Bank of Japan Working Paper Series

# On the Function of the Zero Interest Rate Commitment:

## Monetary Policy Rules in the Presence of the Zero Lower Bound on Interest Rates

Nobuyuki Oda*
nobuyuki.oda@boj.or.jp

Takashi Nagahata**
takashi.nagahata@boj.or.jp

No.05-E-1
Feb 2005

Bank of Japan
2-1-1 Nihonbashi Hongoku-cho, Chuo-ku, Tokyo 103-8660

* Monetary Affairs Department

** Monetary Affairs Department

Bank of Japan Working Paper Series No.05-E-1
February 2005

# On the Function of the Zero Interest Rate Commitment:

## Monetary Policy Rules in the Presence of the Zero Lower Bound on Interest Rates*

Nobuyuki Oda† and Takashi Nagahata‡

**Abstract**

We analyze the effectiveness of monetary policy rules in the presence of the zero lower bound on nominal interest rates, using stochastic simulations.

Specifically, we assume a small structural model composed of the IS and AS curves with both forward-looking and backward-looking agents, and estimate the model using Japanese economic data. Based on the results, we investigate which monetary policy rule is superior, in terms of minimizing social loss, among various policy rules which take the zero lower bound constraint into account. We assume a simple Taylor-type monetary policy rule estimated using the data before the zero interest rate policy was adopted as the baseline policy rule. We then modify this policy rule by adding a policy commitment whereby if deflation occurs the zero interest rate policy will be maintained until the rate of inflation rises beyond a specific level. The analyses indicate that such policy rules can be effective if the contents of the commitment — that is, the conditions for maintaining the zero interest rate policy — are set appropriately based on the economic conditions when the commitment is implemented. We also find that monetary policy rules with a nonlinearity, whereby preemptive monetary easing is appropriately implemented as the nominal interest rate approaches zero, perform well if they are regarded as credible, even without any explicit policy commitment.

**Key Words**: Zero interest rate commitment; Zero lower bound on nominal interest rates; Preemptive monetary easing; Monetary policy rule; Optimal simple rule; Stochastic simulation

**JEL classification**: E52, E58

† Monetary Affairs Department, Bank of Japan (E-mail: nobuyuki.oda@boj.or.jp)

‡ Monetary Affairs Department, Bank of Japan (E-mail: takashi.nagahata@boj.or.jp)

* We would like to express our gratitude to Kosuke Aoki (LSE), Toshiki Jinushi (Kobe University), Kunio Okina (Institute for Monetary and Economics Studies, Bank of Japan), and members at the Monetary Affairs Department of the Bank of Japan for their helpful comments. Regardless, we are solely responsible for any remaining errors in this paper. The views expressed herein are those of the authors and do not reflect those of the Bank of Japan or the Monetary Affairs Department.

## 1. Preface and Summary

Various proposals have been made regarding monetary policy in the presence of the zero lower bound on nominal interest rates. Since the effect of monetary policy declines when facing the non-negativity constraint, there is a wide-shared belief among economists that (1) preemptive monetary easing is important to minimize the likelihood that interest rates will fall to zero, and (2) in cases when interest rates have fallen to zero, "expectations management" which acts on the formation of private-sector expectations toward future monetary policy is important. While macroeconomic theory has played an important role in drawing these general conclusions, there is still room for developing further analyses for policy proposals on the more specific questions of (1) what degree of preemptive monetary easing is appropriate, and (2) how expectations management should be implemented.

There are some prior research papers that have examined the issue of preemptive monetary easing. For example, in three papers that assumed a purely forward-looking structural model, Adam and Billi (2004a) derived the optimal commitment policy, and Nakov (2004) and Adam and Billi (2004b) derived the optimal discretionary policy. Conversely, Kato and Nishiyama (forthcoming) derived the optimal discretionary policy under a purely backward-looking structural model. To better depict the actual conditions of the Japanese economy, our paper conducts analyses assuming a "hybrid" structural model that accommodates both forward-looking and backward-looking agents, and seeks optimal policy under a simple policy rule framework based on observable macroeconomic variables.

Prior research on the importance of expectations management includes Jung, Teranishi, and Watanabe (forthcoming) and Eggertsson and Woodford (2003), of which the latter advocates a kind of price level targeting as the optimal targeting rule for a purely forward-looking structural model. Since the structure in the real economy incorporates various uncertainties, however, there seem to be doubts regarding the feasibility and efficacy of implementing such a specific targeting policy in practice. This leaves the question of what type of options exist for practical expectations management when facing the zero lower bound on interest rates. Analyzing the function of Japan's "policy duration commitment" to maintaining the zero interest rate, which is referred to herein as the "zero interest rate commitment," is adopted in this paper for examining the practical options.[1] In general, the roles of policy commitments include changing the private sector expectations and reducing the uncertainty associated with the conduct of monetary policy. With a zero interest rate commitment, either, or both, of these objectives will be pursued depending on the economic situations.

Keeping these points in mind, in this paper we advance our analysis of practical monetary policy rules in the presence of the zero lower bound on nominal interest rates using stochastic simulations[2]. While the results of these analyses are limited in that they depend on the estimated model, they provide certain policy implications concerning the points of debate for the shift from a zero to a positive interest rate, and conversely for policy conduct under economic conditions where a positive interest rate changes back toward zero.

The remainder of this paper is organized as follows. In section 2, we explain the simulation framework, including the model and the simulation methods. In section 3, we

---

[1] On March 19, 2001 the Bank of Japan (BOJ) announced its decision to introduce and implement the so-called "quantitative easing policy." It has consisted of the maintaining of an ample liquidity supply by using the current account balances (CABs) at the BOJ as the operating policy target and the commitment to maintain ample liquidity provision until the rate of change of the core CPI (nationwide, excluding perishables) becomes positive on a sustained basis. The BOJ also announced that it was ready to increase the amount of purchases of long-term government bonds in order to meet the target on the CABs. It was projected that increasing the CAB targets beyond the level of the required reserve would normally keep the call rate near 0%. Moreover, the commitment regarding future liquidity provision was further clarified in October 10, 2003 with the BOJ committing itself to continue providing ample liquidity until both actual and expected inflation becomes positive. See http://www.boj.or.jp for further details.

Looking back prior to the adoption of this quantitative monetary easing policy, on February 12, 1999 the BOJ announced its decision to guide the uncollateralized overnight call rate, which was then the main target of its money market operations, close to zero. It is the so-called "zero interest rate policy." Thereafter, at a regular press conference in April 1999 the BOJ Governor announced that the Bank will continue with the current policy until deflationary concerns are dispelled.

Both announcements in 1999 and 2001 are understood as having indicated the BOJ's "policy duration commitment" to maintaining the zero interest rate while they are different from each other in terms of the degree in clarifying the contents of the "commitment".

[2] For monetary policy under the zero interest rate constraint, some "unconventional" policy channels may be assumed aside from the "conventional" channel via expectations on the future path of the interest rates. For issues on these channels, which lie outside the scope of this paper, see Oda and Okina (2001), Clouse *et al.* (2003), and Bernanke and Reinhart (2004), for example. As for the effect of the zero interest rate policy and quantitative easing policy in Japan, see Shirakawa (2002), Fujiki and Shiratsuka (2002), Okina and Shiratsuka (2004), Bernanke, Reinhart, and Sack (2004), and Baba *et al*. (2005).

conduct a simulation to evaluate the zero interest rate commitment, such as one implemented in Japan presently, is effective as a permanent monetary policy rule. In section 4, we demonstrate that as a permanent policy rule which takes the zero interest rate constraint into account, a nonlinear policy rule that incorporates preemptive monetary easing based on a simple Taylor-type rule is effective. From section 5, we assume that the economy is initially subjected to some negative shocks and analyze the conditions where a time-restricted commitment to maintain a zero interest rate is effective. Specifically, in section 5 we investigate how the optimal conditions of a zero interest rate commitment may differ depending on initial economic conditions. We also analyze the policy effects when the commitment is made based on a price level measure instead of the inflation rate. Then in section 6, we analyze whether or not the introduction of a zero interest rate commitment is effective in cases where the nonlinear policy rule incorporating preemptive monetary easing has already gained credibility from the private sector. Finally, in section 7 we summarize our analyses, review the points of debate, and consider areas for future research.

The main conclusions obtained in this paper can be summarized as follows.

- A zero interest rate commitment based solely on the inflation rate does not necessarily show good policy performance in promoting economic stability when recognized as a permanent policy rule. Rather, there is a risk that economic stability may be impaired because, when the commitment is in effect, the conditions of the GDP gap are not reflected in monetary policy.
- A nonlinear optimal simple rule, whereby a conventional linear Taylor-type rule is optimally “curved” as the nominal interest rate approaches zero, performs well as a permanent monetary policy rule that takes the zero interest rate constraint into account. This policy rule has the effect of diminishing the “cost” of the zero lower bound via preemptive monetary easing. The desirable shape of the curve can be determined depending on structural parameters such as the target inflation rate and the long-term natural rate of interest. The nonlinear optimal simple rule provides hints regarding (i) how high the interest rate indicated by a normal Taylor-type rule should rise before the transition from zero to positive interest rates, and (ii) the speed at which a normal Taylor-type rule should be reinstated once interest rates have turned positive.

- A zero interest rate commitment can be effective when it is interpreted as a "time-restricted" monetary policy rule that will be terminated after the prevailing deflation is overcome. The content of the zero interest rate commitment – that is, the threshold rate of inflation adopted as a prerequisite for exiting the zero interest rate policy – can be optimally selected based on the economic conditions, such as the size and persistence of the demand and supply shocks, when the commitment is introduced.
- We can assume a zero interest rate commitment based on a price level measure, instead of the inflation rate, but in most cases the policy effect from such a commitment is less than that from a commitment based on the inflation rate that is set optimally.
- On the whole, the policy performance of a Taylor-type rule with an optimally set commitment does not match the performance of the nonlinear optimal simple rule. This suggests that communications to ensure the formation of appropriate private-sector expectations regarding the central bank's stance toward preemptive monetary easing are important. In cases where the zero lower bound on nominal interest rates is encountered before such a policy stance is sufficiently recognized by the private sector, the action of making a zero interest rate commitment may effectively convey the "message" on the monetary policy stance as the qualitative explanations by the central bank do.
- After the nonlinear optimal simple rule has gained complete credibility, except for a few cases with extremely large demand shocks, setting some sort of zero interest rate commitment does not enhance the policy effect. The marginal benefits from adding a zero interest rate commitment are less after the credibility is established than when the credibility is insufficient.
- These analytical results are considered to be qualitatively robust. However, in quantitative evaluations, the conclusions may change depending on the economic environment including the level of the long-term natural rate of interest and the targeted rate of inflation. The conclusions may also change based on the judgments of the relative emphasis that monetary policy should give to inflation stability versus

GDP stability.

## 2. Analytical Method

This section presents the analytical framework of the stochastic simulations conducted from section 3.

**(1) Structural model**

Our analyses assume a small structural model composed of an IS curve (aggregate demand function) and an AS curve (aggregate supply function, Phillips curve) as stipulated below.

• IS curve: $x_t = \phi \cdot E_t x_{t+1} + (1-\phi) \cdot x_{t-1} - \sigma \cdot (i_t - E_t \pi_{t+1} - r_t^n) + g_t$, (1)

• AS curve: $\pi_t = \gamma \cdot E_t \pi_{t+1} + (1-\gamma) \cdot \pi_{t-1} + \kappa \cdot x_t + u_t$, (2)

• Natural rate of interest: $r_t^n = \eta_s \cdot (y_{t+1}^{HP} - y_t^{HP}) + \eta_c$, (3)

• Demand shock: $g_t = \rho_g \cdot g_{t-1} + \varepsilon_t$, (4)

• Supply shock: $u_t = \rho_u \cdot u_{t-1} + \mu_t$, (5)

where $x_t$ is the GDP gap (defined as a percentage deviation from potential real GDP), $\pi_t$ is the inflation rate, and $i_t$ is the nominal short-term interest rate. Here, $\phi$, $\sigma$, $\gamma$, $\kappa$, $\eta_s$, $\eta_c$, $\rho_g$ and $\rho_u$ are structure parameters while $\varepsilon_t$ and $\mu_t$ are white noise.

This is a hybrid structural model incorporating both forward-looking macro variables ($E_t x_{t+1}$, $E_t \pi_{t+1}$) and backward-looking macro variables ($x_{t-1}$, $\pi_{t-1}$) so that the IS and AS curves can both depict effects from expectations and economic persistence. This type of structural model has frequently been used in recent years for monetary policy analyses that emphasize empirical aspects since it can explain the actual economy well while maintaining a micro foundation (e.g., Amato and Laubach [2003]). The natural rate of interest ($r_t^n$) is modeled assuming a linear relationship with the potential GDP growth rate, which is defined as real GDP smoothed by an HP filter ($y_t^{HP}$, logarithmic value). It might be possible to view the long-term natural rate of interest as a constant, but we have set it as a variable for the Japanese economy, which is considered to have undergone structural changes during the estimation period. We assume that the demand shock ($g_t$) and the supply shock ($u_t$) both follow an AR(1) process. We also assume rational expectations concerning the forward-looking macro variables ($E_t x_{t+1}$, $E_t \pi_{t+1}$), presuming that they will return to steady states in the distant future.

As a transmission mechanism of monetary policy, this structural model assumes an inter-temporal substitution effect via policy rate operations. The model does not incorporate any mechanisms whereby quantitative variables, such as the monetary base or the central bank's current account balances (CABs), can influence the IS and AS curves independently under the zero interest rate. Since such a quantitative effect lies outside the scope of our analysis,[3] this paper interprets any commitment to continue quantitative easing as identical to the commitment to maintain a zero percent policy rate, and just uses the general term "zero interest rate commitment."

We utilize the Generalized Method of Moments (GMM) for the estimation. The estimation period runs from 1983 Q1 through 2004 Q1. The data is on a quarterly basis, using the 93SNA figures for real GDP, the year-on-year changes in the CPI (excluding perishables) for the inflation rate, and the uncollateralized overnight call rate for the nominal short-term interest rate. See Chart 1 for the specific data.

The estimation results are presented in Chart 2. The important parameters are all statistically significant, except for the interest rate elasticity ($\sigma$) in the IS curve, which shows a slightly small significance level. The sign conditions are fulfilled for all the parameters. The estimation shows that the ratio of forward-looking to backward-looking economic agents is roughly equal for both curves (52% to 48% for the IS curve and 45% to 55% for the AS curve), and that the AS curve coefficient $\kappa$ ("the slope of the Phillips curve") is relatively small at 0.014.[4]

---

[3] Kimura *et al*. (2002) examine whether, separate from the conventional interest rate channel, increases in the monetary base had any expansionary effect under Japan's quantitative monetary easing policy, and conclude that, if any, such effect was extremely small and uncertain.

[4] Kimura and Kurozumi (2004) is one example of prior research testing a similar model. They use an estimation period running from 1975 Q1 to 1997 Q1. This period differs from the one adopted herein in that it includes the second oil shock and does not include any term with zero interest rates. Their estimated values for the parameters equivalent to $\phi$, $\gamma$, and $\kappa$ in our model are 0.09, 0.65, and 0.20, respectively. In comparison with their results, the characteristics of the conclusions reached in our paper are: (1) a relatively strong forward-looking emphasis in the IS curve, (2) a relatively strong backward-looking emphasis in the AS curve, and (3) a comparatively small slope of the Phillips curve.

As noted above, the IS and AS curves have certain micro foundations, and therefore changes in the expectations regarding the monetary policy rule should not have any influence on the structural model. For that reason, the structural model estimated here can be directly applied to the diverse policy variations assumed for the stochastic simulation analyses from section 3. Finally, for conducting the simulations we make an a priori assumption that the long-term natural rate of interest is 1.5%.[5]

**(2) Monetary policy rule**

In general, most monetary policy analyses assume (1) the optimal discretionary policy rule, (2) the optimal commitment policy rule, or (3) a simple rule[6]. In this paper we assume that, as a baseline case, the economic agents expect that policy will be conducted following a Taylor-type simple rule with some modifications as presented later in subsection 2(5). From section 3, we also conduct analyses for the case when a relatively simple commitment, such as the zero interest rate commitment currently employed by the Bank of Japan, is added to the Taylor-type rule.

Specifically, we estimate the following policy rule for Japan's short-term interest rate before the Japanese economy began to face the zero lower bound.

$$i_t = \phi_i \cdot i_{t-1} + (1-\phi_i) \cdot \{r_t^n + \pi^* + \phi_x \cdot x_t + \phi_\pi \cdot (\pi_t - \pi^*)\}\,. \qquad (6)$$

This is a Taylor-type rule, dictating a positive policy reaction to the GDP gap and

---

5 As supporting evidence for calibrating the future natural rate of interest at 1.5%, the daily average value of the implied forward rate (IFR) starting in 20 years and ending in 30 years is roughly 3.3% on a nominal basis, calculated based on the yield on long-term Japanese government bonds in the third quarter of 2004. Subtracting the target inflation rate of 1.8% estimated in subsection 2(2) results in an interest rate of about 1.5% on a real basis.

6 Optimal discretionary policy is the approach whereby central banks only operate on current-term policy variables (that is, have no influence on the private sector expectations regarding policies in the future), and conduct policy to minimize the loss function. However, since optimal discretionary policy is a function depicted by unobserved variables such as demand and supply shocks, there are some doubts as to whether this policy can actually be grasped by the private sector. Under optimal commitment policy, central banks control private sector expectations by clearly specifying, beforehand, the conditional paths of the monetary policy, to realize a better economy than would have been realized without such a commitment. However, since this rule is also described with unobserved variables, there are some doubts regarding the policy feasibility and credibility. In contrast a simple rule assumes the private sector recognizes that monetary policy will be conducted following a relatively simple mechanism based on observable macro variables. The optimal simple rule can then be derived by setting the parameters in the rule so that the loss function is minimized. The past empirical research indicates that this kind of formulation of monetary policy can explain the actual data.

the inflation gap (defined as the deviation from the target inflation rate). The larger the parameters $\phi_x$ and $\phi_\pi$, the more aggressive the policy reaction to each gap. The interest rate inertia ($\phi_i$) in the rule is used to show that the central bank does not considerably change the policy interest rate from the prior period's level. $\pi^*$ represents the target inflation rate that the central bank deems desirable to achieve over the long term.

We adopt the ordinary least squares method (OLS) for the estimation. The estimation period runs from 1983 Q1 through 1999 Q1, which ends before the beginning of the zero interest rate period in order to estimate the monetary policy reaction function in the form without the zero interest rate constraint.

The estimation results are presented in Chart 3. The coefficient of the inertia is 0.83 and the strength of the policy reaction can be considered as reasonable. The results estimate a long-term target inflation rate of 1.8%.

From section 3 we advance our analyses assuming that the private sector expects monetary policy will be conducted following the policy rule estimated here when we return to economic conditions where we can ignore the zero lower bound on nominal interest rates. For conditions when the zero lower bound on nominal interest rates is significant, we investigate the policy effects from taking the Taylor-type rule estimated here as the basis and adding various types of zero interest rate commitments and/or preemptive monetary easing by making the rule nonlinear.

**(3) Loss function**

The loss of social welfare arising from macroeconomic fluctuations is formulated as the weighted sum of the squares of the inflation gap, the GDP gap, and the interest rate gap which is defined as the deviation from the steady state nominal short-term interest rate. This functional form has been adopted in the literature on monetary policy evaluation. It is also consistent with the loss function which is approximately derived from the economic theory regarding the extent to which the utility of economic agents declines from economic fluctuations.[7] Specifically, the loss function $L_t$ is defined as the sum of the

[7] According to the economic theory, the loss function is formulated as the weighted sum of the squares of the inflation gap, the GDP gap, and the interest rate gap under the standard New Keynesian model of which the AS curve is purely forward-looking. See, for example, Woodford (2003). When the model is modified to allow for the existence of backward-looking agents in the

discount values of the loss in each period (standardized for *n* periods).

$$L_t = E_t\left\{\frac{1}{n}\sum_{\tau=t}^{t+n-1}\beta^{(\tau-t)}\left[\left(\pi_t-\pi^*\right)^2+\lambda_x\cdot x_t^2+\lambda_i\cdot\left(i_t-i^*\right)^2\right]\right\}. \tag{7}$$

The discount factor ( $\beta$ ) is set at 0.995, which means the calibrated discount rate is 2% on an annualized basis. The weight parameters $\lambda_x$ and $\lambda_i$ are interpreted as indicating the shape of the social utility function. These parameters are estimated based on the following assumptions.

We assume that the past monetary policy in Japan has been efficiently implemented based on a sufficient grasp of the weight parameters while the parameters cannot be directly observed. Specifically, the estimated values of the policy response parameters $\phi_i$, $\phi_x$, and $\phi_\pi$ in Equation (6) are assumed to fulfill the condition of minimizing the above-stipulated loss function when the weight parameters $\lambda_x$ and $\lambda_i$ are given. To express these conditions, the loss function Equation (7) is modified as follows.

$$\begin{aligned} L_t\left(\lambda_x,\lambda_i;\phi_{i,}\phi_\pi,\phi_x\right) &= E_t\left\{\frac{1}{n}\sum_{\tau=t}^{t+n-1}\beta^{(\tau-t)}\left[\left(\pi_t-\pi^*\right)^2+\lambda_x x_t^2+\lambda_i\left(i_t-i^*\right)^2\right]\right\} \\ &= E_t[\frac{1}{n}\sum_{\tau=t}^{t+n-1}\beta^{(\tau-t)}\left(\pi_t-\pi^*\right)^2]+\lambda_x\cdot E_t[\frac{1}{n}\sum_{\tau=t}^{t+n-1}\beta^{(\tau-t)}x_t^2]+\lambda_i\cdot E_t[\frac{1}{n}\sum_{\tau=t}^{t+n-1}\beta^{(\tau-t)}\left(i_t-i^*\right)^2] \\ &= a+b\cdot\lambda_i+c\cdot\lambda_x, \end{aligned} \tag{7'}$$

where $a,b,c$ are functions of $\phi_i,\phi_\pi,\phi_x$, defined as $a\equiv E_t[\frac{1}{n}\sum_{\tau=t}^{t+n-1}\beta^{(\tau-t)}\left(\pi_t-\pi^*\right)^2]$, $b\equiv E_t[\frac{1}{n}\sum_{\tau=t}^{t+n-1}\beta^{(\tau-t)}\left(i_t-i^*\right)^2]$, and $c\equiv E_t[\frac{1}{n}\sum_{\tau=t}^{t+n-1}\beta^{(\tau-t)}x_t^2]$. Given the estimated value of the policy parameters $\phi_i,\phi_\pi,\phi_x$, we can calculate the values of *a*, *b*, and *c* and of their partial differential coefficients through stochastic simulations based on the structural model.

The condition that the estimated monetary policy rule was efficient can be described

IS and AS curves, as the model in this paper, the functional form becomes somewhat complicated (Woodford [2003], Amato and Laubach [2003]). This paper does not take into account this change in the functional form and assume the above simple function since such a functional form seems similar to the one perceived in the monetary policy conduct.

Some of the prior research papers set the loss function without including the interest rate fluctuation cost, but Woodford (2003) demonstrates that there are social costs to interest rate fluctuations when the opportunity cost of holding money is explicitly incorporated. From a different perspective, Williams (1999) argues it is appropriate to avert excessive interest rate fluctuations to minimize the interest rate term premium and to maintain the credibility of monetary policy. Thus, this paper adopts the loss function with the interest rate gap.

in this framework by saying that the estimated values of $\phi_i, \phi_\pi, \phi_x$ minimize the loss function $L_t$, given the weight parameters $\lambda_x$ and $\lambda_i$. That is,

$$\frac{\partial L_t}{\partial \phi_i} = \frac{\partial a}{\partial \phi_i} + \lambda_i \frac{\partial b}{\partial \phi_i} + \lambda_x \frac{\partial c}{\partial \phi_i} = 0 ,$$
$$\frac{\partial L_t}{\partial \phi_\pi} = \frac{\partial a}{\partial \phi_\pi} + \lambda_i \frac{\partial b}{\partial \phi_\pi} + \lambda_x \frac{\partial c}{\partial \phi_\pi} = 0 , \qquad (8)$$
$$\frac{\partial L_t}{\partial \phi_x} = \frac{\partial a}{\partial \phi_x} + \lambda_i \frac{\partial b}{\partial \phi_x} + \lambda_x \frac{\partial c}{\partial \phi_x} = 0 , \quad \text{given } \lambda_x \text{ and } \lambda_i .$$

Assuming that the estimated values of $\phi_i, \phi_\pi, \phi_x$ above approximately fulfill these conditions, the values of the unknown parameters $\lambda_x$ and $\lambda_i$ are calculated in reverse. Since the three conditions in Equation (8) cannot simultaneously be fulfilled using the two degrees of freedom, $\lambda_x$ and $\lambda_i$, we calculate the values of $\lambda_x$ and $\lambda_i$ that approximately fulfill the following least-squares problem.

$$\min_{\lambda_x, \lambda_i} \left[ \left( \frac{\partial L}{\partial \phi_i} \right)^2 + \left( \frac{\partial L}{\partial \phi_\pi} \right)^2 + \left( \frac{\partial L}{\partial \phi_x} \right)^2 \right]. \qquad (9)$$

This can be expressed as follows.

$$\begin{cases} \dfrac{\partial}{\partial \lambda_x} \left[ \left( \dfrac{\partial L}{\partial \phi_i} \right)^2 + \left( \dfrac{\partial L}{\partial \phi_\pi} \right)^2 + \left( \dfrac{\partial L}{\partial \phi_x} \right)^2 \right] = 0 , \\ \dfrac{\partial}{\partial \lambda_i} \left[ \left( \dfrac{\partial L}{\partial \phi_i} \right)^2 + \left( \dfrac{\partial L}{\partial \phi_\pi} \right)^2 + \left( \dfrac{\partial L}{\partial \phi_x} \right)^2 \right] = 0 . \end{cases} \qquad (10)$$

Substituting Equation (8) into Equation (10) and using the numerically calculated results of the Jacobian matrix of $\{a, b, c\}$ partially differentiated by $\{\phi_i, \phi_\pi, \phi_x\}$,[8] we arrive at the solution that $\lambda_x$ =0.94 and $\lambda_i$ =0.69.[9] We use this specification of the loss function for

---

[8] The Jacobian matrix is calculated numerically by increasing the respective values of $\{\phi_i, \phi_\pi, \phi_x\}$ estimated in subsection 2(2) by 10% one by one, and then recalculating the respective values of $\{a, b, c\}$.

[9] In the prior literature, the estimation results for the relative weight ($\lambda_x$) in the welfare loss between the fluctuation of the inflation rate and the GDP gap vary greatly depending on the assumptions that are adopted. For example, Sack (2000) conducts empirical analyses on the US economy assuming that monetary policy by the Federal Reserve Board has been optimal discretionary policy, and reports an estimation result of $\lambda_x$ = 0.79. This is relatively close to our

evaluating the policy performance from section 3.

**(4) Framework of the simulation analysis**

From section 3, we conduct stochastic simulations taking the estimated structural model and the assumed monetary policy rule as given and calculate the loss function, by which we evaluate the policy performance. The analysis is to determine what kinds of monetary policy rules should be introduced to effectively stabilize the macroeconomic path. We assume that the probability distribution of demand shocks and supply shocks that have emerged in the past Japanese economy can be applied to the future.

One distinctive characteristic of the simulation is that the existence of the zero interest rate constraint is internalized into the economic structure. In other words, the model incorporates the rational decision-making mechanism whereby economic agents recognize that further monetary easing is not possible when nominal interest rates approach the zero constraint because of a large negative economic shock.[10] Also, since the zero constraint is a nonlinear structure, we conduct the stochastic simulations using a

---

estimation results here for the Japanese economy. Meanwhile, Woodford (2003) adopts settings of $\lambda_x$ = 0.048 and $\lambda_i$ = 0.077 as results analytically derived in line with the economic theory based on the calibration of the deep parameters. Those values are far from the estimation results in this paper, and we revisit this point in section 7.

[10] To mitigate the calculation burden, many prior research papers have adopted the approximation of viewing the zero lower bound on nominal interest rates as an unexpected exogenous policy shock, thus placing the constraint outside the economic structure (Reifschneider and Williams [2000], etc.). In those cases, the economic structure remains within the framework of a log linear approximated model, such as a reduced model from Schur decomposition or a VAR model, and simulations are easily conducted without any forward-looking variables. Under this approach, the negative impact of the zero lower bound is imposed by assuming an external shock that exactly cancels out the amount by which the interest rate is negative. This method, however, assumes irrationality whereby the private sector can never predict the existence of the zero lower bound. In contrast the analyses in our paper explicitly incorporate the mechanism whereby private sector recognizes the existence of the zero lower bound, reflecting it in their expectation formations.

We conducted trial calculations to check the difference between these two methods prior to the model selection. Specifically, we conducted estimations under the reduced-form model from Schur decomposition and under the structural model adopted in our paper until 1999Q1 to calculate the size of the loss function for each economic path of (1) the impulse response to a demand shock (-3.8%), and (2) the impulse response to a supply shock (-1.1%). The calculation results indicate that, compared with the total losses under the structural model adopted in this paper, the total losses under the reduced-form model are 5.7% less for path (1) and 7.5% less for path (2). This may be attributed to the effect whereby the economy is easily stabilized because the private sector holds irrationally optimistic expectations regarding monetary policy when the zero lower bound on nominal interest rates is externalized.

numerical approach.[11] If the economic structure were linear, the certainty equivalence principle could be applied and, therefore, we could derive the optimal policy without considering the uncertainties from additive shocks. However, since the possibility of facing the zero lower bound is an essential point in this paper, we need to take the uncertainty into account. In this case, it is impossible to derive the loss function analytically due to the nonlinearity of the economic structure. This is the main reason we choose a numerical approach for the stochastic simulations. Each simulation was conducted 300 times, in principle[12].

From section 3, in order to evaluate the performance of each monetary policy rule, we adopt the following two methods in accordance with the analytical objectives.

(A) Long-term simulation

This type of simulation starts from a steady state of the economy, and continues indefinitely to evaluate the loss function. In this paper, we adopt an approximation with a time horizon that is limited in the distant future — specifically, 100 quarters (25 years) ahead under Equation (7). This analysis is used for the case assuming that a given monetary policy rule will be implemented permanently.

(B) Short-term simulation

This type of simulation starts from an initial state with relatively large negative economic shocks (demand and supply shocks), and continues over a relatively short time horizon —in our analysis, the time period is 20 quarters (5 years) under Equation (7). This analysis is used for the case assuming that a given monetary policy, such as one with a zero interest rate commitment, will be maintained for a limited period of time.

**(5) Setting the baseline Taylor-type policy rule**

When stochastic simulations are conducted by simply adding a zero interest rate constraint to the Taylor-type rule estimated in subsection 2(2), for a very small number of

[11] The structural model for this simulation includes forward-looking variables. For the calculation of the future economic path under the stochastic simulations, we conducted numerical calculations using a stacked-time algorithm (Hollinger [1996]).

[12] As for the number of simulations, we made trial calculations increasing the number to 5,000 and confirmed that 300 times is sufficient for the analyses in this paper.

economic paths the simulations indicate a divergence toward a deflationary equilibrium. Under the economic theory as demonstrated by Eggertsson and Woodford (2003), with a model that explicitly incorporates money, assuming that money is supplied following an appropriate rule it should be possible to avoid falling into a deflationary equilibrium because of the transversality condition whereby the private sector must use up all the money in the distant future. While our analysis does not explicitly address the role of money, we follow a similar approach to that presented in Reifschneider and Williams (2000)[13] as an alternative means of eliminating the divergence paths. We assume that when economic conditions significantly worsen under the zero interest rate constraint, the economy obtains an easing effect equivalent to having the negative nominal interest rates gained from a Taylor-type rule with no zero lower bound through hypothetical large-scale fiscal expenditures or other measures. The trigger for instituting the hypothetical fiscal policy is set as when the estimated Taylor-type rule suggests interest rates of -3.5% or lower. A nominal interest rate of -3.5% indicates a nearly 7% downward diversion from the long-term equilibrium level (3.3% <1.5% + 1.8%>). In such a case, the assumption that the government would institute large-scale fiscal policy is not unrealistic. Moreover, we confirmed that the results are not greatly influenced in the case where the trigger level of the nominal interest rate is slightly increased and the negative impact of the zero constraint is smaller. It is because the number of paths that would lead to the trigger is extremely small.

Summarizing these settings, the basic form of the monetary policy rule $i_t^{base}$ adopted for the simulation analyses from section 3 can be expressed as Equation (11) below. Hereafter, this is referred to as the “baseline Taylor-type policy rule” or simply the “baseline rule”. See the upper figure of Chart 4 for a graphic representation.

$$\begin{aligned} i_t^{base} &= i_t^{est} && \text{if } \ i_t^{est} \geq 0\%, \\ i_t^{base} &= 0 && \text{if } \ -3.5\% < i_t^{est} < 0\%, \\ i_t^{base} &= i_t^{est} && \text{if } \ i_t^{est} \leq -3.5\%, \end{aligned} \qquad (11)$$

[13] Reifschneider and Williams (2000) adopt the assumption that when nominal interest rates face the zero constraint for over seven continuous years, the influence of the zero constraint can be cancelled out by a hypothetical large-scale fiscal expenditure effect . In our paper we changed this assumption, as detailed above, to reduce the numerical calculation burden.

where $i_t^{est} \equiv \phi_i \cdot i_{t-1}^{est} + (1-\phi_i) \cdot \{r_t^n + \pi^* + \phi_x \cdot x_t + \phi_\pi \cdot (\pi_t - \pi^*)\}$ is the monetary policy rule (Equation (6)) estimated in subsection 2(2).

## 3. Is a Zero Interest Rate Commitment Effective as a Permanent Monetary Policy Rule?

The policy rule $i^{\pi 0com}$ which adds a zero interest rate commitment to the baseline Taylor-type rule is formulated as Equation (12). See Chart 5 for a conceptual diagram.

$$i_t^{\pi 0com} = i_t^{base} \quad \text{if } \pi_t \geq 0\%,$$

$$i_t^{\pi 0com} = 0 \quad \text{if } \pi_t < 0\%. \tag{12}$$

This policy rule is a baseline Taylor-type rule with a commitment whereby the policy rate will become zero whenever the current-term inflation rate is negative, regardless of the conditions of the GDP gap.[14] This can be interpreted as an approximate expression of Japan's present zero interest rate commitment.[15]

In this section, we assume that the term of this policy rule is not limited until the current deflationary phase is overcome, but rather that the rule will continue to be applied permanently. Here the private sector recognizes that the central bank will implement exactly the same zero interest rate commitment if the economy encounters another deflationary phase in the future. In this case, the condition stipulated above applies not only to the exit of the zero interest rate policy but also to the introduction of zero interest rates at the start of a deflationary period.

Is the policy performance relatively better when there is a belief in this kind of policy commitment than when there is no such policy commitment?

---

[14] Under this policy rule, there is a possibility that interest rates will jump to the level indicated by the baseline Taylor-type rule when the zero interest rate commitment is lifted. It would violate the property of the interest rate inertia. To deal with the problem, the actual simulations in this paper are calculated with the property whereby, along with the estimated inertia, the interest rate gradually resumes the level indicated by the Taylor-type rule. In this paper, we apply this property to all the monetary policy rules that contain a zero interest rate commitment.

[15] The Bank of Japan has announced that it is committed to maintaining the quantitative easing policy until the core inflation rate tendency and the prospects for the future core inflation rate both become positive. For further details on this, see the section 2 of the BOJ release "Enhancement of Monetary Policy Transparency" dated October 10, 2003. In our paper, the contents of this announcement are approximated, rather than strictly reflected, in the model to simplify the simulation analyses.

The long-term simulation results in Chart 5 (1) indicate that, compared with the results in Chart 4 (1) when there is no zero interest rate commitment, the loss caused by inflation fluctuations declines (from 0.784 to 0.714), but the loss caused by GDP fluctuations rises (from 1.529 to 1.678) as does the loss from interest rate fluctuations (from 0.879 to 0.964), and thus the total loss expands (from 2.828 to 2.957). In particular, the loss from the GDP fluctuations increases because the decision on continuing or terminating the zero interest rate policy depends solely on the inflation rate, and does not necessarily reflect the economic conditions reflected in the GDP gap.

In actuality, the present zero interest rate commitment in Japan has been depicted as a policy rule for the specific economic term lasting until the economy extricates itself from its present deflationary phase. Our analysis here is based on the assumption of expanding the policy to one whereby the same commitment will be applied forever. The simulation result shows that applying the commitment forever would in fact worsen the policy performance.

We would like to raise the following two issues here, based on the results of the analyses in this section.

(1) Considering the zero lower bound on nominal interest rates, what types of policy rules could be effective on a permanent basis in the extension of the simple rule?

(2)What are the functions of the zero interest rate commitment such as one implemented in Japan presently?

We address the first of these questions by examining a nonlinear optimal simple rule in section 4, and the second question in sections 5 and 6 via policy performance analyses using short-term simulations.

## 4. Preemptive Monetary Easing with the Nonlinear Optimal Simple Rule

As an inference from the analysis in section 3, one conceivable means of adjusting the baseline Taylor-type rule is to reach decisions on whether to continue or terminate the zero interest rate policy considering not only the inflation rate but also the GDP gap. One natural means of combining these two factors would be to refer to the functional form of the estimated Taylor-type rule. We posit an image of such an adjustment in the upper figure of Chart 6. The adjusted policy rule, which corresponds to the estimated baseline rule when sufficiently positive, accelerates the pace at which interest rates are lowered compared with the original rule as rates approach zero, and zero interest rates are introduced when the interest rates under the baseline rule are slightly positive. This can be interpreted as preemptive monetary easing in response to the cost of the zero lower bound constraint. For example, the optimal commitment rule derived in Adam and Billi (2004a) shows a similar form. Many prior papers have expressed this kind of nonlinearity using numerical examples, but we describe the nonlinearity of the policy rule nearby zero using the following parametric nonlinear function (Equations (13) and (14)). We adopt this approach since, while it sacrifices generality to some extent in the optimization, it makes it possible to express a complex nonlinear function using a relatively simple framework.

$$\begin{aligned} i_t^{NL}\left(i_t^{est};a,b\right) &= i_t^{base}\left(i_t^{est}\right)\cdot NL\left(i_t^{est};a,b\right), \qquad if \qquad i_t^{est} \geq 0, \\ &= i_t^{base}\left(i_t^{est}\right) \qquad\qquad\qquad\qquad if \qquad i_t^{est} < 0, \end{aligned} \tag{13}$$

$$\text{where} \quad NL(x;a,b) \equiv 1-\left[1+\exp\{a\cdot(x-b)\}\right]^{-1}. \tag{14}$$

The shape of the nonlinear function $NL(x;a,b)$ is determined by the two positive parameters $a$ and $b$. As shown by the lower part of Chart 6, the shape of the curve can change flexibly in accordance with the parameter settings. Parameter $b$ determines the rough location of the curve center point (more precisely, the point where the value declines by half since $NL(b;a,b) = 1/2$), while parameter $a$ determines the expanse of the curved section (which expands as the value of $a$ declines). We conducted a grid search (Chart 7) to calculate the total loss via a long-term simulation[16] using various settings for these parameters. The optimal parameter settings that minimize the total loss are $a$ = 2.0, $b$ = 1.5, shown in the shaded bloc in the table of Chart 7, and the shape of the corresponding policy

[16] We used a long-term simulation beginning from the steady state to determine the nonlinear optimal simple rule. If one assumes that the economy is hit by some sort of shock at the starting point and conducts the simulation for a limited period of time, different parameters will be selected as optimal than when the simulation begins from a steady state. However, the optimality of the policy rule then depends on specific economic conditions, and because it is not optimal after the economic shock changes, it does not gain credibility over the long term. Strictly calculating the optimal time-consistent policy rule in line with the economic conditions at each time is nothing but seeking the optimal discretionary policy. In this section we give precedence, as described above, to the ease of understanding the policy rule, and adopt the optimal simple rule, instead of the optimal discretionary policy. Looking over the long term, the economy undergoes transitions shifting around the steady state, so the optimized simple rule derived from a simulation beginning from the steady state is likely to gain credibility over the long term.

rule is as presented in the figure at the top of Chart 8. This can be interpreted as having made the baseline Taylor-type rule "optimally" nonlinearized. Since this optimal rule depends on various factors including the model assumed for the analyses and the accuracy of the estimates, we must pay heed to the appropriateness of these assumptions when we quantitatively evaluate the derived results. With this caveat, the characteristics of the shape of the nonlinear optimal simple rule, shown in Chart 8, can be summarized as follows.

- The nominal interest rate is made 0% when the baseline Taylor-type rule indicates a nominal rate of 0.5% or less.
- When the baseline Taylor-type rule indicates nominal interest rates of 1.0%, 1.5%, and 2.0%, this rule results in lower nominal rates of 0.27%, 0.75%, and 1.46%, respectively.
- When the baseline Taylor-type rule indicates a nominal interest rate of 3.0% or higher, this rule almost corresponds to the baseline Taylor-type rule.

Chart 8(1) shows the results of a long-term simulation for this nonlinear optimal simple rule. It indicates that, compared with the baseline Taylor-type rule in Chart 4(1), loss caused by inflation fluctuations declines from 0.784 to 0.716 and loss caused by GDP fluctuations declines from 1.529 to 1.515 while loss from interest rate fluctuations worsens. The total loss declines from 2.828 to 2.774, indicating improved policy performance with the nonlinearity.

In general, insufficient preemptiveness of monetary easing increases the risk that economic stability will be lost owing to the zero lower bound on nominal interest rates, while excessive preemptiveness will inevitably invite economic overheating and inflation. The grid search in this section derived the optimal extent of preemptiveness of monetary easing considering this tradeoff. The result is the level of preemptive monetary easing indicated by the upper figure on Chart 8, shown in comparison with the monetary policy that can be adopted in phases when there is no need to be concerned about the zero lower bound on nominal interest rates. The term preemptive monetary easing as used here refers to both (1) the policy of preemptively strengthening monetary easing while nominal interest rates are positive to avoid coming up against the zero lower bound as well as (2) the policy of prolonging the monetary easing to avert the risk of making the economic conditions worse during phases when interest rates are shifting from zero to a positive level.

The quantitative evaluation above may change if the settings for the target inflation rate and/or the long-term natural rate of interest are altered. To investigate the robustness of the analysis, we altered the target inflation rate from the estimated value of 1.8% to 1.0% and derived a nonlinear optimal simple rule. The optimal parameter settings became $a$ = 3.0, $b$ = 1.5, as shown in Chart 9. The shape of this policy rule is only slightly different compared with the rule when the target inflation rate is 1.8%. It extends the zero interest rate period and hastens the time for return to the normal policy rule.

The analyses in this section assume the private sector recognizes that monetary policy is being conducted following this kind of nonlinear optimal simple rule. The monetary easing effect is strengthened because of such policy expectations. In that sense, communications to promote the formation of private-sector expectations are important for the central bank. If the private sector expectations are uncertain, greater preemptive easing may be required to achieve an equivalent monetary easing effect to that in these analyses.

## 5. Functions of a Time-restricted Zero Interest Rate Commitment

In section 3, our analyses found that the policy performance worsens when the zero interest rate commitment indicated in Equation (12) is interpreted as a policy rule that will be applied permanently in the future. However, the present zero interest rate commitment in Japan, for example, is a time-restricted commitment that will only last until the current deflationary phase ends. If there is a recognition that the commitment will lose effect beyond that time, the commitment may have an economic stabilization effect over the limited term from when the economy falls into deflation with a downward shock until the deflation is overcome. To examine this, we now assume six different types of downward shocks — combinations of demand and supply shocks — for the economic conditions at the point when the commitment is first introduced[17] and conduct short-term simulations

[17] If the kinds of demand and supply shocks that struck the Japanese economy when the zero interest rate policy was introduced in February 1999 (or when the commitment to maintain the policy was announced in April 1999) and/or when the quantitative easing policy was introduced in March 2001 could be identified, those actual policies could be evaluated using the analytical methods adopted herein based on the actual shocks at that time. However, since the issue of discriminating between demand and supply shocks lies beyond the scope of this paper, we proceed with the discussions here assuming these hypothetical but plausible demand and supply shocks.

as explained in subsection 2(4).[18]

For the properties of each shock adopted, the impulse response figures are presented in Chart 10. For cases (1) through (5), the supply shock is fixed with a relatively small burden while the demand shock changes from case (1) with the largest demand shock to case (5) with the smallest. For cases (5) and (6), the demand shock is fixed with a relatively small burden while the supply shock of (5) is small and that of (6) is large.

**(1) Setting the optimal conditions for the zero interest rate commitment**

We begin by comparing the monetary policy performance given the initial shock under the baseline Taylor-type rule (with no commitment) with that under the baseline Taylor-type rule with the zero interest rate commitment indicated in Equation (12). The results of short-term simulations with a time horizon of five years are presented in Charts 4(2) and 5(2), respectively. They show that during phases with a large negative demand shock (the cases where $g$ = -4%, -3% and -2%), the overall loss is less under the policy rule with the zero interest rate commitment (in Chart 5(2)), indicating superior short-term policy performance. This is apparently because the monetary easing, which has a direct effect in erasing the negative demand shock, was strengthened by the commitment, and this reinforced the effect of narrowing the negative GDP and inflation gaps. Meanwhile, since the commitment does not reflect the GDP conditions, it has the negative side-effect of amplifying the GDP fluctuation costs. Overall, a tradeoff emerges: the commitment reduces the inflation fluctuation costs while it increases the GDP fluctuation costs.

The zero rate commitment analyzed above stipulates that the zero interest rate will be maintained "until the inflation rate becomes above 0%," which approximates the present commitment in Japan. We can consider the issue of whether or not this 0% figure

[18] In analyzing the time-restricted zero interest rate commitment in sections 5 and 6, properly it would be desirable to explicitly include a condition in the policy rule whereby the commitment is applied until the end of the initial deflation and is not applied after the end of this deflation. However, that would result in an excessive calculation burden for the simulation. Accordingly, to mitigate the calculation burden in this paper we state a policy rule whereby the commitment will be applied and maintained regardless of the continuation or end of the initial deflation (just as in section 3), and then adopt the method of limiting the loss calculation to a short-term period of five years (short-term simulation). Looking at the distribution of the simulation paths, there are only a few rare cases where the initial deflation is overcome and the economy subsequently falls into another deflation within five years. Thus, we consider that the policy effect of a time-restricted commitment can be approximately evaluated using this method.

is the optimal setting in the analytical framework of this paper. For each of the six different initial economic shocks, Chart 11 presents the policy performance under the short-term simulations for the baseline rule with the zero interest rate commitment, assuming commitments stipulating various threshold rates of inflation (shown as the horizontal axes on the graphs) as the condition for exiting the zero interest rate policy. These results show that the optimal commitment (threshold rate of inflation) which minimizes the total loss varies depending on the initial economic conditions. Specifically, we can draw the following observations from Chart 11.

- As overall tendency, for cases where a large negative demand shock ($g$) is the primary cause for falling into deflation, the total loss declines and the policy performance improves as the threshold inflation rate for exiting the zero interest rate policy is set higher. These tendencies grow in proportion with the extent to which a large demand shock is the primary cause. Case (1) in Chart 11 is an example where a high threshold rate of 1.5% is desirable. On the contrary, if the negative demand and supply shocks are both small, the total loss grows when the threshold inflation rate is set higher. For example, in case (5) it is desirable to set a threshold inflation rate of less than zero.
- Also for cases where a negative supply shock ($u$) is the primary cause, it tends to be better to set a high threshold rate of inflation when the deflation is strong, as in case (6), than when it is weak, as in case (5). However, compared with the conditions where a demand shock is the primary cause, even under a substantially strong negative supply shock, as in case (6), the optimal value of the threshold rate is not all that high at 0%.
- These trends are consistent with the traditional proposition of monetary policy that "for demand shocks it is desirable to cancel out the shock immediately via monetary easing, but for supply shocks the adjustments must be made cautiously giving consideration to the tradeoff between price stability and the stability of the real economy." While negative demand shocks can be directly cancelled out by reinforcing monetary easing via a zero interest rate commitment (an IS curve issue), negative supply shocks must be indirectly erased (an AS curve issue) by pushing up the GDP gap via monetary easing, and thus an excessive commitment entails the risk of excessively overheating the real economy and increasing the total loss.

**(2) Robustness check**

In this section we investigate the robustness of the analyses in subsection 5(1) above from two different aspects.

The first issue concerns the nature of the economic shock. Up until this point we have assumed that future economic shocks will follow the AR(1) process estimated in section 2, but in reality it is also possible that shocks may have a different nature from those of the past. We now assume a higher persistence of the supply shock with an autocorrelation coefficient of 0.45, which is 20% higher than the estimated value of 0.38. We conducted short-term simulations under this setting to see how the optimal zero interest rate commitment may change. We can draw the following observations from the findings which are presented as Chart 12 as compared with the findings in subsection 5(1) above which are presented as Chart 11.

- For initial shock cases (1), (4) and (5), the optimal threshold rate of inflation is the same in Charts 11 and 12. On the other hand, for initial shock cases (2) and (3), the optimal threshold rate of inflation is 0.5% higher in Chart 12, where the supply shock is more persistent than in Chart 11. This is because while the initial size of the shock is the same in the corresponding cases in both charts, as this shock continues for a longer time, the inflation rate and the GDP gap diverge further below their desirable levels and thus reinforcing the monetary easing via a stronger zero interest rate commitment comes to reduce the total loss.
- Focusing on the components of the total loss, there is a tradeoff whereby reinforcing the commitment decreases the loss from the inflation rate fluctuation by hastening the end of the deflation but increases the loss from the GDP gap fluctuation. The loss from the GDP gap fluctuation increases since the commitment conditions are based solely on the inflation rate, and do not reflect the GDP gap conditions. How then is this tradeoff affected when the negative supply shock is more persistent? From one perspective, the longer the shock continues the more desirable it becomes to reinforce policies to revive the economy, that is, to lift the commitment’s threshold rate of inflation. On the other hand, because the period in which the zero interest rate commitment is binding extends as the shock continues longer, the GDP gap fluctuation loss which is a side-effect of the commitment grows, and in this respect it becomes

better to weaken the commitment and lower the threshold rate. Comparing the results in Charts 11 and 12 suggests that in cases (1), (4) and (5) the two forces in this tradeoff almost cancel each other out, and in cases (2) and (3) the former of the two forces is greater than the latter.

The second issue is to investigate the robustness to the setting of the long-term natural rate of interest. In this paper we consistently set the natural rate of interest at 1.5%, but the actual level may be higher or lower than this rate. As an example, we here assume the case whereby the initial demand shock is -1.0% and the initial supply shock is -0.5%, and investigate how the optimal commitment conditions (threshold rate of inflation) change when we conduct the same analyses as in subsection 5(1) above but set the natural rate of interest at 1.0% and 2.0% in addition to 1.5%. Our findings, which are presented as Chart 13, indicate that the optimal threshold rate of inflation is -0.5% when the natural rate of interest is either 1.5% or 2.0%, but that the optimal threshold rate of inflation is 0.0% when the natural rate of interest is 1.0%. This is apparently because when the natural rate of interest is low it is more likely to be caught by the zero lower bound on nominal interest rates, and thus in such cases it becomes more desirable to implement a relatively strong zero interest rate commitment to reinforce the effect in averting the zero lower bound.

**(3) Zero interest rate commitment based on the price level**

Up until this point, our analyses have focused on the type of commitment whereby the conditions for exiting the zero interest rate policy are linked to the inflation rate. However, the theoretical analyses in some of the past literature propose price level targeting as an effective approach to overcoming deflation. It is therefore worthwhile investigating whether or not this kind of approach is also effective for the model of the Japanese economy presumed in this paper. While the definitions of price level targeting are rather broad,[19] we begin our examination using a simple interest rate rule that reflects

---

[19] The term "price level targeting" is used, at least, to indicate the following three different policy approaches. The first, under optimal discretionary policy, adopts the modified loss function in which the inflation fluctuation cost is replaced by the price level fluctuation cost in order to depress the inflation bias. The second, in the optimal simple rule framework, adopts an interest rate rule whereby the inflation gap in the Taylor-type rule is replaced by the price level gap (i.e. the Wicksellian rule). The third, in optimal commitment policy, is the position (known as "optimal targeting policy") of making a commitment whereby policy will always be conducted to satisfy the relational equation between the price level and the GDP gap derived by solving the first-order conditions (FOC).

the price level measure since we base our analyses in this paper within the framework of simple monetary policy rules. Specifically, we replace the inflation gap in our Taylor-type rule with the price level gap, which is defined as the percentage deviation of the price level from the target price level path. This policy rule is known as the "Wicksellian rule" (Giannoni (2000)). We tried to conduct a stochastic simulation under the rule.

However, since there is a significant percentage of backward-looking agents in the structural model, the GDP gap fluctuation increases in reaction to sharply restricting prices, so in many cases the economy then fails to converge when the simulation is conducted. This stands in sharp contrast to the findings in Giannoni (2000) which demonstrated the effectiveness of the Wicksellian rule in a purely forward-looking model.

We then consider the approach of temporarily incorporating some policy effects similar to those from price level targeting only for the restricted term until the prevailing deflation is overcome, instead of adopting price level targeting itself. Specifically, we assume a commitment "to retain the zero interest rate policy until the initial price level is recovered." [20] A conceptual diagram on this commitment is shown in Chart 14. It can be interpreted as a commitment that sets the temporary "target" inflation rate at 0%, whereby the "target" of the price level path stipulates the necessary condition to exit the zero interest rate policy. The temporary "target" inflation rate here could be set differently from the long-term target inflation rate, depending on the economic conditions when the commitment is introduced. To gain a stronger (weaker) monetary easing effect, a positive (negative) temporary "target" can be considered. From the practical perspective of the policy implementation, however, there is a risk that the smooth formation of private sector

---

[20] Mathematically, the price level (in logarithm) at period $t$ equals the price level at period 0 plus the sum of the inflation rates from period 0 to period $t$. The price level gap at period $t$, defined as the deviation from the target price level, corresponds to the gap between the average inflation rate in the latest $t$ periods and the target inflation rate. Therefore, a commitment based on the price level gap at period $t$ can be interpreted as a commitment based on the $t$-period backward moving average inflation rate. One of its characteristics is that the term of averaging ($t$) is variable as time goes by.

In the analysis of this paper, the above mechanism is modeled accurately for the period until the seventh quarter after the commitment based on the price level is implemented. However, for the period after the eighth quarter, the commitment is approximately modeled as one based on the 7-period backward moving average inflation rate. The purpose of the approximation is to reduce the calculation burden in the simulations. We consider that this approximation has no significant distortions on the analyses since there are very few stochastic paths where the commitment is still binding after the eighth period in the simulations here.

expectations would become difficult because of the complexity of the commitment, and that this would diminish the policy effect. For this reason, we basically focused the analyses on the price level based commitment in this specific case, that is on the simply expressed commitment “to retain the zero interest rate policy until the initial price level is recovered.”[21]

We conducted short-term simulations to examine the policy performance of the baseline Taylor-type rule with this price-level commitment under each initial shock. In Chart 15, we compare those results with the performance of the optimally set inflation rate based commitment derived in subsection 5(1). We note the following points from Chart 15.

- There are almost no cases where the policy performance under the commitment based on the price level is better than the policy performance under the optimally set inflation rate based commitment. As an exception, the performance of both is comparable in case (1) of Chart 15, where the loss breakdown indicates that the commitment based on the inflation rate is superior in terms of both the inflation fluctuation loss and the GDP fluctuation loss and inferior in terms of the interest rate fluctuation loss.
- Compared with the commitment based on the inflation rate, the commitment based on the price level has a stronger ability to overcome deflation but it also has a higher risk of overheating the real economy. For this reason, in almost all cases in Chart 15, the commitment based on the price level results in a smaller inflation fluctuation loss but a larger GDP fluctuation loss. On a net basis, since the risk of overheating the economy is greater than the merits gained from rapidly overcoming deflation, the performance of the commitment based on the price level cannot easily overcome the performance of

---

[21] As far as the authors know, no other research has been reported thus far analyzing this type of commitment. Ito and Mishkin(2004), however, proposed a similar policy. They made recommendations for the monetary policy in Japan as follows: (1) A certain type of price level target should be set to overcome deflation; (2) a specific candidate for the targeted path of the price level can be the one which starts at October 1997 and increases with a trend of 1% inflation; and (3) once the targeted price level is achieved, monetary policy should shift to inflation targeting with an announcement of a specific inflation target from the price level targeting. Compared with their proposal, we do not posit the price level as the direct policy target, but rather use it as a necessary condition to exit the zero interest rate policy. Although there are such differences, their objectives can be considered as similar in pursuing a stronger easing effect temporarily by setting a commitment based on the price level.

the commitment based on the inflation rate. This tendency is particularly pronounced in cases (5) and (6) where the supply shock has a high weight.

In order to investigate the robustness of the above analysis, we change the setting of the trend of the "targeted" price level path indicated in the commitment from 0% to 1%. We will not go into detail, but the results of the short-term simulation for this policy rule indicate that the qualitative characteristics of the policy performance are very similar to those above. Quantitatively, in almost all cases of initial shocks, the inflation fluctuation costs are smaller and the GDP and interest rate fluctuation costs are larger for the case of 1% trend of the "targeted" price level path than the case of 0% trend, since the monetary easing effect is stronger for the former case.

**(4) Performance comparison with the nonlinear optimal simple rule**

Next we conducted short-term simulations, given the initial downward shocks, for the nonlinear optimal simple rule derived in section 4, assuming that the rule is sufficiently recognized by the private sector. In Chart 16 we compare those results with the performance of the optimally set inflation rate based commitment derived in subsection 5(1). We note the following points from the results.

- In almost all the cases of initial shocks in Chart 16, the nonlinear optimal simple rule shows better policy performance than the baseline Taylor-type rule with the optimally set inflation rate based commitment.
- Compared with the zero interest rate commitment rule, which aims at overcoming deflation by committing to the inflation rate only, the nonlinear optimal simple rule can take the tradeoff in the response to a negative supply shock into account and realize a well-balanced policy which considers both inflation and the real economy. For this reason, the policy performance of the nonlinear optimal simple rule tends to be relatively good, particularly under conditions such as case (6) where the weight of the supply shock is relatively high.
- As an exception, there is just one case where the zero interest rate commitment rule exhibits superior short-term policy performance. That is case (1) where the economy has experienced a substantially strong negative demand shock. Under this case, a decisive monetary policy that rapidly overcomes the demand shock is deemed desirable, so the conclusion is that an extremely strong commitment (a threshold rate

of inflation of 1.5% for exiting the zero interest rate policy) would have good performance over the short term.

Thus, in most cases we can conclude that rather than introducing a zero interest rate commitment, it is more effective to take a preemptive monetary easing stance as implicated by the nonlinear optimal simple rule. As explained in section 4, however, the private sector may initially not have sufficient information regarding how the simple rule, which is intrinsically linear, becomes nonlinear nearby the zero lower bound on nominal interest rates. It is natural to assume that while the private sector understands the policy rule far from the zero lower bound based on the past policy record, it has only incomplete information regarding the policy rule near zero interest rates, which it has never experienced in the past. In such cases, the dissemination of information from the central bank is considered to be highly significant. While repeatedly providing explanations is important, the introduction of the zero interest rate commitment, as in Japan, may also be significant as a concrete statement that clarifies the central bank's accommodative policy stance to overcome deflation. In other words, the zero interest rate commitment may have the function of conveying the message that monetary policy is being conducted more accommodatively than under the linear baseline Taylor-type rule.

The policy implications of the findings in this entire section can be summarized in the following five points.

(1) In some cases introducing a zero interest rate commitment with optimized settings has a better policy effect than that under the baseline Taylor-type rule.

(2) The conditions for the optimal zero interest rate commitment vary depending on the economic conditions — the size and continuity of the demand and supply shocks — at the time that the commitment is introduced.

(3) While we can assume a commitment based on the price level, such a commitment is superior in strictly limited cases with an extremely large demand shock.

(4) In the process whereby the nonlinear optimal simple rule gains credibility, the introduction of a zero interest rate commitment may have the effect of sending an indirect message on the implementation of preemptive monetary easing to deal with the zero lower bound on nominal interest rates. Similarly, the process of exiting zero

interest rates — or clarifying the conditions for that — may be seen as a policy judgment following the policy rule around zero interest rates and recognized as instructive information for reading the nonlinearity of the rule.

(5) However, in many cases, the baseline Taylor-type rule with a zero interest rate commitment does not have as great a policy effect as the nonlinear optimal simple rule that has gained credibility. Accordingly, aside from the commitment, it is important for the central bank to communicate to the private sector its stance toward preemptive monetary easing nearby the zero lower bound. These communications should go beyond only explaining the conditions under which interest rates will be raised to a positive level, but also include the bank's stance on subsequent interest rate hikes and, conversely, on preemptive monetary easing if a risk emerges that the economy may turn deflationary once again.

## 6. On the Use of Zero Interest Rate Commitments after the Credibility on Preemptive Monetary Easing is Established

Through section 5 we have demonstrated the importance of preemptive monetary easing implicated by the nonlinear optimal simple rule. Does this imply that the significance of introducing a zero interest rate commitment will then dissipate once this policy rule gains credibility?

In general, once a commitment gains credibility, it functions to expand the set of possible economic paths that would not be expected without such a commitment, and improves the performance of the optimal path. To verify whether or not the zero interest rate commitment manifests this kind of function effectively in the future, in this section we assume that the private sector already recognizes the nonlinear optimal simple rule derived in section 4. We conduct short-term simulations, given initial downward shocks, to determine if the nonlinear optimal rule with a zero interest rate commitment based on the inflation rate (see Chart 17 for the conceptual diagram) manifests better policy performance than that without any commitment. The analyses in section 5 assumed that the necessary conditions for exiting the zero interest rate policy were expressed by the zero interest rate commitment and that the sufficient conditions for that were defined by the baseline Taylor-type rule. In contrast, the analyses in this section examine the case where the nonlinear optimal simple rule defines the sufficient conditions for exiting the zero interest rate policy.

The results of the short-term simulation, which are presented in Chart 18, reveal the following points.

- Once the nonlinear optimal simple rule is recognized by the private sector, adding a zero interest rate commitment does not create any additional benefit under many cases of initial shocks, except under cases (1) and (2) where the economy falls into deflation from a large-scale demand shock. In fact, it actually increases the total loss. Specifically, for cases (3) through (6) the total loss is minimized not when a threshold inflation rate is set for exiting the zero interest rate policy, but rather when a pure nonlinear optimal simple rule is set without any commitment. We find the reason for this result by looking at the individual losses across different policy rules in Chart 18. That is, while the marginal price stability effect gained from adding the zero interest rate commitment is relatively small, the risk of overheating the economy that accompanies the reinforcement of price stability is large.

- On the other hand, in cases (1) and (2) the total loss can be decreased by adding a zero interest rate commitment when the optimal conditions are chosen. This is because, when there is a large negative demand shock, a comparatively large benefit is gained from the strong monetary easing effect to erase the shock.

- Under the same demand and supply shocks, the threshold rate of inflation for exiting the zero interest rate policy is lower when the preemptive monetary easing policy is recognized as in Chart 18 (nonlinear optimal simple rule with commitment) than when it is not recognized as in Chart 11 (baseline Taylor-type rule with commitment). For example, in case (2), the threshold rate of the former is -0.5%, but that of the latter is +0.5%. This shows that if the preemptive monetary easing policy is recognized, the additional monetary easing required to deal with deflation can be relatively small.

Based on the above results, it is important to identify whether or not the private sector recognizes the central bank’s stance toward preemptive monetary easing when considering whether or not the zero interest rate commitment should be introduced and how the threshold rate of inflation should be optimally set.

## 7. Concluding Remarks: Summary of Findings and Further Issues

This paper has examined, based on stochastic simulation analyses, what kinds of monetary policy rules are effective assuming a small-scale structural model giving consideration to the zero lower bound on nominal interest rates. We have already summarized the policy implications gained from these analyses at the outset in section 1. In this section, we will now simply review the economic implications of the analytical findings, utilizing conceptual diagrams.

Charts 19 and 20 both indicate that losses from the inflation rate fluctuation and from the GDP gap fluctuation depend on the monetary policy rule, and that the policy optimizing the tradeoff between them is defined as the optimal policy. Among the figures, Chart 19(1) shows that the optimal Taylor-type rule exists at the tangent between the set of Taylor-type rules and the indifference curve regarding the social utility (ie. the loss function). In defining the loss function, the analyses in this paper assume that Japanese monetary policy prior to the zero interest rate period generally followed this optimal Taylor-type rule. Figure 19(2) shows that when a zero interest rate commitment is added to the baseline Taylor-type rule (shown by the black dot), there are some cases where the policy performance improves (shown by the arrows with solid lines) and others where it worsens (shown by the arrows with dotted lines). As demonstrated in subsection 5(1), this depends on whether or not the commitment is set properly in accordance with the conditions of the demand and supply shocks to the economy. Chart 20(1) shows that the policy performance can be improved (shown by the arrows with solid lines) by properly nonlinearizing the baseline Taylor-type rule nearby the zero interest rate. The optimal rule (shown by the black dot) in the set of nonlinear simple rules corresponds to the nonlinear optimal simple rule discovered by the grid search conducted in section 4. Chart 20(2) indicates that when a zero interest rate commitment is added to the nonlinear optimal simple rule (the black dot), the policy performance improves in some cases (shown by the arrows with solid lines) and worsens in others (shown by the arrows with dotted lines). As found in section 6, only in limited cases can the policy performance be improved by adding a zero interest rate commitment after the nonlinear optimal simple rule has already gained credibility. In theory it should be possible to realize the best possible performance at the policy frontier by selecting and implementing the most optimal commitment policy from among all the various options. But in actual practice, there is a strong likelihood that the contents of such a commitment would be so complex that such a commitment would not be feasible. This consideration suggests that finding a commitment superior to the nonlinear optimal simple rule is far from simple.

It should also be pointed out that the relative performance of each policy rule depends on the initial demand and supply shocks. Chart 21 shows the ranking, for each initial shock, of all the policy rules analyzed in this paper in terms of the total loss under a short-term simulation. For example, in case (1) with the largest demand shock, the nonlinear optimal rule with an inflation rate based commitment (where the optimal threshold rate is 0.0%) performs best, and the baseline Taylor-type rule with a price level based commitment performs second best. The third best is the baseline rule with an inflation rate based commitment where the optimal threshold rate is high at +1.5% because of the need to compensate for the lack of recognition of the preemptive monetary easing. In case (2) with the second largest demand shock, the nonlinear optimal rule with an inflation rate based commitment is also the best policy rule as in case (1), but the optimal threshold rate becomes -0.5%, which is lower than in case (1). The ranking of the baseline Taylor-type rule with a commitment based on price level falls to fourth place and the nonlinear optimal rule without any commitment obtains second place. In contrast, in cases (3) and (4) where the initial demand shock is relatively small, the nonlinear optimal rule without any commitment performs best and the one with an inflation rate based commitment is second best. In case (5) where the initial demand shock is very small, any zero interest rate commitment is undesirable regardless of whether the policy stance toward preemptive monetary easing has gained credibility or not. The policy implications from these points are that when approaching the zero lower bound on nominal interest rates the central bank should (i) try to identify the feature of the economic shocks accurately and (ii) judge the extent to which the preemptive monetary easing policy is recognized.

This paper also examined the following two possible functions of a zero interest rate commitment.[22]

(1) The message function of transmitting information to the private sector regarding the contents of the nonlinear monetary policy rule nearby zero interest rates.

(2) As a policy instrument to realize a better economic path that would be impossible to achieve without such a commitment.

The results of our analyses indicate that, when the private sector does not recognize the contents of the nonlinear policy rule, a zero interest rate commitment that is appropriately set in accordance with the economic conditions is highly likely to demonstrate function (2). Under this function the contents of the zero interest rate commitment are directly incorporated into the policy rule, and have the effect of stabilizing the economy. In contrast, under (1) the introduction of the zero interest rate commitment has the potential function of working on the formation of private-sector expectations regarding preemptive monetary easing before facing zero interest rates as well as the policy response just after interest rates return from zero to a positive level. While we do not statistically verify that point in this paper, we believe there is a substantial likelihood that this potential function will become manifest as the understanding of the desirable monetary policy under the zero lower bound permeates.

The analyses in this paper, especially the quantitative contents, are in part dependent on the accuracy of the presumed model and the appropriateness of the assumptions made. Accordingly our analytical findings should be evaluated with some leeway. In this context, we may list the specific points at issue, which require further examinations, as follows.

- Our analyses have adopted the policy judgment criteria regarding the relative importance of inflation stability, GDP stability and interest rate stability by assuming that Japan’s prior monetary policy has been implemented in an optimal manner. In other words, under this approach we have evaluated policy performance assuming that preference in

[22] Aside from above points, another possible effect of a zero interest rate commitment is reducing the mid-term and long-term interest rate risk premium by lowering the uncertainty regarding future monetary policy, and thus realizing a monetary easing effect. This issue, however, lies outside the range of the analyses in this paper. According to empirical analyses in Baba *et al*. (2005), there are cases where such an effect has been observed in Japan during the zero interest rate and the quantitative easing policy period, but the scale of these effects may not have been all that large.

the conduct of prior monetary policy should be unchanged. The counterargument to this notes that there is possibility that Japanese monetary policy has not been conducted optimally in the past, and that a different method could be used to set the policy judgment criteria. One example of such an alternative approach, as explained in Footnote 9, is to calibrate the deep parameters of the economic structure and then derive the loss function from the economic theory. Compared with the settings used in this paper, the conclusions under that method are to give greater emphasis to inflation stability and to place very little emphasis on GDP stability and interest rate stability. In that case, one would expect substantial changes to the analytical findings. One may surmise that the trend would then be toward a high evaluation of the zero interest rate commitment under each case, since qualitatively this would make it possible to ignore the loss of GDP stability that is a side effect of the commitment. In short, when the economic shocks and other settings are given, the conditions for setting the optimal commitment would likely change toward being more accommodative. On the other hand, it is not clear what changes this would bring to the nonlinear optimal simple rule. In general, it is important to discuss further on the best criteria for policy judgment.

- As the ratio of forward-looking to backward-looking agents changes in the structural model, the expectations formation mechanism changes, and this influences the conclusions in our analyses. To address this, it is desirable to check the robustness of the analyses. In particular, the effects of policies that strongly work on the formation of expectations by forward-looking agents — for example, a commitment based on price level — may change significantly depending upon this parameter.

- It would also be desirable to investigate how the optimal monetary policy changes when the leeway for supporting economic conditions with fiscal policy changes as the concern regarding future tax changes depending on the seriousness of fiscal deficits. In such cases, qualitatively, a more accommodative monetary policy might become optimal with more serious fiscal deficits.

- We have assumed a Taylor-type rule as the baseline monetary policy rule in this paper. This choice was made from the assumption that the rule can approximate the expected future monetary policy, since the prior empirical research has reported that the rule well approximates the past policy. While the Taylor-type rule may be considered to involve some features of expectations management, it may be rational to adopt the Taylor-type rule if the rule describes the actual expectations toward monetary policy. Meanwhile, from the theoretical perspective, it is interesting to assume the optimal discretionary policy, instead of the Taylor-type rule, as the baseline rule to evaluate the effects of the zero interest rate commitment since the rule is pure in that it does not involve any expectations effect. In order to investigate this case, we need to derive the solution of the optimal discretionary policy under a hybrid structural model allowing both forward-looking and backward-looking agents and with the zero interest rate constraint taken into account. This is a remaining issue for the future.

## References


Adam, K., and R. M. Billi, 2004a, “Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates,” ECB Working Paper Series, No. 377.

———, and ———, 2004b, “Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates,” ECB Working Paper Series, No. 380.

Amato, J. D., and T. Laubach, 2003, “Rule-of-Thumb Behaviour and Monetary Policy,” *European Economic Review*, 47, pp. 791-831.

Baba, N., S. Nishioka, N. Oda, M. Shirakawa, K. Ueda, and H. Ugai, 2005, “Japan’s Deflation, Problems in the Financial System and Monetary Policy,” *Monetary and Economic Studies*, Vol.23, No.1, Bank of Japan.

Bernanke, B. S., and V. R. Reinhart, 2004, “Conducting Monetary Policy at Very Low Short-Term Interest Rates,” *American Economic Review*, Vol. 94, No. 2, pp. 85-90.

———, ———, and B. P. Sack, 2004, “Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment,” Financial and Economic Discussion Series 2004-48, Federal Reserve Board.

Clouse, J., D. Henderson, A. Orphanides, D. Small, and P. Tinsley, 2003, “Monetary Policy When the Nominal Short-Term Interest Rate Is Zero,” Berkeley Electronic Press, *Topics in Macroeconomics*, Vol.3, No.1, Article 12.

Eggertsson, G., and M. Woodford, 2003, “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 1:2003, pp. 139-233.

Fujiki, H., and S. Shiratsuka, 2002, “Policy Duration Effect under the Zero Interest Rate Policy in 1999-2000: Evidence from Japan’s Money Market Data.” *Monetary and Economic Studies*, Vol. 20, No. 1, Bank of Japan, pp. 1-31.

Giannoni, M. P., 2000, “Optimal Interest-Rate Rules in a Forward-Looking Model, and Inflation Stabilization versus Price-Level Stabilization,” Working Paper, Federal Reserve Bank of New York.

Hollinger, P., 1996, “The Stacked-Time Simulator in Troll: A Robust Algorithm for Solving Forward-Looking Models,” Presented at the Computing in Economics and Finance 1996, Society for Computational Economics.

Ito, T., and F. S. Mishkin, 2004, “Monetary Policy in Japan: Problems and Solutions,” Presented at the US-Japan Conference on the Solutions for the Japanese Economy, sponsored by Columbia Business School and the University of Tokyo, on June 19-20, 2004 (http://www.esri.go.jp/jp/workshop/040622/040622mishkin1.pdf).

Jung, T., Y. Teranishi, and T. Watanabe, forthcoming, “Optimal Monetary Policy at the Zero-Interest-Rate Bound,” *Journal of Money, Credit, and Banking*.

Kato, R., and S. Nishiyama, forthcoming, “Optimal Monetary Policy When Interest Rates Are Bounded at Zero,” *Journal of Economic Dynamics and Control*.

Kimura, T., H. Kobayashi, J. Muranaga, and H. Ugai, 2002, “The Effect of the Increase in Monetary Base on Japan’s Economy at Zero Interest Rates: An Empirical Analysis,” IMES Discussion Paper Series, 2002-E-22, Bank of Japan.

———, and T. Kurozumi, 2004, “Effectiveness of History-Dependent Monetary Policy,” *Journal of the Japanese and International Economics*, Vol.18, No.3, pp. 330-361.

Nakov, A., 2004, “The Zero Bound on Nominal Interest Rates and Optimal Monetary Policy under Discretion,” Unpublished, University of Pompeu Fabra, June 2004.

Newey, W. K., and K. D. West, 1987, “A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix,” *Econometrica*, 55, pp. 703-787.

Oda, N., and K. Okina, 2001, “Further Monetary Easing Policies under the Non-Negativity Constraints of Nominal Interest Rates: Summary of the Discussion Based on Japan’s Experience.” *Monetary and Economic Studies*, Vol. 19, No. S-1, Bank of Japan, pp. 323-369.

Okina, K., and S. Shiratsuka, 2004, “Policy Commitment and Expectation Formation: Japan’s Experience under Zero Interest Rates.” *North American Journal of Economic and Finance*, Vol. 15, No. 1, pp. 75-100.

Reifschneider, D., and J. C. Williams, 2000, “Three Lessons for Monetary Policy in a Low Inflation Era,” *Journal of Money, Credit, and Banking*, Part 2, 32 (4), pp. 936-966.

Sack, B., 2000, “Does the Fed Act Gradually? A VAR Analysis,” *Journal of Monetary Economics*, 46, pp. 229-256.

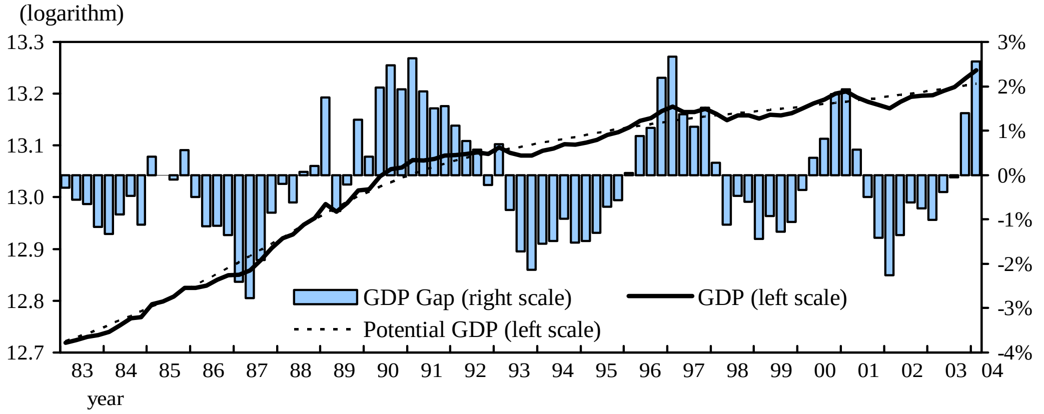
Shirakawa, M., 2002, “One Year under ‘Quantitative Easing’,” IMES Discussion Paper Series, No. 2002-E-3, Bank of Japan.

Williams, J. C., 1999, “Simple Rules for Monetary Policy,” Financial and Economic Discussion Series 1999-12, Federal Reserve Board.

Woodford, M., 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

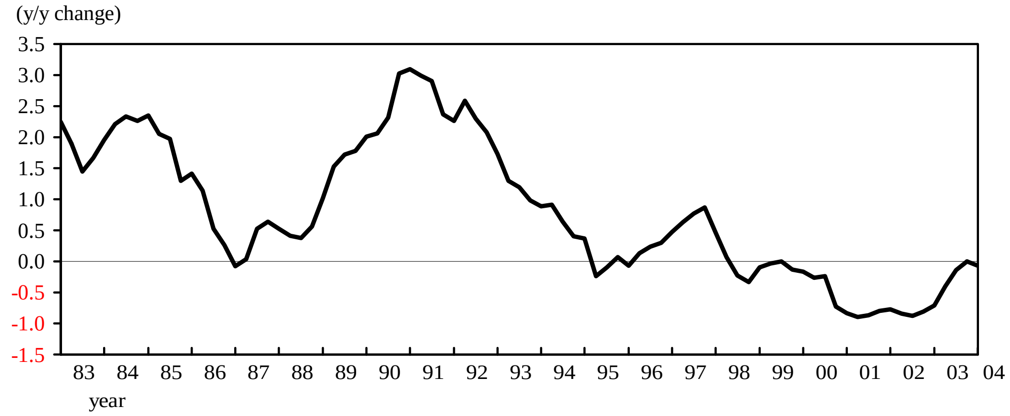
# Macroeconomic Data Used in the Model Estimation

(1) Real GDP ( $y_t$ ), Potential GDP ( $y_t^{HP}$ ), and GDP Gap ( $x_t$ )



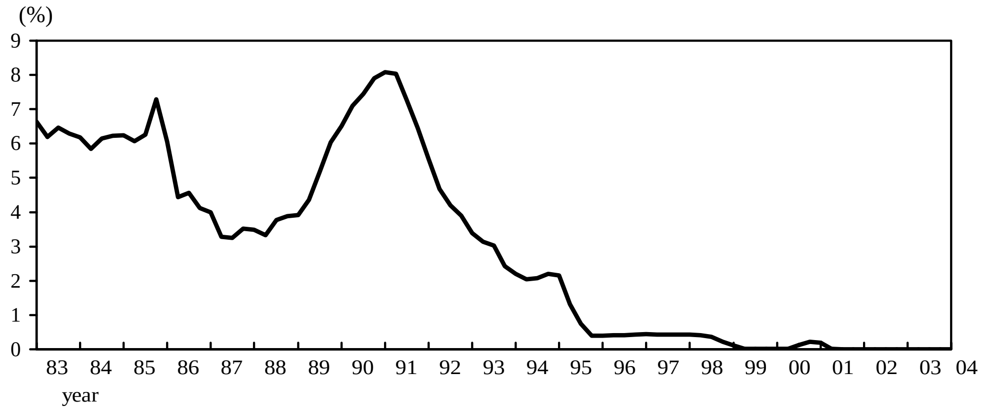

Note: Potential GDP is defined as real GDP smoothed by an HP filter ( $\lambda$ =1,600).
Source: Cabinet Office.

(2) CPI ( $\pi_t$ )




Note: CPI figures are nationwide, excluding perishables.
Source: Ministry of Internal Affairs and Communications.

(3) Call Rate ( $i_t$ )




Note: Uncollateralized overnight call rate; however, collateralized call rate (next-day delivery) figures used prior to 1986 Q4. Quarterly average value of monthly averages.
Source: Bank of Japan.

(Chart 2)

# Estimation of the Structural Model

**[**Structural Model**]**

1. IS Curve: $x_t = \phi \cdot E_t x_{t+1} + (1-\phi) \cdot x_{t-1} - \sigma \cdot (i_t - E_t \pi_{t+1} - r_t^n) + g_t$
2. AS Curve: $\pi_t = \gamma \cdot E_t \pi_{t+1} + (1-\gamma) \cdot \pi_{t-1} + \kappa \cdot x_t + u_t$
3. Natural Rate of Interest: $r_t^n = \eta_s \cdot (y_{t+1}^{HP} - y_t^{HP}) + \eta_c$
4. Demand Shock: $g_t = \rho_g \cdot g_{t-1} + \varepsilon_t$
5. Price Shock: $u_t = \rho_u \cdot u_{t-1} + \mu_t$

**[**Estimation Results**]**

Estimation period: 1983Q1-2004Q1

Estimation method: GMM (Eq.1,2,3), OLS (Eq.4,5)

| | Estimated Value | Standard Error | t-value |
|---|---|---|---|
| $\phi$ | 0.517 | 0.036 | 14.35 |
| $\sigma$ | 0.057 | 0.042 | 1.36 |
| $\gamma$ | 0.454 | 0.028 | 16.02 |
| $\kappa$ | 0.014 | 0.007 | 2.05 |
| $\eta_s$ | 1.425 | 0.368 | 3.88 |
| $\eta_c$ | -1.253 | 1.032 | -1.21 |
| $\rho_g$ | -0.060 | 0.126 | -0.48 |
| $\rho_u$ | 0.381 | 0.116 | 3.29 |

Note: In the GMM estimation, we adjusted for the heteroscedasticity of the error term and the serial correlation following the method in Newey and West [1987]. The number of lags in error terms is 3.

(Chart 3)

# Estimation of the Monetary Policy Rule

**[**Monetary Policy Rule**]**

$$i_t = \phi_i \cdot i_{t-1} + (1-\phi_i) \cdot \{r_t^n + \pi^* + \phi_x \cdot x_t + \phi_\pi \cdot (\pi_t - \pi^*)\}$$

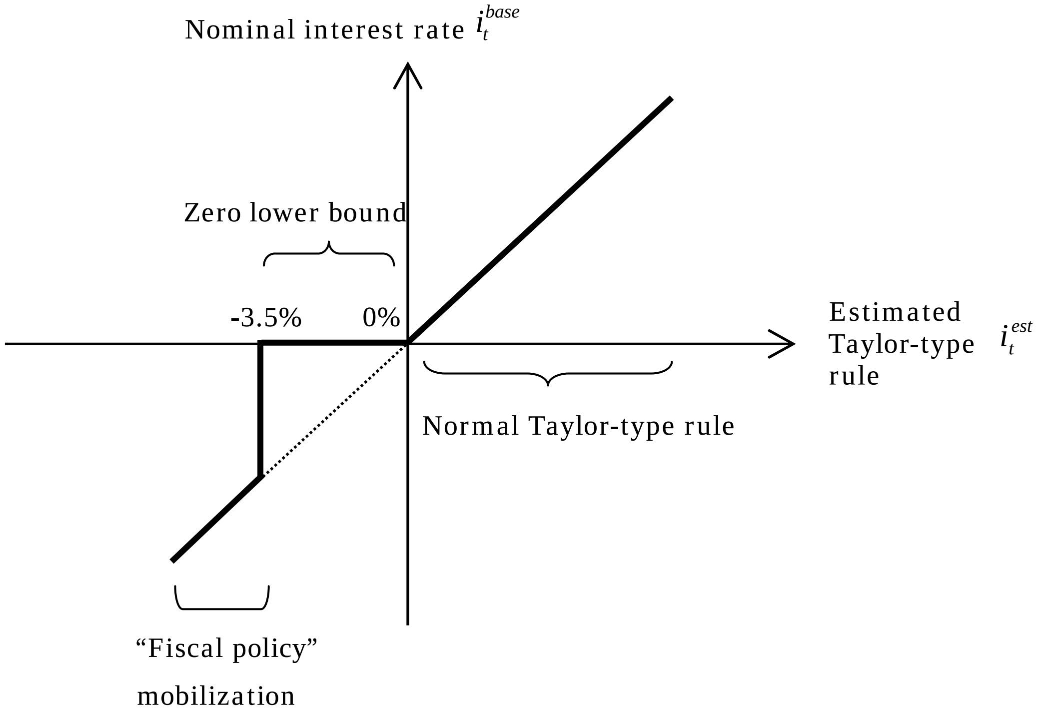
**[**Estimation Results**]**

Estimation Period: 1983Q1-1999Q1

Estimation Method: OLS

| | Estimated Value | Standard Error | t-value |
|---|---|---|---|
| $\phi_i$ | 0.830 | 0.042 | 19.72 |
| $\phi_x$ | 0.755 | 0.351 | 2.15 |
| $\phi_\pi$ | 1.686 | 0.371 | 4.54 |
| $\pi^*$ | 1.805 | 0.548 | 3.30 |

# Baseline Taylor-type Policy Rule




(1) Long-term simulations

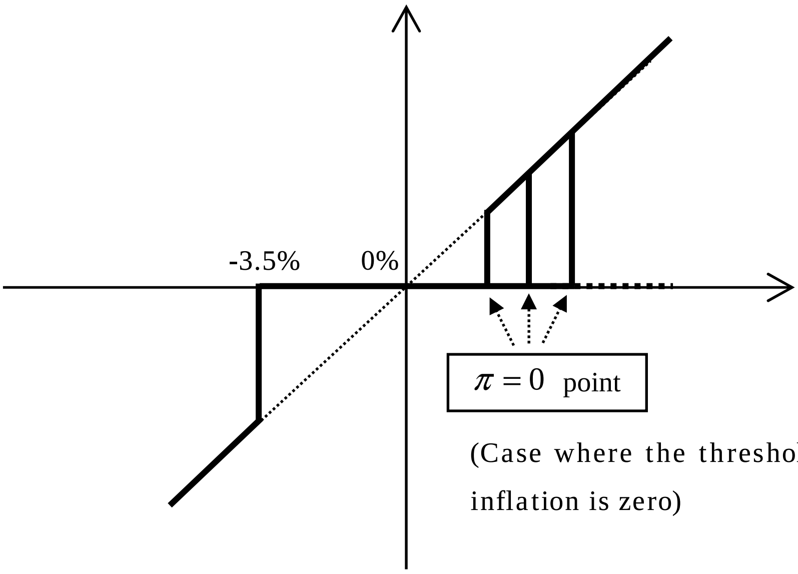
| | | |
|---|---|---|
| Total Loss | | 2.828 |
| Individual Losses | Inflation fluctuation | 0.784 |
| | GDP fluctuation | 1.529 |
| | Interest rate fluctuation | 0.879 |

(2) Short-term simulations

| Initial demand (g) and supply (u) shocks | | g: -4% u: -0.5% | g: -3% u: -0.5% | g: -2% u: -0.5% | g: -1% u: -0.5% | g: -1% u: -1% |
|---|---|---|---|---|---|---|
| Total Loss | | 11.78 | 9.16 | 7.27 | 5.96 | 13.91 |
| Individual Losses | Inflation fluctuation | 2.73 | 2.38 | 2.14 | 1.96 | 5.55 |
| | GDP fluctuation | 6.59 | 4.73 | 3.46 | 2.67 | 5.30 |
| | Interest rate fluctuation | 4.14 | 3.38 | 2.72 | 2.16 | 4.91 |

(Chart 5)

## Baseline Policy Rule with a Zero Interest Rate Commitment



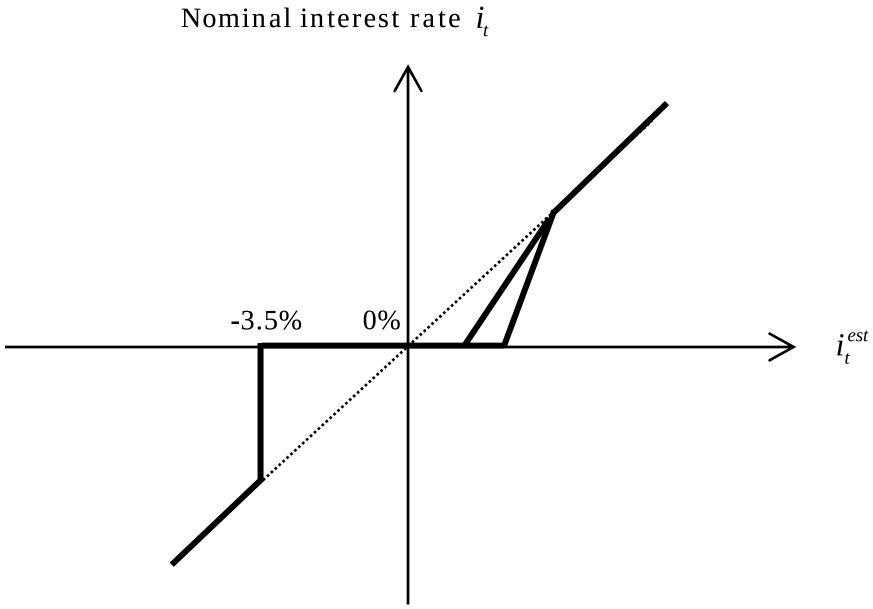

(1) Long-term simulations

| | | |
|---|---|---|
| Total Loss | | 2.957 |
| Individual Losses | Inflation fluctuation | 0.714 |
| | GDP fluctuation | 1.678 |
| | Interest rate fluctuation | 0.964 |

(2) Short-term simulations

| Initial demand (g) and supply (u) shocks | | g: -4%<br>u: -0.5% | g: -3%<br>u: -0.5% | g: -2%<br>u: -0.5% | g: -1%<br>u: -0.5% | g: -1%<br>u: -1% |
|---|---|---|---|---|---|---|
| Total Loss | | 11.73 | 9.00 | 7.26 | 6.20 | 14.04 |
| Individual Losses | Inflation fluctuation | 2.49 | 1.93 | 1.59 | 1.40 | 4.16 |
| | GDP fluctuation | 6.59 | 4.83 | 3.78 | 3.25 | 6.48 |
| | Interest rate fluctuation | 4.41 | 3.67 | 3.07 | 2.54 | 5.49 |

# Examples of Nonlinear Simple Rules

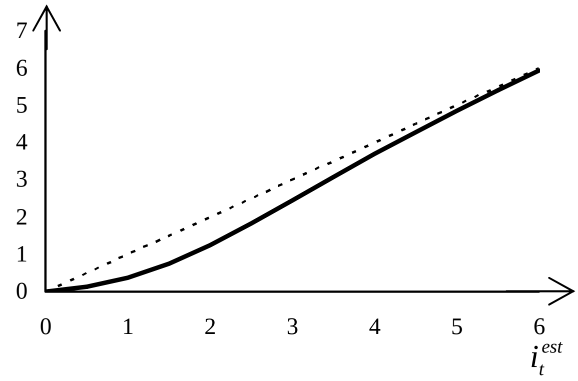



$$i_t^{NL}\left(i_t^{est};a,b\right)=i_t^{base}\left(i_t^{est}\right)\cdot NL\left(i_t^{est};a,b\right), \quad if \quad i_t^{est}\geq 0$$
$$=i_t^{base}\left(i_t^{est}\right), \quad if \quad i_t^{est}<0$$

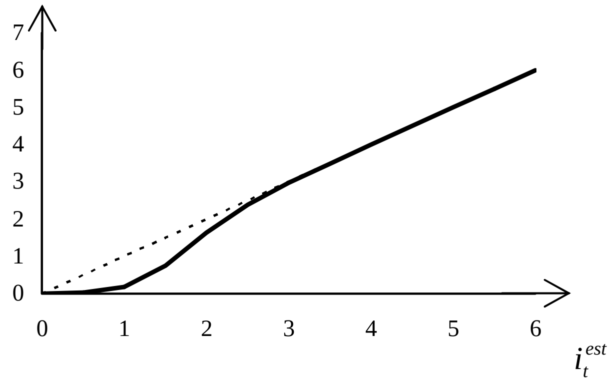
where, $NL(x;a,b)\equiv 1-\left[1+\exp\{a\cdot(x-b)\}\right]^{-1}$.

[ Examples ]

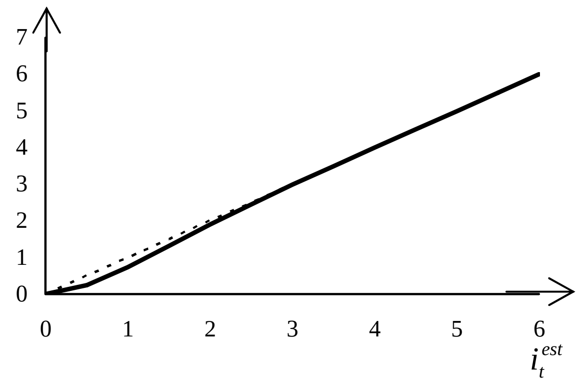
$i_t^{NL}\left(\ i_t^{est};\ 1,\ 1.5\ \right)$



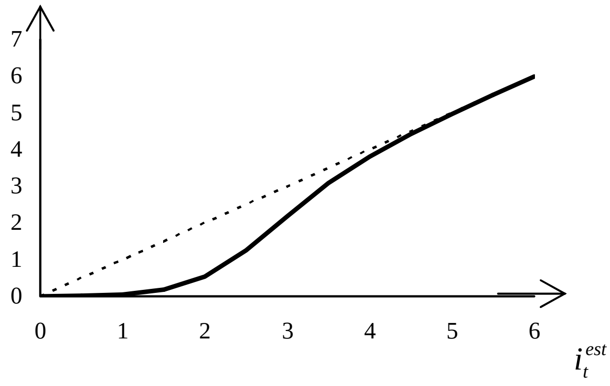

$i_t^{NL}\left(\ i_t^{est};\ 3,\ 1.5\ \right)$




$i_t^{NL}\left(\ i_t^{est};\ 2,\ 0.5\ \right)$




$i_t^{NL}\left(\ i_t^{est};\ 2,\ 2.5\ \right)$



(Chart 7)

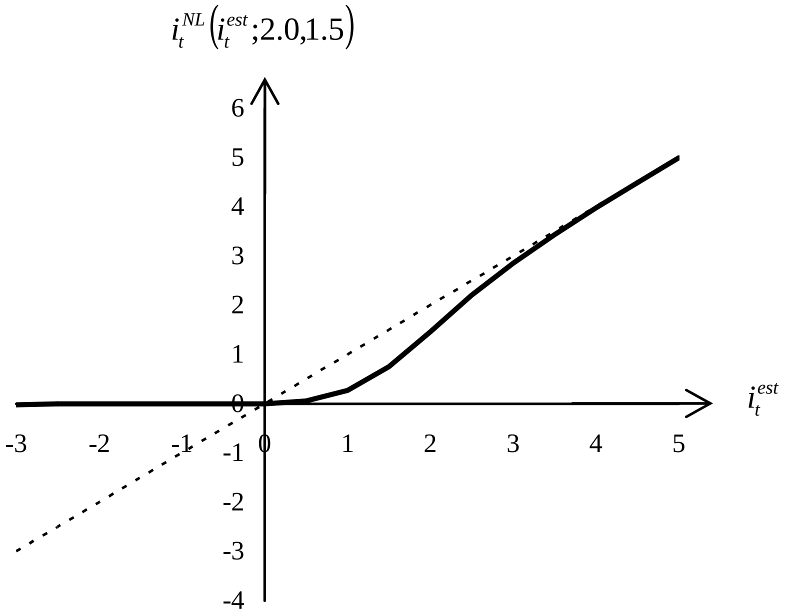
## Grid Search Results with Long-term Simulations: Total Losses under the Policy Rule $i_t^{NL}\left(i_t^{est};a,b\right)$

| | | a | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| b | 1.0 | 2.869 | 2.791 | 2.818 | 2.829 |
| | 1.5 | 2.973 | 2.774 | 2.801 | 2.824 |
| | 2.0 | 3.159 | 2.788 | 2.782 | 2.810 |
| | 2.5 | 3.457 | 2.880 | 2.805 | 2.811 |

Note: The minimum value of the total loss is shadowed.

(Chart 8)

# Nonlinear Optimal Simple Rule



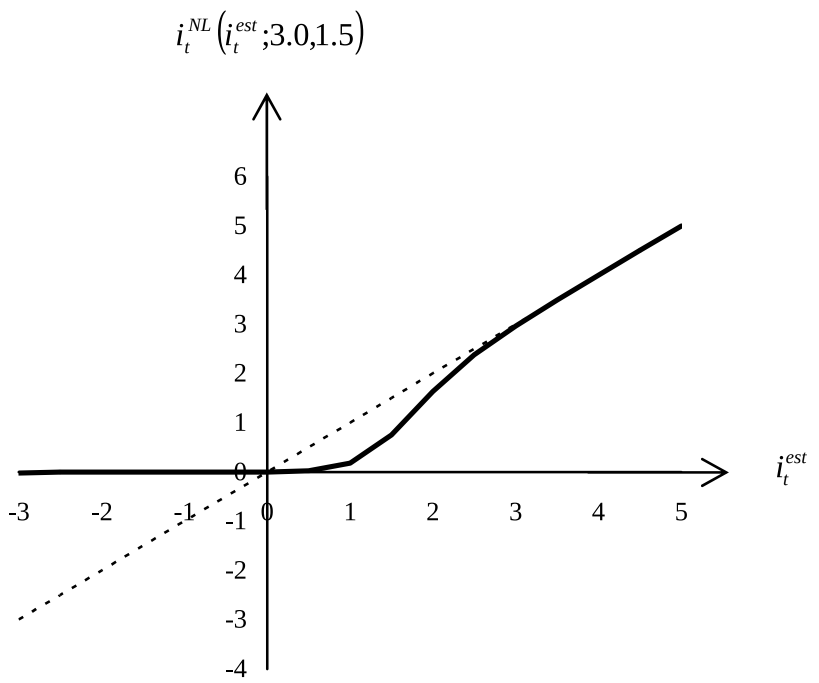

(1) Long-term simulations

| Total Loss | | 2.774 |
|---|---|---|
| Individual Losses | Inflation fluctuation | 0.716 |
| | GDP fluctuation | 1.515 |
| | Interest rate fluctuation | 0.920 |

(2) Short-term simulations

| Initial demand (g) and supply (u) shocks | | g: -5%<br>u: -0.5% | g: -4%<br>u: -0.5% | g: -3%<br>u: -0.5% | g: -2%<br>u: -0.5% | g: -1%<br>u: -0.5% | g: -1%<br>u: -1% |
|---|---|---|---|---|---|---|---|
| Total Loss | | 14.16 | 10.80 | 8.47 | 6.82 | 5.70 | 13.25 |
| Individual Losses | Inflation fluctuation | 2.56 | 2.12 | 1.84 | 1.66 | 1.54 | 4.56 |
| | GDP fluctuation | 8.62 | 6.12 | 4.47 | 3.40 | 2.79 | 5.78 |
| | Interest rate fluctuation | 5.07 | 4.25 | 3.52 | 2.84 | 2.24 | 4.72 |

# Nonlinear Optimal Simple Rule when the Target Inflation Rate is Changed (from 1.81% to 1.0%)




**[** Grid Search Results: Total Losses **]**

| | | a | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| b | 1.0 | 2.99 | 2.76 | 2.77 | 2.79 |
| | 1.5 | 3.19 | 2.79 | 2.75 | 2.76 |
| | 2.0 | 3.51 | 2.92 | 2.81 | 2.78 |
| | 2.5 | 3.98 | 3.20 | 3.01 | 2.95 |

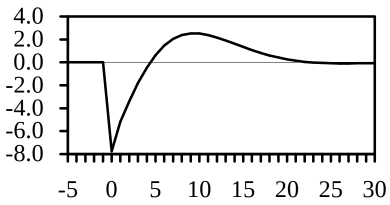
Note: The minimum value of the total loss is shadowed.

(Chart 10)

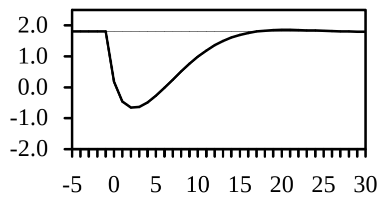
# Impulse Responses Under Each Initial Shock (with Baseline Taylor-type Rule)

(1) Supply shock u:-0.5%, demand shock g:-5%.

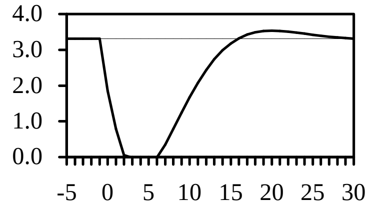
1. GDP gap




2. Inflation rate

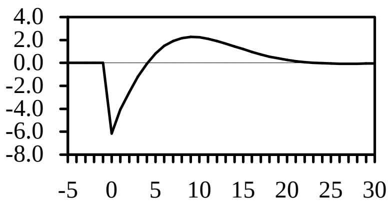



3. Interest rate

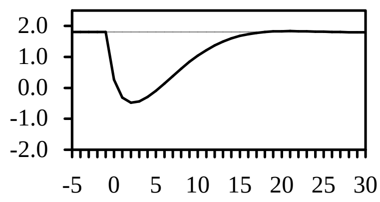



(2) Supply shock u:-0.5%, demand shock g:-4%.

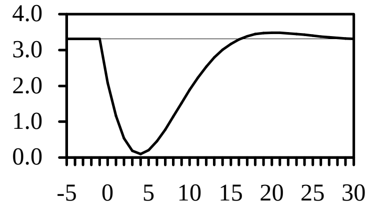
1. GDP gap




2. Inflation rate

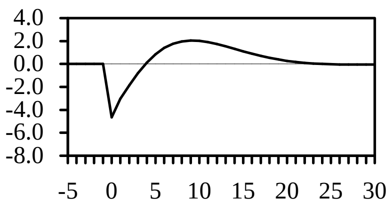



3. Interest rate

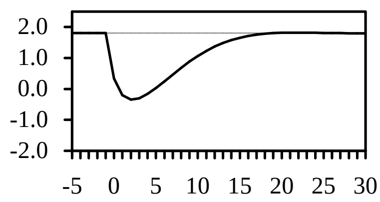



(3) Supply shock u:-0.5%, demand shock g:-3%.

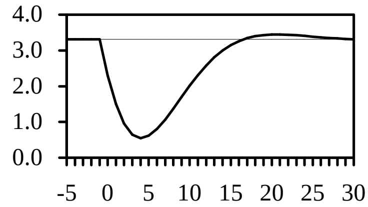
1. GDP gap




2. Inflation rate

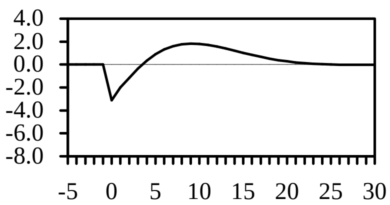



3. Interest rate

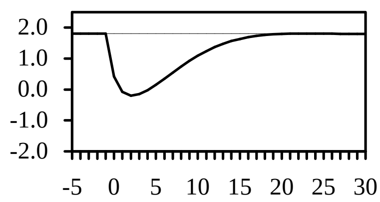



(4) Supply shock u:-0.5%, demand shock g:-2%.

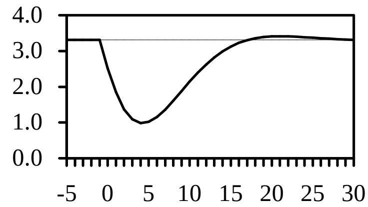
1. GDP gap




2. Inflation rate

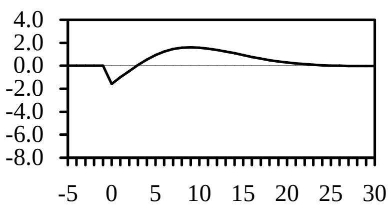



3. Interest rate

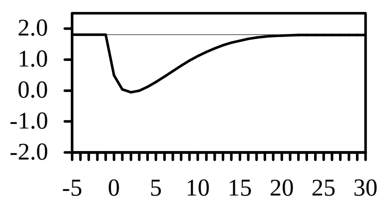



(5) Supply shock u:-0.5%, demand shock g:-1%.

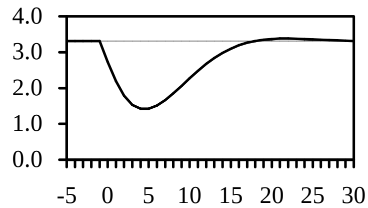
1. GDP gap




2. Inflation rate

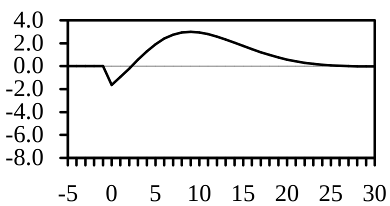



3. Interest rate

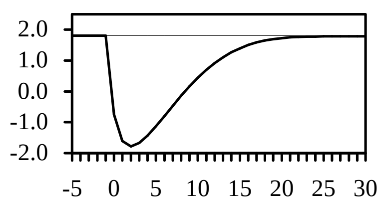



(6) Supply shock u:-1%, demand shock g:-1%.

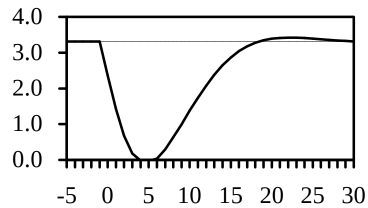
1. GDP gap




2. Inflation rate




3. Interest rate

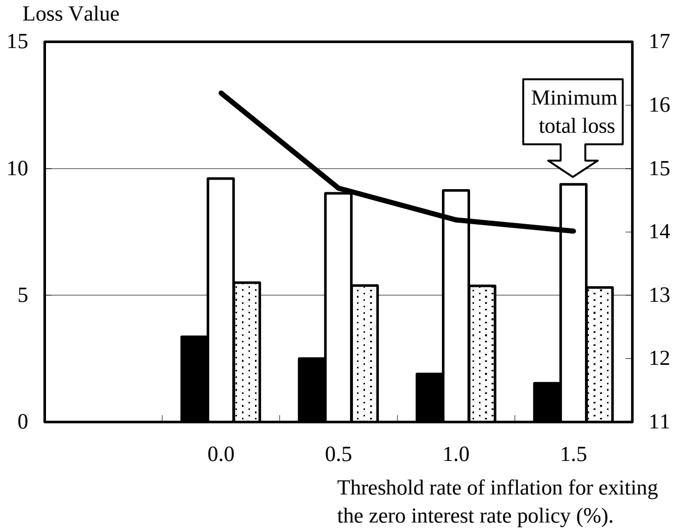


# Short-term Simulations for the Baseline Rule with a Zero Interest Rate Commitment

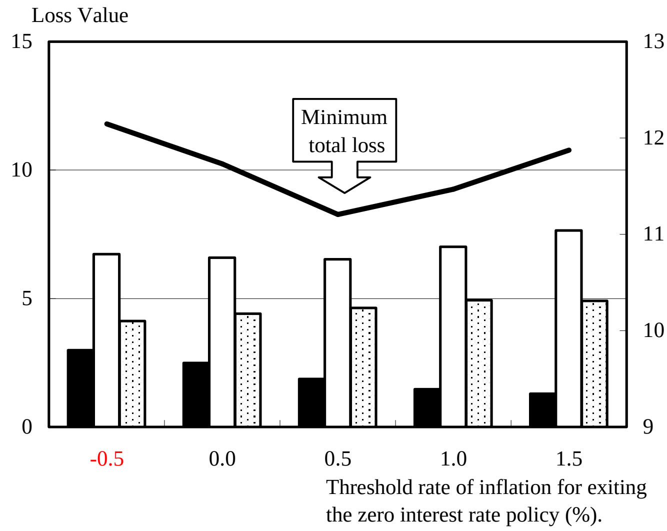
-- Comparison of different commitments under each initial shock

Total loss (right scale)
GDP gap fluctuation loss (left scale)
Inflation rate fluctuation loss (left scale)
Interest rate fluctuation loss (left scale)

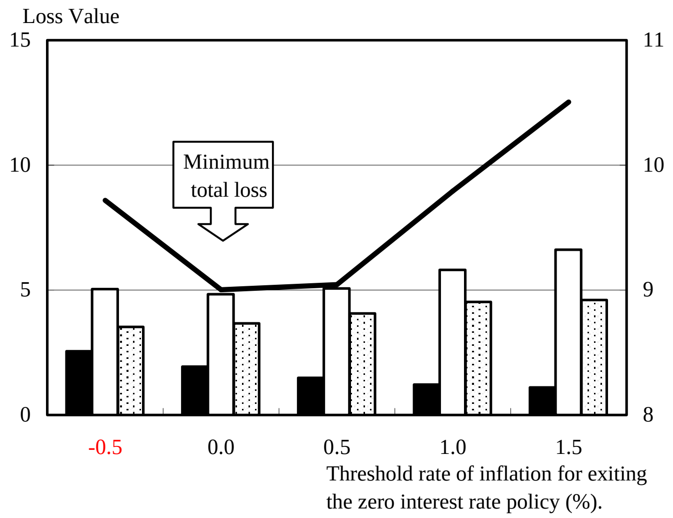
(1) Supply shock u:-0.5%, demand shock g:-5%.



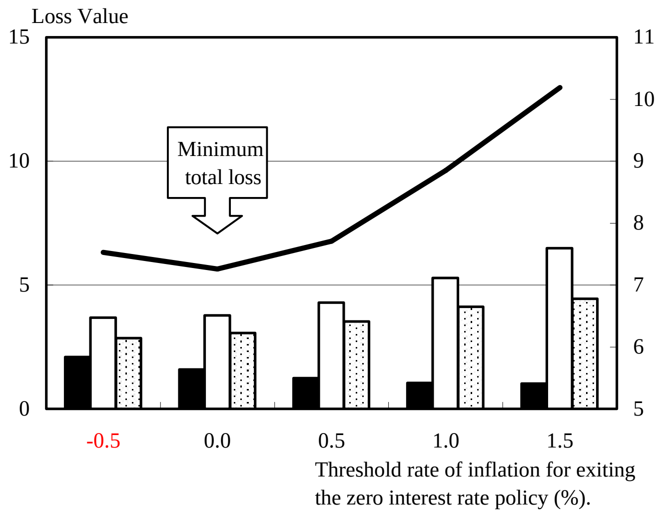

(2) Supply shock u:-0.5%, demand shock g:-4%.



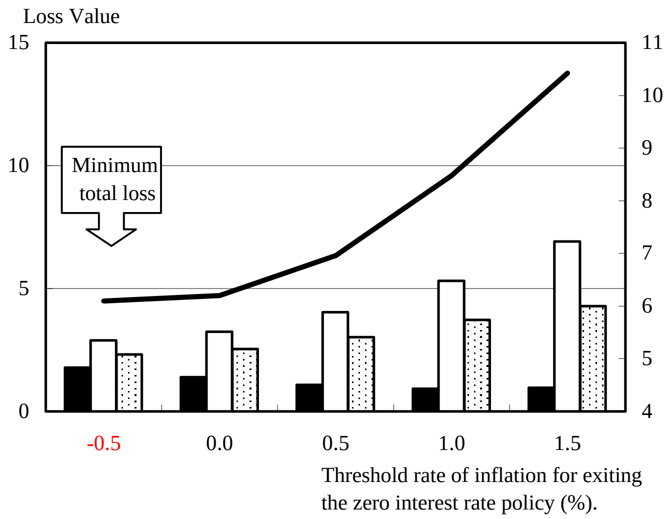

(3) Supply shock u:-0.5%, demand shock g:-3%.



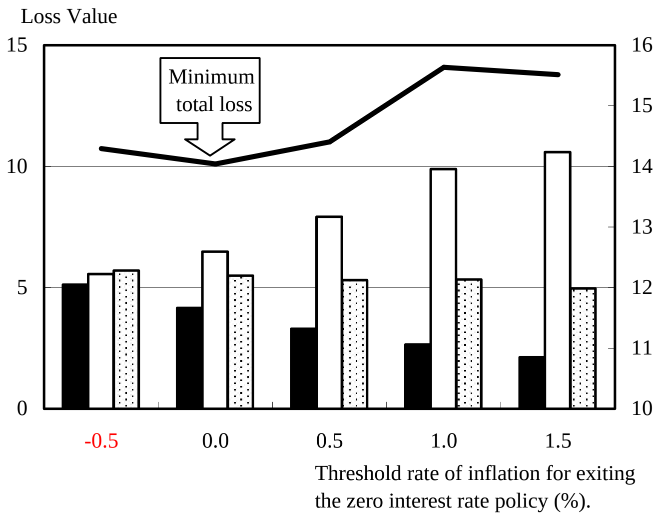

(4) Supply shock u:-0.5%, demand shock g:-2%.




(5) Supply shock u:-0.5%, demand shock g:-1%.




(6) Supply shock u:-1%, demand shock g:-1%.



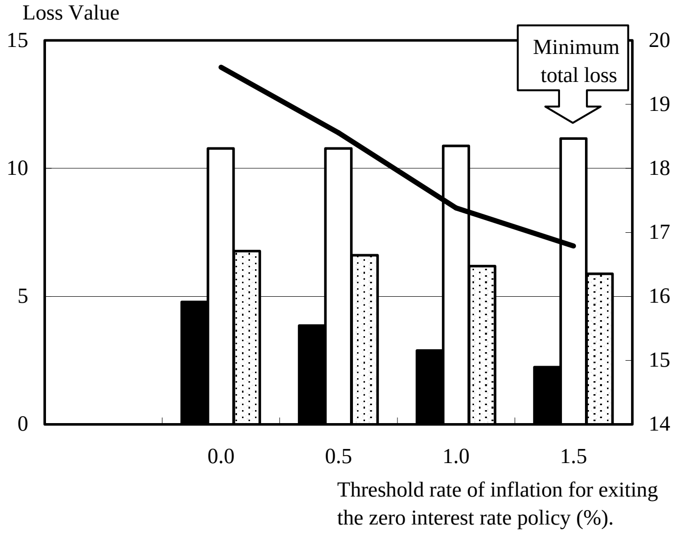
(Chart 12)

# Short-term Simulations for the Baseline Rule with a Zero Interest Rate Commitment

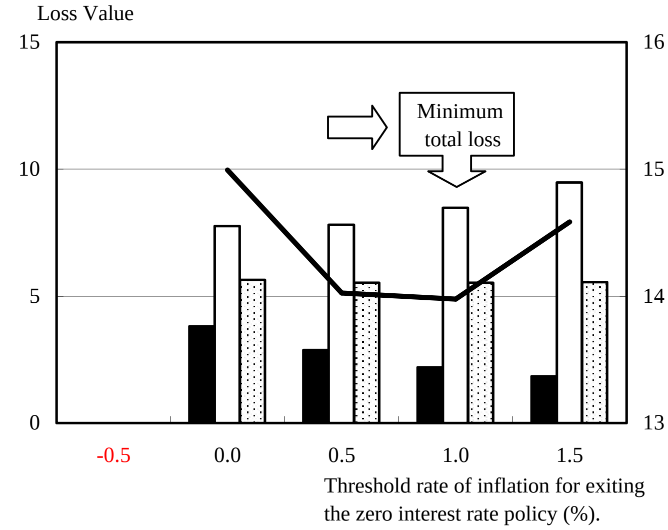
-- When the persistence of supply shocks (auto-correlation coefficient) is increased from 0.38 (the estimated value) to 0.45.

Total loss (right scale)
GDP gap fluctuation loss (left scale)
Inflation rate fluctuation loss (left scale)
Interest rate fluctuation loss (left scale)

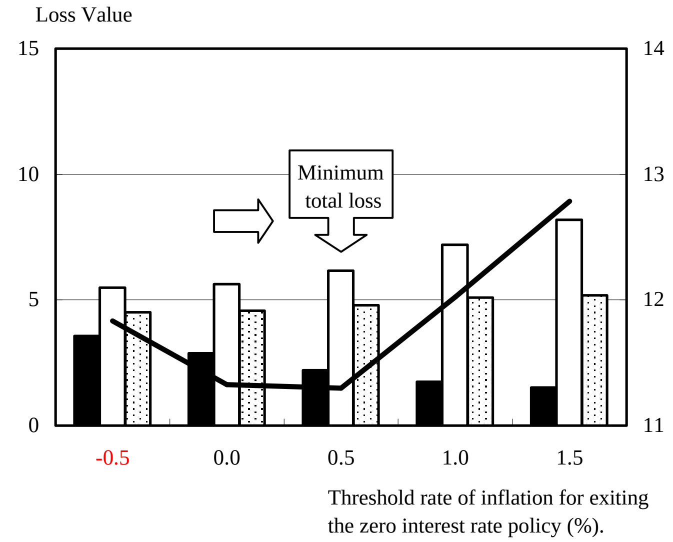
(1) Supply shock u:-0.5%, demand shock g:-5%.



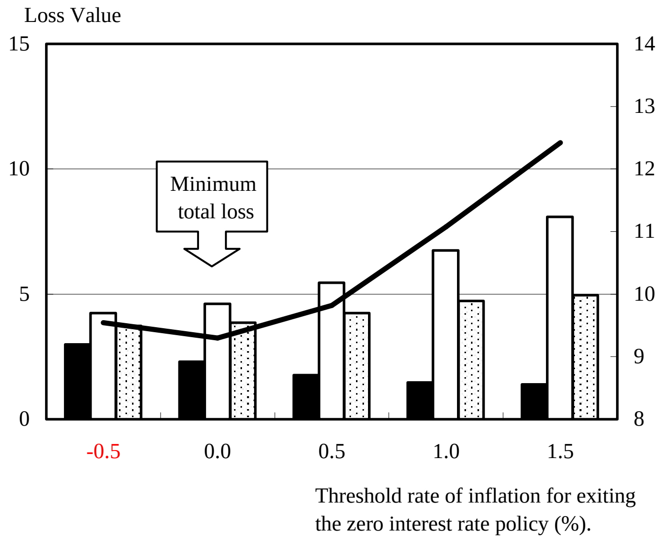

(2) Supply shock u:-0.5%, demand shock g:-4%.



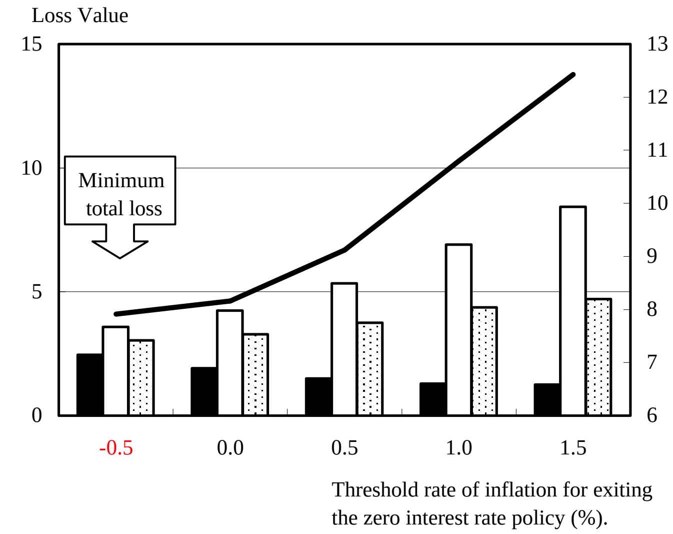

(3) Supply shock u:-0.5%, demand shock g:-3%.




(4) Supply shock u:-0.5%, demand shock g:-2%.




(5) Supply shock u:-0.5%, demand shock g:-1%.



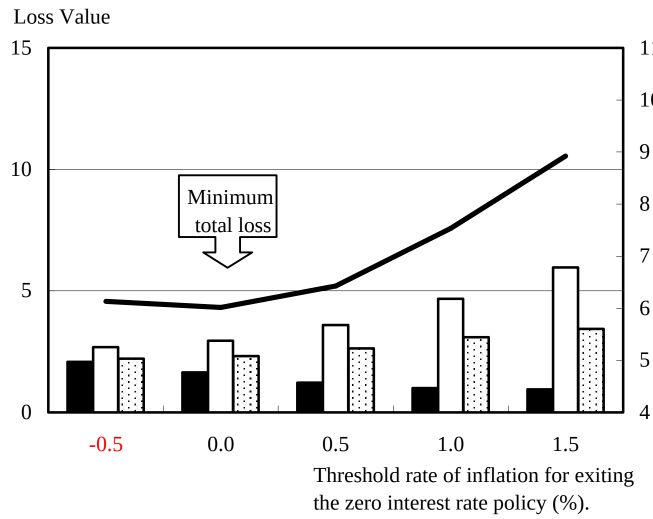
(Chart 13)

## Short-term Simulations for the Baseline Rule with a Zero Interest Rate Commitment when the Setting of the Natural Rate of Interest is Changed

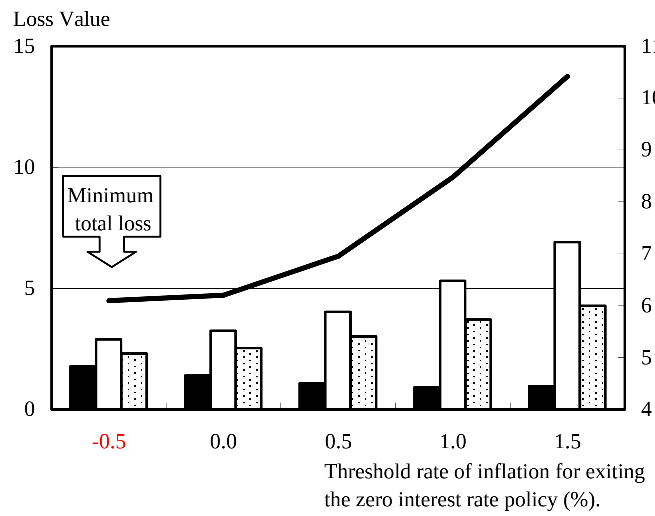
-- Comparison of different commitments under a supply shock of u=-0.5% and a demand shock of g=-1%.

Total loss (right scale)
Inflation rate fluctuation loss (left scale)
GDP gap fluctuation loss (left scale)
Interest rate fluctuation loss (left scale)

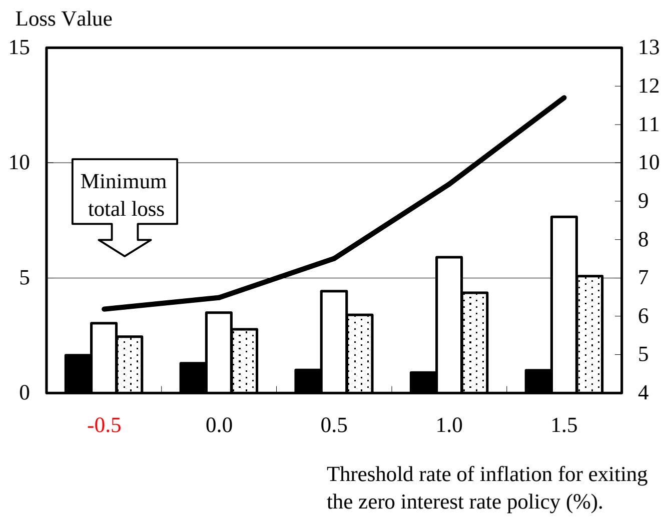
(1) Natural rate of interest: 1.0%




(2) Natural rate of interest: 1.5%



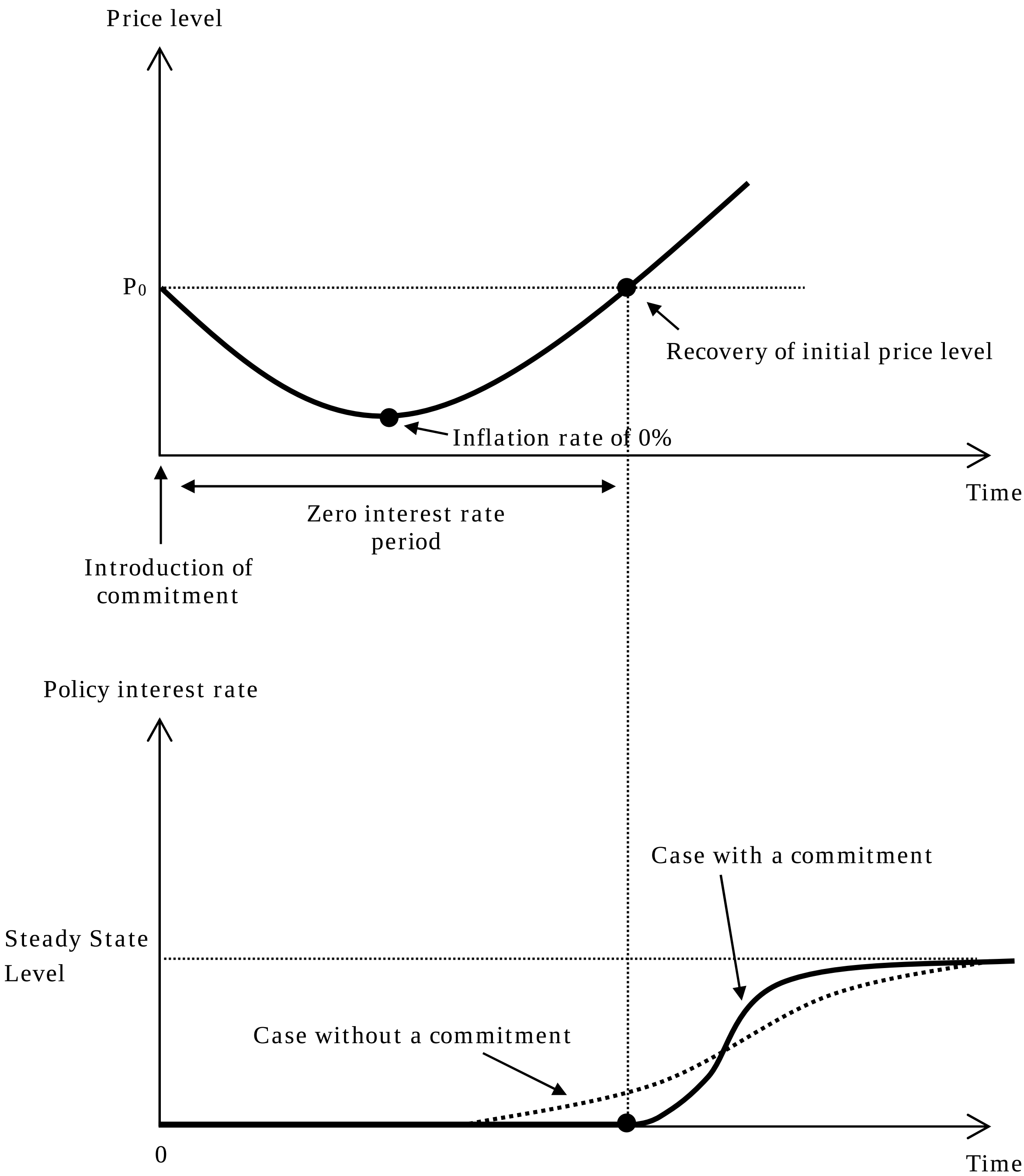

(3) Natural rate of interest: 2.0%



(Chart 14)

Price Level Based Zero Interest Rate Commitment:

A Conceptual Diagram



(Chart 15)

## Short-term Simulations for the Baseline Rule with a Zero Interest Rate Commitment Based on the Price Level (PL)

-- Comparison with the rule with an optimally set inflation rate (IR) based commitment

(1) Supply shock u:-0.5%, Demand shock g:-5%

| | | PL based commit-ment | IR based commit-ment (Threshold $\pi$ : 1.5%) |
|---|---|---|---|
| Total Loss | | 14.00 | 14.01 |
| Individual Losses | Inflation fluctuation | 1.80 | 1.54 |
| | GDP fluctuation | 9.65 | 9.38 |
| | Interest rate fluctuation | 4.54 | 5.30 |

(2) Supply shock u:-0.5%, Demand shock g:-4%

| | | PL based commit-ment | IR based commit-ment (Threshold $\pi$ : 0.5%) |
|---|---|---|---|
| Total Loss | | 11.25 | 11.21 |
| Individual Losses | Inflation fluctuation | 1.50 | 1.87 |
| | GDP fluctuation | 7.43 | 6.53 |
| | Interest rate fluctuation | 4.01 | 4.64 |

(3) Supply shock u:-0.5%, Demand shock g:-3%

| | | PL based commit-ment | IR based commit-ment (Threshold $\pi$ : 0.0%) |
|---|---|---|---|
| Total Loss | | 9.24 | 9.00 |
| Individual Losses | Inflation fluctuation | 1.29 | 1.93 |
| | GDP fluctuation | 5.91 | 4.83 |
| | Interest rate fluctuation | 3.47 | 3.67 |

(4) Supply shock u:-0.5%, Demand shock g:-2%

| | | PL based commit-ment | IR based commit-ment (Threshold $\pi$ : 0.0%) |
|---|---|---|---|
| Total Loss | | 7.91 | 7.26 |
| Individual Losses | Inflation fluctuation | 1.13 | 1.59 |
| | GDP fluctuation | 5.03 | 3.78 |
| | Interest rate fluctuation | 2.96 | 3.07 |

(5) Supply shock u:-0.5%, Demand shock g:-1%

| | | PL based commit-ment | IR based commit-ment (Threshold $\pi$ : -0.5%) |
|---|---|---|---|
| Total Loss | | 7.04 | 6.10 |
| Individual Losses | Inflation fluctuation | 1.02 | 1.78 |
| | GDP fluctuation | 4.59 | 2.89 |
| | Interest rate fluctuation | 2.46 | 2.31 |

(6) Supply shock u:-1%, Demand Shock g:-1%

| | | PL based commit-ment | IR based commit-ment (Threshold $\pi$ : 0.0%) |
|---|---|---|---|
| Total Loss | | 16.40 | 14.04 |
| Individual Losses | Inflation fluctuation | 2.30 | 4.16 |
| | GDP fluctuation | 11.32 | 6.48 |
| | Interest rate fluctuation | 5.02 | 5.49 |

Note: Shadowed areas indicate the smaller loss in the two cases of commitments.

(Chart 16)

# Short-term Simulations for the Nonlinear Optimal Simple Rule

-- Comparison with the rule with an optimally set inflation rate (IR) based commitment

(1) Supply shock u:-0.5%, Demand shock g:-5%

| | | Nonlinear optimal simple rule | IR based commit-ment (Threshold $\pi$ : 1.5%) |
|---|---|---|---|
| Total Loss | | 14.16 | 14.01 |
| Individual Losses | Inflation fluctuation | 2.56 | 1.54 |
| | GDP fluctuation | 8.62 | 9.38 |
| | Interest rate fluctuation | 5.07 | 5.30 |

(2) Supply shock u:-0.5%, Demand shock g:-4%

| | | Nonlinear optimal simple rule | IR based commit-ment (Threshold $\pi$ : 0.5%) |
|---|---|---|---|
| Total Loss | | 10.80 | 11.21 |
| Individual Losses | Inflation fluctuation | 2.12 | 1.87 |
| | GDP fluctuation | 6.12 | 6.53 |
| | Interest rate fluctuation | 4.25 | 4.64 |

(3) Supply shock u:-0.5%, Demand shock g:-3%

| | | Nonlinear optimal simple rule | IR based commit-ment (Threshold $\pi$ : 0.0%) |
|---|---|---|---|
| Total Loss | | 8.47 | 9.00 |
| Individual Losses | Inflation fluctuation | 1.84 | 1.93 |
| | GDP fluctuation | 4.47 | 4.83 |
| | Interest rate fluctuation | 3.52 | 3.67 |

(4) Supply shock u:-0.5%, Demand shock g:-2%

| | | Nonlinear optimal simple rule | IR based commit-ment (Threshold $\pi$ : 0.0%) |
|---|---|---|---|
| Total Loss | | 6.82 | 7.26 |
| Individual Losses | Inflation fluctuation | 1.66 | 1.59 |
| | GDP fluctuation | 3.40 | 3.78 |
| | Interest rate fluctuation | 2.84 | 3.07 |

(5) Supply shock u:-0.5%, Demand shock g:-1%

| | | Nonlinear optimal simple rule | IR based commit-ment (Threshold $\pi$ : -0.5%) |
|---|---|---|---|
| Total Loss | | 5.70 | 6.10 |
| Individual Losses | Inflation fluctuation | 1.54 | 1.78 |
| | GDP fluctuation | 2.79 | 2.89 |
| | Interest rate fluctuation | 2.24 | 2.31 |

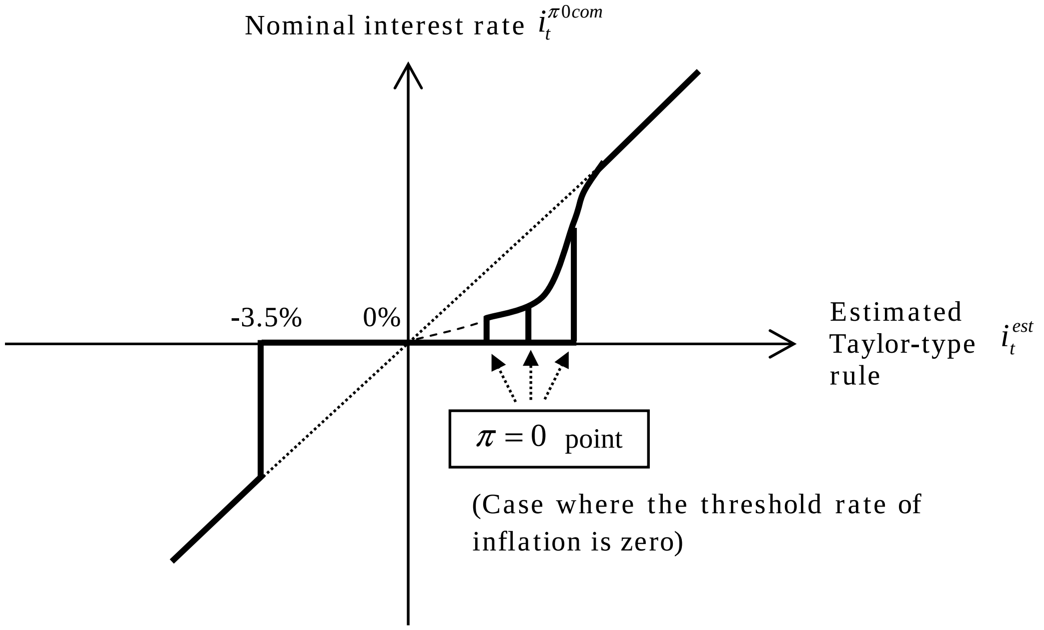
(6) Supply shock u:-1%, Demand Shock g:-1%

| | | Nonlinear optimal simple rule | IR based commit-ment (Threshold $\pi$ : 0.0%) |
|---|---|---|---|
| Total Loss | | 13.25 | 14.04 |
| Individual Losses | Inflation fluctuation | 4.56 | 4.16 |
| | GDP fluctuation | 5.78 | 6.48 |
| | Interest rate fluctuation | 4.72 | 5.49 |

Note: Shadowed areas indicate the smaller loss in the two cases of commitments.

(Chart 17)

# Nonlinear Optimal Simple Rule with a Zero Interest Rate Commitment: A Conceptual Diagram



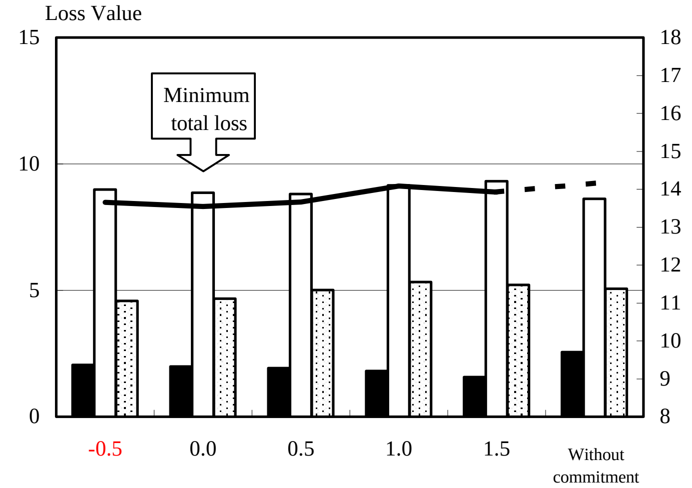
(Chart 18)

# Short-term Simulations for the Nonlinear Optimal Simple Rule with a Zero Interest Rate Commitment

-- Comparison of different commitments under each initial shock

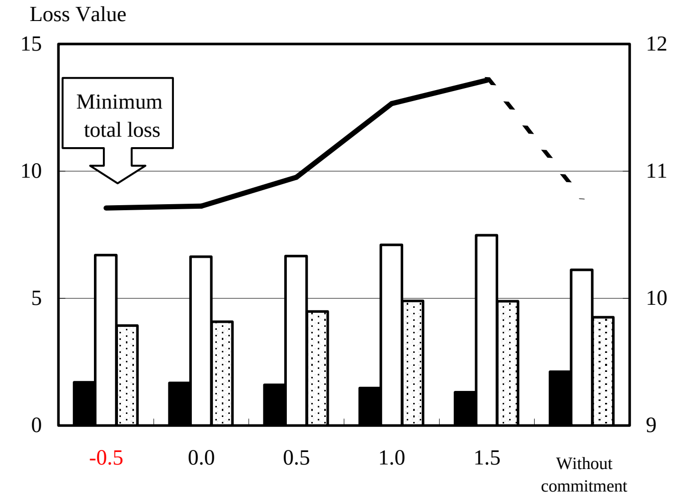
Total loss (right scale)
GDP gap fluctuation loss (left scale)
Inflation rate fluctuation loss (left scale)
Interest rate fluctuation loss (left scale)

(1) Supply shock u:-0.5%, demand shock g:-5%.



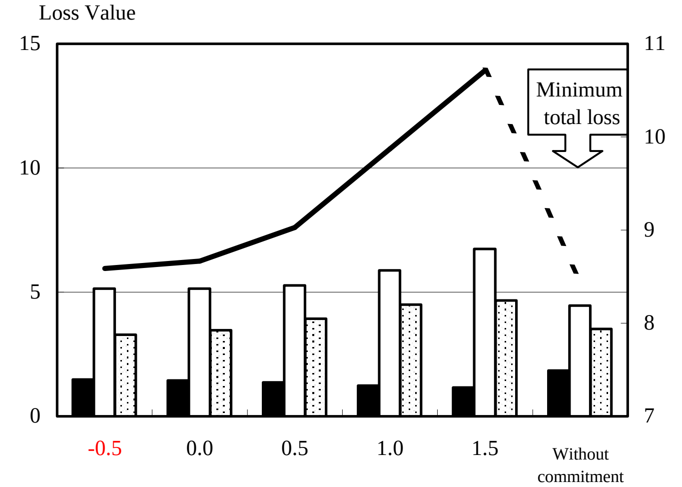

Threshold rate of inflation for exiting the zero interest rate policy (%).

(2) Supply shock u:-0.5%, demand shock g:-4%.



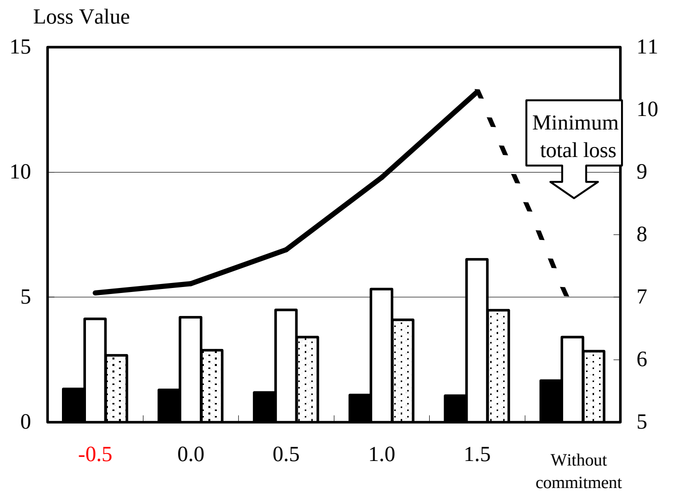

Threshold rate of inflation for exiting the zero interest rate policy (%).

(3) Supply shock u:-0.5%, demand shock g:-3%.



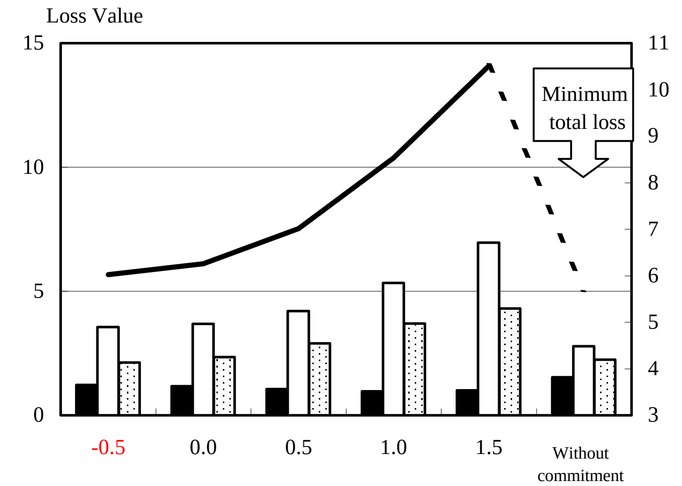

Threshold rate of inflation for exiting the zero interest rate policy (%).

(4) Supply shock u:-0.5%, demand shock g:-2%.



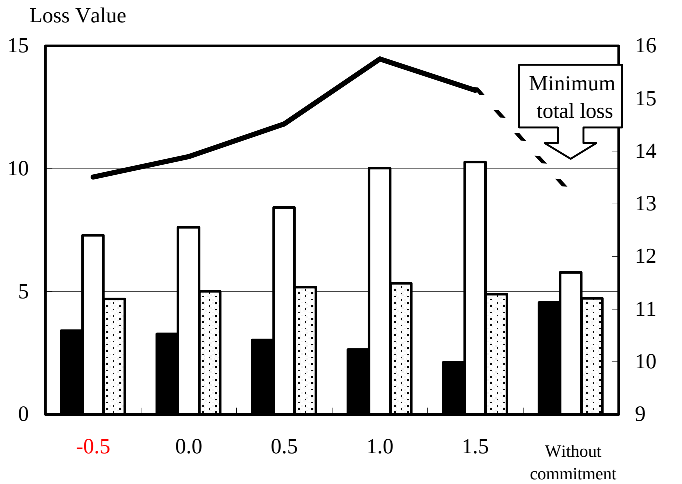

Threshold rate of inflation for exiting the zero interest rate policy (%).

(5) Supply shock u:-0.5%, demand shock g:-1%.




Threshold rate of inflation for exiting the zero interest rate policy (%).

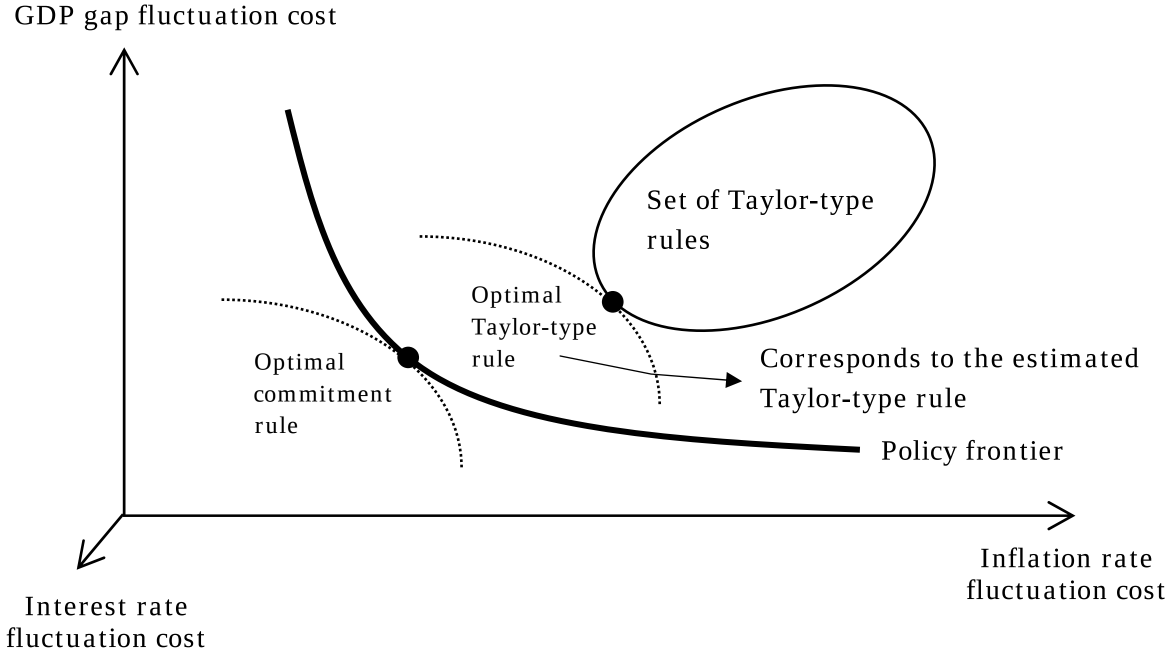
(6) Supply shock u:-1%, demand shock g:-1%.



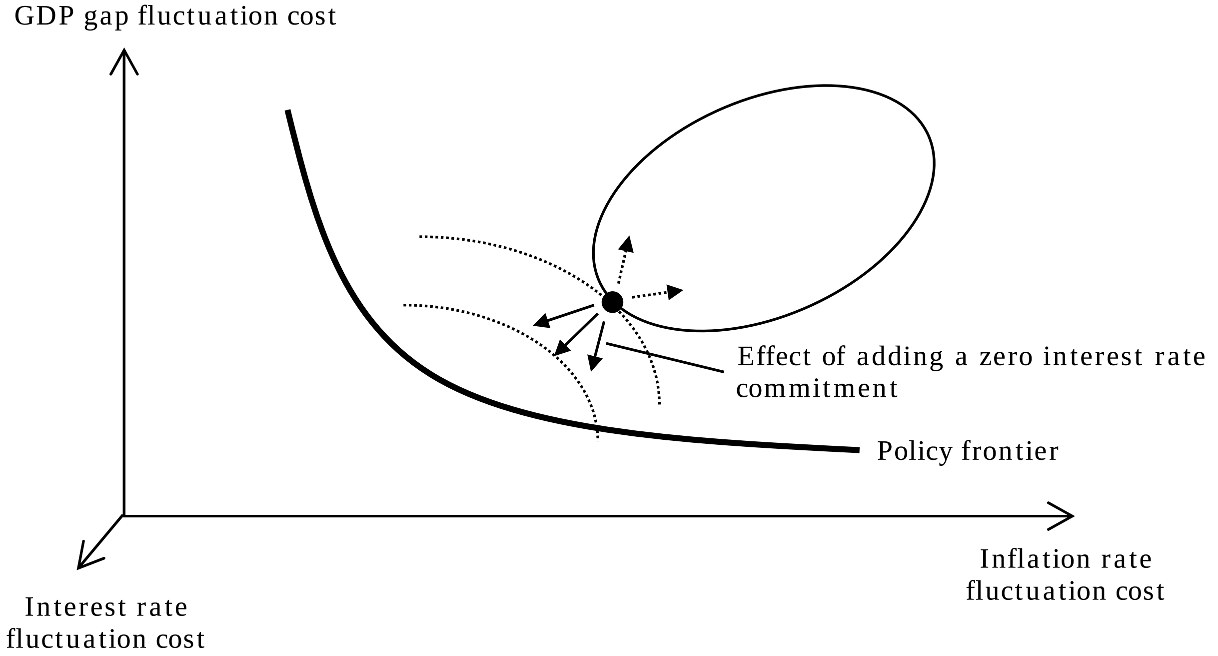

Threshold rate of inflation for exiting the zero interest rate policy (%).

# Summary (1/2)

(1) Estimated Baseline Taylor-type Rule

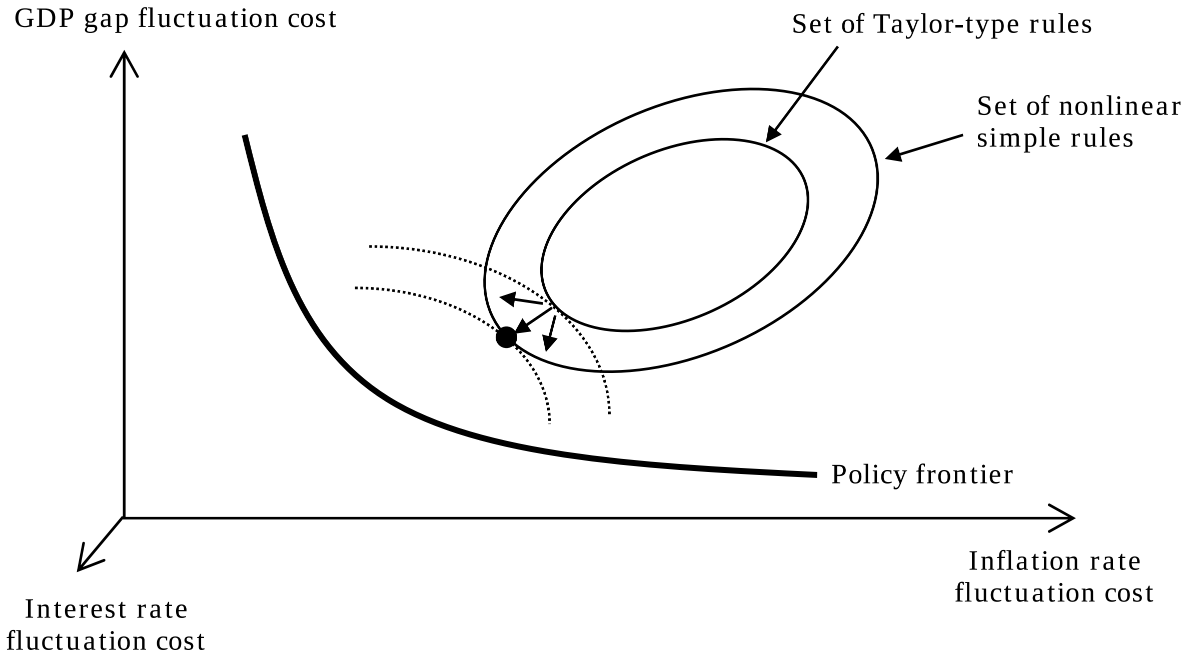



(2) Baseline Rule with a Zero Interest Rate Commitment

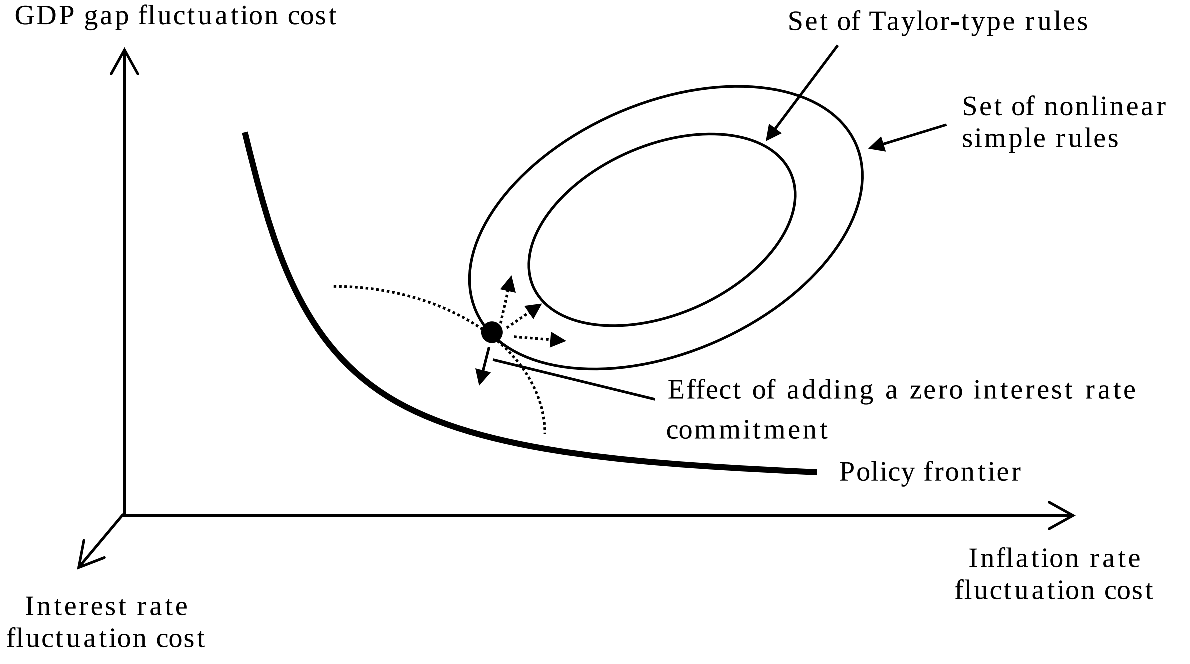


## Summary (2/2)

(3) Nonlinear Optimal Simple Rule




(4) Nonlinear Optimal Simple Rule with a Zero Interest Rate Commitment



(Chart 21)

## The Order of the Performance of the Monetary Policy Rules under Each Initial Shock

Larger Demand Shock

(1) Demand shock g=-5%, supply shock u=-0.5%

| | | Total loss |
|---|---|---|
| Small Loss | Nonlinear optimal rule with a commitment based on the inflation rate (optimal threshold: 0.0%) | 13.54 |
| | Baseline rule with a commitment based on the price level | 14.00 |
| | Baseline rule with a commitment based on the inflation rate (optimal threshold: +1.5%) | 14.01 |
| | Nonlinear optimal rule | 14.16 |
| Large Loss | Baseline Taylor-type rule | -- (*) |

(*) Divergences occurred for the baseline Taylor-type rule simulation.

(2) Demand shock g=-4%, supply shock u=-0.5%

| | | Total loss |
|---|---|---|
| Small Loss | Nonlinear optimal rule with a commitment based on the inflation rate (optimal threshold: -0.5%) | 10.71 |
| | Nonlinear optimal rule | 10.80 |
| | Baseline rule with a commitment based on the inflation rate (optimal threshold: +0.5%) | 11.21 |
| | Baseline rule with a commitment based on the price level | 11.25 |
| Large Loss | Baseline Taylor-type rule | 11.78 |

(3) Demand shock g=-3%, supply shock u=-0.5%

| | | Total loss |
|---|---|---|
| Small Loss | Nonlinear optimal rule | 8.47 |
| | Nonlinear optimal rule with a commitment based on the inflation rate (optimal threshold: -0.5%) | 8.59 |
| | Baseline rule with a commitment based on the inflation rate (optimal threshold: 0.0%) | 9.00 |
| | Baseline Taylor-type rule | 9.16 |
| Large Loss | Baseline rule with a commitment based on the price level | 9.24 |

(4) Demand shock g=-2%, supply shock u=-0.5%

| | | Total loss |
|---|---|---|
| Small Loss | Nonlinear optimal rule | 6.82 |
| | Nonlinear optimal rule with a commitment based on the inflation rate (optimal threshold: -0.5%) | 7.07 |
| | Baseline rule with a commitment based on the inflation rate (optimal threshold: 0.0%) | 7.26 |
| | Baseline Taylor-type rule | 7.27 |
| Large Loss | Baseline rule with a commitment based on the price level | 7.91 |

(5) Demand shock g=-1%, supply shock u=-0.5%

Smaller Demand Shock

Smaller Supply Shock

| | | Total loss |
|---|---|---|
| Small Loss | Nonlinear optimal rule | 5.70 |
| | Baseline Taylor-type rule | 5.96 |
| | Nonlinear optimal rule with a commitment based on the inflation rate (optimal threshold: -0.5%) | 6.03 |
| | Baseline rule with a commitment based on the inflation rate (optimal threshold: -0.5%) | 6.10 |
| Large Loss | Baseline rule with a commitment based on the price level | 7.04 |

(6) Demand shock g=-1%, supply shock u=-1%

| | | Total loss |
|---|---|---|
| Small Loss | Nonlinear optimal rule | 13.25 |
| | Nonlinear optimal rule with a commitment based on the inflation rate (optimal threshold: -0.5%) | 13.51 |
| | Baseline Taylor-type rule | 13.91 |
| | Baseline rule with a commitment based on the inflation rate (optimal threshold: 0.0%) | 14.04 |
| Large Loss | Baseline rule with a commitment based on the price level | 16.40 |

Larger Supply Shock