Bank of Japan Working Paper Series

Fractal Network derived from banking transaction

-- An analysis of network structures formed by financial institutions --

Hajime Inaoka[*]
inaoka@eri.u-tokyo.ac.jp

Takuto Ninomiya[***]
takuto.ninomiya@boj.or.jp

Ken Taniguchi[**]
ken.taniguchi@boj.or.jp

Tokiko Shimizu[**]
tokiko.shimizu@boj.or.jp

Hideki Takayasu[****]
takayasu@csl.sony.co.jp

Bank of Japan
2-1-1 Nihonbashi Hongoku-cho, Chuo-ku, Tokyo 103-8660

No.04-E-04
April 2004

[*] University of Tokyo

[**] Financial Markets Department

[***] Policy Planning Office

[****] Sony Computer Science Laboratories

Abstract

The mechanism of financial transactions provided to society today cannot be determined by a single financial institution, but is determined through a complex structure of mutual cooperation, or a “network”, between several financial institutions. Quantitative analysis of such a “network” structure had not been explored until recently, mainly due to limitations in the data available. This paper analyzes the “network” structure of financial transactions, using the logged data of financial transactions through the BOJ-Net (Current Account of the Bank of Japan), which became obtainable after the introduction of RTGS (Real Time Gross Settlement) in 2001.

This study uses recently developed methods of statistical physics. This field of study provides an analytical framework that treats the complex structure of financial institutions as a structure of elements, or “nodes,” that are connected to one another, through “links.”

Our study shows that the “network” of financial transactions between financial institutions possess fractal structure, similar to that observed in network structures in the natural world (such as river basins) or the structure of the Internet. We also find that financial institutions situated in the middle of the network structure hold more links than those institutions on the periphery of the network, implying that the formed structure is a result of the pursuance of “efficiency” rather than “stability.”

We should pay enough attention to the dynamic nature of the network structure in order to evaluate its stability, since the network structure of financial transactions is not static in nature(the network is not based upon hardware like cables, as in the case of the Internet). Thus, there is a need to confirm the stability of the network structure over time. In this respect, further analysis of “dynamic networks” is worth a try, based on the evidence shown in this paper which confirms a certain degree of robustness within the network.

1. Introduction

Financial institutions, such as banks, play an important role in our society. For example, financial institutions collect funds from various organizations and individuals with excess money, and lend them to organizations and individuals in need of money. Financial institutions also give means of monetary transactions to society through the transfer of funds between agents. We will call the system related to monetary transactions collectively as a "banking system." It goes without saying that it is impossible to form a "banking system" only by a single financial institution. The "banking system" is organized only when several financial institutions cooperate with each other through monetary transactions. Furthermore, the "banking system" must be based upon a stable network of transactions, in order to fulfill its role.

For a better understanding of the factors that affect the stability of the banking system, it is imperative to understand the structure and the mechanism of the system. This study focuses on the element of "cooperation" between financial institutions, concentrating on the relationship of financial institutions through the transfer of funds. In the case of a cash transfer, the relationship can be explained as a delivery (or supply) of funds by one institution, in order to meet the demand of funds of another institution. This demand and supply of funds composes the structure of "cooperation" between financial institutions. A visual image of a complex structure of cash transfers between several financial institutions would look like a giant web. It was our belief that the analysis of this structure would lead to a better understanding of the banking system. Unfortunately, there have been few quantitative observations of fund transfers between financial institutions. One of the reasons is that there has been no means to record every single transaction between financial institutions. In this study, we analyzed the records of fund transfers between financial institutions executed through the BOJ-NET by means of statistical physics. This was possible, due to the introduction of the Real Time Gross Settlement System to the BOJ-NET, where every single transaction has been kept on record.

One of the major aims of statistical physics is an understanding of phenomena in which fluctuations play an important role. Though economic phenomena are such examples, they have not been studied in view of statistical physics until recently. For example, the demand and supply curve, which is a basic concept of economics, expresses the sum of individual demand for goods in society, as a function of price. However, such a scheme takes into account only an average or a sum of a given quantity overlooks stochastic fluctuations of the quantity.

There are important examples of physical phenomena that involve fluctuations. Among such phenomena, Brownian motion is well known in the field of the finance. Though Brownian motion is a phenomenon with a fluctuation, it is considered to be one of the simplest phenomena in the field of statistical physics, because it is explained by a sum of effects of independent events. On the other hand, there are more complex phenomena that involve intricate interaction between elements in a system. One of the examples is "critical phenomena". The critical phenomena have been studied for about half a century. Through these studies, methods to treat fluctuations such as mean-field approximation and a theory of renormalization group have been developed. These methods are applied to various phenomena in the field of the statistical physics today.

In recent years, there has been many studies concerning network structures in the field of statistical physics. A "Network" is a structure composed of nodes connected by links. It has been found that many of the network structures formed in the natural world are "fractal." Fractal is a concept deeply related to the critical phenomena. The structure of the Internet, and the structure of human relations such as co-starring of actors are found to have a fractal nature, and they are called "scale-free networks."

The structure formed by financial institutions is interpreted as a network structure by regarding the financial institutions as nodes and the connection defined by transactions between them as links. We call the network by this definition a "banking network." By means of the methods developed in the studies of network structures in the field of the statistical physics, we may be able to gain knowledge of the structure of the "banking network."

In this study, we analyze the records of the funds transfer between the current accounts at the Bank of Japan. We will present the methods to define the banking network and discuss its structure. We introduce several basic concepts of statistical physics before presenting our work, because these concepts may be unfamiliar to those involved in the field of the economics. We also show various statistics of monetary transactions in appendices because there are few studies examining such statistics.

2. Fractality of network structures

In recent years, the study of "Network structures" has seen significant growth in the field of statistical physics One of the earliest work of this area was done by Barabási et al. They presented that many network structures observed in the natural world show a common feature. The feature is that the number of links connected to a single node follows a power-law distribution. Now it is evident that power-law distributions are observed in the structure of the Internet and in human relations. The network structures of this type have been studied extensively since the discovery of the power-law distributions, and its properties such as the "stability" of the network have already been proven.

The power-law distribution implies the fractality of the network structure. We introduce several basic concepts briefly in this section, because we think that the relation between fractals and power-law distributions may be unfamiliar.

2.1 Fractals and power-law distributions

It is known that there is a close relationship between fractals and power-law distributions. We will explain this through an example of the structure of a river network.

River networks often have a branch-like structure as presented in Fig.1. On a branch-like network, the structure of the main stream and that of the tributaries have similar shapes. When a structure is comprised by sub-structures with the shape similar to that of the whole structure, the geometrical nature is called "self-similarity," and a structure with self-similarity is called a "fractal'."

It is known that the drainage basin area of a river network follows a power law. When an arbitrary point on the river branch is chosen, the drainage basin area for this point is defined by the total area upstream from this point. The power-law distribution $P(x)$ is described by

$$P(x) \propto x^{-\gamma} \qquad (1)$$

where $P(x)$ is the probability density function of a stochastic variable x . The power law of the drainage basin area distribution is interpreted geometrically as follows. The distribution of Eq.(1) follows

$$P(x) = a^{\gamma} P(ax) \qquad (2)$$

where a is a positive parameter. When the stochastic variable x is the drainage basin area, this relation means that the occurrence probability of a tributary with area x is as a^{γ} times big as that of a tributary with area ax. In other words, there are a^{γ} tributaries with area x on average in a tributary with area ax. Eq.(2) expresses the self-similarity of the river network mathematically.

This example does not assure all power-law distributions have geometrical interpretation. However, when a power-law distribution is observed along with a geometrical structure, it may be interpreted as self-similarity.

In the field of statistical physics, a fluctuation of a physical quantity is often observed by means of a probability distribution. The field of finance has also developed methods to treat fluctuations of economic phenomena. For example, there is a method to determine the proper portfolio of stocks by analyzing standard deviations of stock prices. The standard deviation of a physical quantity, however, only describes the magnitude of the fluctuation, which is comparable to the width of the probability density function, and cannot describe the whole form of the probability density function. Statistical physics aims to understand the physical process that forms the fluctuation by observing the function form of the probability density function.

2.2 Cumulative distributions

In this section, we explain cumulative distributions because, in the following sections, statistics of various quantities are presented by cumulative distributions. As in the case of this study, when we have to observe a distribution of a physical quantity, it is easier to observe the cumulative distribution rather than to observe the density distribution, because the graph of the cumulative distribution is usually smoother than that of the probability density distribution. The cumulative distribution $P(\geq x)$ of a stochastic variable x is defined by

$$P(\geq x) = \int_x^{\infty} P(x')\,dx' \qquad (3)$$

where $P(x')$ is the probability density function of x'. Since the cumulative distribution $P(\geq x)$ is an integral of the probability density from x to infinity, $P(\geq a)$ is the probability that variable x takes a value larger or equal to a. $x \geq a$

As we have indicated so far, the distribution of the variable x is usually presented by a probability distribution. In this paper, however, statistics of a quantity is presented by a distribution of number of cases $N(x)$. The cumulative distribution of this case is defined by

$$N(\geq x) = \int_x^{\infty} N(x')\,dx' \tag{4}$$

$N(\geq a)$ is the number of cases in which variable x takes a value larger or equal to a. When variable x takes only a positive value,

$$P(\geq x) = \frac{N(\geq x)}{N(\geq 0)} \tag{5}$$

holds. The distribution $N(\geq a)$ presents the real observational number, while the probability distribution shows the ratio of the number.

Because a cumulative distribution is an integral of a density distribution, in a case of a power-law distribution, the exponent of the cumulative distribution differs by 1 to the exponent of the density distribution.

2.3 Models of network formation

Before the study by Barabási et al., it was believed that the model by Erdös et al could explain the formation of a network structure. The model is based on an idea that a network is formed by a random connection of nodes.

Erdös model forms a network by connecting an arbitrary chosen pair of nodes by a link with probability p. When the number of nodes in a system is N_0, a node in the system forms pairs with other $N_0 - 1$ nodes. The probability that k pairs within $N_0 - 1$ pairs are connected by links and others are not connected is $p^k (1-p)^{N_0 - 1 - k}$. By taking into account the number of combination to choose k pairs out of $N_0 - 1$ pairs, the distribution of the number of links connected to a single node $P(k)$ is calculated as

$$P(k) = \binom{N_0 - 1}{k} p^k (1-p)^{N_0 - 1 - k} \tag{6}$$

When N_0 is large, this distribution is well approximated by a normal distribution with average $(N_0 - 1)p$. The distribution of number of links connected to a single node is called a “degree distribution.”

The results of the Erdös model was not tested until recently. Barabási checked their results by observing various network structures formed in the natural world, and found that degree distributions do not follow Eq.(6) but follow

$$P(k) \propto k^{-\gamma} \tag{7}$$

in many cases. As we mentioned above, a power-law distribution is often related to a fractal structure.

The distribution Eq.(7) implies that the network structures are of fractal nature.

Barabási et al. proposed a model of network formation to explain the fractality of the networks. The model is based on an idea of random growth of a network. The model develops a large network by adding nodes one by one to an initially small network with m_0 nodes. The new node added to the growing network is connected to m links, and the other end of these m links is connected to the growing network. The other m ends are connected to the i th node with probability

$$p = \frac{k_i}{\sum_j k_j} \tag{8}$$

where k_i is the number of links connected to the i th node of the growing network. This means that a node with a large number of links attracts more links. A process in which a large element grows larger is often observed in formation of fractal structures. A network formed by this model presents a power-law degree distribution Eq.(7) and the exponent of the distribution is calculated as γ =2.

In this way, Barabási et al. proved that a model with a growth process of a network could explain the formation of fractal networks.

3. The banking network

We reviewed studies of network structures in the field of statistical physics, and introduced several concepts related to them. As we mentioned in the section above, it has been recognized that there are many fractal networks in the natural world, and our interest brings us back to the possibility of the banking network possessing a fractal nature.

The study of the structure of the banking network provides useful insight from a practical viewpoint. For example, if a financial institution ceases to transfer funds, financial institutions expecting to receive the funds from the troubled one may face difficulty to maintain their function. In a worse case scenario, the malfunction of one financial institution could provoke the malfunction of several financial institutions, creating considerable damage to the entire banking system. In such a situation, if we have an idea of the relationship or network structure of financial institutions in terms of monetary transactions, we may be able to take necessary action in order to prevent systemic contagion.

For this reason, if the structure of the banking network is found to possess fractal properties, we can apply the existing results of recent studies of fractal networks to the banking network.

3.1 Data for analysis

One major method of transaction is the fund transfers of financial institution through the current accounts held at the Bank of Japan. Each major financial institution has a current account with the Bank of Japan and transfers funds between these accounts in order to settle payments. The Bank of Japan maintains an on-line system called the BOJ-NET for funds and Japanese Government Securities (JGSs) settlements between financial institutions. Terminals of the BOJ-NET are installed at financial institutions and the Bank of Japan, including its branch offices. There are a few financial institutions that do not use the BOJ-NET directly. In those cases, however, the transactions are applied to the Bank of Japan by application forms, and in the end, they are executed on the BOJ-NET. Therefore, all the transactions through current accounts at the Bank of Japan are executed on the BOJ-NET and they are electronically recorded on the computer of the online system.

We analyzed the records of transactions executed through the BOJ-NET between June

2001 and December 2002. Especially, we used the records of June 2001 for the analysis of the structure of the banking network and for the statistics presented in the appendix. The other records are used to confirm that the stability of the fractal nature over time.

The main aim of this study is to analyze the structure of the banking network. However, it is needless to say that we cannot completely grasp monetary transactions between financial institutions only through the record of fund transfers through the BOJ-NET.

First, there are means for monetary transactions between financial institutions other than the funds transfers through the BOJ-NET. Second, a part of funds transfers executed to pay for JGSs or corporate bonds are not included in the data set for this analysis. In addition, there are many cases of offsetting transactions. In the cases of offsetting, there may be no record of the transaction or a record with only the net amount.

It is preferable that these limitations be minimized because they may skew the statistics of monetary transactions. However, the records of the transactions through the current accounts are the best possible data we possess at present. Thus, we have assumed that the records well describe the tendency of monetary transactions.

There are several kinds of financial institutions that transfer funds through the current accounts at the Bank of Japan. They include security firms, credit unions, and so on. Hereafter, however, these financial institutions will collectively be called “banks” for simplicity.

A record of a transaction includes the bank codes and the branch codes of the source and the destination of the transaction, the execution time of the transaction, the amount of the funds transferred, and so on. Because we are mainly interested in the network structure of banks in this study, we neglect branches of banks. That is, when branch a of bank A sends money to branch b of bank B, we treat them simply as a transaction from bank A to bank B. There are records of transactions between branches of a same bank in the analyzed data, but these transactions are also neglected.

Including transactions between branches of same banks, there are 546 banks that executed one or more transactions within the period of June 2001. The total number of transactions was about 150 thousand, and the total amount of the money transacted was about 733 trillion yen. When the transactions within the same banks were eliminated, the total number of the transactions amounted about 140 thousand, and the total amount of money transacted was about 710 trillion yen. The maximum amount of one transaction was about 762 billion yen, and the minimum amount was 1 yen.

3.2 Definition of the banking network

Compared to the case of the Internet, it is not obvious how the banking network should be defined from the records of transactions. The Internet is a network composed of hardware such as cables, while the banking network of fund transfers is not composed of any hardware.

We may come up with several ways to define the banking network from the records of the transactions.

The simplest way would be to connect a pair of banks by a link when one or more transactions are executed between the pair during the observation period. However, there is a probability that an unconnected pair may execute a transaction, and if we observe the pair in a longer period, there may be a transaction between the unconnected pair. This indicates that the network structure would depend on the observation period, and the connection itself is not definite.

The next simplest idea may be to connect a pair with a link when the amount of the transferred funds between the pair exceeds a threshold. This may in fact be the most natural idea when we observe a network of monetary transactions. We analyze the total amount of transferred funds between the pairs. The number of banks that executed one or more transactions in the period of June 2001 is, as we mentioned before, 546. Therefore, the number of pairs theoretically formed by 546 banks is 148,785. However, the number of pairs in which one or more transactions are observed in the period is only 7,351. Figure 2 shows the cumulative distribution of the total amount of the funds transferred between those 7,351 pairs. The graph shows a smooth curves in a log-log scale, which means the distribution decays faster than any power law. The graph does not show any characteristic amount of funds suitable for a threshold for the definition of links. Though we can set a threshold at the average of the amount, its physical implication rests unclear.

In this study, we adopt the definition of the banking network as follows. Figure 3 shows the cumulative distribution of the number n of the transactions between the 7,351 pairs. As in the figure, the graph of the distribution in a log-log scale is fit well by a straight line above the kink at about $n \geq 20$. This shows that the cumulative distribution $N(\geq n)$ follows a power law

$$N(\geq n) \propto n^{-1.3} \tag{9}$$

in the range of $n \geq 20$. Because the number of business days in the period of June 2001 is 21, the position of the kink $n = 20$ has a clear meaning of one transaction a day. Thus, we define the banking network by connecting a pair of banks when the number of the transactions between the pair is larger or equal to 21.

3.3 Statistics of the banking network

Though there are 546 banks that executed transactions in the observation period, some of the banks were not considered in our analysis, because pairs with less than 21 transactions were not connected by links. As a result, the banking network is composed of 354 banks. The network structure is presented in Fig.4. A bank with more transactions is placed closer to the center with a few exceptions in the figure. There is a tendency that the banks placed near the circumference have a few links connected to the banks placed close to the center. The number of the pairs connected by links in the figure is 1,727. This is less than 3% of the theoretical number of pairs 62,481. The banking network is, for the most part, composed of unconnected pairs.

Figure 5 shows the cumulative degree distribution of the banking network. The graph follows a power law with an exponent $N(\geq n) \propto n^{-1.1}$ above the kink at n=5. By this graph, it is confirmed that the banking network also has a fractality described by Eq.(7).

The banking network (Fig.4) and the degree distribution (Fig.5) are from the records of the transactions during the period of June 2001. The power law of the degree distribution is always observed in other observation periods. Figure 6 shows the cumulative degree distributions observed in one-month periods from June 2001 to December 2002 with a regular interval of 3 months. Though there is a tendency that the link numbers slightly decrease during the observation period, the degree distributions always follow a power law with a constant exponent. The function forms of the distributions of the number and the total amount of the transactions are also unaffected by the observation period.

4. Nature of scale-free networks

We presented that the banking network has a fractal structure similar to that of the Internet. In the field of the statistical physics, the nature of the scale-free network has already been studied. In this section, we discuss the stability of a scale-free network by introducing the results of such studies, as stability is the most important aspect in an economic perspective.

4.1 Stability of network structures

Barabási et al. discussed the stability of the two types of network structures we introduced in Section 2.3. The stability of a network structure is estimated by the effect caused by the removal of a node from the network structure. The effect is measured by the radius of the network. The radius of a network is defined by the average distance between two nodes arbitrarily chosen of the network. The distance between the two nodes is measured by the number of links traced to reach one node from the other. When there are more than one path, the shortest one is adopted. Barabási et al. measured the radius of a network beforehand, and compared it with the radius of the network after removing a node from the network. When a node is removed from a network, the radius of the network increases in general, because the removal destroys the paths between nodes. By observing the increase of the radius, the effect of the removal of a node to the network is estimated.

Barabási et al. investigated the effect of the removal of nodes for the networks formed by the Erdös model and the Barabási model. They also came up with two methods of removal of nodes. One is to remove a node at random from the network, and the other is to remove the node with the largest number of links. The former is called a "failure" and the latter is called an "attack".

The results are presented in Fig.7. In the case of the Erdös network, the effects of a failure and an attack are nearly identical. This is because the degree distribution of the network is close to a normal distribution and the fluctuation of the link number is small. On the other hand, in the case of the Barabási network, though the effect of a failure is smaller than that of the Erdös network, the effect of an attack is significant. This is because the degree distribution of the Barabási network follows a power law.
The function form of a power-law distribution is asymmetric compared to that of a

normal distribution. This means that large part of links is concentrated to small number of nodes, and there are hub-like nodes in the network. The attack to the hub-like nodes has a significant effect on the network. Thus, the scale-free network is endurable to a “failure" but vulnerable to an “attack”.

4.2 Stability and economy of networks

We discuss further the results in the previous sub-section by considering simplified network structures. The Erdös model forms a network by randomly connecting pairs of nodes.

When the probability of the connection is 1, we get a network structure described in Fig.8(a). We call a network of this type a decentralized network. This network is affected little by the removal of a node because all the nodes in the network are connected directly to the others through links. In this sense, this network is stable. On the other hand, as in the case of the Internet, when the network is composed of hardware, the establishment and maintenance of the links are costly. This type of network proves to be costly, because links that are seldom used must be maintained. Thus, this network is uneconomical.

In the case of the Barabási network, the hub-like nodes in a network form a high concentration of links. By simplifying the network, we imagine a network described in Fig.8(b). We call this a centralized network. There is one hub node in the network and all the other nodes are connected only to the hub node. When one of the peripheral nodes is removed, the effect to the network is negligible.
However, when the hub node is removed, the network completely ceases to function.
In this sense, this network is unstable. On the other hand, since it only has to maintain a small number of links, thus making this network economical, compared to a decentralized network where it is costly to maintain links.

4.3 Comparison of network structures

We now discuss the structure of the banking network comparing it with the network structures discussed above.

We collectively show the exponents of the power-law, degree distributions of various

scale-free networks in Table 1.
All the exponents follow a cumulative distribution

$$N(\geq k) \propto k^{-\tau} \tag{10}$$

As we introduced in Section 2.2, there is a difference of 1 between τ and the exponent τ in Eq.(7) and a relation $\tau = \gamma - 1$ holds. We notice that most of the exponents τ of the degree distributions in the table, including that of the banking network, take values close to 1. The value $\tau = 1$ is considerably smaller than that, predicted by the Barabási model $\tau = 2$.

We discuss the meaning of smaller value of the exponent τ by the graphs in Figure 9. In the figure, the degree distributions of the banking network, the network by the Barabási model, and the two network structures discussed in the previous section are schematically presented.

First, we focus on the degree distributions of the simplified networks we discussed in the previous section. The graph of the decentralized network has a vertical slope at the right and a flat part at the left. This is because all the nodes have a considerable number of links. On the other hand, in the case of the centralized network, the graph has a flat part at the right and a vertical slope at the left. This is because the network has only one hub node with a large number of links, and other nodes have one link each. Though it is meaningless to evaluate the exponents of the power laws of the distributions in these cases, it may be interpreted that the power exponent of the former case is infinity and that of the latter case is zero. That is, a decentralized network, which is a result of the maximized stability, tends to have an exponent of infinity, and a centralized network, which shows maximized economy, tends to have an exponent of zero.

The banking network has a smaller value of the exponent τ than that of the network by the Barabási model. This means that the banking network is more “economic” and less “stable” than the network by the Barabási model.

5. Discussion

We analyzed the structure of the banking network formed by monetary transactions between financial institutions. We found that the banking network shows a power-law, degree distribution, which is also observed in the structure of other networks formed in the natural world. The study of scale-free networks may make it possible to develop a method to maintain the stability of the banking network in the future.

We also present various statistics of monetary transactions between financial institutions in Appendix A. Although this is not the central point of this paper, we consider the statistics to be significant, because the statistics based on the observation of real monetary transactions have not been presented so far.

There are several directions to proceed from this study.

First of all, we analyzed the records of the monetary transactions through the current accounts at the Bank of Japan, because it is the best records that are obtainable at present. However, as we explained in detail in Section 3.1, the records do not cover the whole range of monetary transactions between financial institutions. It is important to widen the coverage of the data.

The "link" of the banking network was defined by the number of transactions between pairs of banks, and the effect of the amount of the transaction were neglected. This meant that a transaction of 1 yen has the same effect of that of a transaction of 100 billion yen. This is intuitively unnatural. There may be a more natural definition of links that takes into account the effect of the amount of transactions.

The dynamics of the banking network is also an important issue.

In this study, we implicitly assume that the network structure defined by the monetary transactions between financial institutions is static and does not change in time. The discussion of the stability of network structures in Section 4 is based on this assumption. It is reasonable to see a network composed of hardware, such as the Internet, as a static network, because, once cables are connected, they are rarely reconfigured. In the case of the banking network, however, when a hub-like bank ceases to function, another bank may play the role of the hub. This means that the network is dynamic.

To check the dynamic aspect of the banking network, we classified financial institutions into groups by a rule in Appendix B, and observed the changes in the classification when a hub bank in a group merged. We found a case where the links of the subsidiary banks to the hub were reconnected to a hub bank of another group. This fact implies that the banking network dynamically ensures its stability through the reconfiguration of links, although the static structure of the network seems unstable.

The study of the dynamic stability of the banking network may lead to the discussion on effective policy to ensure the network stability from an entirely original viewpoint. For example, when a hub bank is at a verge of collapse, we may take a policy to bail it out. Or we may take another policy to support the surrounding banks to reconstruct a relationship with another hub bank smoothly and leave the troubled bank to market forces. The study of dynamic stability in the banking network may enable us to compare the effects of these policies. Other related issues, such as the effect of a network management system on its economy and stability might be also worth exploring.

It is also important to study the timing of settlement. Financial institutions, often in its nature, transact money larger than their own capital. As a result, a bank rarely executes settlement paying large amounts of money successively. Payments of large amounts are made after receiving enough money to execute the settlements. In this way, the timing of funds transfer by a bank is often determined by the timing of receipts from other banks. In fact, the amount of funds transferred is observed to be dependent on times during a day, as in Appendix A. A network structure with time correlation of this kind is new, even to the field of the statistical physics.

The banking network is an intriguing subject to provide new challenges to the field of statistical physics.

Appendix

A. Statistics of funds transfer

Though the aim of this study is the analysis of the banking network, the study yielded various statistics of the monetary transactions between financial institutions as by-products. We wish to present these statistics as an appendix.

The statistics is based on records of fund transfers between the current accounts at the Bank of Japan, during the period of June 2001. The limitations of the records are explained in detail in sec 3.1.

A.1 Temporal distributions

We observe the temporal change in the behavior of financial institutions in terms of fund transfers.

Figure 10 and Figure 11 show the number and total amount of transactions in each day, respectively. In both of these figures, there are two-day gaps in each of the 7 days. These are Saturdays and Sundays, on which financial institutions are closed. The number of business days in the period of June 2001 is 21. The average number of transactions per business day is 6,782, and the average amount of transferred funds per business day is about 33.8 trillion yen. The order of days by the number of the transactions goes 20th, 25th, 29th, and so on, but by amount, the order is 29th, 25th, and 20th.

Figure 12 and 13 show the number and total amount of transactions in each hour, respectively. The number and total amount between, for example, 9 o'clock and 10 o'clock are calculated by summing up the number and the total amount of the transactions between 9 o'clock and 10 o'clock in each business day for the 21 business days. As seen in the figures, both the number and the amount of the transactions concentrate on the hour between 9 o'clock and 10 o'clock. 54% of the transactions are executed and 47% of the total amount are transacted in this hour. There is also a low peak at the hour between 13 o'clock and 14 o'clock.

A.2 Amount of fund transfers of individual banks

We observed the distributions related to the transactions executed by individual financial institutions. For each bank, (a) the total amount of the funds transfer, (b) the total amount of the payment to other banks, and (c) the total amount of the receipt from other banks were calculated, respectively. The distributions are presented in Fig.14.
In distribution (a), we see that the total amount of transactions distributes widely between 100 million yen and 100 trillion yen. The distribution (a) has a characteristic form with the distinctive kink at little short of 10^{14} yen and the vague kinks at 10^{12} and 10^{9} yen. The distributions (b) and (c) have a nearly identical function form with the same characteristics of distribution (a). This does not mean, however, the balance of each financial institution is zero.

We present the distribution of balances in Fig.15. The financial institutions are classified into two groups of positive and negative balances, and the cumulative distributions of the balances for both groups are presented in the figure. The balances distribute between 100 million yen and 10 trillion yen for both the groups.
The maximum value 10 trillion yen is smaller in one order of decade than the maximum value of the total amount of the transactions 100 trillion yen.
A.3 Number of transactions of individual banks

Figure 16 is the cumulative distributions of the number of transactions executed by individual banks. The graph (a) is the distribution of the total number of transactions regardless of the payment or the receipt. The distribution (b) is the distribution of the number of payment, and (c) is that of receipt. In graph (a), the number of transactions ranges from one to a little over 10,000.
Like in the case of the distributions of the amount of transactions, the forms of the distributions (b) and (c) are similar to that of the distribution (a).

B. Classification of banks into groups

We consider a system composed of N banks and classify the banks into groups.

First, we define a distance between two banks. We assume that the number of the transactions between the i th bank and the j th bank $\overline{n}_{ij}$ is theoretically determined by the total numbers of the transactions of the two banks m_i and m_j. One of the simplest definitions of $\overline{n}_{ij}$ is

$$\overline{n}_{ij} = m_i m_j$$

As in this case, when the interaction between two elements is determined by the product of the magnitude of the elements, it is called a mean-field interaction. A generalization of Eq.(11)

$$\overline{n}_{ij} = m_i^{\alpha} m_j^{\alpha}$$

where α is a positive constant, is also a mean-field interaction. By considering the case when there are only two banks in the system, the constant α should be 1/2 because $m_1 = m_2 = \overline{n}_{ij}$.

On the banking network, the actual number of the transactions n_{ij} may not necessarily be proportional to $\overline{n}_{ij}$. We define the distance between the i th bank and the j th bank by

$$l_{ij} = \overline{n}_{ij} / n_{ij}$$

The reciprocal of l_{ij} is the actual number of the transactions normalized by the theoretical number. It is interpreted as the intimacy between the two banks.

Distance between two banks without direct transactions is defined as follows. Though the two banks are not connected by a link, the two belong to the banking network. By tracing the links from one bank to the other, the distance between them is defined by the sum of the length of the links along the path. When there are more than one path between the two banks, the total length of the shortest one defines the distance between the banks.

The banks are sorted by the order of the number of links. And the top N_C banks are

chosen to be the cores of the groups.
We calculate

$$N_G = \sum_{i=1}^{N_C} n_a(d)$$

where $n_a(d)$ is the number of the banks within a radius d from a core a. We define the radius of groups r_G by the distance d that fulfills $N_G = 2N$. The banks are classified into groups so that a bank belongs to the group a when the distance of the bank from the core a is less or equal to r_G.

In the above process there are cases where the bank is placed within r_G from both a core a a and a core b. In other words, the process allows overlaps of groups. The overlaps describe the relation among groups.

The overlap of groups a and b is described by

$$S_{ab} = \sum_{i=1}^{N} \exp\left[-\left(\frac{d_{ai} + d_{bi}}{N_G} \right)^2 \right]$$

S_{ab} is large when a large number of banks belong to both the groups a and b simultaneously. Especially, S_{aa} represents the size of the group a because it depends only on the number of the banks in the group. The independence of a group is related to the size of the group and the overlap with the other groups. We introduce a measure of independence by

$$M_a = S_{aa} - \sum_{b \neq a} S_{ab}$$

This is the effect of the group a itself subtracted by the effects of the overlaps.

References

1:R. N. Mantegna and H. E. Stanley, An introduction to econophysics: correlation and complexity in finance (Cambridge Univ. Press, 1999).

2:Empirical science of financial fluctuations: The advent of econophysics, edited by H. Takayasu (Springer Verlag, 2002).

3:H. E. Stanley, Introduction to Phase Transitions and Critical Phenomena (Oxford University Press, New York, 1987).

4:M. Faloutsos, P. Faloutsos, and C. Faloutsos, Comput. Commun. Rev. **29**, 251 (1999).

5:R. Albert, H. Jeong, and A.-L. Barabási, Nature (London) **401**, 130 (1999).

6:A.-L. Barabási and R. Albert, Sience **286**, 509 (1999).

7:B. B. Mandelbrot, The Fractal Geometry of Nature (Freeman, San Francisco, 1982).

8:R. Albert, H. Jeong, and A.-L. Barabási, Nature **406**, 378 (2000).

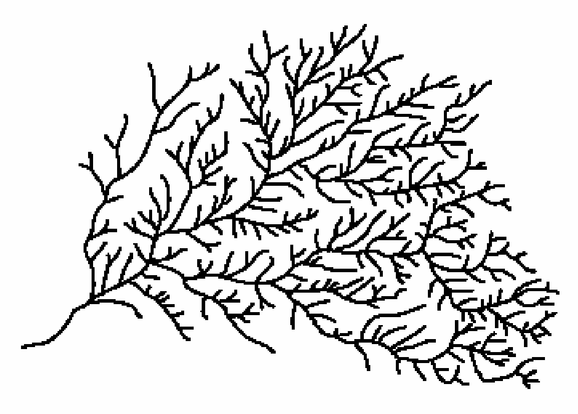

Figure 1. River networks

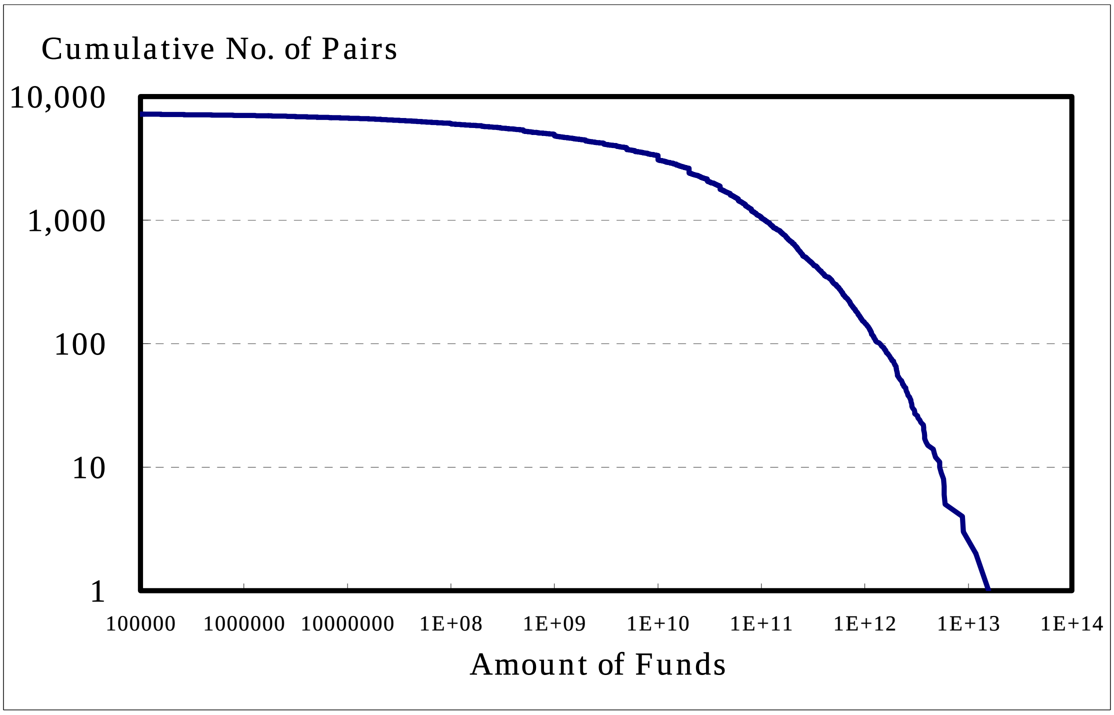

Figure 2. Cumulative distribution of the total amount of funds transferred between 7,351 pairs of financial institutions. X-axis shows the amount of funds. Y-axis shows the cumulative number of pairs.

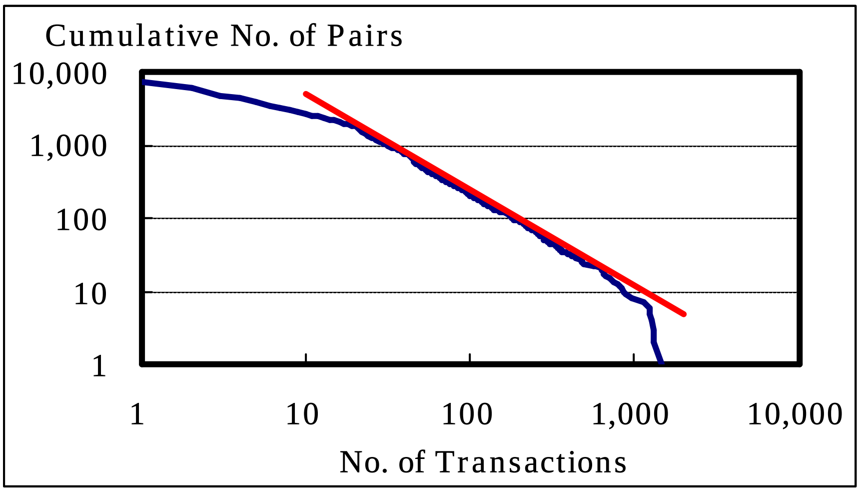

Figure 3. Cumulative distribution of the number of transactions between 7,351 pairs of financial institutions. X-axis shows the number of transactions. Y-axis shows the cumulative number of pairs. The red line is $N(\geq n) \propto n^{-1.3}$.

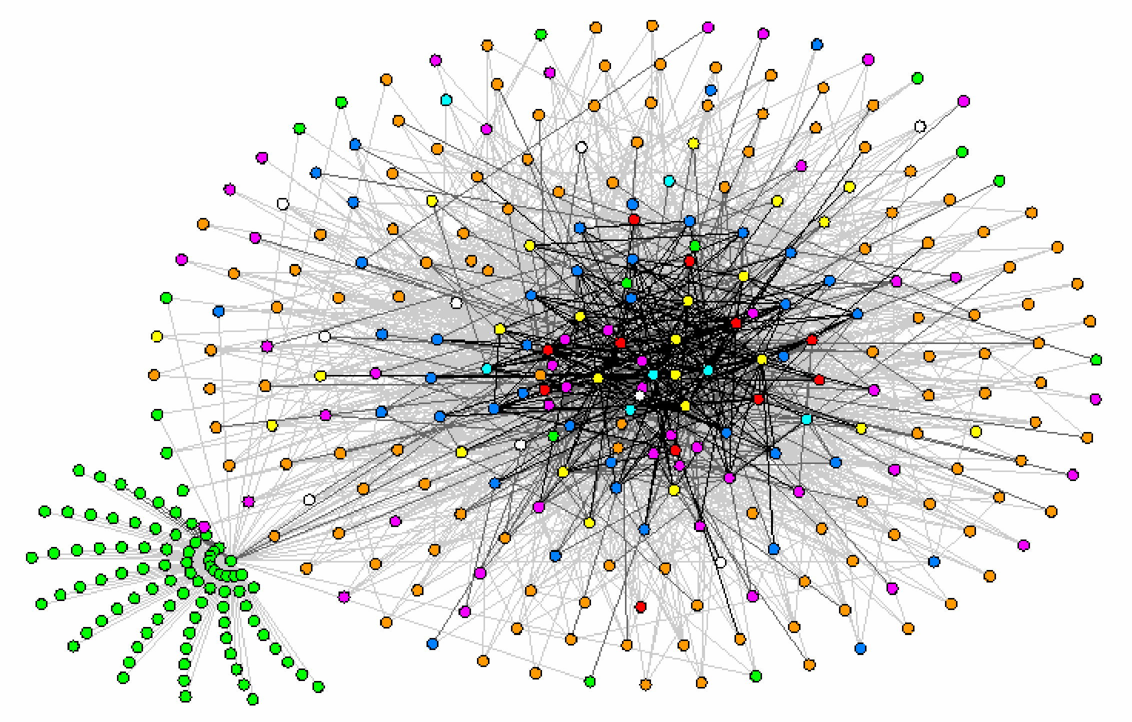

Figure 4. Banking network The bold lines(concentrated in the middle) illustrate the frequency of transactions between pairs.The colored dots represent the following financial institutions. Red: City banks, Pink: Local banks, Yellow: Trust banks, Lime: Shinkin banks, Aqua: Tanshi, Security finance companies, Purple: Foreign banks, Blue: Security firms, White: Others

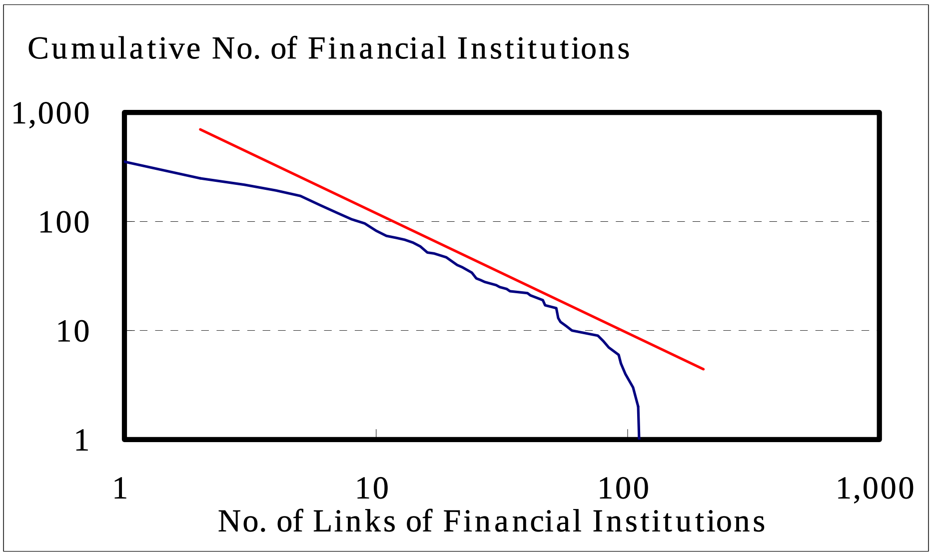

Figure 5. Cumulative distribution of the number of the financial institutions. X-axis shows number of links of financial institutions. Y-axis shows the cumulative number of financial institutions. The red line is $N(\geq n) \propto n^{-1.1}$.

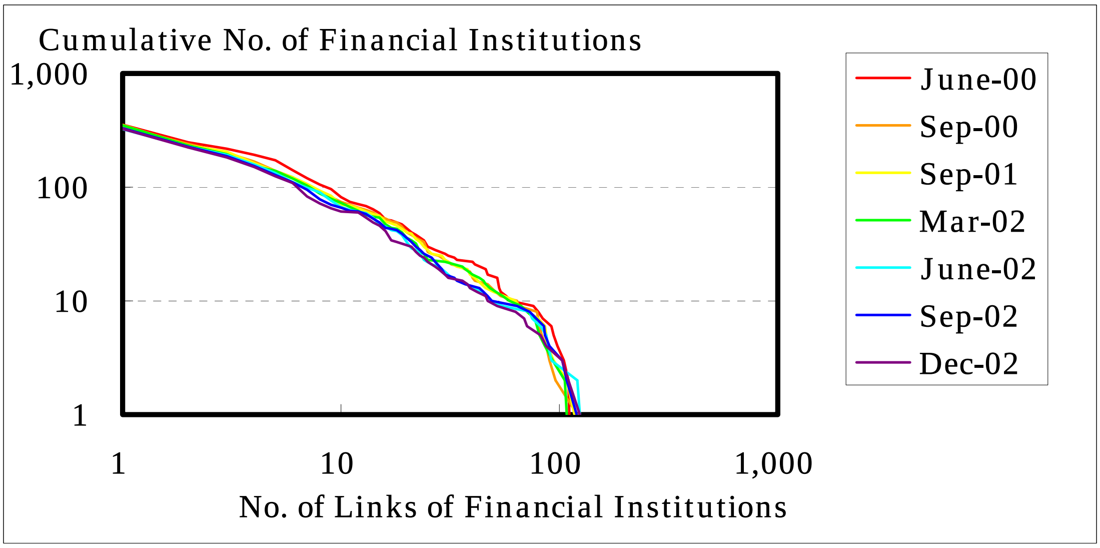

Figure 6. Cumulative distribution of the number of the financial institutions observed in a one-month period. X-axis shows the number of links of financial institutions. Y-axis shows the cumulative number of financial institutions.

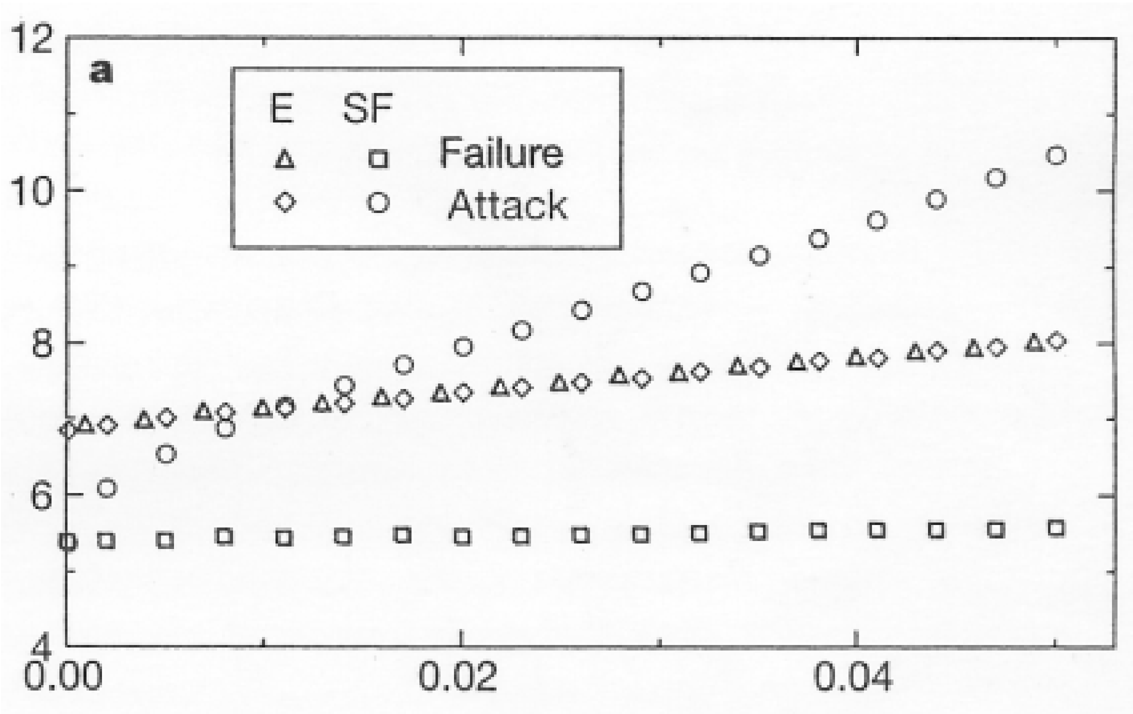

Figure 7. Effect of the removal of nodes for the two types of network. X-axis shows the share of nodes destroyed. Y-axis shows the radius of the network. E: Erdös's network, SF: Barabási's network. Source: reference No.8.

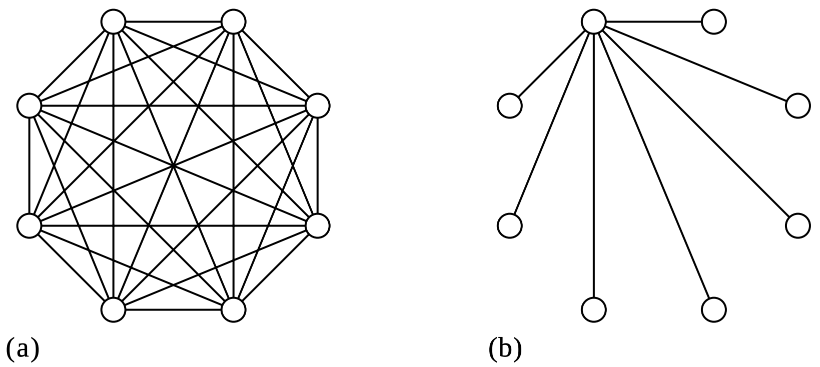

Figure 8. Simplified pattern of network structure (a) Decentralized network. (b) Centralized network.

Banking network	τ =1.1	(Estimation based on this analysis)
Co-acting relationship network	=1.3	
Web-site network	=1.1	
Internet network	=1.5	
Barabási network	=2.0	

Table 1. Degree distributions τ of various scale-free networks

Source: Reference No. 4,5,6

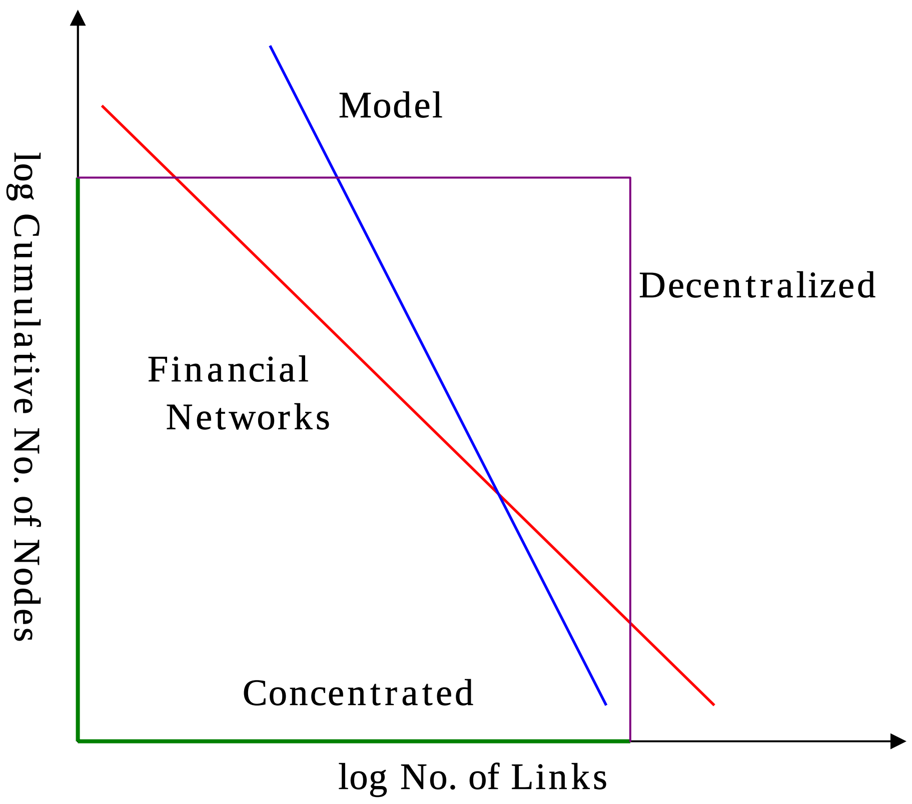

Figure 9. Degree distributions τ of the simplified networks

Appendix A

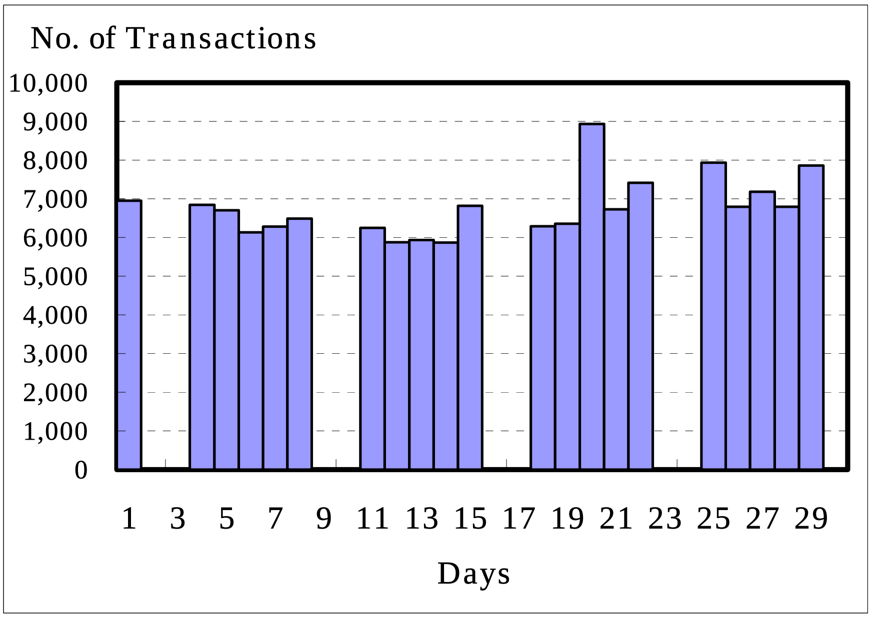

Figure 10. Number of daily transactions

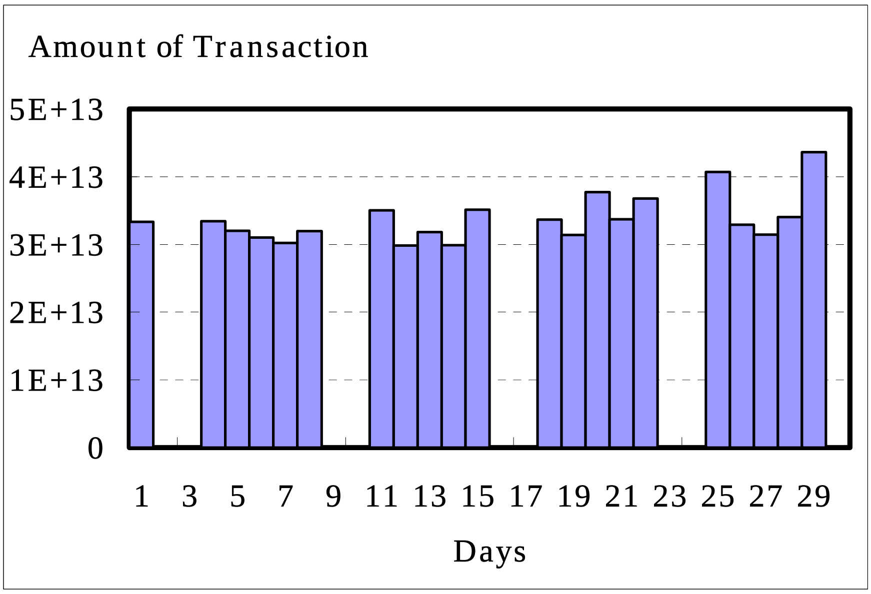

Figure 11. Amount of daily transactions

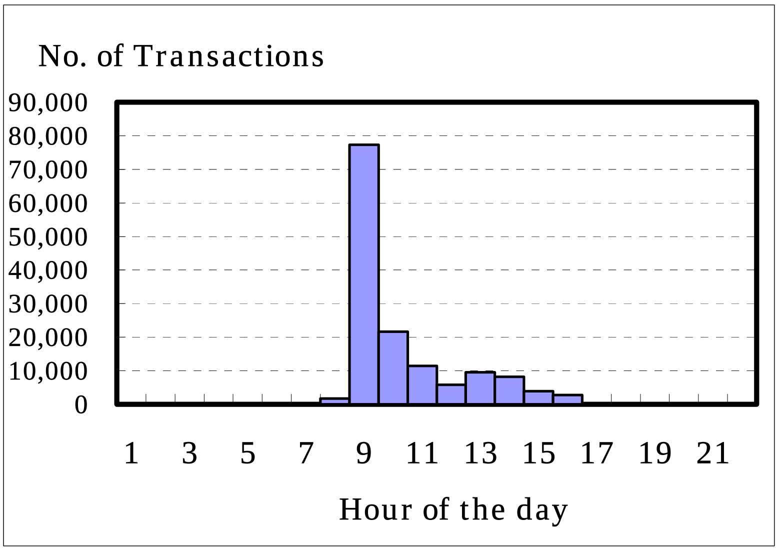

Figure 12. Number of transactions in each hour

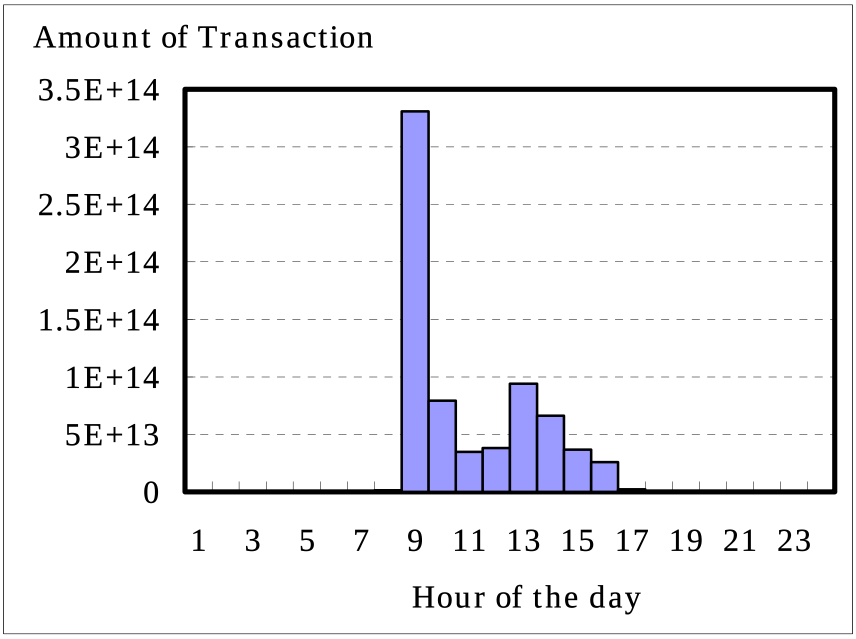

Figure 13. Amount of transactions in each hour

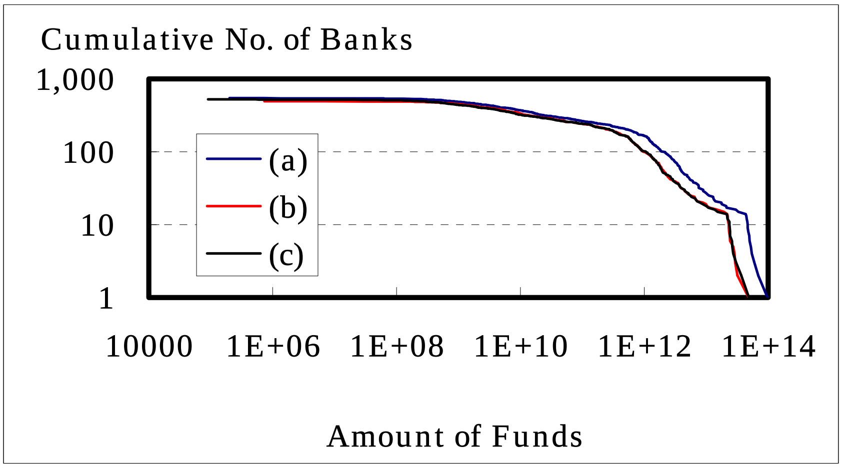

Figure 14. Distribution of amount of funds transfer by individual banks. X-axis is the amount of funds. Y-axis is the cumulative number of banks. (a) Total amount, (b) Payment amount, (c) Receipt amount .

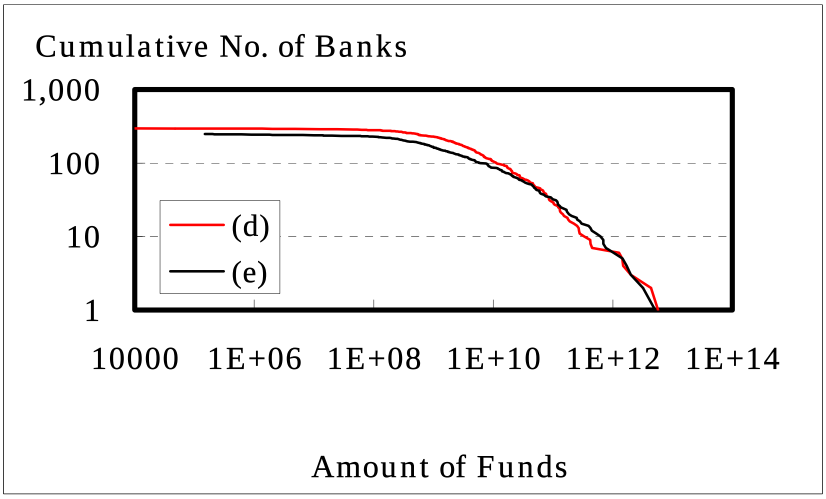

Figure 15. Distribution of balances of individual banks. X-axis is the amount of funds. Y-axis is the cumulative number of banks. (d) Net payment amount, (e) Net receipt amount .

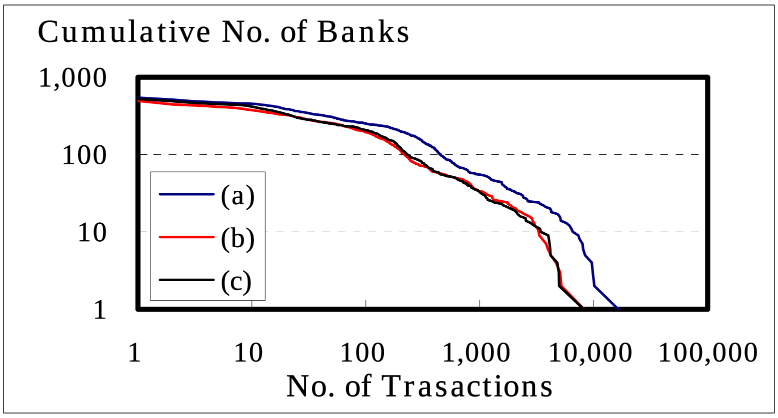

Figure 16. Distribution of number of transactions executed by individual banks. X-axis is the number of transactions. Y-axis is the cumulative number of banks. (a) Total number, (b) Payment transaction, (c) Receipt transaction.