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TIPS: The Trend Inflation Projection System and Estimation Results

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Abstract

In practice, trend inflation is often defined as a common factor extracted from observed inflation rates by removing cyclical effects from business cycles as well as other transitory distortions. Trend inflation can also be interpreted as the infinitely long-term inflation rate expected by private economic agents. If we assume that the central bank's inflation target policy is fully credible, trend inflation will converge to the target inflation rate in the long run. In the short run, however, trend inflation and the target rate can differ due to adaptive, or backward-looking, expectations and changes in the extent to which the inflation target is credible.

Based on these considerations, this paper proposes a simple new methodology for projecting trend inflation, labelled the *Trend Inflation Projection System* (TIPS), where trend inflation is expressed as the weighted average of two components: an adaptive component, where a common trend is extracted from several core inflation measures, and a forward-looking component, namely the target inflation rate. In addition, the weights are allowed to vary over time to capture changes in the degree to which economic agents believe in the inflation target.

The estimation results show that trend inflation in Japan increased dramatically in the first quarter of 2013, when the BOJ raised the target inflation rate, and has continued to rise gradually since then. However, since the second half of 2014, medium- to long-term inflation expectations have shifted downward, meaning that inflation expectations may be formed in a more adaptive manner than in the model. Furthermore, decomposition of the consumer price index (CPI, all items less fresh food) based on the estimated model indicates that although CPI inflation rose from the beginning of 2013 due to the increase in trend inflation, it has decreased again since the second half of 2014 due to transitory factors such as the decline in oil prices.

Keywords: Core Inflation; Trend Inflation; Inflation Target

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1 Introduction

The long-run expected inflation rate, often called trend inflation, is an important driving factor of persistent movements in actual inflation. Many central banks that have adopted inflation targeting conduct monetary policy by adjusting interest rates to bring inflation to its target level. If private agents believe the inflation target is achievable and form their expectations taking the future path of monetary policy and its effects into account, trend inflation should, in theory, converge to the target level as soon as the policy is announced.

In practice, however, this is not always the case. In Japan, the announcement of a 2 percent inflation target rate by the Bank of Japan in Q1 2013, was insufficient for observed medium- to long-term inflation expectations to converge to the target level (Figure 1(a)). There are several possible factors explaining why this is so. One is that private agents tend to form their inflation expectations adaptively and put greater weight on actual inflation observations. Another is that there is heterogeneity among private sector agents in terms of the degree to which they believe that the central bank can achieve the inflation target.

This means that getting a quantitative grasp of these factors based on actual data is extremely important not only for projecting future inflation rates, but also when considering the effectiveness of monetary policies aiming to alter private agents' expectations through the commitment to an inflation target.

A widely employed approach in the literature regarding the estimation of trend inflation is to extract the unobservable trend component from inflation data utilizing econometric methods. Stock and Watson (2007), for example, propose an unobserved component trend-cycle model with stochastic volatility to extract the trend component from inflation data. Mertens (2015) extended Stock and Watson's idea by extracting a trend factor from several different measures of inflation and long-term nominal interest rates.[1]

Meanwhile, Kaihatsu and Nakajima (2015) jointly estimated trend inflation and the

[1] An application of this approach is provided by Garnier et al. (2015), who used it to extract trend inflation for developed countries including Japan.

slope of the Phillips curve for Japan assuming a Markov switching process, while Cogley and Sbordone (2006, 2008), assuming Calvo pricing (Calvo, 1983), proposed to derive the Phillips curve by log-linearizing firms' optimal price setting condition around a time-varying trend inflation rate. They estimated trend inflation using the derived Phillips curve as an identifying restriction in a vector autoregressive (VAR) model.

However, existing approaches used in the literature estimate trend inflation taking its interactions with actual inflation as well as the inflation target rate into account.[2,3]

To fill this gap, we have developed the *Trend Inflation Projection System* (TIPS). In TIPS, trend inflation is defined as the weighted average of adaptive (i.e., backward-looking) and forward-looking expectations. We use the persistent trend factor extracted from various measures of inflation as the adaptive expectation component and the inflation target rate as the forward-looking component. Based on this formulation, TIPS is able to quantify the contribution of these two components to changes in long-run inflation expectations. More concretely, we extract the persistent trend factor from multiple series of core inflation and assume that the coefficient on the inflation target rate, which can be interpreted as measuring the credibility of the inflation target, changes over time.

The remainder of the paper is organized as follows. Section 2 explains the determinants of inflation dynamics, focusing on trend inflation, and describes the details of our model, TIPS. Section 3 then presents the estimation results based on the model and discusses the implications of the results. Next, Section 4 reports simulation results based on the estimated model, while Section 5 concludes.

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[2]Although the large-scale macroeconomic model of the Federal Reserve Board, FRB/US, employs a New Keynesian Phillips curve based on Cogley and Sbordone (2008) to model the inflation rate, the model does not allow to investigate the interaction between actual inflation and the inflation target rate because it assumes that trend inflation follows a random walk (Brayton (2013)).

[3]Until recently, trend inflation in the large-scale macroeconomic model developed by the BOJ, the so-called Quarterly Japanese Economic Model (Q-JEM), had been defined as a latent variable and was estimated using survey data of economists' forecasts. In this setting, it was not possible to explicitly examine the relationship between actual inflation and the inflation target rate, because it was assumed that trend inflation follows a random walk. See Ichiue et al. (2009) and Fukunaga et al. (2011) for details of Q-JEM.

2 Overview of Trend Inflation Projection System

TIPS consist of three building blocks, namely, (1) a New Keynesian Phillips Curve (hereafter NKPC), (2) long-term inflation expectations based on trend inflation, and (3) trend inflation based on the inflation target rate and the persistent component of actual inflation (Figure 2). This section explains each building block in detail.

2.1 Inflation Rate Dynamics in the NKPC

In the canonical dynamic stochastic general equilibrium (DSGE) model, the driving forces of inflation are (i) past realized inflation, (ii) the output gap, and (iii) inflation expectations.[4] Specifically, in this paper, we follow the hybrid NKPC and describe inflation dynamics as follows:

$$\pi_t = \rho\pi_{t-1} + (1-\rho)\pi^e_{t,6\sim 10Y} + \gamma y_t + \epsilon_t, \tag{1}$$

where π, $\pi^e_{6\sim 10Y}$, and y denote the inflation rate, 6-to-10-year-ahead inflation expectations, and the output gap, respectively.

In order to forecast inflation and conduct policy exercises, we need to model long-term inflation expectations. We use the 6-to-10-year-ahead inflation expectations in the Consensus Forecasts as a proxy for long-term inflation expectations. We assume that private agents believe that inflation dynamics in the future follow the same specification as the above hybrid NKPC. Long-term inflation expectations thus are given by the following specification:

$$\pi^e_{t,6\sim 10Y} = \alpha\pi^e_{t-1,6\sim 10Y} + (1-\alpha)\bar{\pi}_t + \beta y^e_{t,6Y} + v_t, \tag{2}$$

[4] The hybrid New Keynesian Phillips curve can be derived assuming monopolistic competition and Calvo pricing. That is, only a limited fraction of firms can adjust the price of their product to its optimal (marginal cost) value, while for firms that cannot adjust the price of their product, the price changes according to an indexation rule (see Christiano et al. (2005) for more details). In this setting, firms that adjust their price take into account not only current marginal costs but also future inflation and marginal costs. Under certain conditions, marginal costs will be proportionate to the output gap, so that ultimately the inflation rate can be expressed as a function of realized inflation, inflation expectations, and the output gap.

where $\pi^e_{6\sim10Y}$, $\bar{\pi}$, y^e_{6Y}, and v represent 6-to-10-year-ahead inflation expectations, trend inflation, the 6-year-ahead expected output gap, and the disturbance term, respectively. It should be noted that we expect the coefficient on the 6-year-ahead expected output gap to be near zero given the average length of the business cycle. Therefore, the main driving force of fluctuations in long-term inflation expectations should be trend inflation.

2.2 Trend Inflation

Trend inflation can also be interpreted as infinitely long-term inflation expectations among private agents. As seen in the previous subsection, when considering fluctuations in long-term inflation expectations, it is essential to consider how trend inflation is determined. In this paper, we focus on two different determinants of trend inflation: adaptive and forward-looking expectations. Under adaptive expectations, the formation of infinitely long-term inflation expectations is based on the trend component of actual inflation. On the other hand, under forward-looking expectations, agents take into account future paths of monetary policy and believe that the inflation target is feasible. In this manner, infinitely long-term inflation expectations coincide with the inflation target rate.[5]

How these expectations formation mechanisms are incorporated in TIPS is explained in detail below. Essentially, TIPS incorporates a feedback loop where actual inflation feeds into trend inflation and vice versa.

2.2.1 Adaptive Expectations Formation[6]

Let us assume that economic agents have no knowledge of the structure of the economy and forecast long-term inflation rates based only on observed inflation rates, extracting the trend component from actual observations. We will use the Beveridge-Nelson

[5] For surveys of the literature on the mechanisms of inflation expectation formation, see Tsuruga and Muto (2008) and Nakazono (2016) for Japan, and Mehra (2002), Trehan (2015), and Fuhrer (2012) for the United States.

[6] We will use the term "adaptive expectations" assuming that inflation expectations are formed from actual observations of underlying CPI inflation. A narrower definition would be to regard adaptive expectations as being determined only by actual observations of the CPI (all items less fresh food).

(hereafter BN) decomposition[7] to extract trend components. In the BN decomposition, observed inflation consists of a permanent and a transitory component. In the BN decomposition, observed inflation consists of a permanent and a transitory component and studies (e.g., Stock and Watson, 2007) show that the permanent component of inflation extracted using BN decomposition forecasts future inflation well.[8] Inflation is decomposed as follows:

$$\pi_t = \tau_t + z_t, \tag{3}$$

where τ and z denote the permanent and transitory components, respectively. Further, we also assume that the permanent component is a random walk:

$$\tau_t = \tau_{t-1} + \xi_t, \tag{4}$$

where ξ is white noise with $E_t\xi_{t+j} = 0$ $(j = 1, 2, ...)$ and $E_t\xi^2_{t+j} = \sigma^2_\tau$, which is assumed to be a constant.

Given these dynamics, we can iterate forward to form k-period-ahead inflation expectations as follows:

$$\pi_{t+k} = \tau_t + \sum_{j=1}^{k} \xi_{t+j} + z_{t+k}. \tag{5}$$

Following Garnier et al. (2015), we assume that the transitory components follow a VAR(2) process given by

$$z_t = \sum_{k=1}^{2} b_k z_{t-k} + e_t, \tag{6}$$

where e denotes the disturbance term.[9] If the expected value of the transitory component, z_{t+k}, converges to zero in the limit, the expected value of inflation will coincide with the

[7] For further details, see Beveridge and Nelson (1981).

[8] There are other models for forecasting inflation, which, for example, use trend components extracted from moving averages of realized inflation rates or Hodrick-Prescott (HP) filtered trends, as well as autoregressive models. The reason that we employ the BN decomposition is that studies such as Stock and Watson (2007) show that it performs well in forecasting inflation.

[9] Two lags for the VAR model are chosen based on the Akaike information criterion (AIC).

permanent component of inflation:

$$\lim_{k \to \infty} E_t[\pi_{t+k}] = \tau_t. \tag{7}$$

Therefore, if fluctuations in inflation are driven by the permanent component, τ, the effect will not dissipate.

2.2.2 Forward-looking Expectations Formation

On the other hand, if private agents form their inflation expectations rationally, then – with inflation targeting – another expectation formation mechanism is more plausible. Specifically, if the central bank commits itself to an inflation target and has the means to achieve the target, private sector agents such as households and firms are likely to believe that this inflation target will be achieved. From a theoretical perspective, infinitely long-term inflation expectations in this case will coincide with the target inflation rate.[10] The implicit assumption underlying this mechanism is that households and firms believe that the structure of the economy is such that central bank policies are indeed effective in achieving the central bank's inflation target. To see why this is the case, let us consider how expectations are formed when private agents assume that the structure of the economy is represented by a basic NKPC.

First, assume that the output gap is described by a hybrid New Keynesian IS curve:

$$y_t = \alpha E_t[y_{t+1}] + (1 - \alpha) y_{t-1} + \rho(i_t - E_t[\pi_{t+1}]) + u_t, \tag{8}$$

where y and π denote the output gap and inflation rate, respectively. Further, i represents the nominal interest rate and u represents exogenous shocks. Second, the inflation rate

[10] As pointed out by Benhabib et al. (2002), it is possible in a rational expectations model for a steady state inflation rate below the central bank's target level to endogenously arise when the zero interest rate lower bound binds. Furthermore, Hills et al. (2016) showed that when the DSGE model is solved without linearization, the stochastic steady state inflation level may be lower than the target rate. However, the literature so far does not incorporate unconventional monetary policies or negative interest rates. If private sector agents' inflation expectations are shaped by the view that the structure of the economy is such that central bank policies are ineffective, it is possible that infinitely long-term inflation expectations may fall below the target level.

is determined by a hybrid NKPC:

$$\pi_t = \beta E_t[\pi_{t+1}] + (1-\beta)\pi_{t-1} + \zeta y_t + \epsilon_t, \tag{9}$$

where ϵ represents exogenous shocks. Finally, the monetary policy rule follows a modified Taylor rule:

$$i_t = \theta i_{t-1} + (1-\theta)[\phi_t y_t + \phi_\pi(\pi_t - \pi_t^*)] + v_t, \tag{10}$$

where π^* and v represent the target inflation rate and exogenous shocks, respectively. In this type of economy, the central bank adjusts the interest rate by responding to the gap between the actual inflation rate and the target rate. As a result of such interest rate adjustments, the output gap changes based on the New Keynesian IS curve, which then feeds into actual inflation through the NKPC, so that eventually inflation approaches the targeted level. The speed of convergence will depend on the parameters, but under reasonable values, infinitely long-term inflation expectations converge to the target as follows:

$$\lim_{k\to\infty} E_t[\pi_{t+k}] = \pi_t^*. \tag{11}$$

2.2.3 Hybrid Expectations Formation

In practice, however, it is difficult to know which of the above two expectation formation mechanisms dominates. Therefore, we will assume two types of agents, namely, agents with adaptive expectations and agents with forward-looking expectations, which each make up a certain share of the population, and define trend inflation as a weighted average of the these two types of agents:

$$\bar{\pi}_t = \delta_t \tau_{t-1} + (1-\delta_t)\pi_t^*, \tag{12}$$

where $1-\delta$ represents the share of forward-looking agents in the economy. In the estimation, we assume that the target inflation rate, π_t^*, is 1 percent from Q1 1990 up to Q4 2012 and 2 percent from Q1 2013 onward.[11]

2.3 Credibility of the Inflation Target

The share of forward-looking agents in the economy ($(1 - \delta)$ in equation (12)) can also be interpreted as a measure of the credibility of the inflation target policy. Although credibility of the inflation target will not change dramatically on a daily basis, it is not necessarily constant over time and could change when the central bank changes the target rate or its regime.

In fact, when the BOJ raised the target inflation rate to 2 percent in 2013, there was a substantial gap between private forecasters' and BOJ Policy Board members' inflation forecasts of 0.6 percentage points (Figure 1(b)). This gap may reflect doubts that the target would be achieved among some private forecasters, who consequently did not incorporate the target into their inflation forecasts. Since the adoption of the inflation target, the gap between inflation forecasts has become smaller, which can be regarded as indicating that the credibility of the inflation target policy has risen over time following the expansion of Quantitative and Qualitative Easing, responses in financial markets, and the BOJ's active communication with the public.

Given these observations, we model credibility as a time-varying parameter as follows:

$$\delta_t = \omega \delta_{t-1} + \kappa D_t + \mu_t, \tag{13}$$

where D is a dummy variable which takes 1 in Q1 2013 and 0 otherwise, and μ is the disturbance term. We estimate the parameters so that long-term inflation expectations are consistent with the 6-to-10-year-ahead inflation expectations from the Consensus Forecasts.[12]

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[11] The reason for assuming a target inflation rate of 1 percent before the first quarter of 2013 is that the median value of the inflation rate Policy Board members considered to be consistent with price stability from a medium- to long-term viewpoint in the "understanding of medium- to long term price stability" introduced in March 2006 was around 1 percent and "the price stability goal in the medium to long term" was set to 1 percent in February 2012.

[12] When the economy consists only of forward-looking agents as in the NK model, medium- to long-term inflation expectations may not reach 2 percent due to, for instance, the zero lower bound on interest

When ω is between 0 and 1, the credibility of the inflation target, $1-\delta$, will approach 1 over time. When ω equals 1, credibility of the inflation target follows a random walk. The parameter ω can also be interpreted as the time it takes for the credibility of the inflation target to reach 1, and we will estimate this parameter from actual data, assuming $0 < \omega < 1$. To capture a possible jump in credibility, we include a dummy variable which takes a value of 1 in Q1 2013, when the BOJ increased its target rate to 2 percent, and 0 otherwise, and estimate κ. The credibility parameter $1-\delta$ is restricted to take a value between 0 and 1 in the estimation.

3 Empirical Analysis

Employing the TIPS model presented in the previous section, we estimate the three blocks of the model and compare the performance of different extraction methods. First, we extract the permanent components from actual inflation employing two different models: a model with four measures of core inflation and a model using only a single CPI to measure inflation. Second, we estimate trend inflation and long-term inflation expectations. Third, we estimate the hybrid NKPC for actual inflation. Fourth, we compare the performance of the two methods for extracting the permanent component of inflation. Finally, we decompose inflation into the different components using the TIPS model.

3.1 Extraction of the Permanent Component

We start by extracting the permanent component of actual inflation, which is necessary to estimate trend inflation, using quarterly data from Q1 1983 to Q1 2016. We decompose actual inflation into a long- and a short-term component and employ two different extraction methods. The first consists of using four core inflation measures to extract the common trend, while the second uses only the CPI for all items less fresh food. The models will be estimated using the Monte Carlo Markov chain (MCMC) approach

rates. In TIPS, this case is captured as a decline in the credibility of the central bank's inflation target.

following Garnier et al. (2015).[13]

3.1.1 Using Four Core Inflation Measures

The four measures of core inflation used to extract the common permanent component are the CPI for all items less fresh food, the CPI for all items less fresh food and energy, the CPI for all items less food and energy, and the trimmed mean. The aim is to exclude temporary distortions in relative prices and focus on core inflation to capture the permanent component.[14] The estimated permanent component is fairly smooth and captures the trend of other core inflation measures quite well, and its movement appears similar to that of the trimmed mean (Figure 3). We also find that the recent decline in the CPI for all items less fresh food is due to the transitory component, mirroring the fall in energy prices. Thus, the permanent component extracted based on the four core inflation measures is less susceptible to changes in relative prices.

3.1.2 Using a Single CPI

We also estimated the model using one measure of core inflation only, namely, the CPI for all items less fresh food. Figure 4 indicates that since the second half of 2014, the CPI for all items less fresh food declined reflecting the sharp drop in oil prices. During this time, the permanent component extracted based on this method declined in the second half of 2014, as well.[15]

Hence, the permanent component extracted from the single CPI is more volatile than the one based on four measures of core inflation.

Based on the role that the permanent component plays in our trend inflation model, one would not expect short-term changes in relative prices to affect trend inflation.

[13] See the Appendix for full details on the estimation procedure.

[14] See Shiratsuka (2015) for the performance of different core inflation measures and Hogen et al. (2015) for the relationship between those different measures and the business cycle. See Shiratsuka (1997) and Mio and Higo (1999) for a more detailed discussion of the trimmed mean and Iwasaki and Kaihatsu (2016) for a discussion of dynamic model averaging techniques to measure underlying inflation.

[15] A possible reason for the decline in the permanent component is that, against the background of Japan's prolonged deflation, temporary changes in relative prices such as a decline in oil prices may have a long-lasting effect through their impact on wage growth.

However, observed 6-to-10-year-ahead inflation expectations in the Consensus Forecasts have gradually decreased since the second half of 2014 with the decline in oil prices. Therefore, it cannot be ruled out that relative price changes may have fed into long-term inflation expectations. Choosing the appropriate method for extracting the permanent component is ultimately an empirical question of the goodness of fit. The estimation results for long-term inflation expectations and actual inflation will be discussed in the following sections.

3.2 Estimation Results of the Long-term Inflation Expectations Model

Using the permanent components from the previous section, we estimate a joint system consisting of the long-term inflation expectations Phillips curve (equation 2) and trend inflation (equations 12 and 13). We will use the 6-to-10-year-ahead inflation expectations from the Consensus Forecasts as a proxy for long-term inflation expectations and the output gap estimates by the Research and Statistics Department of the BOJ.[16]

The 6-year-ahead expected output gap is constructed from an AR(2) model and it is assumed that up to Q4 2012 the target inflation rate was 1 percent, while it is 2 percent from Q1 2013 onward. The model is estimated using MCMC estimation (see the Appendix for details of the estimation method).

The estimation results for the model using the four core inflation measures are shown in Table 1(a). We find that the estimated coefficient on the 6-year-ahead expected output gap (β) is slightly positive but close to zero and not significant, while the autoregressive parameter for credibility (β) is close to unity, implying high persistence. Developments in the credibility measure and the estimated trend inflation are presented in Figure 5. As can be seen, the credibility measure was around 0.8 before Q1 2013 but dropped soon after the introduction of the 2 percent inflation target, then gradually increased again and most recently reached the previous value of around 0.8 again. Meanwhile, the

[16]In the estimation, 6-to-10-year-ahead inflation expectations are regarded as observables and differences from the model-implied values are regarded as measurement errors.

estimated trend inflation rate was around 0.8 percent until the end of 2012, but jumped to 1.5 percent in the first quarter of 2013 and has gradually risen since then, reaching recent values of around 1.6 percent.

The estimated parameters for the model with just one inflation measure, the CPI for all items less fresh food, shown in Table 1(b), are close to those when using four core inflation measures. Moreover, the credibility measure and trend inflation show similar patterns as well. We will discuss which model is more suitable in a later section.

3.3 Estimation Result of the NKPC for Actual Inflation

Next, Table 2 presents the results for the hybrid NKPC (equation 1) with the CPI for all items less fresh food and energy, the CPI for all items less food and energy, and the CPI for all items less fresh food. Inflation is measured in terms of the annualized quarter-to-quarter change in the seasonally adjusted series. In addition, the output gap estimates by the BOJ and 6-to-10-year-ahead expected inflation from the Consensus Forecasts are used. The estimates are obtained using maximum likelihood estimation, and the sample period is from Q1 1990 to Q1 2016. The table shows that all parameters are significant. The autoregressive parameters are relatively high around 0.8, indicating high persistence of inflation. In terms of fit, the model performs relatively well with a standard error of the regression of around 0.6 percentage points on an annualized basis.

3.4 Comparison of the Performance of the Two Models

So far, we have presented the results of the two models, i.e., the model with four core inflation measures and the one with only the CPI for all items less fresh food. In this section, we compare the performance of the two models. We start by comparing the goodness of fit by calculating the marginal likelihood. The results are presented in Table 3 and indicate that the log-marginal likelihood of the model with four core inflation measures is higher than that of the single CPI model, with the log-difference being 1.44. Thus, based on the Jeffreys criterion (Jeffreys, 1961), the former model is more appropriate. Next, we compare the performance of out of sample forecasts for long-term inflation expectations. Specifically, we calculate the root mean squared error (RMSE) of forecasts, using data from Q2 2011 to Q1 2016 (Figure 6). In addition to comparing the two models with each other, their forecast performance is also compared to a simple VAR model consisting of the four core inflation measures, the output gap, and long-term inflation expectations.[17]

We find that the RMSE of the two TIPS models is smaller than that of the VAR model, although the Diebold-Mariano test indicates that the difference is not significant.[18]

The model with the four core inflation measures has a smaller RMSE than the single CPI model, especially over horizons of more than four periods ahead, but the difference is not substantial for shorter horizons of one to two periods.[19]

3.5 Decomposition of Realized and Trend Inflation

We start by reviewing developments in the permanent component of actual inflation. Soon after the financial crisis in 2008, the permanent component of inflation turned negative; however, it gradually improved through 2012 and has been firmly in positive territory since 2013. It slowed somewhat in the second half of 2014, but has recently been on the rise once again (Figure 3).

With these developments in mind, we decompose realized inflation rates and trend inflation to gain an understanding of recent inflation dynamics and the formation of inflation expectations. The results of this decomposition are shown in Figure 7 and indicate that the main driving force underlying trend inflation rates (Figure 7(a)) is

[17] In Figure 6, the number of lags in the VAR is chosen to be five based on the AIC; the results are similar using other specifications such as a lag one VAR based on the Schwarz information criterion.

[18] The reason that the one-period-ahead forecast of the VAR model outperforms the other models probably is differences in the way medium- to long-term inflation expectations data are interpolated. In the VAR estimation, the inflation expectations data before 2014 is converted into quarterly frequency through linear interpolation, whereas in TIPS, Kalman filtering techniques are used.

[19] Comparing the performance of the TIPS model (four core inflation version) and the VAR model using CPI inflation (all items less fresh food and energy), the RMSE of the one-year-ahead forecast is 0.9 percent for both models, so the performance is identical. For longer horizons, say three years, the TIPS model outperforms the VAR model: the RMSE is 0.6 percent for the TIPS model compared to 1 percent for the VAR model.

indeed the inflation target and its credibility, with fluctuations mainly driven by the permanent component. Estimated trend inflation was around 1 percent before Q1 2013 and coincides with 6-to-10-year-ahead forecasts in the Consensus Forecasts. Moreover, as already seen in Figure 7, trend inflation increased after the introduction of the 2 percent inflation target rate in the first quarter of 2013 and has gradually risen to around 1.5 percent. Figure 7(a) also indicates that the permanent component, after bottoming out in 2009, has been positive and has contributed somewhat to the rise in trend inflation.

Next, the decomposition of long-term inflation expectations (Figure 7(b)) shows that following the increase in the inflation target rate in 2013 and the following rise in the credibility of the target, the contribution of the inflation target also increased. Looking closely, we find that while the permanent component has made a negative contribution since 2008, the inflation target and the residual have made a positive contribution, leading long-term inflation expectations to fluctuate around 1 percent. From 2013 onward, long-term inflation expectations started to increase following the increase in the inflation target and the permanent component, but since around mid-2015, they have started to decline, reflecting the negative residual.

Turning to CPI inflation, Figure 7(c) shows a huge decline after 2008, mainly due to the decline in the output gap and the residual. As already seen in Figure 3, in this period the transitory component in the CPI for all items less fresh food was much more volatile than the other components. This suggests that the decline in food and energy prices and the appreciation of the yen may be responsible for the relatively large contribution of the residual during this period seen in Figures 7(c). Since then, CPI inflation gradually increased until the first half of 2014 reflecting the rise in trend inflation and the output gap. However, since the second half 2014, the rate of change in the CPI for all items less fresh food has fallen to close to zero, mainly driven by the contribution of the residual. Meanwhile, core inflation measures that are less affected by fluctuations in food and energy prices as well as the exchange rate such as the CPI for all items less fresh food and energy indicate that inflation slowed somewhat in the first quarter of 2015, but the decline was relatively small and has been reversed driven by increases in trend inflation.

For comparison, we also conducted decompositions of CPI inflation with the TIPS model using the single CPI (Figure 8). The figure shows that the decline in long-term inflation since the second half of FY2014 is due not only to the residuals but also to the permanent component of inflation. On the other hand, in the TIPS model using four core inflation measures, this decline is mainly due to the residual. Therefore, for this period, the single CPI model outperforms the model using four core inflation measures.

4 Impulse Responses

This section examines the impulse responses in the model with the four core inflation measures, which has a slightly higher forecasting performance than the single CPI model, to understand how actual inflation and long-term inflation expectations respond to various exogenous shocks.[20]

In particular, the impulse responses to five shocks, namely, a shock to the core inflation measure itself, to the credibility of the inflation target, to long-term inflation expectations, and to the output gap are examined. In calculating the impulse responses, it is assumed that the output gap follows an exogenous AR(2) process.

4.1 Core Inflation Shocks

In the NKPC framework, inflation is expressed by the output gap, expected inflation, and lagged inflation. In this case, temporary shocks to inflation, stemming, for example, from exogenous regulatory changes, could persist in a sticky manner through the autocorrelation term. In the analysis, we use different core inflation measures for the NKPC and examine the impulse responses to a 0.1 percentage point increase in the core inflation rate. The results are shown in Figure 9.[21]

Specifically, the first panel shows that core inflation rises by 0.1 percentage points on

[20] Since TIPS is a non-linear model, the impulse responses will depend on initial values. The impulse responses assume Q1 2016 to be period 0, and the shocks occur in Q2 2016. The impulse responses are the differences from the baseline forecasts.

[21] In this case, we are considering shocks such as institutional changes in prices, which positively affect core inflation measures and the permanent component of inflation. However, it needs to be kept in mind that the variance of shocks in each of the core inflation measures differs.

impact and then decreases gradually, with the speed of the decrease depending on the persistence of inflation in each of the core inflation measures. On the other hand, long-term inflation expectations rise gradually, driven by the permanent component, reaching a peak of +0.02 percentage points in the 9th quarter and gradually falling back to zero afterwards.

4.2 Credibility Shocks

As shown in Section 3, the autoregressive term related to credibility is close to unity, which implies that the credibility of the inflation target is sticky. In this environment, an exogenous shock to credibility will be long-lasting and could lead to an increase in actual inflation through a rise in long-term inflation expectations. In fact, as shown in Figure 10, which presents the response of long-term inflation expectations to a positive credibility shock, expectations rise to slightly below 0.1 percentage point. Due to this sharp increase in long-term inflation expectations, the CPI for all items less fresh food increases by up to 0.08 percentage points in the 15th quarter. These findings show that credibility shocks are highly persistent and relatively powerful.

4.3 Target Inflation Rate Shock

An increase in the target inflation rate affects core inflation through a change in long-term inflation expectations. Figure 11 shows the impulse responses to a permanent increase in the target inflation rate by 0.1 percentage point. The trend inflation rises by 0.08 percentage points on impact and continues its rise, reaching about 0.09 percentage points in the 12th quarter. Due to this increase, long-term inflation expectations increase by about 0.1 percentage point in the 12th quarter. The core inflation rates increase by 0.04-0.08 percentage points. These results indicate that the effects of an increase in the target inflation rate on core inflation are substantial and permanent due to the rise in long-term inflation expectations.

4.4 Long-term Inflation Expectations Shock

A shock to long-term inflation expectations has a direct effect on core inflation through the hybrid NKPC. Figure 12 shows the impulse responses to a 0.1 percentage point positive shock to long-term inflation expectations. Core inflation rates rise on impact, reaching their peak in the fourth quarter ranging from 0.01 to 0.04 percentage points. The effect then slowly falls back to zero depending on the persistence of inflation in each measure. The permanent component also rises following the actual rise in core inflation, reaching plus 0.02 percentage points in period 4, and gradually diminishes toward the 11th period as core inflation slows. However, the permanent component rises once again after the 11th period. Long-term inflation expectations rise gradually due to rises in the permanent component.

4.5 Output Gap Shock

Finally, we show the impulse responses to an output gap shock. The transmission process is as follows. An increase in the output gap changes core inflation through the NKPC, leading to an increase in the permanent component. This increase in the permanent component raises long-term inflation expectations. However, the size of the impact on inflation depends on the estimated slope parameter of the hybrid NKPC.[22]

Figure 13 shows the impulse responses to a positive output gap shock of 0.1 percent. It should be noted that in all of the exercises, the output gap follows an exogenous AR(2) process. All the core inflation measures rise on impact and reach a peak of 0.04 percentage points at most. The permanent component and long-term inflation expectations also rise, but the magnitude is very small at around 0.01 percentage points at most.

[22] Among the four core inflation measures, the CPI for all items less fresh food is the most responsive to the output gap. This is probably due to the high correlation between the change in exchange rates and the output gap, which we do not model in TIPS.

5 Conclusion

This paper investigated trend inflation, which is a key variable to understand the link between target inflation and long-term inflation expectations. To this end, we built a model to project trend inflation carefully taking into account the expectation formation mechanism and the credibility of the central bank's inflation target policy.

While the Trend Inflation Projection System (TIPS) presented here yields useful insight for the projection of inflation rates and various policy analyses, it is also important to keep the specific nature and limitations of the model in mind. For example, the model does not fully capture the mechanism of inflation expectations formation, nor does it address the determinants of the credibility of the central bank's inflation target. These necessary refinements of the model are left for future work.[23]

[23] For example, a possible direction for extending the analysis is to take the insights of Ehrmann (2015) into account. He investigates the expectations formation process in countries where the inflation rate is lower than the central bank's inflation target rate. He finds that inflation expectations are more affected by realized inflation rates when the inflation rate is below the target than when it is above. Another direction in which to refine our model would be to follow the example of Mertens (2015) and others and use a stochastic volatility model when extracting the permanent component from inflation. Yet another possible direction for extending the analysis would be to take into account the finding of Hattori et al. (2016) that inflation rates tend to approach those of the central bank. Given their finding, we could improve our model by explicitly incorporating interaction effects between the central bank's and private agents' forecasts to investigate further the central bank's role in the inflation expectations formation of private agents.

Appendix: Details of the Model and Estimation Methodology

A1: Beveridge-Nelson Decomposition of Core Inflation Measures

In this paper, following Mertens (2015) and Garnier et al. (2015), we use a state space model to conduct the Beveridge-Nelson decomposition (equation 3, 4 and 6). This Appendix explains the details of model and their estimation methodologies. We assume that n core inflation measures ($\pi_{it}(i = 1, ..., n)$) consist of an unobservable permanent (τ_t) and transitory component (z_{it}), and the transitory component follows a VAR(2) process. Thus we can write each core inflation measure, π_{it}, as follows (equation 3 and 4):

$$\pi_{it} = \tau_t + z_{it}, \text{ and}$$
$$\tau_t = \tau_{t-1} + \xi_t,$$

where we assume that ξ_t follows a normal distribution with variance σ_τ^2 and mean zero. Thus, using vector $\boldsymbol{\pi}_t = (\pi_{1t}, ..., \pi_{nt})'$ and $\mathbf{z}_t = (z_{1t}, ..., z_{nt})'$, we can rewrite the above equations as follows[24]:

$$\boldsymbol{\pi}_t = \mathbf{1}\tau_t + \mathbf{z}_t = A \cdot \mathbf{a}_t,$$

where

[24] While Mertens (2015) assumes that the initial values of the permanent factor are different from core inflation rates, we assume that the initial values are the same across different core inflation measures. Even if we allow initial values of the permanent component to be different among core inflation rates, the estimated dynamics of permanent components do not differ qualitatively.

$$\mathbf{a}_t = \begin{pmatrix} \tau_t \\ z_{1t} \\ \vdots \\ z_{nt} \\ z_{1t-1} \\ \vdots \\ z_{nt-1} \end{pmatrix} \text{,and } A = \begin{bmatrix} \mathbf{1} & I_n & \mathbf{0_{n\times n}} \end{bmatrix}.$$

We define $\mathbf{1}$, I_n, and $\mathbf{0_{k\times l}}$ as a $n \times 1$ vector with 1 for its element, a $n \times n$ identity matrix and a $k \times l$ zero matrix, respectively. The above equation is the observation equation of the state space model. The state equation can be written as follows:

$$\mathbf{a}_t = B \cdot \mathbf{a}_{t-1} + R \cdot \boldsymbol{\varepsilon}_{at},$$

where

$$B = \begin{bmatrix} 1 & \mathbf{0_{1\times 2n}} \\ \mathbf{0_{2n\times 1}} & b \end{bmatrix}, \tag{14}$$

$$b = \begin{bmatrix} b_1 & b_2 \\ I_n & 0_{n\times n} \end{bmatrix}, \tag{15}$$

$$R = \begin{bmatrix} \sigma_\tau & \mathbf{0_{1\times n}} \\ \boldsymbol{\rho}_\tau & \Sigma_z \\ \mathbf{0}_{n\times 1} & 0_{n\times n} \end{bmatrix}, \text{and} \tag{16}$$

$$\boldsymbol{\varepsilon}_{at} = \begin{pmatrix} \xi_t \\ \boldsymbol{\varepsilon}_{zt} \end{pmatrix}. \tag{17}$$

We define $n \times 1$ vector $\boldsymbol{\varepsilon}_{zt} = (\varepsilon_{1t}, ..., \varepsilon_{nt})'$ as idiosyncratic shocks to the transitory component and assume $\varepsilon_{jt} (j = 1, ..., n)$ to follow an independent standard normal distribution. Σ_z represents the standard deviation and correlation structure of transitory shocks and is assumed to be a diagonal matrix with $\sigma_{zi} (j = 1, \ldots, n)$ in its (j, j) element. In addition, we allow permanent component shocks to affect transitory components. An

$n \times 1$ vector $\boldsymbol{\rho}_\tau$ represents the correlations. Here b_k $(k = 1, 2)$ denotes a $n \times n$ coefficient matrix on the k-th lagged vector $(\mathbf{z}_{t-k})$ in the VAR model as follows (equation 6):

$$\mathbf{z}_t = \sum_{k=1}^{2} b_k \mathbf{z}_{t-k} + R_{-1} \boldsymbol{\varepsilon}_{at}, \tag{18}$$

where R_{-1} denotes a matrix representing the correlation structure of shocks in the state equation, which is obtained by dropping the first row from the matrix R.

To estimate the model, we use a Gibbs Sampler of the MCMC method (see Table A-1 and Figure A-1 for more details on priors and posteriors). More concretely, we conducted the following procedure:

(1) Given initial values (or sampled values in the previous steps), draw a sample for state variable vector $\mathbf{a}_t$.

(2) Given other parameter values, draw a sample of coefficients (b_k) on lagged variables in the VAR model for transitory components. In this step, following Cogley and Sargent (2005), we conducted rejection sampling so that the VAR system will be stationary.

(3) Likewise, given other parameter values, draw a sample of the correlation parameter, $\boldsymbol{\rho}_\tau$, between permanent component shock and transitory components from the posterior.

(4) Given other parameter values, draw a sample of variance of transitory component shocks, σ_{zi}^2, from the posterior.

(5) Finally, draw a sample of variance of permanent component shock, σ_τ^2.

(6) Repeat step (1) to (5) for 12,000 times and discard the first 6,000. Calculate the estimators from the last 6,000 samples.

The reason for discarding first 6,000 samples in step (6) is to exclude the effects of initial values on posterior estimation. In step 6, we check the convergence of the MCMC chain by plotting the samples. We also implemented the test proposed by Geweke (1992), where we examined whether two estimated parameters' values based on the first and second half of samples are the same.

The posterior densities are shown in Figure A-1 and A-2. The 90% credible intervals indicate that the correlations between the transitory component disturbance terms and the permanent component shock are significantly negative. The coefficients on lagged variables in the VAR model for the transitory component were significantly positive for the one-lagged variables and not significantly different from zero for the two-lagged variables.

A2: Estimation Method for the Long-term Inflation Expectation model

We use Kalman filter and a Gibbs-Sampler of the MCMC method to estimate the long-term inflation expectations model (equation 2, 12 and 13). The priors for the parameters in the model are shown in Table A-1. More concretely, I estimated the model as follows:

(1) Given other parameter values, draw a sample of δ, which is related to the credibility of the inflation target. In the estimation, 6-to-10 year-ahead inflation expectations from the Consensus Forecast are used as a proxy of the long-term inflation expectations. However, before 2013, the survey was conducted only twice a year. Therefore we assume that the long-term inflation expectation as an unobservable state variable. To impose the restriction of $0 < \delta < 1$ on δ, we conducted a rejection sampling.

(2) Likewise, draw a sample of autoregressive parameters of the long-term inflation expectation, α, and the coefficient on the 6-year-ahead expected output gap, β.

(3) Draw a sample of the autoregressive parameter, ω, of the credibility measure parameter, δ. Here to guarantee the condition of $0 < \omega < 1$, we conducted a rejection sampling.

(4) Draw a sample of the coefficient on the dummy variable, κ, which represents a jump of δ.

(5) Draw a sample of variance of forecast errors for the long-term inflation expectation. Then, draw a sample of variance of measurement errors of the long-term inflation expectation.

(6) Repeat the steps from (1) to (5) for 6,000 times. Discard the first 3,000 sample and use the last 3,000 samples to calculate the estimators.

We check the convergence of MCMC chain in the same way as in the case of the permanent component extraction model.

The posterior densities are shown in Figure A-2. Based on the 90% credible intervals, the estimated coefficient on 6-year-ahead expected output gap, β, is not significantly different from zero. Also the coefficient on the dummy variable, κ, is not significant although most of the samples are positive. Other parameters are estimated to be significantly different from zero.

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Table 1: Estimation Results of Long-term Inflation Expectations

$$\pi^e_{t,6\sim10Y} = \alpha\, \pi^e_{t-1,6\sim10Y} + (1-\alpha)\cdot \bar{\pi}_t + \beta\, y^e_{t,6Y} + v_t$$

$\pi^e_{6\sim10Y}$: 6-to-10-year-ahead inflation expectations, $\bar{\pi}$: trend inflation rate,
y^e_{6Y} : 6-year-ahead expected output gap, v : disturbance term
Sample period: Q1 1990~Q1 2016
Estimation method: MCMC

$$\bar{\pi}_t = \delta_t \cdot \tau_{t-1} + (1-\delta_t)\pi^*_t$$

τ : permanent component, $1-\delta$: credibility measure of the inflation target,
π^* : target inflatation rate(=1(Q1 1990~Q4 2012) or 2 (Q1 2013~Q1 2016))

$$\delta_t = \omega \cdot \delta_{t-1} + \kappa \cdot D_t + \mu_t$$

D : dummy variable $= \begin{cases} 1 \text{ if } t = \text{ Q1 2013} \\ 0 \text{ otherwise} \end{cases}$,
μ : disturbance terrm

(a) Using Four Core Inflation Measures

Parameter	Posterior Median	90% Credible Interval
α	0.80	(0.71, 0.88)
β	0.09	(-0.40, 0.59)
ω	0.97	(0.94, 0.99)
κ	0.11	(-0.05, 0.25)

Log Marginal Likelihood	-34.81

(b) Using CPI (all Items less fresh food)

Parameter	Posterior Median	90% Credible Interval
α	0.79	(0.70, 0.87)
β	0.08	(-0.43, 0.60)
ω	0.97	(0.93, 0.99)
κ	0.12	(-0.03, 0.28)

Log Marginal Likelihood	-36.25

Table 2: Estimation Results of the New Keynesian Phillips Curve

$$\pi_t = \rho\pi_{t-1} + (1-\rho)\pi^e_{t,6\sim10Y} + \gamma\, y_t + \epsilon_t$$

π : inflation rate, $\pi^e_{6\sim10Y}$: 6-to-10-year-ahead inflation expectation,
y : output gap, ϵ : distrubance term
Sample period : Q1 1990~Q1 2016
Estimation method : maximum likelihood

○ Estimated Parameters

ρ

Model	Estimated Value	90% Confidence Interval
CPI (all items less food and energy)	0.89	(0.82, 0.96)
CPI (all items less fresh food)	0.77	(0.66, 0.88)
CPI (all items less fresh food and energy)	0.81	(0.72, 0.90)

γ

Model	Estimated Value	90% Confidence Interval
CPI (all items less food and energy)	0.05	(0.02, 0.09)
CPI (all items less fresh food)	0.09	(0.02, 0.15)
CPI (all items less fresh food and energy)	0.08	(0.03, 0.13)

○ Model Performance

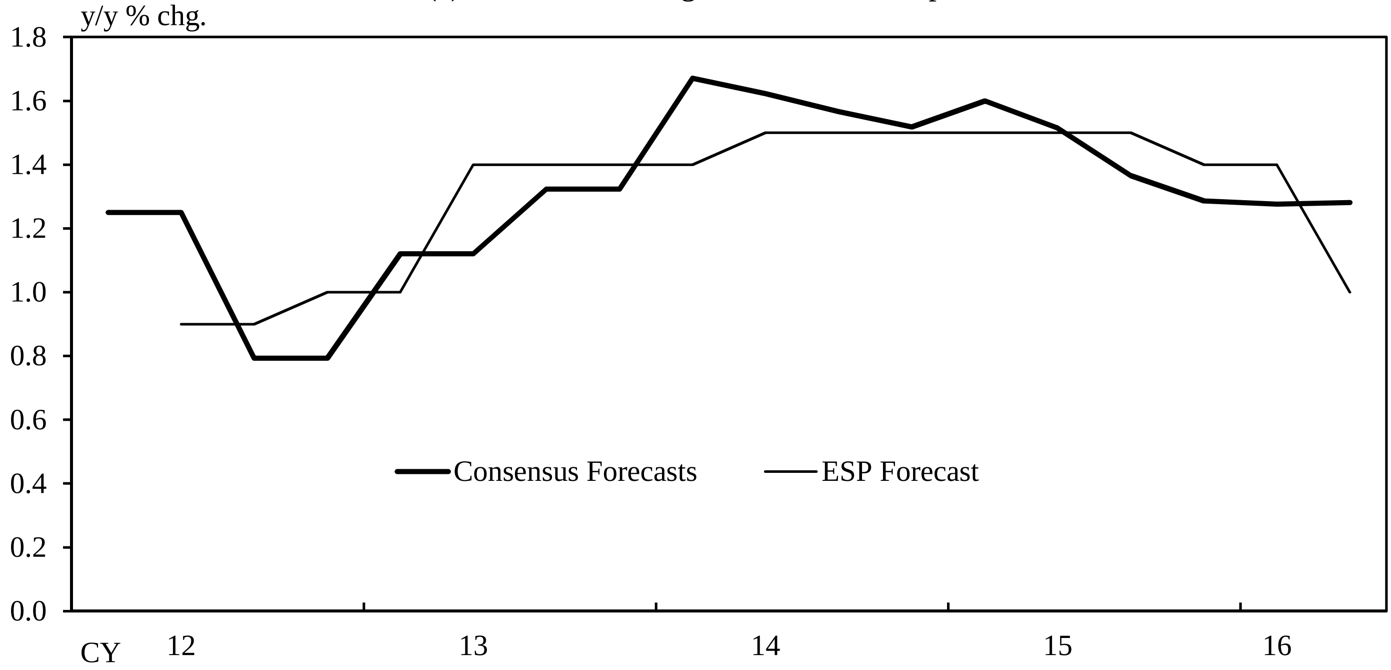
Model	Log Likelihood	Standard Error
CPI (all items less food and energy)	-88.9	0.57
CPI (all items less fresh food)	-123	0.79
CPI (all items less fresh food and energy)	-102.2	0.64

Table 3: Comparison of the Performance of the Models

	Four core inflation measures	Single CPI	Difference (A - B)
Log marginal likelihood	-34.81	-36.25	1.44

Figure 1: Inflation Expectations

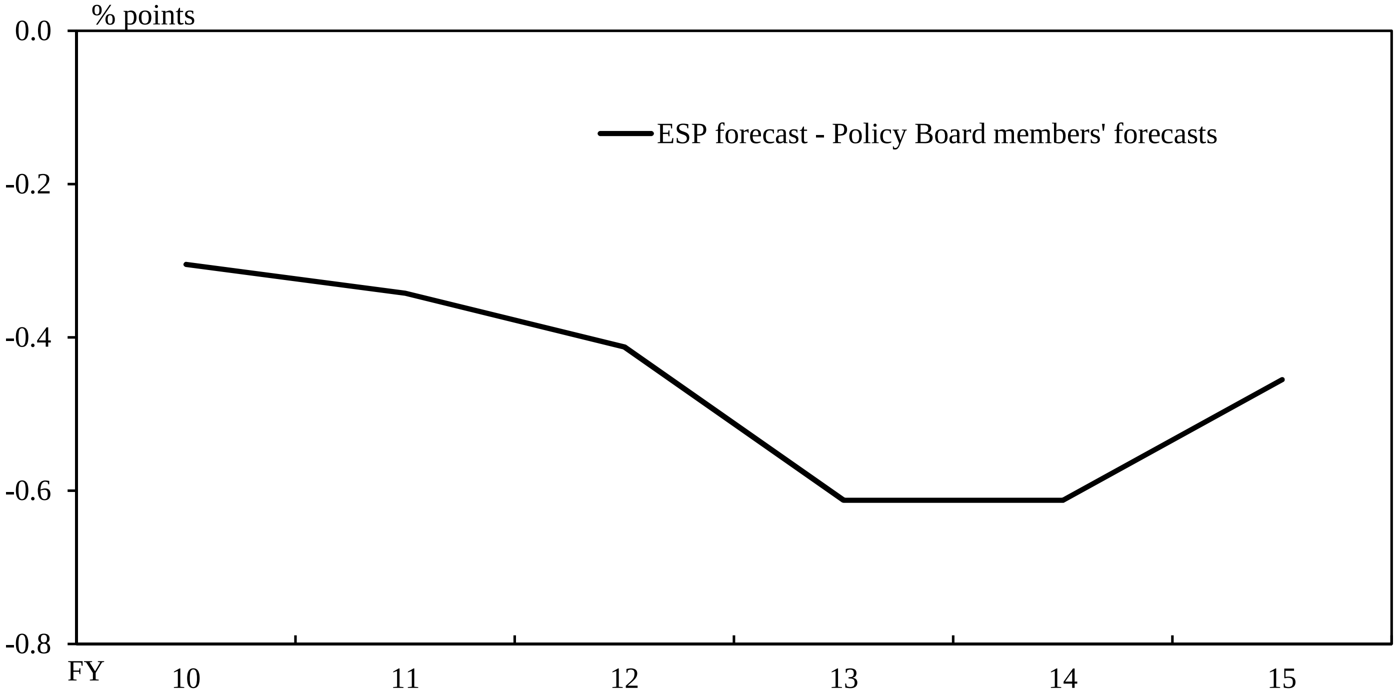
(a) Medium- to Long-term Inflation Expectations



Note: Consensus Forecasts indicates 6-to-10-year-ahead inflation expectations and ESP forecast indicates 7-to-11-year-ahead expectations.

Sources: Consensus Economics Inc., "Consensus Forecasts"; JCER, "ESP Forecast."

(b) Gap between Private Forecasters' and BOJ Policy Board Members' Inflation Forecasts



Notes: 1. The Policy Board members' forecast indicate the median of the CPI (all items less fresh food) forecasts (point estimates). The effects of the consumpution tax hikes are excluded.

2. The gap indicates the average value of the differences of one-year-ahead inflation forecasts in April, July, Octorber and January of each fiscal year.

Sources: JCER, "ESP Forecast"; Bank of Japan.

Figure 2: Overview of TIPS

Three building blocks consisting TIPS:
(i) core inflation (NKPC), (ii) long-term inflation expectations, and (iii) trend inflation.

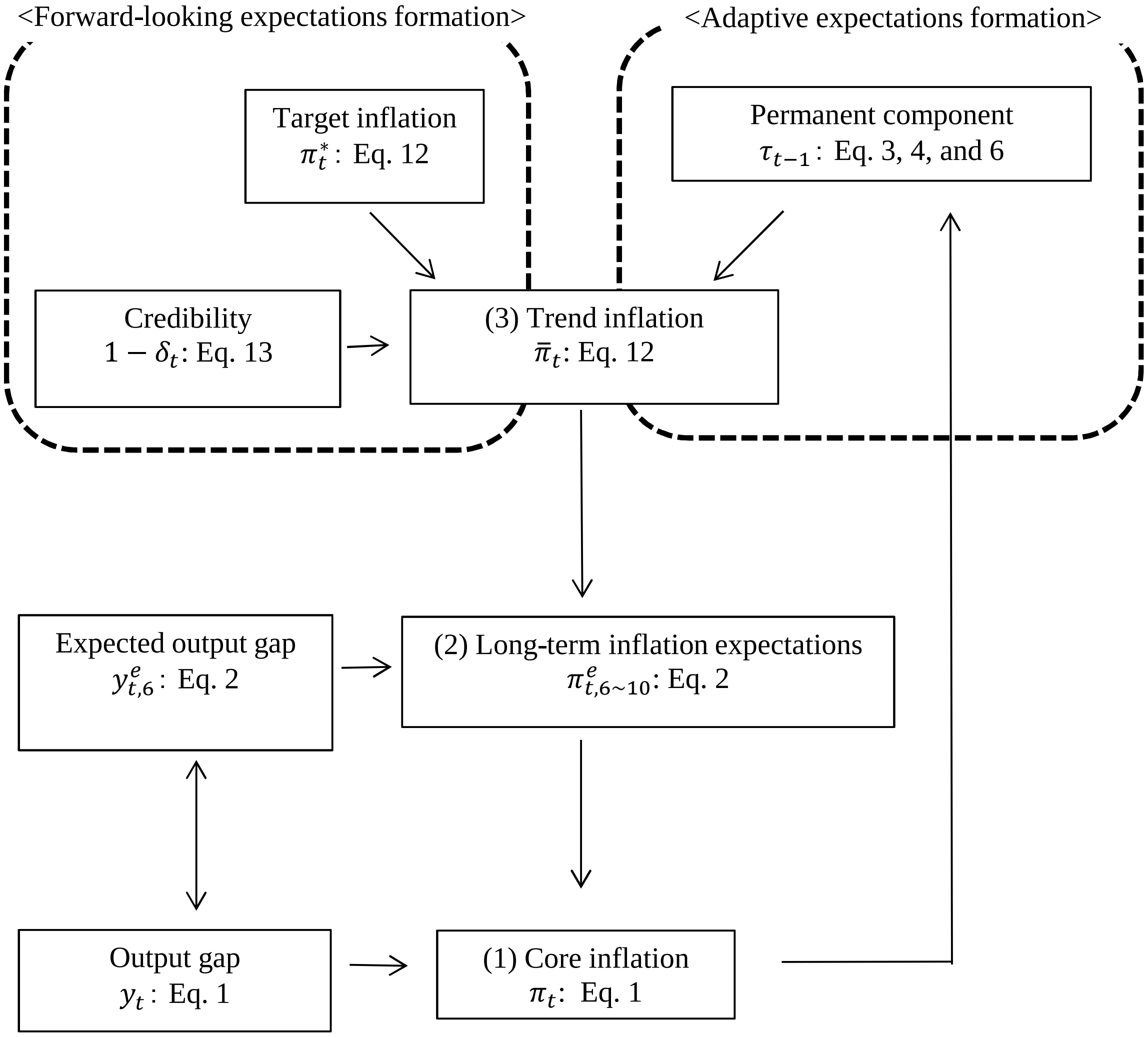
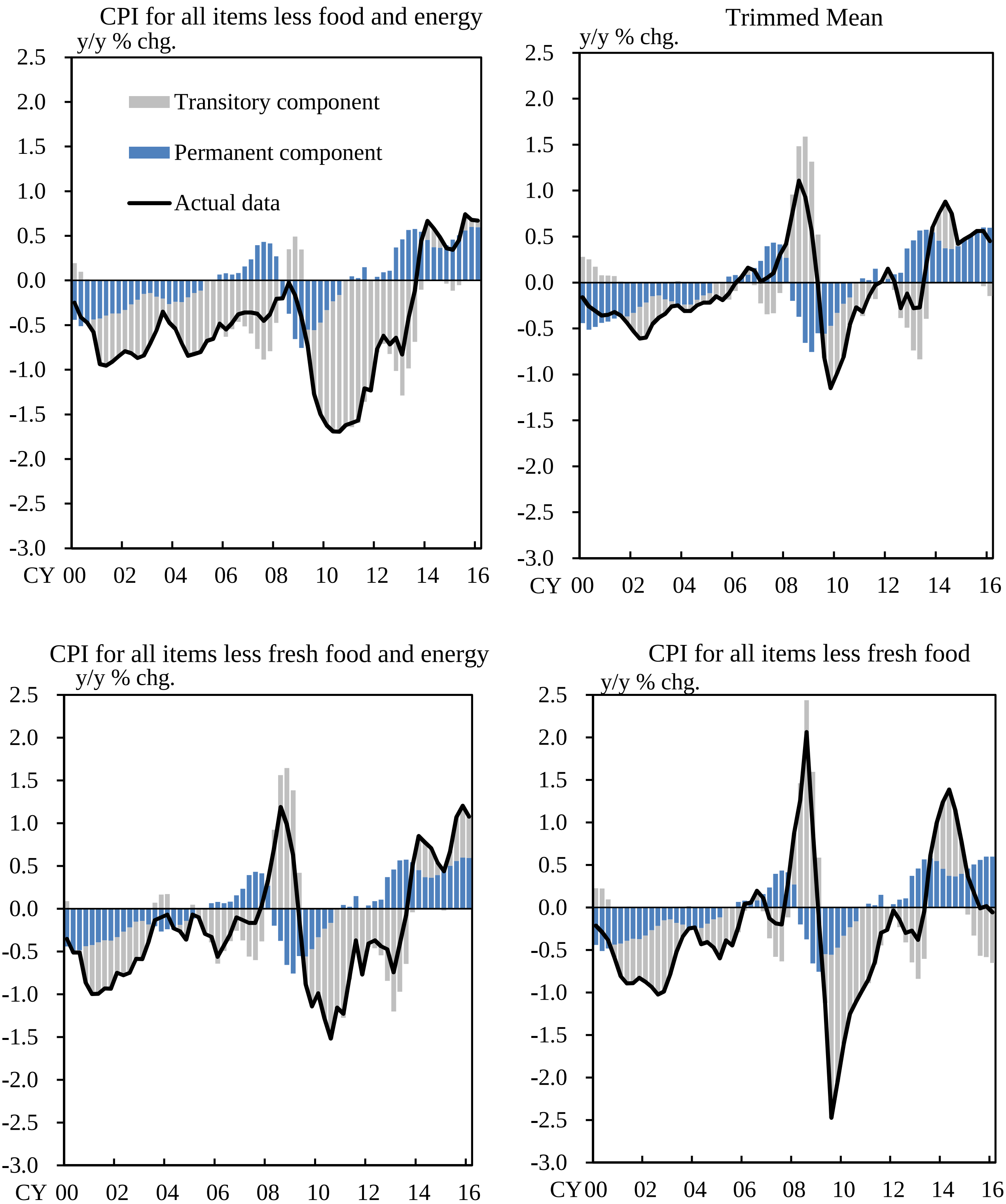


Figure 3: Decomposition of Four Core Inflation Measures



Notes: 1. Each core inflation rate is shown as the moving average of annualized inflation rates of quarter-to-quarter changes in the seasonally adjusted series.

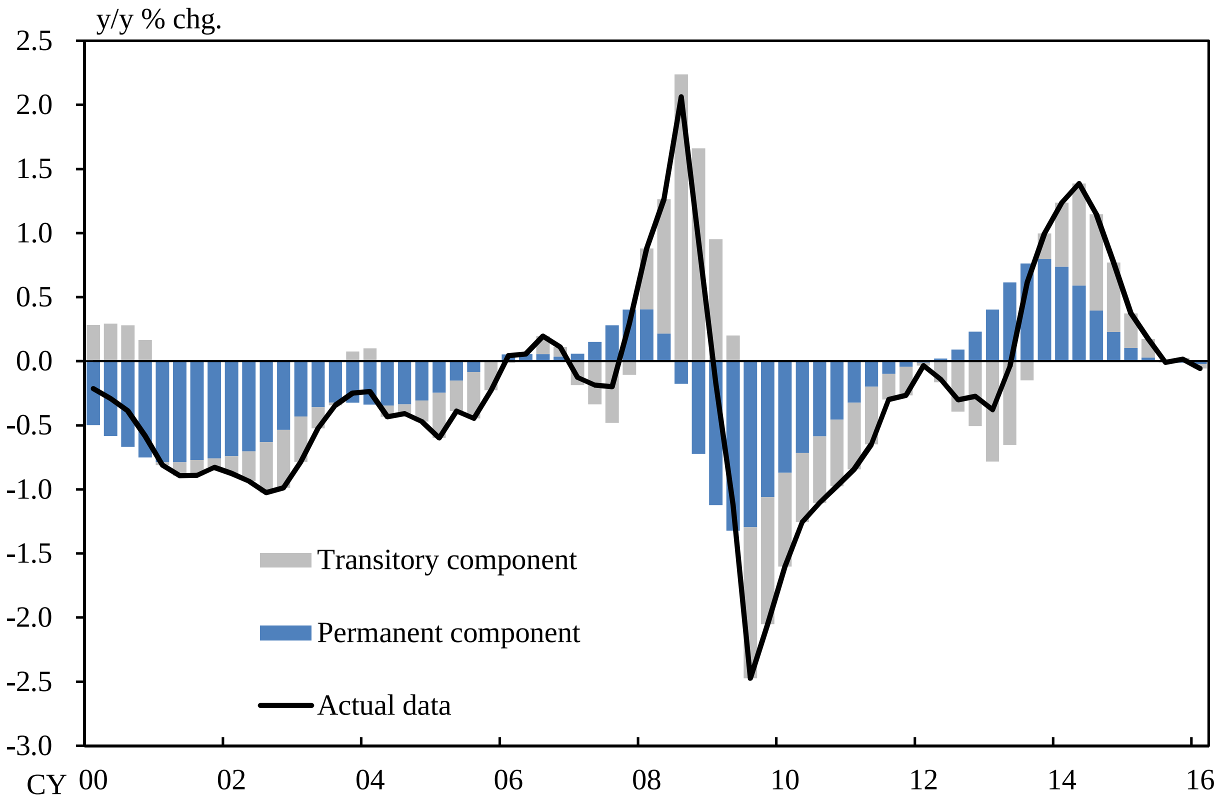
2. Figures for the CPI are adjusted to exclude the estimated effects of changes in the consumption tax rate.

3. The trimmed mean is the weighted average of the year-over-year price changes in individual items making up the CPI. Items are arranged in ascending order of their year-over-year rate of price change and those falling into the upper and lower 10 percent tails by weight are trimmed.

4. Figures for the CPI (less fresh food and energy) in this paper are calculated by the Research and Statistics Department, Bank of Japan.

Source: Ministry of Internal Affairs and Communications.

Figure 4: Decomposition of CPI (all items less fresh food) Using the Single CPI



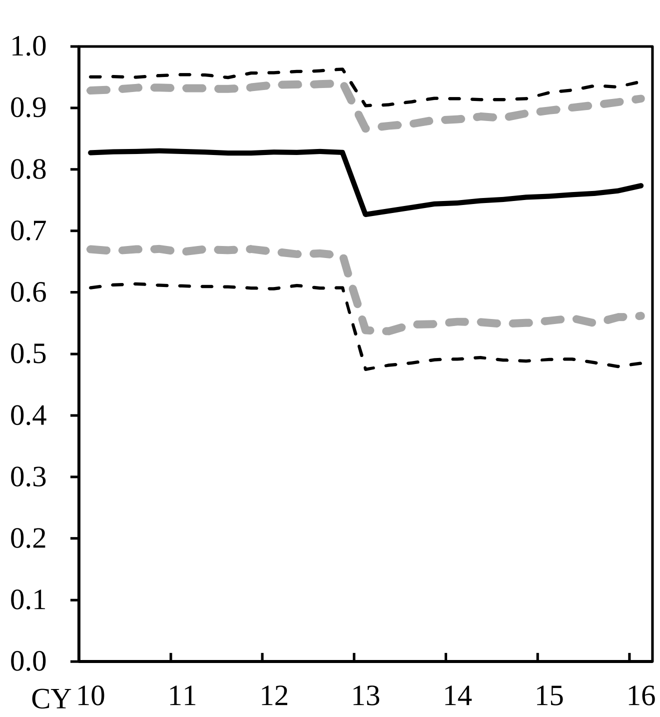
Notes: 1. The change is calculated as the moving average of annualized inflation rates of quarter-to-quarter changes in the seasonally adjusted series.
2. Figures for the CPI are adjusted to exclude the estimated effects of changes in the consumption tax rate.

Source: Ministry of Internal Affairs and Communications.

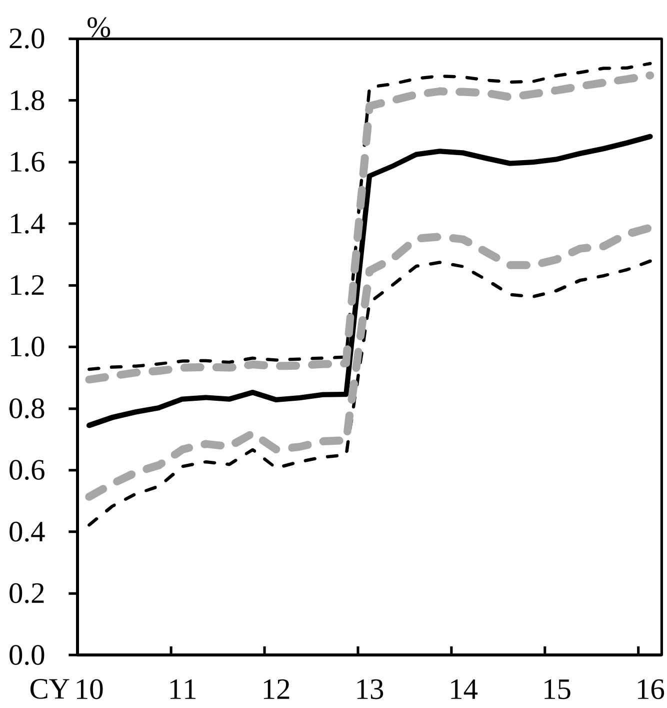
Figure 5: Estimation Result of the Trend Inflation Model

(a) Using Four Core Inflation Measures

Credibility :1-δ

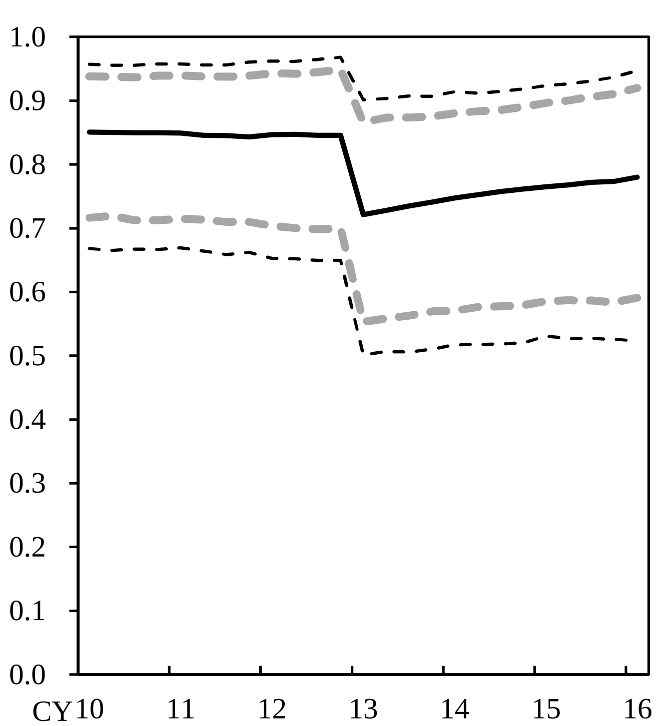


Trend Inflation

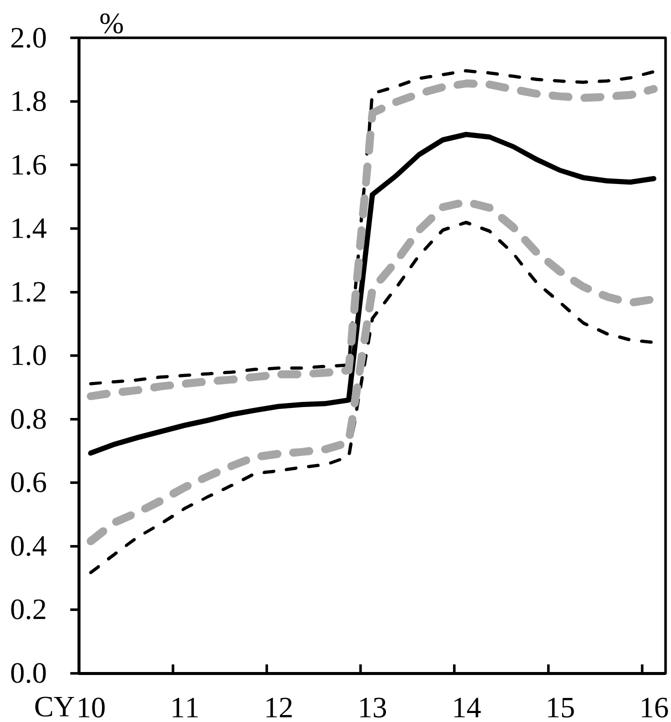


(b) Using CPI (all items less fresh food)

Credibility : 1-δ

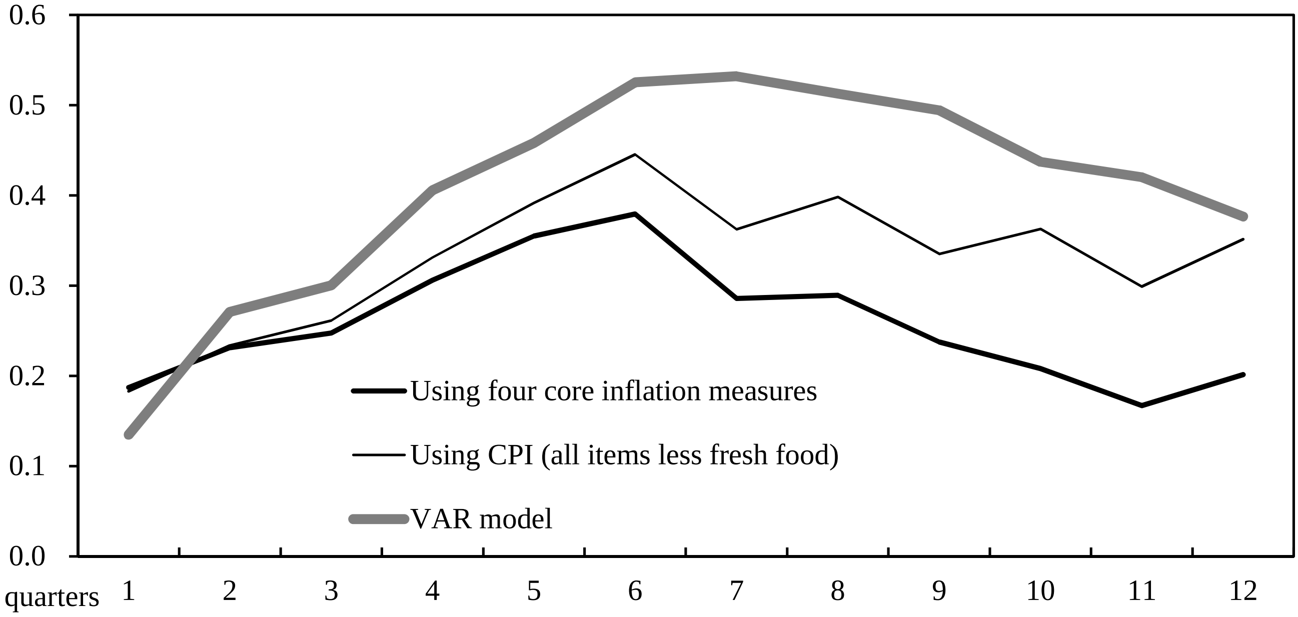


Trend Inflation



Note: The black solid line is the posterior median. The gray bold dotted line and black dotted line indicate 80% and 90% credible intervals, respectively.

Figure 6: Root Mean Squared Errors for the Medium- to Long-term Inflation Expectations

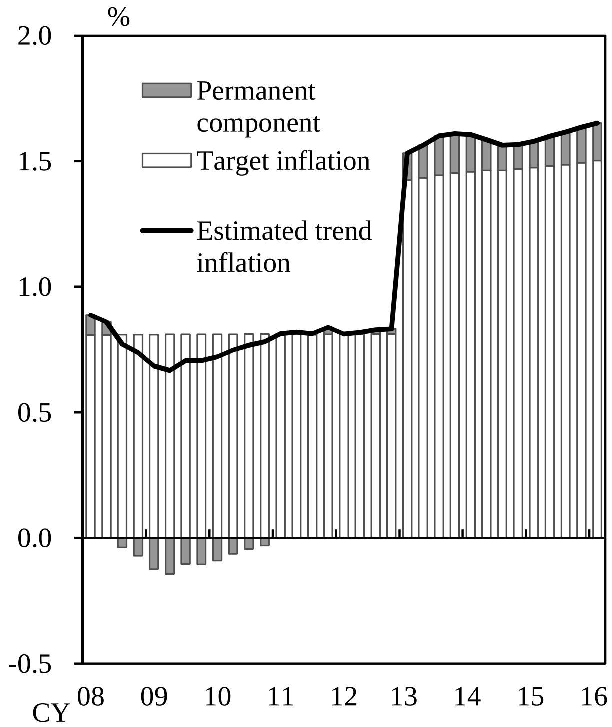


Notes: 1. Out of sample forecast errors are calculated by extending the end of the estimation sample period quarter by quarter from Q4 2010 for each forecast where the initial period of the sample is fixed at Q1 1990.

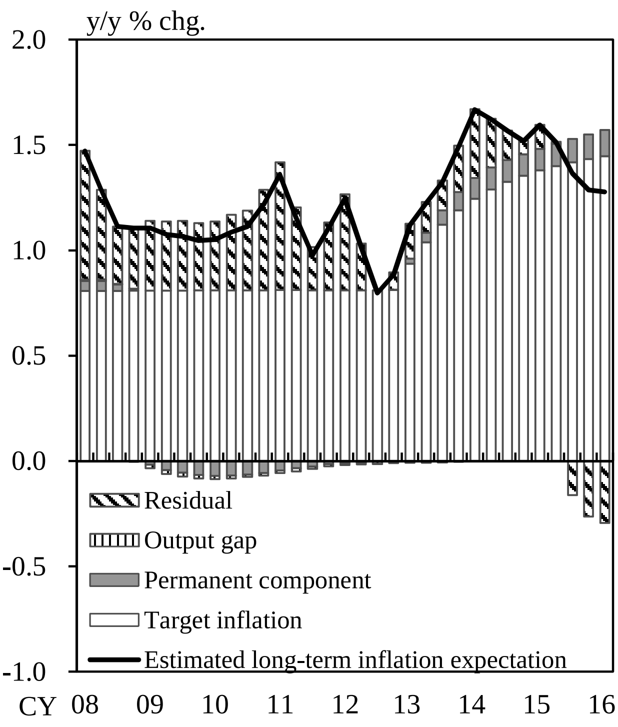
2. The 6 variable VAR model includes four core inflation indices, output gap, and the medium- to long-term inflation expecations. We choose five lags in the VAR model based on the AIC.

Figure 7: Decomposition Using Four Core Inflation Measures

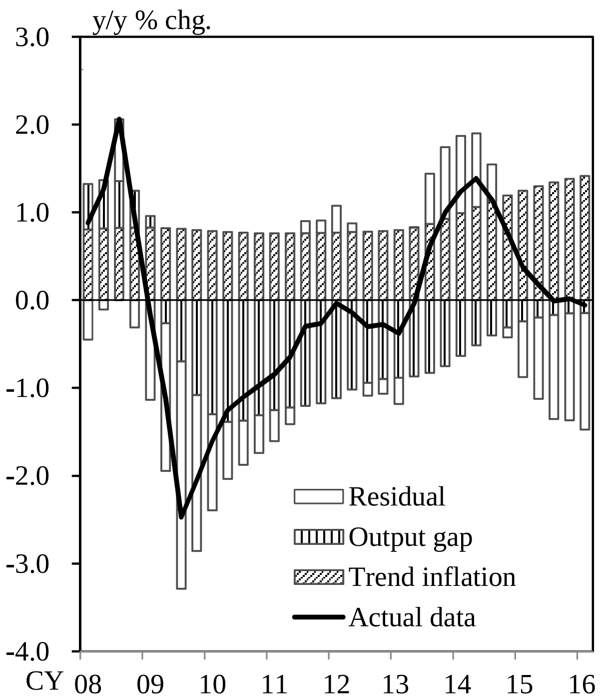
(a) Trend Inflation



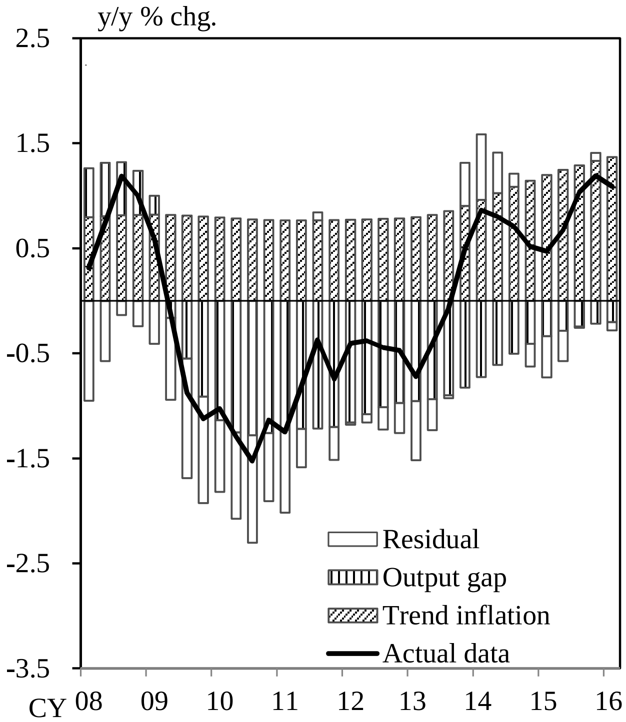
(b) Long-term Inflation Expectation



(c) CPI (all items less fresh food)



(d) CPI (all items less fresh food and energy)



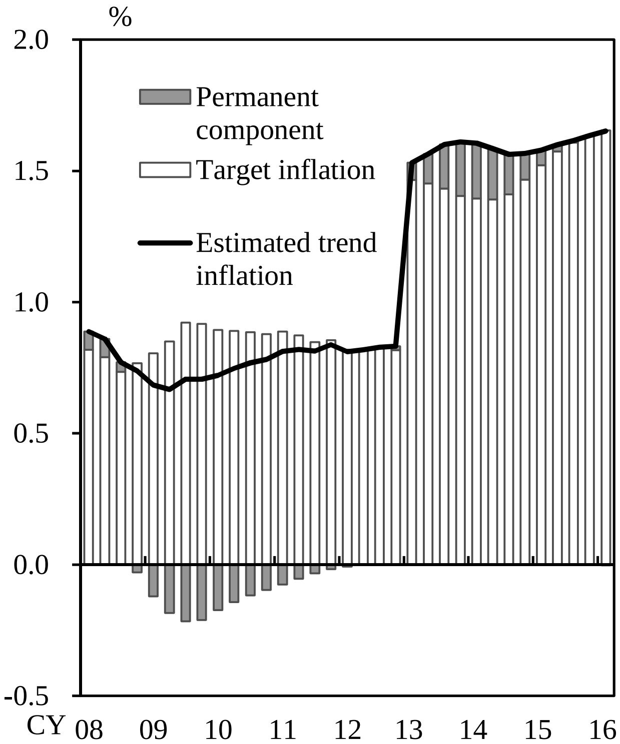
Notes: 1. Actual data indicates a four-quarter moving avearge of the annualized quarter-to-quarter changes in the seasonally adjusted series.

2. Figures for the CPI are adjusted to exclude the estimated effects of changes in the consumption tax rate.

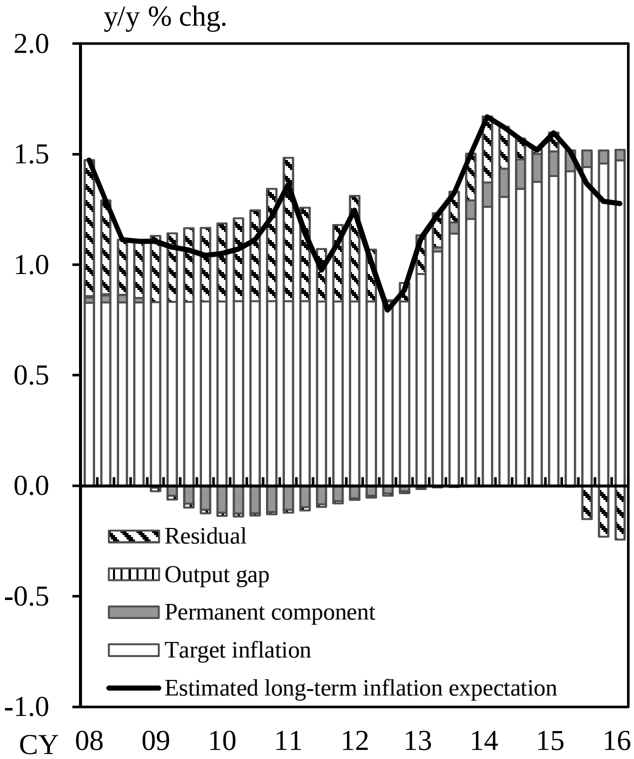
3. In the estimation, 6-to-10-year-ahead inflation expectations are regarded as observables and differences from the model-implied values are regarded as measurement errors

Figure 8: Decomposition Using CPI (all items less fresh food)

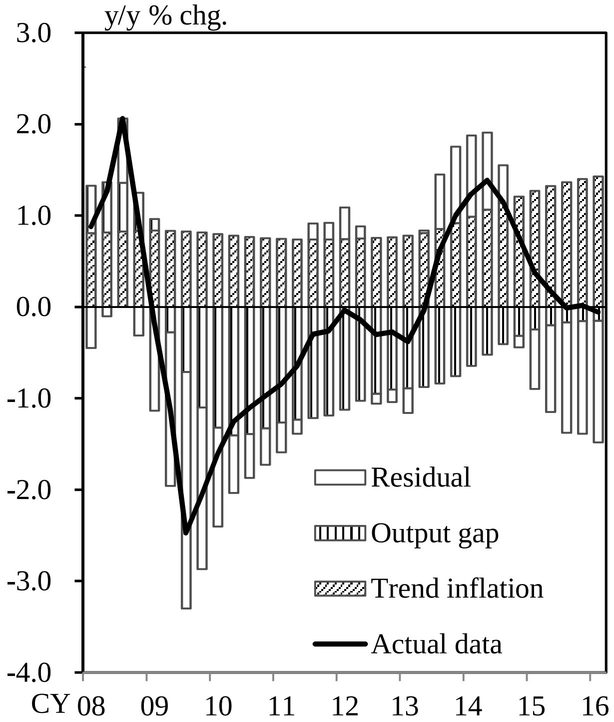
(a) Trend Inflation



(b) Long-term Inflation Expectation

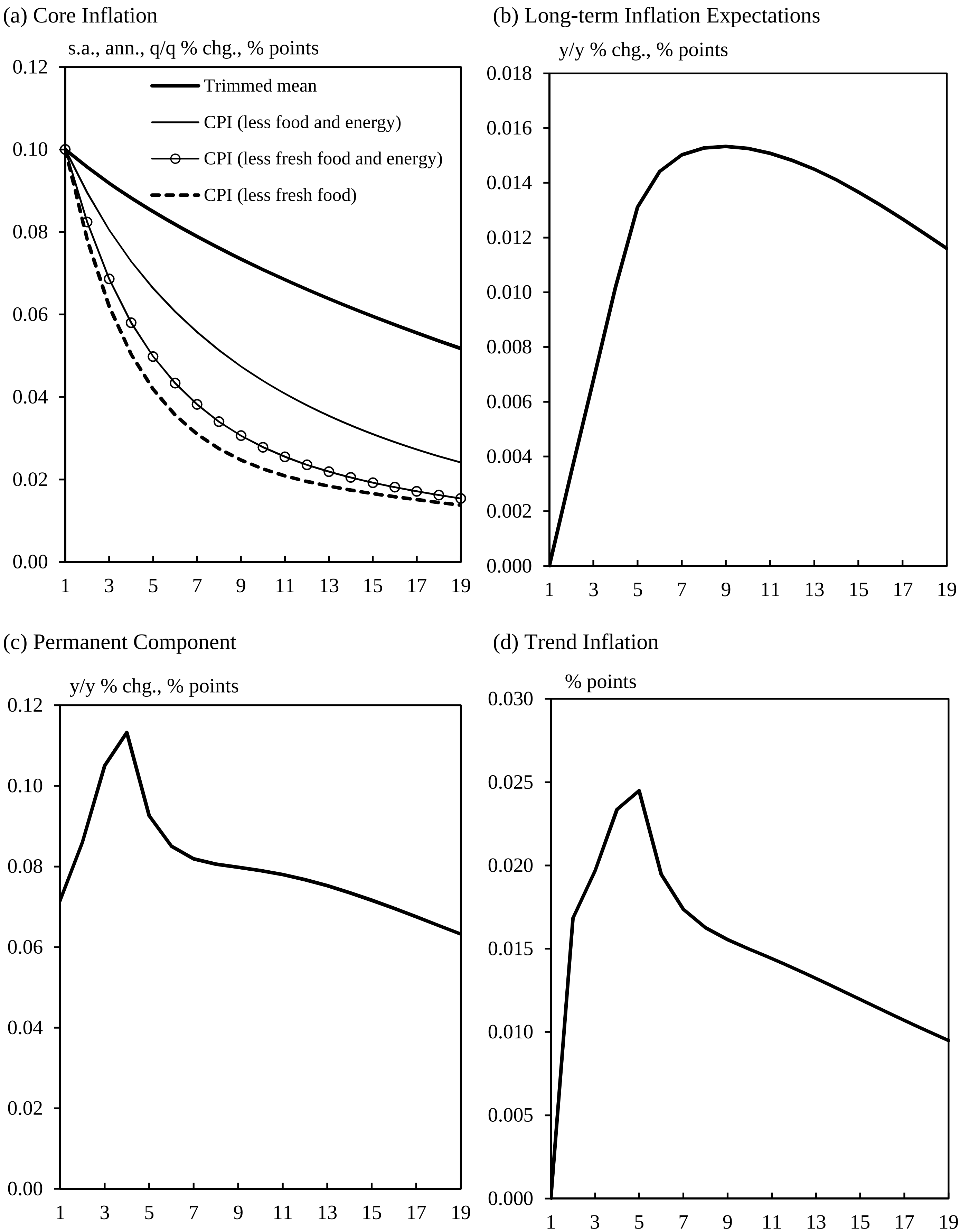


(c) CPI (all items less fresh food)



Notes: 1. Actual data indicates a 4-quarter moving avearge of the annualized quarter-toquarter changes in the seasonally adjusted series.
2. Figures for the CPI are adjusted to exclude the estimated effects of changes in the consumption tax rate.
3. In the estimation, 6-to-10-year-ahead inflation expectations are regarded as observables and differences from the model-implied values are regarded as measurement errors

Figure 9: Impulse Responses to Core Inflation Shocks

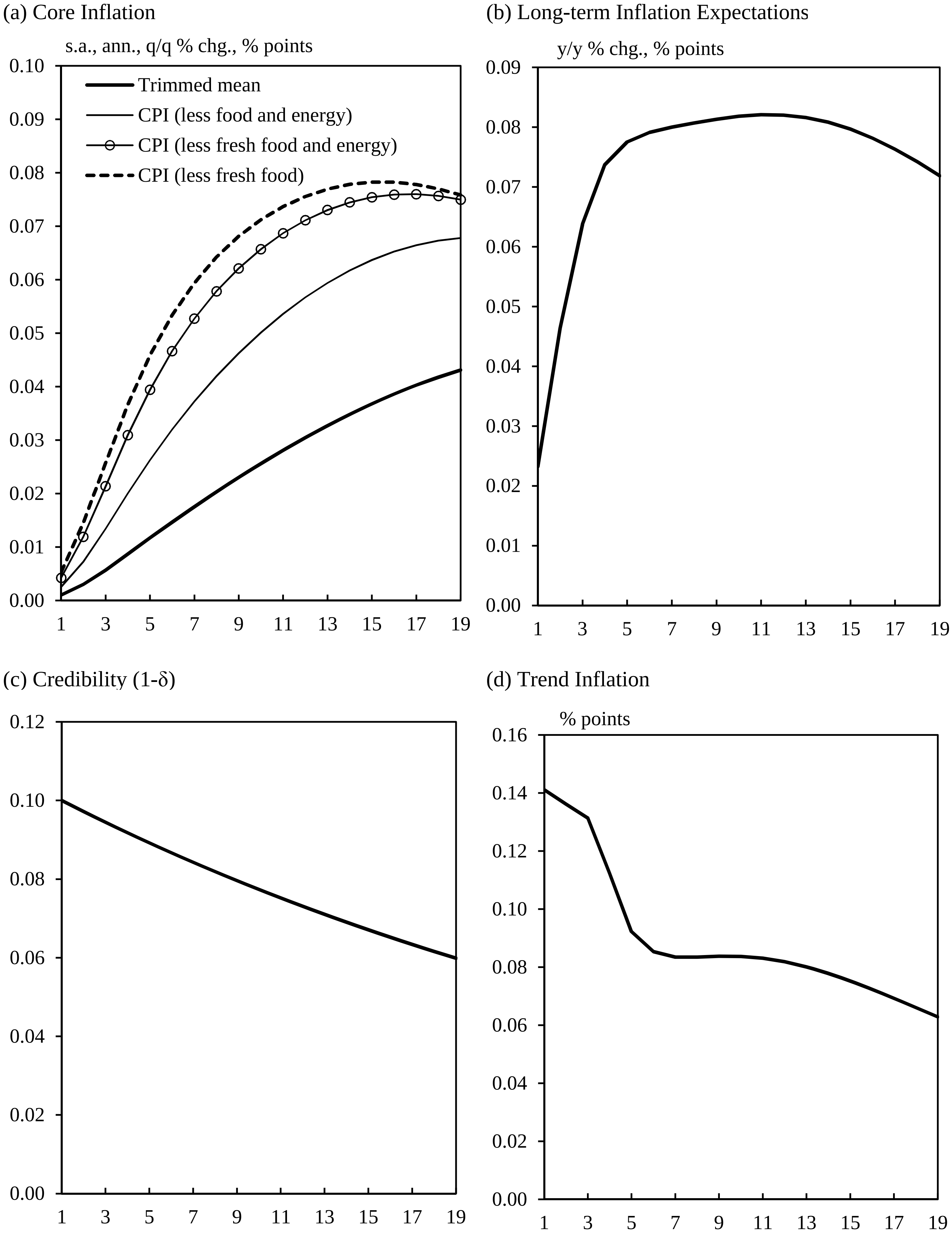


Notes: 1. Responses indicate the difference between the baseline forecast and forecasts when a core inflation rate increases by 0.1% points exogenously in the second quarter of 2016.

2. The period of the shock is set to period 1 in the x-axis and shows the elapsed time in quarters.

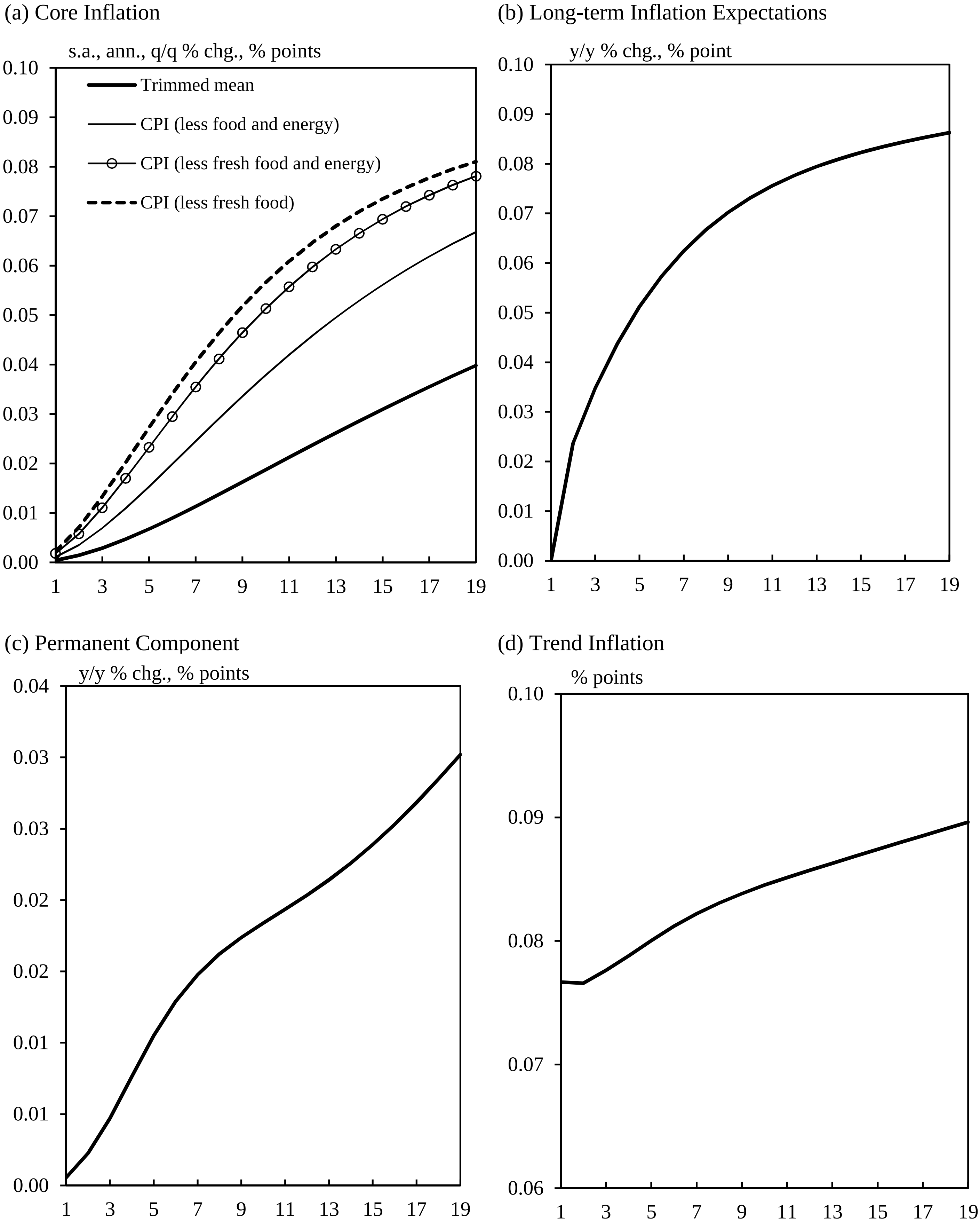
3. Trimmed mean is calculated as the year-on-year change.

Figure 10: Impulse Response to a Credibility Shock (1-δ)



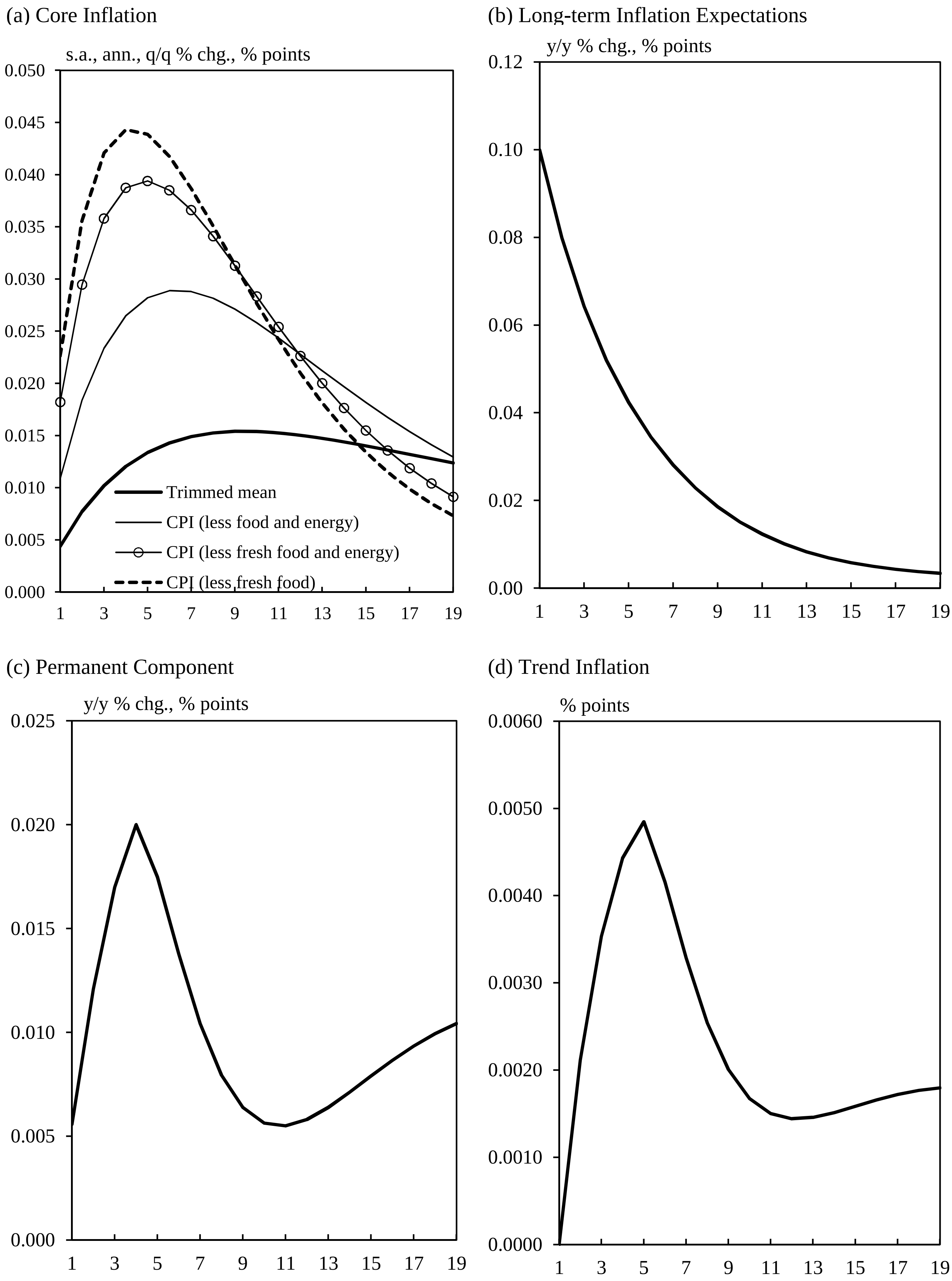
Notes: 1. Responses indicate the difference between the baseline forecast and forecasts when the credibility (1-δ) increases by 0.1 exogenously in the second quarter of 2016.
2. The period of the shock is set to period 1 in the x-axis and shows the elapsed time in quarters.
3. Trimmed mean is calculated as the year-on-year change.

Figure 11: Impulse Responses to a Target Inflation Shock



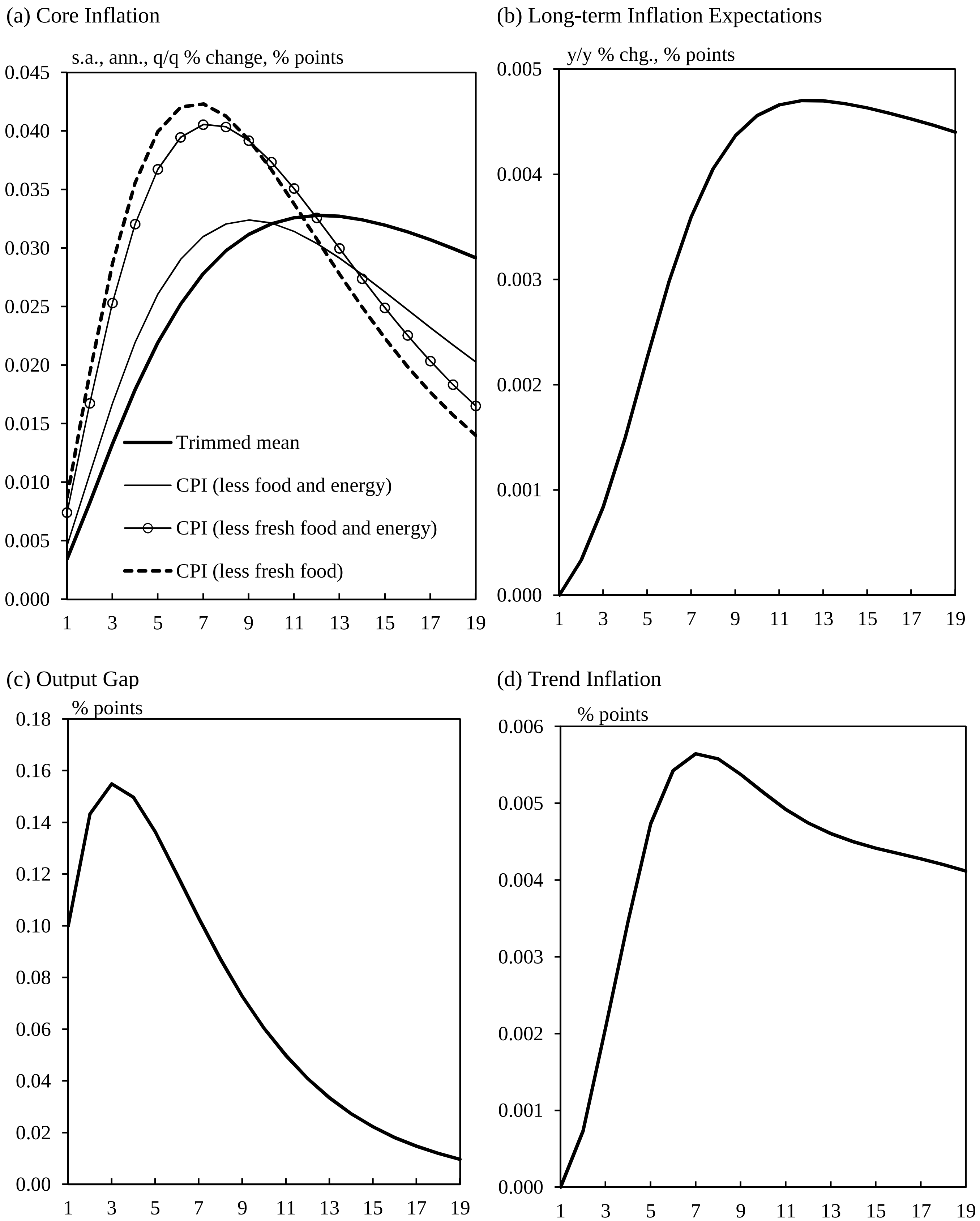
Notes: 1. Responses indicate the difference between a baseline forecast and the forecast when the target inflation rate increased by 0.1% points exogenously in the second quarter of 2016.
2. The period of the shock is set to period 1 in the x-axis and shows the elapsed time in quarters.
3. Trimmed mean is calculated as the year-on-year change.

Figure 12: Impulse Responses to a Long-term Inflation Expectations Shock



Notes: 1. Responses indicate the difference between the baseline forecast and forecasts when the long-term inflation expectation rate increases by 0.1% points exogenously in the second quarter of 2016.
2. The period of the shock is set to period 1 in the x-axis and shows the elapsed time in quarters.
3. Trimmed mean is calculated as the year-on-year change.

Figure 13: Impulse Responses to an Output Gap Shock



Notes: 1. Responses indicate the difference between the baseline forecast and forecasts when the output gap increases by 0.1% points exogenously in the second quarter of 2016.
2. The period of the shock is set to period 1 in the x-axis and shows the elapsed time in quarters.
3. Trimmed mean is calculated as the year-on-year change.

Table A-1: Priors for Estimated Parameters

Permanent Component Extraction Model

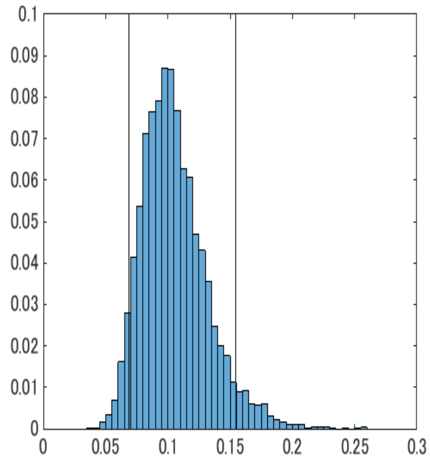
		Mean	Variance
Normal Distribution	ρ_τ : Correlation between permanent component shock and transitory component disturbance term	0	40
	b_1: One-period lagged variables in the VAR model of transitory components	0.5	0.02
	b_2: Two-period lagged variables in the VAR model of transitory components	0	0.02
		Scale	Shape
Inverse Gamma Distribution	σ_{zi}: Variance of transitory component shock	5	2.5
	σ_τ: Variance of permanent component shock	2.5	1

Long-term Inflation Expectation Model

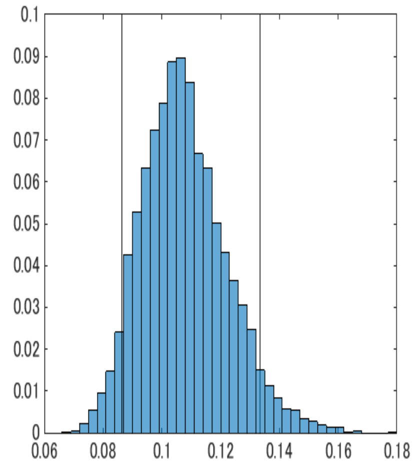
		Mean	Variance
Normal Distribution	α : Lagged term of the long-term expected inflations	0.5	0.01
	β : 6-year-ahead expected output gap	0	0.1
	ω : AR(1) parameter of δ	0.9	0.01
	κ : Coefficient on the dummy variable for δ	0.1	0.01
		Scale	Shape
Inverse Gamma Distribution	Variance of forecast errors for the long-term inflation expectations	0.015	3
	Variance of measurement errors for the long-term inflation expectations	0.0025	3
	Variance of shocks to δ	0.005	3

Figure A-1-1: Posterior Distributions for Extraction of Permanent Components Model

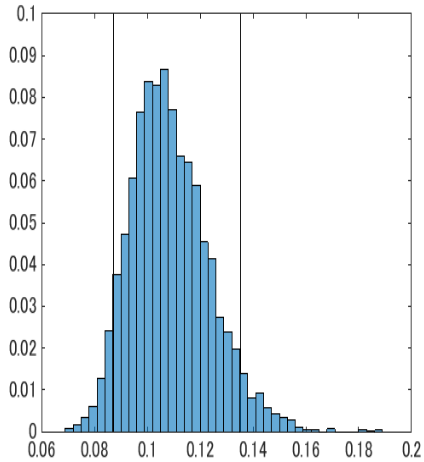
Variance of Permanent Shock



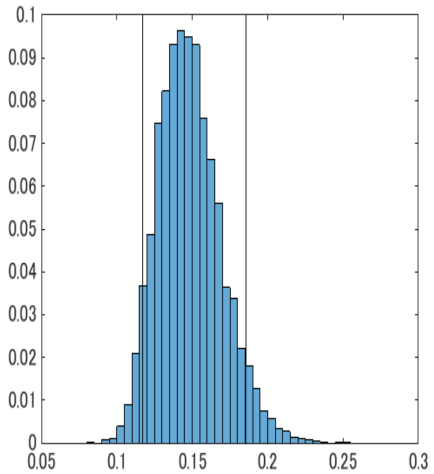
Variance of Transitory Shock
(Trimmed Mean)



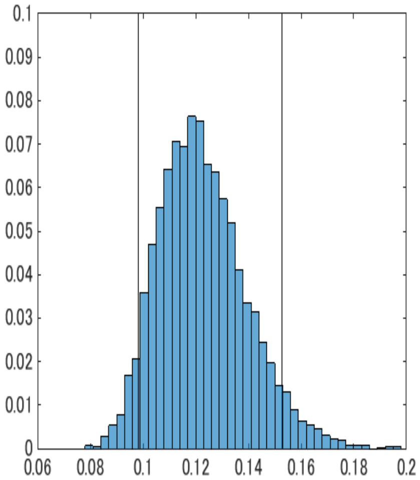
Variance of Transitory Shock
(less food and energy)



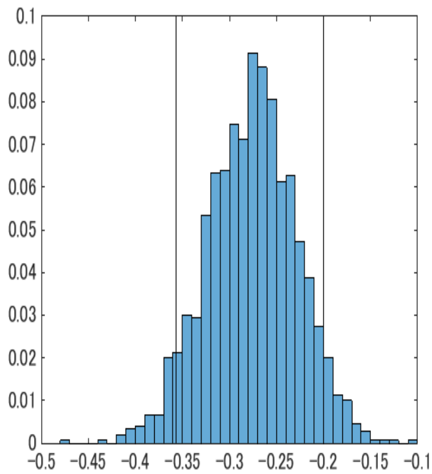
Variance of Transitory Shock
(less fresh food)



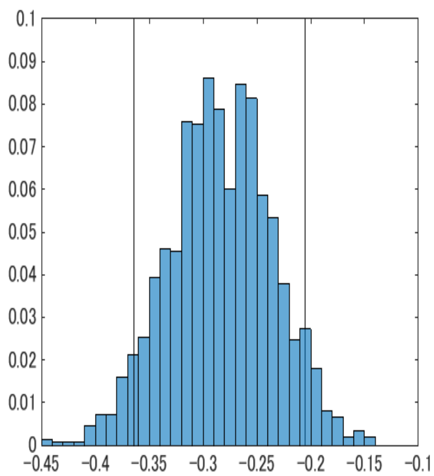
Variance of Transitory Shock
(less fresh food and energy)



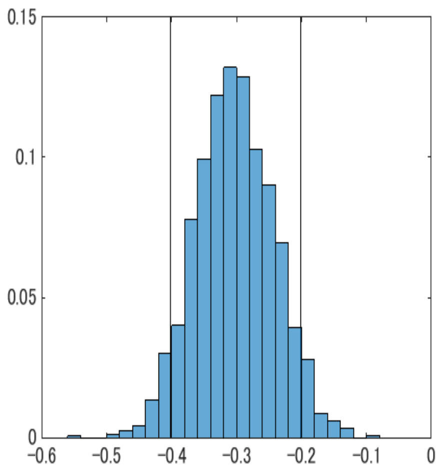
Correlation between Permanent Shock
and Transitory Component Disturbance
（trimmed mean）



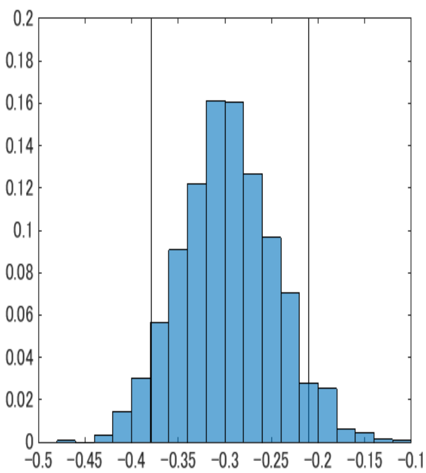
Correlation between Permanent Shock
and Transitory Component
Disturbance（less food and energy）



Correlation between Permanent Shock
and Transitory Component Disturbance
（less fresh food）



Correlation between Permanent Shock
and Transitory Component
Disturbance（less fresh food and energy）



Note: Vertical lines indicate 90% credible intervals.

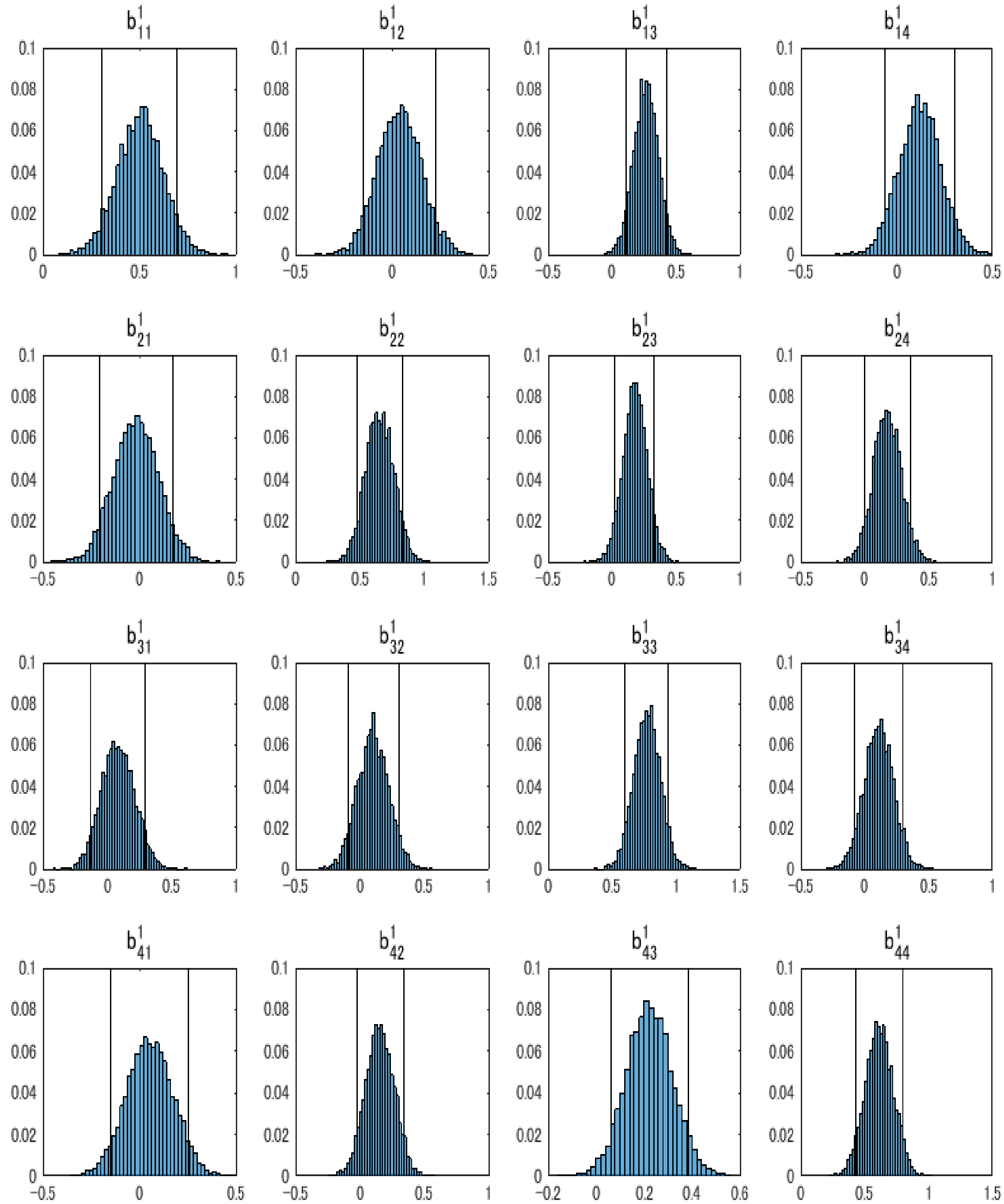
Figure A-1-2: Posterior Distributions for Extraction of Permanent Components Model

Transitory component:

$$z_t = \sum_{k=1}^{2} b_k z_{t-k} + R_{-1}\varepsilon_{at}.$$

z is defined as follows:
$z =$ (trimmed mean, less food and erngy, less fresh food, less fresh food and energy)' .

b_{ij}^k indicates (i,j) element of the coefficient matrix b_k.



Note: Vertical lines indicate 90% credible intervals.

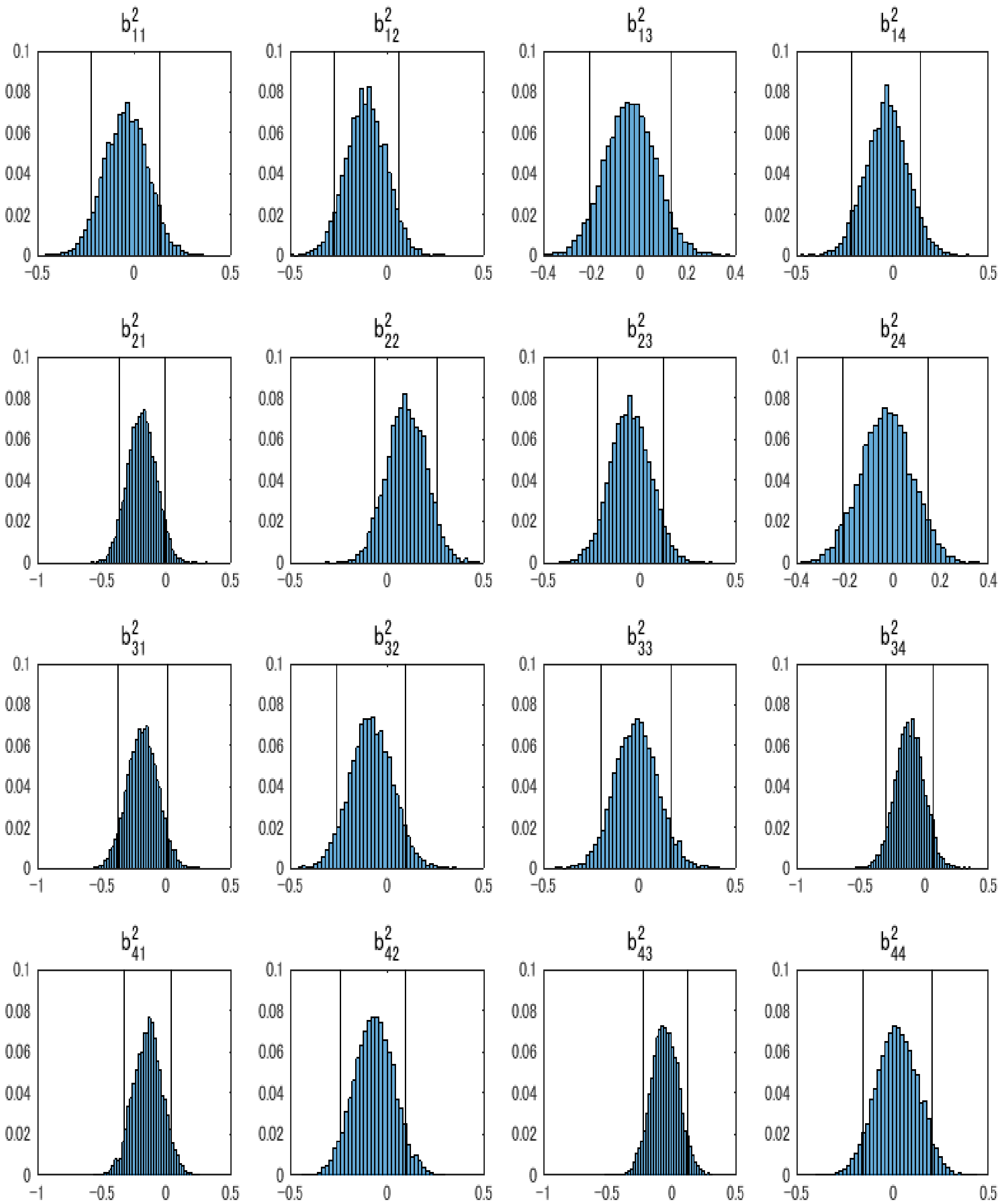
Figure A-1-3: Posterior Distributions for Extraction of Permanent Components Model

Transitory component:

$$z_t = \sum_{k=1}^{2} b_k z_{t-k} + R_{-1}\varepsilon_{at}.$$

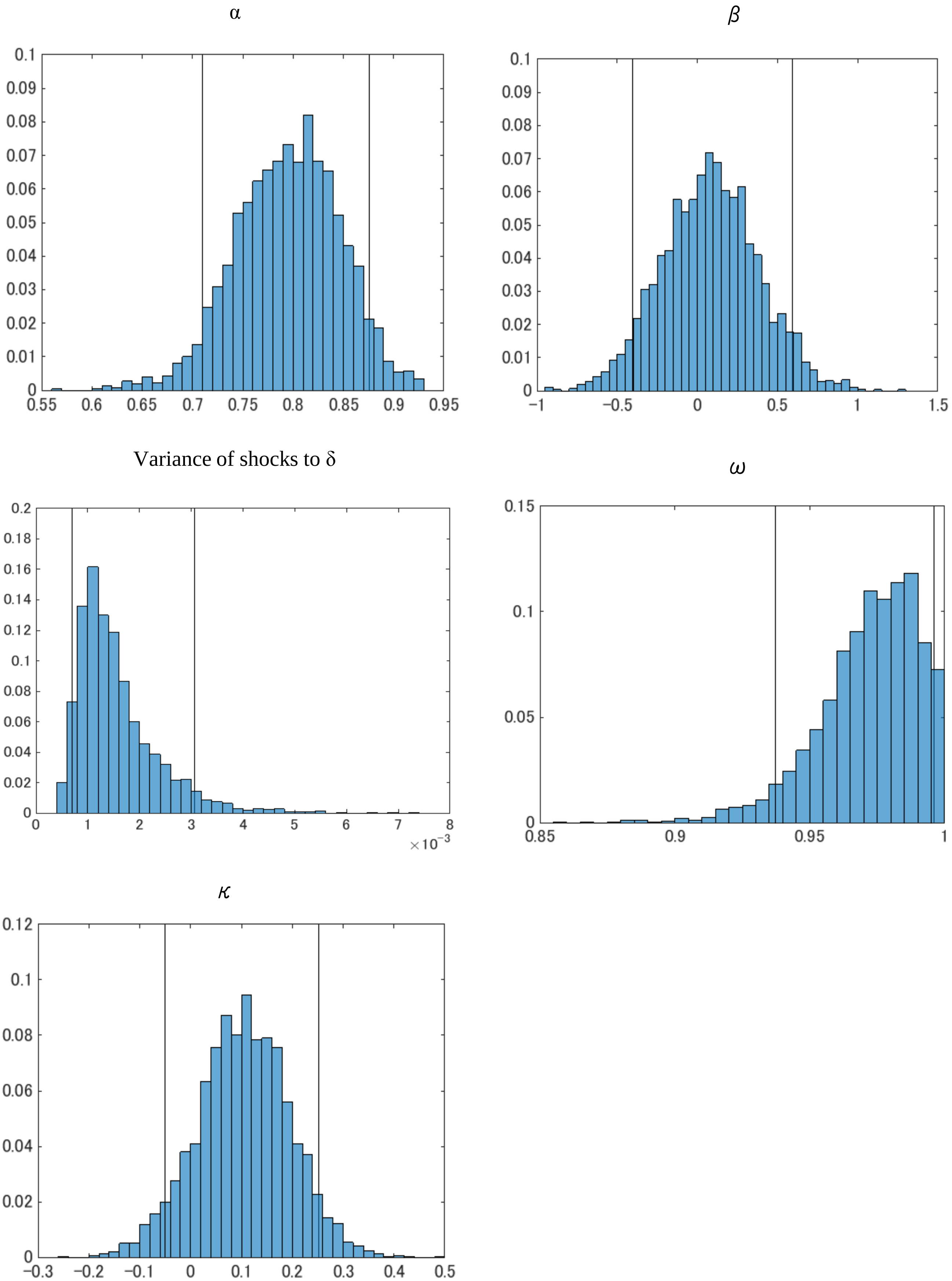
z is difined as follows:
z=(trimmed mean, less food and erngy, less fresh food,less fresh food and energy)' .

b_{ij}^k indicates (i,j) element of the coefficient matrix b_k.



Note: Vertical lines indicate 90% credible intervals.

Figure A-2: Posterior Distributions for Long-term Inflation Expectation Model



Note: Vertical lines indicate 90% credible intervals.