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**Model Uncertainty of Real Exchange Rate Forecast  
over Mid-term Horizons**

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# Model Uncertainty of Real Exchange Rate Forecast over Mid-term Horizons\*

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## Abstract

We investigate the significance of fundamentals variables and uncertainty of appropriate models in one-, two-, four-, and eight-quarter ahead forecasts of quarterly yen-dollar real exchange rates by using 16 fundamentals-based models and the random walk model. Our empirical results show significance of fundamentals variables in two-, four-, and eight-quarter ahead forecasts. Moreover, the reversible jump MCMC approach for uncertainty of appropriate models indicates that appropriate models change over both forecast-time-span and forecast period. This uncertainty could not be fully explained by the hypothesis that real exchange rates are ultimately governed by the true fundamentals-based model.

*Keywords:* Exchange rates, Fundamentals, Prediction, Reversible Jump, Markov Chain Monte Carlo

*JEL Classification:* F31, F37, F47

## 1 Introduction

Earlier works have found that real exchange rates are essentially more unpredictable over shorter horizons. This implies that the short-horizon change of real exchange rates is dominated by noise. Some empirical studies during the recent float have showed little evidence against the hypothesis that log real exchange rates follow a random walk over a shorter horizon. However, if real exchange rates are ultimately governed by economic fundamentals,

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those noisy fluctuations reverse themselves over time. In fact, recent studies have found a tendency for real exchange rates among the major industrialized countries to converge in the long run. See, for example, Frankel (1986), Edison (1987), Edison and Klovland (1987), Kim (1990), Abuaf and Jorion (1990), Ardeni and Lubian (1991), Glen (1992), and Lothian and Taylor (1996). Mark and Choi (1997) reported that fundamentals-based models have more power in real exchange rate prediction over longer horizons. Our paper pushes forward their ideas in respect of the following points.

First, we examine the significance of fundamentals-based models over longer horizons by entertaining more models for real exchange rates comprehensively; that is, seventeen alternative specifications, more than previous studies such as Mark and Choi (1997).<sup>1</sup>

Second, we investigate the uncertainty of appropriate fundamentals-based models for real exchange rates forecast by estimating the posterior probabilities of each model. To understand the process of the determination of real exchange rates, analyzing uncertainty of appropriate models is practically important because uncertainty of appropriate fundamentals-based models over time and over forecast horizons has been frequently observed.

So far, however, the uncertainty of appropriate models has not necessarily been examined statistically. Therefore, we try to provide some insights on this uncertainty of appropriate models and the determination of real exchange rates over mid-term horizons. Possible reasons for the uncertainty are also discussed.

Third, for a given set of competing models, we adopt the reversible jump MCMC method introduced by Green (1995), under which we estimate parameters and model probabilities jointly, instead of the approach that considers the models separately and chooses the best model. This approach is consistent with our research purpose. By extending the MCMC strategy so that the sampler jumps between parameter subspaces of different dimensionality corresponding to different models, we obtain a sample of joint posterior density of the models and model parameters. Based on this method, we can derive the probability of each model over different estimation periods and over different forecast horizons, which may lead to possible reasons of model uncertainty.

Fourth, we evaluate forecast errors under uncertainty of appropriate models by outputs of the reversible jump MCMC. If the appropriate model were uncertain, forecast densities would have larger variance.

In the next section, we investigate the significance of fundamentals variables on forecasting real exchange rates by the ordinary methods used in Mark and Choi (1997). In Section 3, we analyze the uncertainty of appropriate models in prediction of real exchange rates. Section 4 concludes the paper.

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<sup>1</sup>See an Appendix A for model specifications.

## 2 Evaluation of Real Exchange Rate Models by the Ordinary Method

We investigate the significance of fundamentals variables in the mid-term forecast of the yen-dollar exchange rate models based on fundamental macroeconomic variables. Sixteen exchange rate determination models and the driftless random walk model are specified to predict real exchange rates. They are then evaluated by testing the significance of fundamentals variables of each model at different forecast-time-span, i.e., one-, two-, four-, and eight-quarter horizons.

### 2.1 Description of the Models and Data

Our models are mainly grounded on the economic fundamentals suggested by economic theory; some of them are based on those of Mark and Choi (1997). In addition to them, the driftless random walk model – a representative univariate time series model – is also estimated as a point of comparison for our alternative formulations.

For all models, the real exchange rate is employed as a dependent variable which is defined as  $q_t = \ln(S_t P_t^* / P_t)$  at date  $t$  where  $S_t$  is the domestic currency price of one unit of the foreign currency and  $P_t$  and  $P_t^*$  is the domestic and foreign price level, respectively. In our case, the log real exchange rates used as dependent variables for all models are calculated by using GDP deflators. Furthermore, for specifications including interest rate differentials, both short- and long-term interest rates are adopted.

As for data used in models, the detailed definitions of variables and their data sources are given in Appendices B and C. The data set for Japan and the U.S. is on a quarterly basis and covers the period 1975/1Q-2000/4Q.

We adopt the following level autoregressive specification.

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} x_{t-i-j} + e_t, \quad (1)$$

where  $q_t$  is real exchange rate,  $x_t$  is fundamental macroeconomic variables,  $\alpha$  is parameters,  $i$  means  $i$ -quarter ahead forecast, and  $n$  is lag length. This specification is relatively general and includes the error correction model and the differential autoregressive model as special forms.<sup>2</sup>

### 2.2 Backward Averaging Regression

In our exercises of multi-period ahead forecasting, overlapping forecast periods might bring bias in estimating the covariance matrix. In fact, LM tests for serial correlation of estimated errors of each model, including the random walk model, suggest significant serial correlation in each model.

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<sup>2</sup>We adopted four for the lag length after considering the limitation of our sample size.

Therefore, we cannot evaluate the significance of fundamentals-based models and the random walk model statistically. To avoid this bias, we estimate fundamentals-based models by the backward averaging specification, in which possible bias could be eliminated by using additional lagged independent variables.<sup>3</sup> Table 1 shows results of tests for significance of fundamentals variables of each model based on the backward averaging estimation. In the one-quarter ahead forecast, most fundamentals variables of each fundamentals-based model are not significant, which is consistent with the results of Meese and Rogoff (1983a).<sup>4</sup> In two-, four-, and eight-quarter ahead forecasts, however, several fundamentals-based models have significant information on forecasting. Especially, in eight-quarter ahead forecast, fundamentals variables of most models are significant. Thus, the longer the forecast horizon is, the more significant fundamentals variables tend to show.

These results support the hypothesis that fundamentals variables have significant information on the longer-horizon forecast, which is consistent with Mark and Choi (1997).

Based on Table 1, we can also point out the uncertainty of appropriate models. That is, the appropriate models seem to be uncertain, or seem to change over these forecast horizons. If real exchange rates are ultimately governed by the true fundamentals-based model, the influence of the true model is supposed to increase as the forecast horizon become longer. However, this observed uncertainty does not seem to be fully explained by this story. Uncertainty of appropriate fundamentals-based models with significant information on future real exchange rates might also suggest other possibilities.

We statistically analyze this model uncertainty as the probability of each model by the reversible jump MCMC method and implement tests for change of the probabilities.

### 3 Analysis by the Reversible Jump MCMC

We examine the extent to which these fundamentals-based models and the random walk model are adequate by the reversible jump MCMC method. Thus, the reversible jump MCMC method provides the probability of each model.

For a given set of competing models, we adopt the approach under which we estimate parameters and model probabilities jointly, instead of the approach that considers the models separately and chooses the best model. By adopting this approach, we can analyze the possibility of appropriate

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<sup>3</sup>See Mark and Choi (1997) for details.

<sup>4</sup>Meese and Rogoff (1983a) compared the forecasting power of several structural models with that of the simple random walk, various univariate time series models, and a VAR model. Their empirical evidence showed that no model could outperform the random walk at one to twelve-month ahead forecast horizons.

models. To implement this approach, we use the reversible jump MCMC method introduced by Green (1995). By extending the MCMC strategy so that the sampler jumps between parameter subspaces of different dimensionality corresponding different models, we obtain a sample of joint posterior density of the models and model parameters.

Green (1995) proposed a reversible-jump MCMC strategy for generating from the joint posterior,

$$\pi(m, \theta_m | y), \quad (2)$$

based on the standard Metropolis-Hastings approach, where  $y$  is a given set of data,  $m$  indicates a model of a countable set  $M$  of competing models,  $\theta_m$  is a vector of unknown parameters for model  $m$ .

Suppose that the current state of the Markov chain at time  $t$  is  $(m, \theta_m)$ , where  $\theta_m$  has dimension  $d(\theta)$ , and a move is proposed at time  $t + 1$  to a new model  $m'$  with probability  $j(m, m')$  and corresponding parameter vector  $\theta'_{m'}$ .

Then, a vector  $u$  is generated from a specified proposed density  $q(u|\theta_m, m, m')$ , and we set

$$(\theta'_{m'}, u') = g_{m,m'}(\theta_m, u), \quad (3)$$

for a specified invertible function  $g_{m,m'}$  such that  $g_{m',m} = g_{m,m'}^{-1}$ , where

$$d(\theta_m) + d(u) = d(\theta'_{m'}) + d(u'). \quad (4)$$

Green (1995) showed that if the new move is accepted as the next realization of the Markov chain with probability  $a = \min\{1, r\}$ , where

$$r = \frac{\pi(y|m', \theta'_{m'})\pi(\theta'_{m'}|m')\pi(m')j(m', m)q(u'|\theta'_{m'}, m', m)}{\pi(y|m, \theta_m)\pi(\theta_m|m)\pi(m)j(m, m')q(u|\theta_m, m, m')} |J|. \quad (5)$$

with  $J = \partial(\theta'_{m'}, u')/\partial(\theta_m, u)$  means the Jacobian of the transformation, the chain satisfies the required conditions. For details, see Green (1995).

In our investigation, we adopt the strategy proposed by Vrontos, Delaportas, and Politis (2000). They suggest that all the parameters of the proposed model are generated from a proposal distribution. Consequently,

$$\begin{aligned} (\theta'_{m'}, u') &= (u, \theta_m) & (6) \\ \text{with } d(\theta_m) &= d(u'), \\ d(\theta'_{m'}) &= d(u), \\ q(u|\theta_m, m, m') &= q(u|m'), \\ q(u'|\theta'_{m'}, m', m) &= q(u|m), \end{aligned}$$

and Jacobian of the transformation  $J = \partial(\theta'_{m'}, u')/\partial(\theta_m, u) = 1$ . Then, the probability of acceptance of the new move as the next realization of the Markov chain is given by  $a = \min\{1, r\}$ , where

$$r = \frac{\pi(y|m', \theta'_{m'})\pi(\theta'_{m'}|m')\pi(m')j(m', m)q(u'|m)}{\pi(y|m, \theta_m)\pi(\theta_m|m)\pi(m)j(m, m')q(u|m)}. \quad (7)$$

The proposal densities  $q(u|m')$  and  $q(u'|m)$  can be chosen by investigation of a pilot run for each model parameters vector. Estimates by this pilot run are used to construct proposal densities  $q(u|m')$  and  $q(u'|m)$  taken as multivariate normal densities.

As for the probabilities  $j(m, m')$ , we use

$$j(m, m') = (|M| - 1)^{-1} \text{ for all } m, m' \in M, \quad (8)$$

where  $|M|$  is the number of different models that are used in the reversible-jump MCMC algorithm.

We adopt the same specifications for the seventeen models as used in the previous section. We estimate each model parameters and each model probability as follows.

First, we apply 16 fundamentals-based models and the random walk model over different forecast spans – one-, two-, four-, and eight-quarter ahead forecasts – to the yen-dollar real exchange rates, and construct the proposal density of each model by the MCMC method. To analyze the variability of the appropriate model over time, we implement the inch worm regressions; the regressions in which we update the starting period with keeping the sample size.<sup>5</sup> In this stage, a large sample<sup>6</sup> is taken and an initial part<sup>7</sup> of it is discarded. Then, we pick up every 5th sample.<sup>8</sup> These proposals are taken as multivariate normal densities with the mean vector consisting of the sample mean values and covariance matrix equal to the corresponding sample covariance matrix of the parameters in each model.

Second, our exercise of the reversible jump MCMC is implemented over the two sample periods and over different forecast spans; one-, two-, four-, and eight-ahead forecasts. We take 120,000 samples, discard an initial part (20,000 samples), and pick up every 5th sample. Diagnostic tests by Geweke (1992) suggest that the convergence of the Markov chain has been achieved.

In Figures 1 to 4, we illustrate the probabilities of fundamentals-based models and the random walk model on one-, two-, four-, and eight-quarter-ahead forecasts. In every figure, we omitted some fundamentals-base models that have little or zero probabilities. Figure 1 shows the case of one-quarter ahead forecast. As is the case with the previous section and Meese and Rogoff (1983a), the random walk model is dominant.

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<sup>5</sup>We basically adopt 60 quarters as the sample size of our inch worm regression after considering the robustness of regression and possible structural breaks. As for one-quarter ahead forecast, however, we adopt 70 quarters as the sample size of our inch worm regression. This is because we could not obtain stable results regarding the signs of estimated coefficients in the case of smaller sample size.

<sup>6</sup>12,000 sample. This sample size is determined by convergence tests.

<sup>7</sup>2,000 sample.

<sup>8</sup>Diagnostic tests by Geweke (1992) suggest that the convergence of the Markov chain has been achieved. Our decision about picking samples is made so that the sample should be collected to achieve a nearly non-correlated sample.

Figures 2 to 4 show the cases of two-, four-, and eight-quarter ahead forecasts. In these cases, fundamentals-based models seem to have more predictive power than the random walk model. Based on these figures, appropriate models seem to change over time and over forecast horizons.

As to the two-quarter ahead forecast of Figure 2, the fundamentals-based models of the risk premium type, cumulative current accounts with real interest rate differentials, show high probabilities, especially in the later half.

As for the four-quarter ahead forecast of Figure 3, the fundamentals-based models of the risk premium type are dominant in many periods. Thus, in the first half, government finance with real interest rate differential models show high probabilities. In the late half, the risk premium model, cumulative current accounts with real interest rate differential, shows high probability although its probability has been decreasing recently.

Based on Figure 4 which shows the eight-quarter ahead forecast, the fundamentals-based model of the risk premium type model seems still conspicuous relative to other types of models. However, we cannot find any dominant models. Thus, the uncertainty of the model seems to increase, compared with two- and four-quarter ahead forecasts.

Results of the reversible MCMC analysis suggest that appropriate models on real exchange rates seem to change over time and over forecast horizons. One possible reason for this fact is that real exchange rates are ultimately governed by the fundamentals-based model. If real exchange rates are ultimately governed by the true fundamentals-based model, noisy fluctuations reverse themselves overtime. Thus, the longer the forecast horizon become, the larger the influence of the true fundamentals-based model on real exchange rates is supposed to become. However, the observed uncertainty could not be fully explained by this hypothesis even if appropriate models provide significant information on future exchange rates. This is because the probabilities of appropriate models do not increase when the forecast horizon becomes longer, and the probabilities of appropriate model seem to change too largely over time and over forecast horizons. Uncertainty of appropriate fundamentals-based models with significant information on future real exchange rates might suggest other possibilities: expectations of noisy traders on the true fundamentals-based model, changes of extra economic conditions such as political environments, and other factors.

Thus, the real exchange rate is the exchange ratio of not only domestic and foreign goods but also domestic and foreign assets. This property suggests that future expectations of asset prices could affect real exchange rate movements. Therefore, real exchange rates could deviate from true fundamental value even in the longer horizon. Besides, real exchange rates could be affected by noisy expectations of market participants on the true fundamentals-based model, whatever the true fundamentals-based model is. This story might explain the fact that the appropriate models in two- and

four-quarter ahead forecasts – that is, the cumulative current accounts differential models – cannot increase probabilities in eight-quarter horizon. This hypothesis does not necessarily mean that the cumulative current accounts model is not true, but may imply that those models are overvalued in those forecast horizons.

Meanwhile, even if risk-premium type models such as the cumulative current accounts differential model are important, effects of cumulative current accounts on real exchange rate could be affected by the extra economic conditions such as the possibility of trade friction. Under a continuous trade surplus, occurrence or expectation of trade friction may accelerate the appreciation of exchange rates through political pressures. Trade friction between Japan and the U.S. had been observed in the latter half of the 1980s and the first half of the 1990s. This extra economic condition might accelerate the appreciation of the yen and may also lead to overvaluation of the cumulative current accounts differential model.<sup>9</sup>

Uncertainty of appropriate models has another implication on forecasting. Thus, we could derive a more adequate confidence interval for forecasting by considering the uncertainty of models as one factor of forecast errors. Table 2 shows one example in which we indicate ratios of actual real exchange rates observed in 95% confidence intervals of one-quarter ahead forecast by each model, to total actual values.<sup>10</sup> We calculated each 95% confidence interval for forecasting real exchange rate of each time by each model based on outputs of our MCMC simulations (Figure 5), and then derived each ratio of actual real exchange rates observed in 95% confidence intervals of each model.<sup>11</sup>

We can find the ratios by the approach based on a single model do not necessarily exceed 90%, and are sometimes below 70%. Even the ratio by the random walk model does not reach 95%. One explanation may be that the single model approach does not take the uncertainty of appropriate models into account. Meanwhile, the ratio by the reversible jump MCMC approach exceeds 90% and is approximately 95%. This means that the confidence interval by the reversible jump MCMC method can provide a more adequate confidence interval than that by the single model approach.<sup>12</sup>

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<sup>9</sup>To examine statistically the gradual change of probability over time, we should adopt another method such as Markov switching approach, a subject for future study.

<sup>10</sup>The difference of ratio is more apparent in the shorter-horizon forecast.

<sup>11</sup>The forecast period by our inch worm regression is from 1993/4Q to 2000/4Q; that is, 29 quarters.

<sup>12</sup>The ratio by the PPP model is also high. However, the mean forecast error of the PPP model indicating bias of forecast, is 0.190, which is much larger than those of any other models. Absolute values of mean forecast errors of the other models are below 0.038. In this sense, the confidence interval provided by the PPP model is not appropriate even if most of actual values are observed in this interval.

## 4 Implications

The results of the ordinary approach suggest fundamentals variables of fundamentals-based model contain significant information on mid-term forecasts of real exchange rates. By using the reversible jump MCMC method, we analyze the uncertainty of the real exchange rate forecast model. Our results of this approach suggest the appropriate models could change over time and over forecast horizons. If real exchange rates are ultimately governed by the true fundamentals-based model, the influence of the true fundamentals-based model would increase as the forecast horizon becomes longer. However, observed uncertainty cannot be fully explained by this hypothesis, which suggests other possibilities: expectations of noisy traders on the true fundamentals-based model, changes of extra economic conditions such as political environments, and other factors. These other possibilities have important implications for us. Even if a certain fundamentals-based model contains significant information on mid-term forecasts, it does not necessarily mean that such a fundamentals-based model is the true fundamentals-based model.

Uncertainty of appropriate models also has another important implication on forecast. Thus, model uncertainty would increase forecast uncertainty as well as uncertainty of estimated parameters and shocks of every period. Our approach by the reversible jump MCMC method could provide more adequate confidence intervals for forecasts than that by the approach based a single model.

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# Appendix A

## Model specifications

Estimated exchange rate models are briefly categorized into three types: (i) risk premium type, (ii) PPP type, and (iii) monetary model type.

### (i) Risk premium type

Real interest rate differential model

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (drir)_{t-i-j} + e_t$$

Cumulative current accounts differential model

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (dccca)_{t-i-j} + \sum_{j=0}^n \alpha_{3j} (drir)_{t-i-j} + e_t$$

Optimal current accounts differential model<sup>13</sup>

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (dcoca)_{t-i-j} + \sum_{j=0}^n \alpha_{3j} (drir)_{t-i-j} + e_t$$

Cumulative current accounts plus direct investment model

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (dccadi)_{t-i-j} + \sum_{j=0}^n \alpha_{3j} (drir)_{t-i-j} + e_t$$

Government finance differential model

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (dfd)_{t-i-j} + \sum_{j=0}^n \alpha_{3j} (drir)_{t-i-j} + e_t$$

### (ii) PPP type

Purchasing power parity (PPP) model

$$\bar{q}_t = \frac{1}{T} \sum_t^T q_t$$

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<sup>13</sup>For the formulation excluding real interest rate differential,  $\alpha_{3j}$  is assumed to be zero.

Balassa-Samuelson model

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (rpjpn)_{t-i-j} + \sum_{j=0}^n \alpha_{3j} (rpus)_{t-i-j} + e_t$$

Labor productivity model

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (dlp)_{t-i-j} + e_t$$

**(iii) Monetary model type**

Monetary model

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (dmb)_{t-i-j} + \sum_{j=0}^n \alpha_{3j} (dy)_{t-i-j} + e_t$$

General equilibrium model

$$q_t = \alpha_0 + \sum_{j=0}^n \alpha_{1j} q_{t-i-j} + \sum_{j=0}^n \alpha_{2j} (dyp)_{t-i-j} + e_t$$

## Appendix B

### Definition of Variables

- $q$  : ln (real exchange rate)  
 $drir$  : real interest rate differential  
 $dcca$  : (cumulative current accounts/nominal GDP) differential  
 $dcoca$  : ln (optimal cumulative current accounts differential)  
 $dccadi$  : risk premium fatcor  $dccadi \equiv M_j^u B^j + M_g^u B^g + M_e^u B^e + M_c^u B^c$   
 $M^u$  : variance – covariance matrix between five currencies<sup>14</sup>  
 $B^j, B^g, B^e, B^c$  : cumulative current accounts plus direct investment<sup>15</sup>  
 $dfd$  : (government finance deficit or surplus/nominal GDP) differential  
 $rpjpn$  : ln (Japan tradable goods price/non – tradable goods price)  
 $rpus$  : ln (US tradable goods price/non – tradable goods price)  
 $dlp$  : (real manufacturing production/manufacturing employment) differential  
 $dmb$  : ln (monetary base) differential  
 $dy$  : ln (real GDP) differential  
 $dyp$  : ln (real GDP/population) differential

Note: Current accounts related variables are accumulated from 1975/1Q.

## Appendix C

### Data Sources

	Variables	Sources
<i>Exchange rate</i>	nominal yen-dollar	BOJ
<i>Prices</i>	Japan WPI	BOJ
	US PPI	DOL
<i>Interest rates</i>	Japan Government bonds (10 years)	BOJ
	Japan Gensaki (3 months)	JSDA
	US Government bonds (10 years)	IMF, IFS
	US Treasury Bills (3 months)	FRB
<i>Monetary base</i>	Japan	BOJ
	US	FRB
<i>GDP</i>	Japan	CO
	US	DOC
<i>Population</i>	Japan	<i>U.S. Bureau of the Census, International Data Base</i>
	US	
<i>Employees</i>	Japan	MOHLW
	US	DOL
<i>Current Accounts (Direct investment)</i>	Japan	CO, BOJ
	US	DOC
<i>Government finance</i>	Japan	CO
	US	IMF, IFS

Notes:

(i) BOJ: Bank of Japan, DOL: US Department of Labor, JSDA: Japan Securities Dealers Association, CO: Cabinet Office, DOC: US Department of Commerce, MOHLW: Ministry of Health, Labor and Welfare.

(ii) For both optimal current accounts differential model and cumulative current accounts plus direct investment model, all data was taken from IMF, IFS.

Table 1: F-test of Coefficients on Fundamentals-based Explanatory Variables<sup>a</sup>

Model	One-quarter horizon	Two-quarter horizon	Four-quarter horizon	Eight-quarter horizon
Long-run real interest rate differential model	1.625	1.213	1.405	2.810**
Short-run real interest rate differential model	2.683**	3.451**	1.711	4.705***
Monetary model	0.478	0.541	0.753	2.095**
Balassa-Samuelson model	1.591	2.057**	1.356	2.623**
General equilibrium model	0.849	1.160	1.477	4.467***
Labor productivity model	1.000	1.379	1.353	2.992**
Government finance differential model with $r_l$ <sup>b</sup>	1.289	1.319	1.134	2.648**
Government finance differential model with $r_s$ <sup>c</sup>	2.069**	2.569**	1.544	3.228***
Optimal current accounts differential model	0.744	1.042	1.273	3.524**
Optimal current accounts differential model with $r_l$	1.064	1.041	0.970	3.079***
Optimal current accounts differential model with $r_s$	2.328**	2.967***	1.973*	4.070***
Cumulative current accounts differential model with $r_l$	0.951	0.779	1.034	1.499
Cumulative current accounts differential model with $r_s$	1.849*	2.056**	1.459	3.122***
Cumulative current accounts plus direct investment model with $r_l$	1.470	1.673	1.954*	2.422**
Cumulative current accounts plus direct investment model with $r_s$	2.056*	2.477**	1.718	3.247***

\*\*\*, \*\*, and \* means significant at 1%, 5%, and 10%, respectively.

<sup>b</sup> $r_l$  means long-run real interest rate.

<sup>c</sup> $r_s$  means short-run real interest rate.

Table 2: Ratio of Actual Values Observed in 95% Confidence Intervals of Forecasts to Total Actual Values

Model	Ratio
Long-run real interest rate differential model	0.759
Short-run real interest rate differential model	0.724
Monetary model	0.931
Balassa-Samuelson model	0.862
General equilibrium model	0.827
Labor productivity model	0.862
Government finance differential model with $r_l^a$	0.862
Government finance differential model with $r_s^b$	0.897
Optimal current accounts differential model	0.759
Optimal current accounts differential model with $r_l$	0.862
Optimal current accounts differential model with $r_s$	0.793
Cumulative current accounts differential model with $r_l$	0.793
Cumulative current accounts differential model with $r_s$	0.793
Cumulative current accounts plus direct investment model with $r_l$	0.827
Cumulative current accounts plus direct investment model with $r_s$	0.620
PPP model	0.966
Random walk model	0.931
Reversible Jump MCMC	0.966

<sup>a</sup> $r_l$  means long-run real interest rate.

<sup>b</sup> $r_s$  means short-run real interest rate.

Figure 1. Posterior Probabilities of Models (One-quarter horizon)

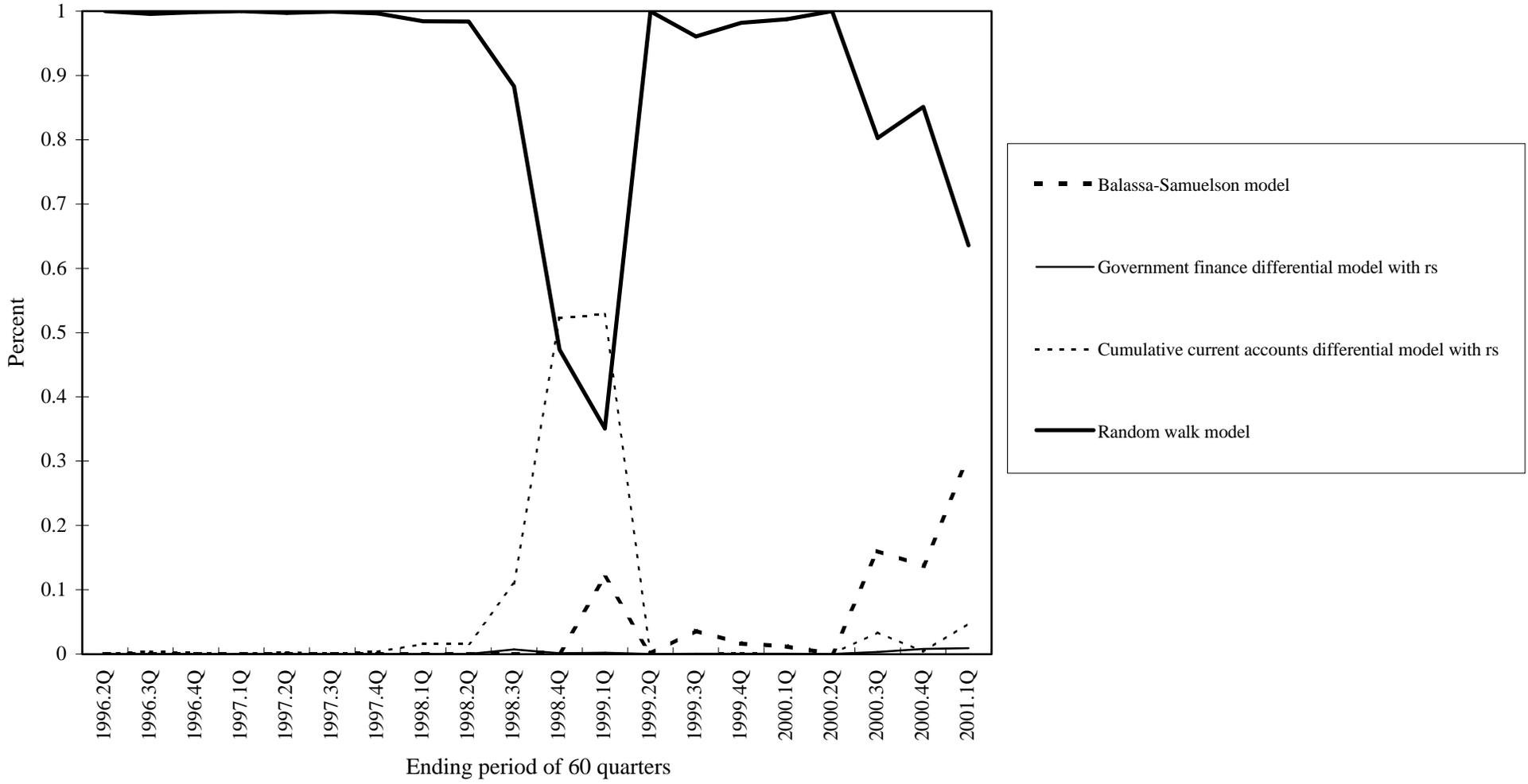
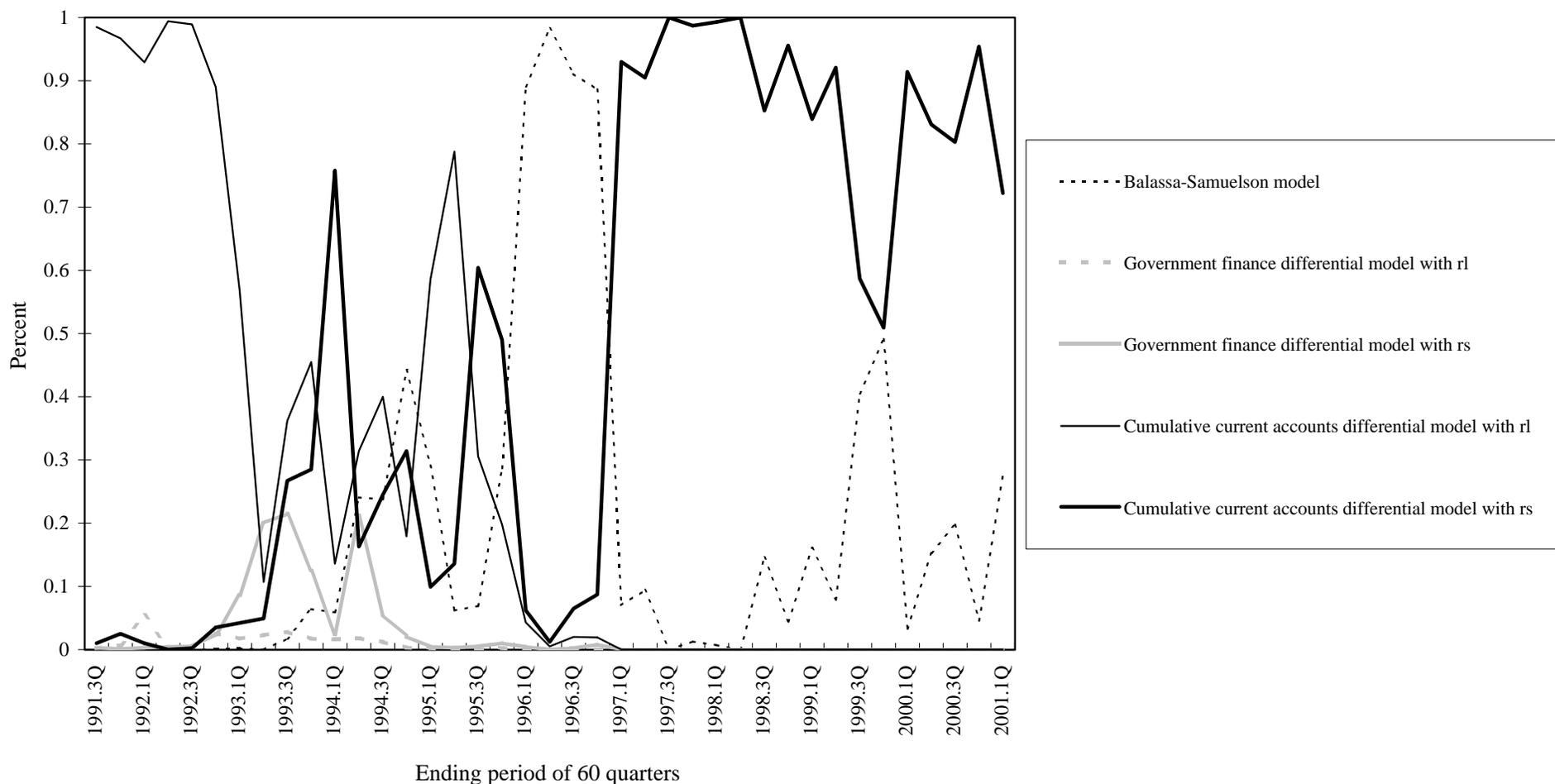
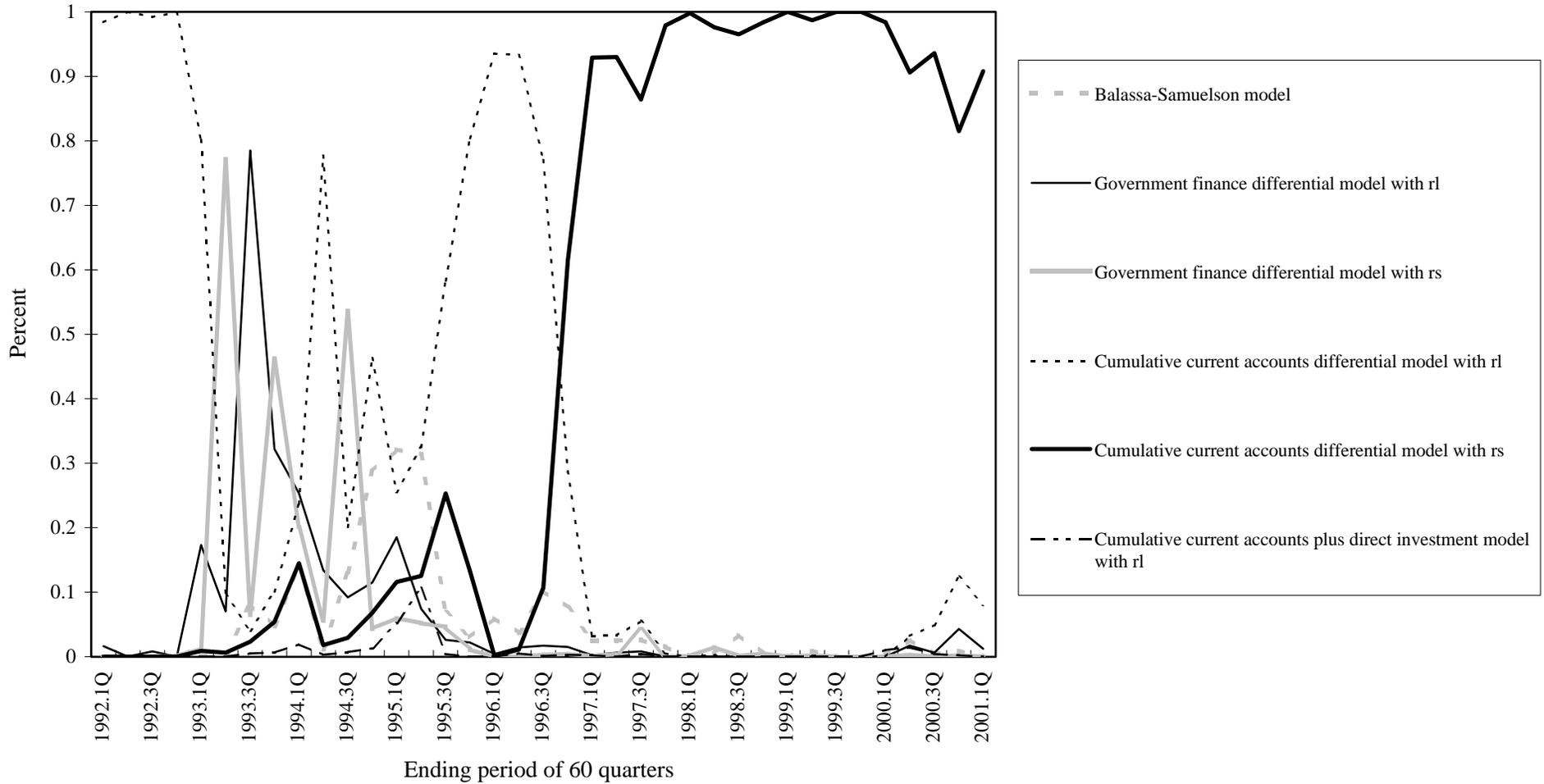


Figure 2. Posterior Probabilities of Models (Two-quarter horizon)



**Figure 3. Posterior Probabilities of Models (Four-quarter horizon)**



**Figure 4. Posterior Probabilities of Models (Eight-quarter horizon)**

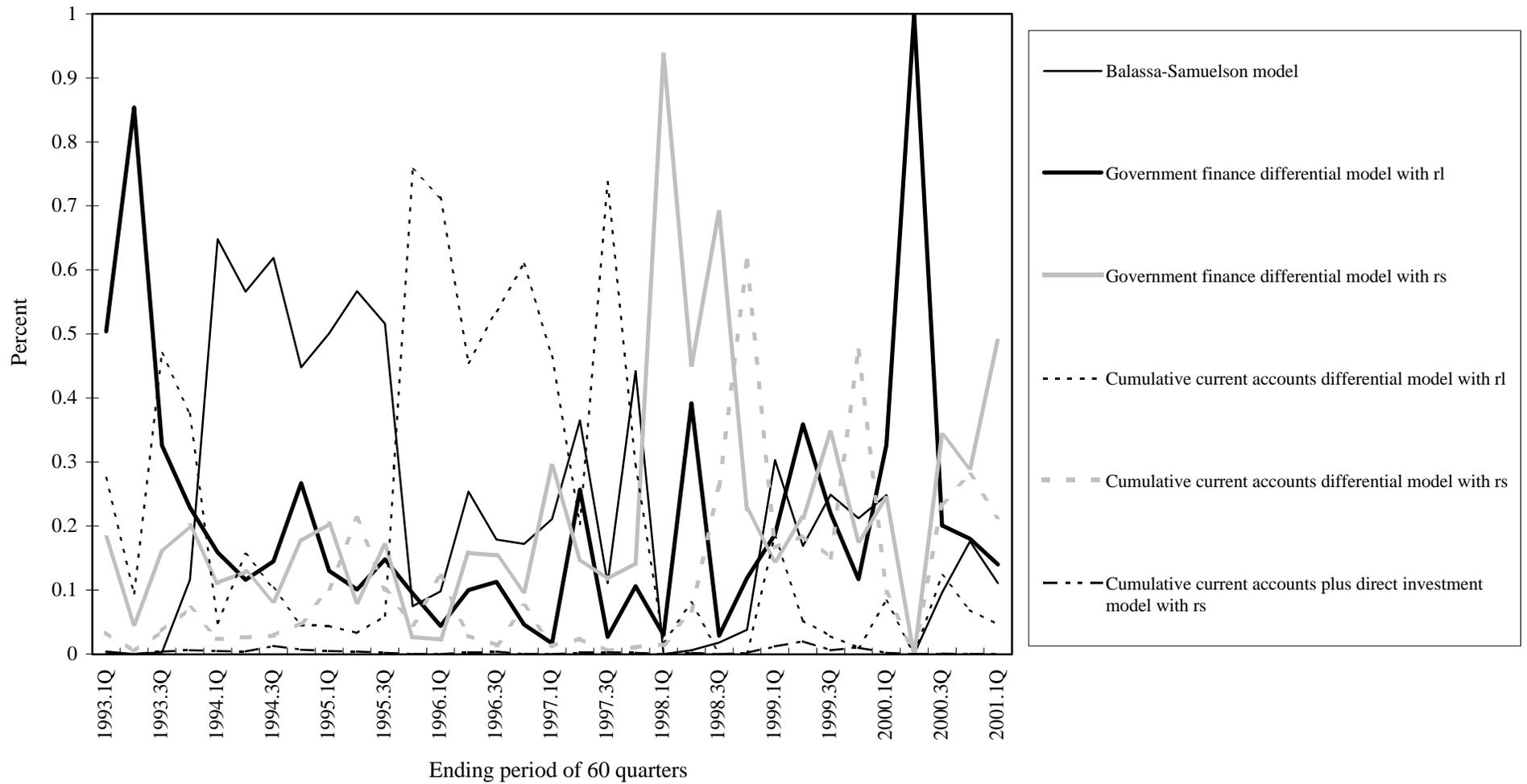


Figure 5. Confidence Intervals of One-quarter ahead Forecast

