Possibilities and Challenges in Economic Capital Management

Risk expressions including VaR and their utilization in management

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Self-Introduction

Mizuho-DL Financial Technology Co.,Ltd.

【Profile】

Subsidiary of Mizuho Financial Group specialize in financial technology

- Address：Ote Center Bldg. 12F Otemachi,Chiyoda-ku,Tokyo
- Capital ：200 million yen
  （Shareholders：Mizuho Corporate Bank, Dai-ichi Mutual Life, Sompo Japan）
- Scope of business: Financial technology development for shareholding group members,as well as general technological consultation for corporate clients

  New financial products/schemes

  Risk management (market, credit, commodity, catastrophe, weather and their consolidation)

  Investment/allocation (portfolio selection, performance measurement)

Contents

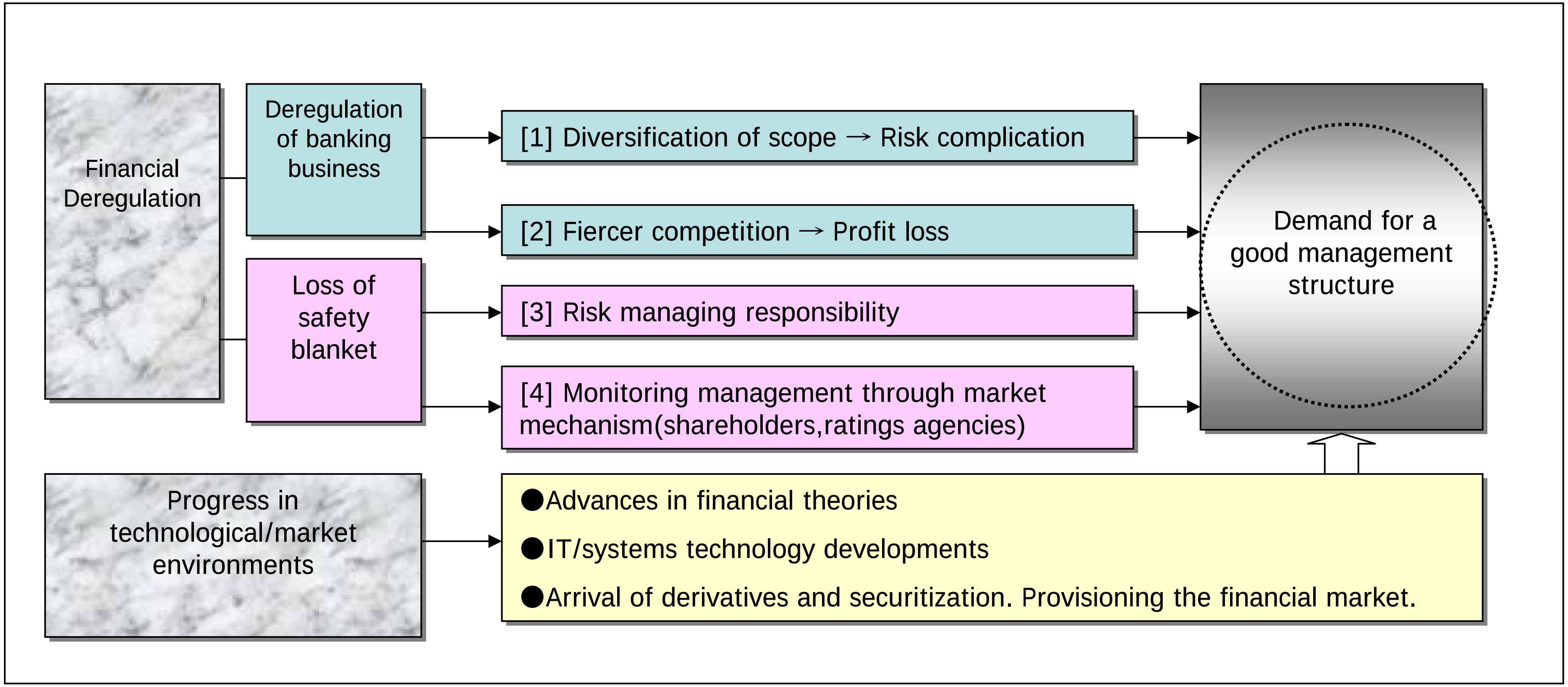
１． Historic setting of risk management

１－１． Where we stand

【points】〇Impact of Financial deregulation [1] diversification of banking operations, [2] intensified competition among and beyond financial sectors.

〇Deregulation paraphrase to loss of governmental protection. Thus, demands for [3] good risk management structures and [4] decent external monitoring.

〇Technological advances and market environments are ready for sorting out a risk management structure.



１－２. Worldwide Re-regulation trend in Mgt. : Outline of New Basel II

BIS I previously

$$\frac{\text{Capital}}{\text{Credit risk} + \text{Market Risk}} \geq 8\%$$

Credit Risk ＝ Risk Asset Amount
Market Risk ＝ Market Risk Volume×12.5

Basel II, essential

１）Minimum capital ratio of 8% is no change. Definition of capital(numerator) remain is the same.

２）Accuracy in credit risk (denominator)

・Scale, dependent risk risk weight

・Accommodate reduction effect

３）Operational risk

４）In average, total required capital stay the same.

５）Emphasize on internal assessment(Pillar2) and market discipline(pillar3）.

Basel II

1st Pillar：Minimum Capital Requirements

$$\frac{\text{Capital（basically same as at present、bud debt allowance adjustment）}}{\text{Credit Risk＋Market Risk（same as at present）＋Operational Risk}} \geq 8\%$$

Credit Risk: Select from among standardized approach, foundation IRB and advanced IRB.

Operational Risk: Loss caused from disruption, system failures, and so on.

2nd Pillar：Supervisory Review Process

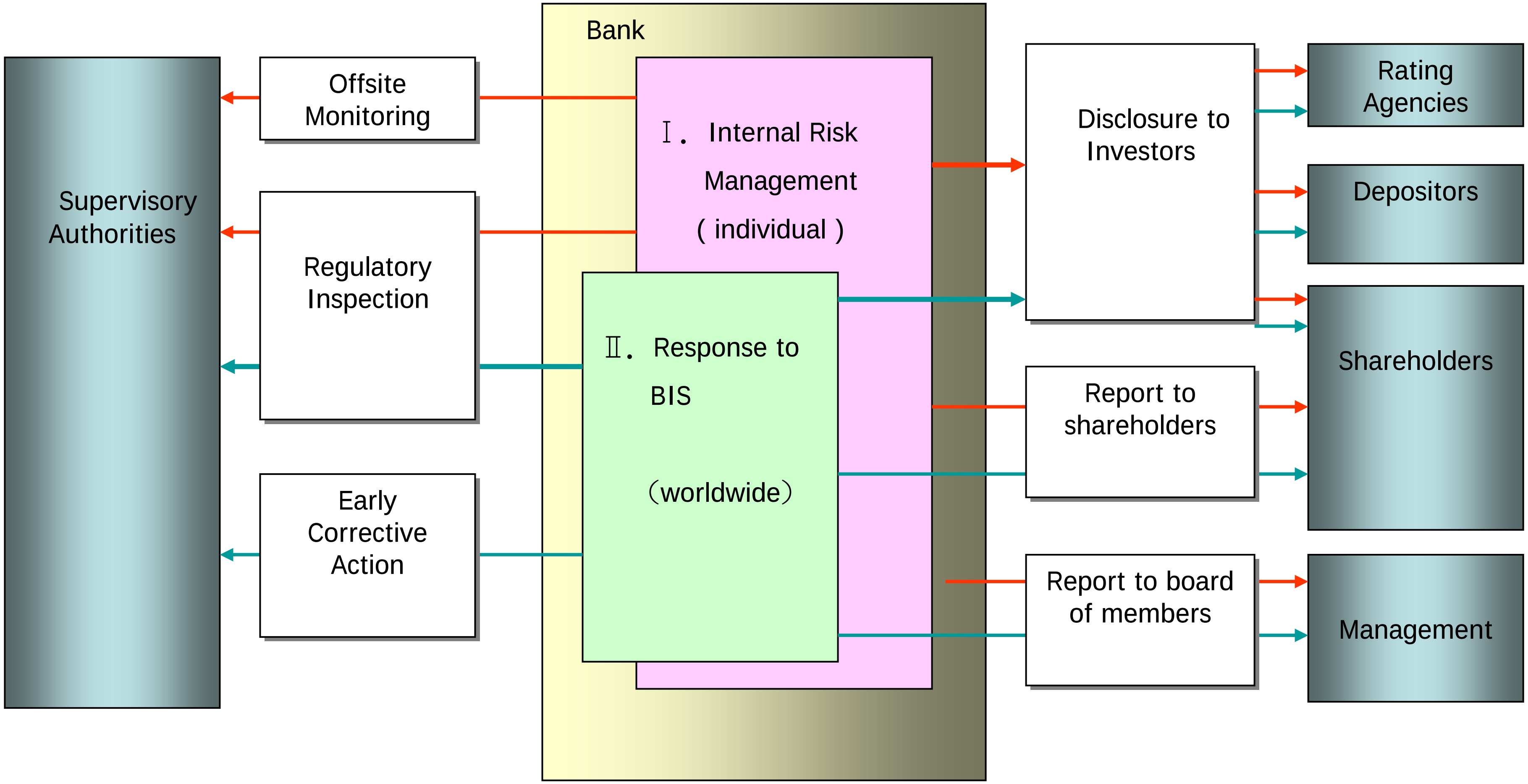
■Supervisor evaluate that individual banks determine higher levels than the minimum r requirement.

■Address specific issues not in Pillar 1, interest rate risk in the banking book, credit concentration risk, Residual risk and stress tests.

3rd Pillar:Market discipline

Leverage the ability of market discipline by enhancing the degree of transparency in bank's public reporting.

１－３．Double perspective Risk Management （=Ⅰ. Internal Risk Management, and Ⅱ. Response to BIS ）



２．Measuring Risk

２－１．Chronicle of risk measurement

１）Investment world before H. Markowitz, risk is something deal within one's "Guts", not something considered to be measured.　(Against The Gods :P. Bernstein)

２）Risk was a major player in two ground breaking theories of financial engineering.

◎Portfolio theory（H. Markowitz : 1952）

・Defined risk as the deviation of return on investment

・ Mathematically, analyze risk reduction effect of diversification

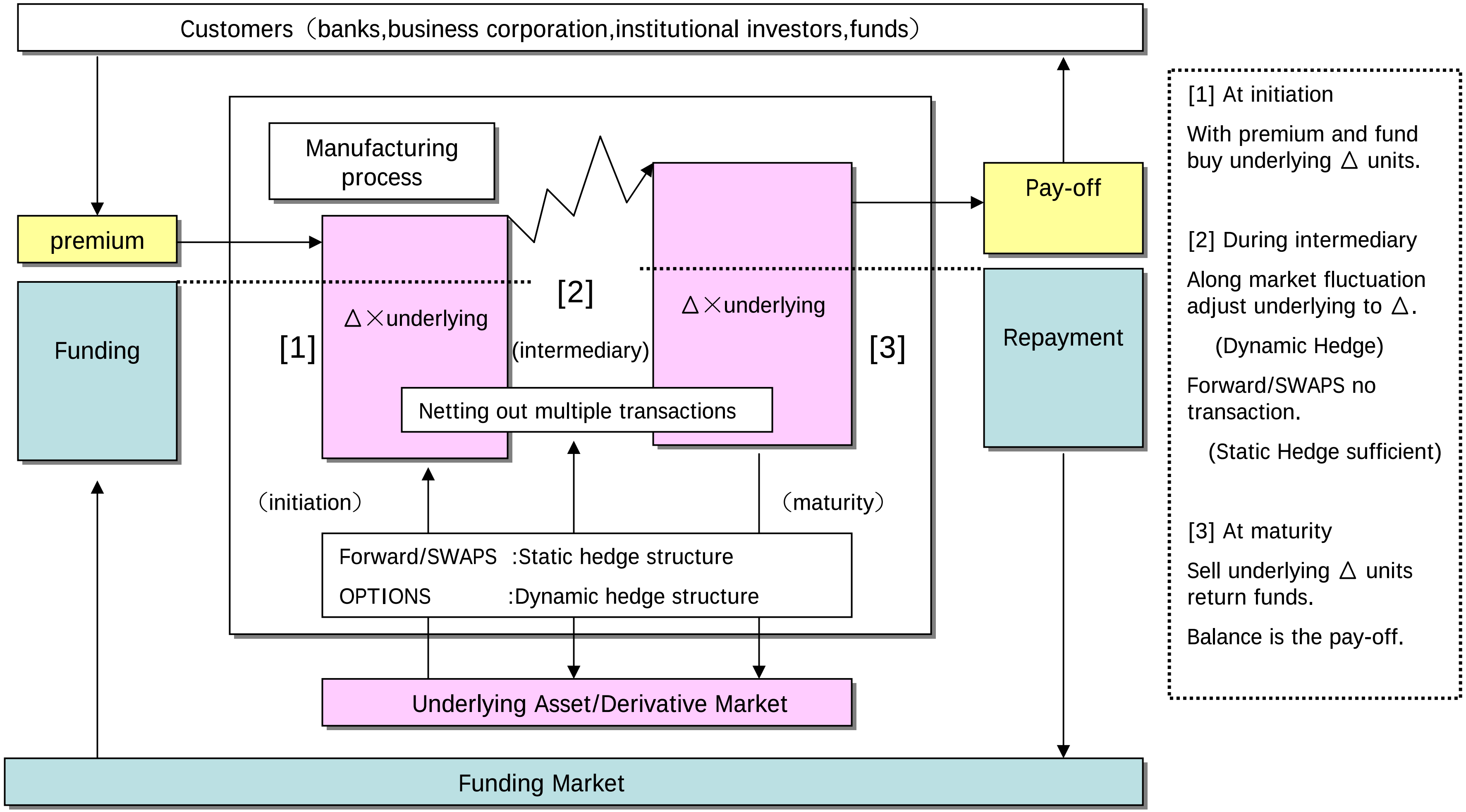
◎Option pricing theory（F. Black & M. Scholes :1973）

・Demonstrate option replication using dynamic hedging method with underlying and risk free assets.

・An option risk is expressed in terms of sensitivity(delta) against underlying movement.

（Note）Delta sensitivity indicator specifies derivative risk

【Cases】■Forward /swap is static hedge, constant delta.
■Option is dynamic hedge, variable delta.



３）The mid-1990s、J. P. Morgan and other banks pushed VaR as a market risk measurement standard.

Basel Committee adopted for trading books.

Henceforth,VaR was accepted as de facto standard for risk measurement.

４）Maximum loss under a given stress scenario can be a measurement of the risk in cases :

◎Of risk regulations on US mortgage banks

◎Of interest rate risk on banking books. Recently BIS II regulation Pillar2.

５）Subsequently, the following areas are active in discussion, research and practical developments in risk measurement after or about year 2000:

[1] Mathematical foundation building on risk measurement

[2] Reduction of a coherent risk measurement to practice

[3] Relationship between risk measure and financial economics

[4] Wide application of risk measurement to financial practices

３． Mathematical foundation of risk measures

３－１． Definitions

■Let Ω be a set of states of nature, is finite.

■X is a random variable that indicates the final net worth $X(\omega)$ of position of each element ω of Ω.

■A risk measure is a real number $\rho(X)$.

Indicates degrees of loss due to the transaction.

■Let $0 \leq p(\omega) \leq 1$ be the probability of the event ω takes place.

$$\sum_{\omega \in \Omega} p(\omega) = 1$$

◎A non-empty set Q of probability measures, such as ρ on the space Ω, is considered as a set of generalize scenarios.

３－２．**Coherent risk measure**

■Risk measure $\rho(X)$ is <u>coherent</u>

when following conditions are met.

１）Subadditivity $\rho(X+Y) \le \rho(X) + \rho(Y) \quad \forall X, Y \in \Gamma$

２）Positive homogeneity $\lambda \ge 0 \Rightarrow \rho(\lambda X) = \lambda \rho(X)$

３）Monotonicity $X \le Y \Rightarrow \rho(X) \ge \rho(Y)$

４）Transition invariance $m \in R \quad \rho(X+m) = \rho(X) - m$

３－３．**Acceptable transaction set**

■A set of transactions A is <u>acceptable</u>

when

１）The acceptance set A contains L_+ .

$$A \supset L_+ = \{X \mid \forall \omega \in \Omega, X(\omega) \geq 0\}$$

２）The acceptance set A satisfies

$$A \cap L\_\_ = \{X \mid \forall \omega \in \Omega, X(\omega) < 0\} = \phi$$

３）The acceptance set A is convex.

$$X \ , Y \ \in \ A \quad \Rightarrow \quad tX + (1-t)Y \in A \qquad 0 \leq t \leq 1$$

４） The acceptance set A is a positively homogeneous cone.

$$X \in A, \quad \lambda > 0 \quad \Rightarrow \quad \lambda \cdot X \in A$$

３－４. **Correspondence between Acceptance Sets and Measures of risks**

- Let A be an acceptable set. A risk measure from A is defined as

$$\rho_{A,r}(X) = \inf\{m \mid mr + X \in A\}$$

r : interest rate of the term

- Let ρ be a given risk measure. Define a subset of financial transactions limited by the measure ρ,

$$A_\rho = \{X \in \Pi \mid \rho(X) \le 0\}$$

Π : set of all financial transactions

- The following relation exist

１）if B is acceptable, then $\rho_{B,r}$ is coherent .

And $A_{\rho_{B,r}} = \overline{B}$, $\overline{B}$ is closure of B

２）if ρ is coherent, then A_ρ is closed and acceptable set.

And $\rho_{A_\rho,r} = \rho$

３－５. **Representation Theorem**

■The risk measure defined on the scenario set Q is defined as

$$\rho_Q(X) = \sup\left\{ E_p\left[-\frac{X}{r} \right] \mid p \in Q \right\}$$

The scenario-based measure ρ_Q is, in fact, a coherent risk measure.

■Conversely, if ρ is coherent risk measure,

there exists a set Q of future scenarios, such that

$$\rho(X) = \rho_Q(X) = \sup\left\{ E_p\left[-\frac{X}{r} \right] \mid p \in Q \right\}$$

４． Some examples of risk measure

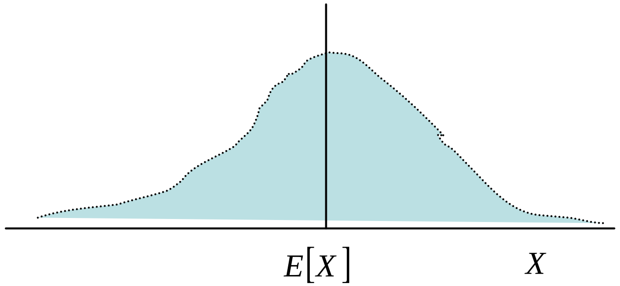
４－１． Variance (STD DEV) based and lower partial moments based

■Given P/L R.V. X of a transaction.

■ Variance (STD DEV) based :

$$V[X] = E\left[(X - E[X])^2\right]$$

$$\sigma[X] = \sqrt{E\left[(X - E[X])^2\right]}$$



■ Lower partial moment based (degree of k) :

$$LPM_k[c, X] = E\left[\max(c - X, 0)^k\right]$$

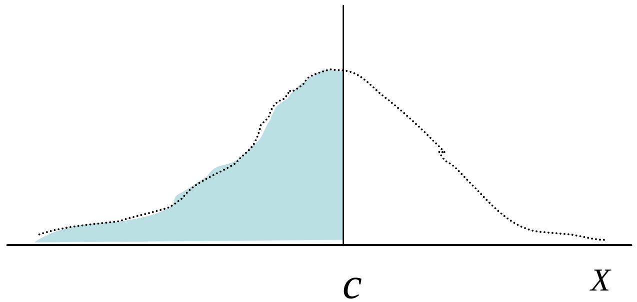
$$NLPM_k[c, X] = LPM_k[c, X] = E\left[\max(c - X, 0)^k\right]^{\frac{1}{k}}$$

Eg ; k＝1 : Expected Regret

$$ER[X] = E[\max(c - X, 0)]$$

Eg ; k＝2 : Semi-Variance

$$SV[X] = E\left[\max(E[X] - X, 0)^2\right]$$



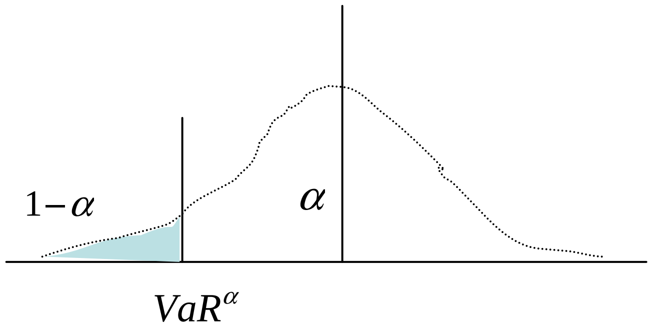
4－2. Value at Risk and Expected Shortfall

■ α percentile

$$x_{(\alpha)} = q_\alpha(X) = \inf\{x \in R : P[X \le x] \ge \alpha\}$$

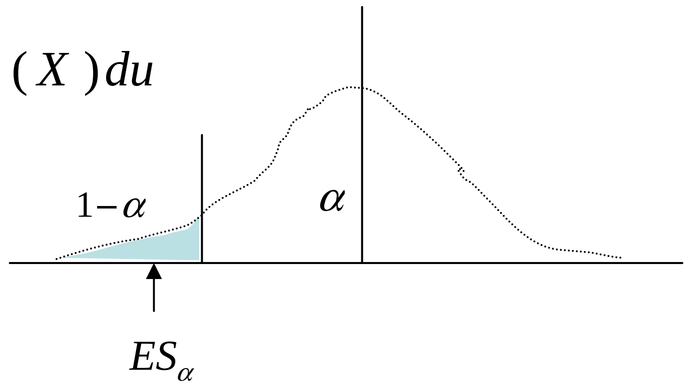
■ VaR (confidence level α)

$$VaR_\alpha(X) = -x_{(1-\alpha)} = q_\alpha(-X)$$



■ES (confidence level α)

$$ES_\alpha[X] = \frac{1}{1-\alpha} E\left[X \cdot 1_{\{X \le x_{(1-\alpha)}\}}\right] = -\frac{1}{1-\alpha}\int_0^{1-\alpha} q_u(X)\,du$$



４－３. **Spectral risk measure**

■ Define spectral risk measure $M_\phi(X)$ as

$$M_\phi(X) = -\int_0^1 x_{(u)} \cdot \phi(u) \cdot du$$

where $\phi \in L([0,1])$, $x_{(u)} = -F_X^{-1}[u]$, $F_X[u] = \Pr[X \le x]$

■ ϕ is a spectral weight function, weighs risk aversion.

if $\|\phi\| = 1$, $\phi > 0$, a decreasing function, ϕ is <u>admissible.</u>

where $\|\phi\| = \int_0^1 \phi(u)du$

■ $M_\phi(X)$ is coherent $\Leftrightarrow$ ϕ is admissible

■Eg ; when $\phi(u) = \frac{1}{\alpha} 1_{\{0 \le u \le \alpha\}}$, $ES_\alpha[X] = M_\phi(X)$

Eg ; when $\phi(u) = \delta(u - \alpha)$, $VaR_\alpha[X] = M_\phi(X)$

５．Practical uses in risk measurement

■Application can be found for example.

１）Financial products pricing

・Insurance valuation principle

・Risk adjusted loan rate

２）Hedge

・Dynamic hedge

・Minimum variance hedge

３）Portfolio optimization

・Portfolio selection : Maximize profit given a risk tolerance.

４）Provision [toward accounting application]

・General provision and credit risk measurement.

５） Methods of aggregate risks and risk capital allocation

・Allocation by risk contribution, and so on.

６）Performance evaluation reflecting risk

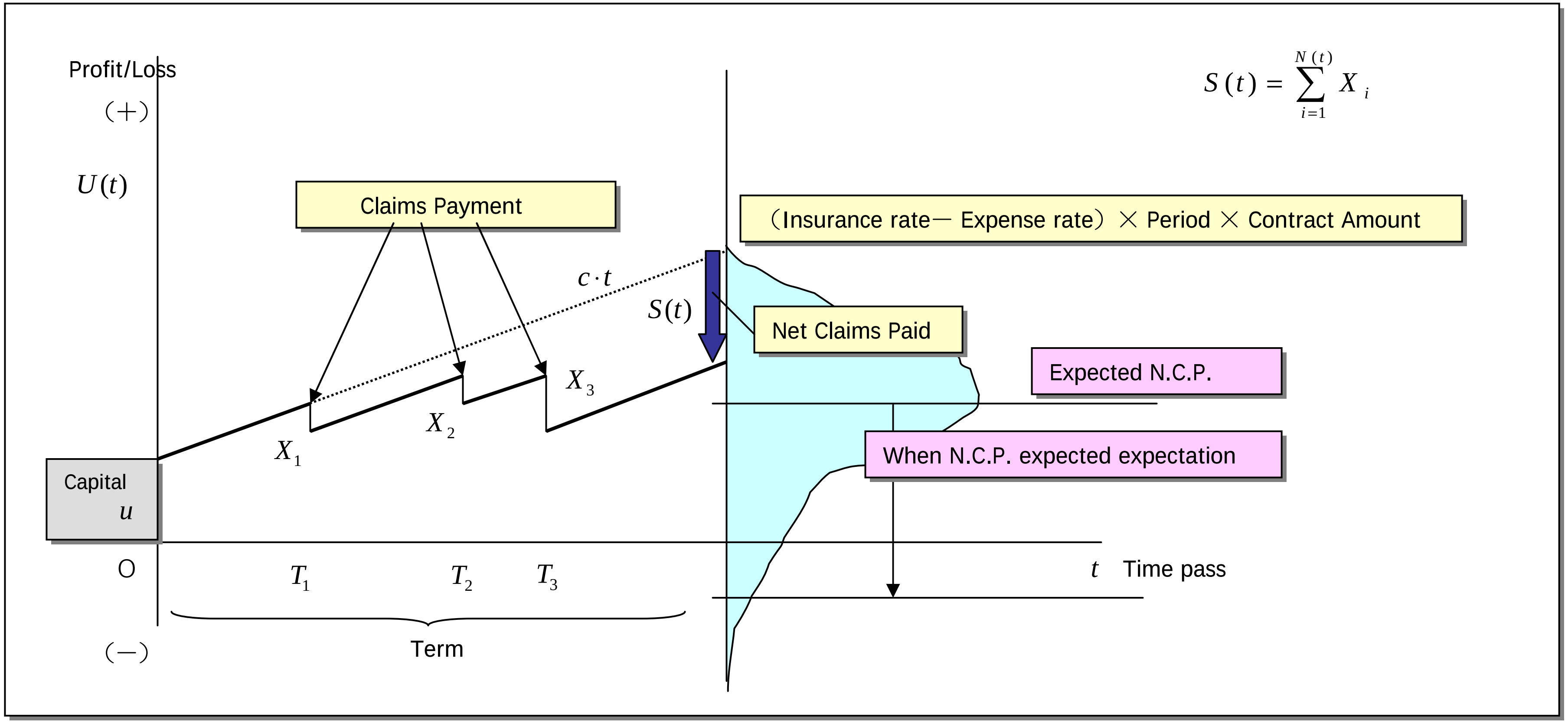
・Risk Adjusted Return on Capital(RARoC)， Shareholder Value added(SVA), and so on.

６． Towards integrated risk management beyond regulatory requirements

６－１． Risk in actuarial theory

○ Insurance payment events are uncertainty in insurance business. Annual premium income $c \cdot t$ is approximately reckonable.

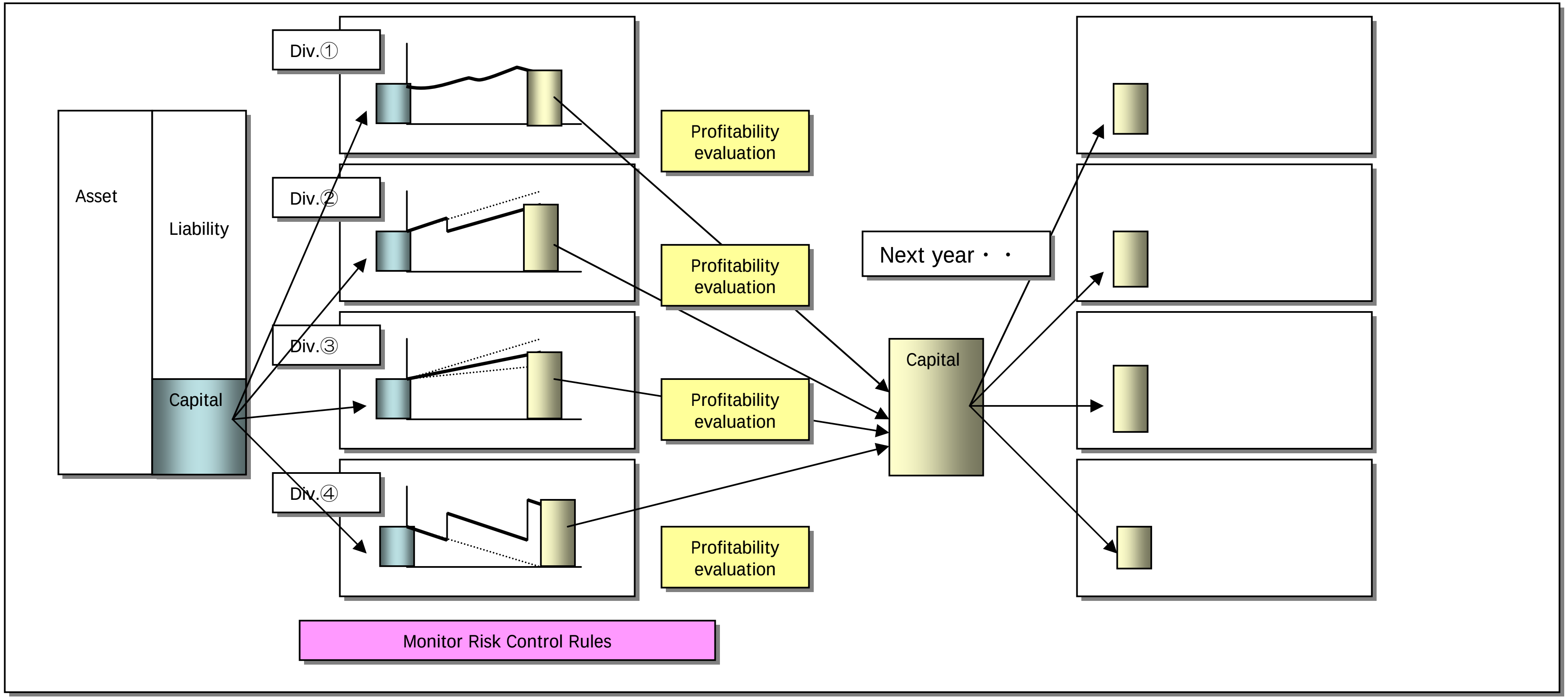
○Net claims paid $S(t)$ per annum is a random variable.



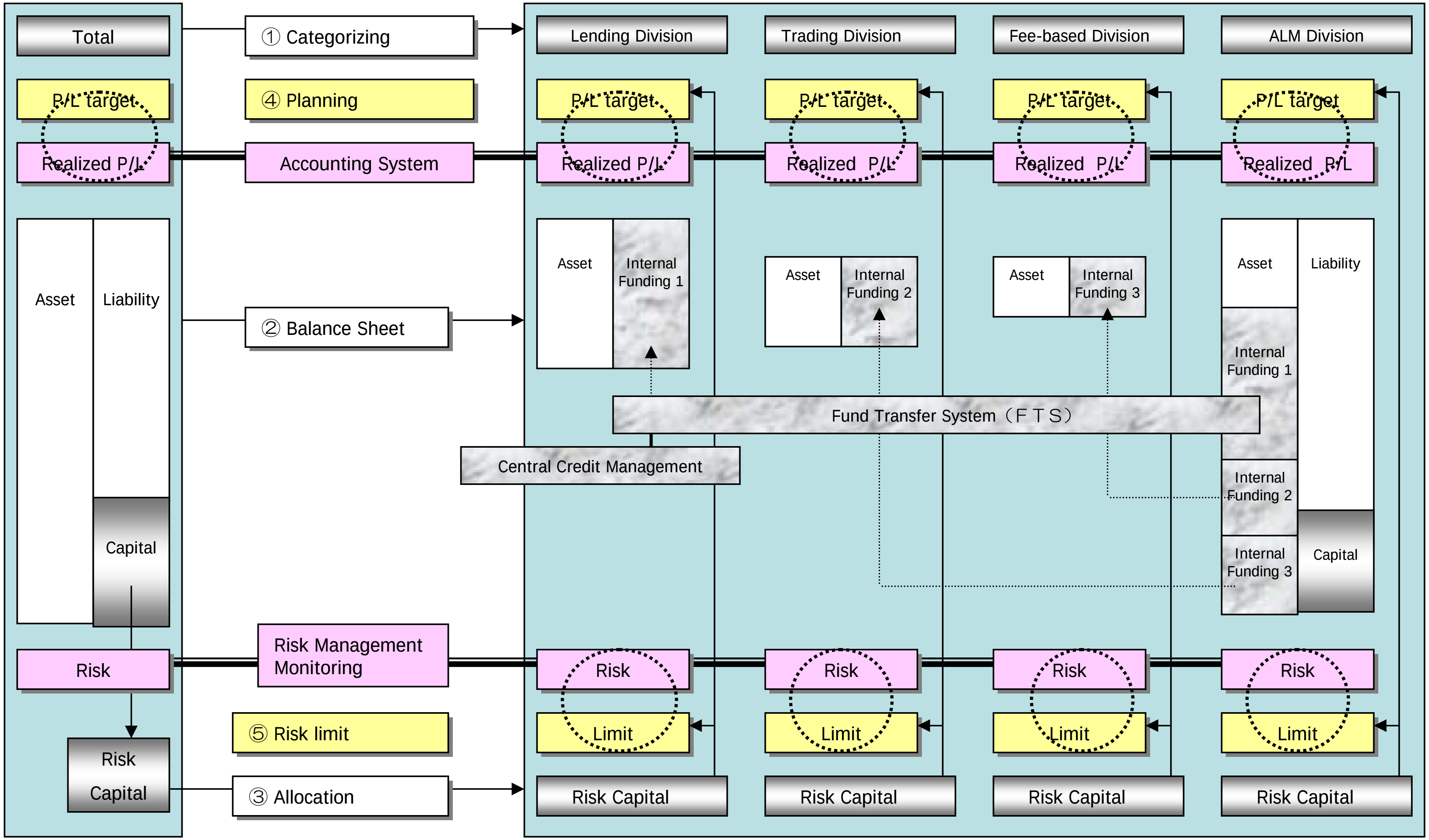
６－２.　Role of Integrated Risk Management in Bank Management

【point】

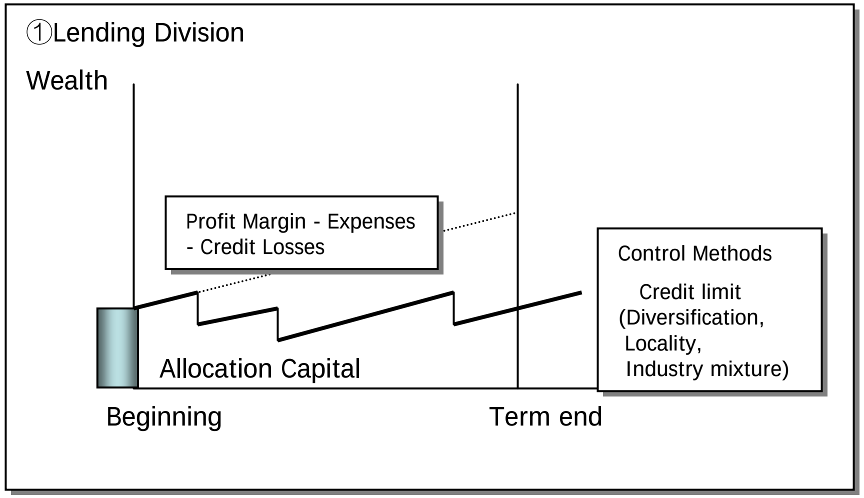
〇Bank is Collection of Risk generating Bodies.
〇Building Profit on Allocated Capital via Controlling Risk
〇Appraisal at End of Accounting Term（ＲＡＰＭ=Risk Adjusted Performance Measurement）
〇Avoid Bankruptcy within a Confidence Level

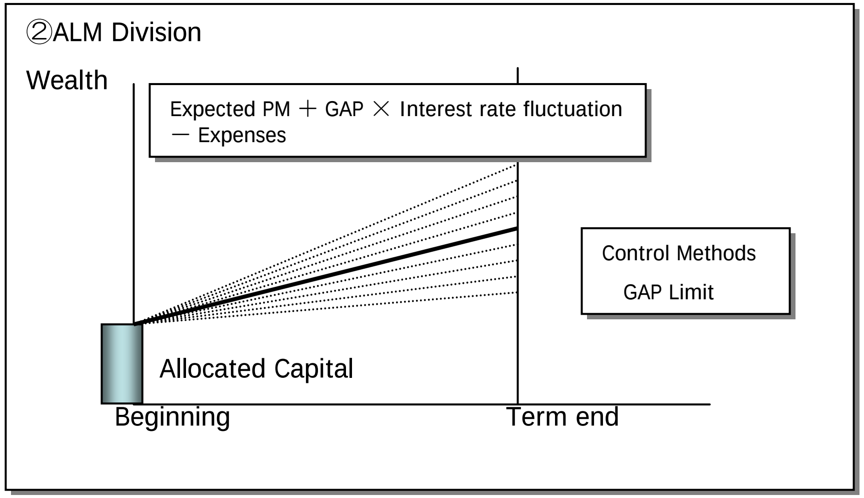


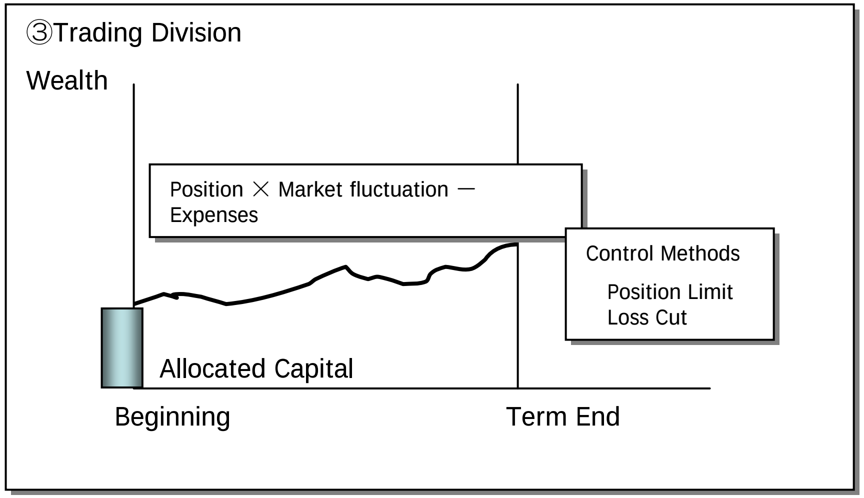
６－３. Management Framework

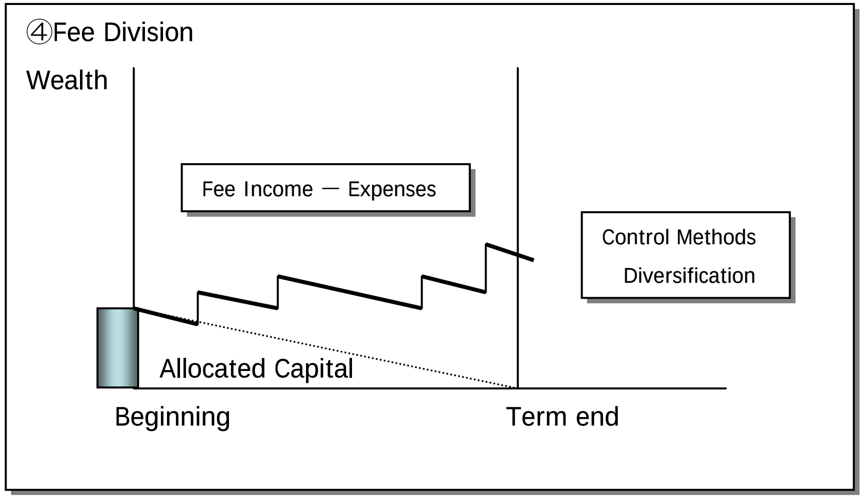


6－4. Profit and Loss(Risk) Dynamics in a typical banking operation





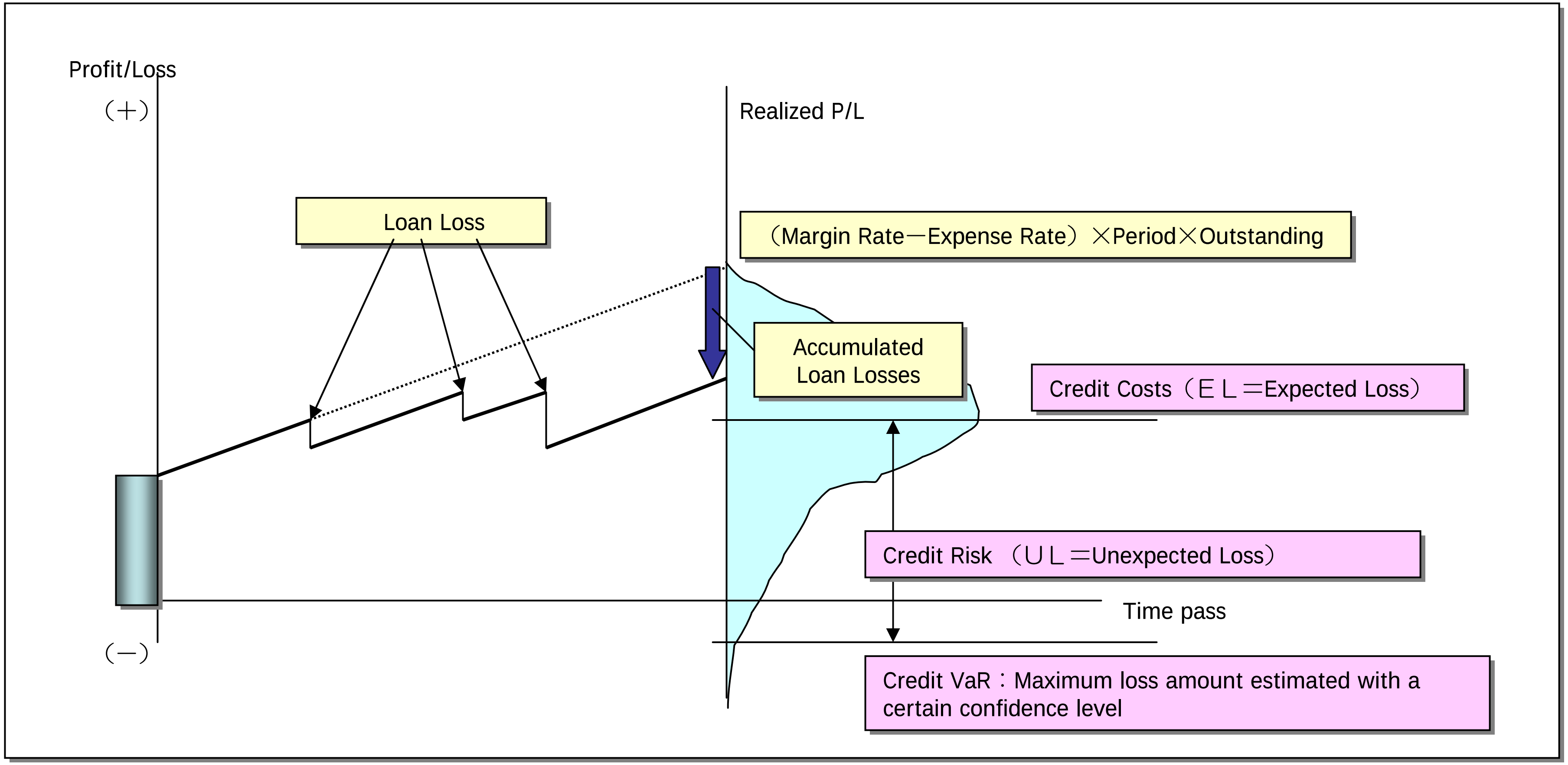




（Note）Lending Division P/L : Anatomy

〇Lending loss is a random variable（＝Credit Risk）. Annual profit margin is computable approximately.

〇Annual P/L(=Net Income Gain – OP. Cost – Loan Loss) is a random variable.



（Note）P/L Process

■Lending Division

$W_{Loan}(0)$ ：Capital Allocation to lending division. = Initially Allocated Capital

$$\tilde{W}_{Loan}(T) = W_{Loan}(0) + \int_0^T d\tilde{W}_{Loan}(t)$$

$$d\tilde{W}_{Loan}(t) = \sum_{i=1}^{N} X_i \cdot \pi_i \cdot dt - C_{Loan} \cdot dt - \sum_{i=1}^{N} X_i (1-\theta_i) \cdot d\tilde{N}_i(t) \qquad (1)$$

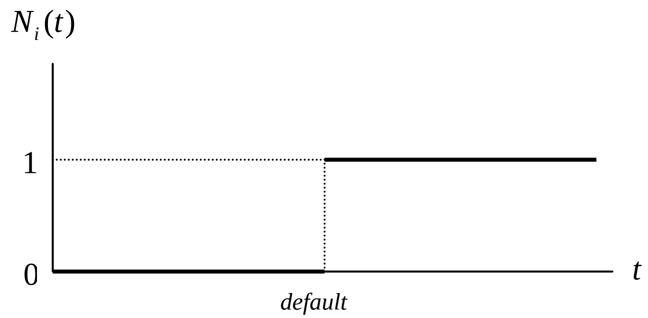
where $X = \sum_{i=1}^{N} X_i$ ：Loan portfolio X_i ：Individual Loan

π_i ：Profit Margin = Loan Rate – Internal Funding Rate

C_{Loan} ：Operating Cost per time unit

θ_i ：Default Recovery Rate

$\tilde{N}_i(t)$ ：Default indicating Jump process



$$\tilde{N}_i(t) = \begin{cases} 1 & default \quad until\text{–}t \\ 0 & not-default \quad until\text{–}t \end{cases}$$

$$P[d\tilde{N}_i(t) = \tilde{N}_i(t+dt) - N_i(t) = 1 \mid N_i(t) = 0] = \lambda \cdot dt \qquad (2)$$

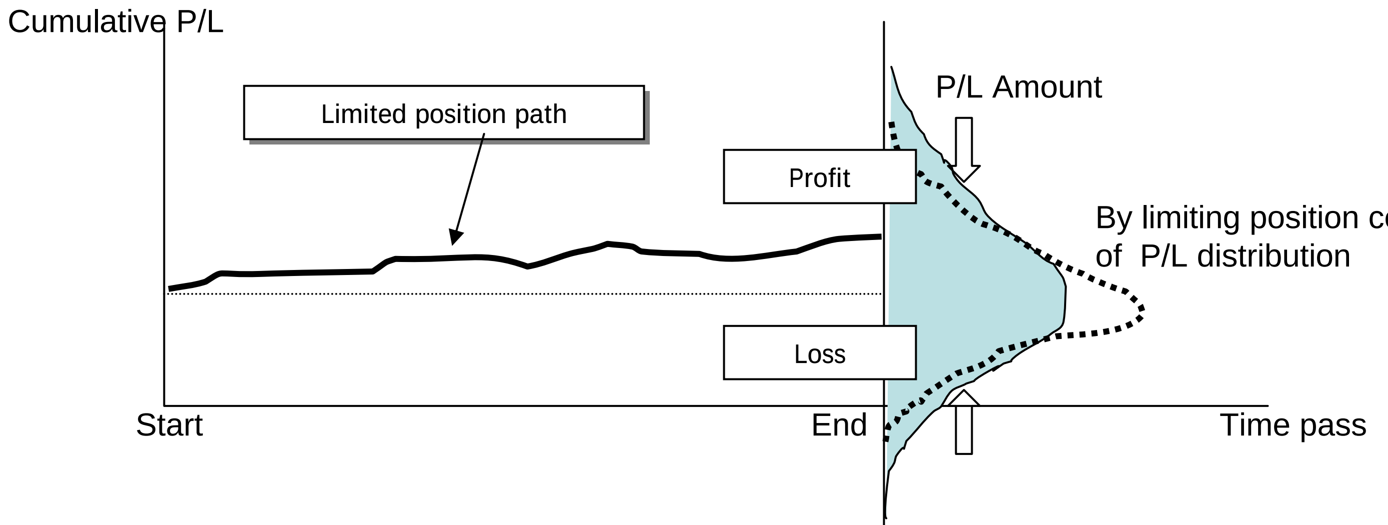
$t = 0$: Initial time,

$$P[\tilde{N}_i(t) = 0 \mid N_i(0) = 0] = e^{-\lambda \cdot t} \approx 1 - \lambda \cdot t \qquad (3)$$

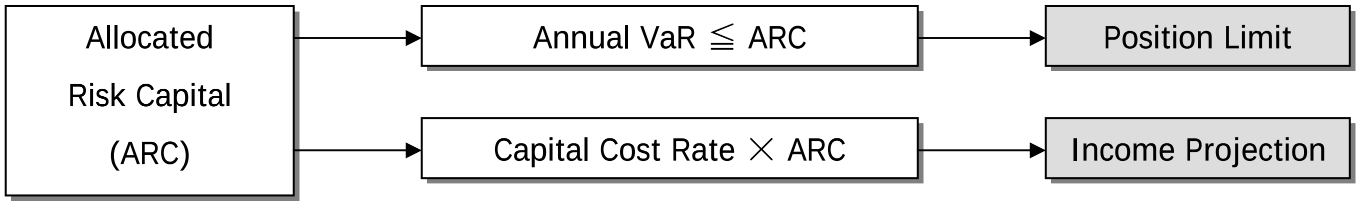
$$P[\tilde{N}_i(t) = 1 \mid N_i(0) = 0] = 1 - e^{-\lambda \cdot t} \approx \lambda \cdot t$$

（Note） Income Projection and Position Limits

（１） Limiting position controls loss events, costing earning opportunities.（see below）



（２） Income projection and risk control plans are, to be consistent, set accordingly with a risk capital allocation.



・Beginning of term : Allocate capital to each department who sets income projection and risk control plan.

・During the term : To make sure risk control rules are kept. (daily monitoring)

・End of term : Outcome (P/L) is evaluated relative to the risk taken.

(Note) Expected Annual P/L and Loan Pricing

■Plan expected annual income over the necessary target for individual contract i

$$\tilde{W}_{Loan}(1) = W_{Loan}(0) + (\sum_{i=1}^{N} X_i \cdot \pi_i - C_{Loan}) \cdot \int_0^1 dt - \sum_{i=1}^{N} X_i \cdot (1-\theta_i) \cdot \tilde{N}_i(1)$$

: Wealth in one year term

$$\begin{aligned} E[\tilde{W}_{Loan}(1) - W_{Loan}(0)] &= \sum_{i=1}^{N} X_i \cdot \pi_i - C_{Loan} - \sum_{i=1}^{N} X_i (1-\theta_i) \cdot E[\tilde{N}_i(1)] \\ &= (\sum_{i=1}^{N} X_i (\pi_i - c_{Loan}(X_i)) - \sum_{i=1}^{N} X_i (1-\theta_i) \cdot \lambda \\ &= \sum_{i=1}^{N} X_i (\pi_i - c_{Loan}(X_i) - (1-\theta_i)\lambda) \end{aligned} \quad (4)$$

(note) $C_{Loan} = \sum_{i=1}^{N} c_{Loan}(X_i) \cdot X_i$

■Denote Γ_i as allocated risk capital for customer i, ρ as the capital cost rate.

It follows

$$X_i(\pi_i - c_{Loan}(X_i) - (1-\theta_i)\lambda) > \rho \cdot \Gamma_i \quad (5)$$

Therefore, the necessary profit margin is

$$\pi_i > c_{Loan}(X_i) + (1-\theta_i)\lambda + \rho \cdot \frac{\Gamma_i}{X_i} \quad (6)$$

■ This expression is aka pricing guideline.

■In above formulation observe basic expressions in credit risk measurements.

$$\tilde{L} = \sum_{i=1}^{N} X_i (1-\theta_i) \tilde{N}_i(1) \qquad EL = E[\tilde{L}] = \sum_{i=1}^{N} X_i (1-\theta_i)\lambda$$

(Note) How to limit the risk (in simplified case)

■To control losses less than given risk capital $W_{Loan}(0)$ within a certain confidence level.

■ Assume that [1] uniform lending amount (X/N), [2] common recovery rate, [3] mutually independent default events.
Annual P/L variance is

$$V[\tilde{W}_{Loan}(1) - W_{Loan}(0)] = \sum_{i=1}^{N} \frac{X^2}{N^2}(1-\theta)^2 V[\tilde{N}_i(1)]$$

$$= \sum_{i=1}^{N} \frac{X^2}{N^2}(1-\theta)^2 \lambda \cdot (1-\lambda) = \frac{X^2}{N}(1-\theta)^2 \lambda \cdot (1-\lambda) \qquad (7)$$

■The confidence level sets loss upper limits to be that ϕ times standard deviation.
Suppose expected loss is covered by the profit margin.

$$UL^2 = \phi^2 \cdot V[\tilde{L}] = \phi^2 \cdot \frac{X^2}{N}(1-\theta)^2 \lambda \cdot (1-\lambda) < W_{Loan}(0)^2 \qquad (8)$$

Thus, the necessary diversification number $N_{\min}$ of companies is expected as

$$N_{\min} > \phi^2 \cdot \frac{X^2}{W_{Loan}(0)^2}(1-\theta)^2 \lambda \cdot (1-\lambda) \qquad (9)$$

６－５. Check points for integrated risk management

（１）**Grasping the total risk amount is the key; Coverage, independence and separation.**

①Coverage Operational risk, risk of fee business, individual risk of stock portfolio, and so on.

②Separation EaR and VaR for banking book, equity risk and credit risk, settlement risk, correlated risk issue, and so on.

③Consistency Is every risk measurement comparable and consistent? (risk occurrence period,confidence level and others)

（２）**Measure risk compare to probability measure.**

①Realized P/L or including unrealized portion for manageable control.

■Which is the choice when compared with risk capital?

②Before or after tax?

■When the profit is adjusted to the risk?

（３）**Measure risk from two essential stand points.**

①Whether risk is controlled under a feasible loss coverage. (Depositors,supervisory authorities,and ratings agencies view points)

②Whether profit sufficient in enough to meat the risk? （Shareholders and ratings agencies view points）

（4）**Let alone measurement, risk controlling structure must be established.**

①Founding on risk capital

・Earning targets （Mark beyond cost of capital for example）

・Transaction limiting protocol （ effectiveness risk control, conformity with target ratings）

②Are individual transaction protocol consistent?

・Position limits and stop-loss rules

・How to incorporate rational on credit limits, diversification, locality and industry mixture.

③Is monitoring working?

・Frequency, reporting lines

・Rule making and revision

・Enforcement, discipline and punishment

（5）**Organize divisional managerial concepts rationally**

①Fund Transfer System

②Central Credit Management （ concentration, transferring between dept. of credit）

③Risk capital allocation （ theory behind ）

④Diversification cross departments and risk categories

⑤Performance evaluation (encouragement, compliance, global vs. local optima, and so on)