Working Paper Series

Deposit Money Creation in Search Equilibrium

Keiichiro Kobayashi[*]

Working Paper 02-4

Research and Statistics Department

Bank of Japan

C.P.O. BOX 203 Tokyo

100-8630 JAPAN

[*] e-mail: kobayashi-keiichiro@rieti.go.jp

Deposit Money Creation in Search Equilibrium*

Keiichiro Kobayashi†

June 4, 2002

Abstract

The endogenous creation of bank credit and of deposit money is modeled. If banks have a limited ability to commit to making interbank loans, then, in order for bank deposits to be accepted as liquid assets, an upper bound is placed upon the size of each bank's asset portfolio, where the bound is determined as a certain multiple of the bank's capital. In our search model, the Central Limit Theorem implies that the multiplier is a non-linear function of the aggregate level of bank assets. Thus when banks have little capital, there emerges an inefficient equilibrium where the production level is low and unemployment exists. In the case where the initial value of bank capital is small and pessimistic macroeconomic expectations prevail, the economy converges on the unemployment equilibrium even if prices are flexible.

Keywords: Limited ability to commit, the interbank market, the Central Limit Theorem, search model.

JEL Classification: C72, E44, E51, G21.

*I wish to thank Hideo Hayakawa, Toshitaka Sekine, Koichiro Kamada, and seminar participants at the Bank of Japan and the Research Institute of Economy, Trade and Industry (RIETI) for many helpful comments. I am also grateful for helpful comments from the anonymous referee. All remaining errors are mine. The views expressed herein are those of the author and not necessarily those of the Bank of Japan or RIETI.

†The Research Institute of Economy, Trade and Industry. E-mail: kobayashi-keiichiro@rieti.go.jp

1 Introduction

This is a simple model to illustrate the macroeconomic role of deposit money creation by banks. We assume the Cash-In-Advance constraint: i.e., that the medium of exchange is restricted to cash in this economy. Our purpose in this paper is to model, within a Cash-In-Advance economic setting, a game theoretic mechanism through which bank deposits are accepted as liquid assets, and to analyze the macroeconomic effect of deposit money creation on the real output of the economy. We show that if banks have incomplete information about withdrawals by depositors and they have no ability to commit beforehand to lend cash to other banks in the interbank market, then the dominant strategy for any bank is to refuse to lend cash to other banks. In this case, the interbank market cannot be formed. If banks have a limited ability to commit, then the condition for the interbank market to be successfully established is that the size of bank lending is no larger than a certain multiple of bank capital where the multiplier is shown to be a function of the aggregate size of bank loans. Thus each bank must limit its lending to a certain multiple of its capital in order for its deposits to be accepted as liquid assets.

When looking at real activity, we use a search model setting (Diamond[1982][1984], Kiyotaki and Wright[1989]) : firms must find trading partners in order to produce final goods, and they need to carry money when searching. The necessity of search induces complementarity among activities by firms. This complementarity and the Central Limit Theorem imply that our *credit multiplier* (the ratio of outstanding bank loans to bank capital) is a non-linear function of the aggregate size of bank loans. Therefore, it is shown that if banks have little capital, there exist multiple equilibria corresponding to different values of the credit multiplier. In the equilibrium corresponding to the small credit multiplier, there is unemployment. Given the appropriate parameter values, the economy converges on the neighborhood of this inefficient equilibrium if the initial value of bank capital is small and pessimistic macroeconomic expectations prevail.

2 Basic Model

In this section, we construct a stylized model to demonstrate the macroeconomic effect of deposit money creation. In the next section, we generalize the model so that we can characterize the general equilibrium completely.

2.1 Economic Environment

The model is an infinite period economy that consists of identical consumers, producing firms, and banks. Time is discrete and goes from zero to infinity. Each consumer is endowed with one unit of labor at the beginning of every period. The only productive input is labor, and the consumer good is the only output. The medium of exchange in this economy is restricted to cash, the amount of which is given exogenously.

Assumption 1 *(Cash-In-Advance Constraint) Firms need to use cash to purchase labor from consumers. Consumers need to use cash to purchase the consumer good.*

Timetable In every period t, events take place in the following order. Details of the events are described in the following subsections unless otherwise noted. Each period t is divided into three subperiods: the beginning (t_b), the intermediate subperiod (t_i), and the end of the period (t_e).

· Subperiod t_b:

(1) Consumers invest their cash C_t in banks. Each consumer is endowed with a unit of labor that is effective only in period t.

(2) Banks engage in corporate lending or investment in other banks. A firm borrows w_t units of cash where w_t is the wage in period t.

(3) Firms deposit the cash in the same bank that they borrowed from[1].

(4) Banks establish membership of the interbank market via a game among banks.

[1]This assumption that firms make deposits in the lender bank is just for simplicity of exposition. Our result does not change qualitatively even if firms do not make deposits in the original lender, but can make deposits in any bank.

· Subperiod t_i:

(5) Firms conduct a search for other firms.

(6) A firm encounters another firm according to a random matching.

(7) After the search activity ends, the following events occur simultaneously (or in random order):

(7-1) In order to pay wages, matched firms either withdraw their deposits, or sell their deposit certificates to other banks. Banks accept the deposit certificates if the issuing banks are members of the interbank market.

(7-2) Firms pay wages to consumers, and produce the consumer good using the labor input.

(7-3) Consumers divide their wage income equally according to an unemployment insurance scheme.

(7-4) Consumers deposit their wage income in their bank accounts.

· Subperiod t_e:

(8) Consumers borrow cash $p_t c_t$ from banks and buy c_t units of the consumer good where p_t is the price of the consumer good in period t.[2]

(9) Firms repay p_t to banks ($p_t > w_t$).

(10) The interbank market opens to settle interbank loans. Banks that issued deposit certificates (issuing banks) pay w_t to the banks holding their certificates (holder-banks).

(11) Banks pay out the remaining surplus ($(1 + r_t)C_t + w_t\lambda_t - p_t c_t$) to consumers.[3]

Consumer There are N consumers who have identical preferences. N is a very large integer. At the beginning of period t ($t = 0, 1, \cdots, \infty$), each consumer has one unit of labor and initial wealth C_t in the form of cash, which she invests in the bank as bank capital. Then a firm employs the consumer with a certain probability (specified later), and the consumer obtains the wage w_t. The consumer may only use her labor to produce the consumer good if she is employed. Otherwise she cannot produce the consumer good, since only firms have access to the production technology. For simplicity, we assume that

[2]See footnote 16.

[3]See footnote 17.

there is unemployment insurance among consumers so that labor-income w_t is equalized among all consumers no matter who is employed. Thus each consumer obtains $w_t \lambda_t$ where λ_t is the endogenous probability of being employed (See equation (2)).

After production has taken place, the consumer chooses her consumption for the current period (c_t). Thus the consumer's problem is the following. Given $\{p_t, w_t, \lambda_t, r_t\}_{t=0}^{\infty}$ and C_0, the consumer solves

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to $p_t c_t + C_{t+1} \leq w_t \lambda_t + (1 + r_t) C_t$, where p_t is the price of the consumer good and r_t is the rate of return on the investment in bank capital, while $u(\cdot)$ is a standard utility function that satisfies $u' > 0$ and $u'' \leq 0$. For simplicity, we also assume that consumers have insurance covering their investments in banks so that each consumer obtains the same return on her investment C_t. Thus r_t is a deterministic number for each consumer.

Firms The number of firms is $\overline{K}$ where $\overline{K}$ is a large integer that satisfies $\overline{K} > N$.[4] We assume that the production technology of one firm is incomplete: a firm is only able to produce a unit of the consumer good if and only if it meets another firm in the market and the two firms exchange information, thus enabling both firms to complete their production technologies. Therefore, a firm needs to search for another firm before it can employ the laborer (= the consumer) to produce the consumer goods. This production technology is a replica of that in Diamond [1982].[5] Although this assumption on the production technology seems too restrictive, it is one way to formulate the complementarity among the activities of economic agents that is commonly observed.

A firm needs to borrow cash from a bank when it starts searching. This is because

[4] The number of firms can be infinite. The existence of the upper limit $\overline{K}$ is not necessary to derive our main results.

[5] We can interpret this technology as a technology of trading like that in the Diamond-model. Suppose that production of the consumer good necessitates a division of labor between two firms. Each firm can produce only an intermediate good, and the final good is produced by combining two intermediate goods. In this case, firms need to search for a trading partner with whom to divide the labor of producing the final good.

of the following assumption.

Assumption 2 *It is only when two firms match that they become able to employ a laborer and so produce the consumer good. The firms must pay the wage in the form of cash before they sell the consumer good. After the firms match, there is no chance before production for them to borrow cash from banks. Firms do not hold cash initially.*

This assumption implies that a firm must borrow w_t units of cash from a bank before it starts searching. As we show in Section 2.2, each bank determines the size of its lending so that condition (10) in Section 2.2 is satisfied. In this case, all banks join the interbank market so that the certificates of deposit issued by banks are accepted as perfectly liquid assets, and all depositors can withdraw their deposits at any time (See Section 2.2). Therefore, a firm borrows cash (w_t) from a bank and deposits the cash in the bank when it starts searching since it regards the bank deposit as a perfect substitute for cash.[6]

Search Technology If L_t firms engage in the search activity in period t, then $q(L_t)L_t$ firms encounter one another according to a random matching process. $q(L_t)$ describes the probability that a firm makes a successful match in this way.

Assumption 3 *The probability* $q(L)$ *is defined such that* $0 \leq q(L) \leq 1$ *and* $q(L)$ *is a strictly increasing function of* L. *If* L (> 0) *is sufficiently small, then* $q(L) = \kappa L + o(L)$ *where* $\kappa > 0$ *and* $\lim_{L \to 0} o(L)/L = 0$.

This assumption that $q(L)$ is strictly increasing in L is justified as follows: firms search for one another by walking randomly around a vast field; in such a case, the probability of encounter is an increasing function of the number of firms.[7] $\text{Min}\{q(L_t)L_t, N\}$ firms

[6] Alternatively, we can assume that firms have a small risk of losing the cash by accident if they carry it during the search activity. In this case, firms strictly prefer making a deposit to holding cash if they believe that the bank deposit is perfectly liquid.

[7] A more realistic explanation for the plausibility of Assumption 3 is the following. A firm searches for a trading partner, who has certain characteristics, by checking the telephone directory. She looks first in the directory for New York, and if she cannot find a trading partner there, then she looks in the directory for New Jersey, and so on for other cities. She repeats this process within a limited time

can employ workers and produce the consumer good. The amount of the consumer good produced in period t is also $\min\{q(L_t)L_t, N\}$.

Banks We assume that there are M banks in the economy. The number of banks M is a very large integer but is negligibly small compared to N:

$$1 \ll M \ll N. \tag{1}$$

This assumption that $M \ll N$ is crucial for the approximation by the Central Limit Theorem (See Section 2.2). Assuming that banks receive an identical amount of capital, each bank has $\frac{N}{M}C_t$ units of cash invested in it by consumers. Given bank capital of $\frac{N}{M}C_t$, a bank can lend w_t to at most $\frac{N}{Mw_t}C_t$ firms. Before starting the search, the debtor firms deposit the cash they borrowed in the bank, since they regard the bank deposit as perfectly liquid, as long as banks satisfy condition (10) in Section 2.2. Thus, the bank has $\frac{N}{M}C_t$ units of cash and $\frac{N}{M}C_t$ units of loans to firms on the asset-side of its balance sheet, and $\frac{N}{M}C_t$ units of deposits made by firms and $\frac{N}{M}C_t$ units of capital on the liability-side. Since the bank has $\frac{N}{M}C_t$ units of cash in hand at this stage, it can lend this cash to other firms and let them in turn make their deposits within its vaults. By iterating this procedure, a bank can expand its balance-sheet (as long as it can satisfy condition (10) in Section 2.2). The withdrawal of deposits is specified later. For now, suppose that banks have made loans to l_t firms and that all deposits, D_t, are liabilities to these same firms. The only condition that a bank must satisfy is $D_t = w_t l_t$, where $w_t l_t$ describes its total lending to firms. In other words, each bank has $w_t l_t$ of loans to firms and $\frac{N}{M}C_t$ of cash on the asset-side, and $w_t l_t$ of deposits and $\frac{N}{M}C_t$ of capital on the liability-side. The $\frac{N}{M}C_t$ units of cash on the asset-side are reserves against the withdrawal of deposits. We assume the following loan contract between banks and firms.

interval. Suppose that it takes a certain time to get hold of a telephone directory, but that it does not take any time to check it. If the number of firms increases in every city, then the number of firms listed in the directory also increases. Then the probability that a firm can find a trading partner within a limited time interval increases. In other words, if there exists increasing returns in the information processing technology of searchers, then the matching probability $q(L)$ is increasing in the number of searchers L.

Assumption 4 *If a firm borrows w_t units of cash from a bank and deposits it immediately in the bank, then this deposit is a demand deposit which receives no interest payment. When the firm makes a deposit, the bank delivers a certificate of deposit with face value w_t to the firm. The issuing bank must pay cash w_t in exchange for the certificate at any time to anybody. After borrowing, the firm starts searching for another firm. When the firm encounters another firm, it withdraws its deposit w_t, pays wage w_t to a worker, and produces one unit of the consumer good using the labor input. After production, the firm sells the good at price p_t. The firm repays p_t to the bank at the end of period t. If the firm does not encounter another firm when searching, then the firm's deposit of w_t and her debt of w_t offset each other at the end of period t.*

Since successful firm encounters are generated at random, the number of firms in debt to a given bank that succeed in employing workers is a random variable. We denote this random variable by $\tilde{m}_t$:

$$\tilde{m}_t = \sum_{i=1}^{l_t} \tilde{z}_{it},$$

where $\tilde{z}_{it}$ is 1 with probability η_t, and 0 with probability $(1 - \eta_t)$, where η_t is the probability that a firm runs into another firm and then successfully employs a worker. Since $M \gg 1$, we can approximate the $\tilde{z}_{it}$s $(i = 1, \cdots, l_t)$ to identically and independently distributed random variables, so that $\tilde{m}_t$ becomes a random variable that follows a binary distribution $b(l_t, \eta_t)$. $E[\tilde{m}_t] = \eta_t l_t$, and $V[\tilde{m}_t] = (1 - \eta_t)\eta_t l_t$. In the equilibrium, $\eta_t = \frac{y_t}{L_t}$ where $y_t = \min\{q(L_t)L_t, N\}$ describes the number of firms that successfully match and hence the level of output, and $L_t = Ml_t$ is the total number of searching firms in the economy.

2.2 Liquidity of Bank Deposit

After the process of random matching is complete, matched firms withdraw their deposits to pay wages to their workers. The total withdrawal from a bank is a random variable $w_t\tilde{m}_t$. Payment of wage w_t to a worker is done in the form of cash. Total wage $w_t y_t$ is divided equally among all consumers as set out in the unemployment insurance scheme.

Thus each worker (a consumer) receives the cash $w_t \lambda_t$ where

$$\lambda_t = \frac{y_t}{N}, \tag{2}$$

and she immediately deposits it in her bank account. We assume that the number of consumers' accounts is equal for each bank. Therefore, each bank obtains new deposits of $\frac{w_t y_t}{M}$ from workers. We can regard a bank as defaulting, i.e. failing to meet the total demand for withdrawals from it[8], if

$$\frac{N}{M} x_t < \tilde{X}_t \equiv \tilde{m}_t - \frac{y_t}{M}, \tag{3}$$

where $x_t \equiv \frac{C_t}{w_t}$, $E[\tilde{X}_t] = \eta_t l_t - \frac{y_t}{M}$, and $V[\tilde{X}_t] = (1-\eta_t)\eta_t l_t$. In the equilibrium where $l_t = \frac{L_t}{M}$, we have $E[\tilde{X}_t] = 0$, and $V[\tilde{X}_t] = (1-\eta_t)\eta_t \frac{L_t}{M}$.

The Central Limit Theorem implies that the random variable $\tilde{W}$ converges in distribution to a random variable that follows the normal distribution $N(0,1)$ as $\frac{L_t}{M}$ becomes a large number, where $\tilde{W}$ is defined by

$$\tilde{W} = \frac{\tilde{X}_t}{\sqrt{\eta_t (1-\eta_t) \frac{L_t}{M}}}.$$

The interbank market and withdrawal by firms When matching firms withdraw w_t from their accounts, the total withdrawal for all banks is $w_t y_t$, while the total deposit that workers make is also $w_t y_t$. Therefore, if there is an interbank market where banks can lend and borrow cash to and from one another, then all banks can meet the total demand for withdrawals by firms.

In this subsection, we derive the main result of our model. In order for banks who have a limited ability to commit and incomplete information about withdrawers to form an interbank market, the variance $V[\tilde{X}_t] = (1-\eta_t)\eta_t l_t$ must not exceed an upper limit that is an increasing function of the bank capital ($\frac{C_t}{w_t}$).

[8] We need to make more technical assumptions about the timing of withdrawals by firms and of deposits by workers, in order to derive condition (3). The technical assumptions make our model unnecessarily complicated without making any qualitative difference to the results. Thus we just assume that a bank defaults if and only if the sum of existing cash ($\frac{N}{M} C_t$) and incoming cash ($w_t \frac{y_t}{M}$) is less than outlay ($w_t \tilde{m}_t$).

The intuition is the following. Consider the game among banks in which they choose either to agree or to refuse to lend cash to one another when their depositors (firms) withdraw w_t. Suppose that a bank (bank i) cannot commit beforehand to lend cash to other banks. In this case, bank i can obtain a non-negative surplus by refusing to lend cash to other banks no matter what the other banks' strategy is: a bank is strictly better off by refusal, if all the other banks refuse to lend cash to one another; at the same time, if some banks agree to lend cash to bank i while bank i refuses to lend cash to other banks, then bank i can use all of its own cash in addition to those banks' cash in order to meet withdrawals by its depositors. Therefore, the dominant strategy for a bank is to refuse to lend cash to other banks. Thus banks that have no ability to commit cannot form an interbank loan market.

Suppose, however, that banks have a limited ability to commit: a bank incurs the penalty ϵ (> 0) if it breaks its precommitment to lend to other banks. In this case, if the surplus that a bank can obtain by refusal is no greater than ϵ, then the interbank market in which all banks agree to lend to all other banks becomes a Nash equilibrium. The condition that the surplus from refusal is no greater than ϵ is shown to be $V[\tilde{X}_t] \leq \phi(\frac{C_t}{w_t})$ where $\phi(\cdot)$ is a positive and increasing function.

In the following, we formalize the intuition described above.

A game of interbank market formation We assume that the following interbank game is played after lending $(w_t l_t)$ is determined, the matchings of firms are realized, and all workers have deposited their wage income in their bank accounts[9].

In the first stage of the game, firms who succeeded in matching form a queue. The order of the firms in the queue is random. The i-th firm $(i = 1, 2, \cdots, \sum_1^M \tilde{m}_k.)$ from the head of the queue goes to the bank $[i]_M$ where $[i]_M = k$ satisfies $1 \leq k \leq M$ and $i = mM + k$ for a non-negative integer m. We assume the following incomplete information for banks:

[9]The deposit of wage income ($\frac{w_t y_t}{M}$ per bank) must be made simultaneously with the withdrawal by firms. But in order to simplify the argument, we assume that all workers deposit their wages *before* firms start to withdraw their deposits. It is easily confirmed that this simplification does not affect the result.

Assumption 5 *A bank cannot observe whether the firms remaining in the queue are the bank's own depositors or not.*

In the second stage of the game, a bank (bank k) either give cash w_t to the i-th firm ($[i]_M = k$) or not. If firm i is a depositor of bank k, then bank k has no choice other than to pay w_t to the firm in exchange for its certificate of deposit. If firm i is the depositor of bank k' ($k' \neq k$), then bank k can choose whether to agree or to refuse to give cash w_t to firm i in exchange for the certificate of deposit issued by bank k'. If bank k accepts the deposit certificate held by firm i, then bank k gives cash w_t to firm i. Bank k sends this deposit certificate to bank k' in the final stage (i.e., the settlement stage) of the game, and bank k' pays cash w_t to bank k in exchange for the deposit certificate[10]. If bank k rejects the deposit certificate held by firm i or bank k runs out of cash before firm i comes, then firm i is left with no cash in hand.

The third stage of the game starts after all firms in the queue have visited some bank. In the third stage, the remaining firms who failed to obtain cash form a second queue. They form the queue in random order. The procedures for the first and second stage are then repeated.

We can make the following observation concerning bank strategy. A bank (bank k) will be weakly better off by refusing to accept deposit certificates issued by other banks as long as its own depositors remain in the queue. This is because bank k can spare cash for withdrawals by its own depositors who remain in the queue by rejecting

[10]Firm i repays $p_t(> w_t)$ to bank k' after it sells the consumer good. Thus it might seems natural to assume that bank k and bank k' split the surplus $p_t - w_t$ equally. However, the assumption that bank k' pays w_t to bank k means that all surplus ($p_t - w_t$) is taken by bank k' (i.e., the original lender to firm i). This assumption is justified as follows: it is plausible to regard bank lending as a form of relationship lending in which the original lender obtains an information rent (See, for example, Diamond and Rajan[2001]). Before the game starts, bank k' provided the relationship lending for firm i. Thus we regard (at least a part of) the surplus $p_t - w_t$ to be the information rent to bank k', which cannot be transferred to bank k.

Although we assume for simplicity that bank k' (the original lender) takes all the surplus, the qualitative nature of our result does not change as long as bank k' takes a larger part of the surplus $p_t - w_t$ than bank k.

other banks' depositors, while successful withdrawals by its own depositors give positive surplus to bank k (the original lender). This observation and the information asymmetry (Assumption 5) justify the following restriction on the choice by banks:

Assumption 6 *A bank that decides to reject a certificate of deposit issued by another bank continues to reject all such certificates for the rest of the game.*

It is easily confirmed that banks always agree to establish an interbank market, i.e., the first best outcome, if they have complete information about withdrawers and they can change their strategies over the sequence of the game.

This three-stage subroutine is repeated until either (1) all cash has been given to firms, (2) all firms in the original queue have obtained cash, or (3) all the remaining cash is possessed by banks that refuse to give cash to firms holding the deposit certificates of other banks, and all remaining firms are depositors of banks who no longer possess cash.

The final stage (i.e., the settlement stage) takes place after the production and sale of the consumer good by firms. Bank k that gave cash w_t to firm i in exchange for the deposit certificate issued by bank k' sends the certificate to bank k', and bank k' pays cash w_t to bank k in exchange for the certificate.

The Prisoners' Dilemma in formation of the interbank market In the above game, banks choose their strategies simultaneously at the beginning of the game. The strategy for bank k is to specify whether it accepts or rejects the certificates of deposit issued by bank k' ($k' \neq k$, $k' \in \{1, 2, \cdots, M\}$). Simultaneous and independent choice of strategy by individual banks makes the situation that we should analyze quite complicated: for example, bank 1 accepts[11] bank 2 but rejects bank 3; bank 2 accepts bank 3 but rejects bank 1; and bank 3 accepts bank 1 but rejects bank 2.

In order to avoid this complication of the analysis, we assume that banks form an interbank market according to the process described below:

[11] We say "bank k accepts (rejects) bank k'," when bank k agrees (refuses) to give cash w_t to anybody in exchange for the deposit certificate issued by bank k'.

Assumption 7 *Let Group* $[1, k-1]$ *denotes the group that consists of banks* 1 *through to* $k-1$ *(*$k = 2, \cdots, M$*) where any pair of the member banks of the group accept each other. The interbank market is formed by the following iteration. (i) Assume that Group* $[1, k-1]$ *acts as a unanimous agent whose objective is to maximize per capita payoff of its member banks; (ii) Bank* k *and Group* $[1, k-1]$ *simultaneously decide whether to accept or to reject each other*[12]*; (iii) If they accept each other, then bank* k *and Group* $[1, k-1]$ *merges and they become Group [1, k]. If this iteration is repeated successfully from* $k = 2$ *to* $k = M$*, then all* M *banks join together to form the interbank market, i.e., Group* $[1, M]$*.*

Now, we show that banks that do not have the ability to commit beforehand cannot form an interbank market.

Lemma 1 *In the case where banks have no ability to commit to accept other banks beforehand, the dominant strategy for bank* k *is to reject Group* $[1, k-1]$ *for all* $k \in \{2, 3, \cdots, M\}$*.*

Proof

Case (i): $k = M$.

The game between bank M and Group $[1, M-1]$ is a simultaneous game with two choices of strategy: A (accept) or R (reject). Note that every bank has fixed its asset size $(w_t l_t)$ beforehand. If both bank M and Group $[1, M-1]$ accept each other, then each bank k $(k = 1, \cdots, M)$ obtains the expected value of the surplus $(p_t - w_t)E[\tilde{m}_t] = (p_t - w_t)\frac{y_t}{M}$. If both choose rejection, then condition (3) and the Central Limit Theorem imply that bank M's expected surplus is

$$\begin{aligned} & (p_t - w_t)\left\{\sum_{m=1}^{\frac{Nx_t}{M}+\frac{y_t}{M}} m \Pr\{\tilde{m}_t = m\} + \left(\frac{Nx_t}{M} + \frac{y_t}{M}\right) \Pr\{\tilde{m} > \frac{Nx_t}{M} + \frac{y_t}{M}\}\right\} \\ \approx \quad & (p_t - w_t)\{E[\tilde{m}_t] - \textstyle\int_{\frac{Nx_t}{M}}^{\infty} \left(\tilde{X}_t - \frac{N}{M}x_t\right) dF_X(\tilde{X}_t)\} \\ = \quad & (p_t - w_t)\{\frac{y_t}{M} - \sigma_t \Psi(\frac{Nx_t}{M\sigma_t})\} \end{aligned} \tag{4}$$

[12] Assume that both bank k and Group $[1, k-1]$ make the decision on the premise that bank i and bank j reject each other for any pair of i and j where $i \in \{1, 2 \cdots, k\}$ and $j \in \{k+1, k+2, \cdots, M\}$.

where

$$\sigma_t \equiv \sqrt{V[\tilde{X}_t]} = \sqrt{(1-\eta_t)\eta_t l_t}, \tag{5}$$

and

$$\Psi(z) = \int_z^\infty (y-z)\frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy. \tag{6}$$

Thus

$$\Psi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} - z(1-\Phi(z)), \text{ where } \Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy. \tag{7}$$

In this case, the condition for default by Group $[1, M-1]$ approximates to

$$\frac{N}{M} x_t < \tilde{X}_{M-1}, \text{ where } \tilde{X}_{M-1} \sim N\left(0, \frac{1}{\sqrt{M-1}}\sigma_t\right), \tag{8}$$

since $\tilde{X}_{M-1} = \frac{1}{M-1}\sum_{k=1}^{M-1} \tilde{X}_k$ where $\tilde{X}_k$ is defined in (3) and is approximately normally distributed $N(0, \sigma_t)$.[13] Thus the surplus for a member bank of Group $[1, M-1]$ can be approximated by $(p_t - w_t)\{\frac{y_t}{M} - \frac{\sigma_t}{\sqrt{M-1}}\Psi\left(\frac{N x_t \sqrt{M-1}}{M\sigma_t}\right)\}$.

If Group $[1, M-1]$ chooses A while bank M chooses R, then bank M obtains $(p_t - w_t)\frac{y_t}{M}$, i.e., the same surplus as in the case where both choose A. This is because all depositors of bank M in the queue obtain cash in exchange for the certificates issued by bank M. At the same time, a member bank of Group $[1, M-1]$ obtains strictly less surplus than in the case where both choose R.

If Group $[1, M-1]$ chooses R while bank M chooses A, then a member bank of Group $[1, M-1]$ obtains strictly greater surplus than in the case where both choose A, while bank M obtains strictly less surplus than in the case where both choose R.

Therefore, the dominant strategy for both bank M and Group $[1, M-1]$ is R.

Case (ii): $k < M$.

Note that banks $k+1, k+2, \cdots, M$ reject banks $1, 2, \cdots, k$. Thus bank k and Group $[1, k-1]$ decide their strategies without influence from banks $k+1, \cdots, M$. If both bank k and Group $[1, k-1]$ accept each other, then the condition for default by Group $[1, k]$

[13] To be more precise, the random variables $\tilde{X}_k$ $(k = 1, 2, \cdots, M)$ are not mutually independent, since $\sum_1^M \tilde{X}_k = 0$. However, we can approximate the distribution of $\tilde{X}_{M-1}$ with that of a normal random variable distributed $N\left(0, \frac{\sigma_t}{M-1}\right)$.

approximates to

$$\frac{N}{M}x_t < \tilde{X}_k, \text{ where } \tilde{X}_k \sim N\left(0, \frac{1}{\sqrt{k}}\sigma_t\right). \tag{9}$$

Thus the payoff for bank k and for a member bank of Group $[1, k-1]$ are the same: $(p_t - w_t)\{\frac{y_t}{M} - \frac{\sigma_t}{\sqrt{k}}\Psi(\frac{Nx_t\sqrt{k}}{M\sigma_t})\}$.

If both bank k and Group $[1, k-1]$ reject each other, then the payoff for bank k is $(p_t - w_t)\{\frac{y_t}{M} - \sigma_t\Psi(\frac{Nx_t}{M\sigma_t})\}$.The payoff for a member bank of Group $[1, k-1]$ is $(p_t - w_t)\{\frac{y_t}{M} - \frac{\sigma_t}{\sqrt{k-1}}\Psi(\frac{Nx_t\sqrt{k-1}}{M\sigma_t})\}$.

If bank k chooses R while Group $[1, k-1]$ chooses A, then bank k obtain the same benefit as in the case when both choose A: i.e., a depositor of bank k who comes across a bank in Group $[1, k-1]$ can get cash in exchange for the deposit certificate of bank k, generating the surplus $p_t - w_t$ for bank k. In addition to this, bank k can use all its cash exclusively to meet the withdrawals made by its own depositors. Thus the surplus for bank k is strictly greater than the payoff in the case when both choose A. A similar logic implies that a member bank of Group $[1, k-1]$ obtains strictly less surplus than in the case when both choose R.

If bank k chooses A while Group $[1, k-1]$ chooses R, then the payoff for a member bank of Group $[1, k-1]$ is strictly greater than in the case when both choose A. Meanwhile bank k obtains strictly less surplus than in the case when both choose R.

Therefore, the dominant strategy for both bank k and Group $[1, k-1]$ is R. (*End of Proof*)

This lemma implies that the game among banks has a unique Nash equilibrium in which all banks choose to reject all other banks. Thus banks that do not have the ability to commit beforehand cannot form an interbank market when they lack complete information concerning withdrawers (Assumption 5).

Formation of the interbank market under limited ability to commit Now we assume that banks have a limited ability to commit:

Assumption 8 *A bank incurs the fine ϵ if it rejects deposit certificates issued by banks* *which it had promised to accept. The fine ϵ is an exogenous parameter for the bank.*

In this case, we can show the following lemma:

Lemma 2 *In the case where banks have a limited ability to commit, the sufficient condition for the existence of an equilibrium in which both bank k and Group $[1, k-1]$ accept each other for all $k \in \{2, 3, \cdots, M\}$ is*

$$\theta_t \equiv \frac{\sigma_t}{\sqrt{2}} \Psi \left(\frac{N x_t \sqrt{2}}{M \sigma_t} \right) \leq \frac{\epsilon}{p_t - w_t}. \tag{10}$$

(*Proof*)

It is sufficient to examine the case where $k = 2$. Both bank 1 and bank 2 obtain $(p_t - w_t)\{\frac{y_t}{M} - \theta_t\}$ respectively by accepting each other. Suppose one bank (bank 1) deviates from this strategy and rejects bank 2, while bank 2 accepts bank 1. In this case, bank 2 gives cash w_t whenever bank 2 comes across a depositor of bank 1, generating the surplus $p_t - w_t$ for bank 1, and bank 1 uses all its cash exclusively to meet the withdrawals made by its own depositors. In this case, the value of bank 1's expected surplus is in between $(p_t - w_t)\{\frac{y_t}{M} - \theta_t\}$ and $(p_t - w_t)\frac{y_t}{M}$. Since bank 1 incurs the fine ϵ by deviation, bank 1 always accepts bank 2 as long as bank 2 accepts bank 1 in the case when condition (10) is satisfied. Therefore, that the both banks accept each other is a Nash equilibrium of the game between bank 1 and bank 2. For the case of bank k and Group $[1, k-1]$ where $k > 2$, the surplus that a bank can obtain by deviation is smaller than θ_t. Thus condition (10) is sufficient. (*End of Proof*)

This lemma guarantees the existence of a Nash equilibrium in which all banks accept all other banks. Note that this is not the unique Nash equilibrium. There exists another Nash equilibrium in which all banks reject all other banks, as long as ϵ is negligibly smaller than $\frac{y_t}{2M}$.

2.3 Equilibrium Lending

Banks determine the volume of lending $w_t l_t$ so that condition (10) is satisfied. Condition (10) can be rewritten as follows since $\Psi(z)$ is a strictly decreasing function.

$$\frac{N}{M}x_t \geq \Psi^{-1}\left(\frac{\epsilon\sqrt{2}}{(p_t - w_t)\sqrt{(1-\eta_t)\eta_t l_t}}\right)\frac{\sqrt{(1-\eta_t)\eta_t l_t}}{\sqrt{2}}. \tag{11}$$

The right hand side is an increasing function of $\sqrt{(1-\eta_t)\eta_t l_t}$. Since $\eta_t = \frac{y_t}{L_t}$ where $y_t = \min\{q(L_t)L_t, N\}$, solving this condition with equality gives the bank's choice of l_t as a function of L_t : $l_t = F(L_t)$.

To simplify the exposition, we assume the following:

Assumption 9 *The fine ϵ is proportional to the surplus from one lending $(p_t - w_t)$ times standard error in the number of withdrawals $(\sqrt{(1-\eta_t)\eta_t l_t})$. Thus*

$$\epsilon = \xi(p_t - w_t)\sqrt{(1-\eta_t)\eta_t l_t},$$

where ξ is a constant.

In this simplified case, condition (11) requires that $\frac{N}{M}x_t$ be proportional to $\sqrt{(1-\eta_t)\eta_t l_t}$. Thus condition (11) can be rewritten as

$$\frac{N}{M}x_t = G\sqrt{\eta_t(1-\eta_t)\frac{L}{M}}, \tag{12}$$

where G is a positive constant and G increases as ξ decreases. The constant G represents the inefficiency of the interbank market, i.e., the difficulty for banks to commit.

In the symmetric equilibrium where all banks choose the same volume of lending, each bank chooses $l_t = \frac{L_t}{M}$ that satisfies (12) in order to be accepted as a member of the interbank market. Given that banks form the interbank market, the bank does not default, but can meet the full demand for withdrawals by depositors even in the case where $\tilde{X}_t$ becomes large enough to satisfy (3).

Since $\eta_t = q(L_t)$ for $L_t \leq L_0$ and $\eta_t = \frac{N}{L_t}$ for $L_t \geq L_0$ where L_0 is determined by $N = q(L_0)L_0$, equilibrium lending is determined by

$$Lq(L)(1-q(L)) = \frac{N^2}{G^2 M}x_t^2 \quad \text{if } L \leq L_0 \tag{13}$$

$$1 - \frac{N}{L} = \frac{N}{G^2 M}x_t^2 \quad \text{if } L_0 \leq L \leq \overline{K}, \tag{14}$$

and $L = \overline{K}$ if neither (13) nor (14) is satisfied by any value of L between 0 and $\overline{K}$. Multiple equilibria emerge for appropriate values of bank capital C_t and w_t. See Figure 1.

Figure 1: Equilibrium Lending

In Figure 1, the parameter values are chosen as follows: $q(L) = \frac{1}{1+\exp\{-\frac{1}{10}(L-70)\}}$, $N = 100$, $M = 50$, and $G = 60$.[14]

When $x_t \equiv \frac{C_t}{w_t}$ is sufficiently large ($x_t = x_1$), then L_t goes to the upper bound: $L_t = \overline{K}$. When $x_t = x_2$, then there exists a unique equilibrium L_h^*. When x_t is chosen to be appropriately small ($x_t = x_3$), then there emerges three equilibrium values for lending: L_h^*, L_m^*, and L_l^* where $L_l^* < L_m^* < L_h^*$. When x_t is sufficiently small ($x_t = x_4$), there exists only one equilibrium L_l^*.

We examine the stability of the various values for equilibrium lending L^*. Since banks determine the volume of lending via $l_t = F(L_t)$ taking L_t as given, we can say that L^* is stable if $F'(L^*) < \frac{1}{M}$. This is because, if $F'(L^*) < \frac{1}{M}$, then l_t goes back to $\frac{L^*}{M}$ when L_t deviates slightly from L^*. We then have the following result concerning stability:

Lemma 3 *The equilibrium value L_l^* that occurs when $x_t = x_3$ or $x_t = x_4$ is stable; L_h^* that occurs when $x_t = x_2$ or $x_t = x_3$ is also stable; L_m^* that occurs when $x_t = x_3$ is unstable.*

See the Appendix for the proof. This lemma implies that, for appropriately small values of $\frac{C_t}{w_t}$, there exist multiple equilibria, which are stable against small perturbations in L.

The Equilibrium Production The search technology implies that the equilibrium output is

$$y_t^* = \min\{q(L_t^*)L_t^*, N\}. \tag{15}$$

In the cases when $x_t = x_1$ or $x_t = x_2$ (Figure1), the equilibrium output satisfies $y^* = N$ and there is no unemployment. In the case when $x_t = x_3$ and $L^* = L_l^*$, output is

[14] In Figures 1, 2 and 3, we normalize L, M and N so that our approximation by the Central Limit Theorem is effective in the figures. Thus we regard, for example, 1 unit of L or N to be equivalent to 1 million firms or consumers respectively, and 1 unit of M to 1000 banks.

$y_l^* = q(L_l^*)L_l^*$ $(< N)$ and there is unemployment. y_l^* is stable against small perturbation in lending L_t^*.

Discussion We have shown that a stable unemployment equilibrium exists if the value of $\frac{C_t}{w_t}$ is small. Our simple model implies that the micro level constraint on bank lending to maintain the liquidity of deposit money (condition (12)) determines macroeconomic performance via a stochastic mechanism, i.e., the Central Limit Theorem.

In this economy, a shortage of bank capital induces unemployment.[15] Such a shortage of bank capital may occur in several ways. One example is through currency crises. When the domestic currency undergoes a sharp devaluation, banks run short of capital if they have assets denominated in the domestic currency while they have liabilities denominated in a foreign currency. Another example might be the bursting of an asset price bubble in the real estate or stock market. If banks have corporate loans the value of which is determined by the assets that debtor firms own, and if debtor's assets consist of real estate and other firms' stocks, then the bursting of an asset price bubble devalues banks' assets abruptly, causing a shortage of bank capital.

One can criticize this story by saying that even if nominal bank capital C_t is impaired, a fall in the wage rate w_t will restore the value of $\frac{C_t}{w_t}$ so that the economy can return to full employment. We need to construct a more complete model in order to analyze the general equilibrium effects of a price change (See the next section). Here we merely suggest one possible defense against this argument: the stickiness of wages compared to currency or asset prices. Exchange rates and asset prices are quite volatile. Thus half of a bank's capital can be wiped out in a very short period of time, while it takes a very long time to reduce the nominal wage by half. As a result, the inefficiency caused by the shortage of bank capital continues for a long period of time.

In the next section, we show that, even if the wage is not sticky, the inefficient equilibrium exists and the economy may stay in its neighborhood forever.

[15] We may interpret C_t as the high-powered money provided by the central bank. However, in our model we do not specify how the central bank controls the amount of C_t, the investment by consumers in bank capital. Further research is necessary to clarify the role of the monetary policy in our model.

3 General Equilibrium and Equilibrium Dynamics

In the previous section, we have shown that the unemployment equilibrium exists if $\frac{C_t}{w_t}$ is sufficiently small. The question in this section is whether a flexible change in the wage w_t can restore the high value of $\frac{C_t}{w_t}$, leading the economy to the full employment equilibrium. In this section we show that, even if w_t is flexible, the value of $\frac{C_t}{w_t}$ that supports the unemployment equilibrium is realized in general equilibrium, and that the economy converges to the bounded neighborhood of the unemployment equilibrium from a wide range of initial values of $\frac{C_0}{w_0}$.

Existence of Unemployment Equilibrium Given the equilibrium output y_t and the equilibrium lending L_t, the consumer solves

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{16}$$

subject to $p_t c_t + C_{t+1} \leq w_t \lambda_t + (1 + r_t) C_t$. The consumers' choice of c_t takes place in subperiod t_e (See Timetable in Section 2). Consumers borrow cash $p_t c_t$ from banks and buy consumer goods[16]; firms that sold the consumer goods repay p_t to banks; banks pay out $(1 + r_t) C_t + w_t \lambda_t - p_t c_t$ to each consumer.[17]

The first order condition for the consumer's problem is

$$\frac{p_t}{p_{t+1}}(1 + r_{t+1}) = \frac{u'(c_t)}{\beta u'(c_{t+1})}. \tag{17}$$

The equilibrium conditions are $c_t = \frac{y_t}{N}$ and $\lambda_t = \frac{y_t}{N}$. The number of unknowns (p_t, c_t, C_{t+1}, r_t, λ_t) is larger than the number of equations (the budget constraint, the first order condition and the equilibrium conditions). Thus there are infinite equilibria. In order to specify the equilibrium, we assume the existence of an asset market:

[16] To be precise, we need also to consider interbank lending in order for banks to meet consumers' demand for borrowing given the supply of firms' repayment. But to simplify the argument, we just assume that the interbank market works well, and that there is no risk of bank default in this stage.

[17] In equilibrium, $r_t C_t = p_t c_t - w_t \lambda_t$. Thus the bank pays out C_t units of cash to each consumer, which is the exact amount of cash that the consumer invested in the bank at t_b.

Assumption 10 *Instead of lending cash to a firm, a bank can invest its cash in another bank's capital, where the rate of return is r_t.*

This assumption in our simple economy is equivalent to assuming that there exists a bond market or a stock market in which consumers or banks can invest their money.

The existence of an asset market implies that arbitrage between the asset market and corporate lending occurs. In this case, we have the following arbitrage condition:

$$1 + r_t = \frac{p_t \eta_t}{w_t}. \tag{18}$$

In the steady state equilibrium where $1 + r_t = \beta^{-1}$ holds, the budget constraint for the consumer problem implies

$$\frac{C_t}{w_t} = \frac{\left(\frac{\beta^{-1}}{\eta_t} - 1\right)\frac{y_t}{N}}{\beta^{-1} - 1} \tag{19}$$

$$= \begin{cases} \left(\frac{\beta^{-1} - q(L)}{\beta^{-1} - 1}\right)\frac{L}{N} & \text{if } L \leq L_0 \\ \\ \frac{\frac{L}{N}\beta^{-1} - 1}{\beta^{-1} - 1} & \text{if } L > L_0 \end{cases} \tag{20}$$

The conditions for equilibrium lending ((13) and (14)) and (19) determine the equilibrium pair, $\frac{C^*}{w^*}$ and L^*. See Figure 2.

Figure 2: Multiple Equilibria

In Figure 2, we set $\beta = 0.95$, while we use the same values as Figure 1 for the other parameters. For a wide range of parameter values, there are two or more equilibria: some equilibria corresponding to unemployment and others to full employment. Note that if the parameter G is sufficiently small, then the unemployment equilibrium vanishes and full employment is always attained in steady state. Figure 3 shows that there is a unique equilibrium in which full employment is attained when G is small. In Figure 3, $G = 7$, while the other parameters are the same as Figure 2.

Figure 3: Efficient Equilibrium

Recall that $\frac{1}{M}L = F(L)$ has two stable solutions: L_l^* and L_h^* when $\frac{C_t}{w_t}$ is small (See Figure 1). We can denote them as $L_l^*(x_t)$ and $L_h^*(x_t)$ where $x_t \equiv \frac{C_t}{w_t}$, since (13) and

(14) imply that L_l^* and L_h^* can be regarded as functions of x_t. Equilibrium lending in the unemployment equilibrium is $L_l^*(x_l^*)$ where x_l^* is the value of $\frac{C^*}{w^*}$ that produces the equilibrium shown in Figure 2 as the intersection of (13) and (19).

Equilibrium Dynamics and Stability of the Steady State We examine whether an economy initially apart from the steady states converges to the steady state equilibrium (x_l^*) specified above.

Suppose that the economy is apart from the steady states at time t. Since the profits of bank shareholders must equal the total bank profit from corporate lending, we have

$$r_t C_t = (p_t - w_t)\frac{y_t}{N} \tag{21}$$

This condition and the budget constraint for the consumer imply that $C_t = C_{t+1}$, which can be rewritten as

$$\frac{p_t}{p_{t+1}} = \frac{\frac{C_{t+1}}{w_{t+1}}\frac{p_t}{w_t}}{\frac{C_t}{w_t}\frac{p_{t+1}}{w_{t+1}}}. \tag{22}$$

Conditions (18) and (21) imply

$$\frac{p_t}{w_t} = \frac{x_t - \frac{y_t}{N}}{x_t\eta_t - \frac{y_t}{N}} \tag{23}$$

where $x_t = \frac{C_t}{w_t}$. Conditions (17) and (18) imply

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{p_t}{p_{t+1}}\frac{p_{t+1}}{w_{t+1}}\eta_{t+1}. \tag{24}$$

Together, conditions (22), (23) and (24) imply

$$u'(c_{t+1})x_{t+1}\eta_{t+1} = \beta^{-1}u'(c_t)x_t\eta_t\left(\frac{x_t - \frac{y_t}{N\eta_t}}{x_t - \frac{y_t}{N}}\right), \tag{25}$$

where $c_t = \eta_t L_t/N$, $\eta_t = q(L_t)$ if $L_t \leq L_0$ and $\eta_t = \frac{N}{L}$ if $L_t > L_0$, and $L_t = L(x_t)$ which is the solution of $\frac{1}{M}L = F(L)$. Note that whether $L_t = L_l^*(x_t)$ or $L_t = L_h^*(x_t)$ is realized depends upon expectations. Since our purpose is to examine the stability of the unemployment equilibrium that corresponds to $L^* = L_l^*(x_l^*)$, we restrict our attention to the case where people's expectations are coordinated to realize $L_t = L_l^*(x_t)$. In this case, $\eta_t = q(L_l^*(x_t))$ and $y_t = L_l^*(x_t)q(L_l^*(x_t))$, since $L_l^*(x_t) < L_0$. Thus (25) is rewritten as

$$J(x_{t+1}) = \beta^{-1}H(x_t)J(x_t), \tag{26}$$

where

$$J(x) \equiv u'\left(\frac{L_l^*(x)q(L_l^*(x))}{N}\right) xq(L_l^*(x)), \text{ and}$$

$$H(x) \equiv \left(x - \frac{L_l^*(x)}{N}\right) / \left(x - \frac{L_l^*(x)q(L_l^*(x))}{N}\right).$$

Difference equation (26) determines x_{t+1} as a function of x_t. We denote this as $x_{t+1} = G(x_t)$. The unemployment equilibrium x_l^* is the solution of $x = G(x)$ (It is easily shown that (19) is derived by setting $x_{t+1} = x_t$, $c_{t+1} = c_t$ and $\eta_{t+1} = \eta_t$ in (25)). We can prove the following proposition.

Proposition 1 *Assume that* $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$. *Assume appropriate parameter values such that*

(a) $1 - \frac{1}{G\sqrt{M\kappa}} > \beta$,

(b) $0 < x_l^* < \overline{x}$, *where* $\overline{x}$ *is defined by* $\frac{dL_l^*(x)}{dx} < \frac{L_l^*(x)}{x}$ *for all* $x \in (0, \overline{x})$,

(c) $x_l^* < \hat{x} < \overline{x}$ *where* $\hat{x}$ *is defined by* $J(\hat{x}) = \beta^{-1}J(x_l^*)$, *and*

(d) $J(x) \neq \beta^{-1}J(x)H(x)$ *for* $x \in (x_l^*, \hat{x})$.

Define $\underline{x}$ *by*

$$J(\underline{x}) = \inf_{x_l^* \leq x \leq \hat{x}} \beta^{-1}J(x)H(x).$$

Then the sequence $\{x_t\}_{t=0}^{\infty}$ *converges to the region* $[\min\{x_l^*, \underline{x}\}, \hat{x}]$ *as* t *goes to infinity, if* $x_0 \in (0, \hat{x})$ *and* x_{t+1} *is determined by (26).*

(*Proof*)

Since $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$, $J(x) = N^{\theta}\{L_l^*(x)\}^{-\theta}xq(L_l^*(x))^{1-\theta}$. It is easy to show that $\{L_l^*(x)\}^{-\theta}x$ is an increasing function of x if $\frac{dL_l^*(x)}{dx} < \frac{L_l^*(x)}{x}$. Therefore, $J(x)$ is an increasing function of x if $0 < x < \overline{x}$.

Next, we compare the values of $J(x)$ and $\beta^{-1}J(x)H(x)$ for values of x close to 0.[18] We can calculate the value of $H(x)$ evaluated at $x = 0$ by L'Opital's rule: $\lim_{x\to 0} H(x) = 1-$

[18] In the following argument, we use the approximation that x and L are small numbers, although we derived the functional form of $L_l^*(x)$ by the Central Limit Theorem assuming that L is a large number. The justification for this approximation that L is small comes from our normalization of the units of L: i.e., that 1 unit of L equals, for example, 1 million firms. Thus we can assume that even a small L is large enough for us to be able to use the Central Limit Theorem to derive $L_l^*(x)$.

$\frac{1}{N}\frac{\partial L}{\partial x}|_{x=0}$. Since $q(L) = \kappa L + o(L)$ for small L, condition (13) implies that $L(x)q(L(x)) \approx \kappa L^2 \approx \frac{N^2}{G^2 M}x^2$. Thus $\frac{\partial L}{\partial x}|_{x=0} = \frac{N}{G\sqrt{M\kappa}}$. Therefore,

$$\lim_{x \to 0} H(x) = 1 - \frac{1}{G\sqrt{M\kappa}}. \tag{27}$$

In the neighborhood of $x = 0$, difference equation (26) can be rewritten as

$$J(x_{t+1}) \approx \beta^{-1}\left(1 - \frac{1}{G\sqrt{M\kappa}}\right) J(x_t). \tag{28}$$

Since $1 - \frac{1}{G\sqrt{M\kappa}} > \beta$ is satisfied, $J(x_{t+1}) = \beta^{-1}J(x_t)H(x_t) > J(x_t)$ for sufficiently small x_t, and therefore $x_{t+1} > x_t$.

We have $\beta^{-1}J(x)H(x) > J(x)$ for all $x \in (0, x_l^*)$, since x_l^* is the smallest positive solution for $\beta^{-1}J(x)H(x) = J(x)$. Since $\beta^{-1}J(x)H(x) < \beta^{-1}J(x)$ for all $x > 0$, the continuity of $J(x)$ and $H(x)$ in the region $0 < x < \overline{x}$ implies convergence to $[\min\{x_l^*, \underline{x}\}, \hat{x}]$. (*end of Proof*)

Therefore, if the parameter values are chosen appropriately, the economy converges to the closed neighborhood of x_l^* from a small value of x_0. Moreover, if $\beta^{-1}J(x)H(x)$ is increasing in x for $\forall x \in (x_l^*, \hat{x})$, then the economy converges to the unemployment equilibrium x_l^* if the initial value $x_0 \in (0, \hat{x})$ and macroeconomic expectations are coordinated so that $L_t = L_l^*(x_t)$ is satisfied. In this case, the unemployment equilibrium is stable.

4 Conclusion

In our simple model, the size of the bank's asset portfolio must not exceed a certain multiple of its capital in order for deposits at the bank to be accepted as liquid assets. The complementarity between firms in the production technology and this microeconomic restriction on deposit money creation produce an unemployment equilibrium.

The policy implication of our model are subtle. In our model, if the initial value of $x_0 = \frac{C_0}{w_0}$ is small and macroeconomic expectations are pessimistic ($L_t = L_l^*(x_t)$), then the economy converges on the neighborhood of the unemployment equilibrium x_l^*. To overcome the weight of pessimistic expectations, it might be effective to augment bank capital with a lump-sum transfer from consumers to banks. If bank capital is augmented and x_t becomes large enough, then L_l^* vanishes and the only value for equilibrium lending that remains is L_h^* (See the case when $x_t = x_1$ in Figure 1).

A further policy implication concerns prudential regulation for maintaining the liquidity of deposit money. In our model, if the parameter G is small, then the inefficient equilibrium vanishes. The parameter G, representing the inefficiency of the interbank market, determines the macroeconomic efficiency of the economy. The factors that determine G include, for example, unambiguity in the stance of the regulatory authorities concerning bank failure. Thus the maintenance of efficient bank regulation is a necessary condition for preventing the emergence of the unemployment equilibrium.

Appendix

Proof of Lemma 2

We will prove that $F'(L_l^*) < \frac{1}{M}$, $F'(L_h^*) < \frac{1}{M}$, and $F'(L_m^*) > \frac{1}{M}$. At $L = L_l^*$, the function $\psi(L) = Lq(L)(1-q(L))$ is increasing. Suppose that L increases infinitesimally, becoming $L_l^* + \delta$. Suppose that, given the social level of lending $L_l^* + \delta$, a individual bank chooses $l_t = \frac{1}{M}(L_l^* + \delta)$. Then $\theta(\frac{1}{M}(L_l^* + \delta), q(L_l^* + \delta)) = \int_{\frac{N}{M}x}^{\infty} \left(\tilde{X}_\delta - \frac{N}{M}x\right) f(\tilde{X}_\delta) d\tilde{X}_\delta$, where $\tilde{X}_\delta \sim N\left(0, \sqrt{\frac{\psi(L_l^*+\delta)}{2M}}\right)$. On the other hand, $\theta(\frac{1}{M}L_l^*, q(L_l^*)) = \int_{\frac{N}{M}x}^{\infty} \left(\tilde{X} - \frac{N}{M}x\right) f(\tilde{X}) d\tilde{X}$, where $\tilde{X} \sim N\left(0, \sqrt{\frac{\psi(L_l^*)}{2M}}\right)$. Since $\psi(L)$ is increasing at $L = L_l^*$, it is the case that $\epsilon = \theta(\frac{1}{M}L_l^*, q(L_l^*)) < \theta(\frac{1}{M}(L_l^* + \delta), q(L_l^* + \delta))$. Since $\theta(l_t, \eta_t)$ is increasing in l_t, l_t must be smaller than $\frac{1}{M}(L_l^* + \delta)$, in order to satisfy (11) given $L_t = L_l^* + \delta$. Thus, $l_t = F(L_l^* + \delta) < \frac{1}{M}(L_l^* + \delta)$, which implies $F'(L_l^*) < \frac{1}{M}$. At $L = L_h^*$, the function $\psi(L) = L\frac{N}{L}\left(1 - \frac{N}{L}\right)$ is increasing. By a similar argument we can show that $F'(L_h^*) < \frac{1}{M}$. Since $\psi(L)$ is decreasing at $L = L_m^*$, the same logic implies that $F'(L_m^*) > \frac{1}{M}$.

References

Diamond, D.W. and R.G. Rajan (2001), "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," *Journal of Political Economy*, 109: 287-327.

Diamond, P. (1984), "Money in Search Equilibrium," *Econometrica*, 52: 1-20.

— (1982), "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 90: 881-894.

Kiyotaki, N. and R. Wright (1989), "On Money as a Medium of Exchange," *Journal of Political Economy*, 97: 927-54.

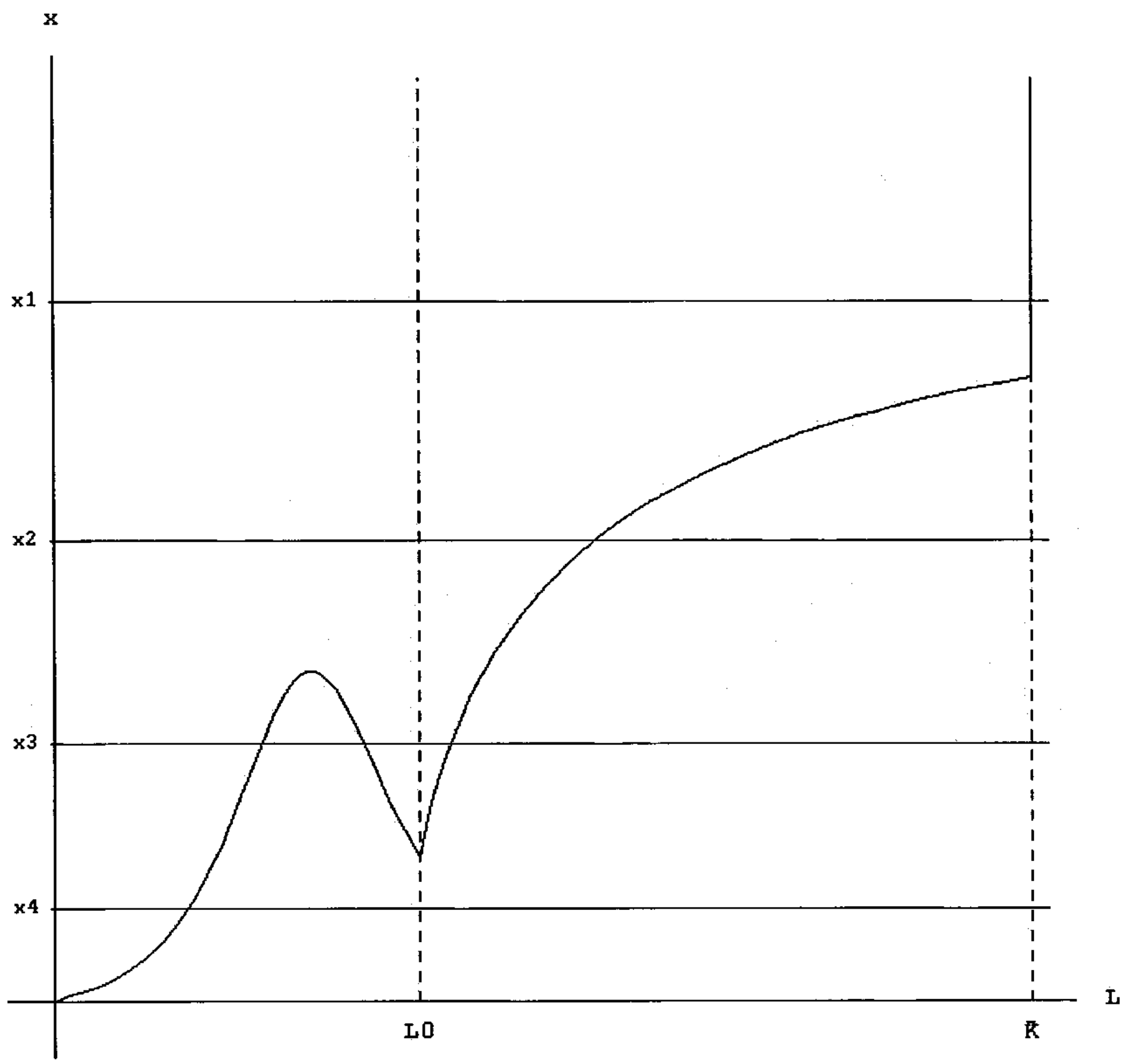

Figure 1

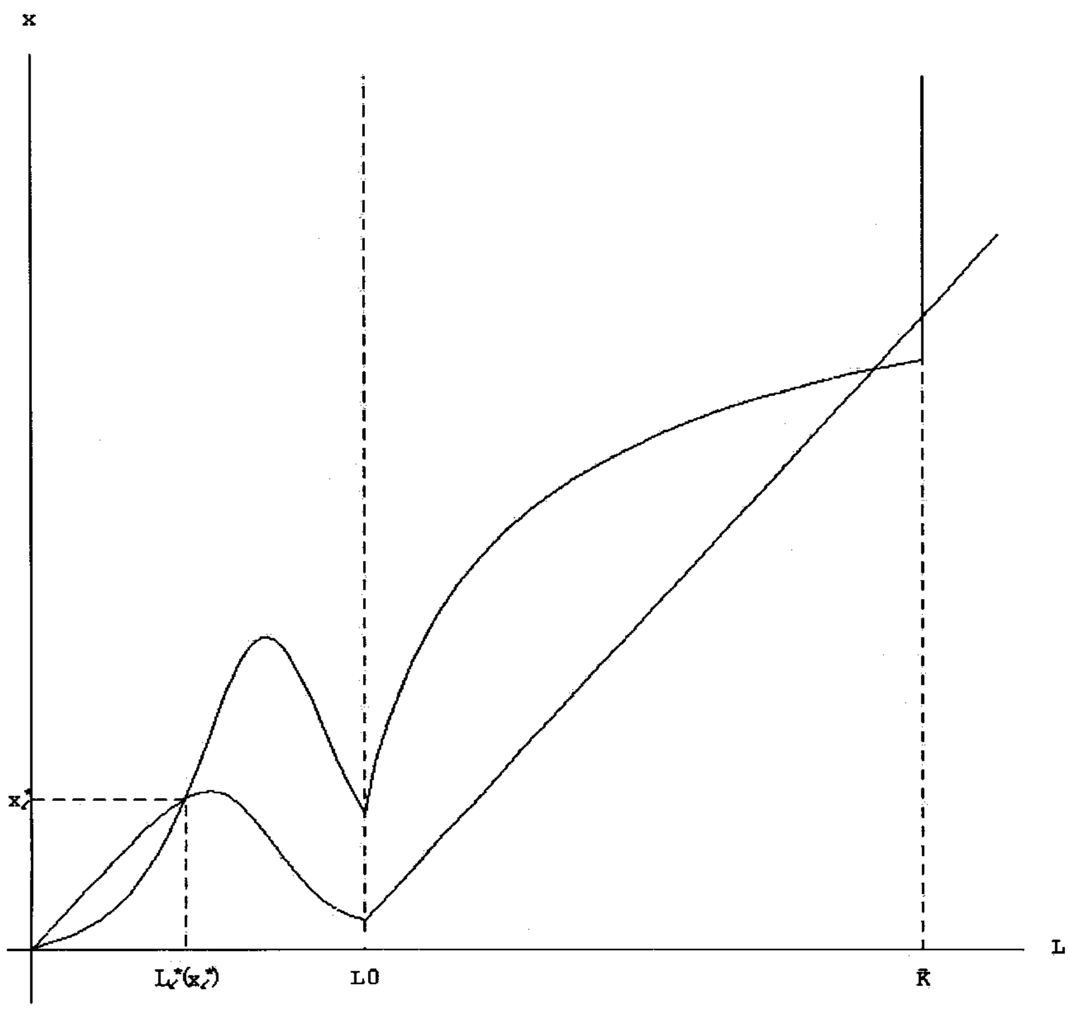

Figure 2

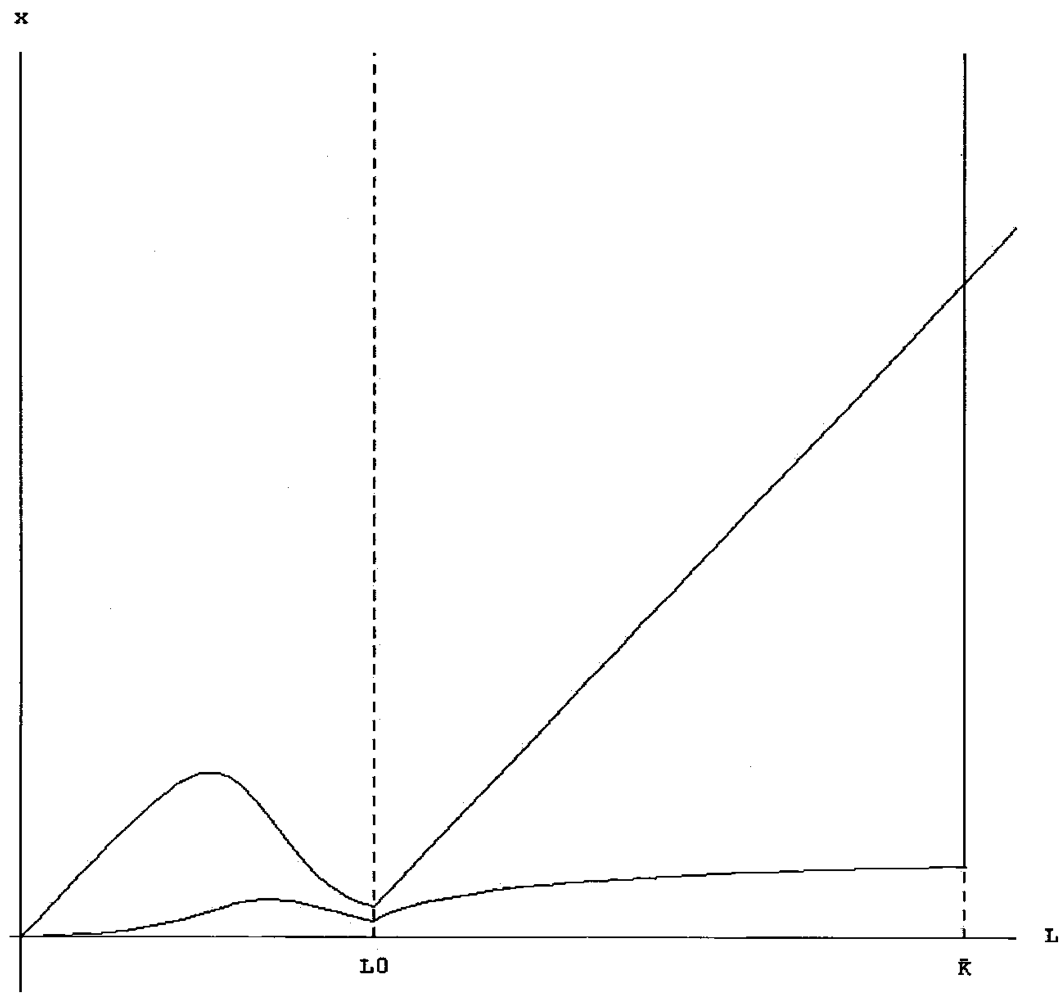

Figure 3