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Abstract

Carbon tax has attracted increasing attention as a means of curbing greenhouse gas (GHG) emissions. While the implementation of carbon taxes necessarily involves consideration of the impact across different sectors and different periods, most existing studies use models which do not provide a detailed account of either sectoral interaction or the dynamic nature of the responses of households and firms. To fill this gap, we construct a New Keynesian multi-sector dynamic stochastic general equilibrium (DSGE) model with an input-output structure of intermediate inputs and an investment network calibrated to Japan's economy. We study the impact over time of carbon tax on different sectors, on aggregate GDP, and on GHG emissions. We then consider the long-term implications through a steady-state analysis, and the short- to medium-term implications by a simulation from 2020 to 2050, under various scenarios with different tax base compositions and announcement timings. We show that the impact on the trade-off between output and GHG emissions is importantly affected by inter-sectoral interactions among firms, and by the intertemporal decisions of households and firms.

JEL classifications: D57, E22, H23, Q54
Keywords: carbon tax, climate change, transition risks, input-output linkages

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1 Introduction

There is a growing international consensus that immediate policy actions should be taken to mitigate climate change. A number of countries have already declared their commitment to achieving net zero greenhouse gas (GHG) emissions by 2050, and they are starting to appraise the policy instruments that might be implemented. Carbon tax policy is a set of taxes imposed on economic activities that cause GHG emissions, such as the use of fossil fuels in the production process, with the aim of curbing GHG emissions by increasing the price of conducting these activities. However, the amount of GHG emissions depends significantly on differences in the type of economic activities across production sectors, or on the technology level of the economic activities at a specific time, due to technological characteristics or the pace of technological advances. For this reason, considerations of the effects of carbon tax implementation necessarily include those of sectoral spillover, typically in the production process, and the intertemporal decisions of firms and households.

Carbon tax is generally considered an effective means of addressing climate change and associated issues, such as more frequent or severe weather events and rising health issues, i.e., physical risks. However, there are concerns about transition risks, particularly the risk that the implementation of carbon taxes could adversely affect how households consume and how firms produce, thus dampening the aggregate and sectoral economies. It is therefore important to carefully conduct both theoretical and quantitative assessments of transition risks, and to refine the policy instruments based on these assessments in order to ensure a smooth transition that does not hamper economic activity.

In this paper, we study the impact on the economy of carbon tax implementation using a medium-scale New Keynesian multi-sector dynamic stochastic general equilibrium (DSGE) model. Our model incorporates elements that are commonly adopted in the standard New Keynesian sticky price model, including forward-looking optimizing households and firms, nominal friction of prices, and adjustment costs of investment. But our model also incorporates two types of sectoral spillovers, namely, spillover through the input-output linkages of intermediate inputs across sectors, similar to the work of Dupor (1999) and Horvath (2000), and spillover through cross-sectional investment networks, similar to the work of Vom Lehn and Winberry (2022). While the transition risk associated with carbon tax implementation is by its nature medium- to long-term and has a disproportionate impact on different sectors, existing studies often employ a model that abstracts from either potential channels of sectoral interaction or dynamic decisions of households and firms, or both. Our aim is to fill this gap. It is important to note that unlike some of the existing studies such as Nordhaus (1994), our model incorporates a mechanism in which the economic activities yield GHG emissions, but it does not incorporate the feedback mechanism from the accumulated GHG emissions to the economic activities, i.e., chronic or acute physical risks that manifest themselves as a result of GHG emissions. This is because our focus is on a better understanding of transition risks, including their inter-sectoral and intertemporal aspects, and because it is difficult to pin down deterministically when and how these physical risks will emerge.

We first use a version of the model that is static but includes detailed sectoral interactions through intermediate inputs and investment goods calibrated to Japan's economy and study the implications of introducing a carbon tax.[1] In the model, carbon taxes are imposed on three types of economic activities: firms' use of fossil fuels as an intermediate input; the production of goods for which the production process causes GHG emissions, such as the production of cement; and the consumption expenditure of a specific good that causes GHG emissions, such as gasoline consumption. Tax rates are set higher for economic activities that cause higher GHG emissions. Throughout the paper, we refer to carbon taxes for each of the three economic activities as intermediate input tax, production process tax, and consumption tax, respectively. When the production process tax or intermediate input tax is imposed, firms reduce the corresponding production of goods or use of fossil fuels as intermediate inputs, which results in changes in the amount of goods available for the production and consumption or composition of intermediate goods for the production process. When the consumption tax is in place, households spend less on the taxed goods and more on other goods, which changes the composition of consumption expenditure, reducing the demand for products of the targeted sector. Changes in economic activities regarding specific goods or sectors are translated to those of other goods or sectors through the input-output structure of the intermediate inputs and the investment network. The same 1% rise in the carbon tax rate may affect the aggregate GDP and GHG emissions differently, depending on the specific type of economic activity on which the tax is imposed. This is because, on the one hand, the way in which a rise in a specific tax rate is translated to changes in the aggregate economy depends on the characteristics of the input-output matrix of intermediate inputs and the investment network, while on the other hand, the way in which a rise in a specific tax rate is translated to changes in GHG emissions depends mostly on the intensity parameter that connects the size of the economic activity to the amount of GHG emissions. For example, an intermediate input tax on "electricity" dampens

[1]It is notable that this exercise is similar to the one conducted by King et al. (2019), although our model differs from theirs by incorporating not only intermediate inputs but also capital stock, and therefore the investment network and addressing the consumption tax and intermediate input tax.

the aggregate economy and reduces GHG emissions significantly, since this sector provides large production inputs to other sectors and at the same time uses a considerable amount of fossil fuels as the intermediate inputs, which in turn produces relatively large GHG emissions. By contrast, a production process tax on "professional, scientific and technical activities" dampens the aggregate economy since the sector plays a large role as a supplier of production inputs. The same tax, however, increases rather than decreases GHG emissions, since GHG emissions from the production process of the sector are limited and a decline in products of this sector increases the production of other sectors that cause higher GHG emissions.

In addition to the static exercise, using the full model that incorporates dynamic decisions of households and firms and is calibrated to Japan's economy, we employ scenarios of carbon prices from 2020 to 2050 provided by the Network for Greening the Financial System (NGFS) and compute how GDP, sectoral value added, and GHG emissions evolve under these scenarios. While this type of exercise, a scenario analysis, is widely accepted as an important tool for evaluating climate-related financial risks, existing exercises are often based on models that abstract from sectoral interlinkages or dynamic aspects. To the best of our knowledge, our work is the first analysis of transition risks based on a full-fledged DSGE model with sectoral interlinkages through intermediate inputs and the investment network.

Our findings are summarized as follows. First, the implementation of carbon tax generally reduces GDP and GHG emissions not only in the short run but also in the long run. Second, the impact of carbon tax on these variables differs depending on various factors. These include the degree of interaction among sectors through the use of intermediate goods inputs and the investment network, advancements in energy-saving or abatement technology in each sector, the level of distortions due to carbon taxes that are already in place, and the timing of the announcement of a carbon tax schedule. In our simulation, for example, we show that, keeping the total amount of cumulative GHG emissions constant until 2050, a GDP decline due to carbon taxation is mitigated when zero tax rates are imposed on the GHGs emitted in the production process, while instead, higher tax rates are imposed on the other two types of activities. Also, the announcement of a carbon tax schedule before actual implementation helps create a smooth transition for the economy by moderating changes in variables, in particular the capital stock. Our results showing that the responses of GDP and GHG emissions depend on these factors implies that there is a combination of different types of carbon tax that minimizes the decline in GDP while achieving a certain amount of reduction in GHG emissions over time. This in turn underscores the importance of careful calibration of carbon tax rates across economic activities and across time.

Related literature. Our study is related to the current literature in three ways. First, it is related to the theoretical and empirical research on transition risks associated with climate change. Our paper is most closely related to King et al. (2019), Devulder and Lisack (2020), Cavalcanti et al. (2021), and Frankovic (2022), which construct a multi-sector static model that incorporates sectoral interaction through intermediate inputs and examine the sectoral impact of carbon taxation.[2] Our paper is in line with these works in stressing the importance of the sectoral transmission of carbon tax and of targeted carbon tax, rather than non-targeted tax, for the purpose of mitigating the trade-offs between GDP and GHG emissions. However, our paper differs from them in also stressing the importance of dynamic decisions through capital formation. Our study is also related to the rapidly growing number of analyses of environmental policy using DSGE models (for a survey of analyses using business cycle models, see Annicchiarico et al. (2022)). For example, Annicchiarico and Di Dio (2015) use a standard New Keynesian model to theoretically examine the macroeconomic impact of setting emission caps and the impact of price rigidities on the effectiveness of such environmental policies. Another theoretical study of the interaction between environmental regulations and macroeconomy is Carattini et al. (2021), which points out that frictions in the financial sector may amplify the effects of environmental regulations on the macroeconomy. Baldwin et al. (2020) focus on the impact of emission reduction policies on brown-sector investment. Our framework differs from these works in the sense that it incorporates both the sectoral interaction through intermediate inputs and the investment network and the dynamic decisions of firms and households in a unified framework to address the unique characteristics of carbon taxation.

Second, our model builds on a business cycle model that features the sectoral interactions that arise from the input-output linkages of intermediate inputs or capital goods production. Studies using this type of model stress that a shock hitting a specific sector may have an aggregate impact, depending on the structure of the input-output matrix and investment network. The studies that focus on intermediate inputs include Dupor (1999), Horvath (2000), Foerster et al. (2011), Acemoglu et al. (2012b), Atalay (2017), and Baqaee and Farhi (2019). Our model is closest to Bouakez et al. (2009), Bouakez et al. (2014), and Pasten et al. (2020), which incorporate the sectoral interaction through intermediate uses into the standard New Keynesian sticky price model and study the transmission of a monetary policy shock. The studies that address the sectoral interaction through linkages of capital production include Foerster et al. (2019) and Vom Lehn and Winberry (2022). Clearly, the difference between our model and these studies is

[2] Blackburn and Moreno-Cruz (2021) also use a model with production linkages, but they analyze the impact of improvements in energy efficiency rather than the impact of carbon taxes.

that we focus on the implications of carbon tax implementation. Carbon tax has some commonalities with sectoral productivity in the sense that its effects are translated to the other sectors.

Lastly, our work contributes to the literature that uses the quantitative model for medium- to long-term scenario analysis under several climate scenarios. De Bruin et al. (2009) modify the Dynamic Integrated model of Climate and the Economy (DICE) originally developed by Nordhaus (1994) to show long-term economic development with different adaptation policies. Popp (2004) extends DICE by endogenizing technological progress in the energy sector and making a long-term projection, using a model where the innovation-related parameters are calibrated to existing empirical studies. There are also studies conducted by policy makers. For Banque de France, Devulder and Lisack (2020) construct a static multi-sector and multi-region general equilibrium model that consists of 55 sectors and three regions calibrated to France, the rest of the European Union, and the rest of the world, and quantitatively assesses how a rise in carbon price translates to a decline in regional and sectoral GDP. Using the outcome of Devulder and Lisack (2020) and that of other models, including Banque de France's rating model, calibrated to two reference risk scenarios given by the NGFS, Allen et al. (2020) provide a methodology by which the associated transition risk facing financial institutions in France can be quantitatively assessed. The European Central Bank (ECB) has also conducted a scenario analysis. The background paper by Dunz et al. (2021) estimates the resilience of non-financial corporations and euro area banks to climate risks, arguing that the medium- to long-term physical risks from climate change outweigh the short-term costs of the transition, and that there are benefits to acting early. Various methods have been developed to analyze the impact of climate change on the economy, and each one has its own advantages and disadvantages. Conventionally computable general equilibrium (CGE) models are used to estimate carbon tax effects (e.g., Meng et al. (2013), McKibbin et al. (2018)). While CGE models generally abstract the dynamic optimization of economic agents and have a rich cross-sectional structure, our model builds explicitly on the dynamic optimization behavior of agents and general equilibrium. However, it does not have, for example, a variety of competing energy sources in the energy sector, or an international perspective.

The remainder of the paper proceeds as follows: Section 2 presents the model description. Section 3 explains our calibration strategy. Section 4 explores the propagation of carbon tax in the presence of production linkages across sectors through intermediate inputs and the investment network. Section 5 presents the simulation result under the NGFS scenarios and describes the implications of sectoral interaction and the dynamic behavior of households and firms in the transmission effects of carbon tax. Section 6 concludes.

2 Model

In this section, we present a multi-sector DSGE model with production linkages across sectors through intermediate inputs and the investment network. The structure of the model is similar to that of Bouakez et al. (2009), Bouakez et al. (2014), and Pasten et al. (2020), which embed the input-output structure into a DSGE model with price rigidity. The economy consists of N sectors. Each product is either used as an intermediate input for production or an input for capital production or is consumed by the households. This framework allows us to capture the knock-on effects of carbon taxes caused by sectoral interconnections in addition to the dynamic adjustment process in the transition to a low-carbon economy.

2.1 Carbon tax

In this paper, we consider the following three types of carbon taxes, à la Devulder and Lisack (2020). Through these three types of carbon taxes, the government addresses all GHG emissions in the economy. These carbon taxes are imposed based on the Scope 1 measure of GHG emissions, so that the sum of the emissions by each sector and by households equates to the total amount of emissions. In the intermediate input stage, a carbon tax ζ_{io}, which we refer to as intermediate input tax, is imposed on GHG emissions from the energy combustion associated with the use of fossil fuels in sector i, where subscript o denotes the petroleum and coal products sector. At the production stage, the carbon tax τ_i, which we refer to as the production process tax, is imposed on sector i's GHG emissions that arise from the production process. For example, in cement production, GHG emissions occur due to the combustion of limestone rather than the consumption of energy, so the tax is imposed on the amount of cement produced. At the final consumption stage, a carbon tax κ_i, which we refer to as the consumption tax, is imposed on GHG emitted from households' consumption of specific goods. For example, when a household receives utility flow from having a car that gets driven, carbon dioxide is emitted due to the combustion of gasoline. The tax is then imposed on the household's purchase of gasoline. Since most of the GHGs emitted by households' consumption activities are estimated to come from the consumption of petroleum and coal products, including gasoline, we assume that only the consumption of these goods is taxed. That is, $\kappa_i = 0$ if $i \neq o$. All tax revenues are rebated lump-sum to the households.

2.2 Model setup

Households. Households earn their income by supplying labor elastically to firms and by holding assets that consist of firms' equity and firms' physical capital. They allocate their income across consumption of N goods and asset investment. Let $C_{i,t}$ denote the consumption of good i at time t and L_t denote the amount of labor supply. There exist shares for each of the firms in sector i, denoted by $S_{i,t}$, and a liquid asset A_t with deterministic return R_t in the economy. Households maximize the following utility function subject to the budget constraint:

$$\max_{\left\{\{C_{i,t}\}_{i=1}^{N}, L_t, A_{t+1}, \{S_{i,t+1}\}_{i=1}^{N}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\log\left(\prod_{i=1}^{N} C_{i,t}^{\alpha_i}\right) - \phi \frac{L_t^{1+\eta}}{1+\eta}\right],$$

subject to

$$P_t A_{t+1} + \sum_{i=1}^{N} Q_{i,t} S_{i,t+1} + \sum_{i=1}^{N} P_{i,t}\left(1+\kappa_{i,t}\right) C_{i,t} \leq R_{t-1} P_t A_t + \sum_{i=1}^{N} R_{i,t-1}^{S} Q_{i,t} S_{i,t} + W_t L_t + \Pi_t,$$

where β is the subjective discount factor, $Q_{i,t}$ is the price of the firm's share at sector i, $R_{i,t}^{S}$ is the return on shares for firms in sector i, and W_t is the nominal wage. Households can allocate their savings across the liquid asset and firms' equity. The parameter α_i denotes good i's share which satisfies $\sum_{i=1}^{N} \alpha_i = 1$. The price of consumption good $C_{i,t}$ is $P_{i,t}$ and the aggregate price index is $P_t \equiv \prod_{i=1}^{N} \left[\left(1+\kappa_{i,t}\right) P_{i,t}\right]^{\alpha_i}$. We define $C_t \equiv \prod_{i=1}^{N} C_{i,t}^{\alpha_i}$ as aggregate consumption. Since labor is assumed to be perfectly mobile across sectors, households decide only the aggregate amount of labor supply L_t. Transfer to households Π_t is the sum of a firm's profit and the government transfer from the carbon tax revenue. Households pay the carbon tax at rate κ_i, for which the rate is pinned down depending on the amount of GHG emissions associated with the consumption of good i.

Firms. In each sector $i = 1, ..., N$, there is a continuum of monopolistically competitive firms. Firm $\omega \in [0, 1]$ in sector i, denoted as i_ω, chooses the price of its differentiated product given the aggregated demand towards the composite of goods produced in the sector. Competitive firms combine these differentiated inputs and produce the composite of the sectoral good using the following production technology: $Y_{i,t} = \left[\int_0^1 Y_{i_\omega,t}^{\frac{\theta-1}{\theta}} d\omega\right]^{\frac{\theta}{\theta-1}}$. The price of the composite is then given by $P_{i,t} = \left[\int_0^1 P_{i_\omega,t}^{1-\theta} d\omega\right]^{\frac{1}{1-\theta}}$. The production technology of firm i_ω is given by the Cobb-Douglas production function with constant returns to scale technology:

$$Y_{i_\omega,t} = \chi_i Z_{i,t} L_{i_\omega,t}^{\gamma_i^L} K_{i_\omega,t}^{\gamma_i^K} \prod_{j=1}^{N} x_{i_\omega j,t}^{\gamma_{ij}}, \tag{1}$$

where $x_{i_\omega j,t}$ is the quantity of the intermediate good produced by sector j used by firm ω in sector i, $Z_{i,t}$ is TFP of sector i, and $\gamma_i^L, \gamma_i^K, \gamma_{ij} \in (0,1)$ are the set of cost share of production inputs that satisfy $\gamma_i^L + \gamma_i^K + \sum_{j=1}^N \gamma_{ij} = 1$ for all i. Following Carvalho and Tahbaz-Salehi (2019), we further assume that the production function includes the normalizing constant defined below:

$$\chi_i = \gamma_i^{L^{-\gamma_i^L}} \gamma_i^{K^{-\gamma_i^K}} \prod_{j=1}^{N} \gamma_{ij}^{-\gamma_{ij}}.$$

Regarding the capital stock, we assume that firms construct sector-specific investment goods from intermediate inputs using the technology:

$$I_{i_\omega,t} = \chi_{ki} \prod_{j=1}^{N} x_{ki_\omega j,t}^{\gamma_{kij}}, \tag{2}$$

where $x_{ki_\omega j,t}$ is the quantity of the intermediate good produced by sector j used by firm ω in sector i for capital production and $\gamma_{kij} \in (0,1)$ is the set of capital production parameters that satisfy $\sum_{j=1}^N \gamma_{kij} = 1$ for all i. As in the production function in (1), $\chi_{ki} = \prod_{j=1}^N \gamma_{kij}^{-\gamma_{kij}}$. Firms need to pay the additional adjustment cost $\Phi^K(I_{i,t}, K_{i,t})$, which we assume as being homogeneous of degree one in $I_{i,t}$ and $K_{i,t}$, as below:

$$\Phi^K(I_{i_\omega,t}, K_{i_\omega,t}) = \frac{\psi_K}{2}\left(\frac{I_{i_\omega,t}}{K_{i_\omega,t}} - \delta\right)^2 K_{i_\omega,t}. \tag{3}$$

Each producer chooses its price of the differentiated goods to maximize profits subject to price adjustment costs defined as follows:

$$\Phi^P(P_{i_\omega,t}, P_{i_\omega,t-1}) = \frac{\psi_P}{2}\left(\frac{P_{i_\omega,t}}{P_{i_\omega,t-1}} - 1\right)^2 Y_{i,t}.$$

As described in the previous subsection, firms bear two types of carbon taxes: the production tax with a rate τ_i and the intermediate tax with a rate ζ_{ij}. Given this environment, a firm's terminal profit is

$$\begin{aligned}
&\pi\left(P_{i_\omega,t-1}, P_{i_\omega,t}, L_{i_\omega,t}, K_{i_\omega,t}, X_{i_\omega,t}\right) \\
&\qquad = P_{i_\omega,t}(1-\tau_i)Y_{i_\omega,t} - W_t L_{i_\omega,t} - \sum_{j=1}^{N} P_{j,t}\left(1+\zeta_{ij}\right)x_{i_\omega j,t} - \sum_{j=1}^{N} P_{j,t}x_{ki_\omega j,t} \\
&\qquad\quad - \mu_{i,t}\Phi^K\left(I_{i_\omega,t}, K_{i_\omega,t}\right) - P_{i,t}\Phi^P\left(P_{i_\omega,t}, P_{i_\omega,t-1}\right),
\end{aligned}$$

where $X_{i_\omega,t} = (x_{i_\omega 1,t}, x_{i_\omega 2,t}, \ldots, x_{i_\omega N,t}, x_{ki_\omega 1,t}, x_{ki_\omega 2,t}, \ldots, x_{ki_\omega N,t})$ and $\mu_{i,t}$ is the price of capital good at sector i. Note that firms pay the dividend to the owner, namely households, from the terminal profit. The decision problem of a firm is

$$\begin{aligned}
V\left(K_{i_\omega,t}, P_{i_\omega,t-1}\right) = \max_{P_{i_\omega,t}, L_{i_\omega,t}, K_{i_\omega,t+1}, X_{i_\omega,t}} &\ \pi\left(P_{i_\omega,t-1}, P_{i_\omega,t}, L_{i_\omega,t}, K_{i_\omega,t}, X_{i_\omega,t}\right) \\
&+ \frac{\lambda_{t+1}}{\lambda_t} V\left(K_{i_\omega,t+1}, P_{i_\omega,t}\right),
\end{aligned}$$

subject to

$$Y_{i_\omega,t} = Y_{i,t}\left(\frac{P_{i_\omega,t}}{P_{i,t}}\right)^{-\theta}, \tag{4}$$

$$K_{i_\omega,t+1} = (1-\delta)K_{i_\omega,t} + \chi_{ki}\prod_{j=1}^{N} x_{ki_\omega j,t}^{\gamma_{kij}}, \tag{5}$$

where λ_t is the Lagrange multiplier associated with the households' budget constraint at time t. As is standard in a monopolistic competitive model, the firm maximizes its profit conditional on the sectoral aggregate demand function in equation (4). The firm's capital depreciates exponentially at rate δ as described in equation (5). For simplicity, we assume the same depreciation rate across sectors.

Taylor rule. The central bank adjusts the nominal interest rate $R_{n,t}$ by the Taylor rule specified below:

$$R_{n,t} = R_{n,t-1}^{\rho_\pi}\left(R^*\pi_t^{\phi_\pi}\right)^{1-\rho_\pi}, \tag{6}$$

where $\pi_t = \frac{\prod_{i=1}^{N}[P_{i,t}]^{\alpha_i}}{\prod_{i=1}^{N}[P_{i,t-1}]^{\alpha_i}}$ and ρ_π denotes the degree of the persistency of the policy rate adjustment. The parameter ϕ_π is the weight attached to stabilizing the inflation rate and R^* is the equilibrium real interest rate. The real interest rate is given by the Fisher equation: $R_t = R_{n,t}/\pi_{t+1}$.

Equilibrium. We impose symmetricity on firms that produce differentiated goods within sectors, and thus omit the subscript ω. The competitive equilibrium for time t =

$1, 2, \ldots$ is characterized by prices $\left(P_{i,t}, W_t, R_t, R^S_{i,t}, Q_{i,t}, R_{n,t}\right)$ and quantities $\left(C_{i,t}, I_{i,t}, A_t, S_{i,t}, K_{i,t}, L_{i,t}, x_{ij,t}, x_{kij,t}\right)$ for $i = 1, \ldots, N$ and $j = 1, \ldots, N$, which clear goods, asset, labor, and capital markets, and economic agents that solve the respective optimization problems given prices. Note that the resource constraint for each good i is given as follows:

$$Y_{i,t} = C_{i,t} + \sum_{j=1}^{N} x_{ji,t} + \sum_{j=1}^{N} x_{kji,t} + \Phi^K \left(I_{i,t}, K_{i,t}\right) + \Phi^P \left(P_{i,t}, P_{i,t-1}\right).$$

See Appendix B for more details of the equilibrium conditions.

Definitions of GDP and value added. Nominal GDP of the economy is defined as

$$\mathrm{NGDP}_t = \sum_{i=1}^{N} \left(P_{i,t} \left(1 + \kappa_{i,t}\right) C_{i,t} + \mu_{i,t} I_{i,t}\right).$$

We define the real GDP growth as the weighted average of real growth rates of consumption and investment:

$$\triangle \log \mathrm{GDP}_{t+1} \equiv \sum_{i=1}^{N} \frac{P_{i,t} \left(1 + \kappa_{i,t}\right) C_{i,t}}{\mathrm{NGDP}_t} \triangle \log C_{i,t+1} + \sum_{i=1}^{N} \frac{\mu_{i,t} I_{i,t}}{\mathrm{NGDP}_t} \triangle \log I_{i,t+1}. \qquad (7)$$

Given the growth rate, it is possible to back up the level of real GDP. Nominal sectoral value added at sector i is

$$\mathrm{NVA}_{i,t} \equiv P_{i,t}(1 - \tau_i) Y_{i,t} - \sum_{j=1}^{N} P_{j,t} \left(1 + \zeta_{ij}\right) x_{ij,t} - \mu_{i,t} \Phi^K \left(I_{i,t}, K_{i,t}\right) - P_{i,t} \Phi^P \left(P_{i,t}, P_{i,t-1}\right).$$

GHG emissions and carbon cycle. The amount of GHG emissions, including that of CO_2 emissions, is determined by the size of the economic activity that causes the emissions and the exogenously given intensity parameters that are denoted as $\xi_{1,i,t}, \xi_{2,i,t}, \xi_{3,t}$ for $i = 1, \ldots, N$. The intensity parameters represent how much of the relevant economic activity is mapped to GHG emissions, and they differ across sectors and activities, reflecting the heterogeneous structure of consumption and the production process. There are three types of GHG emissions—$CO_{2i,t}$, $GHG^f_{i,t}$, and GHG^h_t—depending on how the emissions take place. $CO_{2i,t}$ stands for the total amount of GHG emissions caused by the combustion of fossil fuels in firms' production process in sector i, and it is assumed that the amount is linked to the firms' input of fossil fuels according to the following equation:

$$CO_{2i,t} = \xi_{1,i,t} x_{io,t},$$

where $x_{io,t}$ denotes the amount of fossil fuel products used as the intermediate inputs for producing good i. It is notable that when there is an energy-saving innovation, the value of this intensity parameter becomes smaller, so that less GHG is emitted per the same amount of economic activity. Next, $\mathrm{GHG}_{i,t}^{f}$ stands for the amount of GHGs emitted in the production process of goods i other than the use of fossil fuels:

$$\mathrm{GHG}_{i,t}^{f} = \xi_{2,i,t} Y_{i,t}.$$

Lastly, GHG_{t}^{h} stands for household GHG emissions and is assumed to be linearly proportional to the consumption of fossil fuel goods:

$$\mathrm{GHG}_{t}^{h} = \xi_{3,t} C_{o,t}.$$

The total amount of GHG emissions is then defined as:

$$\mathrm{GHG}_{t} = \sum_{i=1}^{N} \left(CO_{2i,t} + \mathrm{GHG}_{i,t}^{f} \right) + \mathrm{GHG}_{t}^{h}.$$

Following Golosov et al. (2014), we let M_t be the level of atmospheric carbon concentration that is assumed to follow the simple function of past GHG emissions:

$$M_t - \bar{M} = \sum_{s=0}^{T_s} (1 - d_s)\, \mathrm{GHG}_{t-s}, \tag{8}$$

where $\bar{M}$ is the level of pre-industrial atmospheric CO2 concentration and

$$1 - d_s = \varphi_L + (1 - \varphi_L)\, \varphi_0 \,(1 - \varphi)^s .$$

Parameter φ_L denotes the share of GHG emissions that stay in the atmosphere forever, and $(1 - \varphi_L)\, \varphi_0$ is the share of GHG emissions that are not absorbed into the biosphere or the surface of the ocean. The latter decays at the constant ratio φ.

3 Calibration

We calibrate the standard parameters of the model to existing studies and the model parameters related to the sectoral linkages and GHG emissions, including the input-

Parameter	Description	Value
β	Subjective discount rate	0.996
ϕ	Labor disutility	0.200
η	Inverse of Frisch elasticity of labor supply	0.887
γ^L	Labor share	0.307
γ^K	Capital share	0.151
δ	Capital depreciation rate	0.028
θ	Demand elasticity	7
ψ_K	Capital adjustment cost	10.93
ψ_P	Price adjustment cost	6.950
ρ_π	Policy rate adjustment	0.8
ϕ_π	Policy elasticity to inflation	1.5
φ	Parameter on carbon cycle	0.000577
φ_L	Parameter on carbon cycle	0.2
φ_0	Parameter on carbon cycle	0.402

Table 1: Parameter values

output structure and the carbon intensity parameters, to Japan's economy in 2015-2019 whenever possible. The frequency of the model is quarterly.

3.1 Parameter values

Table 1 summarizes the parameter values. We depend greatly on parameter estimates from Okazaki and Sudo (2018) which estimate the parameters of the New Keynesian DSGE model with various frictions for Japan's economy.

Preferences. The parameter α_i is the nominal expenditure share of good i over the household's total expenditure, and the values are taken from the input-output table in 2015 and listed in Table A.2 in Appendix C. The subjective discount factor β is set to be equal to the inverse of the average of the natural rate of interest during 1980-2017 estimated by Okazaki and Sudo (2018).

Production technology. The level of TFP $Z_{i,t}$ is set to unity for $i = 1, ..., N$. The parameter values of the intermediates for input-output network γ_{ij} are computed from sector i's expenditure on the intermediates produced by sector j as a share of sector i's total expenditure from the latest input-output table in 2015. The labor share γ^L and the capital share γ^K of the goods production are taken from the same dataset and are set to 0.307 and 0.151, respectively. Note that the shares of these primary inputs are the same across sectors. The quarterly capital depreciation rate is set at 0.028, taken from Okazaki and Sudo (2018).

Investment technology. The parameter values of adjustment cost ψ_K vary in the literature. The simulated method of moments estimate by Bouakez et al. (2009) is 10.93 while the estimate by Okazaki and Sudo (2018) is 2.275 and Vom Lehn and Winberry (2022) calibrate it to 0.5. Since our model structure is close to Bouakez et al. (2009), we employ their estimate. The parameter values of the production of investment goods γ_{kij} are computed from the fixed capital matrix in 2015.

Carbon cycle. The estimate of total GHG emissions without land-use change in 2019 is 51.5 gigatons of CO_2, taken from UNEP (2021), while that in Japan is 1.212 gigatons of CO_2 in the same period, taken from the National Institute for Environmental Studies, Japan. While our model is calibrated to Japan, given the global nature of the carbon cycle, we multiply the GHG emissions by 51.5/1.212 and feed this into equation (8). The atmospheric carbon concentration in pre-industrial era $\bar{M}$ is set to 2128.78 gigatons of CO_2, following Golosov et al. (2014). Other parameters in equation (8) are also taken from Golosov et al. (2014).

Others. Parameter values of the Taylor rule in equation (6) are set to standard values in the literature (e.g., Luetticke (2021)). All other parameters are taken from Okazaki and Sudo (2018).

3.2 Sectoral GHG emissions and carbon tax rates

Sectoral GHG emissions. The carbon tax burden is determined by the product of the carbon price and GHG emissions. To calibrate the level of carbon taxes on each sector, sectoral GHG emission data are required. Air emissions accounts (AEAs), which record flows of GHG and air pollutants emitted into the atmosphere as a result of economic activity, have the advantage of wide coverage of GHG emissions and the ability to distinguish GHG emissions from fossil fuel use from that of other activities such as industrial processes. As Japan has not officially published AEAs, as of writing, we attempt to compile data on Japan's sectoral GHG emissions, following the "Manual for air emissions accounts" provided by Eurostat (2015). Appendix A presents the details of the methodology for generating the data.

Figure 1 shows the amount of GHG emissions in the total economy by type of economic activity. About 80% of GHG emissions are from firms' use of fossil fuels. Firms' emissions which are not from energy consumption and GHG emissions from households account for about 10% each of total emissions. Figure 2 presents the average sectoral GHG emissions per sectoral value added from 2015 to 2019. The GHG emissions from use of fossil fuels shown in Panel (a) of Figure 2 differ significantly across sectors among which "electricity supply" and "basic metal" are high emission sectors.

As for GHGs emitted in production processes shown in Panel (b) of Figure 2, "agriculture, forestry and fishing" and "non-metallic mineral products" are identified as high emission sectors. Because the burden of carbon tax is higher for industries that emit a larger amount of GHG emissions in our model, these industries, especially "electricity supply," face a higher carbon tax burden when carbon pricing is introduced.

Carbon tax rates. We calibrate the carbon tax rates for each of the fossil fuel use, production, and consumption stages following Devulder and Lisack (2020). Note that carbon price P_t^C is common across stages and taken from the NGFS scenario, as described in Section 5. Each firm emits CO_2 by combusting fossil fuel intermediates from sector o which produces petroleum and coal products. Given the carbon price P_t^C, the total burden of firms in sector i is $P_t^C CO_{2i,t}$. Tax rate $\zeta_{io,t}$ per intermediate expenditure $P_{o,t}x_{io,t}$ is set to cover the expense:[3]

$$\zeta_{io,t}P_{o,t}x_{io,t} = P_t^C \mathrm{CO}_{2i,t}. \tag{9}$$

We assume $\zeta_{oo,t} = 0$ to avoid double taxation. The tax rate on production process $\tau_{i,t}$ of sector i satisfies:

$$\tau_{i,t}P_{i,t}Y_{i,t} = P_t^C \mathrm{GHG}_{i,t}^f. \tag{10}$$

GHG emissions by households are attributed to the consumption of petroleum and coal products. The consumption tax rate on those products thus satisfies the following equation:

$$\kappa_{o,t}P_{o,t}C_{o,t} = P_t^C \mathrm{GHG}_t^h. \tag{11}$$

4 Propagation Mechanism

4.1 Analytical results

Before conducting the quantitative analysis, we show analytically the transmission mechanism of carbon taxes on prices and value added at the sectoral level, using a simplified static model throughout this subsection.[4] In the simplified model, households

[3]Note that the carbon price P_t^C and carbon intensity $\xi_{1,i,t}$ are exogenous, while the amount of GHG emissions $CO_{2i,t}$, intermediate inputs $x_{io,t}$ and price of petroleum and coal products $P_{o,t}$ are endogenous in the economy. Given the relationship $CO_{2i,t} = \xi_{1,i,t}x_{io,t}$, the size of carbon tax rate $\zeta_{io,t}$ that meets the equation below is given as the function of the price of petroleum and coal products $P_{o,t}$. The production process tax rates and the consumption tax rates are computed in the same way.

[4]While similar exercises are conducted in King et al. (2019), our analysis differs from theirs in that it addresses three types of carbon taxes, studies the role of the sectoral interaction through the investment network as well as that through intermediate input usages, and considers the role of the level of the carbon tax rates. Regarding the last point, while they consider the sensitivity with respect to carbon

supply labor of unity inelastically and firms are perfectly competitive, and we only study the non-stochastic steady state of the model where variables are all unchanged.

Impact on sectoral relative prices. We start with the relationship between each of three types of carbon taxes and prices of each good. For derivation of the result in this section, see Appendix D. For the convenience of the analysis, we study how goods price relative to nominal wage $\hat{P}_i \equiv P_i/W$ reacts to a change in tax rates. By substituting the first order conditions into the production function in equation (1), the logarithm of relative price for each sector $i = 1, \ldots, N$ is given as follows:

$$\log \hat{P}_i = -\sum_{j=1}^{N} l_{ij} \log \left(Z_j \left[1 - \tau_j \right] r_K^{-\gamma_K} \right) + \sum_{j=1}^{N} \sum_{k=1}^{N} l_{ij} \gamma_{jk} \log \left(1 + \zeta_{jk} \right). \tag{12}$$

For the analysis below, we define Γ as the parameter matrix for which the (i, j) argument is $\gamma_{ij} + \gamma_i^K \gamma_{kij}$, the weighted sum of cost share parameters of input-output linkages of intermediate inputs γ_{ij} and the investment network $\gamma_i^K \gamma_{kij}$. Parameter l_{ij} is the (i, j) element of matrix L, which is defined as $L = (I - \Gamma)^{-1}$. As explained in Carvalho and Tahbaz-Salehi (2019) and others, l_{ij} denotes the degree of importance of good j for sector i. That is, matrix L is given by $(I - \Gamma)^{-1} = I + \Gamma + \Gamma^2 + \ldots$, which in turn means that matrix L captures all of the impact of changes in input and output prices, including those due to carbon tax rates through production and the investment network, including indirect transactions.

We obtain the sensitivity of a sectoral price to a change in carbon tax, taking the derivative of the logarithm of the relative price with respect to the intermediate input tax ζ_{mo}:

$$\frac{\partial \log \hat{P}_i}{\partial \zeta_{mo}} = \frac{l_{im} \gamma_{mo}}{1 + \zeta_{mo}} \geq 0.$$

The relative price of good i increases with an intermediate input tax rate ζ_{mo}, and how much the price changes depends on the parameter $l_{im}\gamma_{mo}$, which represents the importance of goods m for the production of sector i, l_{im}, multiplied by the cost share of inputs from the petroleum and coal products in the production of goods m, γ_{mo}. That is, the price of goods i rises more with a rise in an intermediate input tax on sector m if sector i uses more of sector m's output and sector m uses more petroleum and coal products.

Next, the sensitivity of relative goods prices of sector i's output to the production

taxes around the state where the tax rates are zero exclusively, we consider the case in which the tax rates are above zero. This is because, as we demonstrate below, the sensitivity with respect to carbon taxes is altered with the level of the tax rates. In other words, even when tax rates increase linearly over time, the impact of an incremental change in carbon tax rates on the economy changes nonlinearly over time.

process tax on sector m is given as

$$\frac{\partial \log \hat{P}_i}{\partial \tau_m} = \frac{l_{im}}{1-\tau_m} \geq 0.$$

Just like the impact of the intermediate input tax rate, the impact of the production process tax is proportional to the argument of matrix L and works in a way similar to how a sector-specific technological shock affects the economy. That is, the price of goods i rises greater if sector i uses more of sector m's output. Lastly, equation (12) does not include the consumption tax rate κ_i, and thus the consumption tax does not affect relative prices.

Impact on relative significance of each sector. Next, we explore the impact of each carbon tax on sectoral weight of value added.[5] Substituting the first order conditions into the market clearing condition, we obtain the equilibrium relationship between the weight of sector i and carbon taxes:[6]

$$w_i^D \equiv \frac{P_i\,(1-\tau_i)\,Y_i}{W + r_K \sum_{j=1}^{N} \mu_j K_j + \Pi} = (1-\tau_i) \sum_{j=1}^{N} \frac{\tilde{l}_{ij}\alpha_j}{1+\kappa_j}. \tag{13}$$

The value added of sector i is equal to $\left(\gamma_i^L + \gamma_i^K\right) P_i\,(1-\tau_i)\,Y_i$; thus, w_i^D is proportional to the sectoral value added as a share of nominal GDP for sector i. Note that $\tilde{l}_{ij}$ is the (i,j) element of $\tilde{L} \equiv \left(I - \tilde{\Gamma}\right)^{-1}$, where matrix $\tilde{\Gamma}$ is defined as

$$\tilde{\Gamma} \equiv \begin{bmatrix} \frac{\gamma_{11}(1-\tau_1)}{1+\zeta_{11}} + \frac{\delta\gamma_1^K[(1+\kappa_1)\gamma_{k11}-\alpha_1](1-\tau_1)}{r_K} & \cdots & \frac{\gamma_{N1}(1-\tau_N)}{1+\zeta_{N1}} + \frac{\delta\gamma_N^K[(1+\kappa_1)\gamma_{kN1}-\alpha_N](1-\tau_N)}{r_K} \\ \vdots & \ddots & \vdots \\ \frac{\gamma_{1N}(1-\tau_1)}{1+\zeta_{1N}} + \frac{\delta\gamma_1^K[(1+\kappa_N)\gamma_{k1N}-\alpha_1]\gamma_{k1N}(1-\tau_1)}{r_K} & \cdots & \frac{\gamma_{NN}(1-\tau_N)}{1+\zeta_{NN}} + \frac{\delta\gamma_N^K[(1+\kappa_N)\gamma_{kNN}-\alpha_N]\gamma_{kNN}(1-\tau_N)}{r_K} \end{bmatrix}.$$

Matrix $\tilde{L}$ summarizes the network parameters of the economy. Note that the (i,j)

[5] This result is an extension of Burres (1994) but is distinct in that carbon taxes directly change the network parameters, and thus affect the sectoral weight of value added.

[6] The impact of a change in carbon tax rates on each sector can be divided into primary and secondary effects. The primary effect is the direct impact of a change in the tax rate on the use of factors of production and the types of goods consumed, while the secondary effect is the change in prices and allocations affected by how the resulting tax revenue is spent. Our model assumes that all tax revenues are transferred to the households, but the secondary effect can change if other assumptions are made about how the government uses the tax revenues. In the analysis of this subsection we focus on the primary effect. From this perspective, the sectoral weights w_i^D are defined as (13): the numerator is the payment for the primary inputs of labor and capital inputs in each sector, and the denominator is the total income of households, including what they receive in exchange for the supply of primary inputs and the tax transfers from the government. When the secondary effects are also taken into account, it is necessary to include in the numerator a term associated with the fiscal policy.

element of $\tilde{\Gamma}$ is the increasing function of the cost share of production inputs produced by sector i in the production of goods j, γ_{ji} and γ_{kji}, which in turn indicates that a higher value of $\tilde{l}_{ij}$ implies that sector j is more dependent on the products produced by sector i. Given the nominal expenditure share of the final goods α_j, therefore, the weight w_i^D is large when more of goods i is used as intermediate inputs or investment goods by other sectors.

Now, we take the derivative of sectoral weights with respect to each of three types of carbon tax. First, differentiating equation (13) by the intermediate tax, we obtain

$$\begin{aligned}\frac{\partial w_i^D}{\partial \zeta_{mo}} &= (1-\tau_i)\sum_{j=1}^{N}\frac{\partial \tilde{l}_{ij}}{\partial \zeta_{mo}}\frac{\alpha_j}{1+\kappa_j}\\ &= -(1-\tau_i)\frac{\tilde{l}_{io}\gamma_{mo}w_m^D}{(1+\zeta_{mo})^2},\end{aligned}$$

The impact on the sectoral weight in sector i depends on how the tax rates change the way that products from sector i are used in the economy, shown in the parameter $\tilde{l}_{ij}$. The decline in output of petroleum and coal products induced by the intermediate input tax ζ_{mo} dampens intermediate input demand and input demand for producing capital goods for goods made by sector i. The decline is pronounced when sector m has a larger share in economy, or equivalently when w_m is high, or when sector m is more dependent on the petroleum and coal products sector, or when γ_{mo} is high. Note that because the sectoral weight w_i^D is defined as (13), the change in weight does not capture the secondary effect, and as a result, the sum of the derivatives $\frac{\partial w_i^D}{\partial \zeta_{mo}}$ across sectors is not zero.

As for the production process tax, the impact on the sectoral weight can be described as:

$$\frac{\partial w_i^D}{\partial \tau_m} = -\frac{w_i^D}{1-\tau_i}\mathbf{1}_{[i=m]} - \frac{1-\tau_i}{1-\tau_m}\sum_{k=1}^{N}\frac{\tilde{l}_{ik}\tilde{\gamma}_{km}w_m^D}{1-\tau_m},$$

where $\tilde{\gamma}_{km}$ is the (k, m) element of $\tilde{\Gamma}$. The first term represents the direct sales loss that arises from the increase in tax rate. The second term is the effect through the change in network parameters $\tilde{\Gamma}$, demonstrating that the greater the importance of sector m to sector i, the greater the rise in the tax rate on sector m dampens the sectoral weight of sector i.

Lastly, the impact of the final consumption tax on sectoral weight of value added is

$$\frac{\partial w_i^D}{\partial \kappa_o} = -(1-\tau_i)\,\tilde{l}_{io}\left[\frac{\alpha_o}{(1+\kappa_o)^2} - \sum_{k=1}^{N}\frac{\delta\gamma_k^K \gamma_{kko} w_k^D}{r_K}\right].$$

The transmission goes from downstream to upstream. Consumption of goods o, i.e., fossil fuel products, decreases in proportion to the tax rate, while a carbon tax does not affect the before-tax price itself because the share of nominal household expenditure on fossil fuel products is constant within the total household consumption basket. This drop in demand, represented in the first term, decreases sales of the petroleum and coal products sector and those of suppliers to that sector, whose size is captured by the parameter $\tilde{l}_{io}$. The second term represents the impact on sectoral weight through investment. When each sector depends greatly on the petroleum and coal products sector for producing capital goods, the negative impact on GDP becomes large, which reduces the denominator of sectoral weight.

4.2 Quantitative example using data for Japan[7]

The importance of a particular sector may differ between countries depending on the precise nature of the input-output matrix of intermediate inputs and the investment network in the country. Intermediate input linkages and the investment network measured by the input-output table and fixed capital matrix in 2015 are presented in Figure 3. Note that sectors in the x-axis are producers and those in the y-axis are purchasers, and the size of the number, indicated by the degree of shade of color, represents γ_{ij} and γ_{kij} for panel (a) and (b), respectively. As for the intermediate input, for most of the sectors, the diagonal elements are shaded, implying that the largest producer of the inputs used in the sector is its own sector. Regarding off-diagonal elements, "electricity supply," "wholesale trade," "transport and postal services," and "professional, scientific and technical activities" are largely used as intermediate inputs by other sectors. The "petroleum and coal products" sector is an important intermediate supplier, in particular for "chemicals," "electricity supply," "gas and water supply, and waste management service," and "transport and postal services."[8] As for the investment network, the diagonal elements are often not shaded, and capital for most industries is purchased from "general-purpose, production and business-oriented machinery," "construction," and

[7]In this subsection and beyond, we depart from the simple model and use the full model presented in Section 2 under model parameters described in Section 3. Note in this subsection, we execute only the steady state analysis.

[8]In this classification, the petroleum and coal products sector includes mining of coal, lignite, crude petroleum, and natural gas.

"professional, scientific and technical activities." We also provide a simple intersectoral network diagram in Appendix C.

The role of the investment network. Relative to intermediate input linkages, not much attention has been paid to the investment network. To see the role of the investment network, in Figure 4, we show for each of 32 sectors the measure of importance as a supplier in the production and investment network, based on data for Japan. Let Γ_K be the matrix for which the (i, j) element is γ_{kij}, i.e., the cost share of goods j in producing investment goods of sector i. The figure shows the difference in $\sum_{i=1}^{N} l_{ij}$ between the two cases. One is the case when Γ_K is calculated from the actual data, and the other is the hypothetical case when Γ_K is the identity matrix, so that only the diagonal elements of Γ_K are non-zero. In either case, the structure of intermediate input is given by the data. Note that the latter case represents an economy where there is no interaction across sectors at the stage of producing investment goods, and all of the capital goods used by a specific sector are produced by that sector. While the "petroleum and coal sector products" sector (sector 7 in the figure) shows up as the pronounced goods supplier when the investment network is absent, it is not so when the investment network is considered. This is because the products of the sector are not widely used for capital goods production. By contrast, the importance of the "general-purpose, production and business-oriented machinery" sector (sector 11 in the figure) and the "professional, scientific and technical activities" sector (sector 29 in the figure) is more pronounced when the investment network is incorporated than otherwise would be the case. This suggests that when the investment network is abstracted, the importance of the fossil fuel producers may be overestimated, while that of the sectors that are important for capital production may be underestimated.

Figure 5 shows the impact on real GDP of a 1% point rise in intermediate input tax rates for all sectors when Γ_K is calculated from actual data or when Γ_K is an identity matrix. In other words, we set $\triangle\zeta_{io} = 0.01$ for $i = 1, ..., N$. Calculation of the impact on real GDP is conducted assuming that the economy is at the zero tax steady state. Note that when an identity matrix is used, the impact of production declines in sectors where petroleum and coal products are used intensively and whose products are not used for investment goods is overestimated when compared with the actual impact. When the investment network is considered explicitly therefore, the impact becomes less significant, compared with the case where the investment network is not considered.

Tax effects on real GDP and GHG emissions. Next, we explore the impact of each carbon taxation on aggregate real GDP and GHG emissions. Figure 6 presents the impact of a 1% rise in the production process tax on each sector at the zero tax steady state.[9] As for the macroeconomic impact, taxes on sectors such as "professional, scientific and technical activities" (sector 29 in the figure) and "real estate" (sector 28 in the figure) have a large adverse impact on real GDP because these sectors are important suppliers of intermediate inputs and investment goods in the production network. However, these sectors do not necessarily emit large amounts of GHGs in their production process, as shown in Figure 2. For example, a production process tax on "professional, scientific and technical activities" increases rather than decreases GHG emissions, since GHG emissions from the production process of the sector are limited, and a decline in the products of this sector increases the production of other sectors that cause higher GHG emissions. By contrast, a production process tax on "agriculture, forestry and fishing" or "food products and beverages" leads to a large decline in GHG emissions with a modest GDP decline, since these sectors provide relatively fewer production inputs to other sectors while they cause relatively larger GHG emissions.

Figure 7 shows the impact on real GDP and GHG emissions of the other two types of carbon tax: intermediate input tax, and consumption tax. The taxes on "electricity supply" (sector 18 in the figure) and "transport and postal services" (sector 23 in the figure) have a large adverse impact on real GDP, reflecting their importance in the production network. With regard to the tax effects on GHG emissions, an intermediate tax on electricity supply and a consumption tax on the petroleum and coal products decrease GHG emissions significantly because these sectors exhibit high carbon intensity.

These results suggest that the impact on real GDP and GHG emissions of carbon tax implementation vary depending on which sectors or which economic activities are taxed. For example, a rise in the production process on "professional, scientific and technical activities" leads to a rise in GHG emissions. It is also important to note that the relative decline in real GDP and GHG emissions differs across sectors or tax types. In the case of intermediate input tax, for example, a rise in the tax on "electricity supply" and "transport and postal services" leads to more or less the same magnitude of real GDP decline, while the impact on GHG emissions is much larger for the former than the latter. These observations imply that, given the target level of GHG emission reduction, the adverse effects on real GDP can be mitigated if the calibration of carbon tax rates, including the scope of sectors or economic activities or the rate of the carbon tax, are conducted so that the trade-offs between GDP and GHG emissions are minimized.

[9] Note that for each of the cases, we assume that other tax rates are zero.

5 Scenario Analysis

This section presents projections of the key macroeconomic variables as well as sectoral variables from 2020 to 2050 under three different climate scenarios provided by the NGFS (2021) using our DSGE model calibrated to Japan's economy, and explores how developments of the variables in the economy are affected by the size and pace of the carbon tax implementation.

5.1 Simulation settings

Scenarios. NGFS provides several scenarios based on a suite of models including IAMs, a macro-econometric model, and earth system models. We employ three main NGFS scenarios: (1) a hot house world scenario, in which no additional climate policies are introduced in 2020 and beyond; (2) a disorderly transition scenario, in which countries maintain their current policies until 2030 and then start taking policy actions toward net zero GHG emissions by 2050; and (3) an orderly transition scenario, in which countries immediately take policy actions in 2020 and beyond to achieve net zero GHG emissions by 2050. These three scenarios have been widely used by central banks and financial authorities to conduct a scenario analysis or climate stress testing.

Figure 8 shows the time paths of the carbon price in Japan, which corresponds to P_t^C in the model, for each of the three scenarios from 2020 to 2050.[10] The carbon price in the hot house world scenario is close to zero throughout the simulation period. In the disorderly scenario, the carbon price is zero until 2030, when it starts rising sharply. By 2050, the price reaches the highest level among the three scenarios. In the orderly transition scenario, the carbon price gradually rises at a modest pace from 2020 to 2050, and its level is below that of the disorderly scenario after the mid-2030s.[11] We assume that for each scenario carbon prices beyond 2050 are fixed at the level in 2050.

Assumptions about carbon intensity. Although the NGFS provides a different declining path of carbon intensity for each of the three scenarios, we assume that for all scenarios, carbon intensity declines at the same pace as assumed in the hot house

[10]The NGFS provides paths of variables for each of the various models it employs. Consequently, there is potentially more than one time path for the same variable. Regarding the time path of the carbon price, we use the path produced by REMIND-MAgPIE, a model that integrates the energy-economy-climate model REMIND with the land-use model MAgPIE. Here, the carbon price is endogenously determined so as to satisfy the emission target set in the model. The carbon prices are denominated in U.S. dollars and are converted into yen in the model, assuming that 110 yen/dollar holds throughout the simulation period.

[11]NGFS (2021) argues that an early increase in the carbon price would encourage the development of green technologies, and thus the carbon price required to curb GHG emissions would be lower in the latter half of the simulation period in the orderly transition scenario.

world scenario.[12] That is, we implicitly assume that technological progress in energy consumption efficiency occurs to the degree that would be achieved when currently implemented policies are unaltered. The change in carbon intensity affects carbon tax rates through equations (9), (10), and (11).

Given these considerations, we derive the carbon tax paths for each type of taxation using the procedure described in Section 3.2. As shown in Figure 9, carbon tax rates rise along with the carbon prices for the orderly and disorderly scenarios. Carbon tax rates differ across sectors, reflecting the heterogeneity of carbon intensity across sectors. For intermediate input tax, industries related to textile products and basic materials tend to face higher tax rates.[13] As for production process tax, the non-metallic mineral products sector pays the highest rate, reflecting the GHG emissions from cement production.

Assumptions about the feedback mechanism from GHG emissions to the output. In contrast to related studies such as Baldwin et al. (2020), Chan (2020), and Carattini et al. (2021), our model abstracts the feedback mechanism from climate change due to accumulated GHGs in the atmosphere to economic activity. Consequently, GHG emissions themselves are treated as an auxiliary variable, bringing about no impact on the decisions of households and firms nor on households' welfare. One justification for this setting is that the simulation period of our analysis is only through 2050, so that, according to the NGFS' estimate, the adverse impact from climate change is relatively contained.

Other assumptions. In addition to the assumptions described above, we impose three implicit additional assumptions, mainly for illustrative purposes. First, although transition risk may manifest itself from the demand side, such as changes in households' preferences, we focus on the supply side—namely, the consequences of implementing carbon taxes exclusively under the premise that households' preferences does not change over time. While changes in households' preferences may also have a significant effect on prices of specific goods through changes towards these goods and thus alter the aggregate output and GHG emissions, such a channel is absent in our simulation.

[12] The assumption that carbon intensity is constant throughout the simulation period leads to a conservative estimation of GHG emissions in that emission abatement technology does not advance in the future. Although this assumption is consistent with the idea that a stress scenario should be severe, we take technological advances into account to the extent considered in the hot house world scenario provided in the NGFS. For example, in the NGFS scenario, the price of renewable energy declines and energy supply shifts from coal to natural gas, which is less carbon intensive, even in the hot house world scenario.

[13] As discussed in Section 3.2, tax rates are set so that the laws of GHG emissions and carbon tax rates stated in equation (9), (10), and (11) hold at the equilibrium. Because tax revenues must equal the carbon price multiplied by GHG emissions, tax rates are generally higher for goods production whose pre-tax prices tend to decline greater with a rise in carbon tax rate.

Second, it is assumed that there are no taxes or subsidies in place in the economy in 2019, and that there are no taxes other than carbon taxation nor subsidies introduced in 2020 and beyond. Consequently, as described above, the presence of positive carbon tax rates distorts the resource allocation at the production and consumption level, reducing the output. Third, we assume a closed economy. As studied in Devulder and Lisack (2020), given increasing cross-border transactions, changes in tax rates in one jurisdiction are easily translated to other jurisdictions, and marginal costs facing domestic firms may be affected indirectly if they use foreign goods as intermediate inputs or capital goods. Related to this, there is the possibility that, as a consequence of technological advances, negative emissions are achieved at a sectoral or national level and some portion of GHG itself becomes tradable and traded across borders. These channels are not incorporated into the simulation.

We specify 2019 as the initial period and 2120 as the end period. We further assume that the economy is at the non-stochastic steady state for both the initial and end periods. To highlight the impact of carbon taxes, we set the TFP $Z_{i,t}$ and other parameters, excluding carbon intensity, to be constant throughout the simulation period.[14] We set the stock of atmospheric carbon in the initial period at 3171.9 gigatons of CO_2, which equates to the estimate for 2020 provided by WMO (2021). As for the hot house world and the orderly scenario, households and firms are assumed to have perfect foresight. That is, there is an announcement regarding the implementation of carbon tax at the beginning of 2020, and households and firms become informed in 2020 of the time paths of carbon prices and carbon intensity from 2020 to 2120. No additional announcements are made in 2021 and beyond, which implies that there are no updates of expectations throughout the simulation period. In contrast, in the disorderly scenario, households and firms are given the time paths of carbon prices and carbon intensity that are equivalent to those in the hot house world scenario at the beginning of 2020. At the beginning of 2030, however, a new announcement is made, and they are informed of new time paths of carbon tax rates that reflect sudden changes in climate policy consistent with the disorderly scenario, and they revise their expectations accordingly, resulting in changes to their economic decisions.

Solution method. We solve the transition path of the economy nonlinearly using the sequence-space Jacobian method proposed by Auclert et al. (2021). This method provides the time paths of endogenous variables that meet the equilibrium conditions

[14] In this simulation, we assume that the input-out matrix and the investment network will be unchanged from 2020 and beyond for the purposes of illustration. Admittedly, it is possible that the production share of intermediate inputs or the investment network itself is altered by advances in environment-related technologies, so that the degree of sectoral interaction may also be changed.

from the specified initial period to the end period, taking into account the time paths of exogenous variables such as the technology level and carbon tax rates and the state variables in the initial period. For climate scenario analyses, the commonly used linear approximation of economic dynamics around the non-stochastic steady state may exhibit a significant approximation error because such climate scenarios span fairly long time horizons and the effects of changes in the economic environment surrounding specific industries, such as the implementation of carbon tax rates, may be significantly large.

5.2 Simulation results

Simulation results. Figure 10 plots the simulated path of macroeconomic variables for each scenario: real GDP, aggregate consumption, aggregate capital, aggregate investment, aggregate labor, utility flow in each period measured by the consumption-equivalent welfare losses, the aggregate GHG emissions, and the stock of atmospheric carbon relative to the level as of 2019. As for the hot house world scenario, the economy continues to stay at the state close to the initial steady state in 2019 and beyond, reflecting the fact that the size of carbon tax rates introduced under the scenario is minor. For the other two transition scenarios, as carbon tax rates continue to increase over the years, real GDP declines to a greater degree. As argued in Section 3, under the premise that no distortionary taxes are in place in 2019, the resource allocation before carbon tax implementation achieves the social optimal, and the implementation would lower aggregate economic activity by shifting firms' production allocations, such as the use of intermediate inputs produced by a specific sector, and households' expenditure allocations, away from the optimal allocations, and thus the degree of distortion becomes greater as the carbon tax rates become higher. For the orderly scenario and the disorderly scenario, given that households and firms take into consideration future changes in production and expenditure structure caused by higher carbon tax rates, they preemptively change the amount of labor inputs and investment immediately after the announcement. Consequently, investment and labor inputs decline by a large amount in 2020, and they continue to do so in the years beyond. As households and firms reduce those economic activities that involve GHG emissions, these emissions become smaller.

Compared with the disorderly scenario, the economy under the orderly scenario sees a larger decline in GDP up to around 2040, and a smaller decline in GDP in 2040 and beyond, indicating that around the year 2040, the cumulative adverse impact of carbon tax implementation under the disorderly scenario exceeds that under the orderly scenario. Notice that the timing of the order of GDP size switch lags the timing of the order of the carbon tax rate switch across scenarios, which is around the year 2035, because the capital stock adjustment takes place only gradually. In terms of GHG emissions, the economy under the orderly scenario sees a smaller stock of atmospheric carbon throughout the simulation period.

It is also worth noting that as the time approaches 2050, an incremental increase in the carbon price has a smaller impact on the aggregate economy and GHG emissions. This is primarily because the level of carbon intensity is reduced by 55% in 2050, and thus the increase in tax rates is moderate compared to 2020. In addition, the impact of intermediate input tax on these variables is a decreasing function of its level, as shown in Section 4. Indeed, while a \$100 increase in the carbon price in 2020, when the rates are zero, reduces the aggregate GDP and GHG emissions by 2.3% and 34.8% in the long run, respectively, an increase in the carbon price by the same amount in 2050 in the orderly scenario reduces the aggregate GDP and GHG emissions by 0.5% and 11.6% in the long run, respectively.

Next, we examine the impact by sector, shown in Figures 11 and 12. For detailed sectoral results for all 32 sectors, see Table A.3 in Appendix E. As expected, there is a large dispersion across sectors in terms of how firms in each of the sectors respond to carbon tax implementation, since the carbon tax affects them differently depending on the production structure and size of the carbon tax rate. The "petroleum and coal products" sector sees the largest decline in value added. Products in the sector are subject to a higher carbon tax rate when used as intermediate inputs or when consumed as final consumption goods. Firms in the sector therefore face a large decline in demand for their products. Because of the drop in current and expected demand from firms in other sectors and from households, firms in the sector start to scale down the size of their labor inputs and capital stock holdings immediately after the announcement. Regarding the capital stock, in the presence of a positive adjustment cost, the decline in demand for the sector-specific capital stock leads to a large fall in Tobin's q.

The "electricity supply" sector sees the largest increase in product price. In contrast to the "petroleum and coal products" sector, firms in the "electricity supply" sector face a large increase in marginal costs due to an increasing tax rate on the use of coal and oil, and therefore raise their product prices to ensure profits. A higher product price reduces demand for products of firms in the sector, which in turn reduces their value added and the primary production inputs. The "non-metallic mineral products" sector also sees a higher product price as a result of the carbon tax implementation. Firms in the sector see increasing carbon tax rates on their goods production and therefore raise product prices to ensure profits. Even sectors that are not directly subject to carbon taxes at the stage of intermediate input use, production or consumption are adversely affected. For example, the "general-purpose, production and business-oriented machinery" sector sees a large decline in value added in periods immediately after the announcements. As shown above, the products of this sector are used as investment goods in various sectors through the investment network. A decline in demand for investment goods at the aggregate economy level therefore affects this sector disproportionately, reducing the price of the products and production inputs of the sector. Tobin's q also falls notably, reflecting a decline in demand.

Comparison with the static approach. In order to see the implications for macroeconomic dynamics of incorporating the intertemporal decisions of households and firms into the model, we compare the time paths of variables generated from our baseline setting shown in Figure 10 with that of variables generated by what we call the static approach. In this approach, we assume that the economy is always at the non-stochastic steady state in each year from 2020 to 2050, and compute the steady state value of the variables, taking as given the size of carbon tax rates in Figure 9.

Figure 13 shows the time paths of the variables under the orderly transition for the two cases. As shown in the figure, the general pattern of the variables is similar in both the baseline setting and the static approach. That is, GDP and its components, the primary production inputs, and GHG emissions all decline from 2019 to 2050 in response to the implementation of carbon tax. It is also seen, however, that the time path of GDP based on the baseline setting evolves to a level above that of GDP generated by the static approach throughout the entire simulation period, which indicates that the static approach somehow overestimates the downward effect of the carbon tax implementation. The reason for this is that the capital inputs under the static approach decline by a greater amount relative to the baseline setting. Under the baseline setting, because of the presence of capital adjustment costs, firms adjust the size of the capital stock only gradually, so that the level of the capital stock in each year is higher than that under the static approach. As a result, the decline in GDP due to the carbon tax implementation is mitigated throughout the simulation period.

Figure 14 shows the cumulative decline of sectoral value added and the sectoral capital stock from 2019 to 2030 for the baseline setting and the static approach. Capital stock declines to a greater degree under the static approach than the baseline setting, which in turn makes the output decline larger. These results imply that, while the static approach is able to capture the general pattern of the transmission of carbon tax implementation, it could overestimate the size of the effect when adjustment costs of capital stock are large and adjustments take a prolonged period of time.

Timing of announcement. How does the timing of a carbon tax announcement affect the transition path? In the baseline analysis, we assume that the carbon taxes go into effect immediately after the schedule is announced. Here, households and firms are informed not only of the current tax rates but of the subsequent sequence of tax rates. However, in reality, such a large tax change is introduced gradually, to ensure smooth implementation. Exploiting the advantage of the dynamic nature of our model, we describe how the transition paths of variables are altered, depending on the timing of the announcement of the tax schedule.

In the baseline simulation for the disorderly scenario, we assume that households and firms receive the announcement of a policy change regarding carbon tax rates at the beginning of 2030. We explore the effect of the early announcement by changing the timing of the announcement while keeping the timing of the implementation unchanged from 2030, as assumed in the baseline simulation. Figure 15 shows the transition paths of the key macroeconomic variables when the announcement is made one year in advance and five years in advance. GDP drops immediately after the announcement, even though the tax has not yet been implemented. As described above, carbon tax implementation distorts resource allocation at the production state, which in turn reduces the return on capital investment. With positive costs for capital stock adjustments and the immobility of capital stock across sectors, firms are better off reducing their capital investment before implementation. The early announcement of carbon tax implementation also reduces GHG emissions before 2030, reflecting the fall in economic activities that are expected to be adversely affected in 2030 and beyond. In 2050, the stock of atmospheric carbon when the policy change is announced five years in advance is 0.16% lower than that of the disorderly scenario, underscoring the effectiveness of early announcement in curbing GHG emissions.

Figure 16 shows the transition paths of the sectoral variables when the announcement is made five years in advance. While the overall impact is similar to what is seen in Figure 15, there is heterogeneity across sectors in terms of how variables react to the announcement, depending on the size of the expected impact of carbon tax implementation. For example, in more-affected sectors such as "petroleum and coal products," Tobin's q drops to a lower level and value added declines by a larger amount than in less-affected sectors such as "real estate." There is also a substitution of capital goods from more-affected sectors such as "petroleum and coal products" to less-affected sectors such as "real estate."

Combination of taxation. As analyzed in Section 4, the impact of carbon taxes on real GDP and GHG emissions varies, depending on which sectors or economic activities are subject to higher carbon taxation. This observation indicates that, with GHG emissions being equal, the adverse impact of carbon taxation on the real economy can be changed, depending on the combination of taxation. In Figure 17, we show the transition paths of macroeconomic variables under three alternative patterns of combination of tax rates in addition to the baseline scenario: only intermediate input tax, both intermediate input tax and consumption tax, and both intermediate and production tax. For the purpose of fair comparison, we keep the period-by-period time path of the accumulated sum of atmospheric GHG emissions across each combination at the same level, allowing tax rates to differ across scenarios. For example, in the first alternative scenario, carbon tax rates on goods production and consumption are zero, and tax rates on the use of fossil fuels as intermediate inputs are higher by the same multiplier for all goods production sectors, so that the total amount of GHG emissions in the economy equates to that under the baseline scenario.

In terms of the size of the decline in real GDP, the taxation pattern that imposes zero tax rates on the production process while also imposing positive rates on the use and consumption of fossil fuels outperforms the other three cases, including our baseline case. Three points are noteworthy. First, as suggested in Section 4, in our model, the main channel through which a positive consumption tax rate affects the economy is changes in households' expenditure share across goods, and not changes in the relative price of intermediate goods or investment goods in the production network, yielding the smallest distortion associated with goods production. By contrast, a positive tax rate on the use of fossil fuels as intermediate inputs or on goods production distorts resource allocation in the goods production process. Second, as shown in Figure 1, the amount of GHG emissions in the production process other than fossil fuel use is quantitatively minor relative to that associated with households' consumption of fossil fuels and firms' use of fossil fuels. In other words, carbon tax needs to be disproportionately higher if carbon tax rates are imposed only on the production process, which in turn yields a larger distortion associated with goods production. Third as shown in Figure 6, taxes on the production process in some sectors may increase rather than decrease GHG emissions, through the substitution effect. This substitution effect increases the tax rate needed to contain the amount of GHG emissions.

Figure 18 shows the difference between variables under the baseline scenario and under each of the three alternative scenarios regarding the type of carbon tax imposed. Note again that the amounts of period-by-period GHG emissions are equalized across scenarios. Under the "intermediate only" and the "intermediate and consumption" scenarios, the time path of GDP is higher than the baseline scenario. The key driver that pushes up GDP under the two scenarios is capital. In these scenarios, a zero tax rate is imposed on the production process so that the distortion of resource allocation at the goods production stage is small, and hence the return on capital investment is higher, which in turn results in higher capital accumulation and a smaller GDP decline. In terms of welfare, the “intermediate and consumption” scenario dominates the baseline from around 2030 and beyond, while other scenarios exhibit lower welfare than the baseline throughout the simulation period. This observation highlights the importance of the composition of tax revenues with respect to the impact of carbon taxes from the welfare perspective.

6 Concluding Remarks

With growing interest in the risks arising from the transition to a green economy, there has been an increase in theoretical research on the impact of carbon tax implementation on economic activity and GHG emissions. In this paper, we have developed a DSGE model with input-output linkage and the investment network; calibrated the model to the input-output table of Japan's economy; derived the implications of carbon tax implementation for GDP, GHG emissions, and sectoral output over time; and quantitatively simulated the consequences of carbon tax implementations under a set of scenarios provided by the NGFS. Our model differs significantly from models that are generally used in the literature, in that it explicitly incorporates both inter-sectoral interactions through the use of intermediate inputs and investment goods, and the effects of intertemporal dynamic decisions of households and firms. Similar to existing studies, our analysis shows that an increase in carbon tax rates reduces economy-wide GHG emissions by raising the costs of carbon-emitting economic activities and changing resource allocations across sectors, goods, and time, while dampening aggregate output and sectoral outputs by distorting the allocation of the production inputs across sectors. In contrast to existing studies however, our analysis also shows that this transmission of carbon tax implementation may be altered, depending on conditions other than the size of carbon tax rates in the economy concerned. These other relevant conditions include the degree of distortion that is already present in goods production before the implementation of the carbon tax, the composition of revenue sources of the carbon tax across sectors and economic activities, and the timing of announcements regarding the schedule of carbon tax implementation. These results highlight the need to carefully design not only the size and composition of any carbon tax but also even the timing of the announcement of its implementation.

There are some caveats regarding the current paper. The first is about how our model treats technological innovations associated with mitigating the effects of climate change. As discussed in existing studies, the implementation of carbon tax may change the medium- to long-term R&D activities of firms, and endogenously alter, for example, the value of carbon intensity parameters in the model (e.g, see Acemoglu et al. (2012a) and Hassler et al. (2021)). Our model abstracts from such a mechanism.

The second caveat is that the current paper focuses on the transmission of carbon tax implementation on economic activity and GHG emissions while omitting consideration of how past and current GHG emissions affect economic activities through, for example, potential changes in labor productivity or in the scale and frequency of natural disasters such as floods. This is partly because, on the one hand, there is a long history in macroeconomics of analysis of sectoral interactions, including those through intermediate input linkages, both in terms of empirical and theoretical analysis, and there is a certain degree of convergence in the analytical framework. On the other hand, there is not necessarily an established view on how to model the feedback from GHG emissions to economic activity. Related to this, our model assumes that households and firms make decisions without considering the externalities that arise from the outcome of these decisions, such as mitigation of climate change resulting from the decision to reduce the use of petroleum. We leave as an important research agenda the analysis of the impact of carbon taxes on economic activity and economic welfare, based on further empirical and theoretical study along these lines.

References

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous (2012a) "The environment and directed technical change," *American Economic Review*, 102 (1), 131–66.

Acemoglu, Daron, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2012b) "The network origins of aggregate fluctuations," *Econometrica*, 80 (5), 1977–2016.

Allen, Thomas, Stéphane Dees, Carlos Mateo Caicedo Graciano, Valérie Chouard, Laurent Clerc, Annabelle de Gaye, Antoine Devulder, Sebastien Diot, Noëmie Lisack, Fulvio Pegoraro et al. (2020) "Climate-related scenarios for financial stability assessment: An application to France," Banque de France Working Paper No. 774.

Annicchiarico, Barbara, Stefano Carattini, Carolyn Fischer, and Garth Heutel (2022) "Business cycles and environmental policy: a primer," *Environmental and Energy Policy and the Economy*, 3 (1), 221–253.

Annicchiarico, Barbara and Fabio Di Dio (2015) "Environmental policy and macroeconomic dynamics in a new Keynesian model," *Journal of Environmental Economics and Management*, 69, 1–21.

Atalay, Enghin (2017) "How important are sectoral shocks?" *American Economic Journal: Macroeconomics*, 9 (4), 254–80.

Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub (2021) "Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models," *Econometrica*, 89 (5), 2375–2408.

Baldwin, Elizabeth, Yongyang Cai, and Karlygash Kuralbayeva (2020) "To build or not to build? Capital stocks and climate policy," *Journal of Environmental Economics and Management*, 100, 102235.

Baqaee, David Rezza and Emmanuel Farhi (2019) "The macroeconomic impact of microeconomic shocks: beyond Hulten's Theorem," *Econometrica*, 87 (4), 1155–1203.

Barrage, Lint (2020) "Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy," *The Review of Economic Studies*, 87 (1), 1–39.

Blackburn, Christopher J and Juan Moreno-Cruz (2021) "Energy efficiency in general equilibrium with input–output linkages," *Journal of Environmental Economics and Management*, 110, 102524.

Bouakez, Hafedh, Emanuela Cardia, and Francisco Ruge-Murcia (2014) "Sectoral price rigidity and aggregate dynamics," *European Economic Review*, 65, 1–22.

Bouakez, Hafedh, Emanuela Cardia, and Francisco J Ruge-Murcia (2009) "The transmission of monetary policy in a multisector economy," *International Economic Review*, 50 (4), 1243–1266.

Burres, David (1994) "Homeomorphism between Leontief and Cobb-Douglas input-output models," *Economics Letters*, 44 (1-2), 49–53.

Carattini, Stefano, Garth Heutel, and Givi Melkadze (2021) "Climate policy, financial frictions, and transition risk," NBER Working Paper No. 28525.

Carvalho, Vasco M and Alireza Tahbaz-Salehi (2019) "Production networks: a primer," *Annual Review of Economics*, 11, 635–663.

Cavalcanti, Tiago, Zeina Hasna, and Cezar Santos (2021) "Climate change mitigation policies: aggregate and distributional effects," Cambridge Working Paper in Economics.

Chan, Ying Tung (2020) "On the impacts of anticipated carbon policies: A dynamic stochastic general equilibrium model approach," *Journal of Cleaner Production*, 256, 120342.

De Bruin, Kelly C, Rob B Dellink, and Richard SJ Tol (2009) "AD-DICE: an implementation of adaptation in the DICE model," *Climatic Change*, 95 (1), 63–81.

Devulder, Antoine and Noëmie Lisack (2020) "Carbon tax in a production network: propagation and sectoral incidence," Banque de France Working Paper No. 760.

Dunz, Nepomuk, Tina Emambakhsh, Tristan Hennig, Michiel Kaijser, Charalampos Kouratzoglou, and Carmelo Salleo (2021) "ECB's economy-wide climate stress test," ECB Occasional Paper.

Dupor, Bill (1999) "Aggregation and irrelevance in multi-sector models," *Journal of Monetary Economics*, 43 (2), 391–409.

Eurostat (2015) "Manual for air emissions accounts, 2015 edition," `https://ec.europa.eu/eurostat/web/products-manuals-and-guidelines/-/KS-GQ-15-009`.

Foerster, Andrew T, Andreas Hornstein, Pierre-Daniel G Sarte, and Mark W Watson (2019) "Aggregate implications of changing sectoral trends," Federal Reserve Bank of San Francisco Working Paper 2019-16.

Foerster, Andrew T, Pierre-Daniel G Sarte, and Mark W Watson (2011) "Sectoral versus aggregate shocks: A structural factor analysis of industrial production," *Journal of Political Economy*, 119 (1), 1–38.

Frankovic, Ivan (2022) "The impact of carbon pricing in a multi-region production network model and an application to climate scenarios," Deutsche Bundesbank Discussion Paper, No.07.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski (2014) "Optimal taxes on fossil fuel in general equilibrium," *Econometrica*, 82 (1), 41–88.

Hashimoto, Ryuichiro and Nao Sudo (2022) "Transmission of flood damage to the real economy and financial intermediation: simulation analysis using a DSGE model," Bank of Japan Working Paper Series, No.22-E-5.

Hassler, John, Per Krusell, and Conny Olovsson (2021) "Directed technical change as a response to natural resource scarcity," *Journal of Political Economy*, 129 (11), 3039–3072.

Horvath, Michael (2000) "Sectoral shocks and aggregate fluctuations," *Journal of Monetary Economics*, 45 (1), 69–106.

King, Maia, Bassel Tarbush, and Alexander Teytelboym (2019) "Targeted carbon tax reforms," *European Economic Review*, 119, 526–547.

Luetticke, Ralph (2021) "Transmission of monetary policy with heterogeneity in household portfolios," *American Economic Journal: Macroeconomics*, 13 (2), 1–25.

McKibbin, Warwick J, Adele C Morris, Peter J Wilcoxen, and Weifeng Liu (2018) "The role of border carbon adjustments in a US carbon tax," *Climate Change Economics*, 9 (01), 1840011.

Meng, Sam, Mahinda Siriwardana, and Judith McNeill (2013) "The environmental and economic impact of the carbon tax in Australia," *Environmental and Resource Economics*, 54 (3), 313–332.

NGFS (2021) "NGFS Climate Scenarios for central banks and supervisors," Network for Greening the Financial System.

Nordhaus, William D (1994) *Managing the global commons: the economics of climate change*, 31: MIT press Cambridge, MA.

Okazaki, Yosuke and Nao Sudo (2018) "Natural rate of interest in Japan," Bank of Japan Working Paper Series, No.18-E-6.

Pasten, Ernesto, Raphael Schoenle, and Michael Weber (2020) "The propagation of monetary policy shocks in a heterogeneous production economy," *Journal of Monetary Economics*, 116, 1–22.

Popp, David (2004) "ENTICE: endogenous technological change in the DICE model of global warming," *Journal of Environmental Economics and management*, 48 (1), 742–768.

UNEP (2021) "Emissions Gap Report 2021: The Heat Is On - A World of Climate Promises Not Yet Delivered," United Nations Environment Programme.

Vom Lehn, Christian and Thomas Winberry (2022) "The investment network, sectoral comovement, and the changing US business cycle," *The Quarterly Journal of Economics*, 137 (1), 387–433.

WMO (2021) "The State of Greenhouse Gases in the Atmosphere Based on Global Observations through 2020," World Meteorological Organization.

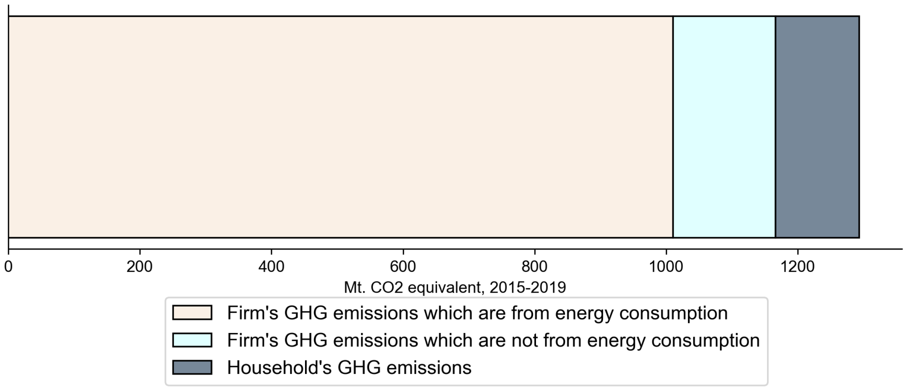
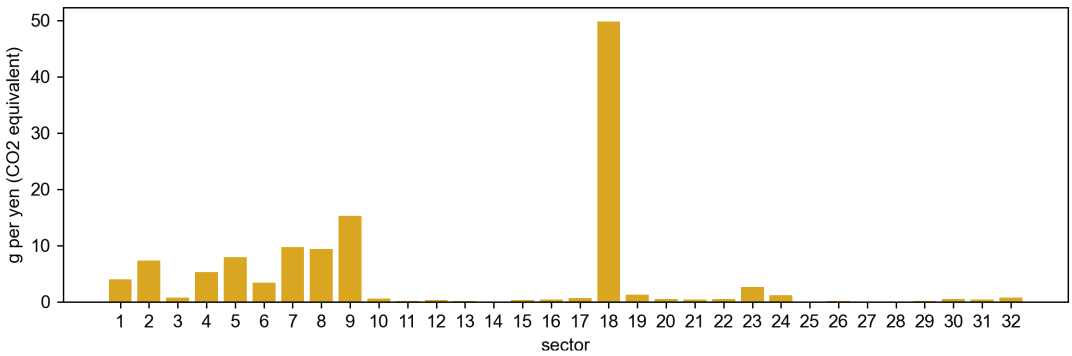
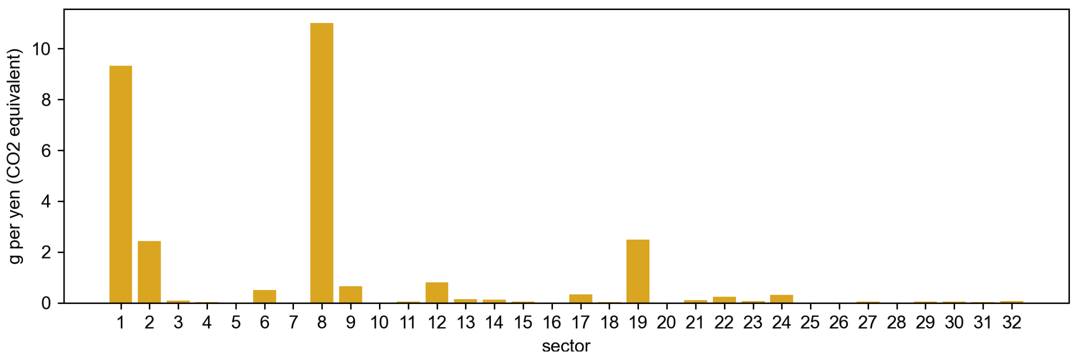


Figure 1: Total GHG emissions



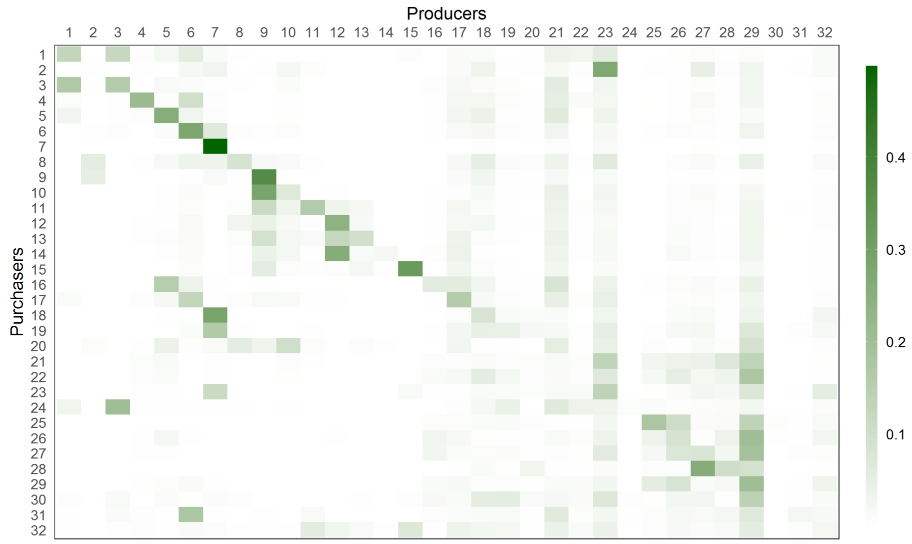
(a) GHG emissions from use of fossil fuels



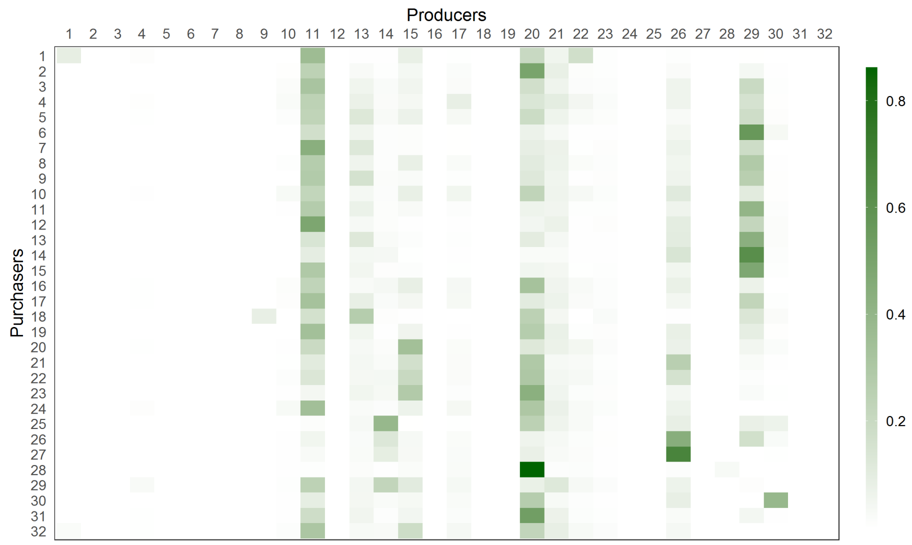
(b) GHG emissions from production process

Figure 2: Sectoral GHG emissions per sectoral value added

Note: Numbers along the x-axis denote sectors as follows: 1. Agriculture, forestry and fishing, 2. Mining, 3. Food products and beverages, 4. Textile products, 5. Pulp, paper and paper products, 6. Chemicals, 7. Petroleum and coal products, 8. Non-metallic mineral products, 9. Basic metal, 10. Fabricated metal products, 11. General-purpose, production and business-oriented machinery, 12. Electronic components and devices, 13. Electrical machinery, equipment and supplies, 14. Information and communication electronics equipment, 15. Transport equipment, 16. Printing, 17. Others, 18. Electricity supply, 19. Gas and water supply, and waste management service, 20. Construction, 21. Wholesale trade, 22. Retail trade, 23. Transport and postal services, 24. Accommodation and food service activities, 25. Communications and broadcasting, 26. Information services, and Image, sound and character information production and distribution, 27. Finance and insurance, 28. Real estate, 29. Professional, scientific and technical activities, 30. Education, 31. Human health and social work activities, 32. Other service activities.



(a) Input-output structure of intermediate inputs in Japan



(b) Input-output structure of investment network in Japan

Figure 3: Input-output structure in Japan

Note: Figures in panel (a) correspond to γ_{ij} in equation (1) and those in panel (b) correspond to γ_{kij} in equation (2). Numbers along the x-axis denote sectors as in Figure 2 .

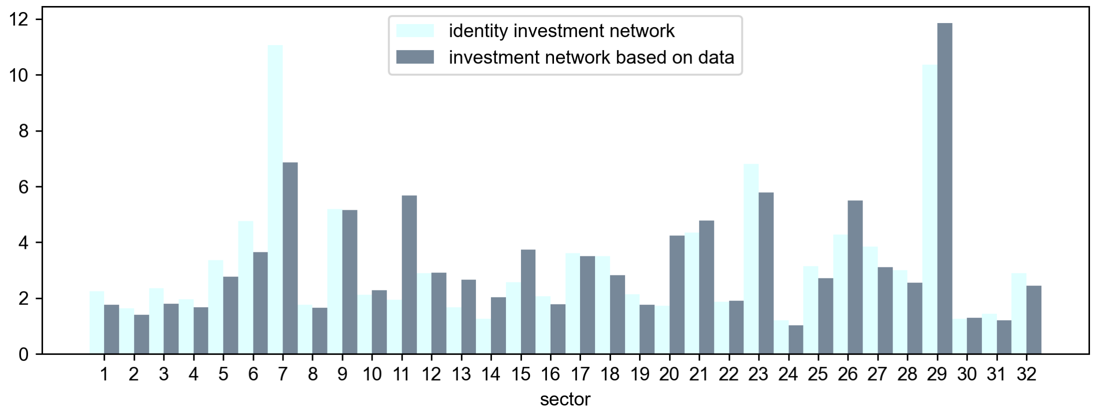


Figure 4: The importance as a supplier in production and the investment network

Note: Figures for sector j are $\sum_{i=1}^{N} l_{ij}$ when Γ_K is an identity matrix and when Γ_K is computed from the data. In either case, the structure of intermediate input is given by the data. Numbers along the x-axis denote sectors as in Figure 2.

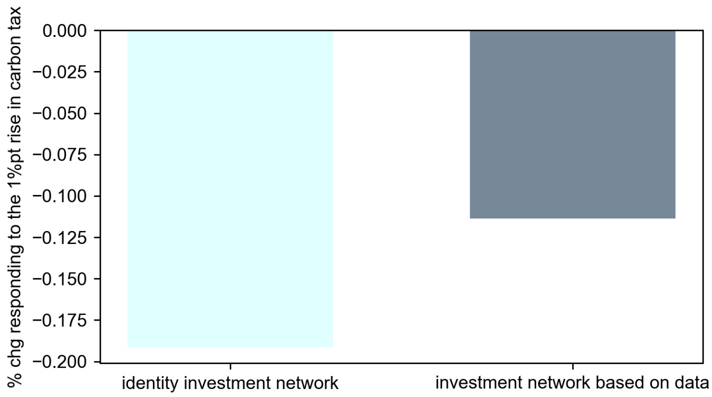


Figure 5: The impact of intermediate input tax on real GDP

Note: Figures are the impact on real GDP when tax rates on intermediate use of fossil fuels are raised by 1%pt for all sectors. The figure for 'identity investment network' is calculated based on the assumption that the matrix of capital goods production function Γ_K is the identity matrix. In either case, the structure of intermediate input is given by the data.

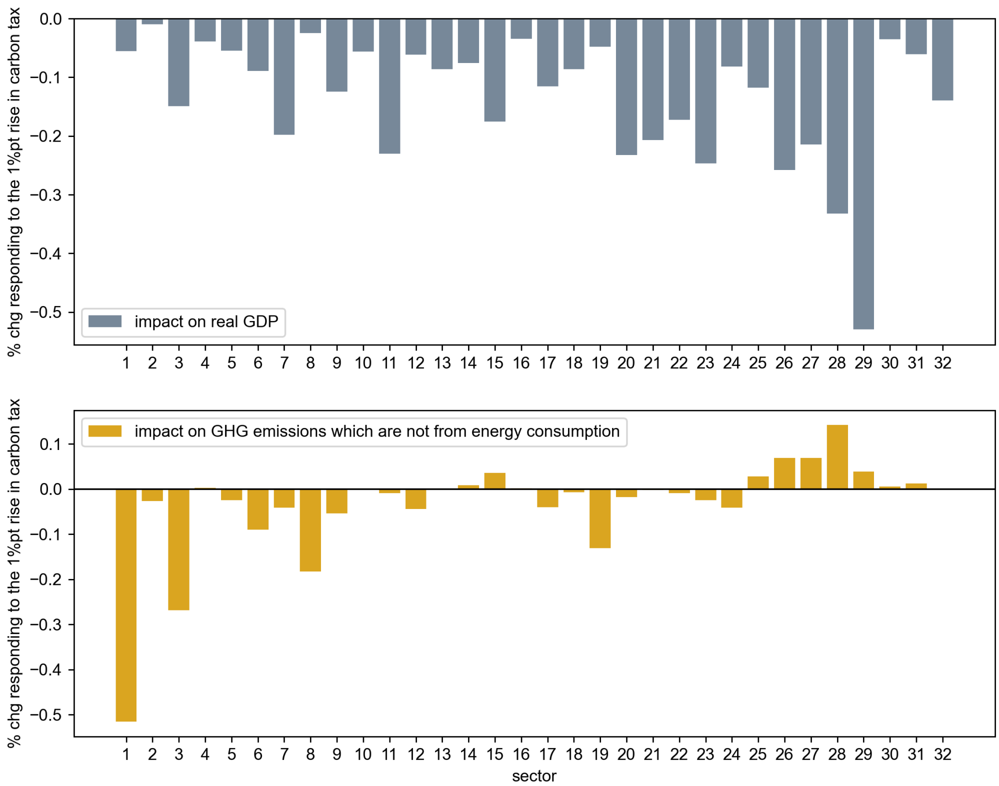


Figure 6: The impact of production process tax on real GDP and sectoral GHG emissions

Note: Figures are the change in aggregate real GDP and the change in sectoral GHG emissions when a 1% tax rate is imposed on the production of each good, or $\tau_i = 0.01$ for each $i = 1, ..., N$ while the other tax rates are zero. Numbers along the x-axis denote sectors as in Figure 2.

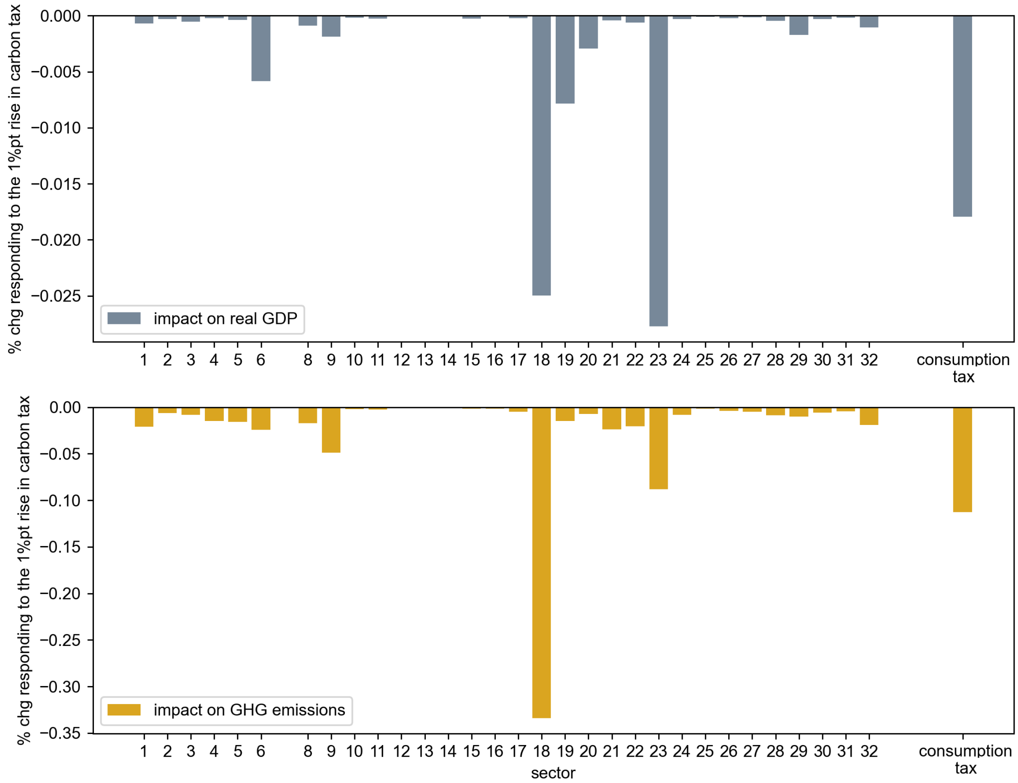


Figure 7: The impact of intermediate input tax and consumption tax on real GDP and aggregate GHG emissions

Note: Figures are the change in aggregate real GDP and the change in aggregate GHG emissions when a 1% tax rate is imposed on the intermediate input of petroleum and coal products used by each sector or final consumption of petroleum and coal products, or $\zeta_{io} = 0.01$ for each $i = 1, \ldots, N$ or $\kappa_o = 0.01$ while the other tax rates are zero. Here, we exclude intermediate input tax on the petroleum and coal products sector to avoid double taxation. Numbers along the x-axis denote sectors as in Figure 2.

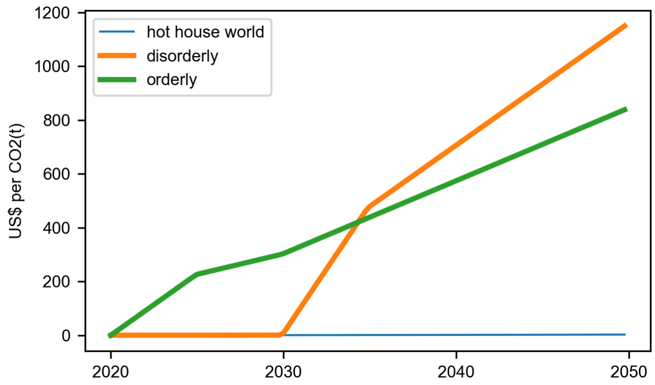


Figure 8: Carbon price path for each scenario

Source: NGFS (2021).

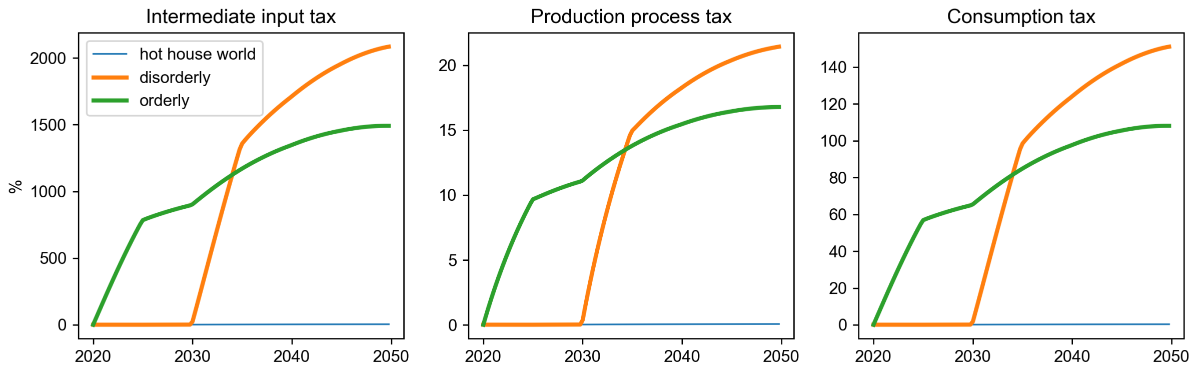


Figure 9: Carbon tax rates across scenarios

Note: Carbon tax rates differ across industries. The series plotted in the figure are the maximum values across sectors.

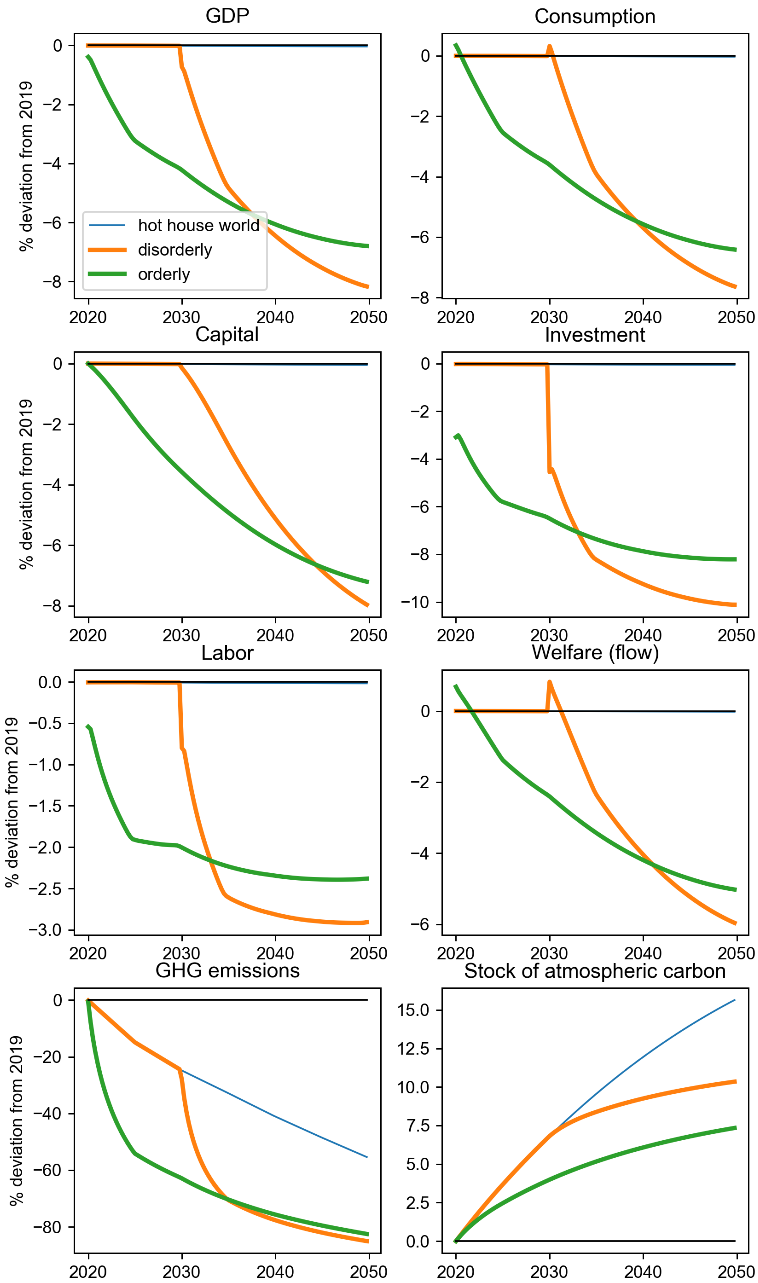


Figure 10: Transition path of macroeconomic variables

Note: To measure welfare, we use consumption-equivalent welfare losses compared with the initial steady state.

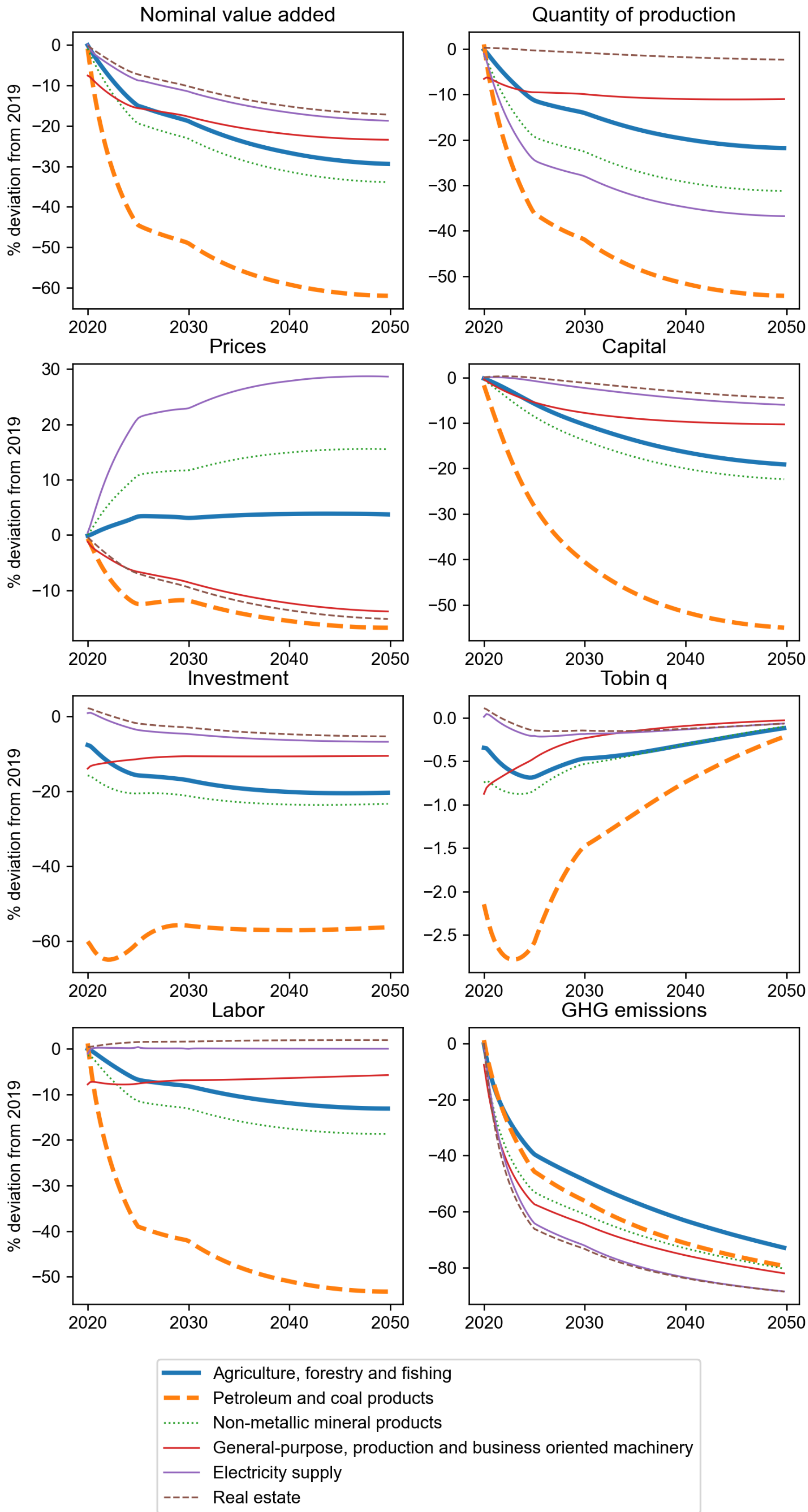


Figure 11: Transition of selected sectoral variables for the orderly scenario

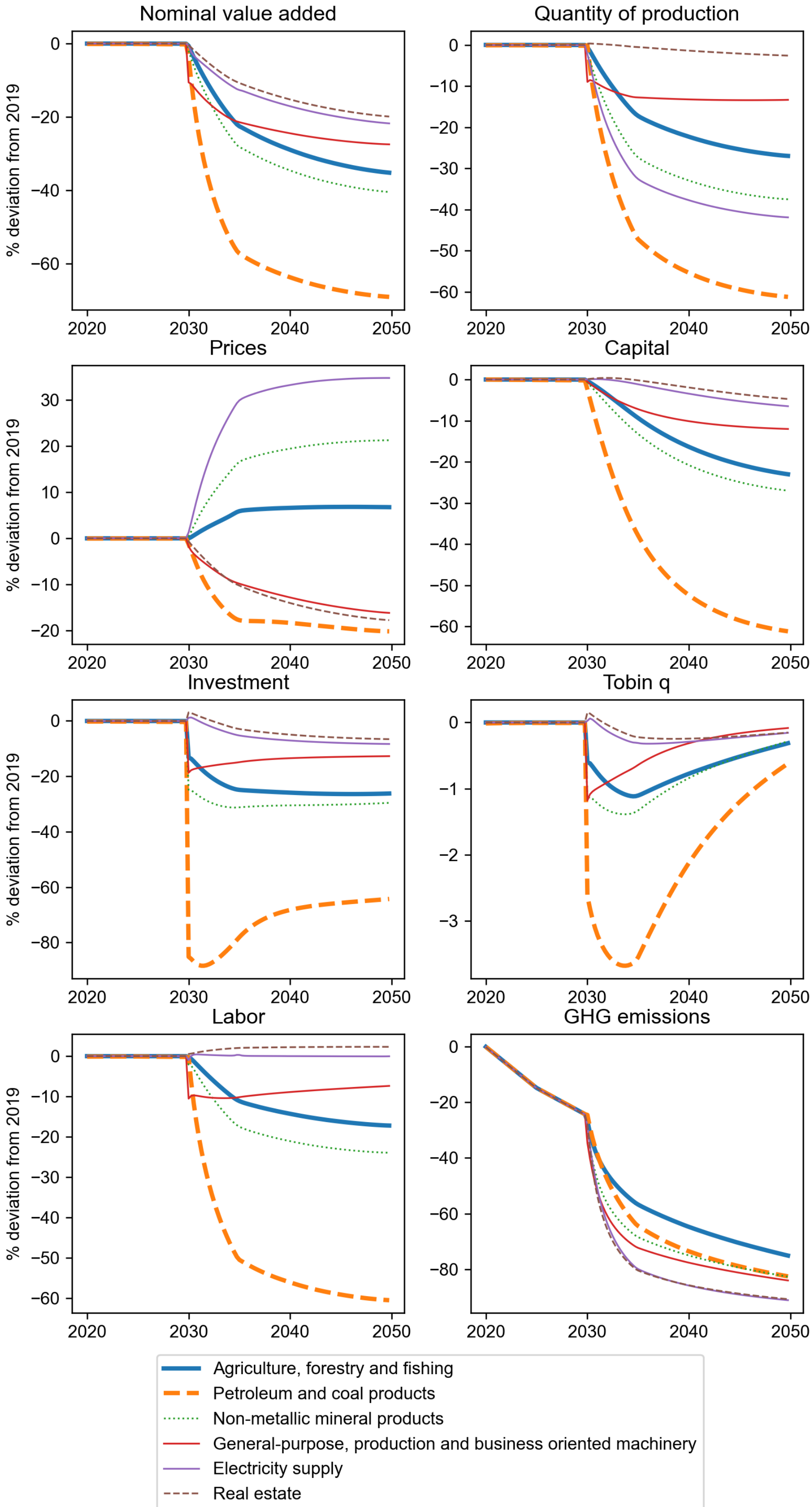


Figure 12: Transition of selected sectoral variables for the disorderly scenario

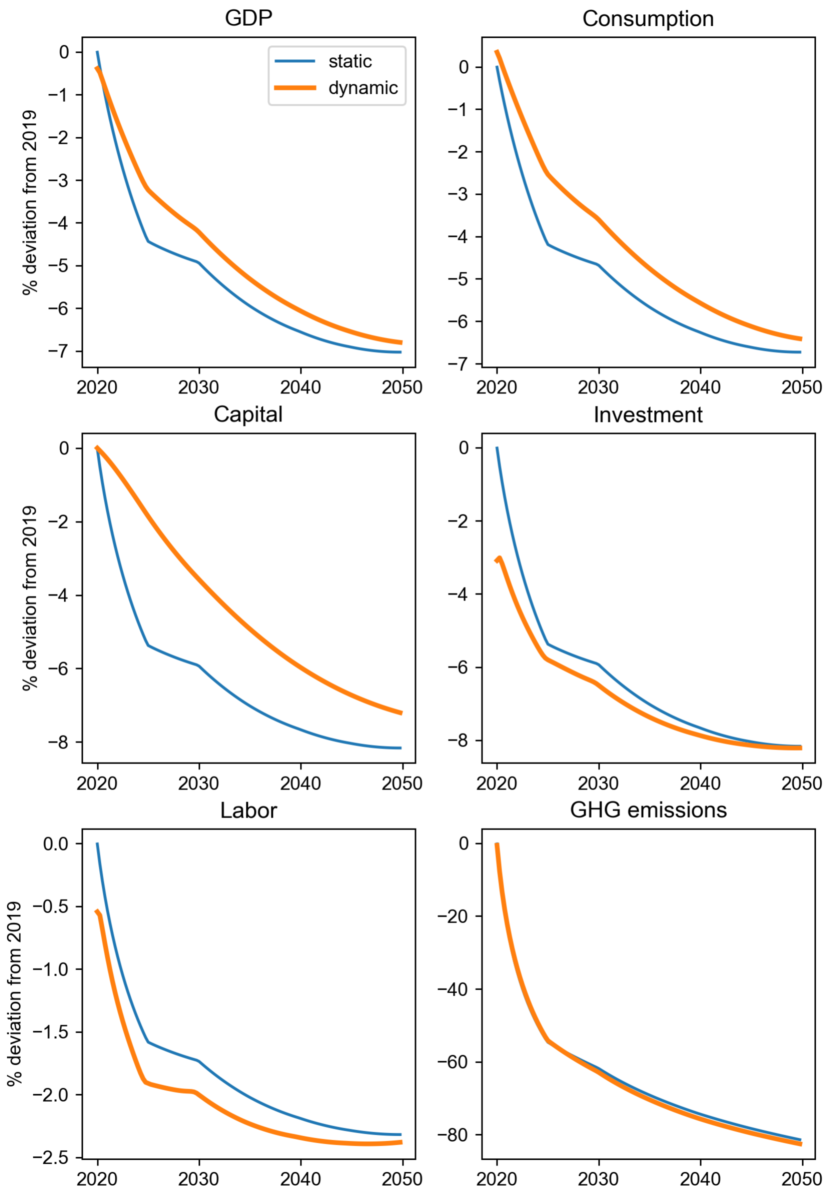


Figure 13: Comparison with the static approach

Note: In the static approach, the economy is in the nonstochastic steady state of the model given the carbon tax rates in every period, while the dynamic approach solves the transition path of the economy assuming perfect foresight.

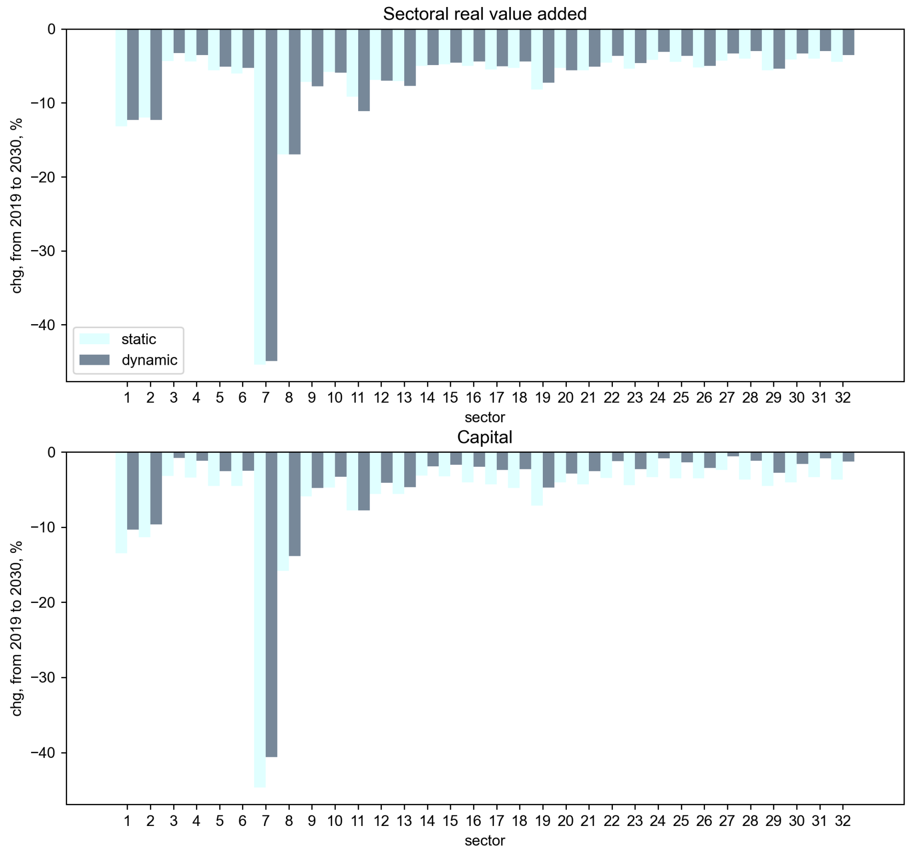


Figure 14: Comparison of sectoral variables with the static approach from 2020 to 2030

Note: In the static approach, the economy is in the nonstochastic steady state of the model given the carbon tax rates in every period, while the dynamic approach solves the transition path of the economy assuming perfect foresight. Numbers along the x-axis denote sectors as in Figure 2.

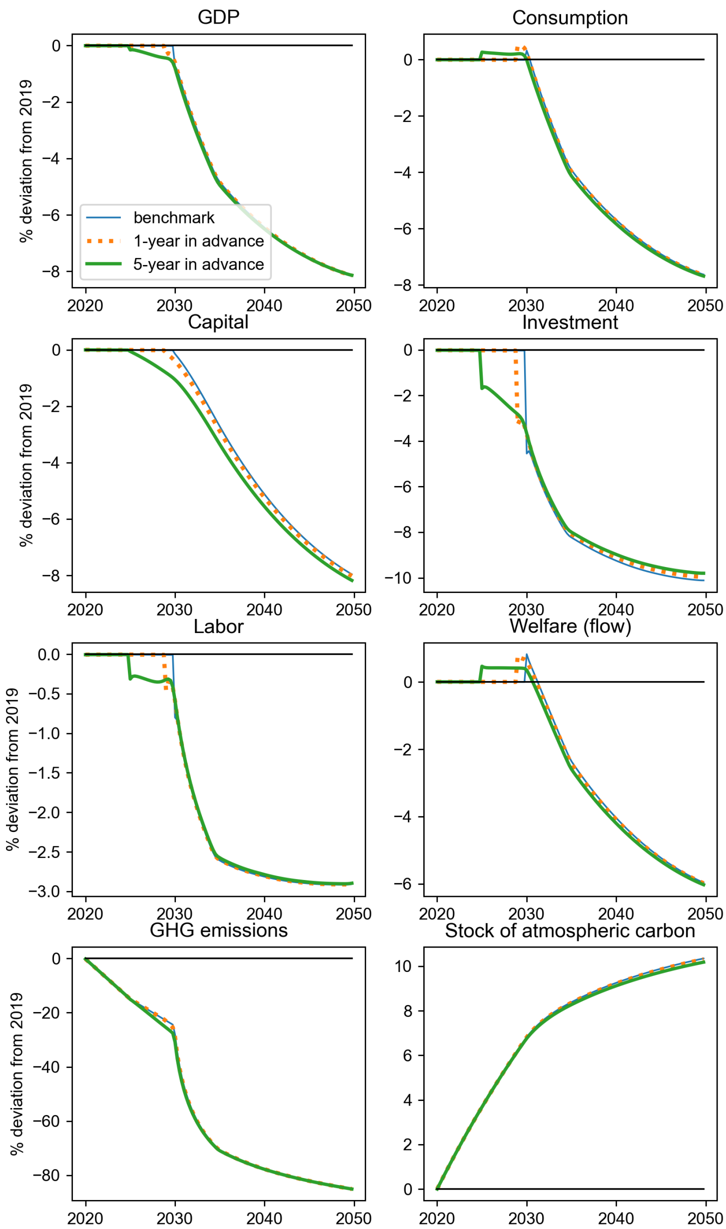


Figure 15: Transition paths across different announcement timings

Note: The benchmark in the figure is the disorderly scenario in Figure 10.

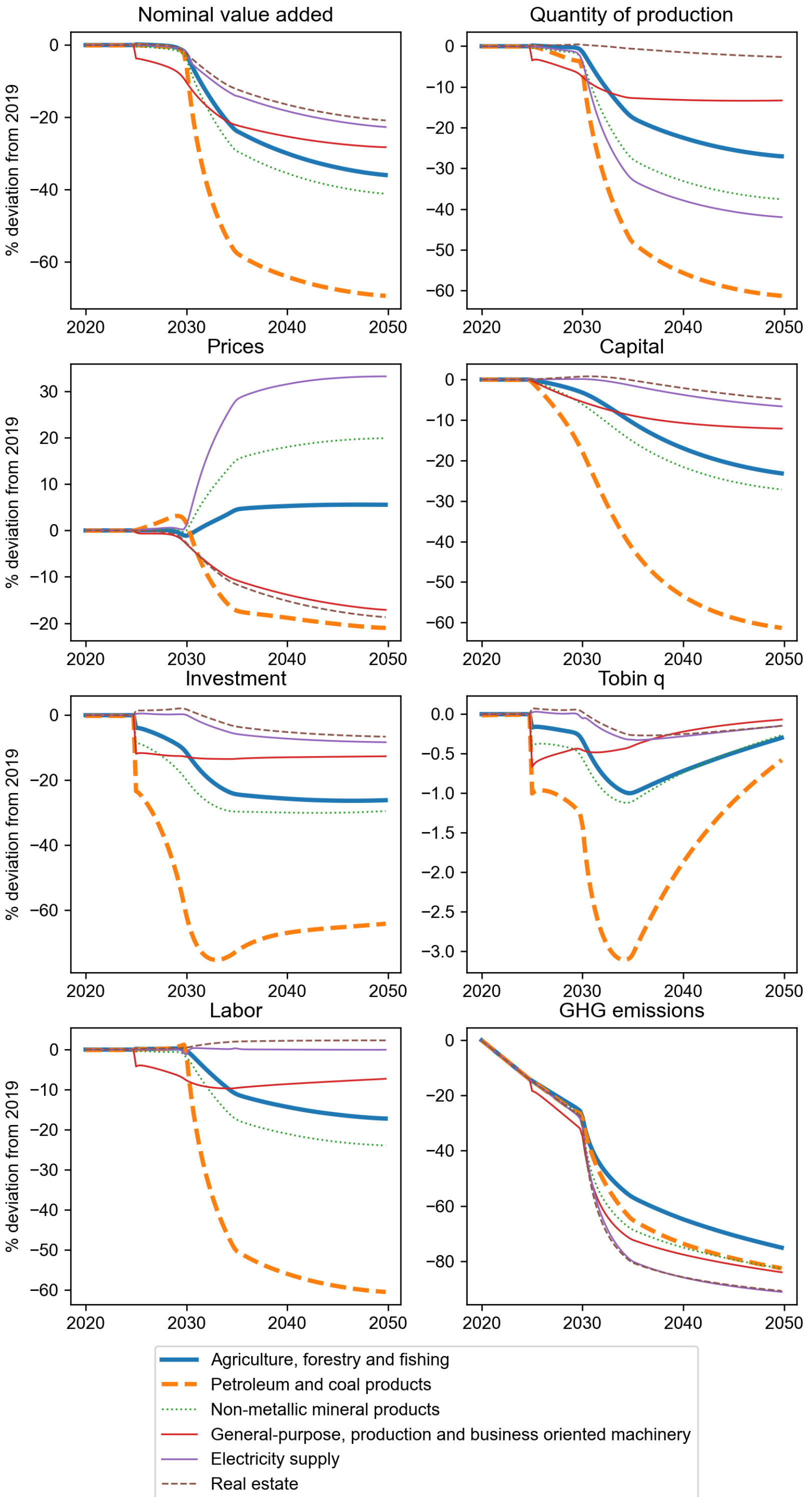


Figure 16: Transition paths of selected sectoral variables when announcement is made five years in advance

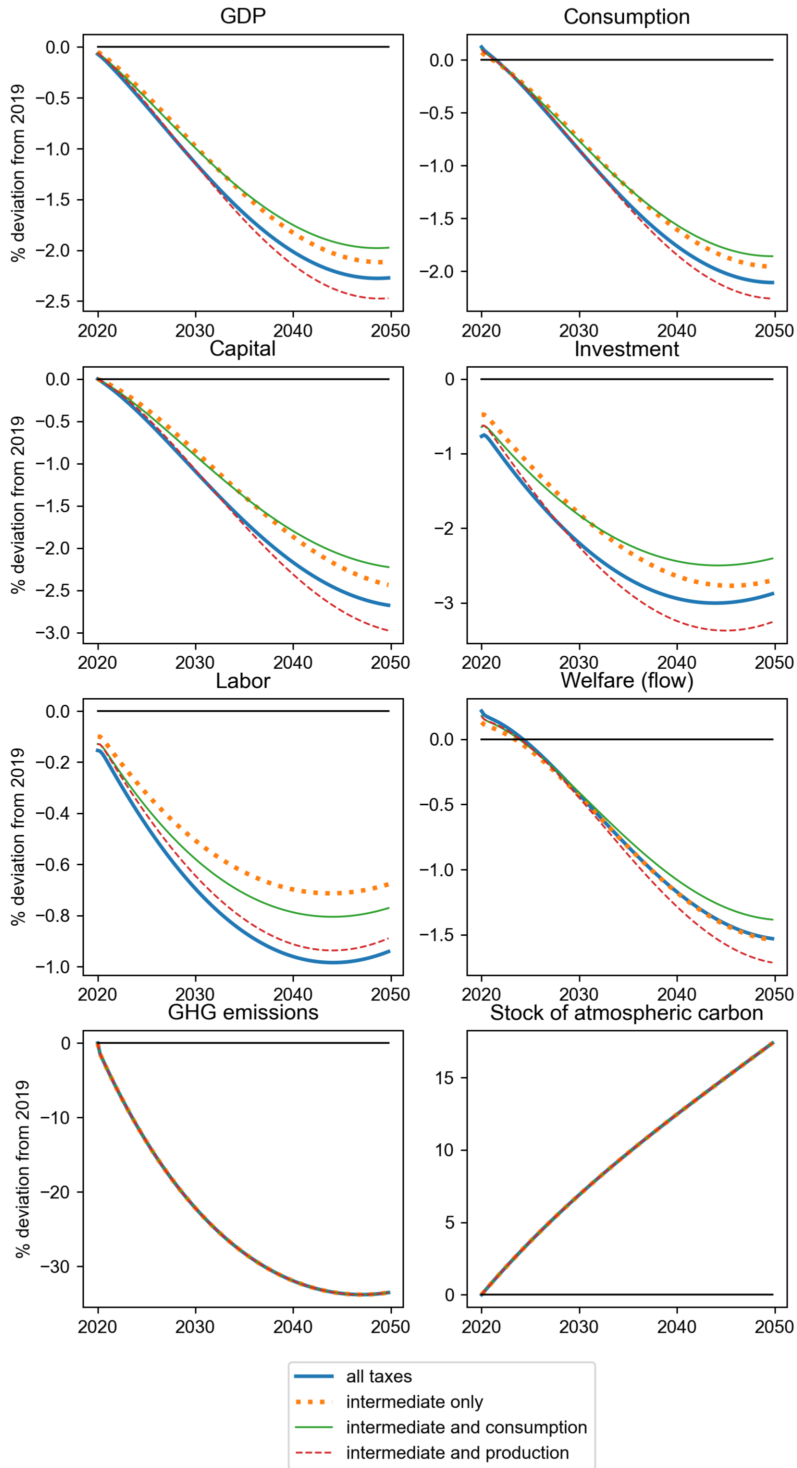


Figure 17: Transition paths across different combinations of taxation

Note: The paths of carbon tax rates are determined so that the time path of the accumulated sum of atmospheric GHG emissions across each combination is kept at the same level. The time path of the stock of atmospheric GHG emissions is exogenously given as a numerical example. Each of the tax rates is determined by the procedure described in Section 3, but the path of carbon price P_t^C is set to satisfy the target level of GHG emissions.

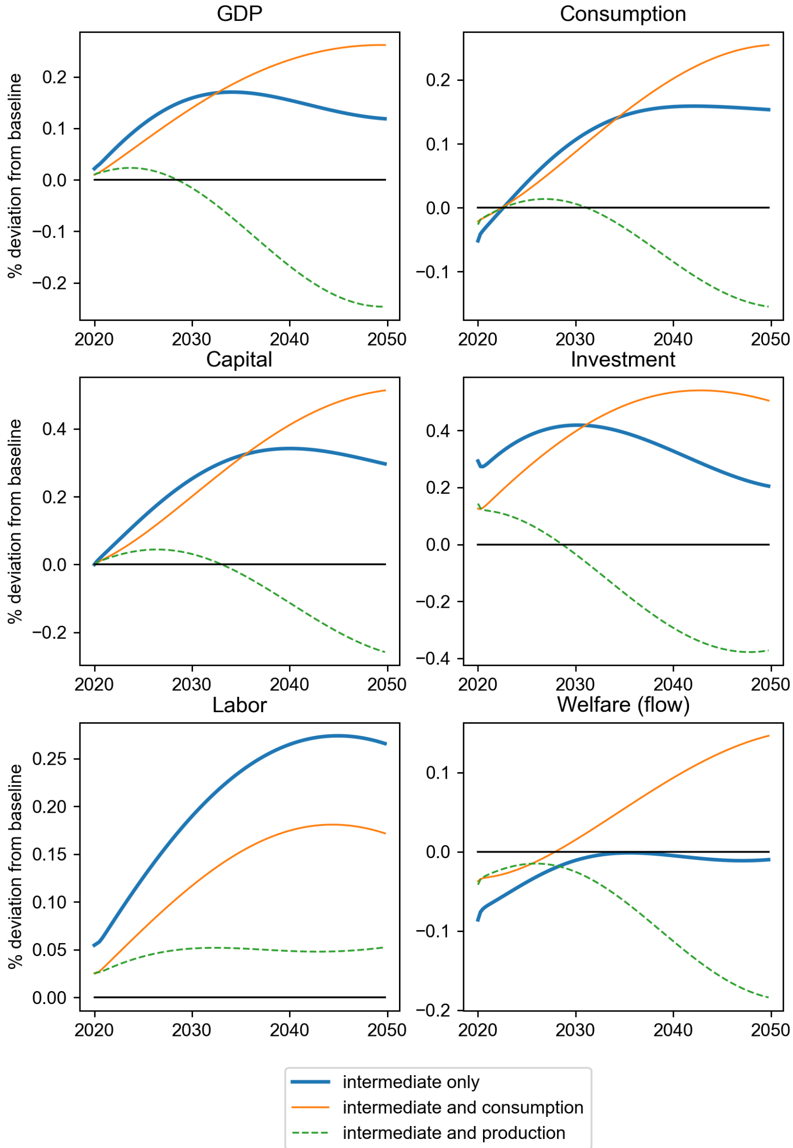


Figure 18: Transition paths across different combinations of taxation: deviation from baseline

Note: The baseline is the "all taxes" case in Figure 17. The paths of carbon tax rates are determined so that the time path of the accumulated sum of atmospheric GHG emissions across each combination is kept at the same level. The time path of the stock of atmospheric GHG emissions is exogenously given as a numerical example. Each of the tax rates is determined by the procedure described in Section 3, but the path of carbon price P_t^C is set to satisfy the target level of GHG emissions.

Appendix

A. Compiling data on Japan's sectoral GHG emissions

A-1. Motivation

In Japan, there are some sets of GHG emissions data with different coverage of GHG emissions and emitters, including the National Greenhouse Gas Inventory Report of Japan, submitted under international conventions, and firm-level reporting data under the Law Concerning the Promotion of Measures to Cope with Global Warming of Japan, but these emissions data do not have all the information necessary to allocate GHG emissions to each economic activity of the whole economy. For example, national emission inventories provide information on GHG emissions of a given country over a year in general, while they are classified by technically delineated processes and sources and do not correspond thoroughly to the classification of economic activities. This obscure linkage between emission flows and economic activities may cause difficulty in understanding the environmental performance of each activity.

From this perspective, air emissions accounts (AEAs), which are widely used in the EU and other countries, seem to have an advantage in use. AEAs record flows of GHGs and air pollutants emitted into the atmosphere as a result of economic activity. As Japan has not officially published AEAs,[15] this paper attempts to compile data on Japan's sectoral GHG emissions, following the "Manual for air emissions accounts" provided by Eurostat (2015). Producing data using the same metrics as Eurostat makes it easy to compare the emissions situations between Japan and the EU.

A-2. Approach

To compile data on Japan's sectoral GHG emissions, an approach based on the national emissions inventory data is mainly applied (the so-called inventory-first approach). The advantages of this approach are (1) the wide coverage of GHG gases[16] and emitters compared to other available emissions data sources in Japan, and (2) the ability to distinguish GHG emissions from fossil fuel use, and from others, such as industrial processes, thanks to the classification structure applied in national emissions inventories.

[15] It should be noted that the National Institute for Environmental Research has published a data book entitled "Embodied Energy and Emission Intensity Data for Japan Using Input-Output Tables (3EID)," whose data are frequently used in the area of Life Cycle Assessment and which complement AEAs.

[16] GHG gases reported in Japan's emissions inventory include carbon dioxide [CO2], methane [CH4], nitrous oxide [N2O], hydrofluorocarbons [HFCs], perfluorocarbons [PFCs], sulfur hexafluoride [SF6], and nitrogen trifluoride [NF3].

The heart of the inventory-first approach is a step to assign an inventory source category (CRF source code) to each emitting production activity followed by the standard industrial classification and private household consumption activity. In this sense, the Eurostat manual provides a table that shows the correspondence between the CRF source codes used in the national emissions inventories and the NACE Rev.2 classification. Following this correspondence table and detailed statements in the National Greenhouse Gas Inventory Report of Japan, we determine the correspondence between the CRF source codes and Japan's industrial classification.

For the CRF source codes that show one-to-one correspondence to the AEA category, the step stated above is enough to assign GHG emissions to economic activities. However, instead of displaying a one-to-one correspondence, some CRF categories provide a variety of economic activities that could possibly relate to the CRF category, or some CRF categories do not specify a certain economic activity because allocation is deemed to be too country-specific. In this case, additional information is needed to determine correspondence, and the General Energy Statistics, the Input-Output Tables, and the Embodied Energy and Emission Intensity Data for Japan are mainly used as information sources.

One typical example of a CRF source code that does not correspond on a one-to-one basis is "road transportation" (CRF source code: 1.A.3.b), as it is possible, in a way, for all entities in a given society to carry out road transportation activities. Steps for that code applied in this paper are as follows. Japan's national emissions inventory indicates that this CRF source code corresponds to two categories in the General Energy Statistics of Japan, which are the final energy consumption by passenger vehicle (code: #811000) and the non-energy use of transportation by passenger vehicle (code: #953000). Data in the General Energy Statistics make it possible to divide emissions from passenger vehicles between households' and industrial use. Emissions from industrial use can be further allocated to each industry using the self-transports matrix in the Input-Output tables. Table A.1 shows the list of CRF codes that require additional information sources to allocate emissions to each industry and households.

It is important to mention that AEAs are originally based on the residence principle, which is the same as the national accounts, instead of the territory principle, which national emissions inventories follow. This means that to produce AEAs based on the inventory-first approach, it is necessary to deduct emissions due to non-resident units operating in the national territory and to add emissions due to resident units operating abroad, as the Eurostat manual indicates. This paper does not make such adjustments, under the assumption that the amount of emissions subject to deduction or addition is relatively small in Japan's case.

CRF code	CRF label	Additional source
1.A.1.c	Total energy, fuel combustion activities, energy industries, manufacture of solid fuels and other energy industries	General Energy Statistics
1.A.2.d	Total energy, fuel combustion activities, manufacturing industries and construction, and pulp, paper and print	General Energy Statistics
1.A.2.g	Total energy, fuel combustion activities, manufacturing industries and construction, and others	General Energy Statistics
1.A.3.b	Total energy, fuel combustion activities, and road transportation	General Energy Statistics Input-Output tables
1.A.4.a	Total energy, fuel combustion activities, other sectors, and commercial/institutional	General Energy Statistics
2.F	Product uses as substitutes for ODS	General Energy Statistics Input-Output tables Embodied Energy and Emission Intensity Data
2.G	Other product manufacture and use	Embodied Energy and Emission Intensity Data

Table A.1: List of CRF codes and additional information sources

B. Model Solution

This section describes the equilibrium conditions of the model in detail. The notation of variables is the same as in Section 2.

Households. We restate the equation described in Section 2 for clarity. Households' maximization problem is

$$\max_{\left\{\{C_{i,t}\}_{i=1}^{N},L_t,A_{t+1},\{S_{i,t+1}\}_{i=1}^{N}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\beta^t\left[\log\left(\prod_{i=1}^{N}C_{i,t}^{\alpha_i}\right)-\phi\frac{L_t^{1+\eta}}{1+\eta}\right],$$

subject to

$$P_tA_{t+1}+\sum_{i=1}^{N}Q_{i,t}S_{i,t+1}+\sum_{i=1}^{N}P_{i,t}\left(1+\kappa_{i,t}\right)C_{i,t}\leq R_{t-1}P_tA_t+\sum_{i=1}^{N}R_{i,t-1}^{S}Q_{i,t}S_{i,t}+W_tL_t+\Pi_t.$$

The first order conditions (FOCs) with respect to $C_{i,t}$, L_t, A_{t+1}, and $S_{i,t+1}$ are

$$\begin{aligned}
[C_{i,t}]:\quad \lambda_t &= \beta^t\frac{\prod_{i=1}^{N}\alpha_i^{\alpha_i}}{P_tC_t},\\
[L_t]:\phi L_t^{\eta} &= \frac{W_t}{P_tC_t}\prod_{i=1}^{N}\alpha_i^{\alpha_i},\\
[A_{t+1}]:\quad \frac{1}{C_t} &= \beta R_t\frac{1}{C_{t+1}},\\
[S_{i,t+1}]:\frac{Q_{i,t}}{P_tC_t} &= \frac{\beta R_{i,t}^{S}Q_{i,t+1}}{P_{t+1}C_{t+1}},
\end{aligned}$$

where λ_t is the Lagrange multiplier of the budget constraint.

Firms. Each good in a sector is produced by monopolistically competitive firms. The production function in the ith goods-producing sector is given by equation (1). Firms produce investment goods from products of each sector using Cobb-Douglas technology, as in equation (2). As stated in Section 2, the firm's value function is

$$\begin{aligned}
V\left(K_{i_\omega,t},P_{i_\omega,t-1}\right)=\max_{P_{i_\omega,t},L_{i_\omega,t},K_{i_\omega,t+1},X_{i_\omega,t}}&\ \pi\left(P_{i_\omega,t-1},P_{i_\omega,t},L_{i_\omega,t},K_{i_\omega,t},X_{i_\omega,t}\right)\\
&+\frac{\lambda_{t+1}}{\lambda_t}V\left(K_{i_\omega,t+1},P_{i_\omega,t}\right),
\end{aligned}$$

subject to

$$Y_{i_\omega,t}=Y_{i,t}\left(\frac{P_{i_\omega,t}}{P_{i,t}}\right)^{-\theta},$$

$$K_{i_\omega,t+1} = (1-\delta)K_{i_\omega,t} + \chi_{ki}\prod_{j=1}^{N} x_{ki_\omega j,t}^{\gamma_{kij}}.$$

We consider the symmetric equilibrium among firms within sectors and thus omit the subscript ω when describing the following equilibrium conditions. We describe the FOCs with respect to each variable. FOCs with respect to $L_{i,t}$, $x_{ij,t}$, and $P_{i,t}$ are

$$[L_{i,t}]: \qquad (P_{i,t}(1-\tau_{i,t}) - \eta_{i,t})\,\gamma_i^L \chi_i Z_{i,t} L_{i,t}^{\gamma_i^L-1} K_{i,t}^{\gamma_i^K} \prod_{j=1}^{N} x_{ij,t}^{\gamma_{ij}} = W_t,$$

$$[x_{ij,t}]: \qquad (P_{i,t}(1-\tau_{i,t}) - \eta_{i,t})\,\gamma_{ij} \chi_i Z_{i,t} L_{i,t}^{\gamma_i^L} K_{i,t}^{\gamma_i^K} \frac{\prod_{j=1}^{N} x_{ij,t}^{\gamma_{ij}}}{x_{ij,t}} = P_{j,t}\left(1+\zeta_{ij,t}\right),$$

$$[P_{i,t}]: \eta_{i,t} = \frac{(1-\tau_{i,t})}{\theta}P_{i,t} - \frac{\psi_P}{\theta}\left(\frac{P_{i,t}}{P_{i,t-1}}-1\right)\frac{P_{i,t}^2}{P_{i,t-1}} + \frac{\lambda_{t+1}}{\lambda_t}\upsilon\left(\frac{P_{i,t+1}^2}{P_{i,t}^2} - \frac{P_{i,t+1}}{P_{i,t}}\right)P_{i,t+1}\frac{Y_{i,t+1}}{Y_{i,t}},$$

where $\eta_{i,t}$ is the Lagrange multiplier with respect to the demand function. Markup for sector i is defined as $\mathrm{mc}_{i,t} = P_{i,t}(1-\tau_{i,t})/(P_{i,t}(1-\tau_{i,t}) - \eta_{i,t})$. The last condition is also known as New Keynesian Phillips curve:

$$\theta(1-\tau_i)\left(\frac{1}{\mathrm{mc}_{i,t}}-1\right) + (1-\tau_{i,t}) - \psi_P\left(\pi_{i,t}-1\right)\pi_{i,t} + \frac{\lambda_{t+1}}{\lambda_t}\theta\upsilon\left(\pi_{i,t+1}-1\right)\pi_{i,t+1}^2\frac{Y_{i,t+1}}{Y_{i,t}} = 0$$

At the steady state,

$$\eta_i = \frac{P_i(1-\tau_i)}{\theta},$$

and consequently steady state markup is $\theta/(\theta-1)$. The envelope condition with respect to price is

$$V_P\left(K_{i,t}, P_{i,t-1}\right) = -P_{i,t}\Phi_2\left(P_{i,t}, P_{i,t-1}\right).$$

FOCs with respect to $K_{i,t+1}$ and $x_{kij,t}$ are

$$[K_{i,t+1}]: \quad \frac{\lambda_{t+1}}{\lambda_t}V_K\left(K_{i,t+1}, P_{i,t}\right) - \mu_{i,t} = 0,$$

$$[x_{kij,t}]: \mu_{i,t}\left(1-\psi_K\left(\frac{I_{i,t}}{K_{i,t}}-\delta\right)\right) = \prod_{j=1}^{N} P_{j,t}^{\gamma_{kij}},$$

where $\mu_{i,t}$ is the marginal cost of producing capital at time t. At the steady state,

$\mu_i = \prod_{j=1}^{N} P_j^{\gamma_{kij}}$. The envelope condition with respect to capital is

$$V_K\left(K_{i,t}, P_{i,t-1}\right) = \gamma_i^K \left(P_{i,t}(1-\tau_{i,t}) - \eta_{i,t}\right) \chi_i Z_{i,t} L_{i,t}^{\gamma_i^L} K_{i,t}^{\gamma_i^K - 1} \prod_{j=1}^{N} x_{ij,t}^{\gamma_{ij}}$$
$$+ \frac{\psi_K}{2}\mu_{i,t} \left\{ \left(\frac{I_{i,t}}{K_{i,t}}\right)^2 - \delta^2 \right\} + (1-\delta)\,\mu_{i,t}.$$

The resulting intertemporal conditions are

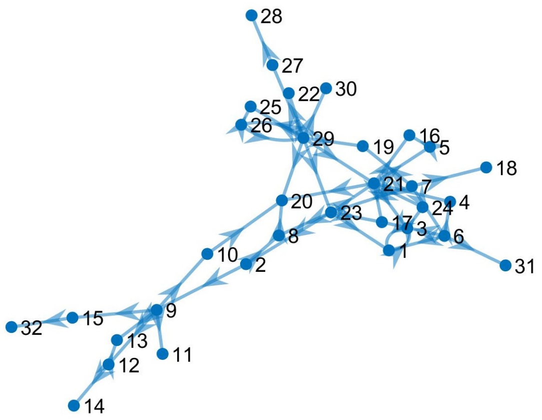
$$\gamma_i^K \frac{(1-\tau_{i,t+1})P_{i,t+1}}{\mathrm{mc}_{i,t+1}} \frac{Y_{i,t+1}}{K_{i,t+1}} + \frac{\psi_K}{2}\mu_{i,t+1} \left\{ \left(\frac{I_{i,t+1}}{K_{i,t+1}}\right)^2 - \delta^2 \right\} = \frac{\lambda_t}{\lambda_{t+1}}\mu_{i,t} - (1-\delta)\,\mu_{i,t+1},$$

for $i = 1, \ldots, N$.

Tobin's q. Marginal $q_{i,t}$ is equal to the marginal value of investment divided by the price of investment goods. From the firm's value function, we obtain Tobin's q straightforwardly,

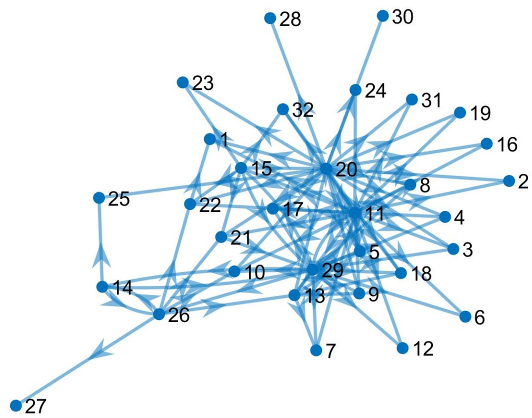
$$q_{i,t} = \frac{\lambda_{t+1}}{\mu_{i,t}\lambda_t} \left[\gamma_i^K \frac{(1-\tau_{i,t+1})P_{i,t+1}}{\mathrm{mc}_{i,t+1}} \frac{Y_{i,t+1}}{K_{i,t+1}} + \frac{\psi_K}{2}\mu_{i,t+1} \left\{ \left(\frac{I_{i,t+1}}{K_{i,t+1}}\right)^2 - \delta^2 \right\} + (1-\delta)\,\mu_{i,t+1} \right].$$

C. Characteristics of sectors



(a) Intermediate input linkages

Note: When the amount of input from one sector to another sector as intermediate exceeds 5% of the total expenditure, a line is drawn between them.



(b) Investment network

Note: When the amount of input from one sector to another sector as input for capital production exceeds 10% of the total input for capital production, a line is drawn between them.

Figure A.1: Sectoral network

Note: Numbers along the x-axis denote sectors as in Figure 2 .

	Industry	Value
1.	Agriculture, forestry and fishing	0.013
2.	Mining (excluding coal, crude oil, and natural gas)	0.000
3.	Food products and beverages	0.093
4.	Textile products	0.015
5.	Pulp, paper and paper products	0.001
6.	Chemicals	0.009
7.	Petroleum and coal products	0.017
8.	Non-metallic mineral products	0.000
9.	Basic metal	0.000
10.	Fabricated metal products	0.001
11.	General-purpose, production and business-oriented machinery	0.000
12.	Electronic components and devices	0.001
13.	Electrical machinery, equipment and supplies	0.011
14.	Information and communication electronics equipment	0.012
15.	Transport equipment	0.022
16.	Printing	0.000
17.	Others	0.012
18.	Electricity supply	0.018
19.	Gas and water supply, and waste management service	0.013
20.	Construction	0.000
21.	Wholesale trade	0.042
22.	Retail trade	0.119
23.	Transport and postal services	0.051
24.	Accommodation and food service activities	0.076
25.	Communications and broadcasting	0.035
26.	Information services, and Image, sound and character information production and distribution	0.001
27.	Finance and insurance	0.060
28.	Real estate	0.221
29.	Professional, scientific and technical activities	0.004
30.	Education	0.020
31.	Human health and social work activities	0.048
32.	Other service activities	0.000

Table A.2: Consumption share α_i by sector

Source: Input-output table in 2015.

D. Carbon tax propagation

In this Appendix, we present the derivation of the analytical result in Section 4.

Impact on relative prices. Substituting the FOCs into the production function in equation (1) and taking the logarithms, we have

$$\log Y_i = \log(\chi_i Z_i) + \gamma_i^L \log \frac{\gamma_i^L P_i (1-\tau_i) Y_i}{W} + \gamma_i^K \log \frac{\gamma_i^K P_i (1-\tau_i) Y_i}{r_K \mu_i} + \sum_{j=1}^N \gamma_{ij} \log \frac{\gamma_{ij} P_i (1-\tau_i) Y_i}{P_j \left(1+\zeta_{ij}\right)}.$$

Arranging this gives us:

$$\log Y_i = \log Z_i + \log P_i (1-\tau_i) Y_i - \gamma_i^L \log W - \gamma_i^K \log r_K \mu_i - \sum_{j=1}^N \gamma_{ij} \log P_j \left(1+\zeta_{ij}\right).$$

The equilibrium price of the capital good of sector i is $\mu_i = \prod_{j=1}^N P_j^{\gamma_{kij}}$. The relative price of sector i is defined as $\hat{P}_i \equiv P_i/W$ and given by:

$$\log \hat{P}_i = -\log\left(Z_i \left[1-\tau_i\right] r_K^{-\gamma_i^K}\right) + \sum_{j=1}^N \gamma_{ij} \log\left(1+\zeta_{ij}\right) + \sum_{j=1}^N \left(\gamma_{ij} + \gamma_i^K \gamma_{kij}\right) \log \hat{P}_j. \quad (14)$$

Now, we have the vector of the equilibrium relative prices using the Leontief inverse matrix which is defined as $L = (I - \Gamma)^{-1}$ where $\Gamma = [\gamma_{ij} + \gamma_i^K \gamma_{kij}]$:

$$\begin{bmatrix} \log \hat{P}_1 \\ \vdots \\ \log \hat{P}_N \end{bmatrix} = L \left(\begin{bmatrix} -\log\left(Z_1 \left[1-\tau_1\right] r_K^{-\gamma_1^K}\right) \\ \vdots \\ -\log\left(Z_N \left[1-\tau_N\right] r_K^{-\gamma_N^K}\right) \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^N \gamma_{1j} \log\left(1+\zeta_{1j}\right) \\ \vdots \\ \sum_{j=1}^N \gamma_{Nj} \log\left(1+\zeta_{Nj}\right) \end{bmatrix} \right),$$

or

$$\begin{bmatrix} \log \hat{P}_1 \\ \vdots \\ \log \hat{P}_N \end{bmatrix} = - \begin{bmatrix} \sum_{j=1}^N l_{1j} \log\left(Z_j \left[1-\tau_j\right] r_K^{-\gamma_j^K}\right) \\ \vdots \\ \sum_{j=1}^N l_{Nj} \log\left(Z_j \left[1-\tau_j\right] r_K^{-\gamma_j^K}\right) \end{bmatrix} + \begin{bmatrix} \sum_{k=1}^N \sum_{j=1}^N l_{1k} \gamma_{kj} \log\left(1+\zeta_{kj}\right) \\ \vdots \\ \sum_{k=1}^N \sum_{j=1}^N l_{Nk} \gamma_{kj} \log\left(1+\zeta_{kj}\right) \end{bmatrix},$$

where l_{ij} is the (i, j) element of L. Using this equation, we derive the derivative of log relative prices with respect to each carbon tax rate:

$$
\begin{aligned}
[\tau_m]: &\ \frac{l_{im}}{1-\tau_m}, \\
[\zeta_{mo}]: &\ \frac{l_{im}\gamma_{mo}}{1+\zeta_{mo}}, \\
[\kappa_o]: &\quad 0.
\end{aligned}
$$

This indicates that the impact of the production process tax is similar to that of a sector-specific adverse technological shock. The intermediate input tax ζ_{mo} increases relative prices and its sensitivity depends on the parameters l_{im} and γ_{mo}, which represent the importance of fossil fuel industry through the intermediate input supplied by sector m.

Impact on real wage. Using the equation of relative prices, nominal labor income is represented as follows:

$$
\log W = \log P_i + \sum_{j=1}^{N} l_{ij} \log\left(Z_j\left[1-\tau_j\right] r_K^{-\gamma_j^K}\right) - \sum_{m=1}^{N}\sum_{j=1}^{N} l_{im}\gamma_{mj}\log\left(1+\zeta_{mj}\right).
$$

By summing with weight α_i for $i = 1, ..., N$, we have

$$
\begin{aligned}
\log\frac{W}{P} = &-\sum_{i=1}^{N}\alpha_i\log(1+\kappa_i) + \sum_{i=1}^{N}\alpha_i\sum_{j=1}^{N} l_{ij}\log\left(Z_j\left[1-\tau_j\right] r_K^{-\gamma_j^K}\right) \\
&-\sum_{i=1}^{N}\alpha_i\sum_{m=1}^{N}\sum_{j=1}^{N} l_{im}\gamma_{mj}\log\left(1+\zeta_{mj}\right), \qquad (15)
\end{aligned}
$$

where $P = \prod_{i=1}^{N}\left[(1+\kappa_i)P_i\right]^{\alpha_i}$. The derivatives of log real wage, $\log W/P$, with respect to carbon tax rates are as follows:

$$
\begin{aligned}
[\tau_m]: &\ -\sum_{i=1}^{N}\frac{\alpha_i l_{im}}{1-\tau_m}, \\
[\zeta_{mo}]: &\ -\sum_{i=1}^{N}\frac{\alpha_i l_{im}\gamma_{mo}}{1+\zeta_{mo}}, \\
[\kappa_o]: &\ -\frac{\alpha_o}{1+\kappa_o}.
\end{aligned}
$$

Impact on sectoral weight. Next, we explore the impact on sectoral weight. Substituting the FOCs into the market clearing condition $Y_i = C_i + \sum_{j=1}^{N} x_{ji} + \sum_{j=1}^{N} x_{kji}$, we have

$$Y_i = \alpha_i \frac{W + r\sum_{i=1}^{N} \mu_i K_i + \Pi}{P_i(1+\kappa_i)} + \sum_{j=1}^{N} \frac{\gamma_{ji} P_j (1-\tau_j) Y_j}{P_i\left(1+\zeta_{ji}\right)} + \sum_{j=1}^{N} \frac{\gamma_{kji}\mu_j I_j}{P_i}. \tag{16}$$

Equation (16) can be written as:

$$Y_i = \alpha_i \frac{\text{NGDP}}{P_i(1+\kappa_i)} + \sum_{j=1}^{N} \frac{\gamma_{ji} P_j (1-\tau_j) Y_j}{P_i\left(1+\zeta_{ji}\right)} + \sum_{j=1}^{N} \frac{\left[(1+\kappa_i)\gamma_{kji} - \alpha_i\right]\mu_j I_j}{P_i(1+\kappa_i)}, \tag{17}$$

where $\text{NGDP} = W + r_K \sum_{i=1}^{N} \mu_i K_i + \Pi$. Using the relationship $\gamma_i^K P_i(1-\tau_i)Y_i = r_K \mu_i K_i = r_K \mu_i I_i/\delta$, nominal investment in sector i is written as

$$\mu_i I_i = \frac{\delta \gamma_i^K P_i (1-\tau_i) Y_i}{r_K}. \tag{18}$$

Multiplying $P_i(1-\tau_i)$ for both sides of equation (17), we have

$$\begin{aligned} P_i(1-\tau_i)Y_i = {} & \alpha_i(1-\tau_i)\frac{\text{NGDP}}{1+\kappa_i} + \sum_{j=1}^{N} \frac{\gamma_{ji}(1-\tau_i)P_j(1-\tau_j)Y_j}{1+\zeta_{ji}} \\ & + \delta(1-\tau_i)\sum_{j=1}^{N} \frac{\gamma_j^K\left[(1+\kappa_i)\gamma_{kji} - \alpha_i\right]P_j(1-\tau_j)Y_j}{r_K}. \end{aligned} \tag{19}$$

We define a sectoral weight of sector i as follows:

$$w_i^D \equiv \frac{P_i(1-\tau_i)Y_i}{\text{NGDP}}.$$

Dividing equation (19) by nominal GDP, we have the equilibrium equation of the sectoral weight as follows:

$$w_i^D = \frac{\alpha_i(1-\tau_i)}{1+\kappa_i} + (1-\tau_i)\sum_{j=1}^{N} \frac{\gamma_{ji} w_j^D}{1+\zeta_{ji}} + \frac{\delta(1-\tau_i)}{r_K}\sum_{j=1}^{N} \gamma_j^K \left[(1+\kappa_i)\gamma_{kji} - \alpha_i\right] w_j^D,$$

or

$$\frac{w_i^D}{1-\tau_i} = \frac{\alpha_i}{1+\kappa_i} + \sum_{j=1}^{N} \tilde{\gamma}_{ij} \frac{w_j^D}{1-\tau_j}, \tag{20}$$

where

$$\tilde{\gamma}_{ij} = \frac{\gamma_{ji}\left(1-\tau_j\right)}{1+\zeta_{ji}} + \frac{\delta\gamma_j^K\left[(1+\kappa_i)\gamma_{kji} - \alpha_i\right]\left(1-\tau_j\right)}{r_K}.$$

Vectorizing the above equations, we derive

$$\begin{bmatrix} \frac{w_1^D}{1-\tau_1} \\ \vdots \\ \frac{w_N^D}{1-\tau_N} \end{bmatrix} = \begin{bmatrix} \frac{\alpha_1}{1+\kappa_1} \\ \vdots \\ \frac{\alpha_N}{1+\kappa_N} \end{bmatrix} + \begin{bmatrix} \tilde{\gamma}_{11} & \cdots & \tilde{\gamma}_{1N} \\ \vdots & \ddots & \vdots \\ \tilde{\gamma}_{N1} & \cdots & \tilde{\gamma}_{NN} \end{bmatrix} \begin{bmatrix} \frac{w_1^D}{1-\tau_1} \\ \vdots \\ \frac{w_N^D}{1-\tau_N} \end{bmatrix},$$

$$\begin{bmatrix} \frac{w_1^D}{1-\tau_1} \\ \vdots \\ \frac{w_N^D}{1-\tau_N} \end{bmatrix} = \left(I - \tilde{\Gamma}\right)^{-1} \begin{bmatrix} \frac{\alpha_1}{1+\kappa_1} \\ \vdots \\ \frac{\alpha_N}{1+\kappa_N} \end{bmatrix}.$$

Letting $\tilde{l}_{ij}$ be the (i, j) element of $\tilde{L} = \left(I - \tilde{\Gamma}\right)^{-1}$, we can write the equilibrium sectoral weight in sector i as

$$w_i^D = (1 - \tau_i) \sum_{j=1}^{N} \frac{\tilde{l}_{ij}\alpha_j}{1 + \kappa_j}.$$

The value added of sector i is equal to $\left(\gamma_i^L + \gamma_i^K\right) P_i(1 - \tau_i)Y_i$; thus, the following results show the sensitivity of the share of sectoral value added to carbon taxes. Note that matrix $\tilde{L}$ is now the function of each type of carbon tax. Taking the derivative of sectoral weight of sector i with respect to each taxation yields the following result:

$$\begin{aligned} [\tau_m] :& -\sum_{j=1}^{N} \frac{\tilde{l}_{ij}\alpha_j}{1 + \kappa_j}\mathbf{1}_{[i=m]} + (1 - \tau_i) \sum_{j=1}^{N} \frac{\partial \tilde{l}_{ij}}{\partial \tau_m} \frac{\alpha_j}{1 + \kappa_j}, \\ [\zeta_{mo}] :& \quad (1 - \tau_i) \sum_{j=1}^{N} \frac{\partial \tilde{l}_{ij}}{\partial \zeta_{mo}} \frac{\alpha_j}{1 + \kappa_j}, \\ [\kappa_o] :& \quad (1 - \tau_i) \left[\sum_{j=1}^{N} \frac{\partial \tilde{l}_{ij}}{\partial \kappa_o} \frac{\alpha_j}{1 + \kappa_j} - \frac{\tilde{l}_{io}\alpha_o}{(1 + \kappa_o)^2} \right]. \end{aligned}$$

Therefore, changes in carbon taxes affect value added through the matrix $\tilde{L}$.

Next, we derive the sensitivity of $\tilde{l}_{ij}$ to the change in the rate of production process tax. The relationship $\tilde{L} = I + \tilde{\Gamma}\tilde{L}$ can be written in the vectorized form:

$$\begin{bmatrix} \sum_{i=1}^{N} \frac{\tilde{l}_{1i}\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\tilde{l}_{Ni}\alpha_i}{1+\kappa_i} \end{bmatrix} = \begin{bmatrix} \frac{\alpha_1}{1+\kappa_1} \\ \vdots \\ \frac{\alpha_N}{1+\kappa_N} \end{bmatrix} + \tilde{\Gamma} \begin{bmatrix} \sum_{i=1}^{N} \frac{\tilde{l}_{1i}\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\tilde{l}_{Ni}\alpha_i}{1+\kappa_i} \end{bmatrix}.$$

The jth row of this equation is

$$\sum_{i=1}^{N}\frac{\tilde{l}_{ji}\alpha_i}{1+\kappa_i}=\frac{\alpha_j}{1+\kappa_j}+\sum_{k=1}^{N}\left(\frac{\gamma_{kj}(1-\tau_k)}{1+\zeta_{kj}}+\frac{\delta\gamma_k^K\left[\left(1+\kappa_j\right)\gamma_{kkj}-\alpha_j\right](1-\tau_k)}{r_K}\right)\sum_{i=1}^{N}\frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i}.$$

Taking the derivative with respect to τ_m for $m=1,...,N$:

$$\sum_{i=1}^{N}\frac{\partial\tilde{l}_{ji}}{\partial\tau_m}\frac{\alpha_i}{1+\kappa_i}=-\left(\frac{\gamma_{mj}}{1+\zeta_{mj}}+\frac{\delta\gamma_m^K\left[\left(1+\kappa_j\right)\gamma_{kmj}-\alpha_j\right]}{r_K}\right)\sum_{i=1}^{N}\frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i}$$
$$+\sum_{k=1}^{N}\tilde{\gamma}_{jk}\sum_{i=1}^{N}\frac{\partial\tilde{l}_{ki}}{\partial\tau_m}\frac{\alpha_i}{1+\kappa_i}.$$

In the vectorized form,

$$\begin{bmatrix}\sum_{i=1}^N\frac{\partial\tilde{l}_{1i}}{\partial\tau_m}\frac{\alpha_i}{1+\kappa_i}\\ \vdots\\ \sum_{i=1}^N\frac{\partial\tilde{l}_{Ni}}{\partial\tau_m}\frac{\alpha_i}{1+\kappa_i}\end{bmatrix}=\begin{bmatrix}-\left(\frac{\gamma_{m1}}{1+\zeta_{m1}}+\frac{\delta\gamma_m^K[(1+\kappa_1)\gamma_{km1}-\alpha_1]}{r_K}\right)\sum_{i=1}^N\frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i}\\ \vdots\\ -\left(\frac{\gamma_{mN}}{1+\zeta_{mN}}+\frac{\delta\gamma_m^K[(1+\kappa_N)\gamma_{kmN}-\alpha_N]}{r_K}\right)\sum_{i=1}^N\frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i}\end{bmatrix}+\tilde{\Gamma}\begin{bmatrix}\sum_{i=1}^N\frac{\partial\tilde{l}_{1i}}{\partial\tau_m}\frac{\alpha_i}{1+\kappa_i}\\ \vdots\\ \sum_{i=1}^N\frac{\partial\tilde{l}_{Ni}}{\partial\tau_m}\frac{\alpha_i}{1+\kappa_i}\end{bmatrix}$$
$$=\begin{bmatrix}-\sum_{j=1}^N\tilde{l}_{1j}\left(\frac{\gamma_{mj}}{1+\zeta_{mj}}+\frac{\delta\gamma_m^K\left[\left(1+\kappa_j\right)\gamma_{kmj}-\alpha_j\right]}{r_K}\right)\sum_{i=1}^N\frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i}\\ \vdots\\ -\sum_{j=1}^N\tilde{l}_{Nj}\left(\frac{\gamma_{mj}}{1+\zeta_{mj}}+\frac{\delta\gamma_m^K\left[\left(1+\kappa_j\right)\gamma_{kmj}-\alpha_j\right]}{r_K}\right)\sum_{i=1}^N\frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i}\end{bmatrix}$$
$$=\begin{bmatrix}-\sum_{j=1}^N\frac{\tilde{l}_{1j}\tilde{\gamma}_{jm}}{1-\tau_m}\sum_{i=1}^N\frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i}\\ \vdots\\ -\sum_{j=1}^N\frac{\tilde{l}_{Nj}\tilde{\gamma}_{jm}}{1-\tau_m}\sum_{i=1}^N\frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i}\end{bmatrix}.$$

We derive the sensitivity of $\tilde{l}_{ij}$ to the intermediate input tax. For $j=o$,

$$\sum_{i=1}^{N}\frac{\partial\tilde{l}_{oi}}{\partial\zeta_{mo}}\frac{\alpha_i}{1+\kappa_i}=-\frac{\gamma_{mo}(1-\tau_m)}{(1+\zeta_{mo})^2}\sum_{i=1}^{N}\frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i}$$
$$+\sum_{k=1}^{N}\left(\frac{\gamma_{ko}(1-\tau_k)}{1+\zeta_{ko}}+\frac{\delta\gamma_k^K\left[(1+\kappa_o)\gamma_{kko}-\alpha_o\right](1-\tau_k)}{r_K}\right)\sum_{i=1}^{N}\frac{\partial\tilde{l}_{ki}}{\partial\zeta_{mo}}\frac{\alpha_i}{1+\kappa_i}.$$

For $j \neq o$,

$$\sum_{i=1}^{N} \frac{\partial \tilde{l}_{ji}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i} = \sum_{k=1}^{N} \left(\frac{\gamma_{kj}(1-\tau_k)}{1+\zeta_{kj}} + \frac{\delta \gamma_k^K \left[\left(1+\kappa_j\right) \gamma_{kkj} - \alpha_j \right] (1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\partial \tilde{l}_{ki}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i}.$$

Vectorizing the equations and solving for $\sum_{i=1}^{N} \frac{\partial \tilde{l}_{ki}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i}$ for $k = 1, \ldots, N$, we have

$$\begin{bmatrix} \sum_{i=1}^{N} \frac{\partial \tilde{l}_{1i}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{Ni}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\frac{\gamma_{mo}(1-\tau_m)}{(1+\zeta_{mo})^2} \sum_{i=1}^{N} \frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \tilde{\Gamma} \begin{bmatrix} \sum_{i=1}^{N} \frac{\partial \tilde{l}_{1i}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{Ni}}{\partial \zeta_{mo}} \frac{\alpha_i}{1+\kappa_i} \end{bmatrix}$$

$$= \begin{bmatrix} -\tilde{l}_{1o} \frac{\gamma_{mo}(1-\tau_m)}{(1+\zeta_{mo})^2} \sum_{i=1}^{N} \frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i} \\ \vdots \\ -\tilde{l}_{oo} \frac{\gamma_{mo}(1-\tau_m)}{(1+\zeta_{mo})^2} \sum_{i=1}^{N} \frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i} \\ \vdots \\ -\tilde{l}_{No} \frac{\gamma_{mo}(1-\tau_m)}{(1+\zeta_{mo})^2} \sum_{i=1}^{N} \frac{\tilde{l}_{mi}\alpha_i}{1+\kappa_i} \end{bmatrix}.$$

Lastly, we derive the sensitivity of $\tilde{l}_{ij}$ to the consumption tax. For $j = o$,

$$\sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} - \frac{\tilde{l}_{oo}\alpha_o}{(1+\kappa_o)^2} = -\frac{\alpha_o}{(1+\kappa_o)^2} + \sum_{k=1}^{N} \left(\frac{\delta \gamma_k^K \gamma_{kko}(1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i}$$
$$+ \sum_{k=1}^{N} \tilde{\gamma}_{jk} \left(\sum_{i=1}^{N} \frac{\partial \tilde{l}_{ki}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} - \frac{\tilde{l}_{ko}\alpha_o}{(1+\kappa_o)^2} \right),$$

$$\sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} = \frac{\left(\tilde{l}_{oo} - 1\right)\alpha_o}{(1+\kappa_o)^2} + \sum_{k=1}^{N} \left(\frac{\delta \gamma_k^K \gamma_{kko}(1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i}$$
$$+ \sum_{k=1}^{N} \tilde{\gamma}_{jk} \left(\sum_{i=1}^{N} \frac{\partial \tilde{l}_{ki}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} - \frac{\tilde{l}_{ko}\alpha_o}{(1+\kappa_o)^2} \right).$$

For $j \neq o$,

$$\sum_{i=1}^{N} \frac{\partial \tilde{l}_{ji}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} - \frac{\tilde{l}_{jo}\alpha_o}{(1+\kappa_o)^2} = \sum_{k=1}^{N} \tilde{\gamma}_{jk} \left(\sum_{i=1}^{N} \frac{\partial \tilde{l}_{ki}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} - \frac{\tilde{l}_{ko}\alpha_o}{(1+\kappa_o)^2} \right).$$

Vectorizing the equations for $k = 1, ..., N$ gives us

$$\begin{bmatrix} \sum_{i=1}^{N} \frac{\partial \tilde{l}_{1i}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{Ni}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \end{bmatrix} = \begin{bmatrix} \frac{\tilde{l}_{1o}\alpha_o}{(1+\kappa_o)^2} \\ \vdots \\ \frac{(\tilde{l}_{oo}-1)\alpha_o}{(1+\kappa_o)^2} + \sum_{k=1}^{N} \left(\frac{\delta \gamma_k^K \gamma_{kko}(1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i} \\ \vdots \\ \frac{\tilde{l}_{No}\alpha_o}{(1+\kappa_o)^2} \end{bmatrix} + \tilde{\Gamma} \begin{bmatrix} \sum_{i=1}^{N} \frac{\partial \tilde{l}_{1i}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} - \frac{\tilde{l}_{1o}\alpha_o}{(1+\kappa_o)^2} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} - \frac{\tilde{l}_{oo}\alpha_o}{(1+\kappa_o)^2} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{Ni}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} - \frac{\tilde{l}_{No}\alpha_o}{(1+\kappa_o)^2} \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} \sum_{i=1}^{N} \frac{\partial \tilde{l}_{1i}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{Ni}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \end{bmatrix} &= \tilde{L} \begin{bmatrix} 0 \\ \vdots \\ \frac{\alpha_o}{(1+\kappa_o)^2} \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ -\frac{\alpha_o}{(1+\kappa_o)^2} + \sum_{k=1}^{N} \left(\frac{\delta \gamma_k^K \gamma_{kko}(1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i} \\ \vdots \\ 0 \end{bmatrix} \\ &= +\tilde{\Gamma} \begin{bmatrix} \sum_{i=1}^{N} \frac{\partial \tilde{l}_{1i}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{Ni}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \end{bmatrix} - \tilde{\Gamma}\tilde{L} \begin{bmatrix} 0 \\ \vdots \\ \frac{\alpha_o}{(1+\kappa_o)^2} \\ \vdots \\ 0 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} \sum_{i=1}^{N} \frac{\partial \tilde{l}_{1i}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{oi}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \\ \vdots \\ \sum_{i=1}^{N} \frac{\partial \tilde{l}_{Ni}}{\partial \kappa_o} \frac{\alpha_i}{1+\kappa_i} \end{bmatrix} = \tilde{L} \begin{bmatrix} 0 \\ \vdots \\ \sum_{k=1}^{N} \left(\frac{\delta \gamma_k^K \gamma_{kko}(1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i} \\ \vdots \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \tilde{l}_{1o} \sum_{k=1}^{N} \left(\frac{\delta \gamma_k^K \gamma_{kko}(1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i} \\ \vdots \\ \tilde{l}_{oo} \sum_{k=1}^{N} \left(\frac{\delta \gamma_k^K \gamma_{kko}(1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i} \\ \vdots \\ \tilde{l}_{No} \sum_{k=1}^{N} \left(\frac{\delta \gamma_k^K \gamma_{kko}(1-\tau_k)}{r_K} \right) \sum_{i=1}^{N} \frac{\tilde{l}_{ki}\alpha_i}{1+\kappa_i} \end{bmatrix}.$$

In summary, the derivatives of sectoral weights with respect to each taxation are as follows:

$$\begin{aligned} [\tau_m] : &\quad -\frac{w_i^D}{1-\tau_i} \mathbf{1}_{[i=m]} - \frac{1-\tau_i}{1-\tau_m} \sum_{k=1}^{N} \tilde{l}_{ik} \tilde{\gamma}_{km} \frac{w_m^D}{1-\tau_m}, \\ [\zeta_{mo}] : &\quad -(1-\tau_i) \frac{\tilde{l}_{io} \gamma_{mo} w_m^D}{(1+\zeta_{mo})^2}, \\ [\kappa_o] : &\; -(1-\tau_i)\, \tilde{l}_{io} \left[\frac{\alpha_o}{(1+\kappa_o)^2} - \sum_{k=1}^{N} \frac{\delta \gamma_k^K \gamma_{kko} w_k^D}{r_K} \right]. \end{aligned}$$

Impact on consumption. For later derivation, we define sectoral weight using household income as

$$w_i^H \equiv \frac{P_i (1-\tau_i) Y_i}{W + r \sum_{i=1}^{N} \mu_i K_i + \Pi}.$$

Using a similar derivation for the case of w_i^D, we have the equilibrium sectoral weight as follows:

$$\frac{w_i^H}{1-\tau_i} = \frac{\alpha_i}{1+\kappa_i} + \sum_{j=1}^{N} \tilde{\gamma}_{ij}^H \frac{w_j^H}{1-\tau_j}, \tag{21}$$

where

$$\tilde{\gamma}_{ij}^H = \frac{\gamma_{ji} \left(1-\tau_j\right)}{1+\zeta_{ji}} + \frac{\delta \gamma_j^K \gamma_{kji} \left(1-\tau_j\right)}{r_K},$$

and $\tilde{\gamma}_{ij}^H$ is the (i,j) element of $\tilde{\Gamma}_H$. Letting $\tilde{l}_{ij}^H$ be the (i,j) element of $\tilde{L}_H = \left(I - \tilde{\Gamma}_H\right)^{-1}$, we have

$$w_i^H = (1-\tau_i)\sum_{j=1}^{N}\frac{\tilde{l}_{ij}^H\alpha_j}{1+\kappa_j}.$$

The derivatives of sectoral weights w_i^H with respect to each taxation are as follows:

$$\begin{aligned}
[\tau_m]: & -\frac{w_i^H}{1-\tau_i}\mathbf{1}_{[i=m]} - \sum_{k=1}^{N}\frac{\tilde{l}_{ik}^H\tilde{\gamma}_{km}^H w_m^H}{(1-\tau_m)^2},\\
[\zeta_{mo}]: & \qquad -(1-\tau_i)\frac{\tilde{l}_{io}^H\gamma_{mo}w_m^H}{(1+\zeta_{mo})^2},\\
[\kappa_o]: & \qquad -(1-\tau_i)\frac{\tilde{l}_{io}^H\alpha_o}{(1+\kappa_o)^2}.
\end{aligned}$$

We find the derivative of consumption C_i with respect to the carbon tax rates. From here on, we assume that γ_i^L and γ_i^K are constant across sectors. From the FOC of households, we have

$$\log P_i(1+\kappa_i)C_i = \log\alpha_i + \log\frac{W + r\sum_{i=1}^{N}\mu_i K_i + \Pi}{P} + \log P.$$

By arranging the above equation, we have

$$\log C_i = \log\alpha_i - \log\hat{P}_i(1+\kappa_i) - \log\frac{W}{P} + \log\frac{W + r\sum_{i=1}^{N}\mu_i K_i + \Pi}{P}. \tag{22}$$

Here,

$$\begin{aligned}
W + r\sum_{i=1}^{N}\mu_i K_i + \Pi &= W + \sum_{i=1}^{N}\frac{r}{r+\delta}\gamma^K P_i(1-\tau_i)Y_i + \Pi\\
&= \left(1 + \frac{r}{r_K}\frac{\gamma^K}{\gamma^L}\right)W + \Pi.
\end{aligned}$$

Real household income can be represented as follows:

$$\frac{W + r\sum_{i=1}^{N}\mu_i K_i + \Pi}{P} = \left(1 + \frac{r}{r_K}\frac{\gamma^K}{\gamma^L}\right)\frac{W}{P} + \frac{W + r\sum_{i=1}^{N}\mu_i K_i + \Pi}{P}w^\Pi,$$

where $w^\Pi \equiv \Pi/\left(W + r\sum_{i=1}^{N}\mu_i K_i + \Pi\right)$. Arranging this and taking the logarithm gives us:

$$\log\frac{W + r\sum_{i=1}^{N}\mu_i K_i + \Pi}{P} = -\log\left(1 - w^\Pi\right) + \log\left(1 + \frac{r}{r_K}\frac{\gamma^K}{\gamma^L}\right) + \log\frac{W}{P}. \tag{23}$$

Substituting (23) into equation (22), we have

$$\log C_i = \log \alpha_i - \log \hat{P}_i (1 + \kappa_i) - \log \left(1 - w^{\Pi}\right) + \log \left(1 + \frac{r}{r_K} \frac{\gamma^K}{\gamma^L}\right).$$

The derivatives of $\log C_i$ with respect to carbon taxes are

$$\begin{aligned}
[\tau_m]:& \quad -\frac{l_{im}}{1 - \tau_m} + \frac{1}{1 - w^{\Pi}} \frac{\partial w^{\Pi}}{\partial \tau_m}, \\
[\zeta_{mo}]:& -\frac{l_{im}\gamma_{mo}}{1 + \zeta_{mo}} + \frac{1}{1 - w^{\Pi}} \frac{\partial w^{\Pi}}{\partial \zeta_{mo}}, \\
[\kappa_o]:& \quad -\frac{\mathbf{1}_{[i=o]}}{1 + \kappa_o} + \frac{1}{1 - w^{\Pi}} \frac{\partial w^{\Pi}}{\partial \kappa_o}.
\end{aligned}$$

The total carbon tax revenue is

$$\begin{aligned}
\Pi &= \sum_{i=1}^{N} \tau_i P_i Y_i + \sum_{i=1}^{N} \zeta_{io} P_o x_{io} + \kappa_o P_o C_o \\
&= \sum_{i=1}^{N} \left(\frac{\tau_i}{1 - \tau_i} + \frac{\zeta_{io}}{1 + \zeta_{io}} \gamma_{io}\right) P_i (1 - \tau_i) Y_i + \frac{\kappa_o}{1 + \kappa_o} \alpha_o \left(W + r \sum_{i=1}^{N} \mu_i K_i + \Pi\right).
\end{aligned}$$

Dividing this by the total household income gives us

$$w^{\Pi} = \sum_{i=1}^{N} \Upsilon_i w_i^H + \frac{\kappa_o}{1 + \kappa_o} \alpha_o,$$

where

$$\Upsilon_i = \frac{\tau_i}{1 - \tau_i} + \frac{\zeta_{io}}{1 + \zeta_{io}} \gamma_{io}.$$

The derivatives of w^{Π} with respect to carbon taxes are

$$\frac{\partial w^{\Pi}}{\partial \tau_m} = \frac{w_m^H}{(1 - \tau_m)^2} + \sum_{i=1}^{N} \Upsilon_i \frac{\partial w_i^H}{\partial \tau_m},$$

$$\frac{\partial w^{\Pi}}{\partial \zeta_{mo}} = \frac{\gamma_{mo} w_m^H}{(1 + \zeta_{mo})^2} + \sum_{i=1}^{N} \Upsilon_i \frac{\partial w_i^H}{\partial \zeta_{mo}},$$

$$\frac{\partial w^{\Pi}}{\partial \kappa_o} = \sum_{i=1}^{N} \Upsilon_i \frac{\partial w_i^H}{\partial \kappa_o} + \frac{\alpha_o}{(1+\kappa_o)^2}.$$

Using these relationships, we find the derivatives of $\log C_i$ with respect to carbon taxes as follows:

$$\begin{aligned}
[\tau_m] &: -\frac{l_{im}}{1-\tau_m} + \frac{1}{1-w^{\Pi}}\left(\frac{w_m^H}{(1-\tau_m)^2} - \sum_{i=1}^{N} \Upsilon_i \left(\frac{w_i^H}{1-\tau_i}\mathbf{1}_{[i=m]} + \sum_{k=1}^{N} \frac{\tilde{l}_{ik}^H \tilde{\gamma}_{km}^H w_m^H}{1-\tau_m}\right)\right), \\
[\zeta_{mo}] &: \qquad -\frac{l_{im}\gamma_{mo}}{1+\zeta_{mo}} + \frac{1}{1-w^{\Pi}}\left(\frac{\gamma_{mo} w_m^H}{(1+\zeta_{mo})^2} - \sum_{i=1}^{N} \Upsilon_i (1-\tau_i) \frac{\tilde{l}_{io}^H \gamma_{mo} w_m^H}{(1+\zeta_{mo})^2}\right), \\
[\kappa_o] &: \qquad -\frac{\mathbf{1}_{[i=o]}}{1+\kappa_o} - \frac{\alpha_o}{1-w^{\Pi}}\left(\sum_{i=1}^{N} \Upsilon_i \frac{(1-\tau_i)\,\tilde{l}_{io}^H}{(1+\kappa_o)^2} - \frac{1}{(1+\kappa_o)^2}\right).
\end{aligned}$$

The first term in each equation represents the impact through the relative price change and the second term represents the mitigation effect provided by the increased tax revenue from the carbon tax.

Impact on investment. Next, we calculate the derivative of firms' investment I_i. From equation (18), we have

$$\begin{aligned}
\log I_i &= -\log \mu_i + \log \frac{\delta \gamma^K}{r_K} + \log w_i^H + \log\left(W + r\sum_{i=1}^{N} \mu_i K_i + \Pi\right) \\
&= -\sum_{j=1}^{N} \gamma_{kij} \log \hat{P}_j - \log \frac{W}{P} + \log \frac{\delta\gamma^K}{r_K} + \log w_i^H + \log \frac{W + r\sum_{i=1}^{N} \mu_i K_i + \Pi}{P}. \quad (24)
\end{aligned}$$

Substituting (23) into equation (24), we have

$$\log I_i = -\sum_{j=1}^{N} \gamma_{kij} \log \hat{P}_j + \log \frac{\delta\gamma^K}{r_K} + \log w_i^H - \log\left(1 - w^{\Pi}\right) + \log\left(1 + \frac{r}{r_K}\frac{\gamma^K}{\gamma^L}\right).$$

The derivatives of $\log I_i$ with respect to carbon taxes are

$$\begin{aligned}
[\tau_m]:& \; -\sum_{j=1}^{N}\gamma_{kij}\frac{\partial \log \hat{P}_j}{\partial \tau_m} + \frac{1}{w_i^H}\frac{\partial w_i^H}{\partial \tau_m} + \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \tau_m},\\
[\zeta_{mo}]:& -\sum_{j=1}^{N}\gamma_{kij}\frac{\partial \log \hat{P}_j}{\partial \zeta_{mo}} + \frac{1}{w_i^H}\frac{\partial w_i^H}{\partial \zeta_{mo}} + \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \zeta_{mo}},\\
[\kappa_o]:& \qquad\qquad \frac{1}{w_i^H}\frac{\partial w_i^H}{\partial \kappa_o} + \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \kappa_o}.
\end{aligned}$$

From the results obtained so far, the derivatives of $\log I_i$ with respect to carbon taxes are

$$\begin{aligned}
[\tau_m]:& -\sum_{j=1}^{N}\frac{\gamma_{kij} l_{jm}}{1-\tau_m} - \frac{1}{w_i^H}\left(\frac{w_i^H}{1-\tau_i}\mathbf{1}_{[i=m]} + \sum_{k=1}^{N}\frac{\tilde{l}_{ij}^H \tilde{\gamma}_{km}^H w_m^H}{(1-\tau_m)^2}\right) + \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \tau_m},\\
[\zeta_{mo}]:& \qquad -\sum_{j=1}^{N}\gamma_{kij}\frac{l_{jm}\gamma_{mo}}{1+\zeta_{mo}} - \frac{1-\tau_i}{w_i^H}\frac{\tilde{l}_{io}^H \gamma_{mo} w_m^H}{(1+\zeta_{mo})^2} + \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \zeta_{mo}},\\
[\kappa_o]:& \qquad\qquad -\frac{1-\tau_i}{w_i^H}\frac{\tilde{l}_{io}^H \alpha_o}{(1+\kappa_o)^2} + \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \kappa_o}.
\end{aligned}$$

Impact on real GDP. From equation (7), we have the sensitivity of real GDP with respect to each carbon tax rate:

$$\begin{aligned}
\sum_{i=1}^{N}\frac{P_i C_i}{\mathrm{NGDP}}\frac{\partial \log C_i}{\partial \tau_m} + \sum_{i=1}^{N}\frac{\mu_i I_i}{\mathrm{NGDP}}\frac{\partial \log I_i}{\partial \tau_m} =& \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \tau_m} - \sum_{i=1}^{N}\frac{P_i C_i}{\mathrm{NGDP}}\frac{\partial \log \hat{P}_i}{\partial \tau_m}\\
&+ \sum_{i=1}^{N}\frac{\mu_i I_i}{\mathrm{NGDP}}\left(-\sum_{j=1}^{N}\gamma_{kij}\frac{\partial \log \hat{P}_j}{\partial \tau_m} + \frac{1}{w_i^H}\frac{\partial w_i^H}{\partial \tau_m}\right),
\end{aligned}$$

$$\begin{aligned}
\sum_{i=1}^{N}\frac{P_i C_i}{\mathrm{NGDP}}\frac{\partial \log C_i}{\partial \zeta_{mo}} + \sum_{i=1}^{N}\frac{\mu_i I_i}{\mathrm{NGDP}}\frac{\partial \log I_i}{\partial \zeta_{mo}} =& \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \zeta_{mo}} - \sum_{i=1}^{N}\frac{P_i C_i}{\mathrm{NGDP}}\frac{\partial \log \hat{P}_i}{\partial \zeta_{mo}}\\
&+ \sum_{i=1}^{N}\frac{\mu_i I_i}{\mathrm{NGDP}}\left(-\sum_{j=1}^{N}\gamma_{kij}\frac{\partial \log \hat{P}_j}{\partial \zeta_{mo}} + \frac{1}{w_i^H}\frac{\partial w_i^H}{\partial \zeta_{mo}}\right),
\end{aligned}$$

$$\sum_{i=1}^{N} \frac{P_i C_i}{\text{NGDP}} \frac{\partial \log C_i}{\partial \kappa_o} + \sum_{i=1}^{N} \frac{\mu_i I_i}{\text{NGDP}} \frac{\partial \log I_i}{\partial \kappa_o} = \frac{1}{1 - w^{\Pi}} \frac{\partial w^{\Pi}}{\partial \kappa_o} - \sum_{i=1}^{N} \frac{P_i C_i}{\text{NGDP}} \frac{\mathbf{1}_{[i=o]}}{1+\kappa_o} + \sum_{i=1}^{N} \frac{\mu_i I_i}{\text{NGDP}} \frac{1}{w_i^H} \frac{\partial w_i^H}{\partial \kappa_o}.$$

The first term in each equation represents the favorable impact on the economy through the carbon tax revenue. The government revenue distributed to households increases as carbon tax rates rise and this partially offsets the adverse impact of taxation. The second term in each equation is the adverse effect on consumption through distortion induced by carbon tax. The size of this effect depends on the importance of the link between sector i and the petroleum and coal sector through sector m. The third term in each equation is the impact on investment and is affected by the change in relative prices and sectoral weight of value added.

As described above, in this model, there are adverse macroeconomic consequences due to the introduction of carbon taxes. A caveat to this conclusion is that the model does not consider potential negative external economies arising from GHG emissions.[17] If we assume a negative external economy, namely the damage to the economy caused by rising temperatures argued in standard integrated assessment models (IAMs) (e.g., Nordhaus (1994)), the optimal rates of the carbon taxes could be higher. In addition, the optimal carbon tax rates could be lower if other distortionary taxes are present as Barrage (2020) argues.

The impact on GDP around the zero tax steady state is as follows:

$$[\tau_m]: \quad w_m^H - \sum_{i=1}^{N} \frac{P_i C_i}{\text{NGDP}} l_{im} - \sum_{i=1}^{N} \frac{\mu_i I_i}{\text{NGDP}} \sum_{j=1}^{N} \gamma_{kij} l_{jm} - \sum_{i=1}^{N} \frac{\mu_i I_i}{\text{NGDP}} \left(\mathbf{1}_{[i=m]} + \frac{w_m^H}{w_i^H} \sum_{k=1}^{N} \tilde{l}_{ik}^H \tilde{\gamma}_{km}^H \right),$$

$$[\zeta_{mo}]: \gamma_{mo} w_m^H - \sum_{i=1}^{N} \frac{P_i C_i}{\text{NGDP}} l_{im} \gamma_{mo} - \sum_{i=1}^{N} \frac{\mu_i I_i}{\text{NGDP}} \left(\sum_{j=1}^{N} \gamma_{kij} l_{jm} \gamma_{mo} + \frac{\tilde{l}_{io}^H \gamma_{mo} w_m^H}{w_i^H} \right),$$

$$[\kappa_o]: \quad \alpha_o - \frac{P_o C_o}{\text{NGDP}} - \sum_{i=1}^{N} \frac{\mu_i I_i}{\text{NGDP}} \frac{\tilde{l}_{io}^H \alpha_o}{w_i^H}.$$

Impact on GHG emissions. Lastly, we derive the sensitivity of GHG emissions with

[17]See Hashimoto and Sudo (2022) for an assessment of the impact of physical damage on Japan's economy.

respect to carbon taxes. The total amount of GHG emissions is the sum of emissions from intermediate fossil fuel consumption $CO_{2i} = \xi_{1,i} x_{io}$, production processes $GHG_i^f = \xi_{2,i} Y_i$, and final consumption $GHG^h = \xi_3 C_o$:

$$\mathrm{GHG} = \sum_{i=1}^{N} (\xi_{1,i} x_{io} + \xi_{2,i} Y_i) + \xi_3 C_o .$$

To calculate the derivative of GHG emissions, the derivatives of each of three economic activities are required.

First, we calculate the derivative of $\log Y_i$. Using the relationship

$$\log w_i^H = \log \frac{P_i (1 - \tau_i) Y_i}{P} - \log \frac{W + r \sum_{i=1}^{N} \mu_i K_i + \Pi}{P},$$

the log of real output of sector i is

$$\log Y_i = \log w_i^H - \log P_i (1 - \tau_i) + \log P + \log \frac{W + r \sum_{i=1}^{N} \mu_i K_i + \Pi}{P}. \tag{25}$$

Substituting (23) into equation (25), we have the equilibrium quantity of output at sector i as:

$$\log Y_i = \log w_i^H - \log \hat{P}_i (1 - \tau_i) - \log \left(1 - w^\Pi\right) + \log \left(1 + \frac{r}{r_K} \frac{\gamma^K}{\gamma^L}\right).$$

The derivatives of $\log Y_i$ with respect to carbon taxes are

$$\frac{\partial \log Y_i}{\partial \tau_m} = \frac{1}{w_i^H} \frac{\partial w_i^H}{\partial \tau_m} - \frac{\partial \log \hat{P}_i}{\partial \tau_m} + \frac{\mathbf{1}_{[i=m]}}{1 - \tau_m} + \frac{1}{1 - w^\Pi} \frac{\partial w^\Pi}{\partial \tau_m},$$

$$\frac{\partial \log Y_i}{\partial \zeta_{mo}} = \frac{1}{w_i^H} \frac{\partial w_i^H}{\partial \zeta_{mo}} - \frac{\partial \log \hat{P}_i}{\partial \zeta_{mo}} + \frac{1}{1 - w^\Pi} \frac{\partial w^\Pi}{\partial \zeta_{mo}},$$

$$\frac{\partial \log Y_i}{\partial \kappa_o} = \frac{1}{w_i^H} \frac{\partial w_i^H}{\partial \kappa_o} + \frac{1}{1 - w^\Pi} \frac{\partial w^\Pi}{\partial \kappa_o}.$$

Next, we explore the sensitivity of x_{io} to the rise in the intermediate input tax rate. Using the relationship

$$P_o (1 + \zeta_{io}) x_{io} = \gamma_{io} P_i (1 - \tau_i) Y_i,$$

the log of x_{io} is represented as

$$\log x_{io} = \log \gamma_{io} + \log \hat{P}_i + \log(1-\tau_i) + \log Y_i - \log \hat{P}_o (1+\zeta_{io}) .$$

The derivatives with respect to carbon taxes are

$$\begin{aligned}\frac{\partial \log x_{io}}{\partial \tau_m} &= \frac{\partial \log \hat{P}_i}{\partial \tau_m} - \frac{\partial \log \hat{P}_o}{\partial \tau_m} + \frac{\partial \log Y_i}{\partial \tau_m} - \frac{\mathbf{1}_{[i=m]}}{1-\tau_m} \\ &= \frac{1}{w_i^H}\frac{\partial w_i^H}{\partial \tau_m} + \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \tau_m} - \frac{\partial \log \hat{P}_o}{\partial \tau_m},\end{aligned}$$

$$\begin{aligned}\frac{\partial \log x_{io}}{\partial \zeta_{mo}} &= \frac{\partial \log \hat{P}_i}{\partial \zeta_{mo}} - \frac{\partial \log \hat{P}_o}{\partial \zeta_{mo}} + \frac{\partial \log Y_i}{\partial \zeta_{mo}} - \frac{\mathbf{1}_{[i=m]}}{1+\zeta_{mo}} \\ &= \frac{1}{w_i^H}\frac{\partial w_i^H}{\partial \zeta_{mo}} + \frac{1}{1-w^\Pi}\frac{\partial w^\Pi}{\partial \zeta_{mo}} - \frac{\partial \log \hat{P}_o}{\partial \zeta_{mo}} - \frac{\mathbf{1}_{[i=m]}}{1+\zeta_{mo}},\end{aligned}$$

$$\frac{\partial \log x_{io}}{\partial \kappa_o} = \frac{\partial \log Y_i}{\partial \kappa_o}.$$

Consequently, the sensitivity of total GHG emissions is as follows:

$$\frac{\partial \mathrm{GHG}}{\partial \tau_m} = \sum_{i=1}^{N}\left(\xi_{1,i} x_{io}\frac{\partial \log x_{io}}{\partial \tau_m} + \xi_{2,i} Y_i \frac{\partial \log Y_i}{\partial \tau_m}\right) + \xi_3 C_o \frac{\partial \log C_o}{\partial \tau_m},$$

$$\frac{\partial \mathrm{GHG}}{\partial \zeta_{mo}} = \sum_{i=1}^{N}\left(\xi_{1,i} x_{io}\frac{\partial \log x_{io}}{\partial \zeta_{mo}} + \xi_{2,i} Y_i \frac{\partial \log Y_i}{\partial \zeta_{mo}}\right) + \xi_3 C_o \frac{\partial \log C_o}{\partial \zeta_{mo}},$$

$$\frac{\partial \mathrm{GHG}}{\partial \kappa_o} = \sum_{i=1}^{N}\left(\xi_{1,i} x_{io}\frac{\partial \log x_{io}}{\partial \kappa_o} + \xi_{2,i} Y_i \frac{\partial \log Y_i}{\partial \kappa_o}\right) + \xi_3 C_o \frac{\partial \log C_o}{\partial \kappa_o}.$$

E. Simulated sectoral value added in 2050

Percent

	Industry	Hot house world	Disorderly	Orderly
1.	Agriculture, forestry and fishing	-0.12	-35.22	-29.36
2.	Mining (excluding fossil fuels)	-0.12	-32.84	-25.63
3.	Food products and beverages	-0.09	-20.53	-17.59
4.	Textile products	-0.08	-20.60	-17.41
5.	Pulp, paper and paper products	-0.09	-22.58	-18.51
6.	Chemicals	-0.09	-23.35	-19.34
7.	Petroleum and coal products	-0.49	-69.03	-61.65
8.	Non-metallic mineral products	-0.14	-40.46	-33.30
9.	Basic metal	-0.10	-25.09	-18.57
10.	Fabricated metal products	-0.09	-22.98	-18.33
11.	General-purpose, production and business-oriented machinery	-0.11	-27.47	-17.16
12.	Electronic components and devices	-0.10	-24.75	-18.83
13.	Electrical machinery, equipment and supplies	-0.10	-24.40	-17.27
14.	Information and communication electronics equipment	-0.09	-21.85	-17.66
15.	Transport equipment	-0.09	-21.53	-17.68
16.	Printing	-0.09	-21.54	-17.60
17.	Others	-0.09	-22.53	-18.41
18.	Electricity supply	-0.07	-21.79	-18.87
19.	Gas and water supply, and waste management service	-0.10	-27.19	-22.60
20.	Construction	-0.09	-22.56	-18.70
21.	Wholesale trade	-0.09	-22.41	-18.18
22.	Retail trade	-0.09	-20.97	-17.81
23.	Transport and postal services	-0.09	-22.00	-18.37
24.	Accommodation and food service activities	-0.08	-20.27	-17.40
25.	Communications and broadcasting	-0.09	-20.69	-17.27
26.	Information services, and Image, sound and character information production and distribution	-0.09	-22.02	-17.59
27.	Finance and insurance	-0.09	-20.42	-17.35
28.	Real estate	-0.08	-19.95	-17.05
29.	Professional, scientific and technical activities	-0.09	-22.45	-17.60
30.	Education	-0.08	-20.26	-17.29
31.	Human health and social work activities	-0.08	-19.93	-17.08
32.	Other service activities	-0.09	-20.63	-17.47

Table A.3: Changes in nominal sectoral value added in 2050 from 2019