The Anchoring of Inflation Expectations in Japan: A Learning-Approach Perspective

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The Anchoring of Inflation Expectations in Japan: A Learning-Approach Perspective

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Abstract

This paper employs a model of learning about long-term inflation to jointly estimate long-term inflation expectations and the degree to which they have been anchored to the 2 percent inflation mark over the last half century in Japan. The estimated model shows that long-term inflation expectations declined to about 2 percent in the late 1980s and remained anchored to the 2 percent mark until the mid-1990s. They fell below 2 percent in the late 1990s, which resulted in a low degree of anchoring until the early 2010s. Following the introduction of the price stability target of 2 percent and the launch of Qualitative and Quantitative Monetary Easing in early 2013, inflation expectations rose until early 2015, but have not yet been anchored to the target. A further VAR analysis demonstrates that markups in domestic goods and services markets are one important reason why expectations have not been anchored at 2 percent since the late 1990s.

Keywords: Inflation expectations; Anchoring; Learning

JEL classifications: D83; D84; E31; E58

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1. Introduction

With central banks around the world increasingly having adopted inflation targets or quantitative inflation benchmarks, a widely held view is that anchoring private agents' inflation expectations to the target or benchmark is essential for price stability.\(^1\) The reason is that because of the key role that inflation expectations play in inflation dynamics the effects of shocks to the economy on inflation are mitigated when inflation expectations are firmly anchored. Therefore, as Kuroda (2016) has pointed out, the anchoring of inflation expectations is a prerequisite for a resilient monetary policy framework.

Despite this crucial importance of anchoring inflation expectations, however, research on Japan, which has experienced weak inflation over the past two decades, has found little evidence that inflation expectations are anchored. For example, using survey data, Gasper et al. (2010) found that among a number of other advanced economies, Japan was the only country in which long-term inflation expectations were far from 2 percent. Meanwhile, Castelnuovo et al. (2003) argued that long-term inflation expectations in Japan are sensitive to short-term expectations, suggesting that inflation expectations are not likely to be anchored. More recently, Ehrmann (2015) highlighted the difficulties involved in raising inflation from below the target rate, and showed that the weak anchoring of inflation expectations in Japan has led to a situation in which inflation dynamics are formed in a rather backward-looking manner. These findings were also confirmed in the Bank of Japan's "Comprehensive Assessment" (Bank of Japan, 2016).

However, little is known about how the anchoring of long-term inflation expectations has evolved over time in Japan. Against this background, the aim of this paper is to estimate long-term inflation expectations in Japan over the past fifty years and measure the degree to which they have been anchored to 2 percent using the theoretical model on the formation of long-term inflation expectations through learning developed by Carvalho et al. (2017). Furthermore, using vector autoregression (VAR)

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\(^1\) See, e.g., Bernanke (2007), Trichet (2009), Yellen (2015), and Kuroda (2016).
analysis, we investigate why expectations are not anchored at 2 percent. An inflation target of around 2 percent has been adopted by many major countries since the 1990s and has become the global norm based on many years of experience in countries around the world. Therefore, in our analysis, we examine long-term inflation expectations in Japan and the degree to which they have been anchored using the 2 percent inflation mark as the yardstick.

In general, inflation expectations are defined as "well anchored" if they exhibit little sensitivity to shocks to the economy. This definition, which was summarized, for example, by Bernanke (2007), forms the basis of many empirical and theoretical studies on the anchoring of inflation expectations. The definition has been widely used in empirical studies employing a single equation approach (e.g., Levin et al., 2004; Beechey et al., 2011; and Gürkaynak et al., 2010) or a VAR approach (e.g., Leduc et al., 2007; Davis and Presno, 2014). These studies examine how medium- to long-term inflation expectations respond to actual inflation or short-term inflation expectations, and inflation expectations are regarded as well anchored if their responsiveness or sensitivity is low. Of particular interest in this context is the model recently developed by Carvalho et al. (2017), which incorporates learning into a price-setting model in order to estimate the sensitivity of inflation expectations. Specifically, they depart from the hypothetical framework of full information rational expectations and incorporate private agents' imperfect knowledge with regard to the inflation target into the model. In the model, private agents form long-term inflation expectations by learning from errors in their short-term inflation forecasts. The degree of learning with regard to long-term inflation from the short-term inflation forecast errors—which is referred to as the

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2 Specifically, Bernanke (2007) stated the following: "Long-run inflation expectations do vary over time. That is, they are not perfectly anchored in real economies; moreover, the extent to which they are anchored can change, depending on economic developments and (most important) the current and past conduct of monetary policy. In this context, I use the term "anchored" to mean relatively insensitive to incoming data. So, for example, if the public experiences a spell of inflation higher than their long-run expectation, but their long-run expectation of inflation changes little as a result, then inflation expectations are well anchored. If, on the other hand, the public reacts to a short period of higher-than-expected inflation by marking up their long-run expectation considerably, then expectations are poorly anchored."
learning gain—can switch between two regimes in the spirit of Marcet and Nicolini (2003): a decreasing gain and a constant gain. The learning gain captures the sensitivity of long-term inflation expectations to short-term forecast errors and decreases the longer forecast errors stay within the decreasing-gain regime, so that it can be interpreted as the degree to which inflation expectations are anchored. An advantage of using the model is that the learning gain is time-varying, so that we can examine how the degree to which inflation expectations are anchored has evolved over time. Estimating the model using U.S. inflation and short-term forecasts from professional surveys, Carvalho et al. find that it accurately fits observed measures of long-term inflation expectations.3

This paper applies the model developed by Carvalho et al. (2017) to Japan in order to examine the degree to which inflation expectations have been anchored and, finding that they have not been well anchored, why this is the case. However, our analysis differs from theirs in a number of respects. First, instead of real marginal costs we use the output gap to describe the inflation dynamics in our model. Second, in addition to data on inflation and short-term inflation forecasts, we treat the output gap as observable, and our observation period spans more than half a century. Treating the output gap as observable increases the amount of data in the estimation and helps to increase the precision of the parameter estimates, for example, measured by the dispersion of the posterior distribution, and makes it possible to compare our results to estimates of the Phillips curve using the output gap, which are widely used for modeling inflation dynamics.

Our empirical results can be summarized as follows. First, similar to Carvalho et al.’s results for the United States, our estimates of long-term inflation expectations in Japan are largely consistent with survey data. Moreover, our estimates closely follow the Synthesized Inflation Expectations Indicators presented in the Bank of Japan’s (2016) Comprehensive Assessment, which suggested that developments in inflation

3 Carvalho et al. (2017) also present results for countries other than the United States, including Japan; however, most of the parameter estimates used in the estimation results for other countries closely follow those for the United States. Moreover, their observation period for Japan is 1991 and onward, which is a much shorter horizon than our observation period, which is from 1966 and onward.
expectations since the launch of the Bank's Quantitative and Qualitative Monetary Easing (QQE) in April 2013 could be divided into three phases: a rise in inflation expectations from April 2013 to summer 2014; largely flat expectations from summer 2014 to summer 2015; and weakening inflation expectations from summer 2015.\(^4\)

Second, our estimates suggest that long-term inflation expectations fell to about 2 percent in the late 1980s due to the reverse oil shock and the appreciation of the yen, so that until the mid-1990s the degree of the anchoring of expectations to 2 percent increased. However, in the late 1990s, inflation expectations fell below 2 percent due to negative pressure not only from the real interest rate gap due to the zero lower bound on nominal interest rates and the decline in the natural rate of interest, but also from markups in domestic markets for goods and services, resulting in the low degree of anchoring of inflation expectations until the early 2010.

Third, our estimates suggest that following the introduction of the price stability target of 2 percent in January 2013 and the launch of QQE in April 2013, long-term inflation expectations rose until early 2015 but have not yet been anchored to the target. This finding is in line with the Bank of Japan's (2016) Comprehensive Assessment. While QQE has had a positive effect on inflation expectations, the decline in crude oil prices and weak global economic conditions from summer 2014 to summer 2016 exerted downward pressure on expectations. In addition, since the late 1990s, markups in domestic markets for goods and services have persistently put downward pressure on inflation expectations. These developments so far have prevented inflation expectation from being anchored to the price stability target.

This paper is also related to the empirical literature on trend inflation in Japan. Estimating a New Keynesian Phillips curve with a regime-switching process, Kaihatsu and Nakajima (2015) show that trend inflation declined until the early 2010s. A similar result is obtained by Gemma et al. (2017), who estimate a generalized New Keynesian Phillips curve that highlights the role of trend inflation in inflation dynamics. Meanwhile, estimating a medium-scale New Keynesian model, Saito et al. (2012) show that trend inflation declined steadily from the late 1990s until the 2000s and that markup

\(^4\) For details of the Synthesized Inflation Expectations Indicators, see Nishino et al. (2016).
shocks played a crucial role in the decline in trend inflation. Further, Nishizaki et al. (2014) identify a number of factors that underpin chronic deflation in Japan, such as the zero lower bound on nominal interest rates, public attitudes toward the price level, and a decline in the natural rate of interest. Finally, using forecaster-level survey data for Japan, Hattori and Yetman (2017) estimate a long-term anchor point to which inflation forecasts of experts in the survey would converge in the absence of shocks. They find that the anchor point fell from the late 1990s onward but has risen in recent years, which is similar to our own empirical finding.

The remainder of the paper is organized as follows. Section 2 describes our model of long-term inflation expectations with learning building on Carvalho et al. (2017). Section 3 estimates the model for Japan and the United States and empirically examines the extent to which long-term inflation expectations are anchored at 2 percent. Section 4 uses VAR analysis to investigate what factors prevent inflation expectations in Japan from being anchored at 2 percent. Section 5 concludes.

2. Model

This section describes our model of long-term inflation expectations with learning building on Carvalho et al. (2017). The model relies on Bernanke's (2007) notion of anchored expectations when considering the anchoring of long-term inflation expectations to a target or benchmark.

2.1. Aggregate Supply Equation

In the model, firms have imperfect knowledge about the central bank's inflation target \( \pi^* \). They therefore form long-term inflation expectations \( \pi_t^e \). Under these circumstances, following Preston (2005) and Carvalho et al. (2017), we can derive the following aggregate supply (AS) equation:  

\[ \text{Following a large number of studies on inflation dynamics, we assume a linear relationship between firms' real marginal cost and the output gap to derive the AS equation (1).} \]
\[ \pi_t - \pi_t^p = \hat{E}_t \sum_{T=t}^{\infty} (\delta \lambda)^{T-t} [\hat{r} y_T + \delta (1 - \lambda) (\pi_{T+1} - \pi_{T+1}^p)] + \mu_t, \]  

where \( \pi_t \) is the inflation rate, \( \pi_t^p \) is a reference inflation rate to which firms index the prices of their products, \( y_t \) is the output gap, \( \mu_t \sim \text{i.i.d.} \ N(0, \sigma_\mu^2) \) is a cost push shock, and \( \hat{E}_t \) denotes firms' subjective beliefs.\(^6\) The parameter \( \delta \in (0,1) \) is the subjective discount factor, \( \lambda \in (0,1) \) is the probability of no price optimization (Calvo, 1983), and \( \hat{r} > 0 \) is a composite parameter. The reference inflation rate is given by

\[ \pi_t^p = \gamma \pi_{t-1} + (1 - \gamma) \pi_{t-1}^e, \]  

where \( \gamma \in [0,1] \) is the relative weight of the inflation rate in the previous period, \( \pi_{t-1} \), and \( \gamma \in [0,1] \) is the degree of price indexation to firms' long-term inflation expectations \( \pi_{t-1}^e \).

In the special case of full information rational expectations \( E_t \), all firms know the inflation target \( \pi^* \), that is, \( \pi_{t-1}^e = \pi^* \) in every period \( t \). In this case, the AS equation (1) reduces to the standard equation in the literature on inflation dynamics:

\[ \pi_t = \frac{\gamma}{1 + \gamma \delta} \pi_{t-1} + \frac{\delta}{1 + \gamma \delta} \hat{E}_t \pi_{t+1} + \frac{\hat{r}}{1 + \gamma \delta} y_t + \frac{1}{1 + \gamma \delta} \mu_t. \]  

For simplicity of analysis, the output gap is assumed to be governed by a first-order autoregression process,

\[ y_t = \hat{a} y_{t-1} + \hat{\varepsilon}_t, \]  

where \( \hat{a} \in (0,1) \) and \( \hat{\varepsilon}_t \sim \text{i.i.d.} \ N(0, \sigma_{\hat{\varepsilon}}^2) \) represents output gap shocks.

### 2.2. Actual Law of Motion of Inflation

To obtain the actual law of motion of inflation, we begin by deriving the rational expectations equilibrium process for inflation. In the special case of full information rational expectations, given the AS equation (3) and the output gap equation (4),

\[^6\text{In this section, we normalize the inflation target } \pi^* \text{ to 0 percent, so both the inflation rate } \pi_t \text{ and long-term inflation expectations } \pi_{t-1}^e \text{ can be viewed as the deviation from } \pi^*.\]
inflation follows the following equilibrium process:

\[ \pi_t = (1 - \gamma)\pi^* + \gamma \pi_{t-1} + \alpha y_{t-1} + \varepsilon_t + \mu_t, \tag{5} \]

where \( \varepsilon_t \equiv \kappa \tilde{e}_t \sim \text{i.i.d. } N(0, \sigma^2_e) \), \( \alpha \equiv \kappa \tilde{a} \), and \( \kappa > 0 \) is a composite parameter.

If firms' information about the inflation target is imperfect, their perceived law of motion of inflation is assumed to be based on the rational expectations equilibrium process (5). Specifically, firms' perceived law of motion of inflation takes the following form:

\[ \pi_t = (1 - \gamma)\pi^*_e + \gamma \pi_{t-1} + \alpha y_{t-1} + \eta_t, \tag{6} \]

where \( \eta_t = \varepsilon_t + \mu_t \) and \( \Gamma \) is a composite parameter.

2.3. Learning about Long-term Inflation

Firms' long-term inflation expectations \( \pi^*_e \) are formed endogenously through learning. Specifically, expectations are assumed to follow a random walk with learning process \( k_t^{-1} \times f_{t-1} \) as the drift term:

\[ \pi_t^* = \pi_{t-1}^* + k_t^{-1} \times f_{t-1}, \tag{8} \]

where \( f_t \equiv \pi_t - \hat{E}_{t-1} \pi_t \) denotes errors in short-term forecasts of inflation and \( k_t^{-1} \) is a time-varying parameter representing the learning gain regarding long-term inflation from the short-term forecast errors. From the short-term inflation forecast obtained from the subjective beliefs of firms' perceived law of motion of inflation (6) \( \hat{E}_{t-1} \pi_t \), and the inflation rate \( \pi_t \) obtained from the actual law of motion (7), the forecast errors are given by

\[ f_t = (1 - \gamma)(\Gamma - 1)\pi^*_e + \eta_t. \tag{9} \]

That is, short-term inflation forecast errors consist of two components: systematic errors
arising from long-term inflation expectations $\pi_t^e$ and the composite shock $\eta_t (= \varepsilon_t + \mu_t)$.

The learning gain parameter $k_t^{-1}$ captures to what extent short-term inflation forecast errors $f_{t-1}$ feed into long-term inflation expectations $\pi_t^e$. As in Carvalho et al. (2017), we assume the learning gain $k_t^{-1}$ follows a Marcet and Nicolini (2003) type regime-switching process:

$$k_t = \begin{cases} k_{t-1} + 1 & \text{if } |\hat{E}_{t-1} f_t| \leq \nu \times \sigma_{\eta}, \\ \bar{g}^{-1} & \text{otherwise}, \end{cases} \quad (10)$$

where $\nu > 0$ is a parameter that regulates how alert firms are to model specifications, which we refer to as the forgiveness parameter, $\sigma_{\eta} (= \sqrt{\sigma_\varepsilon^2 + \sigma_\mu^2})$ denotes the standard deviation of the composite shock $\eta_t (= \varepsilon_t + \mu_t)$, and $\bar{g} > 0$ is a constant. The process (10) consists of two regimes: a decreasing gain and a constant gain regime. In the first regime, the learning gain is decreasing, that is, the current gain $k_t^{-1}$ is smaller than the previous gain $k_{t-1}^{-1}$ as long as the expected forecast error $\hat{E}_{t-1} f_t$ is smaller than or equal in size to the threshold $\nu \times \sigma_{\eta}$. When the absolute size of the expected forecast error exceeds the threshold, the learning gain switches to the second regime, in which the gain is constant. In this regime, long-term inflation expectations $\pi_t^e$ reflect more strongly to the composite shock in the previous period, $\eta_{t-1}$, than in the first regime.

It is worth noting that the first regime defines the zone $|\hat{E}_{t-1} f_t| \leq \nu \times \sigma_{\eta}$ in which the sensitivity of long-term inflation expectations $\pi_t^e$ to the composite shock $\eta_t$ is relatively weak. Using the short-term inflation forecast errors (9), this zone can be rewritten as

$$\pi^* - \frac{\nu \times \sigma_{\eta}}{(1 - \gamma)(\Gamma - 1)} \leq \pi^* + \pi_t^e \leq \pi^* + \frac{\nu \times \sigma_{\eta}}{(1 - \gamma)(\Gamma - 1)}. \quad (11)$$

This zone implies that the learning gain is decreasing, that is, the sensitivity of long-term expectations to shocks to the economy weakens, as long as expectations stays within the range (11). This is in line with the notion of anchored expectations proposed.

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7 When taking the expectations of short term inflation forecast error (9), the aforementioned property on long-term inflation expectations $\hat{E}_{t-1} \pi_t^e = \pi_{t-1}^e$ is used.

8 In the derivation of (11), the normalization assumption ($\pi^* = 0$) is used.
by Bernanke (2007). Therefore, our paper refers to the range represented by (11) as the anchor zone, where the anchoring of long-term inflation expectations to the target is likely to be firm.

In sum, the model consists of the output gap equation (4), the actual law of motion of inflation (7), the learning process regarding long-term inflation (8), the short-term inflation forecast errors (9), the learning gain (10), and the composite shock $\eta_t = \varepsilon_t + \mu_t$.

3. Model Estimation

In this section, we estimate the model presented in the preceding section for Japan and the United States. The estimated model is used to investigate not only developments in long-term inflation expectations over the past half-century but also the degree to which expectations were anchored at a level of 2 percent.

3.1. Methodology and Data

Like Carvalho et al. (2017), we estimate the model using Bayesian methods, but our estimation departs from theirs in several respects. First, as mentioned, we use data on the output gap to estimate the model. Treating the output gap as observable increases the number of observations which helps to increase the precision of the parameter estimates and makes it possible to compare our results to estimates of the Phillips curve using the output gap, which are widely used for modeling inflation dynamics. Second, in order to simplify the estimation procedure, the data on short-term inflation forecasts is constructed by linking data from different sources rather than treating them separately.

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9 See footnote 2.
10 Our estimates of long-term inflation and the learning gain do not differ qualitatively when we treat the output gap as a latent state variable.
11 As for the precision of each estimator, six out of nine structural parameters showed less dispersion in the posterior distribution when treating the output gap as an observable compared to the case when it’s not.
in the observation equations. Third and finally, to mitigate the computational burden, the variance-covariance matrix of the model parameters is estimated using the Hessian matrix.

The estimation for Japan treats three variables as observables: the inflation rate \( \pi_t \), short-term inflation forecasts \( \hat{E}_{t-1}\pi_t \), and the output gap \( y_t \). The observation equations are then given by

\[
\begin{bmatrix}
\pi_t^{\text{data}} \\
\hat{E}_{t-1}^{\text{data}} \pi_t \\
y_t^{\text{data}}
\end{bmatrix}
= \begin{bmatrix}
\pi^* \\
\pi_t - f_t \\
y_t
\end{bmatrix}
+ \begin{bmatrix}
0^1_t \\
0^2_t \\
0^3_t
\end{bmatrix},
\]

where the relation \( \hat{E}_{t-1}\pi_t = \pi_t - f_t \) is used and the \( O_t^i \) terms denote observation errors. The novel feature of the estimation is that data on short-term inflation forecasts are used to extract the forecast errors \( f_t \), which are essential in the learning process with regard to long-term inflation (8). The observation period spans more than half a century from 1966:Q2 to 2017:Q2. The inflation target \( \pi^* \) is set to 2 percent throughout the observation period. As mentioned in the introduction, an inflation target of around 2 percent has become more or less the global standard on the basis of long historical experience in a range of countries. In our analysis, we examine long-term inflation and the degree to which it has been anchored using the 2 percent inflation mark as the yardstick.\(^{12}\)

For Japan, the inflation data \( \pi_t^{\text{data}} \) is the quarter-on-quarter change in the CPI (all items less fresh food). The data on short-term inflation forecasts, \( \hat{E}_{t-1}^{\text{data}}\pi_t \), is constructed by linking data from two sources: from the Japan Center for Economic Research (JCER) for 1966:Q2–1989:Q3, and from Consensus Economics for 1989:Q4–2017:Q2.\(^{13,14}\) Developments in inflation and short-term inflation forecasts in Japan are

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\(^{12}\) The value of the inflation target \( \pi^* \) affects the identification of shocks in the model and thus the estimates of long-term inflation expectations and the degree to which expectations are anchored to the target.

\(^{13}\) The data provided by the JCER is an annual survey of short-term economic forecasts, which asked experts about their near-term outlook. The mean forecasts of CPI inflation are converted into quarterly series under the assumption that quarter-on-quarter changes are equally distributed over the forecast fiscal year.

\(^{14}\) Consensus Economics collects forecasts for the calendar year. We take into account the quarterly
shown in Figure 1(a). The output gap data $y^\text{data}_t$ is constructed by applying the Hodrick-Prescott (HP) filter to real GDP data for 1966:Q2–1982:Q4 and linking the series thus obtained with output gap estimates by the Bank of Japan’s Research and Statistics Department for 1983:Q1–2017:Q2.\textsuperscript{15}

The estimation for the United States is similar to that for Japan, except that the output gap is treated as a latent variable. The inflation data $\pi^\text{data}_t$ is the quarter-on-quarter changes in the CPI (all items), while data on short-term inflation forecasts $\hat{E}^\text{data}_{t-1} \pi_t$ is taken from the Livingston Survey for 1966:Q2–1981:Q2 and the Survey of Professional Forecasters (SPF) for 1981:Q3–2017:Q2.\textsuperscript{16} Developments in these data are plotted in Figure 1(b).

Bayesian methods are used to estimate the model parameters $\theta$. The learning gain process (10) introduces non-linearity in the model, so that the particle filter algorithm provided by Schönh et al. (2005) is employed to calculate the likelihood for the model. The number of particles is set to $N = 2,500$ in the particle filter. The prior distributions of the model parameters $\theta$ are chosen on the basis of Carvalho et al. (2017), as shown in Figure 2(a) for the United States and in Figure 3(a) for Japan.\textsuperscript{17} Estimation of the variance-covariance matrix $\Sigma_{\theta}$ of the model parameters is done by inverting the estimated Hessian matrix $\tilde{H}$: $\hat{\Sigma}_{\theta} = \tilde{H}^{-1}$.\textsuperscript{18} Using the particle filter, the realizations of inflation within the forecast year and calculate short-term inflation forecasts for the remaining quarters under the assumption that short-term forecasts are equally paced in the remaining quarters.

For details on the estimation of the output gap by the Research and Statistics Department of the Bank of Japan, see Kawamoto et al. (2017).

The Livingston Survey asks the price level 6 months ahead. Using this information, quarterly inflation forecasts are constructed under the assumption that the forecasts are the same in the two quarters.

Considering the weak anchoring of inflation expectations in Japan compared to the United States, which is found in the literature, we start with a lower mean value for $\nu$ and a higher mean value for $\bar{g}$ in the prior distribution compared to those of the United States.

Carvalho et al. (2017) employ a simulation method to obtain $\Sigma_{\theta}$ due to unstable Hessians, whereas our estimation uses the approximation method proposed by Higham (1988) to obtain a symmetric positive definite Hessian, which can be easily inverted to provide the estimate $\hat{\Sigma}_{\theta}$.\textsuperscript{18}
prior distributions, and the variance-covariance matrix \( \hat{\Sigma}_\theta \), we employ the Metropolis-Hastings (MH) algorithm to construct the posterior distribution of the model parameters \( \theta \). In the MH algorithm, 200,000 draws are generated.\(^{19}\)

### 3.2. Estimation Results

**Parameter Estimates**

The posterior distributions of the model parameters for the United States and Japan are shown in Figures 2 and 3, respectively. The posterior distributions for the United States are close to those obtained by Carvalho et al. (2017). The posterior mean of the relative weight on past inflation \( \gamma \) is close to zero. This is consistent with the estimation result of Cogley and Sbordone (2008), who find that when drifting trend inflation is introduced in their New Keynesian Phillips curve, intrinsic inflation inertia plays no role in U.S. inflation dynamics.

The posterior distributions for Japan show that the mean estimate of the relative weight on past inflation \( \gamma \) is around 0.4, which demonstrates the presence of intrinsic inflation inertia in Japan. This result is consistent with those obtained by Levin and Piger (2003) and Benati (2008), who report higher persistence of inflation in Japan than in other economies.

Turning to the model parameters pertaining to the anchor zone, the estimate of the standard deviation of each shock \( \sigma_\varepsilon \) and \( \sigma_\mu \) is similar for the United States and Japan, while that of the forgiveness parameter \( \nu \) is larger in the United States. This implies that the anchor zone is wider in the United States than in Japan.

**Estimates of Long-term Inflation Expectations and the Learning Gain**

Panel (a) of Figure 4 displays the estimates of long-term inflation expectations in the

\(^{19}\) Our estimation follows Carvalho et al. (2017) and avoids injecting randomness in the calculation of the likelihood by fixing the following innovations when performing the MH algorithm: random initial conditions, random draws to compute shocks in the prediction step, and random draws in the resampling step.
United States (black solid line) and the 10-year inflation expectations in the SPF (red circles). The figure shows that even though our estimation does not use any data on long-term inflation expectations, such as those from the SPF, our estimates are very close to the long-term inflation expectations in the survey data. Our estimates are also consistent with those obtained by Carvalho et al. (2017). The blue shaded area around the 2 percent inflation mark shown by the green dashed line shows the anchor zone given by eq. (11). The estimated long-term inflation expectations were well above the anchor zone in the 1970s and 1980s, but entered the anchor zone in the second half of the 1990s and have stayed within the zone since. This is consistent with Yellen's (2015) observation that long-term inflation expectations have been anchored since the mid-1990s and have remained stable during and after the financial crisis of 2008. Recall that when long-term inflation expectations lie in the anchor zone, the learning gain is in the decreasing-gain regime, and the degree of anchoring to the 2 percent inflation benchmark increases the longer expectations stay within the zone.

Estimates of the learning gain \(k_t^{-1}\) are shown in panel (b) of Figure 4. Reflecting the developments in long-term inflation expectations, the learning gain entered the decreasing-gain regime in the second half of the 1990s, and the degree of anchoring the inflation expectations to the 2 percent level has increased since then. The probability bands of the learning gain indicate that the financial crisis of 2008 led to a hike in the risk of the de-anchoring of inflation expectations, but this risk has dissipated in recent years.

Next, the estimation results for Japan are shown in Figure 5. Panel (a) of the figure displays the estimated long-term inflation expectations in Japan, \(\pi_t^e\) (black solid line), the 6–10 year inflation expectations in the Consensus Forecasts (red circles), the 2 percent inflation mark (green dashed line), and the anchor zone (blue shaded area). Our estimates of inflation expectations are largely consistent with inflation expectations in the survey data, such as those from Consensus Forecasts, as in the case of the United States. Estimated long-term inflation expectations were well above the anchor zone in the 1970s, but gradually declined to enter the anchor zone in the late 1980s and stayed within the zone until the mid-1990s. However, inflation expectations then fell below the anchor zone in the late 1990s and remained well below the zone until the early 2010s.
Following the introduction of the price stability target of 2 percent in January 2013 and the launch of QQE in April 2013, expectations rose to re-enter the anchor zone in 2014 but then slipped again and have remained slightly below the zone.

Figure 5(b) shows the estimates of the learning gain \( k_{t-1} \). Reflecting the developments in long-term inflation expectations, the learning gain entered the decreasing-gain regime in the late 1980s and the degree of the anchoring of inflation expectations to the 2 percent level increased until the mid-1990s. The gain switched to the constant-gain regime in the late 1990s and stayed in that regime until the early 2010s. The introduction of the price stability target and the launch of QQE in early 2013 led to subsequent swings in the learning gain, that is, switches between the constant- and decreasing-gain regimes.

3.3. Comparison with the Bank of Japan's (2016) Comprehensive Assessment

This section compares our results with those in the Bank of Japan's (2016) Comprehensive Assessment.

*Developments in Long-term Inflation Expectations*

To assess developments in inflation expectations in Japan, the Comprehensive Assessment presents the Synthesized Inflation Expectations Indicators (SIEIs). These indicators are built based on the first principal component extracted from three separate indicators of inflation expectations of firms, households, and experts. As shown in Figure 6(a), the SIEIs indicate that developments in inflation expectations since the launch of QQE can be divided into three phases. In phase 1, which is the phase from the launch of QQE in April 2013 to summer 2014, inflation expectations rose. In phase 2 from summer 2014 to summer 2015, expectations remained largely unchanged. Finally, in phase 3 from summer 2015, inflation expectations weakened.

Figure 6(a) compares our estimate of long-term inflation expectations with the SIEIs. Our estimates are close to the SIEIs and similarly indicate that developments in inflation expectations fall into three phases. Specifically, estimates suggest that long-term
inflation expectations rose following the launch of QQE and entered the anchor zone (phase 1), remained stable for a while in the anchor zone (phase 2), and then slipped, so that they are currently slightly below the anchor zone.

Adaptive Formation of Long-term Inflation Expectations

Another key finding of the Comprehensive Assessment was that long-term inflation expectations in Japan have been formed in a more adaptive manner than those in the United States, the Euro area, and the United Kingdom. The finding was obtained by regressing actual inflation on the 6–10 year inflation expectations in the Consensus Forecasts. The regression results from the Comprehensive Assessment are reproduced in Figure 7(a), which shows that for Japan the estimated coefficient on actual inflation is 0.38 and significant at the 1 percent level, which is much higher than that for the United States, which is around 0.05 and statistically insignificant.

Our estimates of the learning gain are in line with this result in the Comprehensive Assessment, as shown in panel (b) of Figure 7. The learning gain measures the sensitivity of long-term inflation expectations to shocks to the economy, and our estimates indicate that the average learning gain over the corresponding period is 0.29 for Japan, but only 0.02 for the United States.

3.4. Contributions of Structural Shocks to Long-term Inflation Expectations

Our model contains two types of structural shocks: output gap shocks and cost push shocks. This means we can calculate the contribution of the two types of shocks to estimated long-term inflation expectations and actual inflation.

Panel (a) of Figure 8 shows the contribution of output gap shocks and cost push shocks to inflation expectations. The figure shows that the contribution of cost push shocks is larger than that of output gap shocks. On the other hand, panel (b) of the figure presents the contribution of the two shocks to actual inflation. The figure suggests that the contribution of cost push shocks is almost the same as that of output gap shocks.

As just shown, long-term inflation expectations can be decomposed into two
structural shocks in the model: output gap shocks and cost push shocks. However, it is not clear how these shocks relate to the forces which drive fluctuations in the Japanese economy. Therefore, in the next section, we focus on this point and conduct a VAR analysis.

4. VAR Analysis

This section presents our VAR analysis to investigate the reasons that prevent the anchoring of long-term inflation expectations at 2 percent in Japan.

4.1. Methodology and Data

Methodology

The preceding section provided a decomposition of estimated long-term inflation expectations into the contribution of output gap shocks and cost push shocks. The present section presents a VAR analysis designed to further decompose inflation expectations using variables that have a close link with the two types of shocks and characterize economic fluctuations in Japan.

The VAR specification consists of seven variables, which include two variables related to the output gap—the real interest rate gap and world industrial production—and three variables pertaining to cost push shocks—the markup rate, crude oil prices, and the exchange rate—in addition to the contribution of the two types of shocks to long-term inflation expectations obtained in Section 3.4. The real interest rate gap is defined as the gap between the real and the natural rate of interest and is chosen because canonical New Keynesian models include an intertemporal IS equation, where the real interest rate gap is the primary driver of the output gap (e.g., Galí, 2015). World industrial production represents global economic conditions, which influence the output gap through exports and imports. The markup rate, crude oil prices, and the exchange rate all have a direct effect on prices set by firms in Japan, so they are likely to be closely linked to cost push shocks.
Turning to the lag length in the VAR specification, following Kilian (2009) and Kilian and Lutkepohl (2017), we include eight quarter lags so as to capture the second-round effects arising from fluctuations in crude oil prices. Moreover, when calculating the contribution of output gap shocks and cost push shocks to estimated long-term inflation expectations, we assume them to be independent of each other. Thus, the VAR is constructed so that it constitutes two blocks, one containing the three variables related to output gap shocks and another containing the four variables pertaining to cost push shocks. We estimate the VAR by blocks and identify shocks using Cholesky decomposition (see the Appendix for more details).

Data

Figure 9 shows developments in the 5 variables that were added in the VAR. For world industrial production, we use the Composite Leading Indicators from the OECD for 1966:Q2–1990:Q4 and world industrial production (excluding construction) from the CPB Netherlands Bureau for Economic Policy Analysis for 1991:Q1–2017:Q2. The data on crude oil prices is calculated by deflating the WTI index with the U.S. CPI (all items). The exchange rate data is the nominal effective exchange rate (narrow basis) released by the Bank for International Settlements.

Next, for the calculation of the real interest rate gap, we use three series: the nominal interest rate, the expected inflation rate, and the natural rate of interest. The nominal interest rate data is constructed by linking the Bank of Japan's official discount rate for 1966:Q2–1994:Q4, the uncollateralized overnight call rate for 1995:Q1–1998:Q4, and Krippner's (2013) shadow rate for 1999:Q1–2017:Q2. The reason for using the shadow rate is that nominal interest rates in Japan have been at or near the zero lower bound since 1999, and reflecting this situation in Japan and, more recently, elsewhere, some scholars have used shadow rates in an attempt to overcome the empirical difficulties in measuring the effects of monetary policy around the zero lower bound. Our results remain essentially unchanged when using other measures for the shadow rate such as the estimates by Ueno (2017).
bound. For expected inflation, we use the short-term inflation forecasts employed in the model estimation presented in the preceding section. Finally, the natural interest rate data is constructed using the method developed by Laubach and Williams (2003).

Following the literature (e.g., Rotemberg and Woodford, 1995), the markup rate is constructed as the inverse of the labor share and the data is based on the Financial Statements Statistics of Corporations by Industry (Quarterly) from the Ministry of Finance. Specifically, we use the markup rate in retail and services sector, given that price setting in these sectors has a direct effect on the CPI.

4.2. Shocks and Impulse Responses in the Estimated VAR

Before presenting the decomposition of long-term inflation expectations based on the VAR, we examine shocks and impulse responses in the estimated VAR.

Figure 10 shows shocks identified in the estimated VAR. The shocks well capture key events regarding Japan's economy. For example, panel (b) of the figure displays shocks to the real price of crude oil, which exhibited a spike during the oil crisis of 1973, a fall during the reverse oil shock of 1985, and a decline after summer 2014. Panel (c) shows that there were both positive shocks to the exchange rate, such as in 1985, when Japan experienced a rapid appreciation of the yen following the Plaza Accord, as well as

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21 Recent studies using the shadow rate to measure the effects of unconventional monetary policy include Wu and Xia (2016) in a factor-augmented VAR and Iwasaki and Sudo (2017) in local projections. Meanwhile, Francis et al. (2014) propose to overcome the measurement difficulties of monetary policy efficacy associated with the zero lower bound by linking the policy rate and the shadow rate and using a sufficiently long observation period to measure the effects of monetary policy at the zero lower bound.

22 The estimates of the natural rate of interest for Japan using Laubach and Williams's (2003) method are based on Fujiwara et al. (2016).

23 The labor share is calculated as personnel expenses divided by value added (the sum of personnel expenses, ordinary profits, depreciation expenses, and interest expenses). We have conducted an Augmented Dicky-Fuller test on the markup rate (with a constant term and lag selection based on AIC) and the results indicated that null hypothesis of a unit root was rejected at the 10 percent significance level.
negative ones, such as the large depreciation of the yen in late 2012 and late 2014. In panel (d), we can see several adverse shocks to the markup rate in the retail and services sector since the late 1990s stemming from globalization and the rise of e-commerce, which we will return to later. Panel (e) shows that there were several positive shocks to the real interest rate gap from the late 1990s to the early 2010s, indicating the severity of the zero lower bound on nominal interest rates and declines in the natural rate, as well as negative ones following the launch of QQE in early 2013.

Turning to the impulse responses, Figure 11 shows how long-term inflation expectations respond to shocks in the estimated VAR. The signs of the responses are consistent with theoretical predictions. For example, in panel (a), long-term inflation expectations (via the impact on output gap shocks) respond positively to a shock that improves global economic conditions, and negatively to a shock to the real interest rate gap representing monetary tightening. In panel (b), we can also see that long-term inflation expectations (via the impact on cost push shocks) respond positively to shocks that raise the real price of crude oil and the markup rate, and negatively to shocks that represent an appreciation of the yen and monetary tightening.

4.3. VAR-based Historical Decomposition of Long-term Inflation Expectations

Next, we use our VAR analysis to conduct a historical decomposition of long-term inflation expectations in Japan. The results are shown in Figure 12 and have several notable features. First, inflation expectations entered the anchor zone in the late 1980s due to the reverse oil shock and the appreciation of the yen following the Plaza Accord in the mid-1980s, and remained within the anchor zone until the mid-1990s. Second, from the late 1990s through the early 2010s, long-term inflation expectations were below the anchor zone, dragged down not only by the real interest rate gap due to the

24 We checked the robustness of the results of the VAR analysis using the potential growth rate estimated by the Bank of Japan's Research and Statistics Department (see Kawamoto et al. (2017) for details) as an alternative measure for the natural rate of interest. The results are shown in Figure 13, which is hardly different from Figure 12.
zero lower bound on nominal interest rates and the declining natural interest rate, but also by persistent downward pressure on markups in the domestic goods and services market. Third, positive effects from the real interest rate gap owing to the introduction of the price stability target of 2 percent and the launch of QQE in early 2013, along with the depreciation of the yen linked to this, shifted up inflation expectations into the anchor zone amid continuing negative pressure from markups in the domestic goods and services market. Yet, inflation expectations subsequently weakened again and have remained slightly below the zone due to the decline in crude oil prices and weak global economic conditions from summer 2014 to summer 2016.

The persistent negative contribution of markups can be viewed as reflecting a variety of forces that have affected the degree of price competition in Japan over time. For example, the early 1990s were a period when international production chains started to develop and relatively inexpensive products from Asia began to enter the Japanese market. This period also saw substantial deregulation in retail laws, which led to a considerable increase in the number large discount stores and road side chain stores.25 By the mid-1990s, the establishment of international production chains had made considerable headway and newly opened large stores strongly increased their sales shares by selling low-priced products from Asia. These developments affected the degree of competition in Japan's retail and service sectors. For example, according to a survey conducted by the Bank of Japan's Research and Statistics Department (2000), about 90 percent of firms listed on the First Section of the Tokyo Stock Exchange expressed concern that the degree of competition had increased since the mid-1990s due to factors such as deregulation. This increased competition likely made markup pricing more difficult for firms, with some having to squeeze their markups in order to maintain market share. The rise of e-commerce since the 2000s likely also contributed to the

25 The Large-scale Retail Stores Law was successively amended throughout the 1990s. For example, in 1994, owners of large stores were allowed to change tenants within their stores and mid-sized retailers could open a store without reporting to the Ministry of International Trade and Industry (MITI; now Ministry of Economy, Trade and Industry). In the 1980s, the average number of large stores that were opened per year was 560. As a result of deregulation, this number tripled to 1,616 stores in the early 1990s and further increased to 1,922 stores in the late 1990s.
tough competitive environment. According to a report by the Ministry of Economy, Trade and Industry (2017), the share of online purchases in Japan has risen from 0.2 percent in 2000 to 6 percent in 2016. Empirical evidence shows that the increase in the share of online purchases puts downward pressure on prices (Choi and Yi, 2005; Lorenzi and Verga, 2014) due to increased price competition. The increasing trend toward e-commerce therefore likely added to firms' reluctance to raise prices out of fear of losing customers.

5. Concluding Remarks

This paper employed the model of learning with regard to long-term inflation developed by Carvalho et al. (2017) to jointly estimate long-term inflation expectations and the degree to which they have been anchored at 2 percent over the last half century in Japan. In order to investigate why expectations have not been anchored at 2 percent, we also conducted a historical decomposition of the estimated long-term inflation expectations into factors that have affected Japan's economy using a VAR.

The estimated long-term inflation expectations are not only largely consistent with those obtained from survey data but also close to the Synthesized Inflation Expectations Indicators presented in the Bank of Japan's (2016) Comprehensive Assessment. The estimated degree to which inflation expectations are anchored to the 2 percent mark rose from the late 1980s through the mid-1990s following the reverse oil shock and appreciation of the yen in the mid-1980s. However, negative pressure not only from the real interest rate gap due to the zero lower bound on nominal interest rates and the declining natural interest rate but also from markups in domestic goods and services markets led to a fall in long-term inflation expectations in the late 1990s, which resulted in a low degree of anchoring until the early 2010s. Despite continuing negative pressure from markups, the degree of anchoring increased until early 2015 following the introduction of the price stability target of 2 percent and the launch of QQE in early 2013. However, the decline in crude oil prices and weak global economic conditions from summer 2014 through summer 2016 pushed down inflation expectations once again, which as a result have since remained slightly below the anchor zone.
Looking ahead, although the negative pressure from markups will presumably remain, it is likely that upward pressure from the continuation of QQE, the recent rise in crude oil prices, and healthy global economic conditions will start to increase the degree of anchoring of long-term inflation expectations to the price stability target once again.
References

Bank of Japan (2016) "Comprehensive Assessment: Developments in Economic Activity and Prices as well as Policy Effects since the Introduction of Quantitative and Qualitative Monetary Easing (QQE)."


Fujiwara, S., Y. Iwasaki, I. Muto, K. Nishizaki, and N. Sudo (2016) "Supplementary
Paper Series for the "Comprehensive Assessment" (2): Developments in the Natural Rate of Interest in Japan." Bank of Japan Review, No. 16-E-12.


Appendix: Coefficient Restrictions in the VAR

This appendix provides details of the VAR in Section 4 used in the decomposition of long-term inflation expectations. First, we group the seven variables $Y_t$ included in the VAR into two blocks, A and B. Block A consists of three variables: the contribution of the output gap shock to long-term inflation, calculated in Section 3.4, and of two variables related to fluctuations in the output gap, namely, the real interest rate gap and world industrial production. Block B consists of four variables: the contribution of the cost-push shock to long-term inflation, calculated in Section 3.4, and of three variables related to cost push shocks, namely, the markup rate, crude oil prices, and the exchange rate. Second, recall that in Section 3 we assumed that the output gap shock and the cost push shock were mutually independent when calculating the contribution of these shocks to long-term inflation expectations. Therefore, we limit the transmission among the aforementioned two blocks, by imposing the following coefficient restrictions in the reduced form VAR:

$$Y_t = \nu + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + U_t.$$ 

That is, for each element in the coefficient matrix $A_k$ of lag $k = 1, \ldots, p$, we set $A_k(i,j) = a_{k,i,j}$ if variables corresponding to row $i$ and column $j$ are in the same block and otherwise $A_k(i,j) = 0$. This is equivalent to estimating the VAR by blocks. Further, we identify the structural shocks with Cholesky decomposition, ordering the variables in the seven variable VAR as follows; (i) world industrial production, (ii) crude oil prices, (iii) the exchange rate, (iv) the markup rate, (v) the real interest rate gap, (vi) the contribution of output gap shocks to long-term inflation expectations, and (vii) the contribution of cost push shocks to long-term inflation expectations.
Figure 1. Inflation and Short-term Inflation Forecasts

(a) Japan

(b) United States

Sources: Bureau of Labor Statistics; Federal Reserve Bank of Philadelphia; Consensus Economics; Ministry of Internal Affairs and Communications; Japan Center for Economic Research; Bank of Japan, etc.
Figure 2. Estimated Parameters for United States

(a) Prior and Posterior Distribution of Parameters

<table>
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<th></th>
<th>Distribution</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Mean</th>
<th>5 percent</th>
<th>95 percent</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>Gamma</td>
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<td>0.040</td>
<td>0.054</td>
<td>0.043</td>
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<td>$\tilde{g}$</td>
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<td>$\gamma$</td>
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<td>$\sigma_\varepsilon$</td>
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<td>0.306</td>
<td>0.372</td>
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<td>$\sigma_\mu$</td>
<td>Inv.-Gamma</td>
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<td>0.500</td>
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<td>0.248</td>
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<td>$\sigma_{o,1}$</td>
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<td>$\sigma_{o,2}$</td>
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<td>0.500</td>
<td>0.100</td>
<td>0.079</td>
<td>0.124</td>
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(b) Posterior Distribution of Parameters
Figure 3. Estimated Parameters for Japan

(a) Prior and Posterior Distribution of Parameters

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<th>Prior Distribution</th>
<th>Posterior Distribution</th>
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<td></td>
<td>Distribution</td>
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<td>Gamma</td>
</tr>
<tr>
<td>( \tilde{g} )</td>
<td>Gamma</td>
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<td>( \gamma )</td>
<td>Beta</td>
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<td>( \Gamma )</td>
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<td>( \bar{\alpha} )</td>
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<td>( \sigma_{\varepsilon} )</td>
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<td>( \sigma_{\mu} )</td>
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<tr>
<td>( \sigma_{o,1} )</td>
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<td>( \sigma_{o,2} )</td>
<td>Inv.-Gamma</td>
</tr>
<tr>
<td>( \sigma_{o,3} )</td>
<td>Inv.-Gamma</td>
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</table>

(b) Posterior Distribution of Parameters

![Graphs of posterior distributions for different parameters]
Notes: The shaded areas in panel (b) shows the 5-95, 20-80, and 35-65 percentiles of the posterior distribution of the learning gain. The solid line indicates the median of the posterior distribution.
Notes: The shaded areas in panel (b) shows the 5-95, 20-80, and 35-65 percentiles of the posterior distribution of the learning gain. The solid line indicates the median of the posterior distribution.
Sources: Ministry of Internal Affairs and Communications; Japan Center for Economic Research; Cabinet Office; Consensus Economics; Bank of Japan.
Figure 6. Comparison with the Comprehensive Assessment (1)

(a) Long-term Inflation Expectations and Synthesized Inflation Expectations Indicators

(b) Learning Gain

Notes: 1. The Synthesized Inflation Expectations Indicators are obtained through the principal component analysis based on firms', households', and experts' inflation expectations. For details of the indications, see Nishino et al. (2016).

2. The shaded areas in panel (b) shows the 5-95, 20-80, and 35-65 percentiles of the posterior distribution of the learning gain. The solid line indicates the median of the posterior distribution.

Sources: Consensus Economics; QUICK; Bloomberg; Bank of Japan, etc.
Figure 7. Comparison with the Comprehensive Assessment (2)

(a) "Mechanism of Inflation Expectations Formation in Advanced Economies" in the Comprehensive Assessment

\[ \pi_t^e = \theta \pi_{t-1} + (1 - \theta) \pi^* + \epsilon_t \]

<table>
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<th>Inflation expectations 6-10 years ahead</th>
<th>Observed inflation rate in the previous period</th>
<th>Inflation target (2%)</th>
<th>Residual</th>
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</thead>
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<td>Japan</td>
<td>U.S.</td>
<td>Euro area</td>
</tr>
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<td>Estimation using CPI all items as observed inflation indicators</td>
<td>0.38 ***</td>
<td>0.05</td>
<td>0.03 ***</td>
</tr>
<tr>
<td>Estimation using CPI all items less fresh food as observed inflation indicators</td>
<td>0.38 ***</td>
<td>0.08</td>
<td>0.05 ***</td>
</tr>
</tbody>
</table>

(b) Learning Gain

Notes: 1. The asterisks (*** in the table denote statistical significance at 1 percent for each coefficients.
2. The shaded areas in panel (b) shows the 5-95, 20-80, and 35-65 percentiles of the posterior distribution of the learning gain. The solid line indicates the median of the posterior distribution.
Sources: Bank of Japan, etc.
Figure 8. Shock Decomposition

(a) Long-term Inflation Expectations

(b) Inflation

Notes: 1. The figures decompose the difference between the estimated long-term inflation expectations/inflation and 2% by the effect of each shock in the model.

2. The differences between the CPI data and the estimated inflation rates are observation errors.

Sources: Ministry of Internal Affairs and Communications, etc.
Figure 9. Data in the VAR Analysis

(a) Global Economic Condition

(b) Real Price of Crude Oil

(c) Exchange Rate

(d) Markup Rate of Retail and Services Sectors

(e) Real Interest Rate Gap

Sources: BIS; OECD; Federal Reserve Bank of St. Louis; Reserve Bank of New Zealand; CPB; Consensus Economics; Ministry of Finance; Japan Center for Economic Research; Bank of Japan, etc.
Figure 10. Shocks Identified in the VAR

(a) Global Economic Condition

(b) Real Price of Crude Oil

(c) Exchange Rate

(d) Markup Rate of Retail and Services Sectors

(e) Real Interest Rate Gap

(f) Output Gap Shock (Other)

(g) Cost Push Shock (Other)
Figure 11. Impulse Responses

(a) Responses of Long-term Inflation Expectations (via the Impact on Output Gap Shocks)

1. Global Economic Condition
   ann., % points

2. Real Price of Crude Oil
   ann., % points

3. Exchange Rate
   ann., % points

4. Markup Rate
   ann., % points

5. Real Interest Rate Gap
   ann., % points

6. Output Gap Shock (Other)
   ann., % points

7. Cost Push Shock (Other)
   ann., % points

(b) Responses of Long-term Inflation Expectations (via the Impact on Cost Push Shocks)

1. Global Economic Condition
   ann., % points

2. Real Price of Crude Oil
   ann., % points

3. Exchange Rate
   ann., % points

4. Markup Rate
   ann., % points

5. Real Interest Rate Gap
   ann., % points

6. Output Gap Shock (Other)
   ann., % points

7. Cost Push Shock (Other)
   ann., % points

Notes: 1. The figures show the impulse responses of long-term inflation expectations, via the output gap shocks' or the cost push shocks' obtained in section 3.4. Responses are for a plus 1 standard deviation shock.
   2. The shaded areas show the 90% confidence intervals.
Figure 12. Historical Decomposition of Long-term Inflation Expectations

(a) From the Early 1980s Onward

Notes: 1. The figures decompose the difference between the estimated long-term inflation expectations and 2% by the effect of each shock in the VAR.
2. The shaded areas shows the periods when long-term inflation expectations were within the anchor zone.

(b) From 2012 Onward
Notes: 1. This historical decomposition is obtained from the VAR estimated using the potential growth rate of the Bank of Japan’s Research and Statistics Department as an alternative measure of the natural rate of interest.
2. The figures decompose the difference between the estimated long-term inflation expectations and 2% by the effect of each shock in the VAR.
3. The shaded areas shows the periods when long-term inflation expectations were within the anchor zone.