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Phillips Correlation and Trend Inflation under the Kinked Demand Curve

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Phillips Correlation and Trend Inflation under the Kinked Demand Curve

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Abstract

This paper explains the weak 'Phillips correlation' under low trend inflation. This correlation is confirmed empirically but the standard sticky price models fail to account for it. This paper extends the standard sticky price model to the case of the "smoothed off kinked" demand curve, which is typically regarded as a source of the strategic complementarity. Our results suggest that the kinked demand curve can offer an appropriate explanation to fill this gap between the theoretical implication and the empirical facts.

JEL classification: E31, E32

Keywords: sticky prices; trend inflation; kinked demand curve

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1 Introduction

It is a conventional view that the output-inflation correlation, the 'Phillips correlation', is weak under the low trend inflation environment. Ball, Mankiw, and Romer (1988) (hereafter BMR) suggest that the short-run Phillips curve depends on the average rate of inflation, and that it becomes *flatter* when the average rate of inflation declines. Recently, Benati (2007) statistically verifies the BMR's argument, using the data from the post-WWII OECD countries. He concludes that *"the Phillips trade-off Alan Greenspan was facing towards the end of 1980s was not the same as the one faced by Paul Volcker at the beginning of the decade.*"

However, the standard sticky price models, which occupy the predominant position in the recent monetary policy analysis,[1] fail to account for these empirical facts. Notably, Ascari (2004) first finds that the slope of the new Keynesian Phillips curve (NKPC) becomes *steeper* under lower trend inflation. Further, Ascari and Ropele (2007) investigate the general equilibrium outcomes of trend inflation in the standard sticky price models. Their results suggest that the Phillips correlation comes to be *stronger* under low trend inflation than under high trend inflation. These theoretical implications of trend inflation are not consistent with the empirical facts.

A limitation of the standard sticky price models is that it assumes the demand faced by a price-setter has a constant elasticity (CES) form. This assumption implies that the quantity of demand for a fixed-price product grows explosively under positive trend inflation. Then, the expected response of the future demand is highly sensitive to the relative price variations, with the flatter slope of the demand curve. Consequently, firms come to be more reluctant to charge the cost variations onto the current prices, thus making the price level responses more sluggish under the positive inflation environment.

This paper proposes a new mechanism, which can help to capture the desired re-

[1] The sticky price model with the Calvo (1983) type infrequent price adjustment and the monopolistic competition is the workhorse in this literature. It is used to study various topics such as the monetary policy rules (*e.g.,* Goodfriend and King (1997); Clarida, Galí, and Gertler (1999); Woodford (2003)), the optimal monetary policy (*e.g.,* Aoki (2001); Woodford (2003)) and inflation dynamics (*e.g.,* Galí and Gertler (1999); Sbordone (2002)). Most of these research typically assume zero trend inflation at the steady state and log-linearize around zero inflation.

sponses of demand, over the current research on the trend inflation. To be specific, we replace the CES demand curve in the standard sticky price model with the 'smoothed off kinked' demand curve (Kimball (1995)), wherein consumers flee from relatively expensive products but do not flock to inexpensive ones. The kinked demand is expected to avoid the extreme expansion of the demand for the relatively cheap goods and to overcome the discrepancy of the new Keynesian models under trend inflation. Moreover, recent empirical evidence from scanner data gives a support to incorporate the kinked demand curve in the general equilibrium settings (Dossche, Heylen, and den Poel (2006)).

In general, the kinked demand curve is considered to generate the endogenous persistence through the strategic complementarity among the price-setters (Kimball (1995), Bergin and Feenstra (2000) and Dotsey and King (2005)).[2] This paper aims to extend these studies to the trend inflation issues. Namely, employing the Kimball (1995) type kinked demand curve, we derive the NKPC under the a lá Calvo sticky price setting and investigate the effect of trend inflation in a simple general equilibrium models.[3]

We are successful in making clear the following two points; (i) contrary to the existing theoretical results of the typical sticky price models, the coefficient of the NKPC under the kinked demand is increasing with respect to the rise of the trend inflation rate; (ii) embedding our NKPC into a simple dynamic general equilibrium model, we show the greater impulse response of output and the smaller reaction of inflation to a shock under low trend inflation. It is concluded from our analytical results that the kinked demand explains the gap between the theoretical implication of the new Keynesian models and the empirical facts.

Concerning to the flatter slope of the Phillips curve under low inflation, the past

[2]Kimball (1995) first incorporates the smoothed off kinked demand curve in the sticky price model as a source of the real rigidity. Although Kimball (1995) employs the concave demand aggregator in an implicit form, Bergin and Feenstra (2000) adopt the explicit trans-log demand aggregator. Moreover, Dotsey and King (2005) give the specific function form to the Kimball's type concave demand aggregator. All there studies are motivated to generate the endogenous persistence in the sticky price models and do not mention the role of the non-zero trend inflation rate.

[3]Dotsey and King (2005) introduce Kimball (1995)'s demand into the state-dependent pricing model of Dotsey, King, and Wolman (1999).

literature has stressed the role of the time-varying price rigidities. BMR and Romer (1990) claim that the frequency of the price adjustment is lower under the low inflation environment. Recently, Bakhshi, Burriel-Llombart, Khan, and Rudolf (2003) apply the Romer (1990)'s idea to the typical sticky price model and derive the flatter slope of the NKPC under low inflation. Besides, Tobin (1972) and Akerlof, Dickens, and Perry (1996) claims that the unemployment rate increases under the low inflation period because the nominal prices and the nominal wages tend to be more rigid downwards than upwards. Consequently, the Phillips curve is flattering when the inflation rate is near zero.

Our approach complements to these line of research but is different from theirs in that we intend to shed light on the real rigidities from the demand behavior, instead of the rigidities in the price-setting behavior or the wage-setting behavior.

The next section presents the model structure, highlighting the use of Kimball (1995) and Dotsey and King (2005) concave demand aggregator. Section three shows the generalized NKPC. Section four discuss the role of the kinked demand curve. Section five compares the equilibrium dynamics of low trend inflation and that of high trend inflation under the Kimball's demand aggregator. Finally, section six concludes the paper.

2 The model

We construct a simple monopolistic competition sticky price model to clearly illustrate the implication of trend inflation under the "smoothed off kinked" demand curve. In our setup, production is linear in labor input, and consumption and labor effort are separable in utility. Thus, trend inflation and the kinked demand are the key features in this model. To begin with, we specify the Kimball (1995) type demand aggregator, following Dotsey and King (2005). Then, the settings of the economy is present.

2.1 Kimball demand aggregator

We assume that any price-setter indexed by $i \in (0, 1)$ is a monopolistic competitor who produces a differentiated good i. Letting $D(\cdot)$ the demand aggregator, the cost minimization problem of a representative household is,

$$\min_{\{C_t(i)\}} \int_0^1 P_t(i)C_t(i)di \quad s.t. \int_0^1 D\left(\frac{C_t(i)}{C_t}\right)di = 1.$$

where $C_t(i)$, C_t, and $P_t(i)$ is the consumption of a good i, the aggregated consumption and the nominal price of a good i, respectively. C_t is implicitly defined, given the demand aggregator $D(\cdot)$. Kimball (1995) presumes a function $D(\cdot)$ suffices $D(1) = 1, D(\cdot)' > 0$, and $D(\cdot)'' < 0$, for all $C_t(i)/C_t > 0$.

Here, we specify the function form of the aggregator, as in Dotsey and King (2005).

$$D(\tilde{C}_t(i)) = \frac{1}{(1+\eta)\gamma}\left[(1+\eta)\tilde{C}_t(i) - \eta\right]^{\gamma} - \left[1 + \frac{1}{(1+\eta)\gamma}\right] \tag{1}$$

where $\tilde{C}_t(i) \equiv C_t(i)/C_t$, $\gamma \equiv (\epsilon(1+\eta) - 1)/(\epsilon(1+\eta))$, and $\epsilon > 1$.

In this function form, we have one more parameter, η, compared to the Dixit and Stiglitz (1977) type CES function form. This parameter determines the curvature of the demand curve. Furthermore, in the case of $\eta = 0$, $D(\cdot)$ reduces to the CES demand aggregator.

Solving the cost minimization problem gives the following inverse demand function,

$$\frac{C_t(i)}{C_t} = \frac{1}{1+\eta}\left[\left(\tilde{P}_t(i)\frac{P_t}{\lambda_t}\right)^{\frac{1}{\gamma-1}} + \eta\right] \tag{2}$$

where $\tilde{P}_t(i) \equiv P_t(i)/P_t$ and λ_t is the Lagrange multiplier. Denoting $d(\cdot)$ as the inverse function of $D(\cdot)'$, λ_t/P_t suffices $\int_0^\infty D(d(P_t(i)/\lambda_t))di = 1$. Explicitly, λ_t/P_t is expressed as,

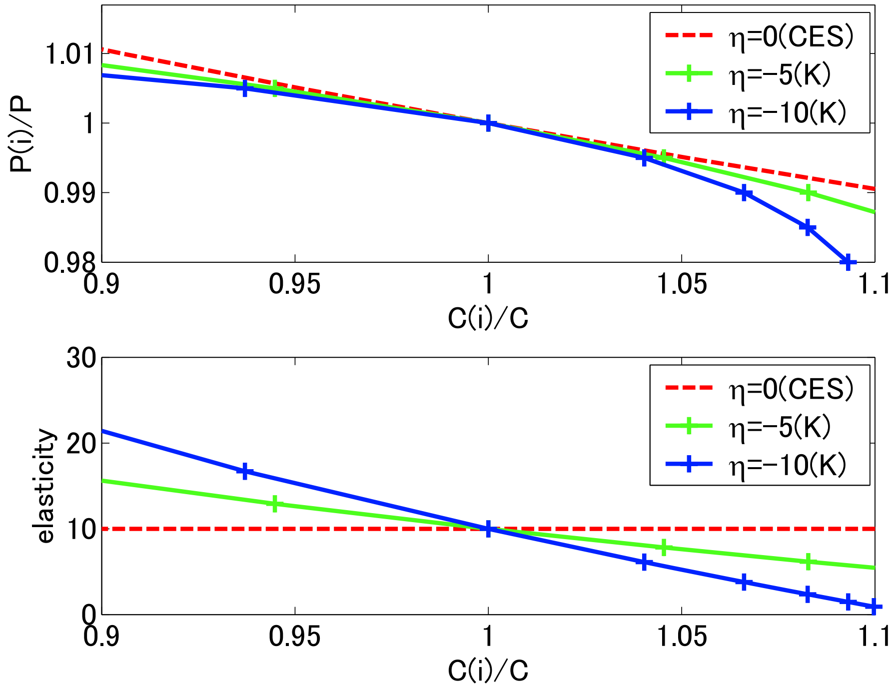
$$\frac{\lambda_t}{P_t} = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\gamma/(\gamma-1)} di\right]^{\frac{\gamma-1}{\gamma}}. \tag{3}$$

An advantage of employing this sort of the demand specification is that it enables to derive the aggregate price index as follows.

$$P_t = \frac{1}{1+\eta}\left[\int_0^1 P_t(i)^{\frac{\gamma}{\gamma-1}} di\right]^{\frac{\gamma-1}{\gamma}} + \frac{\eta}{1+\eta}\int_0^1 P_t(i)di. \tag{4}$$

Eq.(4) means that the aggregate price index is expressed as a sum of the CES type price index and a linear aggregator. Now, we illustrate the demand curve and the price elasticity of demand in Figure 1

Figure 1: Demand Curve: CES vs Kink



Note: K: kinked demand, CES: CES demand.

The upper panel and the lower panel of Figure 1 show the demand curve of this specification and the corresponding price elasticity of demand, respectively. For this calculation, we assume $\epsilon = 10$, which implies the 11-percent price mark-up at the steady state under zero inflation.

When we set $\eta = 0$, the demand curve is equivalent to the CES type. In the case of $\eta = -5$ and -10, the curvature of the demand curve overturns, reflecting the kinked demand property of the Kimball type aggregator. In addition, the price elasticity of the kinked demand is decreasing with respect to the increase of the relative quantity while the CES demand function results in the constant elasticity of demand.

2.2 Households

Next, we set up a representative household's objective function as follows,

$$E_t \sum_{t=0}^{\infty} \beta^t [U(C_t) - V(N_t) + F(M_t/P_t)]$$

where E_t, β, N_t, and M_t are the conditional expectation operator, the subjective discount factor, labor hours and the cash holdings, respectively.

The contemporaneous budget constraint for the representative household is,

$$P_t C_t + E_t D_{t,t+1} B_{t+1} \leq W_t N_t + \Pi_t + B_t + T_t - M_{t+1} + M_t \tag{5}$$

where $E_t D_{t,t+1}$, B_t, W_t, Π_t, and T_t are a price of one period contingent claim bond, the amount of the bond, the nominal wage rate, the dividend from firms and the lump-sum transfer from the government.

Maximizing the expected life-time utility subject to the demand function of eq.(2) and the budget constraint of eq.(5), we yield the first order conditions of the following.

$$\frac{W_t}{P_t} = \frac{V_{nt}}{U_{ct}},$$
$$U_{ct} = \beta E_t \left[U_{ct+1} R_t \frac{P_t}{P_{t+1}} \right],$$
$$F_{M_t/P_t} = -\beta E_t U_{ct+1} + U_{ct}$$

where $R_t = E_t D_{t,t+1}{}^{-1}$, and $D_{t,\tau} = \beta^{\tau-t}(U_{c\tau}/U_{ct})(P_t/P_\tau)$ are the nominal yield of the bonds and the stochastic discount factor. A subscript on U, V, and F denotes the partial derivative of the function.

Hereafter, we parameterize utility functions as $U(C) = \ln(C), V(N) = N$, and $F(M/P) = a_m(M/P)^{1-\gamma_m}/(1-\gamma_m)$, for analytical simplicity.

2.3 Firms

Let the production function of a firm i as follows.

$$C_t(i) = N_t(i).$$

Solving the cost minimization problem yields the real unit cost as $V_t/P_t \equiv v_t = W_t/P_t$.

Firms set their prices in the Calvo fashion such that when a firm gets an opportunity to reset the price at time t, the firm can choose the optimal price to maximize the discounted sum of the future profit. The price-reset probability is denoted as $1-\alpha$ $(0 < \alpha < 1)$. Then, a firm's optimization problem can be expressed as follows.

$$\max_{P_t(i)} E_t \sum_{\tau=t}^{\infty} \alpha^{\tau-t} D_{t,\tau} \left[P_t(i)(1+\tau_c) - V_\tau \right] C_\tau(i) \quad s.t.\ eq.(2)\ and\ eq.(3) \tag{6}$$

where τ_c is the sales tax rate.

Solving the profit maximization problem gives the following optimal relative price equation,

$$\frac{P_t^*}{P_t} \equiv \tilde{P}_t = \frac{1}{\gamma(1+\tau_c)} \frac{E_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \frac{P_t}{P_\tau} \left(\frac{Q_\tau}{P_\tau}\right)^{1/(\gamma-1)} V_\tau}{E_t \sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t} \frac{P_t}{P_\tau} \left[\left(\frac{Q_\tau}{P_\tau}\right)^{1/(\gamma-1)} + \eta \frac{\gamma-1}{\gamma} (\tilde{P}_t)^{-1/(\gamma-1)}\right]} \tag{7}$$

where π_t is the inflation rate, $\tilde{P}_\tau = P_t^*(i)/P_\tau$, and $Q_\tau = (\lambda_\tau/P_\tau)^{-1}$.

3 The generalized NKPC

3.1 The generalized NKPC under the kinked demand

Log-linearizing eq.(7) around the steady state, we obtain the following generalized NKPC.

$$\hat{\pi}_t = \lambda \hat{v}_t + \kappa E_t \hat{\pi}_{t+1} - \omega_1 (1-\overline{\pi})\hat{\phi}_{t+1} - \omega_2 (1-\overline{\pi}^{1/(\gamma-1)})\hat{\psi}_{t+1}, \tag{8}$$

where

$$\begin{cases} \hat{\phi}_t = \left(1-\alpha\beta\overline{\pi}^{-1/(\gamma-1)}\right)\left(\frac{YY}{\gamma-1}\hat{\pi}_t + \hat{v}_t\right) + \alpha\beta\overline{\pi}^{-1/(\gamma-1)}\left(\hat{\phi}_{t+1} - \frac{1}{\gamma-1}\hat{\pi}_{t+1}\right) \\ \hat{\psi}_t = \alpha\beta\overline{\pi}^{-1}\left(\hat{\psi}_{t+1} - \hat{\pi}_{t+1}\right) \end{cases} \tag{9}$$

and $\hat{x}$ denotes the log-deviation from its steady state value, $\overline{x}$. The detail of the derivation and each coefficient $(\kappa, \lambda, \omega_1, \omega_2, YY)$ in eq.(8) are described in Appendix 1.

We can confirm that eq.(8) is equivalent to the standard NKPC under trend inflation (Ascari (2004)) when we set $\eta = 0$. In addition, if the steady state inflation is also zero ($\overline{\pi} = 1$), it reduces to the standard NKPC (Roberts (1995); Clarida, Galí, and Gertler (1999)).

3.2 Parameter settings

In order to examine our NKPC and its coefficients, we specify the model parameters. Table 1 shows the benchmark parameters.

$1-\alpha$, ϵ, η, and β denote the price-reset probability, the price elasticity of demand, the curvature of the demand curve, and the discount factor. First, we set $1-\alpha = 0.4$,

Table 1: Parameters

$1-\alpha$	ϵ	η	β
0.4	10	-10.0	0.99

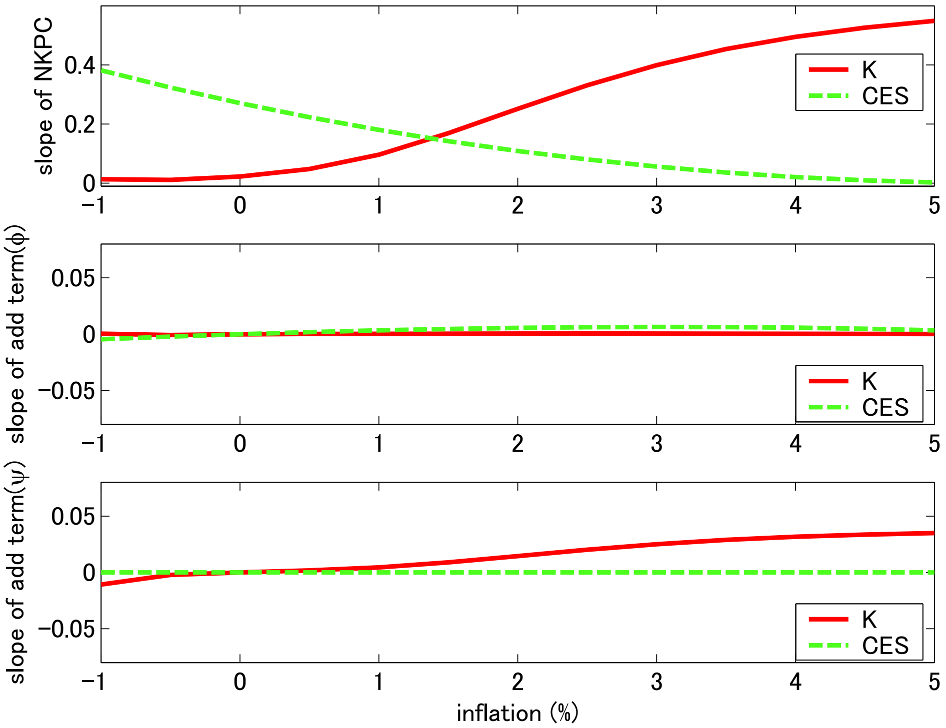
assuming that a firm resets the price 1.6 times a year on average. Since Bank of Japan (2000), who surveys firm's price setting behavior, reports that the average number of times to reset prices is one or two times in a year, our parameter setting is consistent with this evidence. Second, ϵ corresponds to the price elasticity at the zero inflation. $\epsilon = 10$ implies the 11 percent price markup under zero inflation. Third, we specify $\eta = -10$. This value is the medium of Kimball (1995) and Bergin and Feenstra (2000). Chari, Kehoe, and McGrattan (2000) criticize the curvature of the Kimball (1995)'s demand is extraordinary. Namely, Kimball (1995)'s parameterization corresponds to $\eta = -42$ in the present Dotsey and King (2005)'s specification. Relative to Kimball (1995), our parameterization is moderate. Finally, we set $\beta = 0.99$.

3.3 The coefficient of the generalized NKPC and trend inflation

We can point out the two features of our NKPC in eq.(8); (i) each coefficient is the function of trend inflation in addition to the deep parameters; (ii) as is suggested in Ascari (2004) and Ascari and Ropele (2007), the additional terms ($\hat{\phi}_t$) appear at the end of the right hand side of eq.(8). Further, we also have another additional term ($\hat{\psi}_t$), reflecting the introduction of the kinked demand curve. It disappears when the demand curve is CES type ($\eta = 0$). Since these additional terms in eq.(9) include the future variables, the inflation dynamics becomes more front-loading.

The top panel of Figure 2 illustrates the coefficient of the unit cost, in other words, the slope of the NKPC. Under the kinked demand, the slope becomes flatter as the rate of trend inflation declines. Instead, under the CES demand, it becomes steeper as the rate of trend inflation declines, just like Ascari (2004), Bakhshi, Burriel-Llombart, Khan, and Rudolf (2003) and Ascari and Ropele (2007). Therefore, the kinked demand is consistent with the empirical facts of BMR and Benati (2007) at least in the structural function form; the Phillips curve is flatter under the low inflation environment. In the

Figure 2: The coefficients of the NKPC and trend inflation:CES vs Kink



Note: K: kinked demand, CES: CES demand.

latter section, we discuss the intuition behind this difference between the CES demand and the kinked demand.

Next, the bottom two panels of Figure 2 show the coefficients of the additional terms. Both of the parameters are near zero, implying the effect of the additional terms on the inflation dynamics is not substantial.

4 Discussion

A simple intuition for our results follows. Under the CES demand, positive inflation implies that a Calvo firm expects the relatively sensitive demand in the future (Figure 3: left panel). Then, a price-setter is reluctant to reflect the cost changes onto the current price, anticipating the future sensitive demand. Consequently, inflation becomes inresponsive to the cost changes and hence irrespective to the output gap variations.

In contrast, low or negative inflation (deflation) implies that the Calvo firm expects the relatively insensitive demand in the future (Figure 3: right panel). A price-setter inclines to pass-through the cost changes on the price. Hence, inflation comes to be more responsible to the variations in output gap and the slope of the Phillips curve becomes steeper.

Figure 3: Inflation and Demand curve: CES

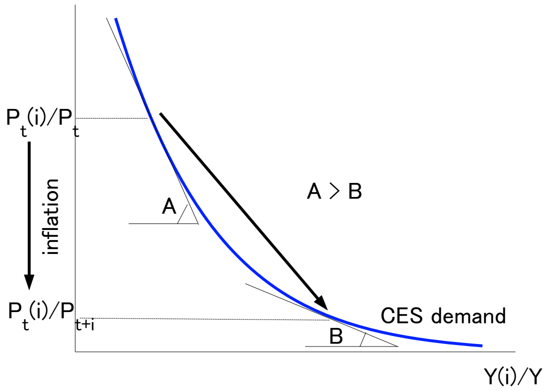
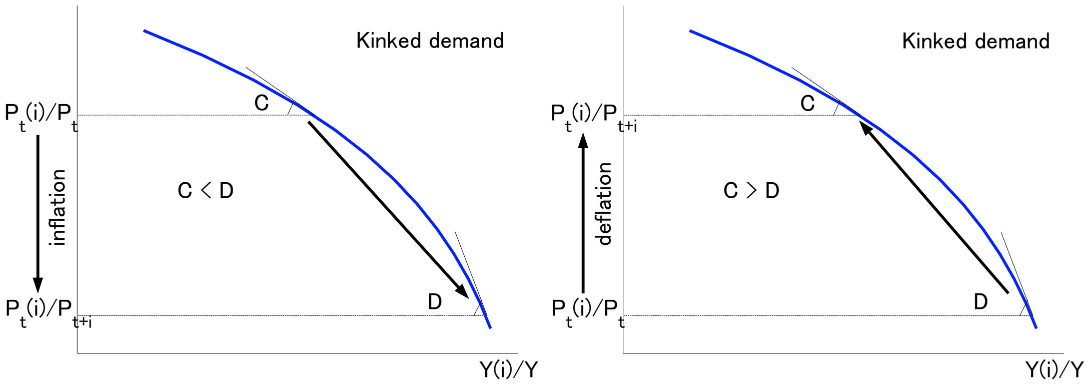


Figure 4: Inflation and Demand curve: Kink



However, the above mechanism turns around under the kinked demand where consumers flee from relatively expensive products but do not flock to inexpensive ones. Positive inflation leads to the insensitive demand and negative inflation results in the responsive demand on average under the kinked demand (Figure 4).

Accordingly, a price-setter is willing to charge the cost changes on the price under positive inflation and is unwilling to do so under negative inflation. Thus, inflation responds more to the cost variations under positive inflation and less under negative inflation. Finally, the slope of the Phillips curve is flatter under low inflation than under high inflation.[4]

5 Equilibrium dynamics

The analysis in the previous section reveals that the replacement of the CES demand with the kinked demand successfully helps to overcome the problem of the standard sticky price model such that the coefficient of the NKPC decreases associated with the rise of the trend inflation rate. However, the empirical facts are the reduced-form general equilibrium outcomes. Therefore, we embed our NKPC in a simple general equilibrium model and examine the Phillips correlation under low and high trend inflation. In order to close the general equilibrium, we employ the money growth rate rule as the monetary policy rule.

5.1 The money growth rate rules

The log-linearized money growth rate rule is specified as $\Delta\hat{m}_{t+1} = \rho\Delta\hat{m}_t + \epsilon_t$ where $\Delta\hat{m}$ and ϵ_t denote the money growth rate and the monetary disturbance, respectively. In the shock simulation, we set $\rho = 0.8$.

5.1.1 System of equations

We can obtain the log-linearized system of the general equilibrium as follows.

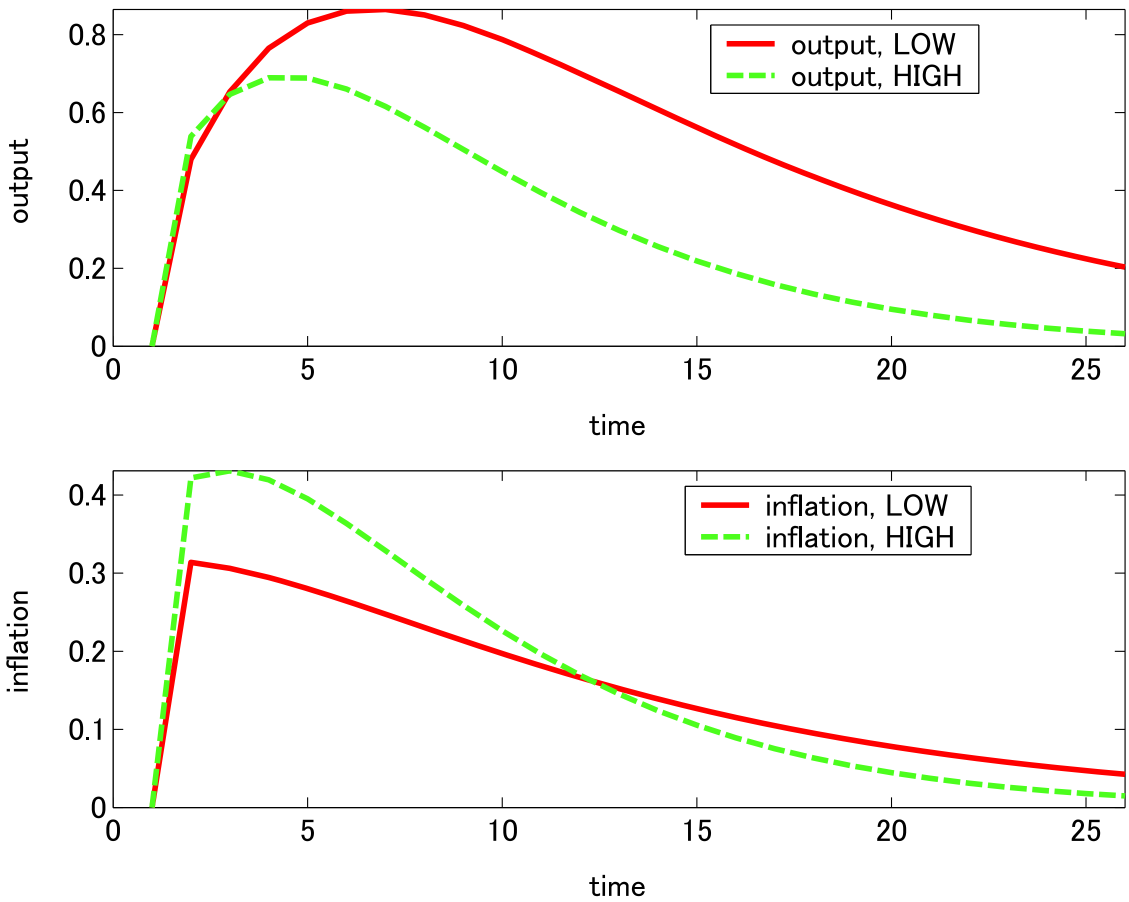
$$\begin{aligned}
\hat{y}_t &= \hat{y}_{t+1} - \hat{r}_t + \hat{\pi}_{t+1} && c - Euler \\
\hat{\pi}_t &= \kappa\hat{\pi}_{t+1} + \lambda\hat{y}_t - \omega_1(1-\overline{\pi})\hat{\phi}_{t+1} + \omega_2(1-\overline{\pi}^{1/(\gamma-1)})\hat{\psi}_{t+1} && NKPC \\
\hat{\phi}_t &= \xi_1\hat{\pi}_t + \xi_2\hat{y}_t + \xi_3\hat{\pi}_{t+1} + \xi_4\hat{\phi}_{t+1} && additional\ term1 \\
\hat{\psi}_t &= \zeta_1\hat{\pi}_{t+1} + \zeta_2\hat{\psi}_{t+1} && additional\ term2 \\
\Delta\hat{m}_{t+1} &= \rho\Delta\hat{m}_t + \epsilon_t && development\ of\ money \\
\hat{r}_t &= \hat{y}_t - \gamma_m(\hat{m}_t - \hat{p}_t) && interest\ rate
\end{aligned}$$

where $\hat{y}_t$, $\hat{r}$ and $\hat{\pi}$ are output gap, interest rate, and inflation rate.

[4] In Appendix 2, we demonstrate that the slope of the NKPC depends on the curvature of the demand curve and becomes steeper as the curvature of the demand curve increases.

5.1.2 Simulation results

Figure 5: money growth rate rules: monetary policy shock



In Figure 5, the upper panel and the lower panel correspond to the impulse responses of output gap and inflation to a one percent positive money growth rate shock, respectively. In each panel, the dotted line represents the case of the high inflation ($6^{1/4}$ percent / quarter) economy and the solid line is the case of the low inflation (0 percent) economy.

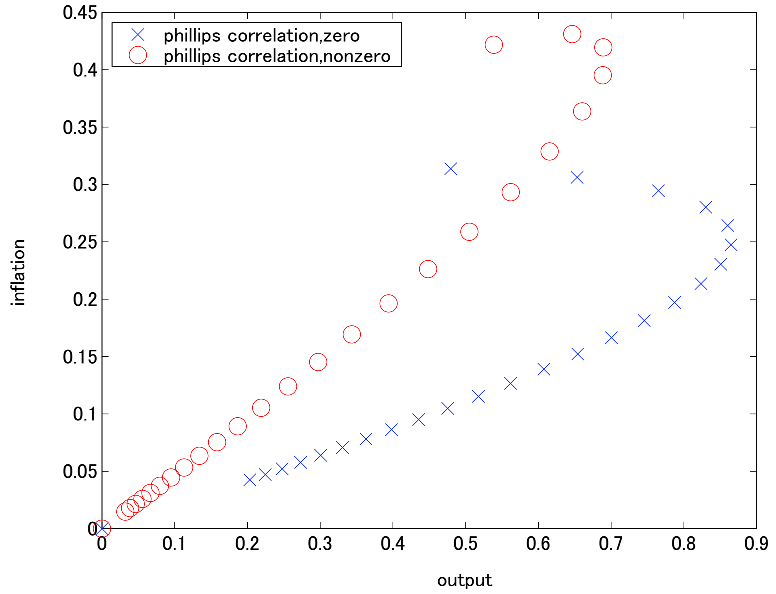
In the figure, we can see the clear difference between the high trend inflation environment and the low trend inflation environment. In the high (non-zero) inflation economy, the impulse response of output gap is positive and large. The inflation also responds in large amount to the shock. The simulation result clearly exhibits the tight linkage between output gap and inflation in the high inflation period.

In contrast, in the low (zero) inflation economy, the impulse response of output gap is greater and more persistent. The inflation rate responds weakly but persistently to the shock. The linkage between the real economic activities and the inflation rate becomes weaker in the low inflation economy.

In addition, the simulation results add a new insight into the literature on the persistent response of output, which is initiated by the Chari, Kehoe, and McGrattan (2000)'s critique of sticky price models. Similar to Bergin and Feenstra (2000) and Dotsey and King (2005), we demonstrate that the concave demand aggregator generates the endogenous persistence in the low inflation environment. However, we also shows that the endogenous persistence disappears in the high inflation environment. Thus, the inner propagation mechanism depends on the level of the trend inflation rate and it disappears as the trend inflation rises.

Finally, Figure 6 illustrates the inflation-output correlation in this simulation. It suggests that the reduced-form Phillips curve is flatter under low trend inflation but that it is steeper under non-zero trend inflation.

Figure 6: The reduced-form Phillips curve: money growth rate rules



Note: Each inflation and output gap is the log-deviation from the steady state.

The result here is consistent with the empirical work by BMR and Benati (2007) although Ascari (2004), Bakhshi, Burriel-Llombart, Khan, and Rudolf (2003) and Ascari and Ropele (2007) suggest that the canonical sticky price model cannot explain this empirical fact. Our analysis is successful in explaining this discrepancy between the standard sticky price models and the empirical evidence. Notably, the mechanism behind our results is different with Bakhshi, Burriel-Llombart, Khan, and Rudolf (2003), which stress the role of time-varying nominal rigidities in the spirit of BMR or Romer (1990). We have shown that the kinked demand can also explain the weak Phillips correlation under low trend inflation.

6 Conclusion

This paper challenges to fill the gap between the implication of the standard sticky price models and the empirical fact about the 'Phillips correlation.' We show that introducing the 'smoothed out kinked' demand curve (Kimball (1995)) helps to explain the flattered Phillips curve under the low trend inflation environment.

It would be useful to extend our work to the other situations of pricing, such as the incomplete exchange rate pass-through. As is suggested in Taylor (2000), the rate of the exchange rate pass-through is decreasing along with the decline of the inflation rate. The mechanism developed here might be helpful in explaining this phenomenon.

In order to shed light on the role of the real rigidity from the demand side and to keep consistency with the existing research by Ascari (2004), Bakhshi, Burriel-Llombart, Khan, and Rudolf (2003) and Ascari and Ropele (2007), we employ the time-dependent Calvo type sticky price models. We do not mean to claim that the Calvo model is innocuous. It is the hotly-debated issue whether the price adjustment mechanism is approximated by the time-dependent rule or the state-dependent rule. Yet, our analytical results suggest that the demand structure is also crucial to study the effect of the trend inflation in the staggered price adjustment models.

In this paper, we do not mention the reason why trend inflation is not zero. However, recent inflation targeting countries adopt the non-zero positive target rate. In addition, the actual data suggests that the long-run inflation rate is not zero around the world. Since the results of the standard model are sensitive to the special assumption of zero inflation at the steady state, it is worthwhile to keep studying the trend inflation issues.

Appendix 1: Derivation of the generalized NKPC

Eq.(7) can be rewritten as follows;

$$\tilde{P}_t = \frac{1}{\gamma(1+\tau_c)} \frac{\phi_t}{\psi'_t + \psi_t \tilde{P}_t^{-1/(\gamma-1)}}, \tag{10}$$

$$\phi_t = Q_t^{1/(\gamma-1)} v_t + E_t \alpha\beta\pi_{t+1}^{-1/(\gamma-1)} \phi_{t+1}, \tag{11}$$

$$\psi'_t = Q_t^{1/(\gamma-1)} + E_t \alpha\beta\pi_{t+1}^{-\gamma/(\gamma-1)} \psi'_{t+1}, \tag{12}$$

$$\psi_t = \eta\frac{\gamma-1}{\gamma} + E_t \alpha\beta\pi_{t+1}^{-1} \psi_{t+1}. \tag{13}$$

Log-linearizing eqs.(10)-(13) gives,

$$\hat{\tilde{p}}_t = \eta_\phi \hat{\phi}_t + \eta_{\psi'} \hat{\psi}'_t + \eta_\psi \hat{\psi}_t + \eta_{\tilde{P}} \hat{\tilde{P}}_t, \tag{14}$$

$$\hat{\phi}_t = \frac{\bar{v}}{\bar{\phi}} \bar{Q}^{1/(\gamma-1)} \left[\frac{1}{\gamma-1}\hat{q}_t + \hat{v}_t\right] + \alpha\beta\bar{\pi}^{-1/(\gamma-1)} \left(\hat{\phi}_{t+1} - \frac{1}{\gamma-1}\hat{\pi}_{t+1}\right), \tag{15}$$

$$\hat{\psi}'_t = \frac{\bar{Q}^{1/(\gamma-1)}}{\bar{\psi}'(\gamma-1)}\hat{q}_t + \alpha\beta\bar{\pi}^{-\gamma/(\gamma-1)} \left(\hat{\psi}'_{t+1} - \frac{\gamma}{\gamma-1}\hat{\pi}_{t+1}\right), \tag{16}$$

$$\hat{\psi}_t = \alpha\beta\bar{\pi}^{-1}(\hat{\psi}_{t+1} - \hat{\pi}_{t+1}), \tag{17}$$

$$\eta_{X\in\{\phi,\psi,\psi',\tilde{P}\}} \equiv \frac{\partial G(\bar{X})}{\partial \bar{X}} \frac{\bar{X}}{G(\bar{X})}, \quad where\ G(\bar{X}) \equiv \frac{\bar{\phi}}{\bar{\psi}' + \bar{\psi}(\bar{\tilde{P}}^{-1/(\gamma-1)})}. \tag{18}$$

where $\hat{x}$ denotes the log deviation of X from the steady state and $\bar{X}$ represents that X is at the steady state.

Now, we also log-linearize the Q_t and the price index of eq.(4).

$$\hat{q}_t = \bar{Q}^{\gamma/(\gamma-1)} \left[-(1-\alpha)\bar{\tilde{P}}^{\gamma/(\gamma-1)} \hat{\tilde{p}}_t + \alpha\bar{\pi}^{-\gamma/(\gamma-1)}\hat{\pi}_t\right], \tag{19}$$

$$\hat{\pi}_t = \frac{1-\alpha}{\alpha} \frac{\left[(1-\alpha)\bar{\tilde{P}}^{\gamma/(\gamma-1)} + \alpha\bar{\pi}^{-\gamma/(\gamma-1)}\right]^{-1/\gamma} \bar{\tilde{P}}^{\gamma/(\gamma-1)} + \eta\bar{\tilde{P}}}{\left[(1-\alpha)\bar{\tilde{P}}^{\gamma/(\gamma-1)} + \alpha\bar{\pi}^{-\gamma/(\gamma-1)}\right]^{-1/\gamma} \bar{\pi}^{-\gamma/(\gamma-1)} + \eta\bar{\pi}^{-1}} \hat{\tilde{p}}_t. \tag{20}$$

Plugging eq.(19) and eq.(20) into eqs.(14)-(17), and eliminating $\hat{\psi}'_{t+1}$, we yield the generalized NKPC as in eq.(8).

Each coefficient in eq.(8) follows.

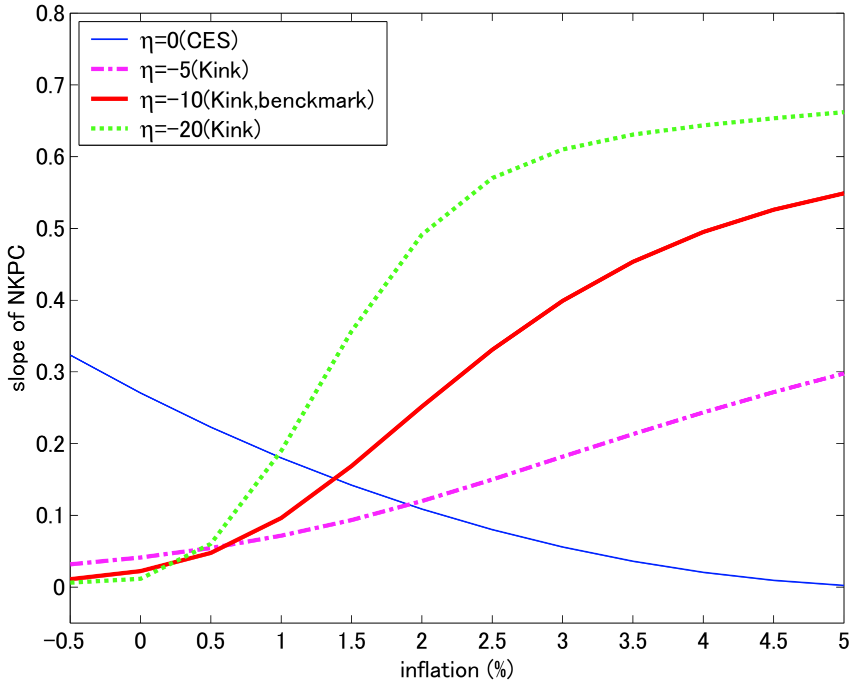
$$
\begin{aligned}
&\lambda \equiv \frac{1-\alpha\beta\bar{\pi}^{-1/(\gamma-1)}}{\gamma-1}\frac{YY}{XX},\\
&\kappa \equiv \frac{\alpha\beta\bar{\pi}^{-\gamma/(\gamma-1)}}{XX(\gamma-1)}\left[\frac{(1-\eta_{\tilde{P}})(\gamma-1)}{ZZ}-\bar{\pi}-\eta_{\psi'}\gamma-\eta_{\psi}\bar{\pi}^{1/(\gamma-1)}(\gamma-1)\right],\\
&\omega_1 \equiv \frac{\alpha\beta\bar{\pi}^{-\gamma/(\gamma-1)}}{XX},\\
&\omega_2 \equiv \frac{\alpha\beta\bar{\pi}^{-\gamma/(\gamma-1)}\eta_{\psi}}{XX},\\
&XX \equiv \frac{1-\eta_{\tilde{P}}}{ZZ}-\frac{YY}{\gamma-1}\left[1+\eta_{\psi'}-\alpha\beta\bar{\pi}^{-1/(\gamma-1)}(1+\eta_{\psi'}\bar{\pi}^{-1})\right],\\
&YY \equiv \bar{Q}^{\gamma/(\gamma-1)}\left[-(1-\alpha)\bar{\tilde{P}}^{\gamma/(\gamma-1)}/ZZ+\alpha\bar{\pi}^{-\gamma/(\gamma-1)}\right],\\
&ZZ \equiv \frac{1-\alpha}{\alpha}\frac{\left[(\bar{Q}\bar{\tilde{P}})^{1/(\gamma-1)}+\eta\right]\bar{\tilde{P}}}{\left[(\bar{Q}\bar{\pi}^{-1})^{1/(\gamma-1)}+\eta\right]\bar{\pi}^{-1}},\\
&\bar{\tilde{P}}^{\gamma/(\gamma-1)} = \frac{1}{1-\alpha}\left\{\left[(1+\eta-\alpha\eta\bar{\pi}^{-1})-\eta(1-\alpha)\bar{\tilde{P}}\right]^{\gamma/(\gamma-1)}-\alpha\bar{\pi}^{-\gamma/(\gamma-1)}\right\},\\
&\bar{Q} = \left[(1-\alpha)\bar{\tilde{P}}^{\gamma/(\gamma-1)}+\alpha\bar{\pi}^{-\gamma/(\gamma-1)}\right]^{-(\gamma-1)/\gamma}.
\end{aligned}
$$

Appendix 2: Sensitivity analysis -the curvature of the demand curve

In the main text, we fix the curvature of the demand curve, η. The sensitivity analysis in Figure 7 demonstrates the effects of alternative η on the slope of the NKPC.

Figure 7 show that the slope of the NKPC reverses when the demand curve is kinked. Further, the slope of the NKPC becomes steeper as $|\eta|$ increases. These feature confirms our discussion in the section 4.

Figure 7: Sensitivity analysis: -the curvature of the demand curve



Note: All parameters except for η are same as in the main text.

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