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Optimal Trend Inflation and Monetary Policy under Trending Relative Prices

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Abstract

In the standard new Keynesian models, the optimal inflation rate is zero while the long-run inflation rate is non-zero positive in many countries. In this paper, we provide a new rationale for the non-zero trend inflation by utilizing the productivity gap between the intermediate-goods sector and the final-goods sector. The productivity gap among the sectors creates the relative price trend of the CPI and the PPI, which is observed in the actual data. Then, we show that the Ramsey-optimal inflation rate of the CPI is positive while that of the PPI is negative. In addition, the efficient allocation cannot be achieved under the productivity gap. Finally, we investigate the optimal monetary policy response to a shock under the trend inflation. Our results suggest that non-zero trend inflation dramatically alters the optimal monetary policy.

*JEL classification:*E31; E32; E52
keywords: Trend inflation; CPI; PPI; Optimal monetary policy; Relative price trend; Sectoral productivity gap

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1 Introduction

Optimal rate of inflation and optimal monetary policy are fundamental but separately considered issues in the theory of monetary policy. Indeed, recent monetary policy research based on the 'new Keynesian synthesis' abbreviates the former by postulating zero inflation at the steady state. This custom is usually justified since the optimal rate of inflation is typically zero as long as researchers except the additional assumptions such as the opportunity cost of money holding.

However, as is presented in Table 1, the long-run inflation rate of the consumer price index, the CPI, is not zero in major developed countries. The standard theory achieves the limited success to explain this non-zero positive CPI inflation as an outcome of the appropriate monetary policy operation. Notably, Schmitt-Grohé and Uribe (2005) show that the optimal inflation rate is -0.4% in the sticky-price medium-scale dynamic general equilibrium model with the zero boundary on nominal interest rates and the distortion from the monopolistic competition, which are often cited as the rationale for setting positive inflation targets. Schmitt-Grohé and Uribe (2005) address: *"This result poses the challenge for future researchers of finding a theoretical explanation for the optimality of positive inflation...*"

Table 1: The long-run CPI inflation rate around the world

	US	UK	GER	ITA	SWE	CAN	JPN	SPN	AUS
1950-2005	3.82	5.77	2.74	6.39	5.14	4.01	3.74	7.65	5.66

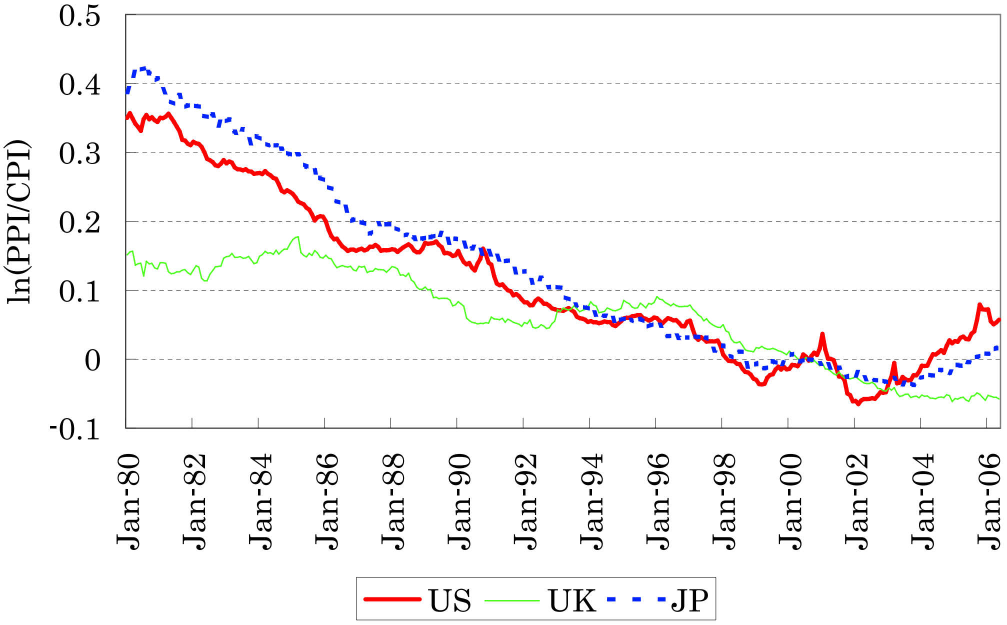
Note: All inflation rates are measured in terms of the consumer price index, the CPI.
Source: "International Financial Statistics," IMF.

The goal of this paper is to provide a new rationale for the non-zero positive CPI inflation rate and to study the optimal monetary policy under the trend inflation. In order to investigate the optimality for the non-zero positive inflation, we focus on the trend in relative prices, which is also observed in the data. Figure 1 illustrates the long-run downward sloping trend in the relative price of the producer price index, the PPI, over the CPI.

According to the new Keynesian view such that the inefficiency loss induced by price stickiness should be minimized, perfect stabilization of the CPI does not always lead to the welfare maximization in the presence of the multiple source of the nominal rigidities. This issue has been studied by Aoki (2001), Erceg, Henderson, and Levin (2000), or Huang and Liu (2005) in the context of the cyclical variations.[1] In these models, a shock creates a

[1] Aoki (2001) points out that the "core inflation" should be stabilized instead of the overall CPI, if the "core" prices are sticky while the "non-core" prices are flexible. Erceg, Henderson, and Levin (2000) maintain that

Figure 1: The relative price between the PPI and the CPI in US, UK, and Japan



Source: International Monetary Fund: "International Financial Statistics"

transitory trade-off between the source of nominal rigidities through the the relative price fluctuations.

Compared to these studies about the cyclical variations around the trend, our primary concern is to study the optimal inflation rate. The relative price development is not just a cyclical phenomenon but also a trending one as is shown in Figure 1. Namely, when the relative price of the CPI and the PPI has a certain trend, it is infeasible to stabilize all individual prices even without stochastic shocks because at least one of the inflation rates must deviate from zero. In such a situation, the central bank has multiple choices. One extreme of these choices is to stabilize the CPI and accept the deflationary PPI. Another extreme is to stabilize the PPI and accept the inflationary CPI. For this reason, the relative price trend can be regarded as a promising candidate to explain the non-zero trend inflation. In fact, we show that the welfare maximizing CPI inflation rate is non-zero positive in the presence of the trending relative prices.

The effect of the relative price trend does not stop here. The relative price trend also affects the optimal monetary policy by inducing the non-zero trend inflation.

Non-zero trend inflation is not problematic for the optimal monetary policy analysis as long as the equilibrium dynamics is independent from the level of the trend inflation. Un-

stabilizing the sticky wage in addition to the sticky price is optimal. Huang and Liu (2005) demonstrate that not only the CPI but also the PPI should be stabilized when both the CPI and the PPI are sticky.

fortunately, however, recent studies (Ascari (2004), Kiley (2004) and Cogley and Sbordone (2006)) point out that the dynamics of inflation hinges upon the level of the steady state inflation rate. These studies suggest that the ”new Keynesian Phillips curve” (NKPC), which is the key equation in the context of the new Keynesian synthesis, changes its form under the non-zero trend inflation. Hence, the relative price trend is expected to change the optimal monetary policy by inducing the non-zero trend inflation, which alters the cyclical dynamics of inflation. In this paper, we explore the optimal monetary policy under the non-zero trend inflation in addition to the optimal rate of inflation.

This paper shares some similarities with recent papers. Khan, King, and Wolman (2003) and Schmitt-Grohé and Uribe (2005) investigate the optimal inflation rate and the optimal monetary policy in the dynamic general equilibrium models. Both of these papers find that the optimal inflation rate is actually negative since the ”Friedman rule” plays a dominant role. Namely, Schmitt-Grohé and Uribe (2005) demonstrate that the zero boundary on nominal interest rates is of no quantitative relevance because the level of the nominal interest rate is high enough to avoid hitting the zero bound.

By contrast, our motivation in this paper is to add a new theoretical reasoning behind the positive CPI inflation in the post-war period, utilizing the relative price trend in the data, which is not explored in their papers. In addition, this paper keeps the analytical tractability and clarifies the effect of non-zero trend inflation on the optimal monetary policy although Khan, King, and Wolman (2003) and Schmitt-Grohé and Uribe (2005)’s results are basically numerical and do not mention the role of the non-zero trend inflation on the optimal monetary policy.

Wolman (2005) is the only exception to study the optimal rate of inflation under the relative price trend in the dynamic general equilibrium models. The main difference of ours from Wolman (2005)’s paper is that (1)we explicitly model an input-output linkage among sectors to make a natural distinction between the CPI and the PPI and (2)we derive not only the optimal rate of inflation but also the optimal monetary policy under the non-zero trend inflation.

Our analytical results reveal that the efficient allocation cannot be achieved in the face of the relative price trend because the inefficiency loss of the relative price dispersions cannot be zero under the trend inflation. This sub-optimal equilibrium allocation exhibits the clear contrast with the one sector sticky price models of Woodford (2003) or the multisector models of Aoki (2001) and Huang and Liu (2005). As for the inflation rate in each sector, the optimal trend inflation of the CPI is positive while that of the PPI is negative. This conclusion supports our view that the relative price trend can be a promising candidate for justifying the positive long-run CPI inflation. The sensitivity analysis of the optimal trend inflation clarifies that prices of the stickier and more demand elastic goods should be stabilized. Finally, optimal monetary policy under non-zero trend inflation shows the decisive contrast to the standard sticky price models. Under the non-zero trend inflation, the central bank can improve the welfare by assigning the disparate roles to the short-run response and the long-run commitment.

The remainder of the paper is organized as follows. Section 2 describes the theoretical model, Section 3 shows what is the price stability under the production chain economy with a relative price trend, Section 4 illustrates the how the optimal monetary policy changes with and without a relative price trend, and Section 5 concludes the paper.

2 The structure of the production chain economy

In this section, we describe the two sector production chain economy, where a large number of identical and infinitely lived households enjoy leisure and consumption of differentiated final goods.

The key feature of this economy lies in the production side. In the intermediate goods sector, a firm produces a differentiated intermediate good from a labor input and sells his or her product to final goods firms. In the final goods sector, a firm produces a differentiated final good out of labor and the composite of intermediate goods. A firm in each sector is a monopolistic competitor in the product market and follows the Calvo (1983) type sticky price setting. Since the production process of a final good is two stages, we call this economy as the two sector production chain economy.

The model here has a similar structure with the one developed by Huang and Liu (2005). The main difference is the existence of intermediate-goods-specific technological progress, which generates a trade-off between stabilizing the two measures of inflation. In addition, it will be shown that positive trend inflation affects the dynamics of the economy around the steady state although Huang and Liu (2005) assume zero trend inflation in both sectors. In the rest of this section, we describe the settings of this economy in detail.

2.1 Households

At the beginning, we specify the utility function for a representative household as follows,

$$E\sum_{t=0}^{\infty}\beta^t[U(C_t)-V(N_t)] \tag{1}$$

where E is an expectation operator, β is the subjective discount factor, C_t is consumption, and N_t is labor hours. In addition, we presume $U(C) = \ln(C)$ to satisfy the King, Plosser, and Rebelo (1988)'s condition for the existence of the balanced growth path.

The consumption good is represented as a Dixit-Stiglitz type composite index of differentiated final goods such that

$$C_t = \left[\int_0^1 Y_{ft}(j)^{(\theta_f - 1)/\theta_f} dj \right]^{\theta_f/(\theta_f - 1)}$$

where $Y_{ft}(j)$ is the amount of a type j final good and $\theta_f > 1$ is the price elasticity of demand for final goods. The contemporaneous budget constraint for the representative household is

$$P_{ft}C_t + E_t D_{t,t+1} B_{t+1} \leq W_t N_t + \Pi_t + B_t + T_t$$

where $E_t D_{t,t+1}$ is a price of one period contingent claim bond, B_t is the amount of the bond, Π is the dividend from firms, and T is the lump-sum transfer from the government.

From the consumer's cost minimization problem, the demand for a final good can be derived as follows,

$$Y_{ft}^d(j) = \left[\frac{P_{ft}(j)}{P_{ft}} \right]^{-\theta_f} C_t \tag{2}$$

where $P_{ft}(j)$ is the price of a final good j and $P_{ft} = [\int_0^1 P_{ft}(j)^{1-\theta_f} dj]^{1/(1-\theta_f)}$ gives the aggregate price index of the final goods. This aggregate price index corresponds to the CPI.

The first order conditions for the household are given by,

$$\frac{W_t}{P_{ft}} = \frac{-V_{nt}}{U_{ct}} \tag{3}$$

$$U_{ct} = \beta E_t \left[U_{c,t+1} R_t \frac{P_{ft}}{P_{f,t+1}} \right] \tag{4}$$

where $U_c = \partial U / \partial C$, $V_n = \partial V / \partial N$, $R_t = E_t D_{t,t+1}{}^{-1}$ is the nominal yield of the bonds and $D_{t,\tau} = \beta^{\tau - t}(U_{c\tau}/U_{ct})(P_{ft}/P_{f\tau})$ is the stochastic discount factor. For analytical simplicity, we assume $V_{nt} = 1$.

2.2 Firms and optimal price setting

2.2.1 Production technology and technology shocks

Labor and a composite of intermediate goods are necessary to produce a type j final good. The production technology is expressed by the following Cobb-Douglas production function, $(0 < \phi < 1)$.

$$Y_{ft}(j) = Y_{mt}(j)^{\phi} (A_{ft} N_{ft}(j))^{1-\phi} \tag{5}$$

where $Y_m(j) = [\int_0^1 Y_m(j,i)^{(\theta_m - 1)/\theta_m} di]^{\theta_m/(\theta_m - 1)}$ is a composite of intermediate goods for a firm j. $N_f(j)$ represents the labor input. $\theta_m > 1$ is the elasticity of substitution among differentiated intermediate goods. A_{ft} is the technological progress in the final goods sector.

To produce a type i intermediate good requires labor as an only input.

$$Y_{mt}(i) = A_{mt}N_{mt}(i) \tag{6}$$

where A_{mt} is the technological progress for the intermediate goods sector and $N_{mt}(i)$ denotes the labor input for a firm i.

Let $Z_{kt} \equiv A_{kt}/A_{kt-1}$ denote the growth rate of the technological progress in k sector ($k \in \{f, m\}$). By assumption, on the non-stochastic balanced growth path, Z_{kt} is constant and equals to $Z_k > 0$. Also, let $\hat{Z}_{kt} \equiv \ln(Z_{kt}/Z_k)$ denote the percentage deviation of the growth rate of technological progress. Then, the evolution of $\hat{Z}_{kt}$ is assumed to be given by,

$$\hat{Z}_{kt+1} = \rho_k\hat{Z}_{k,t} + \epsilon_{kt+1}, \quad \rho_k < 1$$

where we assume $\epsilon_{k,t}$ is i.i.d. normal, mutually independent with finite variance of σ_k^2. The assumptions on the stochastic process are just for the analytical simplicity.

2.2.2 Optimal price setting in the final goods sector

Solving the cost minimization problem yields the factor demand functions,

$$Y_{mt}^d(i) = \phi\frac{V_{ft}}{P_{mt}}\left[\frac{P_{mt}(i)}{P_{mt}}\right]^{-\theta_m}\int_0^1 Y_{ft}^d(j)dj$$
$$N_{ft}^d = (1-\phi)\frac{V_{ft}}{W_t}\int_0^1 Y_{ft}^d(j)dj$$

where $P_{mt} = [\int_0^1 P_{mt}(i)^{1-\theta_m}di]^{1/(1-\theta_m)}$ represents the aggregated intermediate goods price. The aggregated price index corresponds to the PPI. In addition, the unit cost of the final goods production can be derived as follows,

$$V_{ft} = \bar{\phi}P_{mt}^{\phi}\left(\frac{W_t}{A_{ft}}\right)^{1-\phi} \tag{7}$$

where $\bar{\phi} = \phi^{-\phi}(1-\phi)^{-(1-\phi)}$.

Each firm in both sectors is a price taker in the input market and a monopolistic competitor in the product market. Firms set their prices in the Calvo fashion such that when firms get an opportunity to reset their prices at time t, they can choose the optimal price to maximize the discounted sum of the future profit. The price-reset probabilities of a final goods firm and an intermediate goods firm are $1-\alpha_f$ and $1-\alpha_m$, respectively($0 < \alpha_f, \alpha_m < 1$).

The optimization problem of a final goods producer j becomes,

$$\max_{\{P_{ft}(j)\}} E_t\sum_{\tau=t}^{\infty}\alpha_f^{\tau-t}D_{t,\tau}[P_{ft}(j)(1+\tau_f) - V_{f\tau}]Y_{f\tau}^d(j)$$

subject to eq.(2), where τ_f denotes the tax rate.

Solving the profit maximization problem yields the following optimal price function.

$$P^*_{ft}(j) = \frac{\mu_f}{1+\tau_f}\frac{E_t \sum_{\tau=t}^{\infty} \alpha_f^{\tau-t} D_{t,\tau} V_{f\tau} Y_{f\tau}(j)}{E_t \sum_{\tau=t}^{\infty} \alpha_f^{\tau-t} D_{t,\tau} Y_{f\tau}(j)} \tag{8}$$

where $P^*_{ft}(j)$ and $\mu_f = \theta_f/(\theta_f - 1)$ represent the optimal price for a firm j and the price markup ratio, respectively.

2.2.3 Optimal price setting in the intermediate goods sector

Solving the cost minimization problem of an intermediate goods firm and aggregating across firms, we obtain the factor demand function and the unit cost function, as shown in eq.(9) and eq.(10).

$$N^d_{mt} = \frac{1}{A_{mt}} \int_0^1 Y^d_{mt}(i)di \tag{9}$$

$$V_{mt} = \frac{W_t}{A_{mt}} \tag{10}$$

We also suppose the Calvo type price setting in the intermediate goods sector and obtain the optimal price function for an intermediate goods firm i.

$$P^*_{mt}(i) = \frac{\mu_m}{1+\tau_m}\frac{E_t \sum_{\tau=t}^{\infty} \alpha_m^{\tau-t} D_{t,\tau} V_{m\tau} Y_{m\tau}(i)}{E_t \sum_{\tau=t}^{\infty} \alpha_m^{\tau-t} D_{t,\tau} Y_{m\tau}(i)} \tag{11}$$

where $P^*_m(i)$ and $\mu_m = \theta_m/(\theta_m - 1)$ denote the optimal price of a firm i and the price markup ratio, respectively. In both sectors, a firm suffices demand at the posted price.

Given the labor demand in the two sectors, the labor market clearing condition suggests, $N_t = N^d_{ft} + N^d_{mt}$. At the equilibrium, $B_t = 0$ and $T_t = \tau_f P_{ft} C_t + \tau_m P_{mt} Y_{mt}$ hold. Finally, in order to focus on the effect of price stickiness, we eliminate the distortion of the monopolistic competition by setting $1 + \tau_k = \mu_k$, following Woodford (2003).

2.3 The inefficiency loss of the relative price dispersion

Since we do not assume zero inflation at the steady state, while it is normally presumed in the standard new Keynesian literature, we need to take the inefficiency loss of the relative price dispersion under the non-zero trend inflation into account. As is stressed in Goodfriend and King (1997), inefficient price dispersion results in a misallocation of aggregate output across alternative uses of goods. In especial, non-zero trend inflation distorts the equilibrium allocation even at the steady state. We demonstrate it in the following.

First, we specify the inefficiency loss of the relative price dispersion in the intermediate goods sector. Using the production function, the aggregated labor input equation, and the demand function, we rewrite the aggregated labor input in the intermediate goods sector as follows,

$$\begin{aligned} N_{mt} &= \int_0^1 N_{mt}(i)di = \frac{1}{A_{mt}} \int_0^1 Y_{mt}(i)di = \frac{1}{A_{mt}} \int_0^1 \left(\frac{P_{mt}(i)}{P_{mt}} \right)^{-\theta_m} Y_{mt} di \\ &\equiv \frac{1}{A_{mt}} S_{mt} Y_{mt} \end{aligned} \tag{12}$$

where $S_{mt} \equiv \int_0^1 [P_{mt}(i)/P_{mt}]^{-\theta_m} di$.

The last expression of eq.(12) implies,

$$Y_{mt} = \frac{A_{mt} N_{mt}}{S_{mt}} \tag{13}$$

As is demonstrated in Schmitt-Grohé and Uribe (2004), a kind of indices like S_{mt} is bounded below one.[2] From eq.(13), we can see that output is distorted in the case of $S_{mt} > 1$. Therefore, S_{mt} can be regarded as a measure of the inefficiency loss due to relative price dispersion.[3] In fact, the higher S_{mt} is, the more labor is needed to produce a given level of the composite intermediate goods.

Obtaining the distorted output in the final goods sector in a similar way and combining with eq.(13) give the distorted output in the whole economy.

$$C_t = \frac{(A_{mt} N_{mt})^{\phi} (A_{ft} N_{ft})^{1-\phi}}{S_{ft} S_{mt}^{\phi}} \tag{14}$$

where $S_{ft} \equiv \int_0^1 [P_{ft}(j)/P_{ft}]^{-\theta_f} dj \geq 1$.

Eq.(14) shows that if both S_f and S_m are one, the economy can attain the efficient allocation. However, if not, the relative price distortion pushes output inside the production possibility frontier. Hence, we can regarded $S_{ft} S_{mt}^{\phi}$ as a measure of the inefficiency loss from the relative price dispersion.[4]

In order to study the inefficiency loss at the equilibrium, we also derive the dynamics of the inefficiency loss of the price dispersion in a k sector.

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[2]Let $x_{kt}(i) \equiv [P_{kt}(i)/P_{kt}]^{1-\theta_k}$ and plug it into the price index. Then, we obtain $[\int_0^1 x_{kt}(i)di]^{\theta_k/(\theta_k-1)} = 1$. By definition, $S_{kt} = \int_0^1 x_{kt}(i)^{\theta_k/(\theta_k-1)} di$. Finally, applying Jensen's inequality, we can show $1 = [\int_0^1 x_{kt}(i)di]^{\theta_k/(\theta_k-1)} \leq \int_0^1 x_{kt}(i)^{\theta_k/(\theta_k-1)} di = S_{kt}$.

[3]The exact inefficiency loss is expressed as $1 - 1/S_{mt}$.

[4]The exact inefficiency loss is $1 - 1/(S_{ft} S_{mt}^{\phi})$.

$$
\begin{aligned}
S_{kt} &= \int_0^1 \left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta_k} di \\
&= (1-\alpha_k)\left[\frac{P^*_{kt}}{P_{kt}}\right]^{-\theta_k} + \alpha_k(1-\alpha_k)\left[\frac{P^*_{kt-1}}{P_{kt}}\right]^{-\theta_k} + ... \\
&= (1-\alpha_k)\left[\frac{P^*_{kt}}{P_{kt}}\right]^{-\theta_k} + \alpha_k\pi_{kt}^{\theta_k}\left\{(1-\alpha_k)\left[\frac{P^*_{kt-1}}{P_{kt-1}}\right]^{-\theta_k} + \alpha_k(1-\alpha_k)\left[\frac{P^*_{kt-2}}{P_{kt-1}}\right]^{-\theta_k} ...\right\} \\
&= (1-\alpha_k)\left[\frac{P^*_{kt}}{P_{kt}}\right]^{-\theta_k} + \alpha_k\pi_{kt}^{\theta_k} S_{kt-1}
\end{aligned}
\tag{15}
$$

Eq.(15) suggests that $\bar{\pi}_k < \alpha_k^{-1/\theta_k}$ is necessary for S_k to have a stable root, where $\bar{\pi}_k$ denotes the steady state inflation. Hereafter, we limit our attention within the range of $0 < \bar{\pi}_k < \alpha_k^{-1/\theta_k}$.[5]

2.4 The balanced growth path

This economy has two types of permanent shocks. As a result, a number of variables such as output and real wage, will not be stationary along the balanced growth path. We therefore transform variables so as to obtain a set of equilibrium conditions that involve only stationary variables. To be specific, utilizing the cointegration relationships, we detrend C_t and W_t/P_{ft} with $A_{1t} \equiv A_{ft}^{1-\phi}A_{mt}^{\phi}$, P_{mt}/P_{ft} with $A_{2t} \equiv A_{ft}^{1-\phi}A_{mt}^{\phi-1}$, and Y_{mt} with A_{mt}, respectively. We denote the set of equilibrium conditions after detrending as follows,

$$\beta E_t \frac{C_t}{C_{t+1}} \frac{R_t}{\pi_{ft+1}Z_{1t+1}} = 1 \tag{16}$$

$$C_t = \frac{W_t}{P_{ft}} \tag{17}$$

$$v_{ft} \equiv \frac{V_{ft}}{P_{ft}} = \bar{\phi}Q_t^{\phi}\left(\frac{W_t}{P_{ft}}\right)^{1-\phi} \tag{18}$$

$$v_{mt} \equiv \frac{V_{mt}}{P_{mt}} = \frac{W_t}{P_{ft}Q_t} \tag{19}$$

$$N_{ft} = (1-\phi)\frac{v_{ft}}{W_t/P_{ft}} S_{ft}C_t \tag{20}$$

$$N_{mt} = \phi\frac{v_{ft}}{Q_t} S_{ft}S_{mt}C_t \tag{21}$$

$$Y_{mt} = \phi v_{ft} S_{ft} \frac{C_t}{Q_t} \tag{22}$$

[5] Let θ_k=10. Then, in the case of α_k=0.25, 0.5, and 0.8, the upper ceiling of the inflation rate is 90.37%, 31.95%, and 9.034% per year, respectively.

$$Q_t = \frac{\pi_{mt}}{\pi_{ft}}\frac{1}{Z_{2t}}Q_{t-1} \tag{23}$$

$$\frac{P^*_{kt}}{P_{kt}} = \left(\frac{1-\alpha_k\pi_{kt}^{\theta_k-1}}{1-\alpha_k}\right)^{\frac{1}{1-\theta_k}}, \quad k \in \{k, m\} \tag{24}$$

$$S_{kt} = (1-\alpha_k)\left(\frac{P^*_{kt}}{P_{kt}}\right)^{-\theta_k} + \alpha_k\pi_{kt}^{\theta_k}S_{kt-1}, \quad k \in \{f, m\} \tag{25}$$

$$\frac{P^*_{kt}}{P_{kt}} = \frac{\mu_k}{1+\tau_k}\frac{\psi_{kt}}{\phi_{kt}}, \quad k \in \{f, m\} \tag{26}$$

$$\psi_{ft} = E_t\sum_{\tau=t}^{\infty}(\alpha_f\beta)^{\tau-t}\left(\frac{P_{f\tau}}{P_{ft}}\right)^{\theta_f} v_{f\tau} \tag{27}$$

$$\phi_{ft} = E_t\sum_{\tau=t}^{\infty}(\alpha_f\beta)^{\tau-t}\left(\frac{P_{f\tau}}{P_{ft}}\right)^{\theta_f-1} \tag{28}$$

$$\psi_{mt} = E_t\sum_{\tau=t}^{\infty}(\alpha_m\beta)^{\tau-t}\frac{Y_{m\tau}\frac{P_{m\tau}}{P_{f\tau}}}{Y_{f\tau}}\left(\frac{P_{m\tau}}{P_{mt}}\right)^{\theta_m} v_{m\tau} \tag{29}$$

$$\phi_{mt} = E_t\sum_{\tau=t}^{\infty}(\alpha_m\beta)^{\tau-t}\frac{Y_{m\tau}\frac{P_{m\tau}}{P_{f\tau}}}{Y_{f\tau}}\left(\frac{P_{m\tau}}{P_{mt}}\right)^{\theta_m-1} \tag{30}$$

where $Z_{1t} = Z_{ft}^{1-\phi}Z_{mt}^{\phi}$, $Z_{2t} = Z_{ft}^{1-\phi}Z_{mt}^{\phi-1}$, and $Q_t \equiv P_{mt}/P_{ft}$.

Since the assumptions on preference and production technology suffice the balanced growth restriction, we can attain the unique balanced growth path.[6]

3 Inflation gap and efficiency

3.1 Inflation gap in the production chain economy

In the production chain economy, the productivity gap among the sectors determines the inflation gap between the PPI and the CPI.[7] We will show the linkage between the technological progress and inflation, below.

Proposition 1: *On the balanced growth path of the two sector production chain economy where the production functions are expressed as eq.(5) and eq.(6), the relative inflation rate*

[6]To be precise, the balanced growth requires the relative risk aversion in consumption to be unity and the disutility from labor separable. In addition, in the class of constant elasticity of substitution (CES) production functions, the Cobb-Douglas case is the only one permitting a balanced growth path. The existence of the unique balanced growth path is described in Appendix 1.

[7]When we refer to the productivity gap and the inflation gap, we mean the sectoral difference in the rate of the technological progress and inflation.

of the final goods and the intermediate goods suffice the following relationship.

$$\frac{\overline{\pi}_m}{\overline{\pi}_f} = \left(\frac{Z_f}{Z_m}\right)^{1-\phi} \tag{31}$$

where $\overline{x}$ *represents* x *is on the balanced growth path and* $\overline{\pi}_k = \overline{P}_{kt}/\overline{P}_{kt-1}, \overline{P}_{kt}, \overline{P}_{kt-1} > 0, \overline{\pi}_k < \alpha_k^{-1/\theta_k}, \ \ k \in \{f, m\}$.

Proof: Suppose that eq.(31) does not hold. Then,

$$\begin{aligned} \frac{\overline{\pi}_m}{\overline{\pi}_f} > (<) \left(\frac{Z_f}{Z_m}\right)^{1-\phi} \quad & \Rightarrow \frac{Z_m(\overline{\pi}_m/\overline{\pi}_f)}{Z_m^{\phi} Z_f^{1-\phi}} > (<) 1 \\ & \Rightarrow \frac{(\overline{Y}_{mt}/\overline{Y}_{mt-1})(\overline{\pi}_m/\overline{\pi}_f)}{\overline{Y}_{ft}/\overline{Y}_{ft-1}} > (<) 1 \quad for \ t \ > 0 \end{aligned} \tag{32}$$

Eq.(32) implies,

$$\frac{\overline{Y}_{mt}\frac{\overline{P}_{mt}}{P_{ft}}}{\overline{Y}_{ft}} \longrightarrow \infty \ (0) \tag{33}$$

From eq.(33), it is obvious that the economy is not on the balanced growth path. We have proved that the contrapositive of the original statement is true. Therefore the original statement is also true because the two statements are logically equivalent. ■

Proposition1 indicates that the inflation gap between the PPI and the CPI is affected by the sectoral technological progress and the input share of the intermediate goods (ϕ).[8] This result is interesting in a sense that real factors such as production structure affect the nominal aspect of the economy. Further, it implies that the central bank faces a trade-off between the CPI and the PPI stabilization unless the productivity gap is zero ($Z_f = Z_m$).

Proposition1 also suggests that the productivity gap cannot pin down the level of inflation by itself. Thus, the perfectly stabilized CPI with the deflationary PPI and the perfectly stabilized PPI with the inflationary CPI are both consistent with proposition 1, though the perfectly stabilized PPI and CPI cannot be achieved at the same time.

3.2 Can we attain the efficient allocation under the inflation gap?

As is evident from eq.(14), the economy can achieve the efficient allocation when both S_{ft} and S_{mt} are one at the same time. However, as is shown in the proposition 2 below, when

[8]In other words, eq.(31) is the necessary condition for the existence of the balanced growth path. In this paper, following conventions in modern macroeconomics, we study the economy on and around the balanced growth path.

the rate of the technological progress in each sector is different, the economy cannot achieve the efficient allocation.

Proposition 2: *On the balanced growth path of the two sector production chain economy described in the section 2 where the production functions have the expression of eq.(5) and eq.(6) and prices are sticky in both of the sectors, the efficient allocation cannot be achieved unless the technological progress in both sectors has the same trend.*

Proof: Let an inflation rate of a sector k on the balanced growth path be $\bar{\pi}_k$. The relative price of firms who have an opportunity to reset their prices can be derived as follows from the aggregated price index,

$$\left[\frac{\bar{P}^*_{kt}}{\bar{P}_{kt}}\right] = \left[\frac{1-\alpha_k}{1-\alpha_k\bar{\pi}_k^{\theta_k-1}}\right]^{\frac{-1}{1-\theta_k}} \tag{34}$$

From eq.(15) and eq.(34), the inefficiency loss of the price dispersion on the balanced growth path is given by,

$$\bar{S}_k = \frac{1-\alpha_k}{1-\alpha_k\bar{\pi}_k^{\theta_k}}\left[\frac{1-\alpha_k}{1-\alpha_k\bar{\pi}_k^{\theta_k-1}}\right]^{\frac{\theta_k}{1-\theta_k}} \geq 1 \tag{35}$$

Taking the first derivative of eq.(35) with respect to $\bar{\pi}_k$ yields,

$$\frac{\partial\bar{S}_k}{\partial\bar{\pi}_k} = \frac{(1-\alpha_k)^{\frac{1}{1-\theta_k}}\alpha_k\theta_k\bar{\pi}_k^{\theta_k-2}}{(1-\alpha_k\bar{\pi}_k^{\theta_k})^2}\left(\frac{1}{1-\alpha_k\bar{\pi}_k^{\theta_k-1}}\right)^{\frac{1}{(1-\theta_k)}}(\bar{\pi}_f-1)$$

Since $0<\alpha_k<1$, $\theta_k>1$, and $\bar{\pi}_k>0$, $\bar{\pi}_k=1$, $\alpha_k^{1/(1-\theta_k)}$, and α_k^{1/θ_k} fulfill the first order condition ($\partial S_k/\partial\pi_k=0$). However, only $\bar{\pi}_k=1$ gives $\bar{S}_k=1$. Hence, $\bar{\pi}_k=1$ is the unique minimizer of $\bar{S}_k$. Proposition 1 implies that $\bar{\pi}_f\neq\bar{\pi}_m$ unless $Z_f=Z_m$. Thus, at least one of the inefficiency loss is greater than one unless $Z_f=Z_m$.■

This result stands in contrast to what is obtained in the standard one-sector models, where the efficient allocation can be attained by stabilizing inflation (Clarida, Galí, and Gertler (1999); Woodford (2003)). This difference comes from the relative price trend between the CPI and the PPI, which creates a trade-off between two inflation measures. If the central bank stabilizes the CPI, the PPI obliges to have a negative trend. Similarly, if the central bank stabilizes the PPI, the CPI must have a positive trend. Proposition 2 suggests that either inflation or deflation can be a source of the inefficiency loss of relative price dispersion. Hence, the efficient allocation cannot be achieved under a relative price trend.

4 What is the optimal inflation rate? -Ramsey problem

Since the most efficient allocation is not attainable in our setting, it is natural to ask the following question: what is the optimal inflation rate that can achieve the second-best allocation? In order to obtain the optimal trend inflation, which maximizes the social welfare, we solve the Ramsey's optimal taxation problem following Lucas and Stokey (1983), Khan, King, and Wolman (2003), Schmitt-Grohé and Uribe (2004), Levin, Onatski, Williams, and Williams (2006).[9] In order to attain the optimal trend inflation, we derive the steady state inflation rate after solving the social planner's problem.

Here, we also postulate the "timeless perspective" such that the central bank honors commitments made in the past (Woodford (2003)). Then, the optimality conditions associated with the Ramsey equilibrium becomes time invariant.

Now, we define a Ramsey equilibrium as a set of stationary processes,$\{C_t, Y_{mt}, N_{ft}, N_{mt}, Q_t, V_{ft}, V_{mt}, W_t/P_{ft}, R_t, \pi_{ft}, \pi_{mt}, S_{ft}, S_{mt}, P_{ft}, P_{mt}, P^*_{ft}/P_{ft}, P^*_{mt}/P_{mt}, \phi_{ft}, \psi_{ft}, \phi_{mt}, \psi_{mt}\}$ for $t > 0$ that maximize the discounted sum of the representative household utility, eq.(1), subject to the equilibrium conditions, eq.(16)-(30), and the resource constraint, eq.(14), and $R_t \geq 1$, for $t > -\infty$, given exogenous stochastic processes, $\hat{Z}_{ft}, \hat{Z}_{mt}$, values of the variables listed above dated $t < 0$, and values of the Lagrange multipliers associated with the constraints listed above dated $t < 0$.

In our case, it is not an easy task to solve the Ramsey problem analytically. Therefore, we compute the exact solution numerically, following Schmitt-Grohé and Uribe (2005). All the variables at the steady state are shown in Appendix 2.

4.1 Parameter calibration

We begin with calibrating the model's parameters. Table 2 shows the set of benchmark case values.

First, we set the share of the intermediate inputs (ϕ) as 0.653977, using the 2000 Input Output Tables. Specifically, we take the ratio of "total intermediate" to "total industry output" from the manufacturing sector. Next, the price elasticities of both of the sectors (θ_f and θ_m) are 10, which implies approximately 11% price markups. This markup ratio is consistent with the existing studies such as Nishimura, Ohkusa, and Ariga (1999) and

[9] To be precise, we follow the two step method as follows. (1) Given the interest rate, derive the conditions of the equilibrium, (2) solve the social planner's problem in order to maximize the social welfare subject to the conditions of the equilibrium and the resource constraint. Lucas and Stokey (1983) and Khan, King, and Wolman (2003) obtain the Ramsey problem under stationary equilibrium. Schmitt-Grohé and Uribe (2005) derive the Ramsey equilibrium under the two kinds of technological progress. Levin, Onatski, Williams, and Williams (2006) also solve the Ramsey problem in the presence of multiple source of non-stationary shocks.

Table 2: Parameters: benchmark case

ϕ	0.653977	the share of the intermediate inputs in production
θ_f	10	price elasticity of demand in the final goods sector
θ_m	10	price elasticity of demand in the intermediate goods sector
α_f	0.5	the probability not to reset a price in the final goods sector
α_m	0.5	the probability not to reset a price in the intermediate goods sector
Z_f	1.016093	the technological growth in the final goods sector
Z_m	1.037489	the technological growth in the intermediate goods sector
β	$0.99^{1/4}$	the discount factor

Miyagawa, Sakuragawa, and Takizawa (2005).[10] Further, we set the probability not to reset a price in both sectors (α_f and α_m) as 0.5. It suggests that firms reset their prices once a half year. Since Bank of Japan (2000), who surveys firm's price setting behavior, reports that the average number of times to reset prices is one or two times in a year, our parameter setting is consistent with this evidence. The technological progress in the final goods sector is 1.016093 and that in the intermediate goods sector is 1.037489. These numbers correspond to the avarage rate of the labor productivity growth in the machinary sector and the wholesale and retailing sector during 1980 to 2005 in Japan.

4.2 Simulation results: the optimal rate of inflation

Table 3: The optimal rate of inflation: benchmark case

	$\bar{\pi}_f$	$\bar{\pi}_m$
benchmark	1.0105%	-1.9072%

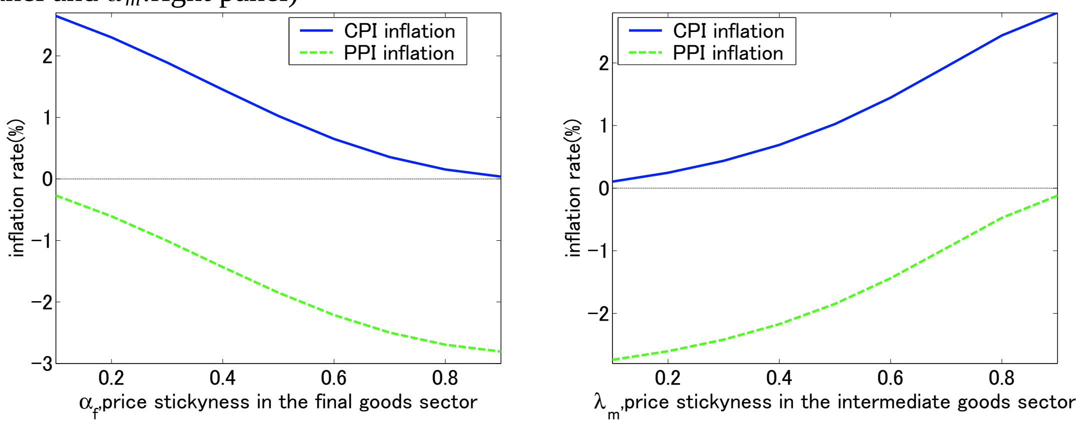
[10] Nishimura, Ohkusa, and Ariga (1999) report 13% as the price markup ratio in Japan and Miyagawa, Sakuragawa, and Takizawa (2005) report 9.1%

Table 3 indicates that the optimal rate of inflation is 1.01% in the CPI and -1.91% in the PPI, respectively. Both inflation rates deviate from zero, suggesting each of the sector should share the inefficiency loss from the existence of the relative price trend. This table also shows that the CPI inflation should be stabilized with much effort since the share of the final goods in overall transactions is greater than that of the intermediate goods.

Yet, our concern is not just to pin down the exact values of optimal inflation. A more interesting question is how the optimal inflation rate behaves with respect to a change in deep parameters. As is remarkably clarified in Aoki (2001) and related studies such as Huang and Liu (2005), Woodford (2003) and Benigno (2004), heterogeneity among sectors plays a crucial role to consider what inflation rate to stabilize. By the sensitivity analysis with respect to the price stickiness and demand elasticity, we can investigate the role of heterogeneity on the optimal rate of inflation while these parameters are symmetric in each sector under the current parameter settings.

In this exercise, our analysis has a clear difference with the existing studies in a sense that we can show the optimal rate of the trend inflation in each sector. Hence, it can present not only what inflation rate to target but also to what level, whereas Aoki (2001) and others focus on the cyclical behavior and just show what inflation rate should be stabilized. It may well to say that we introduce the importance of the sectoral heterogeneity into the analysis of the optimal rate of inflation. Hereinafter, we display the results of sensitivity analysis of the optimal inflation rate in each sector.

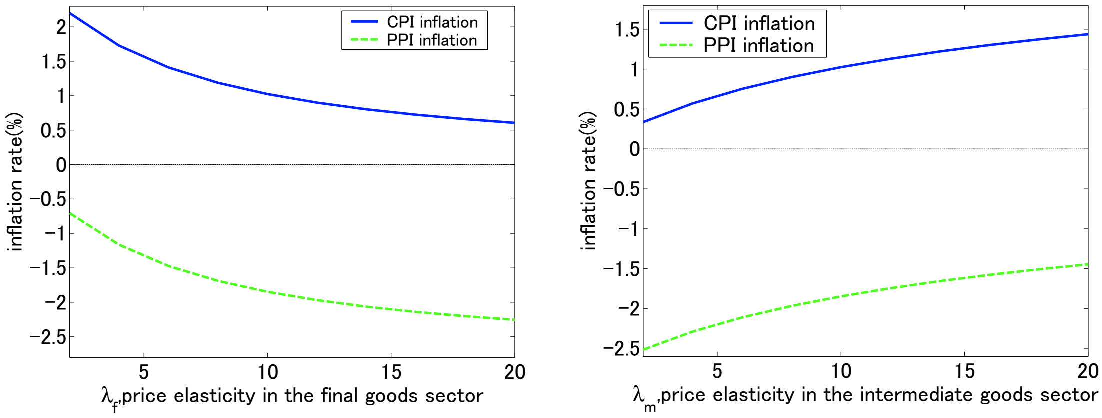
Figure 2: Sensitivity analysis (1): The optimal inflation and the price stickiness (α_f:left panel and α_m:right panel)



The left panel of Figure 2 illustrates the sensitivity of the optimal CPI and PPI inflation to the degree of price stickiness in the final goods sector(α_f). The right panel of Figure 2 shows the sensitivity to the price stickiness in the intermediate goods sector(α_m).

We see from Figure 2 that it is preferable to put more emphasis on stabilizing the stickier price index because inefficiency loss of relative price dispersion is greater in the stickier price sector. These results are consistent with Aoki (2001), Benigno (2004) and Woodford (2003) who study the role of heterogeneity in the price stickiness under the horizontally integrated multisector models. We can say that their suggestion that the stickier price should be stabilized with much effort also holds in a vertically integrated multisector model that provides a natural distinction between the CPI and the PPI.

Figure 3: Sensitivity analysis (2): The optimal inflation and the price elasticity (θ_f:left panel,θ_m:right panel)



Next, Figure 3 demonstrates the sensitivity of the optimal inflation rates to the demand elasticity in each sector (θ_f and θ_m). This figure suggests that it is preferable to put more emphasis on stabilizing the price index with more elastic demand. Intuitively, the higher the price elasticity is, the more sensitive the demand responds to the variations in relative prices and hence the larger the distortion from the relative price dispersion is.[11] When we regard the inefficiency loss of the relative price dispersion as a sort of tax, the result here corresponds to the famous Ramsey rule in the public economics such that the higher tax should be charged on less demand elastic goods.

4.3 Optimal inflation index

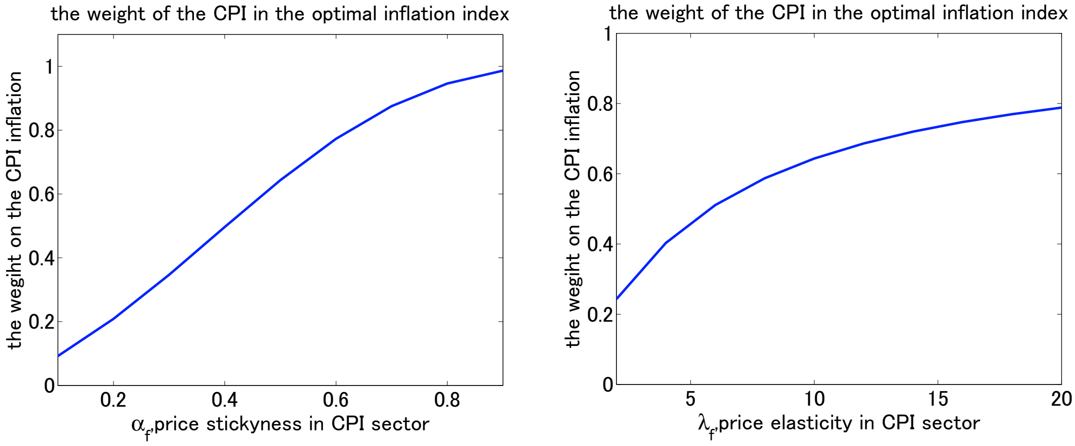
It has been shown that the inflation rate that should be stabilized includes not only the CPI inflation but also the PPI inflation. Since both of the CPI inflation and the PPI inflation are readily available in the data, a simple policy implication is that the central bank should be able to construct an ”optimal inflation index” as following.

$$\pi_t^{opt} = \Upsilon\pi_{ft} + (1 - \Upsilon)\pi_{mt}$$

[11] In other words, ceteris paribus, the Harberger triangle of the higher demand elasticity sector is greater than that of the lower demand elasticity sector.

Utilizing the optimal trend inflation in each price measure, we can derive the optimal weight of the CPI and the PPI, Υ, that approximates the optimal inflation index to zero inflation.

Figure 4: The weight on the CPI in the optimal inflation index



Note1: The left and right panel represents the sensitivity of the CPI weight in the optimal inflation index to the price stickiness and to the price elasticity of the final goods sector, respectively.
Note2: The weight in the figure is $\Upsilon = (\bar{\pi}_m - 1)/(\bar{\pi}_m - \bar{\pi}_f)$.

Figure 4 demonstrates that the weight on the CPI inflation increases with the stickiness of the CPI but it decreases with the price elasticity of the final goods sector. These features reflects the results of the sensitivity analysis in the section 4.2. Under the benchmark parameters, the weight on the CPI inflation is about 0.6.

Thus far, we have obtained the optimal rate of inflation and its properties. The above analysis has an immediate policy implication such that the ”inflation targeting” policy should target not only the CPI, which is conventionally viewed as a reference, but also the PPI. In addition, the analysis provides a justification for the non-zero CPI inflation target without presuming the zero boundary on nominal interest rates or the monetary friction such as the cash-in-advance constraint.

It is worth mentioning that the inflation rate in Table 3 is just a tentative set of the optimal inflation rate in each sector. As is demonstrated in the sensitivity analysis, heterogeneity in the the price stickiness and cyclical sensitivity play crucial roles to pin down the optimal inflation rate. In order to seek for the optimal rates of the CPI inflation and the PPI inflation in practice, it is of importance to acquire the precise estimate of the deep parameters such as the degree of price stickiness, the price elasticity of demand in each sector and the detailed structure of the production chain. Although empirical studies about these parameters have been accumulated, further research effort should be devoted to these subjects.

5 Optimal monetary policy under the trend inflation

In the previous section, we examine how to pin down the optimal inflation rate on the balanced growth path. A next natural question is what is the optimal monetary policy in the presence of the sectoral productivity gap and hence the inflation gap among the price indices.

If trend inflation does not affect the equilibrium dynamics around the balanced growth path, optimal monetary policy would be the same regardless of the optimal inflation rate. However, as is pointed out by Ascari (2004), Kiley (2004) and Cogley and Sbordone (2006), the dynamics of the economy changes depending on the rate of the trend inflation. Therefore, trend inflation is expected to alter the optimal monetary policy. In the following, we study the optimal monetary policy responses under the non-zero trend inflation by log-linearizing the optimality conditions of the Ramsey problem and performing shock simulations around the balanced growth path.[12]

5.1 The level of the trend inflation and the dynamics of inflation

Before carrying out shock simulations, we would like to make clear the reason why the equilibrium dynamics varies depending on the level of the trend inflation. Among others, we focus on the NKPC since the other equilibrium conditions do not alter irrespective of the level of the inflation rate.

5.1.1 The NKPC under the trend inflation

If we allow for the non-zero trend inflation, the log-linearized optimal price function becomes much complicated one compared to the familiar NKPC. In general, as is shown in Ascari (2004) and Cogley and Sbordone (2006), the non-zero trend NKPC is expressed as in eqs.(36)-(37).[13] This generalized NKPC is basically identical to the NKPC of our final goods sector and intermediate goods sector.

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda(\bar{\pi})\hat{v}_t + \lambda(\bar{\pi})\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\hat{\psi}_t \tag{36}$$

$$\hat{\psi}_t = (1-\alpha\beta\bar{\pi}^{\theta})\hat{v}_t + \alpha\beta\bar{\pi}^{\theta}(\theta E_t\hat{\pi}_{t+1} + E_t\hat{\psi}_{t+1}) \tag{37}$$

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[12] Schmitt-Grohé and Uribe (2004) suggest that a first-order approximation to the Ramsey equilibrium conditions provides the almost exact solution of the impulse response.

[13] The effect of the trend inflation becomes weak when we assume the price indexation. However, it still remains except for the case of the perfect indexation where all of firms adjust their prices by indexing to the past or long-run inflation when they cannot obtain the opportunity to reset the prices. In the case of partial indexation, our results do not change qualitatively.

where $\lambda(\bar{\pi}) \equiv (1-\alpha\bar{\pi}^{\theta-1})(1-\alpha\beta\bar{\pi}^{\theta})/\alpha\bar{\pi}^{\theta}$ is the slope of the unit cost. α, β and θ are the probability not to reset the price, discount factor and the price elasticity of demand, respectively. $\hat{x}$ denotes the log-deviation from the steady state ($\bar{x}$). π and v represent the inflation rate and the real unit cost of the final goods sector.

We can confirm that eq.(36) is equivalent to the standard NKPC if the steady state inflation is zero ($\bar{\pi} = 1$).[14] Comparing our NKPC with the standard one, we can point out the following two features: (i)the additional term shows up in the RHS ($\hat{\psi}_t$), and (ii)the coefficients vary depending on the trend inflation rate.

5.1.2 Interpretation of the NKPC (1): the role of the additional term

Eq.(37) is equivalent to the log-linearized version of the numerator of the optimal price function. Rewriting eq.(37) yields;

$$\hat{\psi}_t = \hat{v}_t + E_t \sum_{i=1}^{\infty} (\alpha\beta\bar{\pi}^{\theta})^i (\Delta\hat{v}_{t+i} + \theta\hat{\pi}_{t+i}) \tag{38}$$

Eq.(38) illustrates that $\hat{\psi}_t$ represents the additional loss of the profit induced by the non-zero trend inflation. Specifically, $\hat{\psi}_t$ is the discounted present value of marginal changes in the future unit cost and the variations of demand, where the weights used for discounting depend on the expected future inflation, the discount factor, and the price reset probability. In a nutshell, once we allow for a positive trend inflation, the developments of inflation become forward-looking because firms put their reset prices higher, expecting for the relative prices to be cheaper along with the general-price inflation.

5.1.3 Interpretation of the NKPC (2): the slope of the NKPC

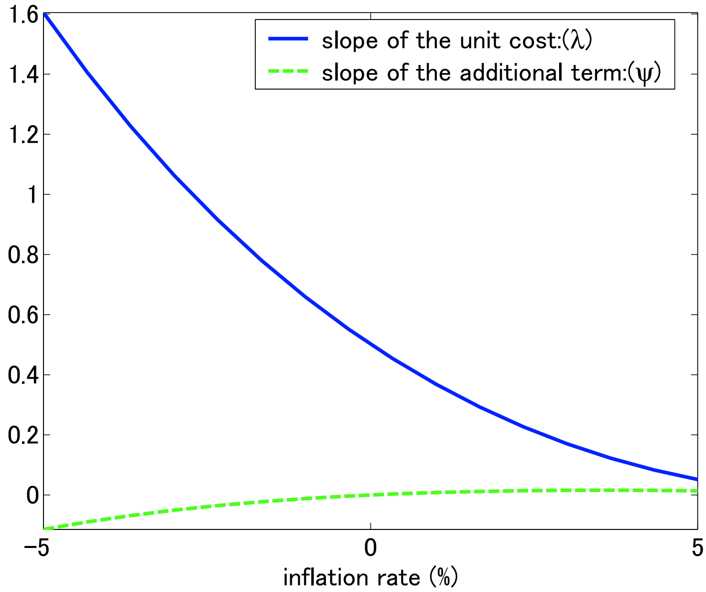
The trend inflation also affects the coefficients of the NKPC. The relationship between the trend inflation and the coefficients of NKPC is illustrated in Figure 5.

First, the slope of the unit cost ($\lambda(\bar{\pi})$) is a decreasing function of the trend inflation.[15] It implies that under the high inflation environment, the response of inflation is less sensitive to the cost variations but under the low inflation environment, the response of inflation is more sensitive. Second, we can see from Figure 5 that the impact of the additional term on the inflation rate is weak with near-zero coefficient.

[14] The standard NKPC is $\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \lambda(1)\hat{v}_t$. See Galí and Gertler (1999) for detail.

[15] In the seminal paper, Ball, Mankiw, and Romer (1988) conjecture that the slope of the Phillips curve is "increasing" function of the inflation. Further, a recent study(Benati (forthcoming)) supports their view by the statistical method. This gap between the theoretical implication and the empirical evidence is a promising future topic.

Figure 5: The coefficient of the NKPC and the trend inflation



As we have seen above, non-zero trend inflation alters the dynamics of inflation. To be specific, positive trend inflation results in the flatter slope of the ”Phillips curve” and negative trend inflation suggests the steeper slope of it. Therefore, in our model, the CPI inflation responds less to the changes of the unit cost while the PPI inflation responds more to the changes of the unit cost since the trend inflation of the CPI is positive and that of the PPI is negative as in the Table 3. Further, the non-zero trend inflation makes the process of inflation more dynamic through the additional term although its impact on inflation is not so strong.

5.2 What is the optimal monetary policy under the trend inflation?

As we have shown, the process of inflation varies depending on the level of the trend inflation. To be specific, in our production chain model with a sectoral productivity gap and hence a relative price trend, the dynamic process of inflation is different in each sector. Then, the optimal monetary policy is also expected to change depending on the sectoral productivity gap. We will show it below by shock simulation under the Ramsey equilibrium.

5.2.1 Parameter settings

In order to examine the effect of the productivity gap, we set the several cases. Table 4 summarizes the case settings.

Fist of all, we consider the standard one sector model with zero trend inflation as a

Table 4: Parameter settings

	trend inflation	ϕ	$\bar{\pi}_f$	$\bar{\pi}_m$	Z_f	Z_m
One sector model	zero	0	0%	0%	0%	0%
Production Chain model	zero	0.654	0%	0%	0%	0%
Production Chain model	non-zero	0.654	0.51%	-0.86%	0%	1%

benchmark case where the share of intermediate input (ϕ) is zero and the technological growth rate in each sector is also zero (the first row of Table 4).

The second case is the production chain model with zero trend inflation where the share of the intermediate input is 0.654 as is presumed in the previous section (the second row of Table 4). By examining this case, we demonstrate the feature of the production chain model under the zero trend inflation.

Finally, we study the production chain model with the non-zero trend inflation (the last row of Table 4). In order to induce the relative price trend between the CPI and the PPI, the rate of the technological progress in the intermediate goods sector is 1% though the rate of the technological progress in the final goods sector is zero.[16] Hence, the technology gap is 1% and the trend inflation of the CPI is 0.51% and that of the PPI is -0.86%. We calculate the optimal inflation rate following the method developed in the previous section.

5.2.2 Simulation results:a technology shock in the final goods sector

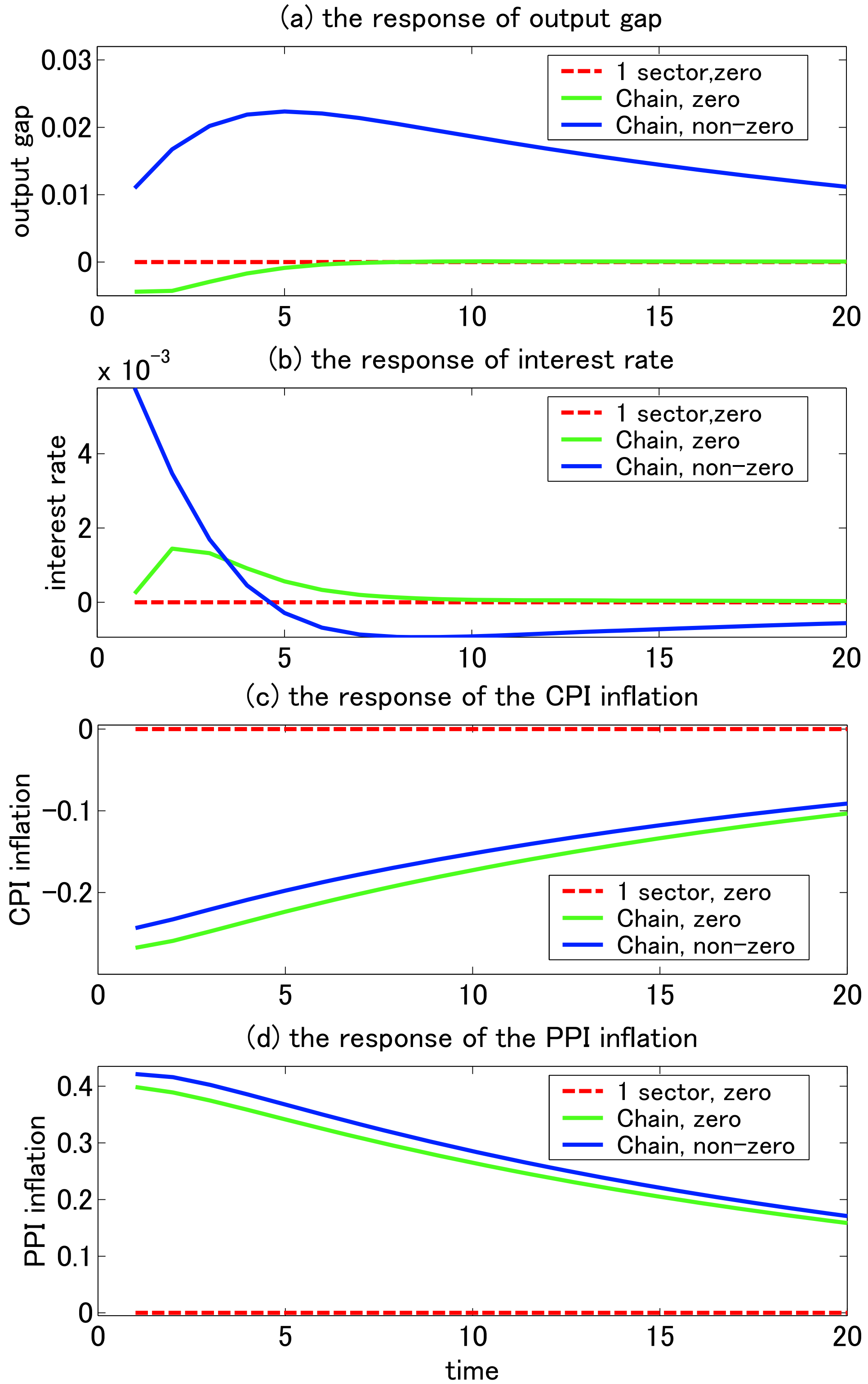
Figure 6 display impulse responses of output gap (panel (a), deviation from the natural level of output), interest rate gap (panel (b), deviation from the natural rate), the CPI inflation and the PPI inflation (panel (c) and (d)) to a one-percentage increase in the technological growth rate of the final goods sector.[17] These figures demonstrate that the responses of output gap and interest rate gap are remarkably different in each case.

In order to understand the effect of the production chain and the trend inflation, we interpret the one sector model with zero inflation first, then move to the production chain model with zero inflation, and finally examine the production chain model with non-zero trend inflation.

[16]The choice of the rate of the technological progress does not alter our basic results below. In our case, the gap of the technological progress is important.

[17]The natural output($\hat{c}_t^*$) and the natural rate($\hat{rr}_t^*$) is derived as follows, letting the lower case letters represent logged variables. First, log-linearizing the consumption Euler equation under the flexible price equilibrium and rewriting it as $\hat{rr}_t^*$, we obtain $\hat{rr}^* = (E_t\hat{c}_{t+1}^* - \hat{c}_t^*)$. Using eq.(3),eq.(7), and eq.(10), the natural output($\hat{c}_t^*$) is written as $\hat{c}_t^* = \phi\hat{a}_{mt} + (1-\phi)\hat{a}_{ft}$. Plugging this into the above consumption Euler equation yields the natural rate($\hat{rr}_t^*$) as $\hat{rr}_t^* = \phi\rho_m\Delta\hat{a}_{mt} + (1-\phi)\rho_f\Delta\hat{a}_{ft}$.

Figure 6: The optimal monetary policy: a shock in the final goods sector



Interpretation (1): one sector model with zero trend inflation

Our production chain model reduces to the standard one sector model (Clarida, Galí, and Gertler (1999); Woodford (2003)) when the share of the intermediate input is zero. The dotted lines in Figure 6 correspond to this case. In the one sector model, it is preferable for the Ramsey planner to raise the nominal interest rate associated with a positive technology shock to cancel out the rise of the natural rate. As a result, the interest rate gap is zero. Since the effect of a technology shock is perfectly cancelled out by raising the interest rate associated with the rise of the natural rate, output gap and both inflation rate are all zero.

Interpretation (2): production chain model with zero trend inflation

In the production chain economy, a technology shock creates an endogenous ”cost push” term, which is exogenously presumed in the standard one sector models (Clarida, Galí, and Gertler (1999)). To see this, we present the log-linearized version of the real unit cost in each sector(eqs.(18)-(19)).

$$\hat{v}_{ft} = (1-\phi)\hat{c}_t + \phi\hat{q}_t \qquad \text{(a unit cost of the final goods sector)} \tag{39}$$

$$\hat{v}_{mt} = \hat{c}_t - \hat{q}_t \qquad \text{(a unit cost of the intermediate goods sector)} \tag{40}$$

where $\hat{c}_t$ and $\hat{q}_t$ denote the output gap and the relative price gap, the log deviation of the relative price (PPI/CPI) from its trend.[18]

In the face of a technology shock in the final goods sector, the relative price gap, $\hat{q}_t$ decreases. Then, ceteris paribus, the marginal cost gap of the final goods sector in eq.(39) falls while the marginal cost gap of the intermediate goods sector in eq.(40) rises. Finally, the CPI inflation falls and the PPI inflation rises in response to the unit cost changes.

To stabilize inflation, it is required for the central bank to attenuate the variations in $\hat{v}_{ft}$ and $\hat{v}_{mt}$ by moving the $\hat{c}_t$. However, the sign of $\hat{q}_t$ is opposite in $\hat{v}_{ft}$ and in $\hat{v}_{mt}$. Hence, a trade-off between the CPI stabilization and the PPI stabilization exists. In addition, to stabilize either the CPI inflation or the PPI inflation, we must give up the stabilization of output, $\hat{c}_t$.

The light green line in Figure 6 represents this case. Under the production chain model with zero trend inflation, it is optimal for the central bank to raise the interest rate gap in response to a shock in the final goods sector. In consequence of the tighter monetary operation relative to the one sector model case, output gap declines. In addition, the CPI inflation falls and the PPI inflation rises.

The optimal monetary policy calls for much tighter money market operation in the production chain economy because a trade-off between the stability of the CPI and the PPI

[18] $\hat{q}_t = \hat{q}_{t-1} + \hat{\pi}_{mt} - \hat{\pi}_{ft} - (1-\phi)(\hat{Z}_{ft} - \hat{Z}_{mt})$

exists. If the central bank raises the interest rate just to offset the rise of the natural rate, it is not enough to prevent the PPI inflation from rising too much though the bank could alleviate the fall of the CPI inflation. As a result, in order to balance the CPI inflation and the PPI inflation, the bank raises the interest rate more than the increase in the natural rate. Finally, interest rate gap rises above zero and output gap declines because of the monetary squeeze.

Interpretation (3): the production chain model with non-zero trend inflation
In the presence of the trend inflation induced by the productivity gap between the intermediate goods sector and the final goods sector, impulse responses (the dark blue line) show a stark contrast with those of the zero trend inflation economy. The interest rate gap rises sharply but becomes negative for a longer period after the temporal hikes. Furthermore, the impulse response of output gap becomes positive and more persistent.

This difference comes from the changes in the process of inflation. As we have seen in the previous subsection, the Phillips curve becomes flatter in the CPI and steeper in the PPI. This difference implies that the nominal rigidity of the CPI increases while that of the PPI decreases. Since the welfare loss is greater in a sector with higher nominal rigidities, it is optimal to pay much effort to stabilize the CPI inflation and to accept the additional deviation of the PPI inflation from its steady state. The mechanism behind the simulation is as following;

- By committing to the long-run monetary easing, the central bank can alleviate the fall of the CPI inflation while she must accept the additional rise of the PPI inflation.
- However, the commitment to the long-run monetary easing results in the current output jump.
- By committing to the short-run monetary tightening, the monetary authority can alleviate the initial jump of current output.
- Since the central bank commits to a long-run monetary easing, the response of output becomes persistent.

In the classic theory of policy, Jan Tinbergen points out that the number of policy instruments should be at least as many as the number of policy targets when a trade-off exists among the targets. In our production chain economy, a technology shock in one sector creates the trade-off between inflation and output because the relative price of the CPI and the PPI works as an endogenous ”cost-push” term. With this situation, Tinbergen’s theorem suggests that the policy planner cannot pursue both of the objectives, stability of inflation and output, at the same time.

However, our simulation results demonstrate that by utilizing the time difference between the current and future interest rate, the monetary authority can relax this trade-off partially. To be precise, by assigning disparate roles to the current interest rate and the future interest rate, the central bank can turn down output gap and the deviations of the inflation rate from the optimum level at the same time. In this sense, the short-run response of the interest rate and the long-run commitment to the future path of the interest rate work as two different policy instruments.

5.2.3 Simulation results:a technology shock in the intermediate goods sector

With respect to a technology shock in the intermediate goods sector, this mechanism works out totally oppositely. In Figure 7, we show impulse responses to a technology shock in the intermediate goods sector.

In the one sector model with zero trend inflation, all impulse responses are zero. In the production chain economy with zero trend inflation, interest rate gap decreases and output gap increases. However, in the presence of the non-zero trend inflation, interest rate gap hikes after tentative monetary easing and hence output gap decreases and becomes persistent.

As we have shown thus far, the optimal monetary policy is subject to the level of the inflation rate. The reason for it comes from the difference in the process of inflation. We can conclude from our results that taking the trend inflation into consideration is important to study the optimal monetary policy analysis.

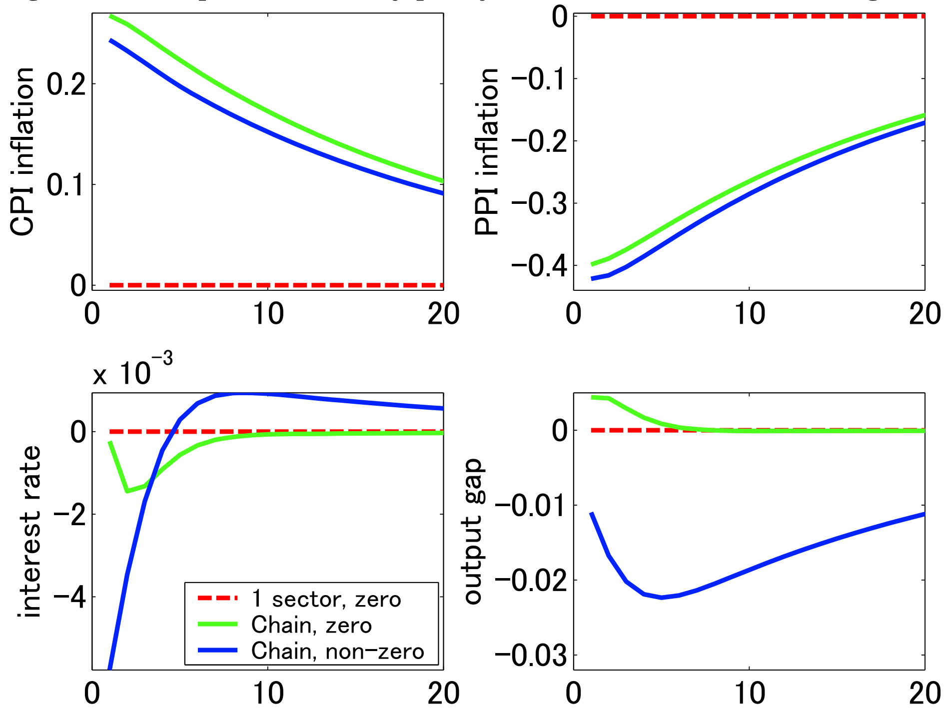
6 Conclusion

In this paper, we study the optimal rate of inflation and the optimal monetary policy simultaneously under the production chain economy where the CPI and the PPI are explicitly modeled. The key assumption here is that there exists a relative price trend between the CPI and the PPI.

In the presence of the relative price trend and the price stickiness in each sector, it is necessary to stabilize not only the CPI, which is conventionally regarded as a price measure to stabilize, but also the PPI. Hence, the optimal inflation is not zero in both of the CPI and the PPI. It may well to say that the relative price trend can be a promising candidate to justify the positive long-run CPI inflation rate in the post-war major countries. Further, the optimal monetary policy is subject to the rate of the trend inflation in each sector.

To be precise, our analytical results reveal that (1) the efficient allocation cannot be achieved in the presence of the relative price trend, (2) under the benchmark set of parame-

Figure 7: The optimal monetary policy: a shock in the intermediate goods sector



ters, the optimal rate of the CPI inflation is 1.01% and that of the PPI inflation is -1.91%, (3) the optimal monetary policy alters depending on the trend inflation. The central bank can attain the better outcome by assignning the disparate roles to the short-run reaction and the long-run commitment under the non-zero trend inflation.

In order to keep the analytical tractability and shed light on the role of the relative price trend, we have focused on nominal rigidities in the CPI and the PPI. We do not mean to claim that the price stickiness in each price measure is the only source of the frictions to be considered. It might be promising to take into account, in addition to these price stickiness, the other frictions such as the opportunity cost of the money holding or the zero boundary on nominal interest rates. Yet, our analytical results suggest that these two would not change our main conclusion: in the presence of the price stickiness in both of the CPI and the PPI, the inflation gap induced by the relative price trend should be shared with both sectors, and hence the relative price trend provides an appropriate rationale for setting positive inflation targets.

Appendix 1:The existence of the balanced growth path

The unique balanced growth path (BGP) exists in the sticky-price, production-chain economy where the rate of technological progress is different among the sectors as long as the

BGP inflation rate in k sector suffices $0 < \bar{\pi}_k < \alpha_k^{-1/\theta_k}$.

To begin with, we seek for the balanced growth rate under the BGP. Then, we show the existence of the BGP, which is consistent with the balanced growth rate.

The balanced growth rate

First of all, we assume that the nominal interest rate is $\bar{R}$, which is time invariant on the BGP. In addition, the growth rate of $\bar{C}_t$ is arbitrary constant(γ_c), by the definition of BGP.

From the consumption Euler equation, the following holds on the BGP,

$$\beta\frac{\bar{R}}{\bar{\pi}_f}\gamma_c = 1 \Rightarrow \bar{\pi}_f = \beta\bar{R}\gamma_c^{-1}$$

Then, the optimal relative price for the final goods sector is expressed as follows.

$$\frac{\bar{P}^*_{ft}}{\bar{P}_{ft}} = \left(\frac{1-\alpha_f\bar{\pi}_f^{\theta_f-1}}{1-\alpha_f}\right)^{\frac{1}{1-\theta_f}} \tag{41}$$

Eq.(41) means the optimal relative price is constant. Then, $\bar{S}_f$ can be expressed as follows, implying $\bar{S}_f$ is time invariant.

$$\begin{aligned}
\bar{S}_f &= (1-\alpha_f)\sum_{t=0}^{\infty}(\alpha_f\bar{\pi}_f^{\theta_f})^t\left(\frac{\bar{P}^*_{ft}}{\bar{P}_{ft}}\right)^{-\theta_f} \\
&= (1-\alpha_f)\sum_{t=0}^{\infty}(\alpha_f\bar{\pi}_f^{\theta_f})^t\left(\frac{1-\alpha_f\bar{\pi}_f^{\theta_f-1}}{1-\alpha_f}\right)^{\frac{-\theta_f}{1-\theta_f}} \\
&= \frac{(1-\alpha_f)}{(1-\alpha_f\bar{\pi}_f^{\theta_f})}\left(\frac{1-\alpha_f\bar{\pi}_f^{\theta_f-1}}{1-\alpha_f}\right)^{\frac{-\theta_f}{1-\theta_f}}
\end{aligned}$$

The definition of Q_t is $Q_t \equiv P_{mt}/P_{ft}$. On the BGP, the growth rate of $\bar{Q}_t$ must be constant. Since $\bar{Q}_t/\bar{Q}_{t-1} = \bar{\pi}_m/\bar{\pi}_f$ and $\bar{\pi}_f$ is time invariant, thus, $\bar{\pi}_{mt}$ must be time invariant. Given the constant $\bar{\pi}_m$, the same operation will provide the time invariant optimal relative price and inefficiency loss for the intermediate goods sector.

$$\frac{\bar{P}^*_{mt}}{\bar{P}_{mt}} = \left(\frac{1-\alpha_m\bar{\pi}_m^{\theta_m-1}}{1-\alpha_m}\right)^{\frac{1}{1-\theta_m}}$$

$$\bar{S}_m = \frac{(1-\alpha_m)}{(1-\alpha_m\bar{\pi}_m^{\theta_m})}\left(\frac{1-\alpha_m\bar{\pi}_m^{\theta_m-1}}{1-\alpha_m}\right)^{\frac{-\theta_m}{1-\theta_m}}$$

Now, using the aggregated production functions, we get

$$\bar{C}_t/\bar{C}_{t-1} = Z_m^{\phi} Z_f^{1-\phi} \equiv Z_1 \tag{42}$$

$$\bar{Y}_{mt}/\bar{Y}_{mt-1} = Z_m \tag{43}$$

Eq.(42) is the γ_c. Further, using the household FOC for the labor supply, we can say $(W_t/P_{ft})/(W_{t-1}/P_{ft-1}) = Z_1$.

On the BGP, $\bar{Y}_{mt}\bar{Q}_t/\bar{C}_t$ is also arbitrary constant, implying

$$\left(\frac{\bar{Y}_{mt}\bar{Q}_t/\bar{C}_t}{\bar{Y}_{mt-1}\bar{Q}_{t-1}/\bar{C}_{t-1}}\right) = 1 \Rightarrow \frac{\bar{\pi}_m}{\bar{\pi}_f} = \frac{Z_1}{Z_m} \equiv Z_2$$

we can also show that the relative share of labor in each sector is constant on the BGP.

$$\frac{\bar{N}_f}{\bar{N}_m} = \frac{1-\phi}{\phi}\frac{\bar{Q}_t A_{mt}}{(\bar{W}_t/\bar{P}_{ft})\bar{S}_m}$$

$$\Rightarrow \frac{\bar{N}_{ft}/\bar{N}_{mt}}{\bar{N}_{ft-1}/\bar{N}_{mt-1}} = \frac{Z_2 Z_m}{Z_1} = 1$$

Further, $\bar{v}_{ft}$ and $\bar{v}_{mt}$ are also time invariant.

$$\frac{\bar{v}_{ft}}{\bar{v}_{ft-1}} = \left(\frac{\bar{Q}_t}{\bar{Q}_{t-1}}\right)^{\phi}\left(\frac{\bar{W}_t/\bar{P}_{ft}}{\bar{W}_{t-1}/\bar{P}_{ft-1}}\right)^{1-\phi}\left(\frac{A_{ft-1}}{A_{ft}}\right)^{1-\phi} = 1$$

$$\frac{\bar{v}_{mt}}{\bar{v}_{mt-1}} = \frac{\bar{W}_t/\bar{P}_{ft}}{\bar{W}_{t-1}/\bar{P}_{ft-1}}\frac{\bar{Q}_{t-1}}{\bar{Q}_t}\frac{A_{mt-1}}{A_{mt}} = 1$$

The existence of the BGP

In order to prove the existence of the BGP, we need to show the existence of unique solution or unique solution path for each variables, which is consistent with the balanced growth rate in the previous subsection.

First, we assume that $\bar{R}$ is the positive constant on the BGP. In addition, the initial relative price Q_0 and the initial level of technological progress $A_{m0} = 1$ are given exogenously. Then, from the consumption Euler equation, we can determine $\bar{\pi}_f$ uniquely as follows.

$$\bar{\pi}_f = \beta\bar{R}Z_1^{-1}$$

Since $\bar{\pi}_f$ is unique and $\bar{v}_{ft}$ is time invariant on the BGP, then, from the optimal price index,

The optimal price index implies the following.

$$\frac{\bar{P}^*_{ft}}{\bar{P}_{ft}} = \frac{\mu_f}{1+\tau_f}\bar{v}_f\frac{1-\alpha_f\beta\bar{\pi}_f^{\theta_f}}{1-\alpha_f\beta\bar{\pi}_f^{\theta_f-1}} \tag{44}$$

Dividing the both side of the price index ($\bar{P}_{ft} = [\int_0^1 \bar{P}_{ft}(j)^{1-\theta_f} dj]^{1/(1-\theta_f)} = [\alpha_f \bar{P}_{ft-1}^{1-\theta_f} + (1 - \alpha_f)\bar{P}_{ft}^{*}]^{1-\theta_f}]^{1/(1-\theta_f)}$) by $\bar{P}_{ft}$ and solving for $\bar{P}_{ft}^{*}/\bar{P}_{ft}$ yields:

$$\frac{\bar{P}_{ft}^{*}}{\bar{P}_{ft}} = \left(\frac{1 - \alpha_f \bar{\pi}_f^{\theta_f - 1}}{1 - \alpha_f}\right)^{1/(1-\theta_f)} \tag{45}$$

From eq.(44) and eq.(45), we obtain:

$$\bar{v}_f = (1 - \Phi_f) \tag{46}$$

where $\Phi_f \equiv \{[\mu_f/(1+\tau_f)][(1-\alpha_f)/(1-\alpha_f\beta\bar{\pi}_f^{\theta_f-1})]^{1/(1-\theta_f)}[(1-\alpha_f\beta\bar{\pi}_f^{\theta_f})/(1-\alpha_f\beta\bar{\pi}_f^{\theta_f-1})]\}^{-1}$.

Now, $\bar{\pi}_m/\bar{\pi}_f = Z_2$ suggests $\bar{\pi}_m = Z_2\bar{\pi}_f$, implying the unique intermediate goods inflation rate. Then, applying the similar operation to the intermediate goods cases, we can attain the following.

$$\bar{v}_m = (1 - \Phi_m) \tag{47}$$

where $\Phi_m \equiv \{[\mu_m/(1+\tau_m)][(1-\alpha_m)/(1-\alpha_m\beta\bar{\pi}_m^{\theta_m-1})]^{1/(1-\theta_m)}[(1-\alpha_m\beta\bar{\pi}_m^{\theta_m})/(1-\alpha_m\beta\bar{\pi}_m^{\theta_m-1})]\}^{-1}$.

Next, the unit cost function of the each sector is expressed as follows.

$$\bar{v}_f = \bar{\phi}\bar{Q}_t^{\phi}\left(\frac{\bar{w}_t}{A_{ft}}\right)^{1-\phi}$$

$$\bar{v}_m = \frac{\bar{w}_t}{\bar{Q}_t A_{mt}}$$

where $w_t \equiv W_t/P_{ft}$,$\bar{\phi} = \phi^{-\phi}(1-\phi)^{-(1-\phi)}$. Then, combining with eq.(46) and eq.(47), and eliminating $\bar{Q}_t$ yields:

$$\bar{w}_t = \frac{(1-\Phi_f)(1-\Phi_m)^{\phi}}{\bar{\phi}} A_{mt}^{\phi} A_{ft}^{1-\phi}$$

Given the initial A_{f0} and A_{m0}, the unique BGP of $\bar{w}_t$ is determined. The household first order condition for labor supply suggests $\bar{C}_t = \bar{w}$ and hence the BGP of the consumption is also determined uniquely.

Once Q_0 is given, the unique path of $\bar{Q}_t$ is also determined uniquely.

$$Q_t = \left(\frac{\bar{\pi}_m}{\bar{\pi}_f}\right)^t \bar{Q}_0$$

Finally, letting $A_{f0} = A_{m0} = Q_0 = 1$ without lack of generality, the aggregated labor demand on the BGP is unique,

$$\bar{N} = \bar{N}_f + \bar{N}_m = \bar{S}_f \bar{v}_f \left[(1-\phi) + \frac{\phi(1-\Phi_f)(1-\Phi_m)^{\phi}\bar{S}_m}{\bar{\phi}}\right]$$

In case of $\bar{\pi} > \alpha_f^{-1/\theta_f}$ or equivalently $\bar{R} > Z_1\alpha_f^{-1/\theta_f}/\beta$, then, $\bar{S}_f < 1$, which implies that output is outside the production possibility frontier and it is not feasible.

Appendix 2:Steady state equilibrium

From eq.(27) and eq.(28) of the final goods sector,

$$\bar{\psi}_f = \frac{\bar{v}_f}{1-\alpha_f\beta\bar{\pi}_f^{\theta_f}}$$
$$\bar{\phi}_f = \frac{1}{1-\alpha_f\beta\bar{\pi}_f^{\theta_f-1}}$$

The optimal price index implies the following.

$$\frac{\bar{P}^*_{ft}}{\bar{P}_{ft}} = \frac{\mu_f}{1+\tau_f}\bar{v}_f\frac{1-\alpha_f\beta\bar{\pi}_f^{\theta_f}}{1-\alpha_f\beta\bar{\pi}_f^{\theta_f-1}} \tag{48}$$

Dividing both sides of the price index ($\bar{P}_{ft} = [\int_0^1 \bar{P}_{ft}(j)^{1-\theta_f}dj]^{1/(1-\theta_f)} = [\alpha_f\bar{P}_{ft-1}^{1-\theta_f} + (1-\alpha_f)\bar{P}^*_{ft}]^{1-\theta_f}]^{1/(1-\theta_f)}$) by $\bar{P}_{ft}$ and solving for $\bar{P}^*_{ft}/\bar{P}_{ft}$ yields,

$$\frac{\bar{P}^*_{ft}}{\bar{P}_{ft}} = \left(\frac{1-\alpha_f\bar{\pi}_f^{\theta_f-1}}{1-\alpha_f}\right)^{1/(1-\theta_f)} \tag{49}$$

From eq.(48) and eq.(49), we obtain,

$$\bar{v}_f = (1-\Phi_f) \tag{50}$$

where $\Phi_f \equiv \{[\mu_f/(1+\tau_f)][(1-\alpha_f)/(1-\alpha_f\beta\bar{\pi}_f^{\theta_f-1})]^{1/(1-\theta_f)}[(1-\alpha_f\beta\bar{\pi}_f^{\theta_f})/(1-\alpha_f\beta\bar{\pi}_f^{\theta_f-1})]\}^{-1}$.

Applying the similar operation with eq.(29) and eq.(30), the optimal price function, and the price index of the intermediate goods sector, we can attain the following.

$$\bar{v}_m = (1-\Phi_m) \tag{51}$$

where $\Phi_m \equiv \{[\mu_m/(1+\tau_m)][(1-\alpha_m)/(1-\alpha_m\beta\bar{\pi}_m^{\theta_m-1})]^{1/(1-\theta_m)}[(1-\alpha_m\beta\bar{\pi}_m^{\theta_m})/(1-\alpha_m\beta\bar{\pi}_m^{\theta_m-1})]\}^{-1}$.

Next, the unit cost function of the each sector is expressed as follows.

$$\bar{v}_f = \bar{\phi}\bar{Q}^{\phi}\bar{w}^{1-\phi}$$
$$\bar{v}_m = \frac{\bar{w}}{\bar{Q}}$$

where $w \equiv W/P_f$, $\bar{\phi} = \phi^{-\phi}(1-\phi)^{-(1-\phi)}$. Then, combining with eq.(50) and eq.(51), and eliminating $\bar{Q}$ yields,

$$\bar{w} = \frac{(1-\Phi_f)(1-\Phi_m)^{\phi}}{\bar{\phi}}$$

Now, the household first order condition for labor supply suggests $\bar{C} = \bar{w}$.

Eliminating $\bar{v}_f$ and $\bar{Q}$ from the labor input in each sector, eq.(20) and eq.(21), we get,

$$\bar{N}_f = \frac{(1-\phi)\bar{\phi}}{(1-\Phi_m)^{\phi}}\bar{S}_f\bar{C}$$
$$\bar{N}_m = \phi\bar{\phi}(1-\Phi_m)^{1-\phi}\bar{S}_f\bar{S}_m\bar{C}$$

The inefficiency losses from the relative price dispersion, $\bar{S}_f$ and $\bar{S}_m$, is expressed as eq.(35).

eq.(23) suggests $\bar{\pi}_m = \bar{\pi}_f Z_2$. Finally, from the consumption Euler equation, eq.(4),

$$\bar{\pi}_f = \beta\frac{\bar{R}}{Z_1}$$

Then, once the central bank chooses $\bar{R}$, the steady state equilibrium is uniquely determined because all the steady state variables can be written as implicit functions of deep parameters and $\bar{R}$.[19]

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[19]Note that $\bar{R} < \frac{Z_1}{\beta}\alpha_f^{-1/\theta_f}$ is necessary for the existence of the solution.

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