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Zero Bound on Nominal Interest Rates and Ex Ante Positive Inflation: A Cost Analysis*

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Abstract

Summers (1991) proposes that a central bank, in conducting monetary policy, should pursue a small but positive ex ante inflation rate even before nominal interest rates hit the zero bound. He insists that the central bank can thus reduce the social costs brought about by negative shocks to the economy. This line of argument, however, does not explicitly consider the social costs arising from the positive inflation rate itself, but emphasizes only the benefits of a reduced risk of hitting the nominal interest rate bound.

In this paper, I consider both the benefits and costs of maintaining a positive ex ante inflation rate as a way of circumventing the discomfort imposed by zero bound constraints. I show that this trade-off between the costs and benefits broadly determines the desirable rate of positive ex ante inflation that the central bank should pursue. By using simple model simulations, I conclude with these two points. Firstly, there in fact exists a desirable rate of positive ex ante inflation whose benefits exceed its costs. By parameterizing the model with average values from past Japanese experience, this finding is also shown to have been applicable to Japan. Secondly, this desirable level of the inflation rate largely depends on the degree of forward-lookingness in the economy, and the size and persistence of shocks generated.

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1 Introduction

Summers (1991) proposes that a central bank can reduce the social costs brought about by negative shocks to the economy, by pursuing a small but positive ex ante inflation rate[1]. This so-called "Summers effect" focuses on the non-negativity constraint on nominal interest rates, i.e. the fact that nominal interest rates cannot take negative values[2]. According to this argument, a central bank should pursue a somewhat positive inflation rate even before nominal interest rates fall to and stay at zero percent, thus keeping the expected inflation rate relatively high as well. Then, even when nominal interest rates hit zero due to possible negative shocks, the central bank can stimulate the economy by letting real interest rates become negative. Also, the central bank can maintain efficiency in its conduct of monetary policy in the sense that the likelihood of exhausting effective policy instruments decreases[3].

An environment of low inflation currently exists throughout the world economy, and especially in the economies of developed countries where it has been caused by the combination of several factors: slower potential economic growth, the increased effectiveness of monetary policy[4], globalization of markets in the post cold war era, and so on. As a result, both expected inflation and nominal interest rates in these economies stay low, increasing the probability that nominal interest rates fall to zero percent following a large negative shock. As for the Japanese economy, the Bank of Japan has now exhausted conventional monetary policy tools, in the sense that it can no longer freely control short-term nominal interest rates, which have been hovering at near zero percent. This is the result of the Bank's lowering interest rates several times in response to large negative shocks since the bursting of the bubble economy. Turning to the US economy, which has maintained relatively steady growth, the

[1] Shiratsuka (2001) is a comprehensive survey over pros and cons of inflation.

[2] Summers (1991) also points out the possibility that negative inflation distorts the resource allocation under the assumption of nominal rigidities.

[3] Conclusions differ among articles as for whether or not a central bank exhausts effective monetary policy tools when nominal interest rates become zero. For example, Woodford (1999a), Reifschneider and Williams (1999), Orphanides and Wieland (1999), Svensson (2001), and Jung et al. (2003) show that a central bank can stimulate the economy under the zero bound on nominal interest rates using commitment framework or other tools.

[4] See Clarida et al. (1999) and Svensson (1999, 2000) for reference.

euphoria led by the Internet revolution (the New Economy) has lost momentum. Therefore the Federal Reserve Board also faces the difficult question of whether its monetary policy can work effectively in a low inflation environment in the face of potentially large negative shocks[5]. Given these actual developments, it is natural to expect that other leading central banks will also pay more attention to the importance of maintaining room for monetary easing in the face of possible negative shocks. For this reason, the Summers effect, which emphasizes the benefit of a small positive ex ante inflation rate, is likely to be of relevance in the discussion of effective monetary policy options in a low inflation environment[6].

However, the Summers effect often emphasizes only the benefit of a reduced risk of hitting zero nominal interest rates and thus suffering social costs from negative shocks. It does not explicitly consider the social costs arising from a positive ex ante inflation rate itself. To justify the existence of the Summer effect, one has to compare the benefit with the cost of a positive ex ante inflation rate and prove that the former is larger than the latter. In other words, the desirable rate of positive inflation that a central bank should pursue is determined by the trade-off between the associated costs and benefits.

In this paper, I introduce a simple theoretical model, in which inflationary expectations in the private sector are assumed to react to the actions of the central bank in pursuit of a positive ex ante inflation rate. Using this model, I carry out both general qualitative simulations and also quantitative simulations for the Japanese economy, presenting the good results of each simulation. I consider these results to have significant implications for the real economy. It should be specifically noted, however, that this paper deals with only desirable

[5]In a speech on November 21, 2002, titled "Deflation: Making Sure It Doesn't Happen Here", Federal Reserved Board (FRB) Governor Ben Bernanke stated that the possibility of the US economy falling into deflation in foreseeable future is extremely small, but that FRB could fight against deflation preemptively, referring to these two points; (1) To preserve positive ex ante inflation as a buffer zone is crucial to reduce the risk of deflation and zero nominal interest rates; (2) FRB could ease its monetary policy aggressively and preemptively when the economy slows down under a low inflation environment. The first point is the main issue discussed in this paper.

[6]Monetary policy implementation under the liquidity trap or the zero bound on nominal interest rates has recently become one of the popular topics in macroeconomics, with many conferences by NBER (National Bureau of Economic Research) and by central banks picking up various theoretical papers in this field. Bank of Japan also held its international conference titled "The role of monetary policy under low inflation" in July 2000.

levels of inflation that are set preemptively, and hence does not have any direct implications with regard to an inflation target for the Japanese economy today, where the zero bound on nominal interest rates has already been encountered. Furthermore, these simulation results for the Japanese economy largely depend on the structure of the simple theoretical model and its parameters, i.e. the quantitative results may change when the model and its parameters are different. Therefore the numerical result for the desired positive inflation rate obtained from the simulations is very rough in nature and should be recognized as having a wide range.

The rest of the paper is organized as follows. Chapter 2 surveys previous studies relating to the zero bound on nominal interest rates, and sets up the theoretical model used in this paper. Chapter 3 shows that the desirable rate of positive ex ante inflation is determined by considering the trade-off between the benefit and the cost of a positive ex ante inflation. Chapter 4 first shows some qualitative simulation results describing the Summers effect, using the model constructed in Chapter 2. Results from quantitative simulations on the Japanese economy are also shown, using the same model but with its parameters adjusted to represent historical averages for the Japanese economy. Chapter 5 discusses two points that emerge from the results of this paper, and Chapter 6 then concludes.

2 Setting up a Model with a Zero Bound on Nominal Interest Rates

2.1 Previous Studies on Models with Zero Interest Rates

This chapter begins by surveying some of the standard literature dealing with monetary policy when faced with the zero bound on nominal interest rates. The previous studies in this field can be classified into two major groups. One concerns itself with preemptive monetary policy, where the issue is what policy options a central bank can take before a large negative shock occurs. Studies on the Summers effect, like the current paper, are included in this group. The other deals with ex post monetary policy options after a negative shock has occurred and nominal interest rates have become zero. Many studies in the latter group insist that what is important for the central bank is to influence inflationary expectations in the private sector through its policy commitment. Below I provide some brief details about each of the two groups in turn.

Notable among the first group of studies are Fuhrer and Madigan (1997), Orphanides and Wieland (1998), Reifschneider and Williams (2000), Hunt and Laxton (2001), Watanabe (1999), and Nishiyama (2003). All of these studies consider implications of the zero bound on nominal interest rates and prove the validity of the Summers effect. The first four studies use relatively large estimated models, run stochastic simulations, and produce numerical estimates of the level of ex ante inflation rate that sufficiently eliminates the possibility of hitting the zero interest rate bound and sufficiently decreases economic fluctuations. The last two studies use small calibrated models consisting of an IS curve and a Phillips curve, and prove the existence of the Summers effect. The following paragraphs provide further details of these papers.

Fuhrer and Madigan (1997) and Orphanides and Wieland (1998) carry out stochastic simulations, using small estimated models of the US economy which include a non-negativity constraint on nominal interest rates. They investigate how high ex ante nominal interest rates should be in order to decrease the volatility of policy target variables such as the output gap and inflation[7]. These studies unanimously conclude that, in light of the size of actual shocks experienced by the US economy in the past, central banks should maintain a small positive ex ante inflation rate if they are to be able to conduct monetary policy effectively. Reifschneider and Williams (2000) runs stochastic simulations for the US economy using the FRB/US model, a large economic model produced by the Federal Reserve Board. They conclude that, if the ex ante inflation rate is below a certain positive percent, nominal interest rates hit the zero bound so frequently that economic fluctuations and the resultant social costs increase. Hunt and Laxton (2001) also carries out simulations for the Japanese economy with

[7]When the nominal interest rates as policy instrument become zero, a central bank cannot fully respond to a large negative shock and resulting economic fluctuations may become large. For example, under the zero bound on nominal interest rates, a central bank cannot shrink the negative output gap, which could be narrowed if the nominal interest rates takes negative values.

Multimod, a multi-country economic model produced by the International Monetary Fund (IMF), adopting a method similar to that of Reifschneider and Williams (2000). They also conclude that ex ante inflation rates below 2 percent allow nominal interest rates to hit the zero bound too frequently, and that the appropriate ex ante inflation rate for the Japanese economy is therefore over 2 percent.

On the other hand, Watanabe (1999) shows the existence of the Summers effect in terms of minimizing social losses, using theoretical models consisting of New-Keynesian Phillips curves, forward-looking IS curves, and central banks which maintain positive inflation targets. Simulations following the method suggested by Jung et al. (2003) demonstrate that a central bank conducting policy under commitment does not require positive ex ante inflation, but that a central bank conducting policy under discretion should preferably maintain a positive ex ante inflation rate of 0.44 percent. Central banks which can credibly commit their policies to economic agents do not need positive ex ante inflation basically because they can stimulate the economy by controlling inflationary expectations even after nominal interest rates have hit zero. Lastly, Nishiyama (2003) uses a backward-looking model to show that the desirable ex ante inflation rate is positive when policy is preemptive.

Among the second group of studies are Woodford (1999a), Svensson (2001), and Jung et al. (2003). Woodford (1999a) insists that policy under commitment is more effective than policy under discretion in an economy facing a liquidity trap, a conclusion which is derived by investigating a model with a forward-looking IS curve and a New-Keynesian Phillips curve with micro foundations[8]. He also shows that monetary policy with inertia is actually optimal, even though his modeling framework does not contain any equations with lagged variables[9]. Svensson (2001) proposes that a central bank can guide the economy out of a liquidity trap by influencing expectations of foreign exchange rates and then by eliminating expected deflation

[8]In monetary policy under discretion, a central bank sets only current nominal interest rates to minimize the current social loss at each period. On the other hand, in monetary policy under commitment, a central bank sets all the current and future paths of the nominal interest rates at a certain period.

[9]When the lag structures are directly included in the model, shocks have persistence and hence monetary policy has inertia.

and lowering real interest rates. His proposal depends on the idea that a central bank can cause the actual exchange rate to depreciate by encouraging expectations of depreciation, which is in effect an argument postulating that uncovered interest rate parity holds. He suggests that one way of generating such expected depreciation is for the central bank to peg a lower level of the exchange rate and then to declare its intention of maintaining this policy framework until domestic prices attain a specific targeted level. Jung et al. (2003) also shows that, even after the economy has fallen into a liquidity trap, the central bank can mitigate deflationary shocks by committing to maintaining its zero interest rate policy until the inflation rate reaches a sufficient positive value[10]. That is to say, the central bank can borrow some of the effects of its future monetary easing by committing to continuing with the zero interest rate policy for a longer period, thus succeeding in lowering current real interest rates to support the economy.

2.2 Setting up a Theoretical Model

Here I use a model extended from the model developed by Clarida et al. (1999), Gali and Gertler (1999), and Woodford (2000), which is used in recent many articles on monetary policy. The economy outside the central bank is represented by two equations, an IS curve and a Phillips curve. The basic forward-looking model in the previous studies includes only lead and current variables and no lagged variables, but here I extend the model by incorporating lags of the output gap and inflation to better reflect the actual Japanese economy as follows:

$$x_t = \mu_1 E_t x_{t+1} + \mu_2 x_{t-1} - \sigma^{-1}\left[(i_t - E_t \pi_{t+1}) - r_t^n\right], \qquad (1)$$

$$\pi_t = \kappa x_t + \nu_1 E_t \pi_{t+1} + \nu_2 \pi_{t-1}, \qquad (2)$$

where π_t is inflation, x_t is the output gap, i_t is the short-term nominal interest rate, and r_t^n is the natural interest rate. σ, κ, μ, and ν are positive parameters. I assume $\mu_1 + \mu_2 = 1$ and $\nu_1 + \nu_2 = 1$.

[10] They show an analytical solution under the non-negativity constraint on the nominal interest rates. To be more specific, they solve the zero bound problem by mixing the Kuhn-Tucker method and Blanchard and Khan (1980) method, the latter of which is usually used to the solution for Real Business Cycle Models.

Equation (1) and (2) represent the hybrid IS curve and the hybrid Phillips curve (or hybrid aggregate supply curve) respectively. Equation (1) states that the output gap in period t is determined by the expected value of the output gap in period t+1, the lag of the output gap in period t-1, and the deviation of the short-term real interest rate from the natural rate of interest in period t[11]. The parameter μ reflects the weight of the effect from the future output gap and the past output gap upon the current output gap, and σ represents the elasticity of the output gap in response to the real interest rate[12].

Equation (2) is the hybrid Phillips curve and this, like the New-Keynesian Phillips Curve, can be derived by extending the staggered price-setting model constructed by Calvo (1983), which assumes monopolistic competition among firms[13]. The extension of Calvo's original model involves the inclusion of the inflation rate in period t-1. The parameter ν reflects the weight of the effect from past and future inflation upon current inflation, and κ represents the elasticity of the output gap in response to inflation[14].

Next, I assume that a period loss function is given by the weighted sum of the squared output gap and inflation as follows[15]:

$$L_t = \pi_t^2 + \lambda x_t^2, \tag{3}$$

where L_t is the period loss function, and λ is a positive parameter which reflects the relative weights placed upon the output gap and inflation. I also assume that the policy instrument by the central bank is the short-term nominal interest rate, and the bank controls its level

[11]The natural rate of interest presented here is the level of the real interest rates which does not accelerate either inflation or deflation, as is shown in equation (1).

[12]Supposing GDP consists of only consumption, as is usual in small theoretical models, this parameter can be interpreted as the inverse of the elasticity of intertemporal substitution (relative risk aversion).

[13]The New-Keynesian Phillips curve constructed by Calvo (1983) includes only expected inflation and the current output gap as the explanatory variables, and does not have the inertia (namely, does not include the lag of inflation as a factor determining current inflation). On the other hand, Fuhrer and Moore (1996) and Gali and Gertler (1999) extend the model without losing the original theoretical foundation to include the lag of inflation in the New-Keynesian Phillips curve.

[14]In the special case of $\mu = 1$ and $\nu = 0$, this Phillips curve becomes the normal New-Keynesian Phillips curve.

[15]Woodford (2002) shows that, when there are transaction frictions that make people hold money balances, the loss function shown as Equation (3) is a quadratic approximation to the expected utility of a representative household. However, unlike the model in this paper, the model assumed by Woodford (2002) does not include the lag of inflation and the output gap.

with a view to minimizing the expected discounted sum of the period losses, which also reflects social costs here[16].

$$L_S = E_0 \sum_{t=0}^{\infty} \delta^t L_t, \tag{4}$$

where δ is the discount rate for both economic agents and the central bank, satisfying $0 < \delta < 1$. Thus the central bank here will not seek to use monetary policy to reduce social loss so far as to achieve a zero output gap and zero inflation. Lastly, I introduce the non-negativity constraint on the short-term nominal interest rate in the model as follows:

$$i_t \geq 0. \tag{5}$$

Therefore the central bank here controls the short-term nominal interest rates with the purpose of minimizing the value of equation (4) within a model consisting of equations (1), (2), and (5) under a certain central bank's behavioral assumption[17].

2.3 Negative Shocks

To make the argument here simple, we consider only large negative demand shocks that act through the natural rate of interest, and we assume that these shocks are deterministic: economic agents and the central bank know the future path of the shocks when the shocks occure. Woodford (1999a) suggests that the natural interest rate shock includes both long-run changes, such as those that affect the potential output gap, and short-run changes, such as changes in government spending. Put differently, both permanent and temporary demand shocks can be captured as natural interest rate shocks. To be more specific, we assume that a large negative shock to the natural rate of interest, denoted by ϵ_0^n, occurs only in period 0, after which the natural rate of interest converges to its steady state, r_∞^t. This specification means that the central bank and economic agents know whole path of the shocks at period

[16]Here I calculate the rule by considering the first order conditions to the IS curve constraint, the Phillips curve constraint and the nominal interest rates constraint under a certain assumption of a central bank behavior.

[17]As the model in this paper does not directly include money, I ignore the policy effect from money. This paper focuses only on the policy effect from the nominal interest rates control.

0.

$$r^n_{t+1} - r^n_\infty = \rho(r^n_t - r^n_\infty) + \epsilon_{t+1}, \quad \epsilon_1 = \epsilon_2 = \cdots = 0,$$

$$\implies \quad r^n_t = \rho^t \epsilon^n_0 + r^n_\infty. \tag{6}$$

The convergence parameter ρ, satisfying $0 \leq \rho < 1$, implies that shocks are temporary when it is less than one and permanent when it takes the value one.

3 Comparing the Cost of Zero Bound on Nominal Interest Rates and that of Positive ex ante Inflation

As mentioned above, when a large negative shock occurs in the economy, the central bank needs enough room for monetary easing, in other words it needs nominal interest rates sufficiently above zero, to mitigate the shock. Otherwise the central bank may find itself deprived of effective policy tools and the resulting economic loss may become significant. To avoid such a situation, it is considered important for central banks to maintain relatively high inflationary expectations and nominal interest rates in anticipation of potential negative shocks. By doing this, they can reduce both the possibility of encountering the zero bound on nominal interest rates and also the overall loss, which is the main point of the Summers effect.

Taking the same broad line, Fuhrer and Madigan (1997), Orphanides and Wieland (1998), Reifschneider and Williams (2000), and Hunt and Laxton (2001) run stochastic simulations using estimated models, and conclude that in both Japan and the United States it is important to maintain a small but positive ex ante inflation rate. The difference is that their conclusions are based not on minimizing the total social loss but on minimizing the variances of variables such as the output gap and inflation[18]. Therefore when considering the desirable level of positive ex ante inflation, they pay attention only to the benefit of positive inflation,

[18] This is based on the Policy Frontier proposed by Taylor (1994). The Policy Frontier is used to evaluate the policy performance, with the policy rule that can realize the minimum variance of endogenous variables, such as inflation and the output gap, is the best.

i.e. the benefit of avoiding the risk of hitting the zero bound on nominal interest rates. Their discussion virtually ignores the cost of positive ex ante inflation[19]. On the other hand, this paper proves that there is a desirable level of positive ex ante inflation and that this is determined by the trade-off between the above costs and benefits[20]. The analysis below demonstrates that the Summers effect survives even when both benefits and costs of positive inflation are considered equally, i.e. a certain small but positive level of ex ante inflation reduces total net social losses.

In the following discussions, I assume for simplicity that the steady-state value of the natural rate of interest is zero and the size of the expected shock to the economy is π^*. Firstly, in the model consisting of equations (1) and (2), I consider the costs involved when a central bank maintains a positive inflation rate $\pi^* > 0$ as part of a preemptive policy to avoid hitting the zero bound on nominal interest rates . I assume that, when economic agents recognize that the central bank's objective function takes the form of equation (9), their expectations of future inflation change and the steady-state level of inflation shifts up in line with the upper inflation bias in equation (9)[21]. This happens because agents believe that the social loss function is the same as the objective function of the central bank. But the point here is that the social loss function does not change from equation (4), and equation (9) remains no more than the central bank's objective function. In other words, agents misunderstand the social loss function. Therefore, when the central bank keeps the ex ante inflation rate at $\pi^* > 0$, the social loss becomes $(\pi^*)^2/(1-\delta)$, which is the cost of maintaining positive inflation.

Secondly, I confirm the benefit of maintaining positive ex ante inflation, which is the decreased likelihood of hitting the zero bound on nominal interest rates. With the central

[19] As pointed out in Kimura and Tanemura (2000), if there is no cost for having positive inflation, the higher the inflation rate is, the better the performance becomes, which is odd . Kimura and Tanemura (2000) insists that there is a certain upper limitation on a desirable level of inflation, because inflation uncertainty increases as level of inflation becomes high.

[20] Watanabe (1999) has similar augments, but it depends on different model and thought.

[21] Watanabe (1999) assumes that the governor of a central bank has preference over positive inflation to create an inflation bias in the central bank.

bank keeping the inflation rate at $\pi^* > 0$, inflationary expectations are affected and equations (1) and (2) change accordingly:

$$x_t = \mu_1 E_t x_{t+1} + \mu_2 x_{t-1} - \sigma^{-1}\left[(i_t - E_t\hat{\pi}_{t+1}) - (r_t^n + \pi^*)\right], \tag{7}$$

$$\hat{\pi}_t = \kappa x_t + \nu_1 E_t \hat{\pi}_{t+1} + \nu_2 \hat{\pi}_{t-1}, \tag{8}$$

where $\hat{\pi}_t \equiv \pi_t - \pi^*$. The central bank here sets the short-term nominal interest rates so as to minimize the value of the following objective function. Here the loss function given by equations (9) and (10) again differs from the social loss function given by equation (4). It merely represents the central bank's objective function as follows:

$$L_S^{'} = E_0 \sum_{t=0}^{\infty} \delta^t \left\{(\pi_t - \pi^*)^2 + \lambda x_t^2\right\} \tag{9}$$

$$= E_0 \sum_{t=0}^{\infty} \delta^t \left\{\hat{\pi}_t^2 + \lambda x_t^2\right\}. \tag{10}$$

Note that the model consisting of equations (1)-(4) is significantly different from the model consisting of equations (7), (8) and (10), in that $r_t^n + \pi^*$ appears in the second term of equation (7) in place of r_t^n in equation (1). In the latter model, the shock π^* which occurs to the natural rate of interest is offset by a pre-set positive inflation of the same size. This is precisely the benefit of keeping ex ante positive inflation.

Having clarified both cost and benefit of maintaining positive ex ante inflation, the next issue is whether the benefit is actually larger than the cost. Only if the benefit exceeds the cost will a central bank have an incentive to maintain positive inflation. Let me emphasize again that equation (10) is merely the central bank's objective function and that evaluation of the social loss should still be carried out using equation (4). Therefore, faced with the same shock, the loss described in equation (4) when a central bank maintains a pre-set positive inflation rate (i.e. when it acts to minimize the objective function given by equation (10)[22]) has to be smaller than the loss that results when the central bank chooses not to maintain

[22]This path is obtained by solving the optimization problem consisting of equations (7)-(10).

a specific positive inflation rate (i.e. it directly minimizes the social loss given by equation (4)[23]). In more concrete terms, starting with the central bank which does not maintain positive inflation, if the total loss caused by the shock π^* is larger than $(\pi^*)^2/(1-\delta)$, the central bank has an incentive to switch policies and maintain a positive inflation rate of π^*. In this case, although the central bank conducts monetary policy according to its own objective function, equation (10), in doing so it is also ultimately acting to minimize the social loss given by equation (4). Whether the benefit is larger than the cost or not depends on the model structure and its parameters[24].

I provide a simple example here. For simplicity, I set $\delta = \kappa = \mu_2 = 0$, $\sigma = \lambda = \mu_1 = 1$. The model then becomes:

$$x_t = E_t x_{t+1} - \left[(i_t - E_t\hat{\pi}_{t+1}) - (r_t^n + \pi^*)\right], \tag{11}$$

$$\hat{\pi}_t = \nu_1 E_t \hat{\pi}_{t+1} + \nu_2 \hat{\pi}_{t-1}, \tag{12}$$

$$L_S^{'} = E_0 \left\{ \hat{\pi}_0^2 + \lambda x_0^2 \right\}. \tag{13}$$

When κ is zero, as shown in equation (12), the inflation rate is independent of the output gap and always stays at its steady state level regardless of shocks to the natural rate of interest. In this setting, consider what happens when the value of π^* is marginally increased from zero and a large negative shock hits the model. π_t always starts off at zero as the inflation rate is always zero, but when $\pi^* > 0$, π_t becomes π^* and the loss $(\pi^*)^2$ is obtained from equation (4). The increase in the loss is given by the squared difference between the increased inflation rate and zero. On the other hand, maintaining the inflation rate at π^* causes the output gap fluctuations to decrease from non-zero values, as the positive inflation rate offsets the shock, reducing its negative effects. The increase in the loss from the former effect is outweighed

[23]This path is obtained by solving the optimization problem consisting of equations (1)-(4) which set π^* at zero.

[24]Whether there exists a positive π^* or not depends on the parameters. In some cases, π^* becomes zero (namely, there is no positive ex ante inflation).

by the decrease in the loss from the latter. This is because the latter is calculated as the square of the decrease in the output gap measured from a non-zero value, not from zero like the former. As a result, maintaining a positive inflation rate of π^* causes the overall loss to decrease. This confirms the existence of $\pi^* > 0$ (illustrated in Chart 1).

Finally, the desirable level of the positive ex ante inflation rate π^*_{opt} is determined at the point where the net benefit, defined as the benefit minus the cost, takes its maximum value. As mentioned above, since the cost of maintaining $\pi^* > 0$ is given by the squared difference between the increased inflation rate and zero, the increase in the cost of raising π^* is a concave, monotonically increasing function of π^*. On the other hand, since the benefit is given by the squared decrease in the output gap measured from a non-zero value, the decrease in the benefit is a convex, monotonically increasing function of π^*[25]. Therefore, the desirable level of the positive inflation rate is uniquely determined (as shown in Chart 1)[26]. And the net decrease in the total loss caused by a positive inflation can be analytically defined as the resulting net benefit of maintaining $\pi^* > 0$.

To summarize, the above discussions have shown that by maintaining a positive inflation rate a central bank can decrease the social loss incurred when a shock occurs. Furthermore, I have shown that the Summers effect exists even when both the cost and the benefit of a positive ex ante inflation rate are explicitly considered.

4 Model Simulations

In this chapter I run some simulations assuming that the central bank conducts monetary policy so as to minimize the loss in each current period by setting only the current short-term nominal interest rate under the certain central bank's behavioral assumption[27] (the details are explained in the Appendix).

[25] When $\pi^* > 0$ becomes sufficiently high, the loss is fully eliminated and the benefit from a positive inflation becomes constant.

[26] Some π^*_{opt} can fully eliminate the possibility of hitting the zero bound on the nominal interest rates. Other π^*_{opt} can not fully eliminate the possibility, but can mitigate the resulting expected loss.

[27] In particular, in the case of $\mu_1 = \nu_1 = 1$, $\mu_2 = \nu_2 = 0$, this solution is defined as the monetary policy under discretion, shown by Clarida et al. (1999). The detail is explained in Jung et al. (2003).

4.1 Qualitative Simulations

Here I concentrate on investigating the qualitative properties of the model. Specifically, I calculate the change in the desirable level of positive inflation when the values of some parameters are changed.

Generally, the desirable level of positive inflation largely depends on the size and the persistence of shocks given, and also on the preferences of economic agents in the model between current and future losses, in other words on the value of the discount rate. So here I change the values of parameters such as μ, which reflects the relative weights attached to the effects of future and past output gaps upon the current output gap, ν, which reflects the relative weights attached to the effects of future and past inflation upon current inflation, δ, which represents the discount rate, ϵ_0^n, which represents the size of the shock, and ρ, which represents the persistence of the shock. Changing each of these parameters in turn, I calculate the respective changes in the desirable level of positive inflation for each case.

In these simulations, I assume $\mu_1 = \mu$ $(0 < \mu_1 \leq 1)$, $\mu_2 = 1 - \mu$, $\nu_1 = \nu$ $(0 < \nu_1 \leq 1)$, $\nu_2 = 1 - \nu$ [28] and set the benchmark value of the parameters as $\mu = \nu = 0.5$, $\delta = 0.99$, $\epsilon_0^n = -0.03$, $\rho = 0.5$. The other parameters are set as $\sigma = 0.157$, $\kappa = 0.024$, $r_\infty^n = 0.005$, $\lambda = 0.021$ [29]. All these parameters are set based on their quarterly figures.

4.2 The Results of the Qualitative Simulations

(1) A change in the dependence of the current economy on the past and the future economies

In this simulation, the desirable level of positive ex ante inflation rises, as μ and ν become smaller (as shown in Table 1). This is because, as μ and ν become smaller, the effects of a shock stay in the economy for longer, which makes it more effective to preemptively

[28] Gali and Gertler (1999) shows that, when the discount rate is set to be one, $\nu_1 + \nu_2$ becomes one, in their models with microfoudations.

[29] I set the parameter values of μ, ν, ρ at the medium value of each parameter range. The parameter values of δ, σ, κ are taken from Woodford (1999a) and the parameter value of λ is taken from Cecchetti and Ehrmann (1999). The parameter values of ϵ_0^n, r_∞^n are set with discretion to show the comprehensive simulation results.

mitigate the shock by maintaining positive ex ante inflation. In other words, the stronger the persistence in the economy, the higher the desirable level of positive inflation becomes. Also, since the model used here has forward-looking properties, the less forward-looking the model, the weaker the momentum bringing it back to its steady-state becomes. Since there will then be less inertia in the economy, the desirable level of positive inflation may fall.

(2) A change in the discount rate[30]

As the discount rate δ becomes smaller, the desirable level of positive inflation rises (as shown in Table 2). This is because, as the discount rate becomes smaller, economic agents relatively pay more attention to losses in the near future than those in the more distant future[31]. More specifically, it becomes more important to preemptively mitigate the first shock by maintaining a positive inflation rate, and it becomes less important to avoid the loss in the far future, namely that in the steady state. Approaching this from a different point of view, the Summers effect may be said to involve the intertemporal redistribution of the social loss: if the discount rate is sufficiently less than one, the central bank can decrease the social loss, which subjectively depends on the discount factor, by transferring the loss from the near future to the far future. On the other hand, if the discount rate is close to one, the intertemporal redistribution effect between the near future and the far future becomes small[32].

(3) A change in the size of the shock

As the size of the shock ϵ_0^n becomes larger, the desirable level of positive inflation rises (as shown in Table 3). This result is straightforward and intuitive: the larger the shock, the higher the possibility of coming up against the zero bound on nominal interest rates, which in turn raises the desirable level of positive inflation in order to counteract this possibility.

(4) A change in the persistence of the shocks

[30] Theoretically the discount rate of the economic agents and a central bank is often set to be the same value as the inverse of the steady-state real interest rate. As a result, the discount rate is set to be near one, as shown in Woodford (1999a).

[31] In other words, the lower the discount rate of the economic agents and a central bank is, the smaller the forward-lookingness on the economy becomes.

[32] In fact, when δ is set to be one, a positive ex ante inflation cannot be obtained in the simulations.

As the persistence of the shock ρ becomes stronger, i.e. as the shock stays longer in the economy, the desirable level of positive inflation rises (as shown in Table 4). This result is consistent with the result obtained in simulation (1) above.

4.3 Quantitative simulations for the Japanese Economy

In this section, by running simulations with specific numerical parameters, I carry out a quantitative investigation into whether monetary policy that takes into account the Summers effect would be effective or not in the Japanese economy. As shown in the previous qualitative arguments, the desirable level of positive ex ante inflation depends largely on the size and the persistence of the shock. However, the actual size and persistence of economic shocks are generally considered subject to constant change. Here, therefore, I run numerical simulations for various shocks of differing size and persistenceusing by using the model introduced above, with parameter values set to reflect Japanese experience and I calculate desirable levels of positive inflation for each case.

Following previous studies, I re-set the model parameters so that they reflect the actual dynamics of the Japanese economy. All data and parameters are quarterly here. Kimura and Kurozumi (2003) obtain statistically significant parameters, using the GMM estimation method to estimate the same hybrid IS curve and the hybrid Phillips curve used in this paper with data running from the first quarter of 1975 to the first quarter of 1997; $\mu = 0.09$, $\sigma = 0.13$, $\kappa = 0.05$, and $\nu = 0.65$[33].

I briefly interpret the estimation results with reference to Kimura and Kurozumi (2003). Firstly, the parameter on the past output-gap in the IS curve is 0.91, meaning that the output gap in the Japanese economy displays a strong degree of inertia. In other words, once a large negative shock occurs, the shock stays longer in the economy. Therefore, we can readily understand the importance of policy that seeks to preempt a large negative shock

[33] Kimura and Kurozumi (2003) uses annual figures for inflation and nominal interest rates and sets the parameters at the annual base. Here I use the parameters transformed into a quarterly basis. Also, Kimura and Kurozumi (2003) assumes that $\mu_1 = \mu$, $\mu_2 = 1-\mu$, $\nu_1 = \nu$, $\nu_2 = 1-\nu$ when they estimate the parameters.

in the Japanese economy[34]. In addition, the parameter on expected inflation in the Phillips curve ν is 0.65, meaning that inflationary expectations have a significant role in determining current inflation in the Japanese Economy. This result is similar to that of Gali et al. (2001), who estimate Phillips curves for the US and EU economies.

I set the natural rate of interest in the steady state at 2 percent per annual; $r^n_\infty = 0.005$[35]. The discount rates for economic agents and the central bank δ are set to be 0.99 for the one and 0.95 for the other[36]. I set λ, which determines the relative weights attached to the output gap and inflation in the loss function of economic agents (and the central bank) to be 0.021, the same as previously. Lastly, I assume three different values for the size of the shock ($\epsilon^n_0 = -0.01, -0.015, -0.02$), and three different values for the persistence of the shock ($\rho = 0, 0.4, 0.8$).

4.4 The results of the quantitative simulations for the Japanese Economy

The simulation results (as shown in Table5 and 6)[37] indicate that the benefit associated with a positive ex ante inflation rate is larger than its cost in the Japanese economy as well. In other words, the existence of the Summers effect is also confirmed for the Japanese economy. As shown in the result of the qualitative simulations, it is also true here that the larger and the longer the shock is, the higher the positive ex ante inflation rate that should be maintained.

Here, as mentioned above, I do not show the unique optimal level of the positive inflation rate for the Japanese economy, because the level largely depends on the structure of the model

[34]Here I explain what these parameter settings mean, without necessarily justifying them. The parameter σ of 0.13 means that the negative output gap expands by 1.5 percent with an annual increase of the real interest rates by 1 percent. The parameter κ of 0.05 means that inflation increases by 0.2 percent as the output gap shrinks by 1 percent.

[35]For instance, in the economy where the growth rate the natural interest rate are high, such as China, a desirable level of the positive ex ante inflation is considered to be near zero. This is because the shocks brought by the natural interest rates are rarely larger than the steady-state level of the natural interest rates and hence the possibility of hitting the zero bound on the nominal interest rates becomes near zero.

[36]In Kimura and Kurozumi (2003), the discount rate of the economic agents and a central bank is set to be one when they estimate parameters.

[37]The table 5 and 6 represent that the shock persistence becomes larger on the left side of the table, and also the size of the negative shock, which occurs at time 0, becomes larger on the low side of the table. The tables show a desirable level of the positive inflation for each case.

and its parameters. Instead, I investigate the level of the ex ante inflation rate suggested by the past output gap shocks to have hit Japan (as shown in Chart 2)[38].

For example, the average output gap since 1999, when the Bank of Japan started the zero interest rate policy, up to 2002, is approximately minus 3.1 percent (as shown in Chart 2). If a shock of this size occurs once, the desirable level of positive ex ante inflation can be calculated from Table 5 to be more or less 0.2 percent, when the discount rate is 0.99 and there is no shock persistence. Note that the optimality of this particular level of the inflation rate holds only to one shock. If the output gap were continuously negative due to a series of natural interest rate shocks, as was in fact the case in the Japanese economy, a higher level of inflation would be required. To give another example, Table 5 shows that, if shocks are more persistent, with the value of ρ set at 0.8, the required level of inflation is about 0.8 percent. The desirable level of positive inflation also differs depending on the discount rate of the central bank, as can be seen in Table 5 and 6.

It is not easy to estimate the size and the persistence of the shocks actually occurring in the economy, and thus the desirable level of positive inflation that should be pursued by the central bank varies depending on the bank's perceptions about such shocks[39]. However, the results here at least demonstrate the existence of a beneficial positive ex ante inflation rate when we look at average experience in Japan in the past.

5 Discussion

5.1 Inflation targeting

In recent years, many central banks overseas have adopted inflation targeting policies to prevent high inflation. Mishkin and Schmidt-Hebbel (2001) surveys the monetary policy

[38]This output gap is calculated by extending the method of Kamada and Masuda (1999). Although they calculate the output gap in terms of maximum potential output, here I estimate the average output gap (in terms of average potential output). According to my estimation, as shown in Chart 2, negative large shocks continued to occur in the Japanese economy after 1992 and the average output gap has recently taken a large negative value.

[39]As pointed out by many previous studies, such as Shiratsuka (2001), very high inflation has various harmful effects upon the economy. Therefore, if the resulting level of a desirable inflation is very high in terms of the Summers Effect, a central bank actually cannot keep it.

frameworks of central banks around the world. According to the survey, as of November 2001, 19 central banks conduct inflation targeting policies and many of them set the target level of the inflation rate at between 1 to 4 percent (year-on-year). The argument is that such inflation targets enable central banks to maintain low and stable inflation. Some other studies also argue that inflation targeting helps to enhance the transparency of monetary policy. In the light of these facts and studies, it seems fair to say that inflation targeting is no longer the exception to the rule, and that it is now one of the standard policy frameworks adopted by central banks[40].

Here I give some thought to inflation targeting in the context of the Summers effect discussed in this paper. When a central bank adopts an inflation target, its main purpose is to prevent high inflation. However, when it sets the level of inflation to be targeted, the non-negativity constraint on nominal interest rates is an important factor. In practice, central banks usually set the target inflation not at zero percent but at a slightly positive value.

In other words, this implies that central banks effectively take the Summers effect into account when setting the target level of inflation. The existence of a desirable level of ex ante inflation has already been demonstrated for the Japanese economy, although the level itself was shown to vary depending on the size of the shocks[41]. An important implication of this paper is that, when a central bank considers introducing inflation targeting, both the cost of the positive inflation as well as the risk of hitting the zero bound on nominal interest rates must be taken into account in determining the target level of inflation.

5.2 "Summers Band"

It is often the case that central banks simultaneously determine both the target level of inflation and also the acceptable range of deviation from this target level. For instance, both the Bank of England and the Reserve Bank of New Zealand adopt either this or a similar policy framework. The Bank of England determines the target level to be 2.5 percent annual

[40] Theoretical studies are also developing, such as Svensson (1999, 2000).

[41] Downward rigidity of nominal wages or uncertainty regarding inflation also plays an important role on determining a desirable level of the positive ex anteinflation.

growth in the RPIX (Retail Price Index excluding mortgage payments) and also sets a range of 1.5 to 3.5 percent around this target level[42]. Establishing a deviation range is generally considered to enhance the transparency and accountability of monetary policy[43].

Here I consider this acceptable deviation range from the point of view of the Summers effect. This is equivalent to considering how the central bank perceives possible negative shocks[44]. As already shown in the qualitative simulations (Tables 3 and 4), as the size of the negative shock becomes larger and its persistence stronger, the desirable level of positive inflation rises. Therefore, when a central bank prepares against a negative shock whose size and persistence are expected to fall within a given range, the desirable level of positive inflation will also take the form of a range corresponding to the expected range of the shock. From the context of the Summers effect, the acceptable range of target inflation can basically be interpreted as establishing in advance the band of room which monetary policy has for manoeuvre, when there exists uncertainty concerning the shocks which the central bank will face. Here, I refer to this range or room for manoeuvre as the "Summers band".

Taking an example from one of the quantitative simulation results for the Japanese economy (namely that shown in Table 5 for a discount rate of 0.99), the range of the Summers band differs depending on the expected size and persistence of the shocks. If the central bank expects shocks to involve about a 5 percent ($\epsilon_0^n = -0.015$) deviation in the output gap and to persist for at least one year or at most for two years, the resulting Summers band for the desirable rate of ex ante inflation is calculated to range from 1 to 2 percent.

6 Concluding remarks

In recent years, many countries have found themselves facing low economic growth and low inflation, and thus the zero bound on nominal interest rates has become a real challenge

[42]When actual inflation deviates from the predetermined range, the Governor of Bank of England has to account for the reasons by an open letter to the Secretary of Treasury.

[43]The lag of the policy effect is taken into account in the deviation range from the targeted inflation.

[44]I assume here that the parameters with respect to the discount rate and the dependence of the current economy on the past and the future economy change very slowly, and a central bank knows these parameters. On the other hand, I assume that a central bank does not have enough information about shocks.

for central banks. Given these circumstances, the questions of what a central bank can and what it should do to prepare against a possible large negative shock are serious ones. With these issues in mind, this paper demonstrates, both qualitatively and quantitatively, that monetary policy that takes into account the Summers effect is one of the effective options available to central banks who wish to take preemptive action before coming up against the binding non-negativity constraint on nominal interest rates.

The implications of this paper are twofold. The first point is that, by maintaining a positive ex ante inflation rate, a central bank can lower the possibility of exhausting its effective policy instruments; in other words it reduces the likelihood that it will come up against the zero bound on nominal interest rates. By doing so, it is able to decrease the overall social loss associated with a negative shock, even when the cost of accepting a positive inflation rate is included. A desirable level of positive inflation whose benefit is larger than its cost is shown to exist, confirming the effectiveness of monetary policy that takes the Summers effect into account. In addition, this optimal level of positive inflation is highly sensitive to the extent to which economic agents are forward-looking as well as to the size and the persistence of the shocks. The second point is that, by setting the parameters in the estimated theoretical model so that they correspond to past Japanese experience, a desirable level of positive ex ante inflation is also shown to have existed for Japan.

Finally, although this paper discusses the desirable level of positive ex ante inflation exclusively from the standpoint of the zero bound on nominal interest rates, other arguments can also be put forward to support the existence of a desirable ex ante inflation rate, based for example on inflation uncertainty or the resource allocation. These arguments provide additional information indispensable for determining the level of positive inflation that is ultimately appropriate. Lastly, in order to reconfirm the quantitative results in this paper, a more sophisticated quantitative investigation into the Summers effect, using large macroeconomic models and other statistical models, would represent a useful direction for future research.

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A Appendix

In this paper, I assume that the central bank conducts monetary policy so as to minimize the loss in each current period. The only tool at its disposal is the current short-term nominal interest rate, and its decision-making is based on current information under the certain central bank's behavioral assumption[45]. I also make the assumption of certainty equivalence in solving the optimization problem. Under this assumption, the central bank can finely control not only the short-term nominal interest rate but also economic agents' expectations of inflation, and there is no uncertainty[46].

First of all, I show the necessary conditions that a central bank has to satisfy when conducting monetary policy. After that, in order to understand the model's properties, I calculate the steady-state solution, and then introduce shocks so as to demonstrate the dynamic solution. Finally the procedure underlying the numerical simulation is described[47].

A.1 Monetary policy by a central bank

In the following discussion, I assume that $\mu_1 = \mu(0 < \mu \leq 1)$, $\mu_2 = 1 - \mu$, $\nu_1 = \nu(0 < \nu \leq 1)$, and $\nu_2 = 1 - \nu$ as in the previous chapters and then describe the central bank's behavior. The model consists of equation (1), which represents the IS curve, equation (2) which represents the Phillips curve, equations (3) and (4), which capture the objectives of the central bank, and equation (5), which represents the non-negativity constraint on nominal interest rates. Then, under the assumption that the central bank sets the current short-term nominal interest rates by using only current information so as to minimize the loss in each current period, the behavior of the central bank can be interpreted via the following Lagrangian optimization

[45] Nishiyama (2002) shows the precise solution under discretion to the model used in this paper. In this paper, I assume a certain central bank behavior, setting a certain rule to a central bank.

[46] This method is developed by Woodford (1999a) and Clarida et al. (1999). It eliminates the misunderstanding between the economic agents and a central bank. Additionally, I assume the deterministic shock here.

[47] In this paper, unlike Orphanides and Wieland (1998), I do not run a stochastic simulation. Although they have to run stochastic simulations to calculate a desirable level of the inflation in terms of minimum variance for variables, I do not need to carry out the same type of simulations, because I calculate a desirable level of the inflation in terms of loss minimization.

problem[48]. Considering the Summers effect does not change this solution method.

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \delta^t L_t + E_0 \sum_{t=0}^{\infty} \delta^t \left\{ 2\phi_{1t} \left[x_t - \mu x_{t+1} - (1-\mu)x_{t-1} + \sigma^{-1}(i_t - \pi_{t+1} - r_t^n) \right] \right\}$$

$$+ E_0 \sum_{t=0}^{\infty} \delta^t \left\{ 2\phi_{2t} \left[\pi_t - \kappa x_t - \nu \pi_{t+1} - (1-\nu)\pi_{t-1} \right] \right\}. \tag{14}$$

where ϕ_{1t} and ϕ_{2t} represent the Lagrange multipliers associated with the IS curve constraint and Phillips curve constraint respectively. The optimization problem when there is the zero bound on nominal interest rates may be considered in terms of a Kuhn-Tucker problem. Therefore I differentiate the Lagrangian to obtain the first-order conditions with respect to π_t, x_t, and i_t. These are the Kuhn-Tucker conditions:

$$\pi_t + \phi_{2t} = 0, \tag{15}$$

$$\lambda x_t + \phi_{1t} - \kappa \phi_{2t} = 0, \tag{16}$$

$$i_t \phi_{1t} = 0, \tag{17}$$

$$\phi_{1t} \geq 0, \tag{18}$$

$$i_t \geq 0, \tag{19}$$

where equations (17)-(19) represent the Kuhn-Tucker conditions regarding the zero bound on nominal interest rates. These conditions state that, while ϕ_1 is non-negative, the nominal interest rate is zero but when ϕ_1 becomes negative, the nominal interest rate becomes non-negative. The above five equations, together with the IS curve and the Phillips curve, are the conditions for the loss minimization.

A.2 The solution in the steady state

Here I show the steady-state solutions. The steady-state solutions of the endogenous variables are calculated by substituting $x_{t+1} = x_t = x_{t-1} = x_\infty$, $\pi_{t+1} = \pi_t = \pi_{t-1} = \pi_\infty$, $i_t = i_\infty$,

[48] Because the economic agents and a central bank perfectly forecast shocks in the present and in the future, expectation operator E is not included in the Lagrangian.

$\phi_{1t} = \phi_{1\infty}$, $\phi_{2t} = \phi_{2\infty}$, and $r_t = r^n_\infty$ into the first-order conditions, the IS curve, and the Phillips curve. There are two steady-state solutions in this model. One is the interior solution given by

$$i_\infty = r^n_\infty, \quad \pi_\infty = 0, \quad x_\infty = 0, \quad \phi_{1\infty} = 0, \quad \phi_{2\infty} = 0. \tag{20}$$

and the other is the corner solution given by

$$i_\infty = 0, \quad \pi_\infty = -r^n_\infty, \quad x_\infty = 0, \quad \phi_{1\infty} = \kappa r^n_\infty, \quad \phi_{2\infty} = r^n_\infty. \tag{21}$$

It is important to note that because I assume that $r^n_\infty > 0$, in the corner solution, the loss becomes non-zero even in the steady-state. Therefore I focus, in the analysis, on the interior solution.

As mentioned above, in the steady state, the nominal interest rate is set to equal the natural rate of interest. In this model, because I assume that shocks occur only from the natural rate of interest, nominal interest rates, which are non-negative ($i_t \geq 0$), are set as follows:

$$i_t = r^n_t. \tag{22}$$

Therefore, when the nominal interest rate is positive ($i_t > 0$), the loss is always zero. Only if the non-negativity constraint on nominal interest rates is binding will any loss occur. Furthermore, when a positive ex ante inflation rate is maintained, nominal interest rates are set as follows:

$$i_t = r^n_t + \pi^*. \tag{23}$$

A.3 Dynamic solution

Now I present the dynamic solution by extending the method proposed by Jung et al. (2003). In this solution, the Kuhn-Tucker conditions are used to deal with the non-negativity constraint on nominal interest rates. At first, under the assumption that the natural interest rate shock converges monotonically to its steady-state value, the non-negativity constraint may straightforwardly be seen to be binding ($i_t = 0$) until some first particular period, denoted by T^B, but not to be binding ($i_t > 0$) afterwards[49]. The entire time interval, from period 0 until the final period, may therefore be divided into two phases in terms of the condition of nominal interest rates as follows:

$$i_t = 0 \quad \text{for} \quad t = 0, 1, \cdots, T^B, \tag{24}$$

$$i_t \geq 0 \quad \text{for} \quad t = T^B + 1, \cdots. \tag{25}$$

I solve the optimization problem by dividing the problem into two cases[50]. One is the case when $t \geq T^B + 1$, where the non-negativity constraint is not binding, and the other is the case when $t < T^B + 1$, where the non-negativity constraint is binding. First I show the solution in the case of $t \geq T^B + 1$. Once the non-negativity constraint becomes non-binding, the Kuhn-Tucker conditions ensure that ϕ_{1t} becomes zero. Then equations (15) and (16) are transformed as follows:

$$\lambda x_t + \kappa \pi_t = 0 \quad \text{for } t = T^B + 1, \cdots. \tag{26}$$

By substituting equation (26) into equation (2), equation (27) is obtained as follows:

$$\pi_{t+1} - \nu^{-1} \left[1 + \frac{\kappa^2}{\lambda} \right] \pi_t + (\nu^{-1} - 1)\pi_{t-1} = 0. \tag{27}$$

The two eigenvalues of the difference equation given by equation (27), ξ_1 and ξ_2 ($\xi_1 \leq \xi_2$), satisfy $0 < \xi_1 < 1$, $1 < \xi_2$. In this case, the solution is uniquely given by

$$\pi_t = 0 \quad \text{for } t = T^B + 1, \cdots. \tag{28}$$

Then, from equations (1) and (26), the paths of the output gap and nominal interest rates are given as follows:

$$x_t = 0 \quad \text{for } t = T^B + 1, \cdots, \tag{29}$$

[49]In other words, a central bank conducts monetary policy according to this behavioral setting.
[50]Because the case where i_t=ϕ_{1t}=0 is also considered, equation (25) includes the equal sign.

$$i_t = r_t^n + \sigma(1-\mu)x_{t-1} \qquad \text{for} \quad t = T^B + 1, \tag{30}$$

$$i_t = r_t^n \qquad \text{for} \quad t = T^B + 2, \cdots . \tag{31}$$

As shown in equations (30) and (31), the path of nominal interest rates consists of two parts. This is because the IS curve includes the lag of the output gap.

Next I show the solution in the case when $t < T^B+1$. When the non-negativity constraint is binding, i_t is zero from the Kuhn-Tucker conditions. Then equations (1) and (2) are transformed to give the following vector equation:

$$X_{t+1} = AX_t + BX_{t-1} + u_t. \tag{32}$$

I assume the initial and terminal conditions are $X_{T^B+1} = 0$ and $X_{-1} = 0$. Here the matrices of equation (32) are defined as follows:

$$X_t = \begin{pmatrix} \pi_t \\ x_t \end{pmatrix}, \qquad A = \begin{pmatrix} \nu^{-1} & -\kappa\nu^{-1} \\ -(\nu\mu\sigma)^{-1} & \mu^{-1} + \kappa(\nu\mu\sigma)^{-1} \end{pmatrix},$$

$$B = \begin{pmatrix} -\nu^{-1}(1-\nu) & 0 \\ (\nu\mu\sigma)^{-1}(1-\nu) & \nu^{-1}(\nu-1) \end{pmatrix}, \quad u_t = \begin{pmatrix} 0 \\ -(\mu\sigma)^{-1}r_t^n \end{pmatrix}. \tag{33}$$

By using equations (28)-(32), the dynamic solution can be calculated, with the precise solution depending on the shock and the values attributed to the parameters.

A.4 Procedure for carrying out the numerical simulations

Here I show how to calculate the T^B subsequently used in computing the dynamic solution, and also how to determine the desirable level of positive inflation π^*_{opt}.

In numerical simulations, firstly it is necessary to identify the T^B that satisfies the following condition obtained from equations (15)-(17) to start calculation:

$$\phi_{1T^B} = -(\lambda x_{T^B} + \kappa\pi_{T^B}) \leq 0. \tag{34}$$

In practice, I search for T^B as follows: (i) T^B is set from one to a sufficiently high value, say 30, after checking $\epsilon_0^n + r_\infty^n < 0$[51]; (ii) I search for the initial value $T^B = c$ that satisfies the

[51] In the case where $\epsilon_0^n + r_\infty^n > 0$, the non-negativity constraint does not bind, namely $T_B = 0$, and the loss is always zero as shown in A.2. As a result, I do not need any calculation.

condition given by equation (34); (iii) finally I attain the solution by calculating the path at $T^B = c - 1$[52].

I turn next to the method for calculating the desirable level of positive inflation π^*_{opt}, which is as follows: (i) I set $\pi^* = 0$ and apply a shock to the model; (ii) I use equations (7), (8), and (10) to calculate the path with given π^*, assuming that the central bank's behavior is characterized as above; (iii) at the same time, I calculate the loss evaluated by equation (4)[53]; (iv) I apply a small positive increase to π^*, pushing it slightly above its pre-set value; (v) I repeat the procedure (ii)-(iv) until the loss evaluated by equation (4) takes the minimum value. The value of π^* associated with the minimum loss is defined as the desirable level of the positive inflation rate π^*_{opt}[54].

[52]However, in the case where $\phi_{1T^B} = i_{1T^B} = 0$, the path calculated in the step (2) is defined as a final solution.

[53]In the case where $\pi^* = 0$, the evaluations given by equation (4) and equation (10) coincide.

[54]In some parameter settings, the loss evaluated by equation (4) becomes the minimum when π^* is zero.

Table 1: A change in the dependence of the current economy on the past and the future economies (μ, ν)

Value of μ, ν ($\mu = \nu$)	0.1	0.5	0.9
Desirable level of positive ex ante inflation (Percent per Annum)	4.8	0.8	0.6

Table 2: A change in the discount rate (δ)

Value of δ	0.99	0.95	0.9
Desirable level of positive ex ante inflation (Percent per Annum)	0.8	3.2	4.0

Table 3: A change in the size of the shock (ϵ_0^n)

Value of ϵ_0^n	-0.01	-0.02	-0.03
Desirable level of positive ex ante inflation (Percent per Annum)	0.2	0.6	0.8

Table 4: A change in the persistence of the shocks (ρ)

Value of ρ	0.3	0.5	0.7
Desirable level of positive ex ante inflation (Percent per Annum)	0.6	0.8	1.2

Table 5: The results of the quantitative simulations for the Japanese Economy (in the case of $\delta = 0.99$, Percent per Annum)

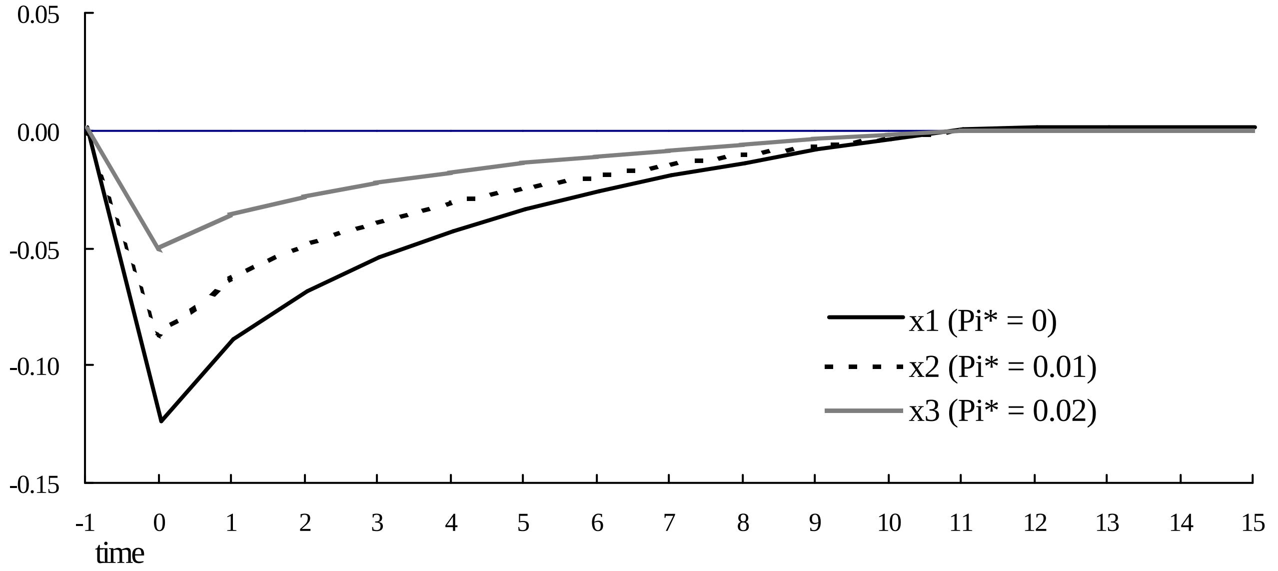
Value of ϵ_0^n \ Value of ρ	0	0.4	0.8
-0.01	0.2	0.4	0.8
-0.015	0.8	1.2	2.0
-0.02	1.2	2.0	2.4

Table 6: The results of the quantitative simulations for the Japanese Economy (in the case of $\delta = 0.95$, Percent per Annum)

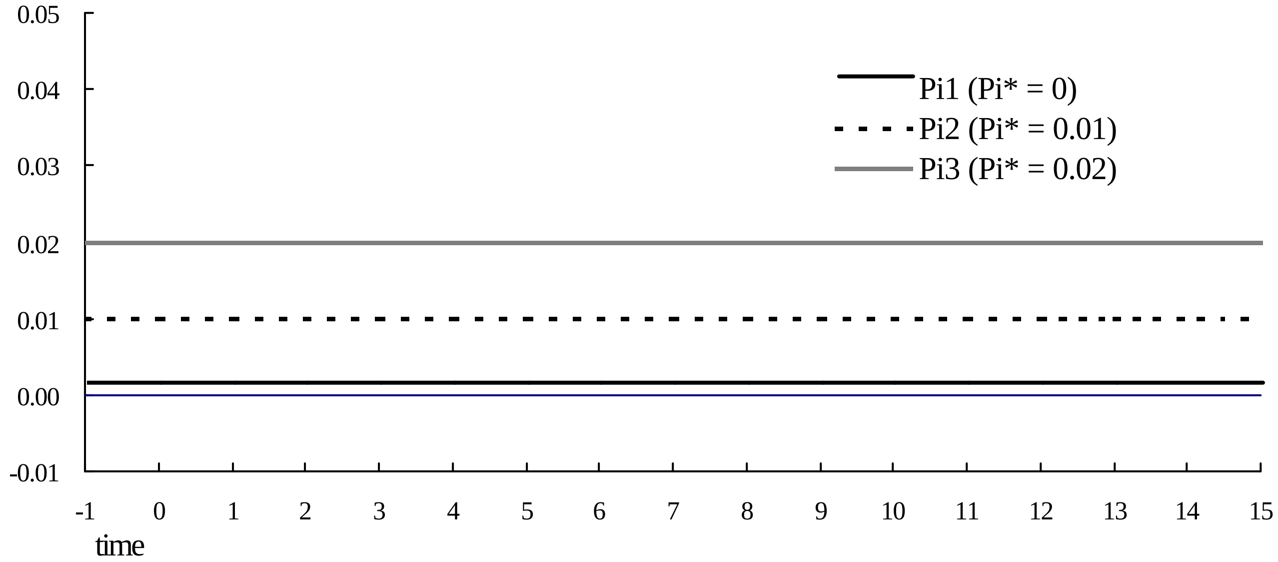
Value of ϵ_0^n \ Value of ρ	0	0.4	0.8
-0.01	0.4	0.8	1.2
-0.015	1.2	2.0	2.8
-0.02	2.0	3.2	4.0

The existence of Pi* and Pi*opt

(1) The output gap response to shock



(2) The inflation response to shock



(3) The costs and benefits of ex ante inflation

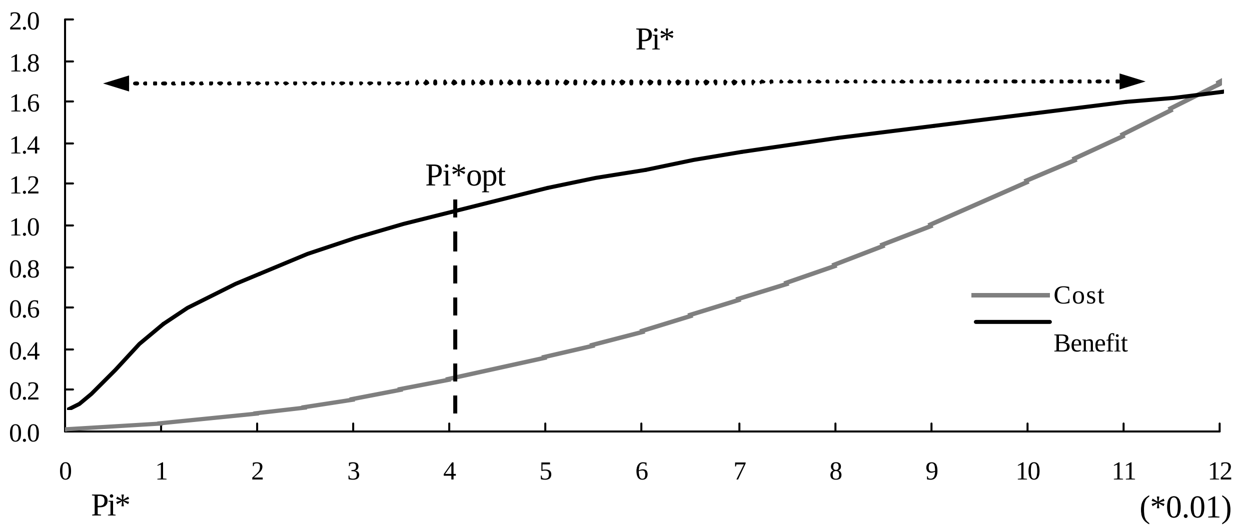
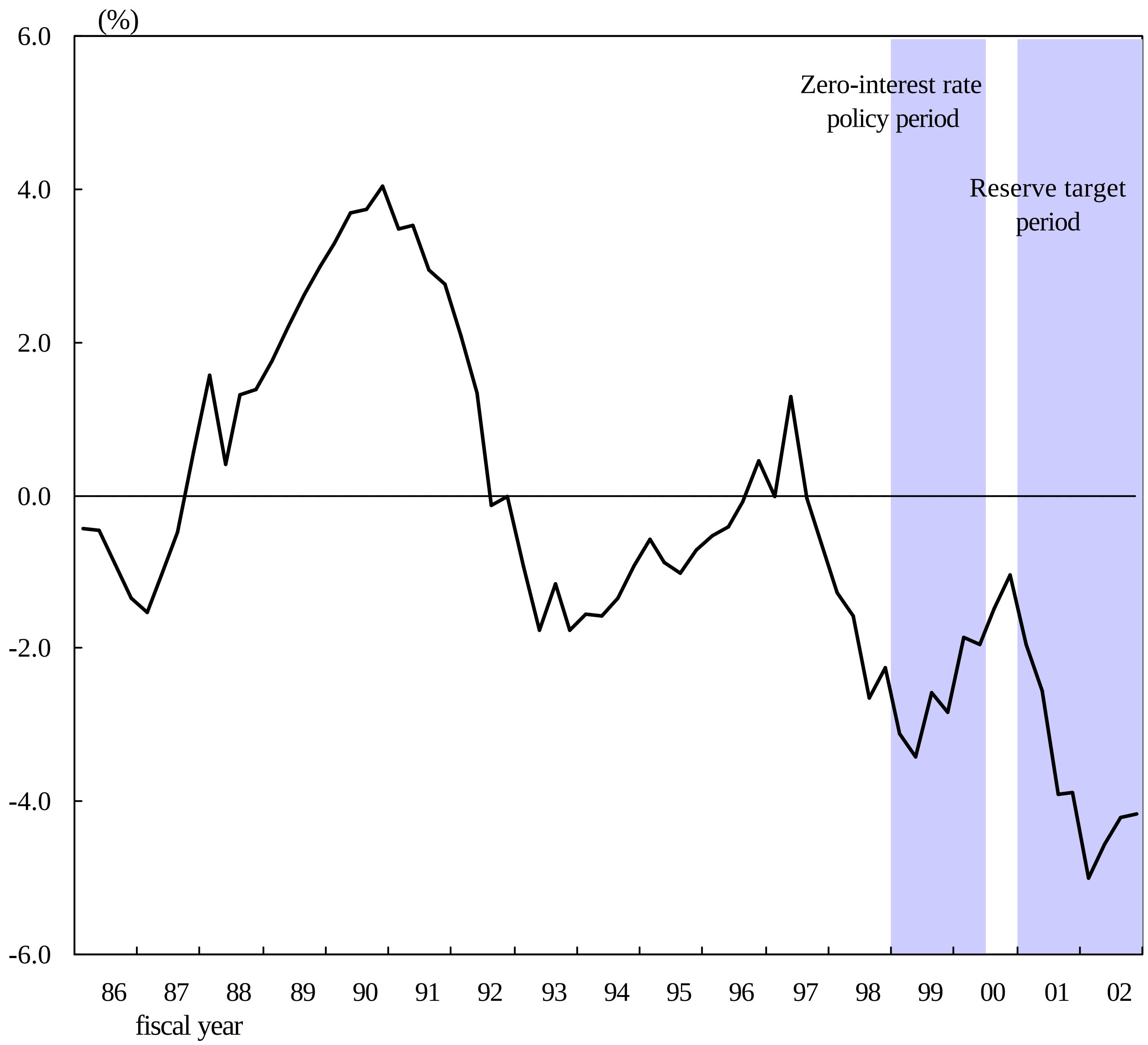


Chart 2

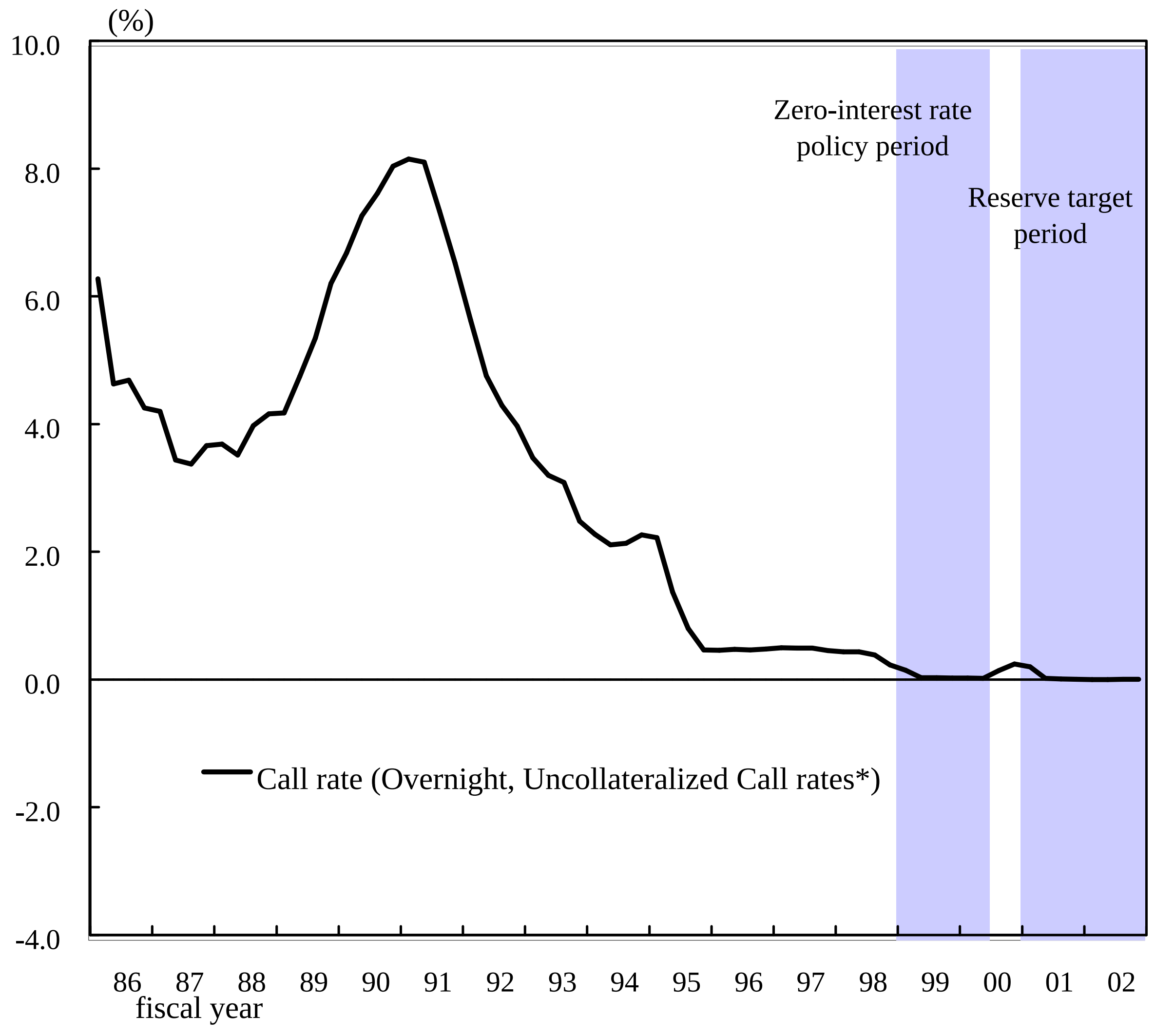
The output gap (Average output gap)



Source: Cabinet Office, "National Accounts", "Capital Stock of Private Enterprises Statistics"; Ministry of Health, Labour and Welfare, "Monthly Labour Survey"; Ministry of Public Management, Home Affairs, Posts and Telecommunications, "Consumer Price Index"; Ministry of Economic, Trade and Industry, "Indices of Industrial Production" etc.

Chart 3

The short-term nominal interest rate



* Quartely average of daily figures

Surce: Bank of Japan.