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How Do Monetary Policy Rules Affect Term Premia?

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Abstract

This paper derives analytical solutions for interest rate term structures in a new Keynesian framework. Theoretically, we consider the conditions for the positive average slope of nominal term structure, and show that the slope of a real one is positive. We then calibrate the model to find the following results. First, term premia represents compensation for risk of time-variation in IS shock rather than cost-push and monetary policy shocks. Second, a small slope of the Phillips curve is needed for a positive slope of the term structure. Third, the term structure of the inflation premium is downward on average. Finally, a less aggressive response to output gap in the monetary policy rule leads to lower term premia. The implication for the recent low long rate is also discussed.

JEL classification: E43; E52

Keywords: Term Structure of Interest Rate, Monetary Policy Rule, New Keynesian Model, Term Premium, Inflation Premium

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1. Introduction

The term structure of interest rates has been widely studied in the finance literature. The pioneer in this field is Vasicek (1977), which was followed by a large number of models. The literature succeeded in finding models with good empirical performance while keeping tractability, often obtaining closed form solutions even for complicated derivative security prices. However, as a caveat, many of the models have no macroeconomic interpretations. Typically, even the dynamics of the short rate are taken to be exogenous.

The monetary economics literature has rapidly developed over the past decade, particularly in applications of the new Keynesian framework. This research employs dynamic general equilibrium models to find that the monetary policy rule—in which the short rate reacts to the output gap and the inflation rate, is crucial for determining the dynamics of the short rate and macroeconomic variables. Recently, several studies have begun to try to integrate the finance and new Keynesian literatures for investigating the relationship between the interest rate term structure and the monetary policy rules.

The pioneers of this kind of study are Piazzesi (2005) and Ang and Piazzesi (2003), both of which examine the properties of the term structure when the short rate is determined by monetary policy rules. That investigation was followed by several studies, such as Wu (2001), Rudebusch and Wu (2003), Hordahl, Tristani and Vestin (2004), and Bekaert, Cho and Moreno (2005). Although those studies succeeded in finding interesting empirical facts, a sufficiently clear understanding on the relationship between macroeconomic variables and interest rate term structure has not yet been obtained. Because these papers emphasized the fit to the data, the models are too large and complicated to obtain clear intuition, or the internal consistency based on the microfoundation is ignored, in particular the stochastic discount factor for calculating the term structure is not consistent with the Euler condition for obtaining the IS curve. Using large and complicated models prevents us from obtaining closed-form solutions, which powerfully contribute to clear intuition. On the other hand, the internally inconsistent models are hard to interpret.

In contrast with these studies, the current paper uses a very simple new Keynesian model, in which the term structure and the IS curve are internally consistent based on the microfoundation. Although some ability to fit the data is sacrificed, the simple model enables us to obtain closed form solutions of the term structure, which is convenient for examining our main interest: how the parameter values in the monetary policy rule affect term premia.

Theoretically we derive two propositions. Proposition 1 considers why the nominal term premium is observed to be positive on average, and whether bondholders receive a positive premium for compensation for risk of time-variation of each shock. Proposition 2 implies that the real term premia are guaranteed to monotonically increase with maturity. In fact, the sign of the slope of the real term structure is still controversial in the literature. Den Haan (1995), among many studies, demonstrates the positive average slope of real term structure, while Ang and Bekaert (2004) argue that the real term structure is almost flat. Our results support the former study.

We also calibrated the model to investigate quantitative implications. The findings are as follows. First, the IS shock explains most of the term premia. On the other hand, the cost-push shock contributes to reduce the premium, and the monetary policy shock has only a tiny effect. Second, the slope of the Phillips curve would have to be small relative to the ones reported in the literature to be consistent with the term structure data. Finally, a less aggressive response to the output gap in the monetary policy rule leads to a lower term premium. This result, interestingly, suggests a possibility for the policy authority to control the term premium.

The subsequent sections are organized as follows. Section 2 reviews a simple new Keynesian model as derived from a microfoundation. Section 3 derives analytical solutions for the nominal term structure, which is internally consistent with the new Keynesian model. The properties of the model are then discussed. Section 4 considers the real rate and the inflation premia. Section 5 calibrates the model and identifies which parameters are important for the term structure. We also discuss the implication of the model for Grrenspan's conundrum, in which the long rate remains low even after the Fed has repeatedly raised the short rate in the recent years. Section 6 concludes.

2. A Simple New Keynesian Model

This section introduces the simplest type of new Keynesian model, with three variables, three shocks, and three equations. The model is derived from the microfoundation as shown in Appendix A. The model is then solved to obtain a close form solution for the dynamics of the three variables. The simplicity of the model represents a great advantage for obtaining intuition for later analyses.

2.1. Three Key Equations in the New Keynesian Model

The key variables in the basic new Keynesian models are output gap x_t, inflation rate π_t, and short rate i_t. The dynamics of these three variables are determined by the three key equations, the IS curve, the Phillips curve, and the monetary policy rule as

$$x_t = E_t x_{t+1} - \frac{1}{\gamma}(\hat{i}_t - E_t \hat{\pi}_{t+1} - \xi_t) \tag{1}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa x_t + u_t \tag{2}$$

$$\hat{i}_t = \phi_x x_t + \phi_\pi \hat{\pi}_t + \omega_t \tag{3}$$

where $\hat{\pi}_t = \pi_t - \overline{\pi}$ and $\hat{i}_t = i_t - \overline{i}$ are defined as the deviations from the steady state values $\overline{\pi}$ and $\overline{i}$. The parameters satisfy $0 < \beta < 1$, $\gamma > 0$, $\kappa > 0$, $\phi_x > 0$ and $\phi_\pi > 1$. γ and β stand for the coefficient of relative risk aversion and the subjective discount factor of the representative household respectively, while κ is the slope of the Phillips curve, which is determined by the degree of the nominal rigidity. The model has three exogenous shocks, which are assumed to obey AR(1) as

$$\xi_t = \rho_\xi \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t} \tag{4}$$

$$u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_{u,t} \tag{5}$$

$$\omega_t = \rho_\omega \omega_{t-1} + \sigma_\omega \varepsilon_{\omega,t} \tag{6}$$

where $0 \le \rho_s < 1$, $\sigma_s > 0$ and $\varepsilon_{s,t} \sim N(0,1)$ i.i.d. for $s = \{\xi, u, \omega\}$. These shocks are called the IS shock, the cost-push shock, and the monetary policy shock, respectively. A positive IS shock can be interpreted as a positive preference shock or a negative technology shock.

The IS curve (1) and the Phillips curve (2) can be derived from microfoundations. In particular, the microfoundation for the IS curve is important for discussing the internal consistency between the IS curve and the interest rate term structure, as shown in Section 3. See Appendix A for the microfoundation.

Monetary policy rule (3) is the famous (though ad hoc) Taylor rule. On the other hand, inertial policy rules, in which the current short rate depends on the lag of the short rate, are also popular in the literature, since an inertial rule is theoretically optimal in the sense that the expected utility of households is maximized. We do not adopt the inertial rule for the following three reasons. First, Rudebusch (1995, 2002) found that the non-inertial policy rule is consistent with term structure data, while the inertial one is not. Second, the optimal rule is difficult to practically implement, and many authors consider simple rules like the Taylor rule as possible alternatives[1]. Third, analytical solutions for term structure cannot be obtained with the inertial rule.

2.2. Dynamics of the Model

The restrictions on the response coefficients in the monetary policy rule $\phi_x > 0$ and $\phi_\pi > 1$ are actually sufficient conditions for determining a unique stationary equilibrium,

$$\begin{pmatrix} x_t \\ \hat{\pi}_t \\ \hat{i}_t \end{pmatrix} = \begin{pmatrix} (1-\beta\rho_\xi)\Omega_\xi & -(\phi_\pi - \rho_u)\Omega_u & -(1-\beta\rho_\omega)\Omega_\omega \\ \kappa\Omega_\xi & [\phi_x + \gamma(1-\rho_u)]\Omega_u & -\kappa\Omega_\omega \\ b_\xi & b_u & b_\omega \end{pmatrix} \begin{pmatrix} \xi_t \\ u_t \\ \omega_t \end{pmatrix} \tag{7}$$

where

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1 See Gali (2003) for discussion on this issue.

$$\mathbf{b} \equiv \begin{pmatrix} b_\xi \\ b_u \\ b_\omega \end{pmatrix} = \begin{pmatrix} [(1-\beta\rho_\xi)\phi_x + \kappa\phi_\pi]\Omega_\xi \\ [\rho_u\phi_x + \gamma(1-\rho_u)\phi_\pi]\Omega_u \\ [\gamma(1-\rho_\omega)(1-\beta\rho_\omega) - \kappa\rho_\omega]\Omega_\omega \end{pmatrix}, \tag{8}$$

and

$$\Omega_s = \frac{1}{(1-\beta\rho_s)[\phi_x + \gamma(1-\rho_s)] + \kappa(\phi_\pi - \rho_s)}. \tag{9}$$

The model with purely forward-looking behavior as (1) and (2) results in a simple form of dynamics (7), in which the three variables are represented as affine functions of the three shocks. Although this parsimonious model will not do as well at fitting data as the larger models often used in the literature, the simple dynamics—which have no state variable other than the three shocks—have a great advantage for providing clear intuition for the relationship between the term structure and macroeconomic variables.

Since Ω_s is positive, b_ξ and b_u are guaranteed to be positive. b_ω is also positive if and only if

$$\kappa < \frac{\gamma(1-\rho_\omega)(1-\beta\rho_\omega)}{\rho_\omega}. \tag{10}$$

If this inequality fails, a positive monetary policy shock lowers the short rate. That is, the direct upward effect of the positive policy shock on the short rate is weaker than the indirect downward effect of the reflections to the reduced output gap and inflation rate. This is because a large slope of the Phillips curve, a large intertemporal elasticity of substitution, or a persistent monetary policy shock makes the transmission mechanism of the monetary policy too strong, and weakens the reacts of the short rate to the policy shock.

We basically regard this odd case as unreasonable and focus on the case in which (10) holds. Thus $\mathbf{b} > \mathbf{0}$ and the short rate is procyclical to all shocks.

We obtained the dynamics of three key variables, which are consistent with the microfoundation. The next section considers the term structure of nominal rates.

3. Term Structure of Nominal Rates

This section considers the nominal term structure. First, we show that the model is mathematically identical to the three-factor Vasicek term structure model, although our model has macroeconomic interpretations in contrast with purely financial models like Vasicek's. Then, we derive the closed form solution for nominal term structure, which is consistent with the microfoundation of the new Keynesian model introduced in Section 2. The solution has convenient forms for identifying the contribution of each shock to the term structure.

3.1. The Stochastic Discount Factor

From the microfoundation for the new Keynesian model introduced in Section 2, we can derive the fundamental asset pricing equation for bond prices,

$$V_t^{(n)} = E_t[Q_{t+1}V_{t+1}^{(n-1)}] \tag{11}$$

for n = 1, 2,… as shown in Appendix A.1. Here $V_t^{(n)}$ is the price of n-period risk-free discount bond, which pays one unit of money at $t+n$. Q_{t+1} is the stochastic discount factor, the log of which satisfies

$$q_{t+1} = \ln\beta - \gamma g - \gamma\Delta x_{t+1} - \pi_{t+1} + \frac{1}{1-\rho_\xi}\Delta\xi_{t+1}, \tag{12}$$

where g is the steady state growth rate of the efficient level of output, as shown in Appendix A.5. The fundamental asset pricing equation (11) is used for obtaining the closed form solution for the interest rate term structure. A special case of the equation, in which $n=1$, is a typical Euler condition for the optimal intertemporal allocation of consumption,

$$V_t^{(1)} = E_t[Q_{t+1}], \tag{13}$$

which is used for deriving the IS curve. Thus the term structure and the IS curve should be internally consistent through the common stochastic discount factor.

From (4)-(7) and (12), we can obtain another form of the stochastic discount factor,

$$q_{t+1} = -i_t + \sum_{s=\{\xi,u,\omega\}}\left[-\frac{1}{2}(\nu_s\sigma_s)^2 + \nu_s\sigma_s\varepsilon_{s,t+1}\right], \tag{14}$$

where

$$\mathbf{v} = \begin{pmatrix} \nu_\xi \\ \nu_u \\ \nu_\omega \end{pmatrix} = \begin{pmatrix} [(1-\beta\rho_\xi)\phi_x + \kappa(\phi_\pi - 1)]\Omega_\xi/(1-\rho_\xi) \\ [-\phi_x + \gamma(\phi_\pi - 1)]\Omega_u \\ [\gamma(1-\beta\rho_\omega) + \kappa]\Omega_\omega \end{pmatrix}. \tag{15}$$

$\nu_s\sigma_s$ can be called the prices of risk corresponding to an innovation of shock s.

From (12) and (14), using $E\Delta x_{t+1} = E\Delta\xi_{t+1} = E\varepsilon_{s,t+1} = 0$, $Ei_t = \bar{i}$ and $E\pi_t = \bar{\pi}$, the steady state value of the short rate can be proved to be represented as

$$\bar{i} = \ln\frac{1}{\beta} + \gamma g + \bar{\pi} - \frac{1}{2}\sum_{s=\{\xi,u,\omega\}}(\nu_s\sigma_s)^2. \tag{16}$$

Equation (15) implies that the prices of risk are time-invariant. In addition, as seen in (7), the short rate is represented as an affine function of the three factors that obey AR(1) processes. These properties of the model are identical with the three-factor Vasicek term structure model, although our model has macroeconomic interpretations in contrast with such purely financial models as Vasicek's. This type of model is known to provide a closed form solution for the term structure, as will be obtained in the next subsection.

3.2. The Nominal Term Structure

Using the fundamental asset pricing equation, the *n*-period nominal interest rate $i_t^{(n)}$, which is defined as the continuously compounded rate satisfying $V_t^{(n)} = \exp(-i_t^{(n)} n)$, is derived as an affine function of the shocks,

$$i_t^{(n)} = \bar{i}^{(n)} + \sum_{s=\{\xi,u,\omega\}} b_s^{(n)} s_t \, , \tag{17}$$

where

$$b_s^{(n)} = b_s \frac{1}{n} \frac{1-\rho_s{}^n}{1-\rho_s} \, , \tag{18}$$

$$\bar{i}^{(n)} - \bar{i} = \sum_{s=\{\xi,u,\omega\}} \left[\frac{\nu_s b_s}{1-\rho_s} \left(1 - \frac{1}{n} \frac{1-\rho_s{}^n}{1-\rho_s} \right) - \frac{1}{2} \left(\frac{b_s}{1-\rho_s} \right)^2 \left(1 - \frac{2}{n} \frac{1-\rho_s{}^n}{1-\rho_s} + \frac{1}{n} \frac{1-\rho_s{}^{2n}}{1-\rho_s{}^2} \right) \right] \sigma_s{}^2 \tag{19}$$

for $n = 1, 2, \ldots$ Note that $i_t^{(1)} = i_t$, $\bar{i}^{(1)} = \bar{i}$, and $b_s^{(1)} = b_s$. See Appendix B for the deviation. Since s_t has an unconditional zero mean, $\bar{i}^{(n)}$ can be interpreted as the unconditional mean of n-period rate.

Several points are worth noting. First, (18) shows that the responses of nominal rates to the shocks have a common form, and (17) and (19) are represented with summations over $s = \{\xi, u, \omega\}$. Thus the effect of each shock on the term structure is represented as common forms, which depend only on four parameters ρ_s, σ_s, b_s and v_s. Therefore, we can identify which shocks are important for determining the term structure on a consistent basis and explain why the contributions of the shocks are different by investigating which of the four parameters are different.

Second, (18) shows that the response of n-period rate to a shock decays as n is longer. In addition, as the autocorrelation of shock ρ_s is larger, the response decays more slowly. Thus more persistent shock has a relatively larger effect on the long rate, and a smaller effect on the term spread between long and short rates.

Third, the nominal term premium $TP_t^{(n)}$ is proved to equal the unconditional average slope as

$$TP_t^{(n)} \equiv i_t^{(n)} - \frac{1}{n}\sum_{\tau=0}^{n-1} E_t i_{t+\tau} = \bar{i}^{(n)} - \bar{i} \ . \qquad (20)$$

Thus, we basically do not distinguish the term premium from the unconditional average slope in later analyses. The term spread $i_t^{(n)} - i_t$ varies around the time-invariant term premium only through the expectations for the future short rates.

Finally, (19) and (20) show that the contribution of a shock to the term premium is proportional to the variance of the innovation $\sigma_s{}^2$. Note that since the variance is the square of the volatility, the term premium is sensitive to the volatility. In addition, (19) seems to suggest the term premium tends to be higher for more persistent shocks, although this is less clear because b_s and ν_s depend on the autocorrelation ρ_s. The numerical exercise in Section 5 displays this tendency.

For simplifying the analysis, let's consider the extreme case with the infinite maturity, as if we consider the longest rate in term structure. The term premium can be represented as

$$TP_t^{(\infty)} = \sum_{s=\{\xi,u,\omega\}} \left[\frac{\nu_s b_s}{1-\rho_s} - \frac{1}{2}\left(\frac{b_s}{1-\rho_s}\right)^2 \right] \sigma_s^{\ 2} \quad . \tag{21}$$

The term premium is represented with a summation of three premia, each of which corresponds to compensation for the risk of time-variation in each shock. Thus, if each of the three premia is positive, we can conclude that the shock contributes to the positive term premium.

The first term in the bracket corresponds to the conditional covariance between the future bond price and the stochastic discount factor. For example, a positive innovation in the IS shock—which may be interpreted as a positive preference innovation, raises the short rate. This is because the positive preference innovation raises the output gap, to which the monetary policy authority reacts by raising the short rate. According to (18), the longer rates also rise in response to the innovation. Thus the innovation lowers the bond price for any maturity. On the other hand, the innovation raises the stochastic discount factor, since a positive preference innovation leads to a higher marginal utility. Consequently, the bond prices and the stochastic discount factor have a negative conditional covariance through the innovation. In that sense, the IS shock is unfavorable for bondholders and a positive premium is required. Similarly, a positive innovation of the monetary policy shock lowers bond prices while suppressing consumption and the inflation rate to raise the stochastic discount factor. Thus, a positive premium is required for the covariance through the monetary policy shock. In contrast, a positive cost-push shock raises the inflation rate, which usually lowers both bond prices and the stochastic discount factor. Thus, the cost-push shock is favorable for bondholders, and a negative premium is usually required.

The second term in (21) is the Jensen term, which always contributes to lower the term premiaince the Jensen term is proportional to the square of $b_s/(1-\rho_s)$, a higher response of the short rate to shock or more persistent shock leads to a larger Jensen term and a lower term premium. In sum, the first term should dominate the second term for each shock to contribute to the positive term premia.

Equation (21) can be rewritten as

$$TP_t^{(\infty)} = \sum_{s=\{\xi,u,\omega\}} \frac{b_s[2(1-\rho_s)\nu_s - b_s]}{2(1-\rho_s)^2}\sigma_s^{\,2} \, . \tag{22}$$

Thus a shock contributes to the positive term premia if and only if

$$2(1-\rho_s)\nu_s - b_s > 0\,. \tag{23}$$

By substituting (8) and (15) into this, we can easily prove the following proposition.

Proposition 1 *The IS shock and the cost-push shock contribute to the positive term premium of infinite maturity rate if and only if*

$$(1-\beta\rho_\xi)\phi_x + \kappa(\phi_\pi - 2) > 0 \tag{24}$$

and

$$-(2-\rho_u)\phi_x + \gamma(1-\rho_u)(\phi_\pi - 2) > 0 \tag{25}$$

respectively. The monetary policy shock always contributes.

Equation (24) implies that aggressive monetary policy rule, in which the response coefficients ϕ_x and ϕ_π are larger, allows the IS shock to contribute to a positive term premium. Note that Proposition 1 considers only the term premium of infinite maturity rate and does not consider the condition that the term premium monotonically increases with the maturity. On the other hand, as will be seen in Section 4, the real term premium is guaranteed to increase monotonically.

For the baseline calibration, introduced in Section 5, (24) holds while (25) fails. In fact, (25) typically will not hold, since at least a large response coefficient to the inflation rate in the monetary policy rule, larger than two, is necessary. Thus the IS shock and the monetary policy shock contribute to the positive term premia, while the cost-push shock contributes to reduce the term premia. The magnitudes of the contributions are examined with calibrated parameters in Section 5.

4. Term Structure of Real Rates and Inflation Premia

This section considers the term structure of real rates. The inflation premia, which are defined as nominal rates minus real rates, are also considered. First, the fundamental asset pricing equation and the stochastic discount factor for real bond prices are derived. Then, using closed form solutions, the properties of the term structure of real rate and inflation premium are discussed parallel to the discussion of nominal rates in Section 3.

4.1. The Fundamental Asset Pricing Equation for Real Bond Prices

The fundamental asset pricing equation for nominal bond prices (11) leads to

$$V_t^{(n)} = E_t[Q_{t+1}Q_{t+2}\cdots Q_{t+n}], \tag{26}$$

which means that the price of an n-period discount bond, whose only payment is one at $t+n$, is the expectation of the payment discounted by the stochastic discount factors. Similarly, the price of the n-period real discount bond, which pays P_{t+n}/P_t at $t+n$, is represented as

$$\tilde{V}_t^{(n)} = E_t[Q_{t+1}Q_{t+2}\cdots Q_{t+n}P_{t+n}/P_t]. \tag{27}$$

This can be rewritten as

$$\begin{aligned}\tilde{V}_t^{(n)} &= E_t[Q_{t+1}E_{t+1}[Q_{t+2}\cdots Q_{t+n}P_{t+n}/P_{t+1}]P_{t+1}/P_t] \\ &= E_t[\tilde{Q}_{t+1}\tilde{V}_{t+1}^{(n-1)}]\end{aligned} \tag{28}$$

where

$$\tilde{Q}_{t+1} = Q_{t+1}\frac{P_{t+1}}{P_t} = \beta\frac{\Psi_{t+1}}{\Psi_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}. \tag{29}$$

Equation (28) is the fundamental asset pricing equation for real bond prices, while $\tilde{Q}_{t+1}$ can be called the real stochastic discount factor.

Using a similar method to that used to derive the log of the nominal stochastic discount factor (14), the log of the real discount factor can be written as

$$\tilde{q}_{t+1} = -r_t + \sum_{s=\{\xi,u,\omega\}}\left[-\frac{1}{2}(\tilde{v}_s\sigma_s)^2 + \tilde{v}_s\sigma_s\varepsilon_{s,t+1}\right], \tag{30}$$

where

$$\tilde{\mathbf{v}} = \begin{pmatrix} \tilde{v}_\xi \\ \tilde{v}_u \\ \tilde{v}_\omega \end{pmatrix} = \begin{pmatrix} [(1-\beta\rho_\xi)\phi_x + \kappa(\phi_\pi - \rho_\xi)]\Omega_\xi /(1-\rho_\xi) \\ \gamma(\phi_\pi - \rho_u)\Omega_u \\ \gamma(1-\beta\rho_\omega)\Omega_\omega \end{pmatrix}, \tag{31}$$

$$r_t = \bar{r} + \sum_{s=\{\xi,u,\omega\}} \tilde{b}_s s_t \ , \tag{32}$$

$$\bar{r} = \ln\frac{1}{\beta} + \gamma g - \frac{1}{2}\sum_{s=\{\xi,u,\omega\}} (\tilde{v}_s \sigma_s)^2 \ . \tag{33}$$

$$\tilde{b}_s = (1-\rho_s)\tilde{v}_s \ , \tag{34}$$

Here r_t is the real short rate which satisfies $r_t = -\ln \tilde{V}_t^{(1)}$ while $\bar{r}$ is its steady state value.

4.2.The Real Term Structure

The real term structure can be obtained in a similar method to that used for the nominal term structure. The n-period real rate $r_t^{(n)}$ can be represented as

$$r_t^{(n)} = \bar{r}^{(n)} + \sum_{s=\{\xi,u,\omega\}} \tilde{b}_s^{(n)} s_t \ , \tag{35}$$

where

$$\tilde{b}_s^{(n)} = \tilde{b}_s \frac{1}{n}\frac{1-\rho_s{}^n}{1-\rho_s} = \frac{\tilde{v}_s(1-\rho_s{}^n)}{n} , \tag{36}$$

$$\bar{r}^{(n)} - \bar{r} = \sum_{s=\{\xi,u,\omega\}} \frac{\tilde{v}_s{}^2}{2}\left(1-\frac{1}{n}\frac{1-\rho_s{}^{2n}}{1-\rho_s{}^2}\right)\sigma_s{}^2 \ . \tag{37}$$

Since $\tilde{b}_s$ and $\tilde{v}_s$ are related in a simple form (34), the unconditional average slope of real rate $\bar{r}^{(n)} - \bar{r}$ has a much simpler form than that of nominal rate.

Several properties are common for the nominal and real rates. First, a response of n-period real rate to a shock decays as n is longer in the same rate as that of nominal rate. Thus the response of inflation premia, which are defined as nominal rates minus real rates, does too. Second, the real term premium $RTP_t^{(n)}$ equals the unconditional average slope as

$$RTP_t^{(n)} \equiv r_t^{(n)} - \frac{1}{n}\sum_{\tau=0}^{n-1} E_t r_{t+\tau} = \bar{r}^{(n)} - \bar{r} \ . \qquad (38)$$

Third, a contribution of a shock to the term premium is proportional to the variance of the innovation $\sigma_s{}^2$. On the other hand, in contrast with nominal rates, (37) and (38) suggest that all shocks are guaranteed to contribute to the positive term premium as the following proposition.

Proposition 2 *All shocks are guaranteed to contribute to positive slopes of the real term premia between any maturities.*

This proposition implies that the real term premia are guaranteed to increase monotonically with maturity. In fact, the sign of the average slope of real term structure is still controversial in the literature. Den Haan (1995), among many studies, demonstrates positive average slope of real term structure, while Ang and Bekaert (2004) argue that the real term structure is almost flat. Our results support the former study.

The inflation premium can be represented as

$$i_t^{(n)} - r_t^{(n)} = \bar{i}^{(n)} - \bar{r}^{(n)} + \sum_{s=\{\xi,u,\omega\}} [b_s^{(n)} - \tilde{b}_s^{(n)}] s_t \ . \qquad (39)$$

Since $b_s^{(n)} \neq \tilde{b}_s^{(n)}$ in general, the inflation premium varies, depending on the shocks. The unconditional average of the inflation premium can be represented as

$$\bar{i}^{(n)} - \bar{r}^{(n)} = \bar{i} - \bar{r} + TP_t^{(n)} - RTP_t^{(n)} \quad . \tag{40}$$

5. Analyses with Calibrated Parameters

In Sections 3 and 4, the closed form solutions for the term structure of nominal rate, real rate, and inflation premium were derived using parameters with macroeconomic interpretations. This section calibrates the model to answer following questions: Which parameters are crucial for the term premia? What value should the parameters take for fitting the term structure data? Why is the slope of term structure positive on average? Do monetary policy rules play an important role for term premia? Which of the real term premia or inflation premia explain the nominal term premia?

5.1. The Calibrated Results

The baseline calibration is reported in Table 1. The parameter values are chosen basically from a range used in the literature to ensure that the model-implied term structure is consistent with the data. For example, $\gamma = 1$ is familiar in standard RBC models, while $\gamma = 0.16$ is suggested by Rotemberg and Woodford (1997). Our calibration $\gamma = 0.4$ is chosen from an interval of those. As an exception, the chosen value for the slope of the Phillips curve $\kappa = 0.005$ is small relative to ones reported in the literature. For example, McCallum and Nelson (2000) reported that κ should be in [0.01, 0.05] to be consistent with their empirical results. For seeing whether these values are consistent with the term structure data, sensitivity analyses—which report the impacts of the changes in parameter values on the term structure—are conducted in the next subsection.

The high autocorrelation of the IS shock is consistent with Gali and Rabanal (2004) and Ireland (2004), whose estimates suggest that a preference shock is highly persistent and dominates the dynamics of the macroeconomic variables. The autocorrelation and volatility of the monetary policy shock come from the residual of the regression of the Taylor rule. Note that the reaction of the short rate to the policy shock is weakened by the reflection to the reduced output gap and inflation rate, as discussed in connection with equation (10). This is the main reason why the policy shock does not play an important role for the term premium.

Figure 1 reports the factor loadings on the nominal rates. This shows that, although the factor loading decays more slowly for more persistent shock as discussed in Section 3, the persistence of the monetary policy shock is insufficient to prevent the factor loading from sharply dropping from 1Q rate to 20Q rate. Thus a positive monetary policy shock raises the short rate more than the long rate, which results in a decrease in the term spread, following the conjecture of Evans and Marshall (1998).

Figures 2 and 3 report the contributions of shocks to the nominal and real term premia, respectively. The figures show that the IS shock explains most of the term premia. One important reason for these results is the higher variance and persistency of the IS shock.

Figure 4 reports the factor loadings of the inflation premia. The factor loadings on the IS shock and monetary policy shock are much smaller than those of the nominal and real rates. Thus, a large part of the inflation premium variation is up to the cost-push shock. The factor loadings can be represented as

$$b_{\xi}^{(n)} - \tilde{b}_{\xi}^{(n)} = \kappa \rho_{\xi} \Omega_{\xi} \frac{1}{n} \frac{1-\rho_{\xi}{}^{n}}{1-\rho_{\xi}} \,, \tag{41}$$

$$b_{u}^{(n)} - \tilde{b}_{u}^{(n)} = \rho_{u} [\phi_{x} - (1-\rho_{u})] \Omega_{u} \frac{1}{n} \frac{1-\rho_{u}{}^{n}}{1-\rho_{u}} \,, \tag{42}$$

$$b_{\omega}^{(n)} - \tilde{b}_{\omega}^{(n)} = -\kappa \rho_{\omega} \Omega_{\omega} \frac{1}{n} \frac{1-\rho_{\omega}{}^{n}}{1-\rho_{\omega}} \,. \tag{43}$$

These equations suggest the small slope of the Phillips curve prevents the IS shock and the monetary policy shock from having large effects on the inflation rate, which results in a smaller effect on the inflation premium.

Figure 5 reports the decomposition of the unconditional average of nominal term premium into those of real term premium and inflation premium. The figure displays that the term structure of inflation premium is downward on average. In fact, the inflation premium for three month rate equals 0.54 percent, while that for infinite maturity equals 0.28 percent.

5.2. Sensitivity Analyses

We'll now turn the focus to the nominal and real term premia and the unconditional mean of inflation premium for infinite maturity, which equal 0.33, 0.59, and 0.28 percent per quarter, respectively, for the baseline parameter values. Table 2 reports changes in the premia corresponding to changes in parameter values. This table enables us to see which of the changes in real term premium or inflation premium explain the changes in nominal term premiaeveral points are worth noting.

First, a larger value of the slope of the Phillips curve κ results in a negative nominal term premium, -0.83 percent, which is inconsistent with the positive unconditional mean of term premia observed in the data. The parameter value of κ is highly controversial in the literature, and our results support a smaller slope of the Phillips curve. This is because a large slope allows the IS shock to have too-large an effect on the inflation rate and the short rate, which results in a higher Jensen term and a lower nominal term premium.

Second, a less aggressive monetary policy responses to the output gap results in a smaller nominal term premium. This is because a less aggressive policy causes the output gap to be more volatile, which results in a larger response of the short rate to the IS shock and a higher Jensen term. On the other hand, the response to the inflation rate is much less important.

Third, even larger autocorrelation and volatility of the monetary policy shock have only a tiny effect, mainly because of the small b_{ω}. This implies that even if the monetary policy authority aggressively deviates from the Taylor rule, the authority does not have to change the short rate significantly, which results in a tiny effect on the term premia.

Fourth, a large autocorrelation and volatility of the cost-push shock strongly suppresses the nominal term premium. Thus more persistent or volatile cost-push shock is inconsistent with the positive slope of the term structure observed in the data.

Finally, since the IS shock is crucial for explaining the term premium, the premium is very sensitive to the autocorrelation and volatility of the IS shock.

In sum, the nominal term premium is sensitive to the slope of the Phillips curve, the monetary policy response to the output gap, and the autocorrelation and volatility of the IS shock. In particular, the large effect of the policy response on the term premia suggests, interestingly, that the policy authority may be able to control the term premia by choosing the monetary policy rule. This effect may be an answer for Greenspan's (2005) conundrum, in which the long rate remains low even after the Fed has repeatedly raised the short rate in recent years. That is, the response to the output gap in the monetary policy rule may become less aggressive, and this reduces the term premium. In fact, the Fed is raising the short rate very slowly, by a "measured" pace, even after U.S. economy has clearly recovered, and this fact may be interpreted as a decline in the response to the output gap in the policy rule. Another possible answer to the conundrum is a less persistent and volatile IS shock. The persistency and volatility of macroeconomic variables such as GDP or inflation rate are known to be lower than before. Stock and Watson (2002), among many studies, attributes this to a change in the property of the shock itself.

6. Conclusion

How do monetary policy rules affect term premia? To answer that question, we used a very simple new Keynesian model in which the term structure and the IS curve are internally consistent based on microfoundations. We succeeded in obtaining closed form solutions for the term structure of nominal and real rates and inflation premia, which are represented by parameters with macroeconomic interpretations, and used them to obtain clearer intuition for the relationship between the term structure and macroeconomic variables.

Theoretically we derive two propositions. Proposition 1 considers the conditions for each shock to contribute to the positive nominal term premium of infinite maturity rate. This proposition provides three implications as follows. First, aggressive monetary policy rules allow the IS shock to contribute to the positive term premiaecond, the cost push shock is difficult to contribute to the positive term premium. Third, the monetary policy shock always contributes to the positive term premium. These results are basically obtained by the property of the conditional covariance between the future bond price and stochastic discount factor, as discussed in Section 3.

Proposition 2 implies that the real term premia are guaranteed to increase monotonically with maturity, and all shocks contribute to the positive real term premium. In fact, the sign of the slope of real term structure is still controversial in the literature. Den Haan (1995), among many studies, demonstrates the positive average slope of real term structure, while Ang and Bekaert (2004) argue that the real term structure is almost flat. Our results support the former study.

We also calibrated the model to investigate quantitative implications. The findings are as follows. First, the IS shock explains most of the term premia. One important reason for this is the higher variance and persistency of the IS shock. On the other hand, the cost-push shock contributes to reduce the premium as predicted in the discussion in connection with Proposition 1. The monetary policy shock has only a tiny effect, because the reaction of the short rate to the policy shock is weakened by the reflection to the reduced output gap and inflation rate. Second, the slope of the Phillips curve would have to be small relative to ones reported in the literature to be consistent with the term structure data. This is because a large slope allows the IS shock to have too-large an effect on the inflation rate and the short rate, which results in a higher Jensen term and a lower nominal term premium. Finally, a less aggressive response to the output gap in the monetary policy rule leads to a lower term premium. This is because a less aggressive policy causes the output gap to be more volatile, which results in a larger response of the short rate to the IS shock and a higher Jensen term. These results may be answers for Greenspan's (2005) conundrum. For example, the response to the output gap in the monetary policy rule may become less aggressive, and this reduces the term premium.

Since the macroeconomics and finance literatures are not yet very well integrated, general equilibrium models—which can explain the dynamics of both macroeconomic variables and asset prices well—require further study. A natural direction of future research is adding another property into the model. The macroeconomic literature often incorporates habit formation, an indexation of price setting, sticky wages, capital accumulation, real frictions such as capital adjustment cost, learning under parameter uncertainty, and so on, into the model. Unfortunately, however, macroeconomics lacks a sufficient consensus on which properties are more important than the others. Examining whether a property of the macroeconomic models can explain term structure data may help identify the most important properties of the model.

Appendix A

This appendix demonstrates how the IS curve (1) can be derived from the microfoundation. The model is typical one in the new Keynesian literature, very similar to Walsh (2003, p.232), although long term bonds are added for discussing the term structure of interest rates.

A.1. Household

The preference of the representative household is defined over consumption goods, real money balances, and a labor supply. The composite consumption good is defined as

$$C_t = \left[\int_0^1 c_{jt}^{(\theta_t - 1)/\theta_t} dj \right]^{\theta_t/(\theta_t - 1)} \qquad \text{(A.1)}$$

where c_{jt} is the individual goods produced by firm j and θ_t is the price elasticity of demand for the individual goods. Here θ_t is an exogenous disturbance, which is the source of the cost-push shock. P_t is the corresponding price index of these goods defined as

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta_t} dj \right]^{1/(1-\theta_t)} \qquad \text{(A.2)}$$

where p_{jt} is the price of individual good produced by firm j. In this setting, the demand for good j can be written as

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta_t} C_t . \qquad \text{(A.3)}$$

The household supplies labor hours h_{jt}, which are used for producing the individual goods c_{jt}.

Households maximize an expected present discount value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \Psi_t \left[\frac{1}{1-\gamma} C_t^{1-\gamma} + \frac{\varpi}{1-\zeta} \left(\frac{M_t}{P_t} \right)^{1-\zeta} - \int_0^1 \frac{\chi}{1+\eta} h_{jt}^{1+\eta} dj \right], \qquad \text{(A.4)}$$

where $0 < \beta < 1$ and all of γ, ζ, η, ϖ and χ are positive. Here Ψ_t is the exogenous preference shock, which is a source of the IS shock. M_t denotes nominal money balances.

The household's budget constraint is

$$P_t C_t + M_t + \sum_{n=1}^{\infty} V_t^{(n)} B_t^{(n)} = \int_0^1 w_{jt} h_{jt} dj + \int_0^1 \Pi_{jt} dj + M_{t-1} + T_t + \sum_{n=1}^{\infty} V_t^{(n-1)} B_{t-1}^{(n)} . \qquad \text{(A.5)}$$

Here w_{jt} and Π_{jt} are the nominal wage per hour and the household's share in the profits of firm j, respectively. The aggregate real output Y_t is defined to satisfy

$$P_t Y_t = \int_0^1 w_{jt} h_{jt} dj + \int_0^1 \Pi_{jt} dj . \qquad \text{(A.6)}$$

$V_t^{(n)}$ is the price of *n*-period risk-free discount bond which pays one unit of money at $t+n$, while $B_t^{(n)}$ is the number of the *n*-period bond held by the household at the end of period *t*. T_t is a nominal lump sum transfer that the household receives from the monetary policy authority in the form of cash. This setting of household's maximization problem is typical in the new Keynesian literature, although the long term bonds are added to the budget constraint for considering the term structure of interest rates.

In the household's decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (A.4) subject to (A.5). One of the first order conditions to the problem is the fundamental asset pricing equation for bond prices (11),

$$V_t^{(n)} = E_t[Q_{t+1} V_{t+1}^{(n-1)}].$$

Here Q_{t+1} is the stochastic discount factor, which satisfies

$$Q_{t+1} = \beta \frac{\Psi_{t+1}}{\Psi_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}}. \tag{A.7}$$

A special case of the fundamental asset pricing equation, in which $n = 1$, is a typical Euler condition (13),

$$V_t^{(1)} = E_t[Q_{t+1}].$$

A.2. The Monetary Policy Authority

The monetary policy authority controls the short rate by obeying a Taylor rule (3). In addition, the policy authority is assumed to just pass out the seigniorage to the representative household as a transfer,

$$T_t = M_t - M_{t-1}. \tag{A.8}$$

Since the bonds are in zero net supply in equilibrium, the budget constraint (A.5) results in $C_t = Y_t$. That is, aggregate consumption and output need not be distinguished from one another in the model.

A.3. Firms

We consider an economy with a continuum of identical firms. For simplicity, we ignore variations in capital so that output of individual goods c_{jt} is a function of exogenous productivity A_t and labor input h_{jt} as

$$c_{jt} = A_t h_{jt} . \tag{A.9}$$

Firms maximize their expected profits

$$E_0 \sum_{t=0}^{\infty} \left[\prod_{\tau=1}^{t} Q_\tau (p_{jt} c_{jt} - w_{jt} h_{jt}) \right] \tag{A.10}$$

subject to three constraints. The first is their production function (A.9) and the second is the demand curve given by (A.3). The third constraint is the friction that each period, only a fraction $1-\alpha$ of all firms are able to adjust their price as a Calvo (1983) sticky price model.

A.4. Efficient Level of Output and the Output Gap

Let's consider the efficient level of output, which is chosen by a social planner who can overcome the friction to adjust the prices. Substituting (A.9) into (A.1) results in

$$C_t = A_t \left[\int_0^1 h_{jt}{}^{(\theta_t - 1)/\theta_t} dj \right]^{\theta_t/(\theta_t - 1)} . \tag{A.11}$$

The social planner maximizes the household's welfare (A.4) subject to (A.11). The efficient level of output Y_t^* is defined as the consumption level that satisfies the first-order conditions to the problem:

$$Y_t^* = \chi^{-\frac{1}{\eta+\gamma}} A_t^{\frac{\eta+1}{\eta+\gamma}} . \tag{A.12}$$

Thus the efficient level of output depends positively on the productivity and can be interpreted as a technology shock.

In fact, under friction, in which all firms are not able to adjust their prices to each period, the actual output can deviate from the efficient level of output. Let's define the output gap x_t as the log difference between these outputs as

$$x_t = y_t - y_t^*, \tag{A.13}$$

where $y_t = \ln Y_t$ and $y_t^* = \ln Y_t^*$. With these settings, a famous linear approximation of the first order condition of the maximization problem (A.10) around the steady state results in the new Keynesian Phillips curve (2) where $\kappa = (1-\alpha)(1-\alpha\beta)/\alpha$ [2]. Here the inflation rate is defined as $\pi_t = \ln P_t / P_{t-1}$.

A.5. The Forward-looking IS Curve

Using (A.13), the log of the stochastic discount factor (A.7) can be written as (12),

$$q_{t+1} = \ln\beta - \gamma g - \gamma\Delta x_{t+1} - \pi_{t+1} + \frac{1}{1-\rho_\xi}\Delta\xi_{t+1},$$

where $g \equiv E\Delta y_t^*$ is the deterministic trend of y_t^* and the IS shock ξ_t is defined as the deviation of $(1-\rho_\xi)(\psi_t - \gamma y_t^*)$ from its steady state. Thus a positive IS shock can be interpreted as a positive preference shock or a negative technology shock.

Since $E_t\Delta\hat{\xi}_{t+1} = -(1-\rho_\xi)\hat{\xi}_t$, log-linearizing the Euler condition (13) around the steady state results in the IS curve (1) where $i_t = -\ln V_t^{(1)}$.

[2] For the deviation, see, for example, Walsh (2003, p.263) or Woodford (2004).

Appendix B

By substituting (14) and (17) into (11),

$$\begin{aligned}
&\exp\left\{-\bar{i}^{(n)}n-\sum_{s=\{\xi,u,\omega\}} b_s^{(n)} s_t n\right\} \\
&= E_t\left[\exp\left\{-\bar{i}+\sum_{s=\{\xi,u,\omega\}}\left(-b_s s_t-\frac{1}{2}(\nu_s\sigma_s)^2+\nu_s\sigma_s\varepsilon_{s,t+1}\right)\right.\right. \\
&\qquad\qquad \left.\left.-\bar{i}^{(n-1)}(n-1)-\sum_{s=\{\xi,u,\omega\}} b_s^{(n-1)} s_{t+1}(n-1)\right\}\right]. \qquad \text{(B.1)}
\end{aligned}$$

Since s_t obeys AR(1),

$$\begin{aligned}
&\exp\left\{-\bar{I}^{(n)}-\sum_{s=\{\xi,u,\omega\}} B_s^{(n)} s_t\right\} \\
&= E_t\left[\exp\left\{-\bar{I}^{(n-1)}-\frac{1}{2}\sum_{s=\{\xi,u,\omega\}}(\nu_s\sigma_s)^2\right.\right. \\
&\qquad \left.\left.+\sum_{s=\{\xi,u,\omega\}}(\nu_s-B_s^{(n-1)})\sigma_s\varepsilon_{s,t+1}-\sum_{s=\{\xi,u,\omega\}}[b_s+\rho_s B_s^{(n-1)}]s_t\right\}\right]. \qquad \text{(B.2)}
\end{aligned}$$

where

$$B_s^{(n)} = b_s^{(n)} n, \qquad \text{(B.3)}$$

$$\bar{I}^{(n)} = (\bar{i}^{(n)}-\bar{i})n. \qquad \text{(B.4)}$$

This, using $\varepsilon_{l,t} \sim N(0,1)$ i.i.d., results in

$$\begin{aligned}
&-\bar{I}^{(n)}-\sum_{s=\{\xi,u,\omega\}} B_s^{(n)} s_t \\
&= -\bar{I}^{(n-1)}-\sum_{l=\{\xi,u,\omega\}}\left\{\nu_s B_s^{(n-1)}-\frac{1}{2}[B_s^{(n-1)}]^2\right\}\sigma_s{}^2-\sum_{s=\{\xi,u,\omega\}}[b_s+\rho_s B_s^{(n-1)}]s_t. \qquad \text{(B.5)}
\end{aligned}$$

Since (B.5) holds for any outcome of ξ_t, u_t and ω_t,

$$B_s^{(n)} = b_s+\rho_s B_s^{(n-1)}, \qquad \text{(B.6)}$$

$$\overline{I}^{(n)} = \overline{I}^{(n-1)} + \sum_{s=\{\xi,u,\omega\}} \left\{ \nu_s B_s^{(n-1)} - \frac{1}{2}[B_s^{(n-1)}]^2 \right\} \sigma_s^{\ 2} . \tag{B.7}$$

Solving (B.6) and (B.7) recursively leads to

$$B_s^{(n)} = b_s \sum_{\tau=0}^{n-1} \rho_s^{\ \tau} = b_s \frac{1-\rho_s^{\ n}}{1-\rho_s}, \tag{B.8}$$

$$\overline{I}^{(n)} = \sum_{s=\{\xi,u,\omega\}} \sum_{\tau=0}^{n-1} \left\{ \nu_s B_s^{(\tau)} - \frac{1}{2} B_s^{(\tau)2} \right\} \sigma_s^{\ 2} . \tag{B.9}$$

Substituting (B.3) and (B.4) into (B.8) and (B.9) results in (17) and (18) respectively.

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Table 1: The Calibrated Parameters

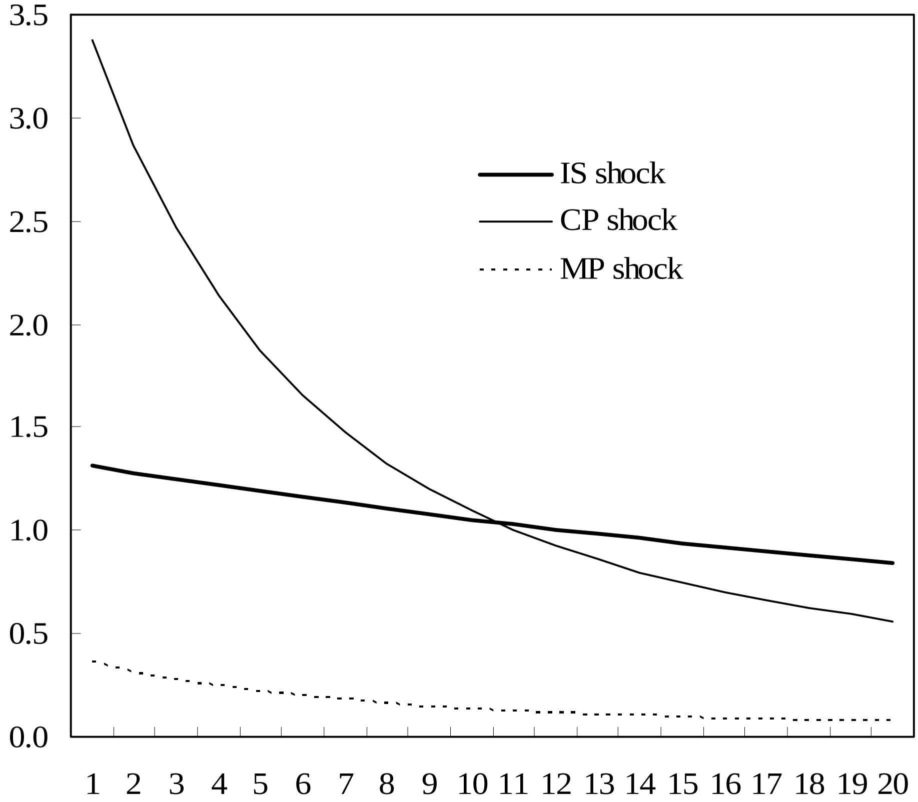
Structural parameters		Implied values	
β	0.99	b_ξ	1.31
γ	0.40	b_u	3.38
κ	0.005	b_ω	0.36
ϕ_x	0.125	ν_ξ	17.47
ϕ_π	1.5	ν_u	0.95
$\bar{\pi}$	0.05	ν_ω	1.75
g	0.007	$\tilde{b}_\xi$	0.90
ρ_ξ	0.95	$\tilde{b}_u$	1.21
ρ_u	0.70	$\tilde{b}_\omega$	0.42
ρ_ω	0.75	$\tilde{\nu}_\xi$	17.91
σ_ξ	0.0060	$\tilde{\nu}_u$	4.04
σ_u	0.0040	$\tilde{\nu}_\omega$	1.67
σ_ω	0.0015	$\bar{i}$	0.012
		$\bar{r}$	0.007

Table 2: The Sensitivity Analysis

	Changes in Parameters	Nominal term premium	Real term premium	Inflation premium
γ	0.50→0.16	0.07	0.07	-0.01
κ	0.005→0.050	-1.16	0.09	-1.20
ϕ_x	0.125→0.025	-0.93	-0.16	-0.73
ϕ_π	1.5→1.1	-0.13	-0.05	-0.09
ρ_ξ	0.95→0.90	-0.31	-0.46	0.13
ρ_u	0.60→0.90	-4.85	0.09	-4.86
ρ_ω	0.75→0.90	-0.00	0.00	-0.00
σ_ξ	0.0060→0.0100	0.73	1.03	-0.25
σ_u	0.0040→0.0100	-0.44	0.07	-0.45
σ_ω	0.0015→0.0100	0.01	0.01	-0.00
The baseline	-	0.33	0.59	0.28

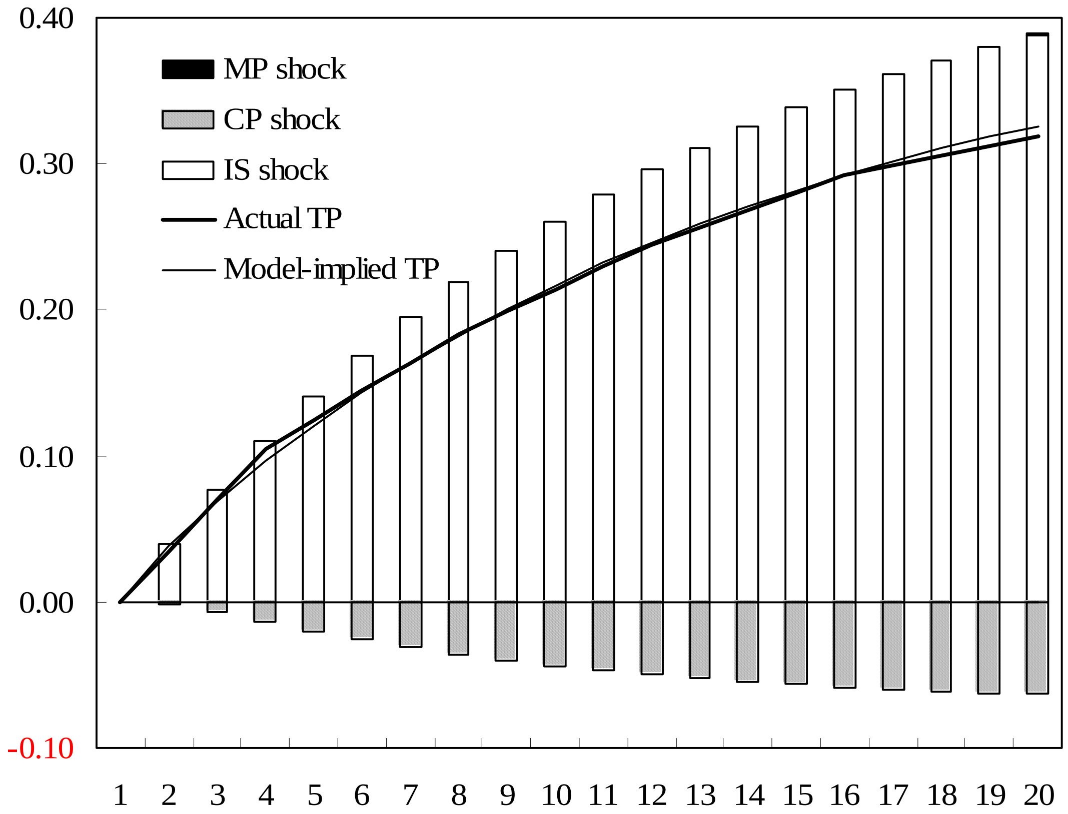
Note: The changes in nominal term premia, real term premia, and the inflation premia (percent per quarter) for infinite maturity when the parameter values change are reported. Note that since the steady state values of short rates also depend on the parameter values, the changes in nominal term premium are not equal to the sum of the changes in real term premium and inflation premium. The baseline reports the premia for the baseline parameter values.

Figure 1: Factor Loading of the Nominal Rate



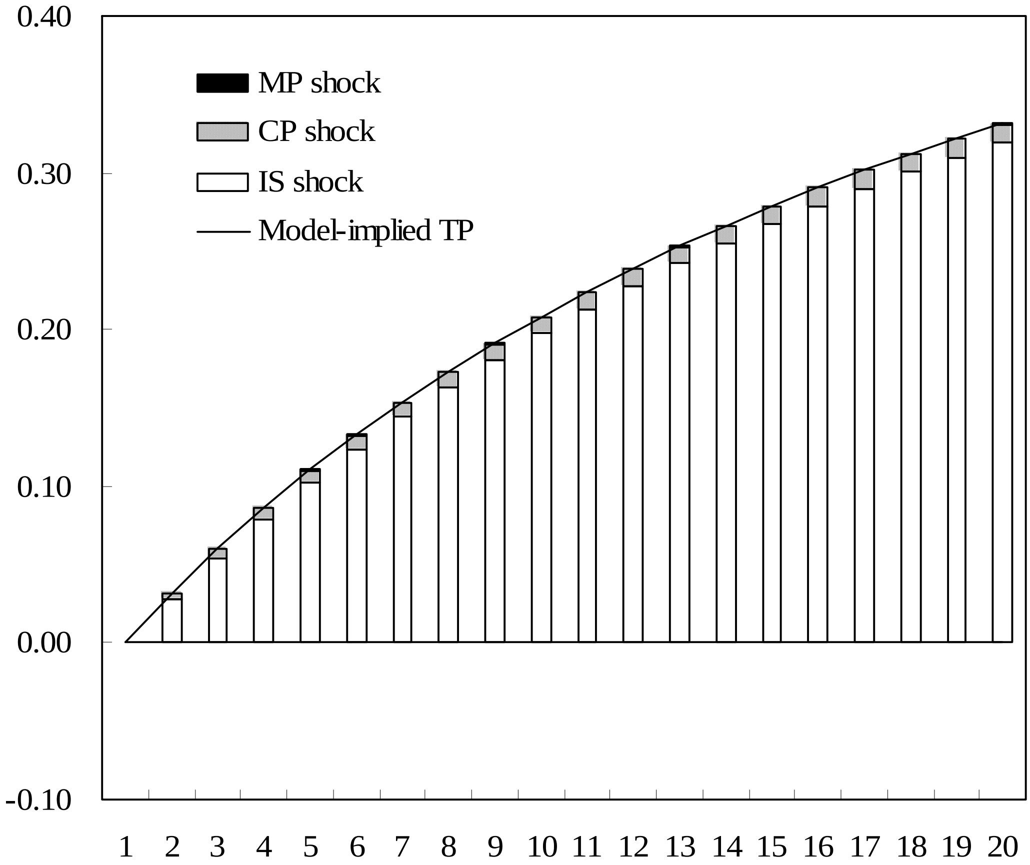
Note: The factor loadings of the nominal rates are reported. The horizontal axes correspond to the maturity (quarters). Thick, thin, and broken lines correspond to the IS shock, the cost-push shock and the monetary policy shock, respectively.

Figure 2: Contributions of the Shocks to the Nominal Term Premium



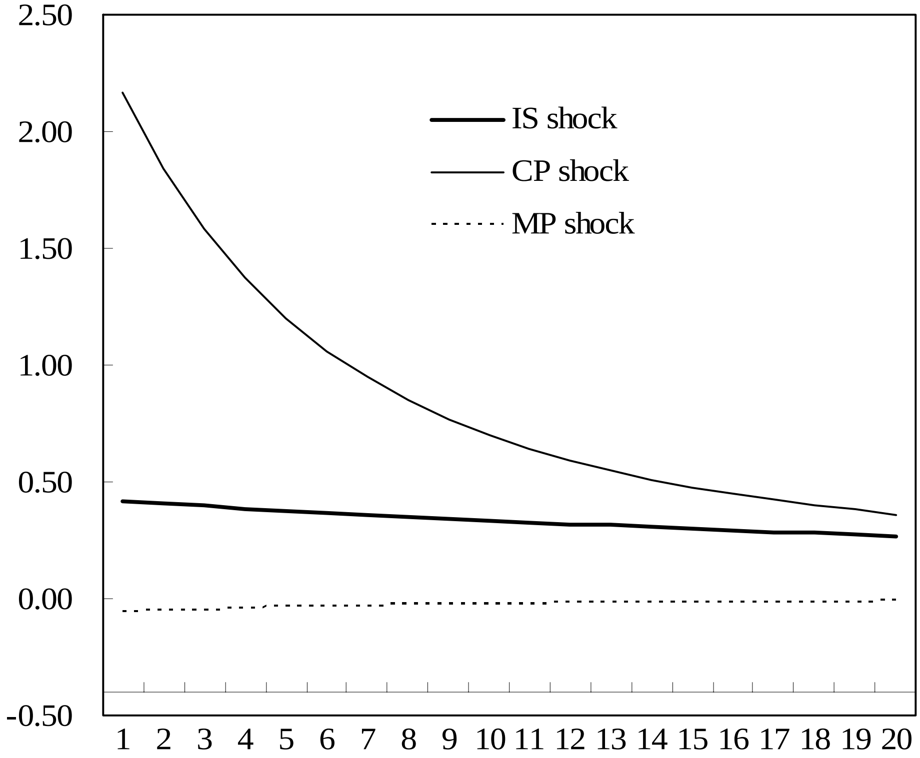
Note: The contributions of the shocks to the nominal term premia (percent per quarter) are reported. The horizontal axes correspond to the maturity (quarters). White, black, and gray bars correspond to the contributions of the IS shock, the cost-push shock, and the monetary policy shock, respectively. Thick and thin lines correspond to the actual term premia and model-implied term premia, respectively. The actual term premia are the average term spreads for 1983:1Q-2002:4Q calculated by using CRSP (Center for Research in Security Prices, Graduate School of Business, the University of Chicago: www.crsp.uchicago.edu. All rights reserved.) Monthly US Treasury Database.

Figure 3: Contributions of the Shocks to the Real Term Premium



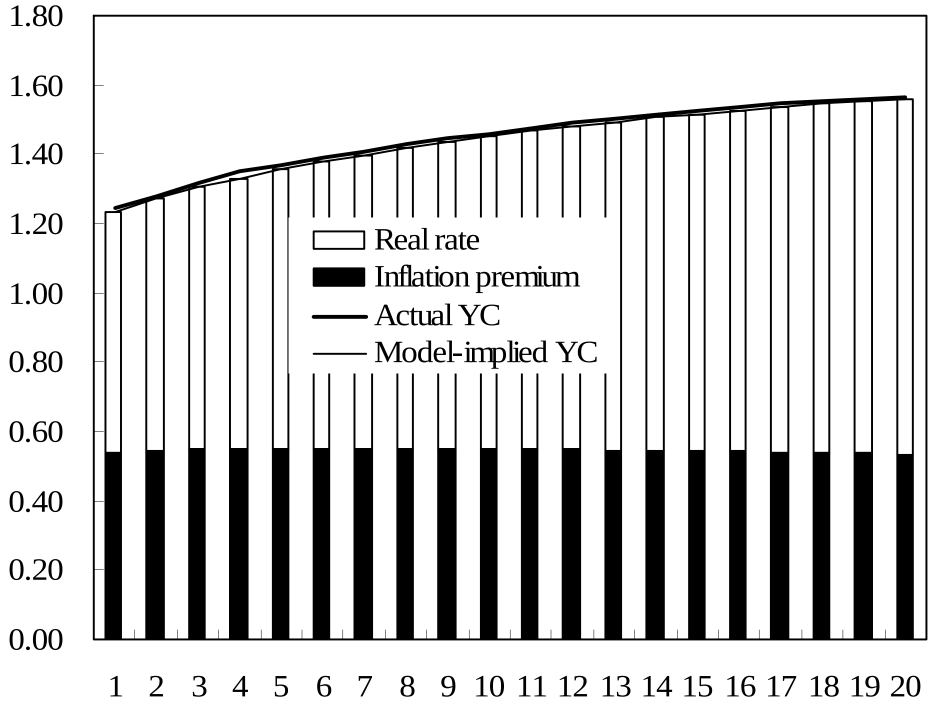
Note: The contributions of the shocks to the real term premia (percent per quarter) are reported. The horizontal axes correspond to the maturity (quarters). White, black, and gray bars correspond to the contributions of the IS shock, the cost-push shock, and the monetary policy shock, respectively. Thin line corresponds to the model-implied term premia.

Figure 4: Factor Loading on the Inflation Premium



Note: The factor loadings of the nominal rates are reported. The horizontal axes correspond to the maturity (quarters). Thick, thin, and broken lines correspond to the IS shock, the cost-push shock, and the monetary policy shock, respectively.

Figure 5: Decomposition of the Nominal Term Structure



Note: The contributions of the real rate and the inflation premium to the nominal rate (percent per quarter) are reported. The horizontal axes correspond to the maturity (quarters). White and black bars correspond to the contributions of the real rate and the inflation premium, respectively. Thick and thin lines correspond to the actual nominal term structure and the model-implied term structure, respectively. The actual term structure is the average term spreads for 1983:1Q-2002:4Q calculated by using CRSP (Center for Research in Security Prices, Graduate School of Business, the University of Chicago: www.crsp.uchicago.edu. All rights reserved.) Monthly US Treasury Database.