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Testing the Purchasing Power Parity Hypothesis:

Re-examination by Additional Variables, Tests with Known Cointegrating Vectors, Monte Carlo Critical Values, and Fractional Cointegration

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Testing the Purchasing Power Parity Hypothesis: Re-examination by Additional Variables, Tests with Known Cointegrating Vectors, Monte Carlo Critical Values, and Fractional Cointegration*

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Abstract

This paper examines the Purchasing Power Parity (PPP) hypothesis of the yen-dollar exchange market by taking into account the following four points: accumulated current accounts as an additional variable, more powerful tests of Horvath and Watson(1995), more appropriate critical values through Monte Carlo experiments, and fractional cointegration. Results show that the PPP hypothesis is not necessarily rejected from the viewpoint of fractional cointegration, although analyses regarding the other three points cannot find any evidence of PPP.

Keywords: PPP, cointegration, Monte Carlo experiments, fractional integration.
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1 Introduction

Many researchers have tested the Purchasing Power Parity (PPP) hypothesis, most recently through cointegration methods. However, these results

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do not necessarily provide conclusive evidence [1]. We examine several possible empirical problems that may affect these ambiguous results of the PPP hypothesis tests. Specifically, we examine the PPP hypothesis of the yen-dollar market by taking into account the following four points: accumulated current account difference as an additional variable, more powerful tests of Horvath and Watson(1995), more appropriate critical values through Monte Carlo experiments, and fractional cointegration.

First, we use accumulated current account differences as an additional variable that could explain the empirical failure of the PPP tests. That is, we consider the accumulated current account difference between the U.S.A. and Japan as a fundamental factor to affect yen-dollar exchange rates. Johansen and Juselius(1992) consider the interest rate differenial as an additional variable. Our approach to include accumulated current account difference in the system can be viewed as an extension of Johansen and Juselius(1992).

Next, we use the Horvath and Watson(1995) testing procedure, which imposes the PPP restrictions and tests for cointegration in a multi-equation setting. This is because several researchers have suggested that low power could affect the results of the test [2]. The Horvath and Watson(1995) procedure has higher power than the usual Johansen method when some of the cointegrating vectors are prespecified.

Third, for our small-sized data, we use more appropriate critical values of the tests through Monte Carlo experiments. This is because Cheung and Lai(1993a,b), and Edison,Gagnon and Melick(1996) report possible finite-sample bias of the cointegration tests due to small samples. We also assess the power of the tests using our data.

Finally, we examine the possibility of fractional cointegration. In the usual framework of cointegration we have two series, say y_{1t} and y_{2t}, which are $I(1)$ and a linear combination of them which is $I(0)$. However, Dueker and Startz(1995) argue that a broad definition of fractional cointegration is that there exists an $I(d-b)$ linear combination of $I(d)$ series with $b \geq 0$, where d is a memory parameter. This definition can provide more information than the $I(1)/I(0)$ framework. That is, if it takes much time for economic agents to react to price differences, the memory parameter of a linear combination can be larger than zero. We consider the fractional cointegration of the residuals from a cointegrating regression.

Results of our analysis show that the PPP hypothesis is not necessar-

[1] See Froot and Rogoff(1995), and Rogoff(1996) for surveys of the PPP hypothesis.

[2] See Frankel(1990), Hakkio and Rush(1991), Lothian and Taylor(1996), and Edison, Gagnon, and Melick(1996).

ily rejected from the viewpoint of fractional cointegration while analyses regarding the other three points do not provide any evidence of PPP.

The paper is organized as follows. In the next section, we test the PPP hypothesis using the usual Johansen procedure and accumulated current account difference between the U.S.A. and Japan as an additional variable. Section 3 reports on tests using the Horvath-Watson procedure and Monte Carlo critical values. Section 4 reports the possibility of fractional cointegration. Section 5 contains some implications.

2 Testing for PPP including additional variables with the Johansen procedure

The Johansen procedure analyses the relationship among stationary or nonstationary variables using the following VAR system[3]:

$$\Delta X_t = \Pi X_{t-1} + \Sigma_{i=1}^{p-1}\Phi_i \Delta X_{t-i} + \mu + \epsilon_t \quad (2)$$

in which X_t is an $n \times 1$ random vector, ϵ_t is $NIID(0, \Sigma_\epsilon)$, and μ is deterministic terms. The long-run relationships are captured in the coefficient matrix of Π. That is, if the rank of Π, denoted r, is between 0 and n, then there are r linear combinations of the variables in the system that are $I(0)$ or cointegrated. As in Engle and Granger(1987), Johansen(1988), and Ahn and Reinsel(1990), it is convenient to write the model in a vector error correction form by factoring the matrix $\Pi = \alpha\beta'$, where α and β are $n \times r$ matrices of full rank and the columns of β denote the cointegrationg vectors. Johansen(1991), and Johansen and Juselius(1990) present two tests for determining the rank, the trace and maximum eigenvalue tests. In addition, Johansen(1991) suggests that tests of restrictions on the coefficients of β have chi-squared asymptotic distributions conditional on the order of cointegration being correct.

For tests of the PPP hypothesis, we use two variables; nominal exchange rate π and price differences dp. The PPP hypothesis means that the two variables (π, dp) are cointegrated with cointegrating vector $(1, -1)'$[4].

[3] This equation can also be expressed as follows.

$$X_t = \Sigma_{i=1}^{p}\Pi_i X_{t-i} + \epsilon_t. \quad (1)$$

[4] Although some of previous studies use a three variable system (*nominal exchange rates*, *domestic price level*, *foreign price level*), we adopt a two variable system. This is because we try to avoid a decrease in the power of the tests

We include two additional variables in the system. One is the interest rate differential between the U.S.A. and Japan, as analyzed in Johansen(1992). The other is the accumulated current account difference between U.S.A. and Japan. These additional variables might explain the empirical failure of PPP tests. Including the accumulated current account difference means that we assume that it could affect exchange rates[5].

In the Johansen method, we implement a two-stage testing procedure. In the first stage, the null hypothesis of no cointegration is tested against the alternative that the data are cointegrated with an unknown cointegrating vector. If the null hypothesis is rejected, a second stage test is implemented with cointegration maintained under both the null and alternative. The null hypothesis is that the data are cointegrated with the specific cointegrating vector implied by the PPP, and the alternative is that the data are cointegrated with an unspecified cointegrating vector.

We define X_t as three types of system; (π, dp) in Case 1, (π, dp, di) in Case 2, and (π, dp, di, dac) in Case 3, where π : exchange rate of yen per dollar, dp : difference of price levels between the U.S.A. and Japan, di : difference in the interest rate between the U.S.A. and Japan, and dca : the difference in accumulated current accounts between the U.S.A. and Japan. Estimation is based on 96 observations fron 1975:4 to 1999:3, with 10 presample observations used for determining the optimum lag length of the testing method. We explain the data in detail in Appendix A.

Table 1 shows the results of the ordinary Johansen tests for cointegration in the first stage, in which the null hypothesis is given by no cointegration[6]. Results suggest there is no cointegration in some cases of the two and three variable systems, and one cointegration in the other cases of the three and four variable systems [7].

due to an increase in the number of variables. Actually, our Monte Carlo experiments show that an increase of variables in systems yields a significant decrease in the power of the tests. See Appendix B.

[5]It is natural for us to consider that trade imbalance presented by accumulated current account affects exchange rates. Johansen and Juselius(1992) consider the interest rate differential as an additional variable. Our approach to include the accumulated current account can be viewed as an extension of Johansen and Juselius(1992).

[6]Lags are determined by step-down testing, beginning with a lag length of 9 and using a 5 percent test for each lag length. We also estimated with lags determined by step-down testing, using a 1 percent test for each lags. The results also reject the PPP hypothesis.

[7]Before Johansen's cointegration test, the stationarity of each variable was tested with the ADF and Phillips-Perron tests. Lags are determined by step-down testing, beginning with a lag length of 9 and using a 5 percent test for each lag length. Both the ADF test and Phillips-Perron tests suggest that all variables are I(1) except di and dca. The difference in short term interest rates di might be $I(0)$ and the difference in accumulated

In the second stage, we test the null hypothesis that the cointegrating vector is equal to (1,-1,0,0) and $\pi - dp = constant$ exists, which means the long-run PPP relation. Results shown in Table 1 suggest that the PPP hypothesis is rejected in all cases. Even the systems of Case 2 and Case 3 with additional variables do not provide any evidence of PPP using the Johansen test.

3 Testing for PPP with the Horvath-Watson procedure

The Johansen procedure used in the previous section has one disadvantage, in that the first stage tests may have low power. In order to solve this problem, we use the procedure proposed by Horvath and Watson(1995) to test for PPP. They propose testing for cointegration with a known cointegration vector, which is given by theory. This approach has an advantage in that the test for cointegration may be significantly more powerful than the test that does not impose the cointegrating vector. We examine the power of the Horvath and Watson procedure in our system and data by applying Monte Carlo experiments[8], which show the higher power of the procedure.

The procedure can be expressed as follows. We make use of the same VAR system (1). In the Horvath and Watson procedure, we impose the following restriction:

$$\Pi = \alpha\beta', \tag{3}$$

where $\beta' = (1\ \text{-}1)'$ in Case 1, $\beta' = (1\ \ \text{-}1\ \ 0)'$ in Case 2, and $\beta' = (1\ \ \text{-}1\ \ 0\ \ 0)'$ in Case 3, to construct Wald statistics. Let $W_{r_0,r_a}(\beta_{o_k},\beta_{a_k})$ define the corresponding Wald statistic under the null and alternative hypotheses: $r_{ok} = rank(\beta_{ok})$, $r_{ou} = r_o - rank(\beta_{ok})$, and similarly for r_{ak} and r_{au}. Subscripts o, a, k and u represent the null hupothesis, the alternative hypothesis, known cointegrating vectors, and unknown cointegrating vectors, respectively.

In implementation of the Horvath and Watson procedure, we use Monte Carlo methods: (1) for avoiding small sample bias; and (2) for searching the parameter space to maximize the chances of rejecting the (false) null hypothesis.

current accounts dca might be I(2). Stationarity tests using a 1 percent test for each lag length give the same results. In spite of these exceptions, we proceed to the cointegration tests because the low power of these tests is well known.

[8] See Appendix B.

Possible small sample bias in the Johansen and Horvath-Watson testing procedures have been reported, for example, by Chen and Lai(1993a,b) and Edison, Gagnon, and Melick(1996). We calculate the critical values of the Horvath and Watson tests for our small sample and confirm it in the two, three, and four variable systems of our data. As for parameter space search, Mood et al.(1974) and Edison et al.(1995) suggest that tests of an appropriate size require a search over the entire parameter space to find the particular member of the parameter space that maximizes the chances of rejecting the (false) null hypothesis [9].

To generate appropriate critical values, we conduct 10,000 trials on the DGP with $\Pi = 0$. Appendix B contains details of the Monte Carlo experiments. We assess the power of the Horvath and Watson tests in the three types of system on our data. Results of the Monte Carlo experiments show that the Horvath and Watson tests are more powerful than tests without a known cointegrating vector, such as the Johansen procedure. In particular, the increase in power in the four variable system is remarkable.

Tables 2 and 3 show the results of the Horvath and Watson test[10]. In every case, the statistics are not significant, even at the 10 percent level[11]. This suggests that there is no cointegration with the cointegrating vector (1 -1) which is consistent with the PPP hypothesis[12].

4 Fractional Cointegration

In the usual framework of cointegration, we have two series, say y_{1t} and y_{2t}, which are $I(1)$ and a linear combination of them which is $I(0)$. Dueker and Startz(1998) argue that a broad definition of fractional cointegration is

[9]These Monte Carlo experiments may yield increases in critical values and reductions in test power, while using the Horvath-Watson procedure may increase the power of cointegration tests. We adopt the Horvath and Watson tests and Monte Carlo critical values simultaneously.

[10]Lags are determined by step-down testing, beginning with a lag length of 9 and using a 5 percent test for each lag length. We also estimated with lags determined by step-down testing, using a 1 percent test for each lags. The results also reject the PPP hypothesis.

[11]In the Horvath and Watson test, we basically choose the largest critical values produced by the simple vector random walk and the estimated data generating process. Even under smaller Monte Carlo critical values in the experiments, however, those statistics are not significant.

[12]We also implement the Horvath amd Watson tests under alternatives with known cointegrating vector $r_{ak} = 1$ and unknown cointegrating vector(s) $r_a = 1, 2, \ldots$. In these tests, the results suggest that there is no cointegration with the cointegrating vector (1 -1), which is consistent with the PPP hypothesis.

that there exists an $I(d-b)$ linear combination of $I(d)$ series with $b \geq 0$. This definition can provide more information than the $I(1)/I(0)$ framework. We analyze fractional cointegration of the residuals from a cointegrating regression [13].

We estimate the memory parameters for all series and a linear combination of the PPP hypothesis. We adopt the estimation method to maximize the local Gaussian likelihood in the frequency domain, as proposed by Phillips(1998).

Table 4 reports the results of fractional integration tests on the four variable system $(\pi, dp(wpi), di(long), dca)$, imposing the cointegration vector $\beta = (1 \ \ \text{-}1 \ \ 0 \ \ 0)'$. Memory parameters of nominal exchange rates and price differences are 1.070 and 1.067, which are so close to one that we cannot reject the null hypothesis that they are one. Meanwhile, the estimated memory parameter of the linear combination is 0.999, which is smaller than those of the nominal exchange rates and price differences. This fact suggests the possibility of fractional cointegration, although we cannot reject the null hypothesis that the memory parameters of the linear combinations are the same as for the nominal exchange rates and price differences statistically. The 95% lower band of the estimated memory parameter is 0.828 and is larger than 0, which suggests that the ordinary PPP hypothesis with the I(0) linear combination of nominal exchange rate and price levels is significantly rejected. Fractional cointegration tests of other systems in our analysis show the same results[14].

We also have to pay attention to the memory parameter of the errors of the system, which are close to zero. We cannot reject the null hypothesis that the memory parameters are equal to zero. This fact suggests that there may be some mechanisms that stabilize the exchange rate in the system. The estimated memory parameter of the differenced variable of accumulated current account difference, which is not unit root nonstationary but is fractionally nonstationary, although other explanatory variables except the PPP linear combination are fractionally stationary. This might

[13] Baillie and Bollerslev(1994), and Chung and Lai(1993c) studied the possibility of fractional contegration in the PPP hypothesis. They also suggest the possibility of fractional cointegration of the PPP hypothesis, although the PPP hypothesis of fractional cointegration is not significant. They use mainly parametric approaches, that is, ARFIMA model, and report a relatively smaller memory parameter of the three variable system (π, *domestic price level*, *foreign price level*) in the yen-dollar market than those in our analysis. This difference might be due to their model specification because they report that their statistics depend on the specification of the parameters.

[14] Fractional cointegration tests using seasonally unadjusted data aso showed the same results.

suggest that the PPP linear combination and accumulated current account differences might be interacting like fractional cointegration, although the theoretical interpretation is complicated and statistical theory for fractional cointegration has not yet been clearly developed. This possibility should be analyzed based on future statistical theory [15].

5 Implications

Our results show that the PPP hypothesis is not necessarily rejected from the viewpoint of fractional cointegration, although analyses regarding the other three points do not find any evidence of PPP.

The PPP constrains the exchange rate between two countries to be proportional to the ratio of the price levels in the two countries. The PPP hypothesis is closely related to the law of one price, which states that the price of a commodity is the same everywhere in the world. However, there are several limits to this theory. Commodities with the same name are not the same everywhere. Transportation costs also account for differences in prices. It takes time for economic agents to react to price differences. Moreover, the law of one price does not apply to nontraded goods. Finally, even if PPP held exactly for each of the traded commodities, the PPP hypothesis can be violated because of differences in the weights given to the different commodities in the construction of price indexes in the two countries. The more interesting question may be not whether the PPP hypothesis holds exactly in the long run, but how fast arbitrage eliminates price differences. In this sense, the PPP should be discussed from the viewpoint of the possibility of fractional integration rather than the strict I(1)/I(0) cointegration.

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A Data Sources

π : ln(nominal exchange rate)
dp : ln(Japan price index) $-$ ln(US price index)
di : $\ln(1 +$ Japan real interest rate/100) $- \log(1 +$ US real interest rate/100)
dca : (Japan accumulated current accounts/GDP)
$-$ (U.S.A. accumulated current accounts/GDP)
Notes : (1) 1+ real interest rate = (1+ nominal interest rate)/(inflation rate one quarter earlier)
(2) Accumulated current accounts are summed from 1973:2Q.

Variables			Sources
Exchange rate		nominal yen/dollar	BOJ
Prices	Japan	WPI	BOJ
	Japan	CPI	Management and Coordination Agency
	U.S.A.	PPI	IMF, IFS
	U.S.A	CPI	IMF, IFS
Interest rates	Japan	Government bonds(10 years)	BOJ
	Japan	*Gensaki*(3 months)	IMF, IFS
	US	Government bonds(10 years)	IMF, IFS
	US	Treasury Bills(3 month)	IMF, IFS
Curent Accounts	Japan		BOJ
	U.S.A.		IMF, IFS
Nominal GDP	Japan		Economic Planning Agency
	U.S.A.		IMF, IFS

Notes :
(1) Seasonally adjusted data except for the exchange rates and interest rates, from 1973/2Q to 1999/3Q
(2) BOJ : Bank of Japan, IFS: International Financial Statistics

B Comparison of Testing Procedures by Monte Carlo Experiments

We conduct two types of Monte Carlo experiments. The first experiment determines the critical values for the Horvath and Watson test statistics to conduct tests of the desired size. The second experiment assesses the power of the tests to discriminate against a false null, given the critical values calculated in the first experiments.

B.1 Size

To calculate critical values, we use both the simple vector random walk processes and the estimated data generating processes for each system, imposing the restriction that $\Pi = 0$. This is because tests of an appropriate size require a search over the entire parameter space to find the particular member of the parameter space that maximizes the chances of rejecting the (false) null hypothesis, as suggested in Mood et al.(1974) and Edison, Gagnon and Melick(1996).

Random samples of 175 observations were created for Y_t, with ϵ_t normally distributed. The first 79 observations were used to initialize the process, leaving 96 observations for the experiments. The number of observations is the same as the data used in our analysis. The Monte Carlo experiments consist of calculating the Horvath and Watson test statistics for 10,000 trials of the process.

Calculated critical values in Table 3 are larger than the asymptotic critical values reported in Horvath and Watson(1995). Although there are differences between the critical values of the simple vector random walk processes and those estimated by the data generating processes, such differences are small. Our critical values of the three variable system are not so different from the Monte Carlo critical values that Edison, Gagnon, and Melick(1997) report in the three variable system of nominal exchange rate, domestic price, and foreign price level.

In each case of our analysis, we use conservatively the largest critical values produced by the simple vector random walk processes and the estimated data generating processes, imposing $\Pi = 0$.

B.2 Power

We conduct power comparisons, that is, we compare the local power of the Horvath and Watson method that imposes the value of the cointegrating vector under the alternative, to the usual corresponding tests that do not use this information. We consider three types of system: Case 1 is a two variable system, Case 2 is a three variable system, and Case 3 is a four variable system. The coefficients β are $(1\ \ \text{-}1)'$, $(1\ \ \text{-}1\ \ 0)'$, and $(1\ \ \text{-}1\ \ 0\ \ 0)'$ for Case 1, Case 2, and Case 3, respectively

First, our discussion focuses on the following simple system to investigate the local power of the tests:

$$\Delta X_t = \alpha\beta' X_{t-1} + \Sigma_{i=1}^{p-1}\Phi_i \Delta X_{t-i} + \mu + \epsilon_t. \tag{4}$$

To investigate the local power of the tests, we suppose that α is local to 0; specifically, we set $\alpha = -c/T$, as follows. When $c = 0$, the system is not cointegrated:

$$\alpha = c/T, c = 0, ..., 31, \tag{5}$$

where T is the number of observations. As in size calculations, random samples of 175 observations were created for Y_t, with ϵ_t normally distributed. The first 70 observations were used to initialize the process, leaving 105 observations for the experiments. The Monte Carlo experiments consist of calculating the Horvath and Watson statistics for 10,000 trials of the process.

In Figure 1, we plot the local power curves. Thus, the $W_{0,1}(0, \beta_{a_k})$ plot shows the power of the test that imposes the true value of the cointegrating vector, and the $W_{0,1}(0,0)$ plot shows the power of the test that does not use this information, such as the Johansen test. The power gains from incorporating the true value of the cointegrating vector are substantial.

We also conduct power comparisons by estimating the data generating processes. Estimated coefficients imposing the PPP restrictions correspond to about C=4 in the power calculation of simple VECM processes. The results of those comparisons also suggest that the Horvath and Watson method have higher powers than the usual method. The powers of the tests jump from 0.05 to 0.10 in Case 1, from 0.06 to 0.11 in Case 2, and from 0.05 to 0.11 in Case 3, respectively, although the actual powers are still low.

Table 1: Johansen test

Two variables model(π, dp)

Price	WPI-PPI	CPI	5% critical value
Trace Test Statistic	16.94**	9.834	15.34(p=0)
	1.974	0.815	3.84 (p≤1)
C. Vector	-1.9	-2.0	
PPP test :LR value	11.50	–	
p-value	(0.0007)	–	
Lag	3	3	

Three variables model(π, dp, di)

Price	WPI-PPI		CPI		
Term of interest rate	long	short	long	short	5% critical value
Trace Test Statistic	43.93***	27.19*	26.61	27.09*	29.38(p=0)
	8.252	8.702	8.940	6.789	15.34 (p≤1)
	1.752	1.850	1.222	1.055	3.84 (p≤2)
C.Vector	-2.2	-2.3	-2.4	-2.6	
PPP test :LR value	32.30	–	–	–	
p-value	(0.0000)	–	–	–	
Lag	3	5	5	5	

Four variables model(π, dp, di, dca)

Price	WPI-PPI		CPI		
Term of interest rate	long	short	long	short	5% critical value
Trace Test Statistic	70.32***	60.15***	85.69***	70.27***	47.21(p=0)
	25.24	26.10	29.20	30.62**	29.38 (p≤1)
	7.765	8.570	6.242	8.708	15.34 (p≤2)
	0.410	0.518	0.092	1.202	3.84 (p≤3)
C.Vector	-3.6	-2.2	-4.3	-3.3	
PPP test :LR value	36.93	22.43	50.96	16.35	
p-value	(0.0000)	(0.0001)	(0.0000)	(0.0003)	
Lag	4	4	3	5	

Notes :

(1) Critical values from Table 15.3 in Johansen(1995).

(2) C.Vector is the cointegrating vector.

(3) Lag length was determined by step-down testing.

(4) *,** and *** means significant at 10 %, 5% and 1%, respectively.

Table 2: Wald statistics

datasets	$n - r_{ou}$	$W_{0,r_a}(0, \beta_{ak})$	β_{ak}	Wald Statistics
$(\pi, di(wpi))$	2	$W_{0,1}(0, \beta_{ak})$	(1 -1)	3.47
$(\pi, di(cpi))$	2	$W_{0,1}(0, \beta_{ak})$	(1 -1)	3.47
$(\pi, dp(wpi), di(long))$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	6.67
$(\pi, dp(cpi), di(long))$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	2.59
$(\pi, dp(wpi), di(short))$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	7.74
$(\pi, dp(cpi), di(short))$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	2.21
$(\pi, dp(wpi), di(long), dca)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	9.60
$(\pi, dp(cpi), di(long), dca)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	5.68
$(\pi, dp(wpi), di(short), dca)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	10.82
$(\pi, dp(cpi), di(short), dca)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	6.60

Note: *,**, and *** means significant at 10%, 5%, and 1%, respectively.

Table 3: MonteCarlo Critical values

				MonteCarlo		
data sets	$n - r_{ou}$	$W_{0,r_a}(0, \beta_{ak})$	β_{ak}	1%	5%	10%
$(SVRW of\ 2\ var.)$	2	$W_{0,1}(0, \beta_{ak})$	(1 -1)	13.90	10.32	8.56
(Asymptotic Values in Horvath and Watson(1995))				(13.73)	(10.18)	(8.30)
$(\pi, dp(wpi))$	2	$W_{0,1}(0, \beta_{ak})$	(1 -1)	13.81	10.40	8.56
$(\pi, dp(cpi))$	2	$W_{0,1}(0, \beta_{ak})$	(1 -1)	14.80	10.98	9.10
$(SVRW of\ 3\ var.)$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	17.06	12.59	10.62
(Asymptotic Values in Horvath and Watson(1995))				(15.41)	(11.62)	(9.72)
$(\pi, dp(wpi), di(long))$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	17.17	12.63	10.57
$(\pi, dp(cpi), di(long))$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	17.55	13.14	11.00
$(\pi, dp(wpi), di(short))$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	16.98	12.69	10.73
$(\pi, dp(cpi), di(short))$	3	$W_{0,1}(0, \beta_{ak})$	(1 -1 0)	17.73	13.13	11.07
(Monte Carlo Values in Edison, Gagnon, and Melick(1994))				(19.24)	(13.49)	(11.11)
$(SVRW of\ 4\ var.)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	22.95	17.42	14.97
(Asymptotic Values in Horvath and Watson(1995))				(17.16)	(13.20)	(11.16)
$(\pi, dp(wpi), di(long), ca)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	24.42	19.07	16.44
$(\pi, dp(cpi), di(long), ca)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	24.49	19.13	16.49
$(\pi, dp(wpi), di(short), ca)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	24.76	18.50	15.94
$(\pi, dp(cpi), di(short), ca)$	4	$W_{0,1}(0, \beta_{ak})$	(1 -1 0 0)	25.10	19.38	16.70

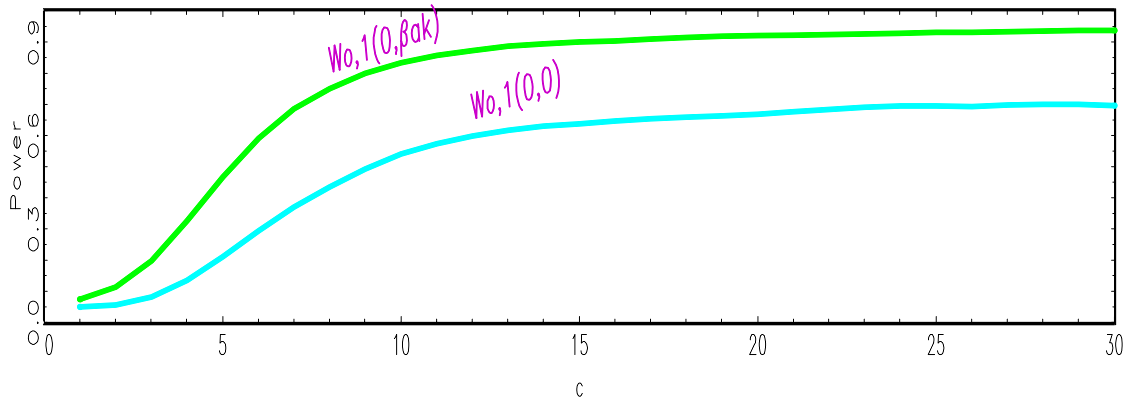
Note: SVRW means a simple vector random walk model.

Table 4: Fractional Cointegration Tests

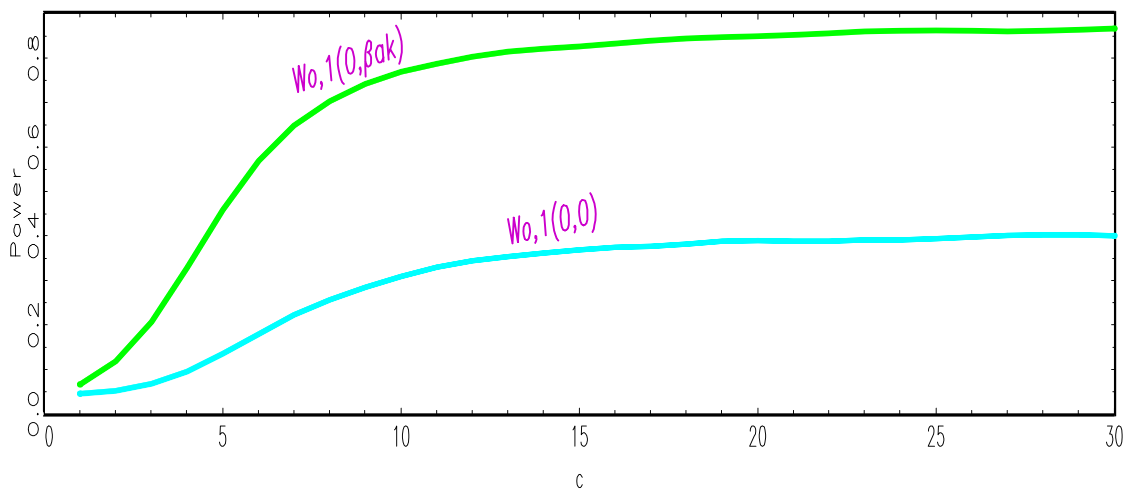
Variables	Memory Parameters	95% lower band	95% upper band
	Dependent Variable		
$\Delta\pi$	0.093	-0.078	0.264
	Independent Variables		
$ECMterm(\pi - dp)$	0.999	0.828	1.171
$((\pi)$	1.070	0.899	1.241)
$((dp)$	1.067	0.896	1.238)
Δdp	0.018	-0.152	0.190
Δdi	-0.119	-0.292	0.056
Δdca	0.793	0.622	0.964
	Estimated Error		
(ϵ)	0.007	-0.168	0.181

Figure 1: Local Power Curves

Case 1: Two Variable System



Case 2: Three Variable System



Case 3: Four Variable System