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Population Aging and the Real Interest Rate in the Last and Next 50 Years

—A tale told by an Overlapping Generations Model—

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POPULATION AGING AND THE REAL INTEREST RATE IN THE LAST AND NEXT 50 YEARS*

--A tale told by an Overlapping Generations Model--

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Abstract

Population aging, along with a secular decline in real interest rates, is an empirical regularity observed in developed countries over the last few decades. Under the premise that population aging will deepen further in coming years, some studies predict that real interest rates will continue to be depressed further to a level below zero. In the present paper, we address this issue and explore how changes in demographic structures have affected and will affect real interest rates, using an overlapping generations model calibrated to Japan's economy. We find that the demographic changes over the last 50 years reduced the real interest rate. About 270 out of the 640 basis points decline in real interest rates during this period was attributed to declining labor inputs and higher saving, which themselves stemmed from the lower fertility rate and increased life expectancy. As for the next 50 years, we find that demographic changes alone will not substantially increase or decrease the real interest rate from the current level. These changes reflect the fact that the size of demographic changes in years ahead will be minimal, but that downward pressure arising from the past demographic changes continue to bite in the years ahead. As Japan is not unique in terms of this broad picture of changes in demographic landscapes over the last 50 years and in the next 50 years, our results suggest that, sooner or later, a demography-induced decline in real interest rates may be contained in other developed countries as well.

JEL classification: E20, J11

Keywords: Declining Real Interest Rates; Population Aging; Overlapping Generations Model

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1 Introduction

Over the last few decades, population aging and a secular decline in real interest rates have gone hand-in-hand in major developed countries, though to varying degrees. Figure 1 shows the time path of real interest rates and 'old-age' dependency ratios in G7 countries over the last 50 years. Real interest rates have seen a continuing decline since the early 1980s, specifically since the end of the Great Inflation of the 1970s, and they have been moving at a rate around zero in recent years. Meanwhile, population aging advanced globally. Dependency ratios have continued to rise since the 1960s, and in some countries the pace has even accelerated since the 1990s. In addition, according to the forecasts by the U.N., or by other domestic authorities, the progress of population aging will continue to deepen in the years ahead. Given the prospective changes in demographic landscapes, some studies, such as Eggertson et al. (2017), point out the possibility that population aging, combined with the effects of other factors, could bring real interest rates down to a value below zero for a prolonged period.

Questions about the influences of aging on macroeconomic variables have long attracted the attention of scholars, including Auerbach and Kotlikoff (1987) on the U.S. economy, and Miles (1999) on the U.K. and European countries. What is different in recent years is that central bank economists ventured into this area of research. Against the backdrop of continuing low interest rates, there is a growing concern among central bankers that the decline in real interest rates may be a signal that the natural rate of interest (or equivalently, the long-run equilibrium real interest rate) – the real interest rate at which economic activity and prices neither accelerate nor decelerate – is declining or will decline in the years ahead, reducing the effectiveness of monetary policy.[1,2] While natural rates of interest are not directly observable, there are suggestive estimates that support these concerns. Figure 2 shows the time path of the natural rate of interest in selected countries estimated based on the methodology developed by Laubach and Williams (2003). Clearly,

[1] The concern is clearly related to discussion about secular stagnation hypothesis. See, for example, Summers (2014, 2015) for the detailed discussion.

[2] Recent voices from central bankers regarding the relationship between population aging and real interest rates or the natural rate of interest include Carney (2017), Draghi (2016) and Fischer (2016).

some cross-country differences notwithstanding, they all exhibit a declining trend and are reaching around zero as of 2017.

In this paper, we address the relationship between population aging and real interest rates by answering the following two questions: Have changes in demographic structures so far affected developments in real interest rates? And, will demographic changes lead to a further decline in real interest rates in the years ahead? To this end, we construct an overlapping generations (OG) model calibrated to Japan's economy that consists of 80 generations of households, a social security system, and the government. Because we are interested in analyzing the changes in the real interest rate that are induced by demographic factors exclusively, our model abstracts from nominal rigidities and short-run structural shocks, such as markup shocks or monetary policy shocks.[3] We choose the OG model as it is suited to precisely exploring the implications of changes in demographic landscapes for factor prices, including the real interest rate, and allocations, isolating the effects of other factors, such as total factor productivity (TFP) or the tax and social security system. We choose Japan's economy, since, as illustrated in Figure 1, Japan is a country where demographic changes are pronounced in terms of their pace and depth when compared with other advanced countries. In the later section, however, we also quantify the effects of demographic changes on real interest rates in the other countries using data from these countries.

With the model, we carry out perfect foresight simulations, similar to those conducted in existing studies, such as Hayashi and Prescott (2002), Chen et al. (2007), Hansen and İmrohoroğlu (2016), and Gagnon et al. (2016). We feed into the model the time paths of exogenous variables, including demographic variables and TFP from 1960 onwards, and compute the equilibrium transition paths of factor prices and allocations during that time. We first show that our model explains the actual data well. That is, the model closely

[3] Along the same line, labor force participation rates are assumed to be constant over time in our model. Consequently, the dynamics of working-age population in the model does not coincide with those of the number of workers in Japan, period by period, because the latter mirror changes in labor force participation rates over the business cycles. It is also notable, however, that as suggested in Figure 6 below, when effects of changes in intensive margin are taken into account as well, the model-generated total labor inputs, namely the product of the extensive and intensive margin of the labor inputs, roughly agree with the data counterparts, at least in terms of the long-run dynamics.

replicates the actual time path of GNP and other key variables from 1960 to 2015. We then simulate the model under various different assumptions about demographic landscapes and other exogenous variables, and study how changes in demographic factors are translated to changes in the real interest rate.

Our answers to the two questions we address are Yes and No, respectively. On the one hand, we find that changes in demographic landscape over the last 50 years, both the decline in the fertility rate and the increase in longevity, have lowered the real interest rate in Japan massively.[4] A decline in the fertility rate directly reduces the working age population and an increase in longevity encourages households' saving, leading to lower labor inputs and higher capital accumulation, which in turn reduces real interest rates. Based on our baseline simulation, 270 out of the 640 basis points decline in the real interest rate from 1960 to 2015 was caused by the changes in the demographic structure. On the other hand, the model also indicates that changes in the demographic landscape from 2016 onwards will have only a minor effect on the real interest rate. Other things being equal, the real interest rate will not increase or decrease much in response to the upcoming changes in demographic factors, staying around the current level. Based on our baseline simulation, the real interest rate will be evolving between 0 and 90 basis points over the next 50 years, and the rates do not depart from that level much even when the demographic landscape is arrested in that of 2015.

Why will the influence of demographic factors on real interest rates wane over the years, and why is the real interest rate not going to decline further in the future? We argue that there are two reasons. Most importantly, the magnitude of demographic change itself will moderate over the course of history. In Japan, over the last 50 years, the fertility rate has dropped from 4.6% in the 1960s to -0.7% in 2015, which results in a decline in the growth rate of the working age population from 2.0% to -1.4% during the period. In the next 50 years, the fertility rate will drop by only 0.5%, which makes the growth rate of the working age population almost unaltered. Similarly, life-expectancy has risen by 12 years over the

[4] To maintain consistency, we define the "fertility rate" as the growth rate of the age-21 population. Admittedly, this definition is different from the common definition of the "total fertility rate," which is the average number of children per woman during her lifetime.

last 50 years, but it will rise by only 4 years over the next 50 years. This "calming down" of population aging suggests that the marginal increase of the downward pressure from now on will be quantitatively minor. It is notable, however, that the effects of the past changes in demographic factors will continue to exert downward pressure on real interest rates. In fact, the second reason is that the effects of the past demographic changes are long-lived and will not disappear in the years ahead, preventing the real interest rates from rising or falling from current levels. This is partly because a rise in longevity is a permanent change and affects households' saving behavior permanently, bringing about a persistent negative effect on the real interest rate. Based on our baseline simulation, for example, the size of the downward pressure on real interest rates due to changes in demographic factors during the 1970s and 1980s is about 130 basis points as of 2015, and this will only shrink to about 90 basis points in 2060.

Lastly, we ask if our findings about the impacts of demographic factors on real interest rates have global implications. Not surprisingly, declining the fertility rate and increasing longevity and a moderation of these demographic changes over time are commonly observed in developed countries. We therefore conduct similar simulation exercises again, this time by feeding into our OG model the demographic variables of the U.S., G7 countries (less Japan and U.S.), and 15 selected OECD countries (less Japan and U.S.). We compute the influence of demographic factors on real interest rates in each of the country groups and compare this with that for Japan. We show that, though some differences are present across these country groups, mirroring differences in demographic developments, the negative effects of demographic factors on real interest rates and the waning influence of these factors in the years ahead are also obtained for the country groups.

Our paper is based on the two strands of the literature. The first strand includes analyses that explore the relationship between demographic structures and real interest rates or the natural rate of interest, including Ikeda and Saito (2014), Fujita and Fujiwara (2016), Rachel and Smith (2015), Gagnon et al. (2016), Carvalho et al. (2016), and Eggertson et al. (2017). Our study is closest to the works by Gagnon et al. (2016) and Eggertson et al. (2017) because they also use a medium-scale OG model as their analytical tools. Similar to our work, Gagnon et al. (2016) document that the changes in demographic structures, in particular those that took place before the 1980s, account for almost all of the decline in real interest rates from 1980 to 2016 in the U.S., and that real interest rates are likely to stay more or less at the current low level in the years ahead. Our paper is also related to studies that explore the relationships between demographic structures and macroeconomic variables, in particular those focusing on Japan's economy using an OG model. These studies include Chen et al. (2007), Braun et al. (2009), Braun and Joines (2015), Muto, Oda, and Sudo (2016, MOS). In particular, our paper is close to MOS (2016) in terms of its focus on population aging in Japan.[5]

The contribution of this paper is to quantify the impact of demographic factors, i.e., changes in the fertility rate and longevity, on real interest rates in the past and next 50 years, using an OG model calibrated to Japan's economy and to examine the implications for other developed countries, including the U.S. Clearly, our paper is not the first paper to address this issue, but it differs from existing studies in the following points. First, our analysis is based on an OG model with heterogenous agents with age-specific characteristics, and contrasts with other model-based approaches on the effects of population aging on the real interest rate in Japan, such as Ikeda and Saito (2014) and Fujita and Fujiwara (2016). The use of an OG model allows us to provide a precise assessment of the distinct role played by changes in the fertility rate and longevity that occur at different periods over the course of history. Second, we show how the transmission of demographic factors to real interest rates is affected by other elements of economic environment. For example, while existing studies with an OG model, including Gagnon et al. (2016) and Eggertsson et al. (2017), disregard the tax system and the social security system, we show that the simulated time path of the real interest rate is biased downward if the social security system is not appropriately modeled. Third, we compare the impact of demographic changes across

[5] Our OG model is built upon the model of MOS (2016). The key departure from theirs is that we broaden our scope of the analysis in time-series dimension and study the 1960s onwards, instead of the 1980s onwards as studied in MOS (2016). As we show below, changes in the demographic landscapes in the 1960s and 1970s are substantially large, and, the effects of these changes influence even today's real interest rates in an important manner. The current paper also differs from MOS (2016) in its focus on the dynamics of real interest rates, whereas MOS (2016) focus on the dynamics of other macroeconomic variables, in particular, those of output growth.

countries and derive global implications about the role of population aging on the real interest rates. This contrasts with existing studies that typically examine the implications for a specific country or region.

The remainder of this paper is organized as follows. Section 2 describes an OG model. Section 3 explains the calibration methodology and the data sources for simulation exercises. Section 4 documents the simulation results under the baseline scenario for exogenous variables including demographic factors. It shows how well the model replicates developments in macroeconomic variables from 1960 to 2015, and how the key variables will develop over the next 50 years. Section 5 gauges the effect of population aging on the real interest rate by conducting simulations under hypothetical scenarios for demographic changes, TFP, and the social security system. Section 6 draws global implications about the effect of population aging using a set of simulations equipped with demographic variables from other developed countries. Section 7 concludes.

2 Model

Outline of our model

Our model is built up on a standard OG model, including that of Auerbach and Kotlikoff (1987). Following MOS (2016), however, we incorporate two additional ingredients into the otherwise standard model: (a) the health insurance system, and (b) a 'bond-in-utility' specification developed in Hansen and İmrohoroğlu (2016). We incorporate (a) because the social security system is considered an important determinant of households' savings behavior, and the ingredient (b) in order to address sizable and persistent positive spreads between the real return on capital and the real interest rates on government bonds in the actual data.[6]

2.1 Demographics

The time period of the model is discrete and annual. The economy consists of 80 generations of different ages denoted by $j = 21, ..., 100$. In each period t, a new generation aged

[6] See Hansen and İmrohoroğlu (2016) for the details of the rationale behind this setting.

21 is born into the economy, while the other existing generations all shift forward by one. The oldest generation, $j = 100$, which we assume to be the maximum age, dies out deterministically in the subsequent period. The growth rate of the new generation (age-21 households) in period t is denoted by ρ_t, which we will hereafter refer to as the fertility rate in the model. Then the age-21 population in period t, expressed as $P_{21,t}$, is given by

$$P_{21,t} = (1 + \rho_t)\, P_{21,t-1}. \tag{1}$$

All households face a mortality risk that is common within the same cohort but may differ across cohorts. They survive to the subsequent period with conditional survival probability $\psi_{j,t}$, which is the probability that households aged $j - 1$ in period $t - 1$ survive to become age j households in period t. Note that $\psi_{101,t} = 0$ by assumption. Then the population, $P_{j,t}$, and cohort share, $\mu_{j,t}$, of age j households in period t are expressed as follows:

$$P_{j,t} = \psi_{j,t} P_{j-1,t-1}, \tag{2}$$

$$\mu_{j,t} = \frac{P_{j,t}}{\sum_{i=21}^{100} P_{i,t}} \quad \text{for } j = 21, ..., 100. \tag{3}$$

For the purpose of the ensuing analysis, we also define the total population P_t as follows:

$$P_t \equiv \sum_{i=21}^{100} P_{i,t}.$$

2.2 Households

Setting

Until mandatory retirement at age $j = 65$, households supply labor to firms and earn wage income according to their age-specific labor efficiency denoted as $\varepsilon_{j,t}$ from age $j =$ 21 to 65, paying the labor income tax and payroll income taxes that cover social security benefits in every period.[7] Households aged $j = 66$ withdraw from the labor force, and those

[7] Although mandatory retirement in Japanese firms is usually set between age 60 and 65, labor force participation rates above age 65 are never zero according to data, though they are not sizable. Following existing studies such as Chen et al. (2007) and MOS (2016), however, we maintain the conventional assumption because detailed and sufficient data on wages, hours, and the number of workers above age 65

aged $j \geq 66$ receive public pension benefits from the social security system. Throughout their lives, households save assets in two forms, capital and government bonds, and receive capital income arising from these assets.

In every period T, newly born households choose sequences of consumption, labor, capital and bond holdings in $t \geq T$, to maximize their expected lifetime utility (discounted by the subjective discount factor β)

$$\sum_{j=21}^{100} \beta^{j-21} \left[\prod_{i=21}^{j} \psi_{i,T-21+i} \right] u\left(c_{j,t}, h_{j,t}, b_{j+1,t+1}\right) \tag{4}$$

subject to the budget constraints over their lifetime

$$\begin{aligned}
&(1+\tau_{c,t})\, c_{j,t} + k_{j+1,t+1} + b_{j+1,t+1} + \lambda_{j,t} m b_{j,t} \\
&= \left[1 + (1-\tau_{k,t})\, r_t^K\right] k_{j,t} + (1+r_t^B) b_{j,t} && (5)\\
&\quad + (1-\tau_{h,t})\,(1-\tau_{s,t}-\tau_{m,t})\, w_t \varepsilon_{j,t} h_{j,t} + \tau_t + \xi_t \qquad \text{for } j \leq 65, \\
&(1+\tau_{c,t})\, c_{j,t} + k_{j+1,t+1} + b_{j+1,t+1} + \lambda_{j,t} m b_{j,t} \\
&= \left[1 + (1-\tau_{k,t})\, r_t^K\right] k_{j,t} + (1+r_t^B) b_{j,t} + pb_{j,t} + \tau_t + \xi_t \qquad \text{for } j > 65. && (6)
\end{aligned}$$

Note that in a period $t \geq T$, these households are at the age of $j = t - T + 21$. $c_{j,t}$ is consumption and $h_{j,t}$ is labor supply. $k_{j,t}$ and $b_{j,t}$ denote capital and bond holdings of age j households at the beginning of period t. w_t is the wage rate, r_t^K is the before-tax real rate of return on private capital, and r_t^B is the after-tax real rate of return on government bonds. $\tau_{c,t}$ is the consumption tax rate, $\tau_{h,t}$ and $\tau_{k,t}$ are the tax rates on income from labor and capital, and τ_t is a lump-sum transfer in period t.[8] ξ_t is a lump-sum transfer associated with accidental bequests, which are left by households who die in the preceding period $t-1$. $\tau_{s,t}$ and $\tau_{m,t}$ are the payroll income tax rates (contribution rates) for public pension and health insurance, respectively. $\varepsilon_{j,t}$ is age-specific labor efficiency for $j \leq 65$,

is not available.

[8]This corresponds conceptually to what is categorized as the sum of the net current transfers and the capital transfers in the System of National Accounts in Japan.

$pb_{j,t}$ is public pension benefits that retirees aged j in period t receive, described later. The age-specific profile of medical costs, denoted by $mb_{j,t}$, is assumed to be exogenously given. As shown in the equations above, only a proportion of the medical expenditures is paid directly by each household. We denote this copay rate by $\lambda_{j,t} \in [0,1]$. We assume that this rate differs across ages and time so that they are consistent with the past and present institutional structure.[9] We assume that a new household born in period t has no initial assets: $k_{21,t} = b_{21,t} = 0$.[10] While no household is able to survive to the maximum age 100, it doesn't fully consume its assets at the age of 100 because of a bequest motive: $k_{101,t} > 0$ and $b_{101,t} > 0$. We also assume that the government collects all accidental bequests including capital income in period $t-1$, and redistributes them equally among all households alive in period t. The total amount of accidental bequests in period t is given by

$$\xi_t = \sum_{j=22}^{101} (1-\psi_{j-1,t-1}) \left[\left\{ 1 + (1-\tau_{k,t-1})\, r_{t-1}^K \right\} k_{j-1,t-1} + (1 + r_{t-1}^B) b_{j-1,t-1} \right] \frac{P_{j-1,t-1}}{P_t}. \quad (7)$$

Following Hansen and İmrohoroğlu (2016), we introduce government bond holdings into the utility function. The basic idea behind this 'bond-in-utility' specification is to incorporate households' preference for the liquidity and safety characteristic of government bonds. The functional form of households' utility is assumed to be separable in terms of its arguments, which are given as

$$\begin{aligned} u\left(c_{j,t}, h_{j,t}, b_{j+1,t+1}\right) &= \log c_{j,t} - \gamma_t \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \eta_t \log(b_{j+1,t+1} + \bar{b}) \quad \text{for } j \leq 65, \quad (8) \\ u\left(c_{j,t}, h_{j,t}, b_{j+1,t+1}\right) &= \log c_{j,t} + \eta_t \log(b_{j+1,t+1} + \bar{b}) \quad \text{for } 65 < j < 100, \\ u\left(c_{j,t}, h_{j,t}, b_{j+1,t+1}\right) &= \log c_{j,t} + \phi(\log b_{j+1,t+1} + \log k_{j+1,t+1}) \quad \text{for } j = 100, \end{aligned}$$

where $\bar{b}$ is the parameter that limits the curvature of the period utility function when

[9] The copay rate is 30% for people below 70 and virtually 0% for people over 69 until 2002. The copay rate is approximately 10% for people over 69 from 2003 to 2007. After 2007, the rate is approximately 20% for people of age 70 to 74 and 10% for people over 74.

[10] The borrowing constraint is not imposed on households' assets in our model. In other words, households are allowed to borrow against their future income.

$b_{j+1,t+1}$ is close to 0, and ν is the Frisch elasticity of households' labor input supply. γ_t and $\eta_{j,t}$ are the time-varying parameters households' preferences for leisure and government bond holding in period t. Higher γ_t or η_t implies that households put a higher value on leisure or government bond holding. For households aged 100, we assume that they have the bequest motive, following Eggertsson et al. (2017) and Lisack et al. (2017). Note that ϕ is the parameter that determines the relative importance of the bequest motive.

Since households receive utility from government bond holding (i.e., $\eta_t > 0$), the first order conditions of age j households in period t yield the following equation regarding the spread Ξ_t of the two asset returns:

$$\Xi_t \equiv (1-\tau_{k,t})r_t^K - r_t^B = \frac{1}{\beta}(1+\tau_{c,t})\eta_t \sum_{j=21}^{100-1} \mu_{j,t} \frac{c_{j,t}}{(b_{j+1,t+1}+\bar{b})}. \tag{9}$$

The above equation shows that the spread Ξ_t is wide when the preference parameter η_t is high or the size of government bond holding relative to the consumption is low. This reflects the fact that households are willing to hold low-return assets only if the marginal utility from holding them is greater.

2.3 Firm

There is a representative firm producing final goods with the Cobb-Douglas production technology. In perfectly competitive spot-markets the firm rents capital and hires labor from households so as to maximize its profit

$$\Pi_t = A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t, \tag{10}$$

where α is the capital share of output, K_t is aggregate capital stock, L_t is aggregate labor input, and R_t is the rental rate of capital stock. A_t denotes TFP, and we assume that the TFP factor grows at the rate of g_t in every period:

$$g_t \equiv (A_t/A_{t-1})^{1/(1-\alpha)}. \tag{11}$$

Note also that the output, or equivalently GNP, in the economy coincides with the size of the aggregate product. That is,

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}.$$

In equilibrium, the factor prices are given by

$$R_t = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} \equiv r_t^K + \delta_t, \tag{12}$$

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha}, \tag{13}$$

where δ_t is the depreciation rate of capital stock. The aggregate demand for capital and labor inputs is equalized to the aggregate supply of these primary inputs, so as to clear the respective markets in every period:

$$K_t = \sum_{j=22}^{100} P_{j-1,t-1} k_{j,t}, \tag{14}$$

$$L_t = \sum_{j=21}^{65} P_{j,t} \varepsilon_{j,t} h_{j,t}. \tag{15}$$

Here, the evolution of the aggregate capital stock is given by

$$K_{t+1} = I_t + (1-\delta_t) K_t. \tag{16}$$

where I_t is aggregate investment. Relatedly, the aggregate saving rate s_t is defined by the following equation.

$$s_t \equiv \frac{I_t - \delta_t K_t}{Y_t}. \tag{17}$$

2.4 Social Security System

The social security system is divided into two sections: public pension and health insurance. The public pension benefit $pb_{j,t}$ provided to age j households in period t depends on their historical wage income, being proportional to the average wage income that households have received during their working years, following Chen et al. (2007). The public pension benefit provided by the social security system to a new retiree in period t is given as

$$pb_{65+1,t} \equiv \frac{\theta_t}{65+1-21} \sum_{i=21}^{65} \{ w_{t+i-65-1} \varepsilon_{j,t} h_{i,t+i-65-1} \prod_{k=1}^{65+1-i} g_{t+i+k-65-1} \}. \tag{18}$$

The public pension benefit that age j households receive is formulated as

$$\begin{aligned} pb_{j,t} &= 0 && \text{for } j = 21, 22, ..., 65, \\ pb_{j,t} &= \frac{\theta_t}{\theta_{t+65+1-j}} pb_{65+1,t+65+1-j}, && \text{for } j = 65+1, ..., 100. \end{aligned} \tag{19}$$

θ_t is the replacement ratio that determines the size of the public pension benefit relative to the past wage. The pension benefit paid to households in the same cohort is constant throughout their lifetime as long as the replacement ratio is unchanged. Note, however, that compared to the cohort a generation before, the next generation receives a different size of benefits depending on the growth rate of TFP.

For both public pension benefits and medical benefits, part of the costs is covered by the government, and the rest is covered by the relevant section of the social security system. Taking as given the coverage ratios of transfers/expenditures financed by the government, which we denote as $\kappa_{s,t}$ and $\kappa_{m,t}$, the social security system adjusts the contribution rates for public pension and health insurance, $\tau_{s,t}$ and $\tau_{m,t}$, in every period in the following manner:

$$\tau_{s,t} = \frac{(1-\kappa_{s,t}) \sum_{j=65+1}^{100} P_{j,t} pb_{j,t}}{w_t \sum_{j=21}^{65} P_{j,t} \varepsilon_{j,t} h_{j,t}}, \tag{20}$$

$$\tau_{m,t} = \frac{(1-\kappa_{m,t}) \sum_{j=21}^{100} P_{j,t} (1-\lambda_{j,t}) mb_{j,t}}{w_t \sum_{j=21}^{65} P_{j,t} \varepsilon_{j,t} h_{j,t}}. \tag{21}$$

2.5 Government

The government raises revenues by newly issuing one-period government bonds and levying taxes on households' consumption, labor income, and capital income, to finance its spending that is the sum of government purchases, transfers/expenditures to the social security system, interest repayments on government bonds, and other lump-sum transfers. Taking the sequences of government revenues and spending as given, government bond issuance is adjusted so that the following consolidated budget constraint holds in every period:

$$(1+r_t^B)B_t + G_t + \kappa_{m,t}\sum_{j=21}^{100} P_{j,t}\left(1-\lambda_{j,t}\right) mb_{j,t} + \kappa_{s,t}\sum_{j=65+1}^{100} P_{j,t}pb_{j,t} + \sum_{j=21}^{100} P_{j,t}\tau_t$$
$$= B_{t+1} + \tau_{c,t}C_t + \tau_{h,t}(1-\tau_{s,t}-\tau_{m,t})w_t L_t + \tau_{k,t} r_t^K K_t, \tag{22}$$

where G_t and B_t are government purchases and government bonds at the beginning of period t, respectively. Note that the supply of government bonds is equalized to the sum of households' bond holdings in each period:

$$B_t = \sum_{j=22}^{100} P_{j-1,t-1} b_{j,t}. \tag{23}$$

2.6 Real Interest Rates in the Model

In this model, households can hold two types of assets, capital stock $k_{j,t}$ and government bonds $b_{j,t}$, and there are two different asset returns r_t^K and r_t^B. Because of the bond-in-utility setting, the return on capital r_t^K is generally higher than the return on government bonds r_t^B.[11] Throughout this paper, we refer the latter as the real interest rate, and study the effects of changes in demographics on this rate. We do so for the purpose of comparison with existing studies on the natural rate of interest, including Laubach and Williams (2003), Gagnon et al. (2016), Fujiwara et al. (2016) and Eggertsson et al. (2017). While spreads between the return on capital and the risk-free rate are universally observed and not unique to Japan, the natural rate of interest in these studies is either estimated using the data of the risk-free rates, such as the call rate or the government bond yield, or analyzed under the premise that the return on capital and the return on government bonds are equalized ex-ante to the level of prevailing risk-free rates.[12]

[11] In fact, the spread between the two assets measured by the System of National Accounts has been sizable in Japan. For example, as of 2015, the after-tax return on capital, measured as the operation surplus divided by the total capital stock is 6.8% whereas the after-tax return on government bonds, measured as the net interest payment by the government divided by government bonds outstanding, is 0.1%.

[12] While the returns on capital r_t^K and the return on government bonds r_t^B are sizably different in level, as we show below, the difference is negligible when comparing the effect of demographic changes on the two rates. This is because, in our simulation setting, thanks to the assumption regarding the government's transfer rule described below, the spread between the two returns Ξ_t is barely affected, if at all, by changes in demographic factors.

3 Data, Calibration, and Assumptions

In our simulation, we compute the equilibrium transition paths of allocations and prices, including real interest rates, from given initial conditions in 1960 toward a steady-state far in the future, taking the sequence of exogenous variables from 1960 to the far future. We feed three classes of inputs into the model: (*i*) structural parameters such as those of the production function, (*ii*) exogenous variables such as fertility rates and tax rates, and (*iii*) the steady state distribution of households' assets. In this section, we describe in detail how to construct these inputs and how to compute the equilibrium path.

3.1 Structural Parameters

The structural parameters of our model, α, β, and ν are calibrated based on information from the sample period running from 1960 to 2015 and their values are listed in Table 1. The capital share of the production function α is 0.362 which is the same value as Hayashi and Prescott (2002). The subjective discount factor β is set so that our benchmark model replicates well the time path of the capital-output ratio during the sample period, and the value of Frisch elasticity ν is set at 0.5 so that our benchmark model captures an average life-cycle pattern of labor inputs during the same period. In addition, the parameter that limits the curvature of the period utility function $\bar{b}$ is set to 0.11, following Hansen and İmrohoroğlu (2016), and the parameter for relative importance of the bequest motive ϕ is set at 18.33, so that it is consitent with the equilibirum condition for households with age 100.[13]

3.2 Exogenous Variables

We conduct perfect foresight simulations from 1960 to 2015 and beyond, by feeding into the model the time paths of exogenous variables from 1960 to infinity and computing the equilibrium prices and allocations. There are four classes of variables; those related to demographics, to production technology, to households, and to the government. Some of

[13] In calibrating the value for ϕ, we use the age-specific expenditure and savings documented in *National Survey of Family Income and Expenditure* for 2014 and obtain the data for $c_{100,t}$ and $k_{101,t}$ by interpolation, and choose ϕ that meets the first order condition of these households.

the exogenous variables are age-specific, shown in Figure 3, and other exogenous variables are shown in Figure 4.

Production technology: Exogenous variables regarding production technology include the depreciation rate of capital stock $\{\delta_t\}_{t=1960}^{\infty}$, and the growth rate of TFP $\{g_t^{1-\alpha}\}_{t=1960}^{\infty}$. For $t \leq 2015$, We use the actual data from 1960 to 2015. For $t \geq 2016$, we use the actual value as of 2015 for the depreciation rate δ_t and the average of actual values from 2010 to 2015 for the growth rate of TFP $g_t^{1-\alpha}$; 6.2% and 1.1%, respectively. Note that the latter value is close to the 1% figure estimated by the Cabinet Office (2017) as the TFP growth rate in the early 2020s.

Households: Exogenous variables regarding households include the preferences for leisure $\{\gamma_t\}_{t=1960}^{\infty}$, these for bond holding $\{\eta_t\}_{t=1960}^{\infty}$, age-specific medical expenditures $\{\{mb_{j,t}\}_{j=21}^{100}\}_{t=1960}^{\infty}$, and labor efficiency $\{\{\varepsilon_{j,t}\}_{j=21}^{100}\}_{t=1960}^{\infty}$. The first variable γ_t is constant from 1960 until the late 1980s. After 1989, it gradually increases over a couple of years, and stays at the constant in the subsequent years.[14] The second variable η_t is set so that the spread Ξ_t generated from the baseline simulation coincides with the data counterpart for $t \leq 2015$.[15] We use the actual data, taken from *Estimates of National Medical Care,* for the medical expenditures $\{mb_{j,t}\}_{j=21}^{100}$ for $1980 \leq t \leq 2015$. For $t \leq 1979$, we calibrate the values with the aggregate figures of the same estimate, using the distribution of ages as of 1980, $\{mb_{j,1980}\}_{j=21}^{100}$. As shown in Figure 3, the medical costs monotonically increase with age. It is also notable that the effects of aging on the costs are more pronounced in recent years. In addition, we use actual data for the age-specific labor efficiency $\{\varepsilon_{j,t}\}_{j=21}^{100}$ for $1988 \leq t \leq 2015$, following the method of Braun et al. (2009). For $t \leq 1987$, we calibrate the values to $\{\varepsilon_{j,1988}\}_{j=21}^{100}$. For $t \geq 2016$, we use the value as of 2015 for the preferences for leisure γ_t and for bond holding η_t, and age-specific labor efficiency $\{\varepsilon_{j,t}\}_{j=21}^{100}$. Furthermore,

[14] We intend to capture the gradual reduction in the statutory workweek stipulated in the Labor Standards Law revised in 1988. This reduction started in 1989 and ended in 1994.

[15] The data counterpart is constructed from SNA for $1980 \leq t \leq 2015$, and by interpolating the real interest rate series constructed following Kamada (2009), for $t \leq 1979$.

in $t \geq 2016$, we assume that $\{mb_{j,t}\}_{j=21}^{100}$ grows at the rate of TFP g_t.

Government: Exogenous variables regarding the government include tax rates $\{\tau_{c,t}, \tau_{h,t}, \tau_{k,t}\}_{t=1960}^{\infty}$, government expenditures $\{G_t/Y_t\}_{t=1960}^{\infty}$, government transfers to households $\{\tau_t/Y_t\}_{t=1960}^{\infty}$, and government transfers to the social security system $\{\kappa_{s,t}, \kappa_{m,t}, \theta_t, \{\lambda_{j,t}\}_{j=21}^{100}\}_{t=1960}^{\infty}$. We use the actual data from 1960 to 2015 for those variables. For $t \geq 2016$, we use the value as of 2015 for the tax rates other than the consumption tax rate $\tau_{c,t}$. The consumption tax rate $\tau_{c,t}$ is assumed to increase to 10% in 2019 and remain unchanged from that rate in the subsequent years.

We assume that the government makes transfers to households in addition to τ_t in period $t \geq 2016$ such that the government debt-to-GNP ratio $\{B_t/Y_t\}_{t=2016}^{\infty}$ is set so that the model-generated value of the spread Ξ_{2016} is unchanged in the following years. Under this transfer rule is set so that the projected real interest rate r_t^B is not affected by the spread term Ξ_t in 2016 onwards, and variations of the real interest rate come from those of the return on capital r_t^K exclusively during this period.[16,17]

Population: Exogenous variables regarding population include the growth rate of the age-21 population (i.e., the 'fertility rate' in our model) $\{\rho_t\}_{t=1960}^{\infty}$, and the conditional survival probability $\{\{\psi_{j,t}\}_{j=21}^{100}\}_{t=1960}^{\infty}$. We use the actual data from 1960 to 2015. For $t \geq 2016$, we use as the benchmark the medium-fertility and medium-mortality variants of population forecasts calculated by the National Institute of Population and Social Security

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[16] Note that as shown in equations (9) and (23), the spread Ξ_t decreases with the amount of government bonds outstanding B_t. When the government issues more bonds to finance its increasing expenditures and transfers due to aging, therefore, the larger government bond causes a decline in the spread and a rise in the real interest rate r_t^B, keeping the real return on capital stock r_t^K constant. Our assumption about the government's transfer rule described here aims to isolate this channel from our analysis. The main reason we choose this approach is because, as shown in panel (10) of Figure 4, the parameter value of η_t has varied substantially from 1960 to recent years. This indicates that, based on our model, a large portion of variations in the spread Ξ_t has been driven by changes in η_t rather than the endogenous effects of government bonds and consumption shown in equation (9).

[17] Similar to ours, related studies on Japan's economy also assume some form of government debt policy in order to ensure the transversality condition of the government over a long horizon. Hansen and İmrohoroğlu (2016) assume that the government follows the trigger rule that raises tax rates when the government debt-to-GNP ratio exceeds a predetermined level. MOS (2016) assume a regime switch of government debt policy that will take place in 2050. In the model, from 2050 and beyond, the government adjusts the lump-sum tax so as to maintain the government bond-to-GNP ratio at its target level.

Research (IPSS). In addition, we use the high and low variants of population forecasts of the IPSS and the figures calculated by the UN for the purpose of the robustness check.

Demographic landscape in Japan

It is useful to sketch here the outline of changes in the demographic landscape over time in Japan. The top two panels in Figure 5 show the time path of life expectancy and the growth rate of the working age population. As shown in Figure 3, the conditional survival probabilities $\{\psi_{j,t}\}_{j=21}^{100}$ have increased monotonically and permanently since 1960 at almost all ages. The life expectancy has risen accordingly from 71 in 1960 to 84 in 2015. Based on the projections of both by the IPSS and the UN, this upward trend will continue in the years ahead, but at a moderate pace, reaching age 88 in 2060. The working age population exhibits a secular decline from the 2000s and continues shrinking in the subsequent years. It has grown on average at an annual rate of 0.5% over the last 50 years, reflecting the high fertility rate during the period, as shown in Figure 4. The growth rate fell to below zero in the late 1990s, and will fall further to an annual rate of -1.1% over the next 50 years. The entry and exit of baby boomers has a significant impact on the working population growth rate. The bumps in the early 1970s and the mid-1990s were the result of the entries of the first and second baby boomers, respectively, and the hollows in the early 2010s and 2040s will a consequence of their exits from the labor market.[18]

The bottom panel shows the distribution of the population by the age of for four different periods: 1960, 1980, 2015, and 2040. The distribution changes shape drastically over time. In the early days, the peak of the distribution was around the age of 20s to 30s, and fewer people appeared above 30s. In addition, the density declines as the ages become higher. In the present day, the distribution has two modes; one peak located around 40s, and the other around the late 60s. In 2040, the distribution has only one mode, located around the late 60s. It is also notable that while the general shape of the distribution in the present day mostly coincides with that in 2040, the shape differs markedly from that

[18] We define the first and second baby boomers as those who were born in 1947–1949 and 1971–1974, respectively, following the convention.

in the early periods. This observation suggests that demographic changes that have taken place so far are quantitatively larger than those that will take place in the years ahead.

3.3 Simulation Procedures

In all of the ensuing simulations, we assume perfect foresight. That is, newly-born households in period t are informed of the sequence of exogenous variables shown in Figures 3 and 4 from t onwards, and optimally determine their life-time consumption and labor supply.

We compute the endogenous variables, namely prices and allocation in the economy, by the shooting algorithm used in existing studies, such as Hayashi and Prescott (2002), Chen et al. (2007), and Hansen and İmrohoroğlu (2016). Given the two steady states, the initial steady state including the initial asset distribution $\{b_{j,1960}\}_{j=21}^{100}$ and $\{k_{j,1960}\}_{j=21}^{100}$,[19] and the ending steady state to which the economy ultimately converges in the infinite future, we extract the sequence of prices and allocation in the economy from 1960 to the infinite future, that satisfies the equilibrium conditions under the given sequence of the exogenous variables. The calibrated ending steady state values of key variables are presented in Table 2.

4 Baseline Simulations

The baseline simulation is conducted by feeding into the model the set of exogenous variables whose values are set as described above, and computing the endogenous paths of prices and allocations.

[19] We do not have a detailed data set on the asset distribution for 1960, which is the initial year in our model. To obtain the initial distribution of assets in 1960, $k_{j,1960}$ and $b_{j,1960}$, we first consider the steady state of an economy where the exogenous variables are constant and their values are calibrated to the actual values of 1960, and compute the life-cycle profiles of capital stock holdings $\bar{k}_j$ and government bond holdings $\bar{b}_j$ in this hypothetical economy. We then multiply the age-specific capital holdings $\bar{k}_j$ at all ages by the same scalar q_k so that the aggregate capital stock-to-output ratio K_{1960}/Y_{1960} computed from the model coincides with the actual data: $k_{j,1960} = q_k \cdot \bar{k}_j$. We do the same to the age-specific government bond holdings $\bar{b}_j$ so that the aggregate government bond outstanding-to-output ratio B_{1960}/Y_{1960} is the same between the model and the data: $b_{j,1960} = q_b \cdot \bar{b}_j$.

4.1 Model's Prediction on Households' Life-cycle Properties

Before analyzing the model's dynamics, here we assess our model's prediction about households' life-cycle profiles with the data. Figure 6 shows the life-cycle profiles of labor supply h_j, and the capital stock holdings k_j for $j = 21, ...100$ generated by our model and in the data.[20] Our model replicates well the general pattern of life-cycle profiles of these variables. Labor supply increases gradually and moderately as households age from 21 to around 50s and starts to decline at a quick pace from the peak to the level close to zero at ages above 50. The gap between the data and the model at ages around 70 reflects the fact that in the data there are households who work after the mandatory retiring age of 65. Capital stock holdings also increase from the age of 21 until the age of around 60s in both the model and the data. In the model, households start to deccumulate capital stock after the retirement age. In the data, by contrast, the capital stock holdings continue to increase, though at a slower pace than earlier ages, possibly due to the presence of the retirement allowance.

4.2 Model's In-sample Performance

We first assess the "in-sample performance" of the model by comparing the endogenous paths of prices and allocation from 1960 to 2015 that are generated from the model with those of the data counterparts. Figure 7 shows the time paths of the key variables of the economy; after-tax real return on capital $(1 - \tau_{k,t})\, r_t^K$, the saving rate s_t, GNP per capita (defined as real GNP divided by total population) Y_t/P_t, the GNP growth rate Y_t/Y_{t-1}, the capital-labor ratio K_t/L_t, the capital share of total assets $K_t/\left(K_t + B_t\right)$, the pension benefits-to-GNP ratio, the medical benefits-to-GNP ratio, and the primary balance-to-GNP ratio. The dashed red lines show the actual data series, and the solid blue lines show the model-generated series.

In summary, our model successfully replicates the actual dynamics of the key macro-

[20] For the data, the life-cycle profiles of capital stock holdings k_j are based on *the National Survey of Family Income and Expenditure.* Note that because there is no detailed data available for a life-cycle profile of government bond holdings b_j, we assume that households at all ages hold government bonds relative to total assets a_j by the same proportion as the aggregate government bond outstanding-to-aggregate total asset ratio. A life-cycle profile of labor supply h_j is constructed from a life-cycle profile of working hours based on the *Basic Survey on Wage Structure,* and the life-cycle profiles of the labor participation rate and labor force based on *the Labor Force Survey.*

economic variables over the sample period. For example, similar to the data, our model generates the "Golden Sixties" or "Japan's miracle," the rapid GNP growth rate seen during the 1960s, and also generates the "Lost decade(s)," the sudden and persistent GNP slowdown starting from the early 1990s. The model also yields the secular decline in return on capital, including its drastic fall in the mid-1970s, which is seen in the actual data. The long-run developments of revenues and expenditures of the social security system and the government, such as the persistent increases in pension and medical benefits, the deterioration in the primary balance of the government, and the accelerating government debt accumulation, are all replicated in the model.

For the real interest rate r_t^B, we compare the time path of the model-generated figure with a set of the natural rate of interest estimated in existing studies.[21] Figure 8 shows the time path of the model-generated series of the real interest rate r_t^B and the after-tax return on capital $(1-\tau_{k,t})\, r_t^K$, and two measures of the natural rate of interest in Japan's economy that are estimated based on the methodology of Laubach and Williams (2003) and Imakubo et al. (2015), and the potential growth rate constructed by the Bank of Japan.[22] While the comparison is only possible from the 1980s onwards, the model-generated real interest rate r_t^B closely tracks the secular decline that commonly appears in these measures, and captures the low level of the rate in recent years.

4.3 Out-of-sample Simulation

Figure 9 shows out-of-sample projections for the real interest rate and GNP from 2016 onwards under the baseline scenario. Roughly speaking, the real interest rate r_t^B will stay at the 2015 level in the years ahead, mirroring the fact that the return on capital r_t^K will stay at the current level. Precisely speaking, however, based on the projections of both the IPSS and the UN, both rates will continue gradually and modestly declining until mid-

[21] Here, we do not compare the model-generated real interest rate r_t^B with the data counterpart. This is because for the period from 1960 to 2015 we calibrate the households' preferences on bond holdings $\{\eta_t\}_{t=1960}^{2015}$ so that the model-generated real interest rate r_t^B captures the movements of the data counterpart, namely the government bond yield based on the SNA.

[22] As discussed in Fujiwara et al. (2016), based on a standard representative agent model, the natural rate of interest coincides with the potential growth rates on a per capita basis, at the steady state, under certain conditions.

2040, when they will reach their nadir, which is about 20 basis points for the real interest rate r_t^B and 450 basis points for the return on capital r_t^K, before ascending gradually in subsequent years.

Japan's GNP Y_t will grow at the rate of about 1% growth for the next ten years, gradually deccelerating to a rate around 0%. From 2040 and beyond, GNP grows at a nearly zero rate. As shown in panel (3) where the growth rate of GNP is decomposed into contributions of TFP A_t, labor inputs L_t, and capital inputs K_t, TFP growth, A_t/A_{t-1}, of 1.1% is almost exactly offset by declines in the growth rate of labor inputs L_t. Although GNP growth declines to a rate around zero in the long run, as shown in panel (4), GNP per capita Y_t/P_t maintains above 1% growth in years ahead. This is primarily because, on a per capita basis, declines in the growth rate of labor inputs L_t are relatively moderate, while capital inputs K_t grow at a positive rate, boosting the growth rate of the output Y_t.

5 Effects of Demographic Changes on Real Interest Rates

How much have demographic changes influenced real interest rates r_t^B so far, and how important will they be in the years ahead? This section answers these two questions with the help of four different types of simulations listed below.

1. Simulations under hypothetical assumptions about demographic structures, $\{\rho_t\}_{t=1960}^{\infty}$ and $\{\{\psi_{j,t}\}_{j=21}^{100}\}_{t=1960}^{\infty}$, where developments in demographic structures are permanently arrested after the end of a specific decade, such as the 1960s, and, in the subsequent years, remain unchanged at the average values in those ten years,

2. Simulations under hypothetical assumptions about demographic structures, $\{\rho_t\}_{t=1960}^{\infty}$ or $\{\{\psi_{j,t}\}_{j=21}^{100}\}_{t=1960}^{\infty}$, where developments in either the fertility rate and longevity are arrested after the end of a specific decade and, in the subsequent years, remain unchanged at the average values in those ten years,

3. Simulations under hypothetical assumptions about TFP growth rates $\{A_t/A_{t-1}\}_{t=1960}^{\infty}$,

4. Simulations based on an alternative model economy where the social security system

is absent

The type 1 simulations intend to identify the timing of demographic changes that are essential in shaping the long-run picture of real interest rate dynamics. In particular, we are interested in comparing the effect on the real interest rate of demographic changes that have taken place over the last 50 years with those that will take place in the next 50 years. The type 2 simulations intend to separately assess the consequences of falling fertility rate and those of increasing longevity, and to determine the key transmission channel within demographic factors. The type 3 and 4 simulations serve as the robustness analysis. We intend to see if what we obtain from the type 1 and 2 simulations regarding the role of demographic factors can fundamentally change when economic environments are also different.

5.1 Interest Rates When Developments of Demographic Structure Arrested

Figure 10 shows the results of the type 1 simulations. In the figure, "Fixed in the Xs" indicates that the fertility rate ρ_t and the conditional survival probabilities $\{\psi_{j,t}\}_{j=21}^{100}$ both take the values assumed under the baseline simulation up to the end of the Xs and, in the subsequent years, take the average values of the previous ten years.[23] The average life-expectancy and the working-age population growth under the alternative assumptions are documented in Table 3.[24]

The top panel shows the time path of the real interest rate r_t^B computed under various assumptions about the timing when demographic changes are arrested. The middle and bottom panels show the real interest rate r_t^B, labor inputs L_t, and capital inputs K_t, in terms of the discrepancy between the variables computed under each of the alternative assumptions about the timing, and those computed under the baseline assumption, namely those shown in Figure 9.

[23] The only exception is the simulation named "Fixed in 2015." In this case, the fertility rate ρ_t and the conditional survival probabilities $\{\psi_{j,t}\}_{j=21}^{100}$ are the actual values up to 2015 and, in the subsequent years, are held constant as of 2015.

[24] Because we hold the fertility rate ρ_t constant but do not hold the working age population growth rate constant, the latter series continues to vary in years after the year X.

Two observations are notable. First, demographic changes have so far depressed the real interest rate r_t^B and will depress the rate in the years ahead. This is seen in the positive gap between the real interest rate generated from the four hypothetical simulations and those generated from the baseline simulation. For example, as of 2015, the real interest rate r_t^B of "Fixed in the 1960s" is 270 basis points higher than the rate computed under the baseline assumption. This gap stands for the quantitative impact on the real interest rate r_t^B of demographic changes from the 1970s and beyond. The sign of this gap is positive across all of the alternative simulations.

Second, the effect on the real interest rate r_t^B of demographic changes that have already occurred is larger than the effect of those that is going to occur in the future. This is seen by comparing the discrepancy series shown in panel (2). For example, the difference between the series of "Fixed in the 1960s" and of "Fixed in 2015" captures the effects of demographic changes from the 1970s to 2015, and the series of "Fixed in 2015" captures the effect of demographic changes in 2016 and beyond. As of 2015, the former effect is about -260 basis points while the latter is about -10 basis points. Even in 2050, while the former is about -220 basis points, the latter is about -70 basis points. Relatedly, it is also notable that the effect of the demographic changes are both persistent and sizable. Using the same numerical example, the effect of demographic changes from the 1970s to 2015 diminishes only by 80 basis points from 2015 to 2060.

Why do demographic changes reduce the real interest rate r_t^B, and why is the effect of those in the past larger than those in the future? In the current model, the dynamics of the real interest rate r_t^B are almost entirely determined by what happens to the return on capital r_t^K, and the latter series is determined by the scarcity of the aggregate capital inputs K_t relative to the aggregate labor inputs L_t, based on the aggregate production function (10).[25] How are then aggregate capital inputs K_t and labor inputs L_t driven by demographic factors? As shown in panels (3) and (4), demographic factors drive the production inputs into opposite directions under the most of the simulations. Namely,

[25] This is because the real interest rate r_t^B in the model is formulated as the return on capital r_t^K minus the spread Ξ_t, and the size of the spread Ξ_t is held constant by the government policy in 2016 and beyond as assumed in Section 3.2.

labor inputs L_t increase and capital inputs K_t decrease when demographic developments are arrested at a specific period.[26] Given the form of the aggregate production (10), such developments in production inputs lead to a lower real interest rate r_t^B, reflecting the relative scarcity of labor inputs L_t.

It is also seen that the effects on production inputs K_t and L_t of demographic changes in the past are larger than those of demographic changes in the future. For example, in panel (3) and (4), the difference between "Fixed in the 1960s" and "Fixed in 2015" and the series of "Fixed in 2015" indicate that, in terms of the magnitude, the impact on production inputs of demographic changes that took place over the last 50 years is larger than that of changes that will occur in the next 50 years.

5.2 Decomposing Effects of Fertility Rates and Longevity

Now we study in detail how each of the two components of demographic factors, the fertility rate ρ_t and the conditional survival probabilities $\{\psi_{j,t}\}_{j=21}^{100}$ affects the real interest rate r_t^B, based on the type 2 simulations. To this end, we first conduct the following four simulations that differ from each other in terms of assumptions about exogenous variables fed into the model: (i) baseline simulation, (ii) baseline simulation except that the fertility rate ρ_t in the 1970s and beyond is set at the average value of the 1960s,[27] (iii) baseline simulation except that the survival probability rates $\{\psi_{j,t}\}_{j=21}^{100}$ from the 1970s and beyond are set at the average values of the 1960s,[28] and (iv) baseline setting except that both the fertility rate ρ_t and the survival probability rates $\{\psi_{j,t}\}_{j=21}^{100}$ from the 1970s and beyond are set to the average values of the 1960s. We then define the measure of "impact of longevity," "impact of fertility rates," and "impact of demographic factors," of a variable X_t as follows.[29]

[26] In the case of "Fixed in the 2000s," labor inputs L_t increase instead of decrease, compared with the baseline. This reflects the fact that the fertility rate ρ_t during the 2000s was on average about 1% lower than the rate in subsequent years starting in 2016 and ending in 2060.

[27] Similar to the simulations above, this specification indicates that $\{\rho_t\}_{t=1970}^{\infty} = 10^{-1}\sum_{s=0}^{9}\rho_{1960+s}$.

[28] Again, similar to the simulations above, this specification indicates that $\left\{\{\psi_{j,t}\}_{j=21}^{100}\right\}_{t=1970}^{\infty} = 10^{-1}\sum_{s=0}^{9}\{\psi_{j,1960+s}\}_{j=21}^{100}$.

[29] Notice that the sum of the "impact of longevity" and "impact of fertility rates" should quantitatively coincide with "impact of demographic factors." However, because our simulations are based on the shooting algorithm and therefore can capture effects arising from the cross-terms of the fertility rate and longevity, these two figures do not coincide exactly, though the difference is quantitatively minor.

- Impact of longevity $\equiv \{X_t \text{ under simulation } (ii)\} - \{X_t \text{ under simulation } (i)\}$,
- Impact of fertility rates $\equiv \{X_t \text{ under simulation } (iii)\} - \{X_t \text{ under simulation } (i)\}$,
- Impact of demographic factors $\equiv \{X_t \text{ under simulation } (iv)\} - \{X_t \text{ under simulation } (i)\}$.

Figure 11 shows the time paths of impact of fertility rates ρ_t, that of longevity $\{\psi_{j,t}\}_{j=21}^{100}$, and impact of demographic factors for the real interest rate r_t^B, the growth rate of GNP Y_t, production inputs L_t and K_t on a per capita basis, and the saving rate s_t. Except for a few years in the early 1960 or 1970s, in all of the simulation periods, demographic factors depress the real interest rate r_t^B, both through the declining fertility rate and increasing longevity. In addition, the two impact series are quantitatively comparable. As of 2015, for example, half of the decline in the real interest rate r_t^B was attributed to changes in the fertility rate, and the other half was attributed to changes in longevity.

Transmission mechanism of changes in fertility rates and longevity to the real interest rate

Fertility rates ρ_t and longevity $\{\psi_{j,t}\}_{j=21}^{100}$ influence the real interest rate r_t^B through distinct channels. A decline in fertility rates ρ_t primarily lowers labor inputs L_t, as it mechanically reduces the labor force population. Its effect on capital inputs K_t, when measured on a per capita basis, is positive in the early part of the sample period, since a shrinking working-age population reduces the total population P_t. In the later part of the sample period, however, the effect on capital inputs K_t becomes negative. This is because a lower fertility rate ρ_t reduces households' income, as shown in the output decline in panel (2), and leads to lower household savings and lower aggregate capital inputs K_t.

Why does the real interest rate r_t^B fall when both production inputs decline? Admittedly, given the functional form of our production function (10), the real interest rate r_t^B should remain constant as long as both production inputs decline at the same rate. In other words, the decline in the real interest rate r_t^B is brought about by the more rapid decline in labor inputs L_t. As shown in Figure 6, when households retire, they reduce their labor inputs discontinuously to zero at that age, but continue holding capital stock due to their consumption smoothing motive. These households' life-cycle behavior results in the heterogenous dynamics of the two production inputs and dampens the real interest rate r_t^B. The downward pressure on the real interest rate r_t^B is pronounced when the fertility rate ρ_t, itself declines, since the decline causes the growth rate of the working-age population to fall, giving a rise to persistently even larger discontinuity in labor supply.

The above mechanism is confirmed in panel (1) of Figure 12 in which we decompose the working-age population growth rate into the contributions of entering and exiting households. We also plot the "impact of fertility rates" on the real interest rate r_t^B in the same panel. While the declining time path of the "impact of fertility rates" roughly tracks the time path of the entire working-age population growth rate in the long-run, the downward pressure, expressed by the path, is clearly pronounced at periods when the number of exiting households prominently increases. For example, at the time when the two baby boomers exit from the labor market during the early 2010s and 2040s, the magnitude of the contribution of exiting households increases, while the "impact of fertility rates" contemporaneously drops markedly, suggesting that the life-cycle mechanism described above operates and reduces the real interest rate r_t^B in these periods.[30]

In contrast, longer longevity decreases labor inputs L_t on a per capita basis, due to the denominator effect as P_t rises, but it increases capital inputs K_t even on a per capita basis. As pointed out in existing studies, such as Chen et al. (2007), Braun et al. (2009), and MOS (2016), when facing increased longevity, households have an added incentive to work and save so as to insure themselves against a longer life after retirement. To see this, we compute the life-cycle profiles of production inputs when longevity is fixed and show them in the bottom panels (2) and (3) of Figure 12. Compared with the alternative case, households work greater and consume their savings more slowly in particular after their retirement. Because of these responses by households, capital K_t accumulates at a larger

[30] It is also notable that, as shown in panel (5) of Figure 11, the saving rate s_t falls as a result of a decline in the fertility rate ρ_t, since the relative size of dis-savers (or equivalently retirees) increases compared with savers (or equivalently incumbent workers). A lower saving rate in principle helps reduce capital inputs K_t and exerts upward pressure on the real interest rate r_t^B. Quantitatively, however, the downward pressure from the declining labor inputs L_t dominates the effect stemming from a low saving rate s_t, reducing the real interest rate r_t^B.

amount relative to labor inputs L_t, and exerts downward pressure on the real interest rate r_t^B.

Effects of future demographic changes

Does the picture of the impact of demographic factors still look the same when we arrest the demographic changes in other periods? In fact, the downward pressure on the real interest rate r_t^B diminishes when we consider the effect of future demographic changes. Figure 13 shows the result of the same set of simulations when the demographic factors, the fertility rate ρ_t and the conditional survival probabilities $\{\psi_{j,t}\}_{j=21}^{100}$, are arrested in 2015. Demographic factors continue to depress the real interest rate r_t^B, mainly through the impact of longevity, but their quantitative impacts are minor compared with what we see in Figure 11. As of 2060, the impact is about -90 basis points, and the bulk of the decline is caused by more capital inputs K_t rather than less labor inputs L_t. The key reason is the size of demographic changes. As shown in Table 3, predicted changes in demographic factors are minor, and this is straightforwardly translated to the minimal impact of demographic factors in the years ahead.

5.3 Sensitivity Analysis: Social Security System

One key difference between existing studies, including Gagnon et al. (2016) and Eggertson et al. (2017), and ours, is that our model explicitly incorporates the tax and social security systems that are both calibrated to the actual Japan economy. Because households' saving behavior can easily change depending on the nature of these institutional environments, particularly that of the social security system, this section explores whether the responses of households and firms to demographic changes are magnified or lessened by the presence of the social security system.

To this end, we construct an alternative model where the social security system is absent, and compare the model-generated endogenous variables based on the alternative model with those based on the baseline model. This alternative model is equivalent to the baseline model except that the public pension benefits, $pb_{j,t}$, and the contribution rates for public pension $\tau_{s,t}$ are set at zero for all j and t. Using this model, we conduct the simulations (i) and (iv) in the previous section and compute the "impact of demographic factors" for selected endogenous variables. Notice that, similar to the discussion above, the "impact of demographic factors" in this case should capture the effect of demographic changes from the 1970s and beyond in a hypothetical economy where the social security system is absent. We then compare the "impact of demographic factors" generated from the alternative model with that generated from the baseline model. Conceptually, the difference between the two should account for the quantitative impact of the social security system on the responsiveness of macroeconomic variables to demographic changes.

Panel (1) of Figure 14 plots the two difference series for the real interest rate r_t^B. It is seen that the depressing effect of demographic factors on the rate are also present when the social security system is absent from the economy, indicating that our simulation results based on the baseline model are qualitatively robust even when the social security system is absent. It is also notable, however, that in an economy without the social security system the depressing effect is meaningfully bigger. In particular, demographic factors continue reducing the real interest rate r_t^B in the years ahead.

The rest of the panels of the figure show why this happens. Panel (2) and (3) show the series of the "impact of demographic factors" for the production inputs, and panel (4) shows the life-cycle profile of capital stock holdings in an economy with and without the social security system. In the former economy, labor inputs L_t decline less and capital inputs K_t increase more in response to population aging. This is because households work longer and save more, accumulating larger assets, so as to smooth consumption over the life-time, in particular, consumption at ages after retirement. By contrast, in the baseline economy where the social security system is present, households save less because they know that a portion of their consumption expenditures is financed by the social security benefits transferred by the government. Consequently, in this economy, accumulation of capital inputs K_t is smaller and the decline in the real interest rate r_t^B becomes moderate.

The above exercise indicates that the nature of the social security system may be important in shaping the dynamics of the real interest rate r_t^B, at least in a country where population aging deepens. Furthermore, under the premise that the social security system generally eases the precautionary motives of households' saving and leads to more capital inputs, like it does in our model, the results of the exercise suggest that measuring the effects of population aging based on a model simulation without appropriate consideration of the social security system of the country of interest may overestimate its negative impact on real interest rates.

5.4 Sensitivity Analysis based on Alternative TFP Assumptions

The analysis above has indicated that the demographic changes over the last 50 years have a material impact on the real interest rate r_t^B and that the future changes will have a minor effect on the rate in the years ahead. In this section, we check the robustness of these results by conducting a series of simulations with alternative assumptions about the time path of TFP growth rates. One motivation for these simulations rests upon the fact that the last 50 years are characterized not only as the era of material changes in the demographic structure, but also as the era of exceptionally high TFP growth. We therefore ask if the high TFP growth witnessed in the past has influenced our estimates of the contribution of demographic changes discussed so far, by separating the effects of TFP growth.

To this end, we compute the "impact of demographic factors" on the real interest rate r_t^B, arresting demographic structures in the 1960s, under the baseline scenario and under five hypothetical scenarios with different assumptions about developments in TFP growth, $\{A_t/A_{t-1} = g_t^{1-\alpha}\}_{t=1960}^{\infty} = -1\%,\ 1\%,\ 2\%,\ 3\%,$ and 5%. Note that in the hypothetical simulations, all of the other exogenous variables are set the same as those under the baseline scenario, and the difference between the two series of the "impact of demographic factors" should account for the quantitative impact of the TFP growth rate on the responsiveness of macroeconomic variables to demographic changes.

Figure 15 shows the "impact of demographic factors" computed under the five different assumptions about time path of TFP, together with that under the baseline scenario. Regarding this set of simulations, it is important to isolate effects arising from changes in production inputs L_t and K_t and those arising from the level of TFP A_t. This is because our measures of the "impact of demographic factors" on the real interest rate r_t^B is calculated by the difference, rather than the ratio, in the real interest rate r_t^B between the two different scenarios, a larger TFP A_t mechanically affects the "impact of demographic factors," even when production inputs are constant.

First we examine how TFP growth affects the responsiveness of production inputs to population aging by computing the "impact of demographic factors," of three variables: Marginal productivity of capital inputs $\alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}$, labor inputs L_t, and capital inputs K_t. When computing these variables, in order to control for the effect of TFP A_t itself, we take the percent deviation between distinct simulations. Panels (2), (3), and (4) show that the responsiveness of the three variables is mitigated when TFP grows at a rapid pace. A decline in labor inputs L_t and an increase in capital inputs K_t are both contained the most when TFP grows at 5%, leading to a modest decline in the return on capital $\alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}$. As shown in panel (1), however, when measured by the difference in the real interest rate r_t^B, such a moderating effect is dominated by the direct effect of TFP level A_t. That is, the impact of demographic factors is the smallest when TFP grows at the slowest pace.

Aside from the working mechanisms, two observations are noteworthy from panel (1) of Figure 15. The first is that TFP growth has not much affected the estimated size of the contribution of demographic factors until recently. That is to say, up to the early 2000, the six lines almost overlap with each other, suggesting that what we have measured as the contribution of the demographic factors so far is not likely to be influenced by TFP growth rates and therefore accurately captures the size and timing of the contribution. The second is that the future effect of TFP on the responsiveness of the real interest rate r_t^B may not be negligible. For instance, as of 2060, the impact of the demographic factors with the highest TFP growth rate is about -320 basis points, while that for the lowest is about -240 basis points. It is also important to note, however, that the general pattern of the time path, namely a bottoming-out of the declining trend in the near future, is obtained so long as the TFP growth rate is contained around 3%.

6 Real Interest Rates in Other Economies

We have shown above that the changes in demographic factors over the last 50 years have induced a gradual decline in the real interest rate r_t^B, which amounted to about 270 basis points as of 2015 in Japan's economy. We have also shown that in the next 50 years these factors will work in such a way that the real interest rate r_t^B will remain more or less at the current level. In this section, we study if these observations hold in other economies.

Demographic landscapes in the U.S., G7, and selected OECD countries

We start by describing demographic landscapes in the U.S., G7 countries, and 15 selected OECD countries,[31] as shown in Figure 16. Note that for the last two country groups, the U.S. and Japan are excluded for comparison purposes. There are some features common with Japan in terms of developments in demographic landscapes. For example, in the U.S., longevity has increased by 6 years over the last 50 years, while it is predicted to increase by only 4 years in the next 50 years. The working-age population has also shown a secular decline over the years. It has grown on average at 1.2% in the years so far, and it is predicted to grow at about 0.3%, in the years ahead. The growth rate decline will bottom out in 2025, with a return to an increase in the rate, albeit gradually, in the following years. Similar observations are made for the G7 and 15 OECD countries though the timing of the bottoming out of the working age population growth rate takes place around 2040 and the bottom is a negative value in these countries.

Admittedly, there are stark differences between these country groups and Japan and within the country groups themselves. Considering increases in longevity, progress has been the fastest in Japan. Japan already achieved the average life expectancy of 80 in the year 2000, whereas the figures in the other country groups were below 80 in that year. In the long run, however, the average life expectancy in the G7 and 15 OECD countries will rise to as high as that of Japan. The exception is the U.S., where even in 2060, average life expectancy will not reach the level of Japan or the other country groups. Considering

[31] The 15 countries include Austria, Australia, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Netherlands, New Zealand, Portugal, Spain, Switzerland, and the U.K.

declines in fertility rates, the decline in the working-age population from 1960 to 2060 takes place again at the fastest rate in Japan. On average, the working-age population growth rate in Japan will drop by 1.6% points from the last 50 years to the next 50 years. By contrast, the counterpart figures are 0.9% points for the U.S., 0.7% points for the G7 countries, and 0.8% points for 15 OECD countries. It is also notable that while the growth rates of working-age population exhibit a common declining trend across countries, the rates in the above country groups will revert back to positive values or values close to zero, which also contrasts sharply with the case of Japan.

Effects of demographic changes on real interest rates

In order to see whether the effects of these similarities dominate those of the differences or vice versa, we conduct simulation exercises using the demographic data of the three country groups. We conduct two types of simulation exercises similar to those shown in Section 5. Namely, we compute the "impact of fertility rates," the "impact of longevity," and the "impact of demographic factors," for the case where developments in demographic variables are arrested in the 1960s and in 2015 for these country groups.[32]

Figures 17 and 18 show the simulation results. In Figure 17, the left panels show the contribution to the developments in the real interest rate r_t^B of changes in demographic factors from the 1960s and beyond, while the right panel shows the contribution of the changes from 2015 and beyond, for the U.S., G7, and selected OECD countries. Figure 18 shows the impact of demographic factors on labor inputs L_t and capital inputs K_t, using the simulation results when the demographic landscapes are arrested in the 1960s.

There are three observations common to all the countries. First, the demographic changes have exerted and will continue exerting downward pressure on the real interest rate r_t^B. As shown in Figure 18, this result is commonly brought about by the fact that aggregate labor inputs L_t decline and aggregate capital inputs increase, in net, due to

[32] Throughout the simulations conducted in this section, we assume that there are no inflows of immigrants and guest workers that are aged above 22 in each of the country groups considered. Figure 19 below shows the working-age population growth rates and dependency ratio that we construct under this assumption, and what are projected by the respective institution including the U.S. For both series, they comove closely over the sample period. See Table 4 for developments in demographic factors under hypothetical scenarios.

the demographic factors in each of the country groups. Second, it is predicted that the additional downward pressure of the demographic factors on the real interest rate r_t^B will die out sooner or later, and the time path of the rate will stabilize. Admittedly, there are some differences across country groups in this respect. As shown in Figure 17, the declining phase in the U.S. seems to have ended already, and it is predicted that the demographic factors will not increase or decrease the rate from the current level. By contrast, the declining phase in G7 and OECD countries will last for the next twenty years and the rate will stabilize only in subsequent years. One reason for this, as illustrated in Figure 16, is that in the latter country groups, working-age population growth will continue to decline and average life-expectancy will grow markedly from now on. Third, the quantitative impact on the real interest rate of the demographic changes that took place over the last 50 years is far greater, compared with that of the demographic changes that will take place in the next 50 years. When measured by the impact on the real interest rate as of 2015, the past demographic changes caused a decline in the rates by a range of 160 to 180 basis points, while the expected future demographic changes will cause a decline in the rates by a range of 30 to 80 basis points. These observations imply that, compared to what has happened so far, any prospective decline in the real interest rates r_t^B will be limited.

7 Conclusion

In this paper, we have quantitatively studied the impacts of population aging on the real interest rate mainly in Japan and also in other countries, using an OG model. In short, our study has confirmed that demographic landscapes have indeed been important elements in shaping the real interest rate movements so far. However, our study also predicts that the role of the factors will become less evident in the years ahead and that other things being equal, the demographic factors alone will not depress or raise the real interest rate substantially from the current level. The basis for this conclusion is the combination of the nature of foreseen demographic changes going forward, in particular in terms of the future pace and size of demographic changes, and the theoretical prediction of our OG model that the effects of past demographic changes are long-lived. Based on our baseline simulation for Japan, while demographic changes over the last 50 years caused a decline of about 270 basis points in the real interest rate as of 2015, they will cause at most a 90 basis points decline in the next 50 years. Qualitatively similar results are obtained for the U.S. and other selected OECD countries.

There are three caveats regarding our analysis. First, our paper focuses on measuring the contributions of demographic factors to variations in the real interest rate, and it is silent about the relative importance of demographic factors compared with other factors. Similarly, it is also beyond the scope of our analysis to examine how other factors, including the TFP growth rate, will evolve in the years ahead and affect the real interest rate.[33] Second, our OG model implicitly assumes that households in the economy have no access to foreign asset markets and that domestic saving and domestic investment coincide. As studied in existing work including MOS (2016), however, open-economy considerations can alter quantitatively how factor prices and allocation react to demographic changes, in particular if capital transactions between countries with different demographic profiles, unlike those considered in this paper, become dominant globally. Third, our analysis assumes that the spread between the return on capital and the real interest rate is not susceptible to changes in demographic factors, and studies essentially how demographic factors affect the return on capital. There is in recent years, however, an increasing body of research on the determinants of the spread associated with safe assets, including government bonds. These areas of interest are left for future research.

[33] In Appendix Figure 1 and 2, we show that prospects for the real interest rate in the future can be importantly affected if prospects for exogenous variables are changed.

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A Appendix: Out-of-sample simulation: alternative assumptions about TFP and retiring age

The model's projections for future macroeconomic variables are not independent from the assumed time path of exogenous variables in the years ahead. In fact, these time paths change, depending in particular on the assumptions made regarding the prospects for TFP growth rates. To show this, we study how time paths of macroeconomic variables are altered when we change assumptions about TFP growth rates $\{g_t^{1-\alpha}\}_{t=2016}^{\infty}$. Appendix Figure 1 shows the case when TFP grows at -1%, 0%, 2%, and 3% in 2016 and beyond.[34] As standard theory predicts, a higher (lower) TFP growth rate results in a higher (lower) return on capital r_t^K and real interest rates r_t^B, compared with the baseline scenario where TFP grows at 1.1%. This is because the productivity of capital inputs is enhanced (dampened) thanks to changes in the technology level A_t and labor inputs L_t. As of 2020, for example, the level of real interest rates r_t^B under assumptions about the growth rate of TFP at -1%, 0%, 2%, and 3% are -0.7%, -0.1%, 1.2%, and 1.8%, respectively, while the level of the real interest r_t^B rate as of 2020 is 0.6% under the baseline scenario. The effects of TFP growth are straightforwardly translated to the growth rate of GNP and that of production inputs. With higher (lower) TFP growth rates, higher (lower) returns to production inputs w_t and r_t^K attract more (less) production inputs, boosting (dampening) the economic growth.

In addition, for the purpose of the comparison, we simulate cases where the retirement age is shifted from the age 65 to ages above, namely 66, 68, and 70.[35] These simulations can also be interpreted as a robustness check analysis on our assumption that all workers exit from the labor force when they are 66. The results are shown in Appendix Figure 2. Qualitatively, such an institutional change broadens the scope of the working age popula-

[34] In this simulation, we assume that all of the exogenous variables including TFP growth rates from 1960 to 2015 $\{g_t\}_{t=1960}^{2015}$ and demographic factors are the same as those assumed in the baseline simulation.

[35] In these simulations, we assume that in each year from 2016, the retirement age is shifted by one year until it reaches the target age, such as 66. Once the retirement age reaches the target age, it stays at that age in the subsequent years.

tion and should increase labor inputs L_t. It is seen, however, that the time path of real interest rates r_t^B, growth rates of production inputs K_t and L_t and GNP Y_t are barely changed quantitatively by these institutional changes. In would appear that the impact of these changes is minor compared with that of TFP.

This analysis indicates that the projected time path of the real interest rates and GNP shown in Figure 9 is altered markedly by assumptions about exogenous variables. This does not imply, however, that the manner in which demographic changes are translated to real interest rates is also altered by changes in the assumptions. In fact, as we show in Section 5 the nature of contributions of demographic factors to real interest rates is not severely affected by the choice about the prospects for TFP growth rates, and the general implications obtained based on the baseline assumptions still hold under alternative assumptions.

Table 1: Structural Parameters

α	β	ν	$\bar{b}$	ϕ
0.362	0.975	0.5	1.1	18.33

Table 2: Steady-State Values of Key Variables

G/Y	Ξ	τ/Y	τ_c	τ_h	τ_k	κ_s	κ_m	$g^{(1-\alpha)}$	ρ
0.120	0.043	0.012	0.10	0.150	0.239	0.336	0.323	1.011	0.00

Table 3: Demographic Variables under the Hypothetical Simulations for Japan

Simulations	Average life expectancy (years)	Working-age population growth rate (%)			
		t = 1980s	t = 2000s	t = 2020s	t = 2040s
Fixed in the 1960s	72.4	3.67	4.51	4.61	4.61
Fixed in the 1980s	78.4	0.81	0.72	1.51	1.52
Fixed in the 2000s	82.5	0.81	-0.27	-1.12	-2.56
Fixed in 2015	85.1	0.81	-0.27	-0.67	-1.27
Baseline	—	0.81	-0.27	-0.66	-1.47

Note: Values of average life expectancy and the working-age population growth rate are those computed under a hypothetical assumption about developments in fertility rates and longevity. They are used in simulations whose results are shown from Figure 10 to 15.

Table 4: Demographic Variables under the Hypothetical Simulations in Other Countries

(1) US

Simulations	Average life expectancy (years)	Working-age population growth rate (%) t = 1980s	t = 2000s	t = 2020s	t = 2040s
Fixed in the 1960s	71.8	3.43	5.15	5.24	5.24
Fixed in 2015	77.9	1.36	1.09	-0.03	-0.25
Baseline	—	1.36	1.09	0.14	0.40

(2) G7 (less Japan and US)

Simulations	Average life expectancy (years)	Working-age population growth rate (%) t = 1980s	t = 2000s	t = 2020s	t = 2040s
Fixed in the 1960s	72.4	0.68	0.57	0.55	0.55
Fixed in 2015	81.1	0.99	0.13	-0.35	-0.44
Baseline	—	0.99	0.13	-0.33	-0.13

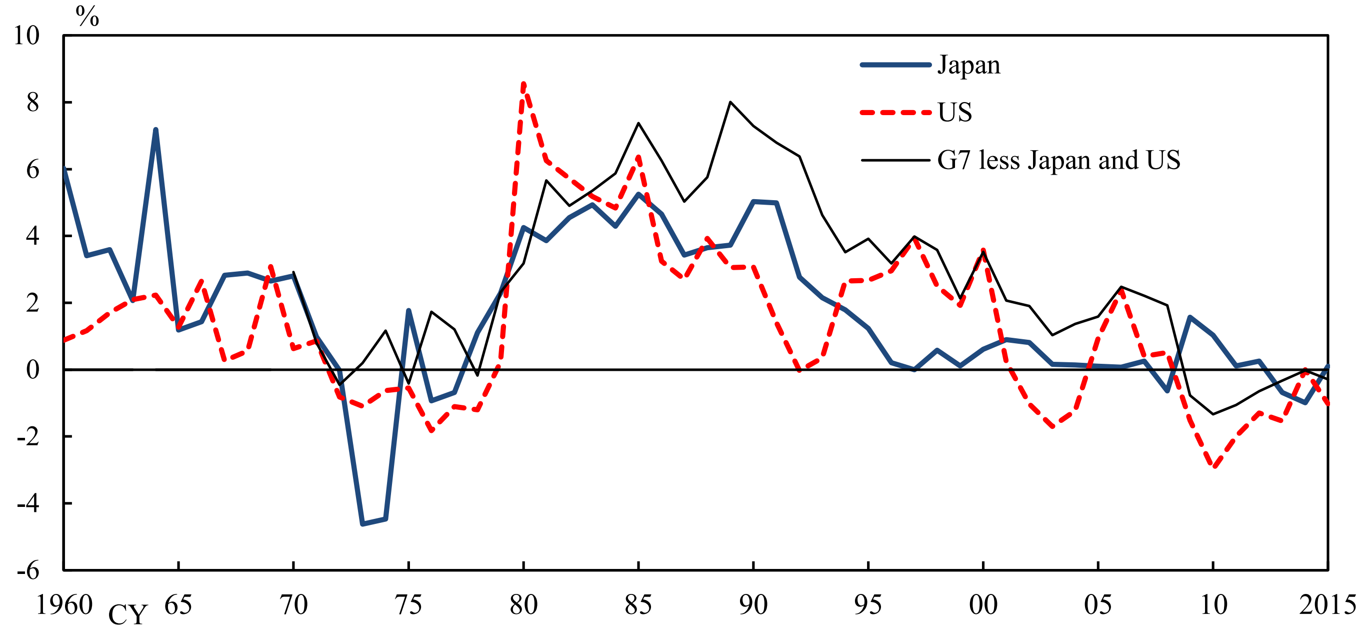
(3) OECD 15 countries (less Japan and US)

Simulations	Average life expectancy (years)	Working-age population growth rate (%) t = 1980s	t = 2000s	t = 2020s	t = 2040s
Fixed in the 1960s	72.5	0.85	0.85	0.83	0.83
Fixed in 2015	81.3	1.07	0.27	-0.29	-0.44
Baseline	—	1.07	0.27	-0.26	-0.16

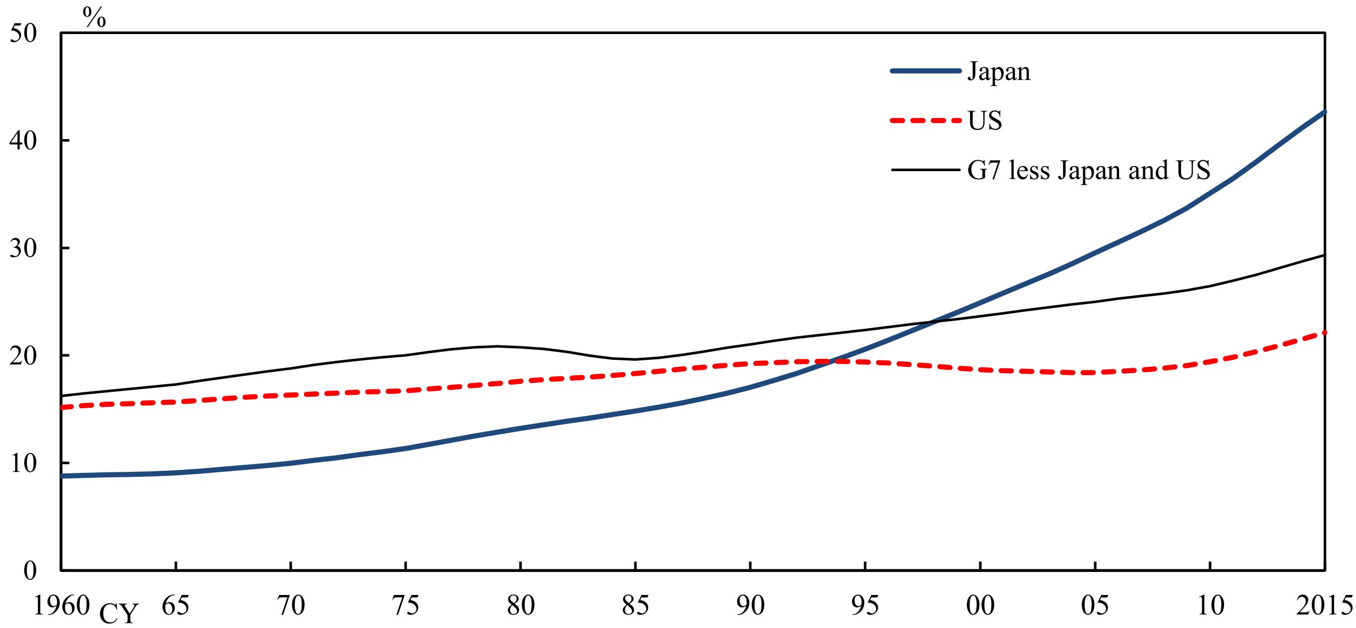
Note: Values of average life expectancy and the working-age population growth rate are those computed under a hypothetical assumption about developments in fertility rates and longevity. They are used in simulations whose results are shown from Figure 17 to 18.

Figure 1: Real Interest Rates and Population Aging

(1) Real interest rates in G7 countries



(2) Dependency ratios in G7 countries



Notes: 1. The real interest rate of five countries in G7 is the weighted average of real interest rates of these countries using the GDP of each of these countries as the weight.

2. The real interest rates are calculated by the nominal interest rates minus the CPI inflation rates.

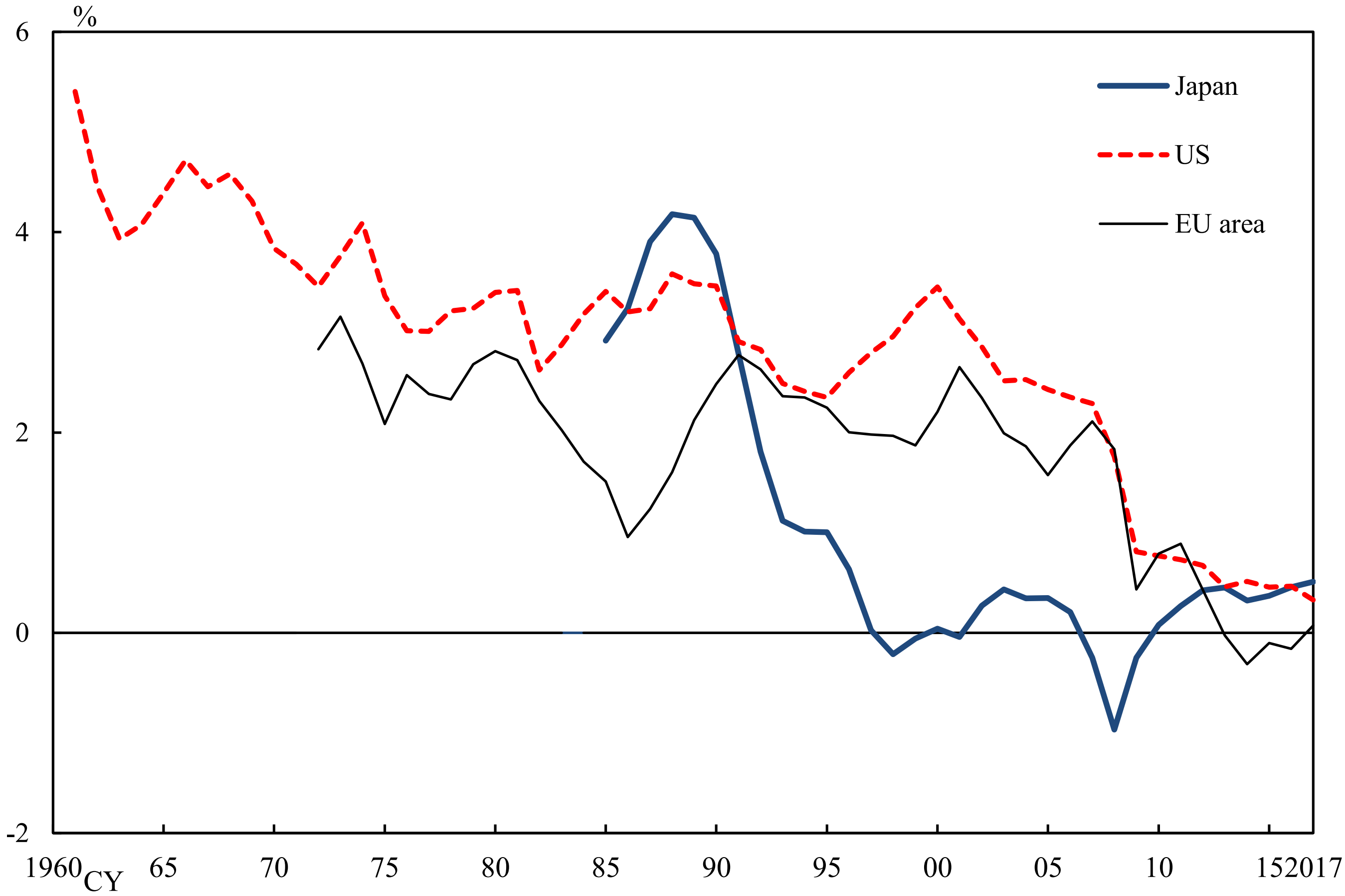
3. Real interest rate of Japan is computed following Kamada (2009).

4. Dependency ratios is defined as the 'old-age' dependency ratio, which is the number of individuals aged above 64 divided by the number of individuals aged 15 to 64.

Sources: IMF, “International Financial Statistics”; World Bank, “World Development Indicators”; UN “World Population Prospects.”

Figure 2: Natural Rate of Interest in Selected Countries

Natural rate of interests estimated following the method of Laubach and Williams (2003)

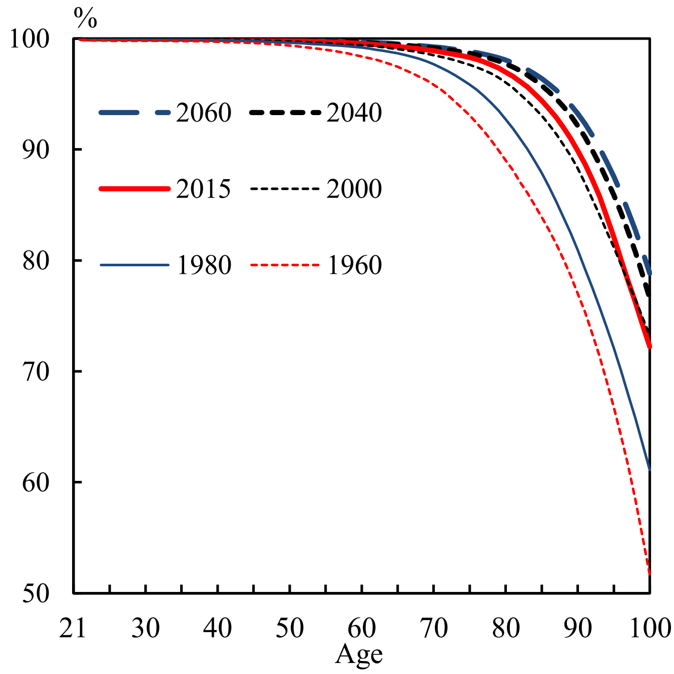


Note: The data for 2017 is the value as of the first half of 2017.

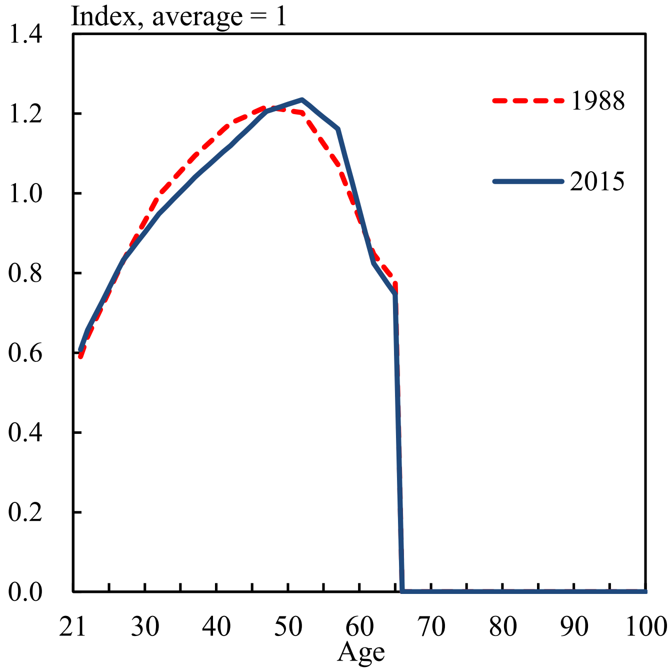
Sources: Holston et al. (2017); Fujiwara et al. (2016).

Figure 3: Age-Specific Exogenous Variables

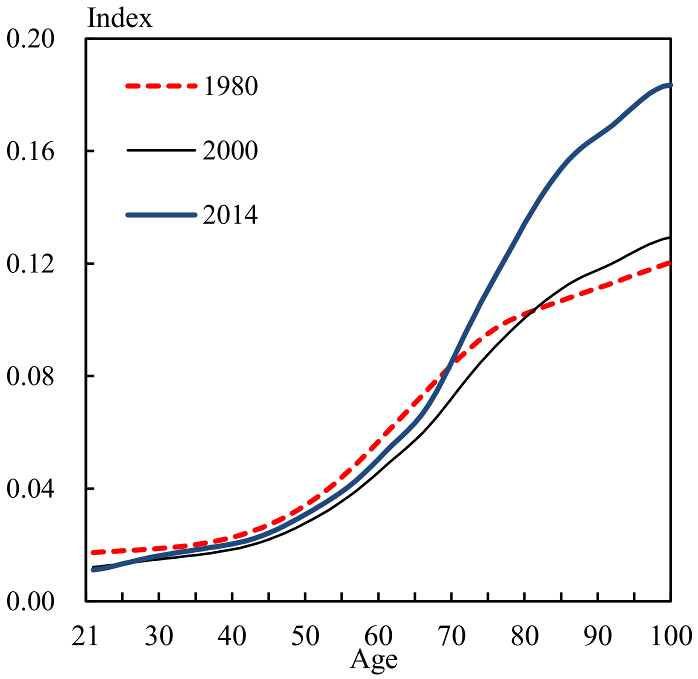
(1) Conditional survival probability



(2) Labor efficiency



(3) Medical costs



Notes: 1. Conditional survival probability is defined as the probability that a household survives in the next year.

2. Labor efficiencies from 1989 to 2014 are interpolated using the values of 1988 and 2015. The values before 1988 are set to the value as of 1988. The values after 2015 are set to the value as of 2015.

3. Medical costs are detrended by the current level of TFP

Sources: National Institute of Population and Social Security Research, “Population Projection for Japan”; Ministry of Health, Labour and Welfare, “Basic Survey of Wage Structure,” “Life Tables,” “National Medical Expenditure.”

Figure 4: Exogenous Variables (1)

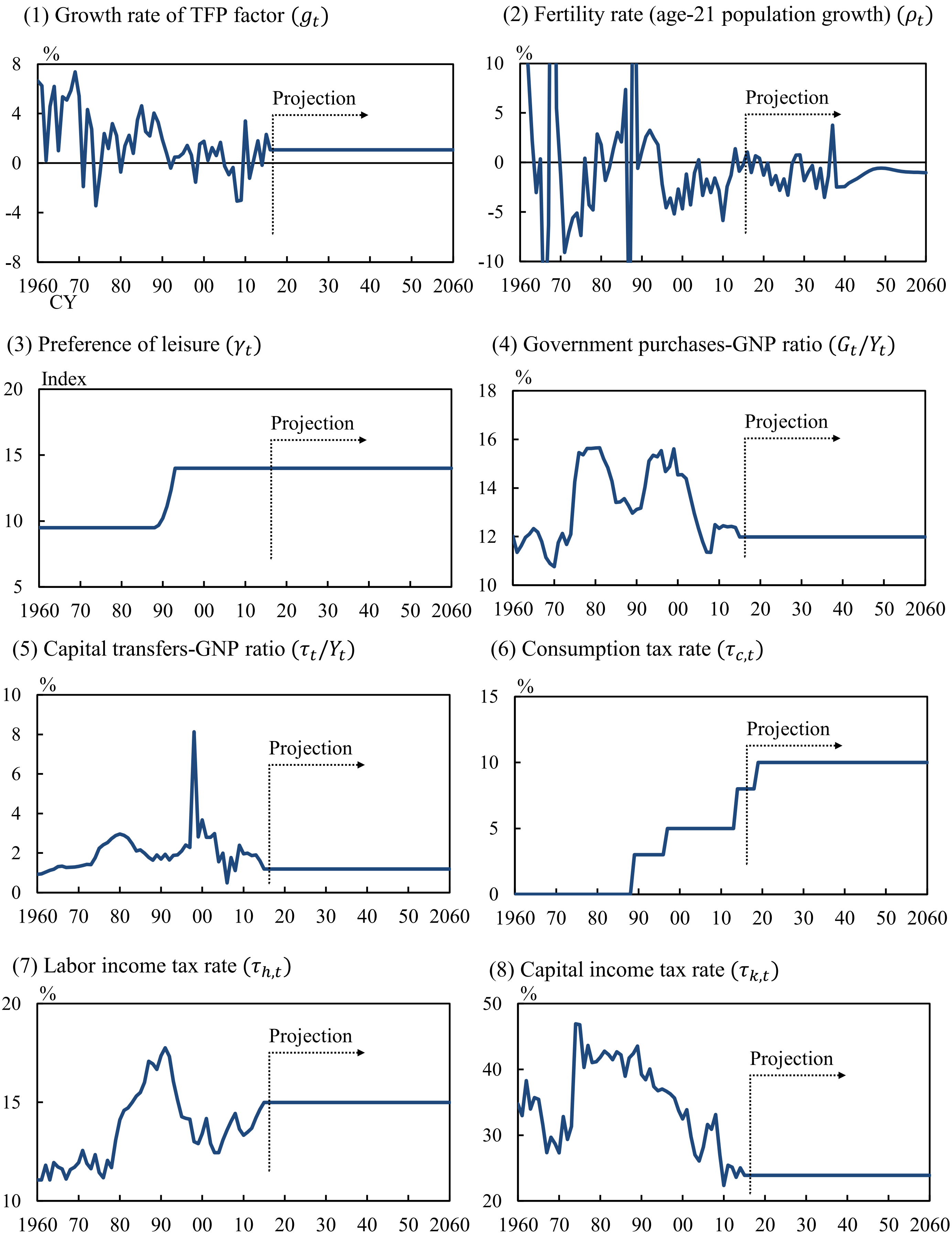
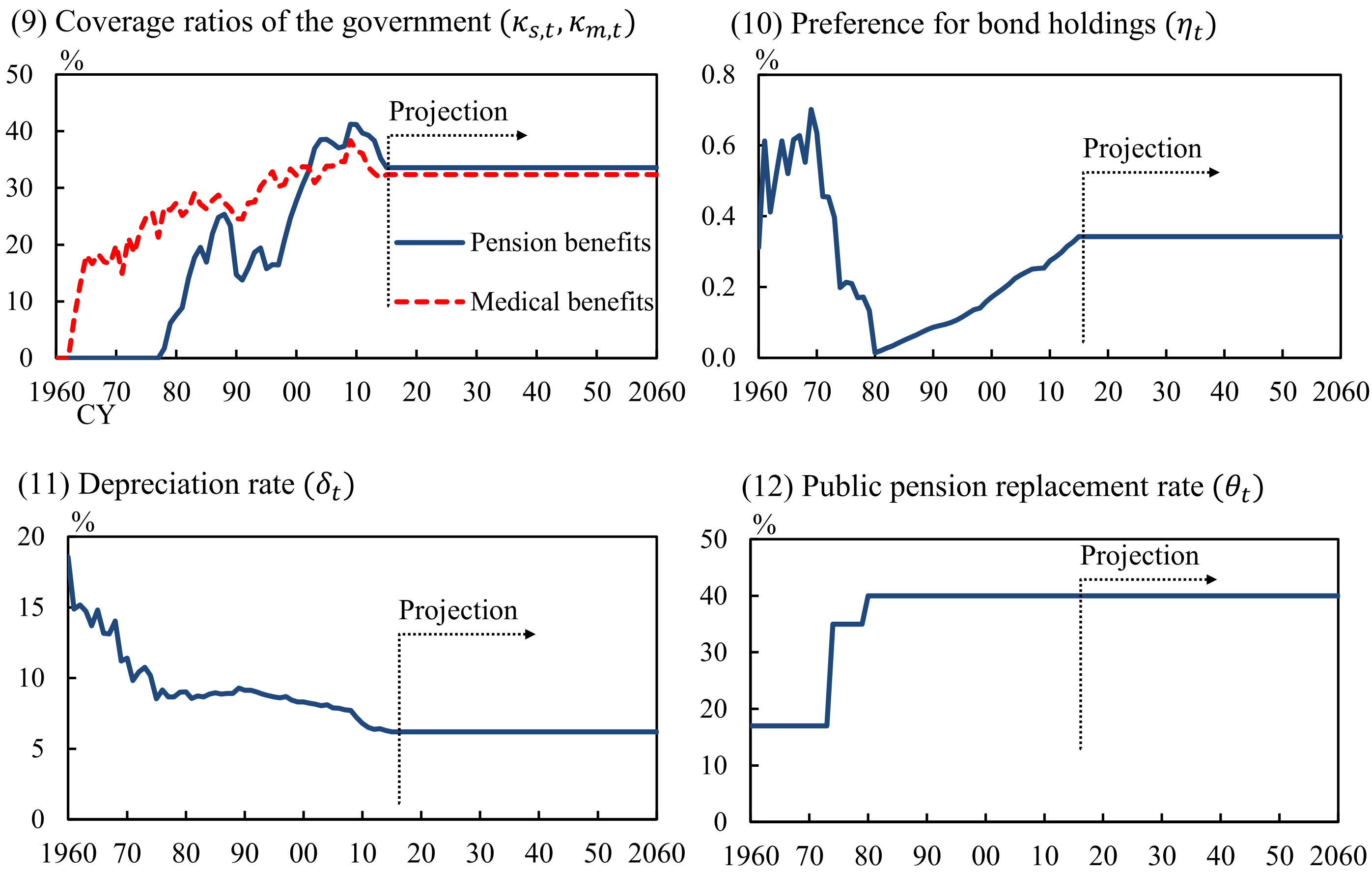


Figure 4: Exogenous Variables (2)

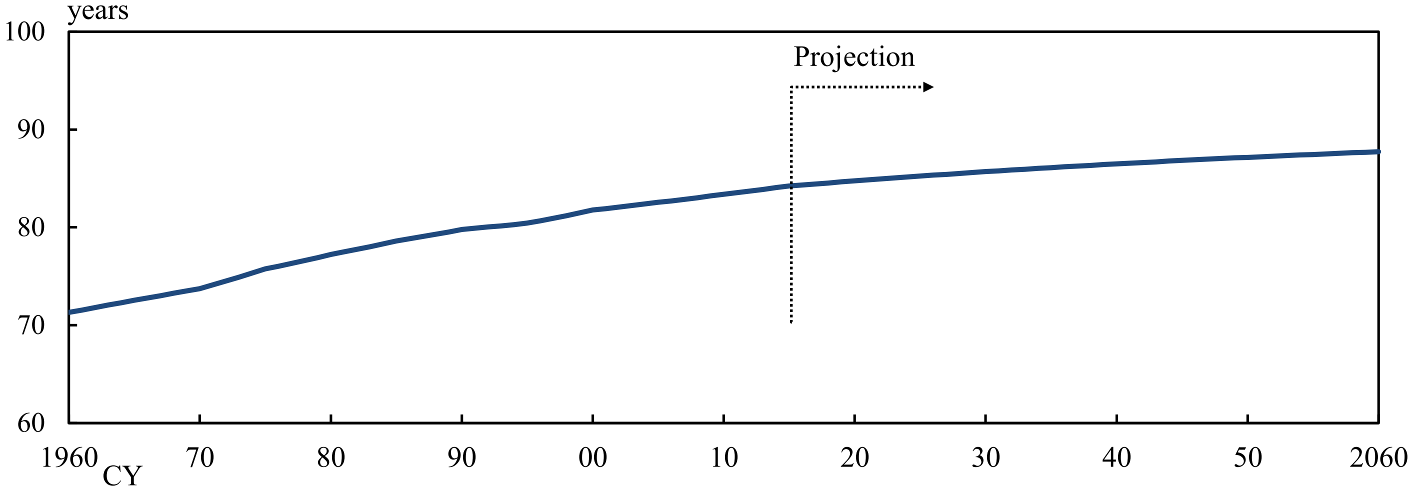


Note: The notation in brackets is the respective exogenous variable used in the main text.

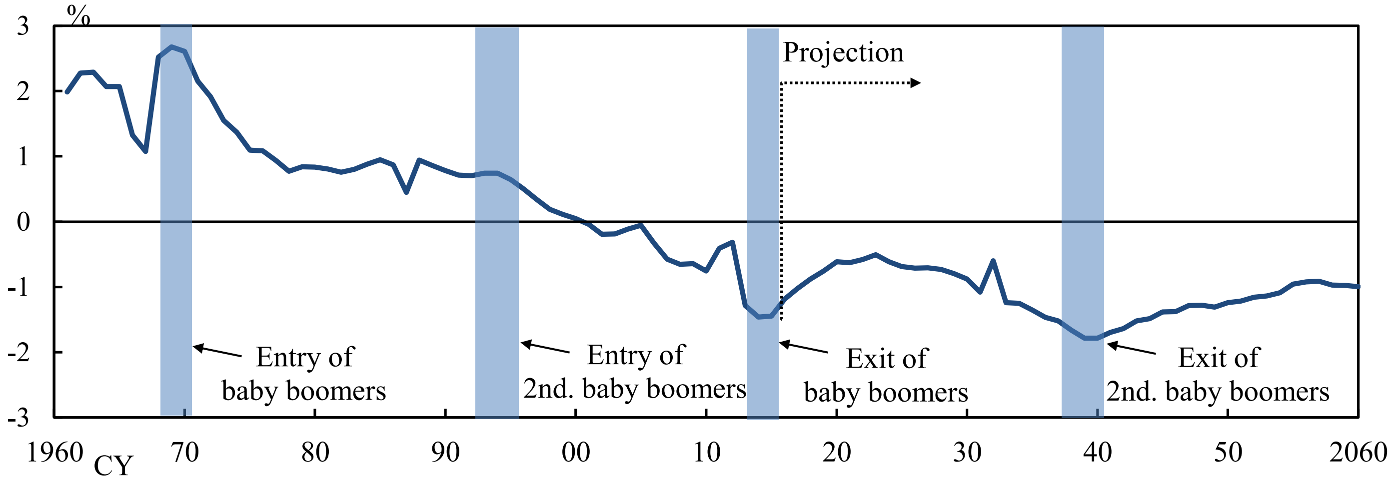
Sources: Cabinet Office, "National Accounts"; Ministry of Health, Labour and Welfare, "Population Census of Japan."

Figure 5: Demographic Landscape in Japan

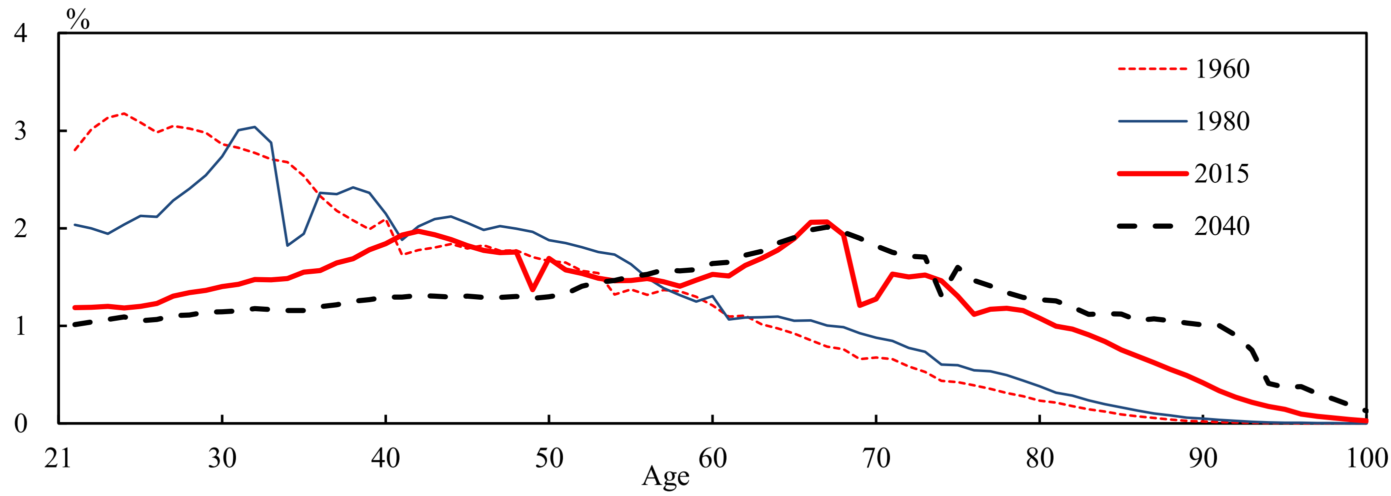
(1) Average life expectancy



(2) Working-age population growth rate



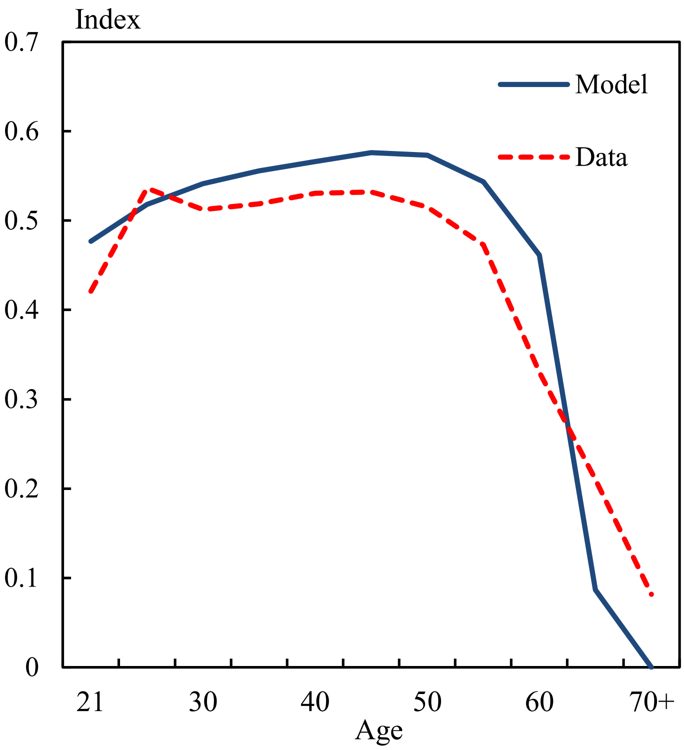
(3) Distribution of population by age



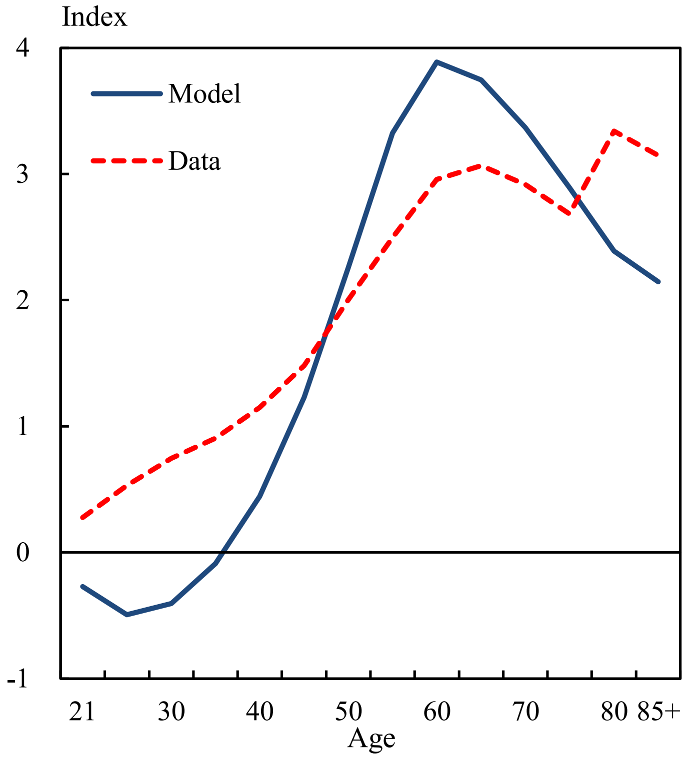
Sources: Ministry of Health, Labour and Welfare, “Population Census of Japan,” “Life Tables”; National Institute of Population and Social Security Research, “Population Projections for Japan.”

Figure 6: Model's Performance on Life-Cycle Profiles

(1) Labor inputs



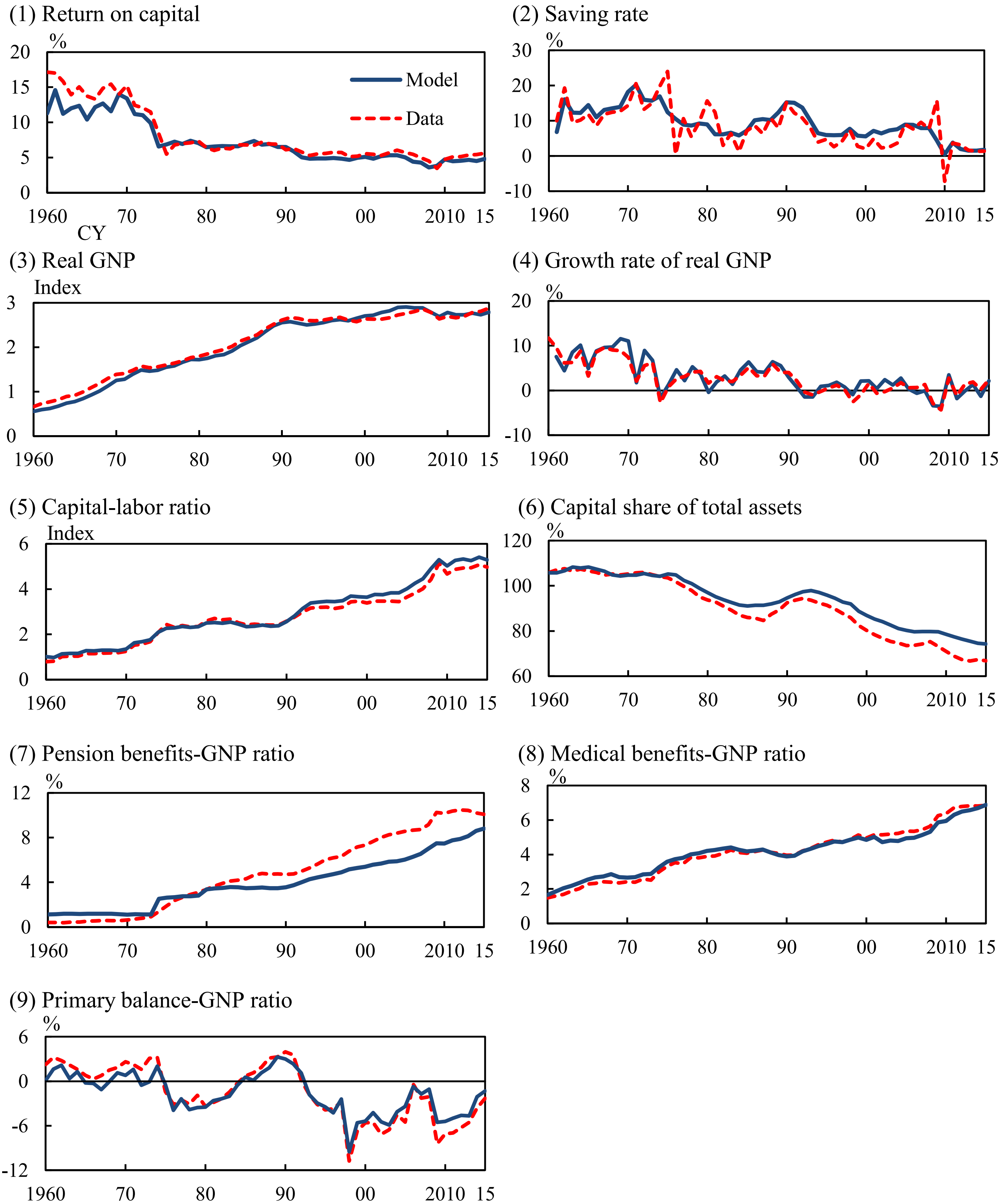
(2) Capital stock holdings



Note: The value of capital stock holdings is detrended by the current level of TFP and the population.

Sources: Ministry of Health, Labour and Welfare, "Basic Survey of Wage Structure"; Ministry of Internal Affairs and Communications, "Family Income and Expenditure Survey."

Figure 7: Model's In-sample Performance



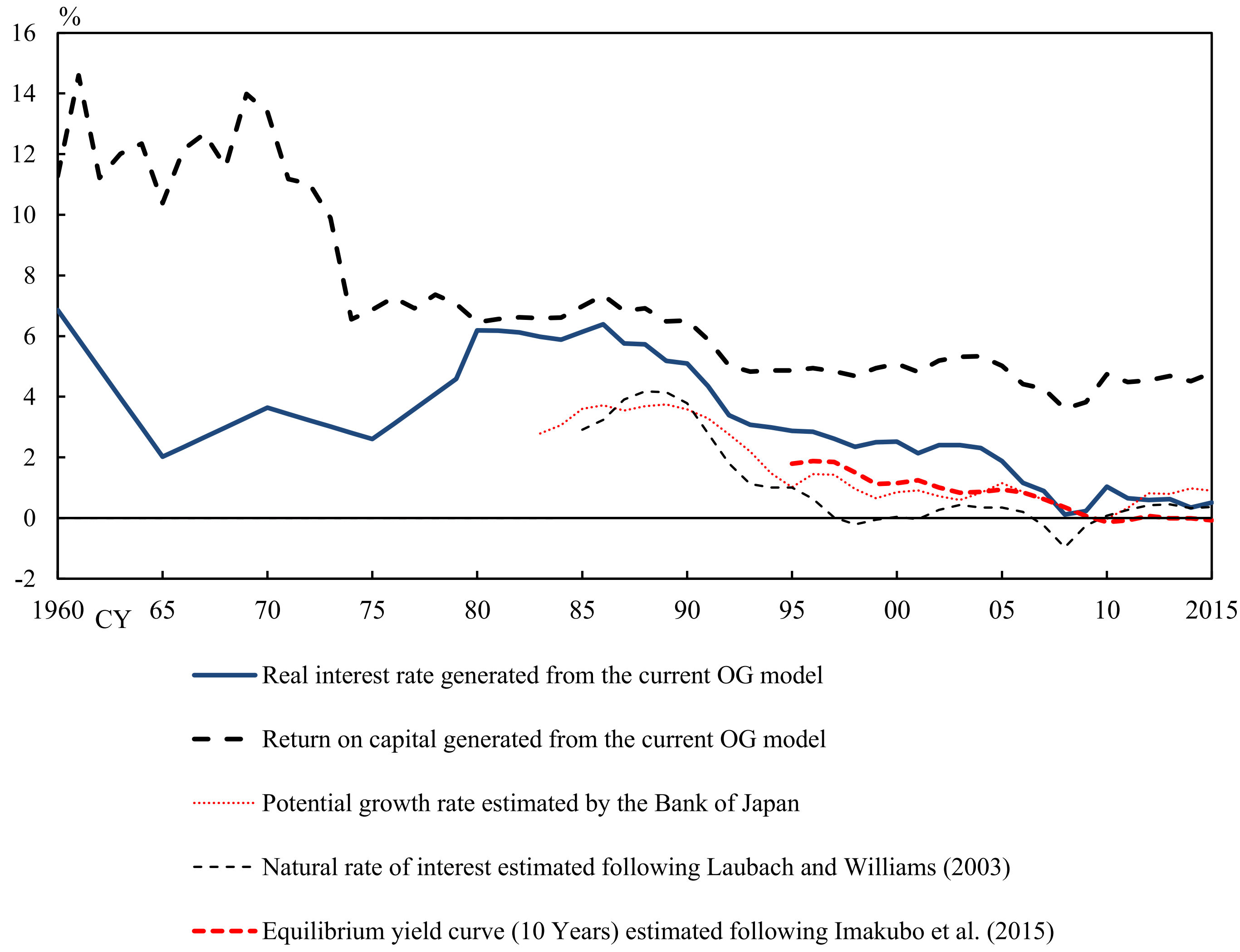
Notes: 1. The value of real GNP and capital stock holdings are detrended by the current level of TFP and the population.

2. The series (1) are on an after-tax basis.

3. The series (3) and (4) are on a per capita basis.

Figure 8: In-sample Simulation of the Real Interest Rate

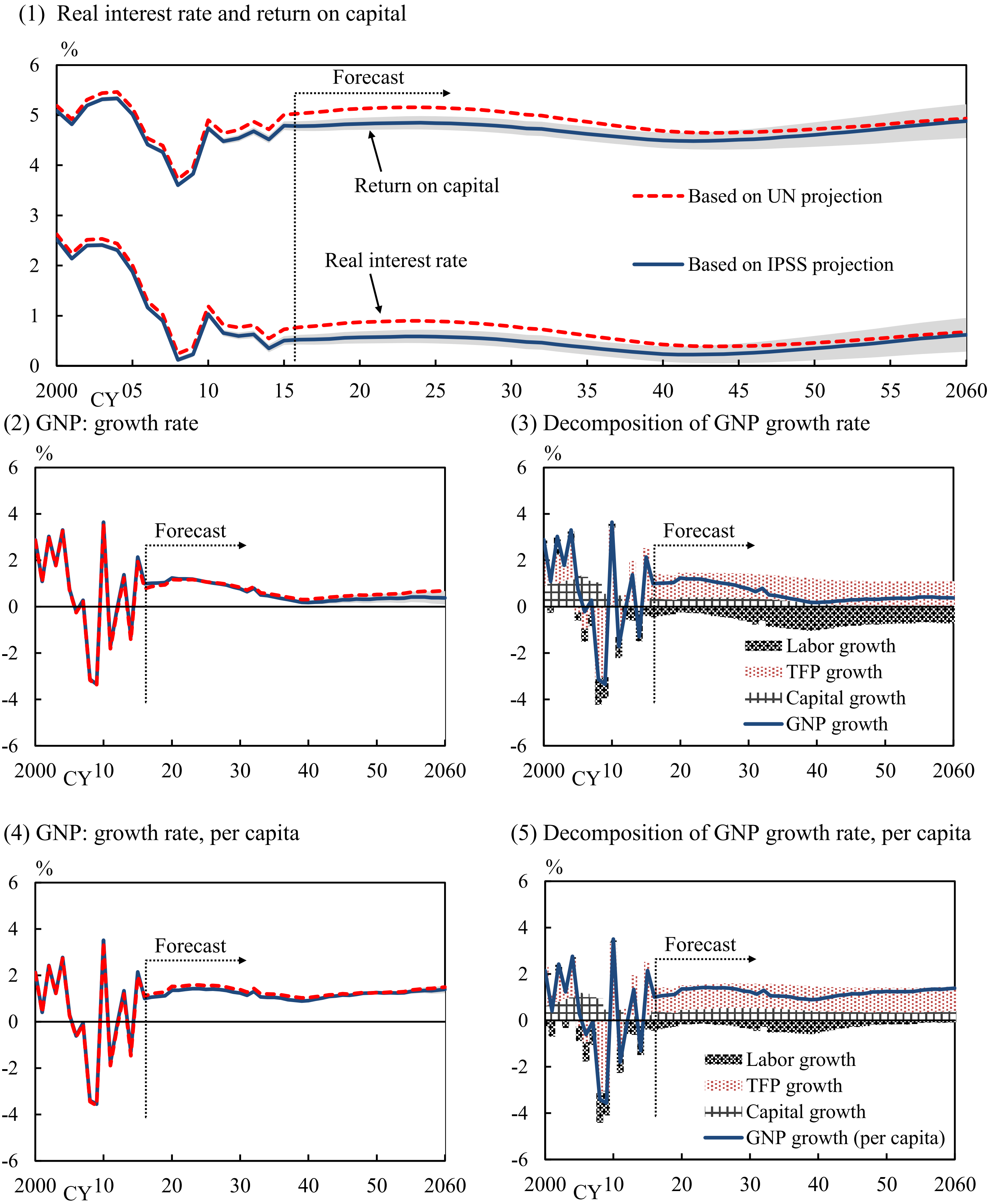
Real interest rate and measures of the natural rate of interest



Note: Return on capital and real interest rate generated from the current OG model are both on after-tax basis.

Sources: Bank of Japan, "Output Gap and Potential Growth Rate"; Imakubo et al. (2015).

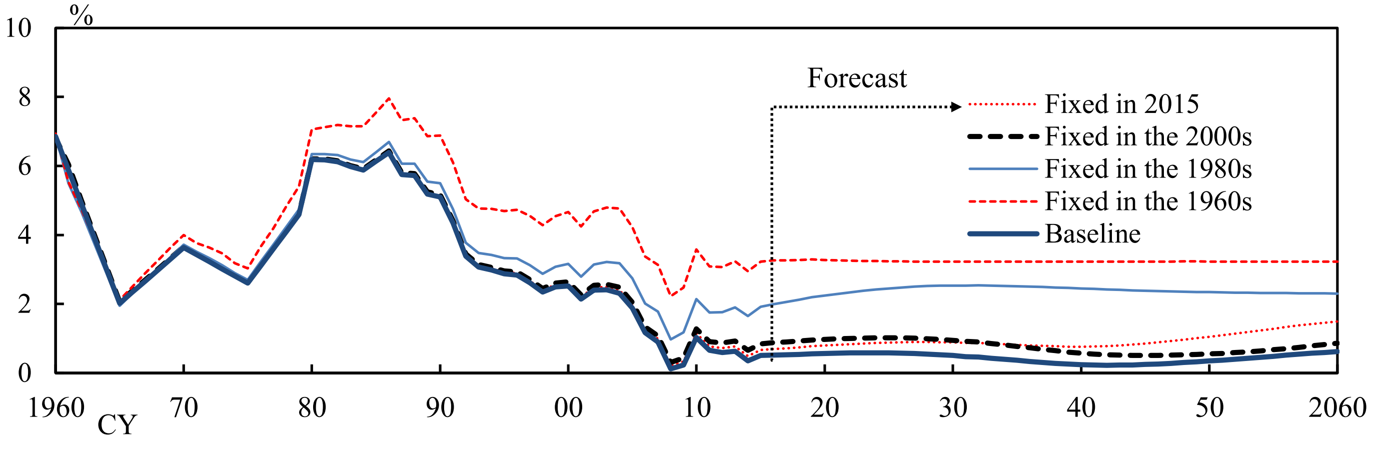
Figure 9: Out-of-sample Simulation of the Real Interest Rate



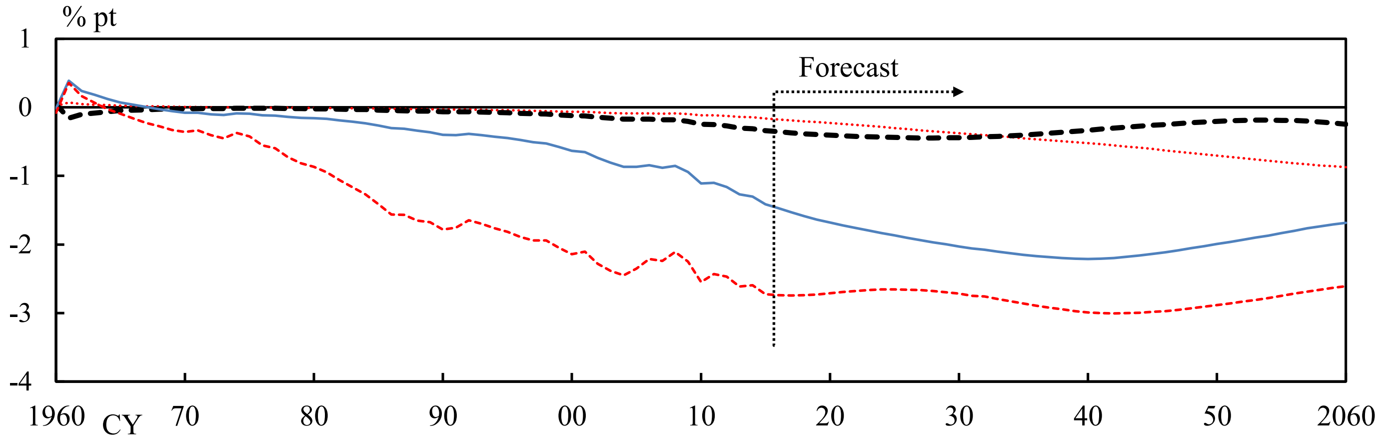
Note: The gray areas in panel (1), (2) and (4) show the range of prediction of macroeconomic variables, based on high and low projections of demographic variations made by the respective institutions.

Figure 10: Simulation under Constant Demographic Landscapes

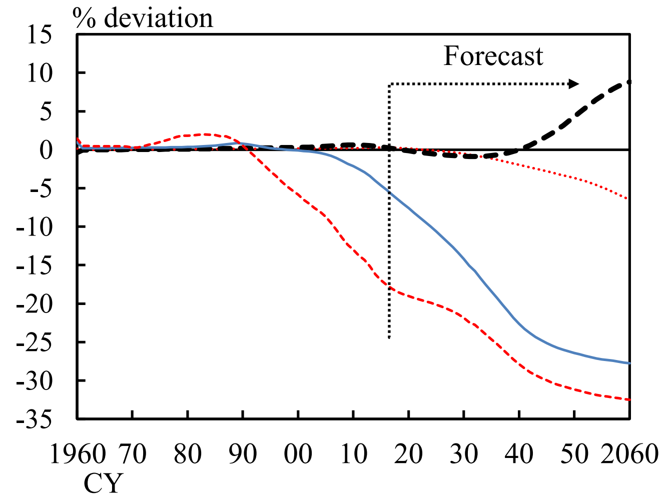
(1) Real interest rate



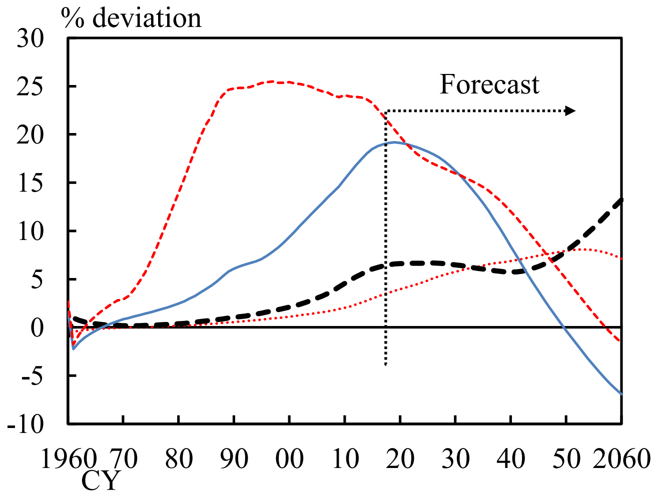
(2) Real interest rate: discrepancy from the baseline



(3) Labor inputs: level
: discrepancy from the baseline



(4) Capital inputs: level
: discrepancy from the baseline



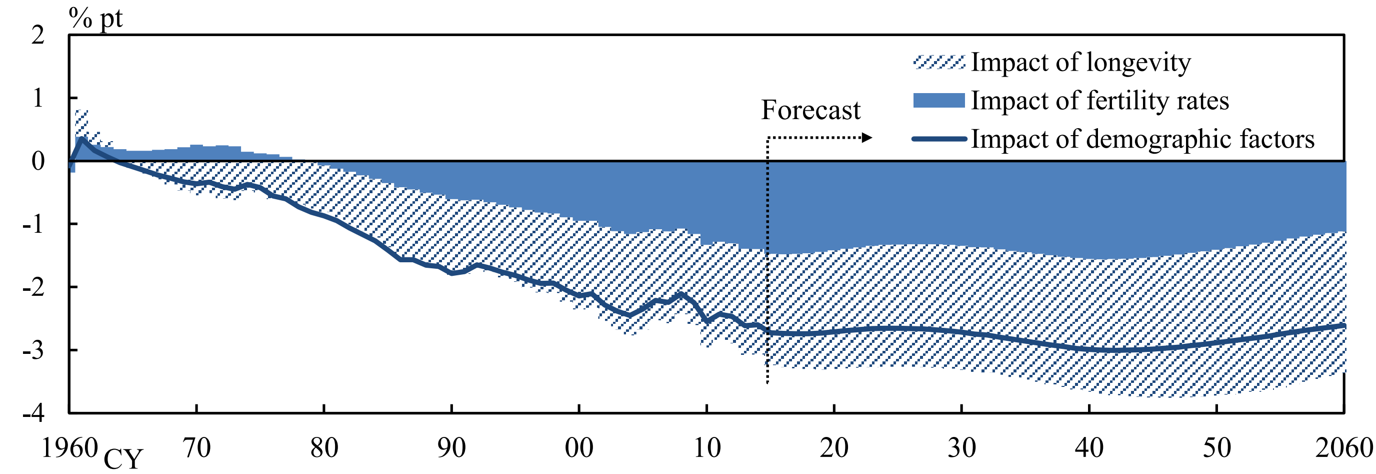
Notes: 1. We measure the impact of demographic factors by the discrepancy between the model-generated variable based on the baseline simulation and that based on the counterfactual simulations, where fertility rate and longevity are held constant at the values of the actual data in each specific decade.

2. The discrepancies in (2) are 'baseline simulations' - 'counterfactual simulations.' The discrepancies in (3) and (4) are the logarithmic differences of 'baseline simulations' -'counterfactual simulations.'

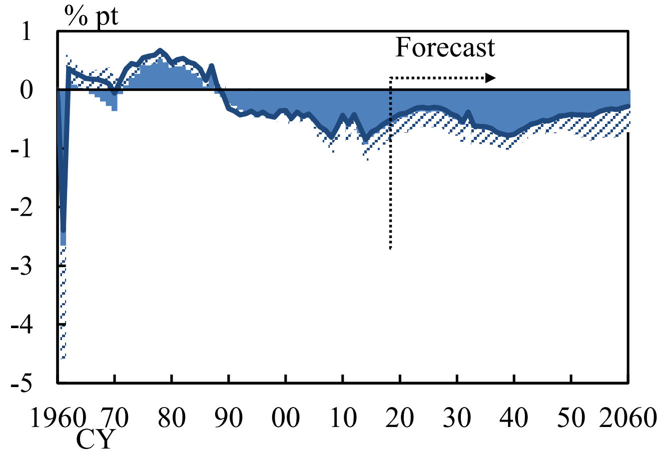
3. The series (3) and (4) are on a per capita basis.

Figure 11: Factors Contributing to Declining Interest Rates (1)

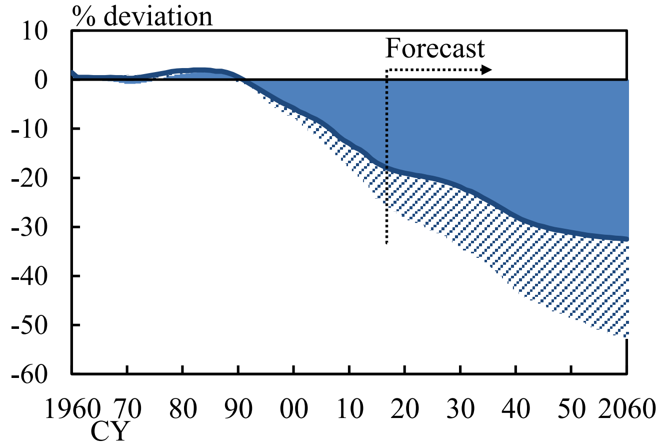
(1) Decomposition of real interest rate



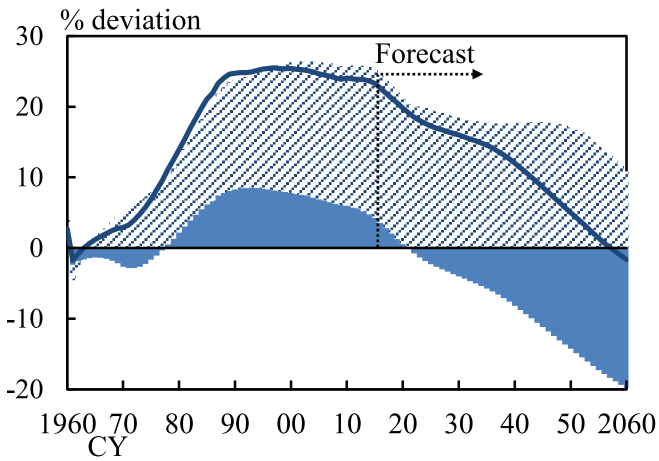
(2) Decomposition of GNP: growth rate



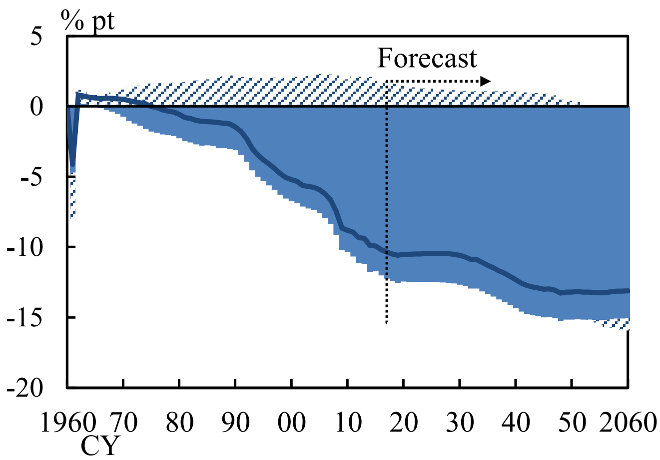
(3) Decomposition of labor inputs: level



(4) Decomposition of capital inputs: level



(5) Decomposition of saving rate



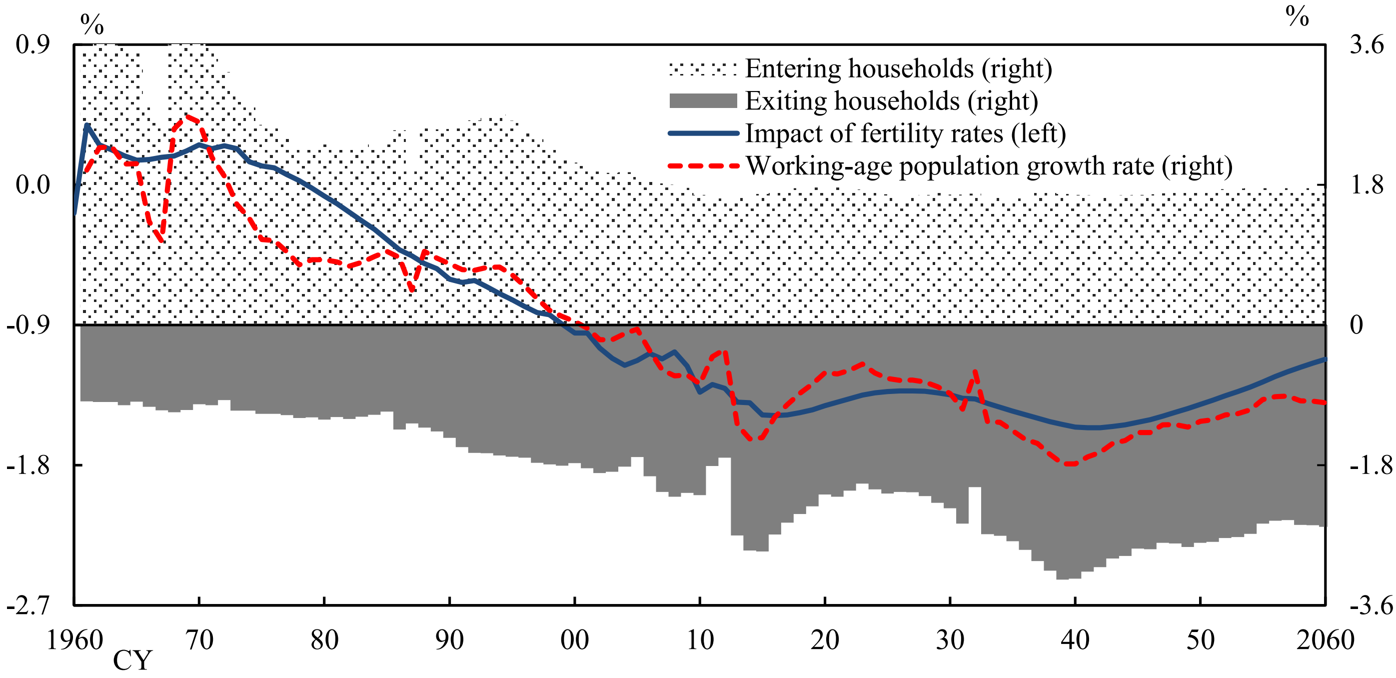
Notes: 1. We measure the impact of demographic factors by the discrepancy between the model-generated variable based on the baseline simulation and that based on the counterfactual simulations, where demographic factors (fertility rate, longevity, or both) are held constant at the values of the actual data in the 1960s.

2. The discrepancies in (1), (2), and (5) are 'baseline simulations' - 'counterfactual simulations.' The discrepancies in (3) and (4) are the logarithmic differences of 'baseline simulations' -'counterfactual simulations.'

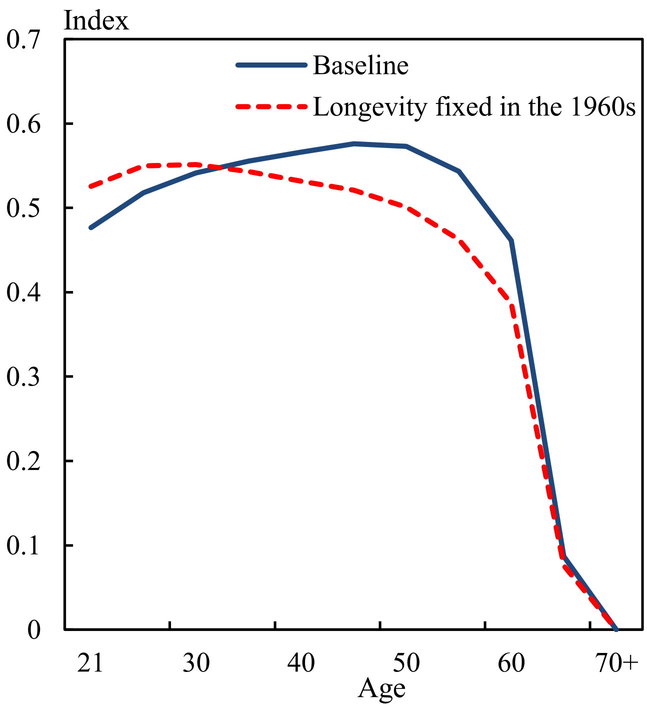
3. The series (2), (3), and (4) are on a per capita basis.

Figure 12: Factors Contributing to Declining Interest Rates (2)

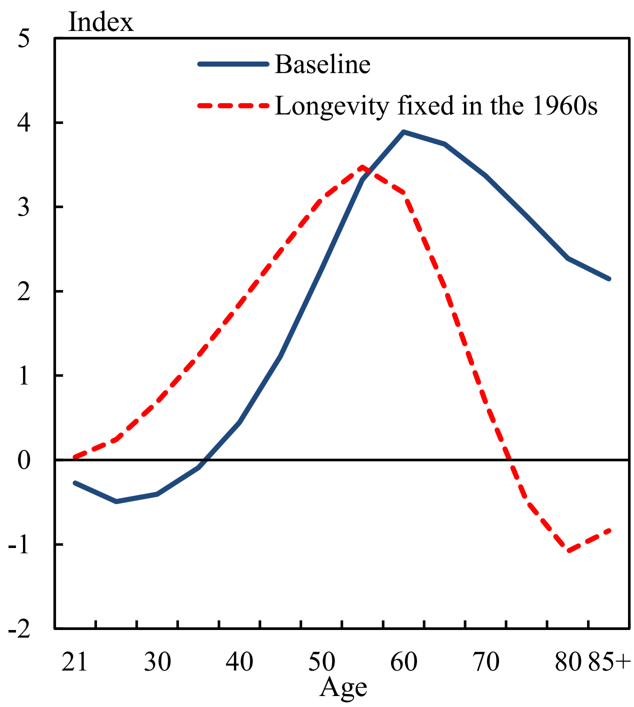
(1) Decomposition of the working-age population growth



(2) Life-cycle profile of labor inputs



(3) Life-cycle profile of capital stock holdings

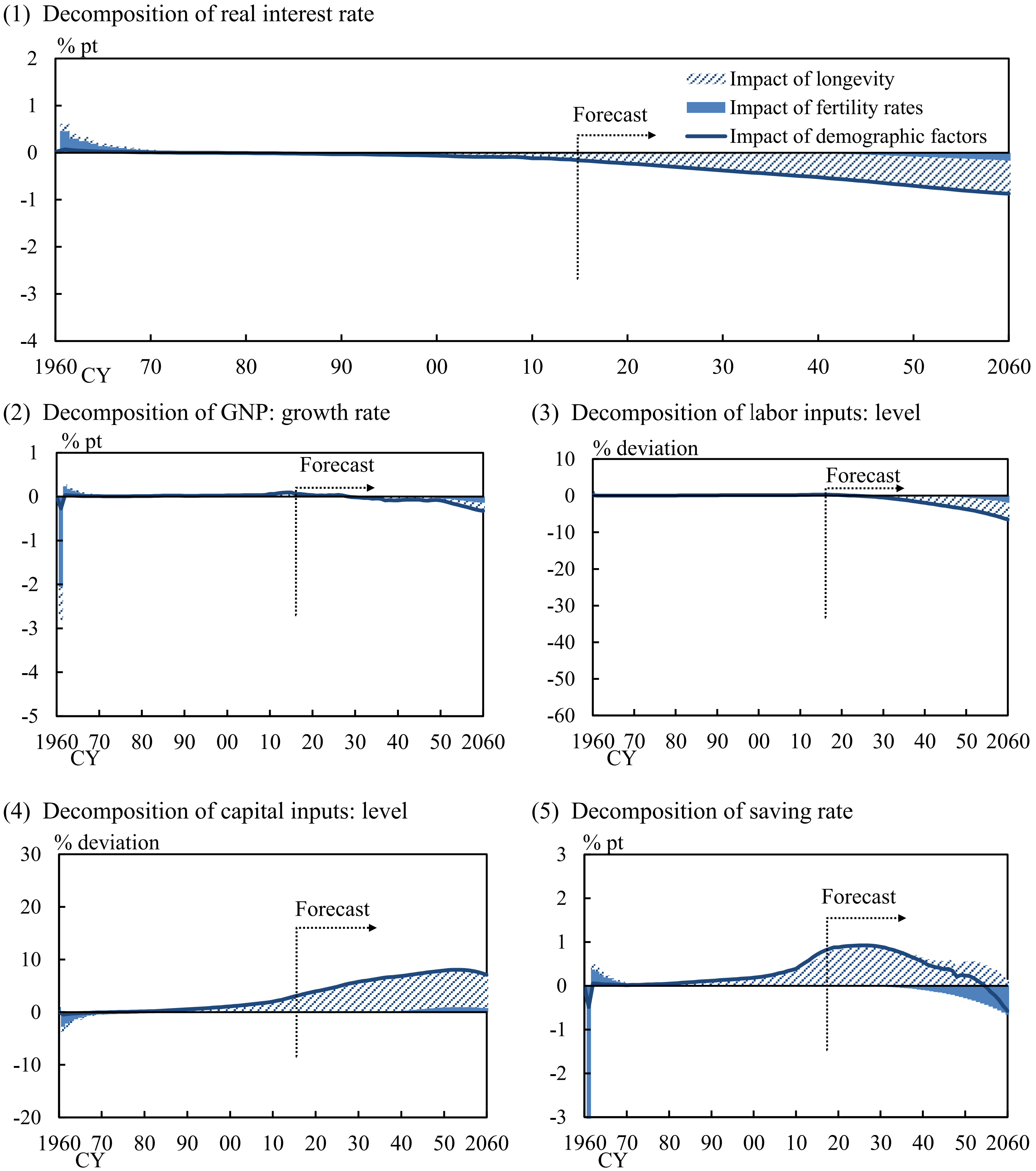


Notes: 1. The life-cycle profile of capital stock holdings is the profile at the terminal steady state in each simulation.

2. The value of capital stock holdings is detrended by the current level of TFP and the population.

Source: Ministry of Health, Labour and Welfare, "Population Census of Japan."

Figure 13: Factors Contributing to Declining Interest Rates (3)



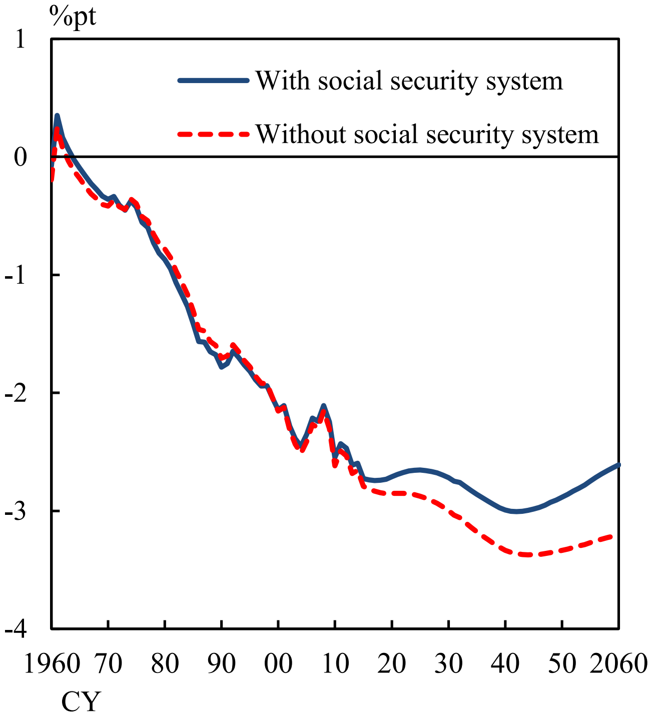
Notes: 1. We measure the impact of demographic factors by the discrepancy between the model-generated variable based on the baseline simulation and that based on the counterfactual simulations, where demographic factors (fertility rate, longevity, or both) are held constant at the values of the actual data in the 2015.

2. The discrepancies in (1), (2), and (5) are 'baseline simulations' - 'counterfactual simulations.' The discrepancies in (3) and (4) are the logarithmic differences of 'baseline simulations' -'counterfactual simulations.'

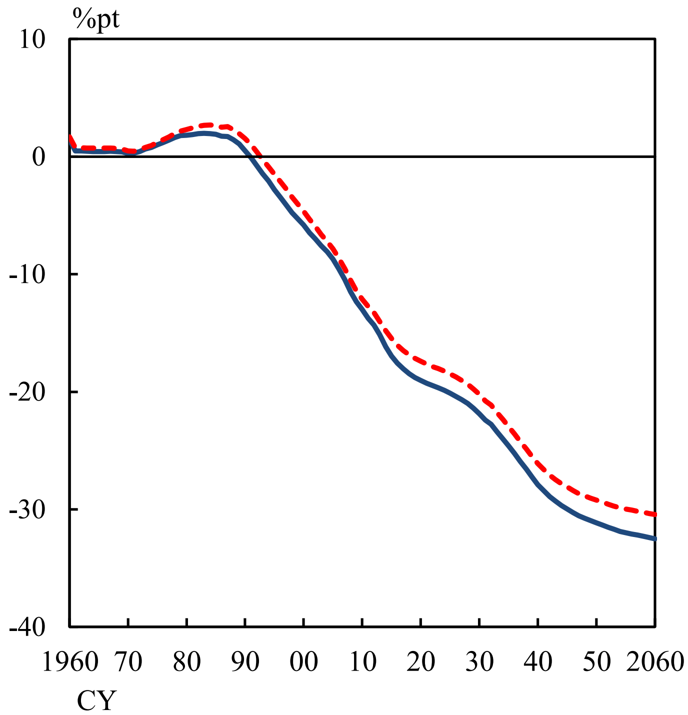
3. The series (2), (3), and (4) are on a per capita basis.

Figure 14: Sensitivity Analysis: Presence of Social Security System

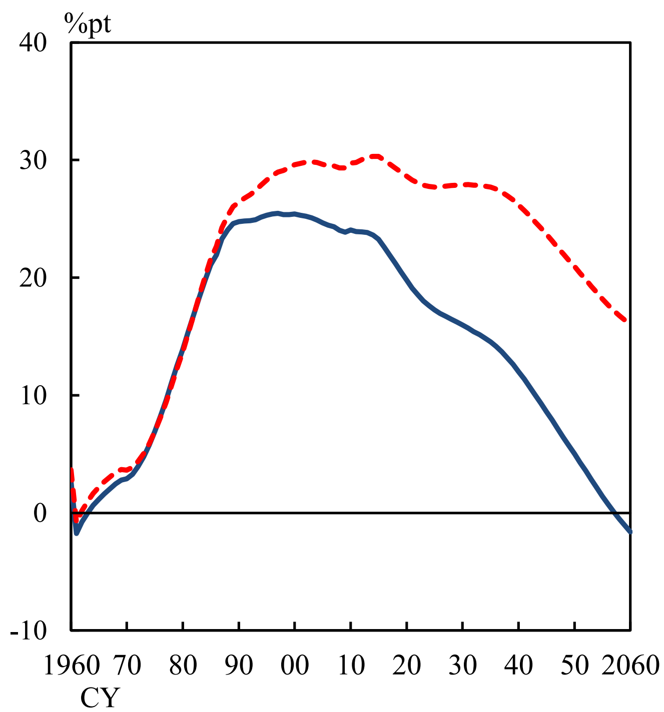
(1) Real interest rate: Impact of demographic factors



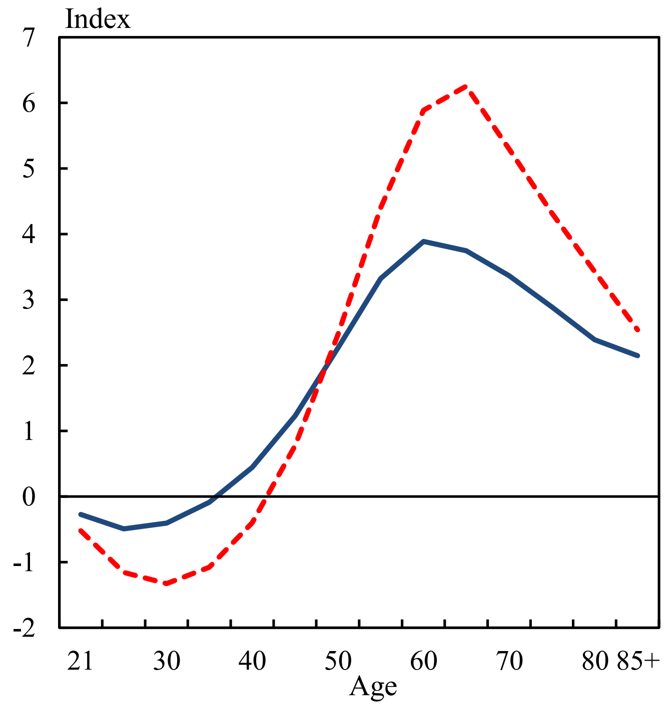
(2) Labor inputs: level
: Impact of demographic factors



(3) Capital inputs: level
: Impact of demographic factors



(4) Life-cycle profile of capital stock holdings



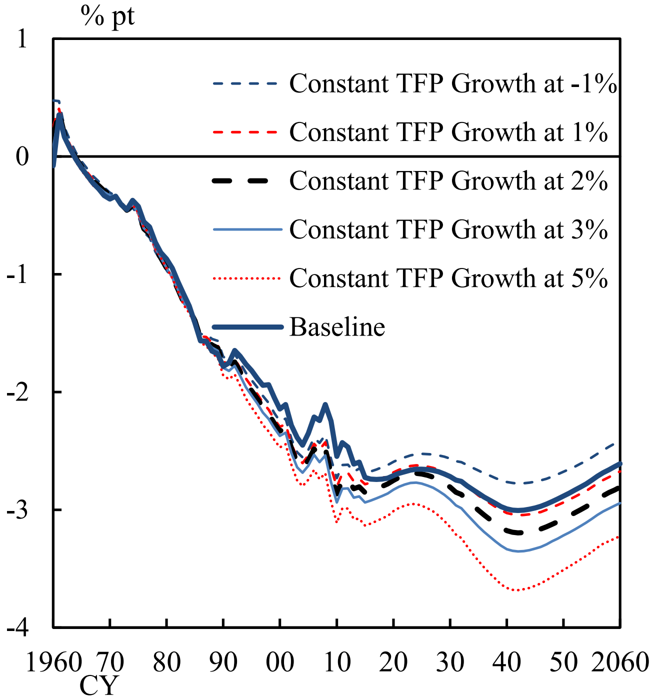
Notes: 1. The value of capital inputs is detrended by the current level of TFP and the population.

2. The life-cycle profile of capital holdings is the profile at the terminal steady state in the baseline simulation in each case.

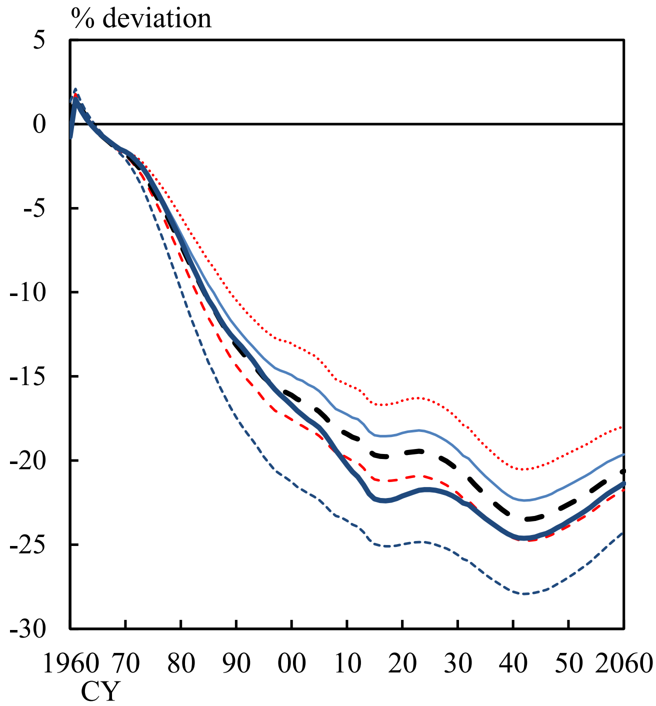
3. The series of (2) and (3) are on a per capita basis.

Figure 15: Sensitivity Analysis: TFP

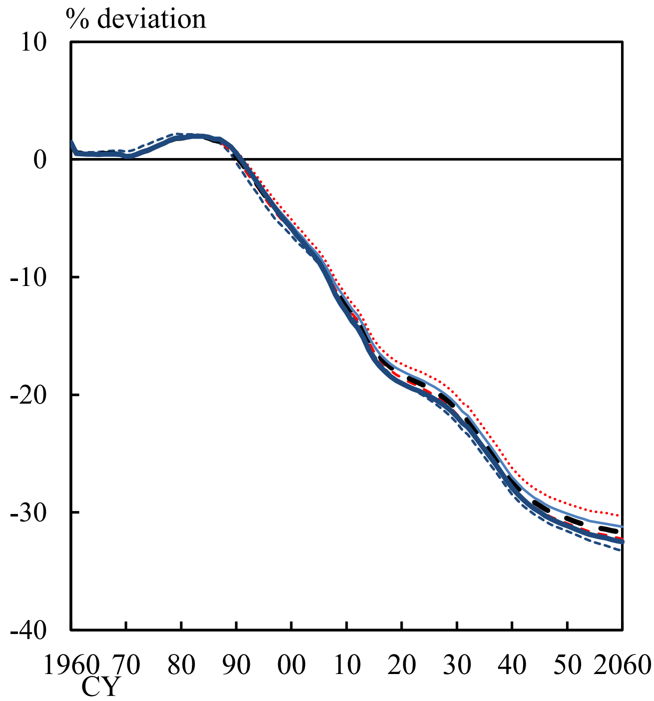
(1) Real interest rate: Impact of demographic factors



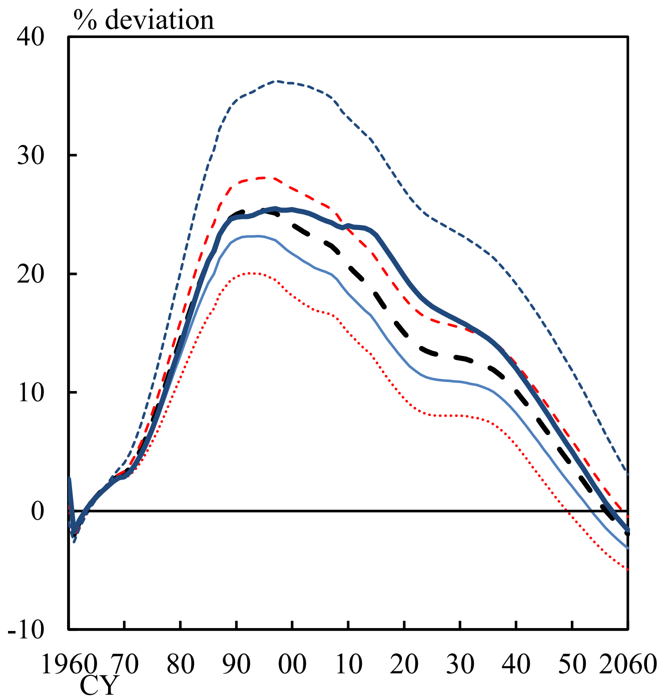
(2) Marginal productivity of capital
: Impact of demographic factors



(3) Labor inputs: level
: Impact of demographic factors



(4) Capital inputs: level
: Impact of demographic factors

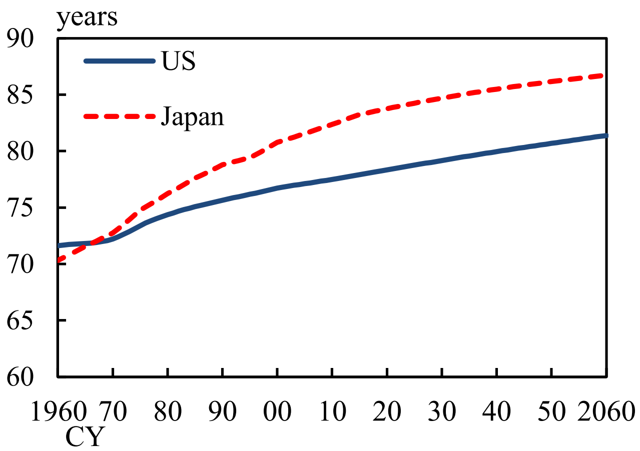


Note: The series (3) and (4) are on a per capita basis

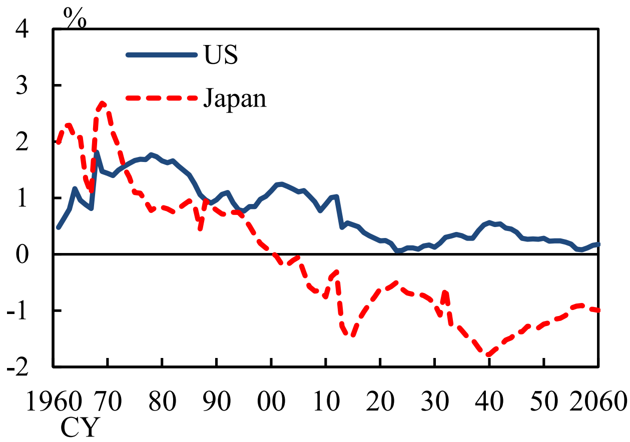
Figure 16: Demographic Changes in Other Countries

(1) US

(a) Average life expectancy

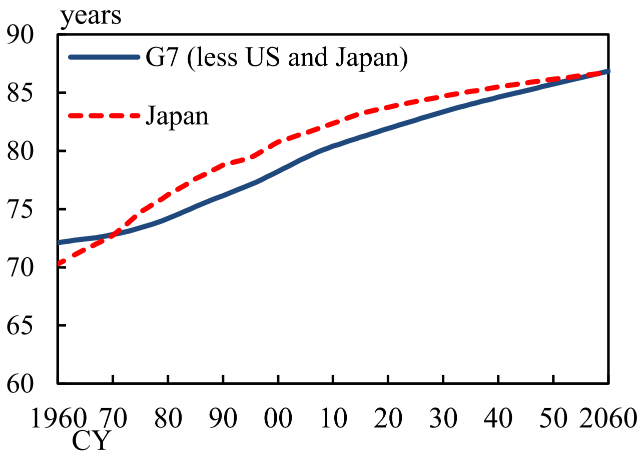


(b) Working-age population growth rate

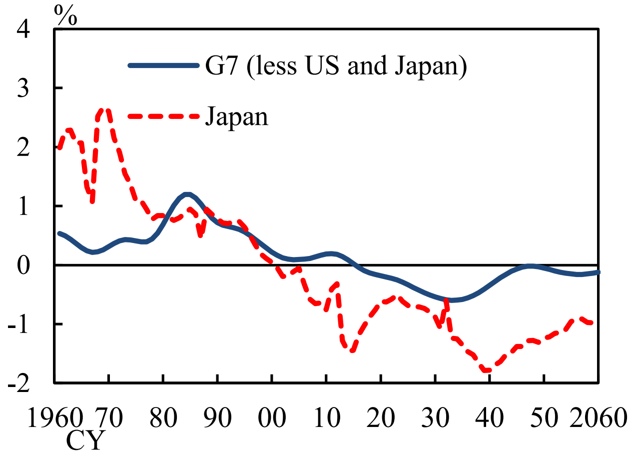


(2) G7 (less US and Japan)

(a) Average life expectancy

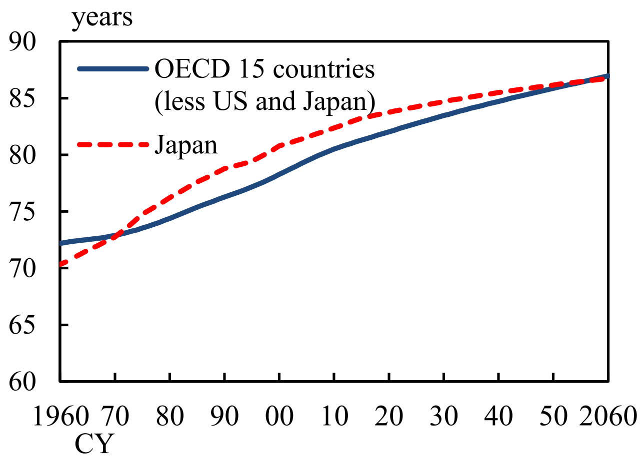


(b) Working-age population growth rate

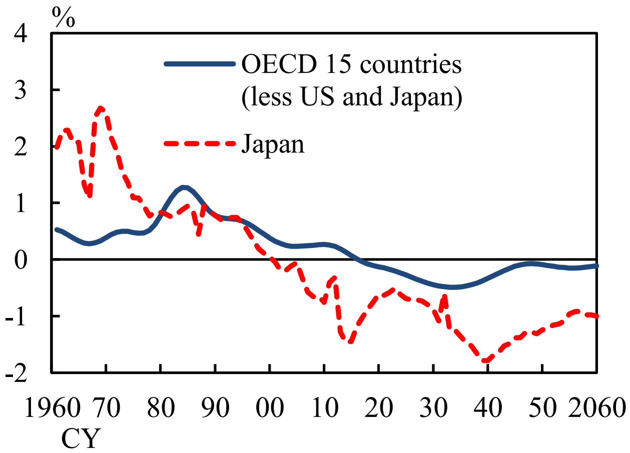


(3) OECD 15 countries (less US and Japan)

(a) Average life expectancy



(b) Working-age population growth rate

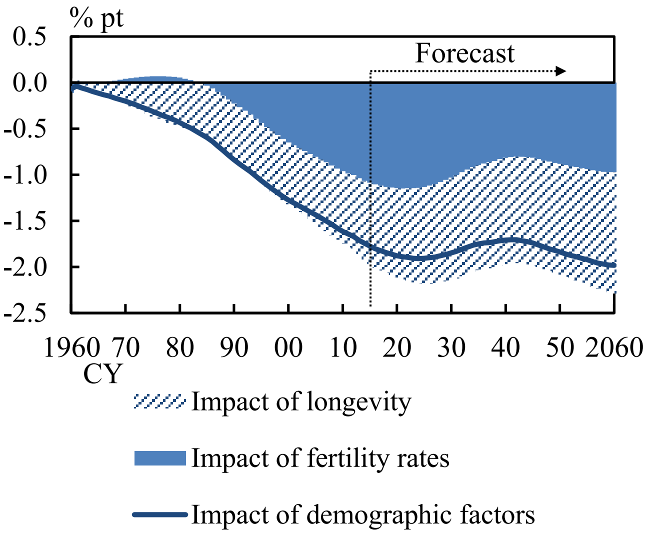


Sources: United Nations, "World Population Prospects"; Census Bureau, "Historical National Population Estimate," and "2014 to 2060 Populations Projections based on Census 2010"; Social Security Administration, "Life Tables for the United States Social Security Area 1900-2100"; Ministry of Health, Labour and Welfare, "Life Tables"; National Institute of Population and Social Security Research, "Population Projections for Japan."

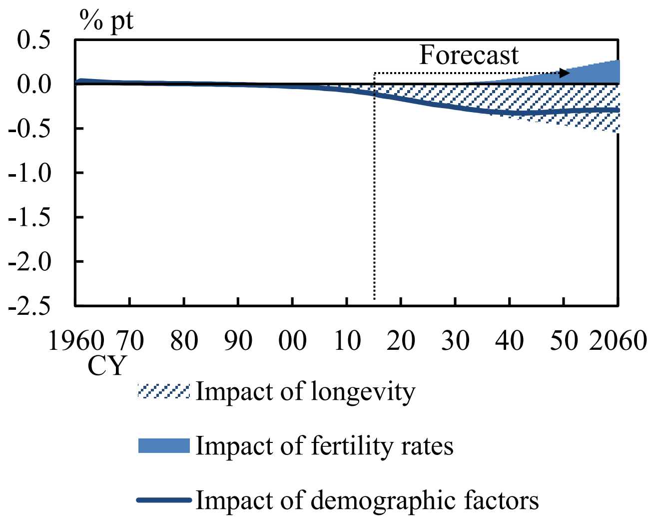
Figure 17: Impact of Demographic Changes on Real Interest Rates in Other Countries (1)

(1) US

(a) Decomposition by demographic factors
(demographic factors fixed in the 1960s)

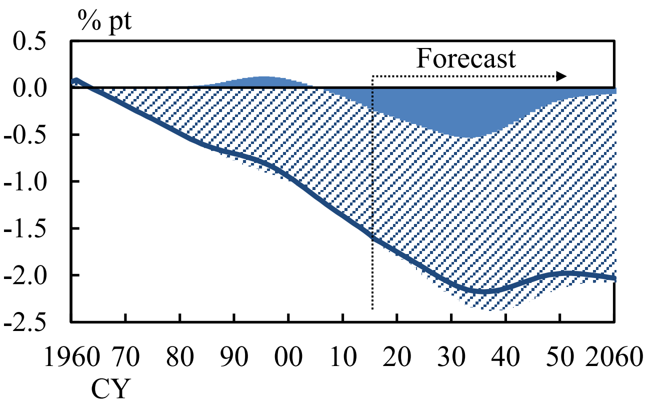


(b) Decomposition by demographic factors
(demographic factors fixed in 2015)

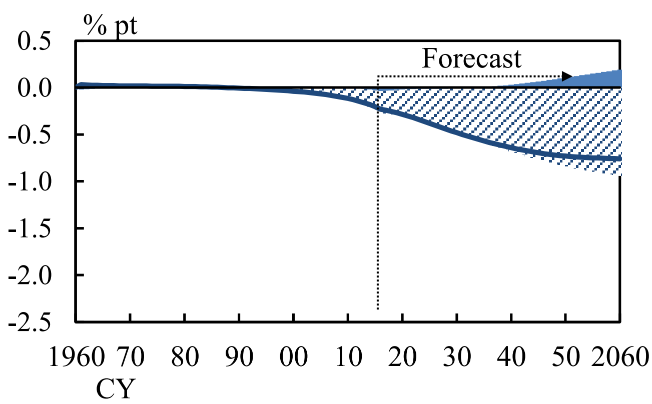


(2) G7 (less Japan and US)

(a) Decomposition by demographic factors
(demographic factors fixed in the 1960s)

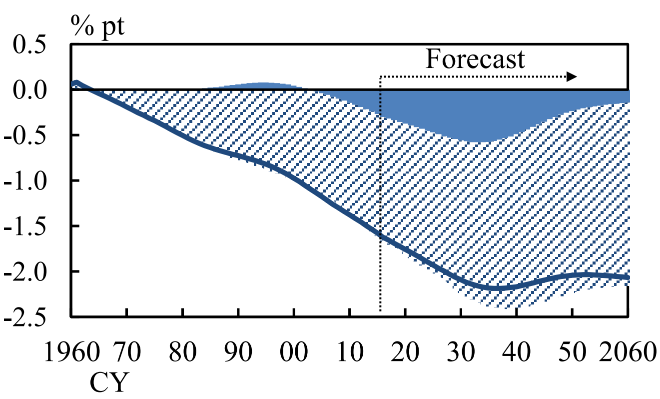


(b) Decomposition by demographic factors
(demographic factors fixed in 2015)



(3) OECD 15 countries (less Japan and US)

(a) Decomposition by demographic factors
(demographic factors fixed in the 1960s)



(b) Decomposition by demographic factors
(demographic factors fixed in 2015)

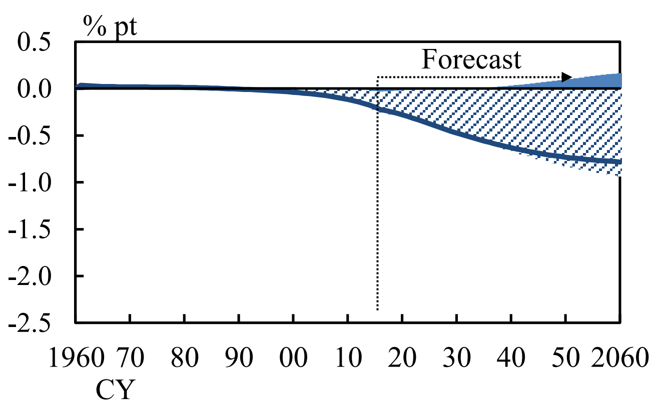
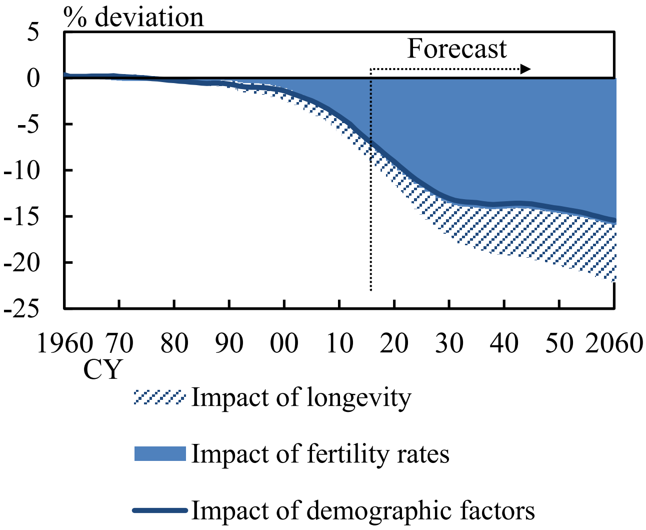


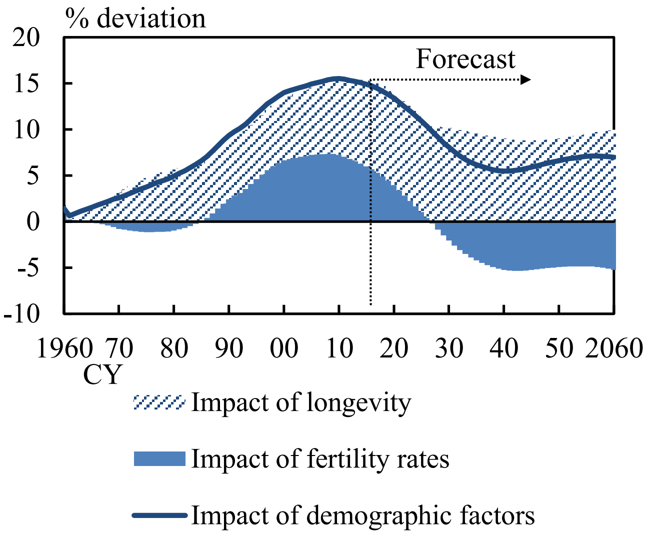
Figure 18: Impact of Demographic Changes on Capital and Labor Inputs in Other Countries (2)

(1) US

(a) Decomposition of labor inputs, level

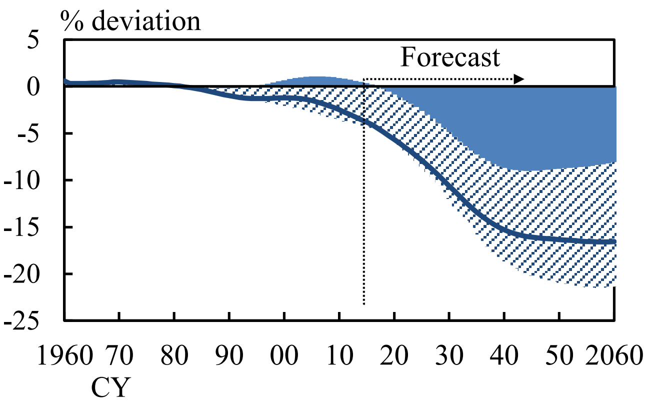


(b) Decomposition of capital inputs, level

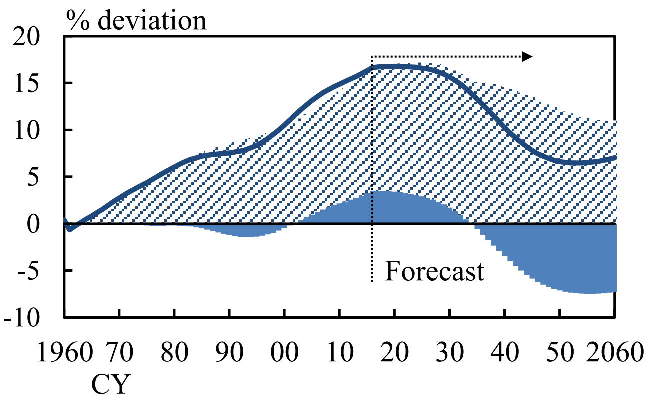


(2) G7 (less Japan and US)

(a) Decomposition of labor inputs, level

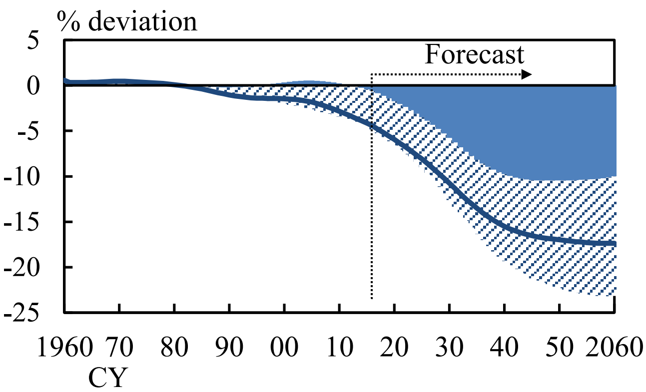


(b) Decomposition of capital inputs, level

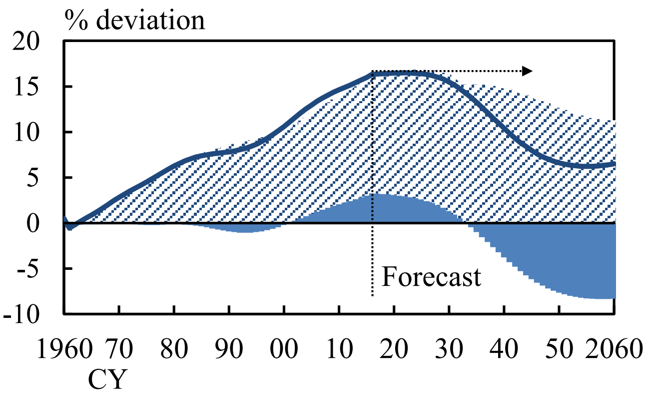


(3) OECD 15 countries (less Japan and US)

(a) Decomposition of labor inputs, level



(b) Decomposition of capital inputs, level



Notes: 1. We measure the impact of demographic factors by the discrepancy between the model-generated variable based on the baseline simulation and that based on the counterfactual simulations, where demographic factors (fertility rate, longevity, or both) are held constant at the values of the actual data in the 1960s.

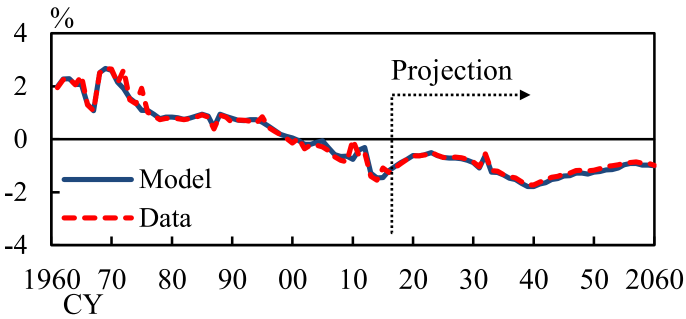
2. The discrepancies are the logarithmic differences of 'baseline simulations' -'counterfactual simulations.'

3. The series are on a per capita basis.

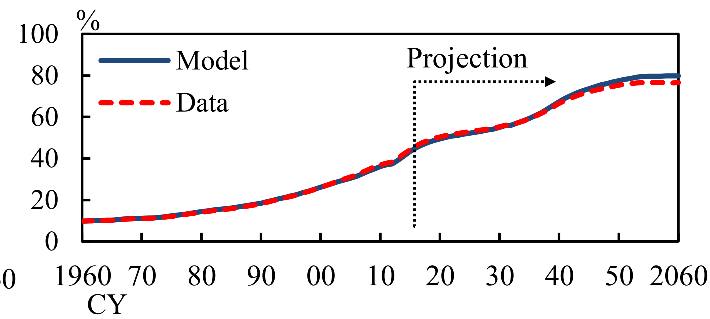
Figure 19: Comparisons of Demographic Variables

(1) Japan

(a) Working-age population growth rate

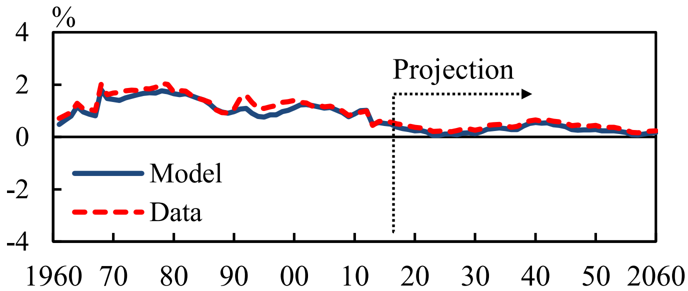


(b) Dependency ratio

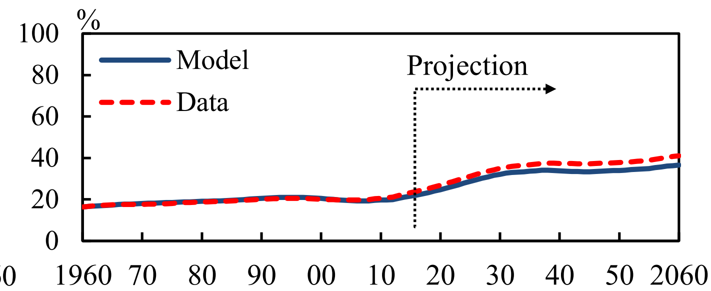


(2) US

(a) Working-age population growth rate

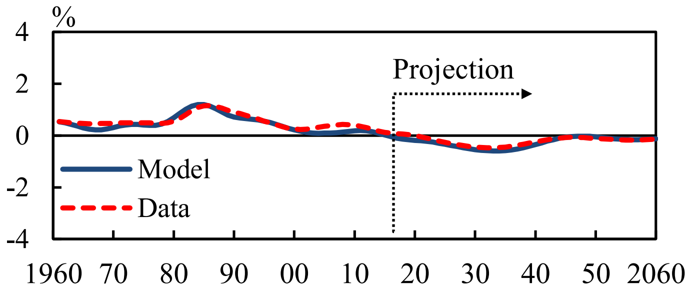


(b) Dependency ratio

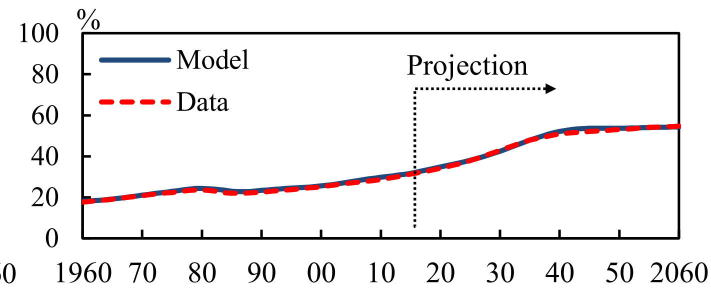


(3) G7 (less US and Japan)

(a) Working-age population growth rate

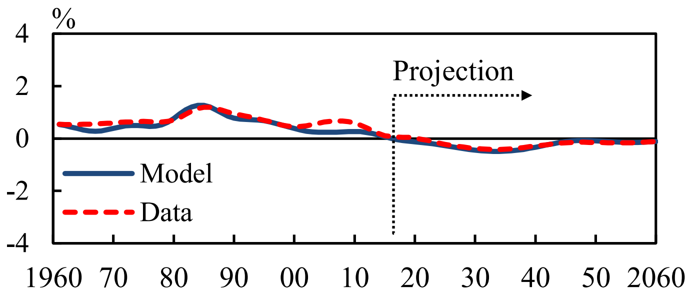


(b) Dependency ratio

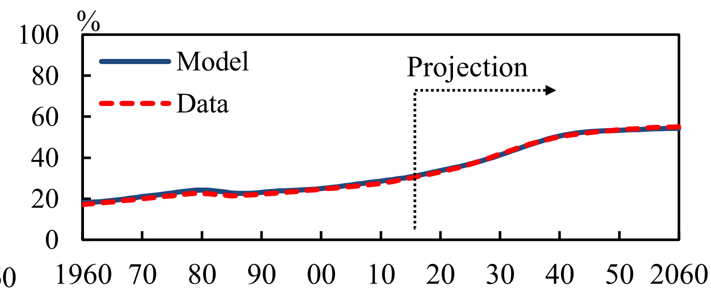


(4) OECD 15 countries (less US and Japan)

(a) Working-age population growth rate



(b) Dependency ratio

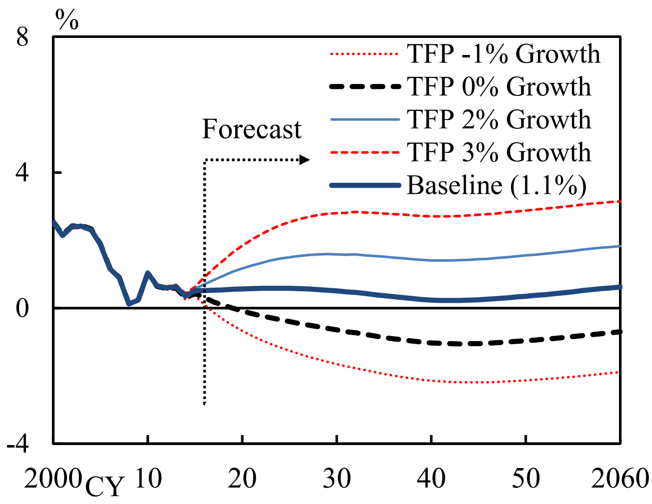


Note: The series of model are model-generated variables based on fertility rates and mortality rates under the baseline simulations. The series of data are the actual figures projected by the respective institutions.

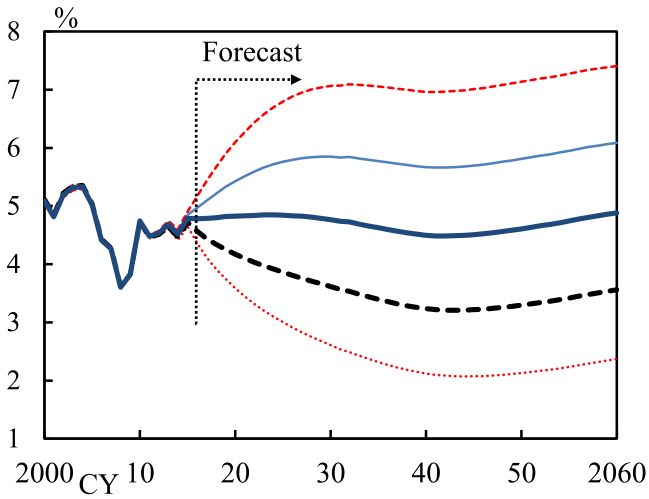
Sources: United Nations, "World Population Prospects"; Census Bureau, "Historical National Population Estimate," and "2014 to 2060 Populations Projections based on Census 2010"; Social Security Administration, "Life Tables for the United States Social Security Area 1900-2100"; Ministry of Health, Labour and Welfare, "Life Tables"; National Institute of Population and Social Security Research, "Population Projections for

Appendix Figure 1: Simulation under Alternative Assumptions on TFP Growth

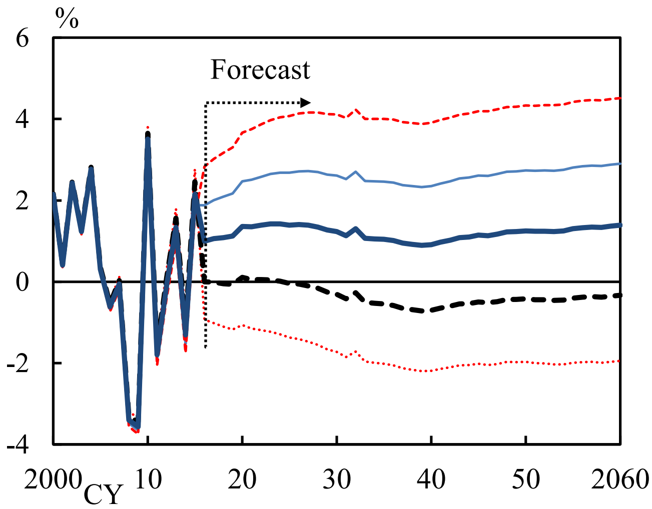
(1) Real interest rate



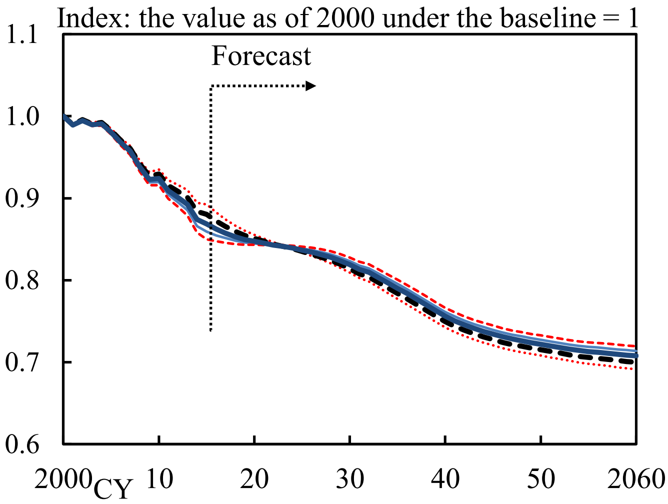
(2) Return on capital



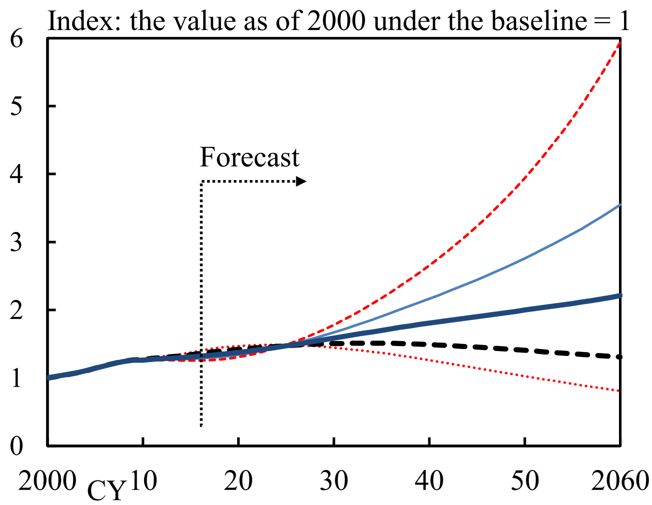
(3) GNP: growth rate



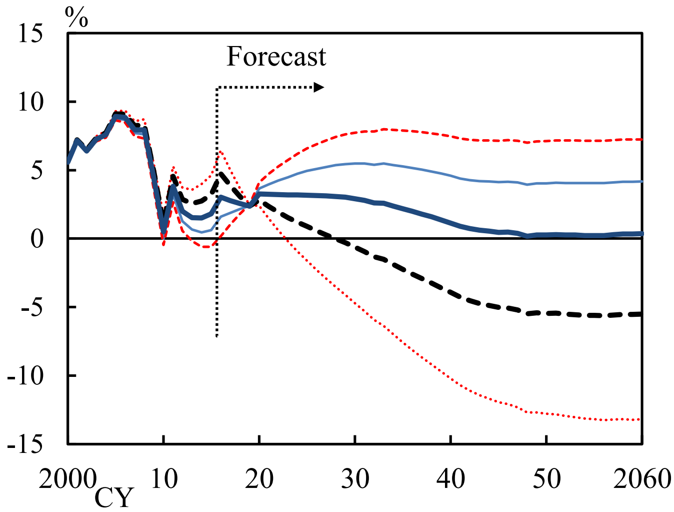
(4) Labor inputs: level



(5) Capital inputs: level

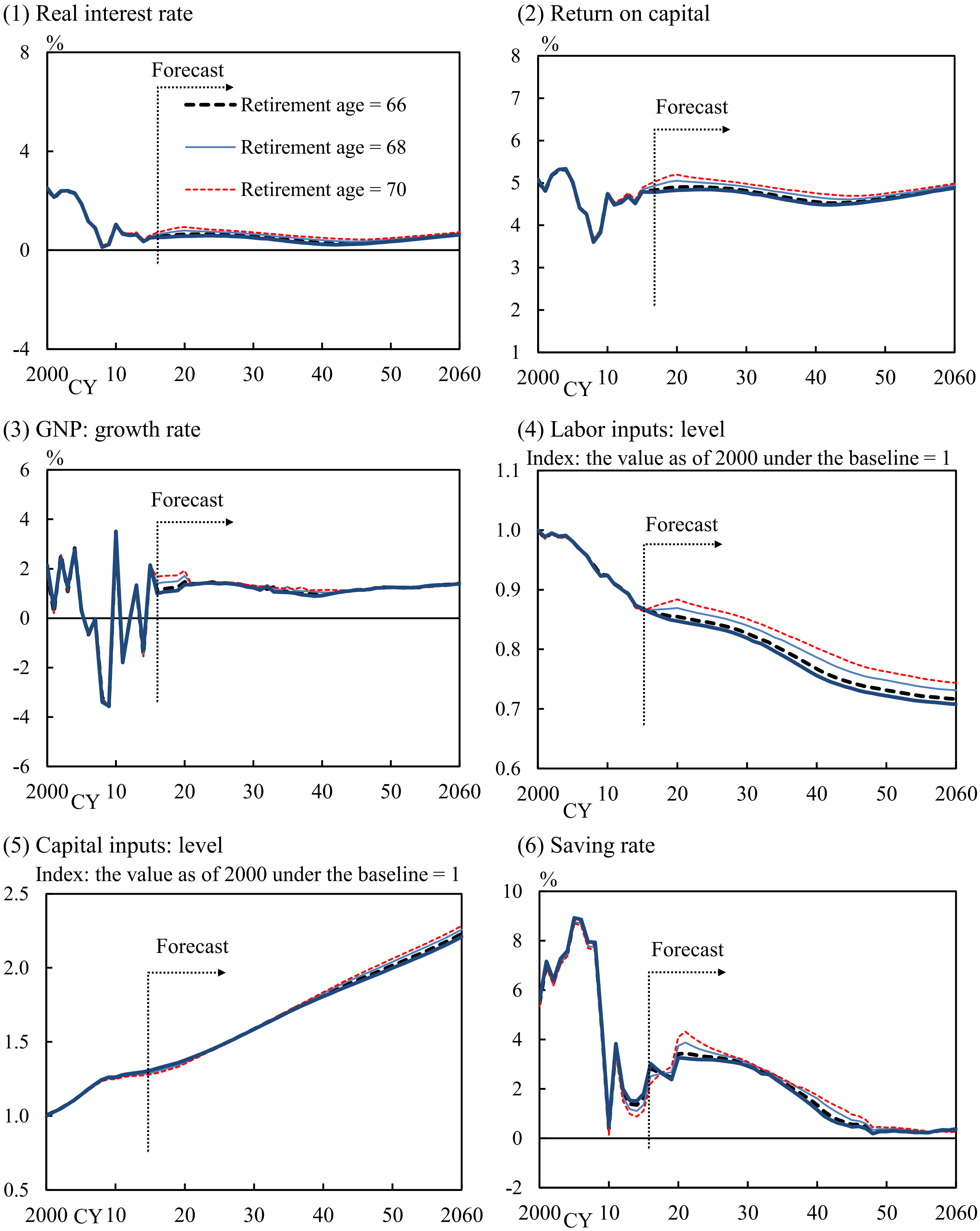


(6) Saving Rate



Note: The series (3), (4), and (5) are on a per capita basis.

Appendix Figure 2: Simulation under Alternative Assumptions on Retirement Ages



Notes: 1: In each simulation, the retirement age from 1960 to 2015 is 65. In the next five years, the retirement age is assumed to increase to 66, 68, and 70 years gradually, and stays constant after the target age is reached.

2. The series (3), (4), and (5) are on a per capita basis.