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Estimating Pipeline Pressures in New Keynesian Phillips Curves: A Bayesian VAR-GMM Approach *

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Abstract

This paper considers a vertical production chain in an otherwise canonical sticky price model, and estimates the New Keynesian Phillips curve with the vertical production stages (PS-NKPC), using the commodity-flow-based U.S. price data. We employ a Bayesian VAR-GMM method and compare the PS-NKPC with the canonical NKPC based on a quasi-marginal likelihood criterion, which is robust under weakly identified parameters. Thus our result adds to the empirical relevance of the so-called "pipeline price pressures" incurred by upstream stages of production. Our estimates suggest that (i) the PS-NKPC performs better than the canonical New Keynesian Phillips curve in terms of quasi-marginal likelihood-based model comparison, and (ii) pipeline price pressures have non-negligible impacts on consumer price inflation as well as producer price inflation.

JEL classification: C11, C26, C52, E31
keywords: New Keynesian Phillips curve, VAR-GMM, Bayesian method, Production chain, Pipeline pressure.

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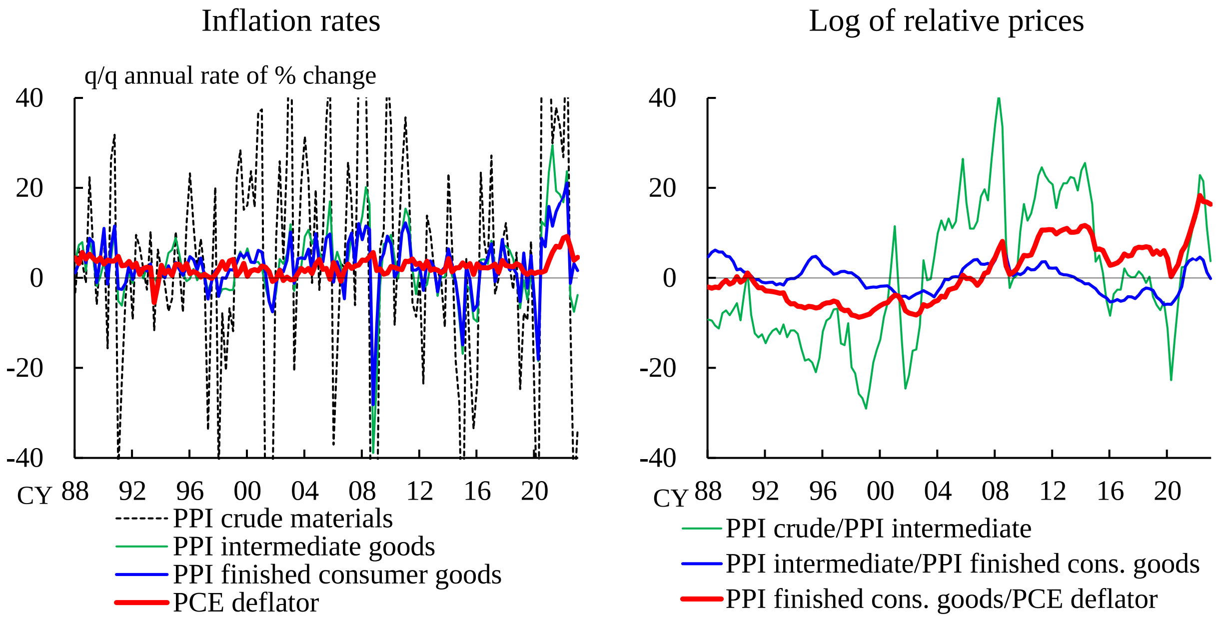
1 Introduction

The smaller volatility of output prices compared with that of input prices (the left of Figure 1 for the case of the U.S.) indicates incomplete pass-through of costs to prices, at least contemporaneously. Another fact is that there seem lags in price adjustment, whose degree is likely varying by stage of pricing in a production chain. In this regard, Blanchard (1987) documents that even small lags of price adjustment in each of the production stages can generate a large degree of price stickiness at the aggregate level. The sluggish price adjustment could also cause a lingering cost-push pressures on pricing of downstream firms, which Smets, Tielens, and Van Hove (2019) calls the "pipeline pressure." This has become evident again since the COVID-19 pandemic when the relative price of upstream to downstream firms kept overshooting. (the right of Figure 1.)

Taking these observations into the New Keynesian Philips curve (NKPC), a modern workhorse for monetary analysis, and obtaining the precise estimates of the pass-through are therefore imminent tasks. Notwithstanding these, theoretical studies, such as Basu (1995) and Huang and Liu (2001), have preceded in this field, leaving empirical works still to be refined. The estimation of the NKPC in early days, such as Roberts (1995) and Galí and Gertler (1999), is agnostic about measuring the impact of cost pressures incurred by suppliers. Clark (1999) estimates the response of upstream and downstream prices to monetary shocks, but his vector autoregression model hinders backing up the structural parameters of the NKPC. The works of Leith and Malley (2007) and Shapiro (2008) are notable, in that they take into account intermediate goods in estimating NKPCs. However, their methods are susceptible to weak identification issues common to all variants of the NKPCs, leaving room for improving the estimation of the structural parameters governing the pipeline pressure.

In this paper, we derive the NKPC with the vertical production chain, denoted as the

Figure 1: Inflation rates of various production stages and pipeline pressures in the U.S.



PS-NKPC, and estimate it using the U.S. PPI and the deflator of Personal Consumption Expenditure (PCE). In deriving the PS-NKPC, we follow the tractable model structure of Huang and Liu (2005) and Wei and Xie (2020), where the production flow goes from processing materials to all the way down to the distribution of consumption goods by retailers. We employ Bayesian VAR-GMM in estimating the PS-NKPC, following the claim made by Gemma, Kurozumi, and Shintani (2023), Kurozumi and Oishi (2022) and Kurozumi, Oishi, and Van Zandweghe (2022) on the merit of the Bayesian approach in comparing various forms of the NKPC.

Our estimation strategy has several notable advantages in pinning down the impact of the pipeline pressure in the NKPC. First, the assumed vertical structure of production in this paper facilitates finding the data counterpart of the model, especially for the inflation rate by production stage, and the variable regarding the pipeline pressures. We can readily point the PPI and PCE inflation, and their relative prices as natural candidates for the data that can be taken to our model.[1] Second, as discussed in Mavroeidis, Plagborg-Møller, and Stock (2014), our Bayesian approach can mitigate the weak identification issues arising from difficulty in forecasting inflation, while avoiding to estimate a full DSGE model, which may exacerbate model misspecification issues. Third, our study enables a robust model comparison between the PS-NKPC and canonical NKPC utilizing quasi-marginal likelihood (QML). Inoue and Shintani (2018) has shown that the Bayesian model comparison is valid even in an environment where some parameters are subject to weak identification, and we take the full advantage of their findings.

Our main findings are two-fold. First, we find that the PS-NKPC fits to the data better than the canonical NKPC according to the QML under various setup in data and models. This result is consistent with Shapiro (2008), which estimates a version of the PS-NKPC utilizing the classical GMM. Our result overrides his result in that our model comparison is robust to the weak identification issues intrinsic to the NKPCs. As a byproduct of the better fit of the PS-NKPC, we find that our estimates of structural parameters under the PS-NKPC formulation are more in line with micro-level evidence. For example, we find that the estimated price stickiness of consumer prices under the PS-NKPC is closer to the micro-level evidence, such as the one reported in Kehoe and Midrigan (2015).

Second, our estimation result points to non-negligible degrees of impact of the pipeline pressure on the inflation rate of each production stage. Although there is a range of uncertainty in the precise value of the share of intermediate inputs, we find that the reduced-form coefficients on the pipeline pressure variables are economically non-negligible. For example, taking the values of the pipeline pressure variable as given, we find that it directly accounts for a full percentage point change in the PCE inflation rate from 2021:Q3 to 2022:Q2, when the disruption in the global supply chain exerted a

[1] Our model is agnostic about the interaction of sectoral price dynamics given by a detailed input-output structure, which is explored in Smets, Tielens, and Van Hove (2019) and Rubbo (2023).

mounting cost-push pressure to consumer prices.[2] Although this number does not account for the whole uptick of the inflation rates during the same period, our model provides a candidate variable that structurally accounts for the fluctuation of inflation, which the traditional output gap alone cannot sufficiently capture.

This paper is related to the previous literature in the following two ways. First, this paper contributes to the literature on the limited-information estimation of Phillips curves. Results of earlier literature that estimates the canonical NKPC (Galí and Gertler, 1999; Galí, Gertler, and López-Salido, 2005) or the PS-NKPC (Shapiro, 2008) have suffered from the weak identification issues, making model comparison a difficult task. To deal with these problems, recent studies employ Bayesian GMM methods for estimating various forms of NKPCs: Inoue and Shintani (2018) compares two forms of the hybrid NKPC, one for assuming the rule-of-thumb (ROT) firms as in Galí and Gertler (1999) and the other for indexation-to-inflation as in Smets and Wouters (2007). Gemma, Kurozumi, and Shintani (2023) estimates a "generalized" version of the NKPC in which they incorporate non-zero trend inflation. They show that incorporating trend inflation into otherwise a standard NKPC generates non-trivial changes in the drivers of inflation, thereby making an improvement of the fit of the NKPC to the data. Kurozumi, Oishi, and Van Zandweghe (2022) evaluates the validity of various forms of expectation formation in the NKPC, and shows that sticky information is a plausible source of inflation inertia in the U.S. Related to this study, Kurozumi and Oishi (2022) finds the best-fit model among variants of the NKPC for Japan is different from that of the U.S. Our paper follows Kurozumi and Oishi (2022) and others in that we utilize a Bayesian method to estimate the NKPC. But our focus is the vertical production chain in an otherwise canonical NKPC, and to find the empirical relevance of the pipeline pressure.

[2] This number omits the indirect effect through the impact on inflation expectation, which should be subject to a range of uncertainty.

Second, therefore our study adds the empirical evidence to the literature on the multi-sector production model and its implications on inflation dynamics. Incorporating a production chain in which a firm uses intermediate goods produced by other firms has been regarded as a source of monetary non-neutrality, and the flat Phillips curve. (Basu, 1995; Huang and Liu, 2001; Nakamura and Steinsson, 2010; Pasten, Schoenle, and Weber, 2020) The cascade effect of the pipeline price pressure, which often prolongs, is also predicted in the large-scale multi-sector production model by Smets, Tielens, and Van Hove (2019).[3] While in general, our result connects to various multi-sector sticky price models, the closest theoretical counterpart to our empirical model is the vertical production chain proposed by Huang and Liu (2001, 2004). In terms of monetary policy, higher welfare can be achieved if a central bank places some weight on stabilizing intermediate goods prices, in addition to on stabilizing the consumption goods prices, especially if the economy has a large share of intermediate inputs (Huang and Liu, 2005; Wei and Xie, 2020; Rubbo, 2023). Most of these findings are presented in the models calibrated to the U.S. data. Our empirical analysis provides some empirical grounds to the theoretical claims.

This paper is organized as follows. Section 2 presents the model setup to derive PS-NKPC. Section 3 describes the Bayesian VAR-GMM method and the data for estimation. Section 4 shows our empirical findings and model selections based on QML. Section 5 concludes.

2 The Model

In this section, we present the PS-NKPC. We follow the model setup of Huang and Liu (2005) and Wei and Xie (2020) to demonstrate stickiness in firms' price setting under a vertical production chain: producing consumption goods requires N vertical stages of

[3] In general, literature on production network emphasizes the importance of intermediate goods in accounting for the business cycle fluctuations. See the review by Carvalho and Tahbaz-Salehi (2019).

processing, in which firms in the stage n of the production chain use the goods produced by the firms in the stage $n-1$ as inputs; prices are sticky and firms in each production stage reset their prices in a Calvo (1983) fashion. In what follows, we assume zero trend inflation for all stages of production. Thus the formulation of the NKPC for each production stage is going to be quite simple, and what makes our model distinct from the canonical NKPC is the calculation of the real marginal cost, the main driver of the inflation in New Keynesian models.[4]

In each production stage, there exists a unit mass of firms, each producing differentiated goods. The individual firm i in the first stage produces intermediate goods using labor as the sole input. The production function is given by

$$Y_{1,t}(i) = A_{1,t} L_{1,t}(i),$$

where $A_{1,t}$ is the productivity level and $L_{1,t}$ is the labor input.

In each subsequent stage, firms in the n-th production stage require the composite of goods produced in the stage $n-1$ and labor as the inputs for production. The production function of the n-th production stage is given by

$$Y_{n,t}(i) = \bar{Y}_{n-1,t}(i)^{\phi_n} (A_{n,t} L_{n,t}(i))^{1-\phi_n},$$

where $\bar{Y}_{n-1,t}(i) = [\int_0^1 Y_{n-1,t}(i,j)^{(\theta-1)/\theta} dj]^{\theta/(\theta-1)}$ is the composite of goods produced in the $(n-1)$-th production stage and θ is the elasticity of substitution.

We assume firms in each production stage are price-takers in the input markets while they are monopolistic competitors in the output market. In each stage, firms set prices following Calvo (1983) manner: a fraction $\alpha \in [0, 1)$ of firms keep their prices unchanged,

4 In the Appendix B, we perform a robustness check, in which we assume non-zero trend inflation for the PCE deflator.

while the rest of firms are allowed to change their prices.

In the case that firm i in the n-th production stage can adjust its price, they choose the optimal resetting price $P^*_{n,t}(i)$ by solving the maximization problem:

$$\max_{P_{n,t}} E_t \sum_{\tau=t}^{\infty} \alpha_n^{\tau-t} D_{t,\tau}(P_{n,t}(i) - \Phi_{n,\tau}) Y^d_{n,\tau}(i),$$

where $D_{t,\tau}$ is the nominal stochastic discount factor between period t and τ, which is common to all production stages.[5] $\Phi_{n,t}$ is the nominal marginal cost and $Y^d_{n,\tau}(i)$ is the demand of firm i's output. The latter two variables are given by

$$\Phi_{n,t} = \begin{cases} \frac{W_t}{A_{n,t}} & n = 1 \\ \left(\bar{P}_{n-1,t}\right)^{\phi_n} \left(\frac{W_t}{A_{n,t}}\right)^{1-\phi_n} & n \geq 2 \end{cases}, \tag{1}$$

$$Y^d_{n,t}(i) = \begin{cases} \phi_n \frac{\Phi_{n+1,t}}{\bar{P}_{n,t}} \left(\frac{P_{n,t}(i)}{\bar{P}_{n,t}}\right)^{-\theta} \int_0^1 Y_{n+1,t}(j)dj & n < N \\ \left(\frac{P_{N,t}(i)}{\bar{P}_{N,t}}\right)^{-\theta} C_t & n = N \end{cases}, \tag{2}$$

where $\bar{P}_{n-1,t} = [\int_0^1 P_{n-1,t}(i)^{(1-\theta)}di]^{(1/1-\theta)}$ is the price for the composite intermediate goods that firms at the n-th production stage take, and C_t is the aggregate consumption. Given the production functions defined above, the marginal cost at the first stage of production is the nominal wage evaluated in unit of effective labor, same as the textbook case of the NKPC. For the subsequent production stages, in contrast, it contains the term regarding upstream goods' prices.

Solving and log-linearizing firm's maximization problem, the inflation rate at each

[5] The exact formula can be obtained by solving the canonical utility maximization problem of households, which we will omit in this paper.

production stage is given by

$$\pi_{n,t} = \beta E_t \pi_{n,t+1} + \lambda_n \hat{mc}_{n,t}, \tag{3}$$

where $\lambda_n \equiv (1-\beta\alpha_n)(1-\alpha_n)/\alpha_n$. Although we have skipped showing the derivation process,[6] this functional form exactly matches that of the canonical forward-looking NKPC. An only tweak is that $\hat{mc}_{n,t}$ is the log-deviation from the steady state of the nominal marginal cost divided by own price of firms at the n-th stage of the production flow:

$$mc_{n,t} = \ln(\Phi_{n,t}/\bar{P}_{n,t}).$$

This seemingly a minor change in the expression of the "real marginal cost" actually offers a rich interpretation of the drivers of inflation, when we account for the expression of the nominal marginal cost (1). For $n \geq 2$, $mc_{n,t}$ can be rewritten as

$$mc_{n,t} = \ln\left(\left(\frac{\bar{P}_{n-1,t}}{\bar{P}_{n,t}}\right)^{\phi_n}\left(\frac{W_t}{\bar{P}_{n,t}A_{n,t}}\right)^{1-\phi_n}\right). \tag{4}$$

The equation (4) demonstrates that, in addition to the wage term, the relative price of the upstream firms to the downstream firms appears as a driver of the inflation. This is the term associated with the pipeline pressure. Note that although only the price of goods produced at one stage above in the production chain enter into the cost function, prices of goods produced at even upper stages can have non-negligible impact on the downstream price, due to the vertical structure of the production chain. In other words, the cost-push shocks hit at multiple stages of the production chain would accumulate, bringing a cascade effect on the inflation of consumption goods, which are produced by

[6] The derivation process can be seen in Huang and Liu (2005) and Appendix F of Wei and Xie (2020).

downstream firms.

We finish the discussion of this section by presenting the exact models that we fit to the data. Let $\hat{g}_{n,t} \equiv \ln\left(P_{n-1,t}/P_{n,t}\right) - \ln\left(P^{ss}_{n-1}/P^{ss}_{n}\right)$ be the log deviation from the steady state of the relative price term. Then it is immediate to arrive at the PS-NKPC as follows:

$$\pi_{n,t} = \begin{cases} \beta E_t \pi_{n,t+1} + \lambda_n \left(\hat{w}_t - \hat{a}_{1,t} - \sum\limits_{i=2}^{N} \hat{g}_{i,t} \right) & n = 1 \\ \beta E_t \pi_{n,t+1} + \lambda_n \left(\phi_n \hat{g}_{n,t} + (1-\phi_n) \left[\hat{w}_t - \hat{a}_{n,t} - \sum\limits_{i=n+1}^{N} \hat{g}_{i,t} \right] \right) & n = 2, ..., N-1 \\ \beta E_t \pi_{N,t+1} + \lambda_n \left(\phi_N \hat{g}_{N,t} + (1-\phi_N)(\hat{w}_t - \hat{a}_{N,t}) \right) & n = N \end{cases}, \tag{5}$$

where $\hat{w}_t$ and $\hat{a}_{n,t}$ are the log-deviation of the real wage and productivity from their corresponding steady state. We define the reduced-form coefficients $\kappa_{mc,n} \equiv \lambda_n(1-\phi)$ and $\kappa_{rp,n} \equiv \lambda_n \phi$ hereafter for convenience. The former is the coefficient on the traditional marginal cost variable, and the latter is on the relative price between the upstream and downstream firms.

Although it is straightforward to derive equations (5) once the marginal cost is defined, there could be measurement issues in $\hat{w}_t$ and especially in $\hat{a}_{n,t}$. To overcome the uncertainty, we present an alternative specification of the PS-NKPC, with a help of using household's optimal condition on the trade-off between labor and leisure, which is a standard condition in the modern macroeconomic theory. Specifically, log-linearizing firms' first order condition around the natural consumptions, *i.e.* the equilibrium consumptions under the flexible price economy, leads to[7]

$$\pi_{n,t} = \begin{cases} \beta E_t \pi_{n,t+1} + \lambda_n \left(\sigma \tilde{c}_t - \sum_{i=2}^{N} \tilde{g}_{i,t} \right) & n = 1 \\ \beta E_t \pi_{n,t+1} + \lambda_n \left(\phi_n \tilde{g}_{n,t} + (1-\phi_n) \left[\sigma \tilde{c}_t - \sum_{i=n+1}^{N} \tilde{g}_{i,t} \right] \right) & n = 2, ..., N-1 \ , \\ \beta E_t \pi_{N,t+1} + \lambda_n \left(\phi_N \tilde{g}_{N,t} + (1-\phi_N) \sigma \tilde{c}_t \right) & n = N \end{cases} \quad (6)$$

where $\tilde{c}_t$ and $\tilde{g}_{n,t}$ is log-deviation of real consumption and relative price from the corresponding equilibrium value under the flexible price economy respectively, and σ is the inverse of the elasticity of intertemporal substitution of households.

We use the following canonical NKPC as the benchmark to compare the performance of the PS-NKPC. One can consider the canonical NKPC corresponds to the case where final stage producers do not require intermediate goods input, *i.e.* $\phi_N = 0$:

$$\pi_{N,t} = \beta E_t \pi_{N,t+1} + \lambda_N (\hat{w}_t - \hat{a}_{N,t}) \quad (7)$$

$$= \beta E_t \pi_{N,t+1} + \lambda_N \sigma \tilde{c}_t. \quad (8)$$

In addition to the purely forward-looking NKPCs, we also consider hybrid NKPCs, in which lagged inflation appears to take into account inflation inertia. We assume a fraction ω_n of the n-th stage producers is a version of rule-of-thumb (ROT) price setters following Galí and Gertler (1999).[8] ROT firms set their price as the average price chosen in the most recent round of price adjustment with a correction based on lagged inflation. That

[7] See Appendix F in Wei and Xie (2020). In deriving the following PS-NKPC, we assume the constant relative risk aversion in consumption and constant marginal utility of leisure for households' preference, i.e. $U_t = \frac{C_t^{1-\sigma}-1}{1-\sigma} - L_t$.

[8] Although researchers tend to choose "indexation to inflation," *à la* Smets and Wouters (2007) as a dominant source of intrinsic inflation inertia, Inoue and Shintani (2018) finds that the ROT price setting fits better to the data in terms of quasi-marginal likelihood. We follow the finding of Inoue and Shintani (2018) and choose the ROT price setting as the candidate model for capturing inflation inertia.

is, hybrid PS-NKPC is given by

$$\pi_{n,t} = \gamma_{b,n}\pi_{n,t-1} + \gamma_{f,n}E_t\pi_{n,t+1} + \kappa_n \hat{mc}_{n,t}, \tag{9}$$

where

$$\begin{aligned}
\gamma_{b,n} &\equiv \frac{\omega_n}{\alpha_n + \omega_n[1-\alpha_n(1-\beta)]}, \\
\gamma_{f,n} &\equiv \frac{\beta\alpha_n}{\alpha_n + \omega_n[1-\alpha_n(1-\beta)]}, \\
\kappa_n &\equiv \frac{(1-\omega_n)(1-\alpha_n)(1-\beta\alpha_n)}{\alpha_n + \omega_n[1-\alpha_n(1-\beta)]}.
\end{aligned}$$

As in the transformation of the equation (3) into the equation (5) and (6), the reduced-form coefficients are denoted as $\kappa_{mc,n} \equiv \kappa_n(1-\phi_n)$ and $\kappa_{rp,n} \equiv \kappa_n\phi_n$ for convenience.

3 Estimation Method and Data

3.1 Baynesian VAR-GMM

Following Gemma, Kurozumi, and Shintani (2023), Kurozumi and Oishi (2022) and Kurozumi, Oishi, and Van Zandweghe (2022), we estimate the PS-NKPCs, along with the traditional NKPCs, employing a Bayesian VAR-GMM approach. This approach consists of the following three features: the expected inflation rates in the NKPCs are derived from Vector Auto Regression (VAR) models; a Bayesian technique is employed to estimate the PS-NKPC and VAR simultaneously; a quasi-marginal likelihood (QML), proposed by Inoue and Shintani (2018), is applied for model selection. The advantages of these three features are well documented in Kurozumi and Oishi (2022), but we reiterate them in below, along with the description of the methodological details.

First, we choose a small set of economic variables including the inflation rate at each production stage, and consider a VAR representation of the set of the variables to derive the expected inflation rates in the NKPCs. In practice, let $\Pi_t \equiv [\pi_{1,t}, \pi_{2,t}, ..., \pi_{N,t}]$ and let $Y_t \equiv [y_{1,t}, y_{2,t}, ..., y_{M,t}]$, where $y_{m,t}$ is the m-th endogeneous variable other than the inflation rates in the system of the VAR. Then, assuming that the VAR has the lag length of k, the VAR system can be conveniently defined as $X_t = AX_{t-1} + \epsilon_t$, where $X_t \equiv [\Pi_t, Y_t, ..., \Pi_{t-k}, Y_{t-k}]^\intercal$, and A contains the block of the VAR parameters. Given the VAR coefficient matrix A, it is straightforward to retrieve the one-period-ahead expected inflation rates by appropriately choosing the rows of $E_tX_{t+1} = AX_t$.

Second, we apply a Bayesian method to estimate the VAR coefficient matrix A, and PS-NKPC parameters $\xi_{nkpc} \equiv [\alpha_1, ..., \alpha_n, \omega_1, ..., \omega_n, \phi_1, ..., \phi_n]^\intercal$ simultaneously.[9] As stressed in Kleibergen and Mavroeidis (2014), the Bayesian approach can mitigate weak identification issues in estimating NKPCs. We take the root of so-called one step VAR-GMM, employed by Guerrieri, Gust, and López-Salido (2010), thereby taking the moment conditions of the PS-NKPC and VAR jointly to the data.[10]

In order to estimate A and ξ_{nkpc} in a Bayesian fashion, we define the vector of moment functions as follows. Let $\xi \equiv [\xi_{nkpc}^\intercal, vec(A)^\intercal]^\intercal$ be the vector of parameters to be estimated in the VAR-GMM specification. And let $g_t(\xi)$ denote the vector of moment functions, which is given by

$$g_t(\xi) = \begin{bmatrix} u_t(\xi) Z_t \\ (X_t - AX_{t-1})X_{t-1} \end{bmatrix}, \tag{10}$$

where $u_t(\xi)$ is the vector of the residuals for the PS-NKPC, and Z_t is the vector of

[9] We fix the discount rate β, and intertemporal elasticity of substitution σ at specified values. See section 3.2 for details.

[10] Alternatively, the VAR and GMM part can be estimated sequentially, as in Sbordone (2002) and Cogley and Sbordone (2008). We do not take this root, and we aim to use full information effectively in estimating all relevant parameters using the Bayesian method.

instruments including a constant of unity.

We use the efficient two-step GMM estimator following Galí and Gertler (1999) and Galí, Gertler, and López-Salido (2005). This estimator is choosen to maximize the objective function $q(\xi) = -(1/2)g(\xi)'Wg(\xi)$, where $g(\xi) = (1/\sqrt{T})\sum_{t=1}^{T} g_t(\xi)$ and W is the optimal weighting matrix using Newey and West (1987) HAC estimator. [11]

We then apply the Block Metropolis-Hastings algorithm to estimate the parameter set $\xi \in \Xi$, following Chernozhukov and Hong (2003). Specifically, the estimators are evaluated based on the quasi-posterior distribution for ξ, which is defined as

$$\frac{\exp(\hat{q}(\xi))p(\xi)}{\int_{\Xi} \exp(\hat{q}(\xi))p(\xi)d\xi},$$

where $p(\xi)$ is the prior distribution for ξ. We evaluate this quasi-posterior distribution using the Markov Chain Monte Carlo (MCMC) simulation. In doing so, the Block Metropolis Hastings algorithm with two blocks, one for NKPC parameters and the other for the VAR coefficients, is applied following Kurozumi and Oishi (2022) among others. [12] In our estimation, the first 10,000 draws are discarded as a burn-in, and the quasi-posterior distribution is derived using the remaining 200,000 MCMC draws.

Finally, we exploit the result of Inoue and Shintani (2018) for model selection under weak parameter identification, and we use the following QML to select a dominant PS-NKPC model:

$$\int_{\Xi} \exp(\hat{q}(\xi))p(\xi)d\xi.$$

Specifically, we compute the QML for each PS-NKPC using the modified harmonic mean

[11] The matrix W is given by $W = [\Gamma_j(\tilde{\xi}) + \sum_{j=1}^{J}(j/J)(\Gamma_j(\tilde{\xi}) + \Gamma_j(\tilde{\xi})')]^{-1}$, where $\Gamma_j(\tilde{\xi}) = [1/(T-j)]\sum_{t=j+1}^{T} g_t(\tilde{\xi})g_{t-j}(\tilde{\xi})'$and J is the lag length chosen by automatic bandwidth selection method of Andrews (1991). This paper employs the first-step consistent estimator for $\tilde{\xi}$.

[12] The details of the algorithm are shown in the Appendix A.

method of Geweke (1999), and then select a model with larger value of QML as a better suited model.

As to which stages of the production flow to include in the estimation of PS-NKPC, we prepare for several specifications. At first hand, one can readily posit the PS-NKPC for the consumer price inflation only, that is the equation for the inflation rate of the last stage of the production chain. In that case, we estimate the equation (3) for $n = N$ only, and the set of inflation variables enters into the VAR, $\Pi_t = [\pi_{N-1,t}, \pi_{N,t}]$. An alternative specification is that the inflation dynamics in each production stage can be approximated by the PS-NKPC (3). Then the aforementioned moment functions (10) should include all the residuals of NKPCs from the 1st to N-th production stage, in which case we should include the whole set of inflation rates in the VAR specification as well.

We should also consider the candidate set of other endogenous variables in the VAR, Y_t. In this paper, we take a very simple approach, only to include the variables relevant to the marginal cost. That is, for (5) we choose $\hat{x}_t = \hat{w}_t - \hat{a}_t$ as the variable included in Y_t, and for (6) we choose $\tilde{c}_t$.

3.2 Fixed parameters and prior distribution

In this subsection, we describe the parameters to be fixed in our estimation, and the prior distributions for estimated parameters. Two parameters are fixed in our estimation: the subjective discount factor $\beta = 0.9975$ and inverse of the elasticity of intertemporal substitution $\sigma = 1.39$ based on the mode of the posterior distribution estimated by Smets and Wouters (2007).

We estimate other parameters in the Phillips curves and coefficients in VAR. The prior distributions of parameters included in the Phillips curves are shown in Table 1. For the probability of no price change at the n-th production stage, α_n, and the fraction

Table 1: Prior distributions for parameters

Parameters in Phillips curve		Distribution	Mean	Std. dev.
α_n	probability of no price change at n-th stage	beta	0.5	0.1
ω_n	fraction of ROT pricing n-th stage firms	beta	0.5	0.1
ϕ_n	share of intermediate goods input at n-th stage	beta	0.5	0.2

of rule-of-thumb pricing for the n-th stage firms, ω_n, the prior distributions are assumed to be the beta distribution with mean 0.5 and standard deviation 0.1. For the share of intermediate goods input at the n-th production stage, ϕ_n, the prior distribution is assumed to be the beta distribution with mean 0.5 and standard deviation 0.2.

3.3 Data

The data for our estimation are based on the U.S. time series with quarterly frequency. We use the producer price index (PPI) constructed by the Bureau of Labor Statistics (BLS) for prices of upstream production stages. The three categorical data resembling to the vertical production chain are available for the PPI with a long time-series: crude material price, intermediate goods price, and wholesale finished consumer goods price.[13] We also use the deflator of personal consumption expenditures (PCE) published by the Bureau of Economic Analysis (BEA) as the price of the final stage, and thereby we assume four stages in the production chain, $N = 4$.

For inflation drivers other than the pipeline pressures, we need to have observations of real wage in unit of efficient labor, which appears in (5), and the log deviation of consumption from its hypothetical flexible price equilibrium level, which appears in (6). For the former, we first construct the real wage by dividing the total employment

[13] The state-of-the-art methodology to capture the pipeline pressure in the U.S. PPI is based on the final demand-intermediate demand (FD-ID) aggregation system. Although the FD-ID system corresponds to the notion of vertical production chain, and covers broader price categories including service prices, the data are available only from November 2009. To ensure long-term time series data, we employ the PPI based on the stage-of-processing aggregation system in estimation.

cost index (ECI) tabulated by BLS by the PCE deflator, and then subtract the utilization-adjusted total factor productivity calculated by the San Francisco Fed. We assume, for simplicity, that the productivity is the same across production stages, *i.e.* $\hat{a}_{n,t} = \hat{a}_t$ for all n. In all, $\hat{x}_{n,t} = \hat{w}_t - \hat{a}_t$ is computed as the log real wage net of the log productivity in terms of the deviation from its sample mean. For the latter, the log deviation of consumption from its hypothetical flexible price equilibrium level is derived by applying Hamilton (2018) filter to the U.S. personal consumption expenditure by BEA.[14]

For instruments Z_t, we include four lags of the inflation rate, and those of the upstream inflation rate, following Shapiro (2008). For instance, we include the PCE inflation and the finished consumer goods PPI inflation in Z_t in estimating the final stage PS-NKPC. We also include two lags of the nominal wage growth measured by total ECI, and the proxy for canonical real marginal costs, $\hat{x}_t$ or$\tilde{c}_t$.

4 Empirical Results

We estimate the PS-NKPCs in two ways. First, we estimate the final stage PS-NKPC and compare them with the canonical NKPC. Second, we estimate the PS-NKPCs of all stages simultaneously, except for the first stage of processing, to compare the parameters across production stages.[15]

4.1 Comparison of PS-NKPC and canonical NKPC

We start from the model comparison between the PS-NKPC and canonical NKPC for the PCE deflator. In estimation, we use the data covering the post Great Inflation

[14] For pipeline pressure variables $g_{n,t}$ to be aligned with the definition of $\hat{x}_{n,t}$ and $\tilde{c}_t$, we construct the log deviation from the sample mean of $g_{n,t}$ for $\hat{g}_{n,t}$, and the Hamilton-filtered series of $g_{n,t}$ for $\tilde{g}_{n,t}$, and use them accordingly depending on specification of the PS-NKPC.

[15] The first stage corresponds to the inflation of crude materials, which is likely to be determined by international markets. This is the out of scope of our study.

Table 2: Log QML of PS-NKPCs and canonical NKPCs

Geweke τ	data	Phillips curve		VAR lag length k=1	VAR lag length k=2
0.5	$\hat{x}_t$	forward-looking	PS-NKPC	-75.64	-110.26
			canonical NKPC	-78.21	-112.25
		hybrid	PS-NKPC	-79.93	-112.55
			canonical NKPC	-80.23	-113.52
	$\tilde{c}_t$	forward-looking	PS-NKPC	-75.41	-113.58
			canonical NKPC	-80.74	-117.80
		hybrid	PS-NKPC	-79.54	-116.75
			canonical NKPC	-84.10	-119.81
0.9	$\hat{x}_t$	forward-looking	PS-NKPC	-75.60	-110.41
			canonical NKPC	-78.21	-112.36
		hybrid	PS-NKPC	-79.90	-112.64
			canonical NKPC	-80.22	-113.63
	$\tilde{c}_t$	forward-looking	PS-NKPC	-75.39	-113.73
			canonical NKPC	-80.75	-117.95
		hybrid	PS-NKPC	-79.52	-116.82
			canonical NKPC	-84.11	-119.96

Notes: This table presents the log QML for each specification of PS-NKPCs and NKPCs. For the data column, $\hat{x}_t$ corresponds to the case where the labor cost part of (4) is defined as $\hat{w}_t - \hat{a}_t$. Instead, $\tilde{c}_t$ corresponds to the case where the log deviation of consumption from its natural level is substituted for the labor cost part of (4). For the Phillips curve column, the "forward-looking" corresponds to the model specification (3) and "hybrid" corresponds to (9). The sample period is from 1988:Q1 to 2019:Q4.

periods from 1988:Q1 to 2019:Q4. As described in the preceding section, the model selection criteria is the QML. Table 2 shows log QML for the PS-NKPCs and canonical NKPCs with the VAR lag length $k = 1, 2$. We use two alternative truncation parameters $\tau = 0.5, 0.9$ proposed by Geweke (1999) to calculate QML. Table 2 reports the result.

Our estimation results mainly point two findings. First and most importantly, the PS-NKPCs have larger QML than the canonical NKPCs under a broad set of combinations in data, model specification and truncation parameters in calculating the QML. This finding suggests that pipeline pressures emerging from the vertical production chain play an important role in accounting for inflation fluctuations. Our result is also consistent with the result of Shapiro (2008), which shows that using real unit input costs as a proxy of the real marginal costs yields a better fit with NKPC models in terms of significance and stability of the coefficients on the marginal cost.

Second, by comparing purely forward-looking NKPCs with hybrid NKPCs, our estimation results show that the forward-looking models have larger QML than hybrid ones universally. Although the extensive studies of Gemma, Kurozumi, and Shintani (2023) and Kurozumi, Oishi, and Van Zandweghe (2022) on the trend inflation and sources of inflation inertia generally support the opposite result, with which we concur, the vastly stylized comparison between the purely forward-looking NKPC and the canonical hybrid NKPC in our study has yielded somewhat a different result. Note, however, that our result is not solely contradicting with Kurozumi, Oishi, and Van Zandweghe (2022), in that they also report that a purely forward-looking NKPC can be favored against a vastly stylized hybrid NKPC *à la* Smets and Wouters (2007) for the post Great-inflation era.

4.2 Quasi-posterior estimates of Phillips curve

Next, we report quasi-posterior estimates of structural parameters and reduced-form coefficients for the PS-NKPC and the canonical NKPC for the PCE deflator.

Table 3 reports the quasi-posterior mean and 95 percent highest quasi-posterior density interval for the structural parameters and reduced-form coefficients for estimated Phillips curves for the PCE deflator. We find that the probability of no price change α_4 in the PS-NKPCs is lower than that of the canonical NKPCs. This result is invariant with the existence of ROT-pricing firms. The probability of no price change for the NKPC in the second column, $\alpha_4 = 0.899$, and that for the PS-NKPC in the third column, $\alpha_4 = 0.837$,

Table 3: Quasi-posterior estimates of Phillips curves for PCE deflator

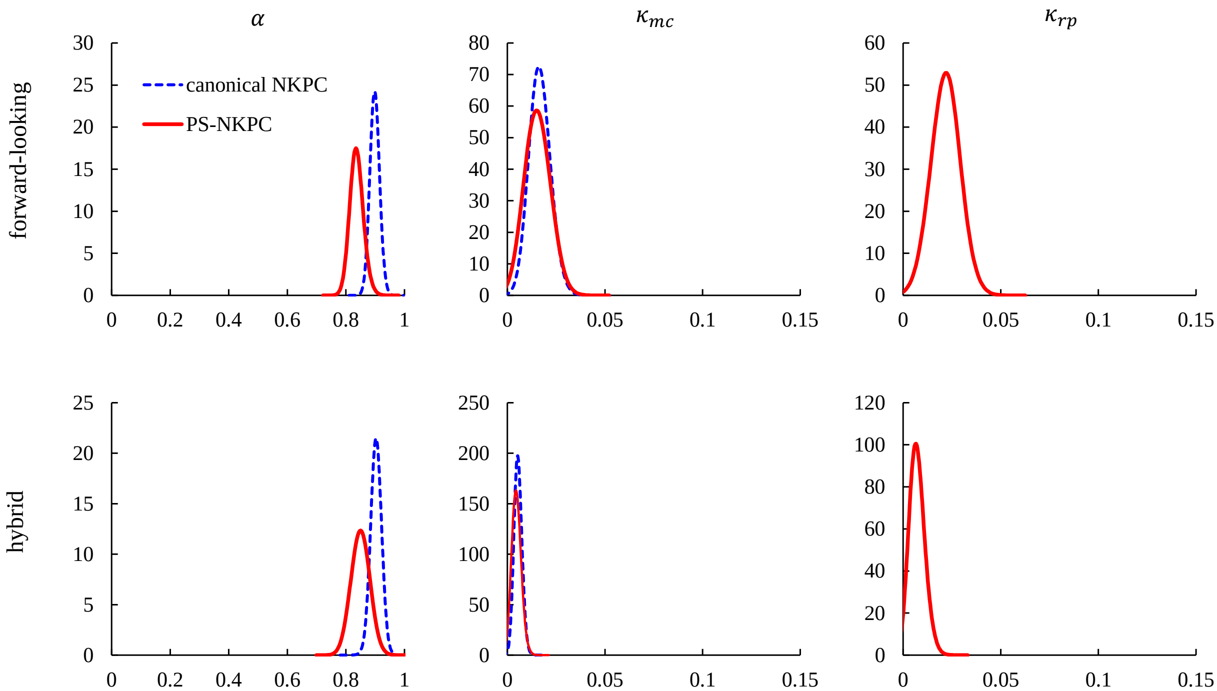
	forward-looking		hybrid	
parameter	NKPC	PS-NKPC	NKPC	PS-NKPC
α_4	0.899	0.837	0.903	0.850
	[0.874, 0.925]	[0.803, 0.877]	[0.874, 0.932]	[0.800, 0.900]
ϕ_4		0.661		0.664
		[0.470, 0.836]		[0.360, 0.880]
ω_4			0.455	0.431
			[0.351, 0.574]	[0.319, 0.545]
$\gamma_{b,4}$			0.334	0.336
			[0.278, 0.392]	[0.271, 0.395]
$\gamma_{f,4}$			0.665	0.663
			[0.607, 0.721]	[0.604, 0.728]
$\kappa_{mc,4}$	0.017	0.015	0.005	0.005
	[0.009, 0.026]	[0.006, 0.026]	[0.003, 0.009]	[0.001, 0.009]
$\kappa_{rp,4}$		0.022		0.007
		[0.010, 0.034]		[0.002, 0.015]

Notes: This table reports the quasi-posterior mean for each parameter in the Phillips curves for the PCE deflator, the final stage of the production flows, using the consumption gap $\tilde{c}_t$ with the VAR lag length of $k = 1$. The numbers in square brackets show 95 percent highest quasi-posterior density interval. The sample period is from 1988:Q1 to 2019:Q4.

imply that the average duration a price is fixed for consumer prices is 29.7 months and 18.4 months respectively. This finding provides a support for the PS-NKPC, because the duration implied in the PS-NKPC is closer to the finding by Kehoe and Midrigan (2015) who reports the average duration of 14.5 months.[16] The left panels of Figure 2 visually make this case: although the posterior density of α_4 of the PS-NKPC somewhat overlaps with that of the canonical NKPC, the density apparently shifts to the left when the PS-NKPC is assumed.

[16] Nakamura and Steinsson (2008) reports a shorter average duration implied in PS-NKPC type specifications, about 7-11 months. However, this number takes temporary price increases into considerations. In contrast, Kehoe and Midrigan (2015) calculates the duration excluding both temporary price increases and decreases.

Figure 2: Quasi-posterior distributions of NKPC parameters



Note: These panels show quasi-posterior distributions for PS-NKPCs and NKPCs using $\tilde{c}_t$ with the VAR lag length of $k = 1$.

The share parameter for intermediate goods in the marginal cost, ϕ_4 is around two-thirds, which implies an important role of intermediate inputs in production process.

Given α_4 and ϕ_4, we can calculate the reduced-form coefficient on the relative price, $\kappa_{rp,4}$. We find that the magnitude of $\kappa_{rp,4}$ implies that the relative price plays a non-negligible role to account for inflation dynamics of the PCE deflator given that pipeline price pressures emerging from upstream production stages are volatile and often persistent, as shown in Figure 1. For example, in the case of the forward-looking PS-NKPC, the large uptick in the pipeline pressures from the production stage of the wholesale finished consumer goods from 2021:Q3 to 2022:Q2 has induced a cumulative of around one percent increase in the PCE deflator in the corresponding periods, in a direct manner. Although this direct effect accounts for just around one-sixth of the

PCE inflation during the same period,[17] this order of magnitude is much larger than the impact generated from output gap variation given our estimates of $\kappa_{mc,4}$, which is in line with traditional estimates. Thus our result suggests that the pipeline pressure is a non-negligible force in accounting for the U.S. inflation dynamics.

Finally, we find no significant difference in the value of the reduced-form coefficients on the canonical real marginal cost variables, $\kappa_{mc,4}$, between the PS-NKPCs and canonical NKPCs, holding other parts of model specification unchanged. These values are somewhat larger than the median estimate shown in Mavroeidis, Plagborg-Møller, and Stock (2014), but they are still within the region of so-called flat Phillips curves reported in previous studies.

4.3 Comparison of PS-NKPCs by production stage

The above estimates only consider the PS-NKPC at the final stage of production. But as our model exposes, the PS-NKPC can be applied to upstream firms as long as they also require intermediate inputs. In that case, the pipeline pressure could flow from the firms at a far upstream stage to the bottom of the production flow gradually. To this end, we jointly estimate the PS-NKPCs for the PPI intermediate goods, wholesale finished goods (PPI final consumption goods), and PCE deflator, each corresponding to $n = 2, 3, 4$. The structural parameters in the Phillips curves and the reduced-form coefficients are reported in Table 4 and Table 5 respectively. The quasi-posterior distributions for some important parameters are shown in Figure 3.

Our findings can be summarized as follows. First, forward-looking models have larger values of QML compared with hybrid models, as in the case of estimating the final stage PS-NKPC alone. This implies that the stylized forward-looking NKPC, when combined

[17] The effect of the pipeline pressure should feed into the inflation expectation term. We omit this effect as there should be uncertainty in, for example, the persistence of pipeline pressures, which is the out of scope of this paper.

Table 4: Structural parameters of PS-NKPCs of all stages

	$\tilde{c}_t$		$\hat{x}_t$	
	forward-looking	hybrid	forward-looking	hybrid
QML	-100.46	-109.28	-98.48	-104.02
α_2	0.766	0.796	0.776	0.794
	[0.732, 0.800]	[0.750, 0.844]	[0.738, 0.813]	[0.741, 0.847]
α_3	0.739	0.725	0.747	0.704
	[0.699, 0.783]	[0.654, 0.802]	[0.706, 0.791]	[0.636, 0.780]
α_4	0.844	0.816	0.855	0.807
	[0.816, 0.875]	[0.763, 0.874]	[0.823, 0.893]	[0.752, 0.866]
ϕ_2	0.369	0.396	0.417	0.372
	[0.147, 0.653]	[0122, 0.741]	[0.152, 0.739]	[0.09, 0.772]
ϕ_3	0.420	0.734	0.407	0.531
	[0.178, 0.648]	[0.411, 0.933]	[0.180, 0.607]	[0.272, 0.697]
ϕ_4	0.557	0.438	0.453	0.344
	[0.451, 0.678]	[0.181, 0.702]	[0.258, 0.705]	[0.100, 0.691]
ω_2		0.471		0.485
		[0.340, 0.499]		[0.404, 0.574]
ω_3		0.426		0.396
		[0.320, 0.564]		[0.300, 0.492]
ω_4		0.336		0.371
		[0.253, 0.423]		[0.285, 0.471]

Notes: This table reports the quasi-posterior mean for each structural parameter in the Phillips curves for the PPI intermediate goods, $n = 2$, wholesale finished goods, $n = 3$, and PCE deflator, $n = 4$. The results are the case with the VAR lag length of $k = 1$. The numbers in square brackets show 95 percent highest quasi-posterior density interval. The sample period is from 1988:Q1 to 2019:Q4.

with the vertical production chain, is a suitable extension of the canonical NKPC for the U.S. inflation dynamics in the post Great-inflation era.

Second, the probabilities of no price change α_n are a bit smaller for upstream firms. For instance, in the case of the forward-looking PS-NKPC using $\hat{x}_t$, the implied durations between price changes for intermediate goods and wholesale finished goods are around 13 months and 12 months respectively, shorter than the case for consumer prices. The estimated durations for PPI are a bit larger than the micro-level evidence shown in Nakamura and Steinsson (2008), which reports the median duration of 7-9 months for the U.S. PPI. Note, however, that our numbers are smaller than the classical GMM estimates of Leith and Malley (2007), which reports the average duration of around 15 months for the U.S. manufacturing industries. Our result reduces the gap between the micro-level evidence and econometric estimates.[18]

The intermediate input shares ϕ_n reported in Table 4 center around 0.4-0.5. But there is a range of uncertainty in the estimates, which is evident from the graphical representation of the quasi-posterior distribution seen in Figure 3. We may require more information, such as on quantity, to obtain precise estimates of the intermediate input shares.[19]

Even with this uncertainty, we find a non-negligible effect of the pipeline pressure in each stage of production, as the estimated $\kappa_{rp,n}$ have a similar order of the magnitude to the case of solo estimation of the final stage PS-NKPC. (See Table 5)

Finally, we find that the coefficients on the canonical real marginal cost $\kappa_{mc,n}$ are larger for upstream firms, consistent with the fact that inflation rates at upstream production

[18] Another factor that can explain the difference is that Nakamura and Steinsson (2008) identifies temporary price increases as regular price changes, which unsurprisingly shortens the duration.

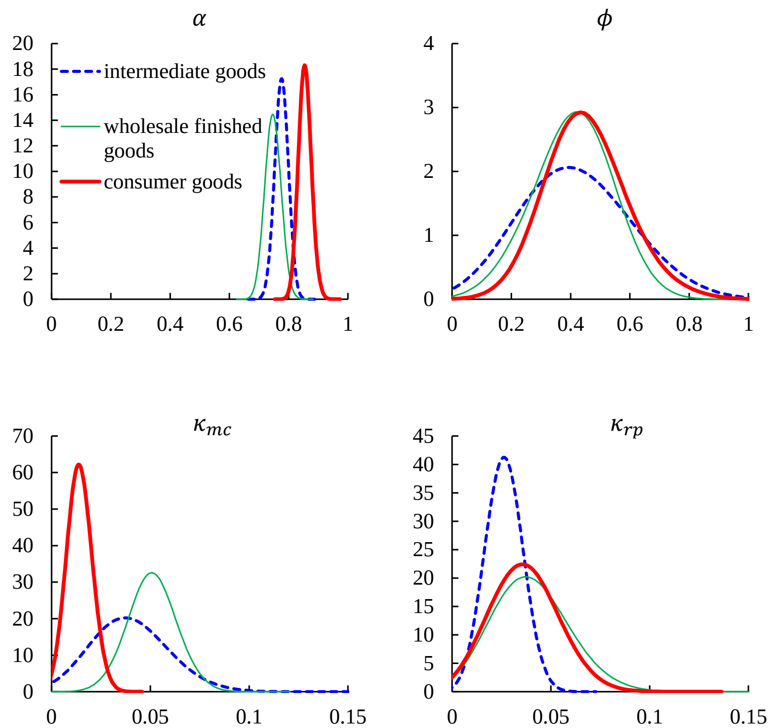
[19] Incorporating the inflation dynamics of upstream firms into the PS-NKPC specification changes the shape of moment functions, the functional form of the VAR, and the set of instruments in estimation. These together could change certain aspects of our estimation results, especially for parameters that are subject to weak identification.

Table 5: Reduced form coefficients in PS-NKPC

	$\tilde{c}_t$		$\hat{x}_t$	
	forward-looking	hybrid	forward-looking	hybrid
$\gamma_{b,2}$		0.343		0.379
		[0.301, 0.381]		[0.335, 0.423]
$\gamma_{b,3}$		0.369		0.359
		[0.305, 0.437]		[0.301, 0.411]
$\gamma_{b,4}$		0.291		0.314
		[0.235, 0.342]		[0.259, 0.370]
$\gamma_{f,2}$		0.656		0.620
		[0.619, 0.698]		[0.577, 0.664]
$\gamma_{f,3}$		0.631		0.640
		[0.562, 0.694]		[0.589, 0.698]
$\gamma_{f,4}$		0.708		0.685
		[0.657, 0.763]		[0.629, 0.740]
$\kappa_{mc,2}$	0.065	0.018	0.040	0.012
	[0.027, 0.108]	[0.005, 0.034]	[0.013, 0.074]	[0.003, 0.025]
$\kappa_{mc,3}$	0.075	0.013	0.051	0.023
	[0.043, 0.110]	[0.003, 0.027]	[0.031, 0.071]	[0.011, 0.036]
$\kappa_{mc,4}$	0.018	0.016	0.014	0.014
	[0.009, 0.028]	[0.005, 0.028]	[0.005, 0.025]	[0.004, 0.030]
$\kappa_{rp,2}$	0.026	0.008	0.026	0.006
	[0.011, 0.041]	[0.002, 0.017]	[0.012, 0.041]	[0.002, 0.013]
$\kappa_{rp,3}$	0.047	0.030	0.040	0.028
	[0.019, 0.078]	[0.008, 0.060]	[0.013, 0.074]	[0.007, 0.055]
$\kappa_{rp,4}$	0.040	0.009	0.036	0.007
	[0.013, 0.072]	[0.002, 0.018]	[0.011, 0.065]	[0.002, 0.013]

Notes: This table reports the quasi-posterior mean for each reduced-form parameter in the Phillips curves for the PPI intermediate goods, $n = 2$, wholesale finished goods, $n = 3$, and PCE deflator, $n = 4$. The results are the cases with the VAR lag length of $k = 1$. The numbers in square brackets show 95 percent highest quasi-posterior density interval. The sample period is from 1988:Q1 to 2019:Q4.

Figure 3: Quasi-posterior distributions of PS-NKPC parameters



Note: These panels show quasi-posterior distributions for forward-looking PS-NKPC using $\hat{x}_t$ with the VAR lag length of $k = 1$.

stages are more volatile.

We should note that our models are agnostic about the impact of trend inflation, as discussed in Gemma, Kurozumi, and Shintani (2023), hence subject to misspecification issues. In the Appendix B, we perform a small set of additional estimations in which the two-percent trend inflation for the PCE deflator is considered. A tentative finding is that considering the trend inflation in the PS-NKPC increases the model fit, but the estimation bias of the structural parameters is potentially small. Further analysis on the trend-inflation, such as the one for intermediate goods inflation in the PS-NKPC context, is warranted and we leave it a future study.

5 Concluding Remarks

In this paper, we estimate the PS-NKPC taking into account the vertical production chain from processing material all the way down to the distribution of consumption goods by retailers. We employ a Bayesian VAR-GMM approach to estimate the PS-NKPC, and then compare it with the canonical NKPC based on a quasi-marginal likelihood criterion, which is robust under weakly identified parameters. The analysis indicates that the PS-NKPC is the better model than the canonical NKPC, suggesting production chains play an important role in accounting for inflation dynamics. We find that the the pipeline price pressures, represented by the the relative price terms between upstream and downstream firms, have a non-negligible impact on the U.S. inflation dynamics.

Some of our results need further exploration. For example, our result raises an issue that inflation data alone is not effective enough to pin down precisely the value for the intermediate input shares. In addition, our model does not incorporate important economic thoughts that would help understanding inflation dynamics, such as those on trend inflation and expectation formation. These omissions could lead to, for example, an imprecise comparison between forward-looking and hybrid NKPCs. Since some of these features can be incorporated by following the methods of existing literature, we expect future studies will follow these venues.

A Block Metropolis-Hastings Algorithm in Bayesian VAR-GMM Estimation

In this appendix, we describe the Block Metropolis-Hastings algorithm to calculate the quasi-posterior distribution of the parameters in Phillips curves and VAR coefficients. The algorithm is based on Kurozumi, Oishi, and Van Zandweghe (2022) who set two blocks, the VAR coefficients and the parameters in Phillips curves. The algorithm takes the following steps.

Initialization

Let ξ_{nkpc} be the Phillips curve parameters, and $vec(A)$ be the VAR coefficient matrix. Initialize A_0 and $\Sigma_{A,0}$ at the quasi-posterior mean and variance of the estimated VAR. Then, initialize $\xi_{nkpc,0}$ at the quasi-posterior mode which is calculated using A_0 and $\Sigma_{\xi,0}$ as the negative of the inverse Hessian of the log quasi-posterior probability density evaluated at $\xi_{nkpc,0}$,

$$\Sigma_{\xi,0} = -\left(\left. \frac{\partial^2 \log(p(\xi_{nkpc}, vec(A_0) \mid Y))}{\partial \xi_{nkpc} \partial \xi'_{nkpc}} \right|_{\xi_{nkpc}=\xi_{nkpc,0}} \right)^{-1}$$

Block Metropolis-Hastings algorithm

For $i = 1$ to N:

1. Draw $vec(\tilde{A})$ from a Gaussian proposal distribution with mean $vec(A_{i-1})$ and variance $c_1\Sigma_{A,i-1}$, where c_1 is the scaling parameter to set an acceptance rate as approximately 25 percent.

Set $A_i = \tilde{A}$ with probability

$$\psi_{1,i} = \min\left\{1, \frac{p\left(\xi_{nkpc,i-1}, vec(\tilde{A})\right) \mid Y)}{p\left(\xi_{nkpc,i-1}, vec(A_{i-1})\right) \mid Y}\right\}$$

and $A_i = A_{i-1}$ with probability $1 - \psi_{1,i}$.

2. Then, draw $\tilde{\xi}_{nkpc}$ from a Gaussian proposal distribution with mean $\xi_{nkpc,i-1}$ and variance $c_2\Sigma_{\xi,i-1}$, where c_2 is the scaling parameter to set an acceptance rate as approximately 25 percent.

   Set $\xi_{nkpc,i} = \tilde{\xi}_{nkpc}$ with probability

$$\psi_{2,i} = \min\left\{1, \frac{p\left(\tilde{\xi}_{nkpc}, vec(A_i)\right) \mid Y}{p\left(\xi_{nkpc,i-1}, vec(A_i)\right) \mid Y}\right\}$$

   and $\xi_{nkpc,i} = \xi_{nkpc,i-1}$ with probability $1 - \psi_{2,i}$.

3. Repeat from step 1 to 2 until $i = N$ where N is the number of total draws including burn-in to be discarded.

B Robustness analysis

We estimate small variants of the PS-NKPC in this appendix. The first is using the U.S. output gap published by the Congressional Budget Office (CBO), instead of using our Hamilton-filtered consumption gap.

Second, we consider the two-percent trend inflation rate in our PS-NKPC, given that the Federal Reserve has made it explicit that the inflation of two percent over the longer run, as measured by the annual change in PCE deflator, is most consistent with their mandate for maximum employment and price stability. We do so by assuming the annualized rate of two-percent trend inflation in the PCE deflator, denoted by $\bar{\pi}$, and reformulate the PS-NKPC of the final stage as follows:

$$\begin{aligned}\pi_{N,t} - \bar{\pi} = {} & \beta\bar{\pi}E_t\left(\pi_{N,t+1} - \bar{\pi}\right) + \bar{\lambda}_N\left(\phi_N \tilde{g}_{N,t} + (1-\phi_N)\sigma\tilde{c}_t\right) \\ & + \bar{\rho}_N E_t\left[\sum_{\tau=1}^{\infty}\left(\beta\alpha_N\bar{\pi}^{\theta-1}\right)^{\tau}\left(\Delta\tilde{y}_{t+\tau} + \theta\left(\pi_{N,t+\tau} - \bar{\pi}\right) - r_{t+\tau}\right)\right],\end{aligned}$$

where

$$\begin{aligned}\bar{\lambda}_N &\equiv \frac{\left(1-\alpha_N\bar{\pi}^{\theta-1}\right)\left(1-\beta\alpha_N\bar{\pi}^{\theta}\right)}{\alpha_N\bar{\pi}^{\theta-1}}, \\ \bar{\rho}_N &\equiv \frac{\left(\bar{\pi}-1\right)\left(1-\alpha_N\bar{\pi}^{\theta-1}\right)}{\alpha_N\bar{\pi}^{\theta-1}},\end{aligned}$$

and where $\Delta\tilde{y}_t$ and r_t are the growth rate of the aggregate output in terms of the deviation from the trend growth and the nominal interest rate respectively.[20] As in the case

[20] In what follows, the elasticity of substitution, θ, is set to 9.32 following Ascari and Sbordone (2014). In addition, we assume $\Delta\tilde{y}_t = \Delta\tilde{c}_t$ and give numbers for $r_{t+\tau}$ from the outside of the PS-NKPC model, using a small vector autoregression model. These procedures facilitate the estimation and comparison with the main result, while reasonably adhering to existing thoughts of macroeconomics.

Table 6: Additional estimates of Phillips curves for PCE deflator

	forward-looking				
	(1)	(2)	(3)	(4)	(5)
gap data	$\tilde{c}_t$	$\tilde{c}_t$	CBO gap	$\tilde{c}_t$	$\tilde{c}_t$
model	NKPC	PS-NKPC	PS-NKPC	NKPC	PS-NKPC
$\bar{\pi}$	zero	zero	zero	two	two
QML	-80.74	-75.41	-78.06	-80.16	-73.42
parameter					
α_N	0.899	0.837	0.860	0.863	0.805
	[0.874, 0.925]	[0.803, 0.877]	[0.825, 0.898]	[0.831, 0.898]	[0.769, 0.848]
ϕ_N		0.661	0.562		0.654
		[0.470, 0.836]	[0.302, 0.807]		[0.437, 0.841]
$\kappa_{mc,N}$	0.017	0.015	0.010	0.016	0.015
or $\bar{\kappa}_{mc,N}$	[0.009, 0.026]	[0.006, 0.026]	[0.003, 0.020]	[0.006, 0.028]	[0.005, 0.028]
$\kappa_{rp,N}$		0.022	0.013		0.021
or $\bar{\kappa}_{rp,N}$		[0.010, 0.034]	[0.005, 0.023]		[0.008, 0.034]

Notes: This table reports the quasi-posterior mean for each parameter in the Phillips curves for the PCE deflator, the final stage of the production flows, with the VAR lag length of $k = 1$. The numbers in square brackets show 95 percent highest quasi-posterior density interval. The sample period is from 1988:Q1 to 2019:Q4.

for the zero-trend-inflation model, we can define the reduced form coefficients for the two-percent-trend model as $\bar{\kappa}_{mc,N} \equiv \bar{\lambda}_N \phi \sigma$ and $\bar{\kappa}_{rp,N} \equiv \bar{\lambda}_N (1 - \phi)$.

The column (3) shows the result when we use the CBO output gap. The QML does not improve, and the coefficient on the output gap and relative price are slightly less potent.

The column (4) and (5) show the case for two-percent trend inflation. The QML improves, especially for the PS-NKPC model. However, the reduced-form coefficients are not meaningfully different from the case of zero trend inflation, shown in the column (1) and (2).

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