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Ippei Fujiwara[†]
ippei.fujiwara@boj.or.jp

Naoko Hara[‡]
naoko.hara@boj.or.jp

Yasuo Hirose[§]
yasuo.hirose@jhu.edu

Yuki Teranishi[¶]
yuuki.teranishi@boj.or.jp

Bank of Japan
2-1-1 Nihonbashi Hongoku-cho, Chuo-ku, Tokyo 103-8660

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[†] Research and Statistics Department
[‡] Research and Statistics Department
[§] Currently at the Johns Hopkins University
[¶] Research and Statistics Department

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Ippei Fujiwara[†], Naoko Hara[‡], Yasuo Hirose[§] and Yuki Teranishi[¶]

Research and Statistics Department, Bank of Japan

2-1-1 Nihonbashi-Hongokucho, Chuo-ku, Tokyo 103-8660, JAPAN

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Abstract

In this paper, we set out the JEM (Japanese Economic Model), a large macroeconomic model of the Japanese Economy. Although the JEM is a theoretical model designed with a view to overcoming the Lucas (1976) critique of traditional large macroeconomic models, it can also be used for both projection and simulation analysis. This is achieved by embedding a mechanism within which "short-run dynamics," basically captured by a vector autoregression model, eventually converge to a "short-run equilibrium," which is defined using a dynamic general equilibrium-type model.

JEL Classification: C30; E10; E17; E50

Key words: Large Macroeconomic Model; Monetary Policy; Numerical Method; Projection;

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The views in this paper should not be taken to be those either of the Bank of Japan, any of its respective monetary policy or other decision making bodies. Further, any errors remain our sole responsibility.

[†]Corresponding author. Email: ippei.fujiwara@boj.or.jp.

[‡]Email: naoko.hara@boj.or.jp.

[§]Currently at the Johns Hopkins University. Email: yasuo.hirose@jhu.edu.

[¶]Email: yuuki.teranishi@boj.or.jp.

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1 Introduction

In this paper, we construct the JEM (Japanese Economic Model), a large macroeconomic model of the Japanese economy, which proves to be a very useful tool for analyzing the current Japanese economic situation as well as projecting the future.

The JEM has two features in common with other modern large macroeconomic models.[1] The first of these is that, since the JEM is a theoretical model designed with a view to overcoming the Lucas (1976) critique of traditional large macroeconomic models for their lack of microfoundations, the macroeconomic dynamics in the JEM is governed by deep parameters which are not affected by policy changes. Therefore, we can conduct realistic and theoretically-consistent policy simulations using model consistent expectations. Explicit treatment of expectations based on models with rigid microfoundations has become one of the most intensively researched areas in macroeconomics. Indeed, Woodford (2003), in the seminal piece of research on monetary policy implementation, states: "successful monetary policy is not so much a matter of effective control of overnight interest rates as it is of shaping market expectations of the way in which interest rates, inflation, and income are likely to evolve over the coming year and later." He thus describes "central banking as management of expectations." This is all the more important in the current situation in Japan, where after hitting the zero bound on nominal interest rates, the Bank of Japan needs to rely more on expectations through the "policy duration effect".[2]

The other feature is the JEM's suitability for projection. This is achieved

[1] This modelling approach is often referred to as the "Core-Noncore Approach." As explained below, the core part is the "short-run equilibrium" model, while the noncore part is the "short-run dynamics." Although this has been the standard approach to date when constructing large scale dynamic macroeconomic models, a new approach has recently emerged (see, for example, Smets and Wouters (2002)), in which an integrated model with persistent shocks is estimated using Bayesian methods.

[2] According to Okina and Shiratsuka (2003), "Even though short term interest rates decline to virtually zero, a central bank can produce further easing effects by a policy commitment. A central bank can influence market expectations by making an explicit commitment as to the duration it holds short-term interest rates at virtually zero. If it succeeds in credibly extending its commitment duration, it can reduce long-term interest rates. We call this mechanism the 'policy duration effect,' following Fujiki and Shiratsuka (2002)."

by embedding a mechanism within which the "short-run dynamics," captured by the Vector Autoregression (VAR) model, eventually converge to the above-mentioned "short-run equilibrium," defined by using a dynamic general equilibrium (DGE)-type model. These short-run dynamics enable the JEM to follow actual economic developments more closely, facilitating prediction and also giving more realistic flavor to the simulation exercises.

In this paper, we will describe the JEM and how it can be used to analyze the Japanese economy. We begin by illustrating the basic structure of the JEM and its underlying philosophy, as well as providing a detailed description, grounded in rigorous macroeconomic theory, of how to derive the equations on which the JEM is based. Then, by looking at the properties of the JEM in response to typical shocks faced by the Japanese economy, we demonstrate that the JEM's shock responses are reasonable both empirically and theoretically, and hence may be considered a very good approximation of the Japanese economy. A further interesting challenge when modelling the current Japanese economy is the need to solve the model when the zero constraint on nominal interest rate is binding. We therefore report our approach to tackling this problem. Moreover, as one of the most significant advantages of the JEM over other theoretical models is its suitability for projections, we also demonstrate how such projections are produced.

This paper is organized as follows. In section 2, we review recent developments in macroeconomic modelling, explaining our initial decision to construct the JEM. We then outline the structure of the JEM in section 3. We go on to show the following derivations: of the short-run equilibrium model, based on the optimizing behavior of economic agents, in section 4; of the steady state in section 5; and of short-run dynamics in section 6. In addition, we also demonstrate in section 7, that the JEM reproduces macroeconomic dynamics quite similar to those observed in the actual Japanese economy. We then turn in section 8 to the ongoing problem facing the Japanese economy, namely the binding of the zero nominal interest rate constraint. Since one of the main advantages of the

JEM is that it can be used for practical simulation, in section 9, we demonstrate how to construct projections that are both theoretically consistent and realistic. Model evaluation is then conducted in section 10. Finally in section 11, the contents of this paper are summarized and possible future extensions of the JEM are discussed.

2 Recent Developments in Macroeconomic Modelling

The past decade has witnessed significant innovations in both the theory surrounding monetary policy implementation and the associated computational issues.[3] Hence, the approaches to analyzing monetary policy that are now available are more scientific than before. Nowadays, research on monetary policy needs to make use of models based on rigorous optimizing behavior so that model-consistent expectations can be derived. However, at the same time, for arguments to be lively and empirically relevant, we also need to employ large macromodels which can be used for projections. In this regard, large macromodels of the "New Neo-Classical Synthesis" type advocated by Goodfriend and King (1997), which have recently become the standard approach in macroeconomic modelling, are indispensable.

The problem of time-inconsistency proposed by Kydland and Prescott (1977), and Barro and Gordon (1983), emphasizes the importance of central bank credibility and had led a number of central banks around the world to employ "inflation target"[4] as a possible counter-measure. In order to examine monetary policy under an inflation targeting scheme, it is essential to employ a model with forward looking behavior and firm microfoundations since an inflation targeting policy operates via agents' expectations. Similar arguments apply to the analysis of fiscal policy, since people naturally expect a fiscal deficit eventually be

[3] See for example, Clarida, Gali and Gertler (1999).

[4] Mishkin and Schmidt-Hebbel (2001) report in detail on the actual implementation of the inflation target schemes world-wide.

resolved using future tax income. For these expectations to be concrete, persuasive and realistic, it is necessary to posit a model with rigid microfoundations. It is only in this way that expectations can be computed in a model-consistent manner, and without resource to exogenous derivation or ad hoc assumption. With noteworthy progress having been made in both computer technology and monetary economic theory during the last decade, central banks have been introducing models with these desirable characteristics, including the QPM (Quarterly Projection Model of the Bank of Canada)[5] and the FPS (Forecasting and Policy System of the Reserve Bank of New Zealand).[6] Even some central banks which choose not to employ inflation targeting nevertheless make use of these new types of macromodel. For example, the FRB/US[7] uses a new-style model that emphasizes the importance of intertemporal substitution.

Traditional macromodels, which are often lumped together under the heading of the "Cowles Commission Approach,"[8] have been criticized in seminal papers by Lucas (1976), insisting on the importance of expectations, and Sims (1980), for its implausible identification. In response to these critiques, the identified VAR and DGE models have been heavily used for macroeconomic analysis. The former is useful for projection and forecasting as well as for impulse response analyses. On the other hand, the latter is more suitable for qualitative analyses, such as policy simulations. This is partly because by maintaining strict "stock-flow consistency," it manages to exclude the possibility that agents can enjoy a "free lunch," i.e. that their decisions on current expenditures have no repercussions for future expenditure. In this way, it eventually ensures a well-defined steady state (where by "well-defined" is meant consistency within the steady state), thus allowing model-consistent expectations to be obtained. In this sense, the VAR and DGE may be seen as complementary modern macroe-

[5]For details, see the series of papers published by the Bank of Canada such as Black, Laxton, Rose and Tetlow (1994), Armstrong, Black, Laxton and Rose (1995), Coletti, Hunt, Rose and Tetlow (1996), Butler (1996).

[6]For details, see Black, Cassino, Drew, Hansen, Hunt, Rose and Scott (1997).

[7]For details, see Brayton, Levin, Tyron and Williams (1997).

[8]This name is taken from Favero (2001) which provides a useful summary of developments in macro-modelling.

conomic methodologies. The new-style macromodels referred to above, however, should ideally possess the properties of both methodologies, since they are proposed as vehicles for both projection and policy simulation. This presents a dilemma which is mitigated by combining the two approaches, the VAR and the DGE, together.[9] This involves setting up a mechanism whereby short-run dynamics, captured by the VAR eventually converge to the short-run equilibrium, which is in turn defined by a DGE-type model. Although this methodology cannot escape the critiques completely,[10] it nevertheless provides an extremely powerful tool for both policy simulation and projection. Here we construct a large-scale dynamic general equilibrium model of this type, which we call the JEM, the Japanese Economic Model. The aim is to improve the analysis of monetary policy, allowing projection that is not only empirically-relevant but also theoretically sound.

3 Outline of the Model

In the JEM, each economic variable evolves through three stages: the "short-run dynamics," the "short-run equilibrium," and the "steady state."

The last of these stages is the steady state where all the real variables grow at the same rate, namely the rate of potential GDP growth. Nominal variables grow at this rate plus the target level of inflation set by the central bank. Before reaching steady state, however, there is an intermediate stage: the short-run equilibrium. Equations determining the short-run equilibrium are derived by extending standard RBC theory, as seen, for example, in King, Plosser and Rebelo (1988). Accordingly, households decide their consumption level according to the permanent income hypothesis, and firms maximizes dividends facing the installation costs while preserving strict stock-flow consistency. Neo-classical dynamics, which involves convergences to the steady state, are depicted in the

[9] This is indeed the nature of the "Core Non-Core Approach."

[10] In particular, the VAR is employed in the JEM more with a view to generating realistic model properties than in response to the Sims critique regarding implausible identification.

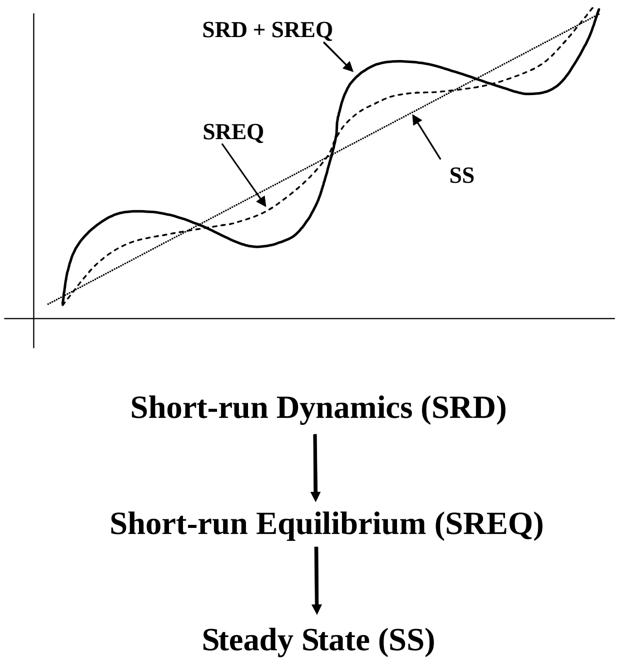

Figure 1: Evolution through Three Stages

short-run equilibrium. Prior to this, however, there is an initial stage of short-run dynamics. Short-run dynamics may be considered in terms of a VAR model around the short-run equilibrium. Such short-run dynamics allow for temporary deviations from equilibrium, as found in the actual macroeconomic data. However, it should be emphasized that all such departures from equilibrium or steady state are temporary in nature, and that all variables are finally made to converge to the steady state (see figure 1).[11] This allows us to attain model consistent expectations and to conduct analysis accordingly.

One of the crucial defects of the theoretically neat DGE model, which has

[11] In contrast to the conventional method based on Blanchard and Kahn (1980), in the analysis below, the uniqueness of the equilibrium path is not guaranteed. Our model is solved using TROLL, which means that the dynamic model is solved numerically by applying the stacked-time method on the Newton-Raphson algorithm. Since the nonlinear model is solved numerically, it is almost impossible to determine the uniqueness of the solution. However, when we linearize the JEM around the steady state, the model seems to be determinate with plausible values for the fudge factor according to the AIM algorithm advocated by Anderson and Moore (1985) and TROLL command, LKROOTS. For details, see Pauletto (1995), Armstrong, Black, Laxton and Rose (1995), or Hollinger (1996).

now become the central tool for analyzing macroeconomic dynamics, is the difficulty of applying it for projection or forecasting purpose. Several measures have been taken to overcome this difficulty and to obtain realistic and persistent responses: in particular, Fuhrer (2000) appeals to habit formation; Roberts (1995) makes use of a new Keynesian Phillips curve; and Rotemberg and Woodford (1997) employ a policy reaction function derived from VAR estimation. The point is that if, as now, there is a pressing need for serviceable analysis of the current state of the economy, the model being employed needs to perform well in projections and forecasting. Analysis, therefore, of movements around the steady state, as seen in a DGE model, may not be close enough to reality since the latter may, after all, find itself far from steady state from time to time. On the other hand, VARs are often used not only for deriving impulse responses but also for projections and forecasts. In short, while in the DGE, we have model-consistent expectations that allow us to conduct policy analysis, the VAR is highly valued for its applicability to projections and forecasts and its ability to reproduce the tendencies usually observed in the data, especially the hump-shaped responses of most macroeconomic variables to shocks. Therefore, by combining these preferred features of the DGE and VAR, we can obtain a powerful dynamic macromodel that can be used not only for projection but also for analysis of monetary policy under zero nominal interest rates, as in Japan today.

Although the JEM is based on macroeconomic models employed at the central banks, we have added several new to allow features in the JEM for more realistic modelling.

- A life cycle income profile as in Faruqee, Laxton and Symansky (1997), is embedded so that fiscal policy in the model should be more non-Ricardian.
- Following Mankiw (1982), and Burda and Gerlach (1992), housing investment is endogenized by considering the housing stock as a durable goods. This induces more real rigidity.

- CES production technology increases generality and allows sensitivity analysis of the effect of the interest rate on investment.
- Monopolistic competition as in Blanchard and Kiyotaki (1987) is introduced in order to achieve a more realistic steady state and theoretical consistency with the existence of relative prices.
- Calibration with a firmer empirical grounding improves confidence in using the JEM for policy analysis.
- The JEM can be used for both projection and policy simulation even under the non-negativity constraint on the nominal interest rate. For this purpose, we employ a new algorithm to solve the model and numerical methods.

In the following sections, we first explain the short-run equilibrium model. We then turn to the steady state, which is attained as the terminal condition of the short-run equilibrium model. Finally, the short-run dynamics, which allow variables to depart temporarily from equilibrium, are described.

4 Short-Run Equilibrium Model

In this section, to understand the basic dynamics in the short-run equilibrium model, we examine each agent's optimizing behavior and use this to derive the equations that drive the model. As deriving all the equations employed in the JEM is not only very time-consuming but also tends to obscure the overall picture,[12] we will focus on the derivation of the basic equations, especially ones derived from households' and firms' optimizing behavior, abstracting from details, such as taxes and deflators.[13]

[12] All the equations employed in the JEM are shown in appendix 1. An *eq* superscript indicates that a variable belongs in the short-run equilibrium model whereas an *ss* superscript indicates a steady state value.

[13] We employ rather different mathematical notations for equations derived from households' and firms' optimizing behavior so that derivations become clearer from those for other equations in the appendix 1.

4.1 Households

The dynamics of the household sectors' decision-making plays the core role in the short-run equilibrium of the JEM.

In the JEM, following the analytical framework advocated by Campbell and Mankiw (1989), there exist two types of consumers: "rule-of-thumb (ROT)" consumers and "permanent-income-hypothesis (PIH)" consumers. ROT consumers simply consume what they earn in each period and save nothing. Therefore, each ROT consumer's consumption equals his individual disposable income. On the other hand, PIH consumers decide their relative expenditure on consumption and housing investment[14] via intertemporal optimization. Introducing two types of consumers allows household expenditure to respond more realistically to shocks, in line with what is often referred to as the "excess sensitivity" or "excess smoothness" observed in consumption dynamics.[15]

Furthermore, the household's equations are based on the Blanchard (1985)-Buiter (1988)-Weil (1989)-Yaari (1965) overlapping generations (OLG) model which is very popular among rational expectations macro models, as it generates a unique steady state consumption level and displays non-Ricardian Equivalence in the equilibrium relationship.[16] Despite its popularity in application, this framework has difficulty in capturing the consumption behavior of retirees and of young liquidity-constrained consumers, i.e. aggregate consumption behavior with demographic changes. Since it fails to engage with these life-cycle considerations, which can play a more substantial role in inducing non-Ricardian equivalence, Evans (1991) claims that the Blanchard-Yaari OLG model expresses just an approximate Ricardian equivalence. Recent works by Faruqee, specifically, Faruqee, Laxton and Symansky (1997), Faruqee and Laxton (2000), Faruqee and Muehleisen (2001) and Faruqee (2002), are among the first attempts to

[14] ROT consumers cannot afford to buy houses.

[15] This is one of the explanations for the rejection of the random walk hypothesis of consumption advocated by Hall (1978). These are well summarized in Muellbauer and Lattimore (1995).

[16] As temporal deviations from equilibrium are admitted in the JEM, it is natural to have non-Ricardian equivalence in the model as a whole.

include life cycle considerations in Blanchard-Buiter-Weil-Yaari OLG model.[17] The model considered here extends Faruqee's work to incorporate durable goods consumption. By considering durable goods consumption as housing investment, we can endogenize the building of the housing stock and this, in turn, results in the persistency of the GDP dynamics observed to some extent in the actual data.

Here, we first look at the derivations of the equations for PIH consumers before turning to the ROT consumers' consumption decision. Finally, by combining these two consumption choices, we attain the aggregate level of consumption and the housing investment function.

4.1.1 PIH Consumers

As the model employed here is the OLG model, we look first at individuals' decision-making.

Individuals

Derivation of Euler Equation Each PIH consumer is assumed to have the additively separable utility[18] from PIH consumption, CFL and the stock value of his house, D, where subscripts a, t denote the individual born at time a's action at time t:[19]

[17] Gertler (1998) makes another attempt to incorporate life cycle behavior within the Blanchard-Buiter-Weil-Yaari framework.

[18] As leisure is fixed, we exclude the disutility generated by working. Furthermore, in contrast with conventional large-scale macroeconomic models, we do not explicitly model money. Implicitly, following the conclusion in Kimura, Kobayashi, Muranaga and Ugai (2002), real balances are included in a utility function with additively separable from. This means that money demand is only passively determined by nominal interest rates and does not have any effect on the economy.

[19] In the standard approach, CFL and D are considered to be non-durable goods consumption and the stock of durable goods, respectively. However, in the JEM, the former is assumed to be consumption and the latter the stock value of housing in line with Bank of England's Model Developing Team (2003).

From an SNA perspective, housing investment can be considered corporate investment. Developers' investment in apartment houses pre-sold to consumers is also counted as housing investment. Hence, Ban, Watanabe, Matsutani, Nakamura, Shintani, Ihara, Kawaide and Takeda (2002) consider housing investment a part of corporate investment. However, most housing investment is in owner-occupied houses, and is carried out purely on the basis of households' intertemporal decision making. For this reason, we regard housing investment as a choice made by households.

$$U_{a,t} = \mathrm{E}_t \sum_{i=0}^{\infty} \left[(1-\gamma)\beta\right]^s \left[\frac{CFL_{a,t+i}^{1-\sigma^{-1}}}{1-\sigma^{-1}} + \theta \frac{D_{a,t+i}^{1-\sigma^{-1}}}{1-\sigma^{-1}} \right], \tag{1}$$

U: utility,

$1-\gamma$: the survival rate,

β: the subjective discount rate,

σ: the elasticity of intertemporal substitution,

θ: the taste parameter.

As only $(1-\gamma)$ consumers will exist in the next period, the utility is necessarily discounted further by this factor.

If we focus more on the flow of durable goods consumption, a budget constraint can be expressed as follows:[20]

$$\begin{aligned} FA_{a,t} &= \frac{1+r_{t-1}}{1-\gamma} FA_{a,t-1} + W_t (1-l) x_{a,t} - CFL_{a,t} \\ &\quad - pd_t \left[D_{a,t} - (1-dh) D_{a,t-1}\right] + RISK_{a,t}, \end{aligned} \tag{2}$$

dh: depreciation rate for housing stock,

pd: the relative price of housing investment goods,[21]

FA: financial assets,

x: the weight defined as below,

$RISK$: the residual income including dividend etc.,[22]

r: (real) interest rate,

W: wage,

l: leisure.

[20] Here, we follow the treatment of durable goods in Obstfeld and Rogoff (1996). At the moment, in order to keep the argument simple, we ignore tax collection.

[21] This is measured by the price of consumption goods.

[22] For example, when the interest rate on net foreign assets holding is higher than that on the domestic capital stock, the profit from this difference is counted as risk income and allocated to PIH consumers. Furthermore, corporate profits are also included in this risk income, since these are also eventually allocated to consumers.

Implicitly, as is popular in the Blanchard-Yaari framework, the existence of a non-profit life insurance company is assumed. As a result, the percentage insurance income equals to γ and competitive lender charges the rate, $\frac{1+r_t}{1-\gamma}$.[23]

Following Faruqee, a weight function is introduced in order to induce the hump shaped profile of labor income, namely the life cycle income profile:

$$x_{a,t} = \frac{\kappa}{(1+\alpha_1)^{t-a+1}} + \frac{1-\kappa}{(1+\alpha_2)^{t-a+1}}, \tag{3}$$

α_1, α_2, κ: parameters.[24]

Now we are ready to derive the structural equations from household decision-making. In order to optimize equation (1) subject to equations (2)and (3), we begin by assuming certainty equivalence[25] and writing down the Lagrangian. The latter is denoted by Φ where the lagrange multiplier is λ:

$$\Phi = \sum_{i=0}^{\infty} \left\{ \begin{array}{cc} & [(1-\gamma)\beta]^i \left[\frac{CFL_{a,t+i}^{1-\sigma^{-1}}}{1-\sigma^{-1}} + \theta \frac{D_{a,t+i}^{1-\sigma^{-1}}}{1-\sigma^{-1}} \right] \\ -\lambda_{a,t+s} & \left(\begin{array}{c} FA_{a,t+i} - \frac{1+r_{t+i-1}}{1-\gamma} FA_{a,t-1+i} \\ -W_{t+i}(1-l)\frac{\kappa}{(1+\alpha_1)^{t-a+1+i}} - W_{t+i}(1-l)\frac{1-\kappa}{(1+\alpha_2)^{t-a+1+i}} \\ +CFL_{a,t+i} + pd_{t+i}\left[D_{a,t+i} - (1-dh)D_{a,t-1+i}\right] + RISK_{a,t+i} \end{array} \right) \end{array} \right\}.$$

From the first order conditions, we obtain the equations below:

$$CFL_{a,t+1} = [\beta(1+r_t)]^{\sigma} CFL_{a,t}, \tag{4}$$

$$CFL_{a,t} = \left(\frac{\iota_t}{\theta}\right)^{\sigma} D_{a,t}. \tag{5}$$

[23]For detailed discussions on the existence of non-profit competitive insurance companies, see Frenkel, Razin and Yuen (1996).

[24]In the JEM, the hump shaped profile of income is expressed not by the two terms indicated here but by three terms.

[25]In the JEM, certainty equivalence, namely perfect foresight, is always assumed when solving the model.

Equation (4) is the consumption Euler equation,[26] and equation (5) is an equation relating consumption and the stock value of housing where ι is the marginal rate of substitution (MRS) between consumption and housing investment defined as below:

$$pd_t - \frac{(1-\gamma)(1-dh)}{1+r_t} pd_{t+1} \equiv \iota_t. \tag{6}$$

Definition of Wealth Here, we first iterate forward the budget constraint in equation (2) so that we can define total wealth, human wealth and financial wealth. Then, the intertemporal budget constraint becomes

$$\begin{aligned} &\sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=0}^{i-1}(1+r_{t+j})} \left(CFL_{a,t+i} + \iota_t D_{a,t+i} \right) = \frac{1+r_{t-1}}{1-\gamma} FA_{a,t-1} \qquad (7) \\ &+ pd_t (1-dh) D_{a,t-1} \\ &+ \sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=0}^{i-1}(1+r_{t+j})} W_{t+i}(1-l) \frac{\kappa}{(1+\alpha_1)^{t+i-a+1}} \\ &+ \sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=0}^{i-1}(1+r_{t+j})} W_{t+i}(1-l) \frac{1-\kappa}{(1+\alpha_2)^{t+i-a+1}} \\ &+ \sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=0}^{i-1}(1+r_{t+j})} RISK_{t+i}, \end{aligned}$$

which is derived with the transversality condition,

$$\lim_{i \to \infty} \left[\frac{(1-\gamma)^i}{\prod_0^i (1+r_{t+i})} \right] FA_{a,t+i} = 0.$$

Further, we define total wealth, human wealth, and financial wealth as follows:

Total wealth: $TW_{a,t} = \sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=a}^{i-1}(1+r_{t+j})} \left(CFL_{a,t+i} + \iota_t D_{a,t+i} \right)$,

Human wealth: $H_{a,t} = H_{a,t}^1 + H_{a,t}^2$,

$$H_{a,t}^1 = \sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=a}^{i-1}(1+r_{t+j})} W_{t+i}(1-l) \frac{\kappa}{(1+\alpha_1)^{t+i-a+1}},$$

$$H_{a,t}^2 = \sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=a}^{i-1}(1+r_{t+j})} W_{t+i}(1-l) \frac{1-\kappa}{(1+\alpha_2)^{t+i-a+1}},$$

[26] Note that this is a standard form of the Euler equation. The existence of the survival rate produces no distortion in the consumption Euler equation.

Financial Wealth: $FA_{a,t} = \sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=0}^{i-1}(1+r_{t+j})} RISK_{t+i}$.

With these definitions of wealth, the intertemporal budget constraint in equation (7) now becomes

$$\begin{aligned} TW_{a,t} &= \sum_{i=0}^{\infty} \frac{(1-\gamma)^i}{\prod_{j=0}^{i-1}(1+r_{t+j})} \left(CFL_{a,t+i} + \iota_t D_{a,t+i} \right) \\ &= \frac{1+r_{t-1}}{1-\gamma} FA_{a,t-1} + pd_t \left(1-dh\right) D_{a,t-1} + H_{a,t} + F_{a,t}, \end{aligned}$$

which is also expressed as an Euler equation.

$$TW_{a,t} = \left(CFL_{a,t} + \iota_t D_{a,t} \right) + \frac{1-\gamma}{1+r_t} TW_{a,t+1}. \tag{8}$$

Marginal Propensity to Consume Now we are ready to derive the marginal propensity to consume, ϕ. First, we conjecture that the solved consumption takes the following form:

$$CFL_{a,t} = \phi_t TW_{a,t}. \tag{9}$$

By substituting equations (4), (5) and (8) into equation (9), we obtain the marginal propensity to consume for each individual PIH consumer:

$$\frac{1}{\phi_t} = \frac{1}{\phi_{t+1}} \beta^{\sigma} \left(1+r_t\right)^{\sigma-1} \left(1-\gamma\right) + 1 + \theta^{\sigma} \iota_t^{1-\sigma}. \tag{10}$$

Aggregation

So far, we have just analyzed individual behavior. However, what we need to discover is aggregate behavior. Here, we aggregate behavior by individual households with different birth dates in order to derive the aggregate consumption and housing investment equation.

In order to obtain the aggregate variables, we first assume that the birth rate is (1-survival rate), namely γ. This means that the population is always the same size,[27] and therefore that the model may be expressed in per-capita bases. Hence, the number of people born at a is

$$N_{a,t} = \gamma \left(1 - \gamma\right)^{t-a}.$$

The individual budget constraint in equation (2) may then be transformed into the aggregate budget constraint:[28]

$$\begin{aligned} fa_t &= (1 + r_{t-1}) fa_{t-1} + w_t \sum_{a=0}^{t} x_{a,t} (1-l) \gamma (1-\gamma)^{t-a} - cfl_t \\ &\quad - pd_t \left[d_t - (1-dh) d_{t-1}\right] + risk_t, \end{aligned}$$

where small capitals denote per capita values.

Since macro level labor income is defined via the first order condition of the production function, to be discussed below, the following condition must be satisfied:

$$w_t \sum_{a=0}^{t} \left(\frac{\kappa}{(1+\alpha_1)^{t-a+1}} + \frac{1-\kappa}{(1+\alpha_2)^{t-a+1}} \right) (1-l) \gamma (1-\gamma)^{t-a} = w_t (1-l).$$

This reduces to the condition specified below, which is the condition the parameters are set to satisfy:

$$1 = \frac{\kappa\gamma}{\alpha_1 + \gamma} + \frac{(1-\kappa)\gamma}{\alpha_2 + \gamma}.$$

Given these parameter settings, households' aggregate budget constraint is now

[27] Specifically, $\sum_{a=0}^{t} N_{a,t} = 1$. The method proposed in Faruqee and Muehleisen (2001) enables the consideration of population dynamics. However, following the Faruqee and Muehleisen method would require us to abandon the per capita economy paradigm. This topic will be covered in future extensions of the JEM.

[28] By definition, FA_t, for example, represents the aggregation of $FA_{a,t}$ across all cohorts.

$$fa_t = (1 + r_{t-1}) fa_{t-1} + w_t (1 - l) - cfl_t - pd_t [d_t - (1 - dh) d_{t-1}] + risk_t. \quad (11)$$

Next, we derive the aggregate equations for human wealth. By definition, they are expressed as follows:

$$\begin{aligned}\sum_{a=0}^{t} h^1_{a,t} \gamma (1-\gamma)^{t-a} &= w_t \sum_{a=0}^{t} \frac{\kappa (1-l)}{(1+\alpha_1)^{t-a+1}} \gamma (1-\gamma)^{t-a} \\ &+ \sum_{a=0}^{t} \frac{1-\gamma}{(1+r_t)(1+\alpha_1)} h^1_{a,t+1} \gamma (1-\gamma)^{t-a+1},\end{aligned}$$

$$\begin{aligned}\sum_{a=0}^{t} h^2_{a,t} \gamma (1-\gamma)^{t-a} &= w_t \sum_{a=0}^{t} \frac{(1-\kappa)(1-l)}{(1+\alpha_2)^{t-a+1}} \gamma (1-\gamma)^{t-a} \\ &+ \sum_{a=0}^{t} \frac{1-\gamma}{(1+r_t)(1+\alpha_2)} h^2_{a,t+1} \gamma (1-\gamma)^{t-a+1}.\end{aligned}$$

These are also expressed as Euler equations:

$$h^1_t = \frac{\kappa\gamma}{\alpha_1 + \gamma} w_t (1-l) + \frac{1-\gamma}{(1+r_t)(1+\alpha_1)} h^1_{t+1},$$

$$h^2_t = \frac{(1-\kappa)\gamma}{\alpha_2 + \gamma} w_t (1-l) + \frac{1-\gamma}{(1+r_t)(1+\alpha_2)} h^2_{t+1}.$$

As for financial wealth, from the definition of financial wealth, the Euler equation for the dynamics of financial wealth becomes

$$fa_t = risk_t + \frac{1-\gamma}{1+r_t} fa_{t+1}. \quad (12)$$

With the definitions above, the aggregate consumption function may be written down as follows:

$$cfl_t = \phi_t \left[(1 + r_{t-1}) fa_{t-1} + pd_t (1 - dh) d_{t-1} + h_t + f_t \right]. \tag{13}$$

The housing stock, meanwhile, is derived via a simple per-capita expression for equation (5):

$$d_t = \left(\frac{\iota_t}{\theta} \right)^{-\sigma} cfl_t. \tag{14}$$

Further, housing investment is defined as the change in the housing stock:

$$ih_t = d_t - (1 - dh) d_{t-1}. \tag{15}$$

4.1.2 ROT Consumers

So far, we have analyzed consumers who make decisions according to intertemporal optimization. Here we introduce ROT consumers, who are liquidity constrained and thus spend all they obtain. It is assumed that in each cohort, $\eta_1, \eta_2 \in [0, 1]$ of consumers are liquidity constrained and cannot borrow. Therefore, the dynamic equations for human wealth are now transformed into

$$h_t^1 = (1 - \eta_1) \frac{\kappa\gamma}{\alpha_1 + \gamma} w_t (1 - l) + \frac{1 - \gamma}{(1 + r_t)(1 + \alpha_1)} h_{t+1}^1, \tag{16}$$

and

$$h_t^2 = (1 - \eta_2) \frac{(1 - \kappa)\gamma}{\alpha_2 + \gamma} w_t (1 - l) + \frac{1 - \gamma}{(1 + r_t)(1 + \alpha_2)} h_{t+1}^2. \tag{17}$$

η should be constrained by the weight function in equation (3). Denoting the oldest age at which a consumer may remain credit constrained by z, then η should satisfy the following conditions:

$$\eta_1 = 1 - \frac{\kappa}{(1 + \alpha_1)^{t+1-z}},$$

$$\eta_2 = 1 - \frac{1-\kappa}{(1+\alpha_2)^{t+1-z}}.$$

As all their incomes are spent, consumption made by ROT consumers crt is determined by

$$crt_t = \eta_1 \frac{\kappa\gamma}{\alpha_1+\gamma} w_t (1-l) + \eta_2 \frac{(1-\kappa)\gamma}{\alpha_2+\gamma} w_t (1-l). \tag{18}$$

Finally, aggregate consumption is now determined by the sum of the consumption levels of the two types of consumer:

$$c_t = crt_t + cfl_t. \tag{19}$$

Equations (6), (10), (11), (12), (13), (14), (15), (16), (17), (18) and (19) constitute the fundamental dynamic equations describing households' decision-making.

4.2 Corporate Sector

The treatment of the corporate sector in the JEM is fairly standard. Following Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987), each firm seeks to maximize its instantaneous dividend in a monopolistically competitive market, subject to the constraints imposed by a CES production function and an installation cost. The latter takes the form of the "time-to-build constraint" advocated by Hall and Jorgensson (1967).

Under these circumstances, the objective of each firm, denoted by j, is to maximize the following dividend:[29]

$$\Pi_{j,t} = (1-tk)\left[\frac{P_{j,t}}{P_t} Y_{j,t} - W_t (1-l) N_{j,t}\right] - pi_t I_{j,t} + dt_t I_{j,t}, \tag{20}$$

tk: tax rate on corporate profits,

Π: dividend,

[29] W denotes the real wage as used in equation (2).

Y: GDP,

pi: relative price of investment goods,

I: investment,

P: price of consumption goods.[30]

As obvious from the equation above, only the consumption goods market is monopolistically competitive, while dt is the rate of depreciation allowance to investment and is defined as

$$dt_t = \frac{dt_{t+1}(1-\delta) + \delta q_t tk}{1+r_t}, \tag{21}$$

δ: the capital depreciation rate,

q: the price of the capital stock which will be defined later as the shadow price.

Since investment is compiled as a stock, investment today not only secures a depreciation allowance for the next period but also into the future. If we multiply both sides of equation (21) by I, the first term becomes the present value of the depreciation allowance obtained from today's investment in the next period while the second term captures the value of the depreciation allowance to the firm in the next period.

Firms' technology takes the form of a CES production function with Harrod Neutral technology:

$$Y_{j,t} = \left\{(1-\alpha)\left[A_t(1-l)N_{j,t}\right]^{\psi} + \alpha\left(KP_{j,t-1}\right)^{\psi}\right\}^{\frac{1}{\psi}}, \tag{22}$$

α: capital share,

ψ: elasticity of substitution between labor and capital stock,

Y: output,

A: technology,

KP: operating stock.

[30] Consumption goods is assumed to be final goods in this subsection.

Each firm faces a time-to-build constraint, meaning that capital stocks cannot be operative right after installation. This can be considered as one form of adjustment cost:[31]

$$KP_{j,t} = (1-b)\,K_{j,t} + bK_{j,t-1},$$

b: parameter.

The standard law of motion for the capital stock is still valid:

$$K_{j,t} = (1-\delta)\,K_{j,t-1} + I_{j,t}.$$

The above two constraints may be integrated and expressed as a single constraint:

$$KP_{j,t} = (1-\delta)\,KP_{j,t-1} + (1-b)\,I_{j,t} + bI_{j,t-1}. \tag{23}$$

As mentioned, monopolistic competition is assumed in the corporate sector. Each firm produces slightly different products. Here, the composite goods are assumed to be the Dixit-Stiglitz aggregate of a multiplicity of differentiated goods indexed by $i \in [0,1]$. Under these settings, the composite consumption and price index are defined as follows:

$$C_t = \left[\int_0^1 C_t\,(i)^{\rho}\,di\right]^{\frac{1}{\rho}},$$

$$P_t = \left[\int_0^1 P_t\,(i)^{\rho}\,di\right]^{\frac{1-\rho}{\rho}}.$$

Following Blanchard and Kiyotaki (1987), a demand function for each goods becomes as follows:

[31] Although in the JEM, the operating capital stock is assumed to be the weighted sum of the physical capital stock for the past eight periods, here, to make the model easily understood, we look only at the case for the previous period.

$$\frac{P_{j,t}}{P_t} = \left(\frac{Y_{j,t}}{Y_t}\right)^{-(1-\rho)}. \tag{24}$$

Combining equations (20), (22), (23) and (24), each firm's optimization problem may be solved by finding the solution to the Lagrangian problem set out below:

$$\Psi = \sum_{i=0}^{\infty} \frac{1}{\prod_{s=0}^{i-1}(1+r_{t+s})} \left\{ \begin{array}{c} (1-tk)\left\langle \begin{array}{c} Y_{t+i}^{1-\rho}\left\{(1-\alpha)\left[A_t(1-l)N_{j,t}\right]^{\psi} + \alpha\left(KP_{t-1}\right)^{\psi}\right\}^{\frac{\rho}{\psi}} \\ -W_{j,t}(1-l)N_{j,t} \end{array} \right\rangle \\ -P_t^I I_{j,t} + dt_t I_{j,t} \\ -q_{t+i}\left[KP_{j,t+i} - (1-\delta)KP_{j,t-1+i} - (1-b)I_{j,t+i} - bI_{j,t-1+i}\right] \end{array} \right\},$$

From the first order conditions, the following equations are derived:

$$pi_t = dt_t + q_t(1-b) + \frac{b}{1+r_t}q_{t+1}, \tag{25}$$

$$\frac{(1+r_t)q_t - (1-\delta)q_{t+1}}{1-tk} = Y_t^{1-\rho}\rho Y_{j,t}^{\rho-\psi}\alpha\left(KP_{j,t}\right)^{\psi-1},$$

$$Y_t^{1-\rho}\rho Y_{j,t}^{\rho-\psi}(1-\alpha)\left[(1-l)N_{j,t}\right]^{\psi-1}A_t^{\psi} = W_{j,t}.$$

Here, we assume a symmetric equilibrium so that $Y_{j,t} = Y_t$, $KP_{j,t} = KP_t$ and $W_{j,t} = W_t$. Then, the latter two are transformed as:

$$\frac{(1+r_t)q_t - (1-\delta)q_{t+1}}{1-tk} = \alpha\rho\left(\frac{Y_{t+1}}{KP_t}\right)^{1-\psi},$$

and

$$W_t = (1-\alpha)\rho A_t^{\psi}\left[\frac{Y_t}{(1-l)N_t}\right]^{1-\psi}.$$

To be consistent with the household equations, we need to express the firm-side equations in per-capita form as well. By definition, as $y_t = \frac{Y_t}{N_t}$ and $kp_t =$

$\frac{KP_t}{N_t}$, the above two equations may be expressed in the following per-capita forms:

$$\frac{(1+r_t)\,q_t-(1-\delta)\,q_{t+1}}{1-tk}=\alpha\rho\left(\frac{y_{t+1}}{kp_t}\right)^{1-\psi}, \tag{26}$$

$$w_t=(1-\alpha)\,\rho A_t^{\psi}\left[\frac{y_t}{(1-l)}\right]^{1-\psi}. \tag{27}$$

The production function, investment, and the time-to-build constraint may also be re-written in per capita forms:

$$y_t=\left\{(1-\alpha)\left[A_t\,(1-l)\right]^{\psi}+\alpha\,(kp_{t-1})^{\psi}\right\}^{\frac{1}{\psi}}, \tag{28}$$

$$k_t=(1-\delta)\,k_{t-1}+i_t, \tag{29}$$

$$kp_t=(1-b)\,k_t+bk_{t-1}. \tag{30}$$

Equations (21), (25), (26), (27), (28), (29) and (30) constitute the fundamental dynamic equations governing firm's decision-making.

4.3 Government

In contrast to some recent works such as Benigno and Woodford (2003), the government sector in the JEM is not considered to be an optimizing agent. There are target levels for government debt[32] and for government expenditure. In order to achieve this target, the government collects tax on labor income, corporate tax, indirect tax, and tariffs.[33]

In each period, government should satisfy the budget constraint:

[32] As explained later, the target is the ratio of government debt to GDP when running the model.

[33] It may seem contrary to the consensus that the government adjusts labor income tax in order to satisfy the government budget constraint instead of indirect tax rates such as consumption tax. However, when conducting projections, we carefully monitor the developments of the income tax rate so that it moves reasonably.

$$gb_t = (1 + r_t)\, gb_{t-1} + g_t - rtd_t - rtk_t - rti_t,$$

gb: government debt,

g: government expenditure,

rtd: revenue from labor tax,

rtk: revenue from corporate tax,

rti: revenue from indirect tax and tariffs.

As with the corporate tax rate, the indirect tax and tariff rates are exogenously set, and the labor tax rate is adjusted so that the government budget constraint should be satisfied in the short-run equilibrium.[34] A brief explanation of the tax collecting system is as follows.

Since the tax on labor income is imposed directly on labor income, revenue is that proportion of labor income:

$$rtd_t = td_t w_t\,(1 - l)\,,$$

td: tax rate on labor income.

The revenue from corporate tax is, similarly, just the corresponding proportion of corporate profits:

$$rtk_t = tk\,[y_t - w_t\,(1 - l) - \delta q_{t-1} k_{t-1}]\,,$$

tk: corporate tax rate.

Revenue from indirect tax and tariffs are collected by including these in deflators. All the equations for deflators, except for those of imports, exports, and inventory, are determined in a similar fashion. Here, therefore, we look only at the identity for the consumption deflator as an example to aid understanding the indirect tax collection system:

[34] In the short-run dynamics, which will be explained later, the amount of government debt is adjusted so that the instantaneous budget constraint is always satisfied.

$$pc_t c_t = (1 + tic)\left[pcd_t\left(c_t - cm_t\right) + (1 + ticm)pcm_t cm_t\right], \tag{31}$$

pc: consumption deflator, tic: indirect tax rate on consumption,

pcd: domestic consumption deflator at factor cost,

cm: imports of consumption goods,

$ticm$: tariff rate on imported consumption goods,

pcm: deflator for imported consumption goods at factor cost.

As a result, indirect tax and tariffs from consumption are defined as:

$$tic * pcd_t\left(c_t - cm_t\right) + tic * pcm_t cm_t + ticm * pcm_t cm_t + tic * ticm * pcm_t cm_t.$$

All these indirect tax and tariff rates are summarized in the average indirect tax rate denoted by tiy. This gives us the identity below:

$$py_t y_t = (1 + tiy_t)\, pfc_t y_t.$$

py: GDP deflator,

pfc: factor cost price for GDP.[35]

pfc is then determined simultaneously, using the above equation and the following:

$$py_t y_t = (1 + tiy_t)\left[\begin{array}{c} pcd_t\left(c_t - cm_t\right) + pihd_t\left(ih_t - ihm_t\right) + pid_t\left(i_t - im_t\right) \\ +pgd_t\left(g_t - gm_t\right) + px_t x_t + pii_t ii_t \end{array}\right],$$

$pihd$: domestic housing investment deflator at factor cost,

ihm: imports of housing investment goods,

pid: domestic investment deflator at factor cost,

[35] This is the deflator excluding indirect taxes and tariffs.

im: imports of investment goods,

pgd: domestic government deflator at factor cost,

gm: imported government goods,

px: exports deflator,

x: exports,

pii: inventory deflator,

ii: inventories.

The right hand side of this equation, excluding $(1 + tiy_t)$, describes nominal GDP minus total indirect taxes. The latter are subtracted because the domestic demand component deflators, such as pcd, exclude indirect taxes and tariffs. For this reason, pfc is taken to be the factor cost price deflator, while tiy is the average indirect tax rate including indirect taxes and tariffs.

As a result, the total revenue from indirect taxes and tariffs may be defined as follows:

$$rti_t = tiy_t pfc_t y_t.$$

4.4 External Sector

As is usually the case for large-scale macromodels, a small open economy is assumed in the JEM. Therefore, in the long-run, it is supposed that the domestic interest rate converges to the world interest rate. Temporary differences between domestic and world interests rates induce financial asset shifts, but net exports are determined so that the identity for the foreign sector is always satisfied.

As explained above, since the core dynamics of the short-run equilibrium in the JEM are driven by the Blanchard-Buiter-Yaari-Weil OLG model rather than the more typical DGE model, the inclusion of a survival rate means that the subjective discount rate can depart from the reciprocal of the steady state real interest rate. The result of this difference is the existence of net exports and non-zero net foreign assets even in the steady state where the domestic and world interest rates are equal.[36]

Let the variables with a $*$ superscript denote those denominated in foreign currencies. We may then express the identity which will always hold for trade and net foreign assets as follows:

$$nfa_t^* = (1 + r_t^*)\, nfa_{t-1}^* + xm_t^*,$$

xm: net exports.

This is transformed into a domestic currency base by defining $nfa_t = nfa_t^* z_t$ giving:

$$nfa_t = (1 + r_t^*)\, \frac{z_{t-1}}{z_t} nfa_{t-1} + xm_t,$$

z: real effective exchange rate.

The real effective exchange rate is determined in order to satisfy the financial market clearing condition:

$$fa_t = pka_t k_t + gb_t + nfa_t,$$

pka: price of capital stock.

Exports and imports are then determined by the following equations. However, as the above identity always need to be satisfied, prices and exchange rates are adjusted either directly or indirectly. Exports are simply determined by the world GDP, y^*, and export prices where the latter are of course sensitive to exchange rates:

$$x_t = x_0 + x_1 y_t^* + x_2 px_t,$$

x_0, x_1, x_2: parameters.

[36] For details on the existence of net trade even in the steady state in the Blanchard-Buiter-Yaari-Weil framework, see Blanchard (1985) and Frenkel, Razin and Yuen (1996).

Imports, on the other hand, are determined in two stages. The same structure is always employed for all the GDP components except for exports and inventories (assumed to be non-tradable). We pick up the case for consumption as an example.

At the first stage, import prices are determined as the weighted average of their past values and prices abroad expressed in terms of the domestic currency:

$$pcm_t = (1 - pcm_1)\, pcm_{t-1} + pcm_1\, (pcrow_t z_t)\,, \tag{32}$$

pcm_1: parameter,

$pcrow$: price of consumption goods in the rest of the world.

Then, the ratio of imported consumption goods to overall consumption is fixed as the relative price of imported goods to domestic goods:

$$cm\_c_t = cm\_c_0 + cms_2\,(1 + ticm)\,\frac{pcm_t}{pcd_t},$$

cm_c_0, cms_2: parameters,

cm_c_t: the ratio of imported consumption goods to overall consumption.

Essentially the underlying dynamics of these equations are similar to those found in open economy models with rigorous microfoundations, established by Obstfeld and Rogoff (1995) and known as the "New Open Economy Macroeconomics (NOEM)".[37]

4.5 Financial Intermediary

Recently, financial market imperfections have been increasingly considered one of the major causes of business cycles, influenced by models such as by the

[37] In the NOEM, monopolistic competition is assumed for both domestic and imported goods, and as a result, the following demand equation for imported goods is derived.

$$cm_t = \zeta \left[\frac{(1 + ticm)\, pcm_t}{pc_t}\right]^{-\vartheta} c_t$$

"financial accelerator model" of Bernanke, Gertler and Gilchrist (1999) and the "credit cycle model" of Kiyotaki and Moore (1997). However, in the JEM, no explicit mechanism for financial market imperfection is embedded. Hence, the financial intermediary is just an artificial entity. The risk-neutral and non-profit financial intermediary's role, therefore, is simply to provide funds and allocate these optimally among different economic agents.

However, looking at movements in actual financial markets, we observe that, in contrast to the predictions of the small open economy model, the domestic real interest rate is not that close to the foreign interest rate, although a tendency towards convergence has been more evident recently. Furthermore, the presence of risk premium means that interest rates governing firm's investment tend to be higher than those for government bonds even when maturities are the same.

Reflecting these stylized facts, the JEM adopts an ad-hoc risk premium[38] which allows us to mimic actual movements in the data. For example, firms are assumed to face an interest rate made up of the risk-free long-term interest rate plus a risk premium:

$$rk_t = rl_t + rk\_rl_t,$$

rk: interest rate for corporate lending,

rl: long-term interest rate,

rk_rl: risk premium on corporate lending rate.

4.6 Prices

So far, we have abstracted the detailed construction of price levels. All demand components have individual deflators, as expressed in equations (31) and (32). Eventually, however, deflators for GDP components always need to satisfy the following condition:

[38] Incorporating time-varying term premium computed from affine transformation of state variables is being examined in our accompanying paper, Fujiwara, Hara, Teranishi, Watanabe and Yoshimura (2004).

$$py_t y_t = pc_t c_t + pi_t i_t + pih_t ih_t + pg_t g_t + pii_t ii_t + px_t x_t - pm_t m_t. \tag{33}$$

5 Steady State

The steady state in the JEM describes a situation in which all real variables are growing at the potential growth rate. Nominal variables grow at this speed plus the target level of inflation set by the central bank. By having this well-defined steady state as a terminal condition, we can include model-consistent expectations in our analytical framework.[39]

The easiest way of understanding the steady state is to think of the steady state value obtained by eliminating time, i.e. the subscript t, from the equations above. For example, the steady state representation of equation (33) is simply

$$py * y = pc * c + pi * i + pih * ih + pg * g + pii * ii + px * x - pm * m.$$

However, if the equation includes lagged variables, the above method is only valid when the potential growth rate is zero or technology growth is zero within the per capita model setting. This is rather unrealistic. Therefore, in the JEM, all real variables are expressed as ratios to potential GDP, yp,[40] allowing us to obtain a well-defined steady state.

For example, dividing both sides of equation (29) by yp gives us:

$$\frac{k_t}{yp_t} = (1-\delta)\frac{k_{t-1}}{yp_t} + \frac{i_t}{yp_t}.$$

If we define $k_t = \frac{k_t}{yp_t}$ as in the JEM, then the relationship between investment

[39]The JEM is solved using TROLL. In TROLL, having set the initial condition and the terminal condition (expressed by the steady state), a large nonlinear model like the JEM is solved by applying a stacked time algorithm to the Newton-Raphson method.

[40]Potential GDP is estimated as in Hirose and Kamada (2001). Further, in a per capita model setting, the potential growth rate depends solely on the technology growth rate.

and the capital stock changes as follows:

$$
\begin{aligned}
k_t &= (1-\delta)\frac{yp_{t-1}}{yp_t}k_{t-1} + i_t \\
&= (1-\delta)\frac{k_{t-1}}{1+ydot_t} + i_t,
\end{aligned}
$$

$ydot$: potential growth rate.

Thus, the steady state relationship between investment and the capital stock becomes:

$$
\frac{ydot+\delta}{1+ydot}k = i.
$$

Similarly, all nominal variables are expressed as ratios to potential GDP multiplied by the GDP deflator, namely nominal potential GDP. By repeating the above approach, we can obtain a well-defined steady state for each variable.

5.1 Growth Accounting

Since all variables in the JEM are thus strictly stationary, the steady state is defined in terms of fixed values. Then, this together with the CES production function with Harrod neutral technology specified in equation (22) guarantees the JEM to be consistent with a balanced growth equilibrium:

$$
1 + ydot_t = (1 + qdot_t)(1 + ndot_t),
$$

$ndot$: Trend population growth rate,

$qdot$: Trend growth in labor-augmenting technical progress.

Both trend population growth and growth in labor augmenting technical progress are exogenous. In order to meet transversality conditions, they are set so that the potential growth rate becomes smaller than the real equilibrium interest rate.

5.2 Deep Parameters

Here, we set out the representative structural parameters which determine the dynamics in both short-run equilibrium and steady state. Note that all the values given here are on an annual basis.

parameter	definition	value
$1-\gamma$	survival rate	0.98
β	subjective discount rate	0.99
σ	intertemporal elasticity of substitution	0.5
θ	taste parameter	0.5
dh	depreciation rate for housing stock	0.06
α	capital share	0.37
ψ	elasticity of substitution	0.001
ρ	demand elasticity	0.2
δ	capital depreciation rate	0.06
r^*	world interest rate	0.01

- The survival rate is set according to the Multimod, the large-scale international macromodel of the IMF.[41]

- The subjective discount rate is determined endogenously in the JEM by setting the steady state net foreign asset position exogenously.

- The intertemporal elasticity of substitution is set according to the estimation results for non-durable goods consumption in Nishiyama (2002).

- The taste parameter which weights the utility from consumption and the stock value of housing is fixed to ensure that the steady state levels of consumption and housing investment, which are already expressed as ratios to potential GDP in the JEM, are empirically reasonable.

- The depreciation rate for housing stock, the capital share and the capital depreciation rate are set at broadly their historical SNA averages.

[41] Details on the current Multimod (MARK III) are described in Laxton, Isard, Faruqee, Prasad and Turtelboom (1998).

- The elasticity of substitution between labor and the capital stock is set so that the production function approximates a Cobb-Douglas function, following the estimation results in Kamada and Masuda (2001).
- The demand elasticity determines the steady state level of corporate profits since its reciprocal is the steady state markup. Therefore, this is basically set at its historical average computed from the SNA data.
- The world interest rate is set rather subjectively to track recent developments in short-term real interest rates in industrialized countries.

6 Short-Run Dynamics

Up to this point, we have introduced the short-run equilibrium (SREQ) model, and the steady state (SS) as the terminal condition of the SREQ. Although the SREQ itself can be used for analyzing the Japanese economy, there are three further points that need to be addressed in order to complete the JEM: 1) the SREQ is unable to account for deviations from equilibrium value frequently observed in actual data; 2) the SREQ framework fails to provide any means of determining the inflation rate; and 3) there is no explicit mechanism for bringing the economy back to steady state, nor is the role of the monetary policy rule in achieving stability specified within the model.

The introduction of the short-run dynamics (SRD) allows variables to deviate temporarily from the equilibrium values determined by agents' optimizing behavior above. Furthermore, the SRD also include a Phillips curve for inflation determination as well as endogenizing monetary policy by incorporating a monetary policy rule. The addition of the SRD to the SREQ completes the JEM. Consequently, all variables evolve through SRD→SREQ→SS, meaning that not only are projection and shock simulation highly realistic but theoretical analysis is also possible as the model retains a well-defined steady state (see figure 1).

In this section, we first provide an outline of these short-run dynamics, and then present the Phillips curve and monetary policy rule that are embedded in the JEM.

6.1 Outline of Short-Run Dynamics

As indeed already mentioned in the introduction, the JEM can be considered as a mixture of VAR and DGE models. The SREQ plays the part of the DGE, while the SRD take on the role of the VAR or Vector Error Correction Model (VECM).

Here, as an example, we look at the short-run dynamics of cfl around its SREQ, which is henceforth denoted by superscript eq:

$$\begin{aligned} cfl_t &= cfl_t^{eq} + cv_1 \left(\frac{\frac{yd_{t-2}}{pc_{t-2}}}{\frac{yd_{t-2}^{eq}}{pc_{t-2}^{eq}}} - 1 \right) \\ &\quad -[cv_{21}(r_{t-2} - r_{t-2}^{eq}) + cv_{22}(rl_{t-2} - rl_{t-2}^{eq})]cfl_{t-2}^{eq} \\ &\quad +cv_3 \left(\frac{nfa_t}{pc_t} - \frac{nfa_t^{eq}}{pc_t^{eq}} \right) - cfladj_t, \end{aligned} \tag{34}$$

cv_1, cv_{21}, cv_{22}, cv_3: parameters,

yd: nominal disposable income.

The second, third and fourth terms on the right hand side determine the extent of the "short-run dynamics" effect on PIH consumption caused by temporary deviations in these economic variables. This may have an impact on short-run consumption behavior, resulting in what are sometimes termed disequilibrium movements.

The term $cfladj$ defines the polynomial adjustment cost (PAC) described by Pesaran (1991) and Tinsley (1993), which is popular in large-scale dynamic general equilibrium models because it allows adjustments with leads and lags to be obtained from optimizing behavior. Concerning consumption dynamics, a second order PAC is employed and $cfladj$ is expressed as follows:

$$
\begin{aligned}
cfladj_t &= cd1\left[cfl_t - cfl_{t-1} - cb1\left(cfl_{t+1} - cfl_t\right)\right] \\
&\quad + cd2\left[\left(cfl_t - cfl_{t-2}\right) - cb1^2\left(cfl_{t+2} - cfl_t\right)\right].
\end{aligned}
$$

$cd1$, $cd2$, $cb1$: parameters.

Combining these together after some manipulation gives us a generalized error correction model which includes leads and lags:

$$
\begin{aligned}
\Delta cfl_t &= -\frac{1}{cd1 + cd2}\left(cfl_t - cfl_t^{eq}\right) + \frac{cd1cb1 - cd2}{cd1 + cd2}\Delta cfl_{t-1} \\
&\quad -\frac{cd2cb1^2}{cd1 + cd2}\Delta cfl_{t+1} - \frac{cd2cb1^2}{cd1 + cd2}\Delta cfl_{t+2}.
\end{aligned}
$$

This specification of the equilibrium (error) correction mechanism has the very favorable property that equation (34) as being derived from agents' optimizing behavior. Denoting the target or desired level of consumption by PIH consumers as cfl^{tar} which is the right hand side of equation (34) excluding the PAC term, then we obtain equation (34) by minimizing the loss defined below:

$$
L = \left\{ \sum_{\tau=0}^{\infty} \left(cfl_{t+\tau} - cfl_{t+\tau}^{tar}\right)^2 + \sum_{i=0}^{n} \varkappa_i \left[A_i\left(L\right) cfl_{t+\tau}\right]^2 \right\}, \tag{35}
$$

$\varkappa$: parameter, $A\left(L\right)$: Lag operator.

In the above specification, n is set at two. The theory behind this loss, L, is that PIH consumers suffer both because of the deviation from their desired level of consumption and from changes in the consumption level. Under these circumstances, PIH consumers try to smooth consumption by gradually narrowing the gap between the present level and their desired level of consumption.

In this sense, although the short-run dynamics may be considered to constitute an ad-hoc non-core approach, they can be still interpreted as being derived from agents' maximizing behavior. In the JEM, economic agents are assumed to conduct two-stage optimization.[42] Agents first derive the equilibrium level by solving the standard optimization problem in the SREQ model. Then, after deciding their target level based on this equilibrium value, they face the loss minimization problem expressed in equation (35). In this sense, if the parameters determining the target level are considered to be deep, the JEM as a whole can escape the Lucas critique since all the parameters employed in the JEM are structural.

We apply this PAC to several variables, although not to all. For example, if the SRD of wages, w is derived by applying the PAC, then it is redundant to further smooth the ROT consumption, crt. By utilizing the PAC appropriately, we can attain a realistic but theoretically consistent long-run dynamic path for each macroeconomic variable.

Most of SRD parameters are estimated using instrumental variables (IV). Parameters for external sectors are mainly calibrated[43] so that the impulse responses to certain shocks in the JEM are similar to those in the VAR.

6.2 Phillips Curve

Inflation dynamics are one of the predominant drivers of short-run dynamics. They induce sticky prices which are thought to be one of the most important factors behind the business cycle. In the JEM, inflation is determined via a Phillips curve for domestically produced goods. Several forms of the new Keynesian Phillips curve[44] which may be considered a Phillips curve with microfoundations, have been introduced in a number of influential pieces of research in this field, such as, for example, the seminal work by Taylor (1979) and Calvo (1983). In the JEM, the hybrid new Keynesian Phillips curve advocated by Fuhrer and

[42] The existence of the bundlers implicitly assumed in the monopolistic competition may suggest the possibility of three stage optimization.

[43] Orcutt (1950) discusses how an aggregation bias, simultaneity bias, and other factors could lead a naive econometrician to find a low trade elasticity even when this elasticity is quite high, and indeed that it is not difficult to obtain a reasonable elasticity which adequately satisfies the Marshall-Lerner condition. Concerning the trade elasticity, Obstfeld (2002) states that "the elasticities are no doubt significantly higher today than they were at the start of the floating-rate period."

[44] Developments in the new Keynesian Phillips curve are well summarized in Roberts (1995).

Moore (1995) is employed. It includes leads and lags of inflation, the sum of the coefficients on which is unity so that the dynamic homogeneity condition or NAIRU condition holds.[45] When this dynamic homogeneity condition holds, we obtain the property that inflation neither accelerates nor decelerates when GDP equals potential GDP, in other words when the output gap is zero. As a result, in the steady state where the output gap is zero, inflation is solely determined by the central bank's adopted target.

In the JEM, the Phillips curve is specified as follows:

$$pdot_t = pdf_1 pdote_t + (1 - pdf_1)\, pdot_{t-1} + pd_0 \left(\frac{y_t}{yp_t} - 1 \right),$$

pdf_1, pd_0: parameters,

$pdot$: inflation rate for domestically produced goods,

$pdote$: expected inflation rate.

Parameters in the above equation are set in line with the estimation results for the hybrid new Keynesian Phillips curve in Japan obtained by Kimura and Kurozumi (2002).

6.3 Monetary Authority

Recent progress in monetary economics during the last decade has been especially noteworthy in the field of optimal monetary policy. With the publication of the seminal paper by Taylor (1993), which established the famous "Taylor Rule," a substantial body of work has been devoted to identifying optimal policy rules either to reduce the variability of the output gap and inflation or to raise the expected utility of the representative agent. All told, monetary policy and monetary policy rules are now considered to play the most critical role in economic stabilization, namely leading the economy to its steady state.

[45] To be strict, the hybrid new Keynesian Phillips curve in this form is derived with microfoundations when the subjective discount rate equals unity, which contradicts the SREQ setting in the JEM. In deriving the hybrid new Keynesian Phillips curve, we implicitly assume that the subjective discount rate can be considered to approximate unity.

In the JEM, the monetary policy rule is given the explicit task of economic stabilization:[46]

$$rn_t = \begin{array}{l} smooth \left[\begin{array}{c} rn_t^{eq} + rsl_4 \left(tpdot_{t+4} - pdottar_{t+4} \right) \\ +rsl_5 \left(tpdot_{t+5} - pdottar_{t+5} \right) \\ +rsl_6 \left(tpdot_{t+6} - pdottar_{t+6} \right) \end{array} \right] \\ \quad + \left(1 - smooth\right) rn_{t-1} \end{array}, \tag{36}$$

rsl_4, rsl_5, rsl_6: parameters,

$smooth$: interest rate smoothing parameter,

rn: call rate,

$tpdot$: weighted average of CPI inflation and $pdot$,

$pdottar$: target level of inflation rate.

Parameters are estimated using instrumental variables.[47] As the lag of the call rate is included, this can be considered a form of the "history dependent monetary policy rule" whose importance is stressed by Giannoni (2000) and Woodford (2003).

7 Diagnostic Simulation

Up to this point, we have focused on establishing the structure of the model. Although as much estimation as possible is employed to obtain the parameters, the model properties should be evaluated in terms of its overall performance. Recently, there has been a tendency to insist upon a "top-down" approach when constructing large-scale macroeconomic models, so that the model as a whole should display reasonable and realistic properties in projections and impulse response analyses. Following this approach, parameters are usually set by cal-

[46] The superior performance of the forecast based rule is verified by Fujiwara, Hara, Hirose and Teranishi (2003).

[47] In monetary DGE models, the instrument rule, which attains the lowest social loss, is usually employed. Although such an optimal rule is obtained in Fujiwara, Hara, Teranishi, Watanabe and Yoshimura (2004), we employ the estimated rule in the basic JEM for better forecasting performances. It is common in the field of large-scale DGE models to apply an empirical rule as the base rule.

ibration. This alternative is to refine the estimation of each equation using cutting-edge econometric techniques. This approach is sometimes referred to as the "bottom-up" approach. However, due to non-exogeneity, simultaneous bias, misspecification and so on, it is almost impossible to obtain reasonable overall model properties using the approach alone.

In the JEM, we make substantive use of the bottom-up approach since we try to obtain parameters by estimation. However, we also pay close attention to the properties of the model as a whole, as we believe that this is where large-scale macroeconomic models have the most to offer. Therefore, several parameters are calibrated.[48] Having done this, we are ready to conduct some diagnostic simulations.

In this section, we conduct several diagnostic shock simulations which are thought to capture the most important disturbances facing the Japanese macroeconomy. To begin with, in order to confirm whether or not the impulse responses obtained in the JEM are consistent with theory and our intuition, we carry out eight shock simulations: 1) a permanent increase in domestic productivity; 2) a permanent decrease in the government's debt-to-income target; 3) a shift in the composition of taxes; 4) a change to the inflation target; 5) an autonomous demand shock; 6) a temporary real exchange rate appreciation; 7) a permanent improvement in the terms of trade; and 8) a monetary policy shock.

At the same time, we have checked whether the shock responses significantly change as the simulation period becomes longer. This is following the concept of "Type III iteration" advocated by Fair and Taylor (1983). Since shock responses do not change significantly with extended horizon, we can conclude the simulation period is long enough to attain convergence.

[48] Recently, there has been increasing interest in methodologies that aim to bridge the gap between these two approaches, as, for example, in Geweke (1999) and Smets and Wouters (2003). Such papers employ Bayesian estimation techniques, enabling them to retain reasonable overall model properties in an estimating context. We would like to employ this method in our future research.

7.1 A Permanent Increase in Domestic Productivity

Figure 2 shows the impulse responses to a permanent increase of 1% in domestic productivity.

The technology shock raises not only output but also the desired capital stock. Hence, investment also increases. As the higher marginal productivity of labor causes wages also to rise, consumption increases. However, consumption evinces life-cycle behavior and investment is bound by the time-to-build constraint, so aggregate demand does not rise as much as output. The result is that the output gap widens. This wider output gap leads to a lower inflation rate. In response to this, the central bank lowers the nominal interest rate. A decrease in the nominal interest rate brings about a lower user cost of capital with the result that investment increases further and the output gap becomes smaller. As for the foreign exchange rate, households need to sell off their net foreign assets so as to support the increase in the domestic capital stock. Consequently, the exchange rate appreciates as net exports necessarily decrease with the decline in the net foreign asset position.

7.2 A Permanent Decrease in the Government's Debt-to-Income Target

Figure 3 shows the impulse responses to a permanent decrease in the government's debt-to-income target of 10%.

In order to decrease debt, the government needs to increase the tax rate on labor income so that the government budget constraint is satisfied. This causes decreases in disposable income and consumption as well as GDP. A decrease in GDP exerts downward pressure on investment and this results in a wider output gap. As a consequence, the inflation rate falls and the central bank lowers the nominal interest rate. On the other hand, a decrease in government debt brings an increase in net foreign assets. This causes the exchange rate to depreciate so that exports increase and imports decrease. These developments in

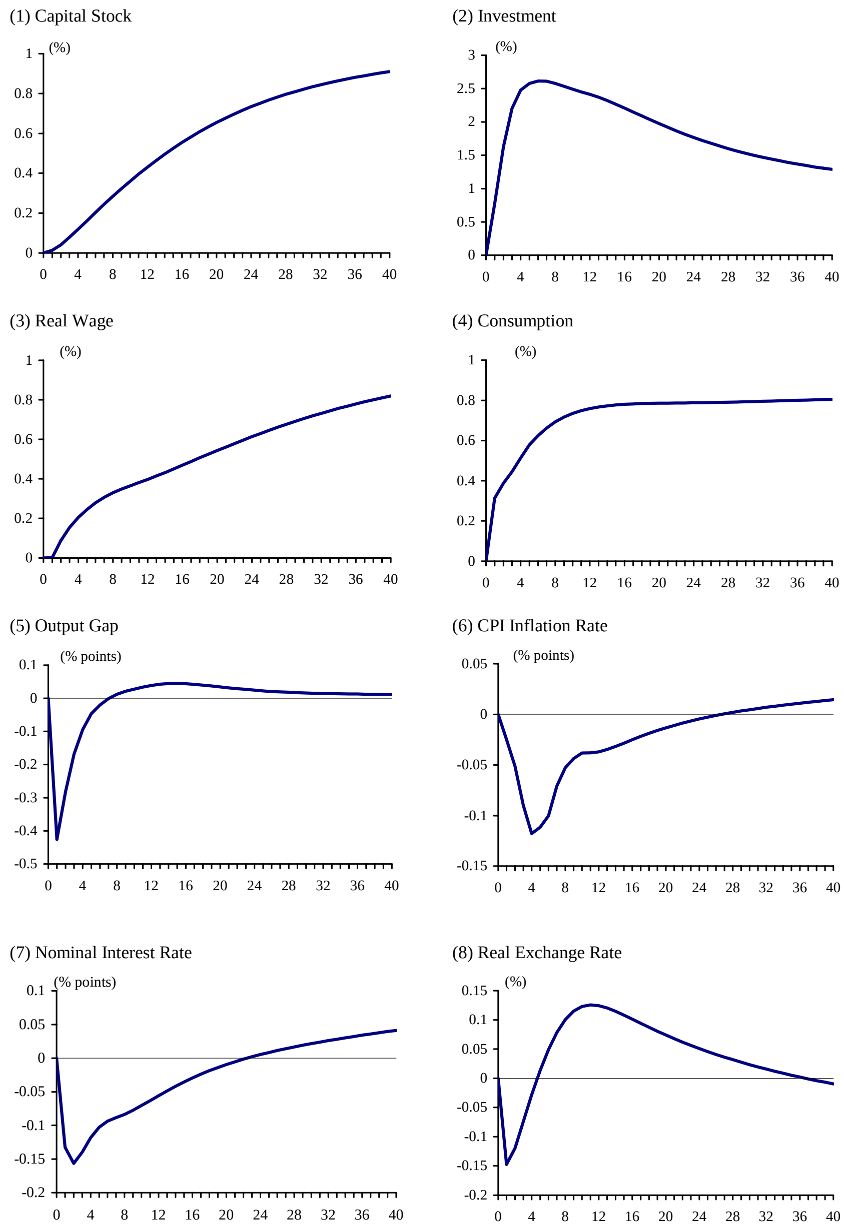

Figure 2: A Permanent Increase in Domestic Productivity

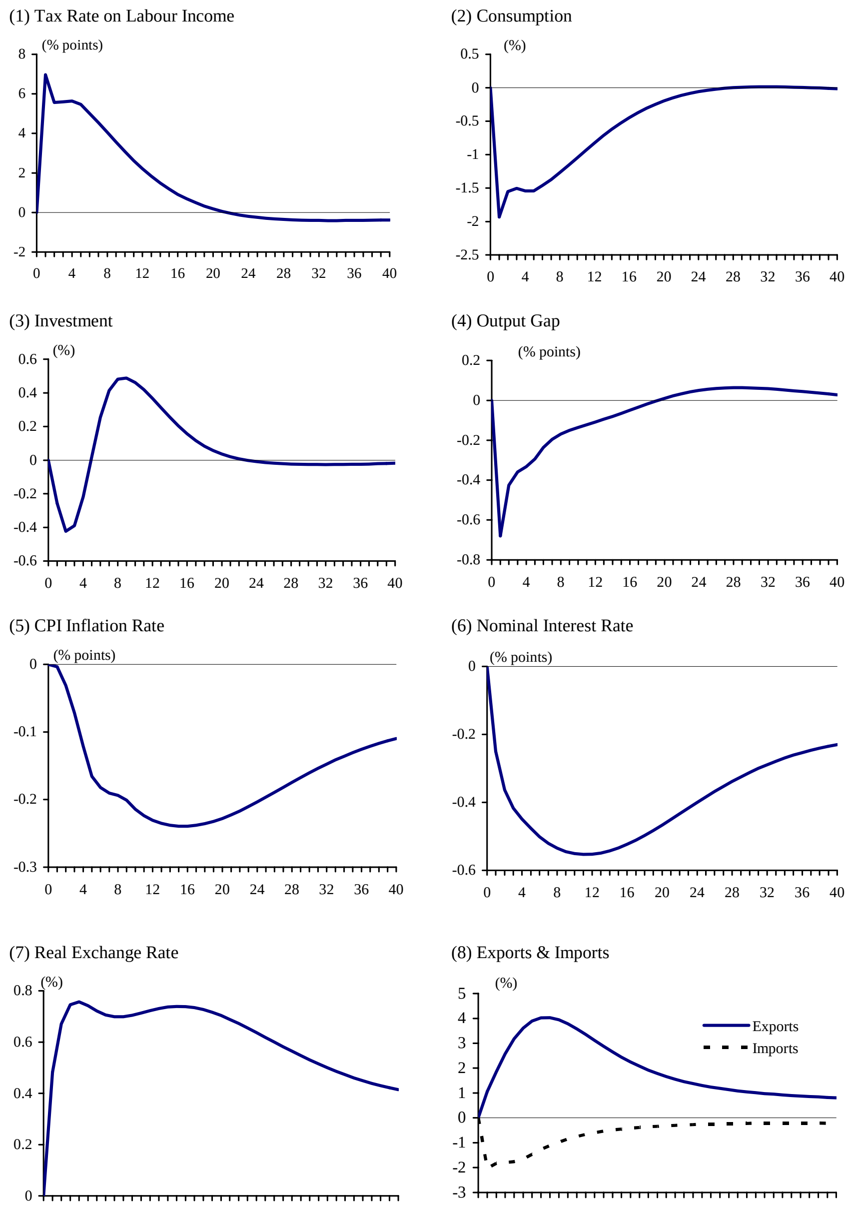

Figure 3: A Permanent Decrease in Government's Debt-to-Income Target

the external sector are further enhanced by the decrease in the nominal interest rate mentioned above, causing an increase in investment as well. Eventually, the government's interest expenses on its debt fall and tax rate on labor income is gradually able to recover to around its level before the shock.

7.3 A Shift in the Composition of Taxes

Figure 4 shows the impulse responses to a permanent increase of 2% in the indirect tax on consumption.

As the government budget constraint must always be satisfied, an increase in the indirect tax reduces the tax rate on labor income. However, as the labor income tax is only gradually adjusted, in the meantime the government lowers its outstanding debt. Hence, the short-run effect from the increase in the indirect tax rate is to reduce consumption. This eventually lowers CPI inflation, following an initial temporary rise due to tax increase, during which the output gap widens. Meanwhile, the initial spurt of inflation causes the central bank to increase the nominal interest rate. After a temporary appreciation, therefore, the exchange rate ends up depreciating.

7.4 A Change to the Inflation Target

Figure 5 shows the impulse responses to a permanent increase of 1% in the inflation target.

Raising the inflation target induces a lower nominal interest rate and therefore, increases investment as a result of lower costs of capital, as well as increasing exports due to the exchange rate depreciation. This in turn increases the output gap, the inflation rate, and inflation expectations. The central bank then reverses its position, raising the nominal interest rate so as to reduce the output gap. Finally, the economy converges to a new steady state in which the inflation rate and nominal interest rate have increased by exactly as much as the inflation target.

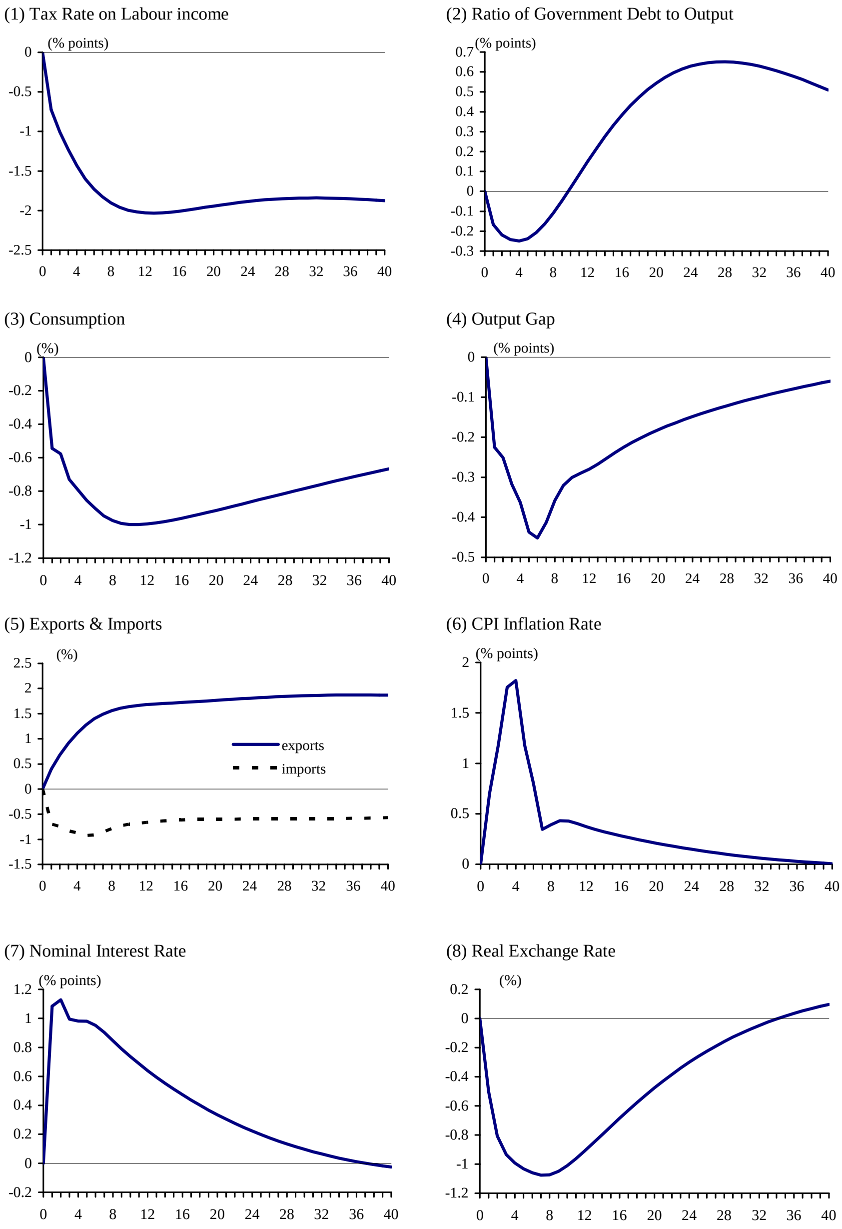

Figure 4: A Shifts in the Consumption of Taxes

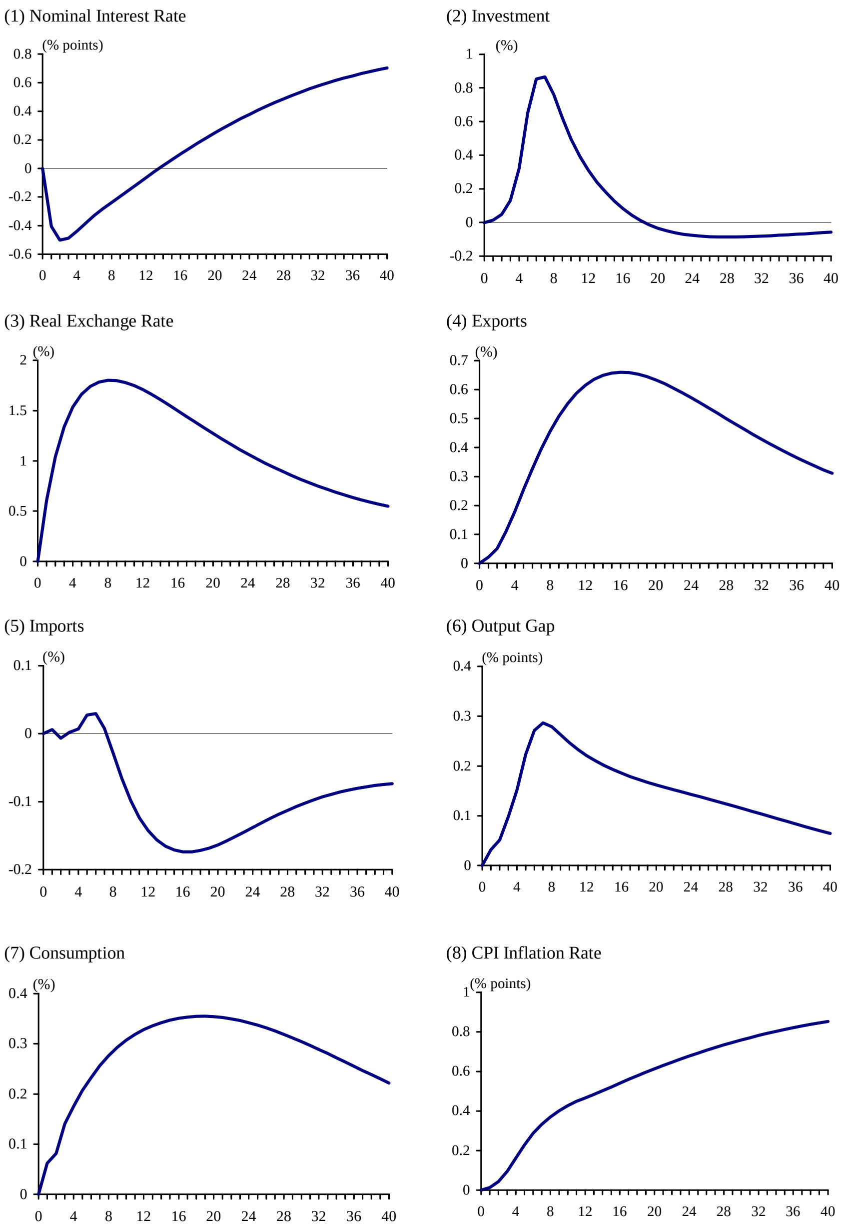

Figure 5: A Change to the Inflation Target

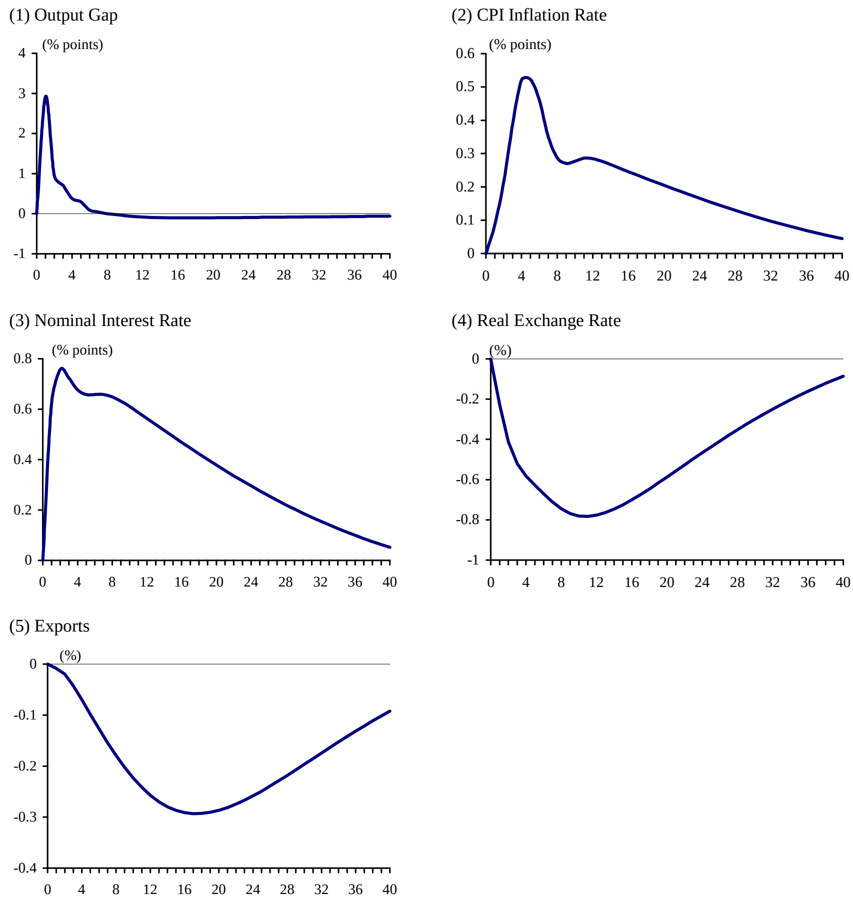

Figure 6: An Autonomous Demand Shock

7.5 An Autonomous Demand Shock

Figure 6 shows the impulse responses to a temporary demand shock to consumption and investment.

As the increases in consumption and investment are just temporary, the production level does not change significantly. The output gap, therefore, becomes positive. Consequently, inflation rises and the central bank raises the nominal interest rate. The exchange rate then appreciates in line with the increase in the nominal interest rate. This causes import prices to decrease putting downward pressure on the CPI. Since net exports fall as a result of the currency appreciation, the output gap contracts. This results in lower inflation and nominal interest rates. Consequently, all the variables return to their initial levels as we would expect following a temporary shock.

7.6 A Temporary Real Exchange Rate Appreciation

Figure 7 shows the impulse responses to a temporary positive shock to the real exchange rate of 1%.

The appreciation in the exchange rate increases imports but decreases exports. These developments result in a widening of the output gap. Furthermore, import prices and therefore CPI inflation fall. This results in a rise in consumption due to an increase in the real purchasing power. Lower inflation decreases the nominal interest rate via the monetary policy rule. This has some limited positive impact on investment but investment is also affected by the lower level of net exports. Overall, investment falls for a while. However, as the shock is only temporary, the economy gradually returns to its initial state, following the same mechanism as above but in the reverse direction.

7.7 A Permanent Improvement in the Terms of Trade

Figure 8 shows the impulse responses to a permanent improvement in the terms of trade: a permanent decrease of 5% in imported goods prices around the globe.

A decrease in the price of imported goods improves the terms of trade and naturally induces domestic deflation. Responding to these developments, the central bank cuts the interest rate by more than the percent change in the CPI inflation rate. This results in a decrease in the domestic real interest rate, so that the real exchange rate depreciates to satisfy the UIP condition.

As for real activities, although the depreciation leads to an increase in exports, net exports decrease because of the increase in domestic purchases of the cheaper imported goods. Real consumption increases as a result of a decrease in consumption deflator. However, there is also a simultaneous and permanent rise in the level of consumption thanks to the increased production capacity that results from a larger capital stock: the lower price of imported capital goods

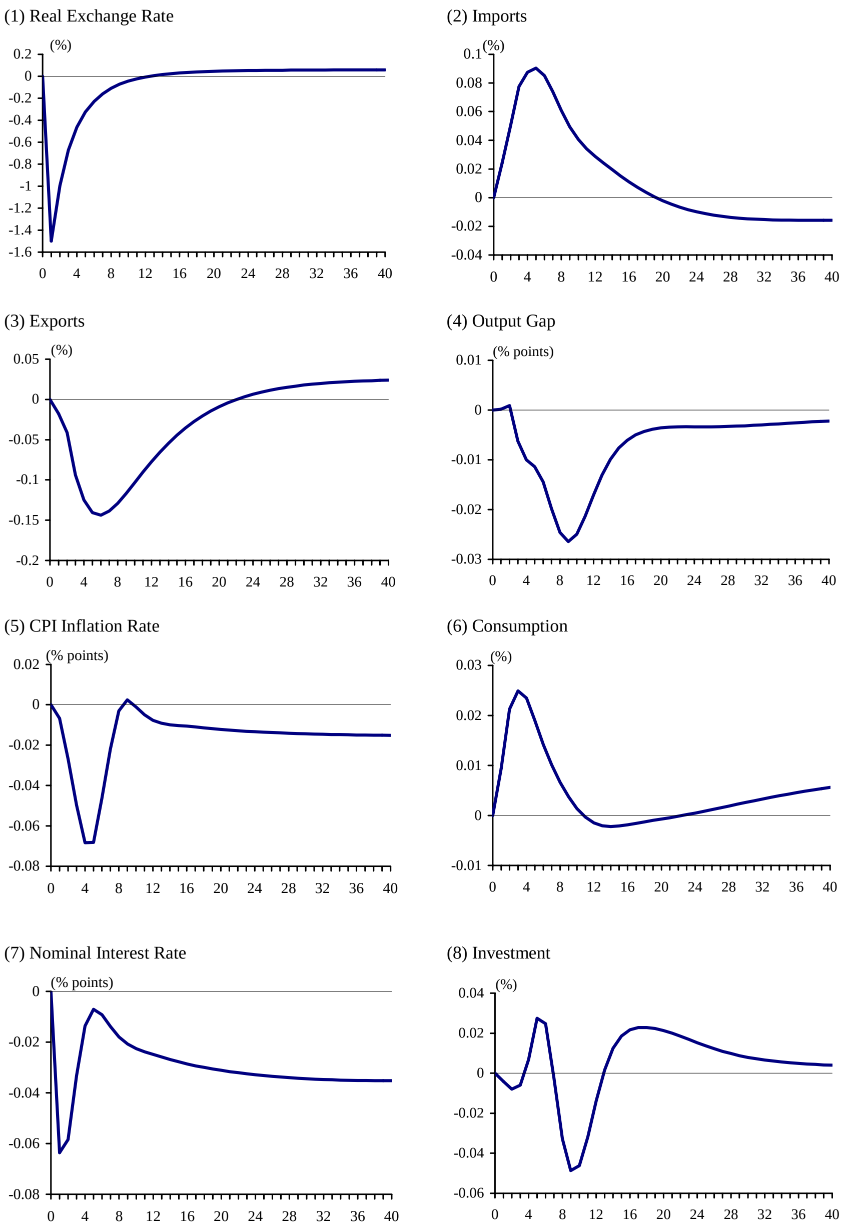

Figure 7: A Temporaty Real Exchange Rate Appreciation

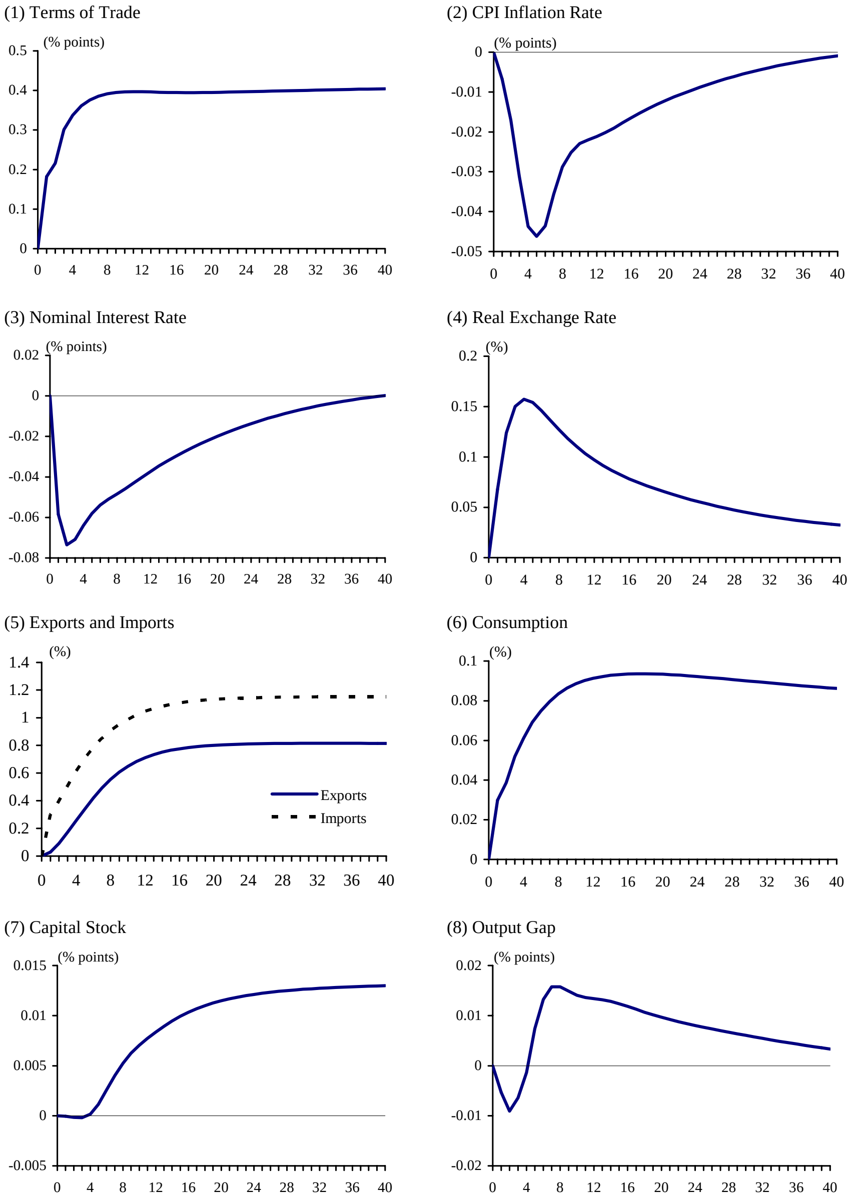

Figure 8: A Permanent Improvement in the Terms of Trade

reduces the cost of capital and therefore increases the desired level of capital stock.

7.8 A Monetary Policy Shock

Figure 9 shows the impulse responses to a temporary increase in the call rate of 1%.

A positive shock to the nominal interest rate increases the cost of capital and therefore reduces investment. It therefore causes the exchange rate to appreciate and exports to decline. Reflecting these developments, the output gap widens and consumption decreases as a result of weak demand, which also induces lower imports. A wider output gap lowers inflation and inflation expectations. This results in a reduction of the nominal interest rate by the central bank. Eventually, investment, exports, and the output gap recover their initial levels.[49]

[49] Our results shows simlilar output composition of monetary transmission mechanism to the one in Fujiwara (2003).

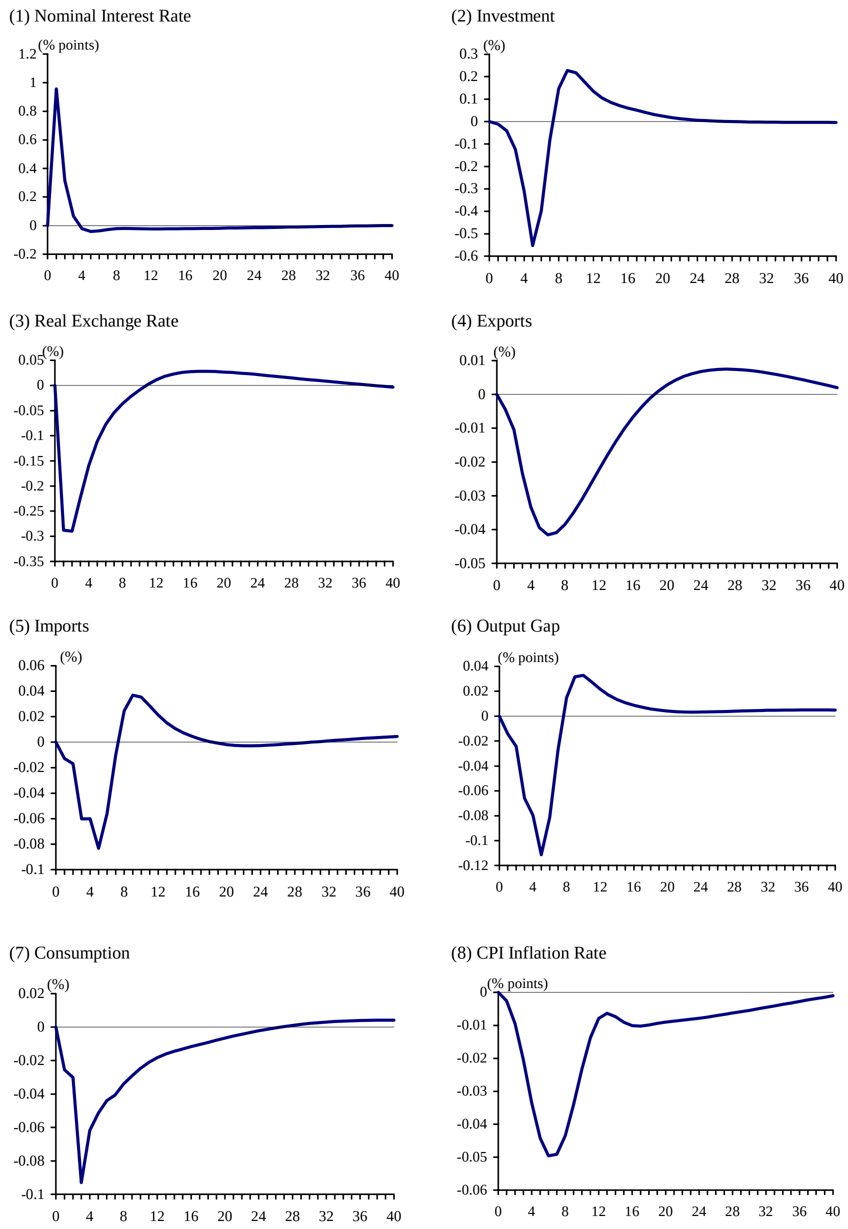

Figure 9: A Monetary Policy Shock

8 Diagnostic Simulation under the Zero Nominal Interest Rate Floor[50]

Since the *raison d'etre* of the JEM is to produce realistic projections and policy simulations for the Japanese economy, the non-negativity constraint on the nominal interest rate should always be considered. Therefore, we here review how the zero floor on the nominal interest rate affects the Japanese economy by simulating a temporary but deterministic shock. The standard cases typically dealt with in the DGE literature are considered: a demand shock[51] and an inflationary shock (cost push shock), both of which can be considered typical shocks[52] occurring in the real economy.

When the JEM is actually used for projection, policy simulation etc., the zero nominal interest rate is introduced by rewriting the equation (36) with a max function as follows:

[50] Pioneering work by Benhabib, Schmitt-Grohe, and Uribe (2002) points to the possibility of multiple equilibria once the zero bound on the nominal interest rate is taken into account. If the central bank follows a simple Taylor rule, the interest rate is raised when the inflation exceeds the target, at the point where the inflation rate is close to its target. Naturally, the interest rate feedback rule and the Fisher equation intersect when the inflation rate equals the target level of inflation. This point may be called the target equilibrium, TE in the standard terminology.
However, at the same time, this together with the existence of non-negativity constraint on the nominal interest rate necessarily implies another point where these two lines intersect. At this second point, the inflation rate is low and possibly negative, the nominal interest rate is zero and monetary policy is passive. This is the so-called BTE, the below target equilibrium, which is sometimes stationary, referred to as the BTSE, and sometimes non-stationary, the BTNE. In a flexible price setting where inflation instantaneously adjusts so that the Fisher equation is always satisfied. Benhabib, Schmitt-Grohe and Uribe (2002) show that although the BTE is indeterminate, the inflation rate and the nominal interest rate close to the TE converge gradually to the BTE.
Although such steady state multiple equilibria are an interesting phenomenon, we choose to follow Jung, Teranishi, and Watanabe (2003) and not to take the BTE into consideration in this paper. Here we solve for the rational expectation solutions using TROLL. In TROLL, when solving the model, we first need to compute the steady state as the terminal condition and then the rational expectations path is computed using a stacked time algorithm. As the BTE will usually be non-stationary, especially in a large macromodel such as the JEM, we cannot designate the BTE as the terminal condition. However, in some cases, even if we set the terminal condition as the TE, no solution is obtained. These developments may be suggesting not an explosive path but that the economy is stuck at the BTE.

[51] A larger shock is applied than in the above experiment so that the zero nominal interest rate floor becomes a binding constraint.

[52] Another typical shock is that on the exchange rate. However, as long as temporary one-off shocks are being considered, the size of shock needed for the zero nominal interest rate constraint to bind is implausibly high. Furthermore, such a simulation finds nothing that was not already suggested by the two experiments above.

$$rn_t = \max \left\{ 0, \begin{array}{c} smooth \left[\begin{array}{c} rn_t^{eq} + rsl_4 \left(tpdot_{t+4} - pdottar_{t+4} \right) \\ + rsl_5 \left(tpdot_{t+5} - pdottar_{t+5} \right) \\ + rsl_6 \left(tpdot_{t+6} - pdottar_{t+6} \right) \end{array} \right] \\ + \left(1 - smooth \right) rn_{t-1} \end{array} \right\}. \qquad (37)$$

With this modification, we can obtain a non-negative call rate and therefore a non-negative nominal interest rate as equation (37) is the core equation of interest rate determination.[53] However, there exists one crucial defect in equation (37). The derivatives of the equation around a zero call rate are not continuous. This may have very important implications when solving the model using TROLL. In TROLL, a large nonlinear dynamic model is solved via the Newton-Raphson method using a stacked time algorithm.[54] When applying the Newton-Raphson algorithm, TROLL obtains the Jacobian matrix as the "Symbolic Derivative," with which derivatives are computed analytically and logically, for example, $\frac{\partial \log(x)}{\partial x} = \frac{1}{x}$. Under this solution system, if the iteration process encounters a point where rn=0 for the largest deviation of inflation from its target, the symbolic derivative is simply incomputable with the result that the Newton-Raphson iteration process simply stops. The model therefore cannot be solved in such cases, even if it conceivably be solved in another way.

One measure to tackle this discontinuous derivative without abandoning the Newton Raphson method entirely is to use dumping. By lessening the Newton gain, the iteration may not fall into the kink as rn=0 even if it falls there without dumping. Another approach to this problem is to approximate equation (37) using some numerical method so that the equation is not only continuous, but also almost non-negative (see appendices 2 and 3).

Recently, with today's significant advances in computer processing power,

[53]The call rate determined in equation (37) is used to compute longer-term interest rates by adding exogenously set risk premiums and according to the term structure defined by the forward looking solution.

[54]For details on the Newton-Raphson method using the stacked time algorithm, see Hollinger (1996).

function approximation is heavily used in solving dynamic programming problems whose value function may not be solved analytically.[55] Of the several methods advocated, we choose to employ function approximation with polynomial interpolation based on the Weierstrass Theorem. This states that:

Any continuous real-valued function f defined on a bounded interval [a,b] of the real line can be approximated to any degree of accuracy using polynomial.

In this paper, equation (37) is approximated by a 10th order polynomial interpolation in order to analyze the impact of the zero floor on the nominal interest rate. As the approximation is extremely long, we will not show it here.[56]

In the following, we present the results both from the max function and from the numerically approximated function.

8.1 An Autonomous Demand Shock

Figure 10 shows the impulse responses to a temporary negative demand shock to consumption and investment.

8.1.1 Without the Zero Nominal Interest Rate Floor[57]

In the short-run, since shocks which decrease consumption and investment are only temporary, production does not decrease significantly. This results in a negative output gap, and the central bank therefore lowers the nominal interest rate as the inflation rate falls. Meanwhile, the exchange rate depreciates in accordance with the decrease in the nominal interest rate. However, as shocks are only temporary, all the variables return to their initial levels.

8.1.2 With the Zero Nominal Interest Rate Floor

As the nominal interest rate cannot fall below zero, the recession produced by the negative shocks is prolonged. The output gap widens and deflation lasts for

[55] For details on the numerical method, see Judd (1998), Marimon and Scott (1999), Ljungqvist and Sargent (2000), Miranda and Fackler (2002).

[56] It can be distributed upon request.

[57] Cases without the zero nominal interest rate floor are basically the same as those shown in section 7.

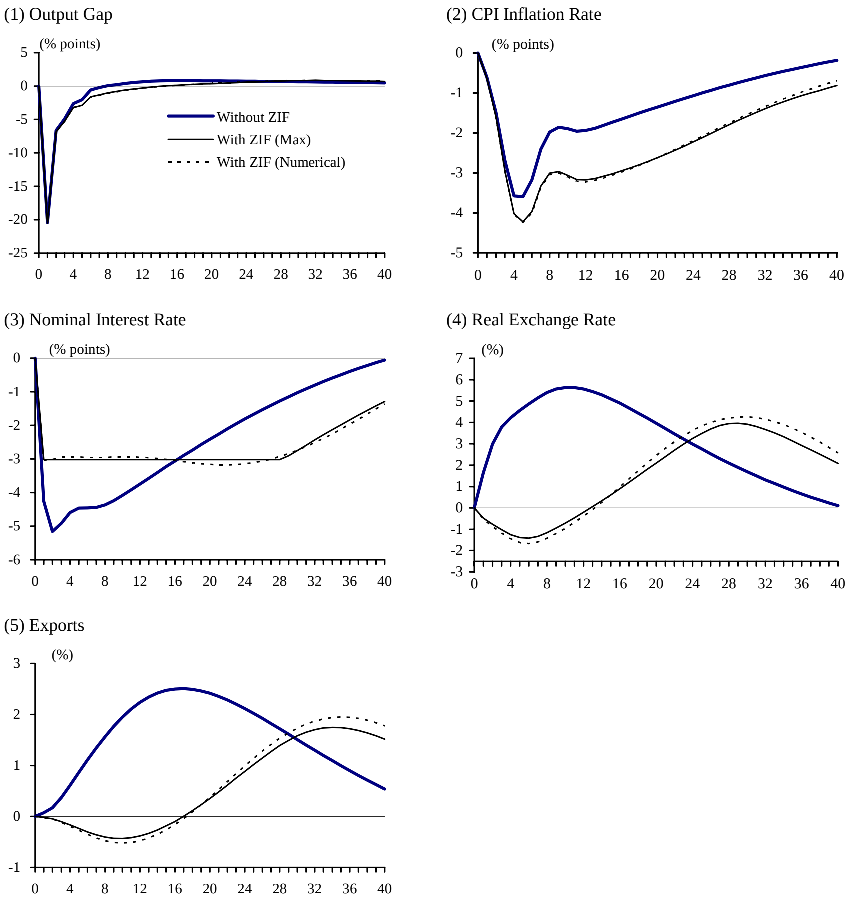

Figure 10: An Autonomous Demand Shock

longer than when a negative nominal rate is allowed. A striking difference can be found in the external sector. With the zero nominal interest rate floor in place, the real interest rate rises as soon as the economy hits this bound. Since the real exchange rate in the JEM is determined via the UIP condition, the result is that the real exchange rate appreciates[58] in direct contrast to the case without the zero floor where it depreciated. Exports then decrease in response.

This underlines the severity of the problem caused by the zero nominal interest floor. Not only is there less freedom alleviate domestic deflationary pressure, but we are also denied a boost from the external sector.

8.2 A Deflationary Shock

Figure 11 shows the impulse responses to a temporary deflationary shock[59] of 1%.

8.2.1 Without the Zero Nominal Interest Rate Floor

Deflationary shocks raise real wages temporarily and therefore consumption increases. This factor and the lowered nominal interest rate caused by deflation enhance investment. Therefore, the output gap becomes positive and imports are boosted. A lower nominal interest rate causes the currency to depreciate. Hence, exports also increase.

8.2.2 With the Zero Nominal Interest Rate Floor

Even if the zero nominal interest rate floor is explicitly included as a constraint, consumption still increases as the mechanism described above remains functional. However, investment and exports suffer because the existence of the lower bound prevents the real interest rate from falling enough to alleviate the deflationary pressure. All told, the economy takes longer to return to its initial state.

[58]Intuition suggests that the currency of a country coming up against the zero nominal interest rate bound is likely to depreciate reflecting its doomed prospects for the future. However, this kind of channel is not embedded in the JEM.

[59]Shocks are applied to *pdot*.

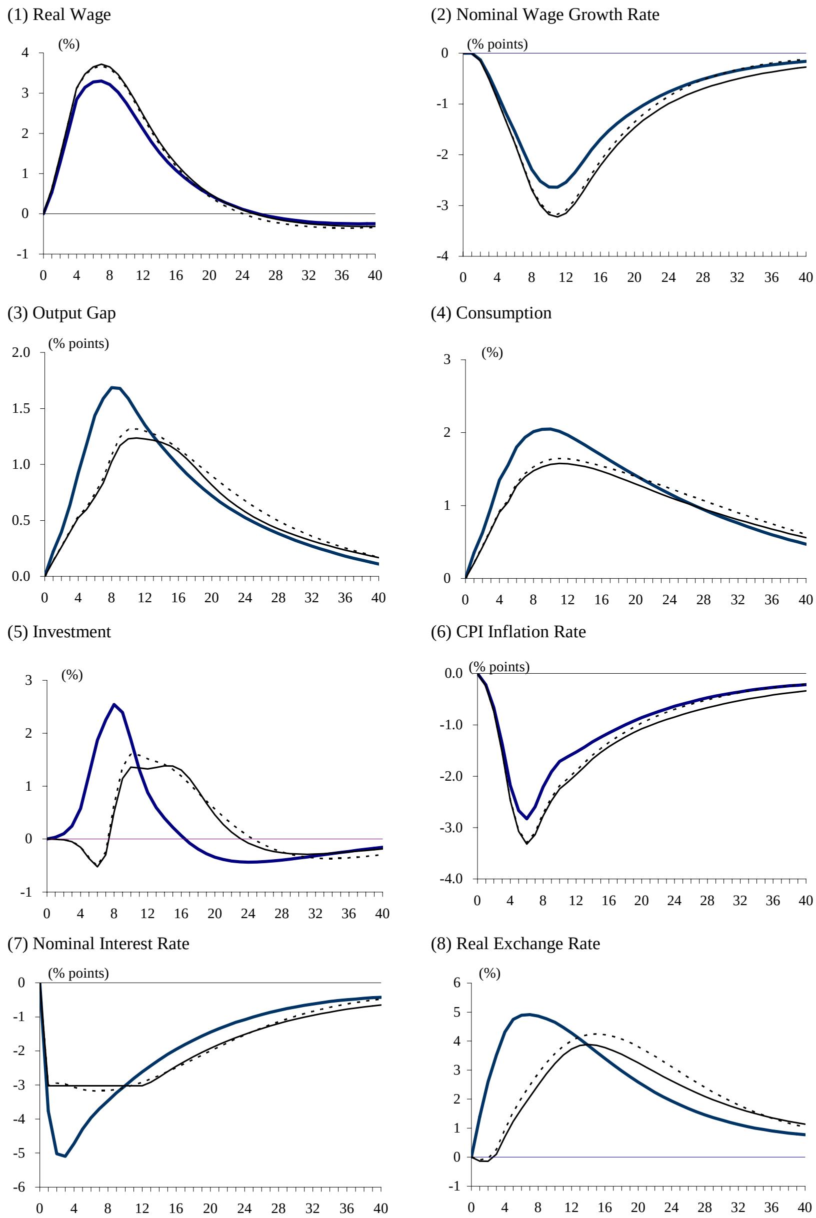

Figure 11: A Deflationary Shock

9 Projection

One of the largest advantages to using the JEM is its capability to produce not only theoretically consistent but also realistic projections. The process of projection may be viewed as one in which the economy is exposed to multiple shocks (mimicking those that have actually occurred in Japan), and we chart the impulse responses as the economy moves back towards its steady state. The following steps are necessary before projections can be made:

- Setting up the database
- Setting the paths of the exogenous variables
- Solving for the steady state
- Proxying a learning mechanism
- Solving the model
- Transforming relative variables into levels

9.1 Setting up the Database

The first step is naturally to set up a database. As described above, all variables except for variables that are defined as rates, such as the interest and inflation rates, are in per capita form and are further normalized by being expressed as ratios to potential GDP. In addition, for analytical convenience, all price variables, namely the deflators, are expressed as ratios to the price of domestically-produced and consumed goods at factor cost.

As for the potential GDP, this is derived as the level of GDP consistent with the time-varying NAIRU, as specified in Hirose and Kamada (2001). Using this definition of potential GDP, the price of domestically-produced and consumed goods at factor cost, and the labor force, we are able to express all variables as relative values in per capita form.

9.2 Setting the Paths of the Exogenous Variables

In contrast with some of the cutting-edge research, such as Benigno and Woodford (2003), since the government sector in the JEM is not an optimizing agent, most of the fiscal variables are exogenous. Exogenous fiscal variables include the corporate tax rate, indirect tax rates, government expenditure, government transfers (including net social security payments), and the size of the government debt. On the other hand, the income tax rate is an endogenous variable determined so as to satisfy the government budget constraint.[60] We set these exogenous variables based on publications by governmental institutions such as the Economic Advisory Council, the Ministry of Finance, the Ministry of Health, Labor and Wealth, etc.

Beside fiscal variables, foreign prices and total factor productivity are also determined exogenously. It would be possible to endogenize total factor productivity by expressing this as a function of some sort of R&D investment or social capital (infrastructure) as summarized in Grossman and Helpman (1991), Aghion and Howitt (1998), and Barro and Sala-i-Martin (1995). However, although theoretically neat, it is uncertain whether the incorporation of endogenous growth theory would make it easier to obtain reasonable projections. For the time being, we continue to treat total factor productivity as exogenous. The incorporation of endogenous growth theory in the JEM is left as a topic for future research.

9.3 Solving for the Steady State

Before solving the model, steady state values are computed by eliminating leads and lags from the JEM. As mentioned, TROLL solves the model as a finite horizon problem with a well-defined steady state. Throughout the projection process, the steady state is treated almost like an exogenous variables.

[60] The tax rate was chosen over government debt to satisfy the government budget constraint solely for reasons of analytical tractability. In any case, since both are monitored to ensure reasonable performance in projection, the decision makes almost no difference so far as the projection itself is concerned.

9.4 Proxying a Learning Mechanism

It has been argued that rational expectations require strong assumptions. Indeed, Evans and Honkapojha (2001) state: "The rational expectations approach presupposes that economic agents have a great deal of knowledge about the economy. Even in our simple examples, in which expectations are constant, computing these constants require the full knowledge of the structure of the model, the values of the parameters, and that the random shock is iid." The pure rational expectation hypothesis is rather unrealistic since agents are considered to possess only "bounded rationality." Hence, in line with the treatment in the FPS[61], we proxy a learning mechanism in the JEM.

Even if we know the steady state, convergence will take some time if we wish to avoid making the strong assumption that agents are perfectly rational. Furthermore, even if the sizes of the shocks currently hitting the economy are known, it is impossible to identify whether these are just transitory or whether they are permanent. Therefore, we assume that agents in the JEM are following a kind of learning process which takes the form of an updating rule.[62] In this updating rule, they begin by observing the economy's past history, simultaneously forecasting the steady state that will be achieved in the long run. As time goes on, agents obtain more information about the economy and use this to confirm their past views. They therefore adjust their desired positions gradually.

Technically, this gradual adjustment or updating rule is achieved by setting the time-varying short-run equilibrium (TVSREQ) paths for several variables such as stocks. These paths are derived by filtering the actual data series with the LRX (Laxton, Rose, and Xie) filter, a modified HP (Hodrick and Prescott) filter, with the assumption that they converge over the projection horizon to their long-run steady state values.

SREQ paths other than the TVSREQ paths set above are computed by

[61] Details are shown in Drew and Hunt (1998).

[62] Similar to incorporating the endogenous growth theory in the JEM, it is left for our future research to embody rigid and formal adaptive learning mechanism, as summarized in Evans and Honkapojha (2001), to the JEM.

simulating the JEM using these TVSREQ paths and the exogenous variables. This simulation ensures overall consistency across all the SREQ paths.

9.5 Solving the Model

Projections are obtained by solving the JEM. Although conducted simultaneously, the process of solving the model can be more easily understood by dividing it into two intuitive parts: computation of the historical innovations and adjustment of forecast errors.

When making projections with the JEM, in order to preserve theoretical consistency, we solve the model in an integrated fashion over both past and future. Critically, the settings of exogenous variables and TVSREQ paths for the future and forecast error adjustments affect the estimates which the model produces for the past. Similarly, changes in estimates of the past will have a simultaneous influence on forecasts of the future. Unlike the traditional Keynesian-style backward-looking model, numerous iterations are required to obtain the consistent projections across both past and future.[63]

9.5.1 Computing Historical Innovations

Since projections can be considered in terms of the impulse responses towards steady state following innovations that actually occurred in the economy, historical innovations need to be computed. These are computed using the exogenous variables, TVSREQ paths and the forecast errors discussed below. Between them these provide the major driving force behind the projection and determine the shape of the convergence dynamics.

9.5.2 Adjusting Forecast Errors

It is more realistic to assume that historical innovations do not disappear right after entering the simulation period.[64] Therefore, historical innovations are set

[63] When conducting simulations however, it is also possible to assume that past evaluations are not affected by future settings.

[64] Setting the TVSREQ paths is also considered to be a form of forecast error adjustment. Even if agents are rational, they may predict that the current gap between the short-run

in some equations and are presupposed to disappear gradually according to an AR process.[65]

9.6 Transforming Relative Variables into Levels

Up until now, projection has been represented in the form of relative values. We need, therefore, to transform these relative values into levels. For real variables, these are just converted into levels by multiplying them by potential GDP. Level conversion of nominal variables is a little more complicated. First, by multiplying the last observation of the price of domestically-produced and consumed goods at factor cost by $(1 + pdot)$, we recover the future level of the price of domestically-produced and consumed goods at factor cost. Using this price level, we are then able to calculate levels for all the nominal variables.

10 Model Evaluation

The diagnostic simulations above demonstrate that the JEM displays reasonable properties when exposed to shocks. This allows us to conclude that the JEM can be used for projection and policy analysis for the Japanese economy. In this section, we look further at how the JEM's impulses hold up against those obtained from VAR.

10.1 Comparison of Impulse Responses against VAR

As proposed by Christiano, Eichenbaum and Evans (1999), the plausibility of a dynamic stochastic general equilibrium model can be evaluated by comparing its impulse responses to those obtained from an identified VAR. Conventionally, since the introduction of the RBC model, lots of attention have been paid to the impulse responses to the technology shock. Indeed, a recent paper by Altig, Christiano, Eichenbaum and Linde (2003) carries out SMM (simulated method

equilibrium values and steady state values will be only slowly adjusted.

[65] Free forecast error adjustments by modellers are banned. Changes in forecast error adjustments need to be reported to the projection meeting.

of moments,) estimation so as to ensure similarity between the impulse responses from their DGE model and those obtained from an identified VAR.

As there are so many parameters to be estimated in the JEM, applying SMM estimation is not very straightforward.[66] We therefore choose not to estimate parameters. Instead, we conduct the "Eye-Ball-Check," in other words, we examine whether we can identify any crucial differences between the JEM impulse responses to the productivity shock detailed above and those obtained from an identified VAR. The VAR estimated by Soejima and Sugo (2003) is employed in order to carry out this comparison.

Soejima and Sugo (2003) estimates a reduced form VECM as follows:

$$\Delta\overline{Z}_t = A\left(L\right)\Delta\overline{Z}_{t-1} + \overline{\alpha}\overline{\beta}'\overline{Z}_{t-1} + \overline{\varepsilon}_t.$$

$\overline{Z}$ denotes the vector of endogenous variables comprising real output $\overline{X}$, real private consumption $\overline{C}$, real money balances $\overline{M}/\overline{P}$, potential output $\overline{Y}^*$, the nominal short-term interest rate $\overline{r}$ and the inflation rate $\overline{\pi}$. All variables are in logs. $\beta' X_{t-1}$ represents three cointegrated relationships, involving long-run consumption and saving,[67] money demand, and the Phillips curve as follows:

$$\overline{C}_{t-1} - \overline{Y}_{t-1} = -\overline{\beta}_{16} r_{t-1} - \overline{\beta}_{17}\pi_{t-1} - \overline{\beta}_{11},$$

$$\overline{M}_{t-1} - \overline{P}_{t-1} = -\overline{\beta}_{22}\overline{Y}_{t-1} - \overline{\beta}_{26}\overline{r}_{t-1} - \overline{\beta}_{21},$$

$$\overline{Y}_{t-1} - \overline{Y}^*_{t-1} = -\overline{\beta}_{37}\overline{\pi}_{t-1} - \overline{\beta}_{31}.$$

Figure 12 compares the impulse responses to a technology shock which raises potential output by 1% in this identified VAR to the impulse responses shown in figure 2.

[66] Amano, McPhail, Pioro and Renninson (2002) evaluate the parameter calibration in the QPM by applying a similar technique.

[67] $\overline{\beta}_{16} = -\overline{\beta}_{17}$ is assumed.

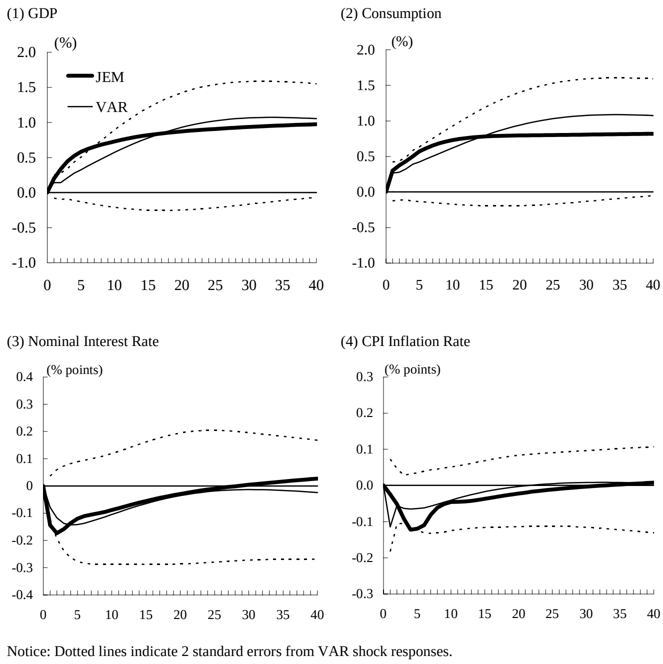

Figure 12: Impulse Responses Comparison with VAR

The JEM's shock responses to the technology shock, often thought the most important factor in causing business cycle fluctuations, are quite similar to those in the VAR. This fact provides further evidence to support the application of the JEM to projection and policy analysis for the Japanese economy from this aspect.[68]

11 Conclusion

In this paper, we have laid out the core structure of the JEM, describing the construction of a theoretical dynamic general equilibrium model which has not only a well-defined steady state but also the ability to produce realistic projections through the addition of short-run dynamics. Diagnostic simulations suggest that the shock responses are reasonable, in the light of historical tendencies

[68] In the future, we would like to examine whether simulated methods of moments and Bayesian estimation are capable of further increasing the JEM's ability to track the actual Japanese economy.

observed in the Japanese economic data. Further, the responses of the JEM to the most fundamental of economic shocks, namely a technology shock, are quite similar to those obtained from a structural VAR with cointegration restrictions. We therefore have confidence in the JEM's suitability as a fundamental model for projection as well as the monetary policy analysis relating to the Japanese economy. However, as no model is perfect for all purposes, it is advisable to pay attention to a *suite of the models*. Combining the insights of the JEM with those attained from a variety of DGE models oriented for different purposes, as well as from identified VAR models, would undoubtedly be the most reliable way of identifying the optimal monetary policy for maximizing social welfare.

Further, innovations in macroeconomics, and especially in monetary economics and international economics, are constantly rendering even the newest macromodels obsolete. We need, therefore, to continuously update our macroeconomic knowledge, and to constantly refine the methods employed in the JEM. At the moment, the following are considered promising directions for future extensions of the model: 1) incorporating demographic dynamics, as in Faruqee (2002) and Faruqee and Muehleisen (2001), or Gertler (1998); 2) re-constructing the rather ad-hoc overseas sector and giving it firmer microfoundations, following Obstfeld and Rogoff (1995) and other advocates of New Open Economy Macroeconomics; 3) estimating the parameters governing the short-run dynamics using Bayesian simulation techniques, as in Geweke (1999) and Smets and Wouters (2003); and 4) embedding learning expectations, as summarized in Evans and Honkapohja (2001).

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Equation List

A Growth accounting

$$ndot_t = ndot_t^{eq} \tag{A1}$$

$$qdot_t = qdot_t^{eq} \tag{A2}$$

$$ydot_t = (1 + ndot_t)(1 + qdot_t) - 1 \tag{A3}$$

$$ydot_t^{eq} = (1 + ndot_t^{eq})(1 + qdot_t^{eq}) - 1 \tag{A4}$$

B Expenditure accounts

B.1 Output

$$y_t = c_t + i_t + ih_t + g_t + ii_t + x_t - m_t \tag{A5}$$

$$py_t y_t = pc_t c_t + pi_t i_t + pih_t ih_t + pg_t g_t + ii_t + px_t x_t - pm_t m_t \tag{A6}$$

$$y_t^{eq} = c_t^{eq} + i_t^{eq} + ih_t^{eq} + g_t^{eq} + ii_t^{eq} + x_t^{eq} - m_t^{eq} \tag{A7}$$

$$py_t y_t = (1 + tiy_t) pfc_t y_t \tag{A8}$$

$$py_t^{eq} y_t^{eq} = (1 + tiy_t^{eq})\, pfc_t^{eq} y_t^{eq} \tag{A9}$$

$$py_t^{eq}y_t^{eq} = pc_t^{eq}c_t^{eq} + pi_t^{eq}i_t^{eq} + pih_t^{eq}ih_t^{eq} + pg_t^{eq}g_t^{eq} + ii_t^{eq} + px_t^{eq}x_t^{eq} - pm_t^{eq}m_t^{eq} \tag{A10}$$

B.2 Consumption

$$c_t = crt_t + cfl_t \tag{A11}$$

$$pc_t crt_t = [\eta_1\beta_1 + \eta_2\beta_2 + \eta_3(1-\beta_1-\beta_2)]\, yd_t \tag{A12}$$

$$\begin{aligned} cfl_t = {} & cfl_t^{eq} + cv_1\left(\frac{\frac{yd_{t-2}}{pc_{t-2}}}{\frac{yd_{t-2}^{eq}}{pc_{t-2}^{eq}}} - 1\right) - [cv_{21}(r_{t-2} - r_{t-2}^{eq}) \\ & + cv_{22}(rl_{t-2} - rl_{t-2}^{eq})]cfl_{t-2}^{eq} + cv_3\left(\frac{nfa_t}{pc_t} - \frac{nfa_t^{eq}}{pc_t^{eq}}\right) - cfladj_t \end{aligned} \tag{A13}$$

$$d_t = ih_t + (1 - deprih_t)\frac{d_{t-1}}{1 + ydot_t} \tag{A14}$$

$$\begin{aligned} ih_t = {} & ih_t^{eq} + ihv_1\left(\frac{\frac{yd_{t-1}}{pih_{t-1}}}{\frac{yd_{t-1}^{eq}}{pih_{t-1}^{eq}}} - 1\right) - [ihv_{21}(r_{t-1} - r_{t-1}^{eq}) \\ & + ihv_{22}(rl_{t-1} - rl_{t-1}^{eq})]ih_{t-1}^{eq} + ihv_3\left(\frac{nfa_t}{pih_t} - \frac{nfa_t^{eq}}{pih_t^{eq}}\right) \\ & - ihadj_t \end{aligned} \tag{A15}$$

$$c_t^{eq} = crt_t^{eq} + cfl_t^{eq} \tag{A16}$$

$$pc_t^{eq}crt_t^{eq} = [\eta_1\beta_1 + \eta_2\beta_2 + \eta_3(1-\beta_1-\beta_2)]yd_t^{eq} \tag{A17}$$

$$pc_t^{eq} cfl_t^{eq} = mpcw_t^{eq} twfl_t^{eq} + \zeta(fa_t^{eq} - fa_{ss}) \tag{A18}$$

$$\frac{1}{mpcw_t^{eq}} = (1-\gamma)\delta^{\sigma} \frac{\left[\frac{pc_t^{eq}}{pc_{t+1}^{eq}}(1+rcon_t^{eq})\right]^{\sigma-1}}{mpcw_{t+1}^{eq}} + 1 + \theta^{\sigma} \iota_t^{eq\ 1-\sigma} \tag{A19}$$

$$\iota_t^{eq} = pih\_pc_t^{eq} - (1-\gamma)\frac{1-deprih_t^{eq}}{1+rcon_t^{eq}} pih\_pc_{t+1}^{eq} \tag{A20}$$

$$\begin{aligned} twfl_t^{eq} = {} & hwfl_t^{eq} + fwfl_t^{eq} + \left(1 + rgb_{t-1}^{eq}\right) \frac{gb_{t-1}^{eq}}{1+ydot_t^{eq}} \\ & + \left(1 + rnfa_{t-1}^{eq}\right) \frac{nfa_{t-1}^{eq}}{1+ydot_t^{eq}} + \left(1 + rk_{t-1}^{eq}\right) \frac{pka_{t-1}k_{t-1}^{eq}}{1+ydot_t^{eq}} \\ & + pih_t^{eq}\left(1 - deprih_t^{eq}\right) \frac{d_{t-1}^{eq}}{1+ydot_t^{eq}} \end{aligned} \tag{A21}$$

$$hwfl_t^{eq} = hwfl1_t^{eq} + hwfl2_t^{eq} + hwfl3_t^{eq} \tag{A22}$$

$$hwfl1_t^{eq} = (1-\eta_1)\,\beta_1 yd_t^{eq} + \frac{(1-\gamma)\,(1+qdot_t^{eq})}{(1+rcon_t)\,(1+\alpha_1)} hwfl1_{t+1}^{eq} \tag{A23}$$

$$hwfl2_t^{eq} = (1-\eta_2)\,\beta_2 yd_t^{eq} + \frac{(1-\gamma)\,(1+qdot_t^{eq})}{(1+rcon_t)\,(1+\alpha_2)} hwfl2_{t+1}^{eq} \tag{A24}$$

$$hwfl3_t^{eq} = (1-\eta_3)\,(1-\beta_1-\beta_2)\,yd_t^{eq} + \frac{(1-\gamma)\,(1+qdot_t^{eq})}{(1+rcon_t^{eq})\,(1+\alpha_3)} hwfl3_{t+1}^{eq} \tag{A25}$$

$$fwfl_t^{eq} = risk_t^{eq} + \frac{(1-\gamma)(1+qdot_t^{eq})}{1+rcon_t^{eq}} fwfl_{t+1}^{eq} \tag{A26}$$

$$ih_t^{eq} = d_t^{eq} - (1-deprih_t^{eq}) \frac{d_{t-1}^{eq}}{1+ydot_t^{eq}} \tag{A27}$$

$$\begin{aligned} fa_t + pc_t^{eq} c_t^{eq} + pih_t^{eq} ih_t^{eq} = {} & yd_t^{eq} + risk_t^{eq} - ii_t^{eq} \\ & + \left(1+rcon_{t-1}^{eq}\right) \frac{fa_{t-1}^{eq}}{1+ydot_t^{eq}} \end{aligned} \tag{A28}$$

$$d_t^{eq} = \left(\frac{\theta}{\iota_t^{eq}}\right)^{\sigma} cfl_t^{eq} \tag{A29}$$

B.3 Investment

$$k_t = (1-depr_t)\frac{k_{t-1}}{1+ydot_t} + i_t \tag{A30}$$

$$k_t^{eq} = (1-depr_t^{eq}) \frac{k_{t-1}^{eq}}{1+ydot_t^{eq}} + i_t^{eq} \tag{A31}$$

$$\begin{aligned} kp_t^{eq} = {} & (1 - ip_1 - ip_2 - ip_3 - ip_4 - ip_5 - ip_6 - ip_7)\, k_t^{eq} \\ & + ip_1 \frac{k_{t-1}^{eq}}{1+ydot_t^{eq}} + ip_2 \frac{k_{t-2}^{eq}}{(1+ydot_t^{eq})^2} + ip_3 \frac{k_{t-3}^{eq}}{(1+ydot_t^{eq})^3} \\ & + ip_4 \frac{k_{t-4}^{eq}}{(1+ydot_t^{eq})^4} + ip_5 \frac{k_{t-5}^{eq}}{(1+ydot_t^{eq})^5} + ip_6 \frac{k_{t-6}^{eq}}{(1+ydot_t^{eq})^6} \\ & + ip_7 \frac{k_{t-7}^{eq}}{(1+ydot_t^{eq})^7} \end{aligned} \tag{A32}$$

$$
\begin{aligned}
kp_t = {} & (1 - ip_1 - ip_2 - ip_3 - ip_4 - ip_5 - ip_6 - ip_7)\, k_t \\
& + ip_1 \frac{k_{t-1}}{1 + ydot_t} + ip_2 \frac{k_{t-2}}{(1 + ydot_t)^2} + ip_3 \frac{k_{t-3}}{(1 + ydot_t)^3} \\
& + ip_4 \frac{k_{t-4}}{(1 + ydot_t)^4} + ip_5 \frac{k_{t-5}}{(1 + ydot_t)^5} + ip_6 \frac{k_{t-6}}{(1 + ydot_t)^6} \\
& + ip_7 \frac{k_{t-7}}{(1 + ydot_t)^7}
\end{aligned} \tag{A33}
$$

$$
ii_t = ii_t^{eq} - iiadj_t \tag{A34}
$$

B.4 Government expenditures

$$
g_t = g_1 g_{t-1} + (1 - g_1) g_t^{eq} + g_3 (u_t - u_t^{eq}) \tag{A35}
$$

$$
g_t^{eq} = g_2 g_{t-1}^{eq} + (1 - g_2)\, g\_y_t^{eq} y_t^{eq} \tag{A36}
$$

$$
gtr_t = gtr_1 gtr_{t-1} + (1 - gtr_1)\, gtr_t^{eq} + gtr_3 (u_t - u_t^{eq}) \tag{A37}
$$

$$
gtr_t^{eq} = gtr_2 gtr_{t-1}^{eq} + (1 - gtr_2)\, gtr\_y_t^{eq} y_t^{eq} \tag{A38}
$$

B.5 External trade

B.5.1 Imports

$$
m_t = cm_t + im_t + gm_t + ihm_t \tag{A39}
$$

$$
m_t^{eq} = cm_t^{eq} + im_t^{eq} + gm_t^{eq} + ihm_t^{eq} \tag{A40}
$$

$$
cm_t = cm\_c_t c_t \tag{A41}
$$

$$cm_t^{eq} = cm\_c_t^{eq} c_t^{eq} \tag{A42}$$

$$cm\_c_t = cm\_c_t^{eq} - cmv_1 \left[(1 + ticm_t) \frac{pcm_{t-1}}{pcd_{t-1}} - (1 + ticm_t^{eq}) \frac{pcm_{t-1}^{ed}}{pcd_{t-1}^{eq}} \right] \tag{A43}$$
$$- cm\_cadj_t$$

$$cm\_c_t^{eq} = cm\_c_0 - cms_2 \left(1 + ticm_t^{eq}\right) \frac{pcm_t^{eq}}{pcd_t^{eq}} \tag{A44}$$

$$im_t = im\_i_t i_t \tag{A45}$$

$$im_t^{eq} = im\_i_t^{eq} i_t^{eq} \tag{A46}$$

$$im\_i_t = im\_i_t^{eq} - imv_1 \left[(1 + tiim_t) \frac{pim_{t-1}}{pid_{t-1}} - (1 + tiim_t^{eq}) \frac{pim_{t-1}^{eq}}{pid_{t-1}^{eq}} \right] - im\_iadj_t \tag{A47}$$

$$im\_i_t^{eq} = im\_i_0 - ims_2 \left(1 + tiim_t^{eq}\right) \frac{pim_t^{eq}}{pid_t^{eq}} \tag{A48}$$

$$gm_t = gm\_g_t g_t \tag{A49}$$

$$gm_t^{eq} = gm\_g_t^{eq} g_t^{eq} \tag{A50}$$

$$gm\_g_t = gm\_g_t^{eq} - gmv_1 \left[(1 + tigm_t)\frac{pgm_{t-1}}{pgd_{t-1}} - (1 + tigm_t^{eq})\frac{pgm_{t-1}^{eq}}{pgd_{t-1}^{eq}}\right] \quad \text{(A51)}$$

$$- gm\_gadj_t$$

$$gm\_g_t^{eq} = gm\_g_0 - gms_2 \left(1 + tigm_t^{eq}\right) \frac{pgm_t^{eq}}{pgd_t^{eq}} \quad \text{(A52)}$$

$$ihm_t = ihm\_ih_t ih_t \quad \text{(A53)}$$

$$ihm_t^{eq} = ihm\_ih_t^{eq} ih_t^{eq} \quad \text{(A54)}$$

$$\begin{aligned} ihm\_ih_t \quad = \quad & ihm\_ih_t^{eq} \quad \text{(A55)} \\ & -ihmv_1 \left[(1 + tiihm_t)\frac{pihm_{t-1}}{pihd_{t-1}} - (1 + tiihm_t^{eq})\frac{pihm_{t-1}^{eq}}{pihd_{t-1}^{eq}}\right] \\ & -ihm\_ihadj_t \end{aligned}$$

$$ihm\_ih_t^{eq} = ihm\_ih_0 - ihms_2 \left(1 + tiihm_t^{eq}\right) \frac{pihm_t^{eq}}{pihd_t^{eq}} \quad \text{(A56)}$$

B.5.2 Exports

$$x_t^{eq} = x\_y_0 + x_1 y_t^* + x_2 px_t^{eq} \quad \text{(A57)}$$

$$x_t = x_t^{eq} + xv_1(px_{t-2} - px_{t-2}^{eq})x_{t-2}^{eq} - xadj_t \quad \text{(A58)}$$

B.5.3 Net exports

$$xbal_t = px_t x_t - pm_t m_t \quad \text{(A59)}$$

$$xbal_t^{eq} = px_t^{eq} x_t^{eq} - pm_t^{eq} m_t^{eq} \tag{A60}$$

$$netx_t = x_t - m_t \tag{A61}$$

$$netx_t^{eq} = x_t^{eq} - m_t^{eq} \tag{A62}$$

$$nfa_t = (1 + rnfa_{t-1}) \frac{nfa_{t-1}}{1 + ydot_t} + xbal_t \tag{A63}$$

$$nfa_t^{eq} = \left(1 + rnfa_{t-1}^{eq}\right) \frac{nfa_{t-1}^{eq}}{1 + ydot_t^{eq}} + xbal_t^{eq} \tag{A64}$$

C Income accounts

C.1 Wage and labor income

$$wa_t = wa_{t-1} \frac{1 + wdot_t}{(1 + pdot_t)(1 + qdot_t)} \tag{A65}$$

$$wa_t^{eq} = \rho \, (1 - \alpha) \, tfp_t^{eq \ \psi} \left(\frac{pfc_t^{eq} y_t^{eq}}{1 - u_t^{eq}} \right)^{1-\psi} \tag{A66}$$

$$wp_t^{eq} = \frac{wa_t^{eq}}{pfc_t^{eq}} \tag{A67}$$

$$wp_t = \frac{wa_t}{pfc_t} \tag{A68}$$

$$wc_t^{eq} = \frac{wa_t^{eq}}{pc_t^{eq}} \tag{A69}$$

$$wc_t = \frac{wa_t}{pc_t} \tag{A70}$$

$$
\begin{aligned}
wctar_t = wc_1 & \left[(1 - wcf_0)wc_t^{eq} + wcf_0 wc_{t+1}^{eq}\right] \\
& + (1 - wc_1)[wcl_1 wctar_{t-1} + wcl_2 wctar_{t-2} + wcl_3 wctar_{t-3} \\
& + (1 - wcl_1 - wcl_2 - wcl_3) wctar_{t-4}]
\end{aligned}
\tag{A71}
$$

$$
wdot_t^{eq} = (1 + pdot_t^{eq})(1 + qdot_t^{eq}) - 1 \tag{A72}
$$

$$
\begin{aligned}
1 + wdot_t = (1 + qdot_t)\{ & 1 + wpe_1(wp_2 pdote_{t-2} + wp_3 pdote_{t-3} \\
& + wp_4 pdote_{t-4} + wp_5 pdote_{t-5} + wp_6 pdote_{t-6} \\
& + (1 - wp_2 - wp_3 - wp_4 - wp_5 - wp_6) pdote_{t-7}) \\
& + (1 - wpe_1)[wp_2 pcdote_{t-2} + wp_3 pcdote_{t-3} + wp_4 pcdote_{t-4} \\
& + wp_5 pcdote_{t-5} + wp_6 pcdote_{t-6} \\
& + (1 - wp_2 - wp_3 - wp_4 - wp_5 - wp_6) pcdote_{t-7}]\} \\
& + wd_2(\frac{wp_{t-1}^{eq}}{wp_{t-1}} - 1) + wd_3(u_{t-1}^{eq} - u_{t-1})
\end{aligned}
\tag{A73}
$$

$$
ylab_t = wa_t(1 - u_t) \tag{A74}
$$

$$
ylab_t^{eq} = wa_t^{eq}(1 - u_t^{eq}) \tag{A75}
$$

C.2 Disposable income

$$
yd_t^{eq} = (1 - td_t^{eq})(ylab_t^{eq} + yd_1 gtr_t^{eq}) + (1 - yd_1) gtr_t^{eq} \tag{A76}
$$

$$
yd_t = (1 - td_t)(ylab_t + yd_1 gtr_t) + (1 - yd_1) gtr_t \tag{A77}
$$

C.3 Risk income

$$
\begin{aligned}
risk_t^{eq} = {} & (rk_{t-1}^{eq} - rcon_{t-1}^{eq})\frac{pka_{t-1}k_{t-1}^{eq}}{1+ydot_t^{eq}} + (rgb_{t-1}^{eq} - rcon_{t-1}^{eq})\frac{gb_{t-1}^{eq}}{1+ydot_t^{eq}} \\
& + (rnfa_{t-1}^{eq} - rcon_{t-1}^{eq})\frac{nfa_{t-1}^{eq}}{1+ydot_t^{eq}} \\
& + pka_t k_t^{eq} - pi_t^{eq} i_t^{eq} - (1-depr_t^{eq})\frac{pk_t^{eq} cu_t^{eq} kp_{t-1}^{eq}}{1+ydot_t^{eq}} \\
& + (1+rk_{t-1}^{eq})\frac{pka_{t-1}k_{t-1}^{eq} - pk_{t-1}^{eq} cu_t^{eq} kp_{t-1}^{eq} - tk_t^{eq} depr_t^{eq} pka_{t-1} k_{t-1}^{eq}}{1+ydot_t^{eq}} \\
& + (1-tk_t^{eq})(pfc_t^{eq} y_t^{eq} - ylab_t^{eq}) - (rk_{t-1}^{eq} + depr_t^{eq})\frac{pk_{t-1}^{eq} cu_t^{eq} kp_{t-1}^{eq}}{1+ydot_t^{eq}} \\
& + check1_t^{eq}
\end{aligned}
\tag{A78}
$$

$$
risk_t = ri_1 risk_{t-1} + (1-ri_1) risk_t^{eq} \tag{A79}
$$

$$
fa_t = pka_t k_t + gb_t + nfa_t \tag{A80}
$$

$$
fa_t^{eq} = pka_t k_t^{eq} + gb_t^{eq} + nfa_t^{eq} \tag{A81}
$$

D Stocks

D.1 Capital

$$
\begin{aligned}
k_t = {} & k_t^{eq} + kv_1\left(\frac{y_{t-4}}{y_{t-4}^{eq}} - 1\right) - [kv_{21}(r_{t-4} - r_{t-4}^{eq}) + kv_{22}(rl_{t-4} - rl_{t-4}^{eq})]k_{t-4}^{eq} \\
& + kv_3(x_{t+1} - x_{t+1}^{eq}) - kadj_t
\end{aligned}
\tag{A82}
$$

$$
cc_t^{eq} = \rho\alpha\left(pfc_{t+1}^{eq} y_{t+1}^{eq}\frac{1+ydot_t^{eq}}{cu_{t+1}^{eq} kp_t^{eq}}\right)^{1-\psi} \tag{A83}
$$

$$cc_t^{eq}\left(1 - tk_t^{eq}\right) = \left(1 + rk_t^{eq}\right)pk_t^{eq} - \left(1 - depr_t^{eq}\right)pk_{t+1}^{eq} \tag{A84}$$

$$dt_t^{eq} = \frac{dt_{t+1}^{eq}\left(1 - depr_t^{eq}\right) + tk_t^{eq} depr_t^{eq} pka_t}{1 + rk_t^{eq}} \tag{A85}$$

$$\begin{aligned}
&\left(1 - ip_1 - ip_2 - ip_3 - ip_4 - ip_5 - ip_6 - ip_7\right)pk_t^{eq} \\
&+ ip_1 \frac{pk_{t+1}^{eq}}{1 + rk_t^{eq}} + ip_2 \frac{pk_{t+2}^{eq}}{\prod_{j=0}^{1}\left(1 + rk_{t+j}^{eq}\right)} + ip_3 \frac{pk_{t+3}^{eq}}{\prod_{j=0}^{2}\left(1 + rk_{t+j}^{eq}\right)} \\
&+ ip_4 \frac{pk_{t+4}^{eq}}{\prod_{j=0}^{3}\left(1 + rk_{t+j}^{eq}\right)} + ip_5 \frac{pk_{t+5}^{eq}}{\prod_{j=0}^{4}\left(1 + rk_{t+j}^{eq}\right)} \\
&+ ip_6 \frac{pk_{t+6}^{eq}}{\prod_{j=0}^{5}\left(1 + rk_{t+j}^{eq}\right)} + ip_7 \frac{pk_{t+7}^{eq}}{\prod_{j=0}^{6}\left(1 + rk_{t+j}^{eq}\right)} \\
&= \left[\left(1 - pk_1\right)pi_t^{eq} + pk_1 pi_{ss}\right] + ke_1(i_t^{eq} - i_{ss}) - dt_t^{eq}
\end{aligned} \tag{A86}$$

$$pka_t = \left(1 - pk_0\right)pka_{t-1} + pk_0 pi_{ss} \tag{A87}$$

D.2 Government bonds and taxes

$$gbtar_t^{eq} = gbtar\_y_t^{eq} y_t^{eq} \tag{A88}$$

$$gbtar_t = gbtar_t^{eq} \tag{A89}$$

$$gb_t^{eq} = gb_{t-1}^{eq} + td_1\left(gb_t^{eq} - gbtar_t\right) + td_2\left(gb_t^{eq} - gb_{t-1}^{eq}\right) \tag{A90}$$

$$
\begin{aligned}
& gb_t + td_t\,(ylab_t + yd_1 gtr_t) + tiy_t pfc_t y_t \\
& + tk_t \left(pfc_t y_t - ylab_t - \frac{depr_t pka_{t-1} k_{t-1}}{1 + ydot_t} \right) \\
& = (1 + rgb_{t-1})\,\frac{gb_{t-1}}{1 + ydot_t} + pg_t g_t + gtr_t
\end{aligned} \tag{A91}
$$

$$
\begin{aligned}
& gb_t^{eq} + td_t^{eq}\,(ylab_t^{eq} + yd_1 gtr_t^{eq}) + tiy_t^{eq} pfc_t^{eq} y_t^{eq} \\
& + tk_t^{eq} \left(pfc_t^{eq} y_t^{eq} - ylab_t^{eq} - \frac{depr_t^{eq} pka_{t-1} k_{t-1}^{eq}}{1 + ydot_t^{eq}} \right) \\
& = \left(1 + rgb_{t-1}^{eq}\right) \frac{gb_{t-1}^{eq}}{1 + ydot_t^{eq}} + pg_t^{eq} g_t^{eq} + gtr_t^{eq}
\end{aligned} \tag{A92}
$$

$$
\begin{aligned}
td_t = {} & tdl_1 td_{t-1} + tdl_2 td_{t-2} + tdl_3 td_{t-3} + tdl_4 td_{t-4} \\
& + (1 - tdl_1 - tdl_2 - tdl_3 - tdl_4)[td_t^{eq} + td_3(gb_{t-1} - gb_{t-1}^{eq}) \\
& - td_4(gbtar\_y_t^{eq} - gbtar\_y_{t-1}^{eq})]
\end{aligned} \tag{A93}
$$

$$
\begin{aligned}
& pc_t^{eq} c_t^{eq} + pih_t^{eq} ih_t^{eq} + pi_t^{eq} i_t^{eq} + pg_t^{eq} g_t^{eq} + px_t^{eq} x_t^{eq} - pm_t^{eq} m_t^{eq} \\
& = (1 + tiy_t^{eq})[pcd_t^{eq}(c_t^{eq} - cm_t^{eq}) + pihd_t^{eq}(ih_t^{eq} - ihm_t^{eq}) + pid_t^{eq}(i_t^{eq} - im_t^{eq}) \\
& + pgd_t^{eq}(g_t^{eq} - gm_t^{eq}) + px_t^{eq} x_t^{eq}]
\end{aligned} \tag{A94}
$$

$$
\begin{aligned}
& pc_t c_t + pih_t ih_t + pi_t i_t + pg_t g_t + px_t x_t - pm_t m_t \\
& = (1 + tiy_t)[pcd_t(c_t - cm_t) + pihd_t(ih_t - ihm_t) + pid_t(i_t - im_t) \\
& + pgd_t(g_t - gm_t) + px_t x_t)]
\end{aligned} \tag{A95}
$$

E Production and the labor market

$$y_t = 0.25 \left\{ (1-\alpha) \left[tfp_t (1-u_t) \right]^{\psi} + \alpha \left(\frac{cu_t kp_{t-1}}{1 + ydot_t} \right)^{\psi} \right\}^{\frac{1}{\psi}} \tag{A96}$$

$$y_t^{eq} = 0.25 \left\{ (1-\alpha) \left[tfp_t^{eq} (1-u_t^{eq}) \right]^{\psi} + \alpha \left(\frac{cu_t^{eq} kp_{t-1}^{eq}}{1 + ydot_t^{eq}} \right)^{\psi} \right\}^{\frac{1}{\psi}} \tag{A97}$$

$$yp_t = 0.25 \left\{ (1-\alpha) \left[tfp_t^{eq} (1-u_t^{eq}) \right]^{\psi} + \alpha \left(\frac{cu_t kp_{t-1}}{1 + ydot_t^{eq}} \right)^{\psi} \right\}^{\frac{1}{\psi}} \tag{A98}$$

$$kp_t = (1 - depr_t) \frac{kp_{t-1}}{1 + ydot_t} + ip_t \tag{A99}$$

$$kp_t^{eq} = (1 - depr_t^{eq}) \frac{kp_{t-1}^{eq}}{1 + ydot_t^{eq}} + ip_t^{eq} \tag{A100}$$

$$\begin{aligned} u_t \quad = \quad & u_t^{eq} - uv_2 \left(\frac{y_{t-1}}{yp_{t-1}} - 1 \right) - uv_3 \left(\frac{y_{t-2}}{yp_{t-2}} - 1 \right) \\ & -uv_4 \left(\frac{y_{t-3}}{yp_{t-3}} - 1 \right) + uv_1 \left(\frac{wp_{t-1}}{wp_{t-1}^{eq}} - 1 \right) - uadj_t \end{aligned} \tag{A101}$$

$$cu_t = \min(cu_t^{eq} - cuadj_t^{eq},\ 1) \tag{A102}$$

F The monetary authority, interest rates, and exchange rates

F.1 Interest rates

$$rn_t = \max \left\{ \begin{array}{c} 0, smooth_t[rn_t^{eq} + rsl_4(tpdot_{t+4} - pdottar_{t+4}) \\ +rsl_5(tpdot_{t+5} - pdottar_{t+5}) + rsl_6(tpdot_{t+6} - pdottar_{t+6})] \\ +(1 - smooth_t)rn_{t-1} \end{array} \right\} \tag{A103}$$

$$pdottar_t = pdottar_t^{eq} \tag{A104}$$

$$pdot_t^{eq} = pdottar_t \tag{A105}$$

$$1 + rn_t^{eq} = (1 + r_t^{eq}) \left(1 + pdot_{t+1}^{eq}\right) \tag{A106}$$

$$1 + rn5_t^{eq} = \left(1 + rn5_{t+1}^{eq}\right) \left(\frac{1 + rt5_t^{eq}}{1 + rt5_{t+1}^{eq}}\right) \left(\frac{1 + rn_t^{eq}}{1 + rn_{t+20}^{eq}}\right)^{\frac{1}{20}} \tag{A107}$$

$$1 + rn5_t = (1 + rn5_{t+1}) \left(\frac{1 + rt5_t^{eq}}{1 + rt5_{t+1}^{eq}}\right) \left(\frac{1 + rn_t}{1 + rn_{t+20}}\right)^{\frac{1}{20}} \tag{A108}$$

$$rnl_t^{eq} = rn5_t^{eq} \tag{A109}$$

$$rnl_t = rl_1(1+rn_t)(1+rt5_t^{eq})+rl_2(1+rn5_t)+(1-rl_1-rl_2)(1+rnl_t^{eq})-1 \tag{A110}$$

$$1 + rn_t = (1 + r_t)(1 + pdot_{t+1}) \tag{A111}$$

$$1 + r5_t^{eq} = \left(1 + r5_{t+1}^{eq}\right) \left(\frac{1 + rt5_t^{eq}}{1 + rt5_{t+1}^{eq}}\right) \left(\frac{1 + r_t^{eq}}{1 + r_{t+20}^{eq}}\right)^{\frac{1}{20}} \tag{A112}$$

$$1 + r5_t = (1 + r5_{t+1}) \left(\frac{1 + rt5_t^{eq}}{1 + rt5_{t+1}^{eq}}\right) \left(\frac{1 + r_t}{1 + r_{t+20}}\right)^{\frac{1}{20}} \tag{A113}$$

$$rl_t^{eq} = r5_t^{eq} \tag{A114}$$

$$1 + rl_t = rl_1(1 + r_t)(1 + rt5_t^{eq}) + rl_2(1 + r5_t) + (1 - rl_1 - rl_2)(1 + rl_t^{eq}) \tag{A115}$$

$$r_t^{eq} = rrow_t^{eq} + rp_t^{eq} \tag{A116}$$

$$rk_t^{eq} = rl_t^{eq} + rk\_rl_t^{eq} \tag{A117}$$

$$rgb_t = rl_t + rgb\_rl_t \tag{A118}$$

$$rgb_t^{eq} = rl_t^{eq} + rgb\_rl_t^{eq} \tag{A119}$$

$$rnfa_t = rl_t + rnfa\_rl_t \tag{A120}$$

$$rnfa_t^{eq} = rl_t^{eq} + rnfa\_rl_t^{eq} \tag{A121}$$

$$rcon_t^{eq} = rl_t^{eq} + rcon\_rl_t^{eq} \tag{A122}$$

$$rp_t = rp_t^{eq} \tag{A123}$$

$$rgb\_rl_t = rgb\_rl_t^{eq} \tag{A124}$$

$$rnfa\_rl_t = rnfa\_rl_t^{eq} \tag{A125}$$

F.2 Exchange rates

$$ze_t = zf_1 z_{t+1} + zl_1 z_{t-1} + (1 - zf_1 - zl_1) z_{t+1}^{eq} \tag{A126}$$

$$z_t = z_1 z_{t-1} + z_2 ze_t \frac{1 + rrow_t + rp_t}{1 + r_t} + (1 - z_1 - z_2) z_t^{eq} \tag{A127}$$

F.3 Inflation expectation

$$\begin{aligned}
pdote_t = {} & [1 - (pde_0 + pde_1 + pde_2 + pde_3 + pde_4 + pde_5 + pde_6 + pde_7 \\
& + pde_8)]\{cpi_1[pdl_1 cpidot4_{t-1} + pdl_2 cpidot4_{t-2} + pdl_3 cpidot4_{t-3} \\
& + (1 - pdl_1 - pdl_2 - pdl_3) cpidot4_{t-4}] + (1 - cpi_1)[pdl_1 pdot4_{t-1} \\
& + pdl_2 pdot4_{t-2} + pdl_3 pdot4_{t-3} + (1 - pdl_1 - pdl_2 - pdl_3) pdot4_{t-4}]\} \\
& + pde_1 pdot_{t+1} + pde_2 pdot_{t+2} + pde_3 pdot_{t+3} + pde_4 pdot_{t+4} \\
& + pde_5 pdot_{t+5} + pde_6 pdot_{t+6} + pde_7 pdot_{t+7} + pde_8 pdot_{t+8} \\
& + pde_0 pdottare_t
\end{aligned} \tag{A128}$$

$$
\begin{aligned}
pcdote_t = [1 - (&pde_0 + pde_1 + pde_2 + pde_3 + pde_4 + pde_5 + pde_6 + pde_7 \\
&+ pde_8)][pdl_1 ncpidot_{t-1} + pdl_2 ncpidot_{t-2} + pdl_3 ncpidot_{t-3} \\
&+ (1 - pdl_1 - pdl_2 - pdl_3) ncpidot_{t-4}] + pde_1 ncpidot_{t+1} \\
&+ pde_2 ncpidot_{t+2} + pde_3 ncpidot_{t+3} + pde_4 ncpidot_{t+4} + pde_5 ncpidot_{t+5} \\
&+ pde_6 ncpidot_{t+6} + pde_7 ncpidot_{t+7} + pde_8 ncpidot_{t+8} + pde_0 pdottare_t \\
&+ pde_9 \left(\frac{tiy_t^{eq} - tiy_{t-4}^{eq}}{1 + tiy_{t-4}^{eq}} \right)
\end{aligned} \tag{A129}
$$

$$
pdottare_t = 0.3 \left(\frac{1}{31} \sum_{j=-15}^{15} cpidot4_{t+j} \right) + 0.7 pdottar_t^{eq} \tag{A130}
$$

F.4 Inflation

$$
pdot_t = pdf_1 pdote_t + (1 - pdf_1) pdot_{t-1} + pd_0 \left(\frac{y_t}{yp_t} - 1 \right) \tag{A131}
$$

$$
(1 + pdot4_t)^4 = (1 + pdot_t)(1 + pdot_{t-1})(1 + pdot_{t-2})(1 + pdot_{t-3}) \tag{A132}
$$

$$
1 + pcdot_t = (1 + pdot_t) \frac{pc_t}{pc_{t-1}} \tag{A133}
$$

$$
\begin{aligned}
1 + pcddot_t = &\left[pcda_1 \left(\frac{\frac{pcd_t}{pcd_t^{eq}}}{\frac{pcd_{t-1}}{pcd_{t-1}^{eq}}} - 1 \right) + 1 \right] \left[pcda_2 \left(\frac{pcd_t^{eq}}{pcd_{t-1}^{eq}} - 1 \right) + 1 \right] \\
&\frac{1 + tic_t}{1 + tic_{t-1}} (1 + pdot_t)
\end{aligned} \tag{A134}
$$

$$1 + pcmdot_t = \left[pcma_1 \left(\frac{\frac{pcm_t}{pcm_t^{eq}}}{\frac{pcm_{t-1}}{pcm_{t-1}^{eq}}} - 1 \right) + 1 \right] \left[pcma_2 \left(\frac{pcm_t^{eq}}{pcm_{t-1}^{eq}} - 1 \right) + 1 \right] \tag{A135}$$

$$(1 + pdot_t) \frac{(1 + ticm_t)(1 + tic_t)}{(1 + ticm_{t-1})(1 + tic_{t-1})}$$

$$1 + pgdot_t = (1 + pdot_t) \frac{pg_t}{pg_{t-1}^{eq}} \tag{A136}$$

$$1 + pgddot_t = (1 + pdot_t) \frac{pgd_t}{pgd_{t-1}^{eq}} \tag{A137}$$

$$1 + pgmdot_t = (1 + pdot_t) \frac{pgm_t}{pgm_{t-1}^{eq}} \tag{A138}$$

$$1 + pidot_t = (1 + pdot_t) \frac{pi_t}{pi_{t-1}^{eq}} \tag{A139}$$

$$1 + piddot_t = (1 + pdot_t) \frac{pid_t}{pid_{t-1}^{eq}} \tag{A140}$$

$$1 + pimdot_t = (1 + pdot_t) \frac{pim_t}{pim_{t-1}^{eq}} \tag{A141}$$

$$(1 + pcdot4_t)^4 = (1 + pcdot_t)(1 + pcdot_{t-1})(1 + pcdot_{t-2})(1 + pcdot_{t-3}) \tag{A142}$$

$$1 + npcdot_t = (1 + pdot_t) \frac{\frac{pc_t}{1 + tic_t}}{\frac{pc_{t-1}}{1 + tic_{t-1}}} \tag{A143}$$

$$(1+npcdot4_t)^4 = (1+npcdot_t)(1+npcdot_{t-1})(1+npcdot_{t-2})(1+npcdot_{t-3}) \tag{A144}$$

$$1+npcddot_t = (1+pdot_t)\frac{pcd_t}{pcd^{eq}_{t-1}} \tag{A145}$$

$$1+npcmdot_t = (1+pdot_t)\frac{pcm_t}{pcm^{eq}_{t-1}} \tag{A146}$$

$$\begin{aligned} cpidot_t = {} & pcdc_0\left(\frac{c_t - cm_t}{c_t}\right)pcddot_t + pcdc_1\left(\frac{c_{t-1} - cm_{t-1}}{c_{t-1}}\right)pcddot_{t-1} \\ & + (1 - pcdc_0 - pcdc_1)\left(\frac{c_{t-2} - cm_{t-2}}{c_{t-2}}\right)pcddot_{t-2} \\ & + pcmc_0\left(\frac{cm_t}{c_t}\right)pcmdot_t + pcmc_1\left(\frac{cm_{t-1}}{c_{t-1}}\right)pcmdot_{t-1} \\ & + pcmc_2\left(\frac{cm_{t-2}}{c_{t-2}}\right)pcmdot_{t-2} \\ & + (1 - pcmc_0 - pcmc_1 - pcmc_2)\left(\frac{cm_{t-3}}{c_{t-3}}\right)pcmdot_{t-3} \end{aligned} \tag{A147}$$

$$\begin{aligned} ncpidot_t = {} & pcdc_0\left(\frac{c_t - cm_t}{c_t}\right)npcddot_t \\ & + pcdc_1\left(\frac{c_{t-1} - cm_{t-1}}{c_{t-1}}\right)npcddot_{t-1} \\ & + (1 - pcdc_0 - pcdc_1)\left(\frac{c_{t-2} - cm_{t-2}}{c_{t-2}}\right)npcddot_{t-2} \\ & + pcmc_0\left(\frac{cm_t}{c_t}\right)npcmdot_t + pcmc_1\left(\frac{cm_{t-1}}{c_{t-1}}\right)npcmdot_{t-1} \\ & + pcmc_2\left(\frac{cm_{t-2}}{c_{t-2}}\right)npcmdot_{t-2} \\ & + (1 - pcmc_0 - pcmc_1 - pcmc_2)\left(\frac{cm_{t-3}}{c_{t-3}}\right)npcmdot_{t-3} \end{aligned} \tag{A148}$$

$$(1 + cpidot4_t)^4 = (1 + cpidot_t)(1 + cpidot_{t-1})(1 + cpidot_{t-2})(1 + cpidot_{t-3}) \tag{A149}$$

$$(1+ncpidot4_t)^4 = (1+ncpidot_t)(1+ncpidot_{t-1})(1+ncpidot_{t-2})(1+ncpidot_{t-3}) \tag{A150}$$

$$tpdot_t = pt_0 cpidot4_t + (1 - pt_0)pdot4_t \tag{A151}$$

F.5 Deflators

$$\begin{aligned} pfc_t^{eq} y_t^{eq} = {} & [pcd_t^{eq}(c_t^{eq} - cm_t^{eq}) + pid_t^{eq}(i_t^{eq} - im_t^{eq}) \\ & + pihd_t^{eq}(ih_t^{eq} - ihm_t^{eq}) + pgd_t^{eq}(g_t^{eq} - gm_t^{eq}) + ii_t^{eq}] \\ & + px_t^{eq} x_t^{eq} + check2_t^{eq} \end{aligned} \tag{A152}$$

$$\begin{aligned} & pcd_t^{eq}\,(c_t^{eq} - cm_t^{eq}) + pihd_t^{eq}\,(ih_t^{eq} - ihm_t^{eq}) \\ & + pid_t^{eq}\,(i_t^{eq} - im_t^{eq}) + pgd_t^{eq}(g_t^{eq} - gm_t^{eq}) + ii_t^{eq} \\ & = c_t^{eq} - cm_t^{eq} + i_t^{eq} - im_t^{eq} + ih_t^{eq} - ihm_t^{eq} + g_t^{eq} - gm_t^{eq} + ii_t^{eq} \end{aligned} \tag{A153}$$

$$\begin{aligned} & pcd_t\,(c_t - cm_t) + pihd_t\,(ih_t - ihm_t) \\ & + pid_t\,(i_t - im_t) + pgd_t\,(g_t - gm_t) + ii_t \\ & = c_t - cm_t + i_t - im_t + ih_t - ihm_t + g_t - gm_t + ii_t \end{aligned} \tag{A154}$$

$$pc_t c_t = (1 + tic_t) \left[pcd_t \left(c_t - cm_t \right) + (1 + ticm_t) \, pcm_t cm_t \right] \tag{A155}$$

$$pc_t^{eq} c_t^{eq} = (1 + tic_t^{eq}) \left[pcd_t^{eq} \left(c_t^{eq} - cm_t^{eq} \right) + (1 + ticm_t^{eq}) \, pcm_t^{eq} cm_t^{eq} \right] \tag{A156}$$

$$\begin{aligned} pcm_t = {} & pcm_t^{eq} + pcmv_1 (z_{t-1} - z_{t-1}^{eq}) pcrow_t^{eq} \\ & + pcmv_2 (pcrow_t - pcrow_t^{eq}) z_t^{eq} - pcmadj_t \end{aligned} \tag{A157}$$

$$pcm_t^{eq} = (1 - pcm_1) \, pcm_{t-1}^{eq} + pcm_1 \left(pcrow_t^{eq} z_t^{eq} + pcm_0 \right) \tag{A158}$$

$$\begin{aligned} pcd_t = {} & pcd_t^{eq} + pcdv_0 \left(\frac{y_t}{yp_t} - 1 \right) + pcdv_1 \left(\frac{y_{t-1}}{yp_{t-1}} - 1 \right) \\ & + pcdv_2 \left(\frac{y_{t-2}}{yp_{t-2}} - 1 \right) + pcdv_3 \left(\frac{y_{t-3}}{yp_{t-3}} - 1 \right) - pcdadj_t \end{aligned} \tag{A159}$$

$$pcd_t^{eq} = pcd_{t-1}^{eq} + 0.75 (pcd_{t-1}^{eq} - pcd_{t-2}^{eq}) \tag{A160}$$

$$pih_t ih_t = (1 + tiih_t) \left[pihd_t (ih_t - ihm_t) + (1 + tiihm_t) pihm_t ihm_t \right] \tag{A161}$$

$$pih_t^{eq} ih_t^{eq} = (1 + tiih_t^{eq}) \left[pihd_t^{eq} (ih_t^{eq} - ihm_t^{eq}) + (1 + tiihm_t^{eq}) pihm_t^{eq} ihm_t^{eq} \right] \tag{A162}$$

$$\begin{aligned} pihm_t = {} & pihm_t^{eq} + pihmv_1 (z_{t-1} - z_{t-1}^{eq}) pihrow_t^{eq} \\ & + pihmv_2 (pihrow_t - pihrow_t^{eq}) z_t^{eq} - pihmadj_t \end{aligned} \tag{A163}$$

$$pihm_t^{eq} = (1 - pihm_1)\, pihm_{t-1}^{eq} + pihm_1 \left(pihrow_t^{eq} z_t^{eq} + pihm_0 \right) \tag{A164}$$

$$\begin{aligned} pihd_t = {} & pihd_t^{eq} + pihdv_0 \left(\frac{y_t}{yp_t} - 1 \right) + pihdv_1 \left(\frac{y_{t-1}}{yp_{t-1}} - 1 \right) \\ & + pihdv_2 \left(\frac{y_{t-2}}{yp_{t-2}} - 1 \right) + pihdv_3 \left(\frac{y_{t-3}}{yp_{t-3}} - 1 \right) - pihdadj_t \end{aligned} \tag{A165}$$

$$pihd_t^{eq} = 0.95 pcd_t^{eq} + 0.05 pid_t^{eq} \tag{A166}$$

$$pi_t i_t = (1 + tii_t) \left[pid_t \left(i_t - im_t \right) + (1 + tiim_t)\, pim_t im_t \right] \tag{A167}$$

$$pi_t^{eq} i_t^{eq} = (1 + tii_t^{eq}) \left[pid_t^{eq} \left(i_t^{eq} - im_t^{eq} \right) + (1 + tiim_t^{eq})\, pim_t^{eq} im_t^{eq} \right] \tag{A168}$$

$$\begin{aligned} pim_t = {} & pim_t^{eq} + pimv_1 pirow_t^{eq} (z_{t-1} - z_{t-1}^{eq}) \\ & + pimv_2 (pirow_t - pirow_t^{eq}) z_t^{eq} - pimadj_t \end{aligned} \tag{A169}$$

$$pim_t^{eq} = (1 - pim_1)\, pim_{t-1}^{eq} + pim_1\,(pirow_t^{eq} z_t^{eq} + pim_0) \tag{A170}$$

$$pg_t g_t = (1 + tig_t)\,[pgd_t(g_t - gm_t) + (1 + tigm_t) pgm_t gm_t] \tag{A171}$$

$$pg_t^{eq} g_t^{eq} = (1 + tig_t^{eq})\,[pgd_t^{eq}(g_t^{eq} - gm_t^{eq}) + (1 + tigm_t^{eq}) pgm_t^{eq} gm_t^{eq}] \tag{A172}$$

$$\begin{aligned} pgm_t = {} & pgm_t^{eq} + pgmv_1 pgrow_t^{eq}(z_{t-1} - z_{t-1}^{eq}) \\ & + pgmv_2(pgrow_t - pgrow_t^{eq}) z_t^{eq} - pgmadj_t \end{aligned} \tag{A173}$$

$$pgm_t^{eq} = (1 - pgm_1)\, pgm_{t-1}^{eq} + pgm_1\,(pgrow_t^{eq} z_t^{eq} + pgm_0) \tag{A174}$$

$$pgd_t = pgd_t^{eq} + pgdv_1 \left(\frac{y_t}{yp_t} - 1\right) - pgdadj_t \tag{A175}$$

$$pgd_t^{eq} = pg\_pi_0 pid_t^{eq} \tag{A176}$$

$$\begin{aligned} px_t = {} & px_t^{eq} + pxv_1 pxrow_t^{eq}(z_t - z_t^{eq}) + pxv_2 \left(\frac{x_t}{x_{t-1}} - 1\right) \\ & + pxv_3(pxrow_t - pxrow_t^{eq}) z_t^{eq} - pxadj_t \end{aligned} \tag{A177}$$

$$px_t^{eq} = (1 - px_1)\, px_{t-1}^{eq} + px_1\,(pxrow_t^{eq} z_t^{eq} + px_0) \tag{A178}$$

$$pm_t m_t = pcm_t cm_t + pim_t im_t + pihm_t ihm_t + pgm_t gm_t \tag{A179}$$

$$pm_t^{eq} m_t^{eq} = pcm_t^{eq} cm_t^{eq} + pim_t^{eq} im_t^{eq} + pihm_t^{eq} ihm_t^{eq} + pgm_t^{eq} gm_t^{eq} \tag{A180}$$

F.6 Foreign prices

$$pcrow_t = pcrow_t^{eq} \tag{A181}$$

$$pirow_t = pirow_t^{eq} \tag{A182}$$

$$pihrow_t = pihrow_t^{eq} \tag{A183}$$

$$pgrow_t = pgrow_t^{eq} \tag{A184}$$

$$pxrow_t = pxrow_t^{eq} \tag{A185}$$

F.7 Other deflators

$$tot_t = \frac{px_t}{pm_t} \tag{A186}$$

$$pc\_py_t^{eq} = \frac{pc_t^{eq}}{py_t^{eq}} \tag{A187}$$

$$pi\_py_t^{eq} = \frac{pi_t^{eq}}{py_t^{eq}} \tag{A188}$$

$$pg\_py_t^{eq} = \frac{pg_t^{eq}}{py_t^{eq}} \tag{A189}$$

$$pih\_py_t^{eq} = \frac{pih_t^{eq}}{py_t^{eq}} \tag{A190}$$

$$px\_py_t^{eq} = \frac{px_t^{eq}}{py_t^{eq}} \tag{A191}$$

$$pm\_py_t^{eq} = \frac{pm_t^{eq}}{py_t^{eq}} \tag{A192}$$

$$pih\_pc_t^{eq} = \frac{pih_t^{eq}}{pc_t^{eq}} \tag{A193}$$

[Variables][69]

c_t: consumption, cc_t: user cost of capital, cfl_t: consumption by forward-looking consumers, $cfladj_t$: polynomial adjustment cost term on consumption by forward-looking consumers, $check1_t$: identity checker 1, $check2_t$: identity checker 2, cm_t: imports of consumption goods, cm_c_t: proportion of consumption goods imported, cm_cadj_t: polynomial adjustment cost term on proportion of consumption goods imported, $cpidot_t$: inflation rate for the CPI, $cpidot4_t$: annual inflation for the CPI, crt_t: consumption by rule-of-thumb consumers, cu_t: rate of capital utilization, $cuadj_t$: polynomial adjustment cost of rate of capital utilization, d_t: housing stock, $depr_t$: depreciation rate on capital, $deprih_t$: depreciation on housing stock, dt_t: rate of depreciation allowance for investment, fa_t: financial assets, $fwfl_t$: financial wealth, g_t: government expenditures, gb_t: government bonds, $gbtar_t$: government debt target, $gbtar\_y_t$: target

[69] Variables with superscript eq are equilibrium values.
Relative prices are against domestically produced and consumed goods at factor cost.
$ydot_t^{eq}$ is the equilibrium trend output growth rate. Variables here are detrended using this trend.

ratio of government bonds to output, gm_t: imports of government goods, gm_g_t: proportion of government goods imported, gm_gadj_t: polynomial adjustment cost term on proportion of government goods imported, g_y_t: target ratio of government expenditures to output, gtr_t: government transfers, gtr_y_t: target ratio of government transfers to output, $hwfl_t$: aggregate human wealth, $hwfl1_t$: human wealth 1, $hwfl2_t$: human wealth 2, $hwfl3_t$: human wealth 3, i_t: corporate investment, ih_t: housing investment, $ihadj_t$: polynomial adjustment cost term on housing investment, ihm_t: imports of housing investment goods, im_iadj_t: polynomial adjustment cost term on proportion of corporate investment goods imported, ihm_ih_t: proportion of housing investment goods imported, ihm_ihadj_t: polynomial adjustment cost term on proportion of housing investment goods imported, ii_t: inventory investment, $iiadj_t$: polynomial adjustment cost term on inventory investment, im_t: imports of corporate investment goods, im_i_t: proportion of corporate investment goods imported, ip_t: investment added to productive capital, k_t: capital stock inclusive of investment not yet productive, $kadj_t$: polynomial adjustment cost term on capital stock, kp_t: production capital, m_t: imports, $mpcw_t$: marginal propensity to consume out of wealth, $ncpidot_t$: inflation rate for the CPI net of indirect tax, $npcdot4_t$: annual inflation rate for the price of consumption net of indirect tax, $ndot_t$: population growth rate, $netx_t$: net imports, nfa_t: net foreign assets, $npcddot_t$: inflation rate for the price of domestic consumption net of indirect tax, $npcdot_t$: inflation rate for the price of consumption net of indirect tax, $npcmdot_t$: inflation rate for the price of imported consumption net of tariff, pc_t: relative price of consumption, pcd_t: relative price of domestic consumption goods, $pcdadj_t$: polynomial adjustment cost term on relative price of domestic consumption goods, $pcdot$: inflation rate for the price of consumption, $pcdote_t$: expected inflation rate for the price of consumption, $pcdot4_t$: annual inflation for the price of consumption, pcm_t: relative price of imported consumption goods, $pcmadj_t$: polynomial adjustment cost term on imported consumption goods, $pcrow_t$: relative price of consumption goods in the rest of the world, pc_py_t: relative price of consumption relative to the price of output, $pdot_t$: inflation rate for the price of domestically-produced and consumed goods at factor cost, $pdote$: expected inflation rate, $pdottar_t$: target inflation rate, $pdottare_t$: expected target inflation rate, $pdot4_t$: annual inflation rate, pfc_t: relative price of output at factor cost, pg_t: relative price of government expenditures, pgd_t: relative price of domestic government goods, $pgdadj_t$: polynomial adjustment cost term on relative price of domestic government goods, pgm_t: relative price of imported government goods, $pgmadj_t$: polynomial adjustment cost term on relative price of imported government goods, $pgmdot_t$: inflation rate for the price of imported government goods, pg_py_t: relative price of government goods relative to the price of output, $pgrow_t$: relative price of government goods in the rest of the world, pi_t: relative price of corporate investment, pid_t: relative price of domestic corporate investment goods, $piddot_t$: inflation for the price of domestic investment goods, $pidot_t$: inflation rate for the price of investment goods, pih_t: relative price of housing investment, $pihd_t$: relative price of domestic housing investment goods, $pihdadj_t$: polynomial adjustment cost term on relative price of domestic housing investment goods, $pihm_t$: relative price of imported housing investment goods, $pihmadj_t$: polynomial adjustment cost term on relative price of imported housing investment goods, pih_pc_t: ratio of relative price of housing investment to that of consumption, pih_py_t: relative price of housing investment goods relative to the price of output, $pihrow_t$: relative price of housing investment goods in the rest of the world, pim_t: relative price of imported corporate investment goods, $pimadj_t$: polynomial adjustment cost term on relative price of imported investment goods, $pimdot_t$: inflation rate for the price of imported goods, pi_py_t: relative price of investment goods relative to the price of output, $pirow_t$: relative price of investment goods in the rest of the world, pk_t: (equilibrium) relative price of capital stock, pka_t: relative price of capital stock, pm_t: relative price of imports, pm_py_t: relative price of imports relative to the price of output, px_t: relative price of exports, $pxadj_t$: polynomial adjustment cost term on relative price of exports, px_py_t: relative price of exports relative to the price of output, $pxrow_t$: relative price of export goods in the rest of the world, py_t: relative price of output, $qdot_t$: trend growth in labor-augmenting technical progress, r_t: 1-quarter real interest rate, $rcon_t$: real interest rate for consumers, $rcon\_rl_t$: real risk premium for consumers, rgb_t: real interest rate on government bonds, rgb_rl_t: real risk premium on government bonds, $risk_t$: risk income, rk_t: real interest rate on capital, rk_rl_t: real risk premium on capital, rl_t: 10-year real interest rate, rn_t: 1-quarter nominal interest rate, $rnfa_t$: real return on net foreign assets, $rnfa\_rl_t$: real risk premium on net foreign assets, rnl_t: 10-year nominal interest rate, $rn5_t$: 5-year nominal interest rate, rp_t: country real risk premium, $rrow_t$: real interest rate in the rest of the world, $rt5_t$: 5-year term premium, $r5_t$: 5-year real interest rate, td_t: net direct labor income tax rate, tfp_t: total factor productivity, tic_t: indirect tax rate on consumption goods, $ticm_t$: tariff rate on consumption goods, tig_t: indirect tax rate on government goods, $tigm_t$: tariff rate on imported government goods, tii_t: indirect tax rate on investment goods, $tiim_t$: tariff rate on corporate investment goods, $tiih_t$: indirect tax rate on housing investment goods, $tiihm_t$: tariff rate on imported housing investment goods, tiy_t: average indirect tax rate, tk_t: tax rate on profits, tot_t: terms of trade, $tpdot_t$: weighted average of inflation rates, $twfl_t$: total wealth, u_t: rate of unemployment, $uadj_t$: polynomial adjustment cost term on rate of unemployment, wa_t: real wage, wc_t: consumer's real wage, $wctar_t$: consumer's target real wage, $wdot_t$: rate of change of nominal wages, wp_t: producer's real wage, x_t: exports, $xadj_t$: polynomial adjustment cost term on exports, $xbal_t$: trade balance, y_t: output, yd_t: real disposable income, $ydot_t$: trend output growth rate, $ylab_t$: real labor income, yp_t: potential output, y_t^*: the world output, ι_t: user cost of capital for housing stock, z_t: real exchange rate measured as the price of foreign currency in units of domestic currency, and ze_t: expected real exchange rate.

Technical Notes[70]

Here, we summarize the technique used in this paper when introducing the non-negativity constraint on the nominal interest rate into the large macroeconomic model.

When we try to solve the large-scale nonlinear dynamic general equilibrium model for cases where the zero nominal interest rate constraint is binding, we are sometimes unable to obtain a solution. However, if the model is linear and satisfies the condition specified in Blanchard and Kahn (1980)[71], there exists a solution even when the zero nominal interest rate constraint is binding. This conclusion is reached in Jung, Teranishi and Watanabe (2003) in a dynamic application of the Kuhn-Tucker Theorem to a dynamic stochastic general equilibrium model.[72]

The reason why the solution may not be obtained in the nonlinear model is as follows. If the model is nonlinear, and if there are equations in the model which can be expressed in the form $x_t = a + \frac{y_t}{z_t}$, then inclusion of the constant means that a shock to z will change the linear relationship between x and y. Therefore, if a large negative shock hits the economy within a nonlinear framework, a solution may not exist (see Figure A-1).

When introducing the zero nominal interest rate constraint into a nonlinear model, the most significant problem is posed by our inability to distinguish why there is no solution. In general, we cannot identify whether the model is insoluble because there exists no solution, or whether it is simply that we have failed to find the solution, even though one exists. Even assuming that the latter is the case, a further difficulty is presented in that we cannot identify the computational problem causing the insolubility. This could be any one of three

[70] Peter Hollinger is highly acknowledged for the analysis in this section for giving us the idea and how to programme it.

[71] If the number of eigenvalues outside the unit circle equals to the number of non-predetermined variables, there exists a unique solution.

[72] As mentioned, TE must be taken as the terminal condition.

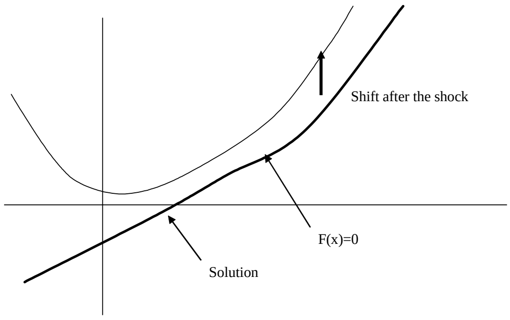

Figure A 1:

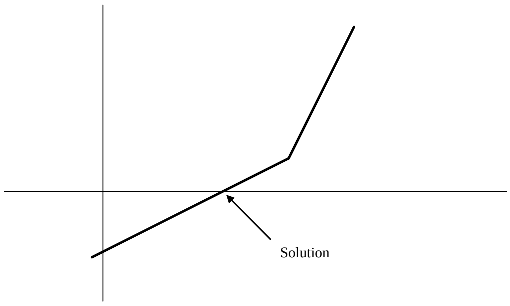

Figure A 2: Discountinuous Derivatives

possibilities: a discontinuous first derivative, stacking into the local maximum (minimum), or the wrong choice of initial value.

Discontinuous Derivatives We first introduce the zero nominal interest rate bound using the max function. Although introducing the zero nominal interest rate bound in this way usually provides us with a solution, there were several cases in which the solution proved unattainable. As TROLL uses the Newton-Raphson algorithm for the stacked matrix when solving the model dynamically,[73] the Newton-Raphson algorithm may collapse when applied to a system of equations which includes a max function whose Jacobian matrix is not continuous. This can be easily understood from Figure A-2.

Figure A-2 shows the iteration process for some arbitrary function with non-

[73]For the details, see Hollinger (1996).

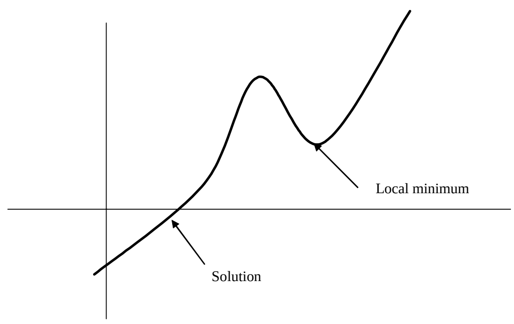

Figure A 3:

linearity stemming from the max function. When the iteration process stacks at the kink where the derivative is discontinuous, the Newton-Raphson algorithm collapses. A possible easy countermeasure is to employ dumping, which, by altering the Newton gain, may prevent the iteration from stopping at the kink. However, our examinations to date suggest that the contribution of dumping is minimal in this context.

Stacked at the Local Maximum (Minimum) Another major computational problem when solving the nonlinear model is the possibility of stacking into the local maximum. As depicted in Figure A-3, with a poor choice of initial value, the solution tends to move towards the local maximum and we may end up without a solution. Again, this problem may be resolved by applying the dumping technique. Further extending the simulation period, which alters the stacked matrix, may enable a solution to be obtained. However, as above, realized gains from such attempts are extremely limited so far.

As mentioned above, in order to avoid the first problem: "discontinuous derivatives," we introduce the fuctionally approximated policy rule with continuous first derivatives described in appendix 3. However, the approximated function necessarily becomes higher order. Therefore, even if we can avoid the risk of stacking at the kink when iterating the Newton-Raphson algorithm, there exist a greater risk of stacking into the local maximum, in other words of ending

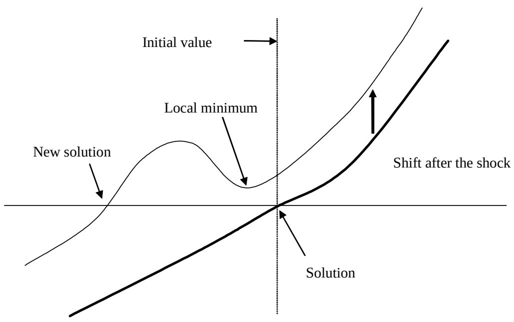

Figure A 4: Wrong Initial Value

up with no solution. Hence, there is considered to be a trade-off between the above two problems.

Choice of the Wrong Initial Value When conducting steady-state simulation, it is obvious that we can obtain a solution when there are no shocks. More concretely, if we use the steady state values as initial values for simulation without any shocks, there always exists a solution as it is input as the initial value. However, as the model is nonlinear, the large shock given to the model alters the linearly approximated dynamics as well. Therefore, as can be seen in Figure A-4, even if there exists a solution, TROLL may report that no solution exists because it stacks at the local minimum if we apply the shock directly without changing the initial value.

To overcome all these problem at once, we introduce a new algorithm for solving the model with binding zero nominal interest rate bounds.[74]

[74] This idea is inspired by Doug Laxton's presentation, "Think Globally, But Take Local Approximation" at the TROLL seminar held in Sevillia.

A Algorithm for Sequentially Updating Initial Values and Functions

The algorithm, "Algorithm for Sequentially Updating Initial Values and Functions," where we are not only sequentially updating the initial values, but also increasing the order of the functional approximation, consists of two parts: Updating the initial value, and updating the function. If we take the steady state simulation as an example, these two parts may be outlined as follows.

A.1 Updating the Initial Value

In this part, we follow the routine below.

i) First, we use the steady state value as the initial value and solve the model with only 1% of the desired shock which is eventually to be applied to the model.

ii) If we succeed in obtaining a solution, this solution is kept. Then another 1% shock is added to the shock applied in i). We keep on updating the initial value as long as a solution is obtained.

iii) If we fail to obtain a solution, we keep the initial value for this failed trial. We then decrease the shock from the level applied in the failed trial, keeping the size of shock only slightly larger than the last successful trial. When the shock being applied in this process reaches the size of the shock given in the failed trial, we go back to ii) again.

iv) The routine comes to an end when either we reach the desired size of shock or the number of failure attempts reaches some arbitrary number, say 100.

A.2 Updating the Function

If we cannot obtain a solution with the max function described in the above iteration, we use another iteration in which the function itself is updated. The process by which the order of approximation is increased is shown in Figure A-5,

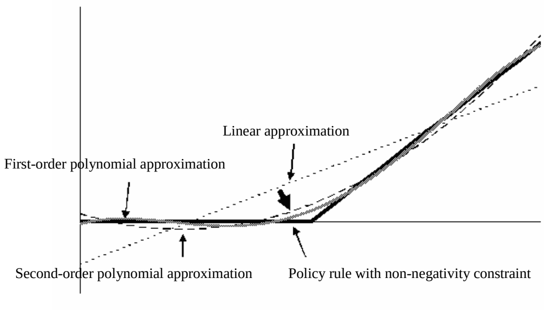

Figure A 5: Updating the Function

where a policy rule with a max function has been updated to become a higher order approximant. This is carried out using the following the routine below.

i) We first replace the max function by a numerically approximated function with a continuous derivative of arbitrary order (usually the 10th-order). If we can obtain a solution with the desired degree of approximation, we recognize this solution as a simulation pass.

ii) If we cannot obtain a solution, we reduce the degree of approximation until a solution is obtained using the above "updating the initial value" iteration. Each solution is used to update the initial value for a higher order of approximation until a solution is obtained for a function with the desired degree of approximation.

iii) If we cannot obtain a solution in ii), we consider that there is no solution or that the solution is unattainable.

By using these two iteration process together, we may be able to overcome the technical difficulty of solving a large model with a non-negativity constraint on the nominal interest rate.

Our attempts to date have always succeeded in obtaining a solution using just the first part of our algorithm. We have not yet been obliged to perform the second iteration process. The benefits attained from the first iteration process are huge. Even if we cannot obtain a solution when we apply the shock all at once, the solution is obtained via this initial value updating.

Appendix 3

Function Approximation with Polynomial Interpolation

If one wants to construct a highly theoretical model in which all the equations are derived from the social planner's optimization problem (in other words a model deriving from "first principle"), this typically requires dynamic programming in order to derive the structural equations. However, it is not usual that the researcher is able to identify the exact form of the value function that enables an analytical solution to be obtained. In these circumstances, it is often beneficial to approximate the value function numerically and then solve the system. Recently, such techniques, which are usually referred to as "Numerical Method," have been heavily applied in economics as in Judd (1998), Marimon and Scott (1999), Ljungqvist and Sargent (2000), and Miranda and Fackler (2002).

In this appendix, we briefly summarize one of the numerous numerical methods available, namely "the function approximation with polynomial interpolation." This is then applied in a simple manner to the policy rule when there is a non-negativity constraint on the nominal interest rate.

A Second Order Approximation of a Quadratic Equation

Here, as an example of a simple functional approximation, we attempt to approximate a quadratic equation expressed as an implicit function:

$$F(x_1, x_2) = 0. \tag{B1}$$

According to the Weierstrass Theorem which states that function approximations with any degree of accuracy can be obtained using a polynomial, the second order approximant of equation (B1), $\widehat{F}$, is shown as below:

$$
\begin{aligned}
& \widehat{F}\left(x_1, x_2\right) && \text{(B2)} \\
= & \left\{\left[\begin{array}{ll} \varphi_{21}\left(x_2\right) & \varphi_{22}\left(x_2\right) \end{array}\right] \otimes \left[\begin{array}{ll} \varphi_{11}\left(x_1\right) & \varphi_{12}\left(x_1\right) \end{array}\right]\right\} \left(\begin{array}{c} c_{11} \\ c_{21} \\ c_{12} \\ c_{22} \end{array}\right) \\
= & c_{11}\varphi_{11}\left(x_1\right)\varphi_{21}\left(x_2\right) + c_{21}\varphi_{12}\left(x_1\right)\varphi_{21}\left(x_2\right) \\
& +c_{12}\varphi_{11}\left(x_1\right)\varphi_{22}\left(x_2\right) + c_{22}\varphi_{12}\left(x_1\right)\varphi_{22}\left(x_2\right),
\end{aligned}
$$

c: parameters.

Polynomials are defined as Chebychev-node polynomials:[75]

$$\varphi_{11}\left(x_1\right) = 1,$$

$$\varphi_{12}\left(x_1\right) = 2 * \frac{x_1 - lb}{lb - ub} - 1,$$

$$\varphi_{21}\left(x_2\right) = 1,$$

$$\varphi_{22}\left(x_1\right) = 2 * \frac{x_2 - lb}{lb - ub} - 1,$$

lb: lower bound of approximation,

ub: upper bound of approximation.

Equation (B2) has four parameters. Therefore, if we pick any four points in $x_1 - x_2$ space, parameters are just identified. How then do we select the four points? When employing the Chebychev polynomial, as here, it is generally

[75]Miranda and Fackler (2002) claims that "Chebychev-node polynomial are very nearly optimal polynomial approximants" according to Rivlin's theorem.

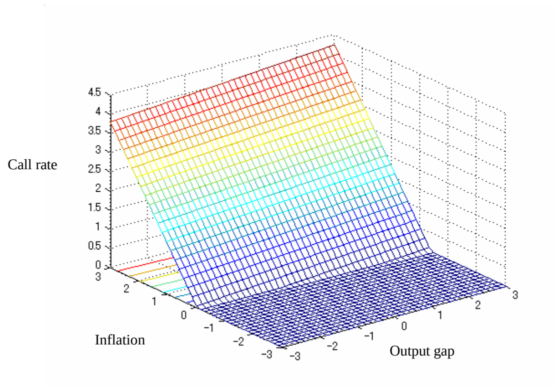

Figure B 1: Taylor Rule

recognized that the most appropriate point selection is given by :

$$x^i = lb + \frac{i-1}{4-1}(ub - lb) \quad \forall i = 1, 2, 3, 4.$$

Constructing the polynomial and defining the points which the polynomial passes through as above, we can derive an approximated equation with any degree of accuracy.

B Application to the Policy Rule

A very simple monetary policy rule[76] is estimated by assuming that the equilibrium nominal interest rate is 1 percent annually. This estimated policy rule is expressed as follows when the zero nominal interest rate bound is imposed:

$$\text{Call rate} = \max\left(0, 0.25 + 1.25 * \text{CPI Inflation} + 0.07 * \text{Output Gap}\right).$$

The shape of this function is demonstrated in Figure B-1.

The shape of the approximant of the 10th degree polynomial interpolation, which is continuously differentiable, is presented in Figure B-2.

[76]Note that this rule is different from the one employed in the JEM.

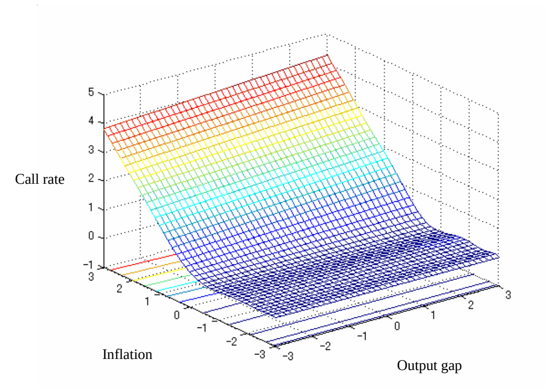

Figure B 2: Approximated Taylor Rule

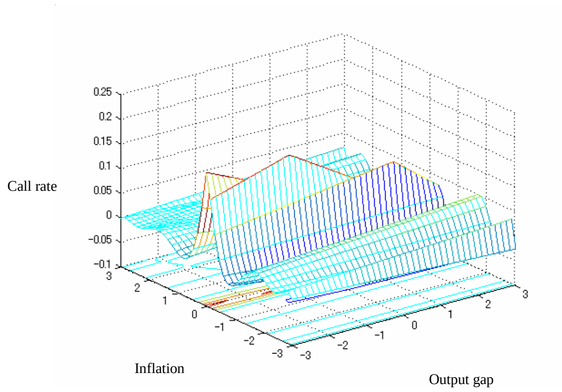

Figure B 3: Difference

Figure B-3 shows the difference between Figure B-1 and Figure B-2. The differences are seen to be minuscule. With the 10th order approximation, we can obtain a very accurate approximation.