# Operational Risk Metrization and Scenario Analysis

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A Workshop on Scenario Analysis
July 18, 2006

A part of this lecture has been derived from "A Theoretical Study and Empirical Analysis of the Operational Risk Metrization Approach," a joint research project with Japan Trustee Services Bank, Ltd. However, it does not mean that the technique described in this paper is Japan Trustee Services Bank's risk metrization method. The views and details given in this paper all belong to the author himself and do not represent the official views of the organization to which the author belongs nor those of Japan Trustee Services Bank.

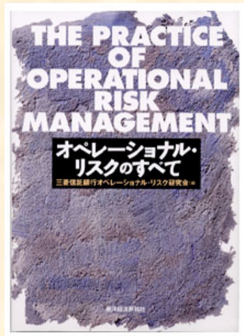
# OUTLINE

- An Outline of the Advanced Measurement Method
- Practical Problems
  - A Review of the Extreme Value Theory-Based Approach
- Is Scenario Analysis Effective？
  - Scenario Building
- An Example of the Scenario Utilization Method Using Extreme Value Theory
  - Application of the POT Approach and the Probability-Weighted Moments Method

# Relationship Between Nakagawa and Operational Risks

- While working at the MTB Investment Technology Institute (presently the Mitsubishi UFJ Trust Investment Technology Institute Co., Ltd.), I proposed analyses of accident data and an operational risk measurement model related to the parent company's operational risks.
  - I also proposed and verified a model equipped with the OperationalRiskBrowser™ developed by Numerical Technologies Incorporated.

- I was responsible for writing Chapter 5 of and an appendix to "The Practice of Operational Risk Management," compiled by the Operational Risk Research Institute, Mitsubishi Trust and Banking Corporation.
  - I have delivered lectures and have written papers concerning the contents of the above book.



# An Outline of the Advanced Measurement Approach (1)

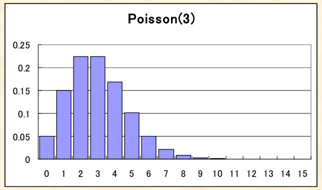
- Advanced Measurement Approach (AMA)
  - This approach is also referred to as the Loss Distribution Approach (LDA).
  - It is here defined as a method of measuring the volume of operational risks, which are to be accounted for in accordance with Basel II, with the 99.9% Value at Risk (VaR) by identifying the distribution of accumulated losses during a certain period of time, using internal accident data (or including external data and scenarios as the case may be).

# An Outline of the Advanced Measurement Approach (2)

- A Rough Flow Up To 99.9 VaR Measurement
  - Accident Frequency Rate (the Number of Accidents during a Given Period):
    - For example, the number of accidents for six months or for one year is assumed to follow the Poisson distribution.
  - Amount of Loss (per Accident) during an Accident:
    - For example, parametric distributions such as the logarithmic normal distribution, the Weibull distribution and the Gamma distribution are applied.
    - An alternative approach is based on extreme value theory (EVT) to express a distribution of excess losses in excess of a certain threshold using the Generalized Pareto Distribution (GPD).
    - An empirical distribution (a non-parametric distribution) is sometimes assumed.
  - By combining the above two factors, a distribution of cumulative losses for a certain period of time is assumed and the 99.9% point of this distribution is obtained as the operational risk VaR.
  - Methods to obtain the accumulated loss distribution include:
    - A method of generating numerous random number scenarios for accumulated losses through the Monte Carlo simulation technique and obtaining a histogram of cumulative losses
    - A method of obtaining a stochastic distribution of cumulative losses with a mathematical technique (approximately) using Panjer's recursion formula and the characteristic function

# An Outline of the Advanced Measurement Approach (3)

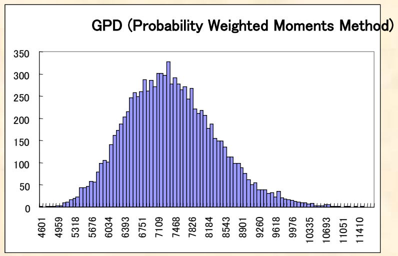
- Frequency Rate Distribution:
  – Poisson Distribution



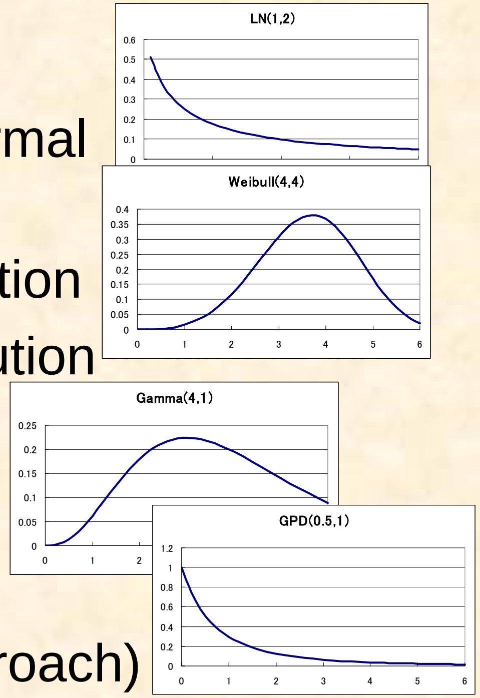

Cumulative Loss Model

Composite Poisson Model

Monte Carlo Simulation




- Loss Distributions:
  – Logarithmic normal distribution
  – Weibull distribution
  – Gamma distribution
  – GPD




  – GPD（POT approach) applies to a high loss portion.
  – A non-parametric method

# An Outline of the Advanced Measurement Approach (4)

- Method of Measurements of Operational Risks with the Advanced Measurement Approach

*In a preliminary step, it is necessary to determine whether the method applies to each business line/event type or measurements are made throughout the bank.

① Determination of a Model:

- Determination of the Accident Frequency Rate and a Distribution of Losses
- Determination of a Method to Calculate a Distribution of Cumulative Losses
- If the POT approach is used, a threshold to distinguish between low losses and high losses should be determined.

② Estimation of Parameters Contained in a Model:

- Average (= Dispersion) Parameter for the Poisson Distribution
- Parameter for a Distribution of Losses

# Practical Problems（1）

- Questions Concerning Determination of a Model:
  - Is it proper to consider that the average accident frequency rate for each period of time is constant? (≒ Is the use of distributions other than the Poisson distribution not possible?)
  - What distribution is a proper distribution of losses?
  - Is it right to consider that the frequency rate, the amount of losses or the continuous amount of losses is independent?
  - When risks are calculated for each cell, what is the dependent relationship between cells?
  - When the POT approach is used, how is a threshold established?

# Practical Problems (2)

- Questions Concerning Estimation of Parameters Contained in a Model:
  - Should the number of accidents be considered on an actual loss basis or should we include deemed accidents?
  - How should we consider the cases where the figures for losses are remaining at an unchanging level?
  - Given that few data are available in the first place, is the accuracy of estimation with the maximum likelihood method assured?
  - If other parameter estimation methods are used, to what extent would these methods allow an estimated value to be obtained?
  - Can we apply the POT approach in a situation when we do not have ample data on large amounts of losses?

# Practical Problems (3)

- As possible solutions to some of the questions:
  - Is the use of scenario analysis effective?
  - Or when we consider combining the advanced method with scenario analysis, can we determine a proper model and an estimation technique as a matter of course?
- From my own personal viewpoint,
  - Scenario analysis and the measurement method using internal loss data should essentially be considered separately and should preferably be treated as complementary to each other.
  - Combination with scenario analysis may not make it easy to apply a parametric method.
  - However, a method of combining scenarios (including scaled scenario of external cases) with internal data and calculating the operational risk 99.9% VaR with a parametric technique may not be totally impossible.

An example of an EVT-based approach to calculate the VaR of operational risks using external data and high-loss data including scenarios only will be explained later.

# Scenario Building (1)

Three Pillars of Basel II:

- Minimum capital requirements - Refinement of risk measurements
- Supervisory review - Banks' own risk identification, assessment, surveillance and management systems
- Market discipline - Reinforcement of information disclosure

- The same holds true for market and credit risks and particularly as for operational risks, we cannot benefit much from solely looking into risk quantification techniques.
- It is important that the bank should formulate a risk management framework and adopt a positive stance toward information disclosure.
- Quotations from the Conclusion by Embrechts-Furrer-Kaufmann(2003):

> Keeping in mind that most serious operational risk losses can not be judged as mere accidents, it becomes obvious that the only way to gain control over operational risk is to improve the quality of control over the possible sources of huge operational losses.
>
> It is exactly here that Pillar 2, and to a lesser extent Pillar 3, becomes extremely important.

# Scenario Building (2)

Purposes of Scenario Building:

- (Directly) to supplement scarce data (particularly on high losses):
  - Are scenarios consistent and affinitive with internal data?
  - Is it sufficient to think in terms of scenario data only without regard to internal data?
- (Indirectly) to identify the sources of risks and their effects in the bank's operational processes:
  - It is this purpose that is essentially important.
  - To build a scenario going beyond the mere objective of risk measurements

Steps of Scenario Building:

1. Collection of data from a bottom up perspective:
   - Data on loss accidents
   - ◎ Statistical data on workloads in each business unit
2. Collection of external cases and consistency checks and scaling
3. Scenario building

# Scenario Building (3)

- Collection of Data from a Bottom Up Perspective:
  - Collecting and organizing internal data is essential for scenario building.
  - Building a proper scenario is impossible without an understanding of the internal situation.
  - Although data on loss accidents should naturally be gathered, what is rather needed to build a scenario is statistical data on the gross workload in each business unit.
  - For example, data on the following components of operations in each business unit should be collected:
    - Items of work for the operation
    - Working hours and years of experience for employees engaged in the operation
    - Number of work processes and working hours per unit of item of work
  - Close relations between operations, if any, need attention.
  - Then,
    - Past cases of accidents must be associated with the above elements; and
    - Occurrence dates of accidents, approximate amounts of direct damage, and long-term indirect effects must be determined.

  These data should be recorded in a loss database.

# Scenario Building (4)

- Collection of External Cases and Consistency Checkups and Scaling:
  - Not only the cases of financial institutions but external cases involving huge losses should also preferably be used as data to build a scenario, including background and results.
    - Human errors may largely depend on the characteristics of financial operations but can also be considered in terms of general factors, such as fatigue and overconfidence.
    - Likewise systems errors may be attributable to the special characteristics of systems used in the financial industry but stability may be considered to show a certain tendency like the so-called bathtub curve.
  - Consistency checkups are conducted in order to examine the extent to which the background and the causes of the external cases collected will occur at the bank.
  - Scaling is carried out when adjustments in the amount of losses in the external cases are desirable after consideration of the bank’s workloads and the amount of its transactions. The conservative position may allow using the large amount of losses as it is.

# Scenario Building (5)

- Scenario Building
  - Even if risk management is to build a scenario, the accounting staff should be held responsible for the building of a scenario.
  - Scaled external loss data are to be compared with the frequency rate.
    - It is necessary to estimate the frequency rate of at least "once in so-and-so years."
    - Data such as systems errors may be associated with the time elapsed since the startup of systems.
    - Events that may occur simultaneously or in succession can be also taken into consideration.
  - For quantitative data, at least the following four items need attention:

    ･Business line ･ Event type ･ Amount of losses･ Frequency rate
  - If possible,
    - Dependent relations, etc.

# Scenario Building (6)

- Scenario Building (Specific Examples and Interpretation)
  - For example, a scenario assuming that “the presumed amount of loss comes to ¥1 billion and this loss is likely to occur once in every twenty years” is established in a cell (a combination of a business line and event type):
    - This scenario is quite different from the assumption that “the presumed amount of loss is ¥50 million and this loss is likely to occur once a year or so.”
    - Supposing that the average number of accidents per year in this cell is 50 cases, it is interpreted that approximately 1,000 accidents are expected to occur in twenty years’ time and one of the accidents (the worst case) causes ¥1 billion in loss.
      - A natural interpretation is that the probability of the amount of loss from the accident exceeding ¥1 billion is 0.1% or so.
      - Conversely speaking, does it mean that the 99.9% VaR for the cell is literally set at ¥1 billion?
    - An ideal approach should allow giving the anticipated frequency of “once in every so-and-so cases processed” or “once in every so-and-so working hours” from the bottom-up analysis, rather than “once in so-and-so years” .
      - A possible approach is to analyze the accident occurrence pattern from the perspective of the study of failure.
      - Is overwork or inattention the cause of error?
      - Is a newly installed system unstable?
      - What about a planned change in the business style?

# Scenario Building (7)

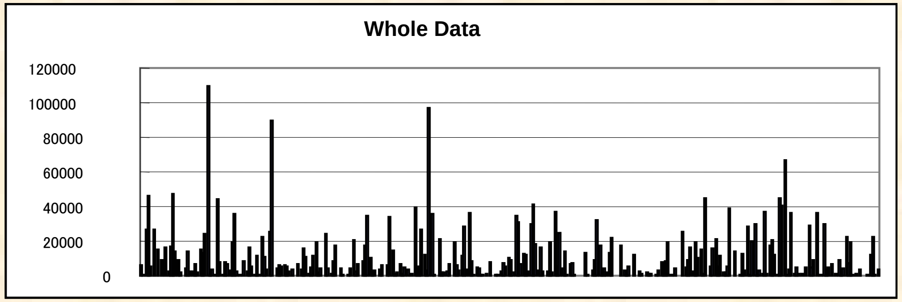
- Scenario Building (How to Use Data)
  - How to use scenario data:
    - A risk audit system that allows you to build a good scenario may well be an important asset for risk management.
    - An extreme case should be considered through stress testing by including correlativity.
    - A distribution of losses per case is estimated in combination with internal data.
      - It is doubtful whether the two factors could be used on an equal footing.
    - A proposed method is (although it is not statistically recognized):
      - First of all, internal data are arranged in ascending order of the amount of loss and a probability based on the corresponding cumulative experience distribution is given to each loss data.
      - Since scenario data are based on the assumption that the anticipated probability corresponding to the amount of loss is given, the pair of the amount of loss and the anticipated probability are fused with internal data.
      - Based on the fused data set, a distribution of losses is estimated. However, attention should be given to the possibility of difference between the scenario data and the estimates.

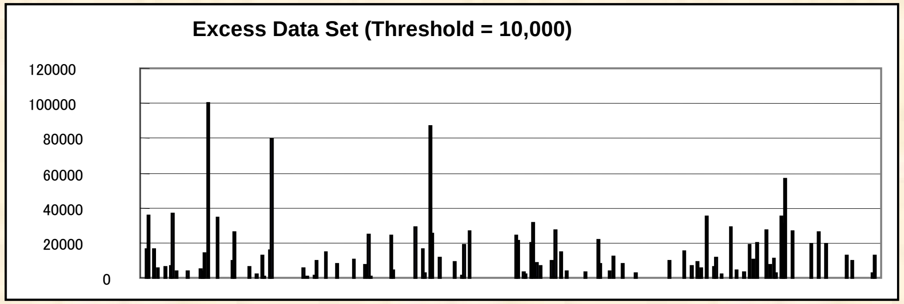
# An Example of the Scenario Utilization Method Using Extreme Value Theory (1)

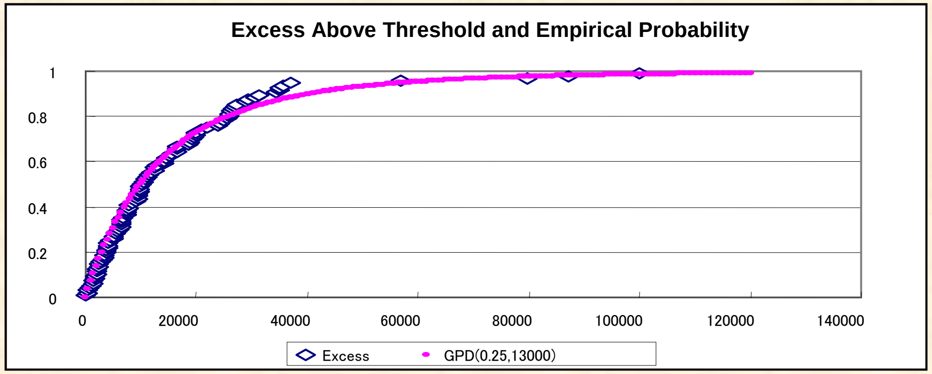
- When VaR is regarded as a measure of risks, a major influence may come not from an accident involving low losses but from a “huge” loss accident, if it occurs infrequently.
- In this case, it may be reasonable to focus critically on the lower right part of the distribution, rather than to build a refined model for the whole distribution of losses.

- Application of Extreme Value Theory (EVT)
- This is a common theme in papers discussing the metrization of operational risks from the perspective of a statistical approach.
- A model that applies a method called the Peak-Over-Threshold (POT) approach is described here.

# An Example of the Scenario Utilization Method Using Extreme Value Theory (2)




A Schematic Description of the POT Approach

Only data in excess of the threshold set at 10,000 relative to a certain data set are handled.

An excess above the threshold of 10,000 is shown.

An empirical distribution of excesses is prepared by arranging excesses in ascending order and assigning appropriate probability to these excesses (◇ in the figure).

This empirical distribution of excesses is approximated by the Generalized Pareto Distribution (GPD).

## An Example of the Scenario Utilization Method Using Extreme Value Theory (3)

- Distribution function in the Generalized Pareto Distribution:

$$G(x;\xi,\beta)=\begin{cases}1-\left(1+\dfrac{\xi}{\beta}x\right)^{-\frac{1}{\xi}} & (\xi\neq 0)\\[2ex] 1-\exp\left(-\dfrac{x}{\beta}\right) & (\xi=0)\end{cases}\qquad \begin{cases}x\geq 0 & (\xi\geq 0)\\ 0\leq x\leq -\dfrac{1}{\xi} & (\xi<0)\end{cases},\ \beta>0$$

  - As $\xi$ increases, the base becomes thicker (even the average does not exist when $\xi$ >1).

- Excess distribution function (relative to *u):*

$X$ : As a probability variable representing the amount of loss from the accident :

$$P(X-u\leq x\mid X>u)=\frac{P(X\leq x+u)-P(X\leq u)}{P(X>u)}$$

## An Example of the Scenario Utilization Method Using Extreme Value Theory (4)

- Theoretically no matter what original loss distribution may be, the excess distribution function relative to a sufficiently high threshold *u* can be approximated by GPD (as long as certain conditions are satisfied).

$$P(X - u \leq x \mid X > u) \approx G(x; \xi, \beta) \qquad \text{for } x > u$$

  - However, a threshold must be determined carefully because the theory does not specify the degree of u.
  - For the sake of accuracy, it is desirable to verify the validity of the empirical distribution of high loss data and GPD by carrying out goodness-of-fit tests such as Kolmogorov-Smirnov and Anderson-Darling tests.

# An Example of the Scenario Utilization Method Using Extreme Value Theory (5)

- What to do with a method of estimating parameters for GPD ･･･
  - Maximum Likelihood Method:
    - If numerous samples are available, an estimate will have a theoretically desirable nature but what if small samples and identical numbers are obtained in clusters?
    - When estimating parameters using a scenario, it is difficult to simply apply this method if scenario data are treated similarly to internal data.
  - Moments Method:
    - This method is often used to estimate parameters when few data are available.
    - This method cannot apply to some distributions in which moments of higher degree do not exist (e.g., when $\xi$ >1 in GPD).
    - When estimating parameters using a scenario, it is difficult to simply apply this method if scenario data are treated similarly to internal data.
  - Least Square Method (calibration rather than estimation):
    - Given an equal weight, entire goodness of fit increases but there may rather be a larger discrepancy to occur at the low-frequency and high-loss portion that is important in risk management.
    - If a scenario is used, this method distinguishes it from internal data by the size of “an empirical cumulative probability.”

## An Example of the Scenario Utilization Method Using Extreme Value Theory (6)

- Probability-Weighted Moments Method
  - This is a method of obtaining a parameter as a solution to an equation, as with the Moments Method, by determining an expected value that is weighted by the exponentiation of the probability of a value exceeding a value calculated from the presumed probability distribution (this probability should be very low) and comparing it with a value derived similarly from actual data.
  - This method is applicable when moments of higher degree do not exist. It is reportedly empirically effective when small samples are given.
    - However, it is difficult to apply this method unless the following expected value can actually be calculated:

$$E[Z(1-F(Z;\theta))^r] \ (\mathrm{Z:\ Probability\ variable,\ F:\ Distribution\ function\ that\ Z\ follows})$$

  - The way “an empirical cumulative probability” is determined has an influence on the results (arbitrariness is involved):
    - It is related to an assessment of the frequency rate of external data.
    - The results of estimation with a scenario incorporated may likely be different from the original scenario.

# Conclusion (1)

- An optimum combination of a distribution and an estimation method cannot be presented in general terms:
  - Supporting with a statistical theory is difficult.
  - There are no absolute criteria.
    → Is it good to remain at the same level as it was or should it be overestimated?

- Difficult estimation of a distribution of losses using external cases and scenarios:
  - Can scaling and an assessment of the incidence rate be implemented properly?
  - The application of a huge loss case may have a very large impact on the estimation process and result in leaving a model out of consideration.

# Conclusion (2)

- There may probably be no complete model.
  - Continuous discussion about an operational risk metric model will lead to gradual improvements in methods.
  - The business process should be scrutinized and, at the same time, the forecasting of errors based on the study of failure, psychology and systems theory must be adopted.
  - In the final analysis, it is necessary to fully analyze the data on accidents both qualitatively and quantitatively. There must be a system under which wisdom is shared to address the issue not as a problem for individual financial institutions but as a problem for the financial industry as a whole.

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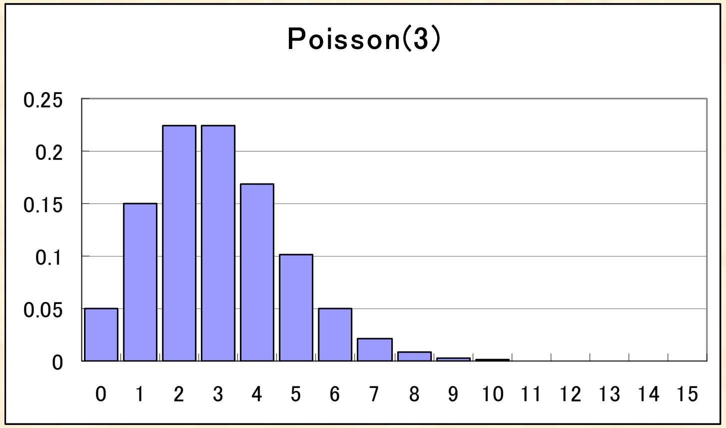
# Reference: Poisson Distribution

- Probability of an event taking place k times:

$$P(k) = e^{-\lambda} \frac{\lambda^{k}}{k!}, \qquad \lambda > 0$$

- The Poisson distribution is used as a demonstrative frequency rate model when the probability of an event is low and the event occurs due to an independent factor or factors.
- The average and dispersion parameters both are expressed by the intensity parameter $\lambda$ .
- Possible extended applications are to make the intensity parameter time-dependent such as $\lambda$ (t) and to use the estimates of external data.
  - Seasonality, the startup of a new business, and the expansion of business operations are taken into consideration.

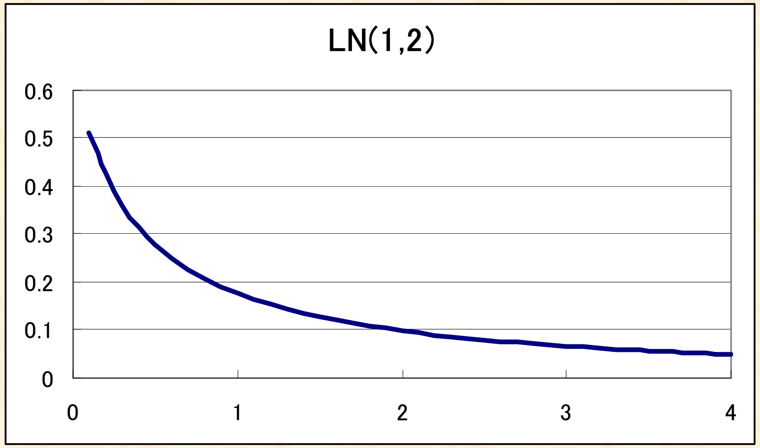


# Reference: Logarithmic Normal Distribution

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma x}\exp\left\{-\frac{(\log x-\mu)^2}{2\sigma^2}\right\}$$

$$F(x;\alpha,\beta) = N\left(\frac{\log x-\mu}{\sigma}\right) \qquad x>0,\quad \sigma>0$$

- This distribution is used, for example, to represent a distribution of future stock prices in financial engineering.
- It is easy to handle for a model that has a distribution with a thick tail.

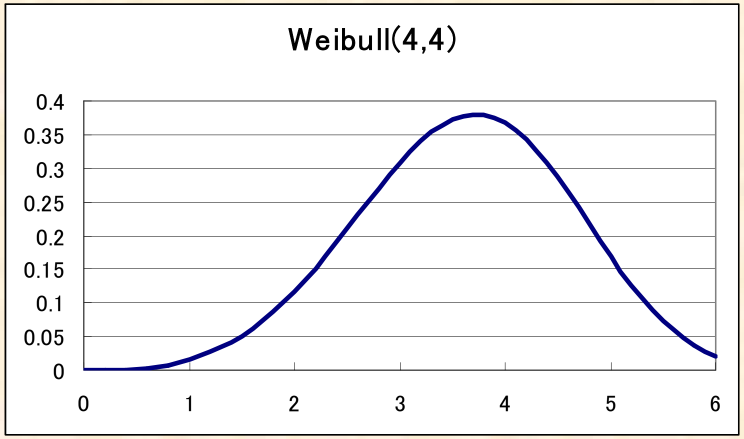


# Reference: Weibull Distribution

$$f(x;\alpha,\beta) = \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}$$

$$F(x;\alpha,\beta) = 1 - \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\} \qquad x \geq 0,\ \alpha > 0,\ \beta > 0$$

- It is used for survival analysis, for example.
- The density function shows a tendency that the greater $\alpha$ is, the steeper the slope becomes, and the smaller $\beta$ is, the greater the degree of change becomes.

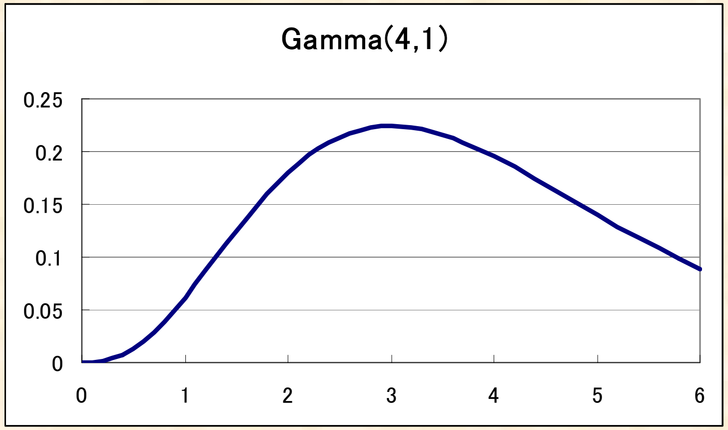


# Reference: Gamma Distribution

$$f(x;\alpha,\beta)=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}\exp\left(-\frac{x}{\beta}\right) \qquad x\geq 0,\ \alpha>0,\ \beta>0$$

$$\Gamma(x)=\int_0^{\infty}e^{-u}u^{x-1}du$$

- It is used for survival analysis, for example.
- The density function shows a tendency that the greater $\alpha$ is, the gentler the slope becomes and the thicker the tail becomes.



# Reference: POT Approach (1)

The distribution function for the whole is given as F(x). (Although F(x) must meet some conditions, a well-known function poses almost no problem.) A threshold is given as u and the excess distribution function for H relative to u is considered:

$$F_u(x) = \frac{F(x+u) - F(u)}{1 - F(u)}$$

Then, when $F_u(x)$ is $u \to x_F$ : a theoretical end,

it is known that a certain $\xi, \beta$ can be uniformly approximated by GPD:

$$G(x;\xi,\beta) = 1 - \left(1 + \frac{\xi}{\beta} x\right)^{-\frac{1}{\xi}} \quad (u < x < x_F)$$

# Reference: POT Approach (2)

Here, for x > u, the distribution function can be expressed by the approximate expression :

$$F(x) \approx (1 - F(u))G(x - u; \xi, \beta) + F(u)$$

When the number of data for the whole is given as N and the number of data exceeding the threshold u is set as n(u), then F(u) can be approximated by $1 - \dfrac{\mathrm{n(u)}}{\mathrm{N}}$

Using this, the function is approximated as :

$$F(x) \approx 1 - \frac{n(u)}{N}\left(1 + \xi \frac{x - u}{\beta}\right)^{-\frac{1}{\xi}} \qquad (x > u)$$

# Reference: POT Approach (3)

◎ An Estimation Approach Using the Probability-Weighted Moments (PWM) Method Relative to GPD

$F(x;\xi,\beta)$ is given as the distribution function for GPD.

$Z$ is the probability variable that follows this distribution function. Thus it should be noted that for the natural number r(the number satisfying $r+1-\xi>0$) the following equation holds true:

$$E[Z(1-F(Z;\xi,\beta))^r]=\frac{\beta}{(r+1-\xi)(r+1)}$$

On the other hand, the data $x_1,\cdots,x_N$ (arranged in ascending order) is assumed to follow GPD. Where,

$$\hat{w}_r=\frac{1}{N}\sum_{n=1}^{N}x_n(1-F(x_n;\xi,\beta))^r$$

# Reference: POT Approach (4)

Here when the uniform random number sequence $\mathrm{U}_1 \cdots, \mathrm{U}_\mathrm{N}$ (arranged in ascending order) on $(0,1)$ is taken, it is known that the following nature holds true (Quantile transformation lemma):

$$(F(x_1;\xi,\beta),\cdots,F(x_N;\xi,\beta)) \overset{(d)}{=} (U_1\cdots,U_N)$$

Therefore, it can be considered as:

$$\hat{w}_r = \frac{1}{N}\sum_{n=1}^{N} x_n(1-U_n)^r$$

Actually, instead of the uniform random number sequence, the plotting points:

$$p_n = \frac{N-n+0.5}{N} \text{ or generally } p_n = \frac{N-n+\delta_n}{N+\gamma_n}$$

are given as appropriate values.

# Reference: POT Approach (5)

Finally,

$$\hat{w}_r = \frac{1}{N}\sum_{n=1}^{N} x_n (1 - p_n)^r$$

is regarded as "the probability weighted moment of the sample", a theoretical value and an equation are generated, and a solution to this equation is given as an estimate.

In the present analysis, $\xi < 1$ is assumed (namely the average exists), and with $r = 0,1$, an equation is created. An estimate can be obtained as :

$$\xi = 2 - \frac{\hat{w}_0}{\hat{w}_0 - 2\hat{w}_1}, \quad \beta = 2 - \frac{2\hat{w}_0\hat{w}_1}{\hat{w}_0 - 2\hat{w}_1}$$