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Has Trend Inflation Shifted?: An Empirical Analysis with a Regime-Switching Model

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Abstract

This paper proposes a new econometric framework for estimating trend inflation and the slope of the Phillips curve with a regime-switching model. As a unique aspect of our approach, we assume regimes for the trend inflation at one-percent intervals, and estimate the probability of the trend inflation being in each regime. The trend inflation described in the discrete manner provides for an easily interpretable explanation of estimation results as well as a robust estimate. An empirical result indicates that Japan's trend inflation stayed at zero percent for about 15 years after the late 1990s, and then shifted away from zero percent after the introduction of the price stability target and the quantitative and qualitative monetary easing. The U.S. result shows a considerably stable trend inflation at two percent since the late 1990s.

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1. Introduction

Trend inflation is the private sector's perception about the level at which inflation is expected to prevail in the medium to long run. The concept of trend inflation is at the core of academic and policy discussions on inflation dynamics. If trend inflation is consistent with the price stability target, the actual inflation rate is expected to converge to the target eventually; otherwise, the actual inflation rate tends to deviate from the target persistently. In this sense, the deviation of trend inflation from the price stability target serves as a practical measure of whether inflation expectations remain anchored in line with the price stability target.

While a number of studies have estimated trend inflation based on a wide variety of econometric models, each study has its own drawbacks.[1] As discussed by Nason and Smith (2008), the most critical challenge in this field of research is that trend inflation is an unobservable state variable. Therefore, how to link trend inflation to other observable macroeconomic variables, such as realized inflation, is the key to successfully estimating it.

A popular approach is a multivariate model of the macroeconomic variables, employing a vector autoregressive (VAR) model.[2] Estimating trend inflation with a VAR model requires an appropriate restriction that identifies trend inflation from macroeconomic variables. As mentioned by Nason and Smith (2008), these strategies are vulnerable to misspecification of the VAR restriction. There is also the issue of bias in the data used for the estimation. Several existing studies directly use survey data on inflation expectations to estimate trend inflation (e.g., Brissimis and Magginas, 2008; Kozicki and Tinsley, 2012), but this method has an intrinsic drawback that the estimate of trend inflation may be distorted by various biases in the survey data.

Alternatively, an unobserved component (UC) model has now become a standard approach to estimate trend inflation. The UC model for trend inflation was first developed by Stock and Watson (2007) and analyzed by many authors such as Cecchetti et al. (2007), Kiley (2008), and Clark and Doh (2011). The authors assume that inflation dynamics can be decomposed into trend and cycle components. Stock and Watson (2007) specify the UC model with stochastic volatility (UC-SV) for innovations of the

[1] A comprehensive survey of estimation of trend inflation can be found in Faust and Wright (2013), and Ascari and Sbordone (2014).

[2] See, e.g., Quah and Vahey (1995), Claus (1997), and Mertens (2011).

trend and cycle components, resulting in a considerably flexible model for describing various changes of the realized inflation. However, the resulting model may actually be too flexible to provide a plausible estimate. In particular, we should note that the estimate can heavily depend on the prior distribution of stochastic volatility.

This paper proposes a new econometric framework for jointly estimating trend inflation and the slope of the Phillips curve, i.e., the coefficient for the output gap, using a regime-switching model. As a unique aspect of our approach, we assume regimes for the trend inflation at one-percent intervals, such as zero, one, or two percent, and estimate the probability of the trend inflation being in each regime. Our framework can estimate the probability, which is assumed to evolve over time, in real time.[3] The concept of a one-percent interval for our trend-inflation model is consistent with the tendency for major central banks to use an integer as the inflation target, and with prior work by Kamada (2013), who showed that households often express their inflation expectations in integers. In addition, the analysis demonstrates that this regime-switching constraint provides robust estimation.[4]

To our knowledge, this paper is the first to explicitly introduce the regime-switching model into trend-inflation dynamics. Our framework can directly address the following question: *What is the probability that the current trend inflation is, say, two percent?* As a further contribution to the literature, the model also allows the slope of the Phillips curve to follow regime-switching. Using this particular approach we can verify whether a change in realized inflation is attributable to trend inflation or the slope of the Phillips curve.[5]

The rest of the paper is organized as follows. In Section 2, we propose the regime-switching trend-inflation model. Section 3 develops an estimation method for the proposed model using the Markov chain Monte Carlo (MCMC) sampling technique. As this section includes a technical description of the estimation procedure, readers who are interested in only empirical results can skip to Section 4. Section 4 provides empirical results on Japanese and U.S. inflation dynamics. Section 5 concludes.

[3] Galati et al. (2011) point out that U.S. inflation expectations may have become less anchored to the target since the global financial crisis.

[4] One alternative to estimating trend inflation is the time-varying parameter model, but this model can be over-parameterized due to its flexible structure as discussed by Koop et al. (2009). To overcome this problem, our method restricts the dynamics of trend inflation by the regime-switching model where the regimes are fixed and intervals are equally spaced.

[5] See, e.g., De Veirman (2009) for the study to examine the flattening of the Phillips curve.

2. The model

2.1. Phillips curve

We consider the following hybrid Phillips curve. This model is referred to as "hybrid" in the sense that the current inflation rate depends on both the lagged inflation and trend inflation that reflects inflation expectations. That is,

$$\pi_t = \sum_{i=1}^{k} \alpha_i \pi_{t-i} + \left(1 - \sum_{i=1}^{k} \alpha_i\right) \mu_t + \beta_t x_t + \varepsilon_t, \ \varepsilon_t \sim N(0, \sigma_t^2), \text{ for } t = 1, \ldots, n, \quad (1)$$

where π_t is the inflation rate; μ_t the trend inflation; x_t the output gap; β_t the slope of the Phillips curve; and ε_t an error term. The coefficients for the lagged realized inflation, $\alpha = (\alpha_1, \ldots, \alpha_k)$, are assumed to satisfy the condition $\left|\sum_{i=1}^{k} \alpha_i\right| \leq 1$.

To understand the characteristics of the trend inflation in equation (1), we consider a steady state where the term $\beta_t x_t$ is zero in equation (1). Because the sum of the coefficients of the lagged inflation π_{t-i} and the trend inflation μ_t is unity, it leads to $\lim_{\tau \to \infty} \mathrm{E}_t \pi_{t+\tau} = \mu_t$.[6] This implies that the conditional distribution of the realized inflation converges to the current trend inflation. In other words, we can interpret that the trend inflation in equation (1) is the level at which inflation is expected to prevail in the long run.

2.2. Regime-switching model

To describe the evolving trend inflation and Phillips-curve slope, there are two main approaches: the time-varying parameter model and the regime-switching model. A number of studies have developed time-varying parameter models in macroeconomic time-series analysis, but Koop et al. (2009) discuss that those models' flexible assumption regarding parameter evolution can lead to over-parameterization. To overcome this problem, Koop et al. (2009) suggest assuming that there can be parameters that change for some periods but not for other periods. This paper avoids the over-parameterization problem by employing a regime-switching model with the assumption that the trend inflation and the Phillips-curve slope shift only occasionally.

[6] We here assume $\mu_t = \mathrm{E}_t \mu_{t+1}$, which implies that the current trend inflation is an optimal forecast of itself in all future periods. As argued in Section 2.2, the trend inflation μ_t is formalized to satisfy this equation.

A variable following a regime-switching model shifts among discrete values over time. A key aspect of our analysis is the one-percent interval for the trend-inflation regimes. For example, the trend inflation is assumed to stay at zero, one, or two percent at a time. This is closely related to the history of inflation targeting with regard to the price stability. There was a natural tendency to set an integer for the inflation target. The majority of inflation targeting countries adopt an integer target of two percent. When it began a quantitative easing policy in 2001, the Bank of Japan announced a commitment to continue quantitative easing until the CPI registers stably a zero percent or an increase year on year. Later, it set the *price stability goal* of one percent in 2012 and the *price stability target* of two percent in 2013. From another perspective, Kamada (2013) analyzes household inflation expectations using data from the *Opinion Survey on the General Public's Views and Behavior* conducted by the Bank of Japan. He reports that households often express expectations in integers. This implies that they tend to think of inflation expectations in integers. These facts fit naturally into our trend-inflation model employing one-percent intervals.

We describe the details of regime-switching in our model as follows. The trend inflation and the slope of the Phillips curve take one of the discrete regimes at each time: $\mu_t \in \{\tilde{\mu}_1, \tilde{\mu}_2, \ldots, \tilde{\mu}_L\}$ and $\beta_t \in \{\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_M\}$. We specify these values as being equally spaced. The regime-switching models employed in macroeconomic analysis usually assume that the regime values are unknown and estimated. In contrast, the regimes in our model are pre-specified.[7]

We assume the first-order Markov process for the process of the regime.[8] We further assume that the trend inflation and the slope shift to a distant regime with a small probability. We define p_μ and p_β as the probability of staying in the current regime:

$$
\begin{aligned}
p_\mu &\equiv \Pr[\mu_t = \tilde{\mu}_i | \mu_{t-1} = \tilde{\mu}_i], \quad \text{for all } i \in \{1, \ldots, L\}, \\
p_\beta &\equiv \Pr[\beta_t = \tilde{\beta}_i | \beta_{t-1} = \tilde{\beta}_i], \quad \text{for all } i \in \{1, \ldots, M\}.
\end{aligned}
$$

[7] See, e.g., Kim and Nelson (1999), and Kim et al. (2014).

[8] This means that the probability of being in the current regime only depends on the previous regime.

Then, we assume transition probabilities:[9]

$$\Pr[\mu_t = \tilde{\mu}_i | \mu_{t-1} = \tilde{\mu}_j] \;=\; 2^{-|i-j|} q_{\mu,j}, \quad \text{for } i, j \in \{1, \dots, L; i \neq j\}$$
$$\Pr[\beta_t = \tilde{\beta}_i | \beta_{t-1} = \tilde{\beta}_j] \;=\; 2^{-|i-j|} q_{\beta,j}, \quad \text{for } i, j \in \{1, \dots, M; i \neq j\}$$

where $q_{\mu,j}$ and $q_{\beta,j}$ are functions of p_μ and p_β that satisfy $\sum_{i=1}^{L} \Pr[\mu_t = \tilde{\mu}_i | \mu_{t-1} = \tilde{\mu}_j] = 1$, and $\sum_{i=1}^{M} \Pr[\beta_t = \tilde{\beta}_i | \beta_{t-1} = \tilde{\beta}_j] = 1$, for all j. We remark that regime-switching of the trend inflation and the slope occurs independently. We estimate p_μ and p_β together with other model parameters and state variables such as μ_t and β_t.

2.3. Stochastic volatility

To obtain a parsimonious model, we explicitly formulate only realized inflation, trend inflation, and the output gap in the Phillips curve (1). Therefore, temporal shocks of other sources of inflation dynamics, such as exchange rate and commodity prices, are all explained by the error term. In this paper, we assume that the variance of the error term evolves over time. A time-varying variance for error terms has become the standard in macroeconomic time series analysis, widely employed in the literature (e.g., Cogley and Sargent, 2005; Primiceri, 2005).[10]

Specifically, for the error term in equation (1), we assume that the log of time-varying variance, $h_t \equiv \log \sigma_t^2$, follows the random-walk process found in the existing studies:

$$h_{t+1} \;=\; h_t + \eta_t, \quad \eta_t \;\sim\; N(0, v^2).$$

This is referred to as stochastic volatility, commonly used in the literature of finance and macroeconomics.[11]

[9] Because we assume symmetric transition probabilities, the equation $\mu_t = \mathrm{E}_t \mu_{t+1}$ in Footnote 6 holds, if we take a sufficiently broad range of regimes.

[10] We remark that there may be some factors which the error term cannot explain, such as a change in the commodity price pass-through.

[11] See, e.g., Shephard (2005), and Cogley and Sargent (2005).

3. Estimation method

3.1. Bayesian analysis and computation

In this paper, we estimate the proposed model based on Bayesian inference using MCMC methods. Because the regime-switching model developed in this paper includes many state variables, the likelihood function of the model is not easily available. It is possible to approximately compute the likelihood based on simulation and filtering method for a given set of parameters, but it requires a computational burden and the maximum likelihood method cannot be applied. The MCMC methods used in this paper is a powerful tool to overcome this difficulty. We describe an outline of the estimation method in this section. Details of the MCMC algorithm are described in the Appendix. As this section includes a technical description of the estimation procedure, readers who are interested in only an empirical result can skip to Section 4.

In an MCMC algorithm, we specify prior distributions for the model parameters and draw samples from the full posterior distribution. We use standard sampling methods for each parameter and state variable as follows. To generate a sample of the trend inflation and the slope of the Phillips curve, we use the multi-move sampler for regime-switching models developed by Carter and Kohn (1994) and Chib (1996). To generate a sample of stochastic volatility, we employ the multi-move sampler for the stochastic volatility model developed by Shephard and Pitt (1997) and Watanabe and Omori (2004). As for the model parameters, we specify conjugate priors and derive the conditional posterior distribution from which we can easily generate the sample.

3.2. Setup for empirical analysis

We describe the estimation setup for empirical analysis using Japan's and U.S. data in Section 4. The trend-inflation regimes are specified as $(-2, -1, 0, 1, 2, 3)$ for Japan, and $(0, 1, 2, 3, 4, 5)$ for the United States, on a percent basis. These values cover the range of realized inflation during the sample period. As for the regime of the Phillips-curve slope, we set $(0.00, 0.05, \ldots, 0.30)$ for Japan, and $(0.00, 0.02, \ldots, 0.12)$ for the United States. These values reflect the results of pre-analysis based on a rolling estimation of a simple regression.[12]

[12] Specifically, we estimated equation (1) where the trend inflation, the Phillips-curve slope, and the error variance are constant.

The AR order k is selected based on the Bayesian information criterion (BIC), namely, $k = 8$ for Japan and $k = 4$ for the United States. The prior specification is set as follows: $\alpha \sim TN_\Omega(0_{k\times 1}, I_{k\times k})$, where TN_Ω denotes the truncated multivariate normal distribution that has positive density only in the domain $\Omega \equiv \{\alpha | \left|\sum_{i=1}^{k} \alpha_i\right| \leq 1\}$; $v^2 \sim IG(5, 0.2)$, where IG denotes the inverse Gamma distribution; $p_\mu \sim B(990, 10)$, and $p_\beta \sim B(990, 10)$, where B denotes the Beta distribution. The priors for p_μ and p_β imply a high probability of staying in the same regime, which is commonly assumed in standard regime-switching models. In MCMC sampling, we generate 10,000 samples after the initial 1,000 samples are discarded as a burn-in period.[13]

In the following analysis, we report estimates of the state variables in two ways. One is a commonly-used smoothed posterior estimate of the state variables, and the other is a "filtered posterior estimate." A smoothed posterior regime probability, $p^s(\mu_t = \tilde{\mu}_i) \equiv \Pr[\mu_t = \tilde{\mu}_i | y]$, is the estimated probability at t with all the information during entire sample period $y \equiv (\pi_1, \ldots, \pi_n)$ given. This smoothed probability, sometimes largely deviating from the real-time one, reflects future information and therefore may take a future change in advance. In order to assess an estimate only conditional on the data up to each time t, we consider a filtered posterior regime probability. Defining $q(\mu_t = \tilde{\mu}_i | \theta, \beta, h) \equiv \Pr[\mu_t = \tilde{\mu}_i | \pi_1, \ldots, \pi_t, \theta, \beta, h]$, where $\theta = \{\alpha, p_\mu, p_\beta, v\}$, $\beta = (\beta_1, \ldots, \beta_n)$, and $h = (h_1, \ldots, h_n)$, we compute its posterior probability:

$$p^f(\mu_t = \tilde{\mu}_i) \equiv \int q(\mu_t = \tilde{\mu}_i | \theta, \beta, h) p(\theta, \beta, h | y) d\theta d\beta dh.$$

In addition, the smoothed posterior estimate of the trend inflation is computed as a weighted average based on the smoothed regime probabilities, i.e., $\sum_{i=1}^{L} \tilde{\mu}_i p^s(\mu_t = \tilde{\mu}_i)$. The filtered posterior estimate is computed similarly using the filtered regime probability $p^f(\mu_t = \tilde{\mu}_i)$.

[13] MCMC streams are fairly clean and stable with quickly decaying sample autocorrelations. The inefficiency factor of the MCMC samples (see, e.g., Chib, 2001) appears to be less than 30, which is notably small compared to standard regime-switching models with unknown regime values. This indicates that the MCMC algorithm for our model provides an efficient sampling scheme.

4. Empirical analysis of Japan's and U.S. inflation dynamics

4.1. Data

We provide an empirical analysis by applying it to inflation developments in Japan and the United States. The analysis for Japan uses quarterly data from 1981/Q1 to 2014/Q4 on the CPI inflation rate, excluding fresh food and adjusted for the effect of the increase in the consumption tax, and the output gap, calculated by the Bank of Japan. For the U.S. analysis, we use the data from 1983/Q1 to 2014/Q4 on the CPI inflation rate, excluding food and energy, and an "unemployment gap," defined as the difference of the seasonally adjusted, civilian over-16 unemployment rate, from its average over the sample period.[14] Figure 1 displays the time series of the data. Note that we reverse the sign of the unemployment gap in line with the output gap. In the estimation, we use the output gap and the unemployment gap with a two-quarter lag.

4.2. Japan's trend inflation

Figure 2 plots the posterior regime probability of trend inflation based on the real-time measure, the "filtered probability"; and the estimate based on all the information from the sample period, the "smoothed probability." The filtered probability indicates that the trend inflation switched from one to zero percent around the second half of the 1990s. From then, the probability of trend inflation being zero percent was highest among the regimes until the end of 2012. During this period, the trend inflation kept staying at zero percent, and the probability of minus-one-percent did not significantly increase, even though the realized inflation settled below zero for several years. For the phases when trend inflation shifts from one regime to another, the smoothed estimates tend to take a future change in advance and are smoother than the filtered estimates.

Figure 3 displays filtered and smoothed posterior estimates, i.e., weighted averages based on the filtered and smoothed regime probabilities, of the trend inflation and the slope of the Phillips curve. The smoothed estimate of the Phillips-curve slope exhibits a gradual decline from the 1980s to the mid-1990s. This flattening Phillips curve is consistent with other empirical evidence in the literature (De Veirman, 2009). From the second half of the 1990s to the end of the sample period, the slope of the Phillips

[14] We use the unemployment gap instead of the output gap estimated by the Congressional Budget Office (CBO), because there is the possibility that the CBO's output gap is negatively overestimated after the recent global financial crisis, as pointed out by Weidner and Williams (2009, 2015).

curve did not change. This indicates that the change in the Phillips-curve slope did not affect the inflation dynamics during this period.[15] Figure 4 plots posterior estimates of the stochastic volatility, which shows a low level of the time-varying variance in the first half of the 2000s and a large hike from 2007 to 2010, probably reflecting global commodity prices and the financial crisis.

Figure 5 plots the realized inflation and its historical decomposition to the components of the model. A feature of Japan's inflation exhibited in this decomposition is a large contribution of the output gap. The fact that the realized inflation stayed below zero, for example, from the second half of the 1990s to the first half of the 2000s, is mostly explained by an increase in negative values of the output gap. In contrast, the trend inflation exhibits little contribution to the realized inflation from the late 1990s to 2012, because the trend inflation was zero percent during this period.

Figure 6 provides another look at the posterior regime probability of the filtered trend inflation, plotting its cross-section distribution at selected times. Panels (1) and (2) show that the most-likely trend inflation shifted from one to zero percent. Panels (2) to (4) summarize the trend-inflation distribution during the zero-trend-inflation periods, where probability was highly concentrated in the zero percent, although the tails showed small variations depending on the phase.

Next, focusing on the development after 2013, the posterior regime probability in Figure 2 shows a clear increase in the probability of one-percent trend inflation, and a rapid hike in the probability of two-percent trend inflation. Behind these upward changes, the probability of zero-percent trend inflation remarkably dropped and has been considerably dominated by one percent. The upward shift is also illustrated by the trend-inflation distribution in Figure 6, Panels (5) and (6). As mentioned above, the slope of the Phillips curve did not change during this period, which is shown in Figure 3. With these characteristics in mind, Figure 5 indicates that the recent increase in inflation is attributable to the rise in the trend inflation and a decrease in the negative values of the output gap.

In sum, the estimated model suggests that the trend inflation stayed at zero percent

[15] Table 1 reports posterior estimates of the parameters. The probability for the Phillips-curve slope staying in the same regime, p_β, is estimated to be higher than the probability for the trend inflation, p_μ. This indicates that the regime-switching occurs less frequently in the slope than the trend inflation in our estimated model. This is also seen in Figure 3 where the slope of the Phillips curve is more stable than the trend inflation.

for about 15 years after the late 1990s, and then shifted away from zero percent in 2013. This dramatic regime change possibly reflects a change in medium- to long-term inflation expectations affected by the introduction of the price stability target in January 2013 as well as the quantitative and qualitative monetary easing in April 2013.

4.3. The U.S. trend inflation

The U.S. realized inflation gradually shifted downward from the 1980s to the 2000s, during the era known as the "Great Moderation." Since then, inflation has fluctuated around two percent. Figure 7 plots the posterior regime probability of trend inflation, which shows that the trend inflation shifted from five percent in the 1980s to two percent in the second half of the 1990s. Surprisingly, the probability of two-percent trend inflation has dominated other probabilities from the second half of the 1990s to the end of the sample period. The filtered probability, which is the real-time measure, exhibits a small drop in the probability of two-percent trend inflation and a hike in the probability of one-percent trend inflation around 2005, probably reflecting disinflation after the burst of the IT bubble. During this period, the Federal Reserve decided to maintain a policy of monetary easing and, as a result, the realized inflation did not decrease to one percent, which may have avoided a decline in trend inflation from two to one percent.

Figures 8 and 9 display posterior estimates, i.e., weighted averages based on the regime probabilities, of the model components. It shows a gradual decline of the trend inflation from the 1980s, and flattening of the Phillips curve, which is similar to the Japan's result. In addition, we find a lower level of the time-varying variance from the 1990s to the first half of the 2000s than that of other periods. Figure 10, which plots the historical decomposition of realized inflation, implies that the U.S. realized inflation has been mainly driven by the trend inflation. In particular, the contribution of the trend inflation has been stable around two percent since the 2000s. This evidence is consistent with existing discussions on anchoring U.S. inflation at this level. It is interesting that, even before the Federal Reserve introduced the price stability target in 2012, our estimates indicate that the trend inflation remained stable at two percent.

4.4. Robustness

We examine how robust our estimates are in terms of the assumption of the equally-spaced regimes. We assess whether the result changes when the interval of the grid regime is altered. The model is estimated using Japan's data with different regime intervals. Specifically, we vary the regime interval of trend inflation as 1, 0.5, and 0.25 percent, and the interval of the slope as 0.5 and 0.25; see the table in Figure 11. Note that the baseline case is the setting used in the analysis of Section 4.2.[16]

Figure 11 depicts posterior estimates of the filtered trend inflation for five regime intervals. There is little difference in posterior mean of the trend inflation from the baseline result. In detail, however, the cases with the trend-inflation interval of 0.5 and 0.25 percent exhibit a slight upward deviation from the baseline around the late 1990s. In this phase, the baseline result shows that the trend inflation shifted down from one to zero percent. The cases with finer intervals can take some mid-point between zero and one percent. Then, the regime change tends to be more gradual, which results in the deviation from the baseline. In addition, posterior standard deviations seem marginally smaller around the early 1990s in the cases with finer intervals. In contrast, comparing the different intervals of the slope, there are almost no differences in the posterior estimates. Focusing on the estimates after the 2000s, the result in each case shows that both the posterior mean and standard deviation are almost the same as the baseline result. Then, we conclude that our model is robust regardless of the assumption of the regimes.

Figure 12 plots the posterior distribution of the trend inflation for five regime intervals. The finer-interval regime gives a smoother approximation of a continuous distribution. The overall shapes of the distribution are almost identical among the different cases. The distributions are graphed for selected time points, 2012/Q4 and 2014/Q4. The estimated regime change through the Bank of Japan's price stability target and the quantitative and qualitative monetary easing is robust. Cases 2 and 5 with the finest intervals show that the probability of negative or zero trend inflation is considerably small.

[16] As in the baseline case, we set the range of the regimes from -3% to 2% for the trend inflation, and from 0 to 0.3 for the slope. We examined broader ranges of the regime, but found only slight changes in the regime probability around the edges of the range.

5. Concluding remarks

This paper proposed a new econometric framework for estimating trend inflation and the slope of the Phillips curve with the regime-switching model. As a unique aspect of our approach, we assume regimes for the trend inflation at one-percent intervals, and estimate the probability of the trend inflation being in each regime. The one-percent interval provides an interpretable and robust estimate.

The empirical result indicates that Japan's trend inflation stayed at zero percent for about 15 years after the late 1990s, and then shifted away from zero percent after the introduction of the price stability target and the quantitative and qualitative monetary easing in 2013. The U.S. trend inflation has been stable at two percent since the late 1990s to the present. This result is consistent with discussions in the literature on anchoring inflation expectations at two percent.

Our modeling framework is so simple that it can be applied to a wide range of models. For example, Kiley (2008) and Garnier et al. (2013) model several different inflation series to identify their common trend. The framework in this paper can be extended to a multivariate inflation model. Such an interesting analysis with an extension of our model remains as a future work.

Appendix. MCMC algorithm

This appendix describes the MCMC algorithm of the proposed model. Define $y = (\pi_1, \dots, \pi_n)$, $\mu = (\mu_1, \dots, \mu_n)$, $\beta = (\beta_1, \dots, \beta_n)$, and $h = (h_1, \dots, h_n)$. We specify prior distributions for the set of model parameters, $\theta \equiv \{\alpha, p_\mu, p_\beta, v\}$ and generate samples from the full joint posterior distribution $p(\theta, \mu, \beta, h|y)$. The components of the algorithm are as follows:

1. Generate $\mu \,|\, \alpha, p_\mu, \beta, h, y$.
2. Generate $\beta \,|\, \alpha, p_\beta, \mu, h, y$.
3. Generate $h \,|\, \alpha, \mu, \beta, y$.
4. Generate $\alpha \,|\, \mu, \beta, h, y$.
5. Generate $p_\mu \,|\, \mu$.
6. Generate $p_\beta \,|\, \beta$.
7. Generate $v \,|\, h$.

Each component of the sampler is explained below.

Generation of regime-switching state variables (Steps 1 & 2)

In Step 1, we use the multi-move sampler for the standard Markov-switching model (e.g., Carter and Kohn, 1994; Chib, 1996) to generate the sample of $(\mu_1, \dots, \mu_n)$ from the joint conditional posterior probability density. Define a state variable $s_t \in \{1, \dots, L\}$ such that $s_t = i$, if $\mu_t = \tilde{\mu}_i$. We further define $\vartheta \equiv (\alpha, p_\mu, \beta, h)$, and $Y_t = \{\pi_\ell\}_{\ell=1}^t$. First, we recursively compute the following two steps:

$$\text{(i) prediction step} : p(s_t|Y_{t-1}, \vartheta) = \sum_{i=1}^{L} p(s_t|s_{t-1} = i, \vartheta)\, p(s_{t-1} = i|Y_{t-1}, \vartheta),$$

$$\text{(ii) filtering step} : p(s_t|Y_t, \vartheta) \propto p(s_t|Y_{t-1}, \vartheta)\, f(\pi_t|s_t, \vartheta),$$

where

$$f(\pi_t|s_t, \vartheta) \propto \exp\left\{ -\frac{(\pi_t - \alpha\pi_{t-1:t-k} - \bar{\alpha}\mu_{s_t} - \beta_t x_t)^2}{2\sigma_t^2} \right\},$$

with $\pi_{t-1:t-k} = (\pi_{t-1}, \dots, \pi_{t-k})'$, and $\bar{\alpha} = 1 - \sum_{i=1}^k \alpha_i$, for $t = 1, \dots, n$. We next generate s_n from $p(s_n|Y_n, \vartheta)$, and then recursively generate s_t for $t = n-1, \dots, 1$, following the probability, $p(s_t|Y_t, \vartheta)\ \times p(s_{t+1}|s_t, \vartheta)$. The same sampling algorithm is used to generate β in Step 2.

Generation of stochastic volatility process (Step 3)

Defining $y_t^* = \pi_t - \alpha\pi_{t-1:t-k} - \bar{\alpha}\mu_t - \beta_t x_t$ yields a form of stochastic volatility model:

$$\begin{aligned} y_t^* &= \exp(h_t/2)e_t, \\ h_t &= h_{t-1} + \eta_t, \\ \begin{pmatrix} e_t \\ \eta_t \end{pmatrix} &\sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & v^2 \end{pmatrix} \right). \end{aligned}$$

We adopt the multi-move sampler for stochastic volatility models developed by Shephard and Pitt (1997) and Watanabe and Omori (2004). See the description of the sampler in Appendix A.2 of Nakajima (2011).

Generation of model parameters (Steps 4-7)

(Step 4) We specify a prior $\alpha \sim TN_\Omega(\alpha_0, \Sigma_0)$, where TN_Ω denotes a truncated multivariate normal distribution that has positive density only in the domain $\Omega \equiv \{\alpha| \left|\sum_{i=1}^k \alpha_i\right| \leq 1\}$. This prior is conjugate and the conditional posterior distribution is $\alpha \,|\, \mu, \beta, h, y \sim TN_\Omega(\hat{\alpha}, \hat{\Sigma})$, where $\hat{\Sigma} = (\Sigma_0^{-1} + W'W)^{-1}$, $\hat{\alpha} = \hat{\Sigma}(\Sigma_0^{-1}\alpha_0 + W'u)$,

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}, \quad \text{and} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix},$$

with $w_t = [(\pi_{t-1} - \mu_t)/\sigma_t, \ldots, (\pi_{t-k} - \mu_t)/\sigma_t]$, and $u_t = (\pi_t - \mu_t - \beta_t x_t)/\sigma_t$.

(Steps 5-6) In Step 5, we set a prior $p_\mu \sim B(a_0, b_0)$, where B denotes the Beta distribution. Then, the conditional posterior distribution is given by $p_\mu \,|\, \mu \sim B(\hat{a}, \hat{b})$, where $\hat{a} = a_0 + n_c$, $\hat{b} = b_0 + n - n_c - 1$, and n_c is the count of the set $\{t \,|\, 1 < t < n-1, \mu_{t+1} = \mu_t\}$. In Step 6, the generation of p_β is computed in the same way.

(Step 7) A conjugate prior $v^2 \sim IG(n_0/2, S_0/2)$, where IG denotes the inverse Gamma distribution, leads to the conditional posterior distribution $v^2 \,|\, h \sim IG(\hat{n}/2, \hat{S}/2)$, where $\hat{n} = n_0 + n - 1$, and $\hat{S} = S_0 + \sum_{t=1}^{n-1}(h_{t+1} - h_t)^2$.

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	Japan		United States	
Parameter	Mean	Stdev.	Mean	Stdev.
α_1	1.1474	0.1104	1.0762	0.1132
α_2	-0.2435	0.1495	-0.0987	0.1348
α_3	-0.0405	0.1406	-0.0993	0.1323
α_4	-0.4981	0.1280	-0.0591	0.0969
α_5	0.6017	0.1318		
α_6	-0.0541	0.1456		
α_7	-0.1643	0.1253		
α_8	0.0495	0.0739		
p_μ	0.9148	0.0155	0.9129	0.0171
p_β	0.9348	0.0100	0.9614	0.0075
v	0.2743	0.0627	0.2496	0.0549

Table 1. Posterior estimates of the parameters.

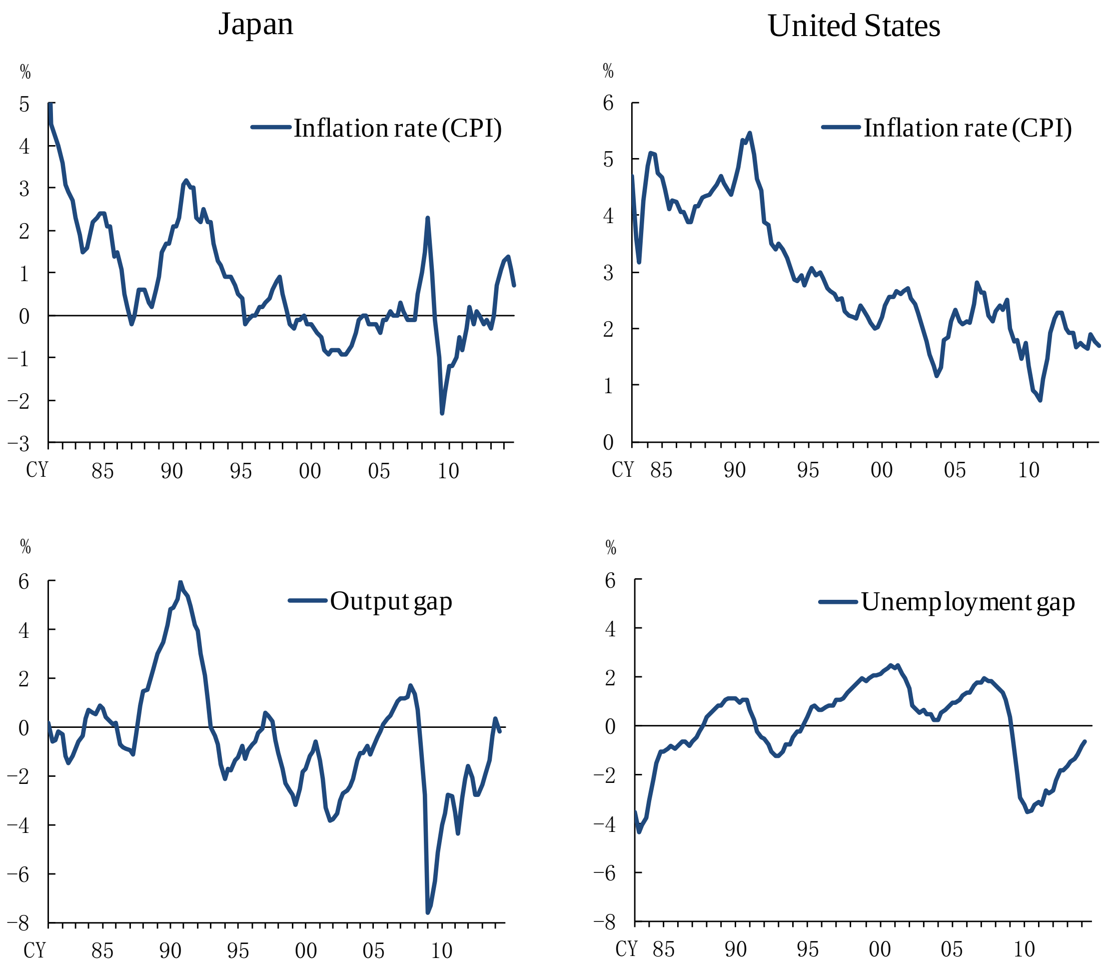


Figure 1. Data for Japan (left) and the United States (right).

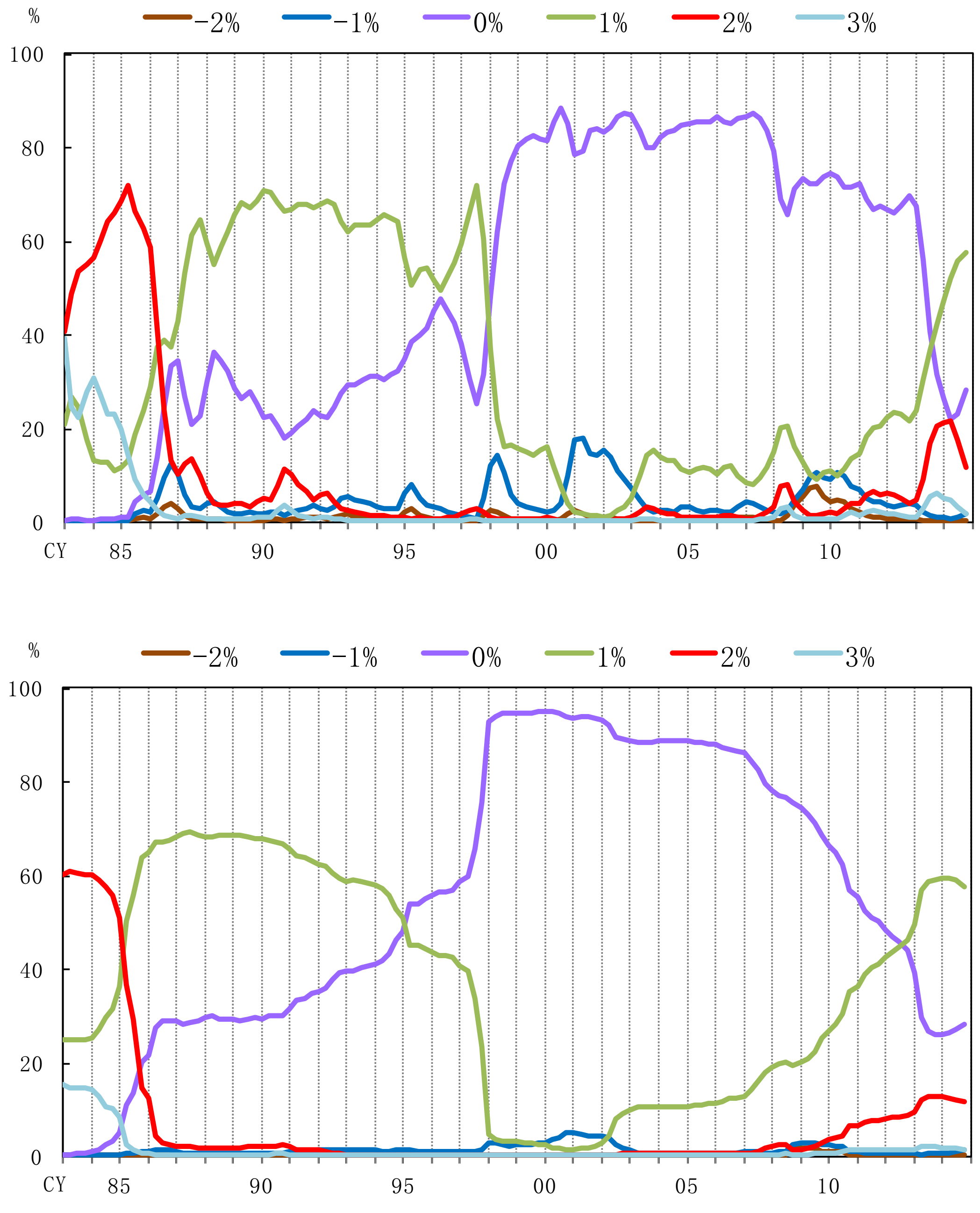


Figure 2. Posterior regime probability of trend inflation, filtered (top) and smoothed (bottom), for Japan.

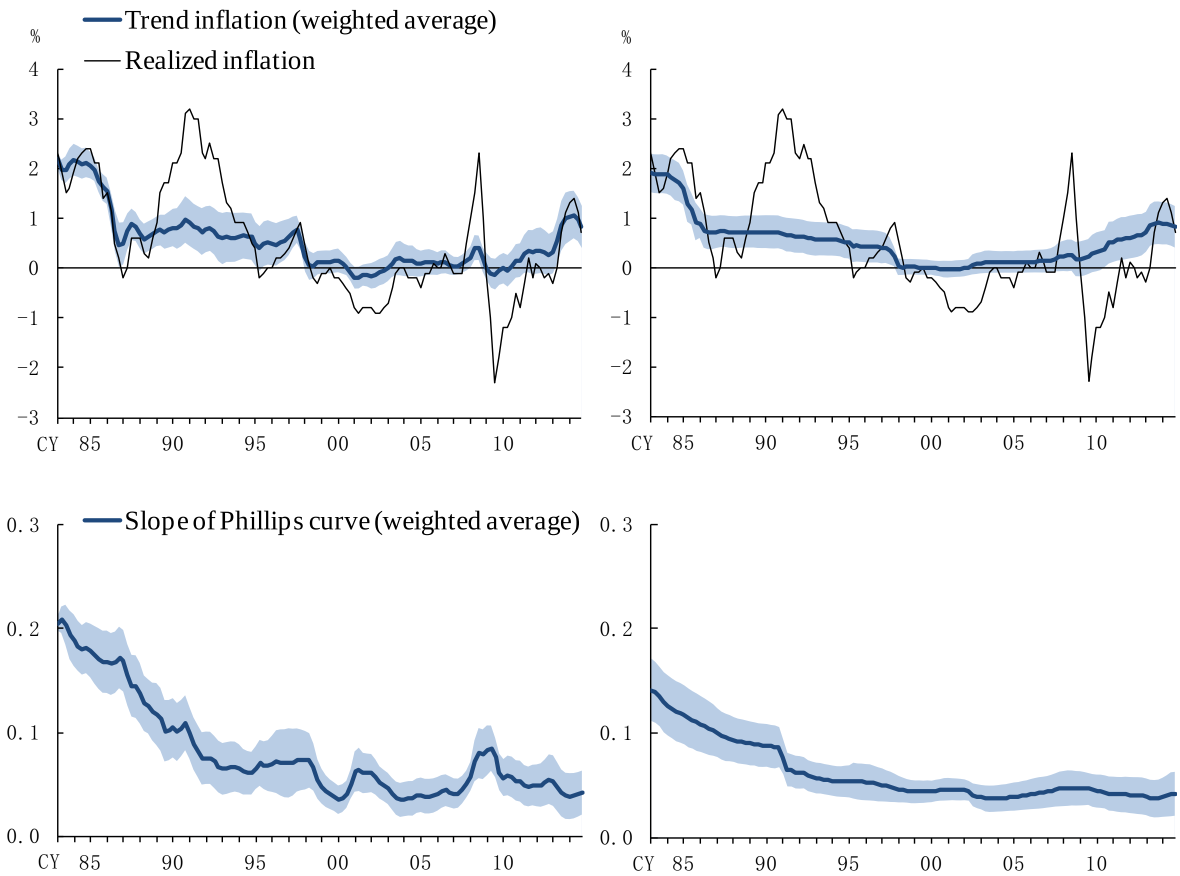


Figure 3. Posterior estimates of trend inflation (top) and the Phillips-curve slope (bottom), filtered (left) and smoothed (right), for Japan. The shaded area indicates the one-standard-deviation band.

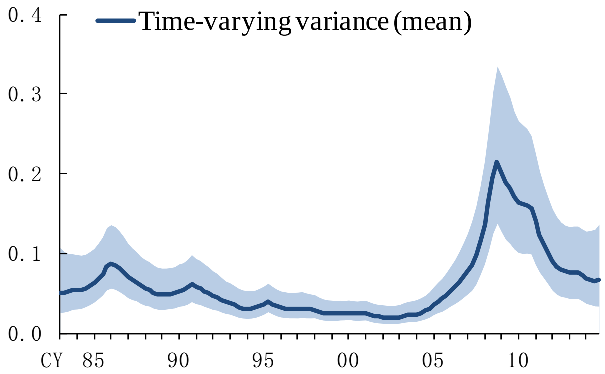


Figure 4. Posterior estimates of smoothed time-varying variance for Japan. The shaded area indicates the one-standard-deviation band.

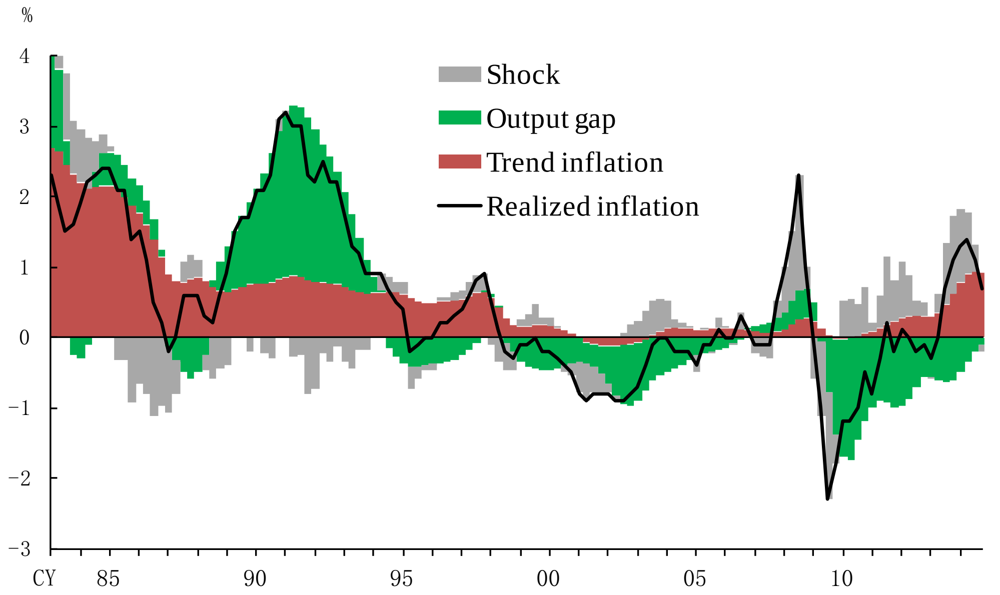


Figure 5. Historical decomposition of realized inflation based on posterior filtered mean for Japan.

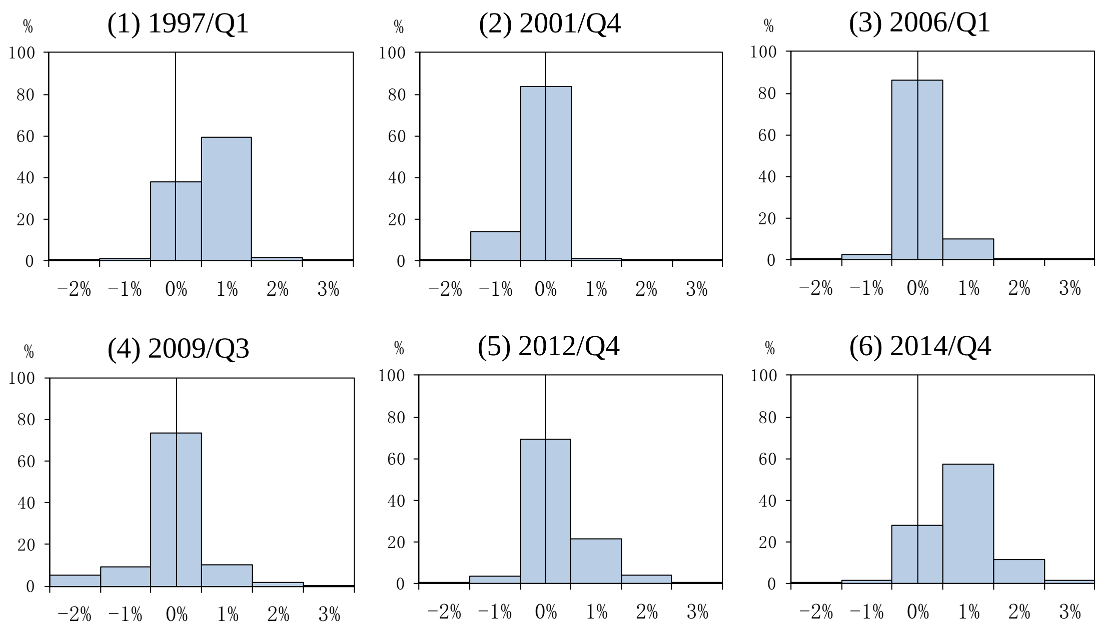
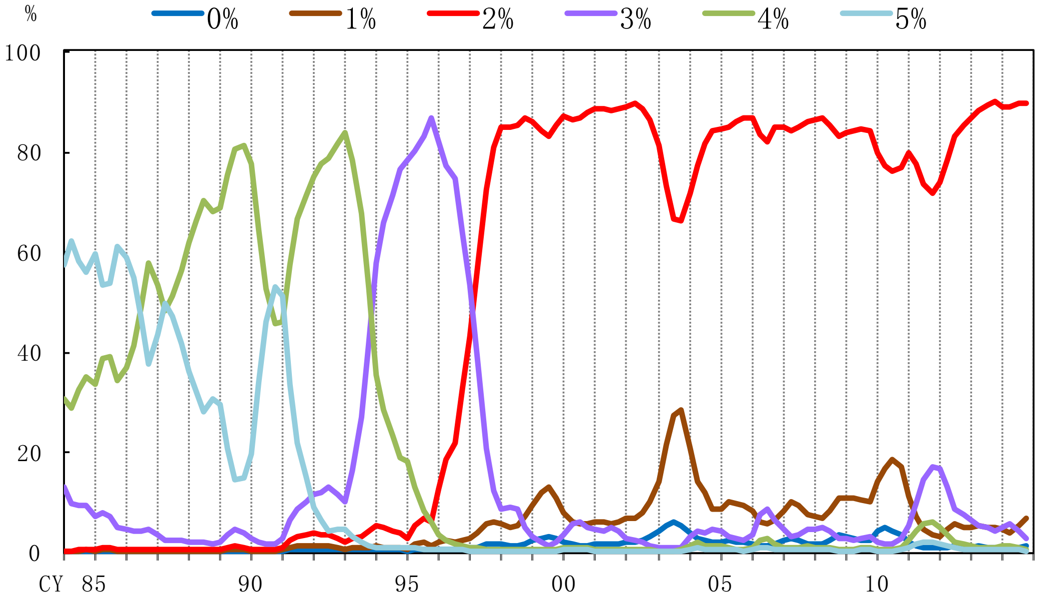


Figure 6. Distributions of posterior regime probability of filtered trend inflation for Japan. The horizontal axis indicates the trend-inflation regimes.



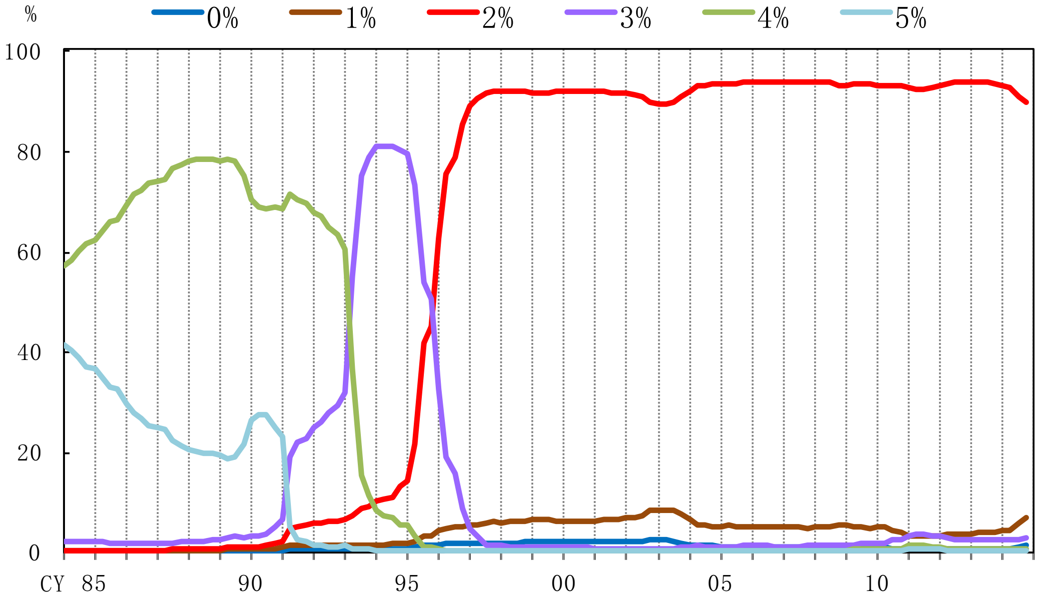


Figure 7. Posterior regime probability of trend inflation, filtered (top) and smoothed (bottom), for the United States.

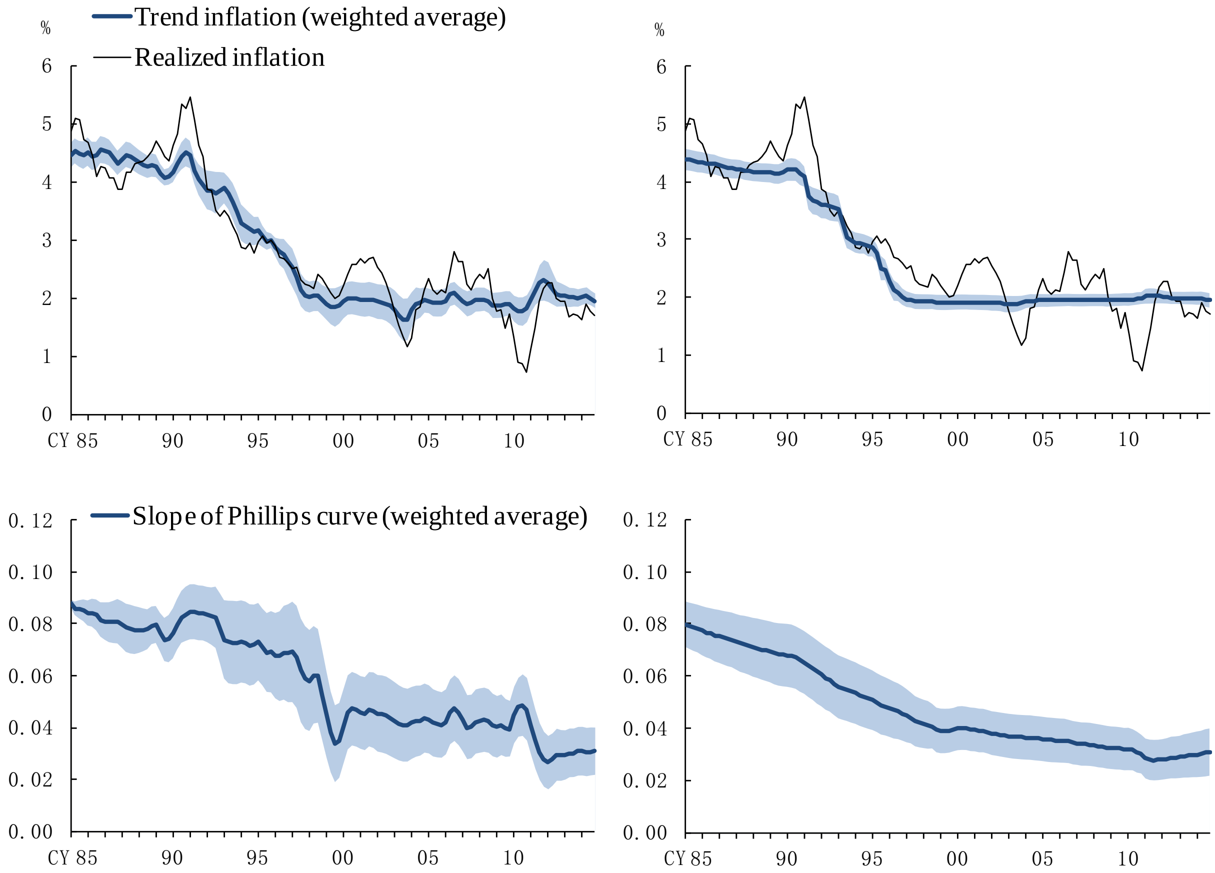


Figure 8. Posterior estimates of trend inflation (top) and the Phillips-curve slope (bottom), filtered (left) and smoothed (right), for the United States. The shaded area indicates the one-standard-deviation band.

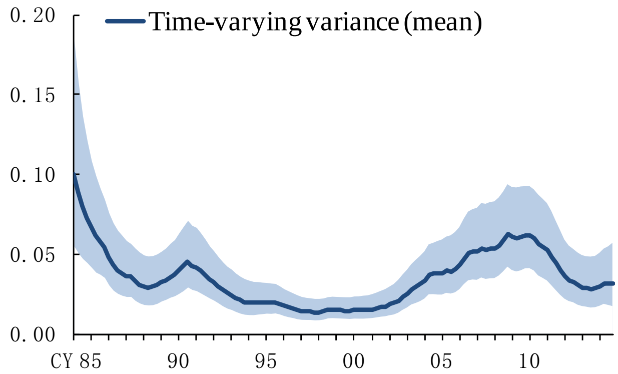


Figure 9. Posterior estimates of smoothed time-varying variance for the United States. The shaded area indicates the one-standard-deviation band.

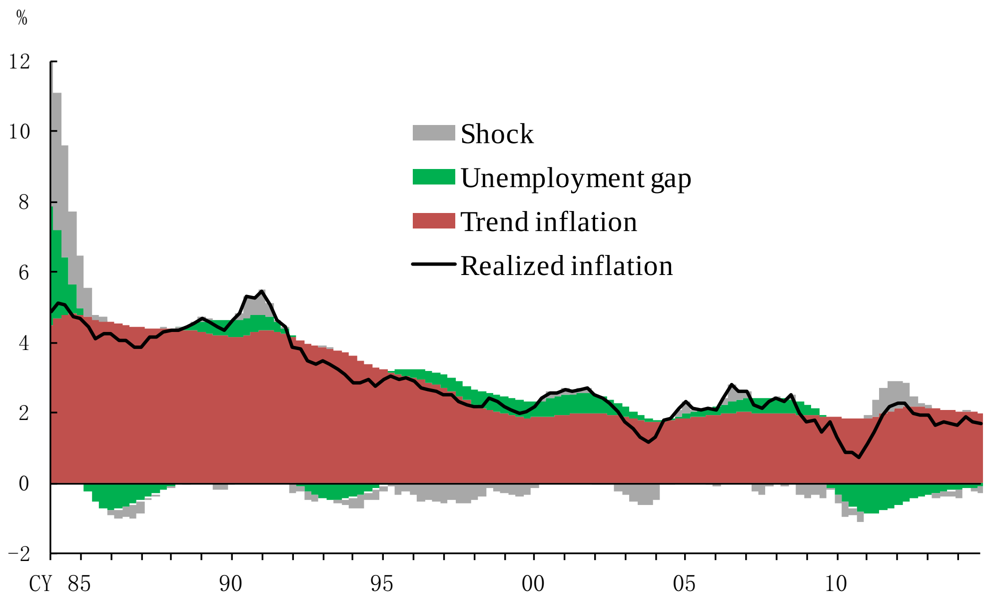
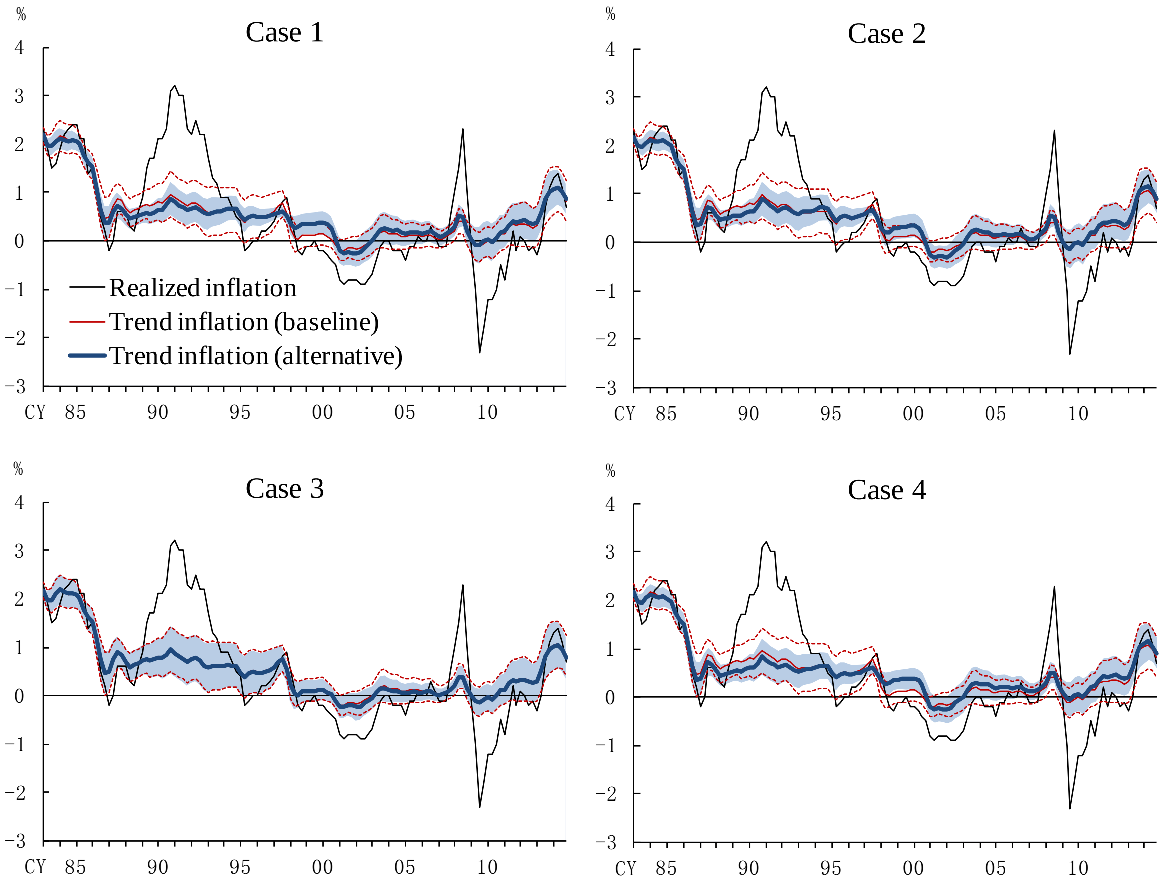
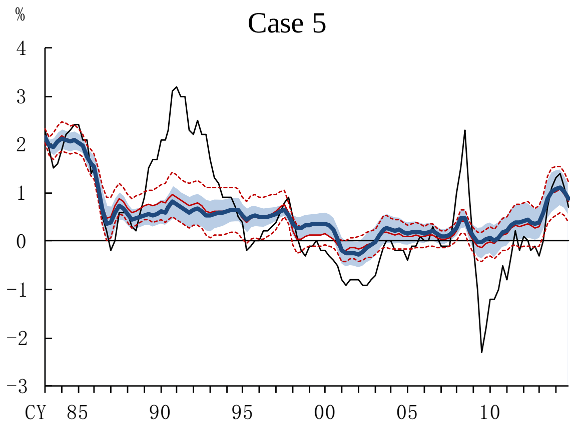


Figure 10. Historical decomposition of realized inflation based on posterior filtered estimates for the United States.





Regime intervals (trend inflation, the slope)	
Baseline	(1%, 0.5)
Case 1	(0.5%, 0.5)
2	(0.25%, 0.5)
3	(1%, 0.25)
4	(0.5%, 0.25)
5	(0.25%, 0.25)

Figure 11. Robustness check: posterior estimates of filtered trend inflation for Japan. The dotted lines and shaded area indicate the one-standard-deviation band for the baseline and alternative models, respectively.

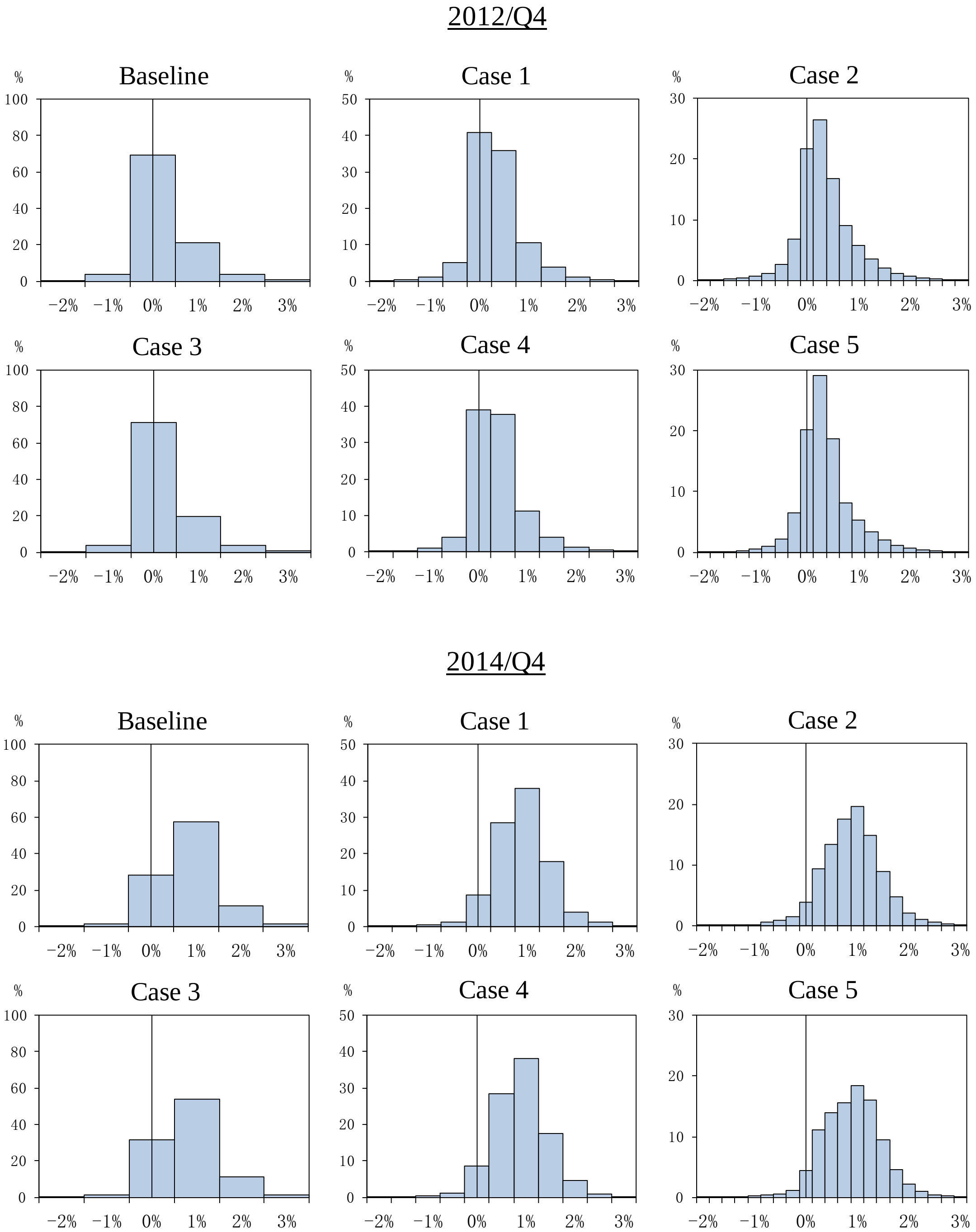


Figure 12. Robustness check: posterior regime probability of filtered trend inflation at 2012/Q4 (top) and 2014/Q4 (bottom) for Japan. The horizontal axis indicates the trend-inflation regime.