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Risk-Taking, Inequality and Output in the Long-Run

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Abstract

We develop a tractable dynamic general equilibrium model with incomplete markets for business risk sharing, which allows for analytical characterization under Epstein-Zin preference with unitary elasticity of intertemporal substitution and Cobb-Douglas technology. Household stationary wealth dispersion is shown to follow a Pareto distribution. In this environment, we conduct comparative statics of stationary output and household inequality when the cost of business risk sharing is reduced. Enhanced risk-taking results in greater long-run outputs and real wage and a lower risk-free interest rate, while its impact on inequality is ambiguous. A quantitative analysis under the parameter values calibrated to Japanese economy shows that elimination of purchase costs for mutual funds leads to an increase in output by 1.3 percent, a decrease in risk-free rate by 15 basis points, and an increase in Gini coefficient of wealth in 2 percentage points.

Keywords: Financial development, risk-free rate, safe asset, Pareto distribution, depository institutions, mutual funds

JEL classification code: E2, G2

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1 Introduction

The Japanese government has promoted a policy agenda of "from savings to asset building" (and its predecessor, "from savings to investment") policy. In an effort to promote economic growth in a rapidly aging society, the main purposes of this agenda are to motivate Japanese households, who have historically held most of their assets in savings deposits, to move into long-term investment in risky equities, as well as to lighten the burden on the public pension system (Financial Services Agency, 2016). The policy efforts include the introduction of defined contribution individual pension plans (DC) and tax incentives for risky portfolios (NISA). These policies are expected to decrease the cost of investment in risky equities and thus induce households to shoulder a larger share of business risk.

In this paper, we investigate the macroeconomic consequences of these policies. We are especially concerned with the quantitative effects of these policies on long-run output and household inequality. By encouraging households to embrace a portfolio which takes on more risk, the policy allows businesses to enjoy easier access to risky funds. The risks borne by households also result in the dispersion of ex-post wealth among ex-ante identical households. The policy also affects financial intermediaries, such as banks, which transform risky assets into risk-less deposits that meet the demand from households for safe assets. Hence, the policy also creates side-effects resulting from a risk-free rate. A general equilibrium model is necessary to evaluate the overall effects of this policy.

To facilitate the analysis, we develop a tractable Bewley-type heterogenous-agent dynamic general equilibrium model which incorporates incomplete markets for risk-sharing. In the model, households can hold three kinds of assets: (i) individual stock, that is, the shares of a firm, (ii) a mutual fund composed of a finite number of individual stocks, and (iii) risk-free asset. The risk-free asset is provided by financial intermediaries which pool a continuum of individual stocks, as well as by the government who issues bonds. The individual stocks conceptually capture individual stock as well as entrepreneurial investment in self-employment. The individual stock is a high-risk, high-return asset, while the mutual fund is a middle risk, middle-return asset and the risk-free asset is a zero-risk, low-return asset. In the model, the returns are different because transaction costs are incurred when households buy the mutual fund or risk-free asset.

Using the model, we analyze the effect of a policy change in which the transaction cost of the mutual fund is decreased. There are two main channels influencing the inequality of households. The first is a portfolio shift from the individual stock to the mutual fund. By decreasing the volatility of household wealth, this channel contributes to the decrease in inequality. The second main channel is a portfolio shift from the risk-free asset to the mutual fund. This channel contributes to the increase in inequality. We conduct quantitative analysis to clarify which channel has a stronger effect and whether inequality widens or shrinks as a result of the policy change. Moreover, we analyze whether the reduction in the transaction cost of the mutual fund promotes physical investment and increases output.

This paper's contribution to existing literature is three-fold. First, we provide a formal general equilibrium analysis of the limited participation in stock markets of Japanese households and the policy agenda which favors household risk-taking. While much has been discussed on this policy issue, no study has employed a dynamic general equilibrium framework to our knowledge. Rather than modeling the limited participation per se as in Attanasio and Paiella (2011) who focused on reconciling consumer theory and asset pricing, we introduce a rate-of-return cost to account for the average risk-taking bahevior in a tractable Bewley model such as in Angeletos and Calvet (2006), and explore general equilibrium implications of the financial costs.

Second, this paper explores to what extent financial developments affect the income and wealth inequality in a Bewley economy. On one hand, studies such as Nirei and Souma (2007), Benhabib et al. (2011), and Jones (2015) have highlighted the importance of rate-of-return risks in accounting for the right tail of income and wealth distributions. On the other hand, authors such as Acemoglu et al. (2006) and Aoki et al. (2017) note that as an economy approaches the world technology frontier, growth requires risky innovation activities. Such an argument lies in the background of the risk-taking policy agenda in the context of Japanese economy. This paper quantitatively assesses the policy's impact on inequality among households.

The third point touches on the broader discussion of how financial developments have affected a risk-free rate. In our paper, we keep banks' intermediary technology unchanged, and ask what happens when the transaction costs of direct risk-taking by households are reduced. The output is increased, in accordance with Obstfeld (1994) who argued that world financial developments enhanced risk-taking through diversification of rate-of-return risks and spurred growth. As a side-effect, we find that the risk-free rate is lowered. This suggests a mechanism in which world financial developments contribute to lowering the risk-free rate, if the banking technology remains unchanged. Moreover, the same mechanism suggests that an economy with lower costs on direct risk-taking exhibits lower demand for risk-free assets. Our analysis quantifies these effects of financial transaction costs on the risk-free rate and safe-asset demand.

The rest of the paper is organized as follows. Section 2 presents the dynamic general equilibrium model. Section 3 characterizes a stationary equilibrium analytically. Section 4 investigates one of the impacts of reduced costs for risk sharing quantitatively. Finally, Section 5 concludes.

2 Model

2.1 Production

The production side of the model is a simplified version of that of Aoki and Nirei (2017). Time t is continuous. There is a continuum of firms indexed by $e \in [0, E]$. Each firm owns physical capital $k_{e,t}$, employs labor $l_{e,t}$, and produces a differentiated good $y_{e,t}$. The production function of the firm is

$$y_{e,t} = z_{e,t} k_{e,t}^{\alpha} l_{e,t}^{1-\alpha},$$

where $z_{e,t}$ is the productivity of the firm. The productivity $z_{e,t}$ follows a geometric Brownian motion

$$dz_{e,t} = \mu_z z_{e,t} dt + \sigma_z z_{e,t} dB_{e,t}.$$

We assume that productivity shocks are idiosyncratic. Thus, the Wiener process $dB_{e,t}$ is uncorrelated with $dB_{e',t}$ for $e' \neq e$.

The firm issues a share at price $q_{e,t}$. Shareholders receive the profit as dividends $d_{e,t}$. The dividend consists of

$$d_{e,t} \equiv (p_{e,t} y_{e,t} - w_t l_{e,t} - \delta k_{e,t}) dt - dk_{e,t},$$

where $p_{e,t}$ is the price of the good the firm produces, w_t is the wage rate, and δ is the capital depreciation rate. The return received by a household who directly holds a unit of the firm's shares can be written as

$$((1 - \xi_q) d_{e,t} + dq_{e,t}) / q_{e,t} = \mu_{q,t} dt + \sigma_{q,t} dB_{e,t}, \tag{1}$$

where ξ_q is the tax rate imposed on the household and where $\mu_{q,t}$ and $\sigma_{q,t}$ are endogenous parameters, which turn out to be independent of the characteristics of firm e.

In order to introduce a mutual fund in this model, we consider that each firm belongs to a group of a finite (countable) number of firms called "neighbors." Each group of firms consists of $n+1$ firms, and thus the measure of neighborhoods is $1/(n+1)$. The set of firms which firm e belongs to, excluding e itself, is denoted by a set $\mathcal{N}_e$. A worker who works at firm e can purchase firm e's shares as well as the shares of other firms in neighborhood $\mathcal{N}_e$. While the worker does not pay a transaction cost when he purchases firm e's shares, he has to pay transaction cost τ_m per dividend $d_{e',t}$ ($e' \in \mathcal{N}_e$) and participation cost $\tau_p q_{e',t}$ per share when he purchases shares of neighbor firms in $\mathcal{N}_e$. We specify that workers purchase equally divided shares of firms in $\mathcal{N}_e$, which we call a mutual fund. Then, the return of the mutual fund is

$$\frac{1}{n} \sum_{e' \in \mathcal{N}_e} [((1 - \xi_q)(1 - \tau_m) d_{e',t} dt + dq_{e',t}) / q_{e',t} - \tau_p dt] = \mu_{m,t} dt + \sigma_{m,t} dB_{\mathcal{N}_e,t}, \tag{2}$$

where $\mu_{m,t}$ and $\sigma_{m,t}$ are endogenous parameters and

$$dB_{\mathcal{N}_e,t} \equiv \frac{1}{\sqrt{n}} \sum_{e' \in \mathcal{N}_e} dB_{e',t}.$$

For the later analysis, we define the price of the shares of firms in $\mathcal{N}_e$, $q_{\mathcal{N}_e,t}$, as

$$q_{\mathcal{N}_e,t} = \frac{1}{n} \sum_{e' \in \mathcal{N}_e} q_{e',t}.$$

Note that workers in $\mathcal{N}_e$ receive a correlated return within the neighborhood. However, the number of workers with correlated returns is at most finite.

2.2 Financial intermediaries and the firm maximization problem

In the model, the returns from a firm's shares are stochastic. The risk arising from holding a mutual fund is not completely diversified away either, because the mutual fund pools only a finite number of firms. In addition to these risky assets, we introduce risk-free assets provided by financial intermediaries. Financial intermediaries supply risk-free bonds by pooling the shares of a continuum of firms and thus completely diversifying the risks. The transformation technology requires transaction cost τ_f per dividend $d_{e,t}$. We assume that $\tau_f > \tau_m$.

Under this setting, the profit maximization problem of a financial intermediary is

$$\max_{\{s^f_{e,t}\}} \mathbb{E}_t \int_0^E \left\{ ((1-\xi_f)(1-\tau_f) d_{e,t} dt + dq_{e,t})\, s^f_{e,t} \right\} de - r^f_t dt \left(\int_0^E q_{e,t} s^f_{e,t} de \right),$$

where $s^f_{e,t}$ is the shares of firm e owned by the financial intermediary and ξ_f is the dividend tax. The tax rate imposed on financial intermediaries ξ_f is different from the tax rate imposed on a household ξ_q when the household directly holds the shares of a firm. The interior solution of the problem is

$$r^f_t q_{e,t} = \mathbb{E}_t \ (1-\xi_f)(1-\tau_f) d_{e,t} dt + dq_{e,t}. \tag{3}$$

We assume that each firm chooses capital and employment to maximize the present value $q_{e,t}$ in (3). From the firm maximization problem, we obtain the following conditions:

$$\mathsf{MPK}_t \equiv r^f_t + \delta = \frac{\partial p_{e,t} y_{e,t}}{\partial k_{e,t}}, \tag{4}$$

$$w_t = \frac{\partial p_{e,t} y_{e,t}}{\partial l_{e,t}}. \tag{5}$$

2.3 Households

We introduce the Blanchard-Yaari type of perpetual youth households (Yaari, 1965; Blanchard, 1985). There is a continuum of households indexed by $i \in [0, 1]$. Households supply one unit of labor inelastically and receive real wage w_t. A household dies without heirs at Poisson rate ν. Households participate in a pension contract in which they obtain a pension payment at rate ν in proportion to their financial wealth, and their financial wealth is relinquished to the pension program upon their death. In each period, the measure ν of new households are born, so that the measure of households is constant. A newly born household has no initial financial wealth. Households discount future utility by the sum of time discount rate ρ and death rate ν.

Each household can hold three kinds of assets, (i) the shares of the firm that the household is employed at and directly purchases, whose return is given in (1), (ii) mutual funds composed of the shares of neighbor firms in $\mathcal{N}_e$, whose return is given in (2), and (iii) risk-free assets, whose return is r_t^f. Risk-free assets consist of the bonds issued by financial intermediaries and by the government, as well as an annuitized value of household future wage income minus taxation. Let $a_{i,t}$ be the total asset of household i who works at firm e. Then,

$$a_{i,t} = s_{i,t}^q q_{e,t} + s_{i,t}^m q_{\mathcal{N}_e,t} + b_{i,t} + h_{i,t}^a,$$

where $s_{i,t}^q$ is the shares of firm e owned by household i, $s_{i,t}^m$ is the shares of firms in neighborhood $\mathcal{N}_e$, $b_{i,t}$ is i's holdings of bonds issued by government or financial intermediaries, and $h_{i,t}^a$ is the after-tax human wealth which evolves as

$$(\nu + r_t^f) h_{i,t}^a = w_t - \psi_t + dh_{i,t}^a/dt$$

where ψ_t is a lump-sum tax. Let $(\theta_{i,t}^q, \theta_{i,t}^m, \theta_{i,t}^f)$ be the shares of the assets (i), (ii), and (iii), where $\theta_{i,t}^q + \theta_{i,t}^m + \theta_{i,t}^f = 1$. The household budget constraint can be written as

$$da_{i,t}/a_{i,t} = \mu_{a,t} dt + \theta_{i,t}^q \sigma_{i,t}^q dB_{e,t} + \theta_{i,t}^m \sigma_{m,t} dB_{\mathcal{N}_e,t},$$

where

$$\mu_{a,t} \equiv \theta_{i,t}^q \mu_{q,t} + \theta_{i,t}^m \mu_{m,t} + \theta_{i,t}^f r_t^f - c_{i,t}/a_{i,t}.$$

In this paper, we focus on a stationary equilibrium. Let a denote a household's total resources available including human asset h. Given prices, households solve the following dynamic programming problem with a continuous-time version of the Epstein-Zin preference (Duffie and Epstein, 1992; Campbell and Viceira, 2002). We assume that the elasticity of intertemporal substitution is one. Then,

$$V(a_t) = \max_{c_t, \boldsymbol{\theta}} \ \mathbb{E}_t\left[\int_t^\infty f(c_s, V(a_s))ds\right],$$

where

$$f(c, V) = (\rho + \nu)(1-\gamma)V\left[\ln(c) - \frac{1}{1-\gamma}\ln((1-\gamma)V)\right].$$

Note that $\rho > 0$ is the rate of time preference and γ is the coefficient of relative risk aversion. The Hamilton-Jacobi-Bellman equation of the household problem is written as follows:

$$(\rho + \nu)V = \max_{c, \theta_q, \theta_m} \ f(c, V) + V_a\mu_a + \frac{1}{2}V_{aa}\left((\theta_q\sigma_q a)^2 + (\theta_m\sigma_m a)^2\right). \tag{6}$$

2.4 Market-clearing conditions

Goods $y_{e,t}$ are aggregated according to

$$Y_t = \left(\int_0^E y_{e,t}^{\frac{\phi-1}{\phi}} de\right)^{\frac{\phi}{\phi-1}}, \ \phi > 1.$$

The aggregate good Y_t is produced competitively and the price of the good is normalized to one. The other aggregate variables are simply summed together. For example, aggregate consumption C_t, aggregate human wealth H_t, aggregate total asset A_t, aggregate stock valuation Q_t, aggregate dividend D_t and physical capital K_t are defined as $C_t = \int_0^1 c_{i,t}di$, $H_t = \int_0^1 h_{i,t}di$, $A_t = \int_0^1 a_{i,t}di$, $Q_t = \int_0^E q_{e,t}de$, $D_t = \int_0^E d_{e,t}de$ and $K_t = \int_0^E k_{e,t}de$. We assume that physical capital is competitively traded between firms. Henceforth, we suppress the subscript i in the household variables such as θ_q and θ_m, because these variables are identical across households as shown below.

Let G_t denote the government purchase of goods. The market-clearing condition for the aggregate good is

$$C_t + \frac{dK_t}{dt} + \delta K_t + \theta_m A_t \tau_p + \left\{\frac{\theta_m A_t}{Q_t}\tau_m + \left(1 - \frac{(\theta_q + \theta_m)A_t}{Q_t}\right)\tau_f\right\}D_t + G_t = Y_t. \tag{7}$$

The labor market-clearing condition is

$$\int_0^E l_{e,t} de = 1.$$

The market-clearing condition for the shares of each firm $e \in [0, E]$ is

$$l_{e,t} s_t^e + \sum_{e' \in \mathcal{N}_e} l_{e',t} s_t^{\mathcal{N}_e} + s_t^f = 1.$$

The market-clearing condition for the risk-free bond is

$$\int_0^1 b_{i,t} di = \int_0^E q_{e,t} s_{e,t}^f de + \Delta_t \tag{8}$$

where Δ_t denotes government debts. The government is indebted with Δ_0 at $t = 0$. The government debt accumulates as

$$\frac{d\Delta_t}{dt} = r_t^f \Delta_t + G_t - (\xi_m(1-\tau_m)\theta_m + \xi_f(1-\tau_f)\theta_f)A_t - \psi_t. \tag{9}$$

By integrating (9) over time, we obtain that the sequence of lump-sum tax ψ_t must honor the following constraint

$$\Delta_t = \int_t^\infty e^{-\int_t^s r_u^f du}(\psi_s - G_s + (\xi_m(1-\tau_m)\theta_m + \xi_f(1-\tau_f)\theta_f)A_s)ds.$$

The aggregate after-tax human wealth is expressed as

$$H_t^a = \int_t^\infty e^{-\int_t^s (r_u^f + \nu)du}(w_s - \psi_s)ds = H_t - \int_t^\infty e^{-\int_t^s (r_u^f + \nu)du} \psi_s ds.$$

This implies that the government debt Δ_t held by households partially cancels out the present value of their future lump-sum tax liability. The aggregate total asset of households satisfies

$$A_t = Q_t + H_t^a.$$

2.5 Equilibrium

A stationary equilibrium is achieved when wage, rate of returns, allocation, household value function and policy functions, and the distribution of total wealth are such that (i) each firm maximizes profit according to (3), (ii) the value and policy functions solve the household dynamic programming problem (6), (iii) markets clear according to (7)–(8), and (iv) the government debt accumulates according to (9).

3 Analytical Results

3.1 Solving stationary equilibrium

In this section, we consider a balanced growth path of the model economy, where the output grows at constant rate g. The preference specification allows the household dynamic programming problem (6) to be solved by linear policy functions, in which the savings rate $s = (a - c)/a$ and portfolio weights θ's are independent of a. The first-order condition for the consumption choice c yields:

$$c = \frac{(\rho+\nu)(1-\gamma)V}{V_a} = (\rho+\nu)a. \tag{10}$$

The first-order conditions for θ_q and θ_m are reduced to:

$$\theta_q = -\frac{V_a}{V_{aa}a}\frac{\mu_q - r^f}{\sigma_q^2} = \frac{\mu_q - r^f}{\gamma\sigma_q^2}, \tag{11}$$

$$\theta_m = -\frac{V_a}{V_{aa}a}\frac{\mu_m - r^f}{\sigma_m^2} = \frac{\mu_m - r^f}{\gamma\sigma_m^2}. \tag{12}$$

Let the relative productivity of firm i be $\tilde{z}_e \equiv z_e^{\phi-1}/\mathbb{E}\left\{z_e^{\phi-1}\right\}$ and define $Z \equiv \mathbb{E}\left\{z_e^{\phi-1}\right\}^{\frac{1}{\phi-1}}$. We can show that the growth rate g coincides with the growth rate of $Z^{\frac{1}{1-\alpha}}$ as

$$g = \left\{\left(\mu_z - \frac{\sigma_z^2}{2}\right) + (\phi-1)\frac{\sigma_z^2}{2}\right\}/(1-\alpha).$$

Then, firm-side variables can be derived as follows:

$$\ell_e = \frac{p_e y_e}{\overline{py}} = \frac{k_e}{\overline{k}} = \frac{q_e}{\overline{q}} = \tilde{z}_e, \tag{13}$$

$$d_e = \overline{d}\tilde{z}_e dt - (\phi-1)\sigma_z\overline{k}\tilde{z}_e dB_e, \tag{14}$$

where

$$\overline{py} \equiv \left(\frac{\alpha\rho}{\mathsf{MPK}}\right)^{\frac{\alpha}{1-\alpha}} Z^{\frac{1}{1-\alpha}}, \tag{15}$$

$$\overline{k} \equiv \left(\frac{\alpha\rho}{\mathsf{MPK}}\right)^{\frac{1}{1-\alpha}} Z^{\frac{1}{1-\alpha}}, \tag{16}$$

$$\overline{q} \equiv \overline{d}\int_t^{\infty}(1-\xi_f)(1-\tau_f)\exp\left\{-\int_t^u (r^f - \mu_d)ds\right\}du, \tag{17}$$

$$\overline{d} \equiv (1-(1-\alpha)(1-1/\phi))\overline{py} - (\delta+\mu_k)\overline{k}, \tag{18}$$

and where $\mu_{k,t}$ and $\mu_{d,t}$ are the expected growth rates of $k_{e,t}$ and $d_{e,t}$, respectively. The $\mu_{k,t}$ and $\mu_{d,t}$ are equal to g along the balanced growth path. Note that the dispersions of the firm variables are solely determined by relative productivity $\tilde{z}$. This property significantly simplifies the computation of transition paths.

Aggregate variables are computed from the variables that are defined from (15) to (18), as $Y = \overline{py}$, $K = \overline{k}$, $Q = \overline{q}$, and $D = \overline{d}$. Dividing the aggregate variables by $Z^{\frac{1}{1-\alpha}}$, we obtain the detrended variables, $\widetilde{Y}_t, \widetilde{K}_t, \widetilde{Q}$, and $\widetilde{D}$. At the stationary equilibrium, these detrended variables become constant.

Variables related to returns in the steady state are computed as follows:

$$
\begin{aligned}
\mu_q &= r^f + \left(\frac{1-\xi_q}{(1-\xi_f)(1-\tau_f)} - 1 \right)(r^f - g), \\
\sigma_q &= (\phi - 1)\sigma_z \left(1 - \frac{1-\xi_q}{(1-\xi_f)(1-\tau_f)} \frac{K_t}{D_t}(r^f - g) \right), \\
\mu_m &= r^f - \tau_p + \left(\frac{(1-\xi_q)(1-\tau_m)}{(1-\xi_f)(1-\tau_f)} - 1 \right)(r^f - g), \\
\sigma_m &= \frac{(\phi - 1)\sigma_z}{\sqrt{n}} \left(1 - \frac{(1-\xi_q)(1-\tau_m)}{(1-\xi_f)(1-\tau_f)} \frac{K_t}{D_t}(r^f - g) \right).
\end{aligned}
$$

The steady state is computed by the following algorithm.

1. Pick a value for r^f.

   (a) Given r^f, MPK is obtained.

   (b) Given MPK, $Y = \overline{py}$, $K = \overline{k}$, $D = \overline{d}$, and $Q = \overline{q}$ are obtained, because they are the functions of MPK.

   (c) Compute $H = w/(\nu + r^f - g)$, where $w = (1-\alpha)(1 - 1/\phi)Y$.

   (d) Compute $A = Q + H^a$.

   (e) Compute μ_q and σ_q.

   (f) Compute θ's.

   (g) Using the variables obtained as above, compute dK/dt from the resource constraint (7).

2. Iterate the loop until the growth rate of aggregate physical capital is equal to the steady state growth rate g, $(dK/dt)/K = g$.

3.2 Pareto distribution of household wealth

First, as a direct application of Aoki and Nirei (2017), we obtain that the stationary wealth distribution follows a double-Pareto distribution.

Proposition 1 *The stationary distribution of $a_{i,t}$ follows a double-Pareto distribution*

$$f(\log a) = \begin{cases} (\nu/\vartheta)e^{-\lambda_1(\log a - \log h)} & \textit{if } a \geq h^a, \\ (\nu/\vartheta)e^{\lambda_2(\log a - \log h)} & \textit{otherwise} \end{cases} \tag{19}$$

where

$$\lambda_1 = \frac{\mu_a}{\sigma_a^2}\left(\frac{\vartheta}{\mu_a} - 1\right), \tag{20}$$

$$\lambda_2 = \frac{\mu_a}{\sigma_a^2}\left(\frac{\vartheta}{\mu_a} + 1\right), \tag{21}$$

$$\vartheta \equiv \sqrt{2\nu\sigma_a^2 + \mu_a^2}. \tag{22}$$

It is established that the double-Pareto distribution emerges as a result of idiosyncratic multiplicative shocks (Reed, 2001; Toda, 2014). The Pareto exponent λ_1 determines the inequality in the right-tail of the wealth distribution. Note that

$$\begin{aligned} \mu_a &= \nu + \theta_q\mu_q + \theta_m\mu_m + \theta_f r^f - (1-s) \\ &= \frac{\mu_q - r^f}{\gamma\sigma_q^2}\mu_q + \frac{\mu_m - r^f}{\gamma\sigma_m^2}\mu_m + r^f - \rho, \\ \sigma_a &= \theta_q\sigma_q + \theta_m\sigma_m \\ &= \frac{\mu_q - r^f}{\gamma\sigma_q} + \frac{\mu_m - r^f}{\gamma\sigma_m}. \end{aligned}$$

We obtain the following comparative statics.

Corollary 1 *The Pareto exponent is negatively related to trend μ_a and diffusion σ_a^2 while it is positively related to ν, i.e., $\partial\lambda_1/\partial\mu_a < 0$, $\partial\lambda_1/\partial\sigma_a^2 < 0$, and $\partial\lambda_1/\partial\nu > 0$.*

The Pareto exponent can be rewritten as

$$\lambda_1 = \sqrt{\frac{2\nu}{\sigma_a^2} + \left(\frac{\mu_a}{\sigma_a^2}\right)^2} - \frac{\mu_a}{\sigma_a^2}. \tag{23}$$

The Pareto exponent in (23) is determined by the death rate ν, the diffusion of household wealth σ_a^2, and the trend-diffusion ratio μ_a/σ_a^2. A lower death rate leads to a smaller Pareto exponent and thus greater inequality in household wealth, because a smaller turnover of households enhances the opportunity for households to accumulate wealth for a longer time period. An alternative interpretation of ν is that it is the birth rate of new households with no financial wealth. Thus, an influx of greater mass at the mode of the distribution $a = h^a$ results in greater equality in the stationary wealth distribution. The effect of the birth rate is counter-balanced by the effect of diffusion σ_a^2, which reduces λ_1 and increases inequality. Thus, the Pareto exponent is determined by the balance between the influx effect ν and the diffusion effect σ_a^2, as in the analysis of Nirei and Souma (2007) and Nirei and Aoki (2016).

The influx-diffusion balance is also adjusted by term μ_a/σ_a^2 in (23). Note that λ_1 is always greater than 1, since otherwise the mean wealth is indefinite. Thus, we observe that the effect of μ_a/σ^2 on λ_1 is negative. This implies that the diffusion effect is somehow mitigated. Also, a greater trend leads to a smaller λ_1. This generates ambiguity in the effect of financial transaction costs on the Pareto exponent through general equilibrium effects. As we see below, a reduction in transaction costs enhances capital accumulation, and thus reduces the steady-state risk-free rate. Thus, the trend of wealth growth may decrease, leading to an increase in wealth equality. In Section 4, we will investigate this issue in detail with numerical analysis.

3.3 Long-run output

In this section, we concentrate on the special case $\xi_q = \xi_f = g = G = \Delta = 0$ to obtain analytical results. Under that environment, we obtain

$$\begin{aligned}
\mu_q &= \frac{r^f}{1-\tau_f}, \\
\sigma_q &= \sigma_z(\phi-1)\left(1-\frac{K}{D}\frac{r^f}{1-\tau_f}\right) \\
&= \sigma_z(\phi-1)\left(1-\frac{\alpha(\phi-1)}{\alpha(\phi-1)+1+\delta/r^f}\frac{1}{1-\tau_f}\right), \qquad (24)\\
\sigma_m &= \sigma_q\chi/\sqrt{n}, \\
\theta_q &= \frac{r^f}{\gamma\sigma_q^2}\frac{\tau_f}{1-\tau_f}, \\
\theta_m &= \frac{r^f}{\gamma\sigma_q^2\chi^2/n}\left[\frac{\tau_f-\tau_m}{1-\tau_f}-\tau_p\right],
\end{aligned}$$

where

$$\chi \equiv \frac{1-\frac{\alpha(\phi-1)}{\alpha(\phi-1)+1+\delta/r^f}\frac{1-\tau_m}{1-\tau_f}}{1-\frac{\alpha(\phi-1)}{\alpha(\phi-1)+1+\delta/r^f}\frac{1}{1-\tau_f}} = \frac{\alpha(\phi-1)(\tau_m-\tau_f)+(1+\delta/r^f)(1-\tau_f)}{-\alpha(\phi-1)\tau_f+(1+\delta/r^f)(1-\tau_f)} \qquad (25)$$

is increasing in r^f.

Then, we obtain

$$\begin{aligned}
\mu_a &= \frac{(r^f)^2}{\gamma\sigma_q^2}\left[\left(\frac{\tau_f}{1-\tau_f}\right)^2+\left(\frac{\tau_f-\tau_m}{1-\tau_f}\right)\left(\frac{\tau_f-\tau_m}{1-\tau_f}-\tau_p\right)\frac{n}{\chi^2}\right] \\
&\quad + r^f\left[1-\frac{n\tau_p}{\gamma\sigma_q^2\chi^2}\left(\frac{\tau_f-\tau_m}{1-\tau_f}-\tau_p\right)\right]-\rho, \\
\sigma_a &= \frac{r^f}{\gamma\sigma_q}\left[\frac{\tau_f}{1-\tau_f}+\left(\frac{\tau_f-\tau_m}{1-\tau_f}-\tau_p\right)\frac{\sqrt{n}}{\chi}\right].
\end{aligned}$$

The risk-free rate r^f is determined from (7). At the steady state, the goods market-clearing condition becomes:

$$\delta K + \kappa D = Y - (\rho+\nu)(Q+H)$$

where

$$\kappa \equiv \frac{\theta_m A_t}{D_t}\tau_p + \frac{\theta_m A_t}{Q_t}\tau_m + \left(1-\frac{(\theta_q+\theta_m)A_t}{Q_t}\right)\tau_f \qquad (26)$$

is the fraction of aggregate dividends lost in financial transaction costs. Let $\varphi \equiv 1-(1-\alpha)(1-1/\phi)$ denote the capital share of income. Then $D = \varphi Y - \delta K$ and $H = (1-\varphi)Y/(r^f + \nu)$. Also, from (15), (16) and (17), we have $Y = ZK^\alpha$ and $Q = D(1-\tau_f)/r^f$. Moreover, we have $Y/(\delta K) = (r^f + \delta)/(\alpha\delta(1-1/\phi))$. Combining these, we obtain an equation that determines r^f:

$$\frac{1-(\rho+\nu)\frac{1-\tau_f}{r^f} - \kappa}{1-(\rho+\nu)\left(\frac{\varphi(1-\tau_f)}{r^f} + \frac{1-\varphi}{r^f+\nu}\right) - \varphi\kappa} = \frac{Y}{\delta K} = \frac{r^f+\delta}{\alpha\delta(1-1/\phi)}. \tag{27}$$

We first show that the financial transaction costs κ and risk-free rate r^f have a monotonic relationship in the above equation.

Lemma 1 *There exists $\bar{\tau}_p > 0$ such that for any $\tau_p < \bar{\tau}_p$, r^f is increasing in κ in Equation (27).*

All proofs are deferred to Appendix A. Given the lemma above, we establish that the financial transaction cost increases the steady-state risk free rate.

Proposition 2 *Suppose that $\xi_f = \xi_m = g = G = \Delta = 0$ and $\tau_p < \bar{\tau}_p$. A decrease in τ_m, a decrease in τ_p, or an increase in n leads to a decrease in r^f in the steady state.*

The risk-free rate directly affects an equilibrium marginal product of capital, and hence determines output and capital as in (15) and (16). Thus, Proposition 2 leads to our main comparative statics result as follows.

Corollary 2 *If τ_m or τ_p is reduced, or if n is increased in the environment of Proposition 2, the steady state values of output Y, capital K, and real wage w strictly increase.*

By reducing the financial transaction costs τ_m and/or τ_p, or by expanding the number n of firms included in a mutual fund, the accumulation of capital is enhanced, which leads to the long-run increase in output and wage.

Household labor income w increases in proportion to output Y. The rate of increase for total dividends $D = \varphi Y - \delta K$ is strictly less than that of Y, because the depreciation cost δK increases more than proportionally to φY. However, the effect on the dividend receipts of households is ambiguous, due to a decrease in transaction costs κ. In the next section, we will numerically investigate the impact of financial costs on the labor share of gross domestic income.

4 Quantitative Results

4.1 Data

To set mean returns and volatility, we use data on annual stock returns of individual companies listed on the Tokyo Stock Exchange for 2011-2015 (based on adjusted closing prices) to estimate time-series averages ($\mu_q = \sum_{t=1}^{T} \mu_{q,t}/T$ and $\sigma_q = \sum_{t=1}^{T} \sigma_{q,t}/T$) of cross-sectional mean ($\mu_{q,t} = \sum_{i=1}^{I} \mu_{q,i,t}/I$) and standard deviation ($\sigma_{q,t}$). We also use data of total annual stock-market returns for 2011-2015 to obtain a time-series average ($\mu_m = \sum_{t=1}^{T} \mu_{m,t}/T$) and standard deviation (σ_m). Nikkei Financial Quest database provides a composite index of the Tokyo Stock Exchange, called TOPIX (including dividends), which tracks all domestic firms of the First Section of the exchange. The bond return is similarly estimated with the yield of 10-year Japanese Government Bond (JGB), which is provided by the Ministry of Finance. The returns are deflated by Japanese GDP deflator time series provided in the World Bank Database. As shown in Table 1, annual real stock returns of individual companies average 25.9%, the expected return of the stock market is 10.4%, and the expected real bond return is 0.6%. Figure 1 shows the time-series of the cross-sectional mean and standard deviation of individual stock returns and of the stock market return. We observe that the mean returns and volatility are stationary during the periods of interest.

We use the Bank of Japan's Flow of Funds accounts to calibrate the household portfolio. It is often pointed out that Japanese households hold most of their financial wealth in the form of risk-free assets such as deposits and defined-benefit pensions. According to the Flow of Funds, $\tilde{\theta}_f = 0.778$ of financial wealth is held in those risk-free assets in 2015, whereas households allocate $\tilde{\theta}_q = 0.089$ of the total financial wealth to equity and stocks and $\tilde{\theta}_m = 0.048$ to mutual funds. The time-series of these shares are shown in Figure 2.

These shares do not exactly correspond to our model shares (θ_f, θ_q, θ_m), because house-

Table 1: Mean and standard deviation of asset returns

Variables	Mean (%)	S.D. (%)		
Individual stocks	25.88	26.76	TSE	2011-15
	25.24	27.78	TSE	2000-15
Aggregate stock	10.44	20.46	TOPIX incl. dividends	2011-15
	3.93	26.34	TOPIX incl. dividends	2000-15
	12.79	28.07	MSCI, Obstfeld (1994)	1976-92
	4.72	21.91	MSCI, Campbell (2003)	1970-99
	2.11	25.90	Aoki et al. (2016)	1995-2014
Bond	0.62	1.91	JGB, 10yr	2011-15
	1.98	1.40	JGB, 10yr	2000-15
	1.39	2.30	Campbell (2003)	1970-99
	0.31	0.69	JGB, 1-2yr, Aoki et al. (2016)	1995-2014

hold total wealth in our model includes after-tax human capital. We obtain our targets for θ_q and θ_m from the data $\tilde{\theta}_q$ and $\tilde{\theta}_m$ through the following procedure. At the end of the 2015 fiscal year, Japanese households hold 1716 trillion yen worth of gross total financial wealth. This amount corresponds to $Q+\Delta$, total financial wealth in our model. Much of the risk-free wealth held by households finance the Japanese government's debt. The outstanding debt of the general government (including local governments and social security funds) amounts to 1207 trillion yen as of 2015. Considering that the general government holds 580 trillion yen worth of financial assets, households expect that the net debt of 627 trillion yen is the amount that will be financed by future taxation, which roughly corresponds to Δ in our model where the Ricardian equivalence approximately holds when ν is small. Thus, we set $\Delta/Q = 627/(1716-627) = 0.58$. Then, we obtain relations between data and model shares as $\tilde{\theta}_q = \theta_q(Q+H)/(Q+\Delta)$ and $\theta_m = \theta_m(Q+H)/(Q+\Delta)$.

The Japanese government debt has grown steadily over two decades. The left panel of

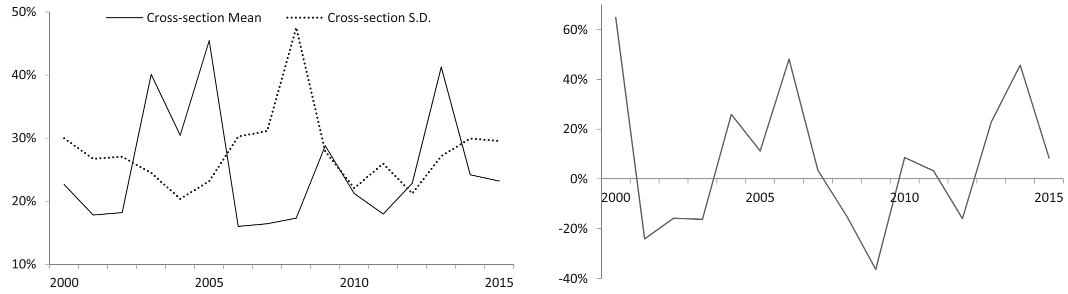

Figure 1: *Left:* Average return and volatility of individual shares of companies listed on Tokyo Stock Exchange. *Right:* Market return of Tokyo Stock Exchange (TOPIX including dividends). Source: Nikkei Financial Quest.

Figure 2: *Left:* Household portfolio share of equity and stocks $\tilde{\theta}_q$. *Right:* Household portfolio share of mutual funds $\tilde{\theta}_m$. The shares represent the asset values divided by household total financial wealth. Source: Flow of Funds, Bank of Japan.

Figure 3 shows the time-series of the net debt of the general government, along with the risk-free asset holdings of households. We observe that the two time-series grow in tandem, if not completely parallel. We interpret this relationship in our model as households absorbing most of the newly issued government bonds by increasing their savings. Thus, an increase in the government debt holdings by households is cancelled out with a reduction in the human wealth which reflects increases in future taxation. Since both government debt and human wealth are risk-free, the household share of risk-free assets is unchanged. This view of Japanese savings behavior corresponds with that of Hayashi (1986) in which the altruistic

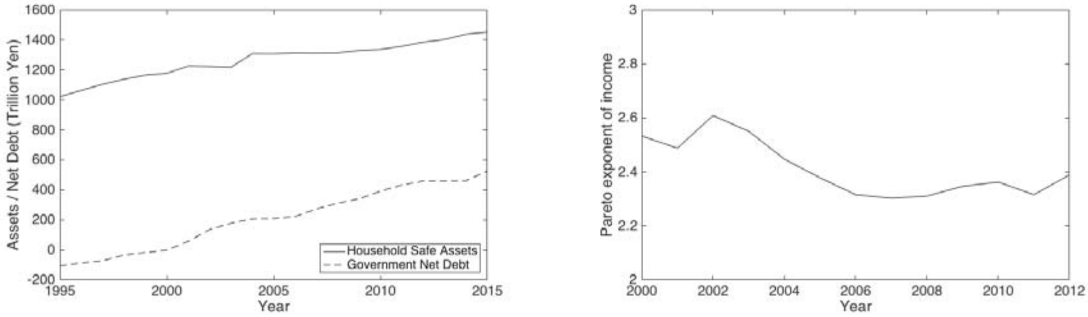

Figure 3: *Left:* Household risk-free assets and government net debt. Source: Flow of Funds and Ministry of Finance. *Right:* Pareto exponent of household income. Source: Moriguchi (2016).

bequest motive plays an important role. Households have a strong incentive to save in our model of perpetual youth since they face a survival probability independent of age, even though the Ricardian equivalence does not hold exactly.

In this paper, we do not include housing or mortgage in household net wealth. We instead treat housing as an imputed consumption included in GDP. Also, we do not consider risky human capital explicitly. In our model, wealthy households who earn a much larger amount of business returns from their portfolio θ_q than their labor income w effectively represent successful entrepreneurs. Hence, we can view θ_q as the portfolio allocated to entrepreneurial endeavor.

For the inequality measure, we use the estimates of the Pareto exponents in Japan provided by Moriguchi (2016). The average Pareto exponent during 2000-2015 is 2.41. The Pareto exponent is stationary during this period as seen in the right panel of Figure 3. In our model, the Pareto exponent of household income coincides with that of wealth. Thus, we use the estimated value interchangeably for income and wealth.

Other parameter values are calibrated as follows. We choose to set the discount rate ρ to the standard value of 0.03. We set ν to 0.02, which implies that the average length of a household is 50 years. In the benchmark model, the coefficient of relative risk aversion γ is set to 5. This value is used in a DSGE model in Caldara et al. (2012) and is also consistent with the empirical estimate of the Japanese households in the survey data by Itō et al. (2017). Following Aoki and Nirei (2017), we set ϕ to 3.33, implying that 30% of firm sales is rent. The depreciation rate δ is set to 0.089, which is taken from Hayashi and Prescott (2002). The capital share in GDP, α, is calculated from SNA as 0.3375.

4.2 Benchmark result

In our calibration, we aim to match the mean returns (r^f and μ_m), risks of an individual stock and a mutual fund (σ_q and σ_m), household portfolio shares (θ_q and θ_m), and the Pareto exponent of wealth distribution (λ_1). The target moments of portfolio variables are estimated for the period 2000-2015 as shown in Table 1 except for σ_q. Table 1 shows that the individual stock in TSE exhibits very modest volatility that is comparable to the market volatility during this period. Lacking alternative measurements, we rely on Moskowitz and Vissing-Jørgensen (2002) for an estimate of volatility of entrepreneurial rate of returns at 0.41 standard deviation. The volatility of mutual funds is set at TSE market volatility, 0.26. Then, we exploit the model prediction that the ratio of entrepreneurial volatility to market volatility is scaled as $\sqrt{n}$, leading to $n = 2.43$.

Other moments, μ_m, θ_q, θ_m, and λ_1, are matched by calibrating four model parameters related to financial transaction costs: σ_z, τ_f, τ_m, and τ_p. The riskiness of business is mostly determined by the size of productivity shock, σ_z. Thus, we use σ_z to match σ_q and μ_m. The cost of transformation of risky stocks to risk-free assets, τ_f, determines the size of overall risks that households take directly through own business and mutual funds. When τ_f is high, households invest less in risk-free assets. The household risk attitude then affects the tail distribution of wealth and income. The greater the risk-taking, the lower the tail index λ_1 and the less equal the distribution. Hence, λ_1 is the main target moment to obtain a calibrated value for τ_f. The costs for mutual funds, τ_m and τ_p, are used to match the portfolio shares θ_m and θ_q.

Our benchmark parameter values are thus determined as $\sigma_z = 0.614$, $\tau_f = 0.123$, $\tau_m = 0.06$, and $\tau_p = 0.022$. Under these values, we match the target moments as shown in Table

2.

Table 2: Calibration targets and endogenous moments

	σ_q	μ_m	σ_m	θ_q	θ_m	λ_1	$\frac{K}{Y}$	$\frac{C}{Y}$	$\frac{A-H^a}{Y}$	r^f
Target	0.411	0.0393	0.2634	0.1057	0.0465	2.41	2.2	0.56	3.3	0.02
Model	0.411	0.0506	0.2712	0.1060	0.0459	2.66	1.8	0.59	4.3	0.04

While it is difficult to find exact empirical counterparts for the transaction costs assumed in the model, we can evaluate their plausibility by examining the other steady state values of the model. In particular, τ_f, τ_m and τ_p determine the size of financial costs κ. The model posits that the portion of goods κ is used to facilitate financial transactions and maturity transformation from stock shares to deposit. Thus, κ is interpreted as the resources employed in the financial sector as in Aoki and Nirei (2017). The steady state value for the share of financial costs in GDP, κ/Y, is 0.068. According to SNA, the share of GDP is 5.2% for the financial sector and 6.6% for the financial and realty sectors excluding housing. Thus, the model value matches well with the actual share of the financial sector in Japan.

It is necessary that τ_f is larger than τ_m in our model, because otherwise the risky mutual fund is dominated in its rate of return by the risk-free asset generated by banks. Our benchmark parameter further assumes that τ_f is much greater than τ_m. This corresponds well with the fact that depository institutions hold large shares in the Japanese financial industry. The trust and administration fee of mutual funds is represented by τ_m, which is charged on the returns of funds in our model. Thus, when the rate of returns of a mutual fund is 4%, $\tau_m = 0.06$ which means that 0.24% of fund value is deducted as trust fees. This value seems plausible for passive index funds purchased by Japanese households. However, Khorana et al. (2009) reports the trust fees in Japan as 1.25%, which would set our τ_m at a higher value 0.31. In addition to τ_m, our model posits that a household must pay transaction fees upon the purchase of mutual funds. These fees, as discussed in detail by Cai et al. (1997), have been historically high in Japan. According to their estimates, upon the purchase of funds during the period between 1981 and 1992, the fees were "typically between 2% and 5% of the investment value", with a turnover ratio of funds of 110% in 1992. Even though the purchase costs and the turnover ratio of mutual funds have declined since then, it is still possible that an annual average rate of this fee, which is represented by τ_p in our model, amounts to 2.2% of the invested value as our benchmark calibration assumes.

Under the benchmark parameter values, the model generates steady-state capita-output ratio $K/Y = 1.8$, the ratio of household financial wealth to output $(A - H^a)/Y = 4.3$, and the consumption-output ratio $(\nu + \rho)A/Y = 0.59$. These ratios at the steady state roughly match with the corresponding statistics in Japan in 2015, which are 2.2, 3.3, and 0.56, respectively. While the wealth-output ratio is matched well under this calibration with standard value of time discount rate $\rho = 0.03$, it is hard to match the observed return of JGB with our risk-free rate. However, we can match the risk-free rate by setting a low value for ρ. Our main numerical results continue to hold under the alternative calibration, as shown in Appendix B.

The low share of mutual funds in household portfolios, at 4.65%, is calibrated in our model by setting the purchase cost τ_p high. This is compatible with the low participation rate of Japanese households in the stock market as documented in Iwaisako (2009) and Fujiki et al. (2012). The share of mutual funds is remarkably lower than that of U.S. households (Financial Services Agency, 2016), even though the gap may be overstated due to statistical difference between two countries (Koike, 2009; Fukuhara, 2016). Kitamura and Uchino (2010) and Itō et al. (2017) suggest that poor financial literacy is a factor contributing to low participation. Aoki et al. (2016) and Yamana (2016) provide structural models to gauge the impact of financial transaction costs in an effort to account for the low participation rate.

4.3 Effects of reduction in financial costs

As we have shown above analytically, a reduction in financial transaction costs encourages risk-taking, enhances capital accumulation and leads to greater outputs and a lower risk-free rate, while its impact on inequality is ambiguous. In this section, we investigate the effect of reduced financial costs quantitatively using the benchmark calibrated model. Tables 3 and 4 show the results.

Table 3: Comparative statics: prices and output

	r^f	w	K/Y	Y	NI
Baseline	0.0446	0.6197	1.7682	1.3369	1.0687
no τ_p	0.0431	0.6233	1.7882	1.3446	1.0825
no τ_m	0.0444	0.6201	1.7701	1.3376	1.0681
$n \times 4$	0.0444	0.6200	1.7699	1.3376	1.0564

Table 4: Comparative statics: inequality and portfolio

	Labor share	λ_1	Gini	P90/P50	θ_q	θ_m
Baseline	0.5799	2.6624	0.4038	1.2554	0.1060	0.0459
no τ_p	0.5758	2.4313	0.4227	1.2872	0.0964	0.1937
no τ_m	0.5805	2.6162	0.4073	1.2606	0.1048	0.0798
$n \times 4$	0.5869	2.5700	0.4090	1.2632	0.1034	0.1766

The first comparative analysis considers the case in which the purchase cost of mutual funds τ_p is eliminated. We observe that the cut in τ_p causes a 15 basis point reduction in the long-run risk-free rate and 1.3 percent increase in the long-run gross domestic income. It also raises the capital-output ratio, and hence the real wage. These results quantify the analytical result in Proposition 2.

The reduction in τ_p enhances production through encouraging risk-taking, but the increased risk-taking leads to inequality in income and wealth among entrepreneurial households. The tail index λ_1 is decreased, which implies an increased dispersion among the wealth of rich households. In fact, the Gini coefficient for the households with assets greater than H^a can be computed as $1/(2\lambda_1 - 1)$. Thus, a decrease in λ_1 increases the Gini coefficient for the wealthy households. We also compute the Gini coefficient of financial wealth for all households, which is shown in Table 4. The disparity of income between the wealthy households and the median households is shown by the ratio of wealth between the 90th percentile and the 50th percentile. The 90/50 disparity increases, because the wealthy households benefit from the wage increase as much as the median household does.

We conduct similar comparative analyses when τ_m, the cost of the returns on mutual funds, is eliminated, and when n, the number of firms included in a mutual fund, is quadrupled (the magnitude is set so that the effect is comparable with the case $\tau_p = 0$). An increase in n diversifies the risk of the mutual funds by a factor of the square root of n. Thus, an increase in n effectively reduces the cost of risk-bearing through mutual funds, and affects household portfolio behavior and the steady state similarly as with cuts in transaction costs. We observe the same responses of the risk-free rate, output, real wage, and most of the inequality measures qualitatively. An exception is that the labor share of national income (NI) is *increased*. This is caused by an increase in capital-output ratio and thus a more-than-proportional increase in capital depreciation. Households who mainly rely on labor income certainly gain from the increased capital and thus real wage. The increase in the labor share is notable for the case of increased n. We did not observe this effect for the elimination of τ_p, since an increasing effect on NI due to the reduction of goods used for transaction costs dominated the effect of the increased capital depreciation in that case.

We conclude this section by discussing whether a reduction of τ_p and τ_m may have contributed to the decrease in risk-free rates, given that the banking technology τ_f has not changed drastically. The costs τ_p and τ_m represent the technology of equity finance, whereas τ_f represents the technology of deposit banks. Noting that risky investments suffer information asymmetry between creditors and debtors, Greenwood and Jovanovic (1990) argued that financial intermediation provides not only risk pooling but also research on investment projects which leads to better allocation of funds and growth. On this ground, Rajan (2005) warned that the investor incentive problem still remains, even though the world with recent financial developments has left "the days of bank-dominated systems with limited competition, risk sharing, and choice." The Japanese banking sector has maintained a steady level of lending which roughly amounts to the size of Japan's GDP. In our model, the high share of risk-free assets of households is upheld by the high costs of participation in the stock markets. In this setup, a reduction of the participation cost will lead to a shift in the portfolio share from the risk-free to risky assets. The portfolio shift enhances aggregate capital accumulation, because average intermediation costs are lower in risky assets than in risk-free assets. As a side effect, the deepening of capital causes a decrease in the risk-free rate. Quantitative analyses showed that the impact on the risk-free rate is modest at below a percentage point under the calibration for the Japanese economy. It is left for future research to determine whether the effect is more pronounced in different environments.

5 Conclusion

This paper quantifies the effect of policies that induce households to shift their portfolios toward business risk-taking. We present a dynamic general equilibrium model with incomplete markets for business rate-of-return risks. We consider two kinds of financial intermediaries, banks and mutual funds. Banks transform a continuum of risky equities into a risk-free asset, which meets household demand for risk-less deposits. Mutual funds pool a finite number of risky equities and provide risky assets, whose risk is reduced through the diversification of independent equities. The banks' transformation services incur relatively high transaction costs compared to mutual funds, but mutual funds charge an additional cost that is paid upon purchase of the funds. We posit that households pay an annual rate of purchase costs.

With this setup, we first show analytically that a reduction in transaction costs for mutual funds increase output and real wage as well as decreasing the risk-free rate in the steady state. This result is intuitive: the reduction in transaction costs encourages household investments in risky business projects and results in the accumulation of business capital. The marginal product of capital decreases as the capital increases, leading to a decline in the risk-free rate given that banks' technology is unchanged. Therefore, financial developments favoring risk-taking result in a decrease in the long-run risk-free rate. Moreover, we derive that the household wealth follows a double-Pareto distribution in the steady state. The Pareto exponent for the right-tail of the wealth and income distributions is determined by the balance between the trend growth and the diffusion of individual wealth process. The effect of reduction in financial transaction costs on wealth dispersion is ambiguous, since the cost reduction affects both the trend and diffusion of wealth process.

We use the model to quantify the effect of cost reduction in the Japanese economy. The model is able to reproduce Japanese household portfolios with reasonable calibration of financial transaction costs. Quantitative analyses show that the reduction in financial costs result in greater inequality in household financial wealth but may increase the labor share of gross national income. Quantitative effects of the reduced financial costs are modest overall. A quantitative analysis under the parameter values calibrated to the Japanese economy shows that elimination of purchase costs for mutual funds leads to an increase in output by 1.3 percent, a decrease in risk-free rate by 15 basis points, and an increase in Gini coefficient of wealth in 2 percentage points.

Appendix

A Proofs

Proof of Corollary 1

From the definition of λ_1 and ϑ, we observe

$$\frac{\partial \vartheta}{\partial \mu_a} = \frac{\mu_a}{\vartheta}, \quad \frac{\partial \vartheta}{\partial \sigma_a^2} = \frac{\nu}{\vartheta},$$
$$\vartheta = \lambda_1 \sigma_a^2 + \mu_a,$$
$$\nu = \frac{\vartheta^2 - \mu_a^2}{2\sigma_a^2} = \frac{\lambda_1^2 \sigma_a^2}{2} + \lambda_1 \mu_a.$$

Using these relations, we obtain that λ_1 is decreasing in μ_a and in σ_a^2 as follows:

$$
\begin{aligned}
\frac{\partial \lambda_1}{\partial \mu_a} &= \frac{1}{\sigma_a^2}\left(\frac{\partial \vartheta}{\partial \mu_a} - 1\right) \\
&= \frac{1}{\sigma_a^2}\left(\frac{\mu_a}{\vartheta} - 1\right) \\
&= \frac{1}{\sigma_a^2}\left(\frac{\mu_a}{\lambda_1\sigma_a^2 + \mu_a} - 1\right) \\
&< 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \lambda_1}{\partial \sigma_a^2} &= -\frac{\lambda_1}{\sigma_a^2} + \frac{\partial \vartheta / \partial \sigma_a^2}{\sigma_a^2} \\
&= \frac{1}{\sigma_a^2}\left(\frac{\nu}{\vartheta} - \lambda_1\right) \\
&= \frac{1}{\sigma_a^2}\left(\frac{\lambda_1^2\sigma_a^2/2 + \lambda_1\mu_a}{\lambda_1\sigma_a^2 + \mu_a} - \lambda_1\right) \\
&= \frac{1}{\sigma_a^2}\left(\frac{-\lambda_1^2\sigma_a^2/2}{\lambda_1\sigma_a^2 + \mu_a}\right) \\
&< 0.
\end{aligned}
$$

Finally, it is immediate that $\partial\lambda_1/\partial\nu > 0$.

Proof of Lemma 1

By modifying Equation (27),

$$
\begin{aligned}
&\alpha\delta(1 - 1/\phi)\left(1 - (\rho+\nu)\frac{1-\tau_f}{r^f} - \kappa\right) \\
&\quad - (r^f + \delta)\left(1 - (\rho+\nu)\left(\frac{\varphi(1-\tau_f)}{r^f} + \frac{1-\varphi}{r^f+\nu}\right) - \varphi\kappa\right) = 0.
\end{aligned}
$$

Taking the total differential of the equation, we obtain

$$
\begin{aligned}
&dr^f \left[\alpha\delta(1-1/\phi)(\rho+\nu)\frac{1-\tau_f}{(r^f)^2} - \left(1 - (\rho+\nu)\left(\frac{\varphi(1-\tau_f)}{r^f} + \frac{1-\varphi}{r^f+\nu}\right) - \varphi\kappa\right)\right. \\
&\left. -(r^f+\delta)(\rho+\nu)\left(\frac{\varphi(1-\tau_f)}{(r^f)^2} + \frac{1-\varphi}{(r^f+\nu)^2}\right)\right] + d\kappa\left[-\alpha\delta(1-1/\phi) + (r^f+\delta)\varphi\right] = 0.
\end{aligned}
$$

This is rewritten as follows.

$$dr^f\left[(\rho+\nu)\frac{1-\tau_f}{(r^f)^2}(\alpha\delta(1-1/\phi)-(r^f+\delta)\varphi)-\left(1-(\rho+\nu)\left(\frac{\varphi(1-\tau_f)}{r^f}+\frac{1-\varphi}{r^f+\nu}\right)-\varphi\kappa\right)\right.$$
$$\left.-(r^f+\delta)(\rho+\nu)\frac{1-\varphi}{(r^f+\nu)^2}\right]+d\kappa\left[-\alpha\delta(1-1/\phi)+(r^f+\delta)\varphi\right]=0.$$

Now, we have $(r^f+\delta)\varphi/(\alpha\delta(1-1/\phi))=\varphi Y/(\delta K)$. This is greater than 1, because capital income net of depreciation cost is positive at the steady state. Thus, $(r^f+\delta)\varphi-(\alpha\delta(1-1/\phi))\geq 0$. Then we obtain that the coefficient for $d\kappa$ is positive.

A sufficient condition for the coefficient for dr^f to be negative is

$$1-(\rho+\nu)\left(\frac{\varphi(1-\tau_f)}{r^f}+\frac{1-\varphi}{r^f+\nu}\right)-\varphi\kappa>0,$$

which is guaranteed if $1-(\rho+\nu)\frac{1-\tau_f}{r^f}-\kappa>0$ using (27) and $Y/(\delta K)>0$. Hence, under this condition, the coefficient for dr^f is negative, and we obtain $dr^f/d\kappa\geq 0$. In what follows, we prove that the condition $1-\kappa>(\rho+\nu)(1-\tau_f)/r^f$ holds for small τ_p.

By the definition of κ in (26) and $D=Qr^f/(1-\tau_f)$, we obtain

$$\begin{aligned}1-\kappa&=1-\left(\tau_f-\frac{A}{Q}(\theta_q\tau_f+\theta_m(\tau_f-\tau_m))\right)-\theta_m\tau_p\frac{A}{Q}\frac{1-\tau_f}{r^f}\\&>1-(\tau_f-(\theta_q\tau_f+\theta_m(\tau_f-\tau_m)))-\theta_m\tau_p\frac{A}{Q}\frac{1-\tau_f}{r^f}\\&=1-\theta_f\tau_f+\theta_m\tau_m-\theta_m\tau_p\frac{A}{Q}\frac{1-\tau_f}{r^f}\end{aligned}\tag{28}$$

where we use $A/Q=1+H/Q>1$ in the second line.

The household budget constraint can be aggregated across households at the steady state as $c/a=\mu_a=\theta_q\mu_q+\theta_m\mu_m+\theta_f r^f$. Substituting policy functions, we obtain

$$\rho+\nu=\theta_q\frac{\tau_f r^f}{1-\tau_f}+\theta_m\left(\frac{\tau_f-\tau_m}{1-\tau_f}r^f-\tau_p\right)+r^f.$$

Rearranging terms gives

$$(\rho+\nu)(1-\tau_f)/r^f=(1-\theta_f\tau_f-\theta_m\tau_m)\frac{\rho+\nu}{\rho+\nu+\theta_m\tau_p}.\tag{29}$$

Note that both the right hand sides of (28) and (29) achieve $1-\theta_f\tau_f-\theta_m\tau_f$ when $\tau_p=0$. Thus, the inequality $1-\kappa>(\rho+\nu)(1-\tau_f)/r^f$ holds when $\tau_p=0$. Moreover, both sides of the inequality are continuous in τ_p. Hence, there exists $\bar{\tau}_p>0$ such that the inequality $1-\kappa>(\rho+\nu)(1-\tau_f)/r^f$ holds for any $\tau_p\in(0,\bar{\tau}_p)$. □

Proof of Proposition 2

We show that κ is increasing in τ_m and decreasing in r^f in Equation (26). First, we modify (26) using optimal portfolio rules for θ's as follows:

$$\begin{aligned}\kappa &= \tau_f - \left(\theta_m\left(\tau_f - \tau_m - (1-\tau_f)\frac{\tau_p}{r^f}\right) + \theta_q\tau_f\right)(1+H/Q)\\ &= \tau_f - \left[n\left(\frac{\tau_f-\tau_m}{1-\tau_f} - \tau_p\right)\left(\tau_f - \tau_m - (1-\tau_f)\frac{\tau_p}{r^f}\right) + \frac{\tau_f^2\chi^2}{1-\tau_f}\right]\frac{r^f(1+H/Q)}{\gamma\sigma_q^2\chi^2} \end{aligned} \tag{30}$$

where

$$\frac{H}{Q} = \frac{r^f}{r^f+\nu}\frac{(1-\alpha)(1-1/\phi)}{1-\tau_f}\frac{1}{\varphi - \alpha\delta(1-1/\phi)/(r^f+\delta)}.$$

We observe that an increase in τ_m reduces the gap in financial costs $\tau_f - \tau_m$, and hence increases κ in (30).

To see the effect of r^f on κ in (30), we start by noting that

$$\frac{d}{dr^f}\left[r^f(1+H/Q)\right] = 1 + \frac{H}{Q}\left[1 + \frac{d\log H/Q}{d\log r^f}\right].$$

We want to sign this derivative. Note that

$$\frac{d\log H/Q}{dr^f/r^f} = 1 + \frac{r^f}{r^f+\delta} - \frac{r^f}{r^f+\nu} - \frac{\varphi r^f}{\varphi(r^f+\delta) - \alpha\delta(1-1/\phi)}.$$

The sum of the first three terms is positive. The last term plus 1 is positive, since

$$1 - \frac{\varphi r^f}{\varphi(r^f+\delta) - \alpha\delta(1-1/\phi)} = 1 - \frac{\varphi r^f}{\varphi r^f + \delta/\phi} = \frac{\delta/\phi}{\varphi r^f + \delta/\phi} > 0.$$

Thus, we obtain $(d/dr^f)(r^f(1+H/Q)) > 0$.

In (30), r^f affects κ through four factors: $-(1-\tau_f)\tau_p/r^f$, $\tau_f^2\chi^2$, $r^f(1+H/Q)$, and $1/(\sigma_q^2\chi^2)$. We have just shown that the third term is increasing in r^f. The fourth term $1/(\sigma_q^2\chi^2)$ is also increasing in r^f from (24) and (25). The first and second terms are also increasing in r^f. Since all of these terms contribute to κ negatively, we obtain that κ is decreasing in r^f in (30). From this, we can prove that, when τ_m increases in (27) and (30), steady state r^f must increase. If we suppose otherwise, then κ in (30) is increased by τ_m as well as by a decrease in r^f. Then, from the previous lemma, r^f must increase in (27). This contradicts with our premise that r^f is decreased. Hence, we prove that r^f is increasing in τ_m.

We also note that τ_p increases κ in (30) while it does not directly affect (27). Proceeding similarly as in the previous paragraph, we show that r^f is increasing in τ_p.

Finally, κ is decreasing in n in (30) while it does not directly affect (27). Thus, r^f is decreasing in n. $\square$

B Alternative calibration

Under the benchmark calibration, the steady-state risk-free rate is higher than the target 10-year JGB rate. This disparity can be amended by setting the time discount rate ρ to be as low as 0.005. Also, we reset the risk aversion parameter to $\gamma = 3$. With parameter values set as $\sigma_z = 0.51$, $\tau_f = 0.37$, $\tau_m = 0.05$, and $\tau_p = 0.014$, we can match the target moments as shown in Table 5.

Table 5: Calibration targets and endogenous moments

	σ_q	μ_m	σ_m	θ_q	θ_m	λ_1	$\frac{K}{Y}$	$\frac{C}{Y}$	$\frac{A-H^a}{Y}$	r^f
Target	0.411	0.0393	0.2634	0.1057	0.0465	2.41	2.2	0.56	3.3	0.0198
Model (main)	0.4110	0.0506	0.2712	0.1060	0.0459	2.66	1.8	0.59	4.3	0.0446
Model (alt.)	0.4075	0.0234	0.2645	0.1047	0.0429	2.67	2.2	0.60	8.6	0.0201

Under these parameter values, the model generates steady-state capita-output ratio at $K/Y = 2.2$ and consumption-output ratio at $(\nu + \rho)A/Y = 0.60$. However, the ratio of household financial wealth to GDP becomes as large as 8.6. The match with the wealth to GDP ratio is in a trade-off with the match with JGB rate.

The comparative statics with reduced transaction costs are shown in Tables 6 and 7. We observe that the qualitative patterns of the comparative statics are maintained from the case with benchmark calibration.

Table 6: Comparative statics: prices and output

	r^f	w	Y	K/Y	NI
Baseline	0.0201	0.6869	2.1639	1.4818	1.0952
no τ_p	0.0194	0.6894	2.1795	1.4872	1.0972
no τ_m	0.0201	0.6870	2.1646	1.4820	1.0953
$n \times 4$	0.0201	0.6871	2.1651	1.4822	1.0954

Table 7: Comparative statics: inequality and portfolio

	Labor share	λ_1	Gini	P90/P50	θ_q	θ_m
Baseline	2.6667	0.6272	0.4031	1.4810	0.1047	0.0429
no τ_p	2.4198	0.6283	0.4221	1.5585	0.0915	0.2002
no τ_m	2.6415	0.6272	0.4051	1.4872	0.1040	0.0635
$n \times 4$	2.6038	0.6273	0.4074	1.4943	0.1029	0.1648

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