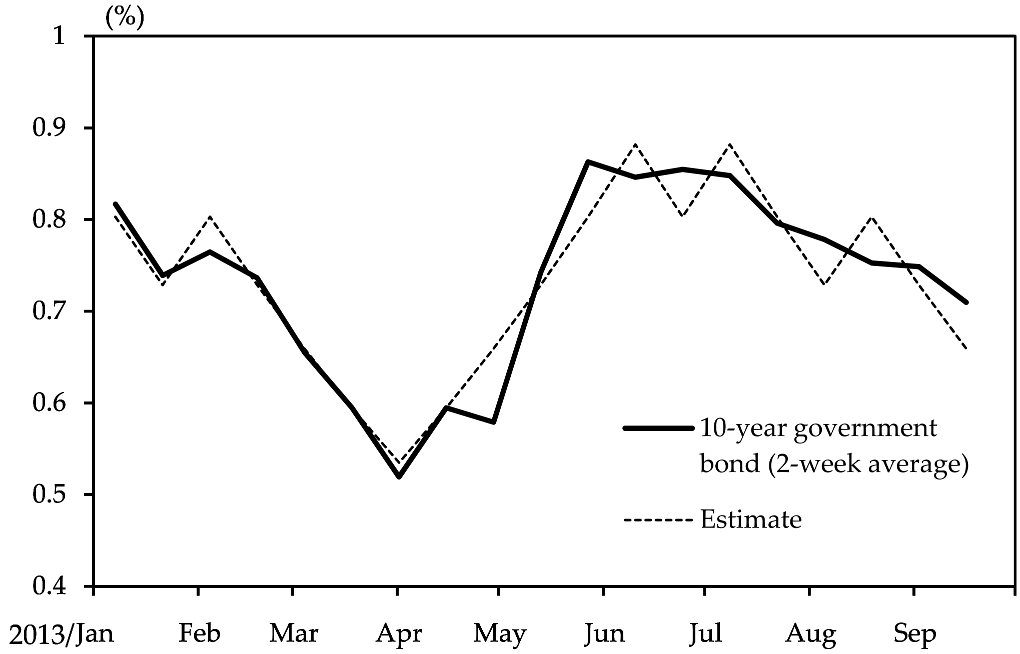


Notes: 1. $n^d = n^s = 10{,}000$, $\mu_H = 1$, $\mu_L = -1$, $\lambda = 0.8$, $q = 0.8$, $\theta_0 = 1$, $P_H = 100$, and $P_L = 86$.

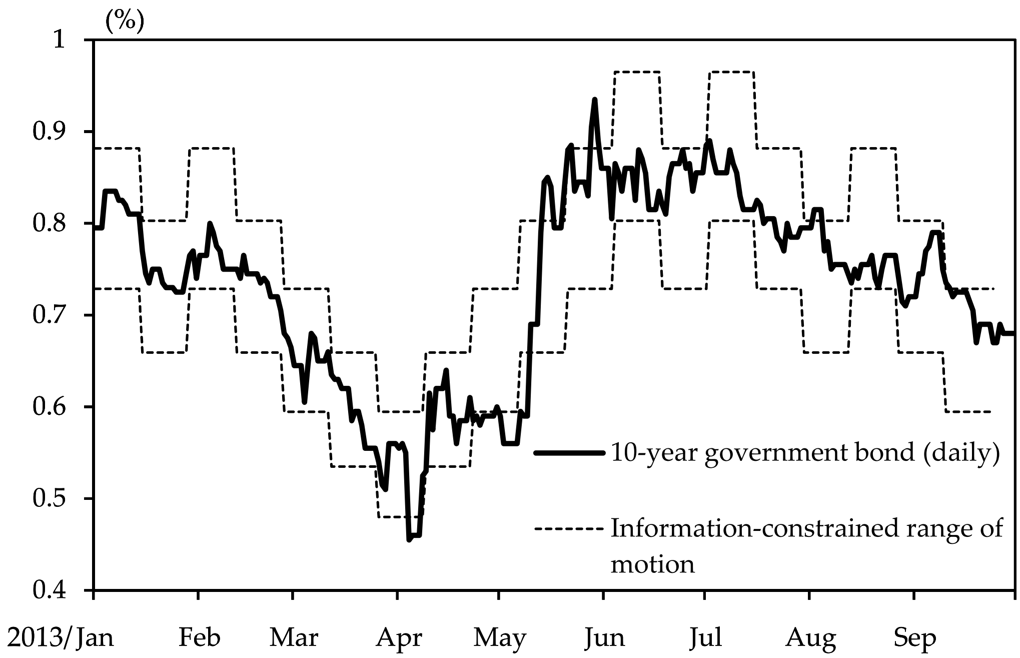
2. Private information is generated from F_H. The simulation is iterated 25,000 times each for the selling and the buying side.

Figure 16. Estimation of the Model with q Constant

(a) Trend of long-term interest rates



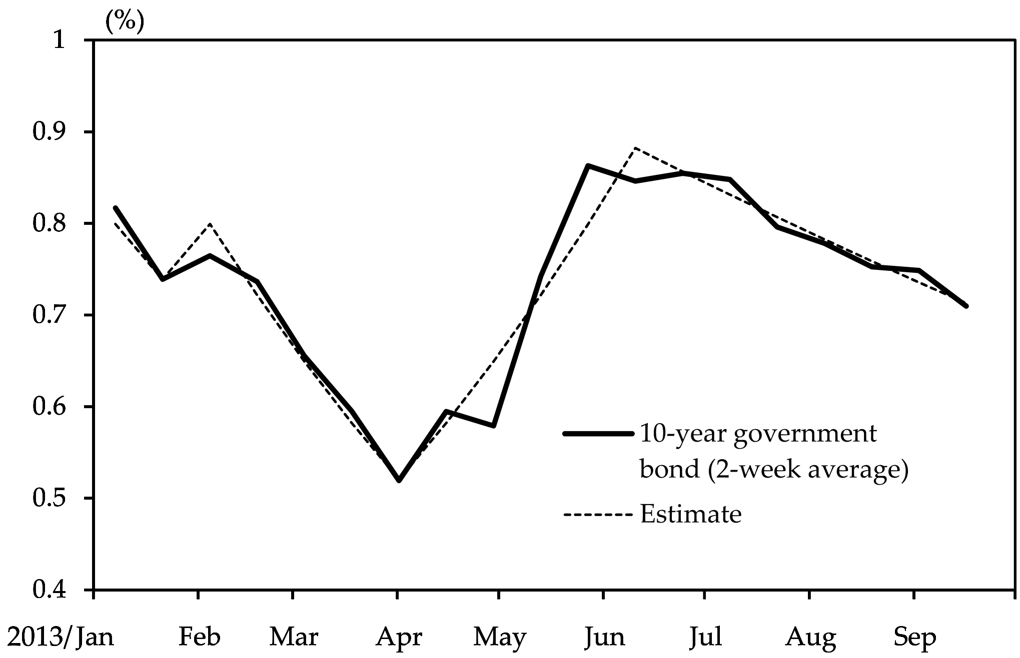
(b) Fluctuations in long-term interest rates



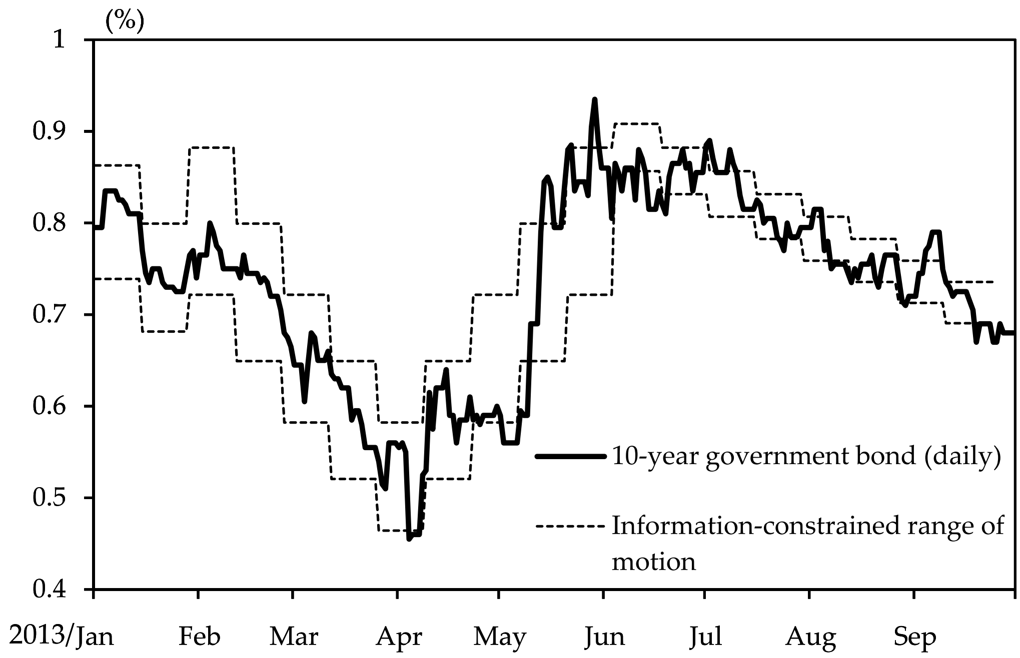
Note: q ,$\theta_0(0)$, and z are estimated with P_H = 100 and P_L = 74.

Figure 17. Estimation of the Model with Time-Varying q

(a) Trend of long-term interest rates



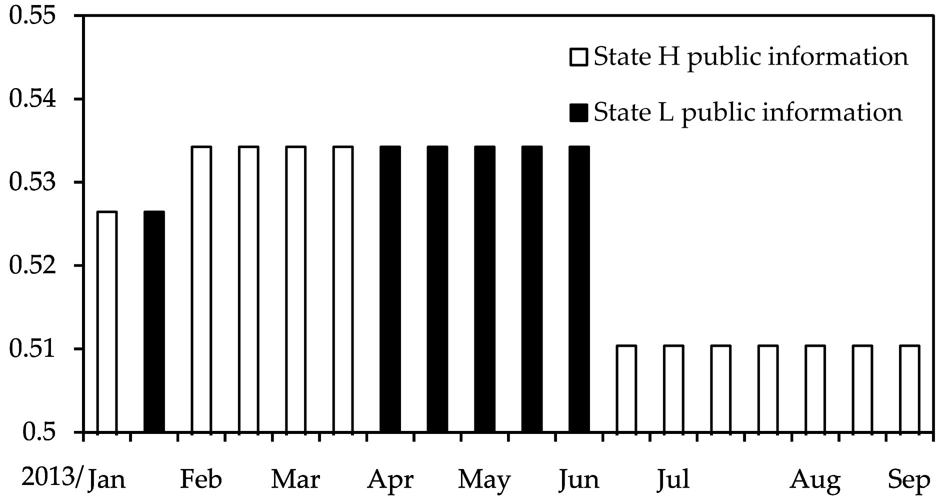
(b) Fluctuations in long-term interest rates



Note: q with two breaks, $\theta_0(0)$, and z are estimated with $P_H = 100$ and $P_L = 74$.

Figure 18. Estimate of Time-Varying q

(a) Estimated q with two breaks



(b) Estimated q varying throughout

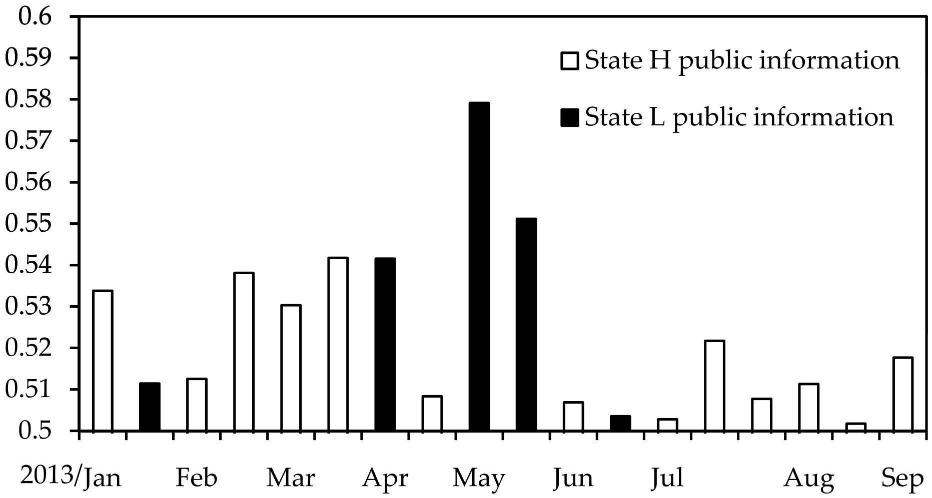
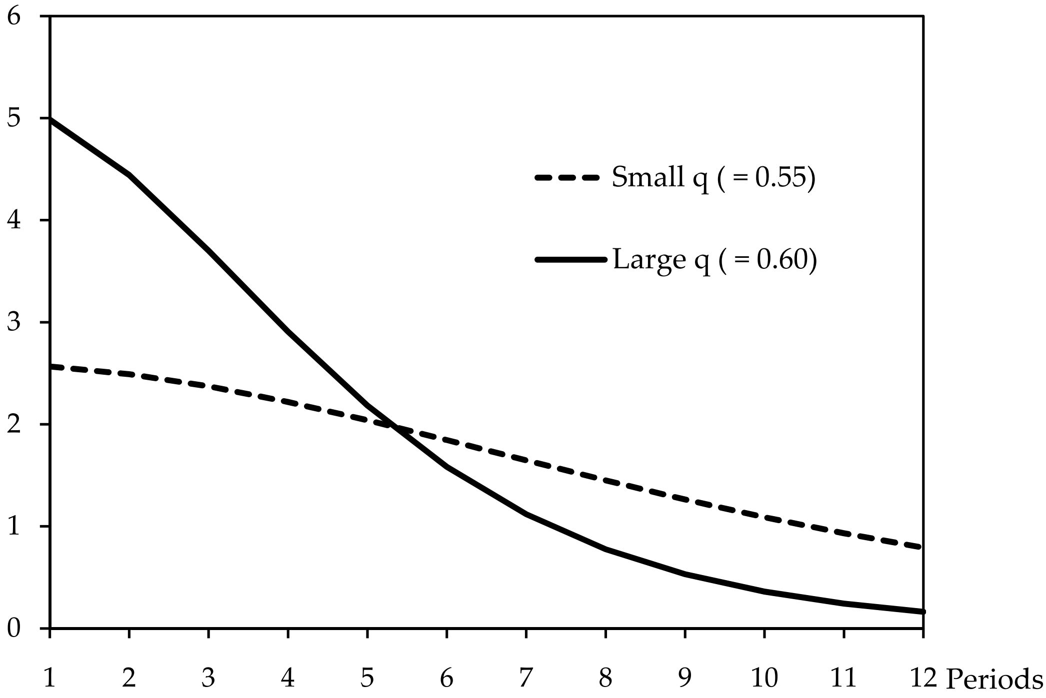


Figure 19. Dynamics of the Width of the Information-Constrained Range of Motion



Note: The calculation starts with $\theta_0 = 1$.