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Inflation and Social Welfare in a New Keynesian Model: The Case of Japan and the U.S.

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Inflation and Social Welfare in a New Keynesian Model: The Case of Japan and the U.S. *

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Abstract

In this paper, we investigate the steady-state inflation rate that maximizes social welfare in a New Keynesian model. We calibrate the model on the Japanese and the U.S. economies, and we solve the model employing a computation method that addresses the non-linear dynamics associated with four major factors affecting the costs and benefits of inflation: (i) nominal price rigidity; (ii) money holdings; (iii) downward nominal wage rigidity (DNWR); and (iv) the zero lower bound of the nominal interest rates (ZLB). The calibrated model suggests the steady-state inflation rate that maximizes social welfare is close to two percent for both Japan and the U.S., though the main driver differs by country: the ZLB for Japan, but the DNWR for the U.S. In addition, around one percentage point absolute deviation from the close-to-two-percent rate induces only a minor change in social welfare. We also find that the lower-end of the range that is acceptable in terms of welfare losses is reduced when we introduce forward guidance in monetary policy through which private agents anticipate a prolonged zero interest rate once the ZLB binds. The estimates of the steady-state inflation rate are subject to a considerable margin of error due to parameter uncertainty in ZLB parameterization.

JEL Classification: E31; E43; E52

Keywords: Inflation; Social welfare; New Keynesian model; Downward nominal wage rigidity; Zero lower bound; Forward guidance

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JANET YELLEN: *Mr. Chairman, will you define "price stability" for me?*

ALAN GREENSPAN: *Price stability is that state in which expected changes in the general price level do not effectively alter business or household decisions.*

JANET YELLEN: *Could you please put a number on that?*

— Transcript of Federal Open Market Committee Meeting, July 2–3, 1996.

1 Introduction

Monetary economists have devoted themselves to the ever-growing debate on the costs and benefits of inflation. Even though the above statement by former Chair of the Board of Governors of the Federal Reserve System Alan Greenspan seems simple enough, assigning a precise number to the inflation rate that is consistent with the notion of price stability is a daunting task. This is because numerous frictions that generate monetary non-neutrality could affect social welfare which is expressed as the economic satisfaction of households and evaluating the precise effect of each channel requires a highly technical approach, both theoretical and computational.

Modern literature on this subject evaluates the steady-state level of inflation rate that maximizes social welfare measured by the representative agent's utility based on micro-founded models. When computing this inflation rate, researchers typically consider one or more of four major factors affecting the costs and benefits of inflation: (1) nominal price rigidity; (2) money holdings; (3) downward nominal wage rigidity (DNWR); and (4) the zero lower bound of nominal interest rates (ZLB). In the wake of the New Keynesian theory around the 1990s, which focuses on nominal price rigidity, a widely accepted view is that zero inflation maximizes social welfare because it eliminates price dispersion among individual goods. In contrast, classical views on the costs and benefits of inflation already existed back around 1970. For example, Friedman (1969) claimed that the inflation rate should be negative so as to keep the nominal interest rate zero and reduce the opportunity cost of holding money. Meanwhile, another classical view reached the opposite conclusion, with Tobin (1972) arguing that positive inflation acts as "the grease of the wheels" in the labor market, as it facilitates real wage adjustment during a recession, in the presence of

DNWR. More recently, Blanchard et al. (2010) have argued that modern economies can hit the ZLB more often than earlier believed, and hence a positive steady-state inflation rate can be justified in this context.

There is considerable difference among previous studies in the estimates of the steady-state inflation rate that maximizes social welfare. Figure 1 shows the distribution regarding the U.S. economy. Two basic facts about the agreements and disagreements over the inflation rate are clear: they are centered around zero, but there is a significant dispersion; there is a tendency for more studies made after the global financial crisis (GFC) to suggest positive inflation. While experience of the ZLB after the GFC has certainly affected the shift in the distribution, more than half of post-GFC studies still claim that zero inflation or deflation is welfare maximizing.[1] One reason for the variation is that each study focuses on different factors affecting the costs and benefits of inflation. While many studies still tend to focus on the analysis of nominal price rigidity, there are relatively few papers investigating DNWR and ZLB, which requires non-linear control systems.

In this paper, we reinvestigate the steady-state inflation rate that maximizes social welfare in a New Keynesian model. Compared with previous studies, our analysis has the following features. First, in order to capture the trade-off regarding the level of the steady-state inflation rate in a balanced manner, our model embeds all the four factors mentioned above that affect the costs and benefits of inflation. Second, we explicitly incorporate the non-linearity of the model, including that arising from non-zero steady-state inflation, DNWR, and ZLB. This methodology allows us to evaluate the inflation rate more accurately than previous studies. Third, we calibrate our model to the Japanese and the U.S. economies, reflecting differences in their economic structure and the periods of ZLB, to analyze the steady-state inflation rate that maximizes social welfare for the two countries. This will show whether the inflation rate might vary across economies depending on differences in their economic structure.

Our main conclusions are summarized as follows. First, our study confirms that positive

[1] A survey by Schmitt-Grohé and Uribe (2010) pointed out that the observed regularity that many central banks in advanced economies have inflation target around two percent is hard to reconcile with the theoretical predictions in favor of zero inflation.

steady-state inflation is welfare maximizing when we calibrate the DNWR and ZLB in our model to the Japanese and the U.S. economies. Specifically, we find that the rate for both countries is close to an annual rate of two percent. Second, the main driver that supports the close-to-two-percent inflation rate in the steady state differs by country: the ZLB is the main driver for Japan, but for the U.S. it is the DNWR. Third, around one percentage point absolute deviation from the close-to-two-percent rate induces only a minor change in social welfare. In the case of Japan, where the adverse effects of the ZLB are relatively large, incorporating a forward guidance measure through which private agents anticipate a prolonged zero interest rate policy once the ZLB binds, reduces the lower-end of the range that is acceptable in terms of welfare losses. Finally, we provide additional analyses that test the robustness of our benchmark results. In particular, we examine the impact of changes in the frequency, duration, and severity of the ZLB, as ZLB parameterization is subject to a considerable margin of error.

This paper joins a wealth of literature on the steady-state inflation rate that maximizes social welfare. As mentioned above, early literature focused on the role of money holdings (*e.g.*, Cooley and Hansen (1989), Schmitt-Grohé and Uribe (2004, SGU hereafter)) and nominal price rigidity (*e.g.*, King and Wolman (1999), SGU (2010)), most of which found that an inflation rate below zero is welfare maximizing.[2] On the other hand, recent studies have investigated the benefits of positive inflation with a particular focus on ZLB (*e.g.*, Coibion, Gorodnichenko, and Wieland (2012, CGW hereafter), Carreras, Coibion, Gorodnichenko, and Wieland (2016, CCGW hereafter), and Kiley and Roberts (2017)) and DNWR (*e.g.*, Kim and Ruge-Murcia (2009), Carlsson and Westermark (2016)). Other studies explore the costs and benefits of inflation from a variety of perspectives, including measurement issues of the inflation rate (SGU (2012)), trends in relative prices among goods (Wolman (2011), Ikeda (2015)), firms' entry and exit (Bilbiie et al. (2014)), and firms' productivity growth (Oikawa and Ueda (2018), Adam and Weber (2019)).

The literature above almost exclusively studies the U.S. economy. With regard to

[2]Recent studies have investigated the consequences of money holdings in different settings. See Oda (2016), for example.

Japan's case, Fuchi et al. (2008) employed a New Keynesian framework, considered all the four factors affecting the trade-off in inflation, and assessed the steady-state inflation rate using a linearized model around zero steady-state inflation. In contrast, our study precisely measures the fluctuations in the economy around non-zero steady-state inflation, and considers the non-linearity imposed by the DNWR and ZLB in a more rigorous manner. In addition, we impose parameter values that generate longer ZLB periods, reflecting the prolonged ZLB experience in Japan. With these refinements, we find that the upper-end of the range of the steady-state inflation rate in Japan within which welfare losses are mitigated from its potential maximum is somewhat higher than in Fuchi et al. (2008).

The remainder of this paper is organized as follows. Section 2 develops our model. Section 3 describes our calibration strategy as well as our computation method. Section 4 provides the baseline results. Section 5 conducts sensitivity analysis on parameter uncertainty. Section 6 is devoted to robustness checks. Section 7 concludes.

2 Model

Our model is built upon the standard New Keynesian model that has been used in analyses of the steady-state inflation rate from a welfare perspective, such as CGW (2012) and SGU (2010). The economy consists of a representative household, monopolistically competitive firms, and a central bank. The household supplies labor service to the production sector, earns wages, consumes, and allocates its wealth to nominal bonds and money. Firms produce differentiated goods and set prices under staggered contracts *à la* Calvo (1983). The central bank sets the policy rate following an interest rate feedback rule.

Our model embeds four major factors affecting the costs and benefits resulting from the level of the steady-state inflation rate: (1) nominal price rigidity; (2) money holdings; (3) DNWR; and (4) the ZLB. Each element works on the steady-state inflation rate that maximizes social welfare in the following ways.

1. Nominal price rigidity

   Under a staggered price setting, where only a proportion of firms can adjust their prices in response to current economic conditions, both inflation and deflation lead to the dispersion of the relative prices among individual goods. The relative price dispersion makes individual goods demand ununiform even when substitutability among goods is symmetric, and results in the misallocation of resources in the economy. Moreover, the deviation of the steady-state inflation rate from zero, as well as the fluctuations around the steady state, increases the cost of nominal price rigidity.[3] Therefore, the presence of nominal price rigidity implies that zero inflation maximizes social welfare.

2. Money holdings

   Holding money brings about a variety of benefits, such as facilitating goods purchases. On the other hand, the opportunity to earn the nominal interest that would be paid on risk-free bonds is lost. In this regard, Friedman (1969) argued that the inflation rate should be negative so as to bring the nominal interest rate, the opportunity cost of holding money, down to zero.

3. DNWR

   A variety of empirical evidence suggests that nominal wages are more rigid downwardly than upwardly. In the presence of DNWR, an adverse shock leads to misallocations in the labor market due to the lack of sufficient real wage adjustments. Tobin (1972) argued that positive inflation acts as the "grease of the wheels" in the labor market, *i.e.*, it facilitates real wage adjustment in a downturn when nominal wages are downwardly rigid.

[3] The point was initially raised by Okun (1971). Recent studies such as Ascari (2004) and Ascari et al. (2018) found that non-zero steady-state inflation generates substantial welfare losses because firms' prices relative to the aggregate price level continue to expand unless firms have the chance to reset their prices.

4. ZLB

   The nominal interest rate is usually bounded at zero.[4] The ZLB can be a constraint on the conduct of monetary policy that controls the nominal interest rate as a policy instrument. To this end, Summers (1991) argued that positive inflation provides the safety margin for cutting the nominal interest rate in a downturn. Blanchard et al. (2010) reinforced this argument claiming that modern economies can hit the ZLB more often than was previously believed.

In the section below, we describe our model settings.

2.1 Household

The representative household receives utility from consuming a composite good C_t and receives disutility from supplying homogeneous labor service H_t. The expected life-time utility is defined below:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \ln\left(C_{t+s}\right) - \frac{1}{1+\frac{1}{\eta}} \chi_{t+s} H_{t+s}^{1+\frac{1}{\eta}} \right\}, \tag{1}$$

where β is the subjective discount factor, η is the Frisch labor supply elasticity, and χ_t is exogenous labor disutility. The household has access to nominal bonds S_t, which carry the gross nominal interest rate $R_{n,t}$ in the next period and are subject to exogenous risk premium Q_t. We assume that nominal money holdings M_t facilitate goods purchases. Specifically, goods purchases are subject to a transaction cost $s(V_t)$, which is a function of the consumption-real balance ratio, or the consumption-based money velocity:

$$V_t \equiv \frac{C_t}{M_t/P_t}, \tag{2}$$

[4] In recent years, some central banks have implemented negative interest rate policies. We investigate the effects of lowering the effective lower bound of the nominal interest rate in Section 6.

where P_t is the price index. The specification of the transaction cost function follows that of SGU (2004):

$$s(V_t) \equiv \delta_1 V_t + \frac{\delta_2}{V_t} - 2\sqrt{\delta_1 \delta_2}, \tag{3}$$

where δ_1, $\delta_2 > 0$ are fixed parameters.[5]

The household's budget constraint is given as follows:

$$(1 + s(V_t))C_t + \frac{M_t}{P_t} + \frac{S_t}{P_t} \leq \frac{M_{t-1}}{P_t} + R_{n,t-1}Q_{t-1}\frac{S_{t-1}}{P_t} + \frac{W_t}{P_t}H_t + \frac{T_t}{P_t} + \Phi_t, \tag{4}$$

where W_t is nominal wage, T_t is lump-sum transfer from government, and Φ_t is firms' real profits distributed to the household.

The household chooses consumption C_t, labor supply H_t, nominal bond holdings S_t, and money holdings M_t, so as to maximize the expected life-time utility (1) subject to the budget constraint (4).

Consumption Euler equation

The first order conditions for consumption and nominal bond holdings yield the consumption Euler equation shown below:

$$\mathrm{E}_t \left[\beta \frac{\Xi_{t+1}}{\Xi_t} \frac{Q_t R_{n,t}}{\Pi_{t+1}} \right] = 1, \tag{5}$$

with

$$\Xi_t = \frac{1}{C_t(1 + s(V_t) + V_t s'(V_t))}, \tag{6}$$

where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate, and Ξ_t denotes the Lagrange multiplier for the household's budget constraint and therefore represents the marginal utility of wealth.

Money demand

From the first order condition for money holdings, the money demand function is given by

$$V_t^2 s'(V_t) = \frac{Q_t R_{n,t} - 1}{Q_t R_{n,t}}. \tag{7}$$

[5]Notice that $s(V_t)$ is non-negative, and is increasing in V_t if and only if V_t is greater than the satiation point $\underline{V} = \sqrt{\delta_2/\delta_1} > 0$.

Equation (7) describes the trade-off regarding money holdings. The benefit is to reduce the transaction cost of goods purchases represented in the left-hand side of (7), whereas the cost is to lose the opportunity to earn the nominal interest as described in the right-hand side of (7).

Wage determination

Regarding wage determination, we assume DNWR. Specifically, we impose

$$W_t \geq \gamma W_{t-1}. \tag{8}$$

The parameter γ governs the degree of DNWR. The higher γ is, the more downwardly rigid nominal wages are. This setup nests the cases of absolute downward rigidity when $\gamma \geq 1$ and full wage flexibility when $\gamma = 0$.[6] Along with the household's first order condition for labor supply, real wages are determined according to the equation below:

$$\frac{W_t}{P_t} = \max \left\{ \chi_t \frac{H_t^{1/\eta}}{\Xi_t}, \gamma \frac{W_{t-1}}{\Pi_t P_{t-1}} \right\}. \tag{9}$$

The first element in the maximum operator represents the marginal rate of substitution of labor supply for consumption, while the second element represents the DNWR in real terms. Notice that real wages can decline up to the inflation rate even though nominal wages are downwardly rigid.

2.2 Firms

There is a continuum of monopolistically competitive firms indexed by i on the unit interval, each of which produces a differentiated good $Y_t(i)$. Firm i uses labor input $H_t(i)$ with a linear production technology:

$$Y_t(i) = A_t Z_t H_t(i), \tag{10}$$

[6] For example, γ can be greater than one in the aggregate economy, when some workers receive wage increases due to indexation to the past wage inflation whereas the other workers' wages are downwardly rigid.

where productivity is common for each firm and consists of a stationary component Z_t and a non-stationary one that grows with a deterministic trend $g = \ln(A_t/A_{t-1})$. The first order condition for cost minimization of labor inputs suggests that firms' real marginal cost MC_t is given by

$$MC_t = \frac{W_t}{P_t}\frac{1}{A_t Z_t}. \tag{11}$$

The output Y_t is given by the CES aggregator of individual outputs:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \tag{12}$$

where θ is the elasticity of substitution across individual goods. Each firm faces the following demand curve:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t, \tag{13}$$

where the corresponding price index is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}. \tag{14}$$

Phillips curve

Firms have monopolistic power over their products and are therefore price setters. We assume that they set their prices under a staggered contract as in Calvo (1983). Specifically, in each period, a fraction $\lambda \in (0,1)$ of firms keeps their prices unchanged, while the remaining fraction $(1-\lambda)$ of firms resets their prices. The reset price B_t maximizes the expected real profits:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \lambda^s \Lambda_{t,t+s} \Phi_{t+s|t}, \tag{15}$$

where $\Phi_{t+s|t}$ is the period real profit at time $t+s$ of the firms that reset their prices at time t,

$$\Phi_{t+s|t} \equiv \frac{B_t}{P_{t+s}} Y_{t+s|t} - MC_{t+s} Y_{t+s|t}, \tag{16}$$

and $\Lambda_{t,t+s}$ is the stochastic discount factor between time t and $t+s$,

$$\Lambda_{t,t+s} \equiv \beta^s \frac{\Xi_{t+s}}{\Xi_t}, \tag{17}$$

subject to the individual goods demand:

$$Y_{t+s|t} = \left(\frac{B_t}{P_{t+s}}\right)^{-\theta} Y_{t+s}. \tag{18}$$

Notice that we drop the firm index i because the optimization problem here is identical across the firms that reset their prices at time t.

The first order condition for the optimization problem above is written in a recursive manner:

$$\frac{B_t}{P_t} = \frac{\Omega_{1t}}{\Omega_{2t}}, \tag{19}$$

where

$$\Omega_{1t} = \frac{\theta}{\theta-1} MC_t \Xi_t Y_t + \lambda\beta \mathrm{E}_t \left[\Pi_{t+1}^{\theta} \Omega_{1t+1}\right], \tag{20}$$

and

$$\Omega_{2t} = \Xi_t Y_t + \lambda\beta \mathrm{E}_t \left[\Pi_{t+1}^{\theta-1} \Omega_{2t+1}\right]. \tag{21}$$

The price index (14) can be rearranged to the equation below:

$$1 = (1-\lambda)\left(\frac{B_t}{P_t}\right)^{1-\theta} + \lambda \Pi_t^{\theta-1}. \tag{22}$$

It is worth noting that taking the first order approximation of the conditions above around the zero-inflation steady state leads to the well-known linearized form of the New Keynesian Phillips curve,

$$\pi_t = \beta \mathrm{E}_t \left[\pi_{t+1}\right] + \kappa \widehat{mc}_t, \tag{23}$$

where $\kappa \equiv \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$, $\pi_t \equiv \ln(\Pi_t)$, and $\widehat{x}$ denotes the log-deviation of variable X from the steady state. In what follows, on the other hand, we explicitly take into account the non-linearity arising from non-zero steady-state inflation. The setting allows us to investigate the welfare consequences of different levels of the steady-state inflation rate.

Aggregate production and price dispersion

By integrating individual production function over firms, the aggregate production is given by

$$Y_t = \frac{A_t Z_t H_t}{D_t}, \tag{24}$$

where the relative price dispersion D_t is defined below:

$$D_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di. \tag{25}$$

By using the definition of the price index, we can derive a recursive formula for the relative price dispersion:

$$D_t = \lambda \Pi_t^{\theta} D_{t-1} + (1 - \lambda) \left(\frac{B_t}{P_t} \right)^{-\theta}. \tag{26}$$

The market clearing conditions for goods and labor markets are given below:

$$Y_t = (1 + s(V_t)) C_t, \tag{27}$$

and

$$H_t = \int_0^1 H_t(i) di. \tag{28}$$

2.3 Central bank

Monetary policy rule

The central bank sets the policy rate following an interest rate feedback rule.[7] We consider two cases for a monetary policy rule. The first case is the so-called Taylor rule,

[7] Another possible subject for study is the solution to the Ramsey problem, *i.e.*, the optimal commitment policy. However, we choose to focus on a simple feedback rule rather than the optimal commitment policy for the following reasons. First of all, SGU (2007) argued that simple policy rules have an advantage in that they can be easily explained to the public. In fact, a number of previous studies found that the actual policy rates set by central banks can be approximated by simple rules (*e.g.*, Taylor (1993), Coibion and Gorodnichenko (2011)). From a technical perspective, moreover, the optimal commitment policy induces additional state variables in the model because the policy maker incorporates agents' expectations when formulating the future policy path, as Khan et al. (2003) discussed. The computational burden to add state variables makes our numerical analysis nearly infeasible given the current state of our numerical method.

in which the central bank sets the nominal interest rate $R_{n,t}$ responding to deviations in the inflation rate Π_t from its steady-state rate Π^* and the output gap Y_t/Y_t^f with interest rate smoothing.[8] Y_t^f denotes the output in the cashless economy under flexible prices and wages. The derivation of Y_t^f is provided in Appendix A. We also consider the zero lower bound. Consequently, the monetary policy rule is given as follows:

$$R_{n,t}^d = \left(R_{n,t-1}^d\right)^{\rho_r} \left\{ R^* \Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\phi_y} \right\}^{1-\rho_r}, \tag{29}$$

and

$$R_{n,t} = \max\left\{R_{n,t}^d\,, 1\right\}, \tag{30}$$

where ϕ_π and ϕ_y are the long-run responsiveness to inflation and that to the output gap, and $\rho_r \in (0,1)$ is the degree of interest rate smoothing. $R^* \equiv g/\beta$ is the natural rate of interest in the steady state. Notice that a higher steady-state inflation rate Π^* ensures more room to cut the nominal interest rate upon an adverse shock, which is the so-called safety margin. It is also worth noting that the size of the safety margin also depends on the level of the steady-state value of the natural rate of interest R^*.

Although the Taylor rule is used extensively in the literature on monetary policy analysis, major central banks in developed economies have conducted unconventional monetary policies to overcome the ZLB problem, especially after the GFC. As an example of such unconventional measures, we investigate the effects of forward guidance. Specifically, we consider the history-dependent rule that was proposed by Reifschneider and Williams (2000, the RW rule hereafter) as the second case. The rule takes the form below:

$$R_{n,t}^b = R^* \Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\phi_y}, \tag{31}$$

$$R_{n,t} = \max\left\{\frac{R_{n,t}^b}{\Gamma_t}, 1\right\}, \tag{32}$$

[8] The steady-state inflation rate Π^* refers to the level of the inflation rate in the deterministic steady state. Therefore, we can think of it as the level of the inflation rate achieved in the long run when any exogenous shocks disappear. Notice that the deterministic steady state does not necessarily coincide with the mean of the stochastic environment in a non-linear setting such as ours. For example, see Kiley and Roberts (2017) regarding this point.

and

$$\Gamma_t = \Gamma_{t-1} \left(\frac{R_{n,t-1}}{R^b_{n,t-1}} \right). \tag{33}$$

Under the RW rule, the central bank keeps track of the gap between the benchmark interest rate $R^b_{n,t}$ that responds to inflation and the output gap, and the actual interest rate $R_{n,t}$. These gaps are accumulated in the term Γ_t, and the nominal interest rate is kept lower than the benchmark interest rate as long as the gap remains, *i.e.*, $\Gamma_t > 1$. In other words, once the economy is constrained at the ZLB, the central bank will continue its low interest rate policy even if the economy begins to soar at future dates.

Money supply

Money is supplied passively to fulfill the money demand of the household. The government makes a lump-sum transfer to the household to balance the consolidated government budget:

$$M_t - M_{t-1} = T_t. \tag{34}$$

2.4 Exogenous processes

We consider three exogenous disturbances: productivity Z_t; labor disutility χ_t; and risk premium Q_t. Among others, fluctuations in the risk premium are the main drivers that bring the economy to the ZLB. Similar specifications are used in previous studies such as CGW (2012) to generate the ZLB episodes.[9] For descriptive purposes, we refer to exogenous variations in the risk premium as "ZLB shocks."

Following CCGW (2016), we consider a regime-switching shock to the risk premium. CCGW (2016) argued that the regime-switching shock is key to replicating the long-lived ZLB episodes observed in the data, while a standard AR(1) shock generates only short-

[9] A rise of risk premium decreases current consumption by raising the rate of return on nominal bonds held by the household relative to the nominal interest rate set by the central bank. CGW (2012) argued that the fluctuations in risk premium have similar effects to net-worth shocks in a model with financial frictions. Moreover, they can be interpreted as exogenous shocks to the aggregate demand of the economy in a parsimonious manner, given the fact that risk premium appears in the consumption Euler equation.

lived ones. Specifically, we assume that the risk premium consists of a regime-switching component Q_t^{rs} and an AR(1) component Q_t^{ar} below:

$$\ln(Q_t) = \ln(Q_t^{rs}) + \ln(Q_t^{ar}). \tag{35}$$

The regime-switching component Q_t^{rs} follows a two-state Markov chain:

$$\ln(Q_t^{rs}) = \begin{cases} -\frac{p_{21}}{p_{12}+p_{21}}\Delta \\ \frac{p_{12}}{p_{12}+p_{21}}\Delta \end{cases}, \tag{36}$$

where $p_{ij} \in (0,1)$ denotes the transition probabilities from Regime i to Regime j for $i,j = 1,2$ with $\sum_{j=1}^{2} p_{ij} = 1$, and $\Delta > 0$ represents the magnitude of the regime-switching shock. Notice that the value in each regime is adjusted such that $\mathrm{E}[\ln(Q_t^{rs})] = 0$. Regime 2 is a recession regime when high risk premiums cause the household to lose the desire to consume in the current period.

The laws of motion of productivity Z_t, labor disutility χ_t, and the AR(1) component of risk premium Q_t^{ar} are given by the following equations:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \epsilon_t^z, \quad \epsilon_t^z \sim i.i.d.N(0,\sigma_z^2), \tag{37}$$

$$\ln(\chi_t) = \rho_\chi \ln\left(\chi_{t-1}\right) + \epsilon_t^\chi, \quad \epsilon_t^\chi \sim i.i.d.N(0,\sigma_\chi^2), \tag{38}$$

and

$$\ln(Q_t^{ar}) = \rho_q \ln\left(Q_{t-1}^{ar}\right) + \epsilon_t^q, \quad \epsilon_t^q \sim i.i.d.N(0,\sigma_q^2), \tag{39}$$

where ρ_z, ρ_χ, $\rho_q \in (0,1)$ are the autoregressive coefficients of the corresponding processes, and ϵ_t^z, ϵ_t^χ, ϵ_t^q are *i.i.d.* exogenous innovations that are normally distributed with mean zero and variance σ_z^2, σ_χ^2, σ_q^2, respectively.

2.5 Equilibrium

An equilibrium consists of a set of prices $\{P_t, W_t, R_{n,t}\}_{t=0}^{\infty}$ and the allocations $\{Y_t, H_t, C_t, D_t, S_t, M_t, T_t, Y_t^f\}_{t=0}^{\infty}$, given exogenous variables $\{A_t, Z_t, \chi_t, Q_t\}_{t=0}^{\infty}$, such that the following conditions are satisfied for all t:

(i) the household maximizes its utility;

(ii) each firm maximizes its profits;

(iii) the central bank sets the policy rate following the feedback rule;

(iv) the consolidated government budget constraint holds;

(v) markets clear.

2.6 Social welfare

We define social welfare as the unconditional expectation of the representative household's utility:

$$\mathrm{E}\left[\ln(C_t) - \frac{1}{1+\frac{1}{\eta}}\chi_t H_t^{1+\frac{1}{\eta}}\right]. \tag{40}$$

In what follows, we consider the cashless economy under flexible prices and wages as the benchmark. Then, we measure welfare losses as the deviations of the social welfare in the model economy from that in the benchmark economy. Formally, we define the consumption-equivalent welfare losses CE, the consumption changes that make the social welfare in the benchmark economy equal to that in the distorted economy, as

$$\mathrm{E}\left[\ln(C_t) - \frac{1}{1+\frac{1}{\eta}}\chi_t H_t^{1+\frac{1}{\eta}}\right] = \mathrm{E}\left[\ln\left((1+CE)C_t^f\right) - \frac{1}{1+\frac{1}{\eta}}\chi_t (H_t^f)^{1+\frac{1}{\eta}}\right], \tag{41}$$

where C_t^f and H_t^f denote respectively the consumption and labor input in the cashless economy under flexible prices and wages. Equation (41) can be rearranged as below:

$$CE = \exp\left\{\mathrm{E}\left[\ln(C_t) - \frac{1}{1+\frac{1}{\eta}}\chi_t H_t^{1+\frac{1}{\eta}}\right] - \mathrm{E}\left[\ln\left(C_t^f\right) - \frac{1}{1+\frac{1}{\eta}}\chi_t (H_t^f)^{1+\frac{1}{\eta}}\right]\right\} - 1. \tag{42}$$

3 Quantitative analysis

3.1 Numerical method

One important issue in quantitative analysis is how we deal with the non-linearity of our model. In this regard, the presence of the DNWR and ZLB introduces kinks into the equilibrium conditions. Therefore, the perturbation method, which is used to solve a wide range of New Keynesian models, cannot be applied to our model. To address this issue, we numerically solve our model using the policy function iteration method of Coleman (1990). The method allows us to explicitly take into account the non-linearity of the model. In addition, the method is applicable to the regime-switching environment. The details of the method are described in Appendix B. Once we solve our model, we conduct stochastic simulations to evaluate welfare losses. At this stage, we approximate the unconditional expectation operator in (42) by taking the mean of the simulated series.

3.2 Calibration

Our calibrated parameter values are illustrated in Table 1.

Differences between Japan and the U.S.

In our calibration, we take into account the differences in economic structure and experience of ZLB episodes between Japan and the U.S. In particular, we focus on the following points:

1. Degree of DNWR

   The degree of DNWR γ is calibrated by applying the method of SGU (2016). They consider that the decline in nominal wages during severe recessions serves as the lower bound of downward wage adjustments. Applying their method to both countries, the calibrated values imply that the degree of DNWR is weaker in Japan than in the

U.S.[10,11] Previous studies on the U.S. economy such as Daly and Hobijn (2014) and Fallick et al. (2016) found that the nominal wages of individual workers were downwardly rigid even in the severe downturn after the GFC. On the other hand, Kuroda and Yamamoto (2005) reported that the DNWR that was measured using the total annual earnings of full-time employees in Japan disappeared after the late 1990s when the Japanese economy experienced a prolonged recession.

2. Steady-state level of natural rate of interest

   We estimate the natural rate of interest for Japan and the U.S. using the Laubach and Williams (2003) model.[12] We use the time average of the estimates after the late 1980's as the steady-state value of the natural rate of interest R^* in the model.[13] The calibrated R^* is lower in Japan than in the U.S.

3. Frequency, duration, and size of ZLB shocks

   The lower the natural rate of interest, the more often the economy is constrained at the ZLB given exogenous shock processes. We consider that the remaining gap between the model and the data in terms of the frequency, duration, and severity of the ZLB episodes is driven by the exogenous ZLB shocks. Specifically, we calibrate the transition probabilities in the Markov chain of the regime-switching component of the ZLB shocks, p_{12} and p_{21}, to match the frequency and duration of the ZLB

[10] To calibrate γ for Japan, we use the average of nominal wage changes in the three-year period beginning at 2000Q4 and that beginning at 2008Q1. Notice that both 2000Q4 and 2008Q1 are the business-cycle peaks of the previous expansions defined by the Cabinet Office of Japan. The average decline of nominal wages in the two periods is –0.89 percent in annual rate, which implies $\gamma = 0.9978$ for Japan. For the U.S., we use the wage changes after the GFC. We take the three-year average of nominal wage changes after 2008Q1, which is equal to +1.79 percent in annual rate. The implied parameter value is $\gamma = 1.0045$. Though Footnote 6 describes the possible case in which γ is greater than one, SGU (2016) indeed reported that γ calibrated to many of the peripheral countries of Europe after the GFC exceeds one.

[11] Note that our measure of wages is the compensation per hour, as is described in the notes of Table 2. Therefore, the calibrated degree of DNWR takes into account adjustments using components other than base pay, such as bonuses. However, the detailed analysis that takes into account the differences in the degree of rigidity among each component of total compensation is left for future research. In this light, it has been pointed out that the base pay of full-time workers are downwardly rigid in Japan.

[12] We employ a calibration strategy given the computation burden to estimate our non-linear model. However, a potential extension is to use an estimation method for a non-linear DSGE model, as suggested by Iiboshi et al. (2018), to obtain a model-consistent estimate for the steady-state value of the natural rate of interest.

[13] The sample period is 1985Q1–2017Q4 for Japan, and 1987Q4–2017Q4 for the U.S.

periods observed in the data.[14] The calibrated values suggest that the Japanese economy receives the ZLB shocks more often (higher p_{12}) and tends to remain in the recession regime for longer periods (lower p_{21}), reflecting the experience of the prolonged ZLB episodes in Japan. The magnitude of the regime-switching shock Δ is calibrated to match the decline in the output gap during the ZLB periods in the data.[15,16]

As for the implications of these differences between Japan and the U.S., the lower degree of DNWR implies that the steady-state inflation rate that maximizes social welfare would be lower in Japan than in U.S., other things being equal. On the other hand, the lower steady-state value of the natural rate of interest and the frequent and long-lasting ZLB shocks lead to a higher steady-state inflation rate in Japan in terms of improving social welfare.

Other parameter values

For utility function, the subjective discount factor β is set to 0.9975.[17] We set the Frisch labor supply elasticity η equal to 0.82 for Japan as in the empirical findings of Kuroda and Yamamoto (2008) and to 1.00 for the U.S. following CGW (2012).[18] The parameters in the money demand function are taken from SGU (2004), that is, $\delta_1 = 0.01110$ and $\delta_2 = 0.07524$. We set the elasticity of substitution across individual goods θ equal to 7

[14] We define the ZLB periods as when the short-term nominal interest rate is below an annual rate of 0.25 percent during the sample period mentioned above. The details of the data series are described in the notes of Table 2. According to the criteria, the periods 1998Q4–2006Q3 and 2009Q1–2017Q4 are classified as the ZLB periods in Japan, compared with 2009Q1–2015Q4 in the U.S.

[15] Specifically, we match the declines of the output gap in Regime 2 of the model with those during the two-year periods since the ZLB periods begin in the data. The length of the data periods broadly corresponds to the length of the recessions around the beginning of the ZLB periods in the data. Regarding the data series, the output gap estimated by the Bank of Japan Research and Statistics Department is employed for Japan. For the U.S., the output gap is the difference between the GDP in the *National Income and Product Accounts* (NIPA) and the potential GDP estimated by Congressional Budget Office.

[16] The calibrated value is larger in the U.S. than in Japan. This is partly because the lower steady-state level of the natural rate of interest in Japan causes ZLB episodes and resulting recessions to Japan even with relatively smaller shocks.

[17] The trend growth rate g is calculated such that $R^* = g/\beta$.

[18] Kuroda and Yamamoto (2008) estimated the Frisch labor supply elasticity using Japanese micro data and found that the estimates fell in a range from 0.67 to 0.97 in different datasets. We use the arithmetical mean of these two values.

following CGW (2012). The degree of price stickiness λ is set to 0.65. This value is based on the frequency of price changes reported by Nakamura and Steinsson (2008). For monetary policy rules, the long-run responsiveness to inflation ϕ_π is 2.50 and that to the output gap ϕ_y is 0.25 with an interest rate smoothing parameter ρ_r equal to 0.90. These values are broadly consistent with the estimates obtained by Sudo and Tanaka (2018) for Japan, and Coibion and Gorodnichenko (2011) for the U.S. Regarding exogenous processes, the parameter values for the persistence of productivity ρ_z, labor disutility ρ_χ, and the AR(1) component of risk premium ρ_q are set equal to 0.90, 0.70, and 0.85, while those for the standard deviation of innovations to productivity σ_z and labor disutility σ_χ are set to 0.0015 and 0.0030, respectively. We set the standard deviation of innovations to the AR(1) component of risk premium σ_q to 0.0025 for Japan and to 0.0020 for the U.S. to match the variations in output for each country.

Model fits

Table 2 shows selected moments of the data and those of the simulated series of the calibrated model of the two countries. Though the model is not fully successful in matching all the business cycle moments in the data, due to the lack of a number of elements introduced in medium-scale DSGE models *à la* Smets and Wouters (2007), the model does capture the salient features of business cycles in both countries including: (1) smaller standard deviation of the inflation rate and the wage inflation rate relative to that of output; (2) moderate persistence of the inflation rate; and (3) positive comovements among variables. On the other hand, the persistence of output and inflation in the model is lower than that in the data. In this regard, we introduce habit formation in consumption to add persistence to the model in Section 6 and assess the robustness of our quantitative results.

4 Results

4.1 Welfare losses under Taylor rule

Figure 2 shows the welfare losses under different levels of the steady-state inflation rate Π^* when the central bank is assumed to follow the Taylor rule (29) and (30). These values are computed based on the stochastic simulations of the model calibrated to each economy. Each line corresponds to the specification that includes some or all the four factors affecting the costs and benefits of inflation described in Section 2. Specifically, we start by computing the welfare losses when only nominal price rigidity is present in the model. Then, we add money holdings, DNWR, and ZLB sequentially, and examine their impact on the steady-state inflation rate that maximizes social welfare.

Several points from the figure are noteworthy. First, in the specification that only includes nominal price rigidity as a source of welfare losses (the black line with diamonds), zero percent inflation in the steady-state maximizes social welfare. This is consistent with the theoretical implication that both inflation and deflation generate welfare losses through the relative price dispersion. Second, when adding money holdings (the blue line with squares), a negative steady-state inflation rate maximizes social welfare because the lower nominal interest rates resulting from the lower steady-state inflation rates reduce the opportunity costs of holding money. Third, incorporating DNWR (the green line with triangles) and ZLB (the red line with circles) leads to a positive steady-state inflation rate that maximizes social welfare because positive inflation reduces the probability that these constraints bind. Fourth, when comparing Japan and the U.S., the consequences of nominal price rigidity and money holdings are quite similar in the two countries. Regarding the benefits of inflation, however, ZLB is the main driver justifying a positive steady-state inflation rate in Japan, whereas DNWR plays a key role in the U.S. These results are consistent with our calibration. In the full model that includes all four factors, the steady-state inflation rate that maximizes social welfare is an annual rate of 1.9 percent for Japan, while slightly higher for the U.S. at 2.3 percent.

4.2 Welfare losses under RW rule

In Figure 3, we show welfare losses under different levels of the steady-state inflation rate when the RW rule (31)–(33) is implemented (the red line with circles). For ease of presentation, we only show the welfare losses when all four factors are introduced.

Under the RW rule, the welfare losses are smaller than those when the central bank follows the Taylor rule (the blue line with squares). This is because the RW rule mitigates the adverse effects of the ZLB by committing to a prolonged low interest rate policy when the economy is constrained at the ZLB. Since the benefits of holding the safety margin in the nominal interest rate provided for the ZLB are weakened under the RW rule, the steady-state inflation rate that maximizes social welfare is slightly reduced, compared with the case under the Taylor rule, to 1.6 percent in Japan and 1.8 percent in the U.S.[19] That being said, the estimates of the inflation rate under the RW rule do not deviate too far from the conventional wisdom of two percent.

4.3 Welfare consequences of shifts in steady-state inflation rate

Figure 3 also evaluates the changes in the welfare losses when the steady-state inflation rate deviates from the estimates in the previous analysis. The bands in the figure indicate the range of the steady-state inflation rates at which the decline in social welfare from its maximum remains within 0.05 and 0.10 percentage points in terms of the consumption-equivalent losses. For reference, 0.05 percent of consumption-equivalent losses amounts to roughly 20 to 30 U.S. dollars per working-age person in each year.[20]

From this figure, we can see that around one percentage point absolute deviation from the close-to-two-percent rate induces only a minor change in social welfare.[21] In the case of Japan, the 0.05 percentage point band implies a range of steady-state inflation rates from 1.2 to 2.8 percent under the Taylor rule. Moreover, implementing the RW rule substantially

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[19]The welfare losses under the RW rule are smaller than in the specification without the ZLB under the Taylor rule (the green line with triangles in Figure 2) for some of the steady-state inflation rates. In this regard, it should be noted that the benchmark interest rate (31) in the RW rule does not include interest rate smoothing. This is largely due to the computational burden of simultaneously incorporating interest rate smoothing and the policy duration effect. This specification of the benchmark interest rate implies expeditious reactions of the nominal interest rate to economic fluctuations compared with the case with interest rate smoothing. Consequently, the RW rule in our analysis has a powerful stabilizing effect.

[20]These values are based on consumption per working-age person, which is calculated by dividing the consumption expenditure in the GDP statistics by the population aged 15 to 64 years old. As of 2017, 0.05 percent of consumption per working-age person is equivalent to 2,001 Japanese yen for Japan, and 32.4 U.S. dollars for the U.S.

[21]The welfare changes within such a range of the steady-state inflation rates are of a similar magnitude to the welfare loss of business cycle that was computed by Lucas (2000). This is consistent with the convention of a standard monetary model where the level of the steady-state inflation rate does not affect the long-run growth.

reduces the lower-end of the range that is acceptable in terms of welfare losses because the RW rule is effective in mitigating the adverse effects of the ZLB even when the steady-state inflation rate is low. To be precise, the range is widened to include steady-state inflation rates from 0.6 to 2.8 percent under the RW rule. Turning to the case of the U.S., the 0.05 percentage point band forms a range of steady-state inflation rates from 1.0 to 3.4 percent under the Taylor rule, and from 0.7 to 3.0 percent under the RW rule.

5 Parameter Uncertainty

Though the baseline calibration reflects key moments of the data that potentially affect the steady-state inflation rate that maximizes social welfare, the uncertainty regarding the parameter values and the resulting effect on the inflation rate may be non-negligible. In particular, parameter uncertainty with respect to ZLB, such as the level of the natural rate of interest and the specification of ZLB shocks, is considerably high because the data offers limited instances of ZLB episodes. In this section, we investigate the potential range of estimates of the steady-state inflation rate arising from such parameter uncertainty.

5.1 Uncertainty regarding natural rate of interest

As is recognized in the literature, the measurement of the natural rate of interest is subject to considerable uncertainty.[22] Since the level of the natural rate of interest along with the inflation rate constitutes the distance of the nominal interest rate from the ZLB, it is one of crucial factors in determining the steady-state inflation rate that maximizes social welfare. While we use the time average of the estimates based on the Laubach and Williams (2003) model as the steady-state value of the natural rate of interest R^* in our baseline calibration, we consider alternative specifications where the value deviates upwardly and downwardly by one standard deviation of the estimated series. Specifically, given that the mean of the estimated natural rate of interest is 0.95 percent with the standard deviation of 1.45 for Japan, we consider the natural rate of –0.50 and 2.40 percent for Japan. The corresponding

[22] See Beyer and Wieland (2019), for example.

alternatives for the U.S. are 0.75 and 2.93 percent, while the baseline estimate is 1.84 percent.[23]

The upper panels of Figure 4 show the welfare losses when assuming the optimistic and pessimistic levels of the natural rate of interest. Here we assume that the central bank follows the RW rule. In the figure, the higher (lower) the steady-state value of the natural rate of interest R^*, the lower (higher) the steady-state inflation rate that maximizes social welfare. In principal, a lower natural rate of interest requires a higher steady-state inflation rate to ensure the same size of safety margin in the nominal interest rate. Besides, the relationship is not necessarily one-to-one: consistent with Andrade et al. (2018), we find that the corresponding changes in the steady-state inflation rate from a welfare perspective are smaller than the changes in R^*. For example, while the standard deviation of the estimated natural rate of interest is 1.45 in Japan, the resulting shift in the steady-state inflation rate from a welfare perspective is around 0.8 percentage points on average in absolute terms. This is because a higher steady-state inflation rate not only brings the benefit of a widening of the safety margin, but it also generates welfare losses through nominal price rigidity and money holdings.

The uncertainty regarding the level of the natural rate of interest forms a considerable range of the steady-state inflation rate that maximizes social welfare for both countries. The implied range for Japan is 1.0 to 2.6 percent, a little wider than that for the U.S., at 1.4 to 2.2 percent.[24] The difference between Japan and the U.S. arises because the calibrated standard deviation of the natural rate of interest is larger for Japan than for the U.S., reflecting the larger time variations in the estimated series of the Laubach and Williams (2003) model. Moreover, since the Japanese economy is calibrated to be at ZLB more frequently, the marginal effects of the changes in R^* are relatively larger.

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[23] The mean of the estimated natural rate of interest is 1.84 percent with the standard deviation of 1.09 for the U.S.

[24] We find that these ranges become a little wider when assuming the Taylor rule. This is because the RW rule mitigates the adverse effects of ZLB, and therefore reduces the variations in social welfare arising from the parameter uncertainty with respect to ZLB. Similar patterns are found in the uncertainty regarding the specification of ZLB shocks.

5.2 Uncertainty regarding ZLB shock

The specification of ZLB shocks is another source of the parameter uncertainty that potentially alters the relationship between social welfare and inflation. To evaluate the uncertainty, we formulate optimistic and pessimistic specifications based on the instances of ZLB periods in each country.[25] Specifically, in the optimistic specification, we assume that there are no regime-switching shocks, *i.e.,* $p_{12} = 0$. Notice that this specification corresponds to the case where ZLB shocks follow AR(1) processes, which is widely used in previous studies, including Fuchi et al. (2008). On the other hand, for the pessimistic specification, we calibrate the size of the regime-switching shock to replicate the largest decline in the output gap during ZLB periods in the data. The data in Japan implies $\Delta = 0.0085$ compared with the baseline calibration of 0.0070. The calibration for the pessimistic case of the U.S. is $\Delta = 0.0143$ as opposed to 0.0135 in the baseline calibration.

The welfare losses under the optimistic and pessimistic specification regarding ZLB shocks are given in the lower panels of Figure 4. Though somewhat smaller than in the case of the uncertainty regarding the level of the natural rate of interest, the specification of ZLB shocks forms the range of the steady-state inflation rate that maximizes social welfare, from 1.1 to 1.9 percent for Japan, and from 1.3 to 1.9 percent for the U.S.

6 Robustness check

In this section, we check the robustness of our quantitative results under two alternative settings. First, we extend the model to incorporate habit formation in consumption, which introduces additional persistence of consumption and other variables. This specification is often used in previous studies including CGW (2012). The analysis basically assesses

[25] Throughout our analysis, we consider ZLB episodes to be driven mainly by a large exogenous shock to the risk premium, which we refer to as a ZLB shock, rather than the endogenous mechanism of the model. This is a parsimonious approach to capture the observed features of ZLB episodes in the data, such as their frequency, duration, and severity. However, one concern is that the presence of ZLB shocks might change the welfare implications of factors in the model other than ZLB. To address this concern, we investigated how different magnitudes of ZLB shock affect the costs and benefits of inflation through factors other than ZLB, using a specification without ZLB. We indeed found that, while a larger ZLB shock amplifies the welfare losses arising from each channel other than ZLB, they are offset against each other, and the steady-state inflation rate that maximizes social welfare remains nearly unchanged.

the sensitivity of our results with respect to the degree of inertia of the model. Second, we investigate the effects of lowering the effective lower bound of the nominal interest rate into slightly negative territory. This is motivated by the fact that several central banks have implemented negative interest rate policies in recent years.

6.1 Habit formation in consumption

To investigate the robustness in terms of adding further persistence to the model, we introduce habit formation in consumption.[26] Specifically, we consider the following preference:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \ln \left(C_{t+s} - hgC_{t+s-1} \right) - \frac{1}{1+\frac{1}{\eta}} \chi_{t+s} H_{t+s}^{1+\frac{1}{\eta}} \right\}, \tag{43}$$

Following CGW (2012), we assume that the household forms the consumption habit based on the own lagged consumption C_{t-1} adjusted for deterministic productivity growth g. h is the degree of habit formation. Then, the Lagrange multiplier in the consumption Euler equation is modified to

$$\Xi_t = \left(\frac{1}{C_t - hgC_{t-1}} - \mathrm{E}_t \left[\frac{\beta hg}{C_{t+1} - hgC_t} \right] \right) \frac{1}{1 + s\left(V_t\right) + V_t s'\left(V_t\right)}. \tag{44}$$

The other equilibrium conditions do not change from the baseline model conditional on the Lagrange multiplier. For monetary policy rule, we consider the case of the Taylor rule.[27] For welfare analysis, we evaluate the unconditional expectation of the utility in

[26] A medium-scale DSGE model typically embeds a variety of elements to capture the persistent dynamics of macro variables observed in data, such as habit formation in consumption and price indexation. On the other hand, we stick to a relatively stylized setting largely due to the computational burden of solving our non-linear model. However, even in our baseline specification without habit formation in consumption, the model has several elements that generate inertia. For example, the price dispersion follows an autoregressive process in a non-linear solution. Moreover, DNWR makes real wages depend on the previous period's level, which leads to the persistence of marginal cost and therefore that of inflation.

[27] With the habit formation in consumption, the output in the cashless economy under flexible prices and wages depends not only on the current exogenous variables but on own lag. For the computational burden of adding another dimension of state space, we modify the monetary policy rule to responding to the deviations of output, instead of those of the output gap, from its steady-state value as below:

$$R_{n,t}^d = \left(R_{n,t-1}^d\right)^{\rho_r} \left\{ R^* \Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y^*}\right)^{\phi_y} \right\}^{1-\rho_r},$$

where Y^* is output in the steady state.

(43) as in the baseline case. Regarding parameter values, the degree of habit formation in consumption h is set equal to 0.7 following CGW (2012) whereas the other parameter values are identical to those in the baseline calibration.

Figure 5 shows the results. The steady-state inflation rate that maximizes social welfare in each specification is close to the baseline result, both qualitatively and quantitatively. The slight decline in the estimates of the inflation rate from the baseline specification arises because the increased persistence due to the consumption habit makes the inflation rate less volatile and therefore leads to the DNWR and ZLB binding less frequently, which reduces the benefits of holding a positive inflation rate as a provision against recession.

6.2 Effective lower bound

Several central banks have implemented negative interest rate policies in recent years. To examine the welfare implications of these policies, we study the effects on the steady-state inflation rate of lowering the effective lower bound into slightly negative territory. In the analysis, we consider a reduction of the lower bound of the nominal interest rate by 0.1 percentage points in annual rate under the Taylor rule. In the model expression,

$$R_{n,t} = \max\left\{R^d_{n,t}\,, \underline{R}\right\}, \tag{45}$$

where $\underline{R}$ is set equal to –0.1 percent in annual rate.

Table 3 compares the steady-state inflation rate that maximizes social welfare under different lower bounds of the nominal interest rate.[28] The table suggests that the estimate of the inflation rate is reduced by 0.1 percentage points from the baseline specification of ZLB when the effective lower bound is reduced. This is simply because the reduction of the lower bound widens the safety margin for cutting the nominal interest rate given the level of the steady-state inflation rate. Meanwhile, it should be noted that our model does

[28] Note that our specification implies that the transaction cost is decreasing in the consumption-real balance ratio if the ratio is lower than the satiation point, and therefore money demand does not diverge even under negative interest rates. We can interpret this as a situation in which there are costs to the physical storing of money.

not incorporate other factors that potentially impact the effectiveness of negative interest rate policies, such as the transmission mechanism through financial intermediaries.

7 Concluding Remarks

In this paper, we investigate the steady-state inflation rate from the perspective of maximizing social welfare in a New Keynesian model that embeds four major factors affecting the costs and benefits of inflation: nominal price rigidity, money holdings, downward nominal wage rigidity, and the zero lower bound. Although many previous studies have examined one or more of these four factors, we examine all four simultaneously and employ a computational methodology that is suitable for addressing the non-linearity induced by non-zero steady-state inflation, DNWR and ZLB.

Most of the previous studies on this topic find that the steady-state inflation rate that maximizes social welfare is close to or below zero percent. This is because many of the studies focus on nominal price rigidity and money holdings. However, when DNWR and ZLB are taken into account in addition to these factors, a positive inflation rate in the steady state can maximize social welfare. In this regard, Janet Yellen, in her remarks during the July 1996 Federal Open Market Committee Meeting, stressed that, among the four factors, DNWR and ZLB are sources that support a positive steady-state inflation rate. Our result suggests that the steady-state inflation rate that maximizes social welfare is close to two percent for both Japan and the U.S. At the same time, we find that around one percentage point absolute deviation from the close-to-two-percent rate induces only a minor change in social welfare. Effective forward guidance can reduce the lower-end of the range that is acceptable in terms of welfare losses. Last but not least, the estimates of the steady-state inflation rate are subject to a considerable margin of error due to parameter uncertainty in ZLB parameterization.

Though we incorporate major factors affecting the costs and benefits of inflation, and employ computation methodologies to address the non-linearity of the model, there may be other factors that potentially affect the steady-state inflation rate that maximizes social welfare. Our model restricts its focus to a closed economy and is therefore agnostic about the issues in an open economy; we also do not take into account the transmission mechanism of monetary policy through financial intermediaries; and finally, our analysis does not consider the full range of unconventional monetary policy measures, such as asset purchases and central bank lending. The necessity of holding a positive inflation rate in the steady state depends on the effectiveness of unconventional monetary policy. In this light, there is further room to examine unconventional measures, such as asset purchases, that might affect the steady-state inflation rate that maximizes social welfare. These issues are left for future research.

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Appendix A Allocation in cashless economy under flexible prices and wages

Under flexible prices and wages, the labor market equilibrium determines the relationship between the marginal product of labor and the marginal rate of substitution of labor for consumption as below:

$$\left(\frac{\theta}{\theta-1}\right)^{-1} A_t Z_t = \chi_t \frac{\left(H_t^f\right)^{1/\eta}}{\Xi_t^f}, \tag{A.1}$$

where the variables with a superscript f denote the endogenous variables in the cashless economy under flexible prices and wages. The right-hand side of (A.1) is the marginal rate of substitution whereas $A_t Z_t$ in the left-hand side is the marginal product of labor. Note that the allocation is still inefficient due to monopolistic distortion of firms represented by the steady-state markup $\frac{\theta}{\theta-1}$. To consider the cashless economy, we disregard the transaction cost of purchasing goods. Along with the market clearing conditions, (A.1) yields:

$$Y_t^f = A_t Z_t \left(\frac{\theta}{\theta-1}\chi_t\right)^{-\frac{1}{1+1/\eta}}, \tag{A.2}$$

$$C_t^f = A_t Z_t \left(\frac{\theta}{\theta-1}\chi_t\right)^{-\frac{1}{1+1/\eta}}, \tag{A.3}$$

and

$$H_t^f = \left(\frac{\theta}{\theta-1}\chi_t\right)^{-\frac{1}{1+1/\eta}}. \tag{A.4}$$

Appendix B Model solution

We employ a version of the policy function iteration method of Coleman (1990) to solve our non-linear model.[29] Specifically, we use the fixed point iteration method. Richter et al. (2014) find that this method has an advantage in terms of the speed of computation

[29] Similar methods are used by Katagiri (2016) and Iiboshi et al. (2018) to solve a New Keynesian model with ZLB.

compared with alternative methods.

The concept of the fixed point iteration method is summarized as follows. A model has the representation:

$$0 = \mathrm{E}_t \left[f \left(\mathbf{S}_t \, , \, \mathbf{X}_t \, , \, \mathbf{S}_{t+1} \, , \, \mathbf{X}_{t+1} \right) \right],$$

where $\mathbf{X}_t$ is a set of jump variables and $\mathbf{S}_t$ is a set of state variables. The inter- and intra-temporal relationships among variables are represented in $f(\cdot)$. $\mathrm{E}_t[\cdot]$ is the expectation operator conditional on the information available at time t. In our baseline model, $\mathbf{X}_t = \left\{ Y_t, H_t, C_t, S_t, M_t, T_t, Y_t^f, P_t \right\}$ and $\mathbf{S}_t = \{D_{t-1}, W_{t-1}, R_{n,t-1}, A_t, Z_t, \chi_t, Q_t\}$. Notice that jump variables and future sate variables can be expressed as function of current state variables in the rational expectation equilibrium. Therefore, the model equations above can be rewritten as below:

$$\begin{aligned} 0 &= \mathrm{E}_t \left[\, f \left(\mathbf{S}_t \, , \, X \left(\mathbf{S}_t \right) \, , \, S \left(\mathbf{S}_t \right) \, , \, X \left(S \left(\mathbf{S}_t \right) \right) \right) \right] \\ &= \mathrm{E}_t \left[\, f \left(\, \Phi \left(\mathbf{S}_t \right) \right) \right], \end{aligned}$$

where $X(\cdot)$ and $S(\cdot)$ are the time-invariant policy functions for jump and state variables, which are summarized in $\Phi(\cdot)$. Notice that the model conditions $f(\cdot)$ and the policy function $\Phi(\cdot)$ are non-linear in general. The policy function iteration method discretizes the state space for $\mathbf{S}_t$ and numerically searches for the mapping $\Phi(\mathbf{S}_t)$ that satisfies the model equations. Consequently, the method is robust to the non-linearity of the underlying function $f(\cdot)$.

Algorithm

The algorithm takes the following steps in each iteration $n = 1, 2, 3...$

1. Formulate the initial guess for the policy functions $\Phi^{(0)}(\mathbf{S}_t)$.

2. Substitute the previous guess $\Phi^{(n-1)}(\mathbf{S}_t)$ into the model equations to obtain the updated policy function $\Phi^{(n)}(\mathbf{S}_t)$. The parameter for updating $\alpha \in (0, 1)$ is set equal to 0.2.[30] In this step, we approximate the future variables by using the linear inter-

[30] Richter et al. (2014) point out that a smaller value of α helps maintain stability of solution especially

polation method between grids and evaluate the expectation operator by numerical integration:

$$\Phi^{*}\left(\mathbf{S}_{t}\right)=\mathrm{E}_{t}\left[f\left(\Phi^{(n-1)}\left(\mathbf{S}_{t}\right)\right)\right]+\Phi^{(n-1)}\left(\mathbf{S}_{t}\right),$$

$$\Phi^{(n)}\left(\mathbf{S}_{t}\right)=\alpha\Phi^{*}\left(\mathbf{S}_{t}\right)+(1-\alpha)\,\Phi^{(n-1)}\left(\mathbf{S}_{t}\right).$$

3. Compute the deviations between the updated and previous policy functions:

$$dist=\max\left|\Phi^{(n)}\left(\mathbf{S}_{t}\right)-\Phi^{(n-1)}\left(\mathbf{S}_{t}\right)\right|.$$

4. Stop iterations if the deviation becomes smaller than the critical value $\epsilon>0$. Otherwise, go back to Step 2. We set $\epsilon=10^{-4}$.

at the beginning of the algorithm whereas it involves a larger number of iterations until convergence. Therefore, they propose to use a small value of α for solving a large model. We set the value of α reflecting the trade-off between speed and stability of computation. However, it should be noted that the convergence criteria of policy function is independent of the value of α.

Table 1: Calibrated parameters

Symbol	Parameter	Japan	U.S.
<u>Steady-state values</u>			
r^*	Natural rate of interest (annual rate)	0.95%	1.84%
<u>Parameters for utility function</u>			
β	Subjective discount factor	0.9975	0.9975
η	Frisch labor supply elasticity	0.82	1.00
<u>Parameters for transaction cost</u>			
δ_1	Parameter in transaction cost	0.01110	0.01110
δ_2	Same as above	0.07524	0.07524
<u>Parameters for price and wage setting</u>			
θ	Elasticity of substitution across individual goods	7.00	7.00
λ	Degree of price stickiness	0.65	0.65
γ	Degree of downward nominal wage rigidity	0.9978	1.0045
<u>Parameters for monetary policy rule</u>			
ϕ_π	Long-run responsiveness to inflation	2.50	2.50
ϕ_y	Long-run responsiveness to the output gap	0.25	0.25
ρ_r	Interest rate smoothing	0.90	0.90
<u>Parameters for exogenous processes</u>			
p_{12}	Transition probability from Regime 1 to Regime 2	3.13%	1.14%
p_{21}	Transition probability from Regime 2 to Regime 1	2.94%	3.45%
Δ	Size of regime-switching shock	0.0070	0.0135
ρ_z	Persistence of productivity	0.90	0.90
ρ_χ	Persistence of labor disutility	0.70	0.70
ρ_q	Persistence of AR(1) component of risk premium	0.85	0.85
σ_z	S.D. of innovations to productivity	0.0015	0.0015
σ_χ	S.D. of innovations to labor disutility	0.0030	0.0030
σ_q	S.D. of innovations to AR(1) component of risk premium	0.0025	0.0020

Table 2: Model fits

Moment	Symbol	Variable	Japan		U.S.	
			Data	Model	Data	Model
Standard deviation ×100	Y	Output	1.42	1.36	1.21	1.12
	C	Consumption	1.00	1.37	1.02	1.13
	H	Labor input	0.82	1.38	1.77	1.14
	π	Inflation rate	0.27	0.52	0.25	0.34
	π^w	Wage inflation rate	1.02	0.86	0.72	0.68
	R	Nominal interest rate	0.62	0.40	0.55	0.32
First-order auto-correlation	Y	Output	0.83	0.58	0.89	0.58
	C	Consumption	0.60	0.58	0.86	0.58
	H	Labor input	0.62	0.54	0.95	0.56
	π	Inflation rate	0.76	0.64	0.77	0.67
	π^w	Wage inflation rate	-0.22	0.19	0.08	0.19
	R	Nominal interest rate	0.95	0.92	0.94	0.93
Correlation with output	Y	Output	1.00	1.00	1.00	1.00
	C	Consumption	0.82	1.00	0.92	1.00
	H	Labor input	0.79	0.94	0.90	0.91
	π	Inflation rate	0.48	0.55	0.15	0.69
	π^w	Wage inflation rate	0.20	0.66	0.05	0.68
	R	Nominal interest rate	0.28	0.08	0.49	0.06

Notes: 1. The data moments are computed in the sample before the ZLB periods begin in each country. The sample period is 1985Q1-1998Q3 for Japan, and 1987Q4-2008Q4 for the U.S.

2. The model moments are those in Regime 1 where a contractionary regime-switching shock is not present. For the simulation, the steady-state inflation rate is set to the mean inflation rate during 1985Q1-2017Q4 for Japan and 1987Q4-2017Q4 for the U.S.

3. For the data series in Japan, the output is the GDP and the consumption is the private consumption in the *System of National Accounts* (SNA), deflated by the consumer price index (CPI, less fresh food). The labor input is the number of employees based on the *Labour Force Survey*, multiplied by hours-worked per employee based on the *Monthly Labour Survey*. The inflation rate is the CPI (less fresh food). The series is adjusted for the introduction of the consumption tax and changes in the rates. The wage inflation rate is constructed from the compensation of employees in the SNA, divided by the labor input. The nominal interest rate is the uncollateralized overnight call rate after 1985Q3, while the collateralized call rate is used before then due to the availability of the data.

4. For the data series in the U.S., the output is the GDP and the consumption is the personal consumption expenditure (PCE) in the *National Income and Product Accounts* (NIPA), deflated by the PCE deflator (less food and energy). The labor input is total hours-worked in the non-farm business sector. The inflation rate is the PCE deflator (less food and energy). The wage inflation rate is the compensation per hour in the non-farm business sector. The nominal interest rate is the effective federal funds rate.

5. The output, consumption, and labor input are on a per working-age person basis. These series are detrended using the Hodrick–Prescott filter.

6. The inflation rate and the wage inflation rate are on a quarter-on-quarter change.

7. The nominal interest rate is in quarterly rate.

Sources: Data series in Japan: Cabinet Office, "System of National Accounts"; Ministry of Health, Labour and Welfare, "Monthly Labour Survey"; Ministry of Internal Affairs and Communications, "Labour Force Survey," "Consumer Price Index"; Bank of Japan, "Call Money Market Data."

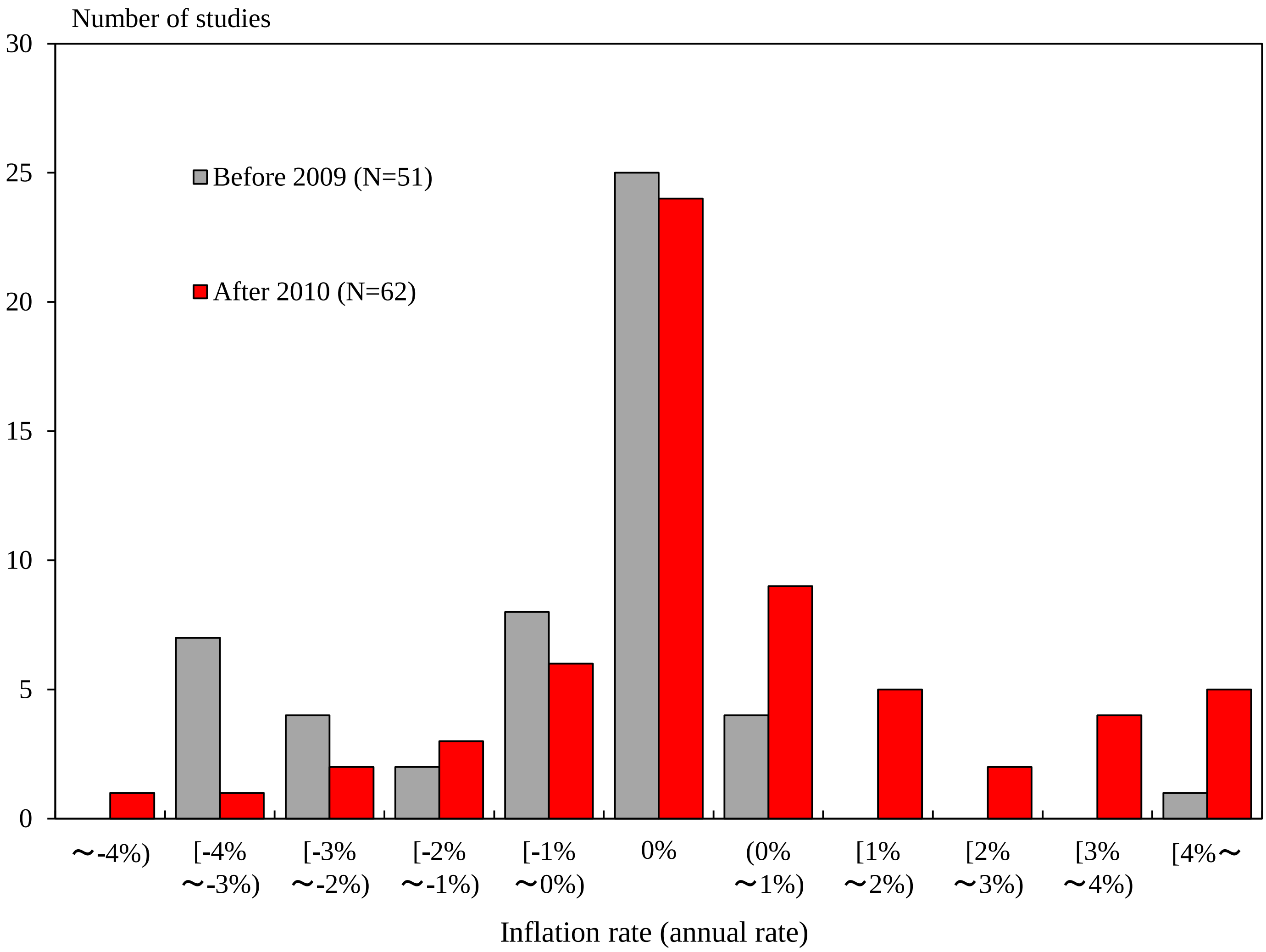
Data series in the U.S.: Bureau of Economic Analysis, "National Income and Product Accounts"; Bureau of Labor Statistics, "Current Employment Statistics"; Board of Governors of the Federal Reserve System, "Selected Interest Rates H.15"; Organization for Economic Co-operation and Development, "Main Economic Indicators." The series are retrieved from FRED provided by the Federal Reserve Bank of St. Louis.

Table 3: Lowering effective lower bound of nominal interest rate

(%, annual rate)

	Steady-state inflation rate that maximizes social welfare	
	Japan	U.S.
ZLB (baseline)	1.9	2.3
Lowering ELB by 0.1 percentage points	1.8	2.2

Figure 1: Steady-state inflation rates that maximize social welfare in previous studies

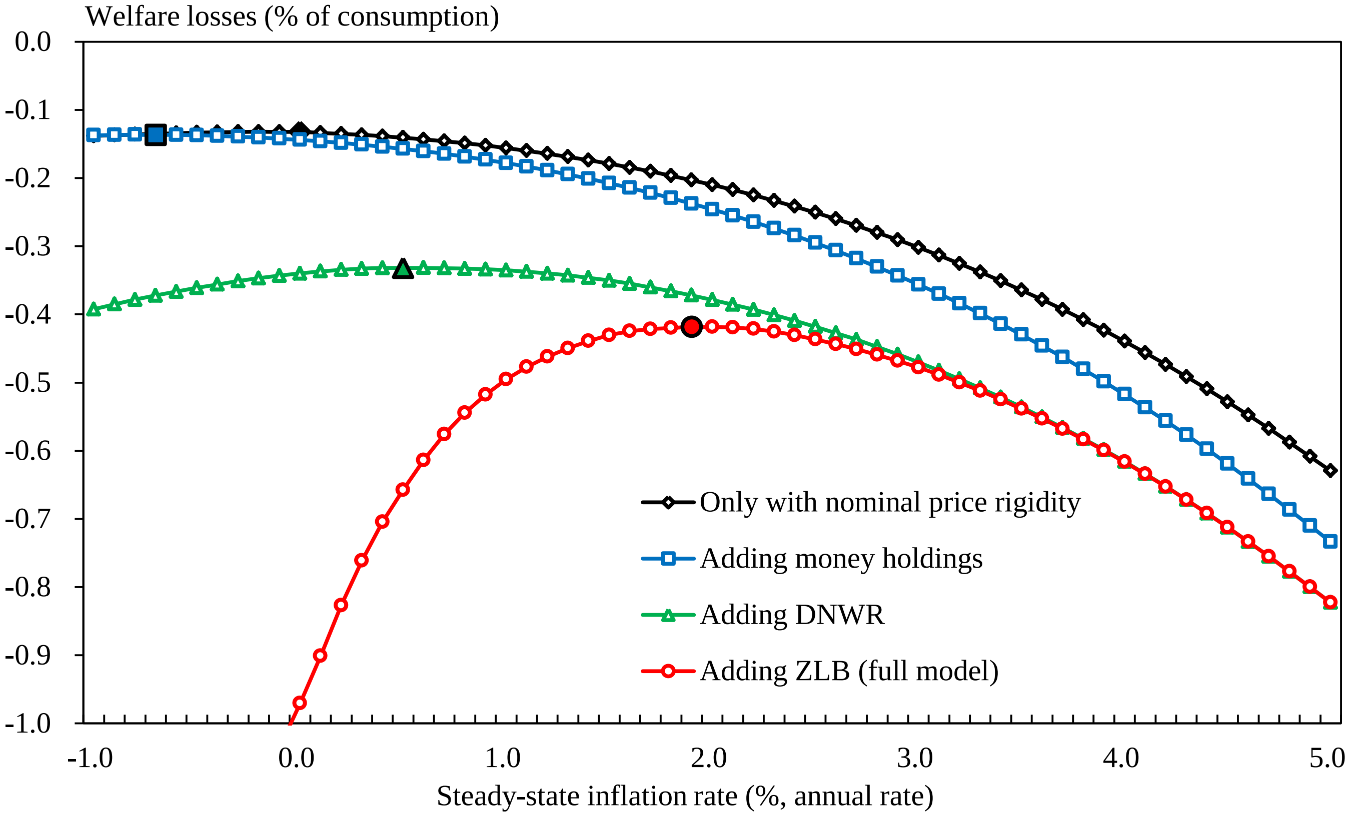


Notes: 1. The figure sums up the results of previous studies conducted between 1989 and 2019 regarding the steady-state inflation rates that maximize social welfare for the U.S. economy.
2. For those studies that give more than one estimate, their average estimate is shown.
3. Square brackets [ ] in the horizontal axis include threshold values, whereas round brackets ( ) do not.

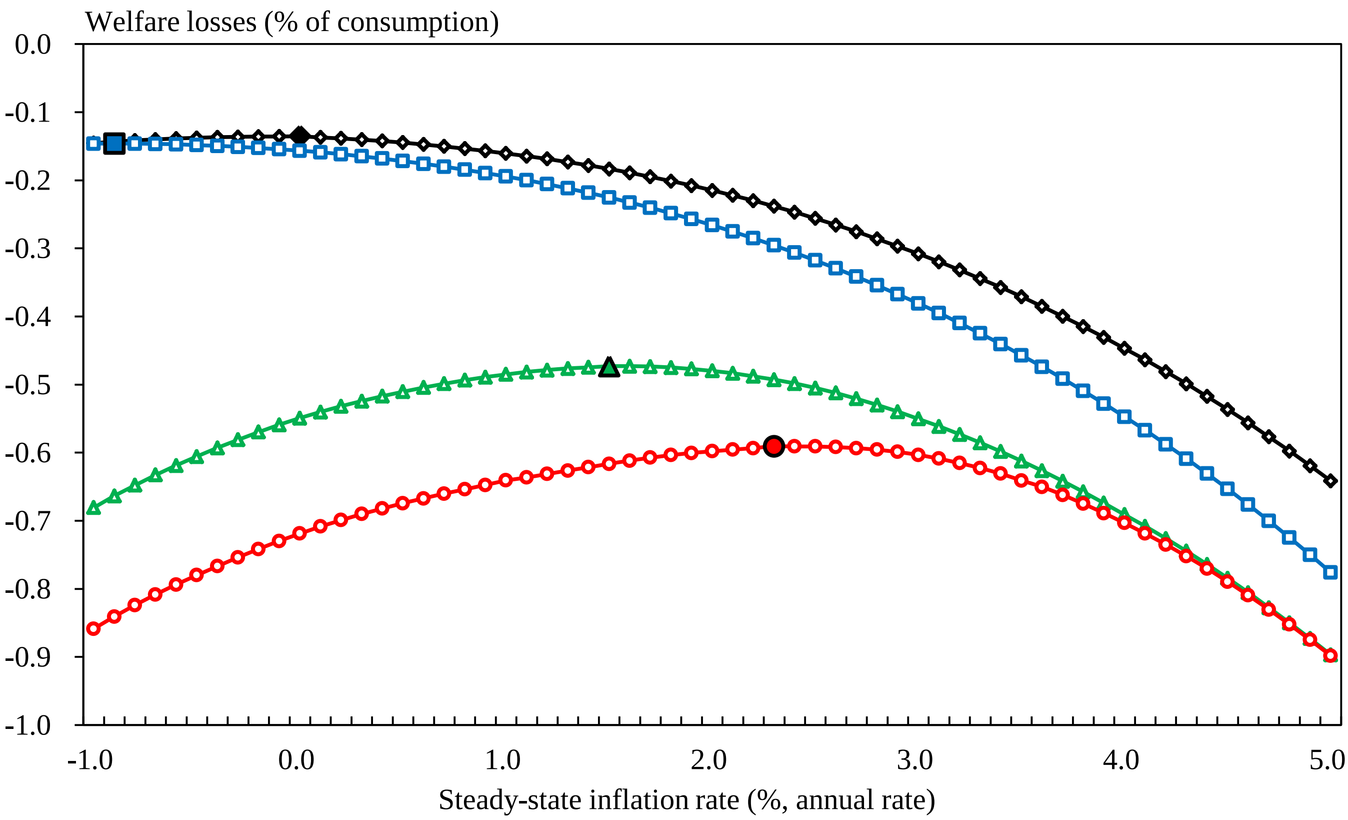
Sources: Diercks (2017) and others.

Figure 2: Welfare losses under Taylor rule

(1) Japan



(2) U.S.

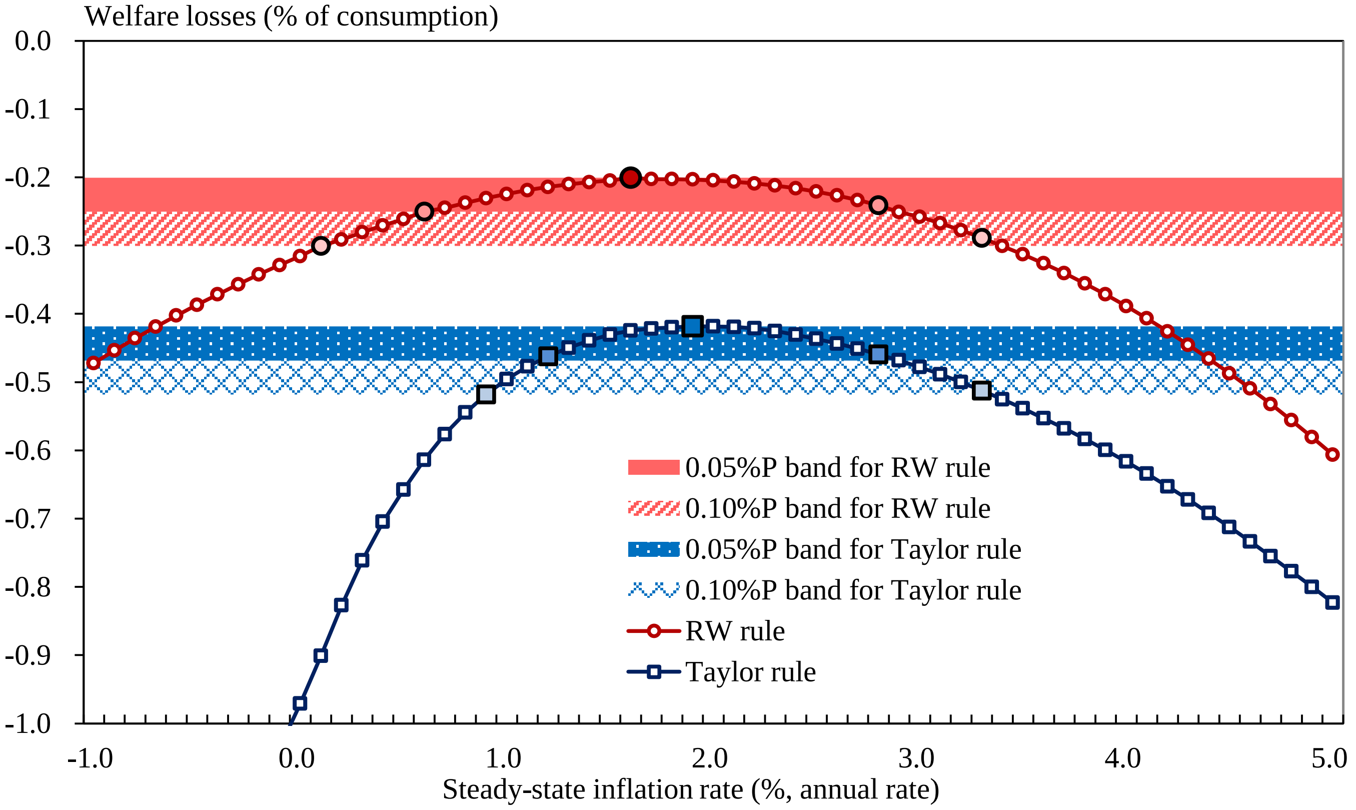


Notes: 1. Welfare losses are the deviations of social welfare from that in the cashless economy under flexible prices and wages. They are measured in terms of the percent of period consumption.

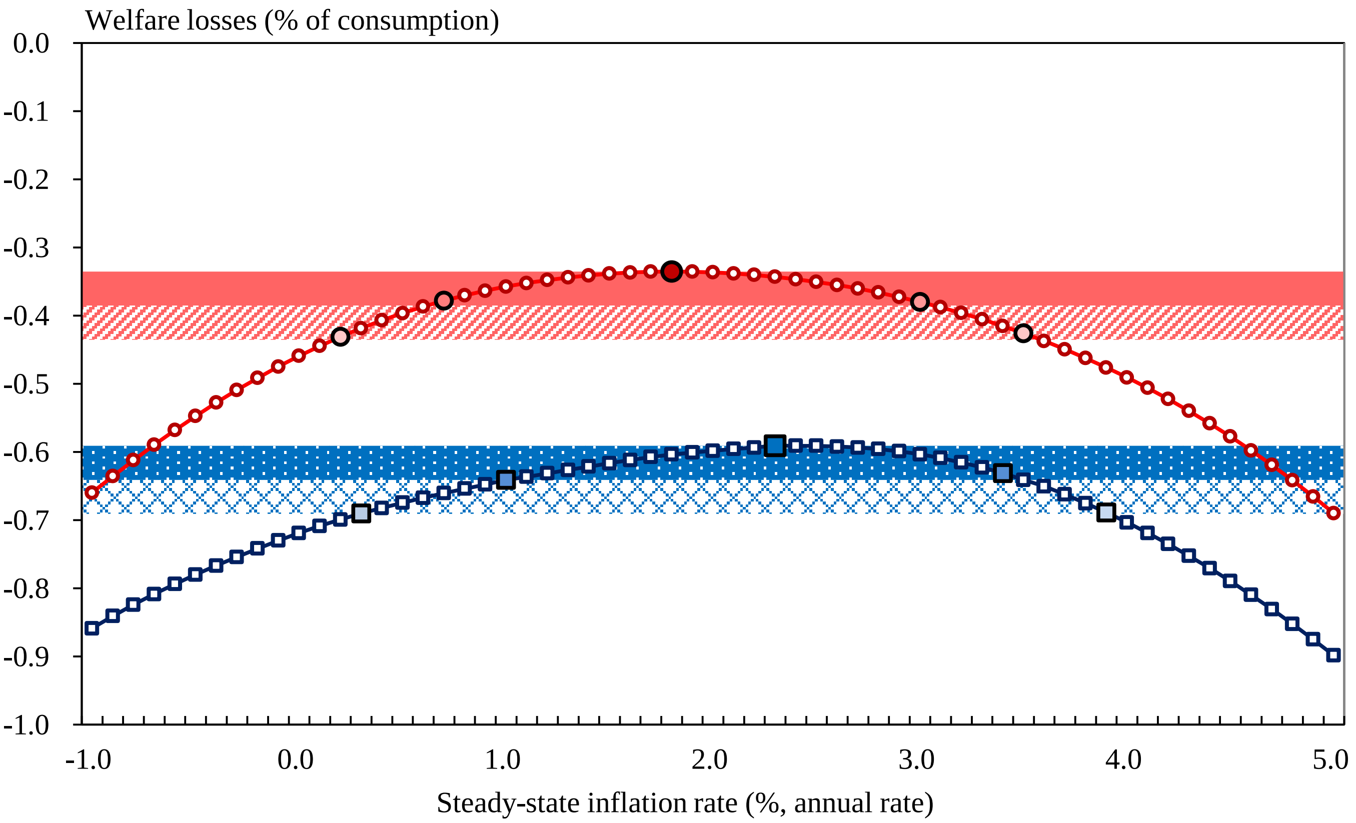
2. The plot point with a filled marker indicates the steady-state inflation rate that maximizes social welfare in each specification (the same hereafter).

Figure 3: Welfare losses under RW rule

(1) Japan



(2) U.S.

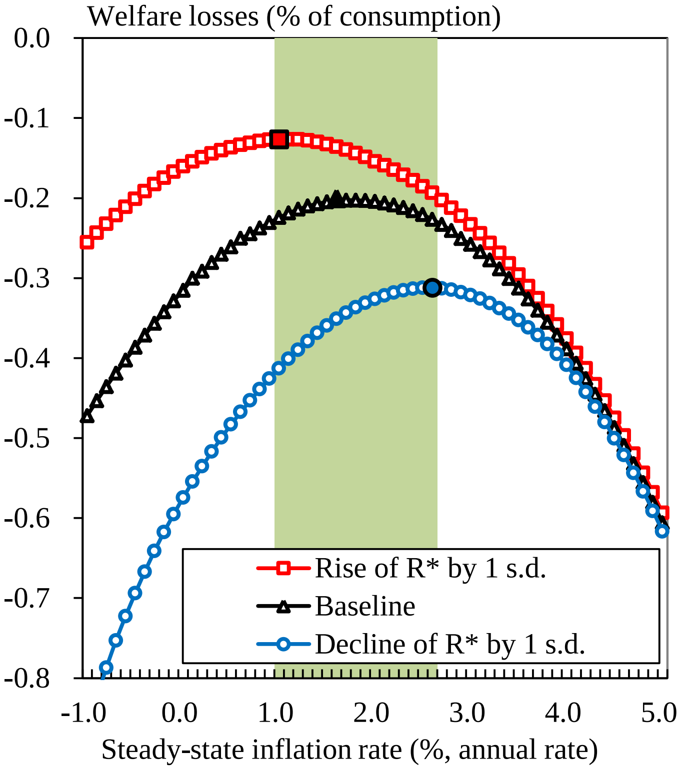


Note: The solid red band and the blue band with pin-dots indicate the range in which the decline in social welfare from its maximum is within 0.05 percentage points, whereas the red band with diagonal lines and the blue band with diagonal grids are those within 0.10 percentage points.

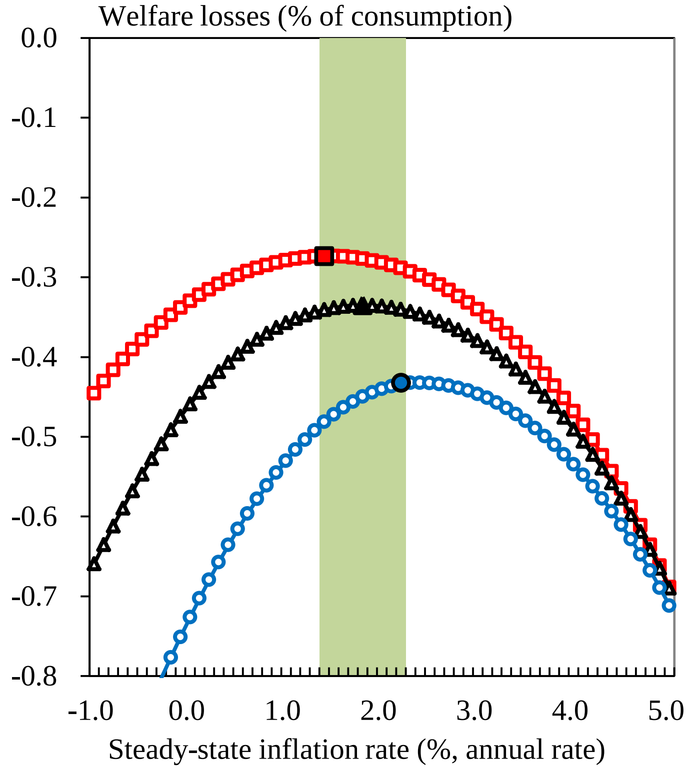
Figure 4: Effects of parameter uncertainty

(1) Uncertainty regarding steady-state level of natural rate of interest

(i) Japan

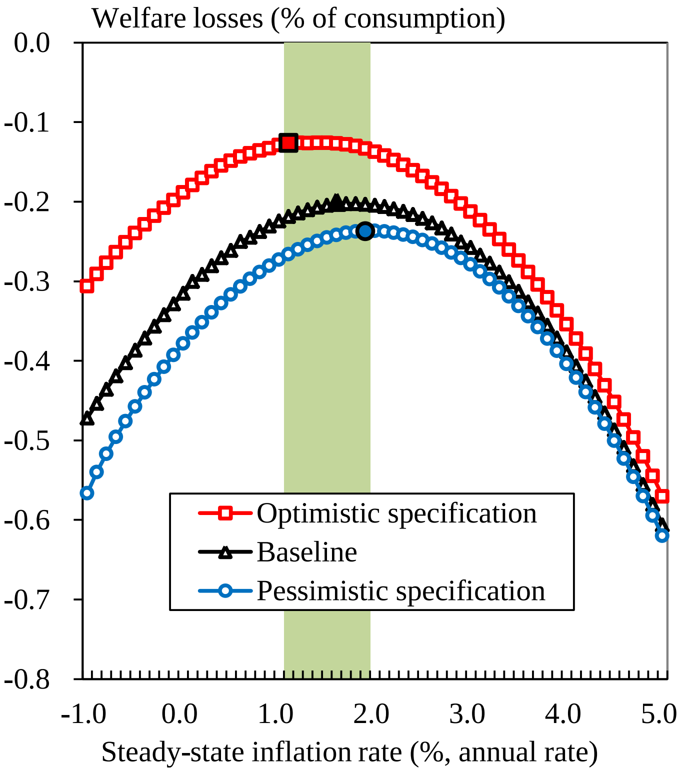


(ii) U.S.

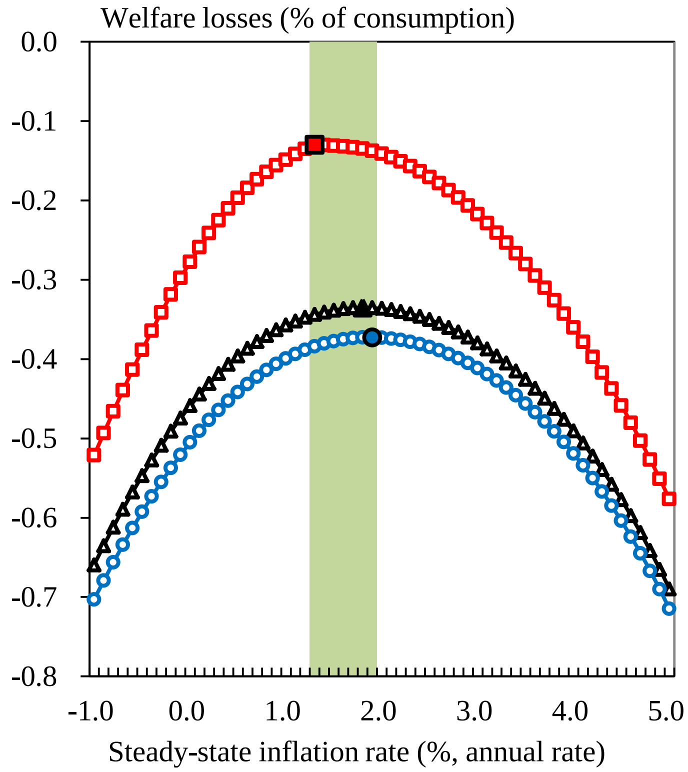


(2) Uncertainty regarding specification of ZLB shocks

(i) Japan



(ii) U.S.



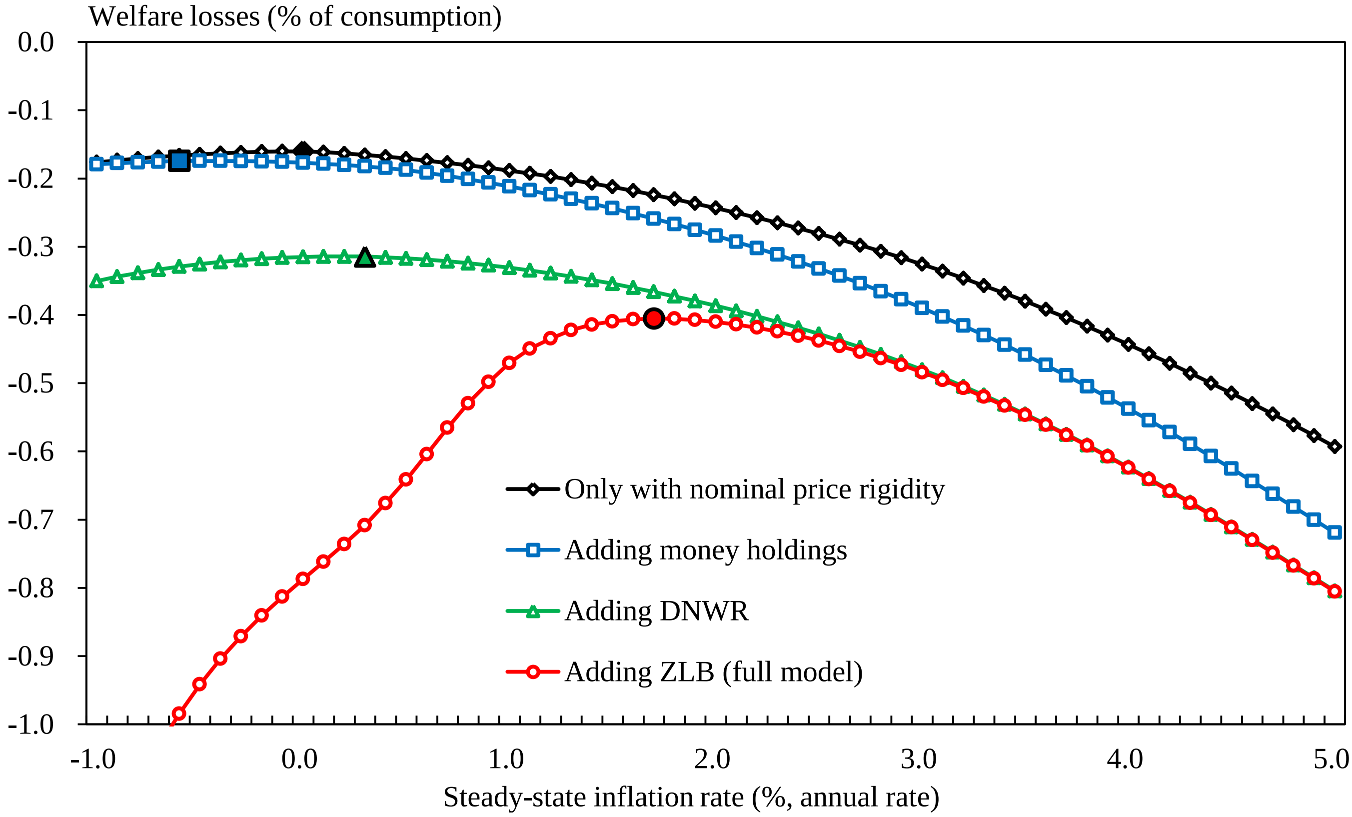
Notes: 1. In (1), we consider the specifications where the steady-state value of the natural rate of interest deviates from the baseline value upward and downward by one standard deviation of the estimated series based on the Laubach and Williams (2003) model.

2. In (2), we formulate optimistic and pessimistic specifications regarding the frequency, duration, and size of the ZLB shocks according to the data.

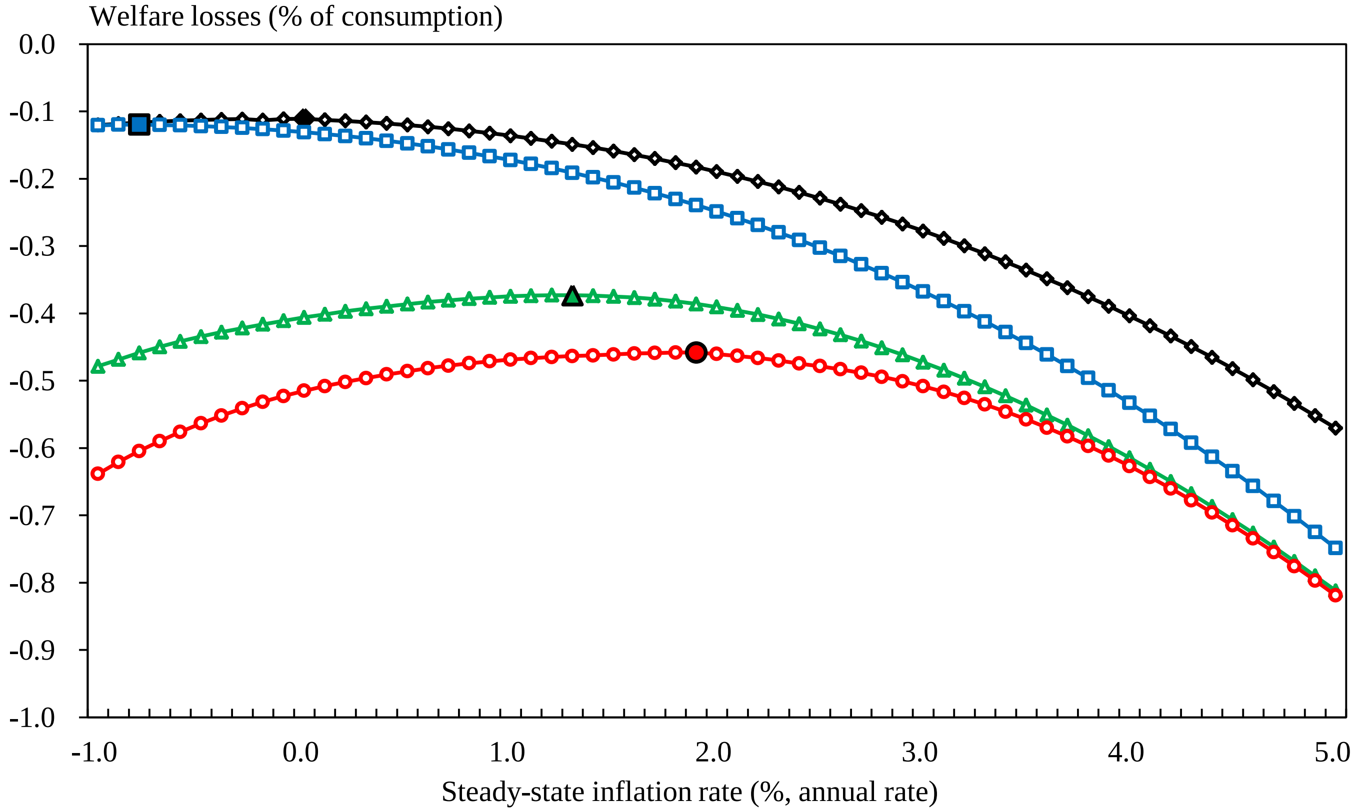
3. The central bank is assumed to follow the RW rule.

Figure 5: Habit formation

(1) Japan



(2) U.S.



Note: The central bank is assumed to follow the Taylor rule.