Bank of Japan Working Paper Series

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Is Skewness Risk Priced in Japanese Corporate Bond Market?

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No.04-E-7
July 2004

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Is Skewness Risk Priced in Japanese Corporate Bond Market?

Shinichi Nishioka* and Naohiko Baba**

Abstract

This paper aims to analyze risk premium of corporate bonds considering skewness as an additional risk factor under a portfolio selection framework. With skewness, risk premium can be expressed as a weighted average of β-risk under the orthodox β-CAPM and γ-risk arising from skewness. We call the pricing model with γ-risk γ-CAPM. The weight between β-risk and γ-risk is determined mainly by the degree of relative risk aversion. Empirical results using Japanese data show that (i) specification tests tend to accept γ-CAPM, rejecting β-CAPM, and (ii) the estimated values of the degree of relative risk aversion are significantly positive on the whole, but become negative when BBB-rated corporate bonds are included in the sub-sample estimation, which covers the period after the adoption of the zero interest rate policy by the Bank of Japan. Also, empirical results using U.S. data show that (iii) the estimated values of the degree of relative risk aversion are much higher than the values estimated by Japanese data, and (iv) the average weight of γ-risk is 10.7 percent in the U.S., compared with 3.2 percent in Japan. This means that γ-risk is priced in U.S. corporate bonds to a larger degree than in the Japanese corporate bonds. These findings imply that Japanese investors have taken excessive credit risk, particularly in BBB-rated corporate bonds under the low interest rate environment.

Keywords: Corporate Bond, Portfolio Selection, Skewness, Co-Skewness, β-CAPM, γ-CAPM

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We benefited greatly from discussions with staff members of the Bank of Japan. We are also grateful to Mr. Yoichi Kita of Nikko Financial Intelligence for providing us with Japanese bond data. Any remaining errors are solely our responsibility. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of Japan.

1. Introduction

Preceding studies about corporate bond pricing generally start from the stylized fact that in the U.S., corporate bond spreads, defined as the difference between corporate bond and government bond yields with the same remaining maturities, are much wider than their corresponding historical default probabilities.[1] These studies usually employ a procedure that (i) estimates the expected default probabilities by methods proposed by Merton [1974], Jarrow, Lando, and Turnbull [1997] and so on, (ii) compares corporate bond spreads and expected default probabilities, and (iii) investigates the relationship between the above differences and various indexes representing market liquidity. In doing so, corporate bond spreads are analyzed independently. Also of interest is analyzing them under a portfolio selection framework, that is, from a risk-return profile of overall bond portfolios.

Corporate bonds differ from government bonds in that they entail default risk, where investors cannot completely recover principle, although they have the same property as fixed income securities. Default risk yields negative skewness, meaning that the distribution of corporate bond returns has a long-tail in the negative zone. Accordingly, unlike equity portfolios whose returns are expected to follow a log-normal distribution in normal circumstances, investors need to consider the risk arising from skewness as well as variance in constructing their bond portfolios. This paper analyzes risk premium of corporate bonds explicitly considering skewness under a portfolio selection framework[2].

After the adoption of the zero interest rate policy (ZIRP) by the Bank of Japan, Japanese bond investors first tried to keep investment profits by taking duration risk, that is, raising the portfolio weight of government bonds with longer maturities. Once a decline in government bond yields extended to the long-term maturity zone, however, some investors stepped up investment in credit risk, which significantly narrowed corporate bond spreads. Narrowing spreads even extended to BBB-rated corporate bonds. Despite favorable conditions for issuing companies, however, issue of corporate bonds have not increased so far. As suggested by Baba et al.[2004], one possible interpretation is that as credit spreads narrowed, investors were crowded out from the market in order of the degree of their risk aversion, lowering market liquidity.

On the other hand, recent studies about credit risk pricing have uncovered an

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[1] Amato and Remolona [2003] find that U.S. BBB-rated corporate bond spreads was about eight times the expected loss given default on average from 1998 to 2002, which they call "the credit spread puzzle." They argue that skewness in the distribution of corporate bond returns calls for an extraordinary large portfolio to achieve diversification, concluding that the spreads are wide because they compensate investors for skewness risk.

[2] Samuelson [1970] proved that the importance of higher moments than the variance is much smaller than that of the expected value and variance, if the distribution of an asset return has the "compactness", which may be viewed as being equivalent to continuity of an asset return. His proposition loses significance, however, when asset returns take sudden jumps in case of defaults of corporate bonds, for instance.

interesting fact: credit risk premium for the same companies can differ between domestic and overseas markets. For instance, CDS (credit default swap) premiums are generally wider than corporate bond spreads for the same Japanese companies. Sugihara et al. [2003] attribute this observation to the fact that overseas investors, main players in the CDS market, evaluate credit risk more rigorously than domestic investors, main players in the domestic corporate bond market. Also, Nishioka and Baba [2004] find that the negative yen funding costs for foreign banks in the foreign exchange swap market stem from the fact that participants in the overseas (dollar) market evaluate the creditworthiness of domestic banks more rigorously than those in the domestic (yen) market. To directly investigate such differences in credit risk evaluation between domestic and overseas markets, we also conduct the same analysis using U.S. data.

The rest of this paper is organized as follows. Section 2 describes the representative investor's portfolio selection problem when considering skewness as an additional risk factor. Section 3 derives the general form of asset pricing model with skewness risk. Section 4 empirically analyzes the asset pricing model with skewness risk using both Japanese and U.S. data. Section 5 summarizes the main findings.

2. Portfolio Selection with Skewness as an Additional Risk Factor

2.1 Review of Skewness and Co-Skewness

Skewness of a stochastic variable, r_i, is defined as the following standardized third moment:

$$skew_i \equiv \frac{E\left[r_i - E[r_i]\right]^3}{\sigma_i^3}. \quad (1)$$

By definition, skewness of a normally-distributed return is zero. Positive/negative skewness indicates a long-tail in the right/left-hand zone of the distribution. Suppose two distributions with identical expected values, say zero for simplicity, and variances. But, they differ in their skewness, one with positive and the other with negative skewness. A stochastic variable with positive/negative skewness has a high probability of a small loss/gain and a low probability of a large gain/loss. The distribution of corporate bond returns is expected to have negative skewness, since corporate bond investors cannot expect large gains due to upper limits on prices and at the same time, they face a low, but non-zero probabilities of large losses in case of default.

Next, co-skewness between returns on the assets i and j is defined as

$$co-skew_{ijj} \equiv E[(r_i - E[r_i])(r_j - E[r_j])^2] = \mathrm{cov}\left(r_i, \sigma_j^2\right). \quad (2)$$

Each co-skewness plays an important role in computing an overall portfolio's skewness. Suppose the portfolio is composed of two assets A and B. Then, the third central moment of the portfolio return can be written as follows:

$$E\left[r_p - E\left[r_p\right]\right]^3$$
$$= E\left[w_A\left(r_A - E\left[r_A\right]\right) + w_B\left(r_B - E\left[r_B\right]\right)\right]^3 \qquad (3)$$
$$= w_A^3\sigma_A^3 skew_A + w_B^3\sigma_B^3 skew_B + 3w_A^2 w_B \operatorname{cov}\left(\sigma_A^2, r_B\right) + 3w_B^2 w_A \operatorname{cov}\left(r_A, \sigma_B^2\right),$$

where r_p, r_A, and r_B denote returns on the portfolio and assets A and B, respectively, and w_A and w_B denote capitalization weights. Equation (3) shows that the third central moment of the portfolio return consists of co-skewness between assets A and B as well as each asset's skewness. A decline in co-skewness decreases the third central moment of the portfolio return, as long as w_A and w_B are positive.[3]

2.2 Portfolio Selection with Skewness Risk

Before deriving the general form of the asset pricing model with skewness risk factor, let us briefly describe a representative investor's optimal portfolio selection in consideration of skewness risk. We assume the model has a risk-free asset and two risky assets, A (corporate bond with skewness) and B (government bond without skewness).[4] Total asset holdings of the investor at $t+1$, W_{t+1}, and portfolio return, r_p, can be written as follows:

$$W_{t+1} = \left(1 + r_p\right)W_t, \qquad (4)$$

$$r_p = r_f + w_A\left(r_A - r_f\right) + w_B\left(r_B - r_f\right), \qquad (5)$$

where r_f denotes the risk-free interest rate, r_A and r_B denote returns on assets A and B, and w_A and w_B denote capitalization weights of assets A and B. Letting $u(W_{t+1})$ be the investor's utility function enables us to write the optimization problem of the investor's expected utility as follows:

$$\max_{w_A, w_B} \quad E_t\left[u\left(W_{t+1}\right)\right]$$
s.t. equations (4) and (5).

The first-order conditions can be written as

$$E_t\left[u'\left(W_{t+1}\right)\left(r_{i,t+1} - r_f\right)\right] = 0. \qquad i = A, B \qquad (6)$$

Now, we specify the investor's expected utility function as follows:

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[3] In markets with many heterogeneous investors, some investors might take short positions in asset i ($w_i < 0$). In such cases, positive co-skewness reduces the portfolio's return's skewness. In aggregation of investors' positions, however, all of the capitalization weights must be positive.

[4] We do not consider equities in portfolio selection. Japanese investors such as major banks, life insurance companies, and pension funds, have decreased their equity holdings virtually with no regard to risk-return profiles in recent years. This is because (i) the government set up a law requiring the banks to reduce equity holdings below their Tier 1 capital (Shareholdings Restriction Law), and (ii) life insurance companies and pension funds have sold equities since the bursting of the IT bubble as part of management policy.

$$E_t[u(W_{t+1})] \equiv E_t[W_{t+1}] - \frac{\lambda}{2} E_t[W_{t+1} - E_t[W_{t+1}]]^2 + \frac{\delta}{3} E_t[W_{t+1} - E_t[W_{t+1}]]^3, \tag{7}$$

where λ and δ denote the weights of each central moment. Equation (7) shows that in addition to each asset's mean and variance, the third central moment influences an investor's utility. The larger the portfolio's third central moment is, the higher the utility. The third central moment in equation (7) can be expanded as follows:

$$\begin{aligned} E_t[W_{t+1} - E_t[W_{t+1}]]^3 = \big(& w_A^3 \sigma_A^3 skew_A + 3w_A^2 w_B \operatorname{cov}(\sigma_A^2, r_B) \\ & + 3w_B^2 w_A \operatorname{cov}(r_A, \sigma_B^2)\big) W_t^3, \end{aligned} \tag{8}$$

where σ_A and σ_B denote standard deviation of assets A and B, $skew_A$ denotes skewness of asset A, and $\operatorname{cov}(\sigma_A^2, r_B)$ and $\operatorname{cov}(r_A, \sigma_B^2)$ denote co-skewness between assets A and B. We assume that the third central moment of asset B is zero, that is, $E_t[r_B - E_t[r_B]]^3 = 0$. Given equation (8), the first-order conditions (6) can be rewritten as

$$\begin{aligned} \frac{\partial u(W_{t+1})}{\partial w_A} = {} & E_t[r_A] - r_f - \lambda[w_A \sigma_A^2 + w_B \rho \sigma_A \sigma_B] \\ & + \delta[w_A^2 \sigma_A^3 skew_A + 2w_A w_B \operatorname{cov}(\sigma_A^2, r_B) + w_B^2 \operatorname{cov}(r_A, \sigma_B^2)] = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{\partial u(W_{t+1})}{\partial w_B} = {} & E_t[r_B] - r_f - \lambda[w_B \sigma_B^2 + w_B \rho \sigma_A \sigma_B] \\ & + \delta[w_A^2 \operatorname{cov}(\sigma_A^2, r_B) + 2w_A w_B \operatorname{cov}(r_A, \sigma_B^2)] = 0. \end{aligned} \tag{10}$$

From equations (9) and (10), we can derive the following demand functions for both assets:

$$\begin{aligned} E_t[r_A] - r_f = {} & \lambda \rho \sigma_A \sigma_B w_B - \gamma w_B^2 \operatorname{cov}(r_A, \sigma_B^2) \\ & + [\lambda \sigma_A^2 - 2\gamma w_B \operatorname{cov}(\sigma_A^2, r_B)] w_A - \gamma \sigma_A^3 skew_A w_A^2, \end{aligned} \tag{11}$$

$$\begin{aligned} E_t[r_B] - r_f = {} & \lambda \rho \sigma_A \sigma_B w_A - \gamma w_A^2 \operatorname{cov}(\sigma_A^2, r_B) \\ & + [\lambda \sigma_B^2 - 2\gamma w_A \operatorname{cov}(r_A, \sigma_B^2)] w_B. \end{aligned} \tag{12}$$

Figures 1 and 2 illustrate the demand functions for corporate bonds (asset A) and government bonds (asset B) described by equations (11) and (12). To clarify the role of two types of co-skewness ($\operatorname{cov}(r_A, \sigma_B^2)$ and $\operatorname{cov}(\sigma_A^2, r_B)$) in bond pricing, let us explain the effect of each co-skewness separately. Figure 1 shows the demand functions when $\operatorname{cov}(\sigma_A^2, r_B) = 0$, while Figure 2 shows the demand functions when $\operatorname{cov}(r_A, \sigma_B^2) = 0$. In both figures, the dotted lines denote the demand functions for corporate bonds under the orthodox mean-variance approach, and the curved lines denote these under the extended approach with skewness risk. Risk premiums are determined at the intersection of supply and demand functions, with supply assumed to be constant for simplicity.

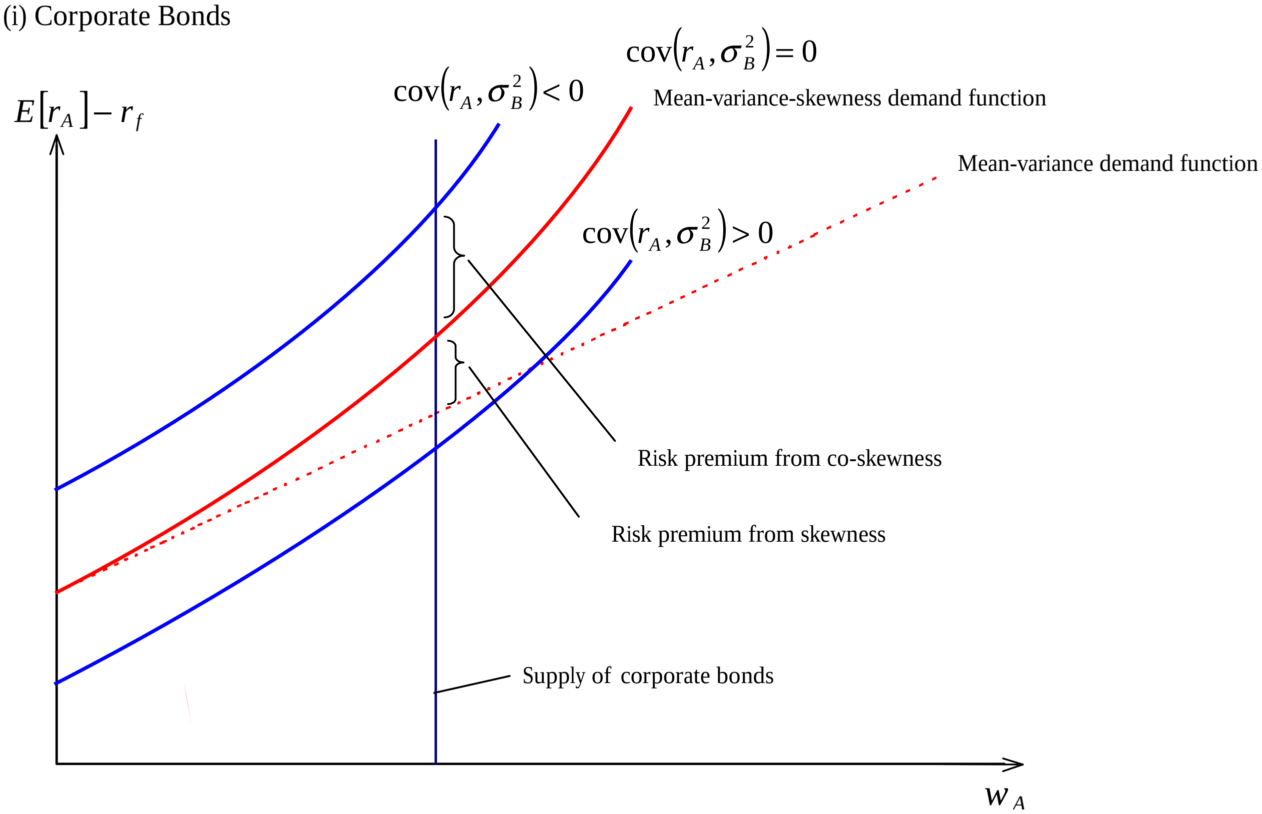
As shown in Figure 1, when $\operatorname{cov}(r_A, \sigma_B^2) < 0$, the demand function for corporate bonds shifts upward, while the slope of the demand function for government bonds becomes steeper than that of the mean-variance demand function. This means investors require larger risk premiums for negative co-skewness since, as shown by equation (8), negative co-skewness lowers a portfolio's return's skewness. On the other hand, as shown in Figure 2, when $\mathrm{cov}(\sigma_A^2, r_B) < 0$, the slope of the demand function for corporate bonds becomes steeper than that of the mean-variance demand function, while the demand function for government bonds shifts upward.

Intuitively speaking, $\mathrm{cov}(r_A, \sigma_B^2)$ ($\mathrm{cov}(\sigma_A^2, r_B)$) is the risk that investors cannot diversify away by lowering the weight of corporate (government) bond holdings, w_A (w_B), while $\mathrm{cov}(\sigma_A^2, r_B)$($\mathrm{cov}(r_A, \sigma_B^2)$) is the risk that they can completely diversify away by lowering w_A (w_B). $\mathrm{cov}(r_A, \sigma_B^2) < 0$ means the government bond volatility lowers the expected return on corporate bonds. The source of the risk is government bond volatility and investors are forced to bear this risk passively on the side of corporate bond holdings. Thus, the risk premium for corporate bonds is always equal to or larger than a certain value, irrespective of the level of w_A. On the other hand, $\mathrm{cov}(\sigma_A^2, r_B) < 0$ means the volatility of corporate bonds lowers the expected return on government bonds. In this case, the source of the risk is corporate bond volatility and investors are able to control the risk by changing w_A. Thus, the risk premium is shown by the slope of the demand curve for corporate bonds compared with the case where $\mathrm{cov}(\sigma_A^2, r_B) = 0$.

Note that we cannot decide *a priori* whether co-skewness has a positive or negative effect on risk premiums for corporate bonds in total. Table 2 shows that, in Japan, co-skewness tends to be negative, meaning co-skewness raises risk premiums for corporate bonds in total.

Figure 1: Demand Functions for Bonds when $\operatorname{cov}(\sigma_A^2, r_B) = 0$

(i) Corporate Bonds



(ii) Government Bonds

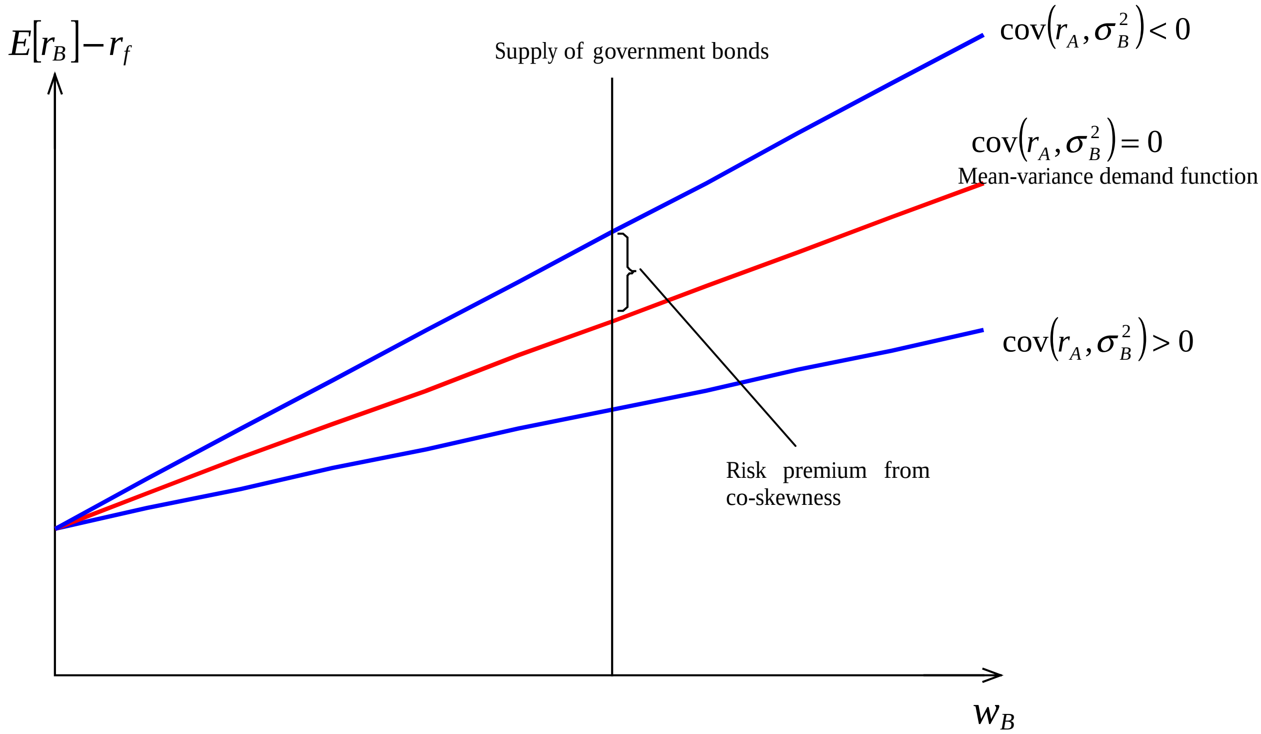
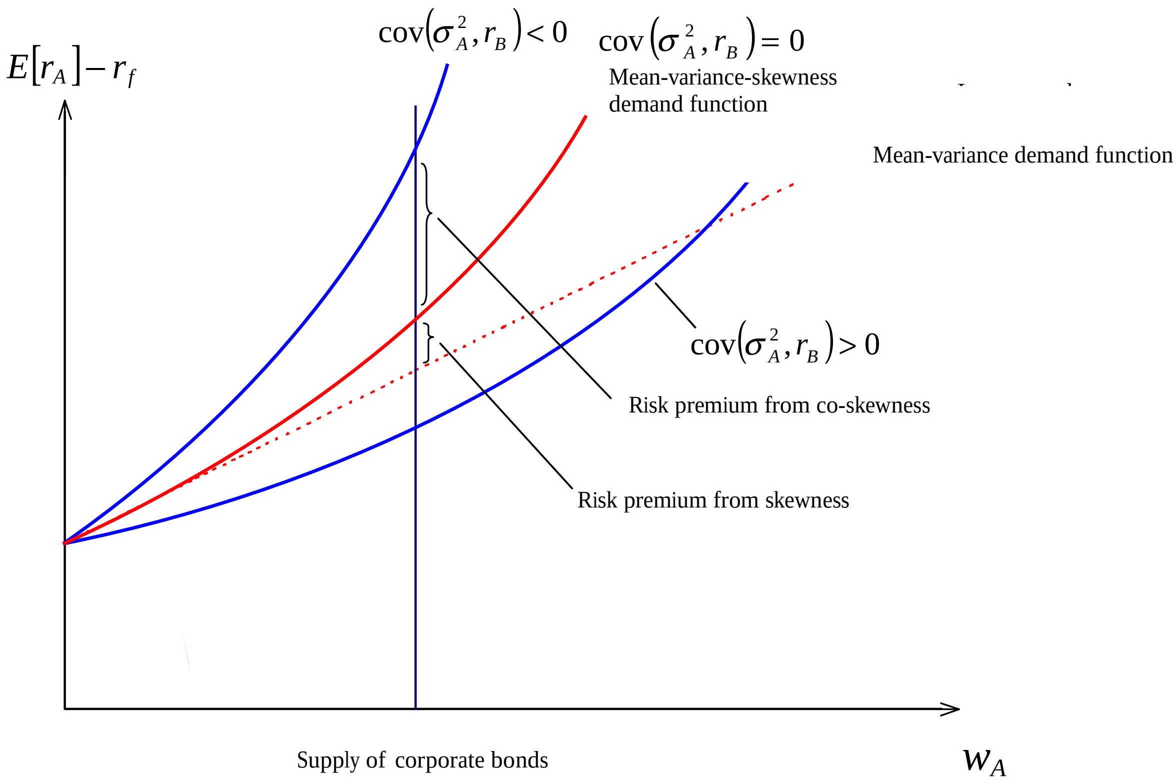
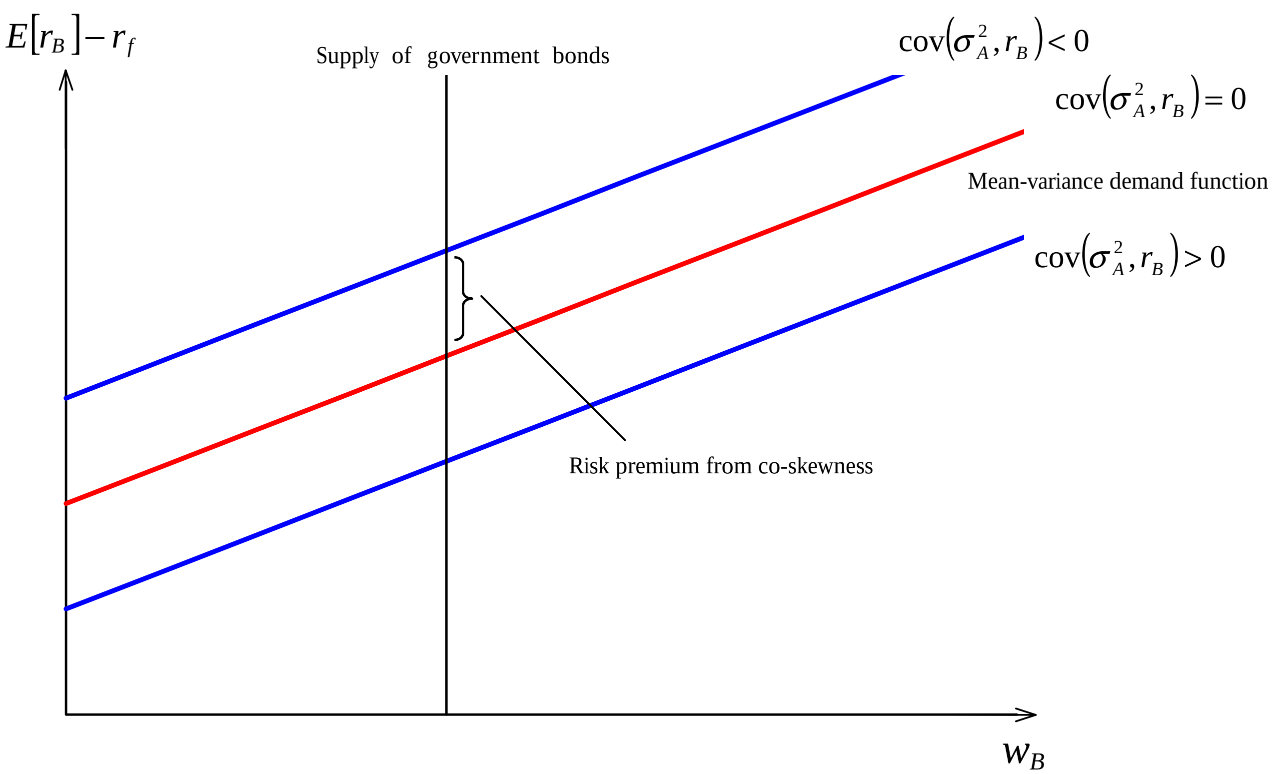


Figure 2: Demand Functions for Bonds when $\text{cov}(r_A, \sigma_B^2) = 0$

(i) Corporate Bonds



(ii) Government Bonds



3. Capital Asset Pricing Model with Skewness Risk

We derive the asset pricing model with skewness risk by generalizing the preceding simple portfolio selection model. We restate the first-order conditions for the optimization problem of the representative investor's expected utility function:

$$E_t[u'(W_{t+1})(r_{i,t+1}-r_f)]=0\,. \qquad i=1,\cdots,N \tag{6}$$

The Taylor expansion of equation (6) centered around $E_t[W_{t+1}]$ up to the second-order using $W_{t+1}=(1+r_{m,t+1})W_t$ yields

$$E_t[r_{i,t+1}]-r_f=-\frac{u''(E_t[W_{t+1}])W_t\,\mathrm{cov}(r_{i,t+1},r_{m,t+1})+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\,\mathrm{cov}(r_{i,t+1},\sigma_m^2)}{u'(E_t[W_{t+1}])+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\sigma_m^2}, \tag{13}$$

where $r_{m,t+1}$ denotes the market return, and σ_m^2 and $\mathrm{cov}(r_{i,t+1},\sigma_m^2)$ are variance of the market return defined as $E_t[r_{m,t+1}-E_t[r_{m,t+1}]]^2$ and co-skewness between the market return and the return on asset i defined as $E_t[(r_{i,t+1}-E_t[r_{i,t+1}])(r_{m,t+1}-E_t[r_{m,t+1}])^2]$, respectively. When asset i corresponds to the market portfolio, equation (13) can be written as

$$E_t[r_{m,t+1}]-r_f=-\frac{u''(E_t[W_{t+1}])W_t\sigma_m^2+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\sigma_m^3 skew_m}{u'(E_t[W_{t+1}])+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\sigma_m^2}, \tag{14}$$

where $skew_m$ denotes market return skewness. Using equations (13) and (14), we can derive the following asset pricing model:[5]

$$E_t[r_{i,t+1}]-r_f=\frac{\sigma_m^2\times\beta_{im}+\frac{1}{2}\kappa\sigma_m^3 skew_m\times\gamma_{im}}{\sigma_m^2+\frac{1}{2}\kappa\sigma_m^3 skew_m}\times\left(E_t[r_{m,t+1}]-r_f\right) \tag{15}$$

where $\beta_{im}\equiv\dfrac{\mathrm{cov}(r_{i,t+1},r_{m,t+1})}{\sigma_m^2}$,

$$\gamma_{im}\equiv\frac{\mathrm{cov}(r_{i,t+1},\sigma_m^2)}{E_t[r_{m,t+1}-E_t[r_{m,t+1}]]^3}=\frac{\mathrm{cov}(r_{i,t+1},\sigma_m^2)}{\sigma_m^3 skew_m}, \text{ and}$$

$$\kappa\equiv\frac{u'''(E_t[W_{t+1}])W_t}{u''(E_t[W_{t+1}])}.$$

β_{im} is referred to as "beta risk", which expresses the risk arising from covariance with the market return, as in the orthodox CAPM, while γ_{im} is referred to as "gamma risk," which expresses the risk arising from co-skewness with the market return. κ denotes the marginal rate of substitution between variance risk and skewness risk, which is negative due to the following property of utility function, $u'(\bullet)>0$, $u''(\bullet)<0$, and $u'''(\bullet)>0$.[6] This means that positive variance lowers an

[5] See Appendix A for more detailed derivation.

[6] These are sufficient conditions for (i) positive marginal utility, (ii) decreasing marginal utility, and (iii) no

investor's utility, while positive skewness increases it. We call equation (15) "γ-CAPM". Note that when $\kappa = 0$, equation (15) corresponds to the orthodox β-CAPM.

Now, let us specify the investor's utility function as the following constant relative risk aversion (CRRA) utility function:

$$u(W_{t+1}) = \frac{1}{1-\alpha} W_{t+1}^{1-\alpha} \quad (\alpha > 0), \tag{16}$$

where α denotes the degree of relative risk aversion. Then, using $\kappa = -(1+\alpha)$, we can rewrite equation (15) as

$$E_t[r_{i,t+1}] - r_f = [w \times \beta_{im} + (1-w) \times \gamma_{im}] \times (E_t[r_{m,t+1}] - r_f), \tag{17}$$

where $$w \equiv \frac{1}{1 - \frac{1}{2}(1+\alpha)\sigma_m skew_m}. \tag{18}$$

Negative values of $skew_m$ ensure $0 < w < 1$, meaning the risk premium of a risky asset can be expressed as a weighted average of β-risk and γ-risk.[7] Given the negative values of $skew_m$, when (i) the degree of relative risk aversion, α, becomes smaller (larger), (ii) volatility of the portfolio return, σ_m, declines (rises), and (iii) skewness of the market portfolio return, $skew_m$, declines (rises), β (γ)-risk weight, w (1- w), rises (declines).

4. Empirical Analysis

4.1 Model Specification

Before estimating equation (17), let us briefly review estimation methods of β-CAPM. The Sharpe-Lintner[8] type model directly estimates equation (17) imposing the constraint, $w = 1$, which typically uses the Treasury Bill (TB) rate as a proxy for a risk-free interest rate. This model assumes the existence of lending and borrowing at a risk-free interest rate. In reality, however, we have difficulty finding a theoretically-consistent proxy because (i) if investors face borrowing constraints, the estimated r_f must be the return on a zero-beta portfolio[9] that lies between the risk-free borrowing and lending rates, and (ii) actual short-term rates such as TB rates are stochastic and are correlated with returns on other financial assets.

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increase in the degree of absolute risk aversion ($\equiv -u''(\bullet)/u'(\bullet)$) with a rise in W, respectively.

[7] Kraus and Litzenberger [1976] and Hwang and Satchell [1999] derived the same pricing equation.

[8] For more details, see Sharpe [1964] and Lintner [1965]. Empirical tests of the Sharpe-Lintner CAPM explore the following three hypotheses: (i) the intercept is zero; (ii) β completely captures the cross-sectional variation of expected excess returns; and (iii) the market risk premium is positive. For a survey of empirical results, see Campbell, Lo, and MacKinlay [1997].

[9] A zero-beta portfolio is defined as a portfolio that has the minimum variance of all portfolios uncorrelated with the market portfolio, while the risk-free interest rate has zero variance by definition. Thus, the return on a portfolio with negative β is lower than the return on a zero-beta portfolio.

In the absence of a risk-free asset, Black [1972] provided a more general version of the CAPM:

$$E_t[r_{i,t+1}] = (1-\beta_{im})E_t[r_0] + \beta_{im}E_t[r_{m,t+1}], \tag{19}$$

where r_0 denotes the zero-beta return. This version assumes that $r_{i,t+1}$ is linearly related to β_{im} and includes a constant term. We apply this method to our model in the following form:

$$\begin{aligned} E_t[r_{i,t+1}] &= \left[1 - \frac{\beta_{im} - \frac{1}{2}(1+\alpha)\sigma_m skew_m \times \gamma_{im}}{1 - \frac{1}{2}(1+\alpha)\sigma_m skew_m}\right] \times r_0 + \frac{\beta_{im} - \frac{1}{2}(1+\alpha)\sigma_m skew_m \times \gamma_{im}}{1 - \frac{1}{2}(1+\alpha)\sigma_m skew_m} \times E_t[r_{m,t+1}] \\ &= const + \frac{\beta_{im} - \frac{1}{2}(1+\alpha)\sigma_m skew_m \times \gamma_{im}}{1 - \frac{1}{2}(1+\alpha)\sigma_m skew_m} \times E_t[r_{m,t+1}]. \end{aligned} \tag{20}$$

We attempt to estimate the parameters, α, β_{im}, and γ_{im} in equation (20). β-CAPM can be written as follows, imposing the restriction, $\alpha = -1$, on equation (20):

$$E_t[r_{i,t+1}] = const + \beta_{im} \times E_t[r_{m,t+1}]. \tag{21}$$

4.2 Estimation Method and Specification Tests

4.2.1 Generalized Method of Moments

We estimate equation (20) by the generalized method of moments (GMM) proposed by Hansen [1982]. GMM is a proper method for our analysis due to its distributional-free property, only requiring some orthogonal conditions. The orthogonal conditions of equation (20) can be written as follows:

$$E\left[r_{i,t+1} - const - \frac{\beta_{im} - \frac{1}{2}(1+\alpha)\sigma_m skew_m \times \gamma_{im}}{1 - \frac{1}{2}(1+\alpha)\sigma_m skew_m} \times r_{m,t+1} \middle| \mathbf{I_t} \right] = 0,$$

$$E[(r_{m,t+1} - \mu_m) r_{i,t+1} - \sigma_m^2 \times \beta_{im} | \mathbf{I_t}] = 0, \quad \text{and}$$

$$E[((r_{m,t+1} - \mu_m)^2 - \sigma_m^2) r_{i,t+1} - \sigma_m^3 skew_m \times \gamma_{im} | \mathbf{I_t}] = 0,$$

where i indicates the asset class of bonds, including government bonds and corporate bonds with credit ratings of AAA, AA, A, and BBB in the Japanese case (AAA/AA, A, and BBB in the U.S. case), and μ_m denotes the expected return on the bond market portfolio. Also, $\mathbf{I_t}$ denotes the vector of information set, as of the beginning of t, referred to as instrumental variables. We use a constant term and the returns on each asset class as the instrumental variables.

4.2.2 Specification Tests

First, we conduct the test of over-identifying restrictions (OI test) proposed by Hansen [1982]. This is a test of a model's overall fit, based on the property that *J*-statistics, loss function of GMM multiplied by the number of the sample period, follows the chi-square distribution with degrees of freedom equal to the number of orthogonal conditions minus the number of estimated parameters. A significant *J*-statistics means the model specification is inappropriate.

The second test is the model selection test proposed by Newey and West [1987]. We set H_0: β-CAPM with a restriction, $\alpha = -1$, on γ-CAPM, and H_1: γ-CAPM without a restriction. Then, we compare the difference in *J*-statistics between β-CAPM and γ-CAPM using the same GMM weighing matrix (that of the unrestricted γ-CAPM), which follows a chi-square distribution with degree of freedom equal to the number of restrictions. A significant *J*-statistics means γ-CAPM is more appropriate than β-CAPM.

Also, the estimated value of the degree of relative risk aversion, α, plays a pivotal role in judging a model's theoretical validity. If H_0: $\alpha = -1$ is not rejected, we accept β-CAPM, while if the estimated value of α is significantly positive, γ-CAPM is judged to be more valid theoretically.

4.3 Data

We use Japanese and U.S. data to estimate equation (20). For Japanese data, we use the Nikko Performance Index with government bonds and corporate bonds with AAA, AA, A and BBB ratings as individual asset classes. The sample period is from January 4, 1996 to April 6, 2004.

For the length of return period, many studies use daily returns since they are likely to satisfy stationarity, as required by GMM. In general, the longer the return period, the less stationary the return. It should be noted, however, that the ideal return period should be consistent with investors' time horizon. Most domestic institutional investors, such as life insurance companies and pension funds, have a time horizon of several years. Thus, we should keep the trade-off between stationarity and an investor's realistic time horizon in mind. Table 1 shows the results of unit root tests (Augmented Dickey-Fuller tests) on some return periods for Japanese and U.S. data: return periods of 20 (1 month), 60, 120, and 250 business days, respectively. All return periods except 250 business days satisfy stationarity. Therefore, we use return periods of 20, 60, and 120 business days in what follows.

Tables 2-1 and 2-2 show basic statistics[10]. The Japanese data suggests the possibility that the underlying assumption of mean-variance approach is not satisfied because the lower the credit ratings, the lower the sample means and the higher the sample variances. Also, the BBB corporate bond return has the smallest (negative) skewness. On the other hand, the risk of corporate bonds with low credit ratings is easier to be diversified away because the lower the credit ratings, the lower the correlation and the negative value of the co-skewness with the market

[10] We only report results for the return period of 60 business days. For other results, see Appendix B.

portfolio return. The U.S. data shows similar properties, except that the BBB corporate bond return has positive skewness.

Table 1: Augmented Dickey-Fuller Test

(i) Japanese Case

		r_m	r_{gov}	r_{aaa}	r_{aa}	r_a	r_{bbb}
20 business days	*t*-value	-5.330***	-5.310***	-5.557***	-5.269***	-4.818***	-3.986***
	Number of lags	20	20	20	20	20	20
60 business days	*t*-value	-5.104***	-5.100***	-5.298***	-4.056***	-4.969***	-4.794***
	Number of lags	14	14	15	5	16	16
120 business days	*t*-value	-3.402**	-3.321**	-3.525***	-3.320**	-3.293**	-3.408**
	Number of lags	11	11	11	10	16	20
250 business days	*t*-value	-2.295	-2.196	-2.888**	-2.097	-2.094	-2.041
	Number of lags	13	13	13	2	1	16

Notes: 1. r_i denotes the return on asset *i*. (m : bond market portfolio, gov : government bonds, aaa : AAA-rated corporate bonds, aa : AA-rated bonds a : A-rated bonds, bbb : BBB-rated bonds)
2. ***, **, and * show that the null hypothesis of a unit root is rejected significantly at the 1, 5, and 10 percent levels, respectively.
3. We chose the number of lags with the smallest AIC (Akaike Information Criteria) from 0 to 20 lags

(ii) U.S. Case

		r_m	r_{gov}	$r_{aaa\,/\,aa}$	r_a	r_{bbb}
20 business days	*t*-value	-6.959***	-6.766***	-7.021***	-7.095***	-7.023***
	Number of lags	20	20	20	20	20
60 business days	*t*-value	-4.592***	-4.731***	-4.620***	-4.701***	-5.067***
	Number of lags	7	3	7	7	1
120 business days	*t*-value	-3.947***	-3.660***	-3.829***	-3.972***	-3.838***
	Number of lags	1	1	0	0	0
250 business days	*t*-value	-2.750*	-2.671*	-2.747*	-2.876**	-2.968**
	Number of lags	4	3	3	3	3

Notes: 1. r_i denotes the return on the *i*-th asset. (m : bond market portfolio, gov : government bonds, aaa/aa : AAA/AA-rated corporate bonds, a : A-rated bonds, bbb : BBB-rated bonds)
2. ***, **, and * show that the null hypothesis of a unit root is rejected significantly at the 1, 5, and 10 percent levels, respectively.
3. We chose the number of lags with the smallest AIC (Akaike Information Criteria) from 0 to 20 lags

Table 2-1: Basic Statistics: Japanese Data (60 Business Days)

(i) Mean, Variance, and Skewness

	r_m	r_{gov}	r_{aaa}	r_{aa}	r_a	r_{bbb}
Mean	0.0328	0.0341	0.0472	0.0321	0.0300	0.0169
Variance	0.0040	0.0048	0.0083	0.0041	0.0041	0.0126
Skewness	-0.4300	-0.5415	-0.0255	-0.1706	-0.1685	-1.4826

(ii) Correlation Matrix

	r_m	r_{gov}	r_{aaa}	r_{aa}	r_a	r_{bbb}
r_m	1.000					
r_{gov}	0.994	1.000				
r_{aaa}	0.974	0.965	1.000			
r_{aa}	0.914	0.877	0.942	1.000		
r_a	0.818	0.766	0.852	0.958	1.000	
r_{bbb}	0.441	0.370	0.471	0.642	0.770	1.000

(iii) Co-skewness Matrix: $co-skew_{iij} \equiv E[(r_i - E[r_i])^2(r_j - E[r_j])] = \mathrm{cov}(\sigma_i^2, r_j)$

	r_m	r_{gov}	r_{aaa}	r_{aa}	r_a	r_{bbb}
σ_m^2	-0.00011	-0.00013	-0.00010	-0.00008	-0.00006	-0.00001
σ_{gov}^2	-0.00015	-0.00018	-0.00017	-0.00013	-0.00010	-0.00003
σ_{aaa}^2	-0.00008	-0.00012	-0.00002	-0.00003	-0.00003	0.00005
σ_{aa}^2	-0.00006	-0.00008	-0.00004	-0.00004	-0.00004	-0.00004
σ_a^2	-0.00005	-0.00006	-0.00005	-0.00004	-0.00004	-0.00010
σ_{bbb}^2	-0.00010	-0.00004	-0.00022	-0.00030	-0.00044	-0.00208

Note: The sample period is from January 4, 1996 to January 9, 2004. The number of observations is 1,974.

Table 2-2: Basic Statistics: U.S. Data (60 Business Days)

(i) Mean, Variance, and Skewness

	r_m	r_{gov}	r_{aaa}	r_a	r_{bbb}
Mean	0.0781	0.0775	0.0830	0.0873	0.0869
Variance	0.0063	0.0086	0.0086	0.0099	0.0125
Skewness	-0.1448	-0.1228	-0.1941	-0.1482	0.3307

(ii) Correlation Matrix

	r_m	r_{gov}	$r_{aaa/aa}$	r_a	r_{bbb}
r_m	1.0000				
r_{gov}	0.9712	1.0000			
$r_{aaa/aa}$	0.9871	0.9499	1.0000		
r_a	0.9638	0.9074	0.9861	1.0000	
r_{bbb}	0.7963	0.6719	0.8237	0.8827	1.0000

(iii) Co-skewness Matrix: $co-skew_{iij} \equiv E[(r_i - E[r_i])^2(r_j - E[r_j])] = \mathrm{cov}(\sigma_i^2, r_j)$

	r_m	r_{gov}	$r_{aaa/aa}$	r_a	r_{bbb}
σ_m^2	0.000072	-0.000097	-0.000098	-0.000107	-0.000076
σ_{gov}^2	-0.000111	-0.000097	-0.000143	-0.000174	-0.000255
$\sigma_{aaa/aa}^2$	-0.000126	-0.000158	-0.000156	-0.000163	-0.000109
σ_a^2	-0.000136	-0.000182	-0.000159	-0.000145	-0.000029
σ_{bbb}^2	0.000100	0.000077	0.000165	0.000245	0.000461

Note: The sample period is from January 1, 1996 to January 27, 2004. The number of observations is 2,349.

4.4 Estimation Results

4.4.1 Japanese Case

Table 3 reports estimation results for the Japanese case. We used the following two sample periods, (a) full sample: January 4, 1996 to April 6, 2004, and (b) sub-sample: April 1, 1999 to April 6, 2004, during which the Bank of Japan conducted the ZIRP and quantitative monetary easing policy. We also estimated the model with or without BBB corporate bonds.

First, both the OI test and the model selection test accept γ-CAPM, rejecting β-CAPM. Second, most estimated values of α are significantly positive, although when BBB corporate bonds are included in the sub-sample estimation, α is significantly negative. This result implies that Japanese investors have taken excessive credit risk particularly in BBB corporate bonds since the adoption of the ZIRP. In fact, most of the Japanese institutional investors, such as life insurance companies and pension funds, set internal limits on investment in low-credit bonds including BBB corporate bonds for their risk-management reasons. Thus, low-credit bondholders in Japan are almost limited to regional financial institutions and retail investors, who do not care much about risk-return profiles of financial assets.

4.4.2 U.S. Case

Table 4 reports estimation results for the U.S. case.[11] First, similar to the Japanese case, the OI test and the model selection test accept γ-CAPM, rejecting β-CAPM. Second, the estimated values of α are significantly positive and are higher when including BBB-rated corporate bonds than when excluding them. This result indicates that U.S. investors have a more cautious attitude towards credit risk than their Japanese investors. It is consistent with the findings by Amato and Remolona [2003].

[11] Table 4 shows the U.S. data satisfies the stationarity.

Table 3: GMM Estimation Results: Japanese Case (60 Business Days)

(i) Full Sample: January 4, 1996 to January 9, 2004, number of observations: 1,974

	Asset Class	Constant	β	γ	α	OI Test	Test of model selection
All Assets	Government Bonds	-0.0021*** (0.0000)	1.0993*** (0.0006)	1.1941*** (0.0051)	0.9302*** (0.0585)	32.8160	2.8593*
	Corporate Bonds						
	AAA	0.0012*** (0.0000)	1.4094*** (0.0019)	0.9672*** (0.0086)			
	AA	0.0019*** (0.0000)	0.9239*** (0.0015)	0.7696*** (0.0053)			
	A	0.0031*** (0.0000)	0.8287*** (0.0021)	0.5957*** (0.0056)			
	BBB	-0.0072*** (0.0000)	0.7632*** (0.0036)	0.1067*** (0.0073)			
Excluding BBB Corporate Bonds	Government Bonds	-0.0019*** (0.0000)	1.0951*** (0.0008)	1.2130*** (0.0062)	0.5691*** (0.0728)	32.7372	1,200.629***
	Corporate Bonds						
	AAA	0.0010*** (0.0000)	1.4097*** (0.0036)	0.9886*** (0.0111)			
	AA	0.0014*** (0.0000)	0.9343*** (0.0032)	0.7847*** (0.0071)			
	A	0.0022*** (0.0000)	0.8469*** (0.0041)	0.6130*** (0.0074)			

(ii) Sub-sample: April 1, 1999 to January 9, 2004, number of observations: 1,174

	Asset Class	Constant	β	γ	α	OI Test	Test of model selection
All Assets	Government Bonds	-0.0014*** (0.0000)	1.0584*** (0.0003)	1.2271*** (0.0034)	-1.8701*** (0.0509)	19.2212	12,466.27***
	Corporate Bonds						
	AAA	0.0060*** (0.0000)	1.1717*** (0.0009)	1.1674*** (0.0031)			
	AA	0.0034*** (0.0000)	0.9387*** (0.0005)	1.0937*** (0.0028)			
	A	0.0076*** (0.0000)	0.7468*** (0.0011)	0.7843*** (0.0018)			
	BBB	0.0051*** (0.0000)	0.8775*** (0.0017)	0.6766*** (0.0018)			
Excluding BBB Corporate Bonds	Government Bonds	-0.0014*** (0.0000)	1.0518*** (0.0002)	1.0841*** (0.0038)	1.5962*** (0.0949)	19.1880	3,641.063***
	Corporate Bonds						
	AAA	0.0063*** (0.0000)	1.1804*** (0.0015)	1.0204*** (0.0037)			
	AA	0.0036*** (0.0000)	0.9363*** (0.0010)	0.9774*** (0.0032)			
	A	0.0080*** (0.0000)	0.7544*** (0.0019)	0.7139*** (0.0023)			

Notes: 1. Figures in parentheses show standard deviations. ***, ** and * denote statistical significance at the 1, 5, and 10 percent levels, respectively.

2. The heteroskedasticity and serial correlation of error terms are corrected by the Newey and West [1987] method. The number of error term lags is assumed to be 60.

3. The OI test shows *J*- statistics proposed by Hansen [1982]. The degree of freedom is 89 with all assets, and 71 without BBB corporate bonds.

4. The test of model selection shows the difference in *J*-statistics between β -CAPM and γ -CAPM. The difference follows a chi-square distribution with one degree of freedom one.

Table 4: GMM Estimation Results: U.S. Case (60 Business Days)

Full Sample: January 2. 1995 to January 27, 2004, number of observations: 2,349

	Asset Class	Constant	β	γ	α	OI Test	Test of Model Selection
All Assets	Government Bonds	-0.0123*** (0.0003)	1.1158*** (0.0392)	1.4356*** (0.2526)	23.9938*** (2.2256)	37.3568	9,782.544***
	Corporate Bonds						
	AAA/AA	-0.0079*** (0.0003)	1.1303*** (0.0397)	1.3824*** (0.2637)			
	A	-0.0080*** (0.0003)	1.1743*** (0.0412)	1.4650*** (0.2730)			
	BBB	0.0012** (0.0005)	1.0826*** (0.0383)	0.9608*** (0.2790)			
Excluding BBB Corporate Bonds	Government Bonds	-0.0120*** (0.0005)	1.1027*** (0.0232)	1.6784*** (0.2290)	13.2465*** (2.4149)	36.3245	463.5407***
	Corporate Bonds						
	AAA/AA	-0.0082*** (0.0005)	1.1208*** (0.0232)	1.6914*** (0.2408)			
	A	-0.0083*** (0.0006)	1.1675*** (0.0246)	1.8318*** (0.2473)			

Notes: 1. Figures in parentheses show standard deviations. ***, ** and * denote statistical significance at the 1, 5, and 10 percent levels, respectively.

2. The heteroskedasticity and serial correlation of error terms are corrected by the Newey and West [1987] method. The number of error term lags is assumed to be 60.

3. The OI test shows *J*-statistics proposed by Hansen [1982]. The degree of freedom is 59 with all assets, and 44 without BBB corporate bonds.

4. The test of model selection shows the difference in *J*-statistics between β-CAPM and γ-CAPM. The difference follows a chi-square distribution with one degree of freedom one.

4.5 Weight of β-Risk and γ-Risk in Corporate Bond Pricing

Table 5 shows the risk weights, w and $1-w$, in equation (17), implied by the above estimation results. In Japan, the average weight of γ-risk, $1-w$, is 3.2 percent, while in the United States, it is 10.7 percent. This means that the U.S. corporate bonds reflect much higher γ-risk than the Japanese corporate bonds. To be precise, although the γ-risk of Japanese corporate bonds is statistically significant, the weight of the γ-risk is almost negligible in its magnitude; Japanese investors seem only care about the β-risk. On the other hand, the U.S. investors care about the γ-risk as well as the β-risk.

Table 5: Risk Weight of β and γ Implied by Estimation Results

	Return Period	Sample Period	Assets	σ_m	$skew_m$	α	w : risk weight of β	$1-w$: risk weight of
Japan	20 business days	Full Sample: Jan. 4, 1996 to Mar.9, 2004	All Assets	0.104	-0.232	0.009	98.8%	1.2%
			Excluding BBB Corporate Bonds			0.802 ***	97.9%	2.1%
		Sub-sample: Apr. 1, 1999 to Mar. 9, 2004	All Assets	0.082	-0.643	-3.456 ***	109.8%	-9.8%
			Excluding BBB Corporate Bonds			-2.780 ***	106.9%	-6.9%
	60 business days	Full Sample: Jan. 4, 1996 to Jan. 9, 2004	All Assets	0.063	-0.430	0.930 ***	97.4%	2.6%
			Excluding BBB Corporate Bonds			0.569 ***	97.9%	2.1%
		Sub-sample: Apr. 1, 1999 to Jan. 9, 2004	All Assets	0.046	-1.012	-1.870 ***	102.2%	-2.2%
			Excluding BBB Corporate Bonds			1.596 ***	94.0%	6.0%
	120 business days	Full Sample: Jan. 4, 1996 to Oct. 8, 2003	All Assets	0.038	-0.098	1.422 ***	99.5%	0.5%
			Excluding BBB Corporate Bonds			1.520 ***	99.5%	0.5%
		Sub-sample: Apr. 1, 1999 to Oct. 8, 2003	All Assets	0.033	-0.441	3.495 ***	96.4%	3.6%
			Excluding BBB Corporate Bonds			10.050 ***	91.5%	8.5%
U.S.	20 business days	Jan. 2, 1995 to Mar. 23, 2004	All Assets	0.141	-0.183	6.451 ***	91.2%	8.8%
			Excluding BBB Corporate Bonds			7.220 ***	90.4%	9.6%
	60 business days	Jan. 2, 1995 to Jan. 27, 2004	All Assets	0.079	-0.145	23.994 ***	87.5%	12.5%
			Excluding BBB Corporate Bonds			13.247 ***	92.4%	7.6%
	120 business days	Jan. 2, 1995 to Oct. 31, 2003	All Assets	0.056	-0.207	41.994 ***	80.2%	19.8%
			Excluding BBB Corporate Bonds			10.093 ***	94.0%	6.0%

Notes: 1. The shadowed zone indicates that α is significantly positive.
2. *** denotes statistical significance at the 1 percent level.

5. Concluding Remarks

We conclude the paper by summarizing the main findings.

(i) If we consider skewness as a risk factor, then risk premium of corporate bonds can be expressed as a weighted average of β-risk under the orthodox β-CAPM and γ-risk arising from skewness, which we call γ-CAPM. The weight is mainly determined by the degree of relative risk aversion.

(ii) Empirical results using both Japanese and U.S. data show that specification tests tend to accept γ-CAPM, rejecting β-CAPM.

(iii) The estimated values of the degree of relative risk aversion are significantly positive on the whole, but become negative when BBB-rated corporate bonds are included in the sub-sample estimation, which covers the period after the adoption of the ZIRP in Japan. Also, empirical results using U.S. data show that the estimated values of the degree of relative risk aversion are much higher than in the values estimated by Japanese data.

(iv) The average weight of γ-risk is 3.2 percent in Japan, while it is 10.7 percent in the United States. This means that γ-risk is much more reflected in the U.S. corporate bonds than in Japanese corporate bonds. These results imply the possibility that Japanese investors have taken credit excessive risk particularly in BBB-rated corporate bonds since the ZIRP was adopted.

Appendix A: Detailed Derivation Process of γ-CAPM

In this Appendix, we show detailed derivation process of γ-CAPM. Approximating equation (6) by the Taylor expansion centered around $E_t[W_{t+1}]$ up to the second-order yields

$$\begin{aligned} u'(E_t[W_{t+1}])(E_t[r_{i,t+1}]-r_f)+u''(E_t[W_{t+1}])E_t[(r_{i,t+1}-r_f)(W_{t+1}-E_t[W_{t+1}])] \\ +\frac{1}{2}u'''(E_t[W_{t+1}])\{E_t[(r_{i,t+1}-E_t[r_{i,t+1}])(W_{t+1}-E_t[W_{t+1}])^2] \\ +(E_t[r_{i,t+1}]-r_f)E_t[W_{t+1}-E_t[W_{t+1}]]^2\}=0 \end{aligned}$$

$$\Leftrightarrow \quad E_t[r_{i,t+1}]-r_f=-\frac{u''(E_t[W_{t+1}])\operatorname{cov}(r_{i,t+1},W_{t+1})+\frac{1}{2}u'''(E_t[W_{t+1}])\operatorname{cov}(r_{i,t+1},\sigma_W^2)}{u'(E_t[W_{t+1}])+\frac{1}{2}u'''(E_t[W_{t+1}])\sigma_W^2},$$

where $\operatorname{cov}(r_{i,t+1},\sigma_W^2)\equiv E_t[(r_{i,t+1}-E_t[r_{i,t+1}])(W_{t+1}-E_t[W_{t+1}])^2]$. Here, letting $r_{m,t+1}$ be market portfolio return and using $W_{t+1}=(1+r_{m,t+1})W_t$ enables us to rewrite the above equation as follows:

$$E_t[r_{i,t+1}]-r_f=-\frac{u''(E_t[W_{t+1}])W_t\operatorname{cov}(r_{i,t+1},r_{m,t+1})+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\operatorname{cov}(r_{i,t+1},\sigma_m^2)}{u'(E_t[W_{t+1}])+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\sigma_m^2}.$$

We can also write

$$E_t[r_{m,t+1}]-r_f=-\frac{u''(E_t[W_{t+1}])W_t\sigma_m^2+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\sigma_m^3 skew_m}{u'(E_t[W_{t+1}])+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\sigma_m^2}.$$

Expanding this equation and rearranging yields

$$\begin{aligned} &E_t[r_{i,t+1}]-r_f \\ &=\frac{u''(E_t[W_{t+1}])W_t\operatorname{cov}(r_{i,t+1},r_{m,t+1})+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\operatorname{cov}(r_{i,t+1},\sigma_m^2)}{u''(E_t[W_{t+1}])W_t\sigma_m^2+\frac{1}{2}u'''(E_t[W_{t+1}])W_t^2\sigma_m^3 skew_m}\times(E_t[r_{m,t+1}]-r_f) \\ &=\frac{\operatorname{cov}(r_{i,t+1},r_{m,t+1})+\frac{1}{2}\frac{u'''(E_t[W_{t+1}])W_t}{u''(E_t[W_{t+1}])}\operatorname{cov}(r_{i,t+1},\sigma_m^2)}{\sigma_m^2+\frac{1}{2}\frac{u'''(E_t[W_{t+1}])W_t}{u''(E_t[W_{t+1}])}\sigma_m^3 skew_m}\times(E_t[r_{m,t+1}]-r_f) \\ &=\frac{\sigma_m^2\frac{\operatorname{Cov}(r_{i,t+1},r_{m,t+1})}{\sigma_m^2}+\frac{1}{2}\frac{u'''(E_t[W_{t+1}])W_t}{u''(E_t[W_{t+1}])}\sigma_m^3 skew_m\frac{\operatorname{cov}(r_{i,t+1},\sigma_m^2)}{\sigma_m^3 skew_m}}{\sigma_m^2+\frac{1}{2}\frac{u'''(E_t[W_{t+1}])W_t}{u''(E_t[W_{t+1}])}\sigma_m^3 skew_m}\times(E_t[r_{m,t+1}]-r_f) \\ &=\frac{\beta_{im}+\frac{1}{2}\frac{u'''(E_t[W_{t+1}])W_t}{u''(E_t[W_{t+1}])}\sigma_m skew_m\gamma_{im}}{1+\frac{1}{2}\frac{u'''(E_t[W_{t+1}])W_t}{u''(E_t[W_{t+1}])}\sigma_m skew_m}\times(E_t[r_{m,t+1}]-r_f). \end{aligned}$$

Appendix B: Statistics and Estimation Results in Case of 20 and 120 Business Days Returns

Table B-1 Basic Statistics: Japanese Data

(i) Mean, Variance and Skewness

		r_m	r_{gov}	r_{aaa}	r_{aa}	r_a	r_{bbb}
20 Business Days	Mean	0.0354	0.0373	0.0536	0.0345	0.0319	0.0222
	Variance	0.0108	0.0130	0.0231	0.0104	0.0091	0.0201
	Skewness	-0.2322	-0.1759	0.2084	0.0144	-0.0974	-0.9514
120 Business Days	Mean	0.0320	0.0330	0.0453	0.0315	0.0294	0.0128
	Variance	0.0015	0.0018	0.0028	0.0016	0.0018	0.0075
	Skewness	-0.0984	-0.0870	0.1683	-0.0354	0.0901	-1.2795

(ii) Correlation Matrix

		r_m	r_{gov}	r_{aaa}	r_{aa}	r_a	r_{bbb}
20 Business Days	r_m	1.0000					
	r_{gov}	0.9958	1.0000				
	r_{aaa}	0.9696	0.9665	1.0000			
	r_{aa}	0.9383	0.9160	0.9440	1.0000		
	r_a	0.8577	0.8250	0.8657	0.9544	1.0000	
	r_{bbb}	0.5516	0.5021	0.5593	0.6874	0.7885	1.0000
120 Business Days	r_m	1.0000					
	r_{gov}	0.9910	1.0000				
	r_{aaa}	0.9725	0.9560	1.0000			
	r_{aa}	0.8690	0.8071	0.9154	1.0000		
	r_a	0.7079	0.6277	0.7741	0.9420	1.0000	
	r_{bbb}	0.1883	0.0959	0.2539	0.5133	0.6871	1.0000

(iii) Co-skewness Matrix: $co-skew_{iij} \equiv E[(r_i - E[r_i])^2(r_j - E[r_j])] = \mathrm{cov}(\sigma_i^2, r_j)$

		r_m	r_{gov}	r_{aaa}	r_{aa}	r_a	r_{bbb}
20 Business Days	σ_m^2	-0.00026	-0.00027	-0.00013	-0.00018	-0.00017	-0.00010
	σ_{gov}^2	-0.00027	-0.00026	-0.00011	-0.00021	-0.00021	-0.00012
	σ_{aaa}^2	0.00015	0.00018	0.00073	0.00030	0.00019	0.00036
	σ_{aa}^2	-0.00008	-0.00010	0.00010	0.00002	-0.00002	0.00000
	σ_a^2	-0.00011	-0.00013	-0.00002	-0.00005	-0.00008	-0.00018
	σ_{bbb}^2	-0.00020	-0.00012	-0.00035	-0.00041	-0.00062	-0.00271
120 Business Days	σ_m^2	-0.000001	-0.000001	0.000000	-0.000001	0.000000	0.000001
	σ_{gov}^2	-0.000001	-0.000001	0.000000	-0.000001	0.000000	0.000001
	σ_{aaa}^2	0.000010	0.000011	0.000025	0.000000	0.000001	0.000001
	σ_{aa}^2	-0.000006	-0.000009	-0.000003	-0.000002	0.000003	0.000011
	σ_a^2	0.000001	-0.000001	0.000003	0.000005	0.000007	-0.000004
	σ_{bbb}^2	0.000027	0.000052	-0.000000	-0.000047	-0.000104	-0.000836

Note: For 20 business days, the sample period is from January 4, 1996 to March 9, 2004, and the number of observations is 2,014, while for 120 business days, the sample period is from January 4, 1996 to October 8, 2003, and the number of observations is 1,914.

Table B-2: Basic Statistics: U.S. Data

(i) Mean, Variance and Skewness

		r_m	r_{gov}	$r_{aaa/aa}$	r_a	r_{bbb}
20 Business Days	Mean	0.0858	0.0870	0.0933	0.0992	0.1019
	Variance	0.0200	0.0263	0.0279	0.0326	0.0420
	Skewness	-0.1827	-0.1593	-0.0927	-0.0369	0.2410
120 Business Days	Mean	0.0738	0.0728	0.0778	0.0815	0.0801
	Variance	0.0031	0.0042	0.0043	0.0047	0.0059
	Skewness	-0.2067	-0.2227	-0.2898	-0.2534	0.2429

(ii) Correlation Matrix

		r_m	r_{gov}	$r_{aaa/aa}$	r_a	r_{bbb}
20 Business Days	r_m	1.0000				
	r_{gov}	0.9747	1.0000			
	$r_{aaa/aa}$	0.9849	0.9570	1.0000		
	r_a	0.9670	0.9244	0.9871	1.0000	
	r_{bbb}	0.8621	0.7747	0.8936	0.9304	1.0000
120 Business Days	r_m	1.0000				
	r_{gov}	0.9734	1.0000			
	$r_{aaa/aa}$	0.9883	0.9555	1.0000		
	r_a	0.9649	0.9143	0.9861	1.0000	
	r_{bbb}	0.7506	0.6226	0.7685	0.8387	1.0000

(iii) Co-skewness Matrix: $co-skew_{iij} \equiv E[(r_i - E[r_i])^2 (r_j - E[r_j])] = \mathrm{cov}(\sigma_i^2, r_j)$

		r_m	r_{gov}	$r_{aaa/aa}$	r_a	r_{bbb}
20 Business Days	σ^2_m	-0.000516	-0.000637	-0.000548	-0.000566	-0.000407
	σ^2_{gov}	-0.000714	-0.000678	-0.000707	-0.000767	-0.000841
	$\sigma^2_{aaa/aa}$	-0.000529	-0.000605	-0.000432	-0.000420	-0.000107
	σ^2_a	-0.000495	-0.000570	-0.000352	-0.000217	0.000288
	σ^2_{bbb}	0.000417	0.000549	0.000884	0.001190	0.002075
120 Business Days	σ^2_m	-0.000035	-0.000046	-0.000048	-0.000052	-0.000032
	σ^2_{gov}	-0.000055	-0.000060	-0.000069	-0.000076	-0.000085
	$\sigma^2_{aaa/aa}$	-0.000063	-0.000076	-0.000081	-0.000085	-0.000059
	σ^2_a	-0.000070	-0.000085	-0.000086	-0.000082	-0.000037
	σ^2_{bbb}	0.000013	0.000001	0.000031	0.000061	0.000109

Note: For 20 business days, the sample period is from January 2, 1995 to March 23, 2004, and the number of observations is 2,389, while for 120 business days, the sample period is from January 2, 1995 to October 31, 2003, and the number of observations is 2,289.

Table B-3-1: GMM Estimation Results: Japanese Case (20 Business Days)

(i) Full Sample: January 4. 1996 to March 9, 2004, number of observations: 2,014

	Asset Class	Constant	β	γ	α	OI Test	Test of model selection
All Assets	Government Bonds	-0.0013*** (0.0001)	1.0898*** (0.0008)	1.3239*** (0.0155)	0.0092 (0.0872)	33.1765	501.0933***
	Corporate Bonds						
	AAA	0.0037*** (0.0002)	1.4143*** (0.0035)	0.8787*** (0.0243)			
	AA	0.0016*** (0.0001)	0.9259*** (0.0025)	0.9168*** (0.0138)			
	A	0.0033*** (0.0001)	0.7975*** (0.0036)	0.8483*** (0.0119)			
	BBB	-0.0065*** (0.0002)	0.7877*** (0.0058)	0.4998*** (0.0158)			
Excluding BBB Corporate Bonds	Government Bonds	-0.0013*** (0.0001)	1.0896*** (0.0012)	1.1311*** (0.0180)	0.8023*** (0.1167)	33.0177	1,074.319***
	Corporate Bonds						
	AAA	0.0037*** (0.0002)	1.4200*** (0.0048)	0.6525*** (0.0314)			
	AA	0.0017*** (0.0001)	0.9289*** (0.0030)	0.7431*** (0.0175)			
	A	0.0034*** (0.0002)	0.8025*** (0.0045)	0.7299*** (0.0158)			

(ii) Sub-sample: April 1, 1999 to March 9, 2004, number of observations: 1,214

	Asset Class	Constant	β	γ	α	OI Test	Test of model selection
All Assets	Government Bonds	-0.0017*** (0.0000)	1.0775*** (0.0010)	1.1814*** (0.0087)	-3.4555*** (0.0816)	19.7953	6,390.534***
	Corporate Bonds						
	AAA	0.0048*** (0.0000)	1.2587*** (0.0019)	1.1517*** (0.0086)			
	AA	0.0047*** (0.0000)	0.9136*** (0.0019)	0.9936*** (0.0073)			
	A	0.0100*** (0.0001)	0.6784*** (0.0029)	0.6860*** (0.0050)			
	BBB	0.0122*** (0.0001)	0.7264*** (0.0042)	0.6309*** (0.0048)			
Excluding BBB Corporate Bonds	Government Bonds	-0.0017*** (0.0000)	1.0654*** (0.0009)	1.0492*** (0.0092)	-2.7799*** (0.0969)	19.7571	908.6672***
	Corporate Bonds						
	AAA	0.0053*** (0.0001)	1.2570*** (0.0024)	0.9887*** (0.0088)			
	AA	0.0051*** (0.0000)	0.9065*** (0.0021)	0.8641*** (0.0077)			
	A	0.0108*** (0.0001)	0.6786*** (0.0037)	0.5941*** (0.0052)			

Notes: 1. Figures in parentheses show standard deviations. ***, ** and * denote statistical significance at the 1, 5, and 10 percent levels, respectively.
2. Heteroskedasticity and serial correlation of error terms are corrected by the method of Newey and West [1987]. The number of lags of error terms is assumed to be 60.
3. OI test shows *J*- statistics proposed by Hansen [1982]. The degree of freedom is 89 with all assets, and 71 without BBB corporate bonds.
4. Test of model selection shows the difference in *J*-statistics between β -CAPM and γ -CAPM. The difference follows chi-square distribution with one degree of freedom one.

Table B-3-2: GMM Estimation Results: Japanese Case (120 Business Days)

(i) Full Sample: January 1. 1996 to October 8, 2003, number of observations: 1,914

	Asset Class	Constant	β	γ	α	OI Test	Test of model selection
All Assets	Government Bonds	-0.0026*** (0.0000)	1.1147*** (0.0006)	0.9639*** (0.0399)	1.4220*** (0.0744)	31.9204	128.3855***
	Corporate Bonds						
	AAA	0.0023*** (0.0000)	1.3504*** (0.0010)	-0.2253*** (0.0618)			
	AA	0.0017*** (0.0000)	0.9303*** (0.0009)	0.9258*** (0.0403)			
	A	0.0039*** (0.0000)	0.8023*** (0.0014)	-0.3472*** (0.0437)			
	BBB	-0.0019*** (0.0000)	0.4864*** (0.0016)	-1.4784*** (0.0315)			
Excluding BBB Corporate Bonds	Government Bond	-0.0026*** (0.0000)	1.1136*** (0.0008)	0.8823*** (0.0482)	1.5198*** (0.0886)	31.8040	1,948.849***
	Corporate Bonds						
	AAA	0.0023*** (0.0000)	1.3503*** (0.0015)	-0.2991*** (0.0738)			
	AA	0.0017*** (0.0000)	0.9313*** (0.0011)	0.8523*** (0.0487)			
	A	0.0037*** (0.0000)	0.8069*** (0.0017)	-0.4140*** (0.0527)			

(ii) Sub-sample: April 1, 1999 to October 8, 2003, number of observations: 1,114

	Asset Class	Constant	β	γ	α	OI Test	Test of model selection
All Assets	Government Bonds	-0.0013*** (0.0000)	1.0507*** (0.0001)	1.1856*** (0.0018)	3.4952*** (0.0897)	18.2716	3,442.073***
	Corporate Bonds						
	AAA	0.0063*** (0.0000)	1.1545*** (0.0004)	1.0050*** (0.0023)			
	AA	0.0031*** (0.0000)	0.9302*** (0.0002)	0.9684*** (0.0016)			
	A	0.0073*** (0.0000)	0.7167*** (0.0005)	0.5751*** (0.0013)			
	BBB	0.0048*** (0.0000)	0.7609*** (0.0009)	-0.0631*** (0.0022)			
Excluding BBB Corporate Bonds	Government Bonds	-0.0013*** (0.0000)	1.0556*** (0.0003)	1.0605*** (0.0031)	10.0503*** (0.1676)	18.2237	21,583.20***
	Corporate Bonds						
	AAA	0.0065*** (0.0000)	1.1662*** (0.0007)	0.8679*** (0.0042)			
	AA	0.0033*** (0.0000)	0.9326*** (0.0004)	0.8699*** (0.0030)			
	A	0.0077*** (0.0000)	0.7215*** (0.0006)	0.5255*** (0.0024)			

Note : See the notes in Table B-3-1.

Table B-4-1: GMM Estimation Results: U.S. Case (20 Business Days)

January 2, 1995 to March 23, 2004, number of observations: 2,389

	Asset Class	Constant	β	γ	α	OI Test	Test of model selection
All Assets	Government Bonds	-0.0101*** (0.0005)	1.1012*** (0.0128)	1.4210*** (0.0914)			
	Corporate Bonds						
	AAA/AA	-0.0063*** (0.0005)	1.1495*** (0.0110)	1.2763*** (0.1117)	6.4511*** (0.6781)	36.6055	310.2246***
	A	-0.0063*** (0.0006)	1.2185*** (0.0114)	1.3210*** (0.1224)			
	BBB	-0.0029*** (0.0007)	1.2343*** (0.0141)	1.0499*** (0.1432)			
Excluding BBB Corporate Bonds	Government Bonds	-0.0099*** (0.0006)	1.0845*** (0.0199)	1.4784*** (0.1104)			
	Corporate Bonds						
	AAA/AA	-0.0064*** (0.0006)	1.1381*** (0.0166)	1.3723*** (0.1408)	7.2203*** (1.3776)	34.4777	239.9090***
	A	-0.0065*** (0.0007)	1.2082*** (0.0165)	1.4306*** (0.1562)			

Notes: 1. Figures in parentheses show standard deviations. ***, ** and * denote statistical significance at the 1, 5, and 10 percent levels, respectively.
2. Heteroskedasticity and serial correlation of error terms are corrected by the method of Newey and West [1987]. The number of lags of error terms is assumed to be 60.
3. OI test shows *J*- statistics proposed by Hansen [1982]. The degree of freedom is 59 with all assets, and 44 without BBB corporate bonds.
4. Test of model selection shows the difference in *J*-statistics between β -CAPM and γ -CAPM. The difference follows chi-square distribution with one degree of freedom one.

Table B-4-2: GMM Estimation Results: U.S. Case (120 Business Days)

January 2, 1995 to October 31, 2004, number of observations: 2,289

	Asset Class	Constant	β	γ	α	OI Test	Test of model selection
All Assets	Government Bonds	-0.0125*** (0.0001)	1.1265*** (0.0214)	1.2833*** (0.0840)			
	Corporate Bonds						
	AAA/AA	-0.0094*** (0.0001)	1.1536*** (0.0222)	1.2935*** (0.0889)	41.9939*** (1.3127)	37.7117	38,039.26***
	A	-0.0092*** (0.0001)	1.1850*** (0.0229)	1.3885*** (0.0918)			
	BBB	0.0078*** (0.0003)	1.0178*** (0.0210)	0.7606*** (0.0927)			
Excluding BBB Corporate Bonds	Government Bonds	-0.0112*** (0.0003)	1.1123*** (0.0131)	1.5638*** (0.1717)			
	Corporate Bonds						
	AAA/AA	-0.0087*** (0.0002)	1.1441*** (0.0126)	1.6336*** (0.1842)	10.0925*** (0.8967)	36.5172	960.7223***
	A	-0.0081*** (0.0002)	1.1780*** (0.0124)	1.7672*** (0.1925)			

Note : See the notes in Table B-4-1.

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