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Monetary Policy Uncertainty and Market Interest Rates

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Monetary policy uncertainty and market interest rates*

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Abstract

This paper examines the relationship between monetary policy uncertainty and the term structure of interest rates. Extending the Ellingsen and Söderström (2001) model, we demonstrate that long-term interest rates are positively related to monetary policy uncertainty, with the magnitude increasing with maturity. Further, we present empirical evidence to show that the theoretical prediction is generally consistent with US data.

Key Words: Monetary policy, term structure of interest rates, GARCH.

JEL Classification: E43, E5

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1 Introduction

Recently, the relationship between monetary policy and market interest rates has been examined intensively by Ellingsen and Söderström (2001). They provided a comprehensive picture depicting how yield curves respond to unanticipated monetary policy changes. Our study extends the Ellingsen-Söderström model so that the second moment implications can be analyzed. Namely, our objective is to investigate how monetary policy uncertainty, defined as the variance of innovations to a monetary policy reaction function, affects long-term interest rates and yield curves. In this paper, we demonstrate that uncertainty about monetary policy positively influences long-term interest rates and that the magnitude increases with maturity. In other words, a yield curve becomes bull flattened in response to a reduction in monetary policy uncertainty.

In terms of empirical analysis, the study most relevant to ours is Favero and Mosca (2001). These authors presented evidence that monetary policy uncertainty significantly declined around 1994 when the FOMC began to release its target level for the federal funds rate. Moreover, their result suggests that if the influence of such monetary policy uncertainty is properly controlled, the pure expectation hypothesis cannot be rejected, especially in the low uncertainty era from 1994-1999. Our measurement of monetary policy uncertainty basically follows the specification of Favero and Mosca. We show that measured monetary policy uncertainty significantly affects longer-term interest rates, as predicted by the model in our paper.

Another issue related to our study is a "conundrum" mentioned in Greenspan (2005). In his testimony to the US Senate, the Federal Reserve Bank's (FRB) Chairman Alan Greenspan said that the "unanticipated behavior of the world bond market remains a conundrum." A number of factors can affect "low" long-term interest rates.[1] Although this study does not intend to provide a definitive answer to the conundrum, our theoretical model and empirical analysis offer a possible explanation, which is that the recent reduction in monetary policy uncertainty is a potential factor affecting the low levels of long-term interest rates and flattened yield curves.

In the following section, we present a simple model that relates monetary policy uncertainty to term structure of interest rates, extending the framework of the inflation forecast targeting introduced by Svensson (1997) and later extended by Ellingsen

[1] For instance, Greenspan (2005) referred to technical factors (*e.g.*, the behavior of mortgage investors) as a possible explanation of the low yields. However, he commented that none of these factors provides a decisive answer to the conundrum.

and Söderström (2001). Section 3 empirically analyzes the relationship between measured monetary policy uncertainty and yield curves to show the theoretical prediction is generally consistent with empirical evidence. Section 4 briefly concludes the paper.

2 The model

Consider a reduced form backward-looking economy, as introduced in Svensson (1997) and Ellingsen and Söderström (2001). Let π_t and x_t denote inflation and detrended output (or the output gap). The economy consists of a dynamic version of IS and AS equations as follows:

$$\begin{aligned} x_{t+1} &= \rho x_t - \delta \left(R_t^{t+1} - E_t \pi_{t+1} + u_t \right) + \varepsilon_{t+1} & (1) \\ \pi_{t+1} &= \pi_t + \alpha x_t + v_{t+1} & (2) \\ u_{t+1} &= \psi u_t - \zeta_{t+1}, & (3) \end{aligned}$$

where equations (1) and (2) represent aggregate demand and aggregate supply functions, respectively. ε and v are *i.i.d.* normal disturbances with variances $\left(\sigma_\varepsilon^2 \ \sigma_v^2\right)$, which we refer to as aggregate demand and aggregate supply shocks, respectively. u_t is a stochastic "natural rate shock." We assume that that the stochastic process of u_t is a simple AR(1) form with $\psi < 1$ as shown in eqn (3). ζ_t is a random distrubance to natural rate shock process, which we discuss in detail later. R_t^{t+1} is a nominal short-term interest rate that is a control variable of a central bank in the economy. Although equation (1) includes a forward-looking variable $E_t \pi_{t+1}$, this can be substituted out through $E_t \pi_{t+1} = \pi_t + \alpha x_t$. Rewriting the IS equation yields:

$$x_{t+1} = (\rho + \alpha\delta)\, x_t - \delta R_t^{t+1} + \delta \pi_t - \delta u_t + \varepsilon_{t+1}.$$

Let the monetary policy rule be a Taylor-type linear function in inflation and output, such that:

$$R_t^{t+1} = q_1 x_t + q_2 \left(\pi_t - \pi^*\right) + \eta_t, \tag{4}$$

where η_t represents an innovation to monetary policy, which we discuss in detail later. It can be shown that if the equation(4) is specified properly, the rule is a solution of a simple linear-quadratic optimal control problem of an optimizing central bank, which has the following objective:

$$\begin{aligned} \min &: \quad E_t \sum_{i=0}^{\infty} \beta^i L_{t+i}, \\ \textit{where} \;\; L_t &= \frac{1}{2} \left\{ x_t^2 + \lambda \left(\pi_t - \pi^*\right)^2 \right\}, & (5) \end{aligned}$$

where β and λ are a subjective discount factor and a preference parameter of the central bank, respectively. Note that π^* is a time-invariant target inflation rate of a central bank. An optimizing central bank follows a Taylor-type policy rule with coefficients:

$$q_1 = \alpha + \frac{\rho\theta_1 + \theta_1 - 1}{\delta\theta_1}, \text{ and } q_2 = 1 + \frac{\theta_1 - 1}{\alpha\delta\theta_1},$$

where θ_1 is a larger root[2] of the characteristic equation $\beta z^2 - \left(\alpha^2\beta\lambda + \beta + 1\right) z + 1$. That is:

$$\theta_1 = \frac{\alpha^2\beta\lambda + \beta + 1 + \sqrt{(\alpha^2\beta\lambda + \beta + 1)^2 - 4\beta}}{2}, \ \theta_1 > 1.$$

For the innovation term, an optimizing central bank set, such that:

$$\eta_t = -u_t.$$

We call η_t a monetary policy innovation, as noted above. Here, we introduce a simple interpretation of η_t, which is, that it is the optimal reaction of a central bank to a stochastic natural rate shock u_t that is observable at period t. As u_{t+j} (for $j > 1$) is uncertain at period t, the term u_{t+j} is the source of monetary policy uncertainty.

In addition, a similar but alternative interpretation of monetary policy uncertainty arising from η_t is possible based on the assumption of asymmetric information between a central bank and market participants. Central banks might have private information on the structure of the economy or their own preferences, *e.g.*, a precise estimate of the natural/neutral level of the real interest rate. If they conduct policy on the basis of such a private information set, their policy rule is likely to appear different to the deterministic part of equation(4). For instance, suppose that a current natural rate shock u_t is unobservable to market participants. In contrast, the central bank observes u_t in advance, and responds to it optimally, such that $\eta_t = -u_t$. In this case, η_t is interpreted as a *forecast error* of market participants[3].

As we specified the stochastic process of u_t as an AR(1) form, naturally η_t follows:

$$\eta_t = \psi\eta_{t-1} + \zeta_t$$

with $\psi < 1$, where ζ_t is a mean zero random disturbance.[4] For ζ_t, we assume *i.i.d.*

[2] See Kato and Nishiyama (2005) for a detailed derivation.

[3] Note that after observing the central bank's response, market participants also know $u_t\,(= -\eta_t)$, as both R_t^{t+1} and $q_1 x_t + q_2\pi_t$ are now in their information set.

[4] The specification is consistent with the well-known observation of so-called interest rate inertia, which is presented in number of empirical studies. Later, in Section 3, we estimate equation(4) with an AR(1) term, ψR_{t-1}^t, added to the right-hand side of the equation. Note that because $\eta_t = \sum_{i=0}^{\infty} \psi^i \zeta_{t-i}$, it can be shown that $R_t^{t+1} = \psi R_{t-1}^t + q s_t + \zeta_t = \mathbf{q}\mathbf{s}_t + \eta_t$, where $\mathbf{s}_t = (s_t, s_{t-1}, \cdots)'$ is a state variable vector and $\mathbf{q} = \left(q, \psi q, \psi^2 q, \cdots\right)$ is a coefficient vector.

$N\left(0,\sigma_\zeta^2\right)$ for the moment. Later, we allow a time-varying conditional variance of ζ_t. In this paper, we specify that a rise in σ_ζ^2 (or $E_t\left(\zeta_{t+j}^2\right)$) implies greater uncertainty stemming from monetary policy.

Obtaining an explicit form for short-term interest rates at period $t+j$ requires several steps, as shown below. First, inserting the monetary policy rule into equation (2) leaves the following relation:

$$\begin{aligned} x_{t+1} &= (\rho+\alpha\delta)\,x_t-\delta\left(q_1x_t+q_2\left(\pi_t-\pi^*\right)+\eta_t+u_t\right)+\delta\pi_t+\varepsilon_{t+1} \\ &= \left(\frac{1-\theta_1}{\alpha\theta_1}\right)\left(\left(\pi_t-\pi^*\right)+\alpha x_t\right)+\varepsilon_{t+1} \\ &= \left(\frac{1-\theta_1}{\alpha\theta_1}\right)s_t+\varepsilon_{t+1}, \end{aligned}$$

where $s_t=\left(\pi_t-\pi^*\right)+\alpha x_t$ is the consolidated state variable of the system. Hence, output and inflation at period $t+j$ can be denoted as:

$$\begin{aligned} x_{t+j} &= \left(\frac{1-\theta_1}{\alpha\theta_1}\right)s_{t+j-1}+\varepsilon_{t+j} \\ \pi_{t+j}-\pi^* &= s_{t+j-1}+v_{t+j}. \end{aligned}$$

Note that the state transition can be expressed as a simple AR(1) form:

$$\begin{aligned} s_{t+1} &= \pi_{t+1}-\pi^*+\alpha x_{t+1} \\ &= s_t+v_{t+1}+\alpha\left[\left(\frac{1-\theta_1}{\alpha\theta_1}\right)s_t+\varepsilon_{t+1}\right] \\ &= \phi s_t+\omega_{t+1}, \end{aligned}$$

where $\phi=1/\theta_1<1$ and $\omega_{t+1}=v_{t+1}+\alpha\varepsilon_{t+1}$. Paying attention to $s_{t+j}=\phi^js_t+\sum_{i=1}^{j}\phi^{i-1}\omega_{t+j+1-i}$, a short-term interest rate at period $t+j$ is written as follows. For $j\geq 1$, :

$$\begin{aligned} R_{t+j}^{t+j+1} &= q_1x_{t+j}+q_2\left(\pi_{t+j}-\pi^*\right)+\eta_{t+j} \\ &= \left[q_1\left(\frac{1-\theta_1}{\alpha\theta_1}\right)+q_2\right]s_{t+j-1}+q_1\varepsilon_{t+j}+q_2v_{t+j}+\eta_{t+j} \\ &\equiv \kappa\phi^{j-1}s_t+\Gamma_{t,j}^{v}+\Gamma_{t,j}^{\varepsilon}+\eta_{t+j}, \end{aligned} \tag{6}$$

where:

$$\begin{aligned}\kappa &= q_1\left(\frac{1-\theta_1}{\alpha\theta_1}\right)+q_2 \\ \Gamma^v_{t,j} &= \kappa\sum_{i=1}^{j-1}\phi^{i-1}v_{t+j+1-i}+q_2 v_{t+j} \\ \Gamma^\varepsilon_{t,j} &= \kappa\alpha\sum_{i=1}^{j-1}\phi^{i-1}\varepsilon_{t+j+1-i}+q_1\varepsilon_{t+j}.\end{aligned}$$

We replace the linear yield curve introduced in Ellingsen and Söderström (2001) with the following compounding version of the expectation hypothesis:

$$\exp\left(nR_t^{t+n}\right)=E_t\exp\left(R_t^{t+1}+R_{t+1}^{t+2}+\cdots+R_{t+n-1}^{t+n}\right). \tag{7}$$

Equation (7) assumes that under the given short-term interest rates determined by the central bank, financial markets calculate longer-term yields according to the continuously compounding interest formula. Based on the equation, we can derive an explicit form of a long-term interest rate with maturity n as follows:

$$\begin{aligned}R_t^{t+n} &= \frac{1}{n}\left[R_t^{t+1}+\kappa\left(\frac{1-\phi^{n-1}}{1-\phi}\right)s_t+\left(\frac{1-\psi^{n-1}}{1-\psi}\right)\psi\eta_t\right] \\ &\quad+\frac{1}{2n}Var\left(\Gamma^v_{t,j}\right)+\frac{1}{2n}Var\left(\Gamma^\varepsilon_{t,j}\right)+\frac{1}{2n}\sum_{i=0}^{n-2}C_n(i)^2\sigma_\zeta^2,\end{aligned} \tag{8}$$

where:

$$C_n(i)\equiv\frac{1-\psi^{n-1-i}}{1-\psi}.$$

For a detailed derivation, see the appendix. This tiny extension allows us to investigate the nature of longer-term interest rates with regard to macroeconomic uncertainty, such as σ_ε^2, σ_v^2 and σ_ζ^2. Our focus is on the relationship between term structure and monetary policy uncertainty, represented by σ_ζ^2.

We are now ready to state the main proposition of this paper. Proposition 1 is a natural consequence of equation(8).

Proposition 1 *For any $n \geq 2$, long-term interest rates with maturity n are positively related to monetary policy uncertainty and the magnitude increases in n.*

Proof. From equation (8), it is evident that for any $n \geq 2$:

$$\frac{\partial R_t^{t+n}}{\partial \sigma_\zeta^2} = \frac{1}{2n} \sum_{i=0}^{n-2} C_n(i)^2 \equiv \Phi(n) \geq 0.$$

Next, it suffices to show that for any integer $n \geq 2$, $\Phi(n+1) - \Phi(n) > 0$.

Expanding the $\Phi(n)$ term yields:

$$\begin{aligned} 2n\Phi(n) &= \sum_{i=0}^{n-2} C_n(i)^2 \\ &= \left(\frac{1-\psi^{n-1}}{1-\psi}\right)^2 + \left(\frac{1-\psi^{n-2}}{1-\psi}\right)^2 + \cdots + \left(\frac{1-\psi^2}{1-\psi}\right)^2 + \left(\frac{1-\psi}{1-\psi}\right)^2. \end{aligned}$$

Hence, the following can be written:

$$\begin{aligned} 2\{\Phi(n+1) - \Phi(n)\} &= \left(\frac{1}{1-\psi}\right)^2 [\frac{1}{n+1}\left((1-\psi^n)^2 + \left(1-\psi^{n-1}\right)^2 + \cdots + (1-\psi)^2\right) \\ &\quad -\frac{1}{n}\left(\left(1-\psi^{n-1}\right)^2 + \left(1-\psi^{n-2}\right)^2 \cdots + (1-\psi)^2\right)]. \end{aligned}$$

Let Ψ_n be:

$$\Psi_n = n(1-\psi^n)^2 - \left\{\left(1-\psi^{n-1}\right)^2 + \cdots \left(1-\psi^2\right)^2 + (1-\psi)^2\right\}.$$

Then, it suffices to show that $\Psi_n > 0$ for any integer $n \geq 2$.

$$\begin{aligned} \Psi_n &= (1-\psi^n)^2 \\ &\quad + (1-\psi^n)^2 - \left(1-\psi^{n-1}\right)^2 \\ &\quad + (1-\psi^n)^2 - \left(1-\psi^{n-2}\right)^2 \\ &\quad + \cdots \\ &\quad + (1-\psi^n)^2 - (1-\psi)^2. \end{aligned}$$

Here, note that $1-\psi^i > 1-\psi^{i-j} > 0$, for any $\psi < 1$ and $i > j > 0$. Therefore, $\Psi_n > 0$ for any $n \geq 2$. The proposition is proved. ■

Corollary 1 *Yield curves become bull-flattened in response to a reduction in monetary policy uncertainty.*

The proposition and the corollary depict the relationship between yield curves and monetary policy uncertainty. Intuitively, Proposition 1 states that the effect of uncertainty accumulates as the maturity increases. However, in the distant future, when the economy converges to the steady state, the effect of uncertainty converges to a constant because the dynamics of the monetary policy innovations are governed by the $AR(1)$ coefficient $\psi < 1$. This is verified by the following:

$$\begin{aligned} \lim_{n\to\infty} \left(\Phi\left(n+1\right) - \Phi\left(n\right)\right) &= \lim_{n\to\infty} \left(\frac{1}{2}\frac{1}{n\left(n+1\right)}\Psi_n\right) \\ &= 0, \end{aligned}$$

because $\lim_{n\to\infty} \Psi_n$ converges to a constant number.

Proposition 1 holds under the assumption of time-invariant monetary policy uncertainty σ_ζ^2. So far, we have maintained this assumption for illustrative purposes. However, for the empirical analysis in the following section, we allow conditional heteroskedasticity for monetary policy uncertainty. In other words, conditional variance $E_t\left(\zeta_{t+j}^2\right)$ is time-varying.

Suppose ζ_{t+j}^2 follows a random walk such that:

$$\zeta_t^2 = \zeta_{t-1}^2 + w_t, \tag{9}$$

where $w_t \sim N\left(0, \sigma_w^2\right)$. Given the specification, the conditional variance of ζ_{t+j} at period t is $E_t\left(\zeta_{t+j}^2\right) = \zeta_t^2 \equiv \sigma_{\zeta,t}^2$. In this case, it is easy to show that Proposition 1 holds by replacing σ_ζ^2 by $\sigma_{\zeta,t}^2$ in equation(8). For more general ARCH specifications of ζ_t^2, we discuss the necessary conditions for the proposition in the appendix.

3 Empirical analysis

First, we estimate a Taylor-type monetary policy rule to obtain forecast errors as a proxy of monetary policy uncertainty. The specification of the policy rule follows Favero and Mosca (2001). As our estimation frequency is monthly, we cannot use the output gap for an explanatory variable in the policy reaction function because it is available only on a quarterly basis. Instead of the output gap, we use monthly changes in payroll employment in the non-farm sector. For inflation and short-term interest rates, we use year-over-year CPI growth rates and euro dollar three-month rates[5]. Moreover, we add a dummy variable $dum1$ in the variance equation to capture the effect of a commitment announced by the FED in August 2003. The FOMC statement, dated

[5] We use the same data set as Favero and Mosca (2001). Payroll employment and CPI data are available from the Bureau of Labor Statistics and euro dollar three-month rates are downloaded via the Bloomberg archive. All the data files are donwloadable at `http://www.ryokato.org/`

August 12 2003, states, "policy accommodation can be maintained for a considerable period." An important question to ask is: did the commitment affect financial markets by reducing uncertainty on monetary policy? The variable $dum1$ comprised zeros for the period from January 1985-July 2003 and ones for August 2003 onwards. As a result, the coefficient on $dum1$ would be estimated as significantly negative if the FED's commitment reduced conditional volatility. The estimation result is shown below in Table 1. In the tables below, variables topped with "$\sim$" are estimator analogues of the corresponding variables in Section 2.

Table 1: GARCH estimation

Monetary policy rule: $R_t^{t+1} = \tilde{\psi} R_{t-1}^t + \tilde{q}_1 \Delta l_t + \tilde{q}_2 \pi_t + q_0 + \tilde{\zeta}_t$					
	$\tilde{\psi}$	$\tilde{q}_1$	$\tilde{q}_2$		S.E.
Spec1	0.97 [0.005]	0.554 [0.069]	0.035 [0.013]		0.236
Spec2	0.97 [0.005]	0.549 [0.062]	0.042 [0.011]		0.237

Variance equation: $\tilde{\sigma}_{\zeta,t}^2 = b_1 \tilde{\zeta}_{t-1}^2 + b_2 \tilde{\zeta}_{t-2}^2 + d_1 \tilde{\sigma}_{\zeta,t-1}^2 + d_2 \tilde{\sigma}_{\zeta,t-2}^2 + b_0 + b_3 \times dum1$				
b_1	b_2	d_1	d_2	b_3
0.169 [0.06]	–	0.787 [0.057]	–	–
0.266 [0.09]	0.271 [0.08]	-0.323 [0.109]	0.610 [0.096]	-0.009 [0.004]

White test for heteroskedasticity

Spec1: w=34.08, p-value=0.00

Note: Sample period is Jan. 1985 to Jan. 2005. Numbers in [ ] are standard errors of each coefficient.

As reported in Favero and Mosca (2001), all the coefficients in the monetary policy rule are estimated to be positive and statistically significant. Further, the estimation result (spec 2) indicates that the null hypothesis of b_3 being equal to zero is rejected. The result implies that the FED's commitment had a non-negligible effect in reducing uncertainty on monetary policy.

Figure 1 shows the conditional variance based on the GARCH estimation (spec 2) from Table 1. We regard the data in Figure 1 as the measured monetary policy uncertainty. Before proceeding to the next estimation table, we introduce an alternative measure of monetary policy uncertainty to assess the validity of the data in Figure 1. In Figure 2, we plot the data of (squared) "unanticipated policy changes" calculated using the methodology suggested by Kuttner (2001). The data in Figure 2 is calculated using the federal funds future rates, which reflect the average forecast of market participants. Roughly speaking, the difference between the federal funds future rate prior to an FOMC meeting and the actual federal funds rate on the very day of the FOMC meeting can be regarded as the forecast error of market participants. As the data in Figure 2 is discontinuous (it can be calculated for a date when an FOMC meeting was held), it is impossible to calculate the correlation between the two sequences. However, even a casual observation confirms that the two measurements of monetary policy uncertainty in each figure have a strong positive correlation, implying that the conditional variance in Figure 1 captures well the financial markets' perception of monetary policy uncertainty.

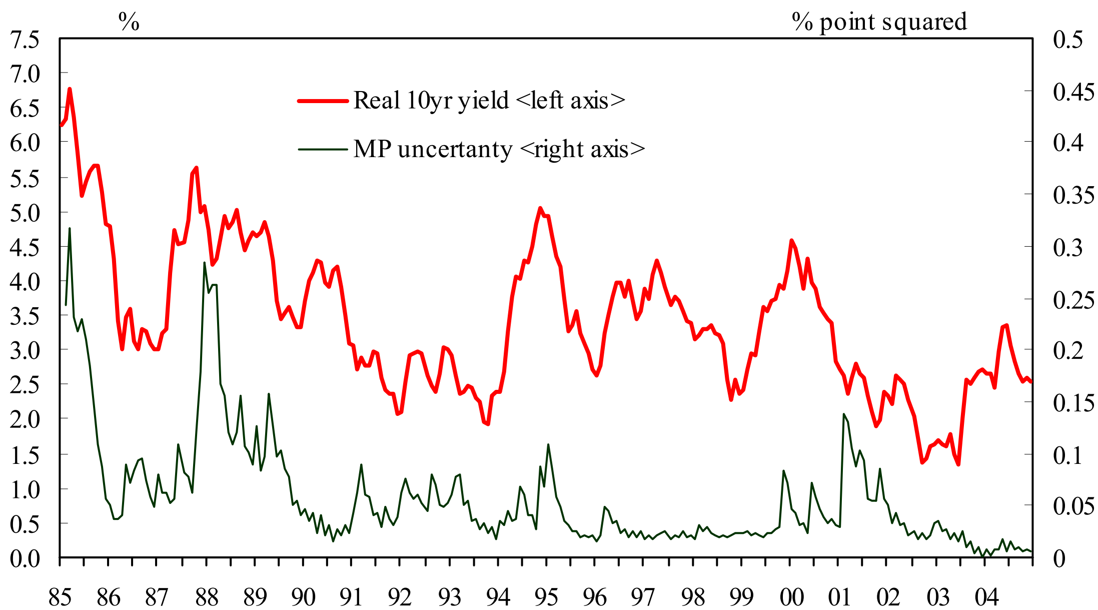

Figure 1: Monetary policy uncertainty

Second, we regress the conditional variance estimated above on real yields with various maturities. The regression is in the spirit of equation(8) in Section 2.[6] Estimation results are shown below in Table 2 and 3.

In the tables, $\tilde{\sigma}^2_{\zeta,t}$ and Δl_t denote the conditional variances derived from the GARCH estimation and the monthly changes in payroll employment in the non-farm sector,

[6]As mentioned in Section 2, both equation (8) in the model and Proposition 1 hold under the assumption of homoskedasticity. See the appendix for the condition under which the proposition holds with a GARCH process.

respectively. R_t^{t+n} is the real yield of an n-year bond, which is calculated as the nominal yields minus the two-year moving average of CPI inflation, a proxy for the expected inflation rate. As shown in the tables, all the coefficients are positive and statistically significant. The outstanding result is that the coefficients on monetary policy uncertainty tend to be larger as the maturity increases. In addition, coefficients on Δl_t become smaller as the maturity increases. All these estimation results are generally consistent with the model in Section 2 and Proposition 1.

Table 2: OLS estimation $\tilde{\sigma}^2_{\zeta,t}$ from GARCH(1,1)

$$R_t^{t+n} = \gamma_1 \tilde{\sigma}^2_{\zeta,t} + \gamma_2 \Delta l_t + \gamma_0 + e_t$$

sample\maturity		1-year	3-year	5-year	10-year
	γ_1	6.15 [1.64]	8.43 [1.32]	9.20 [1.08]	10.12 [0.87]
Jan.85-Dec.01	γ_2	0.28 [0.06]	0.26 [0.05]	0.21 [0.04]	0.18 [0.03]
	S.E.	1.33	1.07	0.87	0.71
	γ_1	8.53 [1.72]	10.44 [1.40]	10.73 [1.14]	11.00 [0.88]
Jan.85-Dec.03	γ_2	0.40 [0.05]	0.36 [0.05]	0.30 [0.04]	0.23 [0.03]
	S.E.	1.42	1.16	0.94	0.73
	γ_1	9.60 [1.69]	11.27 [1.17]	11.29 [1.11]	11.18 [0.85]
Jan.85-Jan.05	γ_2	0.38 [0.06]	0.35 [0.05]	0.28 [0.04]	0.23 [0.03]
	S.E.	1.42	1.16	0.93	0.71

Note: Numbers in [ ] are standard errors of each estimate.

Table 3: OLS estimation $\tilde{\sigma}^2_{\zeta,t}$ from GARCH(2,2)

$$R_t^{t+n} = \gamma_1 \tilde{\sigma}^2_{\zeta,t} + \gamma_2 \Delta l_t + \gamma_0 + e_t$$

sample\maturity		1-year	3-year	5-year	10-year
	γ_1	4.67 [1.66]	6.97 [1.35]	7.80 [1.13]	8.70 [0.94]
Jan.85-Dec.01	γ_2	0.28 [0.06]	0.26 [0.05]	0.22 [0.04]	0.18 [0.03]
	S.E.	1.34	1.10	0.92	0.78
	γ_1	7.16 [1.74]	9.05 [1.43]	9.36 [1.18]	9.60 [0.94]
Jan.85-Dec.03	γ_2	0.41 [0.05]	0.37 [0.05]	0.30 [0.04]	0.23 [0.03]
	S.E.	1.44	1.19	0.98	0.78
	γ_1	8.37 [1.69]	9.97 [1.39]	9.98 [1.14]	9.80 [0.90]
Jan.85-Jan.05	γ_2	0.39 [0.06]	0.35 [0.05]	0.29 [0.04]	0.23 [0.03]
	S.E.	1.44	1.19	0.97	0.77

Note: Numbers in [ ] are standard errors of each estimate.

4 Concluding remarks

This paper demonstrates that long-term interest rates are positively related to monetary policy uncertainty with the magnitude increasing with maturity. Further, we show that the empirical evidence generally supports the theoretical predictions.

A possible concern is that the empirical facts presented in Section 3 may be inconsistent with the well-established theory of precautionary savings à la C-CAPM, which predicts that an increase in consumption volatility leads to lower interest rates because higher uncertainty encourages households to increase savings. In this paper, however, we concentrate on monetary policy uncertainty, which is defined as the (conditional) variance of the forecast errors of a monetary policy rule, whereas C-CAPM studies focus on the volatility of growth rates of consumption as a source of uncertainty. Hence, under the assumption of orthogonality between those two measurements of uncertainty, we claim that, *ceteris paribus*, the main results in the paper are consistent with the theory of precautionary savings or C-CAPM.

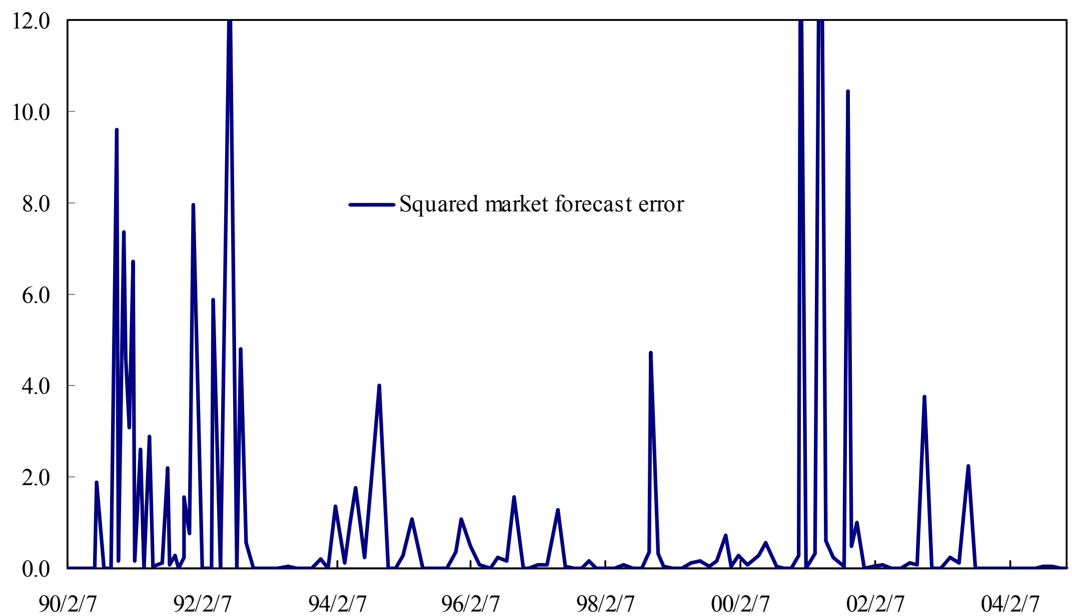

Figure 2: Monetary policy uncertainty–alternative measure

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A Appendix: The explicit form of long-term interest rates

In this appendix, we provide an explicit form of a long-term interest rate with maturity n under a compound version of the (pure) expectation hypothesis (PEH) equation(7). First, we denote short-term interest rates at period $t+j$ as in equation(6):

$$\begin{aligned} R_{t+j}^{t+j+1} &= \kappa\phi^{j-1}s_t + \eta_{t+j} + \Gamma_{t,j}^{v} + \Gamma_{t,j}^{\varepsilon} \\ \eta_{t+1} &= \psi\eta_t + \zeta_{t+1} \\ \zeta_t &\sim i.i.d.\ N\left(0, \sigma_\zeta^2\right) \end{aligned}$$

Writing down all $t+j$ $(1 \leq j \leq n-1)$ period-ahead short-term interest rates (ignoring the $\Gamma^{v}_{t,j}$ and $\Gamma^{\varepsilon}_{t,j}$ terms) assists in understanding the structure, such that:

$$\begin{aligned}
j &= 1 : R^{t+2}_{t+1} = \kappa s_t + \psi \eta_t + \zeta_{t+1} \\
j &= 2 : R^{t+3}_{t+2} = \kappa \phi s_t + \psi^2 \eta_t + \psi \zeta_{t+1} + \zeta_{t+2} \\
j &= 3 : R^{t+4}_{t+3} = \kappa \phi^2 s_t + \psi^3 \eta_t + \psi^2 \zeta_{t+1} + \psi \zeta_{t+2} + \zeta_{t+3} \\
& \quad \cdots \\
j &= n-1 : R^{t+n}_{t+n-1} = \kappa \phi^{n-2} s_t + \psi^{n-1} \eta_t + \psi^{n-2} \zeta_{t+1} + \psi^{n-3} \zeta_{t+2} + \cdots + \zeta_{t+n-1}.
\end{aligned}$$

Collecting the ζ_{t+j} terms results in:

$$\begin{aligned}
\Gamma^{\zeta}_t &= \zeta_{t+1} \left(1 + \psi + \psi^2 + \cdots + \psi^{n-2}\right) \\
&\quad + \zeta_{t+2} \left(1 + \psi + \psi^2 + \cdots + \psi^{n-3}\right) \\
&\quad + \cdots \\
&\quad + \zeta_{t+n-2} (1 + \psi) \\
&\quad + \zeta_{t+n-1} \\
&= \left(\frac{1-\psi^{n-1}}{1-\psi}\right) \zeta_{t+1} + \left(\frac{1-\psi^{n-2}}{1-\psi}\right) \zeta_{t+2} + \cdots + \left(\frac{1-\psi^{2}}{1-\psi}\right) \zeta_{t+n-2} + \zeta_{t+n-1} \qquad (10) \\
&= \sum_{i=0}^{n-2} \left(\frac{1-\psi^{n-1-i}}{1-\psi}\right) \zeta_{t+1+i}.
\end{aligned}$$

Hence:

$$\begin{aligned}
R^{t+1}_t + R^{t+2}_{t+1} \cdots + R^{t+n}_{t+n-1} &= R^{t+1}_t + \kappa \left(\frac{1-\phi^{n-1}}{1-\phi}\right) s_t + \left(\frac{1-\psi^{n-1}}{1-\psi}\right) \psi \eta_t \\
&\quad + \sum_{i=0}^{n-2} \left(\frac{1-\psi^{n-1-i}}{1-\psi}\right) \zeta_{t+1+i} \\
&= R^{t+1}_t + \kappa \left(\frac{1-\phi^{n-1}}{1-\phi}\right) s_t + \left(\frac{1-\psi^{n-1}}{1-\psi}\right) \psi \eta_t \\
&\quad + \sum_{i=0}^{n-2} C(i) \zeta_{t+1+i},
\end{aligned}$$

where:

$$C_n(i) \equiv \frac{1-\psi^{n-1-i}}{1-\psi}.$$

For $n \geq 2$, the variance term is:

$$Var\left(\sum_{i=0}^{n-2} C_n(i) \zeta_{t+1+i}\right) = \left[\sum_{i=0}^{n-2} C_n(i)^2\right] \sigma^2_{\zeta}.$$

Finally, a long-term interest rate with maturity n can be written as:

$$R_t^{t+n} = \frac{1}{n}\left[R_t^{t+1} + \kappa\left(\frac{1-\phi^{n-1}}{1-\phi}\right)s_t + \left(\frac{1-\psi^{n-1}}{1-\psi}\right)\psi\eta_t + \frac{1}{2}\sum_{i=0}^{n-2} C_n(i)^2\sigma_\zeta^2\right].$$

B Appendix: The case of general heteroskedasticity

Suppose that $\zeta_t = \sqrt{h_t}w_t$ where $w_t \sim i.i.d.\ N(0,1)$ and where h_t obeys a GARCH(p,q) specification:

$$h_t = \sum_{i=1}^{p} d_i h_{t-i} + \sum_{i=1}^{q} b_i \zeta_{t-i}^2.$$

Roughly speaking, Proposition 1 requires that the effect of uncertainty accumulates in the future. Hence, under the condition that $\sum_{i=1}^{p} d_i + \sum_{i=1}^{q} b_i = 1$,*i.e.*, the process of ζ_t^2 has a unit root, the changes in monetary policy uncertainty at period t have a permanent effect. Note that the random walk example in equation(9) satisfies this condition. For any GARCH process that satisfies the condition, such as the IGARCH process introduced in Engel and Bollerslev (1986), it is easy to show that Proposition 1 holds by replacing the time-invariant σ_ζ^2 with a time-varying conditional variance $E_t\left(\zeta_t^2\right) = \sigma_{\zeta,t}^2$, where:

$$\sigma_{\zeta,t}^2 = \sum_{i=0}^{p-1} d_i h_{t-i} + \sum_{i=0}^{q-1} b_i \zeta_{t-i}^2.$$

For a GARCH process without a unit root, Proposition 1 does not hold in general. To understand the basic idea, suppose ζ_t^2 follows:

$$\zeta_t^2 = c + \varphi\zeta_{t-1}^2 + w_t.$$

In this case, the *conditional* variance of Γ_t^ζ now becomes:

$$\begin{aligned}
E_t\left(\left(\Gamma_t^\zeta\right)^2\right) &= \left(\frac{1-\psi^{n-1}}{1-\psi}\right)^2 E_t\left(\zeta_{t+1}^2\right) + \left(\frac{1-\psi^{n-2}}{1-\psi}\right)^2 E_t\left(\zeta_{t+1}^2\right) + \\
&\quad \cdots + \left(\frac{1-\psi^2}{1-\psi}\right)^2 E_t\left(\zeta_{t+n-2}^2\right) + E_t\left(\zeta_{t+n-1}^2\right) \\
&= \left(\frac{1-\psi^{n-1}}{1-\psi}\right)^2 \left(\varphi\zeta_t^2 + c\right) + \left(\frac{1-\psi^{n-2}}{1-\psi}\right)^2 \left(\varphi^2\zeta_t^2 + c + \varphi c\right) + \\
&\quad \cdots + \left(\frac{1-\psi^2}{1-\psi}\right)^2 \left(\varphi^{n-2}\zeta_t^2 + c + \varphi c + \cdots + c\varphi^{n-1}\right) \\
&\quad + \left(\varphi^{n-1}\zeta_t^2 + c + \varphi c + \cdots + c\varphi^{n-1} + c\varphi^n\right).
\end{aligned}$$

Now, it is evident that if φ is a small but positive number, so that φ^i quickly approaches zero as i increases, then the cumulative effect of uncertainty is dominated by the diminishing φ^i. Therefore, $\varphi\,(< 1)$ needs to be sufficiently close to one for Proposition 1 to hold in general ARCH cases.