Bank of Japan Working Paper Series

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No.07-E-3
February 2007

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Multi-Sector Menu Cost Model, Decreasing Hazard, and Phillips Curve [†]

Hidetaka Enomoto[‡]

February, 2007

Abstract

This paper generalizes the Golosov-Lucas model (GL model), a single sector menu cost model with idiosyncratic productivity shocks, to multi-sector setting. While the GL model matches some empirical facts, it cannot mimic decreasing hazard functions for price changes, which are observed in many countries. With realistic parameters, the simulation results of the generalized GL model show many features observed in empirical evidences such as decreasing hazard rates. In addition, the simulation results with monetary shocks show flattening of the Phillips curve in a low inflation environment.

Keywords: Menu Cost Models, Hazard Functions, Phillips Curve
JEL Classification: E30

[†]I would like to thank Naohito Abe, Hiroshi Fujiki, Ippei Fujiwara, Mikhail Golosov, Hideo Hayakawa, Masahiro Higo, Hibiki Ichiue, Munehisa Kasuya, Emi Nakamura, Koji Nakamura, Kenji Nishizaki, Kyosuke Shiotani, Shigenori Shiratsuka, Toyoichiro Shirota, Yutaka Soejima, Jón Steinsson, Nao Sudou, Tomohiro Sugo, Takayuki Tsuruga and seminar participants at the Bank of Japan for their helpful comments. I would also like to thank Mototsugu Shintani for his helpful comments on the earlier draft. Views expressed in this paper are those of author and do not necessarily reflect those of the Bank of Japan or its Research and Statistics Department.

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1 Introduction

Price setting behavior is one of the most important research areas in macroeconomics. There are two major categories: time-dependent pricing and state-dependent pricing. In the New Keynesian context, time-dependent pricing such as Calvo (1983) and Taylor (1979, 1980) is popular. While time-dependent pricing is convenient for monetary policy analysis, there is no micro foundation in it and therefore it is subject to Lucas critique.

State-dependent pricing is more appealing since it assumes that firms choose not only the size of price changes, but also the timing of price changes. State-dependent pricing can be split into two categories: menu cost models with idiosyncratic productivity shocks such as Golosov and Lucas (forthcoming) (GL model), and models without them such as Caplin and Spulber (1987) and Dotsey, King and Wolman (1999). Note that, in menu cost models without idiosyncratic productivity shocks, the only source of price changes is monetary shock. Such models are inconsistent with the empirical fact that price changes occur frequently even when inflation is zero.

Golosov and Lucas (forthcoming) introduce idiosyncratic productivity shocks into a menu cost model so that price changes occur even under a zero inflation environment. Even the GL model, however, can't explain decreasing hazard.[1] Decreasing hazard is observed in many countries such as Japan (Saita et al. (2006)), United States (Nakamura and Steinsson (2006a), Klenow and Kryvtsov (2005)), and Euro Area (Álvarez, Burriel, and Hernando (2005)).[2]

To produce decreasing hazard, the model needs to generate time series of individual prices so that a substantial fraction of price changes occur in the first few months after a price change while a considerable fraction of individual prices remain constant in two or three years after a price change. Since the GL model includes only one sector, the model can't generate time series with such heterogeneity.

[1] Decreasing hazard rates imply that a firm will have lower probability of changing its price the longer it has kept it unchanged. By the term "decreasing hazard," people often mean two different things: decreasing hazard across products and decreasing hazard for individual products. Throughout this paper, I use the term "decreasing hazard" to mean a decreasing hazard across products.

[2] The decreasing hazard for individual products found by Nakamura and Steinsson (2006a) is also interesting but I will not focus on this fact. This fact may not be robust since (1) the slope of hazard they estimate is "slightly decreasing or almost flat," and (2) their estimate is subject to a downward bias due to "heterogeneity in frequency of price changes.": Note that Nakamura and Steinsson (2006a) also found that the frequency of price changes is increasing in inflation rates. In addition, note that the database they use to estimate hazard contains prices under the different inflation rates. These two things imply that there is "heterogeneity in frequency of price changes," which can be the source of a downward bias.

Explaining decreasing hazard is important to understand the effects of monetary policy on real economy such as level of production: In the calibrated GL model, money is almost neutral, which may be caused by the model's failure to generate time series of prices with a considerable fraction constant in long-term as seen in Figure 1.

This paper generalizes the GL model to multi-sector setting, where each sector has different menu cost, variance of productivity shocks, and average productivity level.[3]

Simulation results show many desirable features: First, the calibrated model can generate decreasing hazard.[4] Second, the calibrated model predicts that the frequency of price increases responds strongly to inflation while the frequency of price decreases and the size of price increases and price decreases do not. They are consistent with the empirical facts established by Nakamura and Steinsson (2006a), on individual price data. Third, the relationship between inflation and GDP predicted by the model has some interesting features, such as slope of Phillips curve increasing in inflation rates, which are consistent with empirical facts established by Benati (forthcoming).[5]

This paper is organized as follows. Section 2 summarizes the stylized facts this paper tries to explain. Section 3 presents a two-sector menu cost model (Generalized GL Model) where the differences of sectors consist of menu cost, the variance of productivity shocks, and the average productivity level. Section 4 describes the calibration procedure. I also show the predictions of the calibrated model for price changes. Section 5 shows the model's predictions on the relationship between inflation and production. Section 6 concludes.

2 Stylized Facts

The main purpose of this paper is to show that the generalization of the GL model significantly improves the model's ability to mimic the empirical facts about the individual prices and the relationship between inflation and GDP. Here, I summarize the stylized facts with which the predictions of the generalized GL model are consistent.

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[3] Although I show only a two-sector model in this paper, a model with finitely many sectors can be constructed in the same way technically.

[4] For some readers, this accomplishment might seem not to be significant since I calibrate this model to sample moments including the information of decreasing hazard. I can, however, assert that this is good job since previous state-dependent pricing models including the GL model can't be calibrated so as to replicate decreasing hazard.

[5] By the term "Phillips Curve," I mean the relationship between inflation and GDP generated by the model (or observed in data), as seen in Figure 12, throughout this paper.

The first three facts are about the individual retail prices in Japan due to Saita et al. (2006). In 1999-2003 when Japanese economy was in moderate deflation, (1) the average frequency of price changes is 23.1 percent per month, (2) almost half of price changes are price decreases, and (3) the hazard function of price changes is decreasing. The decreasing hazard is also observed in United States (Nakamura and Steinsson (2006a), Klenow and Kryvtsov (2005)), and Euro Area (Álvarez, Burriel, and Hernando (2005)).

The fourth fact is about the individual prices in U.S. due to Nakamura and Steinsson (2006a): (4) The frequency of price increases responds strongly to inflation while the frequency of price decreases and the size of price increases and price decreases do not.[6]

The fifth fact is about the relationship between inflation and GDP. (5) The slope of Phillips curve is increasing in inflation rates according to Benati (forthcoming).[7]

As for these facts, the GL model can't explain (3) and (5) while the generalized GL model can explain all five facts.[8]

3 Model

3.1 Outline of the Model

In this section, I present a two-sector menu cost model with idiosyncratic productivity shocks. I construct the model modifying the GL model. Recall that the GL model can't generate decreasing hazard since decreasing hazard is consistent with heterogeneity in frequency of price changes while the GL model includes only one sector.

In the GL model, frequency of price changes is affected by menu cost, the variance of idiosyncratic productivity shocks, and the level of productivity. Therefore, I present a two-sector menu cost model where the differences of sectors consist of menu cost, the variance of productivity shocks, and the average productivity level.

Outline of the model is as follows: There is a continuum of infinitely lived households, each of which consumes a continuum of goods and services. Each

[6]Golosov and Lucas (forthcoming) provide the international evidence showing that the frequency of price changes is increasing in inflation rates.

[7]As for the relationship between inflation and GDP, the generalized GL model also predicts that high inflation, near zero inflation and deflation imply volatile and low average GDP while moderate inflation implies high and stable GDP, which is consistent with empirical facts about Japan and US data found by Sakura et al. (2005).

[8]The generalized GL model can't explain the seasonality of frequency of price changes pointed out by Nakamura and Steinsson (2006a). This feature, however, seems to be just the out of the scope of this model.

household supplies labor on a competitive labor market. Households are assumed to obtain utility from real cash holdings. Money supply follows a monetary shock process specified later.

There is a continuum of firms, subject to idiosyncratic productivity shocks: 50 percent of firms are "g sector" firms, associated with relatively high frequency of price changes, and the rest of firms are "s sector" firms, associated with relatively low frequency of price changes.[9] Firms in a different sector are subject to idiosyncratic productivity shocks generated by a different shock process. Each firm produces only one of the continuum of consumption goods or services. Each firm sets price of the good subject to a menu cost of re-pricing. Menu cost is measured by labor hour. The length of the labor hour needed to change a nominal price is different between two sectors. Firms use labor to produce the good or service and to re-set nominal prices.

3.2 Two Shocks in the Economy

There are two types of shocks in this economy: a monetary shock and a firm-specific productivity shock. A different sector is subject to firm-specific productivity shocks generated by a different shock process. The log of the money supply is assumed to follow a Brownian motion with drift parameter μ and variance σ_m^2,

$$d\log(m) = \mu dt + \sigma_m dZ_m \tag{1}$$

where Z_m denotes a standard Brownian motion with zero drift and unit variance.

There are firm-specific productivity shocks for "g sector" firms denoted by v_g, and those for "s sector" firms v_s, which are assumed to be independent across firms. The log of a firm-specific productivity shock follows the mean-reverting process:

$$d\log(v_j) = -\eta(\log(v_j) - \log(1 + e_j))dt + \sigma_{v_j} dZ_{v_j} \qquad j = g \text{ or } s \tag{2}$$

where $(1+e_j)$ is the average productivity level of "j sector" firms, and Z_{v_j} is a standard Brownian motion with zero drift and unit variance, distributed independently of Z_m.

[9] I associate "g sector" and "s sector" with data in the way described in the subsection 4.2. Given this, the weights of consumer price index adjusted by Saita et al. (2006) suggest that the fraction of "g sector" firms is about 50 percent.

3.3 State of the Economy and of an Individual Firm

The state of this economy at date t includes money stock m_t, a nominal wage rate w_t and the joint distribution of firms with (p_t, v_t), denoted by $\phi_t(p, v)$, where a firm with (p_t, v_t) denotes a firm with its nominal price p_t and its current productivity shock v_t. The state of an individual "j sector" firm includes p_t and v_{jt} additionally.

3.4 Markets

There is a labor market where firms hire labor from households with w_t. There is also a capital market where claims to the monetary unit are traded. As in Golosov and Lucas (forthcoming), I adopt the convention that

$$\mathrm{E}\left[\int_0^\infty Q_t y_t dt\right] \tag{3}$$

is the value at date 0 of a dollar earnings stream $\{y_t\}_{t=0}^\infty$, a stochastic process defined in terms of m_t.

In equilibrium, the market clearing conditions for consumption, labor, and money are satisfied.

3.5 Consumer

At each date t, each household buys goods and services from every firm distributed according to $\phi_t(p, v)$. The household chooses a buying strategy $\{C_t(\cdot)\}$, where $C_t(p)$ is the number of units of the consumption good that it buys from a firm charging nominal price p at date t. Additionally, the household chooses a labor supply strategy $\{l_t\}$ and a money holding strategy $\{\hat{m}_t\}$, where l_t is the units of labor supplied and $\hat{m}_t$ is dollar balances held at date t.

Let c_t denote Spence-Dixit-Stiglitz consumption aggregate

$$c_t = \left[\int C_t(p)^{1-\frac{1}{\epsilon}} \phi_t(dp, dv)\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{4}$$

A price index P_t is defined as follows:

$$P_t = \left[\int p^{1-\epsilon} \phi_t(dp, dv)\right]^{\frac{1}{1-\epsilon}}. \tag{5}$$

The expected utility of the household over time is expressed as

$$\mathrm{E}\left[\int_0^\infty e^{-\rho t}\left(\frac{1}{1-\gamma}c_t^{1-\gamma} - \alpha l_t + \log\left(\frac{\hat{m}_t}{P_t}\right)\right)dt\right] \tag{6}$$

where the operator $\mathrm{E}(\cdot)$ is defined by the shock processes (1) and (2).[10]

The budget constraint of the household is expressed as

$$\mathrm{E}\left[\int_0^\infty Q_t\left(\int pC_t(p)\phi_t(dp, dv) + R_t\hat{m}_t - w_t l_t - \Pi_t\right)dt\right] \leq m_0 \tag{7}$$

where Π_t is profit income, obtained from the household's holdings of a fully diversified portfolio of claims on the individual firms, plus any lump sum cash transfers. R_t is the nominal interest rate.

The household chooses buying strategy $\{C_t(\cdot)\}$, labor supply strategy $\{l_t\}$, and money holding strategy $\{\hat{m}_t\}$ so as to maximize (6) subject to (7), taking $\{Q_t\}$, $\{R_t\}$, $\{w_t\}$, $\{\Pi_t\}$, $\{\phi_t\}$ as given. The first-order condition for money holdings is expressed as

$$e^{-\rho t}\frac{1}{m_t} = \lambda Q_t R_t, \tag{8}$$

where λ is the Lagrange multiplier independent of time. Note that the equilibrium condition $\hat{m}_t = m_t$ is imposed in the above equation. The first-order condition for consumption is described as

$$e^{-\rho t}c_t^{-\gamma}c_t^{1/\epsilon}C_t(p)^{-1/\epsilon} = \lambda Q_t p, . \tag{9}$$

The first-order condition for labor supply is

$$e^{-\rho t}\alpha = \lambda Q_t w_t, . \tag{10}$$

It can be shown that there is an equilibrium where the nominal interest rate is constant at the level

$$R_t = R = \rho + \mu \tag{11}$$

[10]The linear disutility of labor in (6) can be interpreted as a reflection of the indivisible labor setting proposed by Hansen (1985).

for all realizations of the two shock processes. As Golosov and Lucas (forthcoming) did, I focus on the equilibrium where (11) holds throughout this paper. Then, (8), (10), and (11) imply

$$w_t = \alpha R m_t. \tag{12}$$

(12) implies that $\log(w_t)$ follows the same Brownian motion as the one for monetary shocks. Note that this structure of the model associated with (12) simplifies the analysis significantly.

3.6 Firms

A "j sector" firm faces the demand for the good it produces $C_t(\cdot)$, nominal wage w_t, and a productivity parameter v_{jt}. The production function of "j sector" firms is assumed to be

$$C_t(p) = v_{jt} l_t^f \tag{13}$$

where l_t^f denotes labor used to produce the good.[11] Suppose a firm enters the period with a nominal price p carried over from the past. Then, if this firm leaves price unchanged, its current profit is

$$C_t(p)(p - w_t/v_{jt}). \tag{14}$$

If this firm chooses any price $q \neq p$, its current profit becomes

$$C_t(q)(q - w_t/v_{jt}) - k_j w_t \tag{15}$$

where the parameter k_j is the hours of labor needed for a "j sector" firm to change its nominal price.

Let's think about the present value of a "j sector" firm with its state (p, v_j, w, ϕ_t). I express this present value by $\varphi_j(p, v_j, w, \phi_t)$. This firm chooses a shock-contingent repricing time $T \geq 0$, and a shock-contingent nominal price q to be chosen at T so as to solve

[11] Here, as Golosov and Lucas (forthcoming) and many New Keynesians did in their research, I assume each firm must satisfy household's demand for the good the firm produces.

$$\varphi_j(p, v_j, w, \phi) = \max_T \mathrm{E}\Bigg[\int_0^T Q_t C_t(p)(p - w_t/v_{jt}) dt + Q_T \cdot \max_q [\varphi_j(q, v_{jT}, w_T, \phi_T) - k_j w_T] \Bigg]. \quad (16)$$

Note that, from (4), (9), and (10), the demand function of goods and services is

$$C_t(p) = c_t^{1-\epsilon\gamma} \left(\frac{\alpha p}{w_t} \right)^{-\epsilon}. \quad (17)$$

From the natural normalization $Q_0 = 1$ and (10), I obtain

$$Q_t = e^{-\rho t} \frac{w_0}{w_t}. \quad (18)$$

Then, using (17) and (18), (16) can be expressed as

$$\varphi_j(p, v_j, w, \phi) = \max_T \mathrm{E}\Bigg[\int_0^T e^{-\rho t} \frac{w}{w_t} c_t^{1-\epsilon\gamma} \left(\frac{\alpha p}{w_t} \right)^{-\epsilon} (p - w_t/v_{jt}) dt + e^{-\rho T} \frac{w}{w_T} \cdot \max_q [\varphi_j(q, v_{jT}, w_T, \phi_T) - k_j w_T] \Bigg]. \quad (19)$$

The choice of stopping times T and nominal prices q that attain the right side of (19) is a pricing strategy of a “j sector” firm. In this paper, I analyze a Nash equilibrium of pricing strategies over a continuum of monopolistically competitive firms. In the rest of this section, I will describe how (19) can be analyzed and how this pricing strategy is determined.

Finally, I define $\{\Gamma_{jt}\}$ so that $\Gamma_{jt} dt$ is the fraction of the firms that belong to “j sector” and reprice during the time interval $(t, t + dt)$ in equilibrium. Then, the labor market clearing condition is expressed as

$$l_t = \int \frac{C_t(p)}{v} \phi_t(dp, dv) + k_g \Gamma_{gt} + k_s \Gamma_{st}. \quad (20)$$

The market clearing conditions for consumption goods and services have been incorporated in (19).

3.7 Restatement of Firm's Bellman Equation as a Recursive Form

Note that (19) is not recursive, including the joint distribution ϕ_t implicitly in c_t. This makes (19) difficult to analyze. In addition, to solve this problem numerically, I need to discretize this continuous time model in advance. Therefore, I make two-step approximation as Golosov and Lucas (forthcoming) did: The first approximation is made to keep problem recursive even if σ_m is positive so that the time-invariant Bellman equation is obtained. The details will be discussed in this subsection. The second approximation is made to discretize the continuous time model, which will be discussed in the next subsection. I will analyze the easier case, $\sigma_m = 0$, first, then the harder one, $\sigma_m > 0$.

[The Case of $\sigma_m = 0$]

From (4) and (17), the consumption aggregate can be expressed as

$$c_t = \left[\int \left(\frac{\alpha p}{w_t} \right)^{1-\epsilon} \phi_t(dp, dv) \right]^{1/(\gamma(\epsilon-1))}. \tag{21}$$

Using the change of variable $x = p/w_t$, (21) is rewritten as

$$c_t = \left[\alpha^{1-\epsilon} \int x^{1-\epsilon} \tilde{\phi}_t(dx, dv) \right]^{1/(\gamma(\epsilon-1))}. \tag{22}$$

Now, I assume that there exists an invariant measure $\tilde{\phi}$ and express the corresponding consumption aggregate, given by (22), as $\bar{c}$.[12] Then, I can restate (19) as

$$\varphi_j(p, v_j, w) = \max_T \mathrm{E}\Bigg[\int_0^T e^{-\rho s} \frac{w}{w_s} \bar{c}^{1-\epsilon\gamma} \left(\frac{\alpha p}{w_s} \right)^{-\epsilon} (p - \frac{w_s}{v_{js}}) ds + e^{-\rho T} \frac{w}{w_T} \cdot \max_q [\varphi_j(q, v_{jT}, w_T) - k_j w_T] \Bigg]. \tag{23}$$

Again, using the change of variable $x = p/w$, (23) can be expressed as

$$\frac{1}{w} \varphi_j(wx, v_j, w) = \max_T \mathrm{E}\Bigg[\int_0^T e^{-\rho s} \bar{c}^{1-\epsilon\gamma} (\alpha x_s)^{-\epsilon} (x_s - \frac{1}{v_{js}}) ds + e^{-\rho T} \frac{1}{w_T} \cdot \max_{x'} [\varphi_j(w_T x', v_{jT}, w_T) - k_j w_T] \Bigg]. \tag{24}$$

[12] The existence of this invariant measure can be confirmed numerically in the later calculations.

Finally, setting $\varphi_j(p, v, w) = w\psi_j(x, v)$, firm's Bellman equation becomes a recursive form as follows:

$$\psi_j(x, v_j) = \max_T \mathrm{E}\Big[\int_0^T e^{-\rho t}\bar{c}^{1-\epsilon\gamma}(\alpha x_t)^{-\epsilon}(x_t - \frac{1}{v_{jt}})dt \\ + e^{-\rho T} \cdot \max_{x'}[\psi_j(x', v_{jT}) - k_j]\Big]. \quad (25)$$

[The Case of $\sigma_m > 0$]

A two-shock case is hard to analyze since there must be no invariant measure in this case, implying that the actual policy function is dependent on $\tilde{\phi}_t$. Here, I compute the pseudo-equilibrium proposed by Golosov and Lucas (forthcoming) as an approximation, where each firm is assumed to observe the mean level of c_t correctly but ignore all the fluctuations around this mean level. Under this assumption, the invariant measure $\tilde{\phi}$ and $\bar{c}$ can be obtained using (22) and (25) as in the case of $\sigma_m = 0$, while $\bar{c}$ reflects the effects of σ_m.

Golosov and Lucas (forthcoming) pointed out that the model's property that money is almost neutral keeps the loss in accuracy caused by this approximation little. As seen in the subsequent sections, however, money has more effects on real consumption in the calibrated generalized GL model than in the calibrated GL model. These two things suggest that, if I adopt this pseudo-equilibrium as an approximation, there seems to be more loss in accuracy in the calibrated generalized GL model than in the calibrated GL model. Given the current level of computation ability, there is no feasible alternative method available now.

3.8 Approximating Markov Chains

Here, I show the construction of approximating Markov chains. Define $\tilde{x} = \log(p/w)$ and $\tilde{v} = \log(v)$. Choose some value h as the grid size and define the state space $S = X \times V$. In particular, I take $h = 0.025$ in this paper. Smaller h means higher accuracy of approximation while more computer memory is necessary to take smaller value as h. In addition, I take $\bar{v} = 0.6$ as the common upper bound of $\tilde{x}$ and $\tilde{v}$ and assume that $-\bar{v}$ is the common lower bound of $\tilde{x}$ and $\tilde{v}$.

Based on the description of finite-element methods of Kushner and Dupuis (2001), I can obtain a discrete time and state approximation of the problem (25) as seen in below:

$$\psi_j(\tilde{x}, \tilde{v}_j) = \max \Bigg\{ \Pi(\tilde{x}, \tilde{v}_j)\Delta t + e^{-r\Delta t} \sum_{\tilde{x}', \tilde{v}'_j} \pi_j(\tilde{x}', \tilde{v}'_j | \tilde{x}, \tilde{v}_j) \psi_j(\tilde{x}', \tilde{v}'_j),$$

$$\max_{\xi_j} \Bigg[\Pi(\xi_j, \tilde{v}_j)\Delta t + e^{-r\Delta t} \sum_{\tilde{x}', \tilde{v}'_j} \pi_j(\tilde{x}', \tilde{v}'_j | \xi_j, \tilde{v}_j) \psi_j(\tilde{x}', \tilde{v}'_j) \Bigg] - k_j \Bigg\}, \quad (26)$$

where

$$\Pi(\tilde{x}, \tilde{v}_j) = \overline{c}^{1-\epsilon\gamma} (\alpha)^{-\epsilon} e^{-\epsilon\tilde{x}} [e^{\tilde{x}} - e^{-\tilde{v}_j}] \quad (27)$$

and where π_j is a transition function, defined on $S \times S$, that I define later. The time interval Δt is set by

$$\Delta t = \frac{h^2}{Q} \quad (28)$$

where

$$Q = \sigma_m^2 + \mu h + \sigma_{v_g}^2 + \eta \overline{v} h. \quad (29)$$

If I take time interval Δt small enough, at most one of the variables $\tilde{x}$ and $\tilde{v}$ changes. More specifically, if I take Δt satisfying (28) and if neither $\tilde{x}$ nor $\tilde{v}$ is at its upper bound or lower bound, then π_j of all transitions is zero except for the following transitions:

$$\pi_j(\tilde{x} + h, \tilde{v}_j | \tilde{x}, \tilde{v}_j) = \frac{\sigma_m^2/2}{Q}, \quad (30)$$

$$\pi_j(\tilde{x} - h, \tilde{v}_j | \tilde{x}, \tilde{v}_j) = \frac{\sigma_m^2/2 + \mu h}{Q}, \quad (31)$$

$$\pi_j(\tilde{x}, \tilde{v}_j + h | \tilde{x}, \tilde{v}_j) = \frac{\sigma_{v_j}^2/2}{Q}, \quad (32)$$

$$\pi_j(\tilde{x}, \tilde{v}_j - h | \tilde{x}, \tilde{v}_j) = \frac{\sigma_{v_j}^2/2 + \eta(\tilde{v}_j - e_j)h}{Q}, \quad (33)$$

and

$$\pi_j(\tilde{x}, \tilde{v}_j | \tilde{x}, \tilde{v}_j) = 1 - \frac{\sigma_m^2 + \sigma_{v_j}^2 + \eta(\tilde{v}_j - e_j)h + \mu h}{Q}, \tag{34}$$

if $\sigma_{v_g}^2 \geq \sigma_{v_s}^2, \mu \geq 0$ and $\tilde{v}_j \geq e_j$ are satisfied.[13] At the boundaries, I assume the probability of staying at the current state is increased by the probability of moving out of the boundaries in the next period $(t + \Delta t)$ if $\bar{v}$ were huge. The adaptations of (30) – (34) for the other cases are obvious. I omit them for the brevity.

3.9 Decision Making on Price in Details

As a result of these approximations, firm's problem becomes tractable. If you follow the procedure of the value function iteration as seen in the Appendix A, you can obtain the firm's pricing strategy for each sector.

Note that, by construction, the firm's pricing strategies obtained as a result of calculation in the Appendix A are a Nash equilibrium over a continuum of monopolistically competitive firms: For given joint distributions $\{\tilde{\phi}_t\}$ of prices and productivity levels at current and future dates, each firm's pricing strategy is determined by (26). Conversely, the pricing strategies adopted by all firms define the distributions $\{\tilde{\phi}_t\}$ at future dates, given the initial distribution $\tilde{\phi}_0$.

Here, I explain the basics about firm's price setting behavior under some menu costs using (26). Figure 2 illustrates when each firm changes the nominal price of the good the firm produces if μ is positive and the variance of monetary shocks and that of productivity shocks are both zero. Note that

$$\Pi(\tilde{x}, \tilde{v}_j)\Delta t + e^{-r\Delta t} \sum_{\tilde{x}', \tilde{v}'_j} \pi_j(\tilde{x}', \tilde{v}'_j | \tilde{x}, \tilde{v}_j) \psi_j(\tilde{x}', \tilde{v}'_j)$$

and

$$\Pi(\tilde{x}, \tilde{v}_j)\Delta t + e^{-r\Delta t} \sum_{\tilde{x}', \tilde{v}'_j} \pi_j(\tilde{x}', \tilde{v}'_j | \tilde{x}, \tilde{v}_j) \psi_j(\tilde{x}', \tilde{v}'_j) - k_j$$

are expressed by bold curve and normal curve respectively. In addition, $\hat{\xi}_j$ defined by

$$\hat{\xi}_j(\tilde{v}_j) = \arg\max_{\xi_j} \left[\Pi(\xi_j, \tilde{v}_j)\Delta t + e^{-r\Delta t} \sum_{\tilde{x}', \tilde{v}'_j} \pi_j(\tilde{x}', \tilde{v}'_j | \xi_j, \tilde{v}_j) \psi_j(\tilde{x}', \tilde{v}'_j) \right] \tag{35}$$

[13] Note that $x = p/w$. Therefore, x is expected to decrease after the nominal price is determined if μ is positive.

is expressed by the vertical line. As seen in this figure, each firm changes the nominal price and chooses $\hat{\xi}_j(\tilde{v}_j)$ as a new nominal price when the increase of profit caused by the price change exceeds the size of menu cost ((a), (c) in Figure 2). In other words, there is a "region of inaction" where the firm leaves its nominal price unchanged ((b) in Figure 2).

Figure 3 illustrates how $\hat{\xi}_j$ and "region of inaction" are changed by a negative productivity shock. $\hat{\xi}_j$, the upper bound and the lower bound of "region of inaction" are increased when a negative productivity shock occurs since it induces the increase of marginal cost. Note that "region of inaction" is usually enlarged by the negative productivity shock since the menu cost is assumed to be independent of the quantity sold by the firm, which implies that the effective burden of the menu cost is relatively heavy for firms with low productivity.

4 Predictions of the Calibrated Model for Price Changes

In this section, I calibrate the model and run simulations of the calibrated model for price changes. The predictions can be used to evaluate this model.

Outline of this section is as follows: First, I explain why some specific sample moments are used to calibrate this model. Then, I calibrate this model. To do calibration, I did some simulations to obtain survival rates or hazard functions for price changes. The method of this simulation is straightforward. I describe this method in Appendix B. It is shown that the calibrated model can generate decreasing hazard.[14] Then, I test this model checking whether this model can predict some other empirical facts on individual price data.

4.1 Sample Moments for Calibration

I choose the values of η, σ_{v_j}, and k_j so that the model's predictions on the following sample moments fit the data best: survival rates, the share of price increases in price changes, and the average size of price changes.[15] These sample moments are obtained from the empirical research on individual retail price data in Japan due to Saita et al. (2006). Specifically, in the next subsection, I use the sample moments based on the data from 1999-2003 when Japanese economy was in moderate deflation.

[14]While I also perform sensitivity analysis, I omit the results of them for brevity. If you want to see the results, please send me email. Then, I may provide them to you.

[15]The survival rate of period t is defined as the share of nominal prices unchanged in first t periods after a price change. The hazard rate of period t is defined as the conditional probability that a nominal price is changed in period t given that the price is unchanged in the first $(t-1)$ periods after a price change.

Here, I explain why I use these sample moments to calibrate η, σ_{v_j}, and k_j. First, survival rate is necessary to calibrate the two-sector model since this includes the information about heterogeneity in frequency of price changes across goods.[16]

Second, the share of price increases in price changes is also important because this information prevents overestimates and underestimates of σ_{v_j}. Note that price changes occur due to nominal wage changes or productivity shocks. Since the productivity shocks are assumed to be symmetry, larger σ_{v_j} implies higher degree of symmetry in price changes given the value of μ in the sense that the share of price increases becomes closer to 50 percent. Therefore, if I overestimate the value of σ_{v_j}, then the share of price increases predicted by the model becomes too close to 50 percent.

Third, the average size of price changes is also useful. Suppose that you observe low frequency of price changes. To replicate low frequency, there are two choices in the general case: High k_j or low σ_{v_j}. On the one hand, higher k_j implies larger average size of price changes. On the other hand, lower σ_{v_j} generally implies smaller average size of price changes because low σ_{v_j} implies relatively large effects of price on firm's profit. Intuitive explanation is given in Figure 4.[17] Thus, the average size of price changes suggests why price changes are so frequent (or infrequent).

4.2 Calibration

Here, I specify values of all parameters in the model. As for preference parameters ρ (subjective discount rate), γ (relative risk aversion), and ϵ (elasticity of substitution), I draw on the existing research about these parameters in Japan. Coincidentally, the existing research seems to suggest that the values Golosov and Lucas (forthcoming) used for those parameters in the United States might be applicable in Japan as summarized in Table 1. Therefore, I set $(\rho, \gamma, \epsilon) = (0.01, 2, 7)$. As for α, I draw on Golosov and Lucas (forthcoming) and set $\alpha = 6$, implying that about 37 percent of the unit time endowment is allocated to work.[18]

[16]As seen in Figure 8, the empirical survival rate decreases sharply in the first few months after a price change while it decreases slowly otherwise. This suggests that a substantial fraction of price changes occur in the first few months after a price change while a considerable fraction of individual prices remain constant in two or three years after a price change.

[17]If the variance of productivity shocks is high, then the possibility of changes of productivity level is high. Therefore, the discounted value of the firm's profits reflects the profits consistent with various states of productivity. As a result, the discounted value of profits is not sensitive to the real price (p/w). Given the level of menu cost, this implies that "the region of inaction" is wide and the size of price changes is large.

[18]As a result, $(\rho, \gamma, \epsilon, \alpha)$ in this paper is the same as in Golosov and Lucas (forthcoming). This eases the comparison.

I set the values of parameters of monetary shocks, μ and σ_m, based on the consumer price index from 1999-2003 (Table 2). This is because money was almost neutral in the GL model. In the next section, you can confirm that money is almost neutral in the generalized GL model although the neutrality of money in the generalized GL model is weaker than that in the GL model.

The difference of average productivity between "g sector" and "s sector" is based on SNA data from 1999-2003 (Table 2 and Figure 5). Using the weights of consumer price index adjusted by Saita et al. (2006), I calculate the labor productivity (per hour basis) of goods sector including "agriculture, forestry and fishing," "food products and beverages," and "textiles," and that of service sector including "service activities," "transport and communications" and "electricity, gas and water supply" industries.[19] The definition of each sector is determined so that industries characterized by low frequency of price changes are categorized into service sector. Thus, I can associate goods sector with "g sector" and service sector with "s sector." As a result, the average productivity of "s sector" is assumed to be 10 percent higher than that of "g sector."

After setting these parameters, I perform value function iteration following steps described in Appendix A and obtain hazard rates through simulations following steps described in Appendix B for each set of values $(\eta, \sigma_{v_j}, k_j)$. Note that the share of price increases in price changes, the average size of price increases and that of price decreases for each set of values $(\eta, \sigma_{v_j}, k_j)$ can be calculated based on the invariant joint distributions, which is obtained as a result of value function iteration.[20] Now, I also know survival rates for each set of values $(\eta, \sigma_{v_j}, k_j)$ since I've already known hazard rates. Thus, I can choose the values of η, σ_{v_j}, and k_j so that the model's predictions on these sample moments are fitted best.

The selected values of η, σ_{v_j}, and k_j can be seen in Table 2. As a result of calibration, the variance of productivity shocks of "g sector" is assumed to be larger than that of "s sector." The menu cost of "g sector" is assumed to be smaller

[19]These industries cover about 77 percent of goods and services based on the weights of consumer price data compiled by Saita et al.(2006). About 23 percent of goods and services are not covered mainly because industries in SNA data are defined too roughly. For example, the labor productivity of "precision instruments" industry should not be associated with the labor productivity of firms producing watch. This is because the "precision instruments" industry includes not only firms producing watch consumers use but also firms producing big machineries consumers never use.

[20]Note that, in this analysis, the minimum size of price change is 2.5 percent since I take $h = 0.025$. Because of this, the size of price change may be overestimated somewhat. Although the best way to improve the estimation is to take smaller value for h, it takes long time or may be impossible because of the limitation of the memory capacity. Therefore, I settle for the second best: As the model's predictions on the average size of price changes, I use the values which are the original predictions on the average size minus 1.25 percent.

than that of “s sector.”[21]

The pricing strategies obtained as a result of calibration are summarized in Figure 6 and Figure 7. In these figures, gray grids represent $\hat{\xi}_j$ and the region between two black grids given the productivity level represents “region of inaction.” As long as a firm is in “region of inaction,” then the firm keeps its nominal price level. If a firm with its current productivity level v_j moves in the black grid, then this firm changes its nominal price and chooses $\hat{\xi}_j(\tilde{v}_j)$ as its new nominal price. There are two main features in these two figures: (A) The region of inaction for the firms with low current productivity level is wider than that for the firms with high current productivity level although there are some exceptions .[22] (B) Roughly speaking, the region of inaction for “s sector” firms is wider than that of “g sector” firms.

(A) holds because the effective burden of the menu cost is relatively heavy for firms with low productivity.[23] (B) comes from the assumptions that $k_g < k_s$. It is obvious that larger menu cost implies wider region of inaction.

The sample moments predicted by the calibrated generalized GL model are summarized in Figure 8 and Figure 9. By and large, this calibrated model succeeds in replicating these sample moments: (i) Predicted hazard rates are decreasing in time. (ii) Predicted share of price increases in price changes is about 50 percent. (iii) Both the average size of price increases and that of price decreases predicted by this calibrated model are around 6 percent. (i)-(iii) are consistent with the empirical facts from 1999-2003.[24]

4.3 Predictions of the Model

To test this model in more strict way, I also check whether this model can predict some facts on individual price data: (i) The shape of hazard function is robust in the sense that decreasing hazard is observed in United states and Euro area where

[21]The labor required to adjust prices in this calibrated model is equal to 0.2 percent of overall employment while that in the calibrated one sector GL model is equal to 0.5 percent. The menu cost in this calibrated model is about 0.1-0.2 percent of revenues while that in the calibrated one sector GL model is about 0.5 percent of revenues. Levy et al. (1997) estimate that the menu cost in supermarkets is about 0.7 percent of revenues. Note that the frequency of price changes used here, which is obtained from Saita et al (2006), is calculated based on the data which do not reflect price changes due to the promotional sale. This thing may at least partly explain why the menu cost obtained as a result of calibration seems to be small.

[22]One may find the difference in the shape of “region of inaction” between these two figures. I will explain this point in the next section.

[23]See the subsection 3.9 and Figure 3 for more details.

[24]The frequency of price changes predicted by the calibrated model is close to the observed frequency (Data: 23.1 percent per month, Prediction (after excluding double-counting): 24.8 percent). See Figure 11 and its note.

the average inflation rate is higher than in Japan. (ii) The frequency of price increases responds strongly to inflation while the frequency of price decreases and the size of price increases and price decreases do not.

In this test, only the parameters of monetary shocks are changed to generate different average inflation rates. In concrete, I use the following values as (μ, σ_m): CY1999-2003: (-0.0017, 0.0037), CY1986-1990: (0.0039, 0.0078), CY1980-1985: (0.0079, 0.0077), CY1971-1975: (0.0272, 0.0204).[25] These numbers are obtained from CPI data in the corresponding periods as I did in the calibration part. Since I use data under zero or negative inflation to calibrate this model, this is a strict out-sample test of the model.

The model provides good predictions about (i) and (ii) as seen in Figure 10 and Figure 11. The predicted hazard function is decreasing as long as the average inflation is moderate. This is consistent with the empirical facts that the observed hazard function is decreasing in United States and Euro area. The predictions of the model is also consistent with (ii). The predicted frequency of price decreases doesn't respond strongly to inflation because all price decreases are caused by the idiosyncratic productivity shocks when μ is positive.[26]

5 Monetary Policy Experiments

In this section, I use the calibrated generalized GL model to conduct numerical experiments on the economy's response to various shocks as Golosov and Lucas (forthcoming) did. I focus on the model's predictions about the relationship between inflation and GDP.

5.1 Procedure of Experiments

Here is how I obtain the relationship between inflation and the level of production for different values of (μ, σ_m).

(1) Assume that the initial distribution is the invariant one consistent with (μ, σ_m).

[25] The average annual inflation rates are as follows: CY1999-2003: -0.7 percent, CY1986-1990: 1.6 percent, CY1980-1985: 3.2 percent, CY1971-1975: 11.3 percent.

[26] Suppose $\mu = 0$. Then, all price increases and decreases are caused by the idiosyncratic productivity shocks, which are horizontal movements in Figure 6 or Figure 7. Now, increase the value of μ. Note that positive μ implies that (p/w) tends to decrease, which is downward movement in Figure 6 or Figure 7. Because the downward movement and vertical movements are assumed to be independent, this increase in the value of μ raises strongly the probability of hitting the lower black grids but not changes the probability of hitting the upper black grids in Figure 6 or 7 very much.

(2) Generate the sequences of money with average growth rate μ and the variance σ_m^2 and the sequences of productivity shocks with η and $\sigma_{v_j}^2$. Based on the pricing strategy consistent with (μ, σ_m), calculate the firm's pricing behaviors.

(3) Aggregating the firm's pricing behavior, calculate the inflation rates and GDP defined as $y_t = \int C_t(p)\phi_t(dp, dv)$.

5.2 Results

The typical results of the above experiments can be seen in Figure 12. In this figure, I choose the level of GDP consistent with a stable price environment, i.e. $(\mu, \sigma_m) = (0, 0)$, as a bench mark. The relationship between inflation and GDP predicted by this calibrated model has some interesting features: (i) Roughly speaking, the slope of Phillips curve is increasing in inflation rates, which is consistent with empirical facts established by Benati (forthcoming). (ii) High inflation, near zero inflation and deflation imply volatile and low average GDP while moderate inflation implies stable and high average GDP, which is consistent with empirical facts about Japan and US data found by Sakura et al. (2005). I explain the reasons of (i) and (ii) below.

The region of inaction for "s sector" firms with current productivity low is relatively wide as seen in Figure 6 and Figure 7. Moreover, you can see that $\hat{\xi}_s$ for firms with low productivity is "distorted" in the sense that this wide region of inaction implies downward nominal price rigidity. Given this downward rigidity, it is easy to explain (i): When μ is positive but very low or negative, then money must decrease sometimes. Note that the decrease of money has some real effects although the increase of money has almost no real effects since there is a certain downward rigidity while there is not significant upward rigidity. Therefore, resulting relationship between inflation and GDP exhibits the feature summarized as (i).

The remaining problem is why the region of inaction exhibits downward rigidity and does not exhibit upward rigidity. Note that the region of inaction for "g sector" firms is narrow mainly because of the relatively small menu cost. The region of inaction for "s sector" firms with high productivity is also narrow because the effective burden of the menu cost is relatively light for firms with high productivity.[27]

The region of inaction for "s sector" firms with low productivity is wide since the menu cost is relatively large and current productivity is low. Intuitive explanation of the "distortion" for "s sector" firms with low productivity can be seen

[27] See the subsection 3.9 and Figure 3 for more details.

in Figure 13. This "distortion" is a result of the assumption that the productivity is mean-reverting. If the current productivity is low, then the expected future productivity is higher than the current one because of the mean-reversion. The discounted value of the firm's profits reflects the profits consistent with higher productivity. Note that the profit consistent with higher productivity is maximized at lower relative price (p/w) since higher productivity implies lower marginal cost. Therefore, the typical relationship between discounted value of the firm's profits and the relative price can be expressed as in the right-hand side of Figure 13.

To explain (ii), I need to show why high inflation implies volatile and low average GDP. Volatile GDP comes from large σ_m.[28] Because of large σ_m, the money sometimes decreases. Low average GDP is a result of firm's pricing strategy: As seen in Figure 14, higher μ generally implies higher $\hat{\xi}_j$. Each firm chooses high $\hat{\xi}_j$ since inflation lowers its real price during the period with its nominal price unchanged. Therefore, higher μ implies upward shift of the distribution of real prices. This dampens the demand for goods and services.

Golosov and Lucas (forthcoming) show that the slope of Phillips curve of the calibrated one-sector GL model is always steep. This is because the calibrated one-sector GL model generates prices such that almost all prices change in first one year after a price change, implying that money is nearly neutral. Thus, these predictions about the relationship between inflation and GDP shown in this section are the new implications obtained as a result of the generalization of the GL model[29].

6 Concluding Remarks

I've presented a two-sector menu cost model with idiosyncratic productivity shocks. I split firms into two sectors since the hazard function for price changes implies heterogeneity in frequency of price changes. I name high frequency sector "g sector" and low frequency sector "s sector." This model includes the GL model as a special case since this model becomes the GL model if and only if each sector is identical. I use the sample moments on individual prices in Japan calculated by Saita et al. (2006) to calibrate the menu cost and the variance and autocorrelation of the idiosyncratic shocks. As a result of calibration, "g sector" is characterized

[28] Usually, σ_m is large when μ is large.

[29] Nakamura and Steinsson (2006b) analyze the menu cost model which allows for intermediate goods. They show that the monetary non-neutrality in their multi-sector model is clearer than that in their one sector model. As for the monetary non-neutrality, the analyses presented here suggest that the difference between one sector model and multi-sector model is larger in deflationary or very low inflationary environments. It is curious if the model of Nakamura and Steinsson (2006b) exhibits the same characteristic.

by relatively small menu cost, large productivity shocks, and low average productivity. The behavior of this economy is studied numerically.

As for price changes, the predictions of the generalized GL model are consistent with almost all facts, including decreasing hazard, found by recent research on microdata of individual prices: The only exception I know is the seasonality of the frequency of price changes. Thus, the performance of the generalized GL model to fit the facts on microdata of individual prices is best among the menu cost models since all previous menu cost models couldn't explain decreasing hazard and seasonality of the frequency of price changes.

Then, I use this calibrated model to conduct numerical experiments on the economy's response to various shocks as Golosov and Lucas (forthcoming) did. The relationship between inflation and GDP predicted by this calibrated model has some interesting features: (1) The slope of Phillips curve is increasing in inflation rates, which is consistent with empirical facts established by Benati (forthcoming). (2) High inflation, near zero inflation and deflation imply volatile and low average GDP while moderate inflation implies high and stable GDP, which is consistent with empirical facts about Japan and US data found by Sakura et al. (2005).

These implications may be important to understand why deflation is bad theoretically.[30] To explain Phillips curve relationship during the recent deflation in Japan, there may be three ways: (A) GDP was lowered by some negative shocks, and deflation was caused by low GDP, (B) deflation was caused by monetary shock, and low GDP was caused by deflation, and (C) deflation and low GDP are independent of each other, implying relationship is observed just by accident. This paper may provide theoretical backbone for (B). Note that, however, this paper doesn't prove that (A) is not true: The views such as (A) are just out of the scope of this paper.

In addition, note that the quantified effects of monetary policy in this paper may not be so precise since I use a very simplified model in this paper. To enhance the precision, further research must be done. I end this paper providing the lists for the future research: (i) Divide firms into more sectors. (ii) Generalize this model so as to deal with the difference in the average productivity growth rate of each sector. (iii) Change the specification of the production function into more realistic form. Especially, include capital into the model. (iv) Change the specification of the utility function into more realistic form.

(i) and (ii) is connected since if you divide firms into multi-sector, then you might need to deal with the difference in the average productivity growth rate of

[30] In this model, the increase in GDP means a welfare improvement. This is because firms have some monopolistic power, implying that the production under the price stability is too small in terms of social welfare.

each sector.[31] There is a possibility that the results in this paper will be changed somewhat if you deal with the difference in the average productivity growth rate of each sector carefully. While this extension is desirable, it is difficult since the problem after this extension is no longer recursive.

(iii) is interesting since investment may amplify the fluctuations caused by monetary policy. It is, however, also difficult because of "curse of dimension" that I include capital into the generalized GL model making state space huge. (iv) is necessary to implement reliable welfare analysis. It is known, however, that any change of the form of the utility function used in this paper makes the problem much harder as suggested by Golosov and Lucas (forthcoming). Thus, I need to leave (i)-(iv) as a future research.

Appendix

A. Value Function Iteration

Here, I describe how to obtain the value of $\psi_j(\tilde{x}, \tilde{v}_j)$ using (26).

(1) Guess the value of $\bar{c}$.[32] Guess the joint distribution of firms in "g sector" and that of firms in "s sector" respectively.[33] Guess the value of ψ_j.[34]

(2) Solve (26) based on the guess I made and update the values of ψ_j. If the values of ψ_j are close enough, in the sense of sup norm, to the previous guess of them, go to the next step. Otherwise, go back to the step (1) and use the values of ψ_j as a new guess.

(3) Given the values of ψ_j obtained in the previous step, obtain the pricing strategy (policy function) of "g sector" firms and that of "s sector" firms. Using these strategies and taking account of the effects of π_j, change the joint distribution of firms in "g sector" and that of firms in "s sector." Based on the obtained joint distributions, calculate the value of $\bar{c}$. If these joint distributions and the value of $\bar{c}$ are close enough to the previous guess of them, stop. Otherwise, go back to step (1) and use these joint distributions and the value of $\bar{c}$ as a new guess.

[31] In the two-sector case in Japan, there is no difference in the average productivity growth rate of each sector (Figure 5). This might be because of good luck.

[32] If you have a good guess as a result of calculations for different but similar values of parameters, use it as an initial guess. If you don't have a good guess, calculate the value of $\bar{c}$ in the case of zero variance of all shocks and zero menu cost, and use this value as an initial guess.

[33] If you don't have a good guess, calculate the joint distribution of firms in "g sector" and that of firms in "s sector" in the case of zero variance of all shocks and zero menu cost, and use these joint distributions as an initial guess.

[34] If you don't have a good guess, use zeros as an initial guess.

B. Method of Simulation to Obtain Hazard Functions

Here, I describe how to obtain the hazard rate implied by the generalized GL model with given parameter values.

(1) Generate the sequence of money with average growth rate μ and the variance σ_m^2 and the sequences of productivity with η and $\sigma_{v_j}^2$ starting with grids $(\hat{\xi}_j(\tilde{v}_j), \tilde{v}_j)$ using random numbers and (30) – (34).[35]

(2) Record the timing of price changes given the pricing strategy consistent with the parameter values, the sequence of money and that of productivity. Based on the invariant joint distribution consistent with the parameter values, obtain the weight of firms with each productivity level which change their nominal prices. Using these weights and information of the timing of changes of each price starting with grids $(\hat{\xi}_j(\tilde{v}_j), \tilde{v}_j)$, calculate the hazard rates.

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[36]The English version of this paper is forthcoming.

Table 1: Preference Parameters

(1) Subjective Discount Rate (Quarter) ρ [Golosov and Lucas (2006): 0.01]

Research Paper	Subjective Discount Rate
Kitamura and Fujiki (1997)	0.01
Hayashi and Prescott (2002)	0.006

(2) Relative Risk Aversion γ [Golosov and Lucas (2006): 2]

Research Paper	Relative Risk Aversion
Kitamura and Fujiki (1997)	0.6 - 2.5
Yoshikawa (2001)	1.36
Moridaira and Kamiya (2001)	1

(3) Elasticity of Substitution ϵ [Golosov and Lucas (2006): 7]

Research Paper	Elasticity of Substitution
Nishimura, Ohkusa, and Ariga (1999)	7
Inui and Kwon (2004)	6

Note: Nishimura, Ohkusa, and Ariga (1999) and Inui and Kwon (2004) estimate not the elasticity of substitution but the mark-up. In this model, the mark-up is a function of the elasticity of substitution. Given the relationship, I find the value of the elasticity of substitution which is consistent with the estimated value of the mark-up.

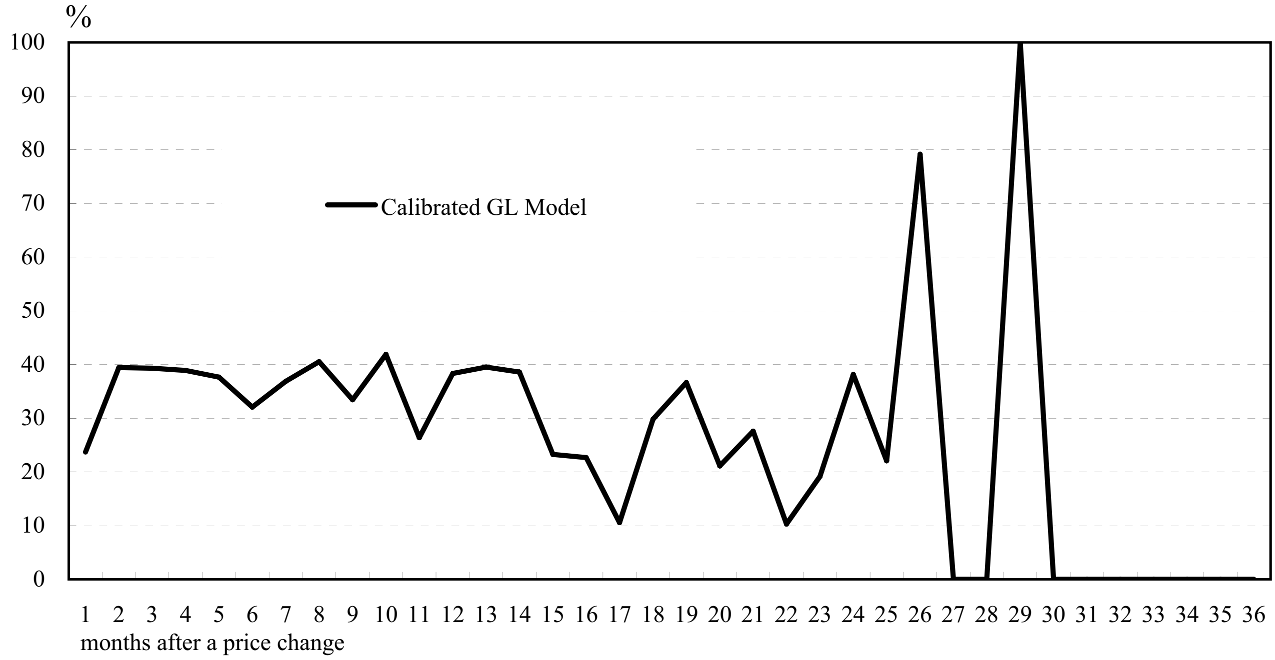
Table 2: Parameters

	Parameters	Value of Parameter	
μ	Average Growth Rate of Money (Quarter)	-0.0017	(0.0064)
σ_m	S.d. of monetary shocks	0.0037	(0.0062)
k_g	menu cost ("g sector")	0.00055	(0.0025)
k_s	menu cost ("s sector")	0.008	(-)
η	Rate of Mean Reversion	0.75	(0.55)
$\sigma^2_{V_g}$	Variance of productivity shocks ("g sector")	0.011	(0.011)
$\sigma^2_{V_s}$	Variance of productivity shocks ("s sector")	0.0005	(-)
e_g	Average productivity of "g sector" firms	-0.05	(0)
e_s	Average productivity of "s sector" firms	0.05	(-)
	Fraction of "g sector" firms	0.5	(1)
	Fraction of "s sector" firms	0.5	(-)

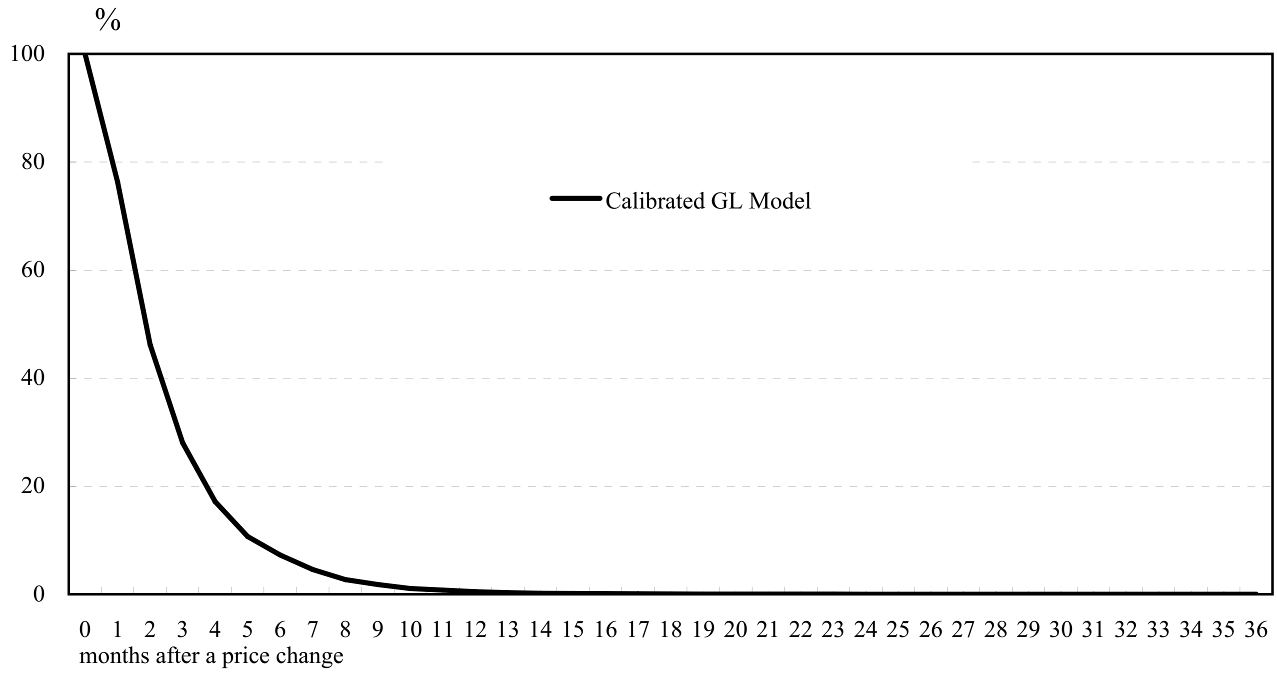
Note: Each value in the parenthesis is the value used by Golosov and Lucas (forthcoming).

Figure 1: Hazard Rates and Survival Rates of Calibrated GL Model

(1) Hazard Rates



(2) Survival Rates



Notes: 1. I use the values of parameters Golosov and Lucas (forthcoming) obtain as a result of their calibration. (I use 0.0062 as σ_m .)

2. As for the definition of hazard rates and survival rates, see subsection 4.1.

3. As for the method of simulation to obtain hazard rates, see Appendix B.

Figure 2: Decision Making on Price

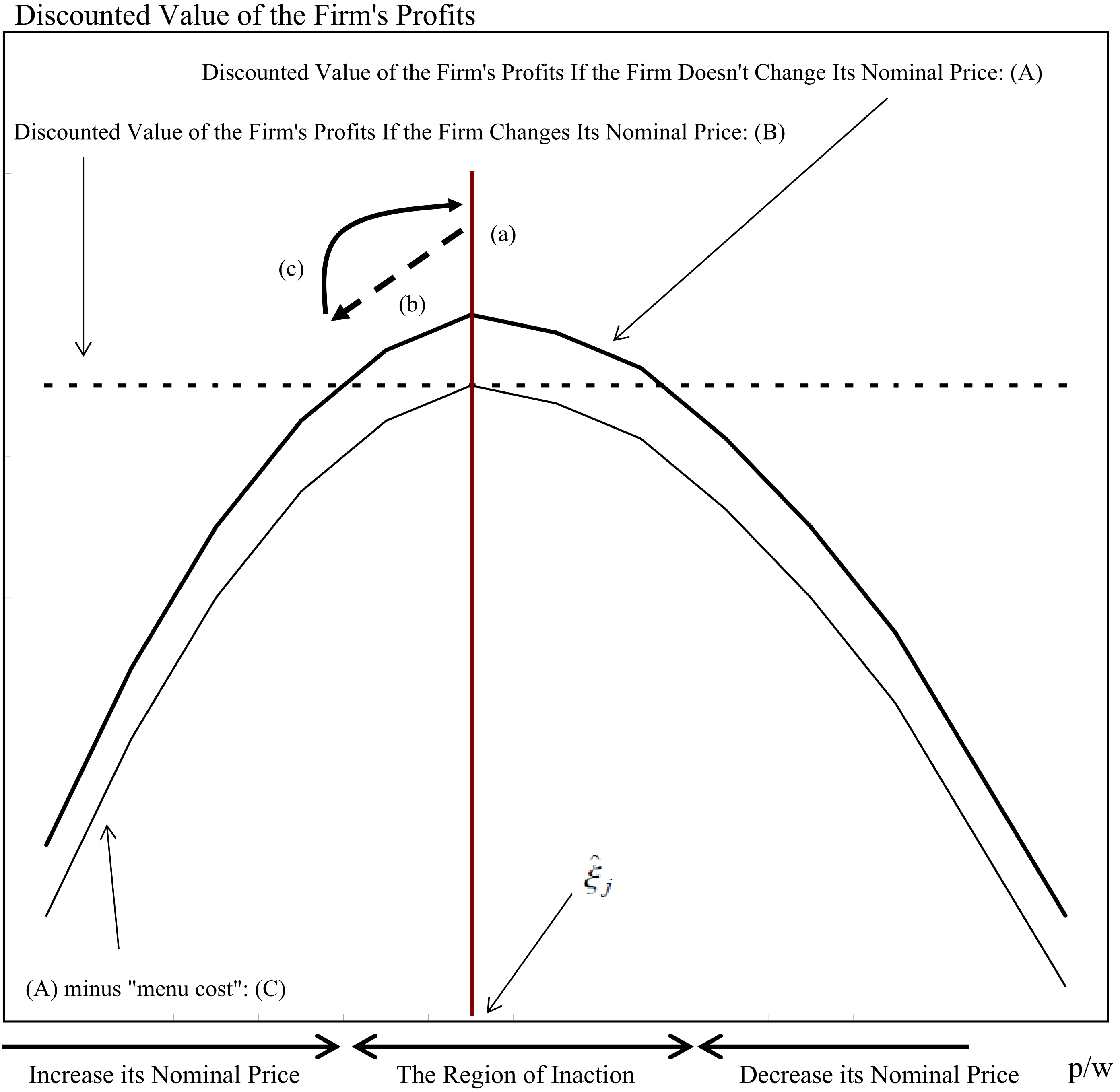


Figure3: The Region of Inaction and Productivity shocks

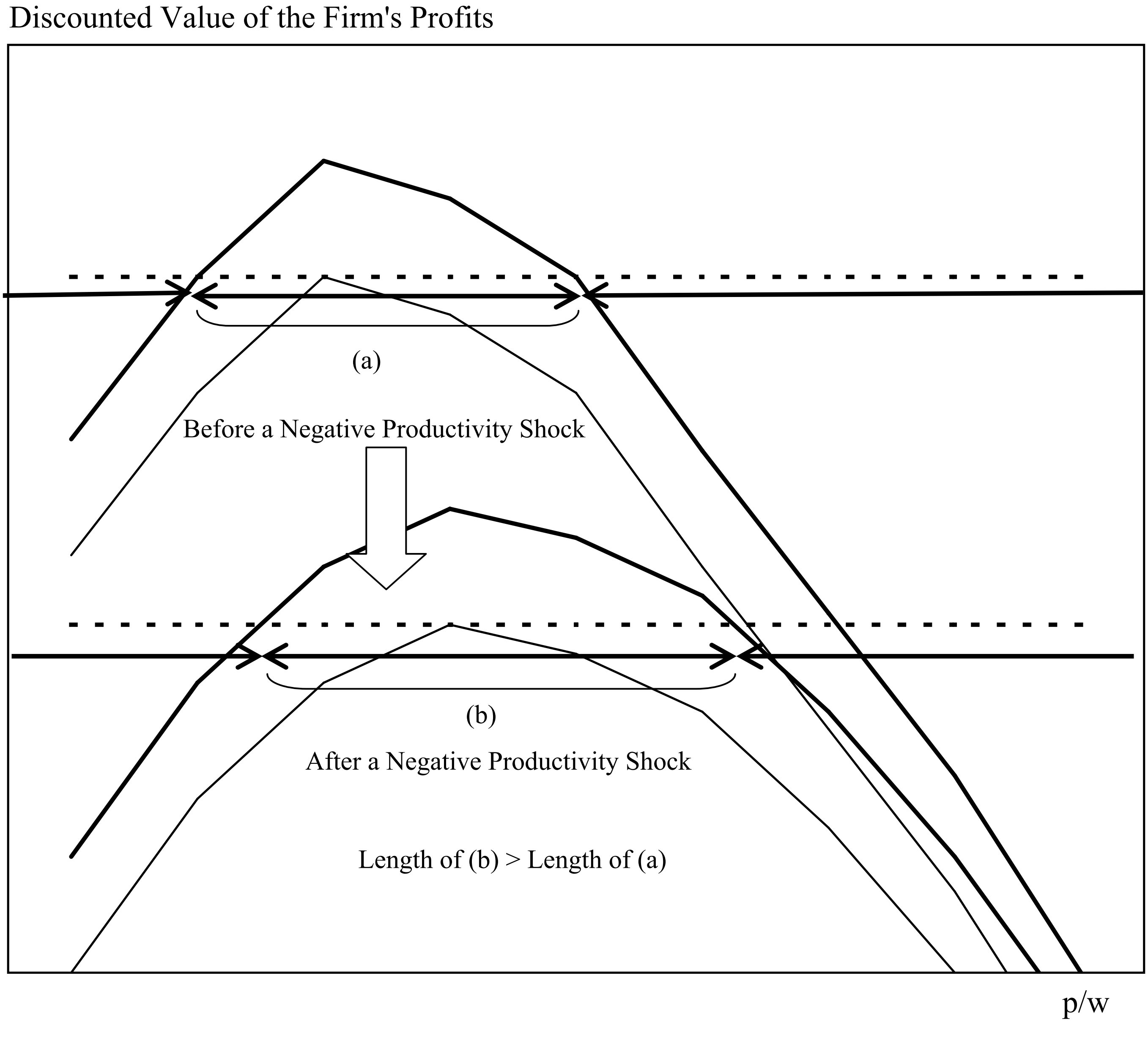


Figure 4: Size of Price Change and Variance of Productivity Shocks

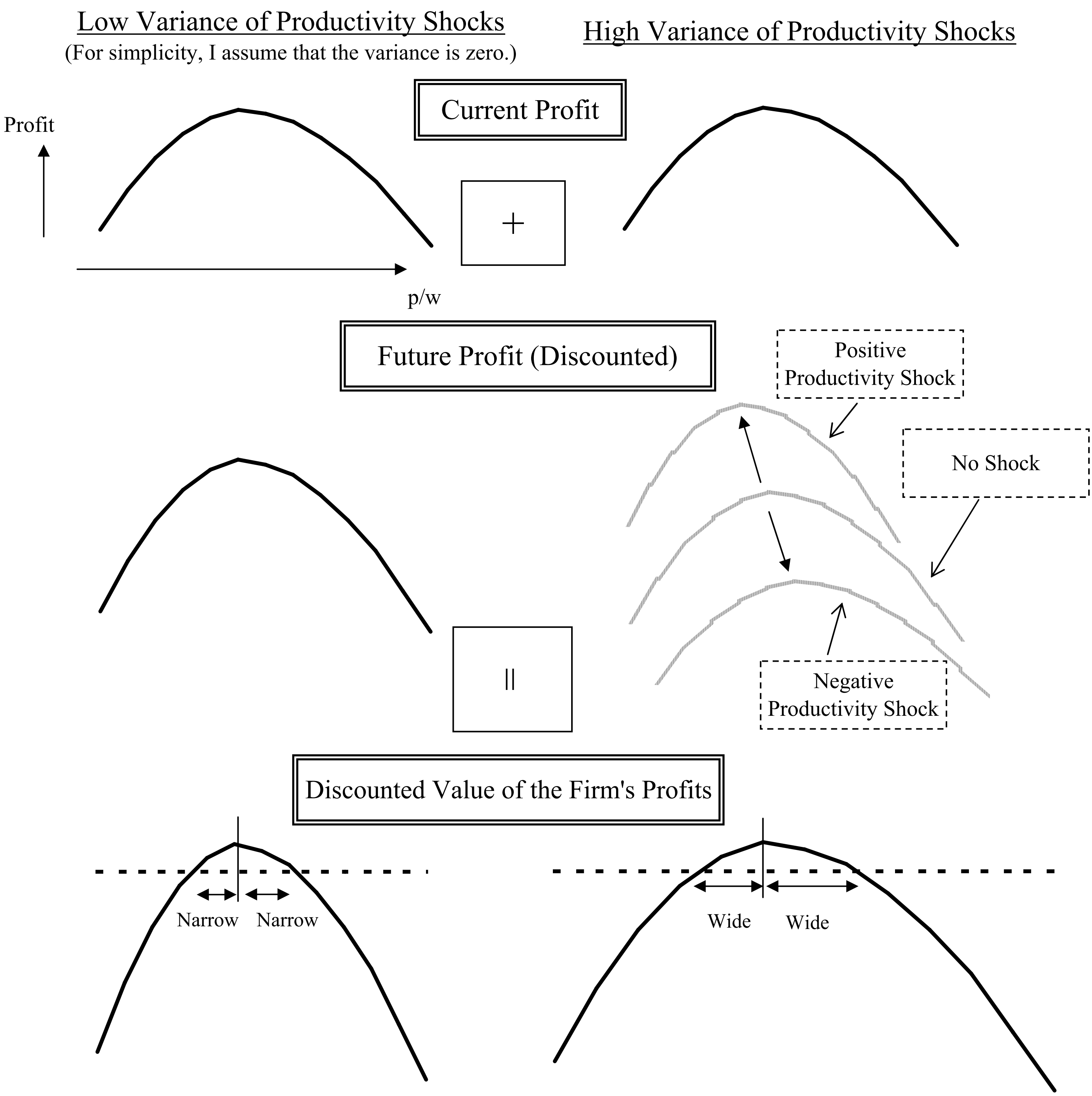


Figure 5: Difference of Average Productivity between goods and services (services/goods)

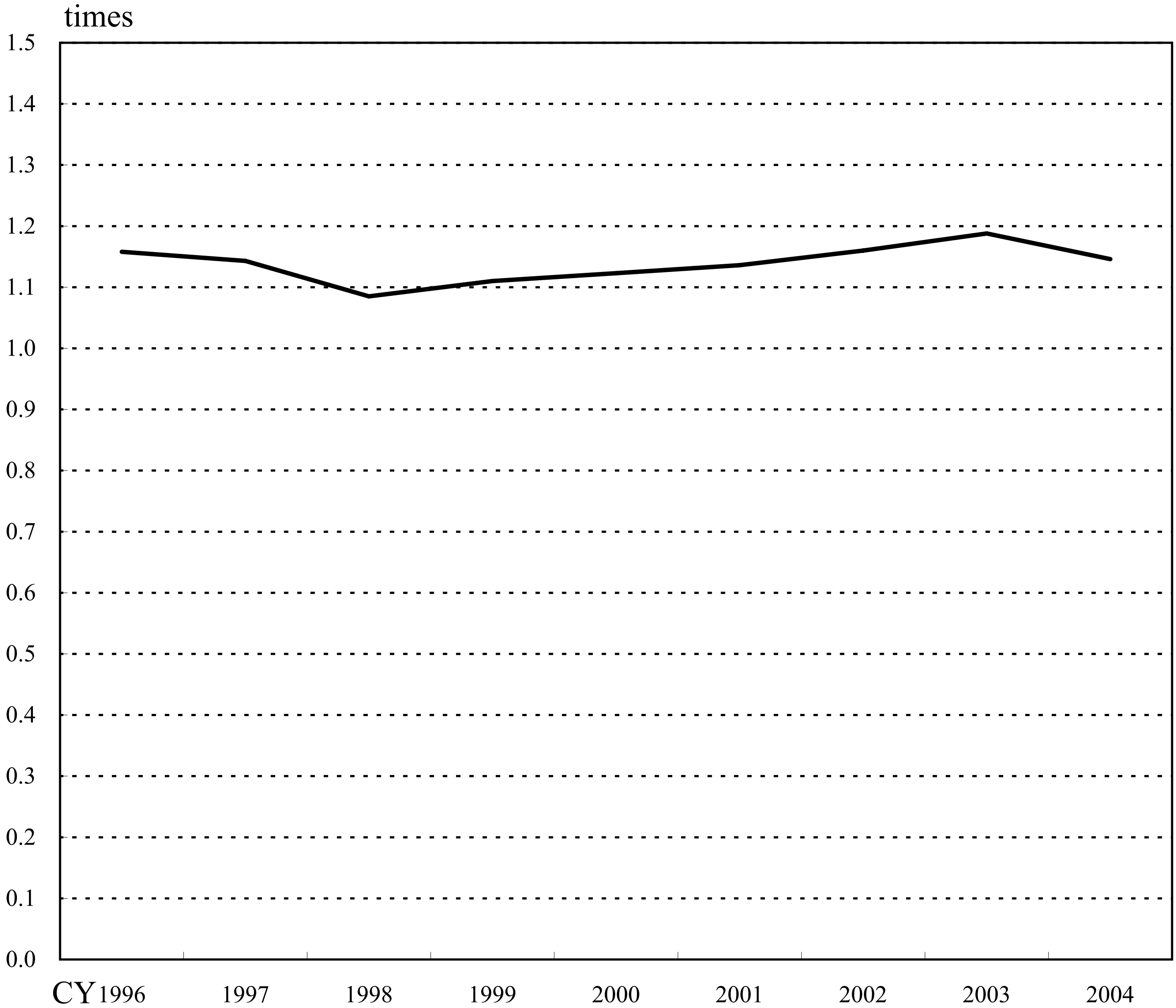


Figure 6: Pricing Strategy ("g sector")

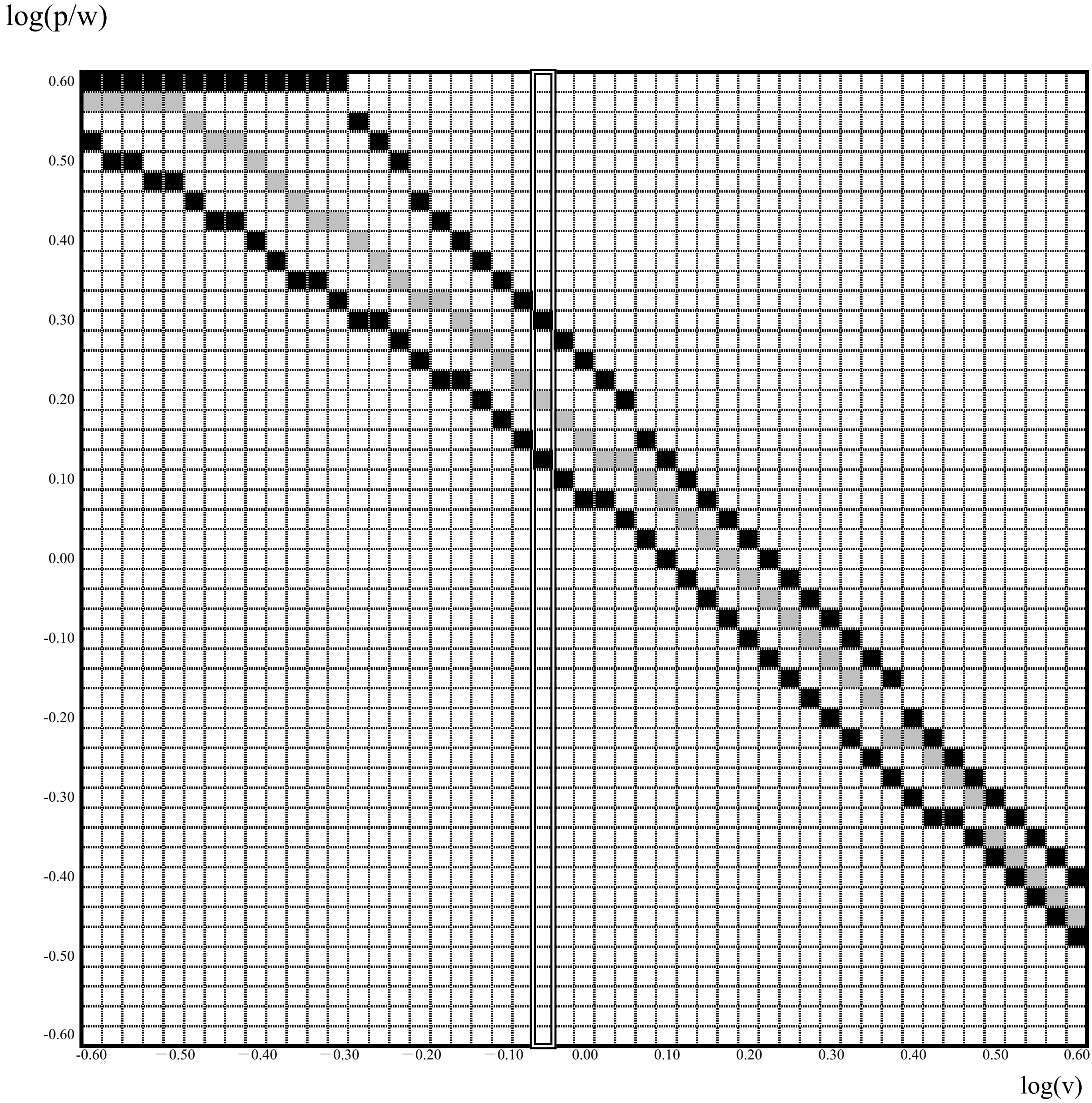


Figure 7: Pricing Strategy ("s sector")

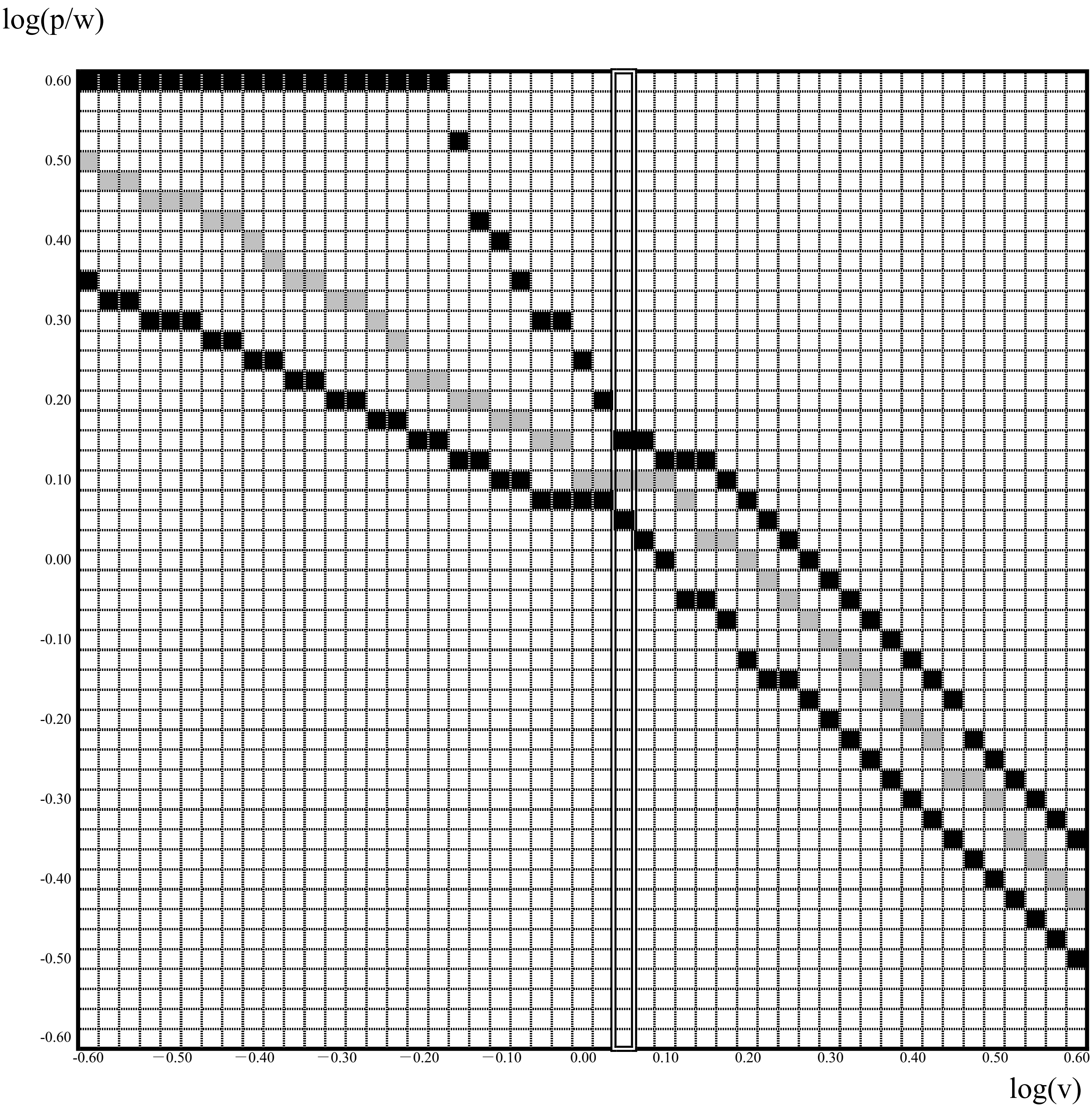
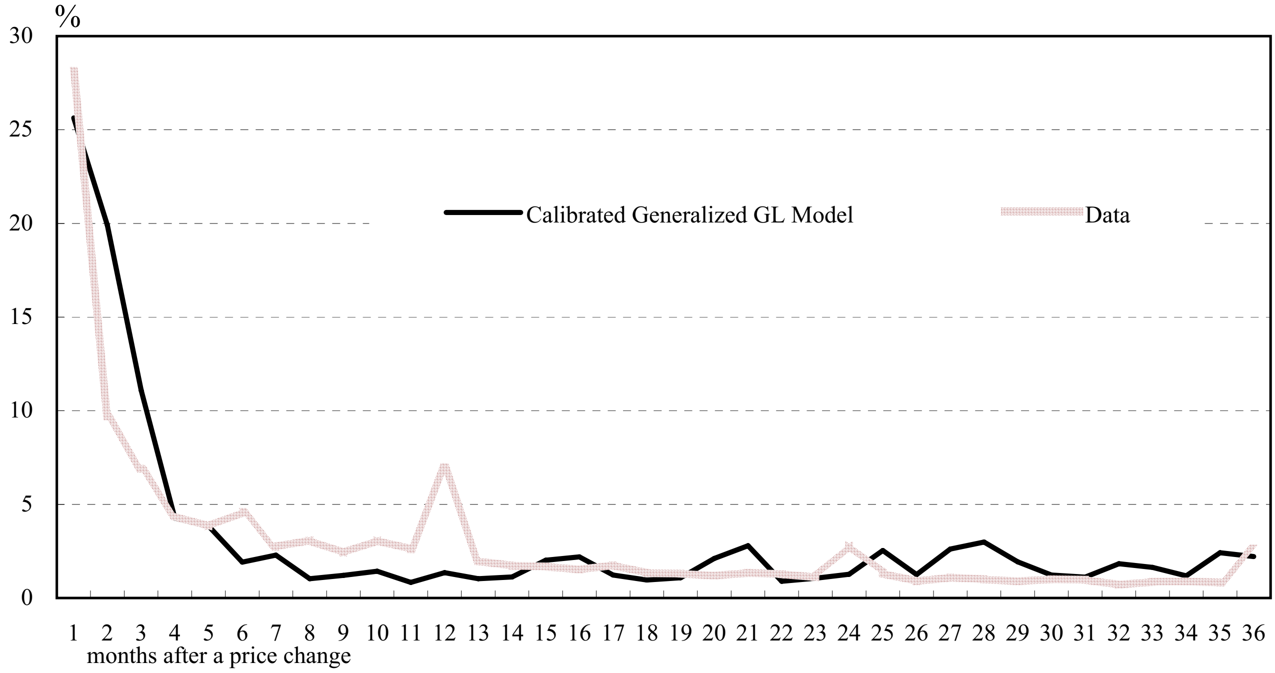


Figure 8: Hazard Rates and Survival Rates of Calibrated Generalized GL Model

(1) Hazard Rates



(2) Survival Rates

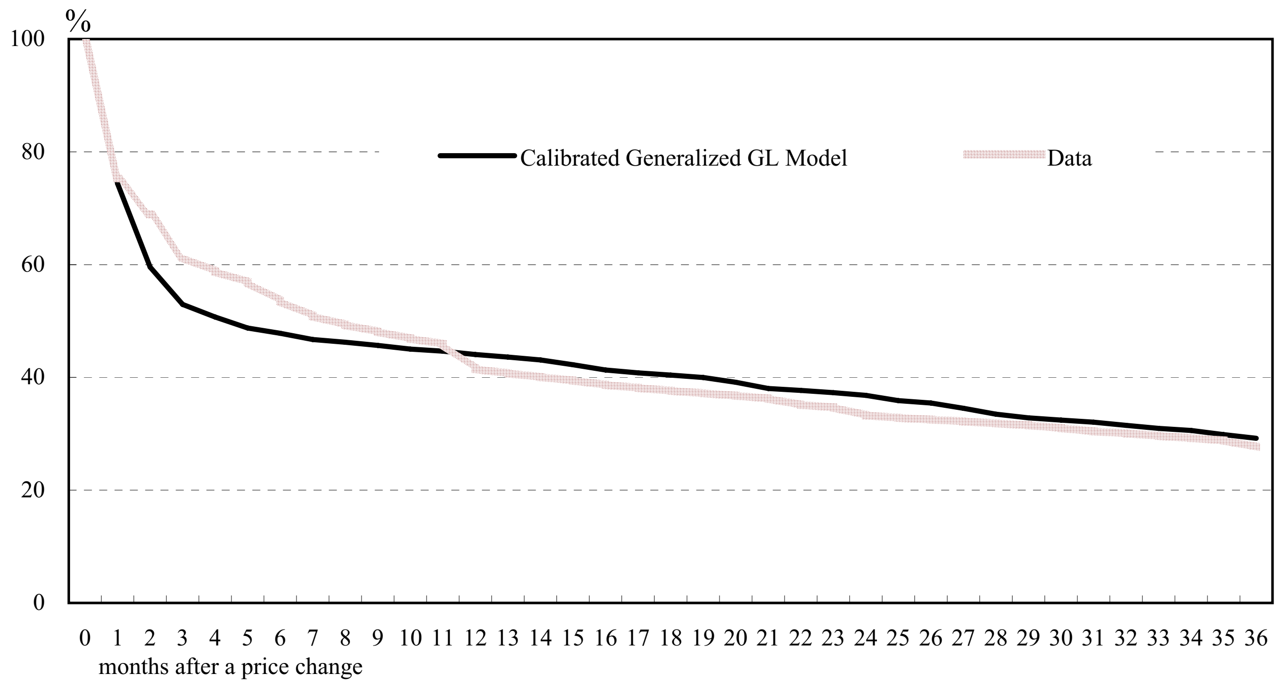
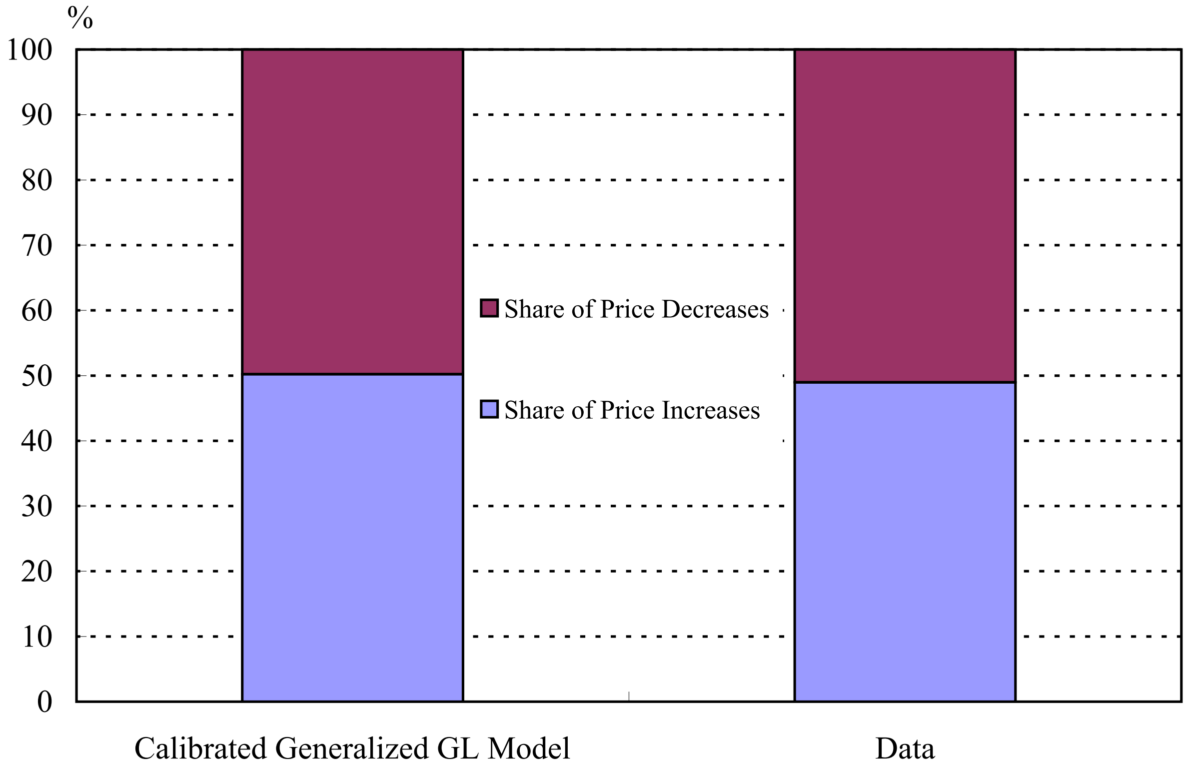


Figure 9: Share of Price Increases (Decreases) and Average Size of Price Changes

(1) Share of Price Increases (Decreases) in Price Changes



(2) Average Size of Price Changes

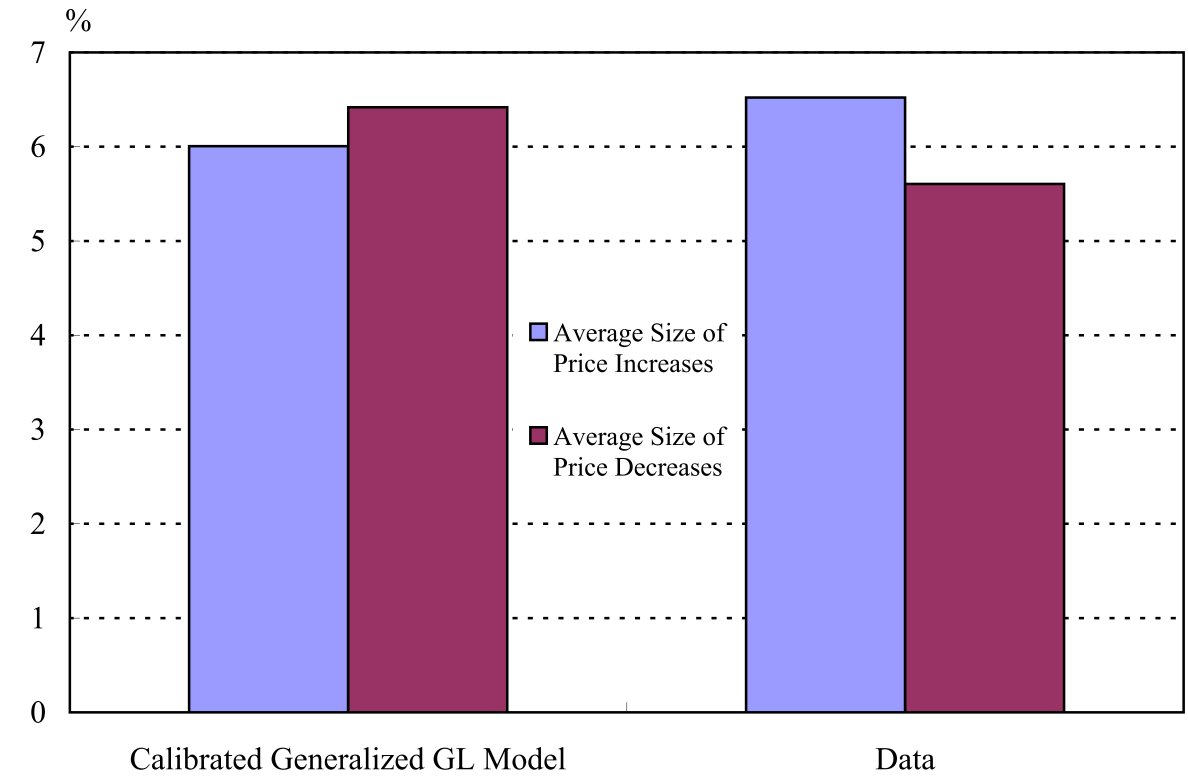
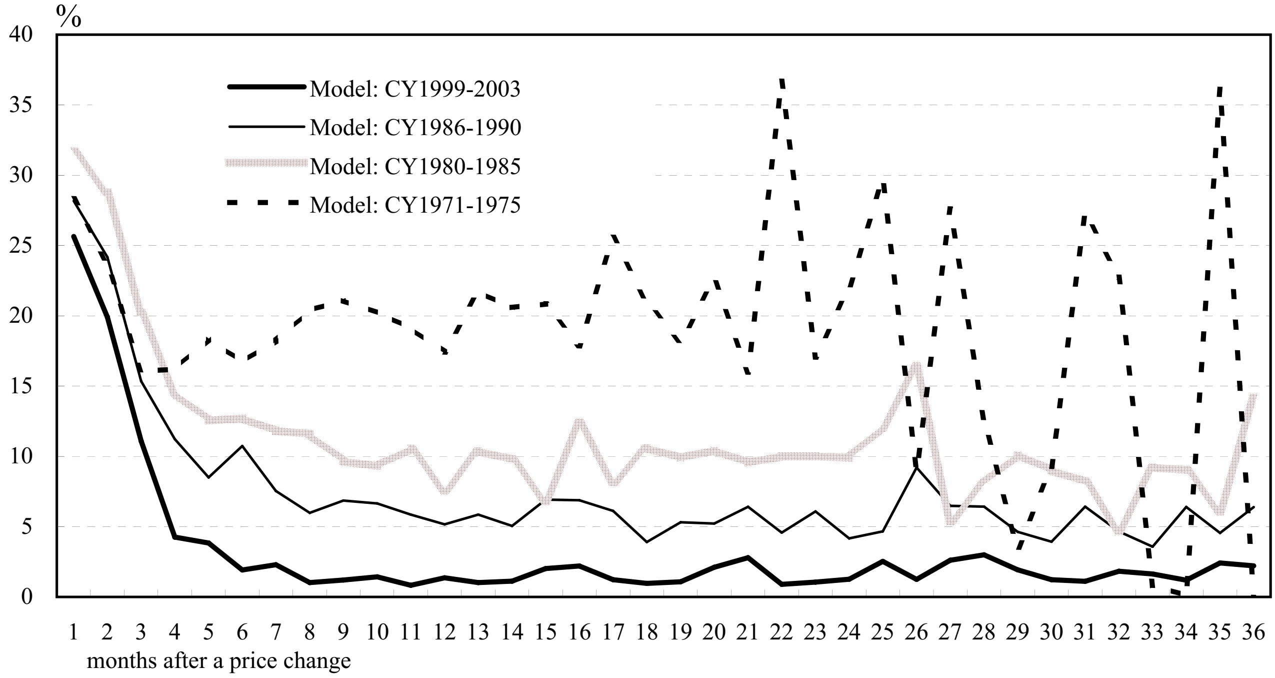


Figure 10: Hazard Rates and Survival Rates under Different Monetary Shock Processes

(1) Hazard Rates



(2) Survival Rates

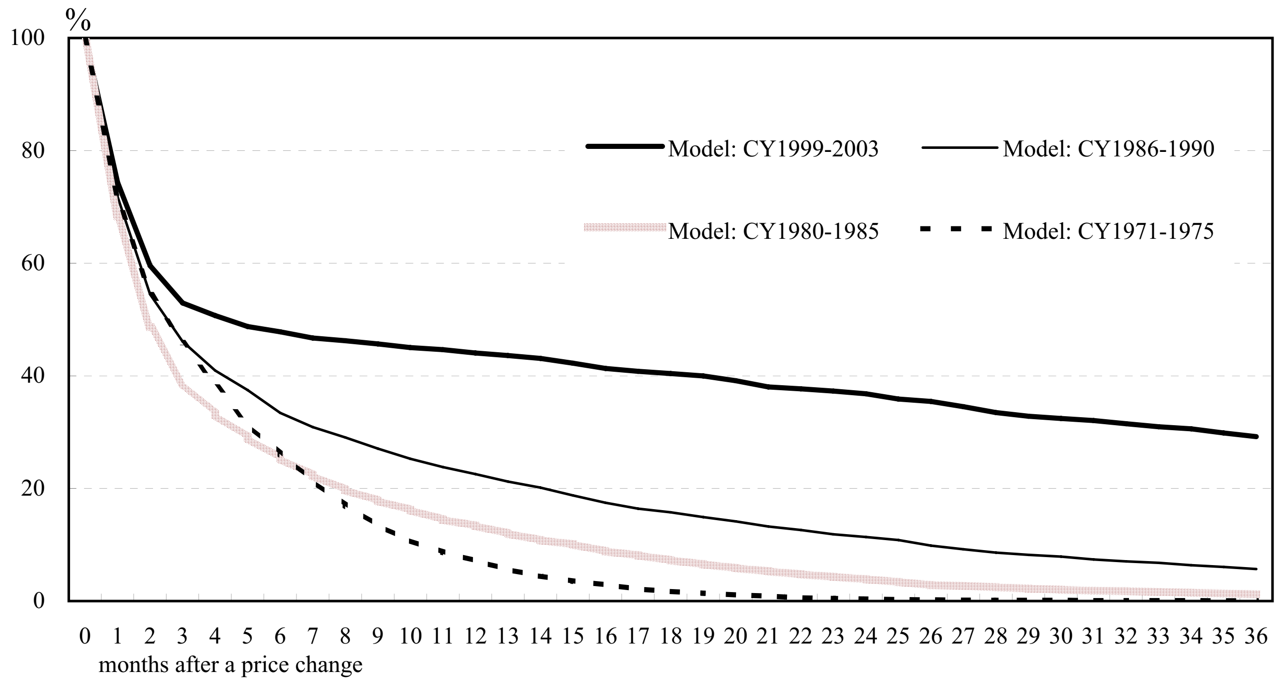
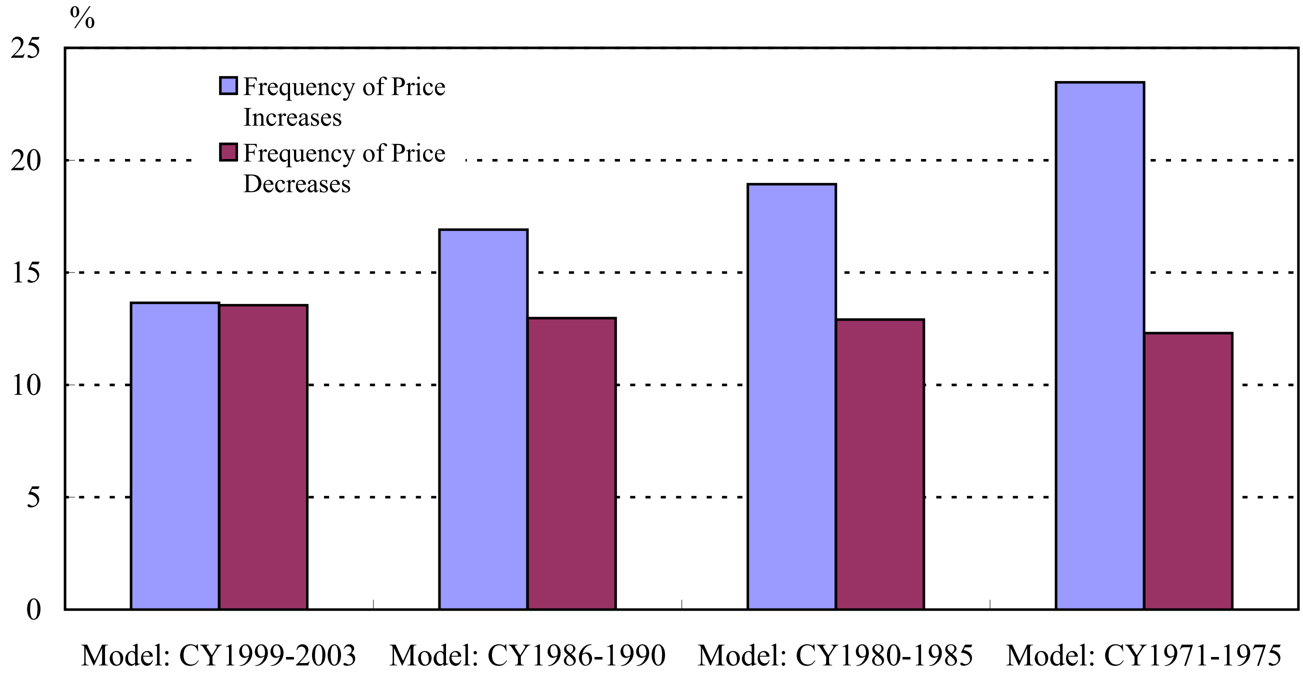
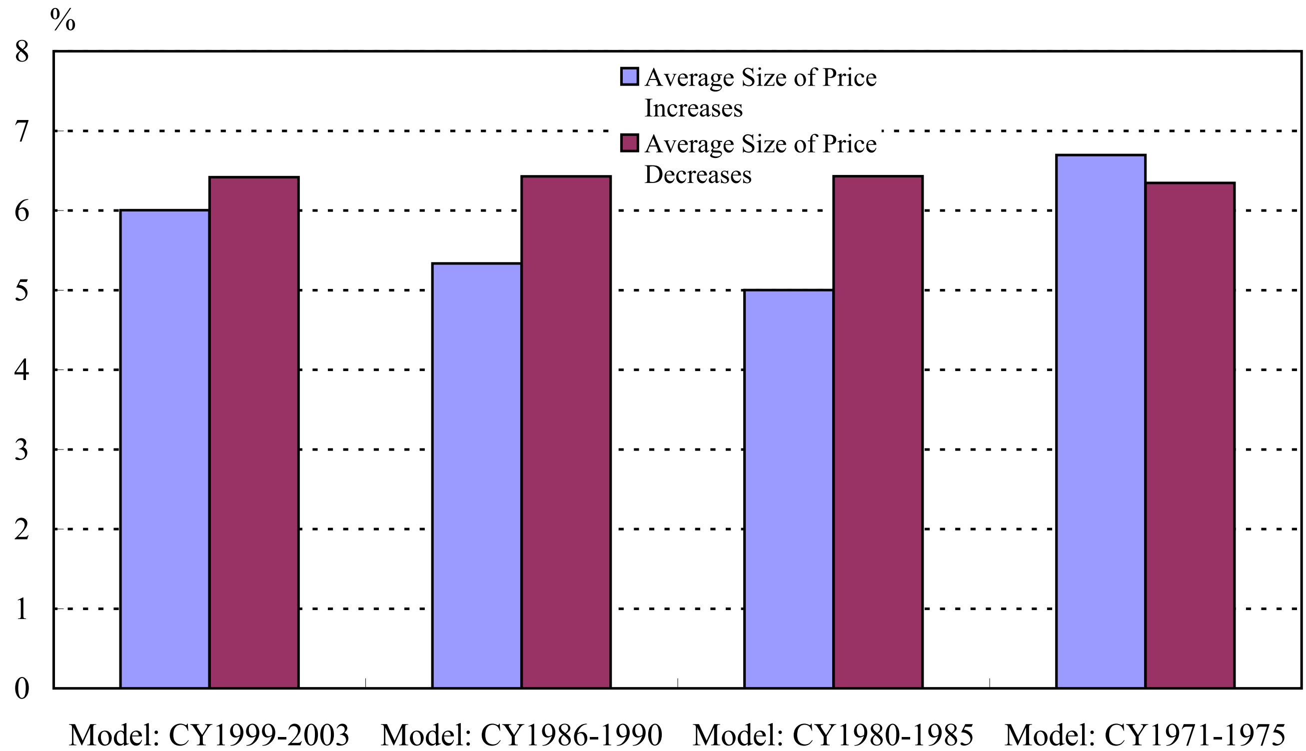


Figure 11: Frequency of Price Increases (Decreases) and Average Size of Price Changes under Different Monetary Shock Processes

(1) Frequency of Price Increases (Decreases)

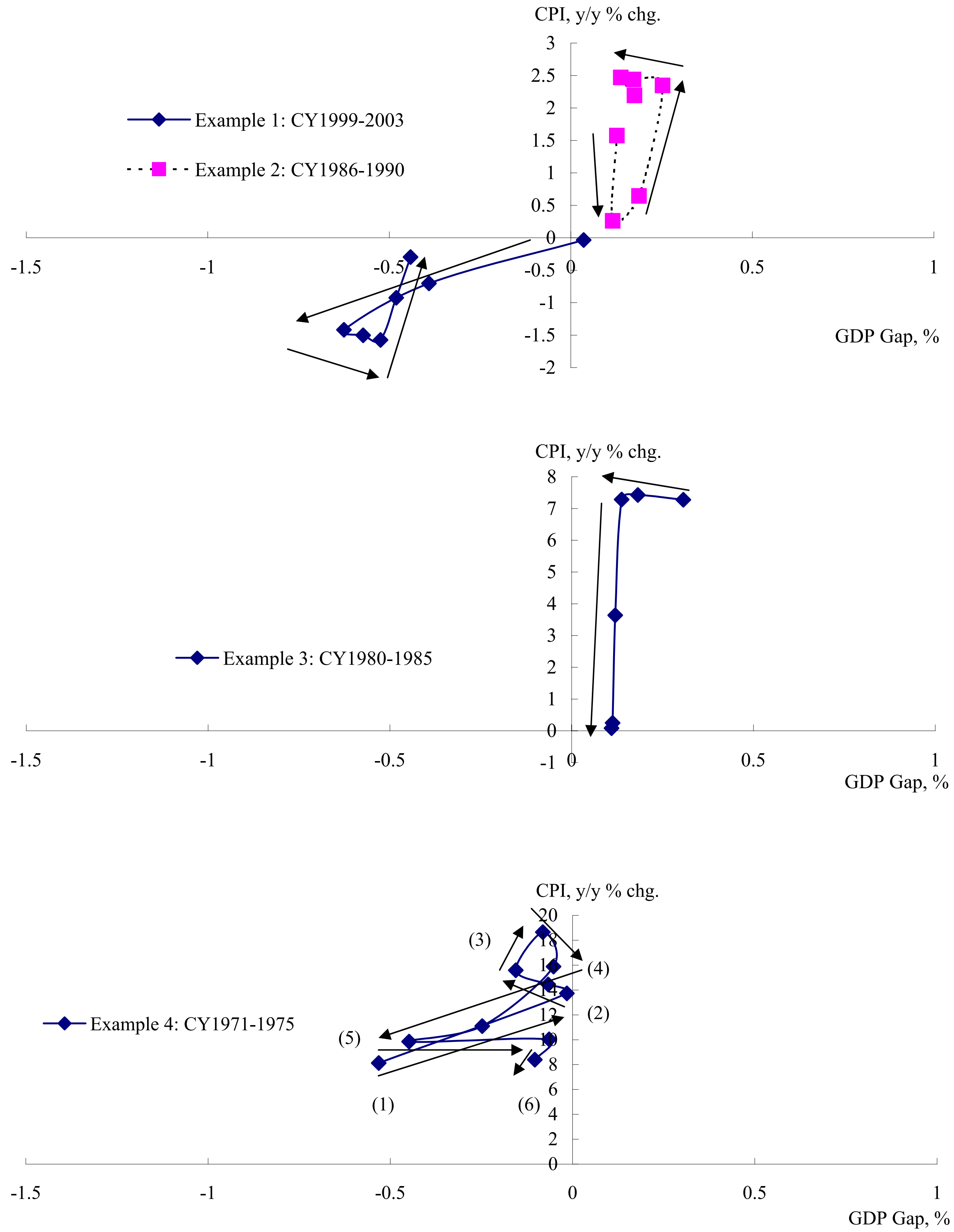


(2) Average Size of Price Increases (Decreases)



Note: The frequency of price changes is calculated based on the invariant distribution. For example, if the time interval is 1/10 month and the frequency of price changes per 1/10 month implied by the invariant distribution is 0.01, then my estimate for the frequency of price changes per month is 0.1. The frequency of price changes in (1) is too high. since there can be sequence of prices which change twice or more within one month. To estimate the measure of double (or more) counting, I perform some simulations. As a result, I found that about 2.5 percent is double counting in the case of CY1999-2003: Before excluding double counting, the frequency of price changes is 27.2 as seen in (1). After excluding double counting, the frequency becomes 24.8.

Figure 12: Phillips Curve



Notes: 1. Each point represents the quarterly value. I obtain each quarterly value by taking the means of the relevant variables over the quarter.

2. GDP Gap is defined as a percentage deviation of the real GDP from the benchmark GDP. Benchmark GDP is defined as GDP consistent with a stable price environment, i.e. (μ , σ_m) = (0, 0)

Figure 13: Nominal Price Rigidity and Mean Reversion of Productivity Shocks

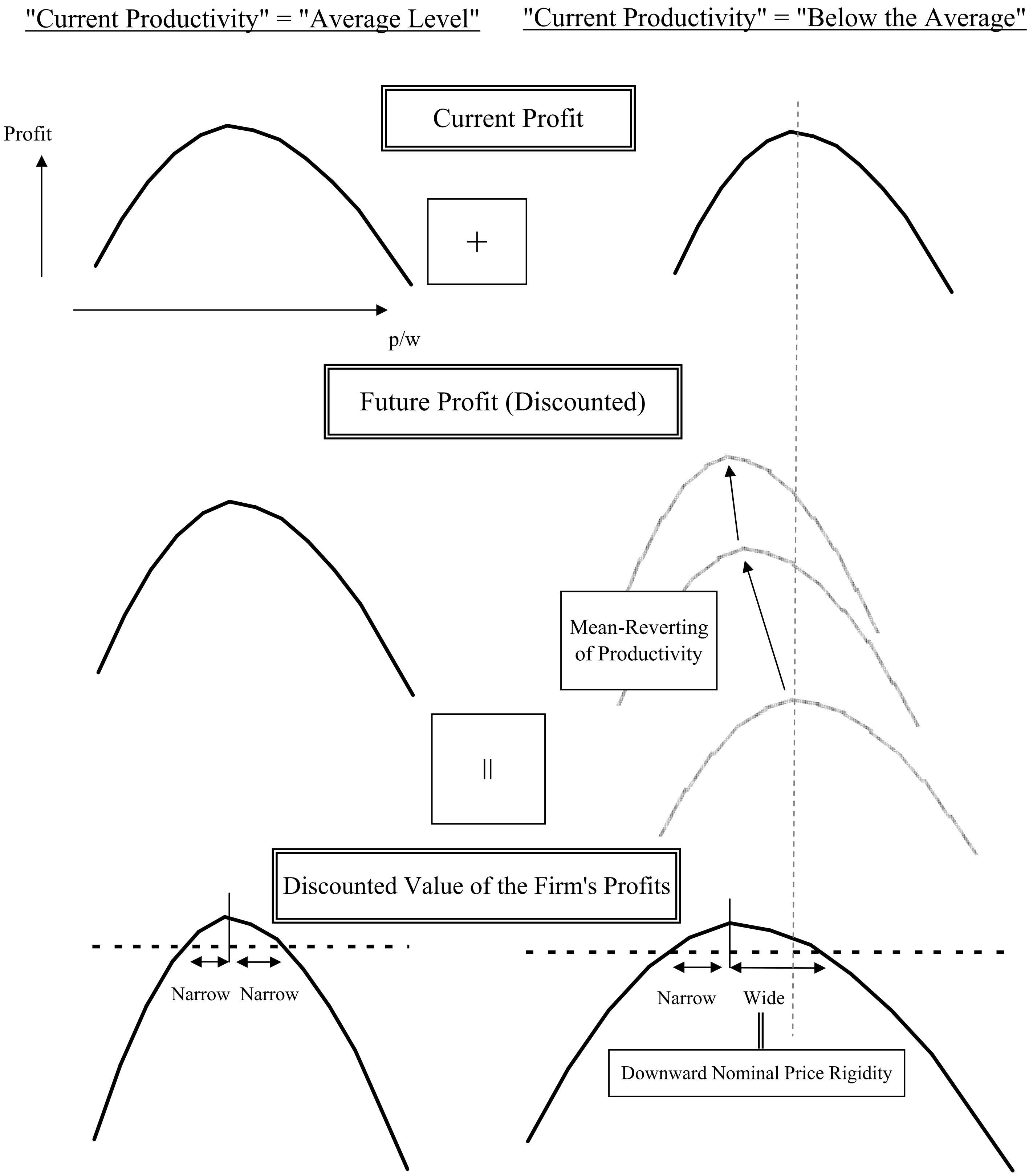
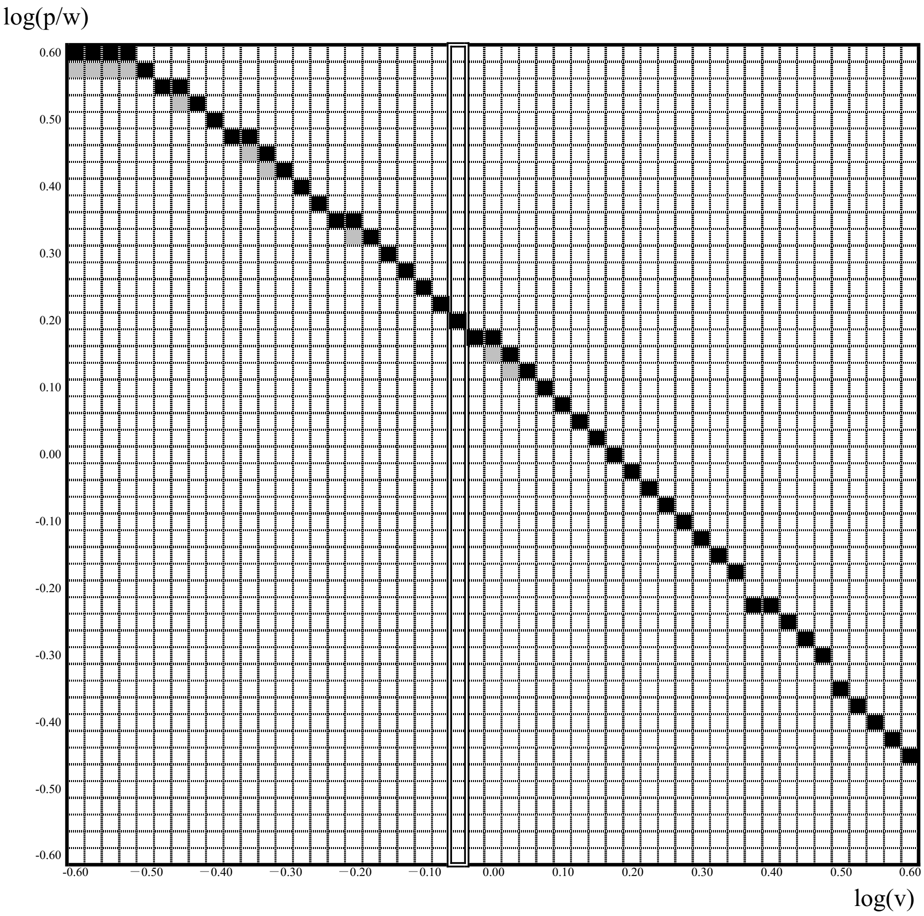


Figure 14: $\hat{\xi}_j$ under Different Monetary Shock Processes

(CY1999-2003: Gray →CY1971-1975: Black)

"g sector"



"s sector"