Bank of Japan Working Paper Series

Why Can the Yield Curve Predict Output Growth, Inflation, and Interest Rates? An Analysis with an Affine Term Structure Model

Hibiki Ichiue*
hibiki.ichiue@boj.or.jp

No.04-E-11
July 2004

Bank of Japan
2-1-1 Nihonbashi Hongoku-cho, Chuo-ku, Tokyo 103-8660

* Research and Statistics Department

Why Can the Yield Curve Predict Output Growth, Inflation, and Interest Rates? An Analysis with an Affine Term Structure Model☆

Hibiki Ichiue*

Department of Economics, University of California, San Diego

Research and Statistics Department, the Bank of Japan

Revised: July, 2004

Abstract

The literature provides evidence that term spreads help predict output growth, inflation, and interest rates. This paper integrates and explains these predictability results by using an affine term structure model with observable macroeconomic factors for U.S. data. The results suggest that consumers are willing to pay a higher premium for a consumption hedge during a higher inflation regime. This causes term spreads to react to inflation shocks, which proves useful for prediction. We also find that term spreads using the short end of the yield curve have less predictive power than many other spreads. We attribute this to monetary policy inertia.

JEL classification: E43; E52

Keywords: Term structure, Monetary policy, VAR

☆ I am especially grateful to my dissertation advisor, James D. Hamilton for his valuable support and comments. I would like to thank Marjorie Flavin, Bruce Lehmann, Alan Timmermann, Keiichi Tanaka, Nobuyuki Oda, Akira Ieda and participants in the presentations at UCSD and the Bank of Japan. I am also grateful to Monica Piazzesi for answering my questions on her papers. The views expressed here are those of the author, and not necessarily of the Bank of Japan.

* Address: 2-1-1 Nihonbashi-Hongokucho Chuo-ku Tokyo 103-8660 Japan
Tel.: +81-3-3279-1111 (Bank of Japan), Fax: +81-3-5255-6758, *E-mail address*: hibiki.ichiue@boj.or.jp

1. Introduction

Many studies in the literature provide evidence that interest rate term spreads contain information about three different future economic variables: output growth, inflation, and interest rates, for various sample periods and countries. But the literatures examining the predictability of these three variables have been quite distinctive. Studies of the predictability of interest rates have been mainly conducted by financial economists testing a popularly-held classic theory, namely the expectations hypothesis[1]. According to this theory, the long rate is equal to the average of expected future short rates plus a time-invariant term premium. However, in spite of its popularity, this hypothesis has typically been rejected. Many economists argue the expectations hypothesis fails because of the assumption of a time-invariant term premium[2]. The literature on the predictability of inflation also has a long history following Fama's (1975) classic study[3]. On the other hand, the history of the literature studying the predictability of output growth is relatively recent. After Stock and Watson (1989) found that the term spread plays an important role in their index of economic leading indicators, many researchers investigated this predictive relationship[4].

Although there is an extensive literature providing evidence and explanations for each of the predictive relationships between term spreads on the one hand, and on the other, output growth, inflation, and interest rates, no paper has yet tried to analyze the interaction between these three relationships. The main purpose of this paper is to integrate these predictability results in an attempt to answer to an important question: why can the term structure predict future movements in economic variables? This study will help us understand the information contained in the term structure of interest rates, and the relationship between the term structure and business

[1] For empirical results of tests of the expectations hypothesis, see, for example, Campbell and Shiller (1991), Hardouvelis (1994), Rudebusch (1995), Campbell, Lo and MacKinlay (1997), Roberds and Whiteman (1999), Bekaert, Hodrick and Marshall (2001), and Cochrane (2001).
[2] The literature provides evidence that the term premium is in fact time-varying. See, for example, Mankiw and Miron (1986), Engle, Lilien and Robins (1987), Engle and Ng (1993), Dotsey and Otrok (1995), and Balduzzi, Bertola and Foresi (1997).
[3] For empirical results on the predictability of inflation, see, for example, Mishkin (1988, 1990a, b, 1991), Fama (1990), Jorion and Mishkin (1991), Estrella and Mishkin (1997), and Kozicki (1997).
[4] For empirical results on the value of term spreads for predicting output growth or recessions, see, for example, Estrella and Hardouvelis (1991), Plosser and Rouwenhost (1994), Haubrich and Dombrosky (1996), Bonser-Neal and Morley(1997), Dueker (1997), Estrella and Mishkin (1997), Kozicki (1997), Bernard and Gerlach (1998), Dotsey (1998), and Hamilton and Kim (2002).

cycle.

We use an affine term structure model (ATSM) with observable economic factors as our main tool, basing our investigation on U.S. data. There have been a number of studies following Ang and Piazzesi's (2003) introduction of this type of model to investigate the relationship between macroeconomic variables and the term structure, for example, Dewachter and Lyrio (2002), Hordahl, Tristani and Vestin (2002), and Wu (2002). These studies depend much on macroeconomic theories to restrict their models so that the results can be interpreted more easily. Furthermore, these models typically use latent variables other than observable variables, and interpret the latent factors as variables such as the monetary policy authority's inflation target.

Conversely, Ang, Piazzesi and Wei (2003) use only observable variables, and they do not use macroeconomic theories other than the no-arbitrage assumption to restrict their model. This type of model can be interpreted either as a VAR with no-arbitrage restrictions or as an ATSM with observable factors that follow a VAR process. In this paper, we call this type of model a VAR-ATSM for convenience. Ang, Piazzesi and Wei use their VAR-ATSM to examine the predictability of output growth using term spreads. We follow this basic idea, which we extend to include the predictive relationships with inflation and short rates[5]. Although their basic idea is very useful for investigating these predictive relationships, some of their assumptions and aspects of their estimation method are not suitable to our purpose here. Ang, Piazzesi and Wei try to identify good forecasting models by comparing the predictive powers, specifically the rolling out-of-sample forecasting performances, of various combinations of regressors. Their parsimonious VAR(1) model and computationally fast, though less efficient, estimation method may be appropriate for such an exercise. Our aim, however, is to shed light on the source of the predictability by analyzing the relationship between impulse response functions and R^2s. Thus we adopt VAR with more lags and a more efficient estimation method, and these contribute to the reliability of the impulse response functions.

We have three main findings. First, the time-varying market price of output growth risk,

[5] Before Ang, Piazzesi and Wei (2003), several papers use term structure models with only latent factors for analyzing predictability using term spreads. For example, Roberds and Whiteman (1999), Dai and Singleton (2002), and Duffee (2002) examine whether empirical results on the predictability of interest rates can be fitted using ATSMs. Hamilton and Kim (2002) use the Longstaff and Schwarz's (1992) term structure model to explain the predictability of output growth. Since these models use only latent factors, however, they have only

which is sensitive to the inflation rate, plays a key role in the predictive relationships. When the inflation rate is higher, consumers are willing to pay a higher premium for a consumption hedge, which may be explained by a simple model with money in the utility function and a monetary policy rule. This causes term spreads to be sensitive to inflation shocks. Since the inflation shock has persistent effects not only on inflation but also on output growth and interest rates, the response of term spreads to the inflation shock helps predict these variables. Second, we also find that term spreads using the short end of the yield curve have less predictive power than many spreads between longer rates. This fact is attributable to the inertial character of monetary policy. Third, it is hard to predict output growth with term spreads at short horizons, because the monetary policy shock affects output growth with a lag while the term structure responds to the shock immediately.

The rest of this paper is organized as follows. Section 2 presents stylized facts from simple OLS results. In Section 3, in order to understand the basic properties of ATSMs, we consider some simple representative models. This section will help to prepare for the more complicated VAR-ATSM introduced in Section 4. Estimation methods and results are considered in Section 5. Here we discuss the relationship between time-varying market prices of risk and the information included in the term structure. In Section 6, we use impulse response functions and model-implied R^2s, which can be obtained from the estimated VAR-ATSM, to explain why term spreads predict well. Section 7 concludes.

2. Simple OLS Results

The empirical studies in the literature examine the predictive power of term spreads for future output growth, inflation, and interest rates using a common econometric method, regressions on the term spreads. However, these regressions do not have exactly the same form. For example, Estrella and Hardouvelis (1991) examine output growth predictability by regressing cumulative output growth, up to *h* quarters ahead, on a fixed term spread between ten-year and

limited value for analyzing the relationships between the term structure and macroeconomic variables.

three-month interest rates:

$$g_{t\to t+h} = \alpha + \beta(r_t^{(40)} - r_t^{(1)}) + \varepsilon_{t+h} \tag{1}$$

where $g_{t_1\to t_2}$ is the output growth rate from t_1 to t_2, and $r_t^{(n)}$ is the n-period nominal discount rate on Treasury bills or bonds at the end of t. On the other hand, Mishkin (1990a) examines inflation predictability by regressing the difference between h-quarter and 1-year cumulative inflation rates on term spreads of matching maturity:

$$\pi_{t\to t+h} - \pi_{t\to t+4} = \alpha + \beta(r_t^{(h)} - r_t^{(4)}) + \varepsilon_{t+h} \tag{2}$$

where $\pi_{t_1\to t_2}$ is the inflation rate from t_1 to t_2. Campbell and Shiller (1991), meanwhile, provide evidence for short rate predictability by using the most popular expectations hypothesis test, regressions of average future short rate changes on term spreads of matching maturity:

$$\frac{1}{h}\sum_{i=0}^{h-1}(r_{t+i}^{(1)} - r_t^{(1)}) = \alpha + \beta(r_t^{(h)} - r_t^{(1)}) + \varepsilon_{t+h-1}\,. \tag{3}$$

All three types of study find that the slope coefficient β is significantly different from zero in many cases, which means that term spreads have predictive power for forecasting movements in macroeconomic variables. Typically they report substantial t-stats and R^2s for these regressions.

As one can easily see, these empirical regressions do not have the same form. For example, (1) and (2) do not use the same regressor. Regression (1) uses a fixed regressor, while the regressor in (2) depends on the forecasting horizon h. In order to analyze the interaction between the predictive relationships, therefore, we need to put the empirical results for predicting the different variables on a consistent basis. For this purpose, we use the regressions below,

$$g_{t+h} = \alpha + \beta(r_t^{(n)} - r_t^{(m)}) + \varepsilon_{t+h}\,; \tag{4}$$

$$\pi_{t+h} = \alpha + \beta(r_t^{(n)} - r_t^{(m)}) + \varepsilon_{t+h}\,; \tag{5}$$

$$r_{t+h}^{(1)} = \alpha + \beta(r_t^{(n)} - r_t^{(m)}) + \varepsilon_{t+h}\,; \tag{6}$$

for various combinations of h, n, and m (h = 1,2,…,12; n, m = 2, 4, 8, 12, 16, 20, and $n > m$), where g_t is the real GDP growth rate from t-1 to t, and π_t is the inflation rate of GDP deflator from t-1 to t. We use discount rate data from CRSP[6]. U.S. quarterly data are used, so we interpret one period as one quarter. g_t, π_t, and $r_t^{(n)}$ are all defined as rates per quarter. The sample period is 1964:1Q-2001:4Q, following Fama and Bliss (1987) who comment that long rate data before 1964 may be unreliable. There are two other properties of the set of regressions (4)-(6) worth commenting on. First, regressands are continuously compounded marginal rates or one-period short rates. Since cumulative rates are the averages of marginal rates, marginal rates are more convenient for specifying which part of the future the term spreads can predict well. Second, we use various forecasting horizons h and term spreads $r_t^{(n)} - r_t^{(m)}$, so we can specify which components of the yield curve predict at which future horizons.

Figures 1 and 2 display the t-stats and R^2s of OLS regressions (4)-(6) for selected term spreads. The 20Q-1Q spread has significant predictive power for output growth, inflation, and short rates, at least for shorter horizons. This result is consistent with the literature, which argues that term spreads between 5-year (or 10-year) and 3-month rates predict well. But surprisingly we found that term spreads without the 1Q rate perform better than the 20Q-1Q spread in many cases. For example, Figure 2 shows that the performance of the 12Q-8Q spread is superior, except for predicting output growth rates at shorter horizons. On the other hand, spreads between short rates, such as the 2Q-1Q spread, are almost useless. Together, these facts seem to imply that term spreads using the short end of the yield curve have less predictive power. This is surprising because the existing literature pays little attention to spreads that exclude the short end of the yield curve, and several studies including Ang, Piazzesi and Wei (2003) argue that the best predictive performance is achieved by maximal maturity difference. Another notable feature of the graphs is the hump-shape traced out by the R^2s of the output growth regressions. This suggests that it is difficult to predict the output growth rate at short horizons.

Why do term spreads have this kind of predictive power? Since the OLS results do not answer this question, we need a more structured model. A useful method for interpreting these

[6] CRSP (Center for Research in Security Prices, Graduate School of Business, the University of Chicago: www.crsp.uchicago.edu. All rights reserved.) Monthly US Treasury Database is used with permission. We can construct discount rates for 1, 2, 4, 8, 12, 16, 20 quarters from the CRSP data. The 1 quarter (3 month) rate is obtained from average rates in the CRSP risk free rates file. The 2 quarter (6 month) rate is constructed by multiplying average-YTM by 12 (there is no data on 9/30/1987, so we interpolate with 3 and 12 month rates). The other rates are obtained from the Fama-Bliss discount bonds file.

OLS results is proposed by Ang, Piazzesi and Wei (2003). They introduce a VAR-ATSM to compare the predictive powers of various combinations of regressors. We follow their basic idea, but extend their analysis so as to include all three predictive relationships, between term spreads on the one hand, and on the other each of output growth, inflation, and short rates. Although their VAR-ATSM is very useful for examining the relationships between macroeconomic variables and the yield curve, some of their assumptions and aspects of their estimation method are not suitable to our purpose. We therefore modify them in Sections 4 and 5. Then, in Section 6, we try to shed light on the source of the predictability by using impulse response functions and R^2s, which can be calculated from the estimates of the VAR-ATSM.

3. Simple Affine Term Structure Models with Observable Factors

Before introducing our VAR-ATSM in the next section, let's consider two simpler ATSMs. Since the complexity of the VAR-ATSM defies easy interpretations, these simpler models provide a useful starting point. A particular complication arises as a result of time-varying market prices of risk, which many classic term structure models assume constant. Since, however, these affect the relationship between short and long rates, i.e. movements in term spreads, they are very important for examining the predictive power of term spreads.

3.1. An ATSM with One Short Rate Factor

Suppose that quarterly data on the short (3-month) rate $r_t^{(1)}$ are characterized by an AR(1) process:

$$r_{t+1}^{(1)} = c_r + \phi_r r_t^{(1)} + \sigma_r u_{r,t+1}, \tag{7}$$

where $u_{r,t+1} \sim N(0,1)$ i.i.d., and $\sigma_r > 0$. Table 1 reports the OLS estimates for (7), which demonstrate the persistence of the short rate ($\phi_r = 0.9037$). Suppose that the stochastic discount factor M_{t+1} follows a conditional log-normal distribution:

$$M_{t+1} = \exp\left(-r_t^{(1)} - \frac{1}{2}\lambda_{r,t}{}^2 - \lambda_{r,t}u_{r,t+1} \right), \tag{8}$$

where

$$\lambda_{r,t} = \gamma_r + \delta_r r_t^{(1)}. \tag{9}$$

In this model, therefore, the market price of risk $\lambda_{r,t}$ is time-varying, depending on the factor $r_t^{(1)}$. In other words, the stochastic discount factor M_{t+1} is affected not only by the exogenous shock $u_{r,t+1}$ but also by the level of the factor $r_t^{(1)}$ through the time-varying market price of risk. Thus the effects of the factor on the yield curve are complicated. Note that if $\delta_r = 0$, i.e. $\lambda_{r,t}$ is time-invariant, this is just the classic Vasicek (1977) model.

Let's assume there is no arbitrage opportunity in the Treasury market. Since this market is one of the largest and most highly liquid markets in the world, the no-arbitrage assumption is extremely reasonable. Under the no-arbitrage assumption, we can use the fundamental asset pricing equation for bond prices,

$$q_t^{(n)} = E_t[M_{t+1}q_{t+1}^{(n-1)}], \tag{10}$$

for n = 1, 2, …, and all t, where $q_t^{(n)}$ is the n-period bond price with $q_t^{(0)} = 1$. Note that (8) and (10) lead to

$$q_t^{(1)} = \exp(-r_t^{(1)}). \tag{11}$$

This is exactly the definition of the relationship between the 1-period bond price and the continuously compounded discount rate. In fact, M_{t+1} is chosen so that (11) holds.

By using the fundamental asset pricing equation (10), we can derive closed forms for the discount rates $r_t^{(n)}$ as affine functions of the factor $r_t^{(1)}$:

$$\hat{r}_t^{(n)} = a^{(n)} + b^{(n)} r_t^{(1)}, \qquad n = 1, 2, \ldots \tag{12}$$

where

$$a^{(n)} = -A^{(n)}/n, \quad b^{(n)} = -B^{(n)}/n, \tag{13}$$

$$A^{(n+1)} = A^{(n)} + B^{(n)}(c_r - \sigma_r \gamma_r) + \frac{1}{2}\sigma_r{}^2 B^{(n)2}, \quad (14)$$

$$B^{(n+1)} = B^{(n)}(\phi_r - \sigma_r \delta_r) - 1, \quad (15)$$

$$A^{(1)} = 0, \quad B^{(1)} = -1^{7}. \quad (16)$$

In (12), the factor loading on the short rate factor $b^{(n)}$ can be interpreted as the sensitivity of longer rates $r_t^{(n)}$ to the short rate $r_t^{(1)}$. From (13), (15), and (16), we can obtain a closed form for $b^{(n)}$:

$$b^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1}(\phi_r - \sigma_r \delta_r)^j . \quad (17)$$

Note that γ_r does not appear in (17). Since the movement of short rates is less volatile than that of long rates for U.S. data, it is reasonable that the absolute value of $b^{(n)}$ decreases as n increases. To satisfy this, we need parameter values such that

$$|\phi_r - \sigma_r \delta_r| < 1. \quad (18)$$

Suppose $\phi_r - \sigma_r \delta_r > 0$, which guarantees $b^{(n)} > 0$. From (17), we can say that the sensitivity of long rates to the short rate is weaker when δ_r is higher. We can relate this claim to the expectations hypothesis. From (7), (12), and (17), we can obtain the term premium:

$$r_t^{(n)} - \frac{1}{n}\sum_{j=0}^{n-1} E_t[r_{t+j}^{(1)}] = a^{(n)} - c_r \frac{1}{n}\sum_{j=0}^{n-1}\sum_{i=0}^{j-1}\phi_r{}^i + \frac{1}{n}\sum_{j=0}^{n-1}[(\phi_r - \sigma_r \delta_r)^j - \phi_r{}^j] r_t^{(1)} . \quad (19)$$

The term premium is therefore constant, i.e. the expectation hypothesis holds, only when $\delta_r = 0$. In this case, movements in long rates $r_t^{(n)}$ depend only on movements in average expected short rates $n^{-1}\sum_{j=0}^{n-1} E_t[r_{t+j}^{(1)}]$. Since $r_t^{(1)}$ follows a persistent AR(1) process, an increase in $r_t^{(1)}$ raises $r_t^{(n)}$. However, when $\delta_r > 0$, a rise in $r_t^{(1)}$ also has a negative effect on $r_t^{(n)}$ through a decrease in the term premium. Therefore, positive δ_r weakens the relationship between short and long

[7] Since this is one of the simplest special cases of VAR-ATSM, it is sufficient to check the proof for the general model introduced in Section 4. For the proof, see Ang and Piazzesi (2003).

rates. Then the sensitivity of the term spread to the factor $r_t^{(1)}$ is stronger when δ_r is larger.

3.2. C-CAPM with money in the utility (MIU) function

Let's consider a C-CAPM, in which the stochastic discount factor follows

$$M_{t+1} = \delta \frac{u_C(C_{t+1}, m_{t+1})}{u_C(C_t, m_t)} \exp(-\pi_{t+1}), \tag{20}$$

where δ is the subjective discount factor, C_t is consumption and m_t is the real money holding at *t*. Suppose that the form of the utility function is

$$u(C_t, m_t) = C_t^{1-\rho} m_t^{\theta}, \tag{21}$$

where $0 < \rho < 1$ and $0 \le \theta < 1$. Then if $C_t = Y_t$[8] in equilibrium, (20) can be rewritten as

$$\begin{aligned} M_{t+1} &= \delta \left(\frac{Y_t}{Y_{t+1}} \right)^{\rho} \left(\frac{m_{t+1}}{m_t} \right)^{\theta} \exp(-\pi_{t+1}) \\ &= \delta \exp(-\rho g_{t+1} + \theta \mu_{t+1} - \pi_{t+1}) \\ &= \exp(\log(\delta) - E_t[\rho g_{t+1} - \theta \mu_{t+1} + \pi_{t+1}] - \rho \sigma_{\varepsilon,g} u_{g,t+1} + \theta \varepsilon_{\mu,t+1} - \sigma_{\varepsilon,\pi} u_{\pi,t+1}), \end{aligned} \tag{22}$$

where μ_{t+1} is the real money growth rate from t to t+1, $\varepsilon_{\mu,t+1} = \mu_{t+1} - E_t[\mu_{t+1}]$, $\sigma_{\varepsilon,g} u_{g,t+1} = g_{t+1} - E_t[g_{t+1}]$, $\sigma_{\varepsilon,\pi} u_{\pi,t+1} = \pi_{t+1} - E_t[\pi_{t+1}]$, $\sigma_{\varepsilon,g} > 0$, $\sigma_{\varepsilon,\pi} > 0$, $u_{g,t+1} \sim N(0,1)$, and $u_{\pi,t+1} \sim N(0,1)$. For simplicity, let's assume $u_{g,t+1}$ and $u_{\pi,t+1}$ are uncorrelated, as we often observe empirically.

First, let's consider a simple case in which $\theta = 0$, i.e. utility is independent of money holding. Since $\rho > 0$, a positive output growth shock has a negative effect on M_{t+1}. This is consistent with a role for bonds as a consumption hedge. That is, when the future output growth rate is higher, consumers feel that future cash flows are less important. Note that both market

[8] We assume this just for simplicity. We can also generalize this model to be consistent with the literature, which shows that the dynamics of the consumption growth rate are smoother than those of the output growth rate, by assuming that the consumption growth rate follows an affine function of the output growth rate with a positive slope coefficient of less than unity. Even in this generalized form, the main properties of the model do not change.

prices of risk, corresponding to the output growth shock $u_{g,t+1}$ and the inflation shock $u_{\pi,t+1}$, are constant ($\rho\sigma_{\varepsilon,g}$ and $\sigma_{\varepsilon,\pi}$ respectively).

Next, let's consider a general case in which $\theta > 0$ and $\varepsilon_{\mu,t+1}$ can be represented as a linear combination of $u_{g,t+1}$ and $u_{\pi,t+1}$ with time-varying weights:

$$\varepsilon_{\mu,t+1} = w_{g,t}u_{g,t+1} + w_{\pi,t}u_{\pi,t+1}, \tag{23}$$

where the weights $w_{g,t}$ and $w_{\pi,t}$ are affine functions of g_t and π_t:

$$w_{g,t} = \omega_g + \omega_{gg}g_t + \omega_{g\pi}\pi_t, \tag{24}$$

$$w_{\pi,t} = \omega_\pi + \omega_{\pi g}g_t + \omega_{\pi\pi}\pi_t. \tag{25}$$

The idea behind (23) is similar to Taylor's rule. But (23) uses the real money growth rate instead of the target short rate, and has time-varying weights. The time-varying weights can be interpreted, for example, as follows. Suppose that the monetary policy authority (the Fed) can perfectly control the real money growth rate μ_{t+1} (i.e. $\varepsilon_{\mu,t+1}$) and can observe $u_{g,t+1}$ and $u_{\pi,t+1}$ before making their policy decision. In response to a surprise increase in the output growth rate ($u_{g,t+1} > 0$), the Fed may accommodate any increase in money demand caused by the output growth shock by allowing the real money growth rate to rise. Conversely, the Fed may suppress the real money growth rate in response to the shock, if they consider that this output growth shock may cause serious inflation in the future. These two plausible stories imply that the weight on the output growth shock $w_{g,t}$ can be either positive or negative. We can also discuss the weight on the inflation shock $w_{\pi,t}$ in a similar way.

With (23)-(25), (22) can be rewritten as

$$\begin{aligned} M_{t+1} = \exp(\log(\delta) - E_t[\rho g_{t+1} - \theta\mu_{t+1} + \pi_{t+1}] \\ -[(\rho\sigma_{\varepsilon,g} - \theta\omega_g) - \theta\omega_{gg}g_t - \theta\omega_{g\pi}\pi_t]u_{g,t+1} \\ -[(\sigma_{\varepsilon,\pi} - \theta\omega_\pi) - \theta\omega_{\pi g}g_t - \theta\omega_{\pi\pi}\pi_t]u_{\pi,t+1}) \ . \end{aligned} \tag{26}$$

Now, in contrast with the simple C-CAPM with $\theta = 0$, the market prices of risk corresponding to $u_{g,t+1}$ and $u_{\pi,t+1}$ are time-varying, depending on g_t and π_t. From (10), (11) and (26), we can obtain

$$r_t^{(1)} = \log(\frac{1}{\delta}) + E_t[\rho g_{t+1} - \theta\mu_{t+1} + \pi_{t+1}]$$
$$- \frac{[(\rho\sigma_{\varepsilon,g} - \theta\omega_g) - \theta\omega_{gg} g_t - \theta\omega_{g\pi}\pi_t]^2 + [(\sigma_{\varepsilon,\pi} - \theta\omega_\pi) - \theta\omega_{\pi g} g_t - \theta\omega_{\pi\pi}\pi_t]^2}{2} . \qquad (27)$$

Although this type of MIU function is often used in the literature, the validity of this theoretical model has been the subject of criticism. Specifically, the utility function may not depend on money directly. Also, the time-separable utility function may be unreasonable due to, for example, habit formation. In Section 4, we will introduce a more general and less restricted model, which nests both models discussed in Section 3.

4. The VAR-ATSM

Now let's introduce the VAR-ATSM used for later analyses. This type of model is used by Ang, Piazzesi and Wei (2003) to examine the predictive power of terms spreads for the output growth rate. We use the VAR-ATSM to examine the predictability not only of output growth, but also of inflation and short rates. The VAR-ATSM can be interpreted as either a VAR model with no-arbitrage restrictions or an ATSM with observable factors that follow a VAR process. Let's start by considering the factor VAR.

We use four variables as factors: the output growth rate g_t, the inflation rate π_t, the short rate $r_t^{(1)}$, and a benchmark term spread s_t. For s_t, we use the term spread between ten-year Treasury bond YTM at the end of quarter t and $r_t^{(1)}$. These four macroeconomic variables are assumed to follow a VAR(4) process,

$$\mathbf{x}_t = \mathbf{c} + \mathbf{\Phi}_1\mathbf{x}_{t-1} + \mathbf{\Phi}_2\mathbf{x}_{t-2} + \mathbf{\Phi}_3\mathbf{x}_{t-3} + \mathbf{\Phi}_4\mathbf{x}_{t-4} + \boldsymbol{\varepsilon}_t , \qquad (28)$$

where $\mathbf{x}_t = (g_t, \pi_t, r_t^{(1)}, s_t)'$ and $\boldsymbol{\varepsilon}_t = (\varepsilon_{g,t}, \varepsilon_{\pi,t}, \varepsilon_{r,t}, \varepsilon_{s,t})'$. Following the VAR literature, let's interpret $r_t^{(1)}$ as a proxy for the monetary policy instrument. Ang, Piazzesi and Wei (2003) use a simpler model than ours. They use a three variable VAR with only one lag, and do not include the inflation rate. The VAR literature, however, usually uses at least four lags for quarterly data, and indicates that the inflation rate plays an important role. Our generalization of Ang, Piazzesi and Wei's model is in line with this literature.

To give a structural interpretation to the VAR, we need identifying assumptions. We use a recursive structure with the variables ordered as $(g_t, \pi_t, r_t^{(1)}, s_t)$. That is,

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\Sigma}\mathbf{u}_t \tag{29}$$

where exogenous shocks $\mathbf{u}_t = (u_{g,t}, u_{\pi,t}, u_{r,t}, u_{s,t})' \sim N(\mathbf{0}, \mathbf{I})$ i.i.d., and $\boldsymbol{\Sigma}$ is lower-triangular with positive diagonal elements. Since significant responses of g_t and π_t to contemporaneous interest rates are implausible, our ordering places them before $r_t^{(1)}$ and s_t. The order of g_t and π_t should not seriously affect the empirical results, since the correlation between $\varepsilon_{g,t}$ and $\varepsilon_{\pi,t}$ is small as shown later. The correlation, however, between $\varepsilon_{r,t}$ and $\varepsilon_{s,t}$ is too large to be ignored. For identifying the last two exogenous shocks $u_{r,t}$ and $u_{s,t}$, typically we need to adopt one of two assumptions: the short rate (the monetary policy authority) does not respond to the term spread (bond market) contemporaneously, or vice versa. Since we often observe long rates moving immediately after changes in monetary policy, the second assumption seems unreasonable. In addition, there is no clear evidence supporting a contemporaneous monetary policy response to the bond market. In fact, the literature provides evidence that the Fed's behavior is inertial: the Fed's responses to new information tend to be delayed. Thus we adopt the first assumption[9]. As will be seen in Section 6, the impulse responses seem to be reasonable, and support our recursive assumption. With this ordering, each component of $\mathbf{u}_t$ can be interpreted as the exogenous shock to the corresponding variable. We call them output growth, inflation, monetary policy, and spread shocks, respectively. Now we may interpret the first three rows of system (28) as an IS curve, a Phillips curve, and a monetary policy rule, respectively. The last row can be interpreted as an endogenous response function for the bond market.

We can rewrite the VAR in (28) into companion form,

---

[9] Most studies in the VAR literature using both short and long rates choose the first assumption. For example, Leeper, Sims, and Zha (1996) discuss this issue in detail, and conclude that the first assumption is less harmful than the second.

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \\ \mathbf{x}_{t-3} \end{bmatrix} = \begin{bmatrix} \mathbf{c} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 & \mathbf{\Phi}_3 & \mathbf{\Phi}_4 \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \\ \mathbf{x}_{t-3} \\ \mathbf{x}_{t-4} \end{bmatrix} + \begin{bmatrix} \mathbf{\Sigma} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}_t \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \tag{30}$$

or

$$\mathbf{X}_t = \tilde{\mathbf{c}} + \tilde{\mathbf{\Phi}}\mathbf{X}_{t-1} + \tilde{\mathbf{\Sigma}}\tilde{\mathbf{u}}_t, \tag{31}$$

where $\mathbf{X}_t = (g_t, \pi_t, r_t^{(1)}, s_t, \ldots, g_{t-3}, \pi_{t-3}, r_{t-3}^{(1)}, s_{t-3})'$ is the 16×1 state vector.

The stochastic discount factor is defined as

$$\begin{aligned} M_{t+1} &= \exp\left(-r_t^{(1)} - \frac{1}{2}\boldsymbol{\lambda}_t{}'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t{}'\mathbf{u}_{t+1}\right) \\ &= \exp\left(-r_t^{(1)} - \frac{1}{2}\boldsymbol{\lambda}_t{}'\boldsymbol{\lambda}_t - \lambda_{g,t}u_{g,t+1} - \lambda_{\pi,t}u_{\pi,t+1} - \lambda_{r,t}u_{r,t+1} - \lambda_{s,t}u_{s,t+1}\right), \end{aligned} \tag{32}$$

where $\boldsymbol{\lambda}_t = (\lambda_{g,t}, \lambda_{\pi,t}, \lambda_{r,t}, \lambda_{s,t})'$ are the market prices of risk. The vector $\boldsymbol{\lambda}_t$ is an affine function of the current economic variables $\mathbf{x}_t = (g_t, \pi_t, r_t^{(1)}, s_t)'$:

$$\boldsymbol{\lambda}_t = \boldsymbol{\gamma} + \boldsymbol{\delta}\mathbf{x}_t, \tag{33}$$

for a 4×1 vector $\boldsymbol{\gamma}$ and a 4×4 matrix $\boldsymbol{\delta}$.

By using the fundamental asset pricing equation (10), we can obtain closed forms for $r_t^{(n)}$ [10]:

$$\hat{r}_t^{(n)} = a^{(n)} + \mathbf{b}^{(n)}{}'\mathbf{X}_t, \qquad n = 1, 2, \ldots \tag{34}$$

where

$$a^{(n)} = -A_n / n, \quad \mathbf{b}^{(n)} = -\mathbf{B}^{(n)} / n, \tag{35}$$

[10] We derive the closed forms for discount rates so that the restriction $\hat{r}_t^{(1)} = r_t^{(1)}$ holds. Since we can calculate YTMs from the discount rates, we could also restrict the model-implied spread $\hat{s}_t$ to be equal to s_t. But since there may be a large measurement error for s_t, we do not use this restriction.

$$A^{(n+1)} = A^{(n)} + \mathbf{B}^{(n)}{}'(\tilde{\mathbf{c}} - \tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{\gamma}}) + \frac{1}{2}\mathbf{B}^{(n)}{}'\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{\Sigma}}'\mathbf{B}^{(n)}, \tag{36}$$

$$\mathbf{B}^{(n+1)}{}' = \mathbf{B}^{(n)}{}'(\tilde{\boldsymbol{\Phi}} - \tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{\delta}}) - \mathbf{e}_3{}', \tag{37}$$

$$A^{(1)} = 0, \quad \mathbf{B}^{(1)}{}' = -\mathbf{e}_3{}', \tag{38}$$

$$\tilde{\boldsymbol{\gamma}} = \begin{bmatrix} \boldsymbol{\gamma} \\ \mathbf{0} \end{bmatrix} \text{ and } \tilde{\boldsymbol{\delta}} = \begin{bmatrix} \boldsymbol{\delta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{39}$$

$\mathbf{e}_j$ is the j th column of the 16×16 identity matrix.

From (35), (37) and (38), we can obtain

$$\mathbf{b}^{(n)}{}' = \frac{1}{n}\mathbf{e}_3{}'\sum_{j=0}^{n-1}(\tilde{\boldsymbol{\Phi}} - \tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{\delta}})^j \quad . \tag{40}$$

This is a quite similar form to (17), and again the term premium is constant only when $\boldsymbol{\delta} = \mathbf{0}$.

5. Estimation

5.1. Estimation methods

The VAR-ATSM has 98 parameters consisting of 78 from the VAR ($\mathbf{c}$, $\boldsymbol{\Phi} \equiv [\boldsymbol{\Phi}_1 \boldsymbol{\Phi}_2 \boldsymbol{\Phi}_3 \boldsymbol{\Phi}_4]$, and $\boldsymbol{\Sigma}$) and 20 in market prices of risk ($\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$). We use GMM to estimate all parameters simultaneously[11]. Moment conditions are constructed by assuming that the three types of error are orthogonal to their instruments. The first of these are the VAR errors,

$$\boldsymbol{\varepsilon}_t = \mathbf{x}_t - (\mathbf{c} + \boldsymbol{\Phi}_1\mathbf{x}_{t-1} + \boldsymbol{\Phi}_2\mathbf{x}_{t-2} + \boldsymbol{\Phi}_3\mathbf{x}_{t-3} + \boldsymbol{\Phi}_4\mathbf{x}_{t-4}), \tag{41}$$

[11] Ang, Piazzesi and Wei (2003) use two-step estimation, in which the VAR parameters are estimated by OLS, and then, given these point estimates, $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are estimated by minimizing the sum of the squared pricing errors of the discount rates. This estimation method has the advantage of having a smaller computational burden than our one-step estimation. On the other hand, since their estimation method does not use efficient weights on the moment conditions, it is less efficient than ours. In particular, their estimates for VAR parameters are unable to attain any of the efficiency gains from the no-arbitrage assumption. Since our later analyses are based on impulse response functions calculated from the estimates of VAR parameters, these

where the instruments are a constant, $\mathbf{x}_{t-1}$, $\mathbf{x}_{t-2}$, $\mathbf{x}_{t-3}$, and $\mathbf{x}_{t-4}$. The second type is the error of the covariance matrix of the VAR,

$$\boldsymbol{\xi}_t = \mathrm{vech}(\boldsymbol{\Sigma}\boldsymbol{\Sigma}' - \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') . \tag{42}$$

We assume that the sample mean of $\boldsymbol{\xi}_t$ is exactly equal to zero. Note that the moment conditions corresponding to (41) and (42) are exactly the same as in OLS. The third type consists of the discount rate pricing errors

$$\mathbf{v}_t = [\nu_t^{(2)}\ \nu_t^{(4)}\ \nu_t^{(8)}\ \nu_t^{(12)}\ \nu_t^{(16)}\ \nu_t^{(20)}]' \tag{43}$$

where

$$\begin{aligned} \nu_t^{(n)} &= r_t^{(n)} - \hat{r}_t^{(n)} \\ &= r_t^{(n)} - (a^{(n)} + \mathbf{b}^{(n)}{}'\mathbf{X}_t) \end{aligned} . \tag{44}$$

We use as instruments a constant, $\mathbf{x}_{t-1}$, and $\mathbf{x}_{t-2}$ for this type of moment. Now we have 132 moment conditions, which are sufficient for identifying 98 parameters. We use the sample period 1964:1Q-2001:4Q, the same as was used for the OLS regressions in Section 2.

We restrict the parameter space with two types of restriction. First, the moduli of the eigenvalues of $\tilde{\boldsymbol{\Phi}}$ are restricted to be less than unity. Since the state vector $\mathbf{X}_t$ follows the VAR(1) process described in (31) with an autocorrelation coefficient matrix $\tilde{\boldsymbol{\Phi}}$, this restriction guarantees the stationarity of $\mathbf{X}_t$. In fact, estimation results show that this restriction does not bind. Second, the moduli of the eigenvalues of $\tilde{\boldsymbol{\Phi}} - \tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{\delta}}$ are restricted to be less than or equal to unity. From (40), the factor loading $\mathbf{b}^{(n)}$ can be considered as the average of $\mathbf{e}_3{}'(\tilde{\boldsymbol{\Phi}} - \tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{\delta}})^j$; j = 0, 1,…, n-1. So this second restriction guarantees that, with maturity n, the factor loading does not diverge. Note that this restriction is the generalization of (18). In our estimation results, only one of the restrictions binds[12].

efficiency gains are crucial.

[12] When a restriction binds, the spectral density matrix at frequency zero is not guaranteed to be the optimal weighting matrix in GMM. To solve this problem, we use the binding restriction to substitute out a parameter in advance. Inference will then be correct when we use the obtained non-restricted GMM to estimate parameters. The estimate and standard error of the substituted parameter are obtained by substituting out another parameter and re-estimating.

5.2. Estimation results

The VAR estimates achieve significant efficiency gains from the no-arbitrage assumption, although point estimates are not so different from the results without the assumption. 42 out of 68 estimates for $\mathbf{c}$ and $\mathbf{\Phi}$ (not reported) are significantly different from zero at a size of 5%, while OLS without the no-arbitrage assumption gives only 17 significant estimates. These efficiency gains contribute to the reliability of the impulse response functions used later.

The estimate of $\mathbf{\Sigma}$ is reported in Table 2. The diagonal elements of $\mathbf{\Sigma}$ are much higher than the others in general, which implies that correlations among the reduced VAR errors are small, but the contemporaneous effect of the short rate shock $u_{r,t}$ on the term spread s_t is too large to be ignored. The output growth shock has the largest volatility, and this is about three times as large as the second largest volatility, that for the inflation shock.

Table 3 reports the estimates for $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$. Seven out of 16 estimates of $\boldsymbol{\delta}$ are significantly different from zero at size of 5%. This result supports the idea that the market prices of risk are indeed time-varying, depending on economic variables. Among these significant parameters, the (1,1) and (1,2) elements of $\boldsymbol{\delta}$, δ_{11} and δ_{12} have the most influence on the term structure. The reason for this is as follows. Given the factors $\mathbf{X}_t$, the term structure depends only on the factor loadings $\mathbf{b}^{(n)}$, which depend on $\tilde{\mathbf{\Phi}} - \tilde{\mathbf{\Sigma}}\tilde{\boldsymbol{\delta}}$ from (40). So the influence of $\boldsymbol{\delta}$ on the term structure depends on $\tilde{\mathbf{\Sigma}}$ (i.e. $\mathbf{\Sigma}$). As we can see in Table 2, the (1,1) element of $\mathbf{\Sigma}$, the volatility of output growth shock, is much larger than the others. So the first row of $\boldsymbol{\delta}$ is the most influential. Among the estimates in the first row, only δ_{11} and δ_{12} are significantly different from zero. In fact, as we will discuss in the next section, δ_{12} plays a key role in the predictive relationships, while δ_{11} does not.

The positive sign of δ_{12} implies that, when the inflation rate π_t is higher, $\lambda_{g,t}$ is higher and bond holders are willing to pay a higher premium for an output growth risk hedge, which results in a lower term premium. Why do they pay a higher premium during a higher inflation regime? A possible explanation can be obtained from the C-CAPM framework with

MIU function discussed in subsection 3.2. Although this C-CAPM contains only output and inflation shocks, we can generalize the model to be consistent with the VAR-ATSM by adding monetary policy and spread shocks in (23) and letting the time-varying weights on shocks depend on all four VAR variables. From (26), $\delta_{12} = -\theta\omega_{g\pi}$. So since $\theta > 0$, $\delta_{12} > 0$ implies $\omega_{g\pi} < 0$. This means that, when the inflation rate π_t is high, the weight on the output growth shock $w_{g,t}$ is small and the Fed is less accommodating toward the output growth shock. This result makes sense if the Fed considers an output growth shock during a high inflation regime likely to cause serious future inflation. In such a situation, when inflation is high, the Fed tends to suppress the real money growth rate in response to an output growth shock. This less accommodating Fed response reduces the correlation between the output growth shock $u_{g,t+1}$ and the real money shock $\varepsilon_{\mu,t+1}$. This reduced correlation causes future marginal utility,

$$u(C_{t+1}, m_{t+1}) = (1-\rho)C_{t+1}{}^{-\rho} m_{t+1}{}^{\theta}, \tag{45}$$

to be more sensitive to the output growth shock, which is a desired property for a consumption hedge. Therefore, consumers are willing to pay a larger premium to hold bonds during a higher inflation regime. We can also discuss the positive sign of δ_{11} in a similar way.

Finally, the J-test supports our estimates with a high p-value of 1.0000[13]. To get a further sense of the robustness of the estimation results, let's compare the model-implied discount rates $\hat{r}_t^{(n)} = a^{(n)} + \mathbf{b}^{(n)}\mathbf{X}_t$ and the sample rates $r_t^{(n)}$. Table 4 reports means and standard deviations of $r_t^{(n)}$ and $\hat{r}_t^{(n)}$, and correlations between them for n = 2, 4, 8, 16, 20. Since they have very similar values for means and standard deviations and the correlations are close to unity, we can conclude that $\hat{r}_t^{(n)}$ approximates $r_t^{(n)}$ very well.

[13] The p-value is calculated from the J-stat (3.2313) and the degrees of freedom (23 = 122 - 98 - 1). Note that since the one of the restrictions on the eigenvalues binds, 1 should be subtracted from the degrees of freedom.

6. Impulse Response Functions and the Predictive Power of Term Spreads

In the previous section, we obtained estimates for our VAR-ATSM with great efficiency gains from the no-arbitrage assumption. Let's use this model to examine the predictive power of term spreads.

From the VAR-ATSM, we can calculate the optimal forecasts conditional on the 16 state variables in $\mathbf{X}_t$. However, our main interest lies not in forecasts conditional on this large number of variables, but in forecasts conditional on the term spread alone, in line with the regressions in (4)-(6). For our purpose, in subsection 6.1, we first consider the relationship between the impulse response functions of variables in regressions (4)-(6) and the R^2s. Since both regressands and regressors can be represented as affine functions of $\mathbf{X}_t$, we can calculate the impulse response functions and the R^2s from parameters in the VAR-ATSM. The relationship between the impulse response functions and the R^2s will be used to shed light on the source of the predictive power of term spreads in subsection 6.2.

6.1. Impulse response functions and model-implied R^2s

Since $\mathbf{x}_t = (g_t, \pi_t, r_t^{(1)}, s_t)'$ follows the VAR process specified in (28), we can calculate the corresponding impulse response functions, and represent the system in VMA(∞) form with identified exogenous shocks. For example, g_t can be represented as

$$g_t = \bar{g} + \sum_{j=0}^{\infty} \psi_{gg,j} u_{g,t-j} + \sum_{j=0}^{\infty} \psi_{g\pi,j} u_{\pi,t-j} + \sum_{j=0}^{\infty} \psi_{gr,j} u_{r,t-j} + \sum_{j=0}^{\infty} \psi_{gs,j} u_{s,t-j}\,, \qquad (46)$$

where $\bar{g}$ is the unconditional mean of g_t, and impulse response functions $\psi_{gg,j}$, $\psi_{g\pi,j}$, $\psi_{gr,j}$, and $\psi_{gs,j}$ are functions of $\mathbf{\Phi}$ and $\mathbf{\Sigma}$. So future output growth g_{t+h} can be represented as

$$g_{t+h} = \hat{g}_{t+h|t} + \sum_{j=0}^{h-1} \psi_{gg,j} u_{g,t+h-j} + \sum_{j=0}^{h-1} \psi_{g\pi,j} u_{\pi,t+h-j} + \sum_{j=0}^{h-1} \psi_{gr,j} u_{r,t+h-j} + \sum_{j=0}^{h-1} \psi_{gs,j} u_{s,t+h-j} \qquad (47)$$

where

$$\hat{g}_{t+h|t} = \overline{g} + \sum_{j=h}^{\infty} \psi_{gg,j} u_{g,t+h-j} + \sum_{j=h}^{\infty} \psi_{g\pi,j} u_{\pi,t+h-j} + \sum_{j=h}^{\infty} \psi_{gr,j} u_{r,t+h-j} + \sum_{j=h}^{\infty} \psi_{gs,j} u_{s,t+h-j} \qquad (48)$$

is the optimal forecast of g_{t+h} conditional on $\mathbf{X}_t$.

Since discount rates $r_t^{(n)} = a^{(n)} + \mathbf{b}^{(n)}{}'\mathbf{X}_t$ and term spreads $r_t^{(n)} - r_t^{(m)}$ are affine functions of $\mathbf{X}_t = (\mathbf{x}_t{}', \mathbf{x}_{t-1}{}', \mathbf{x}_{t-2}{}', \mathbf{x}_{t-3}{}')'$, we can also calculate corresponding impulse response functions, and represent them in VMA(∞) form. For example, $r_t^{(n)} - r_t^{(m)}$ can be represented as

$$r_t^{(n)} - r_t^{(m)} = \overline{r^{(n)} - r^{(m)}} + \sum_{j=0}^{\infty} \kappa_{g,j}^{(n,m)} u_{g,t-j} + \sum_{j=0}^{\infty} \kappa_{\pi,j}^{(n,m)} u_{\pi,t-j} + \sum_{j=0}^{\infty} \kappa_{r,j}^{(n,m)} u_{r,t-j} + \sum_{j=0}^{\infty} \kappa_{s,j}^{(n,m)} u_{s,t-j} , \qquad (49)$$

where $\overline{r^{(n)} - r^{(m)}}$ is the unconditional mean of $r_t^{(n)} - r_t^{(m)}$, and impulse response functions $\kappa_{g,j}^{(n,m)}$, $\kappa_{\pi,j}^{(n,m)}$, $\kappa_{r,j}^{(n,m)}$ and $\kappa_{s,j}^{(n,m)}$ are functions of $\mathbf{\Phi}$, $\mathbf{\Sigma}$, and $\boldsymbol{\delta}$.

Since $\mathbf{u}_t \sim N(\mathbf{0}, \mathbf{I})$ i.i.d., we can calculate the unconditional variances of the VAR variables, optimal forecasts for them, and term spreads. From (46), (48) and (49),

$$\begin{aligned} \sigma_g{}^2 &\equiv \text{var}(g_t) \\ &= \sum_{j=0}^{\infty} \psi_{gg,j}{}^2 + \sum_{j=0}^{\infty} \psi_{g\pi,j}{}^2 + \sum_{j=0}^{\infty} \psi_{gr,j}{}^2 + \sum_{j=0}^{\infty} \psi_{gs,j}{}^2 , \end{aligned} \qquad (50)$$

$$\begin{aligned} \sigma_{\hat{g},h}{}^2 &\equiv \text{var}(\hat{g}_{t+h|t}) \\ &= \sum_{j=h}^{\infty} \psi_{gg,j}{}^2 + \sum_{j=h}^{\infty} \psi_{g\pi,j}{}^2 + \sum_{j=h}^{\infty} \psi_{gr,j}{}^2 + \sum_{j=h}^{\infty} \psi_{gs,j}{}^2 , \end{aligned} \qquad (51)$$

$$\begin{aligned} (\sigma^{(n,m)})^2 &\equiv \text{var}(r_t^{(n)} - r_t^{(m)}) \\ &= \sum_{j=0}^{\infty} \kappa_{g,j}^{(n,m)2} + \sum_{j=0}^{\infty} \kappa_{\pi,j}^{(n,m)2} + \sum_{j=0}^{\infty} \kappa_{r,j}^{(n,m)2} + \sum_{j=0}^{\infty} \kappa_{s,j}^{(n,m)2} . \end{aligned} \qquad (52)$$

Similarly we can calculate the correlations among these variables. The correlation between future output growth g_{t+h} and the current term spread $r_t^{(n)} - r_t^{(m)}$ can be represented as

$$\operatorname{corr}(g_{t+h},\, r_t^{(n)} - r_t^{(m)}) \\ = \sum_{j=0}^{\infty} \frac{\psi_{gg,j+h}\kappa_{g,j}^{(n,m)}}{\sigma_g \sigma^{(n,m)}} + \sum_{j=0}^{\infty} \frac{\psi_{g\pi,j+h}\kappa_{\pi,j}^{(n,m)}}{\sigma_g \sigma^{(n,m)}} + \sum_{j=0}^{\infty} \frac{\psi_{gr,j+h}\kappa_{r,j}^{(n,m)}}{\sigma_g \sigma^{(n,m)}} + \sum_{j=0}^{\infty} \frac{\psi_{gs,j+h}\kappa_{s,j}^{(n,m)}}{\sigma_g \sigma^{(n,m)}} . \tag{53}$$

Since the forecasting error of the optimal forecast $g_{t+h} - \hat{g}_{t+h|t}$ is unable to be predicted by any variable known at time *t*, such as $r_t^{(n)} - r_t^{(m)}$,

$$\operatorname{corr}(g_{t+h},\, r_t^{(n)} - r_t^{(m)}) = \operatorname{corr}(\hat{g}_{t+h|t},\, r_t^{(n)} - r_t^{(m)}) . \tag{54}$$

By squaring the correlation, we can obtain the R^2. For example, the R^2 for regression (4) can be represented as

$$R_{g,h}^{2(n,m)} = \operatorname{corr}(\hat{g}_{t+h|t},\, r_t^{(n)} - r_t^{(m)})^2 . \tag{55}$$

Since the R^2s are functions of parameters in our VAR-ATSM, we can calculate them from the estimates of the parameters. We call these the model-implied R^2s. Equation (54) implies that if $r_t^{(n)} - r_t^{(m)}$ is a good predictor for future output growth g_{t+h}, $r_t^{(n)} - r_t^{(m)}$ should respond to exogenous shocks in a similar way to $\hat{g}_{t+h|t}$. We investigate this by looking at the variance decomposition of $\hat{g}_{t+h|t}$ in the next subsection. Finally, as we can see from (53)-(55), the R^2s depend on the sum of products of the impulse response functions for regressands and regressors. Note that, in (53), indexes for ψ's start from *t+h*, not *t*, because future shocks $\mathbf{u}_{t+1},\ldots,\mathbf{u}_{t+h}$ are unpredictable. This implies that since the ψ's typically decay with the horizon *j*, $r_t^{(n)} - r_t^{(m)}$ is a good predictor if it is responsive to recent shocks, i.e. κ's are large for smaller *j*.

6.2. Why do term spreads have predictive power?

Figure 3 displays the model-implied R^2s from regressions (4)-(6) for three selected term spreads, and is the model-calculated analog of Figure 2. The results show that the model-implied R^2s replicate three properties of the sample R^2s in Figure 2 very well. First, the 12Q-8Q spread performs better than the 20Q-1Q spread, except for output growth predictions at shorter horizons. Second, the 2Q-1Q spread is almost useless. Finally, it is difficult to predict output growth at 1Q ahead. It is therefore reasonable to try to explain the sample R^2s in Figure 2 in terms of the factors that determine the model-implied R^2s in Figure 3. Since the model-implied R^2s are functions of the parameters in our VAR-ATSM, we can analyze how these parameters affect the R^2s.

Figure 4 shows the impulse response functions of the VAR variables g_t, π_t, $r_t^{(1)}$, and s_t to one unit exogenous shocks. These are based on the estimates from the restricted GMM estimation of the VAR-ATSM. In general, these results are consistent with those in the VAR literature. For example, (4-a) and (4-b) show that the short rate, the instrument of the monetary policy authority, responds positively to output growth and inflation shocks. Panel (4-c) demonstrates that the estimated monetary policy shock sharply reduces output growth. This shock also suppresses inflation rates in the long run. These reasonable results imply that estimates of the monetary policy shock are reasonable. Further support is provided by Panel (4-d). As we discussed in Section 3, the most questionable part of our identification strategy may come from the contamination between the monetary policy shock and the spread shock. Panel (4-d) indicates that the estimated spread shock raises output growth and suppresses inflation. Since output growth and inflation should respond to a monetary policy shock in the same direction, the results in (4-d) suggest that the spread shock is not measuring a change in monetary policy.

Figure 5 shows variance decompositions of the optimal forecasts, where the variances of forecasts such as (51) are normalized to unity. As discussed in the previous subsection, this indicates which exogenous shocks should be useful for prediction. Panel (5-a) shows that the output growth shock dominates predictions of output growth at one quarter ahead. Then around 2-4 quarters ahead, the monetary policy shock is the most important. The importance of the inflation shock increases with the forecasting horizon, and this shock finally becomes most influential at 12 quarters ahead. These results are consistent with the impulse response functions in Figure 4. The output growth shock causes a sharp jump in output growth, but only in the short run. The monetary policy shock has a negative effect on output growth, but with 2-4 quarter lags. In the long run, the rise in the short rate induced by the inflation shock is persistent, and this acts to suppress output growth. Panels (5-b) and (5-c) show that the inflation shock is most important for predicting inflation and short rates at most horizons. Accordingly, the response of the term spread to the inflation shock is crucial for specifying the source of its predictive power, especially at longer horizons. Note that, as Figure 4 implies, the effects of exogenous shocks decay with the horizon. So we can also say that good predictors should respond to recent shocks rather than old shocks.

Figure 6 shows impulse response functions of selected discount rates. There are three notable features. First, the effect of the inflation shock on the levels of the discount rates is highly persistent. In fact, the discount rates do not return to zero even after 40 quarters. Since good predictors should respond to recent shocks rather than old shocks, this is an important reason why levels of yield curves do not have great predictive power.

Second, discount rates with different maturities display different responses to recent shocks, while they respond to old shocks in similar ways. This implies that most movements in term spreads are due to recent shocks, because old shocks result in almost parallel shifts of the yield curve. In fact, the upper graphs of Figure 7 illustrate the considerable dependence of both the 20Q-1Q and 12Q-8Q spreads on recent shocks. This is one reason why term spreads have predictive power.

Why do discount rates respond like this? We find that the time-varying market price of risk plays the following important role. As discussed in Section 5, it is the parameters corresponding to the effects of the output growth and inflation rates on the market price of output growth risk, δ_{11} and δ_{12}, that have the most influence on movements in long rates. Of these, only δ_{12} has a supportive role to play in the predictive relationship. As shown in Figure 5, the inflation shock is the crucial element in the predictive relationship, and a positive δ_{12} causes the market price of output growth risk to respond positively to the shock. In contrast to this, a positive δ_{11} reduces predictive power. As shown in (4-b), a positive inflation shock causes a decrease in the output growth rate, which has a negative effect on the market price of output growth risk. Since the effect from δ_{12} dominates the effect from δ_{11}, the market price of output growth risk responds positively and so the term premium responses negatively to the inflation shock.

In evaluating the effect from δ_{12}, we calculated the impulse response functions of discount rates when $\delta_{12} = 0$ and other parameters are unchanged in Figure 8. The main change in the impulse response functions appears in (8-b), which is totally different from (6-b). In (6-b), the responses of longer rates are smaller than those of the short rate, and the difference between their responses almost disappears around 20 quarters ahead. On the other hand, in (8-b), the responses of longer rates are stronger than those of the short rate, and this disparity does not disappear even around 40 quarter ahead. Why are their responses so different? The expectations hypothesis states that the long rate is the average of expected short rates plus a constant term premium. Whether we look at (6-b) or (8-b), the inflation shock continues to raise the short rate up to around 20 quarters ahead. So, according to the hypothesis, the initial responses of long rates with maturities up to 20 quarters should be stronger than the response of the short rate, as illustrated in (8-b). Since δ_{12} is positive, however, the inflation shock raises the market price of output growth risk, and so reduces the term premium. This is why long rates respond less strongly than the short rate in (6-b). The difference between the responses in (6-b) and (8-b) has a significant effect on the predictive power of term spreads. Figure 9 gives model-implied R^2s for the case when $\delta_{12} = 0$. Surprisingly, the R^2s almost disappear. This enables us to conclude that a positive δ_{12}, which can be interpreted in terms of consumers' willingness to pay a higher premium for an output growth risk hedge during a higher inflation regime, is a key explanation for the predictive power of the term spread.

The last notable feature of Figure 6 is the lagged response of the 1Q rate (the monetary policy authority) to output growth and inflation shocks. Panel (6-a) shows that the immediate response of the 1Q rate to an output growth shock is the smallest among the discount rates, although the response of the 1Q rate is largest several quarters ahead. Panel (6-b) shows that the immediate response of the 1Q rate to an inflation shock is smaller than that of the 2Q rate, and almost coincides with the response of the 8Q rate. These results are consistent with inertial behavior by the monetary policy authority, as empirically shown by, among others, Clarida, Gali, and Gertler (2000). The lower graphs in Figure 7 show the impulse response functions of 20Q-1Q and 12Q-8Q spreads to output growth and inflation shocks. The immediate response of the 20Q-1Q spread is much weaker than that of the 12Q-8Q spread because of the slow response of the 1Q rate. Since recent shocks are very important for predictive purposes, we can conclude that this is the reason behind the inferior performance of the 20Q-1Q spread compared to the 12Q-8Q spread. That is, the monetary authority's inertial behavior disturbs the responses of term spreads using the short end of the yield curve to output growth and inflation shocks.

Further support for this view is provided by the correlations between future predicted variables and current term spreads. Since model-implied R^2s are squares of these model-implied correlations, we can use the correlations to analyze why we found the R^2s shown in Figures 2 and 3. Equation (53) has four summed terms, each of which can be interpreted as the contribution of the corresponding exogenous shock to the predictive relationship. Figure 10 shows the contributions of exogenous shocks to the absolute values of the correlations with the 20Q-1Q and 12Q-8Q spreads. The output growth and inflation shocks contribute to the correlations with the 12Q-8Q spread rather than with the 20Q-1Q spread. These differences explain why the 12Q-8Q spread is useful for prediction. This result is consistent with our discussion of the lower graphs in Figure 7.

Another notable feature of Figure 10 is the hump-shaped contribution of monetary policy shocks to output growth predictions. So we can conclude that the hump-shape of the R^2s for the output growth predictions is attributable to the monetary policy shock. That is, the monetary policy shock affects output growth with a lag, while the term structure responds to the shock immediately. This difference in timing makes it harder for term spreads to help forecast output growth at short horizons.

Finally, Figure 11 shows the contributions in the case where $\delta_{12} = 0$. Obviously the sharp drops of R^2s are attributable to the different sign of the contribution of the inflation shock, which is caused by the strong long rate response to the shock.

7. Conclusion

Why do term spreads predict output growth, inflation, and short rates? In answering this question, we used a VAR-ATSM model with four lags and four variables, which is less restricted than similar affine term structure models with observable factors in the existing literature. We succeeded in estimating this model using an efficient method.

We have three main findings. First, the time-varying market price of output growth risk, which is sensitive to the inflation rate, plays a key role in explaining why the term spread helps forecast output growth, inflation, and interest rates. This finding can be interpreted as follows. When the inflation rate is higher, consumers are willing to pay a higher premium for an output growth risk hedge, possibly because, within this higher inflation environment, the Fed's response to an output growth shock is less accommodating and so marginal utility is more sensitive to the shock. This causes term spreads to react to recent inflation shocks, which also proves useful for forming longer-run forecasts. Second, we also found that term spreads using the short end of yield curve have less predictive power than many spreads between longer rates. This fact is attributable to the inertial character of monetary policy. Finally, it is hard to predict output growth with term spreads at short horizons, because monetary policy shocks affect output growth with a lag while the term structure responds to the shock immediately.

References

Ang, A. and M. Piazzesi, 2003. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. Journal of Monetary Economics 50, 745-787.

Ang, A., M. Piazzesi and M. Wei, 2003. What does the yield curve tell us about GDP growth? Working paper, Andersen school, UCLA.

Backus, D., S. Foresi and C. Telmer, 1998. Discrete-time models of bond pricing. Working paper, NBER 6736.

Balduzzi, P., G. Bertola and S. Foresi, 1997. A model of target changes and the term structure of interest rates. Journal of Monetary Economics 39, 223-249.

Bekaert, G., R. J. Hodrick and D. A. Marshall, 2001. Peso problem explanations for term structure anomalies. Journal of Monetary Economics 48, 241-270.

Bernanke, B. S., M. Gertler and M. Watson, 1997. Systematic monetary policy and the effects of oil price shocks. Brookings Papers on Economic Activity 1:1997, 91-142.

Bernard, H. and S. Gerlach, 1998. Does the term structure predict recessions? The international evidence. International Journal of Finance and Economics 3, 195-215.

Bonser-Neal, C., and T. R. Morley, 1997. Does the yield spread predict real economic activity? A multicountry analysis. Economic Review 82, 3rd quarter, 37-53.

Breeden, D. T., 1986. Consumption, production, inflation, and interest rates. Journal of Financial Economics, 16, 3-39.

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay, 1997. The econometrics of financial markets (Princeton University Press).

Campbell, J. Y., and R. J. Shiller, 1991. Yield spread and interest rate movements: a bird's eye view. Review of Economic Studies 58, 495-514.

Christiano, L. J., M. Eichenbaum and C. L. Evans, 1999. Monetary policy shocks: what have we learned and to what end? in J. B. Taylor and M. Woodford, eds., Handbook of macroeconomics, Vol. 1.

Clarida, R., J. Gali and M. Gertler, 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. The Quarterly Journal of Economics 115, 147-180.

Cliff, M. T., 2000. GMM and MINZ program libraries for MATLAB. Krannet Graduate School of Management, Purdue University.

Cochrane, J. H., 2001. Asset pricing (Princeton University Press).

Cochrane, J. H., and M. Piazzesi, 2002. Bond risk premia. Working paper, Andersen school, UCLA.

Dai, Q., and K. J. Singleton, 2002. Expectation puzzles, time-varying risk premia, and affine models of the term structure. Journal of Financial Economics 63, 3, 415-441.

Dewachter, H., and M. Lyrio, 2004. Macro factors and the term structure of interest rates. Journal of Money, Credit and, Banking, forthcoming.

Dotsey, M., 1998. The predictive content of the interest rate term spread for future economic growth. Economic Quarterly, Federal Reserve Bank of Richmond 84, 3, 31-51.

Dotsey, M. and C. Otrok, 1995. The rational expectations hypothesis of the term structure, monetary policy, and time-varying term premia. Economic Quarterly, Federal Reserve Bank of Richmond 81, 1, 65-81.

Dueker, M., 1997. Strengthening the case for the yield curve as a predictor of U.S. recessions. Review, Federal Reserve Bank of St. Louis 79, March/April, 41-51.

Duffee, G. R., 2002. Term premia and interest rate forecasts. Journal of Finance 57, 1, 405-443.

Duffie, J. D. and R. Kan, 1996. A yield-factor model of interest rates. Mathematical Finance 6, 379-406.

Engle, R. F., and D. M. Lilien, and Russell P. Robins, Estimating time varying risk premia in the term structure: the Arch-M model, Econometrica 55, 2, 391-407.

Engle, R. F., and V. K. Ng, 1993. Time-varying volatility and the dynamic behavior of the term structure. Journal of Money, Credit, and Banking 25, 3, 336-349.

Estrella, A., and G. Hardouvelis, 1991. The term structure as a predictor of real economic activity. Journal of Finance 46, 2, 555-576.

Estrella, A., and F. S. Mishkin, 1997. The predictive power of the term structure of interest rates in Europe and the United States: implications for the European Central Bank. European Economics Review 41, July, 1375-1401.

Evans, C. L., and D. A. Marshall, 1998. Monetary policy and the term structure of nominal interest rates, evidence and theory. Carnegie-Rochester Conference Series on Public Policy 49, 53-111.

Fama, E. F., 1975. Short term interest rates as predictors of inflation. American Economic Review 65, 269-282.

Fama, E. F., 1990. Term-structure forecasts of interest rates, inflation, and real returns. Journal of Monetary Economics 25, 1, 59-76.

Fama, E. F., and R. R. Bliss, 1987. The information in long-maturity forward rates. American Economic Review 77, 4, 680-692.

Gordon, D. B. and E. M. Leeper, 1994. The dynamic impacts of monetary policy: an exercise in tentative identification. Journal of Political Economy 102, 6, 1228-1247.

Hamilton, J. D., 1994. Time series analysis (Princeton University Press).

Hamilton, J. D. and D. H. Kim, 2002. A re-examination of the predictability of the yield spread for real economic activity. Journal of Money, Credit, and Banking 34, 2, 340-360.

Harrison, J. M., and D. M. Kreps, 1979. Martingales and arbitrage in multi-period securities markets. Journal of Economic Theory 20, 381-408.

Hansen, L. P., 1982. Large sample properties of generalized method of moments estimators. Econometrica 50, 1029-1054.

Hansen, L. P., and K. Singleton, 1982. Generalized instrumental variables estimation of nonlinear rational expectations models. Econometrica 50, 1269-1286.

Hansen, L. P., and R. Jagannathan, 1991. Assessing specification errors in stochastic discount factor models. Journal of Finance 52, 557-590.

Harvey, C., 1988. The real term structure and consumption growth. Journal of Financial Economics 22, 305-334.

Hardouvelis, G., 1994. The term structure spread and future changes in long and short rates in the G7 countries. Is there a puzzle? Journal of Monetary Economics 33, 255-283.

Haubrich, J. G., and A. M. Dombrosky, 1996. Predicting real growth using the yield curve. Economic Review, Federal Reserve Bank of Cleveland 32, 1st quarter, 26-35.

Hordahl, P., O. Tristani, and D. Vestin, 2002. A joint econometric model of macroeconomic and term structure dynamics. Working paper, European Central Bank.

Jorion, P., and F. S. Mishkin, 1991. A multi-country comparison of term-structure forecasts at long horizons. Journal of Financial Economics 29, 1, 59-80.

Kozicki, S., 1997. Predicting real growth and inflation of monetary policy. Economic Review, Federal Reserve Bank of Kansas City 82, 4th quarter, 39-57.

Longstaff, F. A., and E. S. Schwartz, 1992. Interest rate volatility and the term structure: a two-factor general equilibrium model. Journal of Finance 47, Sep., 1259-1282.

Mishkin, F. S., 1988. The information in the term structure: some further results. Journal of Applied Econometrics 3, 307-314.

Mishkin, F. S., 1990a. What does the term structure tell us about future inflation? Journal of Monetary Economics 25, January, 77-95.

Mishkin, F. S., 1990b. The information in the longer maturity term structure about future inflation. Quarterly Journal of Economics 105, 3, 815-828.

Mishkin, F. S., 1991. A multi-country study of the information in the shorter maturity term structure about future inflation. Journal of International Money and Finance 19, March, 2-22.

Mankiw, N. G. and J. A. Miron, 1986. The changing behavior of the term structure of interest rates. Quarterly Journal of Economics 101, 211-228.

Newey, W. K., and K. D. West, 1987. A simple positive semi-definite heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, May, 703-778.

Pagan, A. R. and J. C. Robertson, 1995. Resolving the liquidity effect. Federal Reserve Bank of St. Louis Review 77, 3, 33-54.

Piazzesi, M., 2001. An econometric model of the yield curve with macroeconomic jump effects. Working paper, Andersen school, UCLA.

Plosser, C. I., and K. G. Rouwenhost, 1994. International term structure and real economic growth. Journal of Monetary Economics 33, February, 135-155.

Roberds, W., and C. H. Whiteman, 1999. Endogenous term premia and anomalies in the term structure of interest rates: explaining the predictability smile. Journal of Monetary Economics 44, 555-580.

Rudebusch, G. D., 1995. Federal Reserve interest rate targeting, rational expectations, and the term structure. Journal of Monetary Economics 35, 2, 245-274.

Rudebusch, G. D., and T. Wu, 2004. A macro-finance model of the term structure, monetary policy, and the economy. FRBSF working paper 2003-17.

Sims, C. A., 1980. Macroeconomics and reality. Econometrica 48, 1, 1-48.

Stock, J. H., and M. W. Watson, 1989. New index of coincident and leading indicators. in O. Blanchard and S. Fischer, eds., NBER Macroeconomic Annual, Vol. 4 (MIT Press).

Taylor, J., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Conference on Public Policy 39, 195-214.

Vasicek, O., 1977. An equilibrium characterization of the term structure. Journal of Financial Economics 5, 177-188.

Walsh, C. E., 1998. Monetary theory and policy (The MIT Press).

Wu, T., 2001. Macro factors and the affine term structure of interest rates. Working paper.

Table 1: Estimated parameters of AR(1) model for the short rate factor

c_r	ϕ_r	σ_r
0.1499 (0.0621)	0.9037 (0.0362)	0.0057

The AR(1) model for the short rate (7) is estimated by OLS. Standard errors are in parentheses. The sample period is 1964:1Q-2001:4Q.

Table 2: Estimates of $\mathbf{\Sigma}$

	Shocks			
	$u_{g,t}$	$u_{\pi,t}$	$u_{r,t}$	$u_{s,t}$
g_t	0.0076	0	0	0
π_t	-0.0001	0.0025	0	0
$r_t^{(1)}$	0.0007	0.0004	0.0023	0
s_t	-0.0002	-0.0002	-0.0014	0.0012

$\mathbf{\Sigma}$ is estimated by GMM, as introduced in Section 5. The sample period is 1964:1Q-2001:4Q.

Table 3: Estimates of $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$

	$\boldsymbol{\gamma}$	$\boldsymbol{\delta}$			
		g_t	π_t	$r_t^{(1)}$	s_t
$\lambda_{g,t}$	-0.50 (0.43)	140* (22)	78* (32)	-26 (26)	-43 (63)
$\lambda_{\pi,t}$	-0.89 (0.90)	-99* (48)	62 (57)	-60 (51)	-177 (114)
$\lambda_{r,t}$	0.25 (0.25)	-23* (10)	-13 (11)	-30* (12)	-45* (17)
$\lambda_{s,t}$	0.67* (0.31)	-46 (29)	28* (14)	-30 (16)	-114* (26)
mean of factor		0.0080	0.0102	0.0159	0.0026
s.d. of factor		0.0089	0.0061	0.0065	0.0032

$\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are estimated by GMM, as introduced in Section 5. The estimates with * are significantly different from zero at 5%. Standard errors are in parentheses. Last two rows report means and standard deviations of g_t, π_t, $r_t^{(1)}$, and s_t. The sample period is 1964:1Q-2001:4Q.

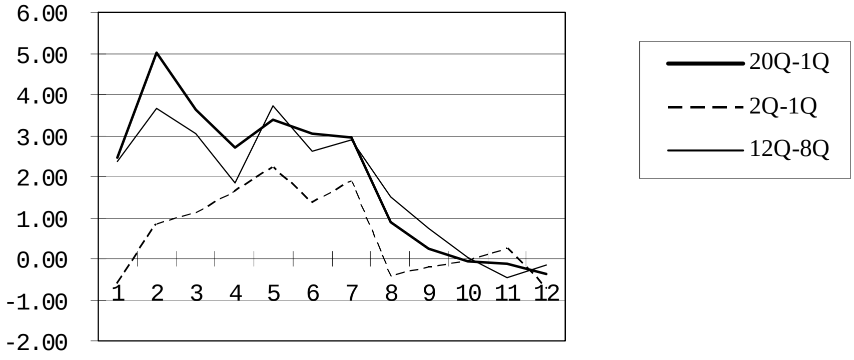
Table 4: The comparison between model-implied rates and sample rates

maturity (n)		2	4	8	12	16	20
mean	$\hat{r}_t^{(n)}$	0.0166	0.0172	0.0177	0.0181	0.0184	0.0186
	$r_t^{(n)}$	0.0166	0.0172	0.0177	0.0181	0.0184	0.0185
s.d.	$\hat{r}_t^{(n)}$	0.0064	0.0062	0.0060	0.0059	0.0059	0.0058
	$r_t^{(n)}$	0.0065	0.0063	0.0062	0.0060	0.0059	0.0058
correlation		0.9927	0.9869	0.9885	0.9902	0.9926	0.9928

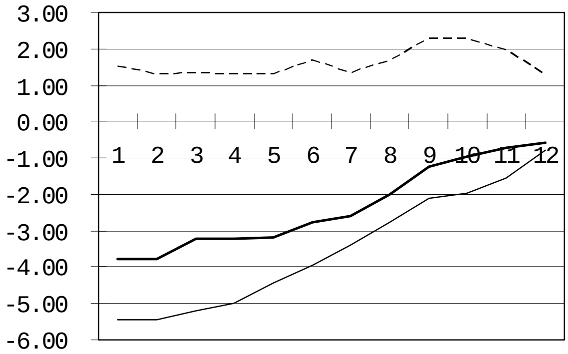
Means and standard deviations of model implied rates $\hat{r}_t^{(n)}$ and sample rates $r_t^{(n)}$ are reported, as well as correlations between them. The sample period is 1964:1Q-2001:4Q.

Figure 1: The *t*-stats of OLS regressions

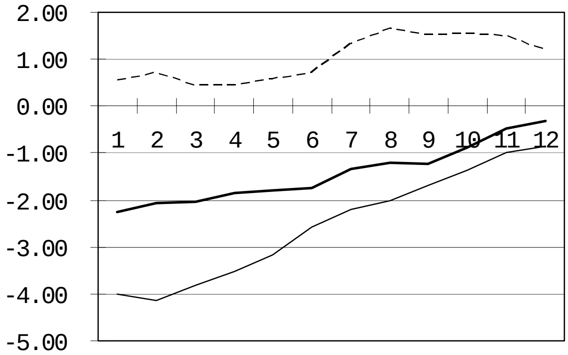
(1-a) Output growth regression



(1-b) Inflation regression

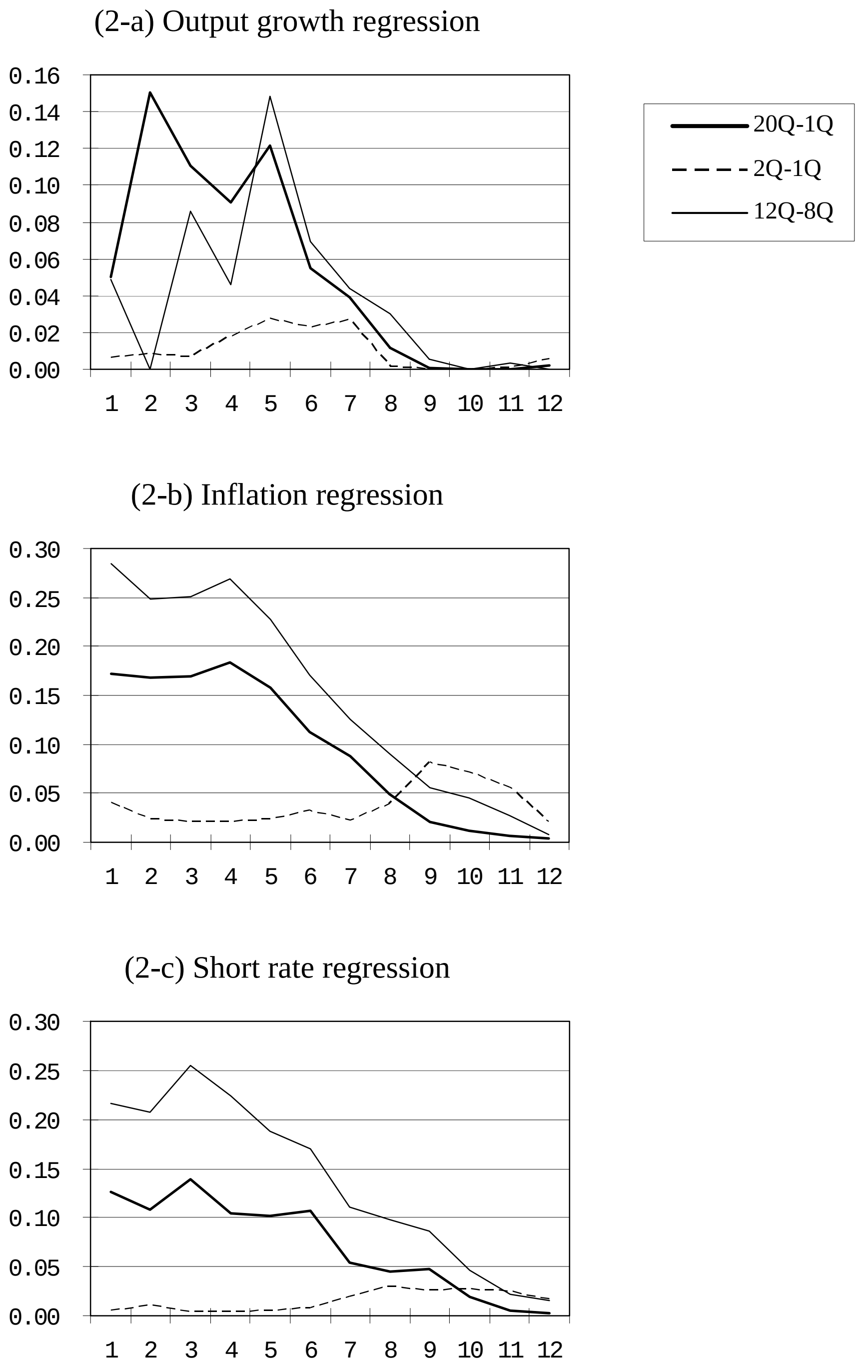


(1-c) Short rate regression



The *t*-stats of OLS regressions (4)-(6) are reported. The horizontal axes correspond to forecasting horizons (quarters). Thick, broken, and thin lines correspond to 20Q-1Q, 2Q-1Q, and 12Q-8Q term spreads respectively. The sample period is 1964:1Q-2001:4Q.

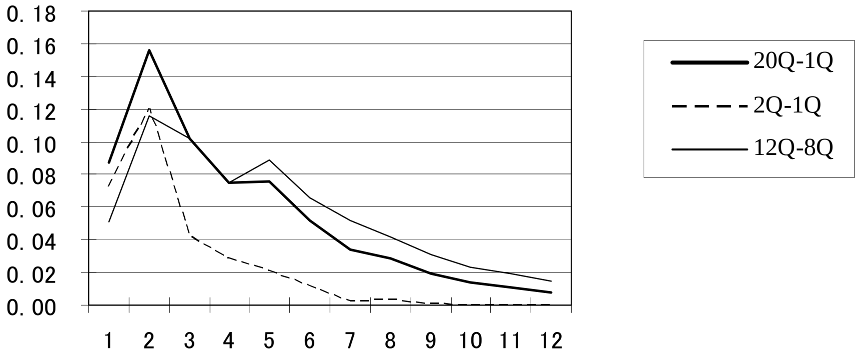
Figure 2: The sample R^2s of OLS regressions



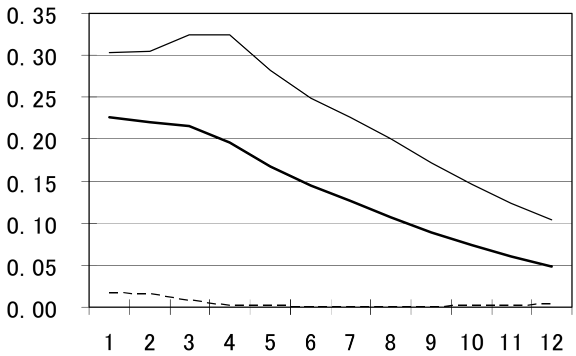
The sample R^2s of OLS regressions (4)-(6) are reported. The horizontal axes correspond to forecasting horizons (quarters). Thick, broken, and thin lines correspond to 20Q-1Q, 2Q-1Q, and 12Q-8Q term spreads respectively. The sample period is 1964:1Q-2001:4Q.

Figure 3: The model-implied R^2s

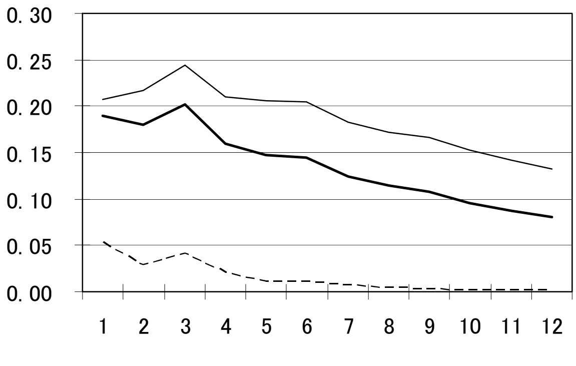
(3-a) Output growth regression



(3-b) Inflation regression

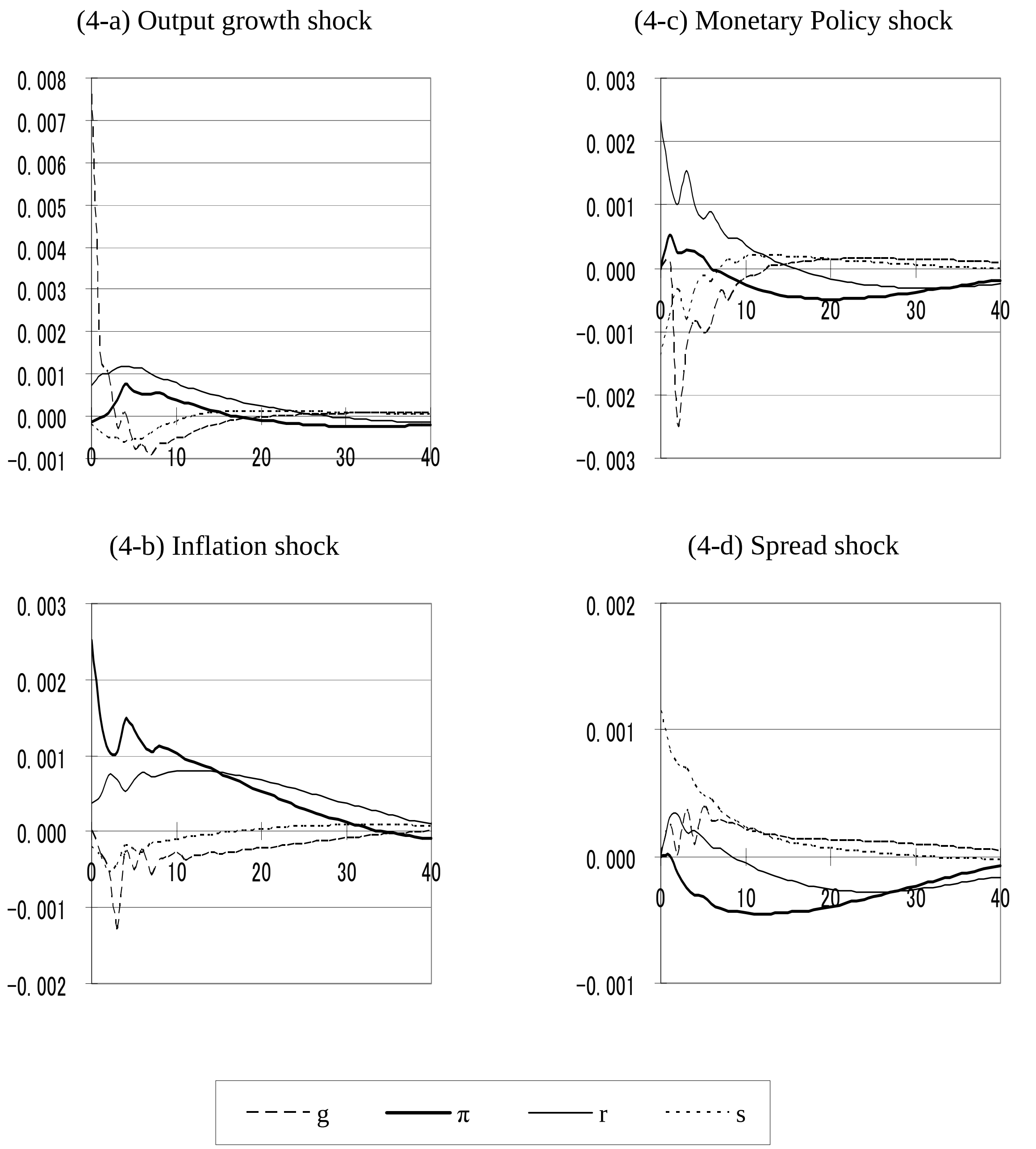


(3-c) Short rate regression



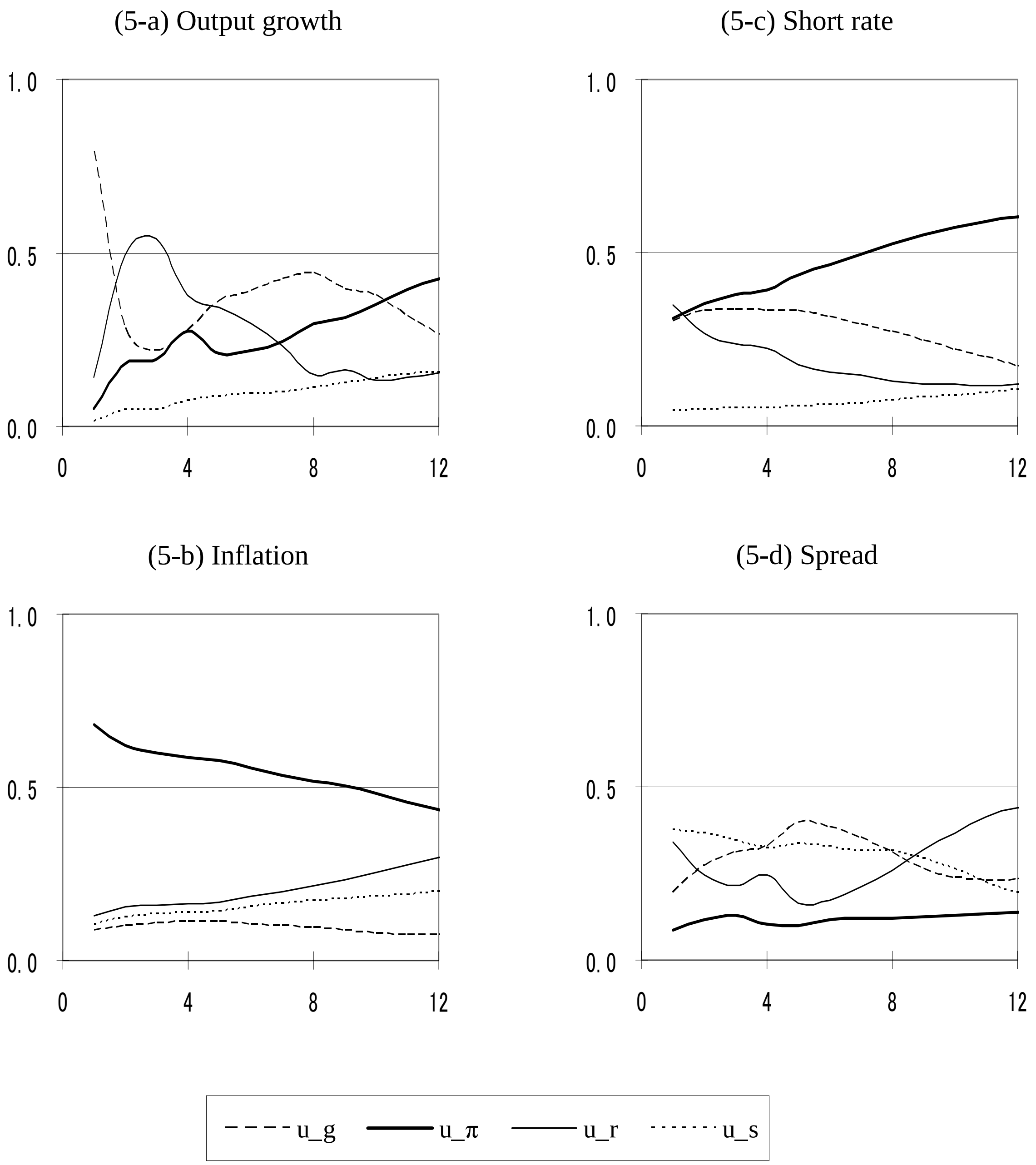
The model-implied R^2s of regressions (4)-(6) are reported. The horizontal axes correspond to forecasting horizons (quarters). Thick, broken, and thin lines correspond to 20Q-1Q, 2Q-1Q, and 12Q-8Q term spreads respectively. The sample period is 1964:1Q-2001:4Q.

Figure 4: The impulse response functions of VAR variables



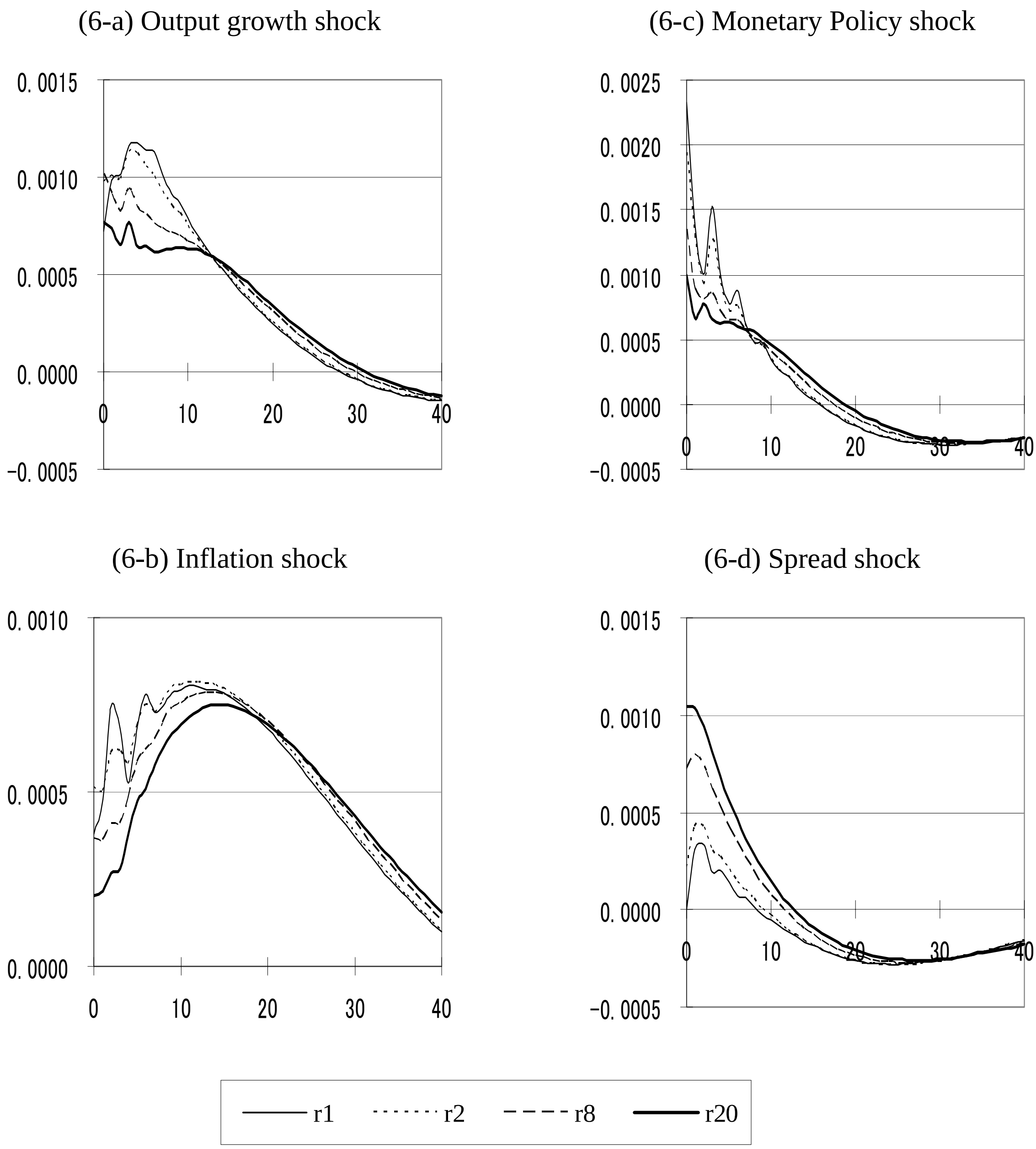
The impulse responses of VAR variables to one-unit exogenous shocks are reported. Broken, thick, thin, and dotted lines correspond to the responses of output growth, inflation, short rates, and term spread respectively. The horizontal axes correspond to horizons (quarters). The sample period is 1964:1Q-2001:4Q.

Figure 5: The variance decompositions of the optimal forecasts for VAR variables



The variance decompositions of the optimal forecasts of VAR variables, in which the variances of the forecasts are normalized to unity, are reported. Broken, thick, thin, and dotted lines correspond to output growth, inflation, monetary policy, and spread shocks respectively. The horizontal axes correspond to forecasting horizons (quarters). The sample period is 1964:1Q-2001:4Q.

Figure 6: The impulse response functions of discount rates

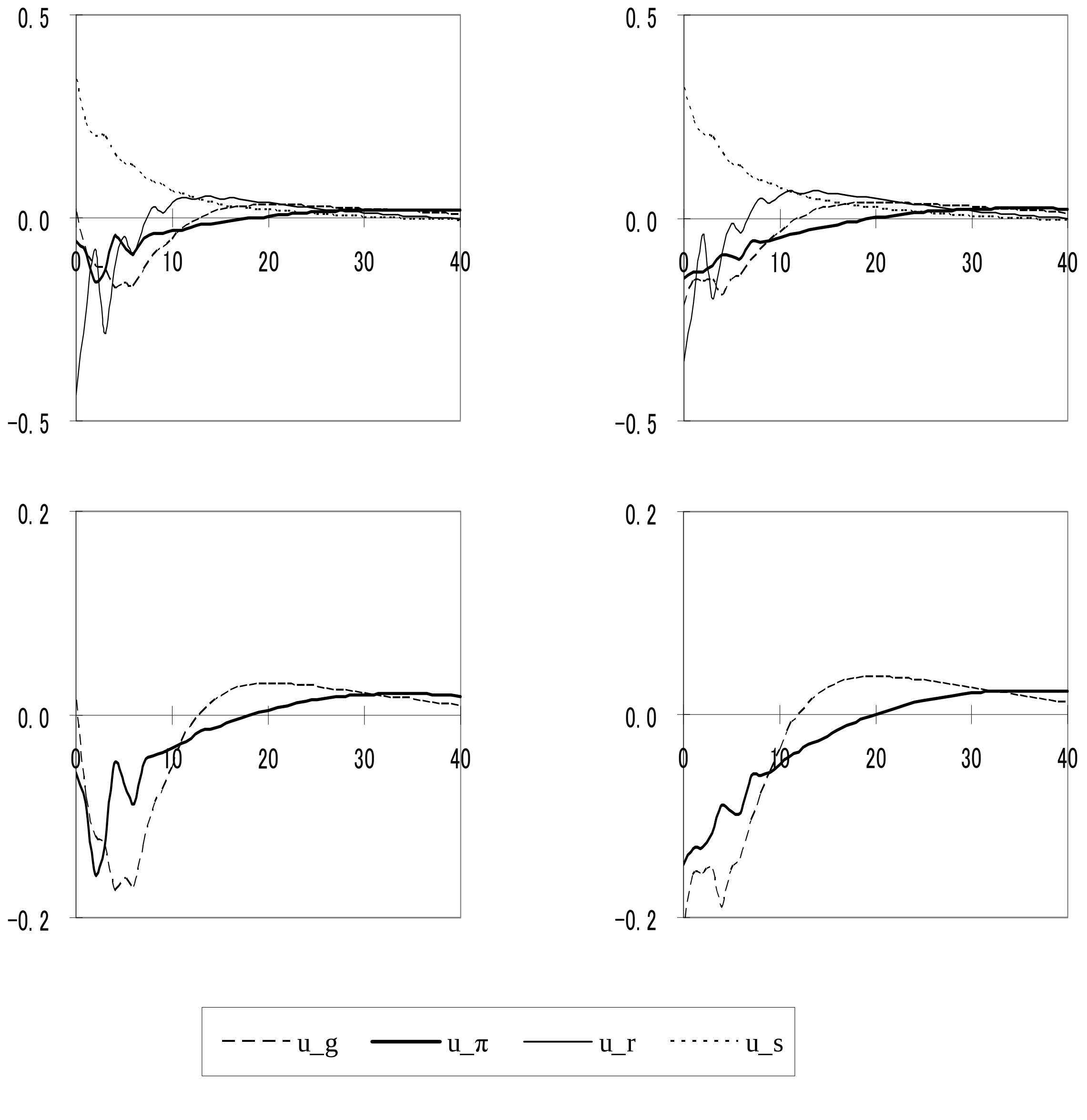


The impulse responses of selected discount rates to one-unit exogenous shocks are reported. Thin, dotted, broken, and thick lines correspond to 1Q, 2Q, 8Q, and 20Q rates respectively. The horizontal axes correspond to horizons (quarters). The sample period is 1964:1Q-2001:4Q.

Figure 7: The impulse response functions of term spreads

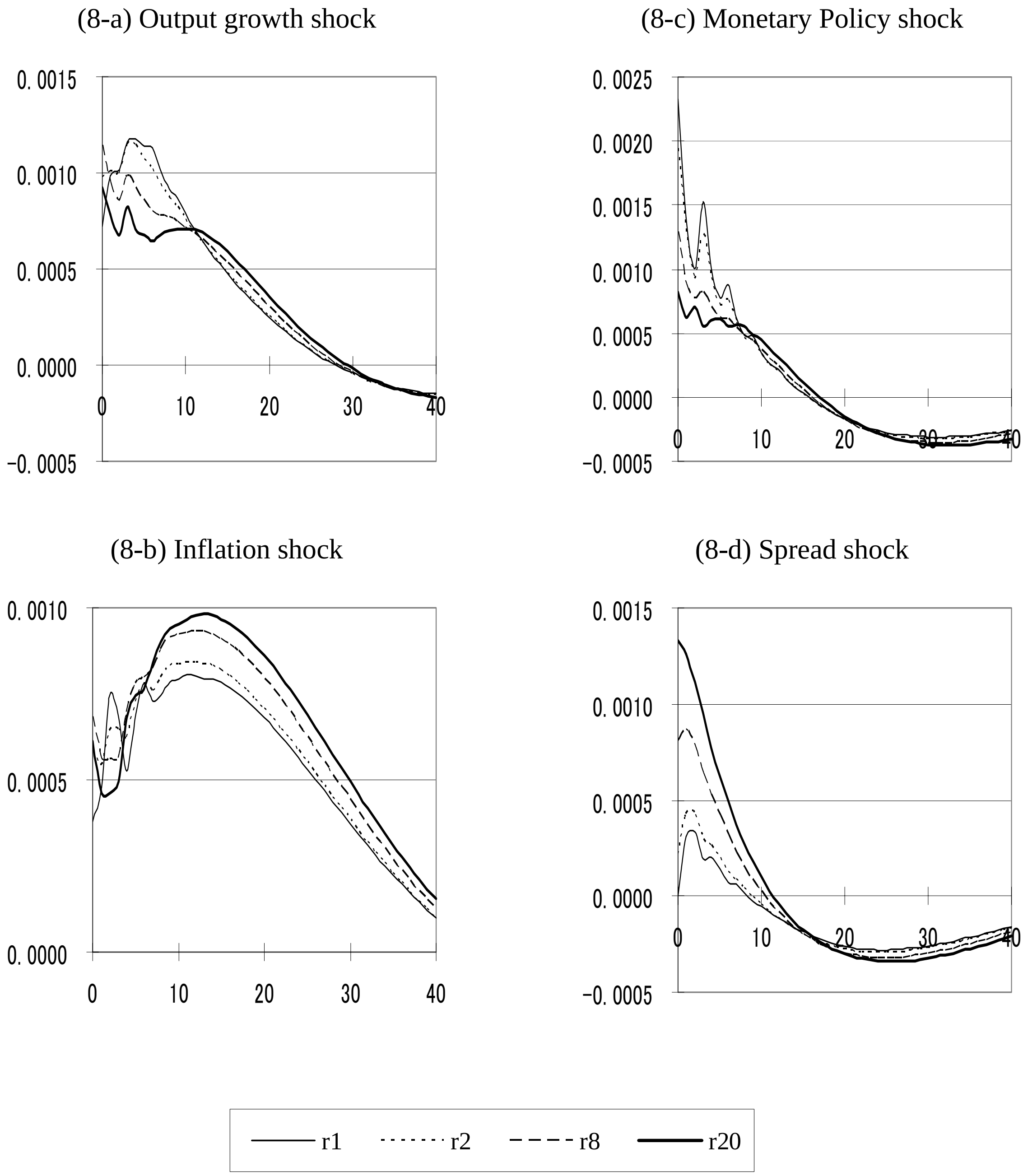
(7-a) The impulse response functions of the 20Q-1Q spread

(7-b) The impulse response functions of the 12Q-8Q spread



The impulse responses of 20Q-1Q and 12Q-8Q spreads to one-unit exogenous shocks are shown in the upper graphs. The scales are normalized so that variances of spreads equal unity. Lower graphs show magnified impulse responses to output growth and inflation shocks. Broken, thick, thin, and dotted lines correspond to output growth, inflation, monetary policy, and spread shocks respectively. The horizontal axes correspond to horizons (quarters). The sample period is 1964:1Q-2001:4Q.

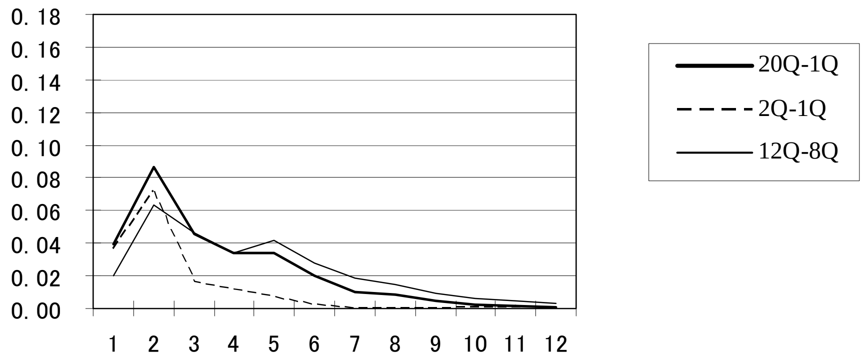
Figure 8: The impulse response functions of discount rates in the case of $\delta_{12} = 0$



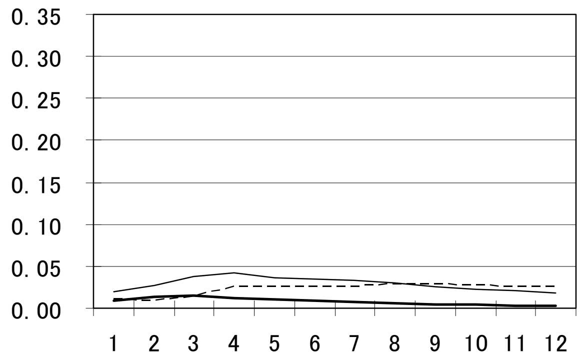
The impulse responses of discount rates to one-unit exogenous shocks are reported. Thin, dotted, broken, and thick lines correspond to 1Q, 2Q, 8Q, and 20Q rates respectively. The horizontal axes correspond to horizons (quarters). The sample period is 1964:1Q-2001:4Q.

Figure 9: The model-implied R^2s in the case of $\delta_{12} = 0$

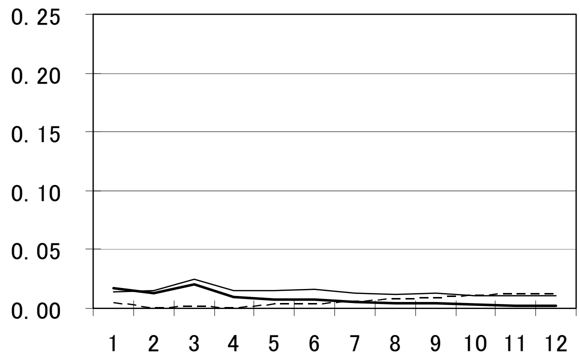
(9-a) Output growth regression



(9-b) Inflation regression

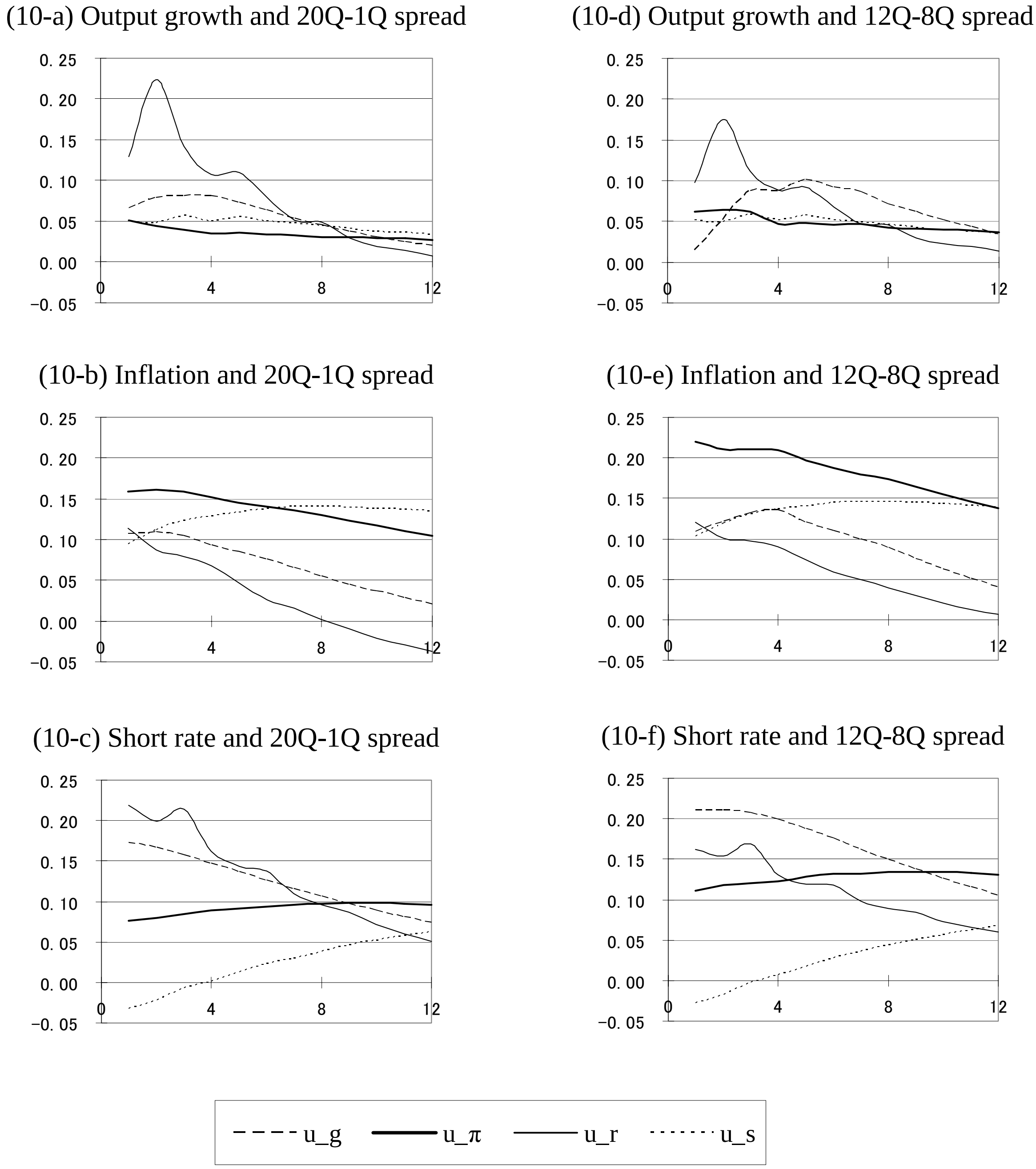


(9-c) Short rate regression



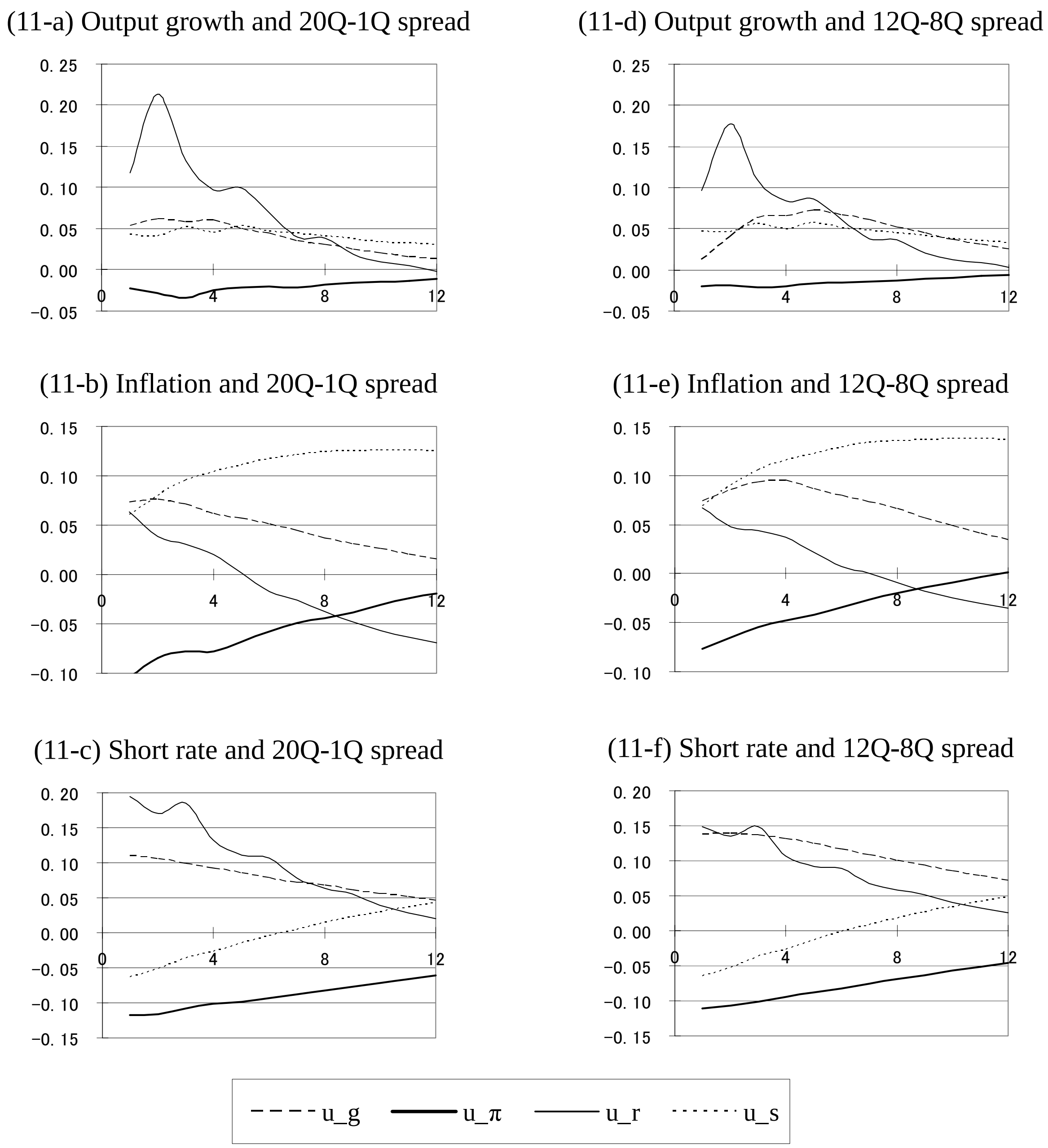
The model-implied R^2s of regressions (4)-(6) are reported. The horizontal axes correspond to forecasting horizons (quarters). Thick, broken, and thin lines correspond to 20Q-1Q, 2Q-1Q, and 12Q-8Q term spreads respectively. The sample period is 1964:1Q-2001:4Q.

Figure 10: Decompositions of correlations between future VAR variables and term spreads



The contributions of shocks to the correlations between future VAR variables and current term spreads are shown. Since the correlations of term spreads with the inflation rate and the short rate are negative, the graphs are flipped for (10-b), (10-c), (10-e), and (10-f). Broken, thick, thin, and dotted lines correspond to output growth, inflation, monetary policy, and spread shocks respectively. The horizontal axes correspond to forecasting horizons (quarters). The sample period is 1964:1Q-2001:4Q.

Figure 11: Decompositions of correlations between future VAR variables and term spreads in the case of $\delta_{12} = 0$



The contributions of shocks to the correlations between future VAR variables and current term spreads are shown. Since the correlations of term spreads with the inflation rate and the short rate are negative, the graphs are flipped for (11-b), (11-c), (11-e), and (11-f). Broken, thick, thin, and dotted lines correspond to output growth, inflation, monetary policy, and spread shocks respectively. The horizontal axes correspond to forecasting horizons (quarters). The sample period is 1964:1Q-2001:4