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Abstract

This study proposes a credit risk model based on purchase order (PO) information, which is called a "PO-based structural model," and performs an empirical analysis on credit risk assessment using real PO samples. A time-series model of PO transitions is introduced and the asset value of the borrower firm is obtained using the PO time-series model. Then, we employ a structural framework in which default occurs when the asset value falls below the debt amount, in order to estimate the default probability of the borrower firm. The PO-based structural model enables us to capture borrower firms' precise business conditions on a real-time basis, which is not the case when using only financial statements. With real PO samples provided by some sample firms, we empirically show the effectiveness of our model in estimating default probabilities of the sample firms. One of the advantages of our model is its ability to obtain default probabilities reflecting borrower firms' business conditions, such as trends in PO volumes and credit quality of buyers.

Keywords: Purchase order information; Credit risk, Structural model

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1 Introduction

Financial institutions have been assessing and monitoring their borrowers' credit risks by measuring financial losses caused by defaults. In financial practice, the credit risks of borrower firms are mainly assessed using their financial statements or stock prices observed in stock markets. However, there are some problems with this traditional method. Specifically, financial institutions cannot recognize changes in a firm's credit risk immediately in the case of using only financial statements, as they are announced infrequently, for example annually or quarterly, and represent nothing more than static information describing business conditions at only one point in time. Thus, monitoring methods using only financial statements do not provide a real-time snapshot of the changes in business conditions. Moreover, market noise on stock prices disturbs the credit risk evaluation using stock prices. In addition, the stock prices of unlisted firms cannot be observed in stock markets at all.

On the other hand, some financial institutions have begun to attempt lending based on information associated with commercial transactions, such as purchase order (PO) information.[1] In this study, we propose a new framework of credit risk assessment with a PO-based structural model, illustrate some empirical results on the proposed model, and consider if the model can contribute to more sophisticated credit risk management.

We provide a credit risk model using PO information, which is called a "PO-based structural model." The input PO information includes attributions of buyers, date of purchase order receipts, and PO amount, which has a format described in Table 1. Our model is a type of structural credit risk model in which a firm's default or bankruptcy occurs as the value of the firm's assets declines to some critical breakpoint, like the face value of the firm's total debt. There has been much research on credit risk assessment with structural models. In most research, the firm value or the value of the firm's total assets is directly modeled by some stochastic process, like a geometric Brownian motion, or some diffusion process, such as Merton (1974). Different from them, Goldstein, Ju, and Leland (2001) and Genser (2006) pay attention to modeling the dynamics of earnings before interest and tax (EBIT) rather than the firm's assets. Then, the asset value can be achieved by the integration of the discounted EBIT from now to bankruptcy or the infinite time horizon. We call this type of structural model an "EBIT-based structural model." On the other hand, Yamanaka (2016) suggested the structural model based on PO information, in which the PO volume transition is first modeled, and then the earnings and asset values are generated according to the PO volumes obtained from the PO volume transition model. As a result, the borrowing firm's business conditions, such as trends in PO volume and credit quality

[1] In this study, commercial transactions indicate manufacturing, purchasing, shipment, and payment and settlement. PO information includes the attributions of buyers, date of purchase order receipts, product attributions, PO amount, shipment date, product prices, and deposit data.

of buyers, reflect the estimated probabilities of default (PDs) immediately. We call this type of structural model a "PO-based structural model."

We propose an extended version of the PO-based structural model of Yamanaka (2016) and demonstrate some empirical results of the model. The model of Yamanaka (2016) assumes that the PO arrivals continue without cessation. However, there are often intermittent PO arrivals in some kind of businesses, for example, construction and manufacture of plant equipment. Thus, we extended the model of Yamanaka (2016) in order to treat intermittent PO arrivals. In particular, we introduce a model of the probability of PO arrivals and a model of sizes of received PO volumes and combine them to obtain a time-series model of PO volume transaction. Then, we demonstrate the applicability of the model to practical credit risk monitoring with an empirical analysis using real PO samples provided by some sample firms.

The rest of this paper is organized as follows. Section 2 illustrates PO sample data provided by the sample firms in this study. Section 3 introduces a PO-based structural model. Then, Section 4 illustrates some empirical results on estimating the PDs of the sample firms. Section 5 concludes.

Table1: Data format of the sample PO information

Buyer	Date of PO receipts	Product number	Unit price / thousand yen	Quantity	PO amount / thousand yen
Firm A	January 2018	XXX-000001	682.4	240	163776
Firm B	January 2018	XXX-000003	1023.1	30	30693
Firm C	January 2018	XXX-000002	823.5	30	24705
Firm A	February 2018	XXX-000004	218.9	2140	468446
Firm B	February 2018	XXX-000003	1023.1	50	51155
Firm A	March 2018	XXX-000001	682.4	930	634632
Firm C	March 2018	XXX-000005	253.8	2670	677646
Firm A	April 2018	XXX-000003	1023.1	50	51155
Firm B	April 2018	XXX-000003	1023.1	30	30693
Firm C	April 2018	XXX-000006	728.1	130	94653

Note : All information in this table is fictional.

2 Data

The sample data for our study are the monthly PO records of the three sample firms: Kojima Industries Corporation, Hikari Kikai Seisakusho Co., Ltd., and another sample firm that we call "Firm α" in this paper.[2]

Kojima Industries Corporation is an unlisted firm that manufactures interior and exterior automobile components. The main buyers of the firm are leading auto manufacturers, such as Toyota Motor Corporation, Toyota Auto Body Co., Ltd., Toyota Motor East Japan, Inc., Hino Motors, Ltd., and Daihatsu Motor Co., Ltd.[3] Hikari

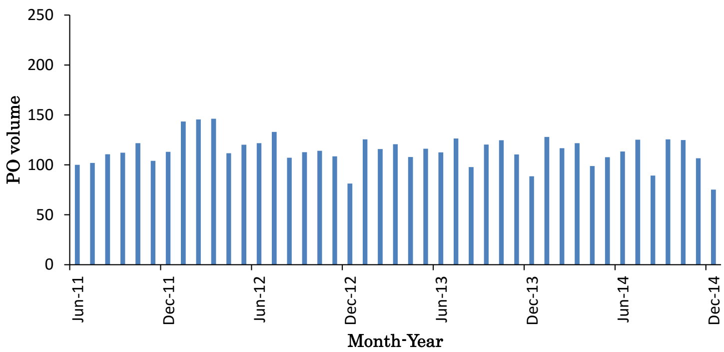
[2] The original data provided by Kojima Industries Corporation are daily. There are no data for the date of PO arrivals, and thus, that we set it to the date 5 business days before delivery. Then, we aggregate the same month PO records and use them in the analysis.

[3] The data source is the Kojima Industries Corporation website (`http://www.kojima-tns.co.`

Kikai Seisakusho Co., Ltd. is an unlisted firm that manufactures special-purpose grinding machines and cutting tools, including indexable inserts and drill bodies. The main buyers of the firm are major manufacturers providing industrial materials.[4]

Figures 1, 2, and 3 show the transition of monthly PO volumes in our samples. We recognize that the transition of PO volumes for Kojima Industries Corporation is moderate and there is some seasonality in PO volumes; for example, there is a relative decrease in PO volumes every August and December. On the other hand, there are extreme PO volume fluctuations for Hikari Kikai Seisakusho Co., Ltd., since the POs on grinding machines arrive intermittently and its volumes are quite large. PO volumes for Firm α have been increasing gradually since 2009.

Figure1: Time-series plots of Kojima Industries Co. monthly PO volumes



Note : Total PO volumes from all buyers, with the volume in June 2011 equal to 100.

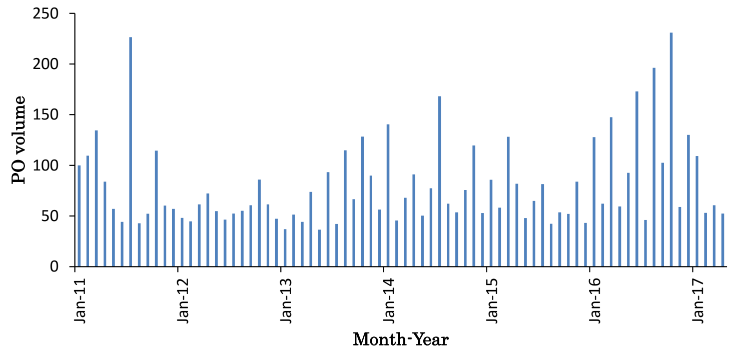
We model the POs of each buyer. Hereafter we number each buyer in order of the size of PO volumes. Here, we separately treat every PO sample of buyers in the top ranks and the others.[5] For instance, we model the PO transitions of the buyers for Kojima Industry Corporation that are ranked in the top nine PO volumes ($i = 1, 2, \cdots 9$). In addition, we model the transition of aggregated PO volume for the remainder and we label the entity $i = 10$. For Hikari Kikai Seisakusho Co., Ltd., we model the PO volume time-series transitions on grinding machines and cutting tools, since the features of PO volumes are quite different between these two types of products. Then, each buyer separated by the product type is labeled by the ranking of the size of its volumes. As a result, we model the PO volume transitions of the

jp/) as of March 26, 2018.

[4] The data source is the Hikari Kikai Seisakusho Co., Ltd. website (`http://www.hikarikikai.co.jp/`) as of March 26, 2018.

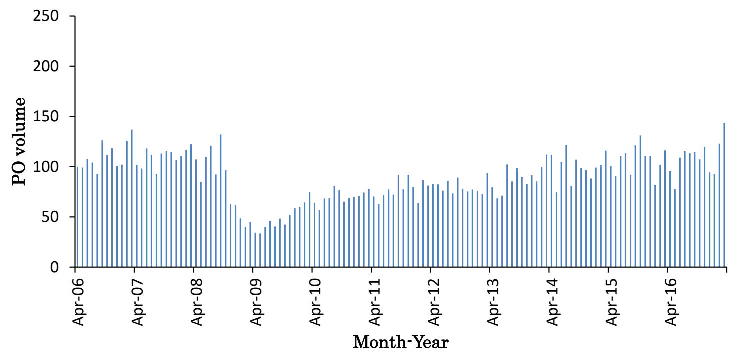
[5] We select the top buyers so that the sum of their PO volumes accounts for 80% and over of total PO volumes and there are at least eight buyers.

Figure2: Time-series plots of Hikari Kikai Seisakusho Co., Ltd. monthly PO volumes



Note : Total PO volumes from all buyers, with the volume in January 2011 equal to 100.

Figure3: Time-series plots of Firm α monthly PO volumes



Note : Total PO volumes from all buyers, with the volume in April 2006 equal to 100.

top nine buyers ($i = 1, 2, \cdots 9$) and aggregated remainder of grinding machines and cutting tools ($i = 10, 11$). For Firm α, we model the POs of the buyers that are ranked in the top 50 PO volumes ($i = 1, 2, \cdots 50$) and that of the remainder ($i = 51$).

3 Model

This section provides a structural model for credit risk assessment based on PO information, which is called the PO-based structural model. We provide a simple modeling framework that clearly shows the steps to derive asset value from PO information. Therefore, our model is simpler than the actual practical accounting processes of generating firm assets from a PO.

The target firm for credit risk assessment is the seller-side of purchase orders. First, we construct a time-series model of PO volume transition for each buyer. There are two components of a time-series model of PO volume transition: a model of the probability of receiving a PO at each time and a model of the size of the PO volume received. Then, the proceeds of sales are calculated by the sum of the POs with a time lag between receiving POs and collecting the proceeds of sales. Moreover, we calculate the cost of producing and supplying products. Then, we obtain profits or losses by calculating the difference between the sales and costs. Finally, we calculate the asset value by the discounted present value of future profits and losses. The firm defaults when the asset value falls below the debt amount.

3.1 Model of received PO volume

In this subsection, we introduce a model for time-series transition of PO volumes. We consider the discrete time space $\mathcal{T} = \{0, 1, 2, \cdots, \infty\}$ associated with monthly PO samples described in Section 2. We denote the set of corresponding buyers by $\mathcal{I} = \{1, 2, 3, \cdots, I\}$. We denote the time at which the j-th PO from buyer i arrives by $h^i(j)$. Then, the time sequence of PO arrivals is $H^i = \{h^i(1),\ h^i(2),\ \ldots\} \subseteq \mathcal{T}$. We write $\{O^i_t\}_{t \in \mathcal{T}}$ for PO volumes ordered by buyer $i \in \mathcal{I}$ at time t, which are a stochastic process.[6] Here, $O^i_t = 0$ in the case of $t \notin H^i$.

In order to identify PO arrival time $h^i(j)$, we model the probability of a PO arrival at time t $\Pr(h^i(j) = t | h^i(j-1) < t)$. In particular, we employ a logit model as follows:

$$\Pr(h^i(j) = t | h^i(j-1) < t,\ \{x^i_k(t)\}_{k=1}^K) = \frac{1}{1 + \exp(-(A^i + \sum_{k=1}^K B^i_k x^i_k(t)))} \tag{1}$$

Here, $\{x^i_k(t)\}_{k=1}^K$ are the explanatory variables and A^i and $\{B^i_k\}_{k=1}^K$ are coefficients.

Now, we illustrate some estimation results for the PO arrival probability model (1) for buyers of Hikari Kikai Seisakusho Co., Ltd. The in-sample span is from January 2011 to April 2016. The target buyers in the model estimation are PO time-series of 5 buyers ($i = 5, 6, 7, 8, 9$), whose historical PO has arrived intermittently. We employ some candidate explanatory variables as follows: " number of PO arrival times in last 1 year," "number of PO arrival times in last 3 months," "number of PO arrival times in last 6 months," "presence or absence of PO arrival in last month," and "time-span from the last PO arrival $(t - h^i(j-1) - 1)$." Then, we execute the model selection using the Akaike information criterion (AIC). As a result of the model selection, we select the model that has "number of PO arrival times in last 1 year." Table B-3 in AppendixB shows the estimated parameter values. These results shows a positive

[6] In our model, we model PO volume transactions for each buyer. It is possible to model transactions separately by products.

estimated value of coefficients B^i of "number of PO arrival times in last 1 year." While the result is not statistically significant, it implies that the probability of PO arrival at time t increases when the number of PO arrival times in the last year increases.

In order to specify the size of PO volumes when the PO arrives, we introduce a time-series model with a 1-month difference of log-PO volumes, that is, $R^i_j = \log(O^i_{h^i(j)}) - \log(O^i_{h^i(j-1)})$. [7] PO volumes among buyers are correlated if buyers belong to the same category of businesses. We consider this in the model. Specifically, we employ the time-series model of $R^i_j = \log(O^i_{h^i(j)}) - \log(O^i_{h^i(j-1)})$, described as follows:

$$R^i_j = \alpha_i + \sum_{\ell=1}^{L} \beta^i_\ell \tilde{x}^i_\ell(h^i(j)) + \sigma_i \left(\rho_i W_j + \sqrt{1-\rho_i^2}\epsilon_{i,j} \right) \tag{2}$$

Here, $W_j \sim N(0,1)$, $\epsilon_{i,j} \sim N(0,1)$ and these random variables are independent of time j and buyer i, where $N(m,v)$ is normal distribution with mean m and variance v. $\{\tilde{x}^i_\ell(h^i(j))\}_{\ell=1}^L$ are the covariates and α_i, $\{\beta^i_\ell\}_{\ell=1}^L$, σ_i, and ρ_i are parameters to be estimated. This model captures the correlation of PO volumes by common factor W_j, and the strength of the correlation is specified by the factor loading ρ_i. Idiosyncratic risks are captured by $\epsilon_{i,j}$.

We employ candidate covariates listed in Table 2. Then, we execute model selection using the AIC. Finally, we select the model that has "R^i_{j-12}" ($\tilde{x}^i_1(h^i(j))$) and "average of $\{R^i_{j-1}, R^i_{j-2}, \ldots R^i_{j-12}\}$" ($\tilde{x}^i_2(h^i(j))$); these covariates are significant.[8]

Table2: Candidate explanatory variables of the difference of log-PO volumes

$\tilde{x}^i_1$: R^i_{j-12}	$\tilde{x}^i_2$: average of $\{R^i_{j-1}, R^i_{j-2}, \ldots R^i_{j-12}\}$
$\tilde{x}^i_3$: average of $\{R^i_{j-1}, R^i_{j-2}, R^i_{j-3}\}$	$\tilde{x}^i_4$: average of $\{R^i_{j-1}, R^i_{j-2}, \ldots R^i_{j-6}\}$
$\tilde{x}^i_5$: R^i_{j-1}	$\tilde{x}^i_6$: average of $\{R^i_{j-11}, R^i_{j-12}, R^i_{j-13}\}$
$\tilde{x}^i_7$: average of $\{R^i_{j-12}, R^i_{j-13}, R^i_{j-14}\}$	

The estimated parameters of the PO model are described in Tables 3, 4, and 5. We recognize that the estimated value of coefficients β^i_1 for Kojima Industry Corporation is larger than that of the other sample firms. This result implies the existence of seasonality of PO volume transitions for Kojima Industry Corporation. The estimated values of factor loading ρ_i imply the existence of correlation in PO volumes in our

[7] We apply the Dickey–Fuller test for the unit root test to the sample data of the difference of log-PO volumes. The results show that none of the time series of the difference of log-PO volumes has a significant unit root at the 5% significance level.

[8] We conduct the Kolmogorov–Smirnov (K–S) test, which tests the goodness of fit of a realized series of residuals and associated distribution $N(0,1)$. From the K–S test, we confirm that the amount of PO volumes of a buyer that is not rejected at less than the 5% significance level. In addition, the p-values of the Ljung–Box test show that there is no significant auto-correlation of the residuals, and the model is not rejected.

Table3: Statistics of estimated parameters of the PO model for Kojima Industries Corporation

	α^i	β_1^i	β_2^i	σ^i	ρ^i
mean	-0.0085	0.5659	-5.6699	0.1628	0.7410
standard deviation	0.0249	0.2136	2.4290	0.0514	0.1269
maximum value	0.0362	1.0484	-2.3035	0.2905	0.9762
minimum value	-0.0438	0.3021	-10.4084	0.1104	0.6098

Table4: Statistics of estimated parameters of the PO model for Hikari Kikai Seisakusyo Co., Ltd.

	α^i	β_1^i	β_2^i	σ^i	ρ^i
mean	-0.1055	0.2816	-4.8882	1.2612	0.3046
standard deviation	0.1981	0.3129	1.7821	0.9883	0.4188
maximum value	0.0177	0.9354	-3.4014	2.2782	1.0000
minimum value	-0.5371	0.0183	-8.6621	0.1956	-0.1361

Table5: Statistics of estimated parameters of the PO model for Firm α

	α^i	β_1^i	β_2^i	σ^i	ρ^i
mean	0.0203	0.2750	-5.0536	0.6840	0.2784
standard deviation	0.0571	0.1481	2.4209	0.3368	0.1610
maximum value	0.2908	0.6807	-1.0835	1.7353	0.5746
minimum value	-0.1266	-0.0646	-10.5890	0.2852	-0.0609

sample data. Tables B-1 , B-4 , and B-6 in AppendixB show the estimated parameter values.

PO volumes $\{O^i_{h^i(j)}\}_{t\in\mathcal{T}}$ are obtained from $\{R^i_j\}$ by

$$O^i_{h^i(j)} = \left\{O^i_{h^i(j-1)} \times \exp(R^i_j)\right\} 1_{\{t\le T_i\}} \tag{3}$$

Here, T_i is the time to break off business connections with buyer i. We assume the break-off of business connections occurs only when the buyer defaults. Then, T_i is the default time of buyer i.

Default models of buyers with default correlations among buyers are considered to capture the risk of losing POs. Buyer defaults are modeled using a Merton-type one-factor model. The PD until time $t+1$, under the condition that buyer i survives at least until time t, is obtained as the probability that the credit quality X^i falls below default barrier Q^i. Credit quality X^i is given by

$$X^i = \tilde{\rho}_i \tilde{W} + \sqrt{1 - \tilde{\rho}_i^2}\tilde{\epsilon}_i, \tag{4}$$

$$\tilde{W} \sim N(0,1), \ \tilde{\epsilon}_i \sim N(0,1) \tag{5}$$

and these random variables are independent of time. Then, the PD of buyer i is

$$\Pr(X^i < Q^i) = \Phi(Q^i) \tag{6}$$

where $\Phi(\cdot)$ is the cumulative normal distribution. The default correlation among buyers is captured by common factor $\tilde{W}$, and the strength of the default correlation is specified by $\tilde{\rho}$. The default correlation between buyer i and j is obtained by calculating $\tilde{\rho}_i \tilde{\rho}_j$ with the model. The joint PD is calculated by

$$\Pr(X^1 < Q^1, \ X^2 < Q^2, \ \cdots, \ X^I < Q^I) = \int_{-\infty}^{\infty} \prod_{i=1}^{I} \Phi\left(\frac{Q^i - \tilde{\rho}_i w}{\sqrt{1 - \tilde{\rho}_i^2}}\right) \phi(w) \mathrm{d}w \tag{7}$$

where $\phi(\cdot)$ is the density of normal distribution.

Default barriers Q^i are estimated according to eq.(6) with the historical PDs of the associated credit rating of buyer i. For no-rated buyers, we assume the ratings as BBB. We estimate the default correlations among buyers with the stock price data for the buyers. For non-listed buyers, we employ TOPIX sector indexes of the corresponding sector. We estimate the parameters by minimizing the sum of the square difference between the historical correlation matrix of stock prices and the correlation matrix obtained by factor loadings.[9] Estimates of factor loading are shown in Tables B-2 , B-5 , and B-7 in AppendixB. They show that there are default correlations among some buyers, since $\tilde{\rho}_i \tilde{\rho}_j$ exceed 0.4 with several pairs of (i, j).

3.2 Calculation of Earnings

Next, we calculate proceeds of sales obtained from associated PO volumes. We assume that the firm receives proceeds of sales amounting to PO volumes after some time lags if there are no problems with product delivery and sales collection. If there are canceled POs or buyer defaults before sales collections, cash below the PO amount is collected by cancellation charges and recovery-given default. Thus, proceeds of sales at time t are given by

$$S_t = \sum_{i=1}^{I} \left(O_{t-m}^i 1_{\{t < T_i\}} + (1 - LGD^i) O_{t-m}^i 1_{\{t \geq T_i\}} \right). \tag{8}$$

[9] Using asset correlation is desirable, but we employ stock correlation for the data restriction.

Constant $m \geq 0$ is the time lag between the arrival of the POs and collection of proceeds of sales.[10] Constant LGD^i is the rate of loss that occurs when the POs of buyer i are canceled or buyer i defaults.

As we assume the break-off of business connections occurs only when the buyer defaults, T_i indicates that the default time of buyer i and LGD^i equals the loss rate given the default. We set the value of LGD^i conservatively as $LGD^i = 1$. We set $m = 2$ for Kojima Industries Corporation and $m = 5$ for others.[11]

To obtain profits and losses from proceeds of sales, we calculate operating costs, which we give as a function of PO volumes.

$$C_t = f\left(\{O^i_{t-g}\}_{i\in\mathcal{I}}\right). \tag{9}$$

Here, function $f : \mathbb{R}^I \to \mathbb{R}$ is a cost function, and constant g is the time lag between PO arrival and corresponding cost defrayment.[12]

We simply assume that the function of operating costs is given by the linear function.

$$f\left(\{O^i_{t-g}\}_{i\in\mathcal{I}}\right) := a\sum_{i=1}^{I} O^i_{t-g}1_{\{t<T_i\}} + b. \tag{10}$$

Here, a is a constant parameter indicating the variable cost ratio and b is a constant parameter indicating fixed cost. To estimate the parameters of the above function (10), we use the historical annual proceeds of sales (PO volumes) from the firm profit and loss statements (P/L). With the estimated operating cost function, according to eq.(9), we obtain operating costs. Here, we suppose that operating costs arise when the account of sales is raised. We set the time lag between receiving the POs and occurrence of operating costs to 0 months ($g = 0$) for Kojima Industries Corporation, 3 months ($g = 3$) for Hikari Kikai Seisakusho Co., Ltd., and 1 month ($g = 1$) for Firm α. Operating profits and losses P_t are obtained by the difference in proceeds of sales and operating costs. Earnings before tax (EBT) are obtained by adding operating profits and losses, non-operating profit and losses, and extraordinary profit and losses.

$$EBT_t = P_t + \bar{P}_t = S_t - C_t + \bar{P}_t$$

Here, $\bar{P}_t$ is the sum of non-operating profit and losses and extraordinary profit and losses at time t and a given.[13] Then, net earnings are obtained by adjusting tax

[10] The time lag between the arrival of the PO and the realization of sales depends on products and seasons. However, we set the time lag as constant for simplicity.

[11] We set average time lag based on interviews with sample firms and compare the trade accounts receivable described in the balance sheets and monthly PO volumes. For example, we set $m = 2$ in the case that trade accounts receivable are equivalent to 2 months' PO volumes, for example, trade accounts receivable are 10 and monthly POs are 5.

[12] A negative (positive) value implies cash in (out).

[13] In our empirical study in Section 4, we do not consider extraordinary profit and losses.

payments to EBT:

$$E_t = EBT_t 1_{\{EBT_t<0\}} + (1-G)EBT_t 1_{\{EBT_t \geq 0\}} \tag{11}$$

Here, constant G denotes the corporate tax rate and we set $G = 0.4$.

3.3 Valuation

We assume that there are no dividends to shareholders, and the net earnings are internally retained and then added to non-business assets. If net earnings are negative, non-business assets decrease by the same amount. The additional asset value resulting from net earnings from time 0 to time t is given by $\sum_{s=0}^{t} E_s$. Furthermore, retained net earnings are not invested in the business. $\tilde{V}_t$ represents non-business assets except obtained earnings. Then, asset value at time t is obtained by the sum of the present value of future earnings, non-business assets, and obtained earnings:

$$V_t = \bar{V}_t + \tilde{V}_t + \sum_{s=0}^{t} E_s$$

where

$$\bar{V}_t = \sum_{s=t}^{\infty} \frac{\mathbb{E}\left[P_s|\mathcal{F}_t\right]}{(1+r)^{s-t}} \tag{12}$$

Here, r is the weighted average cost of capital (WACC) of the sample firms. $\mathbb{E}\left[\cdot|\mathcal{F}_t\right]$ denotes conditional expectation under the information $\mathcal{F}_t$ obtained by time t.

For the empirical analysis in Sec 4, we calculate asset values as follows. First, we calculate the preset value of earnings by

$$PV_t = \sum_{s=t}^{M-1+t} \frac{\mathbb{E}[P_s|\mathcal{F}_t]}{(1+r)^{s-t}} + \frac{\mathbb{E}[P_{M+t}|\mathcal{F}_t]}{r(1+r)^{M-1}} \tag{13}$$

In our empirical analysis in Sec 4, we set $M = 12$. AppendixC shows the process for calculating PV_t. Then, we adjust the obtained present value by the book value of assets:

$$\bar{V}_t = PV_t \times \frac{BV_0}{PV_0} \tag{14}$$

Here, BV_0 is the book value of business assets at $t = 0$. This adjustment is necessary because the present value calculated by (13) becomes quite large in some simulation scenarios in our empirical analysis. We assume that the value of extra asset $\tilde{V}_t$ is constant and set the value by the non-business asset value obtained from the last balance sheet. We obtain the WACC for the sample firms using the capital asset pricing model.

Then, the default time is the first time that the asset value falls below the debt amount:

$$\tau = \inf\{t \in \mathcal{T}\backslash\{0\} | V_t < D_t\}$$

In the case of no time point t, such that $V_t < D_t$, we set $\mathcal{T} = \infty$. Here, D_t denotes the amount of debt at time t and is updated when the latest financial statements of the sample firms are disclosed.

4 Case Study

This section illustrates the empirical results for estimating the PDs of sample firms.

4.1 Settings

We calculate the 1-year forward PD based on our model for every month of the out-of-sample span. The out-of-sample span is from January 2014 to December 2014 for Kojima Industries Corporation, from May 2016 to April 2017 for Hikari Kikai Seisakusho Co., Ltd., and from April 2016 to March 2017 for Firm α.

We estimate the PDs using Monte Carlo simulation. We simulate the future PO volumes, calculate the corporate values, and obtain the 1-year forward PDs of the sample firms. The simulation is executed with 100,000 trials every month. We count the number of default trials in which the net capital becomes negative, and the PDs are obtained by the ratio of the number of default trials to the number of all trials. For the debt amount D_t, we employ current liabilities on the balance sheets of the sample firms.

We set the PO arrival probability to $\Pr(h^i_j = t | h^i_{j-1} < t) = 1$ ($^\forall t \in \mathcal{T}$) for the buyers of which PO arrival is successive in the in-sample span. For the buyer whose PO arrival is intermittent in the in-sample span, we employ the PO arrival probability obtained by the probability model. In detail, we set $\Pr(h^i_j = t | h^i_{j-1} < t) = 1$ for all buyers of Kojima Industries Corporation and Firm α and the top three buyers for Hikari Kikai Seisakusho Co., Ltd.[14] In addition, a buyer ($i = 4$) for Hikari Kikai Seisakusho Co., Ltd. orders constantly in the out-of-sample, so that we assume $\Pr(h^i_j = t | h^i_{j-1} < t) = 1$.

For some buyers with few PO samples, we have difficulty estimating the model. In that case, we obtain the future PO volumes of the buyers by the in-sample average of PO volumes. The total PO volumes of the remaining buyers[15] are given by log-normal distribution estimated with the in-sample data.

We estimate the model parameters at the beginning of the out-of-sample period and use the estimated parameter values for the whole out-of-sample period. The case

[14] Although there is a risk of stopping received PO except the defaults of buyers in the future, we do not consider that.

[15] $i = 10$ for Kojima Industries Corporation, $i = 10, 11$ for Hikari Kikai Seisakusho Co., Ltd., and $i = 51$ for firm α.

of re-estimating the parameters in every time point of the out-of-sample period is reported in AppendixA.

4.2 Results

Figures 4, 5, and 6 show the estimated 1-year forward PDs under the realized PO scenario. The range of estimated PDs is 0.027–0.107% for Kojima Industries Corporation, 0.013–0.026% for Hikari Kikai Seiakusho Co., Ltd., and 0.000–0.002% for Firm α. For all the sample firms, the level of estimated PDs is quite low, mainly due to the high credit quality (low default risk) of the buyers and the stability of the PO volume of Kojima Industries Corporation, the high profit level of Hikari Kikai Seisakusho Co., Ltd., and the diversified buyer portfolio(Figure7) of Firm α.

In addition, we recognize that the estimated PDs increase (decrease), reflecting the decrease (increase) in the PO volumes, in Figure 4, 5. Here, we check the correlation coefficient between PO volumes and PD for both firms. It is -0.79 for Kojima Industries Corporation and -0.63 for Hikari Kikai Seisakusho Co., Ltd., so that there is negative correlation between PO volumes and PD.

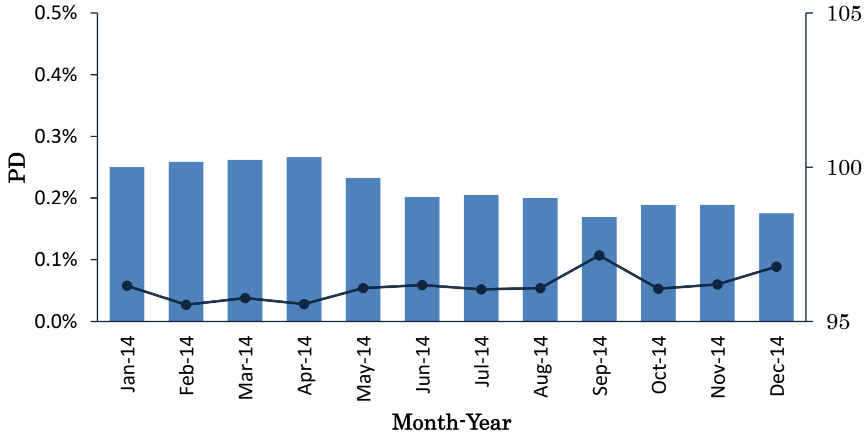
For the comparison, we estimate PD with the alternative model, which uses only financial statements for input. The model is provided by Teikoku Databank, Ltd. (TDB), which is one of the largest corporate credit research companies in Japan. The estimated PD by the TDB model is 0.213% for Kojima Industries Corporation, 0.153% for Hikari Kikai Seiakusho Co., Ltd., and 0.710% for Firm α. Thus, the estimated PD obtained by our model is lower than that of the TDB model. These results show that positive factors for the business circumstances of the sample firms, such as the high credit quality of buyers and diversification of buyers, are reflected in the model.[16]

In order to confirm that the changes in PO information have effects on estimated PD, we estimate the PDs under the stressed PO scenarios. Figure 8 shows the estimated PDs under the stressed PO scenarios in which the credit rating of buyers declines in December 2014, the PO volume decreases (increases) by 5% in November 2014. Figure 8 shows that the down-grades of buyers immediately cause an increase of estimated PDs, and decreases (increases) in PO volumes also immediately cause an increase (decrease) of estimated PDs. These results show that credit risk modeling based on PO information enables financial institutions to monitor the credit risk affected by a change in business conditions of borrower firms, such as PO volumes and the credit quality of buyers, on a real-time basis.

In addition, we calculate the impact of PO concentration on specific buyers to estimated PD. We calculate the conditional PD in the case in which the top share buyer of Firm α defaults under the scenario in which the PO from the top share buyer A increases. As a result, the conditional PD increases when the PO from the top share buyer increases (Figure 9).

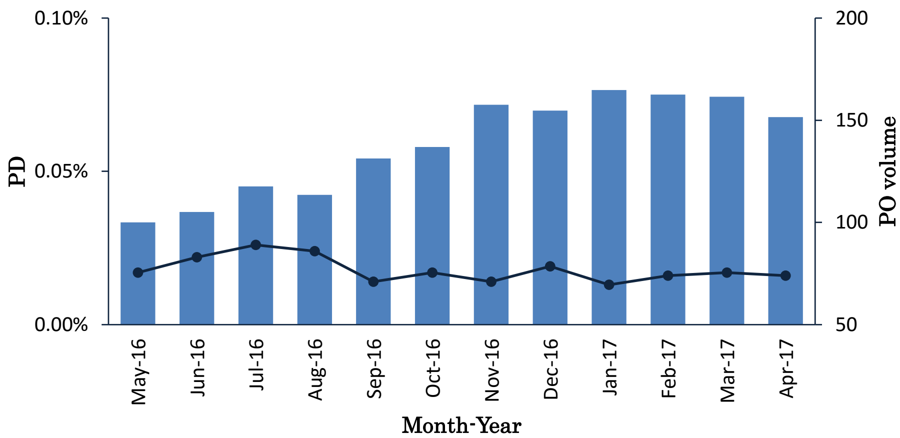
[16] It is possible that the PD estimate is lower in our model, since there are not many stresses that have a great influence on managing businesses in the in-sample data.

Figure4: Estimated 1-year PDs (solid line) and cumulative 1-year PO volumes observed each month (bar plots) for Kojima Industries Corporation



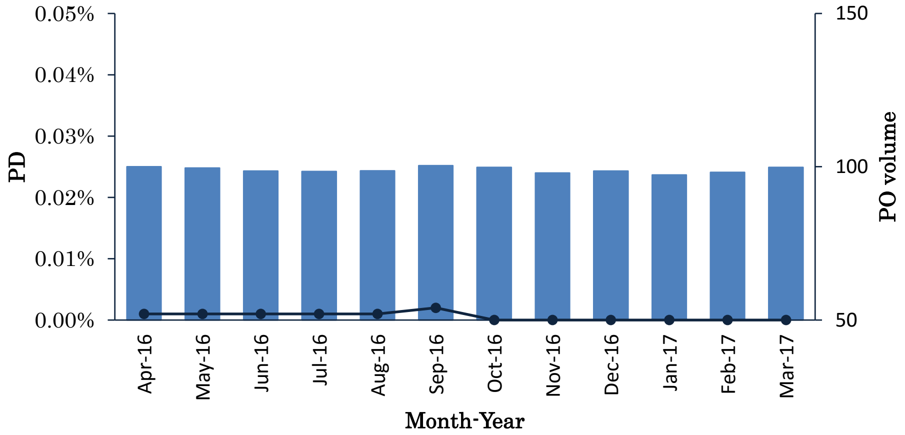
Note : The plotted cumulative 1-year PO volumes are standardized by setting the cumulative 1-year volume in June 2011 as 100.

Figure5: Estimated o1year PDs (solid line) and cumulative 1-year PO volumes observed each month (bar plots) for Hikari Kikai Seisakusho Co., Ltd.



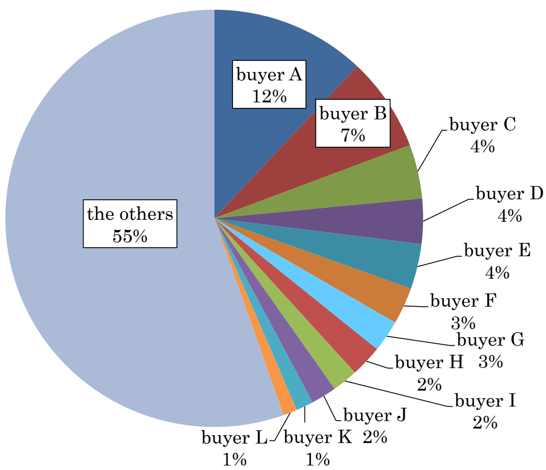
Note : The plotted cumulative 1-year PO volumes are standardized by setting the cumulative 1-year volume in May 2016 as 100.

Figure6: Estimated 1-year PDs (solid line) and cumulative 1-year PO volumes observed each month (bar plots) for Firm α



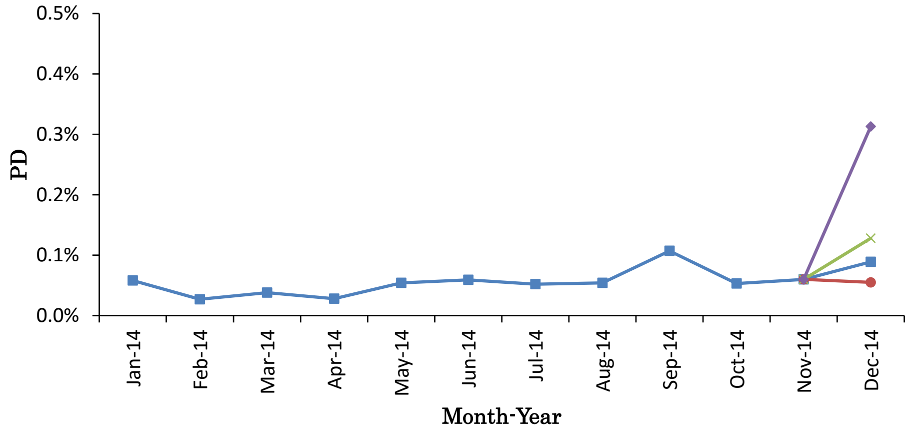
Note : The plotted cumulative 1-year PO volumes are standardized by setting the cumulative 1-year volume in April 2016 as 100.

Figure7: PO volume share of each buyer for Firm α



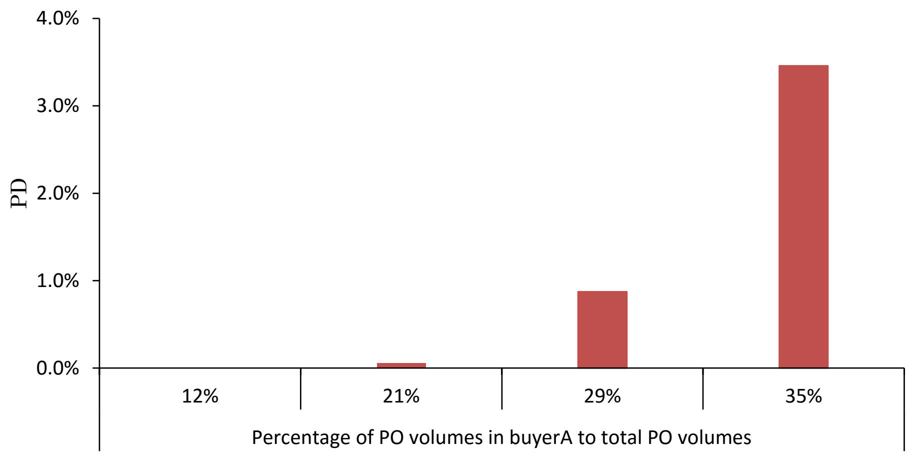
Note : PO volumes mean the sum of PO volumes in the in-sample term. The maximum share is about 12% and the sum of PO volumes of buyers whose share is less than 1% accounts for 55%.

Figure8: Estimated 1-year PDs under scenarios in which last PO volumes increase (decrease) by 5% and buyers are downgraded



Note : The solid line with a rhombic shows estimated PDs under the stress scenario of simultaneous one-rank downgrade of the buyers in December 2014. The solid line with a cross shows estimated PDs under the stress scenario of a 5% decrease of the PO volumes in November 2014. The solid line with a square shows the results of Figure 4. The solid line with a dot shows the estimated PDs under the stress scenario of a 5% increase of the PO volumes in November 2014.

Figure9: Change of estimated 1-year PDs for Firm α in the case of a PO volume increase from buyer A



Note : The conditional PD is calculated under scenarios in which the PO volumes double, triple, and quadruple, that is, the share of PO volumes from buyer A becomes 21%, 29%, and 35% and then buyer A defaults.

Table6: All default scenarios simulated in April 2017

Scenario label number	The factor of default
31404, 44324, 52588, 77289	Default of all of the top 8 buyers
10488, 98520	Default of 7 of the top 8 buyers
18754, 38621	Default of 7 of the top 8 buyers
8197	Default of 7 of the top 8 buyers
61110	Default of 7 of the top 8 buyers
35167	Default of 6 of the top 8 buyers
69402	Default of 6 of the top 8 buyers
197, 34790, 44375	Default of 5 of the top 8 buyers
46370	Default of 4 of the top 8 buyers

Note : Even though default number of buyers is the same in many scenarios, we separate the scenario number in accordance with the combination of default buyers.

We are able to confirm the situations of defaults of the sample firms by checking the default scenarios. Table 6 shows all the default scenarios in which Hikari Kikai Seisakusho Co., Ltd. defaults. This result implies that the Hikari Kikai Seisakusho Co., Ltd. defaults only in the case of joint defaults of several buyers.

5 Conclusion

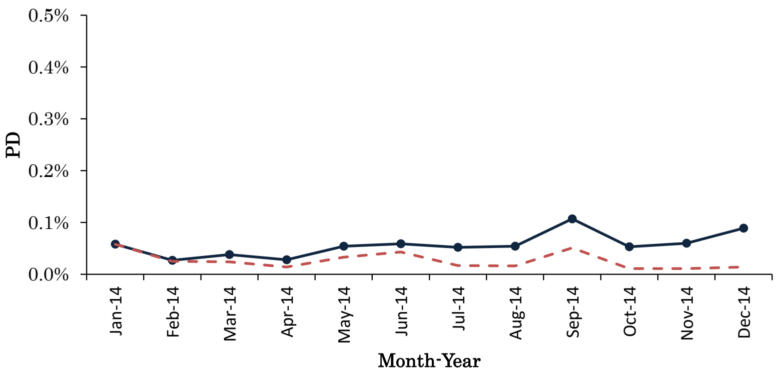
We proposed a structural credit risk model based on PO information and presented a case study on credit risk assessment using the model. In order to deal with intermittent PO arrivals, we constructed a time-series model of PO volume transaction that is a combination of the model of the probability of PO arrivals with the model of sizes of received PO volumes. Specifically, we employed the logit model for PO arrival probability and auto-regressive time-series model for the size of arrived PO. In this model, we introduced correlation of PO volumes among corresponding buyers and default risks of buyers. We obtained the distribution of asset value by simulation with our model and estimated the PDs of the sample firms. The results show that the credit risk model based on PO information enables financial institutions to monitor credit risk affected by changes in borrower firms' business conditions, such as PO volumes and their buyers' credit quality, on a real-time basis. In addition, we confirmed that the concentration risk of received PO, namely, increasing the share of PO volumes from a specified buyer, increases the PD.

Although we constructed a type of structural credit risk model in this study, we consider it possible to establish a model that grasps the correlation with PD and credit score statistically if we were to obtain more data of purchase order receipts and PD records. The results of the case study reveal that the credit risk assessment using PO information enables us to realize credit risk assessment reflecting firms' business conditions on a real-time basis. If borrower firms supply PO information to financial institutions regularly, the borrower firms could be offered appropriate business support from financial institutions on a timely basis. We hope that risk assessment is exercised multidirectionally, employing the risk assessment method using PO information, in the future.

AppendixA Model Sensitivity on the in-sample span

We check the model sensitivity regarding the span of the in-sample data. In particular, we reestimate our model with the increase of in-samples associated with updating the risk assessment time point. Figure A-1 , A-2 , and A-3 show the estimated PD obtained with the reestimated model. We can recognize that the PD is estimated stably even though the model parameters are renewed by adding new in-sample data.

FigureA-1 : Estimated 1-year PDs in the case of fixed parameters (solid line) and renewed parameters (broken line) for Kojima Industries Corporation



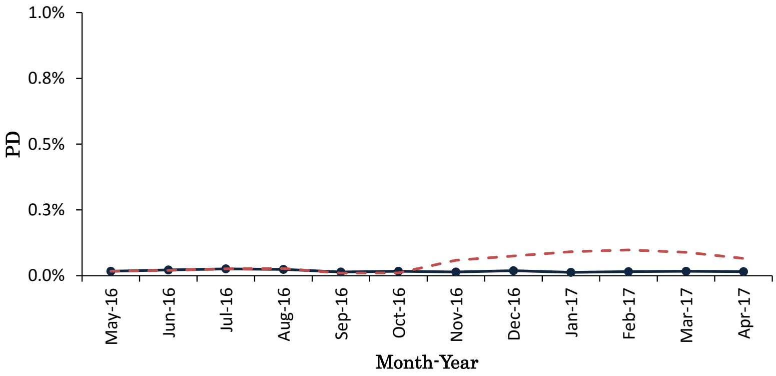
AppendixB Estimated parameter values

We describe the model parameter values for the analysis in this section.

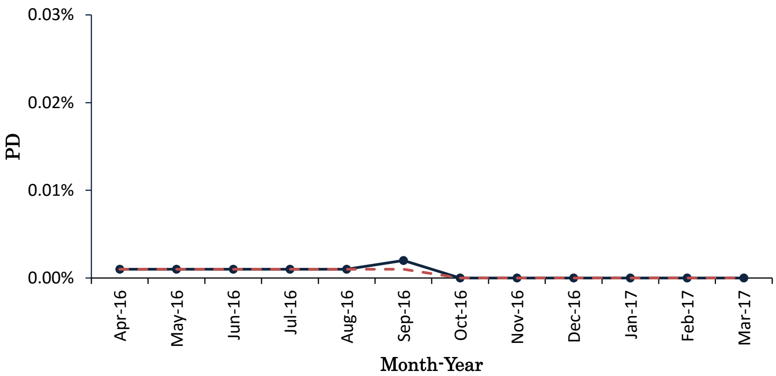
■Kojima Industries Corporation We execute the estimation of the parameters of the PO volume model with the free statistical software package `R`. In particular, we use the intrinsic function "`lm`" to obtain parameter values. Table B-1 shows the estimated values of coefficients of model (2). Table B-2 shows the estimated factor loading of model (4).

■Hikari Kikai Seisakusho Co., Ltd. We execute the estimation of the PO arrival probability model (1) with the intrinsic function "`glm`" of `R`. Table B-3 presents this result. Table B-4 shows the estimated parameter value of model (2) and Table B-5 shows

FigureA-2 : Estimated 1-year PDs in the case of fixed parameters (solid line) and renewed parameters (broken line) for Hikari Kikai Seisakusho Co., Ltd.



FigureA-3 : Estimated 1-year PDs in the case of fixed parameters (solid line) and renewed parameters (broken line) for Firm α



TableB-1 : Estimated parameter values of model (2) for Kojima Industries Corporation

i	1	2	3	4	5	6	7	8	9
α^i	-0.0293	-0.0438	0.0066	0.0362	-0.0191	0.0168	-0.0188	0.0070	-0.0323
	(0.0292)	(0.0432)	(0.0271)	(0.0452)	(0.0281)	(0.0365)	(0.0693)	(0.0449)	(0.0365)
β_1^i	0.7521	0.4752	1.0484	0.3021	0.3189	0.5465	0.5952	0.5337	0.5206
	(0.2022)	(0.2544)	(0.1355)	(0.2101)	(0.1785)	(0.2812)	(0.2923)	(0.2644)	(0.1933)
β_2^i	-8.7806	-4.6080	-6.3292	-6.2232	-2.3035	-4.5993	-10.4084	-4.2583	-3.5181
	(2.7175)	(2.0594)	(1.8379)	(2.6480)	(1.5058)	(2.4208)	(3.1692)	(2.3652)	(2.4506)
σ^i	0.1197	0.1681	0.1104	0.1687	0.1187	0.1502	0.2905	0.1871	0.1521
	(0.0042)	(0.0086)	(0.0027)	(0.0073)	(0.0032)	(0.0046)	(0.0164)	(0.0099)	(0.0043)
ρ^i	0.8139	0.9762	0.6098	0.6411	0.6564	0.8023	0.6433	0.8965	0.6293
	(0.3362)	(0.3478)	(0.3257)	(0.3270)	(0.3277)	(0.3355)	(0.3271)	(0.3417)	(0.3265)

Note : Bracketed figures are estimated errors.

TableB-2 : Estimated factor loading of model (4) for Kojima Industries Corporation

i	1	2	3	4	5	6	7	8	9
$\tilde{\rho}_i$	0.8495	0.8767	0.8767	0.8253	0.8360	0.7114	0.7153	0.9024	0.8266
	(0.3080)	(0.3114)	(0.3114)	(0.3048)	(0.3064)	(0.2983)	(0.3005)	(0.3085)	(0.3055)

Note : Bracketed figures are estimated errors.

the estimated factor loading of model (4).

TableB-3 : Estimated parameter values of model (1) for Hikari Kikai Seisakusho Co., Ltd

i	5	6	7	8	9
A^i	−0.5328	−2.9269	−0.7082	−1.4960	−1.9683
	(1.0148)	(0.7497)	(0.8914)	(1.4150)	(0.5404)
B^i	0.1625	2.7371	1.6385	3.2658	4.3424
	(2.5187)	(8.3329)	(1.4812)	(2.0000)	(1.2240)

Note : Bracketed figures are estimated errors.

TableB-4 : Estimated parameter values of model (2) for Hikari Kikai Seisakusho Co., Ltd

i	1	2	3	5	7	8
α^i	0.0101	−0.0281	0.0151	−0.5371	−0.1103	0.0177
	(0.0521)	(0.0364)	(0.0278)	(0.7158)	(0.4697)	(0.4136)
β_1^i	0.1405	0.3523	0.2069	0.9354	0.0364	0.0183
	(0.1404)	(0.1589)	(0.1454)	(0.2505)	(0.2076)	(0.2100)
β_2^i	−5.1437	−3.9765	−3.4014	−8.6621	−3.6390	−4.5065
	(1.6169)	(1.3223)	(1.1895)	(3.4605)	(1.9816)	(2.3560)
σ^i	0.3715	0.2560	0.1956	2.2780	2.2490	2.2170
	(0.0236)	(0.0113)	(0.0061)	(1.2585)	(1.2449)	(1.2874)
ρ^i	0.1569	0.0042	−0.1361	0.7506	1.0000	0.0522
	(0.5687)	(0.5574)	(0.5659)	(1.0465)	(3.0533)	(0.5586)

Note : Bracketed figures are estimated errors. The buyers that are not mentioned in this table do not have enough PO information to model (2), so that we set their future PO volume to the PO volume average during the in-sample term.

TableB-5 : Estimated factor loading of model (4) for Hikari Kikai Seisakusho Co., Ltd.

i	1	2	3	4	5	6	7	8	9
$\tilde{\rho}_i$	0.9479	0.9479	0.9479	0.8630	0.9479	0.9768	0.9479	0.8829	0.9768
	(0.3501)	(0.3501)	(0.3501)	(0.3565)	(0.3501)	(0.3488)	(0.3501)	(0.3354)	(0.3488)

Note : Bracketed figures are estimated errors.

■Firm α Table B-6 shows the estimated parameter values of model (2). Table B-7 shows the estimated factor loading of model (4)

TableB-6 : Estimated parameter values of model (2) for Firm α

i	1	2	3	4	5	6	7	8	9	10	11	12	13
α^i	−0.0007	0.0075	0.0071	0.0577	0.0152	0.0023	0.0905	−0.0001	−0.0030	0.0284	0.0065	−0.0104	0.0030
	(0.0410)	(0.0373)	(0.0313)	(0.0819)	(0.0659)	(0.0413)	(0.0881)	(0.0706)	(0.0363)	(0.0651)	(0.0948)	(0.0320)	(0.0339)
β_1^i	0.2481	0.1350	−0.0431	0.0921	0.2580	0.2558	0.4203	0.2017	0.0493	0.0748	−0.0646	0.2636	0.6124
	(0.0830)	(0.1009)	(0.0964)	(0.0995)	(0.0868)	(0.0908)	(0.0946)	(0.0871)	(0.0980)	(0.1087)	(0.1402)	(0.0974)	(0.0934)
β_2^i	−5.2194	−2.5264	−1.8117	−5.2050	−5.9571	−3.1703	−8.0340	−4.4793	−1.0835	−4.4849	−2.0417	−2.9849	−9.1956
	(0.8476)	(0.8115)	(0.6511)	(0.9598)	(0.9517)	(0.9060)	(1.1435)	(0.8970)	(0.4748)	(0.9932)	(1.1979)	(0.8222)	(1.1903)
σ^i	0.4228	0.3855	0.3240	0.8425	0.6812	0.4269	0.9002	0.7296	0.3748	0.6722	0.5635	0.3296	0.3511
	(0.0231)	(0.0200)	(0.0181)	(0.1038)	(0.0765)	(0.0298)	(0.1138)	(0.1187)	(0.0204)	(0.1068)	(0.1081)	(0.0256)	(0.0205)
ρ^i	0.4987	0.4484	0.3637	0.1736	0.3591	0.4479	0.1697	0.2508	0.4043	0.3773	−0.0456	0.5330	0.2824
	(0.3196)	(0.3180)	(0.3158)	(0.3127)	(0.3157)	(0.3180)	(0.3126)	(0.3137)	(0.3168)	(0.3162)	(0.3118)	(0.3207)	(0.3142)
i	14	15	16	17	18	19	20	21	22	23	24	25	26
α^i	0.0049	−0.0184	0.0057	0.0197	0.0476	−0.0114	−0.0130	−0.0414	−0.1266	−0.0090	0.0514	−0.0076	−0.0078
	(0.0555)	(0.0445)	(0.0477)	(0.0699)	(0.1089)	(0.0278)	(0.0664)	(0.0676)	(0.1419)	(0.0540)	(0.1678)	(0.0651)	(0.0277)
β_1^i	0.1370	0.1647	0.2021	0.1888	0.4218	0.2859	0.2232	0.3458	0.1740	0.3988	0.3772	0.4377	0.3019
	(0.1049)	(0.0880)	(0.0981)	(0.0995)	(0.0833)	(0.0921)	(0.0957)	(0.0950)	(0.1003)	(0.0931)	(0.0891)	(0.0904)	(0.0927)
β_2^i	−5.0541	−3.3931	−3.4993	−4.2983	−6.8196	−1.9879	−3.5264	−4.1188	−5.9825	−4.7029	−7.7721	−10.5890	−3.4888
	(1.0008)	(0.8118)	(0.8677)	(0.9435)	(1.0156)	(0.6611)	(0.8618)	(0.9609)	(1.0878)	(0.9940)	(1.0482)	(1.1448)	(0.8321)
σ^i	0.5738	0.4601	0.4934	0.7225	1.1233	0.2856	0.6861	0.6959	1.4618	0.5587	1.7353	0.6728	0.2862
	(0.0475)	(0.0363)	(0.0321)	(0.0674)	(0.2304)	(0.0147)	(0.1453)	(0.0746)	(0.3525)	(0.0482)	(0.4638)	(0.0746)	(0.0119)
ρ^i	0.3621	0.2220	0.4996	0.2492	0.1857	0.5746	0.3415	0.2300	0.2173	0.3956	0.2295	−0.0609	0.4796
	(0.3158)	(0.3133)	(0.3196)	(0.3136)	(0.3128)	(0.3223)	(0.3153)	(0.3134)	(0.3132)	(0.3166)	(0.3134)	(0.3119)	(0.3190)
i	27	28	29	30	31	32	33	34	35	36	37	38	39
α^i	0.0460	0.0176	0.0188	0.0235	0.1117	0.0546	−0.0096	−0.0284	−0.0023	0.0024	0.0036	0.1466	0.0344
	(0.1106)	(0.1201)	(0.0487)	(0.0973)	(0.0900)	(0.0571)	(0.0306)	(0.0575)	(0.0385)	(0.0315)	(0.0464)	(0.0832)	(0.0517)
β_1^i	0.4513	0.4802	0.2668	0.1944	0.2366	0.2910	0.4092	0.1693	0.6807	0.2100	0.1994	0.2292	0.4069
	(0.0852)	(0.0892)	(0.0881)	(0.1317)	(0.0939)	(0.0968)	(0.0917)	(0.1109)	(0.0767)	(0.0981)	(0.0951)	(0.0990)	(0.1066)
β_2^i	−7.4555	−7.8601	−2.4527	−4.3622	−7.9111	−7.6892	−1.4957	−4.1929	−7.3559	−3.1474	−1.7057	−4.776	−6.6051
	(1.0483)	(1.1275)	(0.7098)	(1.1848)	(1.1583)	(1.1378)	(0.6359)	(0.9447)	(1.1160)	(0.8090)	(0.5986)	(1.0276)	(1.3359)
σ^i	1.1432	1.2422	0.5036	0.7277	0.9186	0.5799	0.3158	0.5906	0.3982	0.3256	0.4803	0.7916	0.5183
	(0.1716)	(0.2053)	(0.0515)	(0.0783)	(0.1458)	(0.1146)	(0.0163)	(0.0531)	(0.0177)	(0.0148)	(0.0335)	(0.1314)	(0.0804)
ρ^i	0.0860	0.1403	0.1747	0.1174	0.1818	0.0108	0.4843	0.2700	0.3818	0.5667	0.4373	0.2142	0.0338
	(0.3120)	(0.3123)	(0.3127)	(0.3122)	(0.3128)	(0.3118)	(0.3191)	(0.3140)	(0.3163)	(0.3220)	(0.3177)	(0.3131)	(0.3118)
i	40	41	42	43	44	45	46	47	48	49	50		
α^i	0.0069	0.0242	0.0009	0.0696	0.2908	0.0787	0.0204	−0.0137	−0.0181	0.0594	−0.0212		
	(0.0276)	(0.0704)	(0.0809)	(0.1076)	(0.2177)	(0.0507)	(0.1168)	(0.0690)	(0.0308)	(0.0756)	(0.1273)		
β_1^i	0.3225	0.2056	0.1672	0.3422	0.3393	0.1674	0.5377	0.2645	0.2785	0.4241	0.3143		
	(0.0835)	(0.0973)	(0.0961)	(0.0913)	(0.1811)	(0.0948)	(0.0914)	(0.0964)	(0.0934)	(0.0885)	(0.0893)		
β_2^i	−1.3373	−4.0765	−3.7173	−5.8500	−4.9848	−5.4007	−10.0222	−7.4867	−4.1261	−9.2895	−7.9505		
	(0.5605)	(0.8855)	(0.8211)	(1.0122)	(1.7730)	(0.9544)	(1.2064)	(1.0788)	(0.9069)	(1.1489)	(1.0694)		
σ^i	0.2852	0.7270	0.8363	1.1069	1.1070	0.5111	1.2086	0.7128	0.3170	0.7773	1.3165		
	(0.0115)	(0.0672)	(0.0999)	(0.2522)	(0.2864)	(0.0435)	(0.2339)	(0.0642)	(0.0193)	(0.0940)	(0.2382)		
ρ^i	0.4370	0.3221	0.1866	0.3775	0.2585	0.1312	0.0717	0.0958	0.4239	0.1252	0.2000		
	(0.3177)	(0.3149)	(0.3128)	(0.3162)	(0.3138)	(0.3123)	(0.3119)	(0.3120)	(0.3173)	(0.3122)	(0.3130)		

Note : Bracketed figures are estimated errors.

TableB-7 : Estimated factor loading of model (4) for Firm α

i	1	2	3	4	5	6
$\tilde{\rho}_i$	0.8616	0.8702	0.5877	0.7802	0.7802	0.8997
	(0.1573)	(0.1574)	(0.1558)	(0.1568)	(0.1568)	(0.1576)
i	7	8	9	10	11	12
$\tilde{\rho}_i$	0.7489	0.8065	0.8223	0.6893	0.5877	0.8807
	(0.1566)	(0.1569)	(0.1570)	(0.1563)	(0.1558)	(0.1574)
i	13	14	15	16	17	18
$\tilde{\rho}_i$	0.9634	0.8193	0.7372	0.2752	0.3957	0.8083
	(0.1580)	(0.1570)	(0.1565)	(0.1548)	(0.1551)	(0.1569)
i	19	20	21	22	23	24
$\tilde{\rho}_i$	0.8807	0.9634	0.7242	0.8129	0.5877	0.7802
	(0.1574)	(0.1580)	(0.1565)	(0.1570)	(0.1558)	(0.1568)
i	25	26	27	28	29	30
$\tilde{\rho}_i$	0.5294	0.8807	0.8400	0.6461	0.7824	0.8193
	(0.1555)	(0.1574)	(0.1572)	(0.1560)	(0.1568)	(0.1570)
i	31	32	33	34	35	36
$\tilde{\rho}_i$	0.7870	0.7413	0.8807	0.7707	0.7666	0.8807
	(0.1568)	(0.1566)	(0.1574)	(0.1567)	(0.1567)	(0.1574)
i	37	38	39	40	41	42
$\tilde{\rho}_i$	0.9634	0.8616	0.8329	0.8807	0.8616	0.2550
	(0.1580)	(0.1573)	(0.1571)	(0.1574)	(0.1573)	(0.1547)
i	43	44	45	46	47	48
$\tilde{\rho}_i$	0.9634	0.5877	0.8807	0.6560	0.4947	0.8807
	(0.1580)	(0.1558)	(0.1574)	(0.1561)	(0.1554)	(0.1574)
i	49	50				
$\tilde{\rho}_i$	0.5324	0.8121				
	(0.1555)	(0.1570)				

Note : Bracketed figures are estimated errors.

AppendixC Calculation of business asset value

We illustrate the calculation method for business asset PV_t of eq. (13).

In order to calculate business asset value

$$PV_t = \sum_{s=t}^{M-1+t} \frac{\mathbb{E}[P_s|\mathcal{F}_t]}{(1+r)^{s-t}} + \frac{\mathbb{E}[P_M|\mathcal{F}_t]}{r(1+r)^{M-1}},$$

we calculate the expectation of PO and survival probability of buyers, considering

$$\mathbb{E}[P_s|\mathcal{F}_t] = \sum_{i=1}^{I} \mathbb{E}[O^i_{s-m}|\mathcal{F}_t]\Pr(s < T_i|\mathcal{F}_t) - a\sum_{i=1}^{I} \mathbb{E}[O^i_{s-g}|\mathcal{F}_t] - b$$

for $s = t, t+1, \cdots, M$. The survival probability is derived from eq. (6). In the case of $u \le t$, the expectation of PO $\mathbb{E}[O^i_u|\mathcal{F}_t]$ is given by the realized value of O^i_u. In the case of $u > t$, we derive

$$\begin{aligned}
\mathbb{E}[O^i_u|\mathcal{F}_t] = {} & \mathbb{E}[O^i_u|h^i(N^i_t+1) = u,\ \mathcal{F}_t]\Pr(h^i(N^i_t+1) = u|\mathcal{F}_t) \\
& + \mathbb{E}[O^i_u|h^i(N^i_t+2) = u,\ \mathcal{F}_t]\Pr(h^i(N^i_t+2) = u|\mathcal{F}_t) \\
& \cdots \\
& + \mathbb{E}[O^i_u|h^i(N^i_t+(u-t)) = u,\ \mathcal{F}_t]\Pr(h^i(N^i_t+(u-t)) = u|\mathcal{F}_t),
\end{aligned}$$

where N^i_t denotes the number of PO arrivals before t $(N^i_t = \sum_{j=1}^{\infty} 1_{\{h^i(j)\le t\}})$. Probability $\Pr(h^i(N^i_t+\ell) = u|\mathcal{F}_t)$ is given by $\Pr(h^i(N^i_t+\ell) = u|\mathcal{F}_t) = {}_{u-t-1}\mathrm{C}_{\ell-1} p_i^{\ell-1}(1-p_i)^{(u-t-1)-(\ell-1)}p_i$, where p_i denotes the constant PO arrival probability derived from eq.(1). Here, we assume that $l-1$ times PO are generated from time t to time $u-1$ and then, lth time PO occurs at time u. ${}_L\mathrm{C}_\ell$ denotes the number of combinations of choosing ℓ from L. Conditional expectation of PO volume $\mathbb{E}[O^i_u|u = h^i(N^i_t+n),\ \mathcal{F}_t]$ is

$$\mathbb{E}[O^i_u|u = h^i(N^i_t+n),\ \mathcal{F}_t] = \exp\left(m^i_{u|n,\ t} + \frac{\nu^i_{u|n,\ t}}{2}\right)$$

where $m^i_{u|n,\ t}$ and $\nu^i_{u|n,\ t}$ denote expectation and variation of $\log(O^i_u)$, respectively, under information $\{u = h^i(N^i_t+n), \mathcal{F}_t\}$. Considering model (2), we obtain

$$\log(O^i_{h^i(j)}) = \log(O^i_{h^i(j-1)}) + \alpha_i + \sum_{\ell=1}^{L} \beta^i_\ell \tilde{x}^i_\ell(h^i(j)) + \sigma_i\left(\rho_i W_j + \sqrt{1-\rho_i^2}\epsilon_{i,j}\right)$$

Then, expectation $m^i_{u|n,\ t}$ is obtained recursively by

$$
\begin{aligned}
m^i_{u|n,\ t} &:= \mathbb{E}[\log(O^i_u)|u = h^i(N^i_t + n),\ \mathcal{F}_t] \\
&= \alpha^i + (1 + \frac{\beta^i_2}{12})\mathbb{E}[\log(O^i_{h^i(N^i_t+n-1)})|\mathcal{F}_t] + \beta^i_1\mathbb{E}[\log(O^i_{h^i(N^i_t+n-12)})|\mathcal{F}_t] \\
&\quad + (-\beta^i_1 - \frac{\beta^i_2}{12})\mathbb{E}[\log(O^i_{h^i(N^i_t+n-13)})|\mathcal{F}_t] \\
&= \alpha^i + (1 + \frac{\beta^i_2}{12})m^i_{u|n-1,\ t} + \beta^i_1 m^i_{u|n-12,\ t} + (-\beta^i_1 - \frac{\beta^i_2}{12})m^i_{u|n-13,\ t}
\end{aligned}
$$

In the case of $n \leq 0$, $m^i_{u|n,\ t}$ is confirmed under $\mathcal{F}_t$. In addition, variance $\nu^i_{u|n,\ t}$ is obtained as

$$
\begin{aligned}
\nu^i_{u|n,\ t} &:= \mathbb{V}[\log(O^i_u)|u = h^i(N^i_t + n),\ \mathcal{F}_t] \\
&= (1 + \frac{\beta^i_2}{12})^2\mathbb{V}[\log(O^i_{h^i(N^i_t+n-1)})|\mathcal{F}_t] + (\beta^i_1)^2\mathbb{V}[\log(O^i_{h^i(N^i_t+n-12)})|\mathcal{F}_t] \\
&\quad + (-\beta^i_1 - \frac{\beta^i_2}{12})^2\mathbb{V}[\log(O^i_{h^i(N^i_t+n-13)})|\mathcal{F}_t] + \sigma^2_i \\
&= (1 + \frac{\beta^i_2}{12})^2\nu^i_{u|n-1,\ t} + (\beta^i_1)^2\nu^i_{u|n-12,\ t} + (-\beta^i_1 - \frac{\beta^i_2}{12})^2\nu^i_{u|n-13,\ t} + \sigma^2_i
\end{aligned}
$$

Here, $\mathbb{V}$ denotes variance. In the case of $n \leq 0$, $\nu^i_{u|n,\ t} = 0$ under $\mathcal{F}_t$.

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