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Optimal Monetary Policy Rule under the Non-Negativity Constraint on Nominal Interest Rates

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Abstract

We identify a monetary policy rule that remains optimal even in the presence of the non-negativity constraint on nominal interest rates. This rule also compensates for any past shortfalls in monetary easing during the zero interest rate period.

Keywords: optimal monetary policy rule; non-negativity constraint on nominal interest rates

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1 Introduction

What rule should a central bank follow when monetary policy is restricted by the non-negativity constraint on the nominal interest rates? The Japanese economy has been faced with short-term nominal interest rates of virtually zero for about five years, and a similar situation now threatens many developed countries, including the United States. Research on optimal monetary policy within such an environment is therefore increasingly important.

In this paper, we search for optimal interest rate rules that remain robust even under the non-negativity constraint. Our main finding is that the only interest rate rule that remains optimal whether or not the non-negativity constraint binds is one that does not include lagged nominal interest rates. This rule therefore provides the best guide for when to terminate the zero interest rate policy. By retaining information on the past in the form of endogenous variable such as inflation and the output gap, instead of in the form of lagged nominal interest rates, this rule can maintain optimal history-dependency.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 introduces the non-negativity constraint on the nominal interest rates and uses the Kuhn-Tucker solution in order to demonstrate the optimal monetary policy rule. Section 4 concludes our discussion.

2 The model

We use the model developed by Clarida, Gali, and Gertler (1999) and Woodford (2003). The economy outside the central bank is represented by three equations: an "IS curve", a "Phillips curve", and a shock to the natural interest rate.

$$x_t = E_t x_{t+1} - \sigma \left[(i_t - E_t \pi_{t+1}) - r_t^n \right], \tag{1a}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}, \tag{1b}$$

$$r_{t+1}^n - r_\infty^n = \rho(r_t^n - r_\infty^n) + \epsilon_{t+1}, \quad \epsilon_1 = \epsilon_2 = \cdots = 0,$$

$$\Longrightarrow \quad r_t^n = \rho^t \epsilon_0^n + r_\infty^n. \tag{1c}$$

Eq. (1a) represents the forward-looking IS curve. This IS curve states that the output gap in period t, denoted by x_t, is determined by the expected value of the output gap in period t+1 and the deviation of the short-term real interest rate, the nominal interest rate i_t minus the expected rate of inflation $E_t\pi_{t+1}$, from the natural rate of interest in period t, denoted by r_t^n. Eq. (1b) is a forward-looking Phillips curve. This Phillips curve states that inflation in period t depends on the expected rate of future inflation in period t+1 and the output gap in period t. Here σ, κ, and β are parameters, satisfying $\sigma > 0$, $\kappa > 0$, and $0 < \beta < 1$. Eq. (1c) describes an adverse shock to the economy. To make the problem simple, we consider only the situation in which the economy is hit by a large-scale negative demand shock caused by a shift in the natural rate of interest. We assume that this large negative shock to the natural rate of interest, denoted by ϵ_0^n, occurs only in period 0. Afterward the natural rate of interest gradually converges to its steady-state, $r_\infty^n = i^*$, according to the convergence parameter ρ, which satisfies $0 \leq \rho < 1$. We also assume that, prior to the shock, the economy in the model is in steady-state, where x_t and π_t are zero and i_t is i^*.

Next, we present the central bank's intertemporal optimization problem. We assume that the central bank's policy instrument is the short-term nominal interest rate. The central bank solves an intertemporal optimization problem in period 0, considering the expectation channel of monetary policy, and commits itself to the computed optimal path. This is the optimal solution from a timeless perspective defined by Woodford (2003). The central bank chooses the path of the short-term nominal interest rate, starting from period 0, so as to minimize welfare loss L_S, where

$$L_S = E_0 \sum_{t=0}^{\infty} \beta^t L_t. \tag{1d}$$

The period loss function is given by

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_i (i_t - i^*)^2, \tag{1e}$$

where λ_x and λ_i are positive parameters.

3 Optimal monetary policy rule under the zero bound on nominal interest rates

As shown by Giannoni and Woodford (2002), from a timeless perspective, central banks can conduct optimal monetary policy using several different simple interest rate rules. They show that under the model consisting of Eqs.(1a)-(1e) the optimal monetary policy is achieved by the following interest rate rule:

$$A : i_t = \rho_1 i_{t-1} + \rho_2 \triangle i_{t-1} + \phi_\pi \pi_t + \phi_x \triangle x_t + (1 - \rho_1) i^*, \tag{2a}$$

where $\rho_1 = 1 + \frac{\kappa\sigma}{\beta} > 1$, $\rho_2 = \beta^{-1} > 1$, $\phi_\pi = \frac{\kappa\sigma}{\lambda_i} > 0$, $\phi_x = \frac{\sigma\lambda_x}{\lambda_i} > 0$. We call this Rule A. Giannoni and Woodford (2002) also show that this optimal interest rate rule can take several different forms. For example, we can re-write Rule A as follows:

$$B : i_t = \eta_1 i_{t-1} + P(L)(\phi_\pi \pi_t + \phi_x \triangle x_t) + (1 - \eta_1) i^*, \tag{2b}$$

$$C : i_t = Q(L)(\phi_\pi \pi_t + \phi_x \triangle x_t) + i^*, \tag{2c}$$

where $P(L)$ and $Q(L)$ are functions of the lag operator that satisfy the following conditions:

$$P(L) \equiv 1 + \eta_2 L + \eta_2^2 L^2 + \cdots ,$$

$$Q(L) \equiv 1 + \frac{\eta_1^2 - \eta_2^2}{\eta_1 - \eta_2} L + \frac{\eta_1^3 - \eta_2^3}{\eta_1 - \eta_2} L^2 + \cdots ,$$

where η_1 and η_2 are parameters, satisfying $\eta_1 > 1$ and $0 < \eta_2 < 1$ ($\eta_1 + \eta_2 = 1 + \beta^{-1} + \kappa\sigma\beta^{-1}$ and $\eta_1\eta_2 = \beta^{-1}$). We call these rules Rule B and Rule C respectively. Although they are differently formulated, these three rules manifest the same history-dependency and have the same optimal equilibrium when the zero bound on the nominal interest rates is not imposed.

However, we do not know whether these rules remain optimal in the presence of the non-negativity constraint. Next, therefore, we investigate whether the central bank's policy remains optimal if the non-negativity constraint on the nominal interest rates, $i_t \geq 0$, is simply introduced to the interest rate rules given by Eqs.(2a)-(2c) as follows.

$$D : i_t = Max(0,\ \rho_1 i_{t-1} + \rho_2 \triangle i_{t-1} + \phi_\pi \pi_t + \phi_x \triangle x_t + (1-\rho_1) i^*), \tag{2d}$$

$$E : i_t = Max(0,\ \eta_1 i_{t-1} + P(L)(\phi_\pi \pi_t + \phi_x \triangle x_t) + (1-\eta_1) i^*), \tag{2e}$$

$$F : i_t = Max(0,\ Q(L)(\phi_\pi \pi_t + \phi_x \triangle x_t) + i^*). \tag{2f}$$

We call these rules Rule D, Rule E, and Rule F respectively. We can then show the following *lemma*:

Lemma 1: Rule F is the only rule that remains optimal whether or not the non-negativity constraint on the nominal interest rate is binding.

A sketch of the proof follows. We first extend the model of Giannoni and Woodford (2002) to include the zero bound on the nominal interest rates and suggest an optimal solution using the Kuhn-Tucker theorem. Then, we check whether the rules given by Eqs. (2a)-(2c) remain optimal or not.

The optimization problem is represented by a Lagrangian of the form:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ L_t - 2\phi_{1t} \left[x_{t+1} - \sigma(i_t - \pi_{t+1} - r_t^n) - x_t \right] - 2\phi_{2t} \left[\kappa x_t + \beta \pi_{t+1} - \pi_t \right] - 2\phi_{3t} i_t \right\} \right\},$$

where ϕ_1, ϕ_2, and ϕ_3 represent the Lagrange multipliers associated with the IS constraint, the Phillips curve constraint, and the nominal interest rate constraint, respectively. We differentiate the Lagrangian with respect to π_t, x_t, and i_t to obtain the first-order conditions:

$$\pi_t - \beta^{-1}\sigma\phi_{1t-1} + \phi_{2t} - \phi_{2t-1} = 0, \tag{3a}$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0, \tag{3b}$$

$$\lambda_i(i_t - i^*) + \sigma\phi_{1t} - \phi_{3t} = 0, \tag{3c}$$

$$i_t\phi_{3t} = 0, \tag{3d}$$

$$\phi_{3t} \geq 0, \tag{3e}$$

$$i_t \geq 0. \tag{3f}$$

Eqs. (3d), (3e), and (3f) are Kuhn-Tucker conditions for the non-negativity constraint on the nominal interest rates. The above six conditions, together with the IS (Eq. (1a)) and Phillips (Eq. (1b)) equations, are the conditions governing the loss minimization. We eliminate ϕ_1 and ϕ_2 from Eqs. (3a), (3b), and (3c) to obtain an equation with respect to ϕ_3 and i.

$$\phi_{3t} - \lambda_i i_t = -\lambda_i\big(Q(L)\,(\phi_\pi\pi_t + \phi_x\triangle x_t) + i^*\big). \tag{4a}$$

When the non-negativity constraint is not binding (i.e., $\phi_{3t} = 0$), it is easy to reconfirm that Rule C is optimal from Eq. (4a). When the non-negativity constraint is binding (i.e., $i_t = 0$), Eq. (4a) becomes:

$$\phi_{3t} = -\lambda_i\big(Q(L)\,(\phi_\pi\pi_t + \phi_x\triangle x_t) + i^*\big). \tag{4b}$$

From the Kuhn-Tucker conditions, while ϕ_3 is positive, the nominal interest rate is zero. Once ϕ_3 becomes non-positive, the nominal interest rate becomes non-negative.

We now observe that Eq. (4b) is equivalent to the right hand side of Eq. (2c) multiplied by $-\lambda_i$. It is thus straightforward to demonstrate that Rule C remains optimal even in the face of the non-negativity constraint. This is because both Rule F and ϕ_3 dictate the same timing for termination of the zero interest rate policy. On the other hand, neither Rule A nor Rule B is optimal because both Rules D and E suggest termination dates that differ from that dictated by ϕ_3.

From the *lemma*, we can also identify the following relations between the termination dates suggested by the different rules:

$$T^D \neq T^E \neq T^F = T^{opt},$$

where T^D, T^E, and T^F are the termination dates for the zero interest rate policy suggested by Rules D, E, and F, respectively, and T^{opt} is the optimal termination date obtained from ϕ_3. Since T^F differs from T^D and T^E, the latter are not optimal. Intuitively, this is because Rules D and E lose important information about history-dependency by allocating zero values to i_{t-1} and i_{t-2}. However Rule F preserves optimal history-dependency, even in the presence of the non-negativity constraint, by using past information contained in endogenous variables such as x_t and π_t, instead of lagged nominal interest rates. As a result, the central bank can compensate, as required, for shortfalls in past monetary easing during the zero interest rate period, by continuing with the zero interest rate policy for longer.

4 Concluding remarks

In this paper, we show that policy rules which ignore the non-negativity constraint on the nominal interest rates do not always remain optimal when this constraint is introduced. These rules usually suggest termination dates for the zero interest rate policy that are not optimal.

However, we find that the rule that omits the lagged nominal interest rates remains optimal even when the zero interest rate bound is imposed. Since it retains optimal history-dependency, this rule can fully compensate for insufficient past monetary easing during the zero interest rate period.

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