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Consumers' Price Beliefs, Central Bank Communication, and Inflation Dynamics*

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Abstract

Many developed economies in recent years have been characterized by a tight labor market and a low inflation environment, a phenomenon referred to as "missing inflation." To explain this phenomenon, we develop a dispersed information model in which consumers' search for cheaper prices affects firms' pricing behavior. The model shows that firms are reluctant to pass through cost increases because they fear a disproportionate decline in their sales. A history of low and stable inflation amplifies this effect by decreasing consumers' inflation beliefs. In this case, enhancement of the central bank's communication regarding its inflation target more firmly anchors consumers' inflation beliefs and makes the Phillips curve flatter, while enhancement of the central bank's communication about the current aggregate price level has the opposite effect.

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1 Introduction

Inflation dynamics have received considerable attention in the macroeconomic literature. Recently, the "missing inflation puzzle" — the coexistence of a tight labor market and low inflation in many developed economies — has been of central interest to economists and has led them to reconsider the relationship between inflation and real activities.[1] One hypothesis regarding the missing inflation puzzle is that private-sector inflation expectations have been anchored by central bank communication (Bernanke 2010; Yellen 2013). If the central bank announces its inflation target and people believe that the central bank is able to achieve it, then their inflation expectations will be anchored to the target level, and inflation itself will be close to the target as a result. In fact, in recent years, this phenomenon of anchored expectations and missing inflation has been observed simultaneously in many advanced economies.[2] However, the experience of some countries is not consistent with the hypothesis that missing inflation is the result of well-anchored inflation expectations. A prominent example is Japan. In 2013, the Bank of Japan increased its inflation target from around 1 percent to 2 percent. Despite this policy change and subsequent aggressive monetary easing, inflation in Japan has remained below the target. Japanese firms have been very reluctant to increase prices even when their production costs (such as wages and input prices) have increased. This raises the question why Japanese firms have been reluctant to raise prices.

The answer that we propose in this study is inspired by Blinder et al.'s (1998) survey study, which showed that firms hesitate to increase prices for fear of losing their customer stock.[3] Our hypothesis is that firms' fear of losing customers is closely linked to consumers' beliefs about the prices charged by other firms, or the aggregate price level in general. Suppose that a shop manager is wondering whether to increase the prices of his products. If his customers believe that prices are low and unchanged everywhere else, they will be more likely to start looking for cheaper prices elsewhere, so that the manager will be more likely to lose customers.[4] In contrast, if customers believe that prices are going to increase everywhere, they will not be more likely to search for cheaper prices elsewhere, and as a result it is easier for the shop manager to increase prices. Consumers' beliefs about aggregate prices depend not only on the central bank's inflation target but also, more importantly, on their

[1] See, e.g., Krugman (2014), Constancio (2015), and Blanchard, Cerutti, and Summers (2015).

[2] See International Monetary Fund (2013) and Stevens (2013).

[3] Moreover, various studies argue that inertial price adjustment is due mainly to consumer behavior (the demand side) rather than factors on the firm side (the supply side). See, for example, Nakamura and Steinsson (2011) and L'Huillier and Zame (2015).

[4] A large number of studies capture firms' fear of losing customers by incorporating the customer base into their models (Phelps and Winter 1970; Bils 1989; Kimura 2013; Gourio and Rudanko 2014; and Gilchrist et al. 2017). By contrast, this study considers firms' fear of losing customers by introducing consumer price search into the model.

experience regarding past inflation. Indeed, some empirical research using disaggregated data indicates that people do not necessarily pay attention to the central bank's inflation target.[5] In addition, numerous studies have shown that consumers' expectations depend on their own inflation experience (Marcet and Sargent 1989; Malmendier and Nagel 2016; Diamond, Watanabe, and Watanabe 2018).

In this paper, we formalize the above hypothesis. We analyze theoretically how people's inflation experience anchors their beliefs about the aggregate price level and affects current aggregate inflation, particularly when their beliefs are not firmly anchored to the inflation target. We focus our analysis on the interaction between consumers' search behavior under information frictions and the pricing decisions of firms worried about losing customers.[6] We also draw some implications for central bank communication.

Framework. We develop a general equilibrium model in which consumers' search for cheaper prices affects firms' pricing behavior. In order to explain price search, we consider a dispersed information model in which consumers do not necessarily know all prices in the economy unless they choose to conduct a price search.

The economy consists of a unit mass of islands and there are three types of agents: households, firms, and a central bank. On each island, there are a representative household and a firm. In order to make the model tractable, we assume that a household consists of a worker and a consumer. The worker supplies labor, while the consumer makes decisions on spending and price search. A key variable in our model is the consumer's beliefs on the current aggregate price level, which in the remainder of the analysis we refer to as "*price beliefs.*" There are aggregate and island-specific productivity shocks. Goods are homogeneous across islands and to simplify our exposition, we assume that there is one representative good in the economy. However, the market for that good is segmented across islands. Due to island-specific productivity shocks, the price of the good differs across islands. A consumer on a certain island observes the price of the good on that island without incurring any costs but incurs search costs to find out the price of the good on another island. In other words, the consumer does not have perfect information about the prices on other islands (and hence about the aggregate price level). By observing the price on his island and other signals about the aggregate price level (to be specified later) and combining these observations with his prior beliefs about the general price level, the consumer updates his beliefs on the aggregate price level. We assume Bayesian updating. Based on his updated beliefs, the consumer makes his price search decision. The firm on each island sets its price taking the consumer's

[5] See Kumar et al. (2015) and Coibion, Gorodnichenko, and Kumar (2018).

[6] Coibion, Gordnichenko, and Weber (2019), Abe and Ueno (2016), and Ichiue, Koga, Okuda, and Ozaki (2019) show that the anchoring effects of the inflation target may be reduced by a lack of information.

search decisions as given. The central bank controls aggregate nominal income to stabilize inflation and the output gap.

Main results. Given the setting just described, assume, for example, that the price of the good on a certain island increases. If the consumer on that island believes that the prices on other islands remain low, he is likely to incur the search cost and look for a cheaper price. We show that this consumer behavior creates a quasi-kink in the demand curve facing the firm. In other words, the demand for the good that the firm produces may decrease disproportionately if it increases its price. Facing such a quasi-kinked demand curve, the firm is reluctant to increase its price even if it experiences an increase in its production cost, as shown by Kimball (1995).

One of the contributions of our study is that it shows that, since the quasi-kinked demand curve in our model stems from the consumer's search behavior, the location and the degree of the kink in the demand curve are endogenously determined by the consumer's beliefs about the aggregate price level, which in turn is determined by his prior belief and the signals about the aggregate price level he receives. More importantly, the kink depends on monetary policy, because it affects the consumer's price beliefs. The kink also depends the consumer's beliefs about the central bank's inflation target.

Our model implies that when consumers have experienced a long period of low and stable inflation, the slope of the Phillips curve — the equilibrium relationship between aggregate inflation and changes in aggregate production costs — becomes flat. This implies that prices do not increase significantly even if costs increase. This is our explanation for the "missing inflation." We show that this result holds when consumers are confident about their beliefs about the central bank's inflation target (and believe that the central bank's inflation target is lower than it actually is) and they do not have precise information about the aggregate price level.

The intuition behind this result is as follows. If consumers do not have precise information on the current aggregate price level and are confident about their beliefs about the central bank's inflation target, their inflation expectations remain low, reflecting their inflation experience. This implies that they keep believing that prices of goods everywhere else remain almost unchanged. Therefore, they are more likely to search for cheaper prices if they see a price increase. Facing this consumer behavior, firms are reluctant to pass through an increase in their production costs to the prices they charge. Because consumers' prior beliefs reflect past realizations of inflation, their inflation experience anchors current inflation.

The model also explains why consumers have little incentive to pay attention to changes in the central bank's inflation target when they experience a long period of low and stable inflation. If they observe that inflation is low and stable, they infer that the underlying inflation target is also low and stable and become confident in their beliefs about the inflation target. In line with the rational inattention hypothesis (Sims (2003)), they then regard the benefits of collecting more information on the inflation target as small. Consumers can also learn about changes in the inflation target from changes in the prices they observe. However, those who are confident in their beliefs about the central bank's inflation target attribute changes in prices mainly to changes in fundamentals rather than a misperception of the central bank's target. These results imply that consumers' inflation experience can persistently affect their beliefs about aggregate inflation and can cause prolonged missing inflation.

In order to draw policy implications from our model, we conduct two counterfactual simulations. The first simulation focuses on changes in the monetary policy stance, i.e., changes in the relative policy weights on the stability of inflation around its target and the stability of the output gap. When the central bank puts greater weight on inflation stabilization, the Phillips curve becomes flatter, and as a result the central bank will have to accept a more volatile output gap in order to stabilize inflation. This effect is stronger when consumers' price beliefs rely to a larger extent on their past inflation experience. The second simulation focuses on the effects of central bank communication vis-a-vis the general public.[7] Specifically, we examine two cases: (1) communication about the inflation target, and (2) communication about the current aggregate price level. First, communication about the inflation target partly anchors consumers' price beliefs to the price level consistent with the target, so that consumers have less incentive to conduct price search, as long as the increases in prices they observe are smaller than the price increases implied by the inflation target. This makes it easier for firms to adjust their prices to the level consistent with the inflation target. However, this communication makes the Phillips curve flatter, which implies that the sensitivity of inflation to the central bank's policy actions become smaller. This, in turn, implies that the central bank may have to accept large fluctuations in output gap in order to stabilize inflation in response to economic shocks. Second, communication on the current aggregate price level leads consumers' beliefs about the aggregate price level to comove with the actual aggregate price level. The reason is that such communication decreases the effects of past inflation experience on consumers' price beliefs. This makes it easier for firms to increase their prices when their costs increase. In other words, the Phillips curve will become steeper, which implies that the impact of monetary policies on inflation increases. However, consumers' price beliefs will become more sensitive to other economic shocks. In that sense, the central bank's communication policy may face a trade-off between

[7] The effects of central bank communication vis-a-vis the public on the economy have attracted growing attention from policy makers. See, for example, Blinder (2009), Haldane (2017), and Haldane and McMahon (2018).

anchoring private-sector expectations and strengthening the effectiveness of its policies in terms of steering inflation. Finally, we also analyze the feedback effects of communication on the degree of uncertainty facing the central bank regarding economic shocks. We show that communicating the inflation target makes it more difficult for the central bank to make precise inferences about productivity shocks underlying price developments but makes it easier for the central bank to make precise inferences about mark-up shocks. On the other hand, communicating the aggregate price level has the exact opposite effects.

Related literature. This study is closely related to the following three strands of literature. First, our research contributes to the literature on the role of consumers' imperfect information in macroeconomic fluctuations.[8] Lorenzoni (2009) shows that noise shocks hitting consumers represent a source of demand shocks and analyzes the effects of such shocks on output and inflation. L'Huillier (2019) and Matejka (2015) argue that firms strategically refrain from price adjustments in order to hide private information from their consumers. In contrast, our study proposes the view that firms are reluctant to change their prices because a firm's price change does not fully transmit its private information about the general price level to consumers.

Second, our paper provides micro-foundations for "real rigidities." The seminal studies by Ball and Romer (1990) and Kimball (1995) highlighted the importance of the shape of the demand curve for firms' pricing behavior. See, for example, Dotsey and King (2005) and Yun and Levin (2011) for an overview of recent developments in this literature on New Keynesian models with a quasi-kinked demand curve. While most of the studies in this literature assumed the existence of a quasi kink in the demand curve in their models, our model provides micro-foundations for a quasi-kinked demand curve based on consumer search under imperfect information.[9]

Third, our research is closely related to the literature on the causes of the flattening of the Phillips curve. The existing literature proposes a variety of hypotheses such as better anchoring of inflation expectations, non-linearity of the Phillips curve originating from menu costs (Ball, Mankiw, and Romer 1988; Ball and Mazumder 2011), and changes in market structures (Sbordone 2009; Riggi and Santoro 2015). The distinct feature of our hypothesis is that the source of missing inflation is consumers' price beliefs reflecting their past inflation

[8]Note that most imperfect information models assume informational frictions on the firm side. See, for example, Mankiw and Reis (2002), Woodford (2003), Ui (2003, 2019), Nimark (2008), Angeletos and La'O (2009), and Mackowiak and Wiederholt (2009).

[9]The earliest contribution to the literature on a kinked demand curve under imperfect information is Negishi (1979), since he mentions the relationship between information frictions and kinked demand. Dupraz (2018) also shows that scarcity of information leads to a kinked demand curve, using a general equilibrium model with consumer search; however, for tractability of the model he assumes that firms' expectations about the prices the other firms choose are not fully rational.

experience.

Outline. The remainder of this study is organized as follows. Section 2 introduces our dispersed-information version of the general equilibrium model with consumer search, while Section 3 characterizes the equilibrium. Section 4 analyzes the mechanism behind missing inflation. To derive policy implications, Section 5 conducts counterfactual simulations, while Section 6 concludes. The appendices contain step-by-step derivations of our results.

2 An Economy of Islands with Dispersed Information

Our model is constructed as follows. The economy consists of a continuum of islands with mass one. Each island is denoted by $i \in [0, 1]$. On each island, there are a representative household and a firm. We assume that the representative household consists of a consumer and a worker. The consumer makes the household's consumption decisions, while the worker makes the labor supply decisions. We assume that the consumer and the worker do not share information with each other within any given period. We make this assumption in order to examine the implications of imperfect information for consumption decisions in a tractable manner. Goods are homogeneous across islands, and to simplify our exposition we assume that there is one representative good in the economy. We also assume that the market for the good is segmented across islands. Due to heterogeneity in production costs (explained in Section 2.2), the price of the good differs across islands.

The consumer purchases the good from only one firm. The first candidate is the firm on the same island, but the consumer has the option to change from the firm on his own island to a randomly selected firm from another island ($i' \in [0, 1]$). We refer to this option as *price search*. This setting allows us to capture the situation that the *extensive margin*, i.e., the number of consumers that eventually purchase the good at a particular firm matters for the firm's profits. For the sake of analytical simplicity, we assume that the consumer can use that option only once and, once he does, he must purchase the good from the other firm at any price. The worker supplies homogeneous labor to firms on any island. In other words, we assume that the labor market is not segmented. Finally, we assume that the central bank controls aggregate nominal demand to minimize its loss function subject to the structure of the economy.

In order to explain imperfect information, we assume that the consumer on each island does not know the prices charged on other islands when deciding to purchase the good. All he knows are the price charged on his island and the aggregate price level in the economy in the past. He also receives signals on the current aggregate price level as well as the inflation target set by the central bank. In order to simplify the analysis, we assume that workers on all islands have perfect information but cannot communicate with consumers. We also assume that firms have perfect information on all variables in the economy except consumers' search costs. The central bank observes prices but cannot observe the realized values of underlying economic shocks or the signals regarding the aggregate price level and consumers' beliefs with regard to its inflation target. We then focus on the role of informational rigidities on the part of consumers in inflation dynamics and the interaction between consumers' behavior and the central bank's policies. Our model is a static model and prices are flexible. However, consumers' prior beliefs on the current aggregate price level play the role of generating a dynamic link between the sequence of equilibrium prices. In the following, we first derive the equilibrium price by taking consumers' priors as given. We then characterize the endogenous evolution of the priors.

2.1 The Household

The representative household on an island consists of a consumer and a worker. The consumer makes the household's consumption purchase decisions, while the worker makes the household's labor supply decisions. The utility of the representative household allocated to island i is given by

$$U(C_{i,t}, s_{i,t}, \phi_{i,t}, N_{i,t}) \equiv \ln C_{i,t} + s_{i,t} \ln \phi_{i,t} - N_{i,t}, \tag{1}$$

where $C_{i,t}$ and $N_{i,t}$ respectively denote the consumption and labor supply of the household on island i at time t. In what follows, variables with subscript t denote those variables at time t. Variable $s_{i,t} \in \{0, -1\}$ is an indicator that takes 0 if the consumer does not perform a price search and takes -1 if he does perform a price search. The term $\ln \phi_{i,t}$ represents the cost of price search for the consumer on island i. It is assumed that search costs are heterogeneous across islands. The consumer makes his search and consumption decisions taking his income as given, while the worker makes labor supply decisions taking the consumption level as given.

Let us begin with the search decision. The representative consumer on island i observes the price of the good on island i but does not directly observe the price of the good on other islands unless he performs a price search. When the consumer does not perform a search, he buys the good on island i. Let $C_{i,t}^{i}$ denote the level of consumption in this case. On the other hand, when he does perform a search, he is randomly allocated to another island, $i' \in [0, 1]$ and buys the good on that island. Let $C_{i,t}^{i'}$ denote the level of consumption when he is allocated to island i'. To summarize,

$$C_{i,t} = \begin{cases} C_{i,t}^{i} & \text{if } s_{i,t} = 0, \\ C_{i,t}^{i'} & \text{if } s_{i,t} = -1. \end{cases} \tag{2}$$

Next, we describe the consumer's budget constraint. The nominal income of the household on island i is the sum of the worker's wage income and dividend payments from firms. The nominal labor income is $N_{i,t}W_t$, where W_t is the economy-wide nominal wage (discussed below). The dividends paid by firms are equal to their profits. We assume that the aggregate profits of firms across all islands, Π_t, are equally distributed to all consumers. Therefore, the nominal income of the household on island i, $I_{i,t}$, is given by

$$I_{i,t} \equiv N_{i,t}W_t + \Pi_t. \tag{3}$$

The price the consumer on island i pays for the good depends on whether or not he performs a price search. If he does not perform a price search, he faces price $P_{i,t}$. Here, $P_{i,t}$ represents the nominal price of the good the firm on island i charges. Therefore, his consumption level in this case is given by

$$C_{i,t}^{i} = \frac{I_{i,t}}{P_{i,t}}. \tag{4}$$

On the other hand, if he does perform a price search, he will be allocated to another island, i' ($i' \neq i$), and faces a different price, $P_{i',t}$. Here, $P_{i',t}$ is the price that the firm on island i' charges for the good. His consumption level in this case is

$$C_{i,t}^{i'} = \frac{I_{i,t}}{P_{i',t}}. \tag{5}$$

We assume that the consumer makes his search decision in order to maximize the following expected utility:

$$\mathbb{E}[U(C_{i,t}, s_{i,t}, \phi_{i,t}, N_{i,t})|\Omega_{i,t}^{c}] = \mathbb{E}\left[\ln C_{i,t} + s_{i,t}\ln \phi_{i,t} - N_{i,t}|\Omega_{i,t}^{c}\right], \tag{6}$$

where $\Omega_{i,t}^{c}$ denotes the information set of the consumer initially allocated to island i. When he makes his search decision, he takes the labor supply decision made by the worker as given. In equation (6), expectations are taken over $C_{i,t}$. When the consumer makes his search decision, he observes the price level on island i, namely $P_{i,t}$, but observes neither the nominal income $I_{i,t}$ nor the prices on other islands $P_{i',t}$ $(i' \neq i)$. The information set of the household on island i is defined later in Section 2.4.

Let us now turn to the worker. The worker makes his labor supply decision taking as given the search decision made by the consumer and hence the price of the good. We assume that labor is mobile across islands and that the labor market is perfectly competitive. Recall also that we assume the worker has perfect information. Maximizing utility (1) with respect to $N_{i,t}$ subject to (4) and (5) yields the standard optimal labor supply decision:

$$1 = \begin{cases} \frac{1}{C_{i,t}^{i}} \frac{W_t}{P_{i,t}} & \text{if } s_{i,t} = 0, \\ \frac{1}{C_{i,t}^{i'}} \frac{W_t}{P_{i',t}} & \text{if } s_{i,t} = -1. \end{cases} \tag{7}$$

2.2 The Firm

The production function of the firm on island i at time t is given by

$$Y_{i,t} = A_{i,t} N_{i,t},$$

where $A_{i,t}$ represents the level of technology on island i at time t, which is heterogeneous and log-normally distributed across islands. The purpose of introducing heterogeneous technology shocks is to introduce heterogeneity in productivity in order to generate price dispersion across islands. $N_{i,t}$ denotes the labor input on island i. Let $D_{i,t}(P_{i,t})$ denote the demand for the good produced by the firm on island i. As shown later in Section 3.2, the demand for the firm's good decreases in $P_{i,t}$. The firm's nominal profit is given by

$$\Pi_{i,t} \equiv P_{i,t} Y_{i,t} - W_t N_{i,t} = P_{i,t} D_{i,t}(P_{i,t}) - W_t \frac{D_{i,t}(P_{i,t})}{A_{i,t}}. \tag{8}$$

The firm chooses $P_{i,t}$ to maximize (8) taking W_t as given. The functional form of $D_{i,t}(P_{i,t})$ is explained in detail in Section 3.2. For simplicity, we assume the firm has perfect information except with regard to consumers' search costs.

For later purposes, it is convenient to define here the firm's nominal marginal cost, which is given by

$$MC_{i,t} = \frac{W_t}{A_{i,t}}. \tag{9}$$

Since in subsequent sections we will analyze the log-linearized version of the model, let us express (9) in linearized form. Consider an equilibrium in which all exogenous variables are constant and identical across islands. For example, $A_{i,t} = \overline{A}$ for all i and all t. In this equilibrium, all the real variables such as $C_{i,t}$, $N_{i,t}$, $Y_{i,t}$ will be constant and identical across all islands for all t. Denote these real endogenous variables as $\overline{C}$, $\overline{N}$, and $\overline{Y}$. We denote the log-deviation of a real variable from its steady state value by small letters with a "hat" on top. For example,

$$\hat{a}_{i,t} \equiv \ln(A_{i,t}/\overline{A})$$

denotes the log deviation of the productivity on island i from its steady state value $\overline{A}$. We define $\widehat{c}_{i,t}$, $\widehat{n}_{i,t}$, $\widehat{y}_{i,t}$ in a similar manner. Needless to say, nominal variables, such as $P_{i,t}$ and W_t, are not necessarily stationary. Let us denote the logarithm of these nominal variables by small letters. For example, $p_{i,t}$ denotes the log of the nominal price charged by the firm on island i for the good:

$$p_{i,t} \equiv \ln P_{i,t}. \tag{10}$$

Then, equation (9) can be expressed in terms of log variables as follows:

$$mc_{i,t} = w_t - \widehat{a}_{i,t},$$

where $mc_{i,t} \equiv \ln MC_{i,t}$ and $w_t \equiv \ln W_t$. We assume that $\widehat{a}_{i,t}$ is normally distributed, i.e.:

$$\widehat{a}_{i,t} = \widehat{a}_t + \widehat{e}_{i,t}, \tag{11}$$

$$\widehat{a}_t \sim \mathcal{N}(0, \mathbb{V}_a), \tag{12}$$

$$\widehat{e}_{i,t} \sim \mathcal{N}(0, \mathbb{V}_e). \tag{13}$$

Here, $\widehat{a}_t$ represents aggregate productivity shocks and $\widehat{e}_{i,t}$ represents island-specific productivity shocks, which are the source of price dispersions across islands through the dispersion in production costs. Variances $\mathbb{V}_a$ and $\mathbb{V}_e$ are assumed to be constant over time.

2.3 The Central Bank

Finally, let us describe the behavior of the central bank. Since we analyze the log-linearized model in later sections, let us define the objective of the central bank using logarithmic variables. Define the aggregate price level of the economy as

$$p_t \equiv \int_0^1 p_{i,t} di, \tag{14}$$

where $p_{i,t}$ is defined by equation (10). Similarly, define

$$\hat{y}_{i,t} = \ln(Y_{i,t}/\overline{Y}), \tag{15}$$

where $\overline{Y}$ is the level of output in the steady state, and define aggregate output as

$$\hat{y}_t \equiv \int_0^1 \hat{y}_{i,t} di. \tag{16}$$

We assume that the central bank minimizes the following loss function:

$$\max \mathbb{E}_{CB|t} \left[-\lambda \left(\pi_t - \pi_t^* \right)^2 - (1-\lambda) \left(\widehat{y}_t - \widehat{a}_t \right)^2 \right], \quad 0 \leq \lambda \leq 1, \tag{17}$$

where $\pi_t = p_t - p_{t-1}$ is the inflation rate and π_t^* is the central bank's inflation target at time t. $\mathbb{E}_{CB|t}$ denotes the expectation operator conditional on the central bank's information set at time t. Since $\hat{a}_t$ corresponds to changes in natural output, term $\widehat{y}_t - \widehat{a}_t$ in equation (17) represents the output gap. Parameter λ represents the weight on inflation stabilization. Following Erceg and Levin (2003), we assume that the central bank's inflation target is stochastic. Further, we assume that the inflation target π_t^* follows a random-walk process given by

$$\pi_t^* = \pi_{t-1}^* + \delta_t, \tag{18}$$

where

$$\delta_t \sim \mathcal{N}(0, \mathbb{V}_\delta) \tag{19}$$

represents shocks to the inflation target. The variance $\mathbb{V}_\delta$ is assumed to be constant over time.

Following the literature,[10] for tractability we assume that the central bank uses control over nominal output M_t as its policy instrument, where M_t is defined as

$$M_t \equiv P_t Y_t.$$

Let m denote the log of nominal output. In log form, we obtain

$$m_t = \hat{y}_t + p_t. \tag{20}$$

The central bank controls m_t to minimize (17) subject to the structure of the economy (defined below).

[10] See, for example, Lucas (1973), Woodford (2003), and Angeletos and La'O (2009).

2.4 Information and the Sequence of Events

This subsection defines the information set of households, firms, and the central bank. As explained above, a household consists of a consumer and a worker, each with different information, but they do not communicate with one another within a period. At time t, the consumer on island i observes the price the firm on island i charges ($p_{i,t}$) but does not observe the prices firms on other islands charge ($p_{i',t}$, $i' \neq i$) unless he performs a price search. If he does perform a price search, he observes the price charged by the firm on an island he newly visits. We assume that he also observes a noisy signal on the aggregate price level, which is defined as the average price of the good charged by firms across all islands. Note that since for simplicity we focus on a representative good, in our model the average price of the good corresponds to the aggregate price level (consumer price index). Therefore, in what follows, we use the terms "the average price" and "the aggregate price level" interchangeably.

The noisy signal that consumer i receives at time t is given by

$$p_{s,i,t} = p_t + \epsilon_{p,i,t}, \tag{21}$$

where

$$\epsilon_{p,i,t} \sim \mathcal{N}(0, \mathbb{V}_{\epsilon_p})$$

represents normally distributed idiosyncratic noise. An interpretation of this idiosyncratic noise term is that it represents the consumer's misunderstanding of the aggregate price level. Although in practice central banks and governments make data on aggregate price measures publicly available, the public may not pay sufficient attention to such information.[11] For instance, members of the public may not take sufficient time to interpret the data or may not look at the data altogether. Such lack of attention can lead to a misreading of developments in the price level. In our model, the idiosyncratic noise term in equation (21) represents the consumer's idiosyncratic misperceptions of the aggregate price level. If the consumer pays more attention to available data, this reduces the extent of misperceptions regarding the price level, and the degree to which the consumer pays attention is represented by the inverse of the variance $\mathbb{V}_{\epsilon_p}$. A smaller variance means that the consumer pays more attention to data and hence his misperceptions regarding developments in the price level are smaller.

We also assume that the consumer does not directly observe the inflation target of the central bank π_t^*. Instead, he observes a noisy signal on the inflation target π_t^*, which is given by

$$\pi_{s,t}^* = \pi_t^* + \epsilon_{\pi,t}^*, \tag{22}$$

[11] See, for example, Angeletos and Lian (2018), who interpret the idiosyncratic noise as idiosyncratic variation in the interpretation of publicly available data.

where

$$\epsilon^*_{\pi,t} \sim \mathcal{N}(0, \mathbb{V}_{\epsilon^*_\pi}) \tag{23}$$

is normally distributed noise. The signal $\pi^*_{s,t}$ is common across consumers, so that it can be interpreted as a signal that is based on media reports. In sum, the set of variables observable to the consumer on island i at time t when he makes his search decision $s_{i,t}$ is[12]

$$\Omega^c_{i,t} = \left\{ \{p_{i,j}, p_{s,i,j}, \pi^*_{s,j}\}^t_{j=0}, \{p_j, y_j\}^{t-1}_{j=0}, \right\}. \tag{24}$$

The set of unobservable variables consists of the history of aggregate and idiosyncratic productivity ($\{\widehat{a}_j, \widehat{a}_{i,j}, \widehat{e}_{i,j}\}^t_{j=0}$), the target inflation rate and innovations in the target rate ($\left\{\pi^*_j, \delta_j\right\}^t_{j=0}$), the current aggregate price and output (p_t, y_t), and noise in the measurement equations ($\{\epsilon_{p,i,j}, \epsilon^*_{\pi,j}\}^t_{j=0}$). In sum, the set of unobservables is

$$\left\{ \left\{ \widehat{a}_j, \widehat{a}_{i,j}, \widehat{e}_{i,j}, \pi^*_j, \delta_j, \epsilon_{p,i,j}, \epsilon^*_{\pi,j} \right\}^t_{j=0}, p_t, y_t \right\}.$$

Further, we assume that the consumer knows the law of motion of the inflation target (equations (18) and (19)) and the law of motion of productivity (equations (11), (12), and (13)). Consumer i has three observation equations: equations (21), (22), and the equation that determines the equilibrium price on island i as a function of the underlying unobservables (the functional form of which will be derived in subsequent sections). These constitute the state-space representation of the consumer's filtering problem regarding the unobservables. We specifically focus our analysis on the filtering of the inflation target π^*_t and the aggregate price level p_t.

Since all stochastic variables are assumed to follow normal distributions, consumer i's prior and posterior distributions about π^*_t are normally distributed. We will use the following notations. The prior distribution about π^*_t at time t (i.e., before the consumer observes all the time-t variables) is denoted by

$$\pi^*_t \sim_i \mathcal{N}(\pi^*_{i,t|t-1}, \mathbb{V}_{\pi^*,t|t-1})$$

where $\pi^*_{i,t|t-1}$ and $\mathbb{V}_{\pi^*,t|t-1}$ respectively denote the mean and variance (imprecision) of consumer i's beliefs about the inflation target. At time t, this prior is given. The posterior mean and variance are denoted as $\pi^*_{i,t|t}$ and $\mathbb{V}_{\pi^*,t|t}$ and will be derived in subsequent sections. The general formula of the recursive updating of $\pi^*_{i,t|t}$, $\mathbb{V}_{\pi^*,t|t}$, $\pi^*_{i,t|t-1}$, and $\mathbb{V}_{\pi^*,t|t-1}$ is given by a

[12]Note that $\left\{\{p_{i',j}\}_{i'\neq i}\right\}^{t-1}_{j=0}$ is redundant information for the consumer's decisions as long as $\{p_j, y_j\}^{t-1}_{j=0}$ is included in his information set.

Kalman filter and is provided in Appendix C.

As in standard filtering problems, the prior mean ($\pi^*_{i,t|t-1}$) and variance ($\mathbb{V}_{\pi^*,t|t-1}$) at time t are determined by the history of equilibrium prices and signals up to time $t-1$ and are therefore predetermined at time t. We start by focusing our analysis on the equilibrium in which

$$\pi^*_{i,t|t-1} = 0 \tag{25}$$

and

$$\mathbb{V}_{\pi^*,t|t-1} = \mathbb{V}_{\pi^*}. \tag{26}$$

What we have in mind in assumption (25) is a situation in which past realizations of inflation until time t have been low, so that consumers believe that the central bank's inflation target is low, namely, zero per cent.[13] The prior variance $\mathbb{V}_{\pi^*}$ represents the degree to which consumers are convinced by their prior mean. We will conduct some comparative statics to investigate the effects of $\mathbb{V}_{\pi^*}$ on equilibrium inflation.

In order to focus our analysis on the price search and goods purchasing decision of households, we assume, as already mentioned, that workers have perfect information. We also assume that firms have perfect information except with regard to consumers' search costs, $\ln \phi_i$. We denote the information set of the firm on island i as $\Omega^f_{i,t}$. Regarding the central bank, we assume that it observes prices $\{p_{i,t}, p_t\}$ but is not able to observe the realized values of shocks and the signals that consumers receive with regard to aggregate prices and its inflation target ($\{p_{s,i,t}, \pi^*_{s,t}\}$). We denote the central bank's information set as Ω^{CB}_t.

The sequence of events is as follows. (1) At the beginning of the period, all shocks including changes in the central bank's inflation target π^*_t occur and consumers observe the noisy signal $\pi^*_{s,t}$. (2) Firms set their prices $p_{i,t}$, taking the nominal income ($I_{i,t}$) as given. As a result, the aggregate price p_t is determined and the signals about the aggregate price, $p_{s,i,t}$, are generated. (3) Consumers make their search decision based on the information set $\Omega^c_{i,t}$ and hence their consumption decision, taking workers' labor supply and hence their income as given. (4) The central bank decides its monetary policy m_t. (5) Workers decide their labor supply. (2)-(5) occur simultaneously.

3 Equilibrium

In this section, we briefly describe the equilibrium conditions. Appendix A provides the detailed derivation of each condition. In what follows, the steady state refers to the equilibrium

[13] For simplicity, we assume that consumers' prior belief on the inflation target is zero ($\pi^*_{i,t|t-1} = 0$). However, our results remain intact if we assume $\pi^*_{i,t|t-1} \neq 0$.

of the economy, in which all exogenous disturbances are zero; that is,

$$A_{i,t} = A_t = \overline{A},\ \epsilon^*_{\pi,t} = 0,\ \epsilon_{p,i,t} = 0,\ \delta_t = 0.$$

3.1 The consumer

Labor supply. The first order condition for the optimal labor supply (equation (7)), the definition of the worker's nominal income (equation (3)), and the consumer's budget constraint (equations (4) and (5)) imply that, in equilibrium,

$$N_{i,t} = N_t = 1 - \frac{\Pi_t}{W_t} \qquad \text{for all } i.$$

Therefore, the nominal income is identical across workers on all islands (i.e., $I_{i,t} = I_t$ for all i),[14] and, as a result, nominal consumption expenditure is identical across consumers regardless of whether they engage in price search:

$$C^i_{i,t}P_{i,t} = C^{i'}_{i,t}P_{i',t} \qquad \text{for all } i,\ i'.$$

Therefore,

$$C^i_{i,t}P_{i,t} = C^{i'}_{i,t}P_{i',t} = C_tP_t$$

where C_t and P_t are the aggregate consumption and price level.

Therefore, equation (7) implies that the equilibrium nominal wage satisfies

$$W_t = C_tP_t.$$

In log form, this can be written as

$$w_t = \widehat{c}_t + p_t.$$

Bayesian updating of price beliefs. Before characterizing the consumer's search decision, we need to compute his beliefs on the prices on other islands. Given the information set defined above, consumer i updates his belief on the aggregate price level p_t, which provides useful information for his search decision.

Note that current price ($p_{i,t}$) and the price signal ($p_{s,i,t}$) are determined simultaneously with the consumer's updated beliefs. The reason is that the consumer observes $p_{i,t}$ and $p_{s,i,t}$ to update his beliefs, and the firm's pricing decisions in turn depend on the consumer's beliefs

[14] Therefore, the aggregate nominal income equals the aggregate wage, i.e., $I_{i,t} = N_{i,t}W_t + \Pi_t = W_t\left(1 - \frac{\Pi_t}{W_t}\right) + \Pi_t = W_t$.

in equilibrium. For this reason, it is convenient to first compute $\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$, that is, the consumer's belief regarding the central bank's inflation target conditional on the signal $\pi_{s,t}^*$ (equation (22) but not conditional on the current price $(p_{i,t})$ and the current price signal $(p_{s,i,t})$. Under assumptions (25) and (26), Bayesian updating implies that $\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$ is given by

$$\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] = \frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_{s,t}^* + \frac{\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}0, \tag{27}$$

where $\mathbb{V}_{\epsilon_{\pi^*}}^{-1}/\left(\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}\right)$ and $\mathbb{V}_{\pi^*}^{-1}/\left(\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}\right)$ are the relative precision of the signal $\pi_{s,t}^*$ and the prior about the inflation target, i.e., zero.[15]

Next, we compute the consumer's beliefs on the aggregate price level, p_t. Again, it is convenient to first compute $\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$, that is, the consumer's belief on the aggregate price conditional on $\pi_{s,t}^*$ and $\Omega_{i,t-1}^c$ but not conditional on $p_{i,t}$ and $p_{s,i,t}$. Appendix A.8 shows that $\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$ is given by

$$\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] \equiv \alpha\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] + p_{t-1}, \tag{28}$$

where α is defined in Appendix A.8.

Next, we compute $\mathbb{E}\left[p_t|\Omega_{i,t}^c\right]$, which takes contemporaneous observables ($p_{i,t}$ and $p_{s,i,t}$) into account. We can define the updating of the belief about p_t as a signal extraction problem from the three signals $\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$, $p_{i,t}$, and $p_{s,i,t}$. The observation equation for $p_{s,i,j}$ is given by equation (21). The static relationship between $p_{i,t}$ and p_t is endogenously determined in equilibrium (which is given in Appendix A.8). Finally, note that $\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$ is an unbiased predictor of p_t. Therefore, $\mathbb{E}\left[p_t|\Omega_{i,t}^c\right]$ is given by

$$\mathbb{E}\left[p_t|\Omega_{i,t}^c\right] = \left(1 - \omega_\alpha - \omega_\beta\right)p_{i,t} + \omega_\alpha p_{s,i,t} + \omega_\beta\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right], \tag{29}$$

where ω_α and ω_β are the weights that minimize the imprecision of the posterior belief. These weights are characterized in Appendix A.8). We call $\mathbb{E}\left[p_t|\Omega_{i,t}^c\right]$ the "price beliefs."

Price search. The consumer takes the worker's labor supply decisions and hence his nominal income (I_t) as given. Note that

$$\ln C_{i,t} + s_{i,t}\ln\phi_{i,t} = \ln\left(C_{i,t}\left(\phi_{i,t}\right)^{s_{i,t}}\right).$$

[15] Moreover, the expectations about $\epsilon_{\pi,t}^*$, $\mathbb{E}\left[\epsilon_{\pi,t}^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$ that affect the price of the good on each island, is also affected by the signal $\pi_{s,t}^*$. Specifically,

$$\mathbb{E}\left[\epsilon_{\pi,t}^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] = \frac{\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_{s,t}^* + \frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}0.$$

The consumer makes his search decision to maximize

$$\mathbb{E}\left[\ln\left(C_{i,t}\left(\phi_{i,t}\right)^{s_{i,t}}\right)|\Omega_{i,t}^{c}\right] = \max\left\{\mathbb{E}\left[\ln\left(\frac{I_t}{P_{i',t}}\left(\phi_{i,t}\right)^{-1}\right) \mid \Omega_{i,t}^{c}\right], \ln\left(\frac{I_t}{P_{i,t}}\right)\right\}. \tag{30}$$

In equation (30), the first element of the max function is the expected utility when the consumer performs a price search, while the second element represents the expected utility when he does not. Consumer i performs a price search if and only if the expected price on island i relative to the average of prices on other islands is sufficiently high to compensate for the search cost. More specifically, consumer performs a price search if and only if[16]

$$\mathbb{E}\left[\ln\frac{P_{i,t}}{P_t}|\Omega_{i,t}^{c}\right] > \ln\phi_{i,t}. \tag{31}$$

3.2 The firm

Demand curve for the good incorporating the extensive margin of demand. Firm i is the monopoly supplier of the good on island i. When it chooses its price, $P_{i,t}$, it must take into account the fact that the consumer on island i may leave the island to perform a price search if the price the firm charges is too high. The search decision of consumer i (equation (31)) can be rewritten as

$$\widetilde{P}_{s,i,t} > \phi_{i,t}. \tag{32}$$

where $\widetilde{P}_{s,i,t}$ represents the consumer's posterior belief about the relative price on island i to the average price level, which is given by[17]

$$\widetilde{P}_{s,i,t} \equiv \exp\left(\mathbb{E}\left[\ln\frac{P_{i,t}}{P_t}|\Omega_{i,t}^{c}\right]\right).$$

The nominal demand per consumer is given by I_t. The consumer who was originally on island i leaves the island if inequality (32) holds. Otherwise, the consumer remains on island i. Moreover, we assume that some of the consumers who left other islands in search of a better price are randomly allocated to island i, and their number is given by $\widetilde{\varphi}_t \in [0,1]$. In other words, $\widetilde{\varphi}_t$ represents the number of new consumers who moved from other islands to island i. Firm i takes $\widetilde{\varphi}_t$ as given.

[16] For the derivation, see Appendix A.1.

[17] Note that because $\widetilde{P}_{s,i,t} = \exp\left(\ln P_{i,t} - \mathbb{E}\left[\ln P_t|\Omega_{i,t}^{c}\right]\right) = \exp\left(\left(\omega_\alpha + \omega_\beta\right)\ln P_{i,t} - \omega_\alpha \ln P_{s,i,t} - \omega_\beta \mathbb{E}\left[\ln P_t|\pi_{s,t}^{*}, \Omega_{i,t-1}^{c}\right]\right)$, $\partial\widetilde{P}_{s,i,t}/\partial P_{i,t} = \left(\partial\widetilde{P}_{s,i,t}/\partial\ln P_{i,t}\right)\left(\partial\ln P_{i,t}/\partial P_{i,t}\right) = \left(\omega_\alpha + \omega_\beta\right)\widetilde{P}_{s,i,t}/P_{i,t}$ holds.

Given the search decision of the consumer (31), firm i's demand function is given by

$$D_{i,t}\left(P_{i,t}\right) \equiv \left[z(P_{i,t}) + \widetilde{\varphi}_t\right] \frac{I_t}{P_{i,t}}, \tag{33}$$

where $z(P_{i,t})$ is an indicator variable such that

$$z(P_{i,t}) = \begin{cases} 1 & \text{if } \widetilde{P}_{s,i,t} \leq \phi_{i,t}, \\ 0 & \text{otherwise.} \end{cases}$$

The term $z(P_{i,t}) + \widetilde{\varphi}_t$ in equation (33) represents the number of potential customers for firm i, that is, the *extensive margin* of demand, while $I_t/P_{i,t}$ is the amount of real consumption per consumer, i.e., the *intensive margin.* The important property of this demand function is that demand decreases disproportionately if price $P_{i,t}$ exceeds the threshold value that leads the consumer to perform a price search (given by equation (32)). In other words, the demand function involves a kink at the level of price $\widetilde{P}_{i,t}$ that satisfies

$$\widetilde{P}_{s,i,t} = \phi_{i,t}.$$

Demand function (33) is a type of so-called quasi-kinked demand curve. Importantly, the kink depends on the consumer's belief about the relative price on island i to the average price level $\widetilde{P}_{s,i,t}$.

Optimal pricing. Next, we derive the optimal pricing decision of the firm on island i. We assume that the firm does not know the search cost parameter $\phi_{i,t}$ that determines the threshold price that triggers the consumer's price search. For tractability, we assume that $\phi_{i,t}$ follows a uniform distribution:

$$\phi_{i,t} \sim \mathcal{U}\left[\underline{\phi}, \overline{\phi}\right]. \tag{34}$$

The firm's profit function is given by (8). Given (33) and (34), firm i chooses P_i to maximize the expected profit (8), taking $W_{i,t}$ and I_t as given. More specifically, its maximization problem is

$$\max_{P_{i,t}} \mathbb{E}\left[\Pi_{i,t}|\Omega_{i,t}^f\right] = \max_{P_{i,t}} \ \left(P_{i,t} - MC_{i,t}\right)\left[\Pr\left(\widetilde{P}_{s,i,t} \leq \phi_{i,t}\right) + \mathbb{E}\left[\widetilde{\varphi}_t|\Omega_{i,t}^f\right]\right] \frac{I_t}{P_{i,t}}.$$

Here, $\mathbb{E}\left[\widetilde{\varphi}_t|\Omega^f_{i,t}\right] \in [0,1]$ is the expected number of new customers for firm i that come from other islands $i' \in [0,1]$. It is identical across firms and we simply denote it by $\varphi_t \equiv \mathbb{E}\left[\widetilde{\varphi}_t|\Omega^f_{i,t}\right]$.

In Appendix A.2, we show that the firm's optimal price is given by[18]

$$P_{i,t} = \mu\left(\widetilde{P}_{s,i,t}\right) MC_{i,t}, \tag{35}$$

where

$$\mu\left(\widetilde{P}_{s,i,t}\right) \equiv \frac{\left(\overline{\phi} + \left(\overline{\phi} - \underline{\phi}\right)\varphi_t\right) - (1-\omega_\alpha-\omega_\beta)\widetilde{P}_{s,i,t}}{(\omega_\alpha+\omega_\beta)\widetilde{P}_{s,i,t}}.$$

An important property of the optimal price (equation (35)) is that the firm's mark-up $\mu\left(\widetilde{P}_{s,i,t}\right)$ is not constant. Specifically, the mark-up is a decreasing function of the price on island i:

$$\frac{\partial\mu\left(\widetilde{P}_{s,i,t}\right)}{\partial\widetilde{P}_{s,i,t}} \equiv -\frac{\left(\overline{\phi} + \left(\overline{\phi} - \underline{\phi}\right)\varphi_t\right)}{(\omega_\alpha+\omega_\beta)\widetilde{P}^2_{s,i,t}}(\omega_\alpha+\omega_\beta)\frac{\widetilde{P}_{s,i,t}}{P_{i,t}} = -\frac{\left(\overline{\phi} + \left(\overline{\phi} - \underline{\phi}\right)\varphi_t\right)}{\widetilde{P}_{s,i,t}P_{i,t}} < 0.$$

This implies that the firm adjusts its price by less when its costs increase. In other words, it is reluctant to pass through an increase in its costs to its price.

This result follows from the quasi-kinked demand curve that the firm faces. To examine this result in detail, we calculate the elasticity of expected demand with respect to the firm's own price. Because expected demand $\mathbb{E}\left[D_{i,t}\left(P_{i,t}\right)|\Omega^f_{i,t}\right]$ is expressed as

$$\mathbb{E}\left[D_{i,t}\left(P_{i,t}\right)|\Omega^f_{i,t}\right] = \left[\frac{\overline{\phi}}{\overline{\phi}-\underline{\phi}} + \varphi_t - \frac{1}{\overline{\phi}-\underline{\phi}}\widetilde{P}_{s,i,t}\right]\frac{I_t}{P_{i,t}},$$

the elasticity is given by

$$\begin{aligned}\varepsilon_{D,P_{i,t}} &\equiv \frac{\partial\mathbb{E}\left[D_{i,t}\left(P_{i,t}\right)|\Omega^f_{i,t}\right]/\mathbb{E}\left[D_{i,t}\left(P_{i,t}\right)|\Omega^f_{i,t}\right]}{\partial P_{i,t}/P_{i,t}} \\ &= -1 - \frac{(\omega_\alpha+\omega_\beta)\widetilde{P}_{s,i,t}}{\left(\overline{\phi} + \left(\overline{\phi} - \underline{\phi}\right)\varphi_t\right) - \widetilde{P}_{s,i,t}}.\end{aligned}$$

In Figure 1, we plot $\varepsilon_{D,P_{i,t}}$ (y-axis) against $P_{i,t}$ (x-axis) for three values of ω_β: $\omega_\beta = 0$, 0.5, and 1.[19] Weight ω_β in equation (29) represents the degree to which the consumer's posterior beliefs about the aggregate inflation depend on his own prior beliefs. As will be

[18] There also exists $P_{i,t} \to \infty$ as a corner solution because of the assumption that the consumer performs a price search *only once*. However, we focus on the interior solution, since the corner solution is neither robust nor realistic.

[19] We employ the parameter values listed in Table 1.

shown later, the consumer's prior beliefs reflect his past inflation experience. For example, if he experiences low and stable inflation for a long period of time, the prior mean of his expectations with regard to aggregate inflation is low and the precision of his prior is high, which implies that he believes that inflation will remain low in subsequent periods. A tight prior implies a higher value of ω_β. For simplicity, we assume $\omega_\alpha = 0$, that is, there is no signal about the current aggregate price level. We also assume a situation in which the consumer receives no signal about the central bank's inflation target. These two assumptions imply that $\mathbb{E}\left[\ln P_t | \pi^*_{s,t}, \Omega^c_{i,t-1}\right] = \ln P_{t-1}$ holds. We normalize the aggregate price level in the previous period by setting $P_{t-1} = 1$. The figure shows that the expected demand with a high ω_β exhibits the characteristics of a quasi-kinked demand curve in the sense that the price elasticity becomes higher as the price increases.[20] An increase in ω_β makes the updating of the consumer's price beliefs slower. Therefore, if the current price on island i, $P_{i,t}$, is higher than P_{t-1}, the consumer will be more likely to believe that the relative price of $P_{i,t}$ to P_t is too high and hence start to search for a lower price on other islands.

[Figure 1 about here]

Finally, in Appendix A.3 we show that log-linearization of the optimal pricing equation (35) yields

$$p_{i,t} = (1-\widetilde{\kappa})\left(\frac{1}{\omega_\alpha+\omega_\beta}\widehat{x}_{1,t} + \frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_{s,i,t} + \frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t}, \Omega^c_{i,t-1}\right]\right) + \widetilde{\kappa} mc_{i,t}, \tag{36}$$

where

$$\begin{aligned} \widetilde{\kappa} &\equiv \left[1 + \frac{(\omega_\alpha+\omega_\beta)\left(\overline{\phi}+1-\underline{\phi}\right)}{(\omega_\alpha+\omega_\beta)+\left(\overline{\phi}-\underline{\phi}\right)}\right]^{-1}, \\ \widehat{x}_{1,t} &\equiv \ln\left(\left(\overline{\phi}/\left(\overline{\phi}-\underline{\phi}\right)+\varphi_t\right)/\left(\overline{\phi}/\left(\overline{\phi}-\underline{\phi}\right)+\overline{\varphi}\right)\right). \end{aligned} \tag{37}$$

Equation (36) shows that, as usual, the optimal price $p_{i,t}$ depends on the marginal cost ($mc_{i,t}$); in addition, it also depends on information about the aggregate price level, $p_{s,i.t}$, information about the aggregate price level formed by beliefs on the inflation target $\mathbb{E}\left[p_t|\pi^*_{s,t}, \Omega^c_{i,t-1}\right]$, and the probability of obtaining new customers $\widehat{x}_{1,t}$. An important property of the optimal pricing equation (36) is that the sensitivity of $p_{i,t}$ with regard to $mc_{i,t}$ depends on the consumer's beliefs. Comparative statics show that

$$\frac{\partial\widetilde{\kappa}}{\partial\omega_\alpha} < 0, \quad \frac{\partial\widetilde{\kappa}}{\partial\omega_\beta} < 0.$$

[20] Although the level of the elasticity also differs for different values of $\omega_\beta = 0$, 0.5, and 1, it is the *slope*, not the level, that affects the degree of kink in the demand curve.

The result $\frac{\partial \widetilde{\kappa}}{\partial \omega_\beta} < 0$ is of particular interest. Parameter ω_β represents the importance of prior beliefs in the consumer's belief formation (see equation (29)). A higher value of ω_β implies that the consumer's price beliefs depend more on his prior beliefs. This situation emerges when he is more convinced that the price level today will be similar to the price level in the past. Equation (36) implies that the firm's price becomes less sensitive to changes in its marginal cost when ω_β becomes larger. Intuitively, when the consumer is more convinced that the price level today will be similar to the price level in the past, he is more likely to leave the island when he observes an increase in the price on that island ($p_{i,t}$) because he believes that prices everywhere will be as low as before. In other words, he believes that the observed change in $p_{i,t}$ more likely reflects a change in the relative price ($p_{i,t}/p_t$) than in the aggregate price level (p_t). As a result, the firm becomes reluctant to increase its price when its marginal costs increase out of fear of losing its customers. Similarly, ω_α becomes higher if the consumer's signal on the aggregate price level (21) is more accurate. The consumer then believes that the observed change in $p_{i,t}$ more likely reflects a change in the relative price ($p_{i,t}/p_t$) than in the aggregate price level (p_t). This decreases $\widetilde{\kappa}$.

3.3 Aggregation

Before characterizing the behavior of the central bank, it is useful to obtain the aggregate supply equation.

Marginal costs. From the optimality conditions of the consumer, the market clearing condition, and (20), we obtain

$$w_t = \widehat{c}_t + p_t = \widehat{y}_t + p_t = m_t.$$

From the optimality conditions of the firm, we obtain

$$\begin{aligned} mc_t &= w_t - \widehat{a}_t, & (38)\\ mc_{i,t} &= w_t - \widehat{a}_t - \widehat{e}_{i,t}. & (39) \end{aligned}$$

Finally, by combining these conditions, we obtain

$$\begin{aligned} mc_t &= m_t - \widehat{a}_t, & (40)\\ mc_{i,t} &= m_t - \widehat{a}_t - \widehat{e}_{i,t}. & (41) \end{aligned}$$

Aggregate price. Given these marginal costs and the condition of optimal pricing at the firm level (36), we show in Appendix A.4 that the aggregate price level is given by

$$p_t = (1-\kappa)\left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\left(\pi_t^* + \epsilon_{\pi^*,t}\right) + p_{t-1}\right) + \kappa(m_t - \widehat{a}_t), \tag{42}$$

where

$$\begin{aligned} \kappa &\equiv \frac{\widetilde{\kappa}}{1-(1-\widetilde{\kappa})\left[1+\left(\overline{\phi}-\underline{\phi}\right)\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}\right]/\left(\overline{\phi}+1-\underline{\phi}\right)} \\ &= \frac{\overline{\phi}-\underline{\phi}+\omega_\alpha+\omega_\beta}{(\overline{\phi}-\underline{\phi})(\omega_\beta+1)+\omega_\alpha+\omega_\beta}. \end{aligned} \tag{43}$$

Parameter κ represents the sensitivity of the aggregate price to monetary policy. An important property of (42) is that κ depends on the consumers' beliefs (ω_α and ω_β). The comparative statics show that[21]

$$\frac{\partial \kappa}{\partial \omega_\alpha} = \frac{(\overline{\phi}-\underline{\phi})\omega_\beta}{\left\{(\overline{\phi}-\underline{\phi})(\omega_\beta+1)+\omega_\alpha+\omega_\beta\right\}^2} > 0 \tag{44}$$

and

$$\frac{\partial \kappa}{\partial \omega_\beta} = -\frac{\left(\overline{\phi}-\underline{\phi}\right)\left(\overline{\phi}-\underline{\phi}+\omega_\alpha\right)}{\left\{(\overline{\phi}-\underline{\phi})(\omega_\beta+1)+\omega_\alpha+\omega_\beta\right\}^2} < 0. \tag{45}$$

Similar to the comparative statics of equation (37), equation (45) implies that the aggregate price becomes less sensitive to monetary policy when ω_β becomes larger. This is caused by firms' fear of losing their customers. Instead of $\frac{\partial \widetilde{\kappa}}{\partial \omega_\alpha} < 0$, here we obtain $\frac{\partial \kappa}{\partial \omega_\alpha} > 0$. This property results from the coordination among firms regarding their pricing decision that emerges in equilibrium. Recall that m_t affects the marginal cost of all firms. Therefore, all firms have an incentive to increase their prices when m_t increases. A higher value of ω_α means that consumers have more accurate information about the aggregate price level. This implies that consumers know that all other firms also increase their prices. Then each firm does not need to fear losing its customers if other firms also increase their prices. This induces coordinated price changes among firms and therefore κ becomes larger.

Next, using equations (20) and (42) as well as the definition of inflation $\pi_t = p_t - p_{t-1}$, the relationship between real activity $\widehat{y}_t$ and inflation, i.e., the *Phillips curve,* is given by

$$\pi_t = \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\left(\pi_t^* + \epsilon_{\pi^*,t}\right) + \frac{\kappa}{1-\kappa}(\widehat{y}_t - \widehat{a}_t). \tag{46}$$

[21] We thank Takashi Ui for suggesting these comparative statics.

In what follows, in order to introduce the policy trade-off between inflation stabilization and output gap stabilization facing the central bank, we assume that there exist mark-up shocks μ_t in the aggregate supply equation:

$$\mu_t \sim \mathcal{N}(0, \mathbb{V}_\mu).$$

Therefore, the Phillips curve (46) is modified as follows:

$$\pi_t = \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \left(\pi_t^* + \epsilon_{\pi^*,t}\right) + \frac{\kappa}{1-\kappa}(\widehat{y}_t - \widehat{a}_t) + \mu_t. \tag{47}$$

We use this equation as the aggregate supply equation of the model economy.

3.4 The central bank

The central bank controls m_t to minimize (17) subject to the structure of the economy represented by equation (47). Appendix A.5 shows that the optimal value of m_t is given by

$$\begin{aligned} m_t \;&=\; \mathbb{E}\left[\widehat{a}_t | \Omega_t^{CB}\right] + \left(1 - \Xi \frac{\mathbb{V}_{\pi^*}^{-1} + (1-\alpha)\,\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\pi_t^* \qquad (48) \\ &\quad + \Xi\left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \mathbb{E}\left[\epsilon_{\pi^*,t} | \Omega_t^{CB}\right] + \mathbb{E}\left[\mu_t | \Omega_t^{CB}\right]\right) + p_{t-1}, \end{aligned}$$

where

$$\Xi \equiv \frac{-\lambda\kappa\,(1-\kappa) + (1-\lambda)\,(1-\kappa)^2}{\lambda\kappa^2 + (1-\lambda)\,(1-\kappa)^2}.$$

Note that the central bank takes the consumers' information set as given, although its policy actually affects the consumers' information set. The interpretation of this optimal monetary policy is straightforward: the central bank fully reacts to changes in the estimates of aggregate technology shocks $\mathbb{E}\left[\widehat{a}_t | \Omega_t^{CB}\right]$ but responds only partially to changes in the estimates of other variables ($\mathbb{E}\left[\epsilon_{\pi^*,t} | \Omega_t^{CB}\right]$, $\mathbb{E}\left[\mu_t | \Omega_t^{CB}\right]$).

We next characterize the signal extraction process of the central bank about shocks to fundamentals as well as noise ($\epsilon_{\pi^*,t}$, $\widehat{a}_t$ and μ_t). The Phillips curve (47) can be written in terms of the price level as

$$\begin{aligned} p_t \;&=\; \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \left(\pi_t^* + \epsilon_{\pi^*,t}\right) + p_{t-1} + \frac{\kappa}{1-\kappa}(\widehat{y}_t - \widehat{a}_t) + \mu_t, \qquad (49) \\ &=\; (1-\kappa)\left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \left(\pi_t^* + \epsilon_{\pi^*,t}\right) + p_{t-1} + \mu_t\right) + \kappa(m_t - \widehat{a}_t). \qquad (50) \end{aligned}$$

Equation (49) represents the observation equation for the central bank. Appendix A.6 shows the details of the filtering problem to derive $\mathbb{E}\left[\widehat{a}_t|\Omega_t^{CB}\right]$, $\mathbb{E}\left[\epsilon_{\pi^*,t}|\Omega_t^{CB}\right]$, and $\mathbb{E}\left[\mu_t|\Omega_t^{CB}\right]$ in equation (48). Equation (48) can then be expressed only in terms of exogenous variables (namely, $\epsilon_{\pi^*,t}$, $\widehat{a}_t$, and μ_t) as

$$\begin{aligned} m_t &= \left(1+\frac{1-\kappa}{\kappa}\frac{\lambda\kappa^2-(1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2}\frac{\mathbb{V}_{\pi^*}^{-1}+(1-\alpha)\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\pi_t^* \\ &\quad+\left(w_{\widehat{a}}+\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}}+w_\mu\right)\frac{\lambda\kappa^2-(1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2}\right) \\ &\quad*\left(\widehat{a}_t-\frac{1-\kappa}{\kappa}\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t}+\mu_t\right)\right), \end{aligned} \tag{51}$$

where $w_{\epsilon_{\pi^*}}$, $w_{\widehat{a}}$, and w_μ indicate the Kalman gain on learning about each variable, and their sum equals one.[22] By substituting (51) into (50), we obtain the final form of the inflation dynamics of this economy:

$$\pi_t=\Xi\pi_t^*+\Psi\left[(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t}-\kappa\widehat{a}_t+(1-\kappa)\mu_t\right], \tag{52}$$

where[23]

$$\begin{aligned} \Xi &\equiv 1-\frac{(1-\lambda)(1-\kappa)^2}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2}\left(1-\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right), \\ \Psi &\equiv 1-w_{\widehat{a}}-\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}}+w_\mu\right)\frac{\lambda\kappa^2-(1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2}. \end{aligned}$$

We have now characterized inflation dynamics π_t as a function of the inflation target and fundamental shocks (π_t^*, $\epsilon_{\pi^*,t}$, $\widehat{a}_t$, μ_t). In what follows, we use equation (52) to analyze the impact of consumer price search on inflation dynamics. Before doing this, the remaining task is to characterize the equilibrium information structure, that is, the Kalman gain parameters $(\omega_\alpha, \omega_\beta)$. These gain parameters are determined endogenously in equilibrium.

3.5 Endogenous Information Structure

Finally, we characterize the endogenously determined parameters of the information structures $(\omega_\alpha, \omega_\beta)$. The optimal ω_α and ω_β are the minimizers of the imprecision of beliefs about the aggregate price level, $\mathbb{V}\left[p_t|\Omega_{i,t}^c\right]$. For the derivation of ω_α and ω_β, see Appendix A.8.

[22] For the derivation, see Appendix A.6.
[23] For the derivation, see Appendix A.7.

We have now characterized the consumers' Bayesian updating and search decisions, the firms' pricing decisions, and the central bank's monetary policy. One key feature of our model is the information structure, that is, the fact that the information value of each signal is dependent not only on the variance of fundamental shocks, but also on other agents' actions. In the following exercises, we use numerical simulations to solve for the equilibrium and examine the implications of consumer price search for inflation dynamics. Appendix B summarizes the complete economic system for the simulations.

4 Solving the Missing Inflation Puzzle

This section examines under what circumstances the economy exhibits missing inflation.

Benchmark parametrization. To begin with, we provide an overview of the benchmark parametrization used for the numerical simulations.

Table 1: List of Calibrated Parameter Values

Parameter		
$\overline{\phi}$	Upper bound of ϕ_i	1.65
$\underline{\phi}$	Lower bound of ϕ_i	0.82
λ	Relative weight on inflation stabilization	0.5
$\mathbb{V}_e$	Variance of island-specific productivity shocks	$(0.02)^2$
$\mathbb{V}_a$	Variance of aggregate productivity shocks	$(0.01)^2$
$\mathbb{V}_\mu$	Variance of mark-up shocks	$(0.02)^2$
$\mathbb{V}_{\pi^*}$	Uncertainty of the inflation target for consumers	$(0.001)^2$
$\mathbb{V}_{\epsilon_p}$	Imprecision of the signal about the aggregate price level	$(0.01)^2$
$\mathbb{V}_{\epsilon_{\pi^*}}$	Imprecision of the signal about the inflation target	$(0.01)^2$

The interpretation of the calibrated parameter values is as follows. $\overline{\phi} = 1.65$ indicates that the maximum of the cost of price search measured in terms of consumers' utility level is $\ln(1.65) \approx 0.5$, while $\underline{\phi} = 0.82$ means that the minimum is $\ln(0.82) \approx -0.20$.[24] The cost of price search is high if the consumer feels a certain degree of loyalty to his own island, while it could be negative if the consumer has a preference for visiting new islands (stores). $\lambda = 0.5$ implies that the central bank cares about inflation stabilization and output gap

[24] The mean of ϕ_i is around 1.24, implying that the price search changes the level of utility by $\ln(1.24) \approx 0.09$.

stabilization equally. $\mathbb{V}_e > 0$, $\mathbb{V}_a > 0$, and $\mathbb{V}_\mu > 0$ indicate that the aggregate price level fluctuates and island-specific prices are heterogeneous. $\mathbb{V}_{\pi^*} > 0$ means that the consumer is uncertain about the inflation target. The assumption $\mathbb{V}_{\epsilon_p} = \mathbb{V}_{\epsilon_{\pi^*}} = (0.01)^2$ implies that consumers have virtually no useful signal on the aggregate price level and the central bank's inflation target. In particular, $\mathbb{V}_{\epsilon_p} = (0.01)^2$ implies that consumers put almost zero weight ($\omega_\alpha \approx 0$) on the measure of the aggregate price level when they update their price expectations (equation (29)). We make these assumptions in order to highlight the role of the tightness of consumers' prior beliefs on the equilibrium aggregate price level. Bearing Japan's missing inflation experience in mind, we set consumers' priors about the inflation target as below the target and, for simplicity, we assume they are zero.

In what follows, unless we mention specific parametrizations, we employ the parameter values listed in Table 1.

Consumers' beliefs and the flattening of the Phillips curve. Figure 2 shows the relationship between ω_β in equation (29) and κ in equation (42), given that $\omega_\alpha = 0$. Recall that ω_β represents the degree to which consumers' posterior beliefs about the aggregate price level depend on their own prior beliefs. A tight prior belief implies a higher value of ω_β. Parameter κ represents the sensitivity of aggregate inflation to monetary policy actions (y-axis). Figure 2 shows that there is a negative relationship between ω_β and κ, confirming our comparative statics shown in (45). This implies that a tighter prior belief makes the Phillips curve flatter.

[Figure 2 about here]

Next, we examine why ω_β might take a large value. Figure 3 shows the comparative statics for the relationship between ω_β and the variances $(\mathbb{V}_{\pi^*}, \mathbb{V}_e)$ of the two variables, namely, consumers' prior beliefs about the central bank's inflation target π_t^* and idiosyncratic productivity shock $\widehat{e}_i$. Panel (a) plots $\mathbb{V}_{\pi^*}$ on the x-axis and ω_β on the y-axis. It shows that ω_β increases as consumers' perceived uncertainty about the inflation target $\mathbb{V}_{\pi^*}$ decreases. Note that a smaller value of $\mathbb{V}_{\pi^*}$ means that consumers believe that they have already formed relatively accurate beliefs on the central bank's inflation target. Consumer then do not update their beliefs about the target, which implies that their updated beliefs about the target remain close to their prior mean belief. In other words, the information value of the prior is larger and this fact results in a larger ω_β. Panel (b) plots the variance of island-specific productivity shocks $\mathbb{V}_e$ on the x-axis and ω_β on the y-axis. The figure shows that there is a positive relationship between them. A larger variance of island-specific productivity shocks means that fluctuations in prices are more likely to be caused by island specific shocks rather than aggregate shocks. Therefore, island-specific price $p_{i,t}$ is less informative about the aggregate price p_t. In other words, the information value of $p_{i,t}$ becomes small, thus leading to a large ω_β.

To summarize, the exercises shown in Figure 2 imply that when consumers do not update their price beliefs, the Phillips curve becomes flatter. This occurs when consumers view the uncertainty regarding the central bank's inflation target as low and the variance of idiosyncratic shocks as large (Figure 3).

[Figure 3 about here]

Consumers' lack of benefit of being attentive to the inflation target. The exercises shown in Figures 2 and 3 assume that the variance of the noise contained in the signal about the central bank's inflation target is very large ($\mathbb{V}_{\epsilon_{\pi^*}} = (0.01)^2$). This assumption implies that consumers do not have much information on the inflation target. We have so far exogenously assumed that consumers have scarce information about the inflation target. In practice, consumers can easily obtain information about the central bank's inflation target. In terms of our model, this implies that $\mathbb{V}_{\epsilon_{\pi^*}}$ could be treated as an endogenous variable. The literature on rational attention models provides a formal theoretical framework to analyze how agents choose the precision of their signals. In our model, a smaller value of $\mathbb{V}_{\epsilon_{\pi^*}}$ means that consumers pay more attention to the inflation target. While formally modeling agents' attention is beyond the scope of this study, we briefly show based on our model that under our benchmark calibration consumers may have little incentive to acquire and process information about the inflation target. The reason is that in our model paying more attention to the inflation target does not help consumers to forecast the aggregate price level with greater precision.

Specifically, our interest is in the effects of an increase in the imprecision of the signal ($\mathbb{V}_{\epsilon_{\pi^*}}$) on the imprecision of the posterior belief about the aggregate price level ($\mathbb{V}\left[p_t|\Omega^c_{i,t}\right]$), that is, $\partial\mathbb{V}\left[p_t|\Omega^c_{i,t}\right]/\partial\mathbb{V}_{\epsilon_{\pi^*}}$.[25] Figure 4 shows how $\mathbb{V}\left[p_t|\Omega^c_{i,t}\right]$ changes in response to changes in $\mathbb{V}_{\epsilon_{\pi^*}}$ under two different values for consumers' perceived uncertainty about the inflation target ($\mathbb{V}_{\pi^*} \in \{(0.001)^2, (0.01)^2\}$). The figure confirms that $\mathbb{V}\left[p_t|\Omega^c_i\right]$ increases sharply in response to an increase in $\mathbb{V}_{\epsilon_{\pi^*}}$ when $\mathbb{V}_{\pi^*}$ is large, while it is almost invariant when $\mathbb{V}_{\pi^*}$ is small. In other words, the benefit of paying attention to the inflation target (i.e., making $\mathbb{V}_{\epsilon_{\pi^*}}$ smaller) is small in the sense that $\mathbb{V}\left[p_t|\Omega^c_{i,t}\right]$ does not decrease. This implies that consumers do not have a large incentive to obtain precise information on the inflation target as long as they believe that their beliefs regarding the inflation target are relatively precise (i.e., $\mathbb{V}_{\pi^*}$ is small).

[Figure 4 about here]

[25] For the functional form, see Appendix A.8.

Sluggish updating of consumers' beliefs about the inflation target. We next show that consumers may not change their beliefs about the inflation target quickly even when the target changes. In order to demonstrate this, we need to examine how the consumer on island i revises his beliefs about the inflation target. In our model, the consumer updates his beliefs about the inflation target based on aggregate inflation as follows.[26] Aggregate inflation is given by equation (52). Dividing both sides of equation (52) by Ξ, we obtain

$$\frac{1}{\Xi}\pi_t = \pi_t^* + \frac{1}{\Xi}\Psi\left[(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} - \kappa\widehat{a}_t + (1-\kappa)\mu_t\right] \tag{53}$$

Note that the aggregate inflation rate π_t on the left-hand side of (53) is observable to the consumer at the end of the period, while the variables π_t^*, $\epsilon_{\pi^*,t}$, and $\widehat{a}_t$, μ_t are unobservable. Therefore, we can interpret equation (53) as representing a noisy signal on the central bank's inflation target, π_t^*. In other words,

$$\frac{1}{\Xi}\pi_t \sim \mathcal{N}\left(\pi_t^*, \mathbb{V}_{\pi_t^*|\pi_t}\right), \tag{54}$$

where

$$\mathbb{V}_{\pi_t^*|\pi_t} \equiv \frac{\Psi^2}{\Xi^2}\left[(1-\kappa)^2\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}} + \kappa^2\mathbb{V}_a + (1-\kappa)^2\mathbb{V}_\mu\right].$$

Under this information structure, Appendix C.1 shows that consumer i's beliefs about the inflation target are given by

$$\mathbb{E}\left[\pi_t^*|\Omega_{i,t}^c\right] = \left(1-\chi_\alpha-\chi_\beta\right)\pi_{i,t|t-1}^* + \chi_\alpha\pi_{s,t}^* + \chi_\beta\frac{1}{\Xi}\pi_t.$$

We then examine how the tightness of the consumer's beliefs about the inflation target (i.e., $\mathbb{V}_{\pi_t^*}$) affects the sensitivity of his beliefs about the inflation target to changes in the target ($\partial\mathbb{E}\left[\pi_t^*|\Omega_{i,t}^c\right]/\partial\delta_t$), which is given by $\chi_\alpha+\chi_\beta$.[27] Figure 5 plots $\mathbb{V}_{\pi^*}$ on the x-axis and the sensitivity $(\chi_\alpha+\chi_\beta)$ on the y-axis. We consider two cases: $\mathbb{V}_{\epsilon_{\pi^*}} = \mathbb{V}_{\epsilon_p} = (0.01)^2$ and $\mathbb{V}_{\epsilon_{\pi^*}} = \mathbb{V}_{\epsilon_p} = (0.001)^2$. The first, represented by the solid line, is an extreme case in which signals on the inflation target and aggregate inflation are not informative, while the second, represented by the gray line, is the case in which those signals are informative. The solid line shows that when the signals are not informative (large $\mathbb{V}_{\epsilon_{\pi^*}}$ and $\mathbb{V}_{\epsilon_p}$) and the consumer

[26] The idea that consumers learn about changes in fundamentals from fluctuations in aggregate variables including prices was first introduced by Angeletos and Werning (2006), Gorodnichenko (2008) and Amador and Weill (2010, 2012).

[27] It should be noted that $p_{i,t}$ and $p_{s,i,t}$ become redundant information for consumers if they observe π_t. For the functional form, see Appendix C.1.

believes that his beliefs/consumers believe that their beliefs about the inflation target are precise (small $\mathbb{V}_{\pi^*}$), they do not adjust their beliefs about the inflation target in response to changes in the target. The reason is that they believe that fluctuations in the price level are to a large extent caused by fundamental shocks and noise ($\epsilon_{\pi^*,t}$, $\widehat{a}_t$, μ_t) rather than their misperception of the inflation target π^*_t. The gray line indicates that consumers' beliefs about the inflation target are relatively more sensitive to changes in the target if the signals are informative, but the sensitivity is still low if they regard their beliefs about the target as being precise.

These exercises indicate that consumers barely change their beliefs about the inflation target if they regard their beliefs to be precise. They do not revise their beliefs significantly in response to changes in macroeconomic variables. This offers an explanation for the prolonged missing inflation. The remaining question is why they believe that the inflation target is low and fairly stable. In what follows, we argue that their inflation experience in the past plays a crucial role.

[Figure 5 about here]

Consumers' inflation experience and beliefs. In our model, consumers receive two signals on the central bank's inflation target — a direct signal about the target, and aggregate inflation, which contains information about the target. In Japan, inflation has been low and stable for a considerable period of time. The long period of low and stable inflation may have made consumers prior beliefs about the inflation target low and tight. This generates a link between consumers' inflation experience and their priors about the inflation target and current aggregate inflation.

Appendix C.2 shows that consumer i's prior beliefs about the inflation target are given by

$$\pi^*_{i,t|t-1} = \sum_{j=1}^{\infty} \left(1 - \chi_\alpha - \chi_\beta\right)^{j-1} \left(\chi_\alpha \pi^*_{s,t-j} + \chi_\beta \frac{1}{\Xi} \pi_{t-j}\right).$$

Consumer i's prior beliefs are a linear combination of the history of inflation rates $\{\pi_{t-j}\}_{j=1}^{\infty}$ and signals on the inflation target $\left\{\pi^*_{s,t-j}\right\}_{j=1}^{\infty}$. Figure 6 shows how the consumer's uncertainty about the inflation target is affected by the variability of inflation and of the inflation target. In Figure 6, we change the variance of fundamental shocks ($\mathbb{V}_a$, $\mathbb{V}_\mu$, and $\mathbb{V}_\delta$) in order to change the variability of inflation. For details, see Appendix B and C.1.

Figure 6 shows that stable inflation, reflecting stable fundamentals and a stable inflation target, reduces consumers' uncertainty about the inflation target $\mathbb{V}_{\pi^*}$. Note that if stable inflation reflects stable fundamentals, consumers assume that their beliefs about the inflation target based on inflation dynamics are precise. If stable inflation reflects a stable inflation target, consumers believe that the target will remain more or less unchanged. Panels (a), (b), and (c) plot the variability of inflation along with $\mathbb{V}_a$, $\mathbb{V}_\mu$, and $\mathbb{V}_\delta$ on the x-axis and $\mathbb{V}_{\pi^*}$ on the y-axis, respectively,[28] and all the panels show a positive relationship. That is, stable inflation results in small uncertainty about the inflation target among consumers. Panel (d) plots consumers' uncertainty about the target against the variance of the target and shows that there is a positive relationship between them.

[Figure 6 about here]

Next, we discuss the effects of a history of low inflation on the level of consumers' priors, $\pi^*_{i,t|t-1}$. Under our assumption that consumers have scarce information ($\mathbb{V}_{\epsilon_{\pi^*}} \to (0.01)^2$), $\chi_\alpha \approx 0$ holds, so that we approximate prior beliefs simply by the linear combination of the history of inflation rates:

$$\pi^*_{i,t|t-1} \approx \chi_\beta \sum_{j=1}^{\infty} \left(1-\chi_\beta\right)^{j-1} \frac{1}{\Xi} \pi_{t-j}.$$

Therefore, a history of low inflation leads to low priors. Figure 7, which is based on this equation, indicates how changes in $\chi_\beta \in \{0.1, 0.25, 0.5\}$ affect the weights for past inflation rates ($\frac{1}{\Xi}\pi_{t-1}, \frac{1}{\Xi}\pi_{t-2}, ..., \frac{1}{\Xi}\pi_{t-10}$), that is, the updating process of consumers' beliefs about the inflation target. The panels imply that a history of more stable inflation rates results in slower updating of consumers' beliefs (a smaller χ_β), so that the beliefs depend more on past inflation experience.

[Figure 7 about here]

5 Discussion: Policy Implications

An important property of our model is that the slope (κ) of the Phillips curve (42) depends on consumers' price beliefs, which in turn depend on monetary policy. This implies that κ is determined endogenously by monetary policy. This section draws some implications from the model for monetary policy. We first analyze the effects on κ of changes in the relative policy weight between output stabilization and inflation stabilization, and derive the implications for the trade-off between inflation variability and output gap variability. Further, we then analyze the effects of central bank communication about the inflation target and the current level of the aggregate price on inflation dynamics. We continue to use the benchmark parametrization shown in Table 1.

[28] With regard to the relationship between the variability of inflation and ($\mathbb{V}_a$,$\mathbb{V}_\mu$, $\mathbb{V}_\delta$), see Appendix C.2.

5.1 Monetary Policy Stance

Monetary policy and inflation dynamics. In our model, the central bank's loss function is given by equation (17), where λ represents the relative weight between inflation stabilization and output gap stabilization. A larger λ means that the central bank puts more emphasis on stabilizing inflation rather than the output gap. We examine the impact of changes in λ on consumers' beliefs ω_β and ultimately on inflation dynamics. Figure 8 shows how changes in λ affect ω_β and the slope of the Phillips curve, κ. According to equation (29), a consumer's price beliefs depend on three factors: the price he observes on his island ($p_{i,t}$), the price signal ($p_{s,i,t}$), and his beliefs about the inflation target ($\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$) that affects $\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$ in equation (29). Parameter ω_β represents the importance of $\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$ when the consumer updates his price beliefs. A larger ω_β means that his price beliefs are affected to a larger degree by his beliefs about the inflation target. Note that under our calibration $\mathbb{V}_{\epsilon_{\pi^*}} = (0.01)^2$, equation (27) implies that the weight for $\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$ is virtually equal to zero, which is consumers' prior mean belief about the inflation target. We assume that the prior mean itself is equal to past realizations of aggregate inflation. Therefore, ω_β in this case measures the degree to which the consumers' price beliefs depend on their prior beliefs about the inflation target. Panel (a) shows that ω_β is monotonically increasing in λ. The intuition behind this is simple. The more the central bank attempts to stabilize inflation, the more consumers expect the effects of aggregate shocks on inflation to be alleviated through the central bank's policy, so that consumers expect inflation to be closer to their beliefs about the inflation target. Panel (b) shows that inflation becomes less sensitive to changes in marginal costs (and monetary policy) as λ increases. As is shown in Panel (a), if λ is large, consumers expect that inflation will be close to their perceived inflation target. Firms are then more reluctant to adjust their prices in response to changes in their production costs. The reason is that they know that consumers are more likely to perform a price search once they increase their prices. To summarize, the model implies that there is a negative relationship between the relative weight the central bank puts on inflation λ and the slope of the Phillips curve.

[Figure 8 about here]

Consumers' beliefs and the trade-off between inflation variability and output gap variability. Next, we analyze how consumers' beliefs affect the conduct of monetary policy. For this purpose, we compute the efficiency frontier of the trade-off between inflation variability and output gap variability. In our model, a cost push shock creates a trade-off between inflation stabilization and output gap stabilization. For each value of $\lambda \in [0, 1]$, we compute the equilibrium inflation and output gap that minimize the central bank's loss function, and compute the corresponding variances of inflation and output gap.[29] Plotting the variance for each value of λ, we obtain the "efficiency frontier," which is shown in Figure 9. The value of $\lambda = 1$ implies that the policy chosen by the central bank is to set the variance of inflation equal to zero in exchange for accepting a large variance of the output gap. As λ decreases, the central bank achieves a smaller variance of the output gap at the expense of a larger variance of inflation. Therefore, as shown in the figure, the efficiency frontier is downward sloping.[30] One determinant of the efficiency frontier is the slope of the Phillips curve, κ. In our model, κ is endogenously determined as a function of consumers' beliefs and the monetary policy stance represented by λ. In order to highlight how the endogeneity of κ affects the trade-off, we also draw the efficiency frontier assuming that κ is exogenously fixed. This frontier is represented by the gray line in the figure. The figure indicates that, compared with the efficiency frontier with κ exogenously fixed, the efficiency frontier curve rotates clockwise once the endogeneity of κ is taken into account. The intuition behind this result is as follows. A larger λ means that the central bank puts more emphasis on stabilizing inflation, which makes κ smaller (see Figure 8). A flatter Phillips curve implies a steeper slope of the efficiency frontier, because, facing a flatter Phillips curve, the central bank has to accept a larger variability of the output gap in order to reduce the variability of inflation. This implies that a central bank that wishes to achieve more stable inflation around its target has to accept larger variability of the output gap. In other words, inflation dynamics become less sensitive to changes in marginal costs, and monetary policy action involves changes in output gap.

[Figure 9 about here]

5.2 Central Bank Communication and Inflation Dynamics

In our model, consumers do not have perfect information about the aggregate state of the economy. They face uncertainty regarding the central bank's inflation target and the aggregate price level. This explains why they perform a price search, which in turn means that the slope of the Phillips curve is endogenously determined by consumers' price beliefs. The policy question that naturally arises is how central bank communication with the public affects inflation dynamics. In this section, we consider two kinds of communication: (1) providing consumers with information about the inflation target, and (2) providing them with information about the current aggregate price level. In our benchmark calibration, we assumed $\mathbb{V}_{\epsilon_p} = (0.01)^2$ and $\mathbb{V}_{\epsilon_{\pi^*}} = (0.01)^2$. These assumptions imply that consumers do not receive any informative signals about the aggregate price level and the inflation target. We

[29] We assume an inflation target of 2 percent ($\pi_t^* = 0.02$).

[30] For the functional form of the variability of inflation and output gap, see Appendix D.1.

model central bank communication by choosing smaller values of $\mathbb{V}_{\epsilon_p}$ and $\mathbb{V}_{\epsilon_{\pi^*}}$. For example, a small $\mathbb{V}_{\epsilon_{\pi^*}}$ means that the central bank provides the public with precise information about the inflation target and consumers pay attention to this information.

Communication on the inflation target. We start by analyzing the effect of communication about the central bank's inflation target. We maintain the assumption that the central bank does not communicate with consumers about the aggregate price level ($\mathbb{V}_{\epsilon_p} = (0.01)^2$), so that the price signal ($p_{s,i,t}$) is not informative. Using equations, (27), (28) and (29), the price beliefs of consumer i are given by

$$\mathbb{E}\left[p_t|\Omega^c_{i,t}\right] = (1-\omega_\alpha-\omega_\beta)\, p_{i,t} + \omega_\alpha p_{s,i,t} + \omega_\beta \left(\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \left(\pi^*_t + \epsilon_{\pi^*,t}\right) + p_{t-1} \right). \tag{55}$$

When $p_{s,i,t}$ is not informative, we can approximate $\omega_\alpha \approx 0$. Equation (55) then reduces to

$$\mathbb{E}\left[p_t|\Omega^c_{i,t}\right] \approx (1-\omega_\beta)\, p_{i,t} + \omega_\beta \left(\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \left(\pi^*_t + \epsilon_{\pi^*,t}\right) + p_{t-1} \right). \tag{56}$$

Using equation (50) and the definition $\pi_t \equiv p_t - p_{t-1}$, inflation is given by

$$\pi_t = (1-\kappa) \left(\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \left(\pi^*_t + \epsilon_{\pi^*,t}\right) + \mu_t \right) + \kappa(m_t - \widehat{a}_t - p_{t-1}), \tag{57}$$

where κ is defined as in equation (43). Equation (56) implies that the degree to which price beliefs are anchored to π^*_t is represented by $\partial \mathbb{E}\left[p_t|\Omega^c_{i,t}\right]/\partial \pi^*_t = \omega_\beta \alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}} / \left(\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}\right)$. The focus of our analysis here is on the degree to which aggregate inflation is anchored to the target, $\partial \pi_t/\partial \pi^*_t = (1-\kappa)\, \alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}} / \left(\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}\right)$, and the sensitivity of inflation to monetary policy action, $\partial \pi_t/\partial m_t = \kappa$. Because ω_β is endogenous, we numerically evaluate the effects of a decrease in $\mathbb{V}_{\epsilon_{\pi^*}}$ on $\partial \mathbb{E}\left[p_t|\Omega^c_{i,t}\right]/\partial \pi^*_t$, $\partial \pi_t/\partial \pi^*_t$ and $\partial \pi_t/\partial m_t$. Figure 10 presents the results. Panel (a) shows how $\partial \mathbb{E}\left[p_t|\Omega^c_{i,t}\right]/\partial \pi^*_t$ (y-axis) depends on the imprecision of information about the inflation target, $\mathbb{V}_{\epsilon_{\pi^*}}$ (x-axis), and confirms that more precise information on the inflation target leads to a tighter anchoring of price beliefs to the inflation target. Panel (b) shows the relationship between $\partial \pi_t/\partial \pi^*_t$ (y-axis) and $\mathbb{V}_{\epsilon_{\pi^*}}$ (x-axis). The panel shows that anchoring of consumers' price beliefs to the inflation target leads to anchoring inflation to the target. Panel (c) shows that the sensitivity of aggregate inflation to monetary policy actions ($\partial \pi_t/\partial m_t$) is dampened by the anchoring effects in Panel (b), while Panel (d) shows that the sensitivity of inflation to the noise contained in the signal of the inflation target ($\partial \pi_t/\partial \epsilon_{\pi^*,t} = (1-\kappa)\, \alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}} / \left(\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}\right)$) increases. To sum up, successful communication on the inflation target allows the central bank to effectively anchor inflation to the target. On the other hand, inflation becomes less sensitive to monetary policy actions. This implies that when inflation changes in response to economic shocks, the central bank has to change its policy more aggressively to offset the effects of those shocks on inflation.

[Figure 10 about here]

Communication on the aggregate price level. In our model, consumers do not know the aggregate price level. This explains why they perform price search and is the source of the quasi-kinked demand curve, i.e., the fact that a price increase may lead to a disproportionate loss of customers. As a result, firms become reluctant to increase their prices when their marginal costs increase. This mechanism results in a flatter Phillips curve. Therefore, it is of interest to examine what effects it would have if the central bank provided the public with information on the aggregate price level. In order to simplify the analysis, we still maintain the benchmark assumption that the central bank provides little information about the inflation target ($\mathbb{V}_{\epsilon_{\pi^*}} = (0.01)^2$). In this case, consumer i's price beliefs are given by

$$\mathbb{E}\left[p_t|\Omega^c_{i,t}\right] \approx (1 - \omega_\alpha - \omega_\beta)\, p_{i,t} + \omega_\alpha p_{s,i,t} + \omega_\beta p_{t-1}.$$

Now consumer i's price beliefs depend not only on the price charged by the firm on his own island ($p_{i,t}$) but also on the signal about the aggregate price level ($p_{s,i,t}$). As the imprecision ($\mathbb{V}_{\epsilon_p}$) of $p_{s,i,t}$ decreases, ω_α increases, but ω_β decreases, which in turn makes inflation more sensitive to monetary policy actions. This property is shown in Figure 11. Panel (a) plots ω_β on the y-axis and the imprecision of the signals on the aggregate price level, $\mathbb{V}_{\epsilon_p}$, on the x-axis. The panel shows that the degree to which consumers' price beliefs depend on their prior beliefs declines as the signals about the aggregate price level become more accurate. Panel (b) presents the relationship between the sensitivity of aggregate inflation to the target $\partial\pi_t/\partial\pi^*_t$ (y-axis) and $\mathbb{V}_{\epsilon_p}$ (x-axis). It shows that more precise signals on the aggregate price level result in less anchored inflation dynamics. Panel (c) plots $\mathbb{V}_{\epsilon_p}$ on the x-axis and the sensitivity of aggregate inflation to monetary policy actions, $\partial\pi_t/\partial m_t$, on the y-axis. The panel indicates that there is a negative relationship between these two variables. A key lesson from this exercise is that as the central bank communicates better with the public regarding the aggregate price level, the effects of its policies on inflation become larger, while inflation becomes more sensitive to economic shocks. This allows the central bank to achieve its inflation target without sacrificing the stability of output gap if it holds precise information on changes in fundamentals.

[Figure 11 about here]

Feedback effects of communication. In our model, the central bank also faces uncertainty. Even though it observes aggregate inflation, it does not directly observe the underlying economic shocks. However, it needs to infer those shocks in order to conduct monetary policy optimally (see equation (48)). Since inflation is an endogenous variable that is determined as a function of fundamental shocks, the central bank can infer those shocks by observing inflation. However, inflation also depends on consumers' price beliefs, which are in turn affected by the central bank's communication. This implies that the information content of inflation depends on the central bank's communication policy. Therefore, it is interesting to consider how the central bank's communication policy affects the degree of uncertainty the central bank itself faces.

To do so, for expository purposes, we express (49) as follows:

$$\begin{aligned}\pi_t &= \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \left(\pi_t^* + \epsilon_{\pi^*,t}\right) + \frac{\kappa}{1-\kappa}(\widehat{y}_t - \widehat{a}_t) + \mu_t \\ &= \gamma_{\pi^*}\pi_t^* + \gamma_{\epsilon_{\pi^*}}\epsilon_{\pi^*,t} + \gamma_{\widehat{y}}\widehat{y}_t - \gamma_{\widehat{a}}\widehat{a}_t + \mu_t,\end{aligned}$$

where $\gamma_{\pi^*} = \gamma_{\epsilon_{\pi^*}} \equiv \alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1} / \left(\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}\right)$ and $\gamma_{\widehat{y}} = \gamma_{\widehat{a}} \equiv \kappa / (1-\kappa)$ are the sensitivities of the inflation rate to each of the variables. Given this equation, the variance of the observable aggregate variables,

$$\mathbb{V}\left[\pi_t - \gamma_{\pi^*}\pi_t^* - \gamma_{\widehat{y}}\widehat{y}_t\right],$$

can be decomposed into the variances of the unobserved variables :

$$\mathbb{V}\left[\gamma_{\epsilon_{\pi^*}}\epsilon_{\pi^*,t} - \gamma_{\widehat{a}}\widehat{a}_t + \mu_t\right] = \gamma_{\epsilon_{\pi^*}}^2 \mathbb{V}_{\epsilon_{\pi^*}} + \gamma_{\widehat{a}}^2 \mathbb{V}_a + \mathbb{V}_\mu.$$

Importantly, since the central bank observes aggregate inflation and its own inflation target, it can fully identify linear combinations of fundamental shocks and noise ($\gamma_{\epsilon_{\pi^*}}\epsilon_{\pi^*,t} - \gamma_{\widehat{a}}\widehat{a}_t + \mu_t$), but it cannot identify ($\epsilon_{\pi^*,t}$, $\widehat{a}_t$, μ_t) separately. The sensitivity of inflation to each of them determines the efficiency of the central bank's information extraction about the unobservable variables. Figure 12 shows how the central bank's communication changes this sensitivity and the central bank's information extraction problem.[31] Panel (a) shows the effects of communicating the inflation target on the informativeness of aggregate inflation on the underlying unobservable economic variables. The degree of communication is represented by the imprecision of the signal about the target ($\mathbb{V}_{\epsilon_{\pi^*}}$). More precise communication on the target is represented by a smaller value of $\mathbb{V}_{\epsilon_{\pi^*}}$. Panel (a) plots $\mathbb{V}_{\epsilon_{\pi^*}}$ on the x-axis and the contribution of productivity ($\hat{a}_t$) and the markup shock ($\hat{\mu}_t$) to the variability of inflation caused by the

[31] Regarding the imprecision of the central bank's beliefs about shocks to each of the fundamentals, see Appendix D.2.

unobservable fundamentals ($\gamma_{\widehat{a}}^2 \mathbb{V}_a / \mathbb{V}\left[\gamma_{\epsilon_{\pi^*}} \epsilon_{\pi^*,t} - \gamma_{\widehat{a}} \widehat{a}_t + \mu_t\right]$, $\mathbb{V}_\mu / \mathbb{V}\left[\gamma_{\epsilon_{\pi^*}} \epsilon_{\pi^*,t} - \gamma_{\widehat{a}} \widehat{a}_t + \mu_t\right]$) on the y-axis.[32] The panel shows that the contribution of $\widehat{a}_t$ increases as $\mathbb{V}_{\epsilon_{\pi^*}}$ increases, while that of μ_t decreases. This is because a larger $\mathbb{V}_{\epsilon_{\pi^*}}$ results in less anchored inflation dynamics. Therefore, as $\mathbb{V}_{\epsilon_{\pi^*}}$ increases, fluctuations in inflation are more likely to be caused by $\widehat{a}_t$. Panel (b) shows that, as a result, if the central bank communicates its inflation target better to the public (i.e., $\mathbb{V}_{\epsilon_{\pi^*}}$ is smaller), it can infer $\widehat{a}_t$ less precisely from observing aggregate inflation, while it can infer μ_t more precisely. Panels (c) and (d) conduct the same exercise with regard to communication on aggregate prices. Panel (c) shows that because inflation dynamics are less sensitive to shocks under a larger $\mathbb{V}_{\epsilon_p}$, the contribution of $\widehat{a}_t$ to the variability of inflation is now decreasing in $\mathbb{V}_{\epsilon_p}$. Finally, panel (d) shows that, in contrast to the case with communication about the target, communication about aggregate prices makes it easier for the central bank to extract information on $\widehat{a}_t$ and more difficult to extract information on μ_t. To sum up, the two types of communication have the opposite effect on the central bank's information extraction problem.

[Figure 12 about here]

6 Concluding Remarks

In this study, we developed an island model with dispersed information in which consumers' search for cheaper prices affects firms' pricing behavior. The model offers an explanation for the missing inflation puzzle in Japan. We highlight the role of consumers' inflation experience in their belief formation process, search decision, and ultimately firms' price-setting. Firms are reluctant to pass through an increase in costs, since they fear a disproportionately large fall in their sales. This large fall in sales is caused by consumers' search for cheaper prices. In particular, when consumers believe that prices are low and stable everywhere else, they have a greater incentive to look for cheaper prices. A history of low and stable inflation amplifies this effect by anchoring consumers' price beliefs to a lower level. We also draw implications for monetary policy and the central bank's communication policy. It should be noted that our model can also account for the missing disinflation observed in the United States. Suppose, for example, that inflation has been close to 2 percent and consumers therefore expect inflation of 2 percent. Under these circumstances, consumers believe that prices on average are going to increase by 2 percent, firms do not have an incentive to decrease prices even when their production costs decrease (e.g., during a recession). Because households do not necessarily observe the prices that other firms charge and believe that

[32] Note that the contribution of $\gamma_{\epsilon_{\pi^*}}^2 \mathbb{V}_{\epsilon_{\pi^*}}$ to the variability of inflation caused by the unobservable variables is small for any reasonable parameterization.

prices are going to increase on average by 2 percent, firms do not lose their customers unless they increase their prices by more than 2 percent. Moreover, decreasing their prices will not help firms to attract new customers, since customers may not notice the price decrease.[33]

Our model can be extended in multiple directions. One extension would be to develop a dynamic model with nominal rigidities to explore the role of information frictions in a forward-looking setting. Such rigidities generate additional mechanisms, for example through the Euler equation representing the intertemporal substitution of consumption. Another extension would be to analyze the optimal monetary policy and communication policy from the perspective of maximizing consumers' utility. A promising approach for identifying optimal communication is the framework of information design, which enables us to investigate all the possible signal structures (Kamenica and Gentzkow 2011; Bergemann and Morris 2013). Needless to say, it is also important to empirically examine our theoretical predictions. Such extensions could provide further useful insights to gain a better understanding of inflation dynamics and contribute to the debate on the effects of central banks' communication on developments in the macroeconomy.

[33] For more on the missing disinflation puzzle, see Coibion and Gorodnichenko (2015) and Gilchrist et al. (2017). In both the United States, aggregate inflation rates were broadly consistent with the inflation target before the global financial crisis. Consumers' beliefs therefore were likely to be anchored to their inflation experiences, which were consistent with the inflation target.

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A Derivation of the Equilibrium Conditions

A.1 Price Search

The condition to satisfy the consumer's optimal decision-making with regard to price search is that he performs price search if and only if the following condition holds:

$$\begin{aligned}
\mathbb{E}\left[\ln \frac{I_t}{P_{i',t}} \left(\phi_{i,t}\right)^{-1} |\Omega_{i,t}^c\right] &> \mathbb{E}\left[\ln \frac{I_t}{P_{i,t}} |\Omega_{i,t}^c\right] \Leftrightarrow \mathbb{E}\left[\ln \frac{1}{P_t} \left(\phi_{i,t}\right)^{-1} |\Omega_{i,t}^c\right] > \ln \frac{1}{P_{i,t}} \\
&\Leftrightarrow \mathbb{E}\left[\ln \frac{P_{i,t}}{P_t} |\Omega_{i,t}^c\right] > \ln \phi_{i,t} \Leftrightarrow \exp\left(\mathbb{E}\left[\ln \frac{P_{i,t}}{P_t} |\Omega_{i,t}^c\right]\right) > \phi_{i,t}.
\end{aligned}$$

A.2 Optimal Pricing

For notational simplicity, denote $X_{1,t} \equiv \overline{\phi} / \left(\overline{\phi} - \underline{\phi}\right) + \varphi_t$, and $\overline{X}_2 \equiv \left(\overline{\phi} - \underline{\phi}\right)^{-1}$. The profit function is

$$\begin{aligned}
\mathbb{E}\left[\Pi_{i,t} |\Omega_{i,t}^f\right] &= \left(P_{i,t} - MC_{i,t}\right) \left[\Pr\left(\widetilde{P}_{s,i,t} \leq \phi_{i,t}\right) + \mathbb{E}\left[\widetilde{\varphi}_t |\Omega_{i,t}^f\right]\right] \\
&= \left(P_{i,t} - MC_{i,t}\right) \left[X_{1,t} - \overline{X}_2 \widetilde{P}_{s,i,t}\right] \frac{I_t}{P_{i,t}} \\
&= X_{1,t} I_t - MC_{i,t} X_{1,t} \frac{I_t}{P_{i,t}} - \overline{X}_2 \widetilde{P}_{s,i,t} I_t + MC_{i,t} \overline{X}_2 P_{i,t}^{-1} \widetilde{P}_{s,i,t} I_t.
\end{aligned}$$

The optimality condition is

$$\begin{aligned}
0 &= MC_{i,t} X_{1,t} I_t P_{i,t}^{-2} - \overline{X}_2 \frac{\partial \widetilde{P}_{s,i,t}}{\partial P_{i,t}} I_t - MC_{i,t} \overline{X}_2 P_{i,t}^{-2} \widetilde{P}_{s,i,t} I_t + MC_{i,t} \overline{X}_2 P_{i,t}^{-1} \frac{\partial \widetilde{P}_{s,i,t}}{\partial P_{i,t}} I_t \\
\Leftrightarrow \quad P_{i,t} &= \frac{X_{1,t} - \overline{X}_2 \left(\widetilde{P}_{s,i,t} - P_{i,t} \frac{\partial \widetilde{P}_{s,i,t}}{\partial P_{i,t}}\right)}{P_{i,t} \overline{X}_2 \frac{\partial \widetilde{P}_{s,i,t}}{\partial P_{i,t}}} MC_{i,t}.
\end{aligned}$$

A.3 Linear Approximation of Optimal Pricing

We approximate optimal pricing (35) as follows. Using the notation $p_{i,t} \equiv \ln\left(P_{i,t}\right)$, $\widetilde{P}_{s,i,t} \equiv \exp\left(\ln P_{i,t} - \mathbb{E}\left[\ln P_t |\Omega_{i,t}^c\right]\right)$, $mc_{i,t} \equiv \ln\left(MC_{i,t}\right)$, and $\widehat{x}_{1,t} = \ln\left(X_{1,t} / \overline{X}_1\right)$, we have

$$\frac{\partial \widetilde{P}_{s,i,t}}{\partial P_{i,t}} = \frac{\partial \widetilde{P}_{s,i,t}}{\partial \ln P_{i,t}} \frac{\partial \ln P_{i,t}}{\partial P_{i,t}} = \left(\omega_\alpha + \omega_\beta\right) \frac{\widetilde{P}_{s,i,t}}{P_{i,t}},$$

so that

$$
\begin{aligned}
\ln P_{i,t} &= \ln\left(\frac{X_{1,t}-\overline{X}_2\widetilde{P}_{s,i,t}+\overline{X}_2P_{i,t}\frac{\partial\widetilde{P}_{s,i,t}}{\partial P_{i,t}}}{P_{i,t}\overline{X}_2\frac{\partial\widetilde{P}_{s,i,t}}{\partial P_{i,t}}}\right)+\ln MC_{i,t}\\
&= \ln\left(\frac{X_{1,t}-\overline{X}_2\left(1-\omega_\alpha-\omega_\beta\right)\widetilde{P}_{s,i,t}}{\overline{X}_2\left(\omega_\alpha+\omega_\beta\right)\widetilde{P}_{s,i,t}}\right)+\ln MC_{i,t}\\
&= \frac{\overline{X}_{1,t}}{\overline{X}_{1,t}-\overline{X}_2\left(1-\omega_\alpha-\omega_\beta\right)}\left(\ln\left(X_{1,t}/\overline{X}_1\right)-\ln P_{i,t}+\mathbb{E}\left[\ln P_t|\Omega^c_{i,t}\right]\right)+\ln\left(MC_{i,t}\right)\\
&= \frac{\overline{X}_1}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}\widehat{x}_{1,t}-\frac{\overline{X}_1}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}p_{i,t}\\
&\quad+\frac{\overline{X}_1}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}\left(\left(1-\omega_\alpha-\omega_\beta\right)p_{i,t}+\omega_\alpha p_{s,i,t}+\omega_\beta\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right)+\ln\left(MC_{i,t}\right)\\
&= \frac{\overline{X}_1}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}\widehat{x}_{1,t}-\frac{\overline{X}_1}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}\left(\omega_\alpha+\omega_\beta\right)p_{i,t}\\
&\quad+\frac{\overline{X}_1}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}\omega_\alpha p_{s,i,t}+\frac{\overline{X}_1}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}\omega_\beta\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]+\ln\left(MC_{i,t}\right)\\
\Leftrightarrow p_{i,t} &= \frac{\overline{X}_1}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}\left(\widehat{x}_{1,t}-\left(\omega_\alpha+\omega_\beta\right)p_{i,t}+\omega_\alpha p_{s,i,t}+\omega_\beta\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right)+mc_{i,t}\\
&= \left(1-\widetilde{\kappa}\right)\left(\frac{1}{\omega_\alpha+\omega_\beta}\widehat{x}_{1,t}+\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_{s,i,t}+\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right)+\widetilde{\kappa}mc_{i,t},
\end{aligned}
$$

where $\widetilde{\kappa}\equiv\left[1+\frac{\overline{X}_1\left(\omega_\alpha+\omega_\beta\right)}{\overline{X}_1-\left(1-\omega_\alpha-\omega_\beta\right)\overline{X}_2}\right]^{-1}$.

A.4 Aggregate Prices

We first characterize φ_t, which is endogenously determined by firms' prices. φ_t represents the proportion of firms that set their prices higher than the threshold value that leads consumers to perform price search. Therefore,

$$
\varphi_t=\left(1-\int_{i\in[0,1]}\frac{\overline{\phi}-\widetilde{P}_{s,i,t}}{\overline{\phi}-\underline{\phi}}\right)=\frac{1}{\overline{\phi}-\underline{\phi}}\left[\int_{i\in[0,1]}\widetilde{P}_{s,i,t}-\underline{\phi}\right].
$$

We approximate this around the steady state $\widetilde{P}_{s,i,t}=1$ (under $A_{i,t}=A_t=\overline{A}$, $\epsilon^*_{\pi,t}=0$, $\epsilon_{\pi,i,t}=0$) as follows:

$$
\begin{aligned}
\ln\varphi_t &= \ln\left(\frac{1}{\overline{\phi}-\underline{\phi}}\right)+\ln\left[\int_{i\in[0,1]}\widetilde{P}_{s,i,t}-\underline{\phi}\right]\\
&\Leftrightarrow\ \ln\left(\varphi_t/\overline{\varphi}\right)=\frac{1}{1-\underline{\phi}}\left[\left(\omega_\alpha+\omega_\beta\right)\int_{i\in[0,1]}p_{i,t}-\omega_\beta p_{s,i,t}-\omega_\beta\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right],
\end{aligned}
$$

where $\overline{\varphi} \equiv \left(1-\underline{\phi}\right) / \left(\overline{\phi}-\underline{\phi}\right) \in (0,1)$. Thus, $\overline{X}_1 = \left(\overline{\phi}+1-\underline{\phi}\right) / \left(\overline{\phi}-\underline{\phi}\right)$.

Next, we have

$$\begin{aligned}
\ln\left(X_{1,t}/\overline{X}_1\right) &= \ln\left(\frac{\overline{\phi}}{\overline{\phi}-\underline{\phi}}+\varphi_t\right) \Leftrightarrow \widehat{x}_{1,t} = \frac{1-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\ln\left(\varphi_t/\overline{\varphi}\right) \\
&= \frac{1}{\overline{\phi}+1-\underline{\phi}}\left[\left(\omega_\alpha+\omega_\beta\right)p_t - \omega_\alpha p_{s,i,t} - \omega_\beta \mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right],
\end{aligned}$$

where $p \equiv \int_{i\in[0,1]} p_i$.

We then obtain the equilibrium prices as follows. Individual prices are

$$\begin{aligned}
p_{i,t} &= \left(1-\widetilde{\kappa}\right)\left(\frac{1}{\omega_\alpha+\omega_\beta}\widehat{x}_{1,t}+\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_{s,i,t}+\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right)+\widetilde{\kappa}mc_{i,t} \\
&= \left(1-\widetilde{\kappa}\right)\left(\begin{array}{c}\frac{1}{\overline{\phi}+1-\underline{\phi}}\left[p_t-\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_{s,i,t}-\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right] \\ +\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_{s,i,t}+\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\end{array}\right)+\widetilde{\kappa}mc_{i,t} \\
&= \left(1-\widetilde{\kappa}\right)\left(\frac{1}{\overline{\phi}+1-\underline{\phi}}p_t+\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\left(\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_{s,i,t}+\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right)\right)+\widetilde{\kappa}mc_{i,t} \\
&= \left(1-\widetilde{\kappa}\right)\left(\frac{1}{\overline{\phi}+1-\underline{\phi}}p_t+\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\left(\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_{s,i,t}+\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right)\right) \\
&\quad +\widetilde{\kappa}(m_t-\widehat{a}_t-\widehat{e}_{i,t}),
\end{aligned}$$

and the aggregate price $p_t \equiv \int_{i\in[0,1]} p_{i,t}$ is

$$\begin{aligned}
p_t &= \left(1-\widetilde{\kappa}\right)\left(\frac{1}{\overline{\phi}+1-\underline{\phi}}p_t+\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\left(\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_t+\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right)\right)+\widetilde{\kappa}\int_{i\in[0,1]}mc_{i,t} \\
&= \left(1-\widetilde{\kappa}\right)\left(\frac{1}{\overline{\phi}+1-\underline{\phi}}p_t+\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\left(\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_t+\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]\right)\right)+\widetilde{\kappa}mc_t \\
\Leftrightarrow\ p_t &= \frac{\left(1-\widetilde{\kappa}\right)\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\beta}{\omega_\alpha+\omega_\beta}}{1-\left(1-\widetilde{\kappa}\right)\left[1+\left(\overline{\phi}-\underline{\phi}\right)\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}\right]/\left(\overline{\phi}+1-\underline{\phi}\right)}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right] \\
&\quad +\frac{\widetilde{\kappa}}{1-\left(1-\widetilde{\kappa}\right)\left[1+\left(\overline{\phi}-\underline{\phi}\right)\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}\right]/\left(\overline{\phi}+1-\underline{\phi}\right)}(m_t-\widehat{a}_t) \\
&= \left(1-\kappa\right)\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]+\kappa(m_t-\widehat{a}_t),
\end{aligned}$$

where $\kappa \equiv \frac{\widetilde{\kappa}}{1-\left(1-\widetilde{\kappa}\right)\left[1+\left(\overline{\phi}-\underline{\phi}\right)\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}\right]/\left(\overline{\phi}+1-\underline{\phi}\right)}$.

Finally, from

$$\begin{aligned}
\pi^*_{s,t} &= \pi^*_t + \epsilon_{\pi^*,t}, \\
\mathbb{E}\left[p_t | \pi^*_{s,t}, \Omega^c_{i,t-1}\right] &= \frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \pi^*_{s,t} + p_{t-1} = \frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \left(\pi^*_t + \epsilon_{\pi^*,t}\right) + p_{t-1},
\end{aligned}$$

we obtain

$$p_t = (1-\kappa)\left(\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \left(\pi^*_t + \epsilon_{\pi^*,t}\right) + p_{t-1}\right) + \kappa(m_t - \widehat{a}_t).$$

A.5 Monetary Policy: Part 1

The central bank's monetary policy is obtained from the following optimization problem:

$$\begin{aligned}
& m_t \equiv \arg\max \mathbb{E}\left[-\lambda\left(\pi_t - \pi^*_t\right)^2 - (1-\lambda)\left(\widehat{y}_t - \widehat{a}_t\right)^2 | \Omega^{CB}_t\right] \\
= \;& \arg\max \mathbb{E}\left[\begin{array}{c}
-\lambda\left(\begin{array}{c} (1-\kappa)\left(\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*}+\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\left(\pi^*_t + \epsilon_{\pi^*,t}\right) + p_{t-1} + \mu_t\right) \\ +\kappa(m_t - \widehat{a}_t) - p_{t-1} - \pi^*_t \end{array}\right)^2 \\
-(1-\lambda)\left(\begin{array}{c} m - (1-\kappa)\left(\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*}+\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\left(\pi^*_t + \epsilon_{\pi^*,t}\right) + p_{t-1} + \mu_t\right) \\ -\kappa(m_t - \widehat{a}_t) - \widehat{a}_t \end{array}\right)^2 |\Omega^{CB}_t
\end{array}\right] \\
= \;& \mathbb{E}\left[\widehat{a}_t | \Omega^{CB}_t\right] + \frac{\lambda\kappa\left(1 - (1-\kappa)\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*}+\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right) + (1-\lambda)(1-\kappa)^2 \frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*}+\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\pi^*_t \\
& + \frac{-\lambda\kappa(1-\kappa) + (1-\lambda)(1-\kappa)^2}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\left(\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\mathbb{E}\left[\epsilon_{\pi^*,t} | \Omega^{CB}_t\right] + \mathbb{E}\left[\mu_t | \Omega^{CB}_t\right]\right) + p_{t-1} \\
= \;& \mathbb{E}\left[\widehat{a}_t | \Omega^{CB}_t\right] + \left(1 - \frac{-\lambda\kappa(1-\kappa) + (1-\lambda)(1-\kappa)^2}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\frac{\mathbb{V}^{-1}_{\pi^*} + (1-\alpha)\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right)\pi^*_t \\
& + \frac{-\lambda\kappa(1-\kappa) + (1-\lambda)(1-\kappa)^2}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\left(\frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\mathbb{E}\left[\epsilon_{\pi^*,t} | \Omega^{CB}_t\right] + \mathbb{E}\left[\mu_t | \Omega^{CB}_t\right]\right) + p_{t-1}.
\end{aligned}$$

A.6 Monetary Policy: Part 2

We start by characterizing the information extraction by the central bank.[34] The central bank's observation equation is the Phillips curve (equation (49)). The filtering problem is to optimally attribute fluctuations in the aggregate price p_t (excluding the effects of π^*_t and $\widehat{y}_t$) to the unobservable variables ($\epsilon_{\pi^*,t}$, $\widehat{a}_t$, μ_t). Following Amador and Weill (2010, 2012) and Veldkamp (2011), we can compute the posterior mean of the unobservable variables in the following way. We can interpret equation (49) as representing the signals on $\epsilon_{\pi^*,t}$, $\widehat{a}_t$ and μ_t.

[34] Aoki and Kimura (2009) and L'Huillier and Zame (2014) employ a similar approach to the one we use here and consider a situation in which the central bank learns about shocks to fundamentals from changes in aggregate variables.

These are respectively given by

$$\pi_t = \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \left(\pi_t^* + \epsilon_{\pi^*,t}\right) + \frac{\kappa}{1-\kappa}(\widehat{y}_t - \widehat{a}_t) + \mu_t$$

$$\Leftrightarrow \epsilon_{\pi^*,t} = -\pi_t^* + \frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \left(\pi_t - \frac{\kappa}{1-\kappa}(\widehat{y}_t - \widehat{a}_t) - \mu_t\right)$$

$$\Leftrightarrow \widehat{a}_t = \widehat{y}_t + \frac{1-\kappa}{\kappa}\left(-\pi_t + \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \left(\pi_t^* + \epsilon_{\pi^*,t}\right) + \mu_t\right)$$

$$\Leftrightarrow \mu_t = \pi_t - \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \left(\pi_t^* + \epsilon_{\pi^*,t}\right) - \frac{\kappa}{1-\kappa}(\widehat{y}_t - \widehat{a}_t).$$

Note that π_t^* and $\widehat{y}_t$ are observable to the central bank. Therefore, the information values of the signals are

$$\epsilon_{\pi^*,t} \sim \mathcal{N}\left(-\pi_t^* + \frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_t - \frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\frac{\kappa}{1-\kappa}\widehat{y}_t, \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2 \left[\left(\frac{\kappa}{1-\kappa}\right)^2 \mathbb{V}_a + \mathbb{V}_\mu\right]\right),$$

$$\widehat{a}_t \sim \mathcal{N}\left(\widehat{y}_t + \frac{1-\kappa}{\kappa}\left(-\pi_t + \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_t^*\right), \left(\frac{1-\kappa}{\kappa}\right)^2 \left[\left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \mathbb{V}_\mu\right]\right),$$

$$\mu_t \sim \mathcal{N}\left(\pi_t - \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_t^* - \frac{\kappa}{1-\kappa}\widehat{y}_t, \left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \left(\frac{\kappa}{1-\kappa}\right)^2 \mathbb{V}_a\right).$$

Since we assume that $\epsilon_{\pi^*,t}$, $\widehat{a}_t$ and μ_t are independent of each other, the central bank's posterior mean of each variable is given by

$$\mathbb{E}\left[\epsilon_{\pi^*,t}|\Omega_t^{CB}\right] = \frac{\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^{-2}\left[\left(\frac{\kappa}{1-\kappa}\right)^2 \mathbb{V}_a + \mathbb{V}_\mu\right]^{-1}}{\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^{-2}\left[\left(\frac{\kappa}{1-\kappa}\right)^2 \mathbb{V}_a + \mathbb{V}_\mu\right]^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\left(\epsilon_{\pi^*,t} + \frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\left(-\frac{\kappa}{1-\kappa}\widehat{a}_t + \mu_t\right)\right),$$

$$\mathbb{E}\left[\widehat{a}_t|\Omega_t^{CB}\right] = \frac{\left(\frac{1-\kappa}{\kappa}\right)^{-2}\left[\left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \mathbb{V}_\mu\right]^{-1}}{\left(\frac{1-\kappa}{\kappa}\right)^{-2}\left[\left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \mathbb{V}_\mu\right]^{-1} + \mathbb{V}_a^{-1}}\left(\widehat{a}_t - \frac{1-\kappa}{\kappa}\left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} + \mu_t\right)\right),$$

$$\mathbb{E}\left[\mu_t|\Omega_t^{CB}\right] = \frac{\left[\left(\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \left(\frac{\kappa}{1-\kappa}\right)^2 \mathbb{V}_a\right]^{-1}}{\left[\left(\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \left(\frac{\kappa}{1-\kappa}\right)^2 \mathbb{V}_a\right]^{-1} + \mathbb{V}_\mu^{-1}}\left(\mu_t + \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} - \frac{\kappa}{1-\kappa}\widehat{a}_t\right).$$

Define

$$w_{\epsilon_{\pi^*}} \equiv \frac{\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^{-2}\left[\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a+\mathbb{V}_\mu\right]^{-1}}{\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^{-2}\left[\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a+\mathbb{V}_\mu\right]^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}},$$

$$w_{\widehat{a}} \equiv \frac{\left(\frac{1-\kappa}{\kappa}\right)^{-2}\left[\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\mathbb{V}_\mu\right]^{-1}}{\left(\frac{1-\kappa}{\kappa}\right)^{-2}\left[\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\mathbb{V}_\mu\right]^{-1}+\mathbb{V}_a^{-1}},$$

$$w_\mu = \frac{\left[\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a\right]^{-1}}{\left[\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a\right]^{-1}+\mathbb{V}_\mu^{-1}}.$$

Expectations can then be simplified as follows:

$$\mathbb{E}\left[\epsilon_{\pi^*,t}|\Omega_t^{CB}\right] = w_{\epsilon_{\pi^*}}\left(\epsilon_{\pi^*,t}+\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\left(-\frac{\kappa}{1-\kappa}\widehat{a}_t+\mu_t\right)\right),$$

$$\mathbb{E}\left[\widehat{a}_t|\Omega_t^{CB}\right] = w_{\widehat{a}}\left(\widehat{a}_t-\frac{1-\kappa}{\kappa}\left(\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t}+\mu_t\right)\right),$$

$$\mathbb{E}\left[\mu_t|\Omega_t^{CB}\right] = w_{\widehat{\mu}}\left(\mu_t+\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t}-\frac{\kappa}{1-\kappa}\widehat{a}_t\right).$$

Finally, by substituting the expectations above into (48), we obtain (51):

$$\begin{aligned}
m_t &= p_{t-1}+w_{\widehat{a}}\left(\widehat{a}_t-\frac{1-\kappa}{\kappa}\left(\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t}+\mu_t\right)\right)\\
&+\left(1-\frac{-\lambda\kappa(1-\kappa)+(1-\lambda)(1-\kappa)^2}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2}\frac{\mathbb{V}_{\pi^*}^{-1}+(1-\alpha)\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\pi_t^*\\
&+\frac{-\lambda\kappa(1-\kappa)+(1-\lambda)(1-\kappa)^2}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2}w_{\epsilon_{\pi^*}}\left(\epsilon_{\pi^*,t}+\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\left(-\frac{\kappa}{1-\kappa}\widehat{a}_t+\mu_t\right)\right)\\
&+\frac{-\lambda\kappa(1-\kappa)+(1-\lambda)(1-\kappa)^2}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2}w_\mu\left(\mu_t+\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t}-\frac{\kappa}{1-\kappa}\widehat{a}_t\right)
\end{aligned}$$

$$
\begin{aligned}
&= p_{t-1} + \left(1 + \frac{1-\kappa}{\kappa}\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\frac{\mathbb{V}_{\pi^*}^{-1} + (1-\alpha)\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\pi_t^* \\
&\quad + \left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\widehat{a}_t \\
&\quad - \frac{1-\kappa}{\kappa}\left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} \\
&\quad - \frac{1-\kappa}{\kappa}\left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\mu_t.
\end{aligned}
$$

A.7 Shocks, Noises, and Inflation Dynamics

By substituting (51) into (42), we obtain

$$
\begin{aligned}
p_t &= (1-\kappa)\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\left(\pi_t^* + \epsilon_{\pi^*,t}\right) + p_{t-1} + \mu_t\right) + \kappa(m_t - \widehat{a}_t) \\
&= (1-\kappa)\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\left(\pi_t^* + \epsilon_{\pi^*,t}\right) + p_{t-1} + \mu_t\right) - \kappa\widehat{a}_t + \kappa p_{t-1} \\
&\quad + \kappa\left(1 + \frac{1-\kappa}{\kappa}\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\frac{\mathbb{V}_{\pi^*}^{-1} + (1-\alpha)\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\pi_t^* \\
&\quad + \kappa\left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\widehat{a}_t \\
&\quad - \kappa\frac{1-\kappa}{\kappa}\left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} \\
&\quad - \kappa\frac{1-\kappa}{\kappa}\left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\mu_t \\
&= p_{t-1} + \left(\kappa + (1-\kappa)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\frac{\mathbb{V}_{\pi^*}^{-1} + (1-\alpha)\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} + (1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\pi_t^* \\
&\quad + \kappa\left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2} - 1\right)\widehat{a}_t \\
&\quad - (1-\kappa)\left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2} - 1\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} \\
&\quad - (1-\kappa)\left(w_{\widehat{a}} + \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2} - 1\right)\mu_t
\end{aligned}
$$

$$
\begin{aligned}
&= p_{t-1} + \left(1 - \frac{(1-\lambda)(1-\kappa)^2}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\left(1 - \frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\right)\pi_t^* \\
&\quad -\kappa\left(1 - w_{\widehat{a}} - \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}} w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\widehat{a}_t \\
&\quad +(1-\kappa)\left(1 - w_{\widehat{a}} - \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}} w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} \\
&\quad +(1-\kappa)\left(1 - w_{\widehat{a}} - \left(\frac{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}} w_{\epsilon_{\pi^*}} + w_\mu\right)\frac{\lambda\kappa^2 - (1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2 + (1-\lambda)(1-\kappa)^2}\right)\mu_t.
\end{aligned}
$$

A.8 Endogenous Information Structures

Consumers' expectations can be decomposed into the true value and noise terms as follows:

$$
\begin{aligned}
\mathbb{E}\left[p_t|\Omega_{i,t}^c\right] &= (1-\omega_\alpha-\omega_\beta)\,p_{i,t} + \omega_\alpha p_{s,i,t} + \omega_\beta\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] \\
&= (1-\omega_\alpha-\omega_\beta)\left(p_t + (p_{i,t}-p_t)\right) \\
&\quad +\omega_\alpha\left(p_t + (p_{s,i,t}-p_t)\right) + \omega_\beta\left(p_t + \left(\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] - p_t\right)\right) \\
&= p_t + (1-\omega_\alpha-\omega_\beta)(p_{i,t}-p_t) + \omega_\alpha(p_{s,i,t}-p_t) + \omega_\beta\left(\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] - p_t\right).
\end{aligned}
$$

Moreover, given equation (52), $\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] = \mathbb{V}_{\epsilon_{\pi^*}}^{-1}/\left(\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}\right)\pi_{s,t}^*$, and $\mathbb{E}\left[\epsilon_{\pi^*,t}|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] = \mathbb{V}_{\pi^*}^{-1}/\left(\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}\right)\pi_{s,t}^*$, $\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]$ is expressed as follows:

$$
\begin{aligned}
\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] &= p_{t-1} + \Xi\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] + \Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\mathbb{E}\left[\epsilon_{\pi^*,t}|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] \\
&= p_{t-1} + \Xi\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_{s,t}^* + \Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\frac{\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_{s,t}^* \\
&= p_{t-1} + \left[\Xi + \Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right]\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_{s,t}^* \\
&= p_{t-1} + \left[\Xi + \Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right]\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right]
\end{aligned}
$$

Therefore, α for $\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] \equiv \alpha\mathbb{E}\left[\pi_t^*|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] + p_{t-1}$ is identified as follows:

$$
\alpha = \Xi + \Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \Leftrightarrow \alpha = \frac{\Xi}{1 - \Psi(1-\kappa)\frac{\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}.
$$

The noise terms $(p_{i,t}-p_t)$, $(p_{s,i,t}-p_t)$, and $\left(\mathbb{E}\left[p_t|\pi_{s,t}^*, \Omega_{i,t-1}^c\right] - p_t\right)$ can be expressed as follows:

$$
\begin{aligned}
(p_{i,t}-p_t) &= (1-\widetilde{\kappa})\frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha + \omega_\beta}\epsilon_{\pi,i,t} - \widetilde{\kappa}\widehat{e}_{i,t}, \\
(p_{s,i,t}-p_t) &= \epsilon_{\pi,i,t}.
\end{aligned}
$$

$$
\begin{aligned}
& \left(\mathbb{E}\left[p_t | \pi^*_{s,t}, \Omega^c_{i,t-1}\right] - p_t\right) \\
= \;& p_{t-1} + \left[\Xi + \Psi\left(1-\kappa\right) \frac{\alpha \mathbb{V}^{-1}_{\pi^*}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right] \frac{\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \pi^*_{s,t} - p_t \\
= \;& \left[\Xi + \Psi\left(1-\kappa\right) \frac{\alpha \mathbb{V}^{-1}_{\pi^*}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right] \frac{\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \pi^*_{s,t} - \Xi \pi^*_t \\
& - \Psi \left[\left(1-\kappa\right) \frac{\alpha \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \epsilon_{\pi^*,t} - \kappa \widehat{a}_t + \left(1-\kappa\right) \mu_t\right] \\
= \;& \left[\left(\Xi + \Psi\left(1-\kappa\right) \frac{\alpha \mathbb{V}^{-1}_{\pi^*}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right) \frac{\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} - \Xi\right] \pi^*_t \\
& + \left[\Xi - \Psi\left(1-\kappa\right) \alpha \frac{\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right] \frac{\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} \epsilon_{\pi^*,t} + \Psi \kappa \widehat{a}_t - \Psi\left(1-\kappa\right) \mu_t.
\end{aligned}
$$

The first and second equations include the common noise term $\epsilon_{\pi,i,t}$, and we therefore eliminate $\epsilon_{\pi,i,t}$ from the first equation as follows:

$$
\begin{aligned}
& \left(p_{i,t} - p_t\right) - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta} \left(p_{s,i,t} - p_t\right) = -\widetilde{\kappa} \widehat{e}_{i,t} \\
\Leftrightarrow \;& \frac{p_{i,t} - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta} p_{s,i,t}}{1 - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta}} - p_t = - \frac{\widetilde{\kappa}}{1 - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta}} \widehat{e}_{i,t}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\mathbb{V}_1 \;\equiv\;& \mathbb{E}\left[\left(\frac{p_{i,t} - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta} p_{s,i,t}}{1 - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta}} - p_t\right)^2 | \Omega^c_{i.t-1}\right] = \left(\frac{\widetilde{\kappa}}{1 - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta}}\right)^2 \mathbb{V}_e, \\
\mathbb{V}_2 \;\equiv\;& \mathbb{E}\left[\left(p_{s,i,t} - p_t\right)^2 | \Omega^c_{i.t-1}\right] = \mathbb{V}_{\epsilon_p}, \\
\mathbb{V}_3 \;\equiv\;& \mathbb{E}\left[\left(\mathbb{E}\left[p_t | \pi^*_{s,t}, \Omega^c_{i,t-1}\right] - p_t\right)^2 | \Omega^c_{i.t-1}\right] \\
=\;& \left[\left(\Xi + \Psi\left(1-\kappa\right) \frac{\alpha \mathbb{V}^{-1}_{\pi^*}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right) \frac{\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}} - \Xi\right]^2 \mathbb{V}_{\pi^*} \\
& + \left[\Xi - \Psi\left(1-\kappa\right) \alpha \left(1 - \frac{\alpha \mathbb{V}^{-1}_{\pi^*}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right)\right]^2 \left(\frac{\mathbb{V}^{-1}_{\epsilon_{\pi^*}}}{\mathbb{V}^{-1}_{\pi^*} + \mathbb{V}^{-1}_{\epsilon_{\pi^*}}}\right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \Psi^2 \kappa^2 \mathbb{V}_a + \Psi^2 \left(1-\kappa\right)^2 \mathbb{V}_\mu.
\end{aligned}
$$

Since the noises of the signals ($\left[1 - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta}\right]^{-1} \left[p_{i,t} - \left(1-\widetilde{\kappa}\right) \frac{\overline{\phi} - \underline{\phi}}{\overline{\phi} + 1 - \underline{\phi}} \frac{\omega_\alpha}{\omega_\alpha + \omega_\beta} p_{s,i,t}\right]$,

$p_{s,i,t}$, $\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right]$) are independent of each other,

$$
\begin{aligned}
\mathbb{E}\left[p_t|\Omega^c_{i,t}\right] &= \frac{\mathbb{V}_1^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}\left(\frac{p_{i,t}-(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}p_{s,i,t}}{1-(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}}\right) \\
&\quad+\frac{\mathbb{V}_2^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}p_{s,i,t}+\frac{\mathbb{V}_3^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right] \\
&= \left[\frac{\mathbb{V}_1^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}\frac{1}{1-(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}}\right]p_{i,t} \\
&\quad+\left[\frac{\mathbb{V}_2^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}-\frac{\mathbb{V}_1^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}\frac{(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}}{1-(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}}\right]p_{s,i,t} \\
&\quad+\frac{\mathbb{V}_3^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}\mathbb{E}\left[p_t|\pi^*_{s,t},\Omega^c_{i,t-1}\right],
\end{aligned}
$$

and thus,

$$
\begin{aligned}
\omega_\alpha &= \frac{\mathbb{V}_2^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}-\frac{\mathbb{V}_1^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}\frac{(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}}{1-(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}} \\
\omega_\beta &= \frac{\mathbb{V}_3^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}.
\end{aligned}
$$

Note that the imprecision $\mathbb{V}\left[p_t|\Omega^c_{i,t}\right]$ is given as the inverse of the sum of the precision of signals:

$$
\mathbb{V}\left[p_t|\Omega^c_{i,t}\right]=\frac{1}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}.
$$

B The Economic System and Economic Conditions

With the exogenous variables $(\lambda,\overline{\phi},\underline{\phi},\mathbb{V}_e,\mathbb{V}_a,\mathbb{V}_\mu,\mathbb{V}_{\pi^*},\mathbb{V}_{\epsilon_{\pi^*}},\mathbb{V}_{\epsilon_p})$, the inflation dynamics of this economy are as follows:

$$
\pi_t=\Xi\pi^*_t+\Psi\left[(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t}-\kappa\widehat{a}_t+(1-\kappa)\mu_t\right],
$$

where

$$
\begin{aligned}
(1)\ \Xi &\equiv 1-\frac{(1-\lambda)(1-\kappa)^2}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2}\left(1-\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right),\\
(2)\ \Psi &\equiv 1-w_{\widehat{a}}-\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}w_{\epsilon_{\pi^*}}+w_\mu\right)\frac{\lambda\kappa^2-(1-\lambda)\kappa(1-\kappa)}{\lambda\kappa^2+(1-\lambda)(1-\kappa)^2},\\
(3)\ \kappa &\equiv \frac{\widetilde{\kappa}}{1-(1-\widetilde{\kappa})\left[1+\left(\overline{\phi}-\underline{\phi}\right)\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}\right]/\left(\overline{\phi}+1-\underline{\phi}\right)},\\
(4)\ \widetilde{\kappa} &\equiv \left[1+\frac{(\omega_\alpha+\omega_\beta)\left(\overline{\phi}+1-\underline{\phi}\right)}{(\omega_\alpha+\omega_\beta)+\left(\overline{\phi}-\underline{\phi}\right)}\right]^{-1},\ (5)\ \alpha=\frac{\Xi}{1-\Psi(1-\kappa)\frac{\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}},\\
(6)\ w_{\epsilon_{\pi^*}} &\equiv \frac{\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^{-2}\left[\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a+\mathbb{V}_\mu\right]^{-1}}{\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^{-2}\left[\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a+\mathbb{V}_\mu\right]^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}},
\end{aligned}
$$

$$
\begin{aligned}
(7)\ w_{\widehat{a}} &\equiv \frac{\left(\frac{1-\kappa}{\kappa}\right)^{-2}\left[\left(\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\mathbb{V}_\mu\right]^{-1}}{\left(\frac{1-\kappa}{\kappa}\right)^{-2}\left[\left(\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\mathbb{V}_\mu\right]^{-1}+\mathbb{V}_a^{-1}},\\
(8)\ w_\mu &= \frac{\left[\left(\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a\right]^{-1}}{\left[\left(\alpha\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a\right]^{-1}+\mathbb{V}_\mu^{-1}},\\
(9)\ \mathbb{V}_1 &= \left(\frac{\widetilde{\kappa}}{1-(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}}\right)^2\mathbb{V}_e,\ (10)\ \mathbb{V}_2=\mathbb{V}_{\epsilon_p},\\
(11)\ \mathbb{V}_3 &= \left[\left(\Xi+\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}-\Xi\right]^2\mathbb{V}_{\pi^*}\\
&\quad+\left[\Xi-\Psi(1-\kappa)\alpha\left(1-\frac{\alpha\mathbb{V}_{\pi^*}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\right]^2\left(\frac{\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\Psi^2\kappa^2\mathbb{V}_a+\Psi^2(1-\kappa)^2\mathbb{V}_\mu,\\
(12)\ \omega_\alpha &= \frac{\mathbb{V}_2^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}-\frac{\mathbb{V}_1^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}\frac{(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}}{1-(1-\widetilde{\kappa})\frac{\overline{\phi}-\underline{\phi}}{\overline{\phi}+1-\underline{\phi}}\frac{\omega_\alpha}{\omega_\alpha+\omega_\beta}},\\
(13)\ \omega_\beta &= \frac{\mathbb{V}_3^{-1}}{\mathbb{V}_1^{-1}+\mathbb{V}_2^{-1}+\mathbb{V}_3^{-1}}
\end{aligned}
$$

Note that we have 13 endogenous variables $(\Xi,\Psi,\kappa,\widetilde{\kappa},\alpha,w_{\epsilon_{\pi^*}},w_{\widehat{a}},w_\mu,\ \mathbb{V}_1,\ \mathbb{V}_2,\ \mathbb{V}_3,\omega_\alpha,\omega_\beta)$ and 13 non-redundant conditions.

C Updating of Beliefs on the Inflation Target

C.1 Learning from Prices

In period t, there exist three types of information about the central bank's inflation target π_t^*:

$$(1)\ \pi_t^* \sim_i \mathcal{N}(\pi_{i,t|t-1}^*, \mathbb{V}_{\pi^*,t|t-1}),$$

$$(2)\ \pi_{s,t}^* = \pi_t^* + \epsilon_{\pi,t}^*, \epsilon_{\pi,t}^* \sim \mathcal{N}(0, \mathbb{V}_{\epsilon_\pi^*})$$

$$(3)\ \frac{1}{\Xi}\pi_t = \pi_t^* + \frac{1}{\Xi}\Psi\left[(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} - \kappa\widehat{a}_t + (1-\kappa)\mu_t\right] \sim \mathcal{N}\left(\pi_t^*, \mathbb{V}_{\pi_t^*|\pi_t}\right),$$

where

$$\mathbb{V}_{\pi_t^*|\pi_t} \equiv \frac{\Psi^2}{\Xi^2}\left[(1-\kappa)^2\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}} + \kappa^2\mathbb{V}_a + (1-\kappa)^2\mathbb{V}_\mu\right].$$

We conjecture that each consumer does the linear filtering of the aggregate price as follows:

$$\mathbb{E}\left[\pi_t^*|\Omega_{i,t}^c\right] = \left(1-\chi_\alpha-\chi_\beta\right)\pi_{i,t|t-1}^* + \chi_\alpha\pi_{s,t}^* + \chi_\beta\frac{1}{\Xi}\pi_t.$$

We then find the optimal weights for filtering $(\chi_\alpha, \chi_\beta)$. We transform the signals with correlated noises $(\pi_{s,t}^*, \frac{1}{\Xi}\pi_t)$ into those with independent noises as follows:

$$(2)\ \pi_{s,t}^* = \pi_t^* + \epsilon_{\pi,t}^*, \epsilon_{\pi,t}^* \sim \mathcal{N}(0, \mathbb{V}_{\epsilon_\pi^*})$$

$$(3')\ \left(\frac{\frac{1}{\Xi}\pi_t - \frac{1}{\Xi}\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_{s,t}^*}{1-\frac{1}{\Xi}\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}\right) = \pi_t^* + \left(\frac{\frac{1}{\Xi}\Psi\left[-\kappa\widehat{a}_t + (1-\kappa)\mu_t\right]}{1-\frac{1}{\Xi}\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}\right).$$

Therefore, the optimal filtering with signals (1), (2), and (3') is defined as follows:

$$\begin{aligned}
\mathbb{E}\left[\pi_t^*|\Omega_{i,t}^c\right] &= \frac{\widehat{\mathbb{V}}_1^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}}\pi_{i,t|t-1}^* + \frac{\widehat{\mathbb{V}}_2^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}}\pi_{s,t}^* \\
&\quad + \frac{\widehat{\mathbb{V}}_3^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}}\frac{\frac{1}{\Xi}\pi_t - \frac{1}{\Xi}\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\pi_{s,t}^*}{1-\frac{1}{\Xi}\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}} \\
&= \frac{\widehat{\mathbb{V}}_1^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}}\pi_{i,t|t-1}^* + \frac{\widehat{\mathbb{V}}_3^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}}\frac{1}{1-\frac{1}{\Xi}\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}\frac{1}{\Xi}\pi_t \\
&\quad + \left(\frac{\widehat{\mathbb{V}}_2^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}} - \frac{\widehat{\mathbb{V}}_3^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}}\frac{\frac{1}{\Xi}\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}{1-\frac{1}{\Xi}\Psi(1-\kappa)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}\right)\pi_{s,t}^*,
\end{aligned}$$

where

$$\begin{aligned}
\widehat{\mathbb{V}}_1 &= \mathbb{V}_{\pi^*,t|t-1}, \widehat{\mathbb{V}}_2 = \mathbb{V}_{\epsilon_{\pi^*}}, \\
\widehat{\mathbb{V}}_3 &= \left(\frac{\frac{1}{\Xi}\Psi\kappa}{1-\frac{1}{\Xi}\Psi\left(1-\kappa\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}\right)^2\mathbb{V}_a + \left(\frac{\frac{1}{\Xi}\Psi\left(1-\kappa\right)}{1-\frac{1}{\Xi}\Psi\left(1-\kappa\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}\right)^2\mathbb{V}_\mu.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\chi_\alpha &= \frac{\widehat{\mathbb{V}}_2^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}} - \frac{\widehat{\mathbb{V}}_3^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}}\frac{\frac{1}{\Xi}\Psi\left(1-\kappa\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}{1-\frac{1}{\Xi}\Psi\left(1-\kappa\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}, \\
\chi_\beta &= \frac{\widehat{\mathbb{V}}_3^{-1}}{\widehat{\mathbb{V}}_1^{-1}+\widehat{\mathbb{V}}_2^{-1}+\widehat{\mathbb{V}}_3^{-1}}\frac{1}{1-\frac{1}{\Xi}\Psi\left(1-\kappa\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}}.
\end{aligned}$$

Finally, from $\pi_t^* = \pi_{t-1}^* + \delta_t$ and

$$\begin{aligned}
\mathbb{E}\left[\pi_t^*|\Omega_{i,t}^c\right] &= \left(1-\chi_\alpha-\chi_\beta\right)\pi_{i,t|t-1}^* + \chi_\alpha\left(\pi_t^*+\epsilon_{\pi^*,t}\right) \\
&\quad +\chi_\beta\left(\pi_t^* + \frac{1}{\Xi}\Psi\left[\left(1-\kappa\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} - \kappa\widehat{a}_t + \left(1-\kappa\right)\mu_t\right]\right),
\end{aligned}$$

we obtain

$$\partial\mathbb{E}\left[\pi_t^*|\Omega_{i,t}^c\right]/\partial\delta_t = \chi_\alpha + \chi_\beta.$$

C.2 Implications of Consumers' Inflation Experience

This appendix shows the formation process of $\pi_{i,t|t-1}^*$ and $\mathbb{V}_{\pi^*,t|t-1}$ in a general form. Given $\pi_{i,t|t-1}^*$ and $\mathbb{V}_{\pi^*,t|t-1}$, $\pi_{i,t|t}^*$ and $\mathbb{V}_{\pi^*,t|t}$ are formed as follows:

$$\begin{aligned}
\pi_{i,t|t}^* &= \mathbb{E}\left[\pi_t^*|\Omega_{i,t}^c\right] \\
&= \left(1-\chi_\alpha-\chi_\beta\right)\pi_{i,t|t-1}^* + \chi_\alpha\left(\pi_t^*+\epsilon_{\pi^*,t}\right) \\
&\quad +\chi_\beta\left(\pi_t^* + \frac{1}{\Xi}\Psi\left[\left(1-\kappa\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\epsilon_{\pi^*,t} - \kappa\widehat{a}_t + \left(1-\kappa\right)\mu_t\right]\right) \\
&= \left(\chi_\alpha+\chi_\beta\right)\pi_t^* + \left(\chi_\alpha + \chi_\beta\frac{1}{\Xi}\Psi\left(1-\kappa\right)\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)\epsilon_{\pi^*,t} \\
&\quad -\chi_\beta\frac{1}{\Xi}\Psi\kappa\widehat{a}_t + \chi_\beta\frac{1}{\Xi}\Psi\left(1-\kappa\right)\mu_t + \left(1-\chi_\alpha-\chi_\beta\right)\pi_{i,t|t-1}^* \\
&= \Upsilon_{\pi^*}\pi_t^* + \Upsilon_{\epsilon_{\pi^*}}\epsilon_{\pi^*,t} - \Upsilon_{\widehat{a}}\widehat{a}_t + \Upsilon_\mu\mu_t + \widehat{\Upsilon}\pi_{i,t|t-1}^*,
\end{aligned}$$

where

$$
\begin{aligned}
\Upsilon_{\pi^*} &\equiv \chi_\alpha + \chi_\beta, \Upsilon_{\epsilon_{\pi^*}} \equiv \chi_\alpha + \chi_\beta \frac{1}{\Xi} \Psi (1-\kappa) \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}, \\
\Upsilon_{\widehat{a}} &\equiv \chi_\beta \frac{1}{\Xi} \Psi \kappa, \Upsilon_\mu \equiv \chi_\beta \frac{1}{\Xi} \Psi (1-\kappa), \widehat{\Upsilon} \equiv 1 - \chi_\alpha - \chi_\beta.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\pi^*_{i,t|t} &= \Upsilon_{\pi^*} \pi^*_t + \Upsilon_{\epsilon_{\pi^*}} \epsilon_{\pi^*,t} - \Upsilon_{\widehat{a}} \widehat{a}_t + \Upsilon_\mu \mu_t + \widehat{\Upsilon} p^*_{i,t|t-1} \\
&= \sum_{j=0}^{\infty} \widehat{\Upsilon}^j \left(\Upsilon_{\pi^*} \pi^*_{t-j} + \Upsilon_{\epsilon_{\pi^*}} \epsilon_{\pi^*,t-j} - \Upsilon_{\widehat{a}} \widehat{a}_{t-j} + \Upsilon_\mu \mu_{t-j} \right).
\end{aligned}
$$

Note that $\pi^*_{i,t|t}$ can be expressed as a function of the signals $\left\{ \pi^*_{s,t-j}, \pi_{t-j} \right\}_{j=1}^{\infty}$ as follows:

$$
\begin{aligned}
\pi^*_{i,t|t} &= \left(1 - \chi_\alpha - \chi_\beta\right) \pi^*_{i,t|t-1} + \chi_\alpha \pi^*_{s,t} + \chi_\beta \frac{1}{\Xi} \pi_t \\
&= \sum_{j=0}^{\infty} \left(1 - \chi_\alpha - \chi_\beta\right)^j \left(\chi_\alpha \pi^*_{s,t-j} + \chi_\beta \frac{1}{\Xi} \pi_{t-j} \right).
\end{aligned}
$$

Therefore, from $\pi^*_t = \pi^*_{t-1} + \delta_t$ and $\delta_t \sim \mathcal{N}(0, \mathbb{V}_\delta)$, consumer's prior beliefs about the inflation target are given by

$$
\pi^*_{i,t|t-1} = \sum_{j=1}^{\infty} \left(1 - \chi_\alpha - \chi_\beta\right)^j \left(\chi_\alpha \pi^*_{s,t-j} + \chi_\beta \frac{1}{\Xi} \pi_{t-j} \right).
$$

From the equation about the formation of $\pi^*_{i,t|t}$,

$$
\begin{aligned}
\mathbb{V}_{\pi^*,t|t} &= \mathbb{V}\left[\pi^*_t | \Omega^c_{i,t} \right] \\
&= \left(\chi_\alpha + \chi_\beta \frac{1}{\Xi} \Psi (1-\kappa) \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \right)^2 \mathbb{V}_{\epsilon_{\pi^*}} \\
&\quad + \left(\chi_\beta \frac{1}{\Xi} \Psi \kappa \right)^2 \mathbb{V}_a + \left(\chi_\beta \frac{1}{\Xi} \Psi (1-\kappa) \right)^2 \mathbb{V}_\mu + \left(1 - \chi_\alpha - \chi_\beta\right)^2 \mathbb{V}_{\pi^*,t|t-1},
\end{aligned}
$$

holds and we have $\mathbb{V}_{\pi^*,t|t-1} = \mathbb{V}_{\pi^*,t-1|t-1} + \mathbb{V}_\delta$. Therefore, $\mathbb{V}_{\pi^*}$ is obtained as the value which satisfies the following condition:

$$
\begin{aligned}
\mathbb{V}_{\pi^*} &= \left(\chi_\alpha + \chi_\beta \frac{1}{\Xi} \Psi (1-\kappa) \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \right)^2 \mathbb{V}_{\epsilon_{\pi^*}} \\
&\quad + \left(\chi_\beta \frac{1}{\Xi} \Psi \kappa \right)^2 \mathbb{V}_a + \left(\chi_\beta \frac{1}{\Xi} \Psi (1-\kappa) \right)^2 \mathbb{V}_\mu + \left(1 - \chi_\alpha - \chi_\beta\right)^2 \mathbb{V}_{\pi^*} + \mathbb{V}_\delta.
\end{aligned}
$$

Note that from the inflation dynamics

$$\pi_t = \Xi \pi_t^* + \Psi \left[(1-\kappa) \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \epsilon_{\pi^*,t} - \kappa \widehat{a}_t + (1-\kappa) \mu_t \right],$$

the variability of inflation (unconditional on the realized inflation target π_t^*) is

$$\mathbb{E}\left[(\pi_t - \pi_{t-1})^2 \right] = \Xi^2 \mathbb{V}_\delta + 2\Psi^2 \left[(1-\kappa)^2 \left(\frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \kappa^2 \mathbb{V}_a + (1-\kappa)^2 \mathbb{V}_\mu \right].$$

D Policy Implications

D.1 The Trade-off Between Inflation Variability and Output Gap Variability

For a fixed π_t^*, the variability of the inflation rate is

$$\begin{aligned} &\mathbb{E}\left[(\pi_t - \pi_t^*)^2 | \Omega_t^{CB} \right] = (\Xi - 1)^2 (\pi_t^*)^2 \\ &+\Psi^2 \left[\left((1-\kappa) \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \right)^2 \mathbb{V}_{\epsilon_{\pi^*}} + \kappa^2 \mathbb{V}_a + (1-\kappa)^2 \mathbb{V}_\mu \right]. \end{aligned}$$

The output gap in this economy is given by (49):

$$\begin{aligned} \widehat{y}_t - \widehat{a}_t &= \frac{1-\kappa}{\kappa} \left(\pi_t - \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} (\pi_t^* + \epsilon_{\pi^*,t}) - \widehat{\mu}_t \right) \\ &= \frac{1-\kappa}{\kappa} \begin{pmatrix} \left(\Xi - \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \right) \pi_t^* \\ + \left([\Psi (1-\kappa) - 1] \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \right) \epsilon_{\pi^*,t} \\ + [\Psi (1-\kappa) - 1] \widehat{\mu}_t - \Psi \kappa \widehat{a}_t \end{pmatrix} \end{aligned}$$

The variability of output gap is

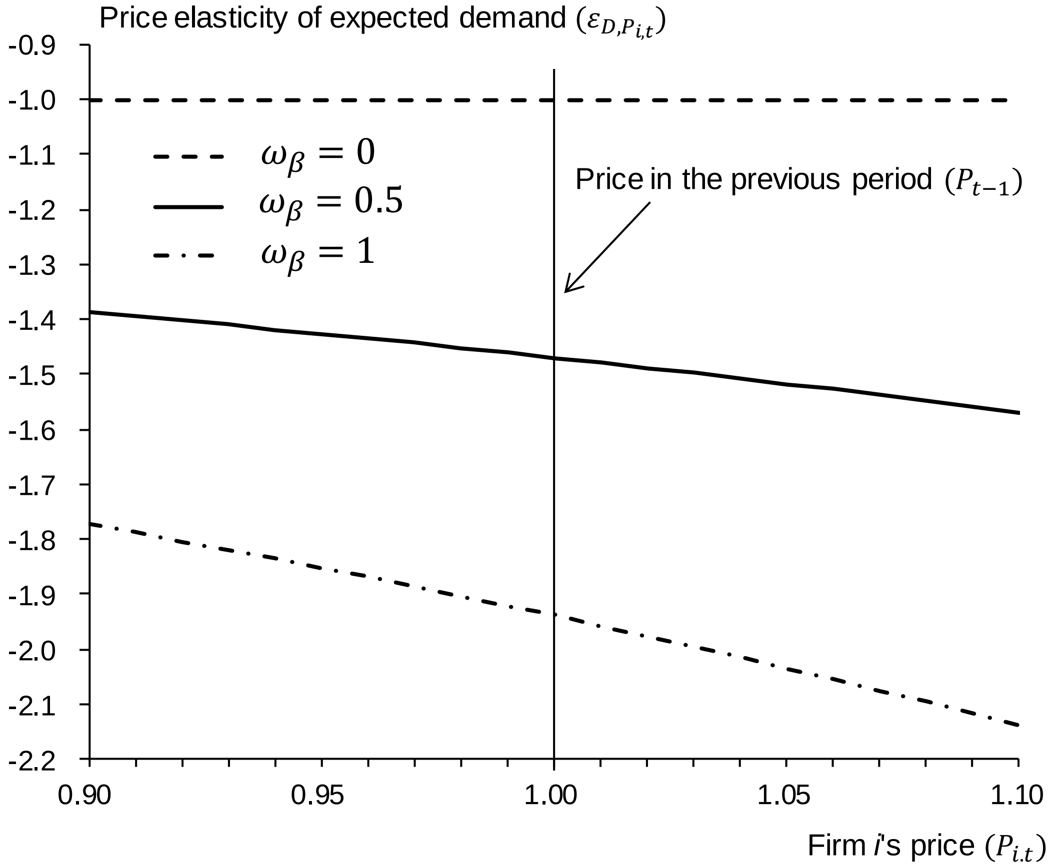
$$\mathbb{E}\left[(\widehat{y}_t - \widehat{a}_t)^2 | \Omega_t^{CB} \right] = \left(\frac{1-\kappa}{\kappa} \right)^2 \begin{pmatrix} \left(\Xi - \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \right)^2 (\pi_t^*)^2 \\ + \left([\Psi (1-\kappa) - 1] \frac{\alpha \mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1} + \mathbb{V}_{\epsilon_{\pi^*}}^{-1}} \right)^2 \mathbb{V}_{\epsilon_{\pi^*}} \\ [\Psi (1-\kappa) - 1]^2 \mathbb{V}_\mu + \Psi^2 \kappa^2 \mathbb{V}_a \end{pmatrix}.$$

D.2 Information Extraction by the Central Bank

From Appendix A.6, the imprecision of the central bank's beliefs about fundamental shocks is given by

$$
\begin{aligned}
\mathbb{V}\left[\epsilon_{\pi^*,t}|\Omega_t^{CB}\right] &= \left[\left(\frac{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^{-2}\left[\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a+\mathbb{V}_\mu\right]^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}\right]^{-1}, \\
\mathbb{V}\left[\widehat{a}_t|\Omega_t^{CB}\right] &= \left[\left(\frac{1-\kappa}{\kappa}\right)^{-2}\left[\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\mathbb{V}_\mu\right]^{-1}+\mathbb{V}_a^{-1}\right]^{-1}, \\
\mathbb{V}\left[\mu_t|\Omega_t^{CB}\right] &= \left[\left[\left(\frac{\alpha\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}{\mathbb{V}_{\pi^*}^{-1}+\mathbb{V}_{\epsilon_{\pi^*}}^{-1}}\right)^2\mathbb{V}_{\epsilon_{\pi^*}}+\left(\frac{\kappa}{1-\kappa}\right)^2\mathbb{V}_a\right]^{-1}+\mathbb{V}_\mu^{-1}\right]^{-1}.
\end{aligned}
$$

Figure 1: Price elasticity of expected demand



Notes: The parameters are $\overline{\phi} = 1.65, \underline{\phi} = 0.82, \omega_\alpha = 0,\ \varphi_t = 0.5,$ *and* $P_{t-1} = 1.$

Figure 2: Consumers' price beliefs and inflation dynamics

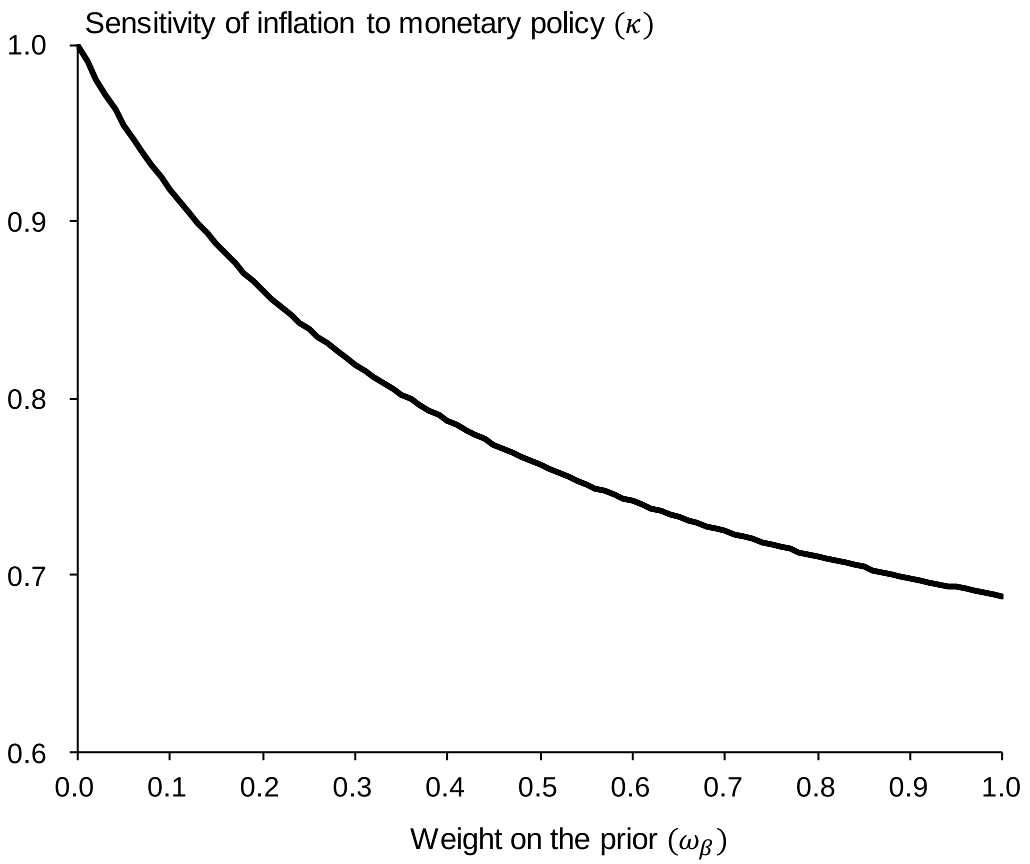
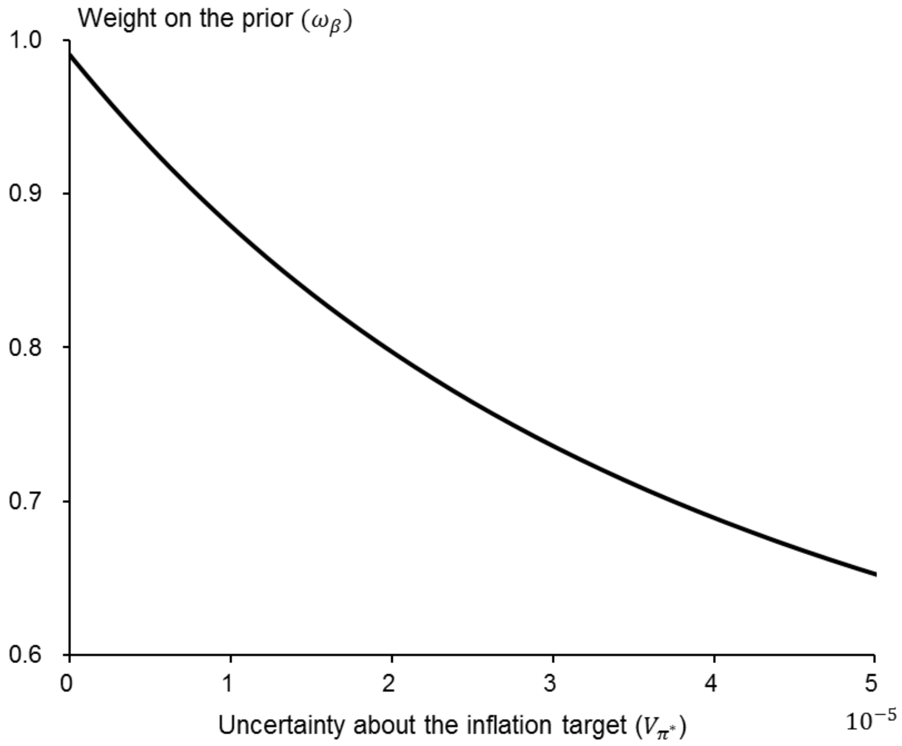


Figure 3: Uncertainty and weight on the prior

(a) Inflation target



(b) Island-specific productivity shocks

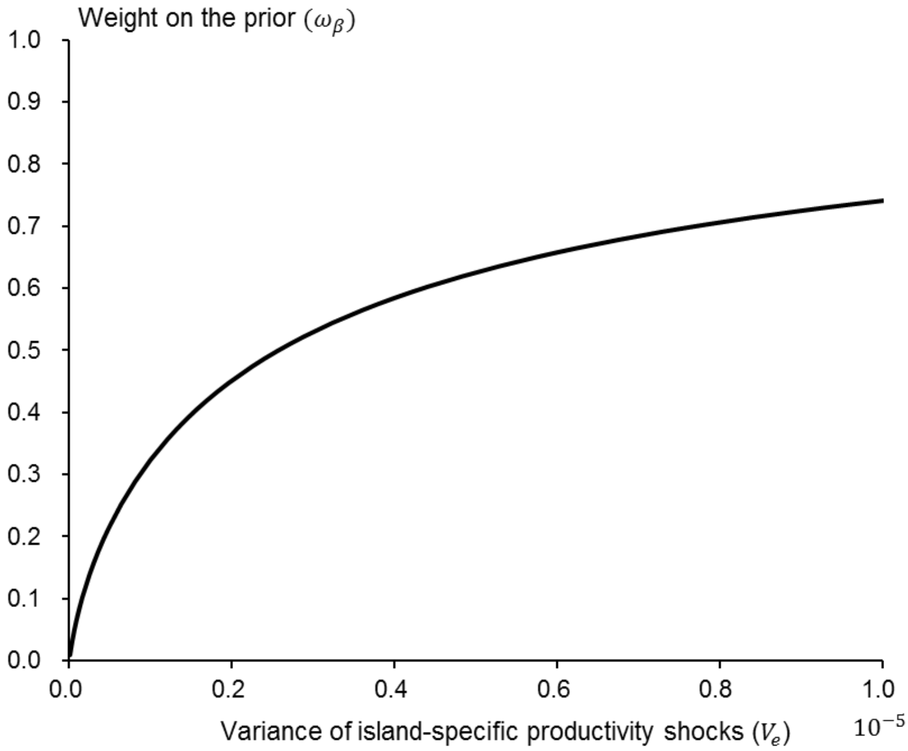


Figure 4: Consumers' uncertainty about their price beliefs

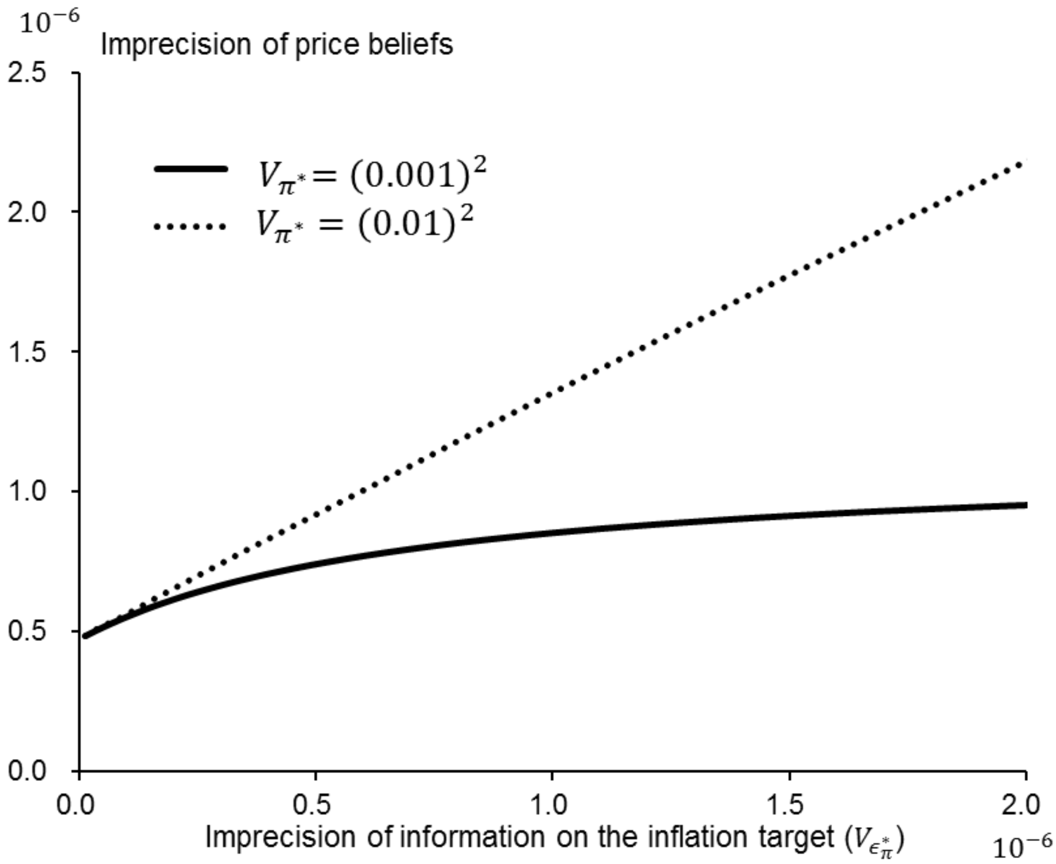


Figure 5: Sensitivity of consumers' beliefs on the inflation target

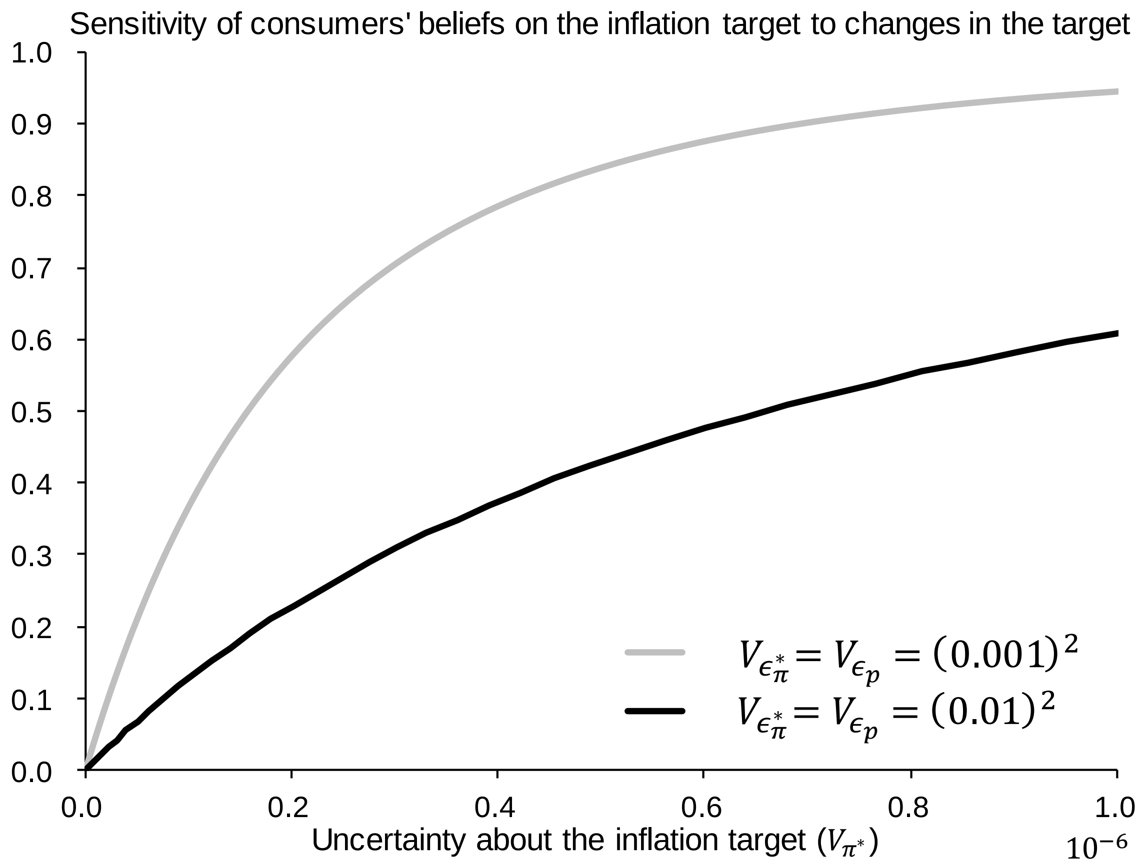
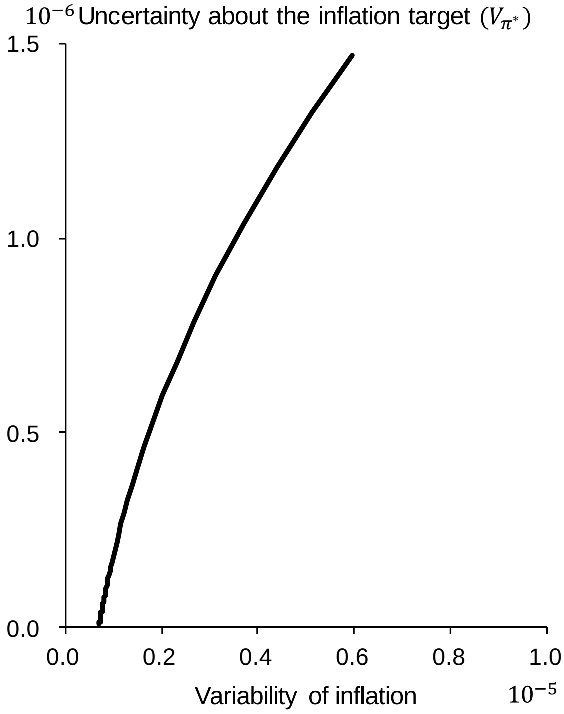
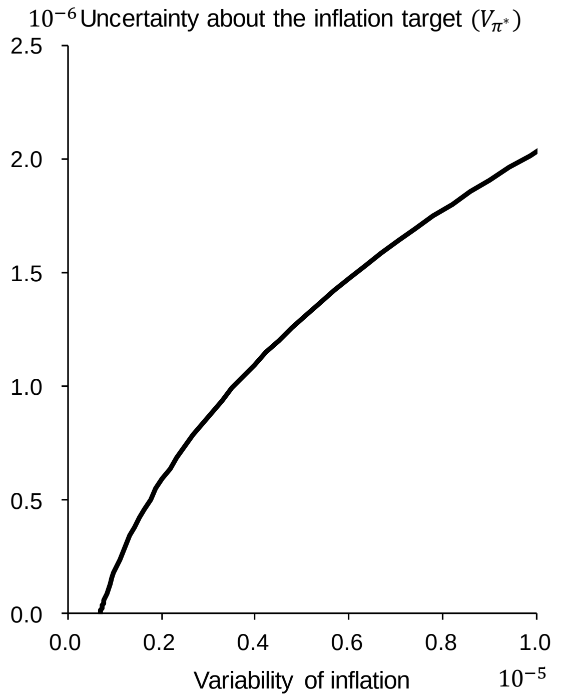


Figure 6: Inflation experience and prior beliefs (1)

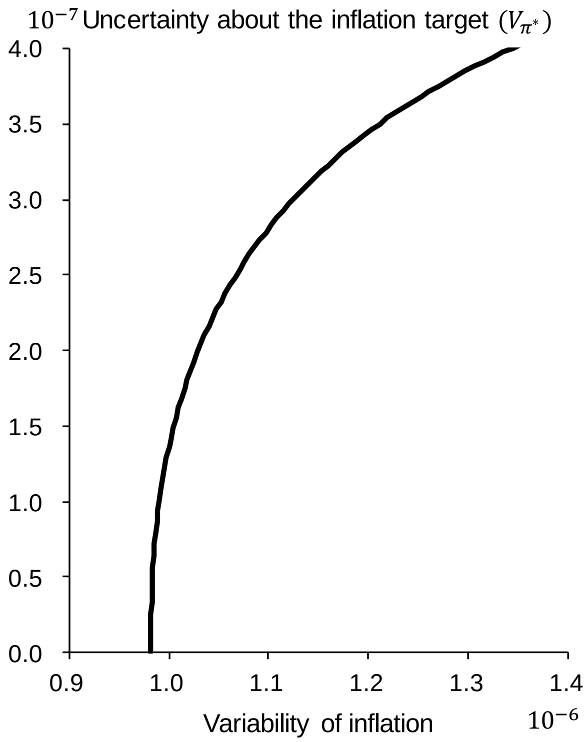
(a) Aggregate productivity shocks



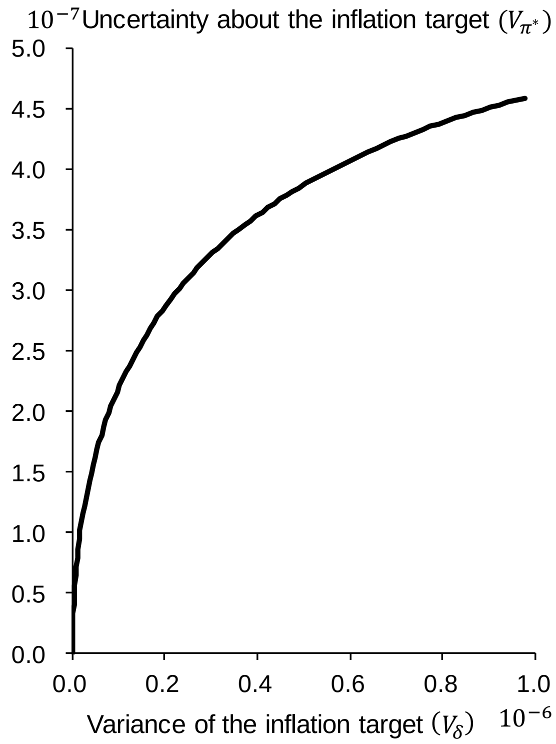
(b) Mark-up shocks



(c) Variance of inflation target (1)



(d) Variance of inflation target (2)



Notes: The parameters are $V_\delta = (0.001)^2$ *and* $V_{\varepsilon_p} \to \infty$.

Figure 7: Inflation experience and prior beliefs (2)

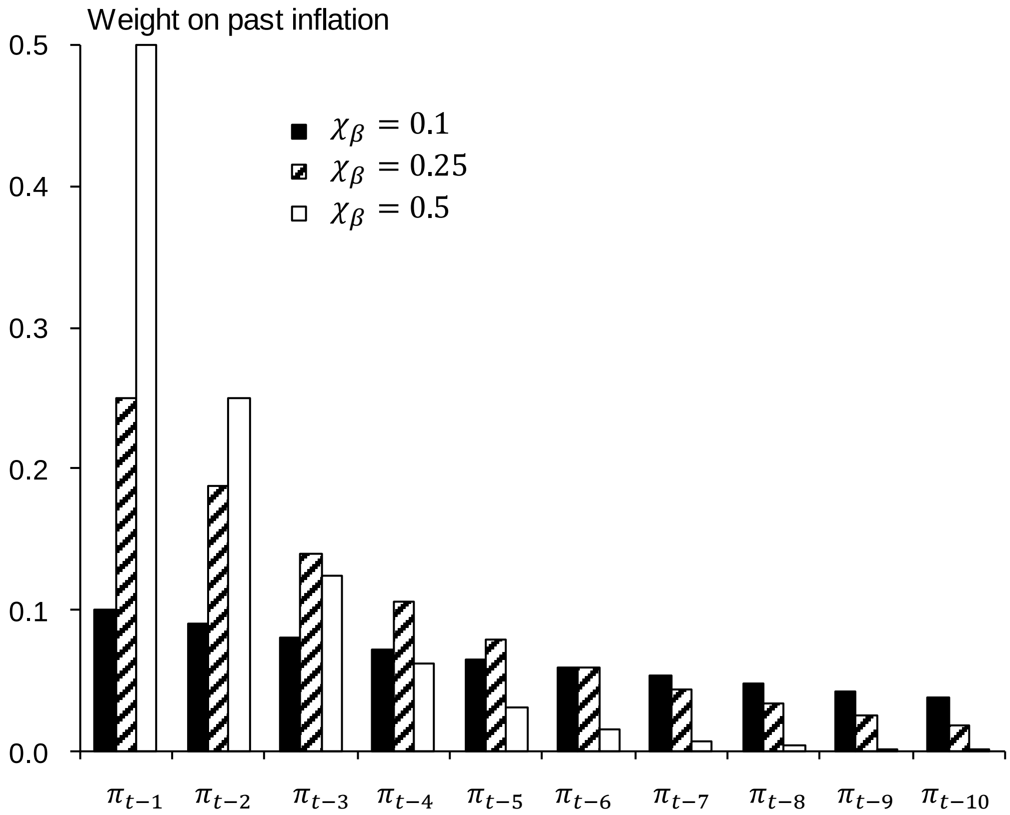
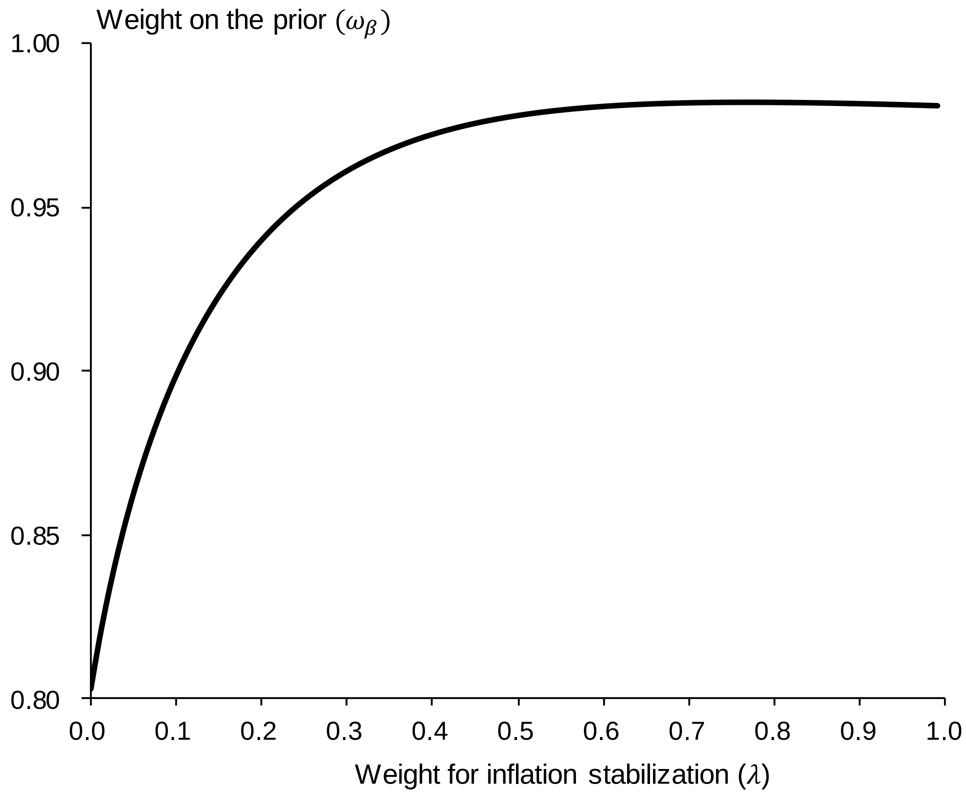


Figure 8: Monetary policy stance and inflation dynamics (1)

(a) Weight for inflation stabilization and consumers' beliefs



(b) Weight for inflation stabilization and inflation dynamics

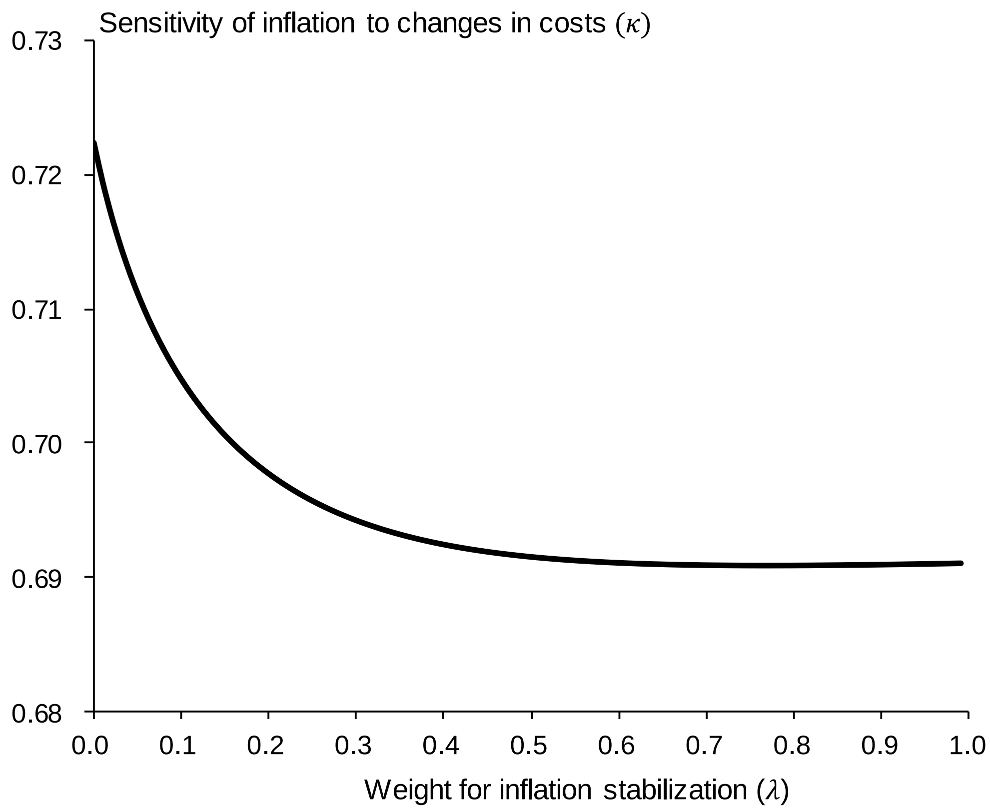


Figure 9: Monetary policy stance and inflation dynamics (2)

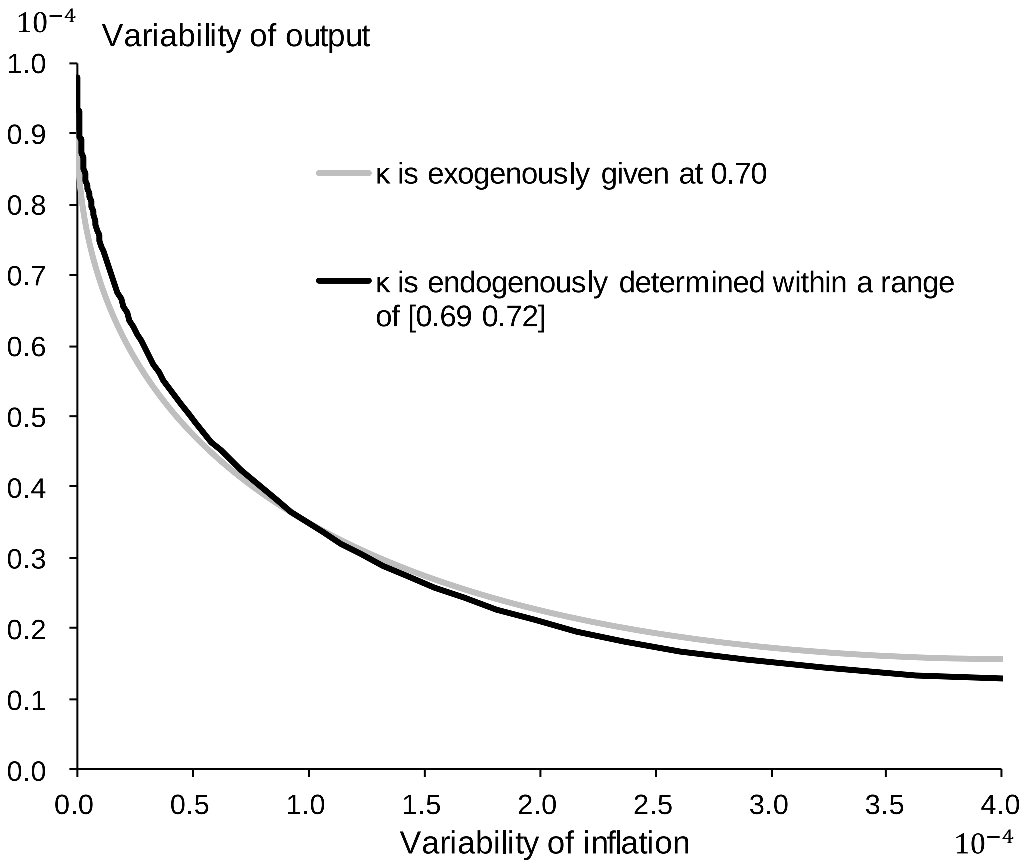


Figure 10: Effects of communication on inflation target

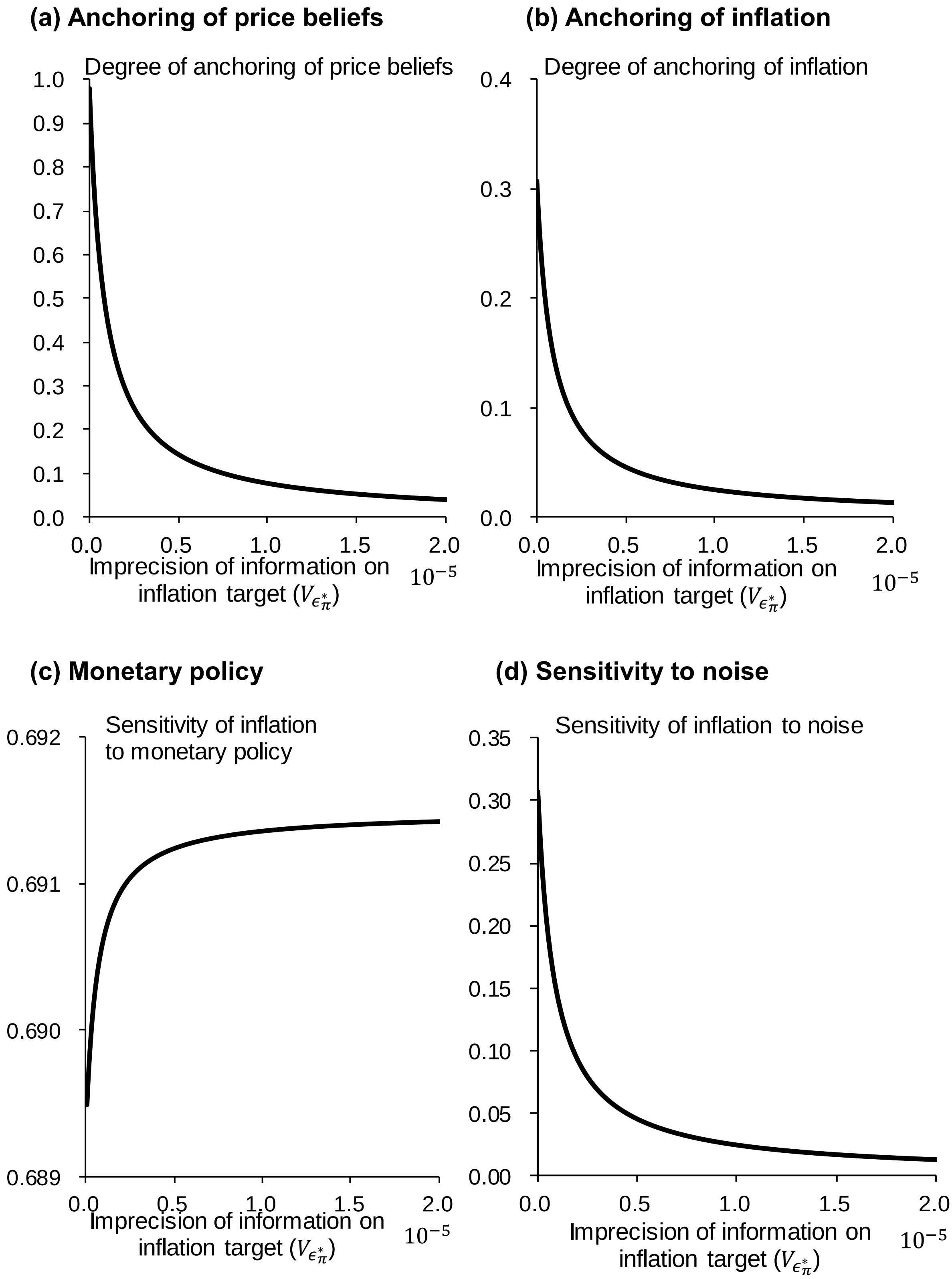
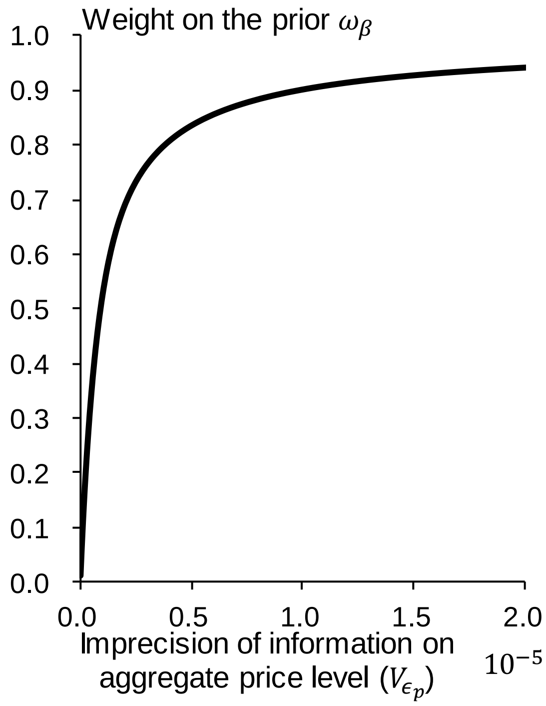
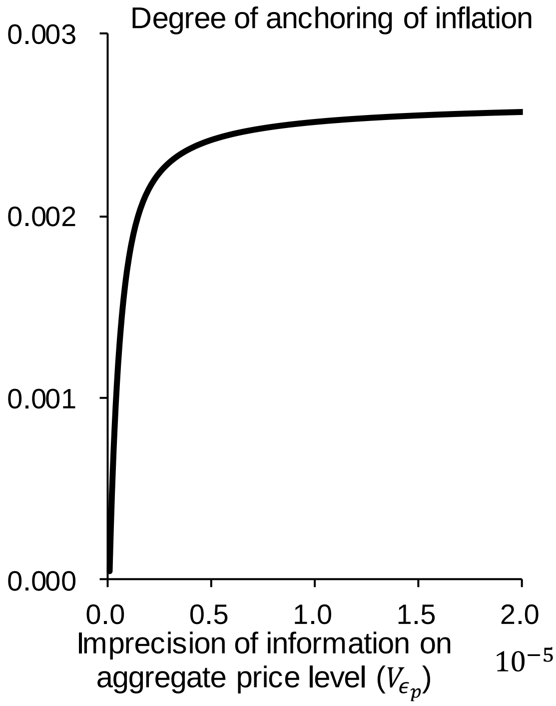


Figure 11: Effects of communication on aggregate price level

(a) Consumers' beliefs



(b) Anchoring of inflation



(c) Monetary policy

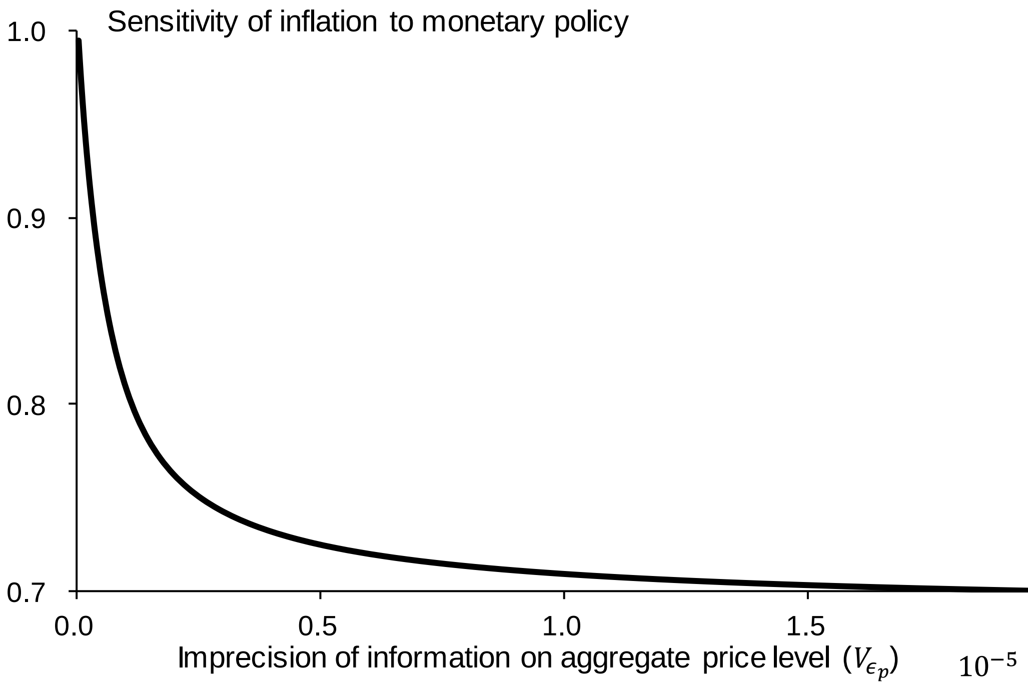
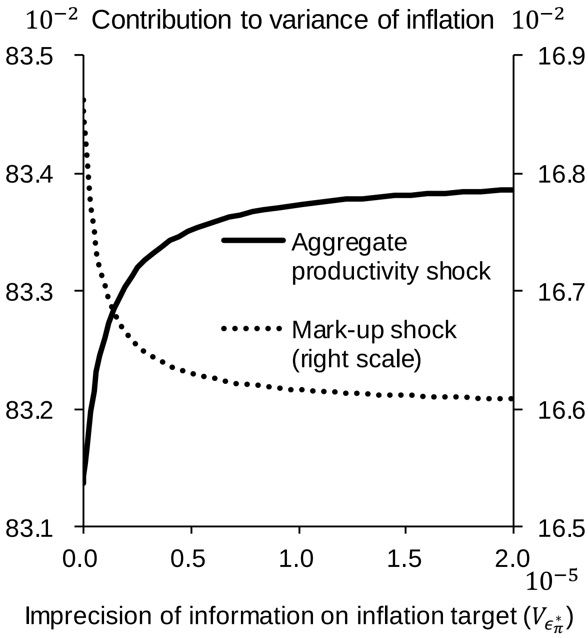
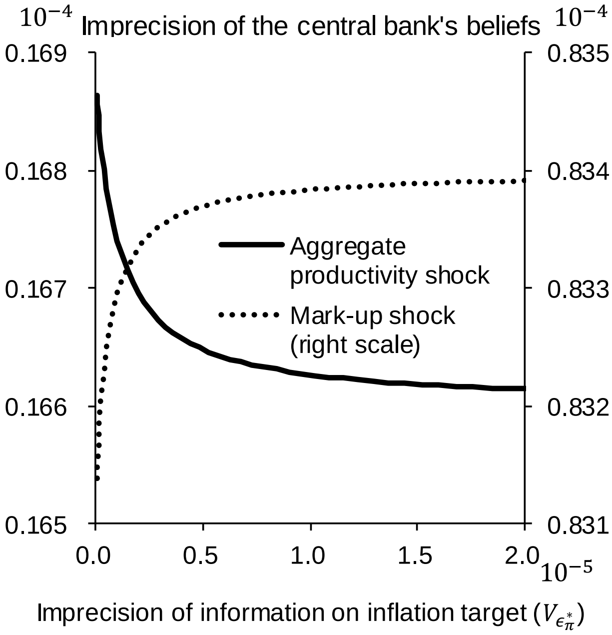


Figure 12: Communication and information extraction

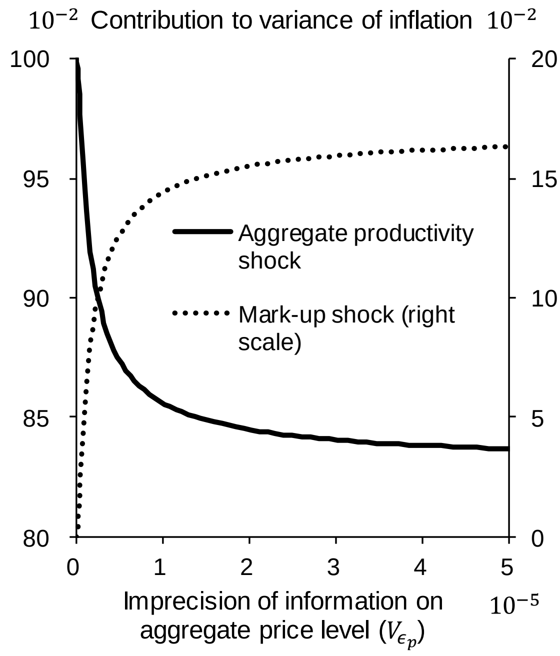
(a) Communication on inflation target and inflation dynamics



(b) Communication on inflation target and the central bank's beliefs



(c) Communication on aggregate price level and inflation dynamics



(d) Communication on aggregate price level and the central bank's beliefs