Bank of Japan Working Paper Series

The Costs and Benefits of Inflation: Evaluation for Japan's Economy

Hitoshi Fuchi*
hitoshi.fuchi@boj.or.jp

Nobuyuki Oda**
nobuyuki.oda@boj.or.jp

Hiroshi Ugai***
hiroshi.ugai@boj.or.jp

No.07-E-10
May 2007

Bank of Japan
2-1-1 Nihonbashi Hongoku-cho, Chuo-ku, Tokyo 103-8660

* Monetary Affairs Department, Bank of Japan.

** Monetary Affairs Department, Bank of Japan.

*** Monetary Affairs Department (currently, Okayama Branch), Bank of Japan.

The Costs and Benefits of Inflation: Evaluation for Japan's Economy♦

Hitoshi Fuchi*, Nobuyuki Oda**, and Hiroshi Ugai***

May 2007

Abstract

This paper quantitatively evaluates a steady-state inflation rate that is considered desirable from the perspective of social welfare, using a model describing the Japanese economy.

Specifically, it begins by setting out points concerning the costs and benefits that accompany inflation. We build a model capable of evaluating the effects, on social welfare, of several of these points: the opportunity cost of holding money, the zero lower bound on nominal interest rates, price stickiness and the downward wage rigidity. Building on this, we conduct a stochastic simulation that quantitatively evaluates the social loss with different steady-state inflation rates. We also analyze the range of changes in the steady-state inflation rate that minimizes the social loss when we change the model settings.

Key words: Inflation; Social loss; Monetary policy; Zero lower bound on nominal interest rates; Downward wage rigidity; Price stickiness; Opportunity cost of holding money

JEL classification: E31, E52, E58

♦ We would like to thank Kosuke Aoki (London School of Economics), seminar participants at the Bank of Canada, and many colleagues at the Bank of Japan, including Hiroshi Fujiki, Ippei Fujiwara, Takeshi Kimura, Shigenori Shiratsuka, Tomohiro Sugo, Kozo Ueda, and Isao Yamamoto, for their helpful comments. The opinions expressed here, as well as any remaining errors, belong to the authors and should not be ascribed to the Bank of Japan or the Monetary Affairs Department.

* Monetary Affairs Department, Bank of Japan (E-mail: hitoshi.fuchi@boj.or.jp)

** Monetary Affairs Department, Bank of Japan (E-mail: nobuyuki.oda@boj.or.jp)

*** Monetary Affairs Department (currently, Okayama Branch), Bank of Japan (E-mail: hiroshi.ugai@boj.or.jp)

1. Introduction

What are the costs and benefits of inflation to the Japanese economy?

If money is merely a veil and price changes have no impact on the real economy, there should be no cost or benefit regardless of how high or low the inflation rate may be. However, in the real world, there are many factors that prevent money from being a veil. This in turn means that inflation will indeed produce both costs and benefits to social welfare.

During the 1970s, for example, prices rose significantly and Japan experienced significant damage as a result of the inflation, creating a majority opinion that controlling inflation was desirable for social welfare. However, when prices began to fall in the late 1990s, there was renewed interest in the potential benefits of inflation. Interest in inflation thus tends to be biased towards the costs or benefits of the time, but inflation does in fact simultaneously produce both. The optimal inflation rate should be the rate at which the two balance each other.

This paper attempts to quantitatively evaluate the steady-state inflation rate that minimizes the social loss in light of the nature of the Japanese economy, taking into account the costs and benefits of inflation comprehensively. The "steady-state inflation rate" focused on in this paper is the growth rate of general prices in a steady state, sometimes referred to as the "long-run rate of inflation" or the "trend inflation rate." It can be interpreted as the inflation rate affected over the long-term by a framework of the monetary policy conduct. Its optimal value can be understood as the inflation rate that the central bank should target over the long term.

From a similar perspective as ours, Teranishi [2003] formerly analyzed the desired long-run rate of inflation for the Japanese economy. The analysis in this paper covers more points concerning the costs and benefits of inflation, uses a larger-scale model, and identifies the optimal inflation rate based on the social loss function derived more consistently with the model than Teranishi [2003].

There are many points of contention over the costs and benefits produced by different steady-state inflation rates, which we will set out in Section 2. Four of them will be given particular focus in our analysis. The first is the opportunity cost of holding money, which generates a cost in conjunction with inflation. The second is the "safety margin" against the zero lower bound on interest rates, which produces a benefit in conjunction with inflation. The third is the distortion in relative prices caused by price stickiness, which generates a cost in conjunction with both inflation and deflation. Finally, we will consider the downward rigidity of wages, which produces a benefit in conjunction with inflation up to a certain point. In this paper we combine all the four factors into a single model, enabling us to make a general evaluation of the costs and benefits of different steady-state inflation rates. We then use this model to quantitatively evaluate the steady-state inflation rate that minimizes the social loss.

This paper is organized as follows. Section 2 surveys a number of points of contention concerning the costs and benefits resulting from inflation. From there, we narrow down the costs and benefits to be included in our quantitative evaluation. In Section 3, we build an economic model, with which we can compare the costs and benefits, and derive a social loss function that is theoretically consistent with the model. Section 4 calculates the social loss under the steady-state economy as a starting point of our analyses, and identifies a steady-state inflation rate that minimizes the social loss. The analysis in Section 5 uses a stochastic simulation under the zero lower bound on interest rates in order to expand our analysis to account for a social loss resulting from deviations from the steady state. In Section 6, we analyze the extent to which the results of the preceding section will change when the underlying assumptions are modified. Section 7 contains our conclusions.

2. Points concerning Costs and Benefits of Inflation

This section sets out points on how a steady-state inflation rate generates costs and benefits in light of social welfare. Subsection (1) contains a survey of prior research on individual points of contention. Subsection (2) narrows down those points to be included in our quantitative evaluation. Readers only interested in the analytical content of the paper are encouraged to skip subsection (1) and move directly to (2).

(1) Survey of arguments regarding costs and benefits of inflation

Prior research on the social-welfare costs and benefits of inflation has looked at a number of different points: (a) the opportunity cost of holding money, (b) the zero lower bound on interest rates, (c) price stickiness, (d) the downward wage rigidity, (e) identification of changes in relative prices, (f) non-inflation-neutral taxation, (g) the permanent effects of unexpected price changes, and (h) endogenous technological progress. We will review them in order.

(a) The opportunity cost of holding money

Nominal interest is equivalent to the opportunity cost of holding interest-free money. This cost generates a social loss because, from the perspective of the consumer holding money, it is dead weight in the consumer’s surplus. From this perspective only, social welfare is maximized when nominal interest rates are zero. This is known as the "Friedman Rule" after the research of Friedman [1969]. If nominal interest rates are zero and real interest rates are, as is usual, positive, the inflation rate is negative. In other words, under the Friedman Rule, deflation is desirable. Nonetheless, empirical analysis reports that the social loss resulting from the opportunity cost of holding money is small (Lucas [2000], Shiratsuka [2001]).

The Friedman Rule is concerned with maximizing the utility of holding money and takes no account of the impact of the steady-state inflation rate on real consumption. In fact, Friedman advocated the natural rate hypothesis in which, in a steady state, the inflation rate has no impact on the real economy.

In contrast to the natural rate hypothesis there is the argument that the steady-state inflation rate will impact real consumption through the opportunity cost of holding money.

From a historical perspective, Tobin [1965] used a growth model that assumed a constant savings rate to conclude that a high monetary growth rate and inflation rate would have a positive effect on real consumption. According to Tobin, inflation would, through the mechanism of the opportunity cost of holding money, exert pressure to reduce money holdings and, on the assumption of a constant savings rate, encourage the substitution of real investments for money holdings. An increase in real investments results in a positive impact on real consumption because of the increase in output capacity. This is known as the "Tobin Effect."

Such a behavioral equation with an *a priori* assumption of a constant savings rate is rarely used today. In its place, standard practice is to analyze economic behavior based on intertemporal optimization. Sidrauski [1967], in his pioneering work in this area, used an optimization model featuring a rigid labor supply and a utility function with money to find that in a steady state, the monetary growth rate and inflation rate had no impact on the real economy. This is known as the "long-run superneutrality of money."

However, it was shown that a steady-state inflation rate could have an impact on real variables depending on the assumptions underlying the model. One example is Brock's [1974] model that assumes a flexible labor supply; another is Stockman's [1981], which assumes that money holdings are required for investment expenditures. Wang and Yip [1992] conclude that the impact of the monetary growth rate and inflation rate on the real economy is positive if money holdings and real consumption are substitutable and negative if they are complementary.[1] Most money demand models, including the

[1] This conclusion is derived based on the optimization behavior of households with no overlapping generations. With this assumption the long-run Fisher effect, or the inflation neutrality of real interest rate, holds even if the long-run neutrality of money does not hold. In contrast, it is known,

cash-in-advance model and shopping-time model, feature a complementary relationship between money holdings and real consumption, and therefore generate effects that are the exact opposite of the Tobin effect, which means that they are substitutable. In other words, according to these models, inflation has a negative impact on real consumption. This "reversed Tobin effect" is also known as the "direct effect of money." When this effect is present, a higher inflation rate in a steady state will result in higher nominal interest rates through the Fisher relationship, which will restrain money holdings and reduce consumption. Thus, the loss of social welfare is realized because consumption declines.

(b) The zero lower bound on nominal interest rates

When nominal short-term interest rates approach the zero lower bound, economic stability becomes more vulnerable because room for further monetary easing is lost. To avoid the zero bound it is appropriate for the monetary policy to maintain a slightly high expected inflation rate. This concept was advocated by Summers [1991].

There are many conceivable ways to increase the expected inflation rate. One would be policies that increase the steady-state inflation rate. If there is a one-to-one relationship between steady-state inflation rate and nominal interest rates, the higher the steady-state inflation rate, the higher the nominal interest rate. In such circumstances, a central bank would have room to lower the policy rate in the event of a negative shock to the economy, reducing the potential of hitting the zero lower bound (Woodford [2003], Nishiyama [2003]). From such a perspective, analyses on the desired steady-state inflation rate were conducted for the US economy by Fuhrer and Madigan [1997], Orphanides and Wieland [1998], and Reifschneider and Williams [2000] and for the Japanese economy by Hunt and Laxton [2001].

(c) Price stickiness

When prices are sticky, changes in general price levels generate menu costs that cause relative prices to change.

A Calvo-type price-setting model explains this in more concrete terms. Under this model, the opportunity to set the optimal price is given for each firm as a probability. Firms that can reset a price will set the optimal price so as to maximize the discounted

---

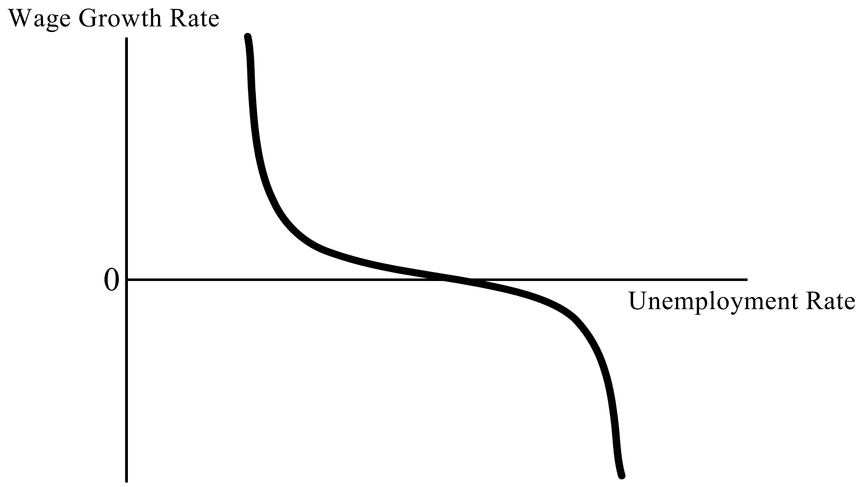
for a model with overlapping generations, that the long-run superneutrality of money does not hold and even the long-run Fisher effect may not hold (Weiss [1980], Weil [1991]). In such a case, the steady-state inflation rate can have an effect on the real interest rate, which leads to different implications on monetary policy from those derived by an usual analysis assuming the long-run Fisher effect.

present value of profits. Those unable to reset a price will maintain prices at the previous-term levels. In this economy, these two types of firms in light of their price setting exist in each time, and their relative prices differ from each other. This difference in relative prices generates a difference in relative volumes of outputs, resulting in the loss of social welfare because the optimal level of outputs, which reflects the productivity of the economy and preferences of households, is not realized. This type of social loss is minimized when inflation is zero (Rotemberg and Woodford [1998], Woodford [2003]).[2]

(d) Downward wage rigidity

There is a point of contention which focuses on the downward rigidity of wages, discussed by Tobin [1972], as a benefit factor of inflation. When the wage growth rate is lowered within a moderate range, the supply and demand balance in the labor market slackens and unemployment worsens. A further drop in wages, when the growth rate is substantially negative, will worse the unemployment rate no further. Figure 1 conceptually illustrates this relationship. The tradeoff between wages and unemployment is a long-term, nonlinear relationship, not being extinguished in a steady state.

Figure 1: A Nonlinear Tradeoff between Wages and Unemployment (Conceptual Diagram)



[2] It is possible to assume an indexation in price setting, instead of the above-mentioned assumption that firms not given the opportunity to reset a price will maintain prices at the previous-term levels. With the indexation pricing, firms unable to set the optimal price will reset prices according to a given rule. In such a case, the social loss is not necessarily minimized when inflation is zero. For example, Erceg, Henderson, and Levin [2000] developed a model for an indexation of upward price revisions in accordance with the steady-state inflation rate. In this model, the social loss is minimized when inflation is at the steady-state inflation rate.

In a steady state, if there is a one-to-one relationship between the inflation rate and the wage growth rate, a higher inflation rate means a higher wage growth rate. Thus, an unemployment rate is lowered, with the wage growth rate in a moderate range. This is referred to as the "grease effect" of inflation. The effect was studied in recent years by Akerlof, Dickens and Perry [1996] and Groshen and Schweitzer [1999]. Studies on the Japanese economy include Kimura and Ueda [2001] and Kuroda and Yamamoto [2003a, 2003b, 2005].

(e) Costs of identifying changes in relative prices

The basic function of pricing is to effectively communicate information on relative prices, which is required by economic agents for their decision-making (Friedman [1977]).

In an economy where the cost of collecting and processing information is not negligible, inflation makes it difficult to identify changes in relative prices. Inflation will therefore over the long term reduce real output by distorting the decision-making of economic agents (Issing [2004]). The same is true of deflation. In this sense, whether inflation or deflation, the larger the absolute value of the steady-state rate of price changes, the greater the social welfare costs it generates. To avoid these costs, it is desirable to achieve a steady-state inflation rate that creates a state in which "economic agents no longer take account of the prospective change in the general price level in their economic decision making" (Greenspan [1996]).

(f) Effect of non-inflation-neutral taxation

If taxes are not lump-sum but have an influence on resource allocation, the long-run inflation rate may also have an effect on resource allocation if taxes are not neutral with respect to the inflation rate. Such an effect was empirically studied by Feldstein [1999] based on national tax systems in each country. Similar techniques were applied to Japan by Ueda [2001].

(g) Permanent effects of unexpected price changes

A stochastic change in the steady-state inflation rate can be interpreted as generating unexpected price changes. Unexpected price changes are widely recognized to have a temporary impact on the real economy. There are also theories indicating that unexpected price changes can have a permanent impact; examples include theories on the hysteresis effect of employment, the new open-economy macroeconomics, and debt deflation.

Blanchard and Summers [1986, 1987] is the seminal work on the hysteresis effect of employment. In terms of the ability of workers to negotiate with employers, existing workers, who are insiders, are thought to have greater negotiating power than the unemployed, who are outsiders. Employment levels in the previous period will have a substantial impact on decisions of current employment, and therefore, a shock which hits the employment levels once can influence the levels permanently. In such cases, unexpected price changes can have a permanent impact on employment levels.

Obstfeld and Rogoff [1995, 1996] mark the beginning of research into the new open-economy macroeconomics. Unexpected price changes can have a permanent impact on the real economy through changes in the trade balance and foreign credit/debt.

One of the earliest studies into debt deflation was Fisher [1933]. If a debt contract is signed on a nominal basis, falling prices will increase the real amount outstanding on the debt and decrease net assets. When financial intermediation functions are imperfect and there are restrictions on fund raising, declines in net assets hurt the real economy. A number of theoretical models have been developed to study credit channels and financial accelerators based on the imperfection of financial markets. However, few of these theories delve into the relationship with general price deflation. One can intuitively grasp that an unexpected decline in prices will result in a transfer of income from the debtor to the creditor. This will give the debtor a higher debt burden and worsen the financial position. The debtor's probability of default will consequently increase, which will increase the expected value of default costs.[3] Thus, unexpected price declines will carry additional real costs not seen with price gains.

(h) Effects of endogenous technological progress

In general, the long-term real economic growth rate is determined by the rate of technological progress and the rate of population growth. Standard theory on monetary policy assumes that the rates of both technological progress and population growth are exogenous, so the inflation rate has no impact on the long-term real economic growth rate. On the other hand, endogenous growth theory, which attempts to incorporate the mechanism of technological progress, leaves open the potential for the long-term real economic growth rate to be influenced by the inflation rate.

Theoretical analyses have been reported on the effect of inflation on the long-term real economic growth rate by incorporating endogenous growth theory into existing arguments on the real effects of nominal variables. Examples include De Gregrio [1993],

---

[3] The income of the creditor will increase. However, the increase in the debtor's probability of default is an asymmetrical cost, and thus the debtor's costs will be higher in comparison to those of the creditor.

which incorporates endogenous growth theory into the reversed Tobin effect of money, and van der Pleog and Alogoskoufis [1994], which incorporates monetary policy and endogenous growth theory into a Blanchard [1985]-type overlapping generations model. In these theoretical models, there is almost never a one-to-one relationship between the inflation rate, interest rates and the money growth rate, and they therefore provide different insights than can be achieved with standard theory, which is based on one-to-one relationships. However, the role of monetary policy within the context of endogenous growth theory remains virtually unexamined (Chang and Lai [2000]).

Turning to empirical research, studies have been done since the 1990s on the impact of the inflation rate on long-term real economic growth. De Gregrio [1992] and Fischer [1993] identify negative correlations between the inflation rate and the real economic growth rate. Bruno and Easterly [1995], Barro [1997], and Judson and Orphanides [1999] later report observing a negative correlation between the inflation rate and the real economic growth rate only in case inflation rates are higher than a certain level. Meanwhile, Bullard and Keating [1995] find that the long-run inflation rate will influence real economic levels, but not the long-term real economic growth rate.

(2) Selection of points included in quantitative evaluation

In this subsection, we narrow down points from among those discussed in the previous subsection to be included in our quantitative evaluation. In the end, this paper selects the above-mentioned points (a)-(d) for the evaluation while it passes over the points (e)-(h). Our reasons are as follows.

First, behind the problem with identification of changes in relative prices laid out in (e), is the existence of information costs, but it is not an easy task to model costs of information collection and processing for quantitative evaluation. Therefore, this paper assumes perfect information.

Regarding (f), taxation can change according to the nation’s choices at the time. We therefore elect not to discuss the costs and benefits of inflation based on any particular tax system even if the current taxation is non-inflation-neutral.

With respect to (g), it is in fact difficult to systematically control unexpected price changes by monetary policy, and indeed it becomes almost impossible the more rational the formation of expectations by economic agents.

Finally, regarding (h), when technological progress is endogenous and the steady-state inflation rate has an effect on real economic growth, the steady-state inflation rate would potentially have a relatively large impact on social welfare. However, the theory that the long-run inflation rate influences the real economic growth rate is a wide departure from standard economic theory, which is based on the natural rate hypothesis. In addition, as noted above, recent empirical researches find no significant relationship between the inflation rate and economic growth as long as the inflation rate is not substantially high. It is therefore difficult to conceive of serious welfare costs being generated as long as inflation or deflation is not extreme.

That leaves the points (a) through (d) for our quantitative evaluation in the following sections. Our basic strategies for evaluation are summarized below.

(a) The opportunity cost of holding money

This point is included in our quantitative evaluation because the Friedman Rule holds true from the perspective of consumer surpluses in conjunction with holding money.

On the other hand, with regard to the Tobin effect (or the reversed Tobin effect) of the opportunity cost on the real economy, the direction of the effect is uncertain since its theoretical basis depends on the way in which money-holding behavior is formulated. We have therefore excluded it from our quantitative evaluation.[4]

(b) The zero lower bound on nominal interest rates

This point is included in our quantitative evaluation because the problem can actually occur, as experienced by the Japanese economy between the end of the 1990s and mid-2006.

The handling of the zero lower bound will depend on the specifications of a monetary policy rule. Within the context of optimal monetary policy, Jung, Teranishi and Watanabe [2005] and Adam and Billi [2006, 2007] add the zero-lower-bound constraint on the central bank when it minimizes the loss function. On the other hand, it is also common for monetary policy to be expressed as a simple policy reaction function. Indeed, monetary policy has traditionally been expressed as an exogenous supply of money. For example, Krugman [1998], which is the simplest observation of the zero lower bound, assumes that the central bank supplies money exogenously. When expressing monetary policy in terms of a simple interest-rate rule, as is consistent with the actual policy conduct, it is standard to add a non-negativity constraint for the policy rate. This paper elects to express the zero lower bound in the form of a non-negativity constraint on the policy reaction function. We also account for the effect of “obtaining an advance on future monetary easing” by affecting people’s expectations, in the same way as Reifschneider and Williams [2000], when a zero lower bound is hit (for details see Section 5 (2)).

[4] For example, it is known that the reversed Tobin effect is extinguished over both the short and long terms if one assumes the additive separability of consumption and money in the money-in-the-utility function.

(c) Price stickiness

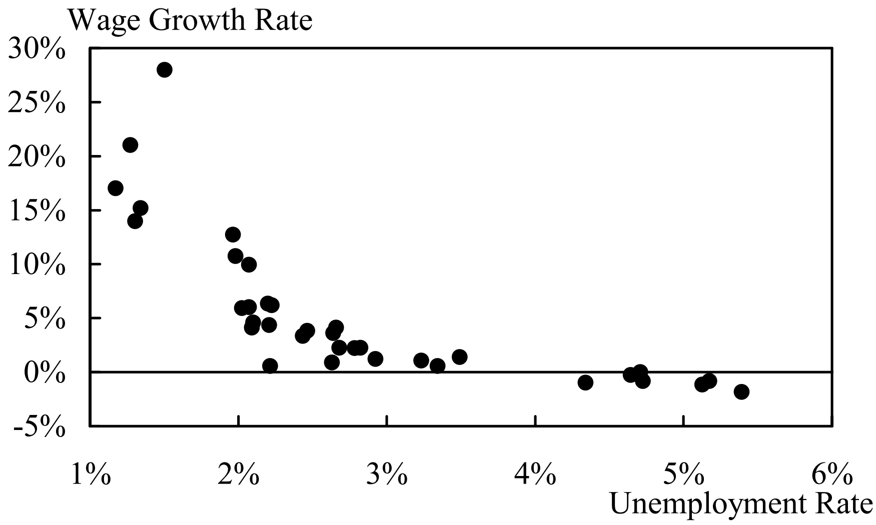
There are many different models of price stickiness, but ease of handling has made Calvo-type price-setting models the most common choice in recent years. This paper also assumes Calvo-type price-setting to derive a New Keynesian-type Phillips curve. We quantitatively evaluate the effect of price stickiness, using this Phillips curve and the loss function which is consistent with our model.

(d) Downward wage rigidity

According to Tobin [1972], the downward wage rigidity means that the Phillips curve is not vertical even over the long term. This nature goes against the traditional natural rate hypothesis. We will comment on this point briefly.

According to empirical analysis based on cross-sectional data from Japan, full downward wage rigidity can be confirmed from the 1992-1997 data, but is not observed after 1998 (Kuroda and Yamamoto [2005]). Examining macro data chronologically, the Japanese economy experienced a gradual decline in the wage growth rate during the 1990s, coupled with worsening unemployment. Even in 1998, when full downward rigidity could no longer be observed, there was no improvement in the employment rate. Both the wage growth rate and the unemployment rate bottomed out in 2002 and have since seen modest improvements. Data from 1970 to 2004 can be plotted simply, as shown in Figure 2, to create a nonlinear curve that illustrates the tradeoff relationship between the wage growth rate and the unemployment rate.

Figure 2: The Wage Growth Rate and Unemployment in Japan



(Note) The wage growth rate is defined as the annual change in compensation of employees per person. The unemployment rate is defined as the ratio of the unemployed in labor force.

The tradeoff observed in Figure 2 may not necessarily be a structural tradeoff, but the observed facts illustrated above are to a considerable degree consistent with the scenario described by Tobin [1972].

We therefore believe it appropriate to accept the fact of downward wage rigidity and the consequent long-term tradeoff in Japan. Nonetheless, we also acknowledge, under the assumption of the natural rate hypothesis, the wage Phillips curve is often considered to be vertical over the long term. Therefore, in Section 6 (2) we include analysis based on the long-term vertical Phillips curve even though it is not supported empirically.

3. Setup of Model

This section builds a structural model to express the costs and benefits of inflation subject to quantitative evaluation and derives a social loss function that is consistent with the model.

We posit a small-scale, closed-economy model. The private sector consists of households, retailers, and producers. Households of indefinite continuity consume goods and hold money. Consumption goods are individually differentiated. Retailers purchase intermediate goods from producers, convert them to individual consumption goods and sell them to households with a sticky price. Producers invest labor to produce intermediate goods. For simplicity, the production function does not include capital stock and there is no investment. Producers control wages and employment based on the labor efficiency of workers. We derive the downward rigidity of wages based on the assumption that workers are more reluctant to work, and lower their labor efficiency, when their wages are declining.

(1) Households

A household is expressed by the index $h \in [0,1]$. Its initial utility function, $U_0(h)$, is expressed as follows:[5]

[5] Unlike the real-business-cycle models or New Keynesian models, both of which usually observe flexible labor supplies, our model does not take account of the disutility of labor. This is due to the difference in the interpretation of unemployment. Models that account for the disutility of labor deem unemployment to be "leisure," which creates utility for workers in a sense. In other words, unemployment is voluntary. In such a model, the divergence between the marginal rate of substitution for consumption and leisure and the marginal productivity of labor is considered to be a social loss. On the other hand, our model does not account for the disutility of labor because it does

$$U_0(h) = E_o \sum_{t=0}^{\infty} \beta^t \{u(c_t(h), \xi_t) + v(m_t(h))\}, \tag{1}$$

where, u and v are the utility functions of consumption and money holdings, respectively, for each term. β is the subjective discount factor. $c_t(h)$ is the consumption index of household h at time t. $c_t(h)$ is defined as shown below as the Dixit-Stiglitz index of individual goods consumption, $c_t(h,z)$, expressed with individual-goods index $z \in [0,1]$:

$$c_t(h) \equiv \left[\int_0^1 c_t(h,z)^{\frac{\theta_t - 1}{\theta_t}} dz \right]^{\frac{\theta_t}{\theta_t - 1}}.$$

In this equation, θ_t is a variable expressing the price elasticity of individual-goods demand, as will be shown in Equation (5) below. $m_t(h)$ represents the real money balance held by household h.[6] ξ_t is a preference shock. The lowercase variables here are all figures divided by a trend (see details below).[7]

---

not interpret unemployment as being "leisure" for the worker or as creating utility. Under this interpretation, all reduction in consumption as a result of unemployment is deemed to be a social loss.

[6] We assume a utility function where consumption and money are additively separable. This means that there will be no Tobin effect or reversed Tobin effect.

[7] A consumption variable in the utility function in Equation (1) is written with a lowercase letter. It means that, in our assumption, consumption divided by a trend gives households utility. This assumption differs from that of the standard growth model, which assumes that consumption prior to the division by a trend gives households utility.

This paper adopts such an assumption in order to make the model consistent with the actual macroeconomic data in Japan. To be specific, empirical evidence indicates that in Japan's economy the intertemporal elasticity of substitution of consumption is much smaller than one, i.e., the coefficient of relative risk aversion is substantial. In addition, when a model in which consumption in the utility function incorporates a trend is assumed, it is generally the case that the theoretical real interest rate includes not only the rate of time preference but also the product of the consumption growth rate and the coefficient of relative risk aversion. Therefore, as in the case of Japan's economy where the coefficient of relative risk aversion is substantial, the theoretical real interest rate is likely to become far higher than the actual rate.

It is to avoid this problem that we assume the households who attain the utility from the consumption and the money holdings that are divided by a trend. It can be seen, in the development of the model, that with our assumption the real interest rate is independent of the coefficient of relative risk aversion. Thus, the above-explained problem will not occur. Therefore, the assumption is deemed appropriate for describing actual Japan's economy.

The budget constraint equation of household z is expressed as:

$$\int_0^1 P_t(z)C_t(h,z)dz + M_t(h) - M_{t-1}(h) + B_t(h) - R_t B_{t-1}(h) = W_t(h)l_t(h) + T_t(h) + \Gamma_t, \quad (2)$$

where $C_t\,(h,\ z)$, $M_t\,(h)$ and $B_t\,(h)$ represent individual goods consumption, nominal balance of money holdings, and nominal balance of outstanding bonds. These variables are controlled by households in utility maximization behavior. Uppercase variables such as these are variables prior to division by a trend. In relation to the lowercase variables already seen, with a deterministic trend D_t, consumption would, for example, be $C_t\,(h,\ z) = c_t\,(h,\ z)\ D_t$. In the pages that follow, unless otherwise specified, uppercase variables represent values prior to division by trend; lowercase, after division.

In Equation (2), $P_t\,(z)$ is the individual-goods price, R_t the gross rate of nominal interest, and $W_t\,(h)$ nominal wages, all of which are givens from the perspective of households. In this context, $l_t\,(h)$ is the number of employed persons in household h. As will be discussed later, producers allocate employment to households, so from the perspective of households, $l_t\,(h)$ is a given. Additionally, $T_t\,(h)$ is transfer income from a consolidated government funded by seigniorage, and Γ_t is dividends paid from the profits of producers and retailers.

To facilitate the maintenance of a notional "representative agent," it is assumed that $T_t\,(h)$ is transferred so as to eliminate differences in wage income $W_t\,(h)\ L_t\,(h)$ among households:

$$W_t(h_1)L_t(h_1) + T_t(h_1) = W_t(h_2)L_t(h_2) + T_t(h_2)\ for\ \forall h_1, h_2 \in [0,1].$$

$T_t\,(h)$ is transferred as a lump sum, and thus has no impact on household utility maximization behavior. On the assumption that the initial money balance and bond balance are the same for all households, each household has the same income. The consumption, money holding and bond holding chosen by individual households are all homogeneous each term. Below we will omit the household index h.

Assuming that, under the budget constraint Equation (2), households control goods consumption, money holdings and bond holdings so as to maximize the utility of Equation (1), the first-order conditions can be derived as:

$$E_t \beta \frac{u_c(c_{t+1}, \xi_{t+1})}{u_c(c_t, \xi_t)} \frac{P_t D_t}{P_{t+1} D_{t+1}} R_t = 1, \quad (3)$$

$$\frac{v_m(m_t)}{u_c(c_t, \xi_t)} = \frac{R_t - 1}{R_t}, \text{ and} \tag{4}$$

$$\frac{c_t(h,z)}{c_t} \equiv \left(\frac{P_t(z)}{P_t}\right)^{-\theta_t}, \tag{5}$$

where, price index P_t is defined as the Dixit-Stiglitz index of individual goods prices $P_t(z)$ as:

$$P_t \equiv \left[\int_0^1 P_t(z)^{1-\theta_t} dz\right]^{\frac{1}{1-\theta_t}}. \tag{6}$$

Equation (3) and (4) can be interpreted as the Euler equation and the money demand function, respectively. Equation (5) is a demand function of individual goods. The θ_t on its right side can be interpreted as the price elasticity of individual-goods demand.

"Zero-inflation steady state" is defined, in the same way as Woodford [2003], as the state in which the prices of all individual goods $P_t(z)$ are the same within a term and do not change over the periods. Below we derive linear equations for the model, using a log-linear approximation around the zero-inflation steady state.

First, Equation (3) leads to the following equation under the zero-inflation steady state:

$$\beta\gamma^{-1}R = 1,$$

where, value R, which is R_t with the subscript t removed, expresses the value of R_t in the zero-inflation steady state. This practice holds true below for other variables as well. γ_t is the trend gross growth rate, D_t / D_{t-1}, and γ is its value in the steady state.

Defining the discount factor as $\delta \equiv \beta\gamma^{-1}$, it can be seen that $R = \delta^{-1}$. The log-based net real interest rate is defined as $r \equiv -\log\delta$, leading to $R = \exp r$.

The log-linear approximation to the Euler equation around the zero-inflation steady state is expressed as:

$$\hat{c}_t = E_t\hat{c}_{t+1} - \sigma\left(\hat{i}_t - E_t\hat{\pi}_{t+1} - \hat{\gamma}_{t+1}\right) - \hat{g}_t. \tag{7}$$

The circumflex accents (^) above the variables indicate deviation from the zero-inflation steady state on a logarithmic basis. For example, $\hat{c}_t \equiv \log c_t - \log c$. The same applies to all other variables below unless otherwise specified. However, interest and inflation rates are defined as:

$$i_t \equiv \log R_t,\ \hat{i}_t \equiv \log R_t - \log R \text{, and}$$

$$\hat{\pi}_t \equiv \log P_t - \log P_{t-1}.$$

The parameter σ is the intertemporal elasticity of substitution of consumption defined as $\sigma \equiv u_c / u_{cc} c$, where $u_c \equiv u_c(c,\xi)$ and $u_{cc} \equiv u_{cc}(c,\xi)$ are the value of the partial derivative at the zero-inflation steady state. Similarly, where parentheses for a function are omitted below, the function expresses the values at the zero-inflation steady state. The final term on the right side of Equation (7), $\hat{g}_t$, is demand shock, which is defined as:

$$\hat{g}_t \equiv \frac{u_{c\xi}\xi}{u_{cc}c}\left(\hat{\xi}_t - E_t\hat{\xi}_{t+1}\right).$$

Next, the money demand function, Equation (4), leads to the following equation at the zero-inflation steady state:

$$\frac{v_m}{u_c} = 1 - \delta. \tag{8}$$

Taking its log-linear approximation, we obtain:

$$\hat{m}_t = \frac{\delta}{1-\delta}\chi\left\{\left(\delta^{-1} - 1\right)\left(\sigma^{-1}\hat{y}_t - \frac{u_{c\xi}\xi}{u_c}\hat{\xi}_t\right) - \hat{i}_t\right\},$$

where $\chi \equiv v_m / v_{mm} m$. Note also that $\delta^{-1} - 1$ is equivalent to net interest rate and is a very small value. Therefore, the first term in the middle brackets is smaller than the second term, $-\hat{i}_t$. By abstracting the first term, we obtain the following approximation to the money demand function:

$$\hat{m}_t = -\frac{\delta}{1-\delta}\chi\hat{i}_t. \tag{9}$$

Note that consumption and money have additive separability in the utility function and money has no direct effect. Because of this, the money demand function is not expressed in the model, but will be used to derive the social loss function, as described later.

(2) Retailers

Retailers purchase intermediate goods from producers, who will be described later on, in competitive markets, convert them to differentiated individual goods *z*, and sell them to households on monopolistic competitive markets.[8] It is assumed that retailers encounter Calvo-type price rigidity in selling the individual goods.

Retailer *z* maximizes the following profit function:

$$\sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \alpha^j \left(P_t^* - P_{t+j}^m\right) C_{t+j}(z),$$

where $\Lambda_t \equiv \beta^t u_c(c_t, \xi_t) P_t^{-1} D_t^{-1}$, which is the nominal discount factor, α is the probability of being unable to revise prices, P_t^* is the price set if prices can be revised, and P_{t+j}^m is the price of the intermediate goods. By substituting the individual-goods demand function, Equation (5), into the above equation, we obtain:

$$\sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \alpha^j \left(P_t^* - P_{t+j}^m\right) \left(\frac{P_t^*}{P_{t+j}}\right)^{-\theta_t} C_{t+j} .$$

The first-order condition for profit maximization can be derived as:

$$E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \alpha^j \left(\theta_t \frac{P_{t+j}^m}{P_t^*} - (\theta_t - 1)\right) \left(\frac{P_t^*}{P_{t+j}}\right)^{-\theta_t} C_{t+j} = 0 . \tag{10}$$

In addition, the price index defined by Equation (6) is subject to the following transition equation, depending on the percentage of producers able and unable to revise prices:

$$P_t = \left[\alpha P_{t-1}(z)^{1-\theta_t} + (1-\alpha) P_t^{*1-\theta_t}\right]^{\frac{1}{1-\theta_t}} . \tag{11}$$

[8] Retailers are separated from producers to avoid complexity arising from problems such as strategic complementarities.

The above equations on retailers' behavior can be expressed as deviation from the zero-inflation steady state as follows. We define the relative prices as:

$$p_t^* \equiv \frac{P_t^*}{P_t} \text{ and } p_t^m \equiv \frac{P_t^m}{P_t},$$

and then their values in the zero-inflation steady state are:

$$p^* = 1 \text{ and } p^m \equiv \frac{\theta - 1}{\theta}, \tag{12}$$

both of which are constant over time. Equation (10) can be expressed as a log-linear approximation around the zero-inflation steady state as:

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\hat{p}_{t+j}^m - \frac{1}{\theta - 1}\hat{\theta}_{t+j} + \sum_{k=1}^{j} \pi_{t+k} - \hat{p}_t^* \right) = 0 .$$

The same equation also holds true for term t + 1. Combining the term t and the term t + 1 equations, we arrive at:

$$\hat{p}_t^* = (1 - \alpha\beta)\hat{p}_t^m + \alpha\beta E_t(\pi_{t+1} + \hat{p}_{t+1}^*) + \frac{1 - \alpha\beta}{\theta - 1}\hat{\theta}_t . \tag{13}$$

Furthermore, Equation (11) can be expressed as a log-linear approximation to produce:

$$\hat{p}_t^* = \frac{\alpha}{1 - \alpha}\hat{\pi}_t .$$

By substituting this into Equation (13), we obtain:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \hat{p}_t^m + \hat{\mu}_t , \tag{14}$$

which is a so-called New Keynesian-type Phillips curve. In Equation (14), λ is the slope of the Phillips curve and $\hat{\mu}_t$ is a price shock, defined respectively as:

$$\lambda \equiv \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \text{ and } \hat{\mu}_t \equiv \frac{(1 - \alpha)(1 - \alpha\beta)}{\theta - 1}\hat{\theta}_t .$$

(3) Producers

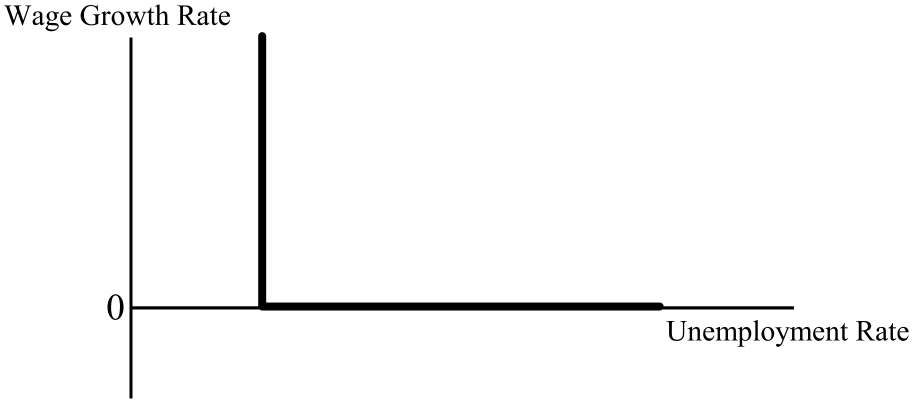
We provide a summary of our model on producers in this subsection. Details of derivation of the model are relatively complex, and thus have been consigned to Appendix A.

We build a model of the downward rigidity of wages, relying upon the efficiency wage model, which is considered the standard theory of unemployment, and by positing workers whose labor efficiency declines due to a decline in nominal wages. The efficiency wage model assumes that labor efficiency changes according to the level of wages and other factors, and that producers adjust wage levels to maximize their profits. When demand in the labor market is short, producers adjust both employment levels and wages. Workers are allocated jobs at this time. We consider labor to be the only factor of production and do not deal with capital stock or any other factor. It is also assumed that the production function is a linear function of labor.

We will begin by examining a case without any downward rigidity of wages, where the unemployment rate is maintained at a constant level regardless of the wage growth rate. In other words, there is no tradeoff between wages and unemployment. This is because at this point in time we have introduced no nominal friction, and thus there is no relationship between wages, which are a nominal variable, and unemployment, which is a real variable.

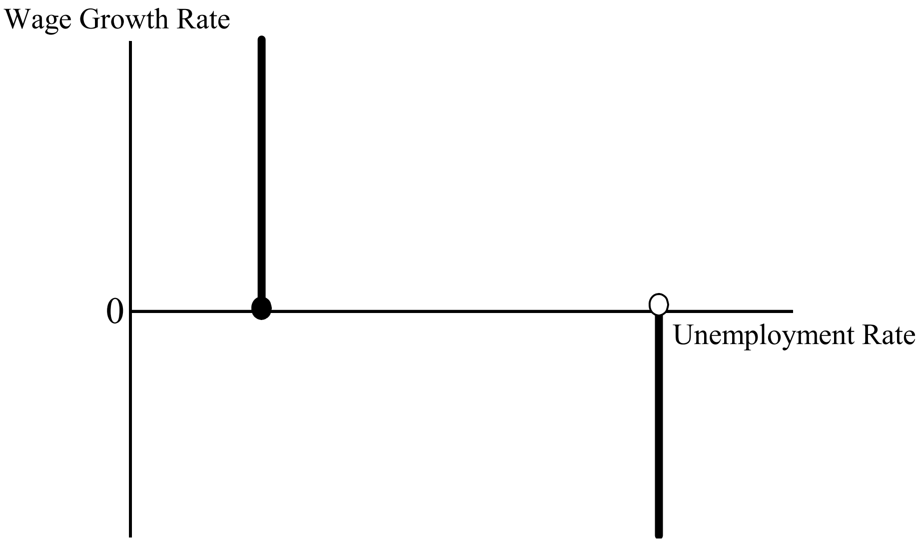
Next, we consider a contrastive case in which there is full downward rigidity of wages. Figure 3 illustrates the tradeoff relationship between wages and unemployment of the case. It shows an "L"-shaped curve, where the curve is vertical when the wage growth rate is positive and is flat when the rate is zero.

Figure 3: A Tradeoff between Wages and Unemployment under Full Downward Rigidity of Wages



It seems, however, that a model like this, with full downward wage rigidity, is not suited to the Japanese economy, which experienced wage declines in the 1990s. Therefore, rather than assume full downward rigidity, we assume the discontinuous labor efficiency function[9] in which the labor efficiency is lower when wages are in decline than it is when wages are in growth. It partly takes into account the downward rigidity, as shown in Figure 4.

Figure 4: A Tradeoff between Wages and Unemployment When Labor Efficiency is Lower with Wages Declining

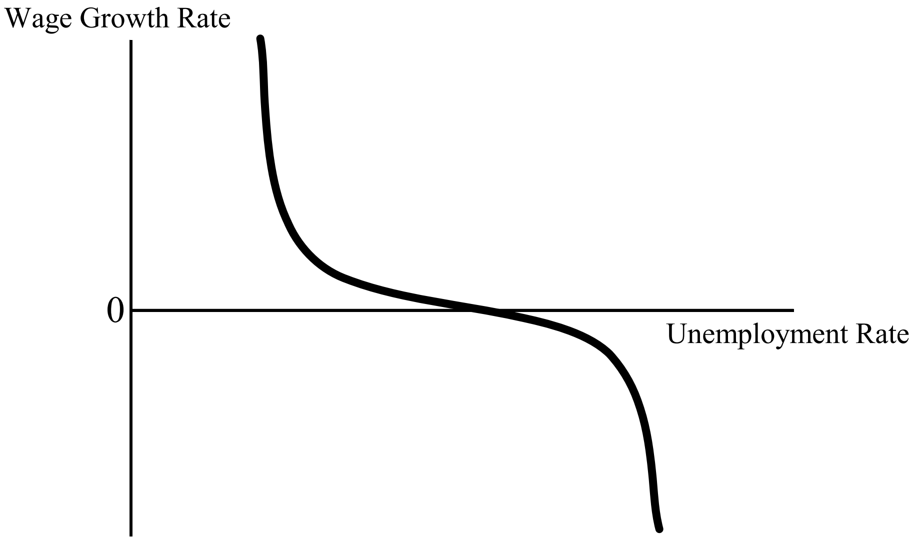


On the other hand, the actual data from Japan in Figure 2 indicates that the tradeoff relationship between wages and unemployment slopes downwards to the right when the wage growth rate is around zero. One hypothesis to explain this relationship is that there may be unevenness in the wage growth rate because of individual shocks to individual producers. Under this assumption, the higher the average wage growth rate, the higher the share of producers with growing wages, and therefore the lower the unemployment rate. For this case, it is easy to empirically estimate the relationship if we assume that the wage growth rate at individual producers follows a normal distribution. By adjusting the labor efficiency function from these perspectives so that there is a distribution of the wage growth rate at individual producers, we obtain the final model, presented in

[9] Elsby [2006], working from the perspective of behavioral economics, incorporates the risk-avoidance behavior of workers into a model and argues that workers will reduce labor efficiency if wages decline.

Appendix A, which leads to a tradeoff relationship between wages and unemployment such as that illustrated in Figure 5.

Figure 5: A Tradeoff between Wages and Unemployment When There Are Idiosyncratic Shocks



The curve indicating the tradeoff relationship above is subject to the cumulative normal distribution function. When the average wage growth rate is well into positive territory, wages are rising at the majority of producers, so the unemployment rate remains virtually unchanged near its floor value and the curve is close to vertical. As the average wage growth rate declines, the percentage of producers with declining wages increases. The unemployment rate for the labor market including producers with declining wages is relatively high, which results in higher overall unemployment rates. When the average wage growth rate is well into negative territory, wages are declining at the majority of producers, so the unemployment rate remains virtually unchanged near its upper-bound value and the curve is again close to vertical.

The equation below expresses this mathematically:

$$\upsilon_t = -\left(\upsilon^{upper} - \upsilon^{lower}\right) CN\left(\frac{\varpi_t}{\sigma_\varpi}\right) + \upsilon^{upper}, \tag{15}$$

where υ_t represents the unemployment rate, which is defined using the logarithmic value of the employment rate l_t as $\upsilon_t \equiv -\log l_t$, CN is the cumulative normal distribution function, and ϖ_t is the average wage growth rate defined in terms of logarithmic changes. The parameter υ^{upper} is the upper bound for the unemployment rate; υ^{lower}, the lower bound; and σ_{ϖ} is the standard deviation of the distribution of individual wages.

The zero-inflation steady state in Equation (15) is:

$$\upsilon = -\left(\upsilon^{upper} - \upsilon^{lower}\right) CN\left(\frac{\varpi}{\sigma_{\varpi}}\right) + \upsilon^{upper}.$$

Furthermore, Equation (15) can be given a linear approximation to produce:

$$\hat{\upsilon}_t = -\eta \hat{\varpi}_t, \tag{16}$$

where $\hat{\upsilon}_t \equiv \upsilon_t - \upsilon$ and $\eta \equiv \left(\upsilon^{upper} - \upsilon^{lower}\right) CN'\left(\varpi / \sigma_{\varpi}\right)$.

The equations above do not distinguish between long and short terms; they indicate the existence of a tradeoff between wages and unemployment even over the long term. The natural rate hypothesis does not, therefore, hold true in this case.

On the other hand, under the natural rate hypothesis, the impact on economic agents of nominal variables is the money illusion, which is extinguished over the long term. This feature does not match the above-mentioned characteristics of long-term differences in labor efficiency in terms of whether wages fall or not. Therefore, as an alternative specification of our basic model with the long-term tradeoff, we will also derive a specification with a short-term tradeoff based on the natural rate hypothesis. That is, we adjust the labor efficiency function so that it is influenced by short-term changes and assume that workers form their expectations in a backward-looking way. This setup produces the following tradeoff between wages and unemployment:

$$\varpi_t = \varpi_{t-1} - \left(\upsilon_t - \upsilon\right),$$

where the steady-state unemployment rate, v, is constant regardless of the inflation rate—in other words, it is the non-accelerating inflation rate of unemployment (NAIRU). This equation can also be expressed in terms of deviation from the zero-inflation steady state as:

$$\hat{\varpi}_t = \hat{\varpi}_{t-1} - \hat{\upsilon}_t. \tag{17}$$

In addition, we can derive the following relationships from producer behavior. The real unit labor costs s_t, defined as $s_t \equiv W_t l_t / P_t Y_t$, matches the relative price of the intermediate goods p_t^m. With the trend-divided value of labor productivity A_t, denoted as $a_t \equiv A_t / D_t$, the relationship between output and the unemployment rate is $y_t = a_t l_t$. The steady-state wage growth rate ω is equivalent to the trend growth rate of output, $\log \gamma$. In this paper we define the output gap, $\hat{x}_t$, as the logarithmic deviation between actual output and production capacity[10]: $\hat{x}_t \equiv \hat{y}_t - \hat{a}_t$. The relationship between the output gap, the employment rate and the unemployment rate is: $\hat{x}_t = \hat{l}_t = -\hat{\upsilon}_t$.

(4) Market clearing conditions

The goods market clearing requires that:

$$\int_0^1 Y_t(f)df = \int_0^1 \int_0^1 C_t(z,h)dzdh .$$

Because we consider a model assuming a closed economy and no investment, output and consumption match in the equilibrium.

The money and bond markets clearing requires that:

$$\int_0^1 M_t(h)dh = M_t^s \quad \text{and} \quad \int_0^1 B_t(h)dh = 0 ,$$

where M_t^s denotes the money supply. Bonds issued by the consolidated government are zero in the equilibrium.

(5) Overall model

Let us summarize the equations derived in subsections (1) through (4). We will express the macroeconomic variables here in terms of logarithmic deviation from the zero-inflation steady state.

- IS curve

$$\hat{x}_t = E_t \hat{x}_{t+1} - \sigma\left(\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{\gamma}_{t+1}\right) + \hat{g}_t + E_t \Delta\hat{a}_{t+1} . \tag{18}$$

- Dynamics of the unit labor cost

$$\hat{s}_t = \hat{s}_{t-1} + \hat{\varpi}_t - \hat{\pi}_t - \Delta\hat{a}_t . \tag{19}$$

---

[10] The output gap in this paper is defined based on the utilization rate of production factors, which has long been used in economic analysis in practice. It is different from the output gap, defined as a logarithmic deviation between actual output and the natural rate of output, which is typically used in an analysis with a New Keynesian-type model.

- New Keynesian-type Phillips curve

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \hat{s}_t + \hat{\mu}_t . \quad (20)$$

- The tradeoff between the wage growth rate and the output gap

$$\hat{x}_t = \eta \hat{\varpi}_t . \quad (21)$$

- The alternative relationship between the wage growth rate and the output gap: NAIRU version

$$\hat{\varpi}_t = \hat{\varpi}_{t-1} + \hat{x}_t . \quad (22)$$

Equation (18) is derived from Equation (7) by using the relationships: $\hat{y}_t = \hat{c}_t$ and $\hat{x}_t \equiv \hat{y}_t - \hat{a}_t$. Equation (19), which indicates the relationship between the unit labor cost, the inflation rate and the wage growth rate, is obtained by transforming the equation defining the unit labor cost into its logarithmic deviation expression in combination with the relationship: $y_t = a_t l_t$. Equation (20) is derived from Equation (14) by using the relationship: $\hat{p}_t^m = \hat{s}_t$. Equation (21) and (22) are obtained, respectively, from Equation (16) and (17) by using the relationship: $\hat{x}_t = -\hat{\upsilon}_t$.

The steady state with a positive inflation rate is expressed as a deviation from the zero-inflation steady state. This deviation is expressed using an overbar on the variables. For example, $\overline{\pi}$ is the deviation of the steady-state inflation rate from zero inflation. Equations (18) and (19) imply one-on-one relationships, at the steady state, between the inflation rate and both nominal interest rates and nominal wage growth rates:

$$\bar{i} = \overline{\pi} \quad \text{and} \quad \overline{\varpi} = \overline{\pi} .$$

Similarly, Equation (21) implies for the steady state:

$$\overline{x} = \eta \overline{\varpi} . \quad (23)$$

However, the tradeoff relationship between wages and unemployment is essentially nonlinear. Therefore, instead of Equation (23), we will use the following equation, which is derived from Equation (15) without the log-linear approximation, for the purpose of quantitative evaluation.

$$\overline{x} = \left(\upsilon^{upper} - \upsilon^{lower}\right)\left(CN\left(\frac{\varpi + \overline{\varpi}}{\sigma_{\varpi}}\right) - CN\left(\frac{\varpi}{\sigma_{\varpi}}\right)\right) . \quad (24)$$

This means that the level of the output gap will vary according to the steady-state inflation rate. On this point, the natural rate hypothesis found in Friedman [1969] does not hold true. By nature it derives exclusively from the downward rigidity of wages and does not depend on other factors that would disprove the natural rate hypothesis, for example, the direct effects of money or effects felt through markup rates.[11] From the perspective of the natural rate hypothesis, Equation (21) can be replaced with a NAIRU-type Equation (22), which results in a steady state of:

$$\bar{x} = 0 .$$

The natural rate hypothesis holds true here because the output gap in the steady state is constant and not dependent on the inflation rate.

(6) Social loss function

We derive the social loss function by calculating a second-order approximation to the utility function of households in the same way as Woodford [2003]. Details of the derivation process are given in Appendix B. The function of the total social loss, TSL_0, is ultimately expressed with the following formula:

$$TSL_0 = E_0 \sum_{t=0}^{\infty} \beta^t SL_t \ , \tag{25}$$

$$SL_t = \frac{1}{2}\frac{\theta}{\lambda}\hat{\pi}_t^2 - \hat{x}_t + \frac{1}{2}\left(\sigma^{-1} - 1\right)\hat{x}_t^{\,2} + (1-\delta)k\left\{\frac{\delta}{1-\delta}\chi\hat{i}_t + \frac{1}{2}\left(\chi^{-1} - 1\right)\left(\frac{\delta}{1-\delta}\chi\hat{i}_t\right)^2\right\}. \tag{26}$$

The social loss for each term, SL_t, is expressed as a second-order Taylor series approximation around the zero-inflation steady state.[12] On the right side of Equation

---

[11] The steady-state markup rate $\bar{s}^{-1}$ depends on $\bar{\pi}$. The explanation of this has been omitted because it is not important to our model. For more details see King and Wolman [1996].

[12] Note that this social loss function, expressed as a second-order approximation, includes linear terms of the output gap and the nominal interest rate.

Generally, when a structural model is expressed as a first-order approximation as in this paper, the precision of macro variables is also the first order. In such a case, if the formula of the second-order approximation for the social loss function is purely quadratic, including no linear term of a macro variable, the precision of the social loss will be in fact the second order. On the other hand, when the formula for the social loss includes linear terms of macro variables, as in this paper, the precision of the valuation will fall to the first-order level. This problem is pointed out in Dr. Kosuke Aoki's comments to the paper.

(26), we have adjusted the constant so that the social loss is zero in the zero-inflation steady state. Moreover, the coefficients for macroeconomic variables are normalized so that the one for the first-order term of the output gap is set at -1. This means that a 0.01 increase in SL_t will result in a social loss equivalent to 1% of the real GDP. Note that k is the value of the ratio of outstanding money to GDP in the zero-inflation steady state.

Below is an explanation of the individual terms on the right side of Equation (26). The first term is an inflation-rate term that expresses the "loss from price changes." The derivation process shows that it is the loss generated by distortions in relative prices among individual goods as a result of price changes. The fact that the inflation-rate term has only a quadratic term and no linear term is to indicate that the loss is minimized at zero inflation.

The second and third terms represent output-gap terms that express the "loss from the output level." The coefficient for the linear term is negative, indicating that the loss is lower at a higher output level. The coefficient for the quadratic term is positive, indicating that changes in output levels will generate losses.

Finally, the fourth term is an interest-rate term that expresses the "loss from money holding." The coefficient for the linear term is positive, indicating that the higher the interest rate the greater the opportunity cost of holding money and the greater the loss of consumer surplus. This term can be interpreted as an expression of the Friedman Rule.

To resolve the problem thoroughly, it is worth considering the methodology of Benigno and Woodford [2005], with which the structural model can be expressed as a second-order approximation. In this paper, however, because the calculation burden of stochastic simulations is quite heavy, the priority should be given to reducing the complexity of the model, thereby employing a first-order approximation for the structural model. Issues arising from the above-mentioned problem are addressed as follows.

First, concerning the terms of the nominal interest rate, which are related with the loss from money holding, the coefficient of the linear term is relatively small compared with that of the quadratic term. Thus, the approximation error is insignificant. On the other hand, concerning the terms of the output gap, which are related with the loss of the output level, the coefficient of the linear term is not necessarily small. In a typical analysis based on a New-Keynesian model, the potential problem of approximation error is eliminated by assuming that the efficient level of output is achieved around the zero-inflation steady state, through the adjustment of production volume with subsidies from the government, thereby there is no linear term of the output gap. This paper, however, considers that it is not appropriate to use such an assumption for the purpose of simplification in evaluating the optimal steady-state inflation rate, and therefore allows production volume to be lower than the efficient level of output in the zero-inflation steady state, i.e., there is a linear term of the output gap in the social loss function. As a result, a second-order error in the output level could not be negligible in evaluating social loss. Taking this into consideration, this paper uses the original nonlinear Equation (24), instead of the first-order approximation, in calculating the deviation of the output level in the arbitrary-inflation steady state from that in the zero-inflation steady state, so as to reduce the problem of the approximation error.

The quadratic term in the fourth term represents the loss from interest-rate changes.

4. Evaluation of Social Loss in the Steady State

(1) Concept of social loss in the steady state

In this section and later, we quantitatively evaluate the social loss for Japan's economy, using the economic model and the social loss function constructed in the previous section.

In so doing, we can divide up the social loss into those that occur in the steady state and those that are the result of deviation from the steady state. If the economic structure is in the range of linear approximation and monetary policy does not face the zero lower bound of interest rates, the model as a whole becomes linear, and thus the social losses generated by deviation from the steady state do not depend on the level of the steady-state inflation rate. Therefore, when comparing social losses with different steady-state inflation rates, there is no need to consider the social losses generated by deviation from the steady state; it is sufficient to compare steady-state social losses. On the other hand, if there is a zero interest rate bound, the model becomes nonlinear and the social losses generated by deviation from the steady state will depend on the steady-state inflation rate.

In light of these perspectives, this section measures the social losses in the steady state and compares them under different steady-state inflation rates. This analysis is a starting point to the overall evaluations that follow in later sections. The analysis holds true when the economic structure is within the range of linear approximation around the zero inflation and the nonlinearity from the zero interest rate bound can be ignored. Hereafter, we consider that the economic structure is indeed in the range of the approximation as long as the inflation rate is in the range of 0-3%,[13] and conduct our analyses in this setting. We handle the nonlinearity from the zero interest rate bound in Sections 5 and 6.

(2) Social loss function for the steady state

The social loss function for a term, expressed as Equation (26), can be written in steady states as:

[13] The size of approximation errors that arise from abstracting the higher-order terms is in the degree of 0-3% of the terms that are not abstracted in the approximation when the inflation rate is in the range of 0-3%. We consider that such errors can be ignored for the purpose of our analysis.

$$\overline{SL} = \frac{1}{2}\frac{\theta}{\lambda}\bar{\pi}^2 - \bar{x} + \frac{1}{2}\left(\sigma^{-1} - 1\right)\bar{x}^2 + (1-\delta)k\left\{\frac{\delta}{1-\delta}\chi\bar{i} + \frac{1}{2}\left(\chi^{-1} - 1\right)\left(\frac{\delta}{1-\delta}\chi\bar{i}\right)^2\right\}. \quad (27)$$

In steady states, the one-to-one relationship between interest and inflation rates results in $\bar{i} = \bar{\pi}$. $\bar{x}$ is as shown below if a tradeoff is recognized between wages and unemployment:

$$\bar{x} = \left(\upsilon^{upper} - \upsilon^{lower}\right)\left(CN\left(\frac{\varpi + \bar{\pi}}{\sigma_{\varpi}}\right) - CN\left(\frac{\varpi}{\sigma_{\varpi}}\right)\right). \quad (28)$$

As an alternative to Equation (28), we also use a NAIRU-type wage Phillips curve as:

$$\bar{x} = 0.$$

Therefore, Equation (27) is a function of $\bar{\pi}$. It is possible to find the optimal value $\bar{\pi}^*$ which minimizes the social loss function. Specifically, $\bar{\pi}^*$ satisfies the FOC as:

$$\left.\frac{d\overline{SL}}{d\bar{\pi}}\right|_{\bar{\pi}=\bar{\pi}^*} = 0.$$

We can express the solution of $\bar{\pi}^*$ analytically if we make a log-linear approximation to Equation (28) as $\bar{x} = \eta\bar{\pi}$.[14] However, we do not use this approximation below. We keep the nature of non-linearity in the evaluations of $\bar{\pi}^*$ by numerically plotting the function $\overline{SL}$ so that we can evaluate it more accurately.

It is necessary to set plausible parameters to quantitatively evaluate the social loss. In this paper, we set parameter values to reflect the nature of the Japanese economy wherever possible.

(3) Data

The data used to set parameters has term increments of years. This is because producers in Japan do not sufficiently adjust wages and employment in monthly or

[14] When we make a log-linear approximation to Equation (28), the solution of the optimal value, $\bar{\pi}^*$, can be derived as:

$$\bar{\pi}^* = \frac{\eta - k\delta\chi}{\theta\lambda^{-1} + \eta^2\left(\sigma^{-1} - 1\right) + k\delta^2(1-\delta)^{-1}\chi(1-\chi)}.$$

When we use a NAIRU-type wage Phillips curve as an alternative to Equation (28), it can be expressed as:

$$\bar{\pi}^* = \frac{-k\delta\chi}{\theta\lambda^{-1} + k\delta^2(1-\delta)^{-1}\chi(1-\chi)},$$

which is negative.

quarterly increments, making a term of at least one year necessary. In addition, we use fiscal year data because Japan's adjustments to wages and employment tend to follow the standard fiscal year that begins in April and ends the next March. In the pages that follow, "year" refers not to the calendar year beginning in January but the fiscal year beginning in April.

Unless stated otherwise, the sample period is from 1981 to 2004. This period was chosen to exclude the impact of the oil crises, which continued to be felt prior to 1980. Data that includes the 1970s is discussed in Section 6.

For output Y_t we use real GDP; for price index P_t, the GDP deflator; for nominal wages W_t, compensation of employees per person; for labor productivity A_t, per-employee real GDP. The gross interest rate R_t is based on the overnight call rate, where the collateralized call rate is used up to 1985 and the uncollateralized call rate in 1986 and beyond.

The output gap $\hat{x}_t$ is measured as the deviation of unemployment rate υ_t from its sample average because $\hat{x}_t$ corresponds to $\hat{\upsilon}_t$ in the model.

For the real unit labor cost s_t, we use labor's share of national income. Labor's share is calculated by multiplying per-employee wages by number of employees and dividing by nominal GDP. $\hat{s}_t$ is measured as the logarithmic deviation from the sample average of labor's share.

Finally, money is measured as M1 because it is non-interest-bearing financial assets held by the private sector.[15]

(4) Parameter settings

We set the discount factor δ at 1/1.03 because the sample average of the ex-post real short-term interest rate is 3%. Regarding the labor productivity, which we measure as a ratio of real GDP to number of employees, the average growth rate for the sample period is 1.8%. Therefore, the trend of the gross rate of productivity growth, γ, is set at 1.018. The subjective discount factor β is set as $\beta = \delta\gamma = 1.018/1.030 \cong 0.982$. The wage growth rate ω, defined based on the logarithmic change, in the zero-inflation steady state is set as $\varpi = \log\gamma \cong 0.018$, which is the logarithmic trend of the productivity growth.

Referring to Galí and Gertler [1999], we calculate the probability, α, that retailers will not be able to revise prices by estimating the log-linear Phillips curve:

[15] M1 includes ordinary deposits, which do earn a small amount of interest. However, since the amount is negligible, we deem them to be non-interest-bearing for the purposes of this paper.

$$E\left[\alpha\pi_t - \alpha\beta\pi_{t+1} - (1-\alpha)(1-\alpha\beta)\hat{s}_t\right] = 0 .$$

We use the instrumental variables method for the estimation since the equation includes a forward-looking variable π_{t+1}.[16] The instrumental variables are the constant term, $\hat{\pi}_{t-1}$, $\hat{s}_{t-1}$, and υ_{t-1}. β is fixed at the value set above. The estimate of α is 0.58 with standard error of 0.14. Based on the result, we set $\alpha = 0.6$. This means that the average interval of price revisions, $1/(1-\alpha)$, is 2.5 years. Based on this, the slope of the Phillips curve, λ, is set at 0.27.

Next, we calculate values of the interest-rate elasticity of money demand, χ, intertemporal elasticity of substitution of consumption, σ, and the ratio of money balance to GDP in the zero-inflation steady state, k, by estimating the following equation, which is based on the money demand function in Equation (4):

$$\frac{M_t}{P_t Y_t} = k' \left(\frac{i_t - 1}{i_t} \right)^{-\chi} .$$

This is derived from Equation (4) by assuming that the elasticity is constant and that χ is equal to σ. We adopt the latter assumption because the failure of χ and σ to match would make the steady state untenable for the ratio of money balance to GDP. k' is a constant, which we estimate by the nonlinear least squares method. Estimates are 0.116 for χ with standard error of 0.003, and 0.185 for k' with standard error of 0.005.[17] Based on these estimations, for the purpose of evaluating the social loss we set the parameters, $\chi = \sigma = 0.12$,[18] and $k = k'(1-\delta)^{-\chi} = 0.28$.

Working from Equation (12) and Equation (39), the elasticity of demand for individual goods θ has the following relationship with the steady-state value of real unit labor costs s:

$$\theta = \frac{1}{1-s} .$$

The sample period average of real unit labor costs is $s = 0.68$. From this, we set θ at 3.1.

---

[16] According to Galí and Gertler [1999], GMM estimation would be preferred. This analysis, however, uses the instrumental variables method because of the small size of the sample. There is no significant change in our results when GMM estimation is used.

[17] Shiratsuka [2001] estimated χ at 0.11, a result similar to ours.

[18] This is close to Kimura and Kurozumi's [2004] result, estimating that $\sigma = 0.15$.

Regarding the tradeoff relationship between wages and unemployment, we estimate the following equation:

$$\upsilon_t = -\left(\upsilon^{upper} - \upsilon^{lower}\right)CN\left(\frac{\varpi_t}{\sigma_{\varpi}}\right) + \upsilon^{upper} .$$

The next table contains the results of the estimation with the least squares method, and the parameters set based on these results.

Table 1: Estimation Result (1981-2004) of a Tradeoff between Wages and Unemployment and Parameter Settings

	Estimated value	(Standard error)	Parameters set
υ^{upper}	0.0571	(0.0030)	0.057
υ^{lower}	0.0247	(0.0015)	0.025
σ_{ω}	0.0116	(0.0029)	0.012

This result means that, when the wage growth rate is zero, the unemployment rate will be 4.1%, the average of the estimated upper and lower bounds of the unemployment rate.

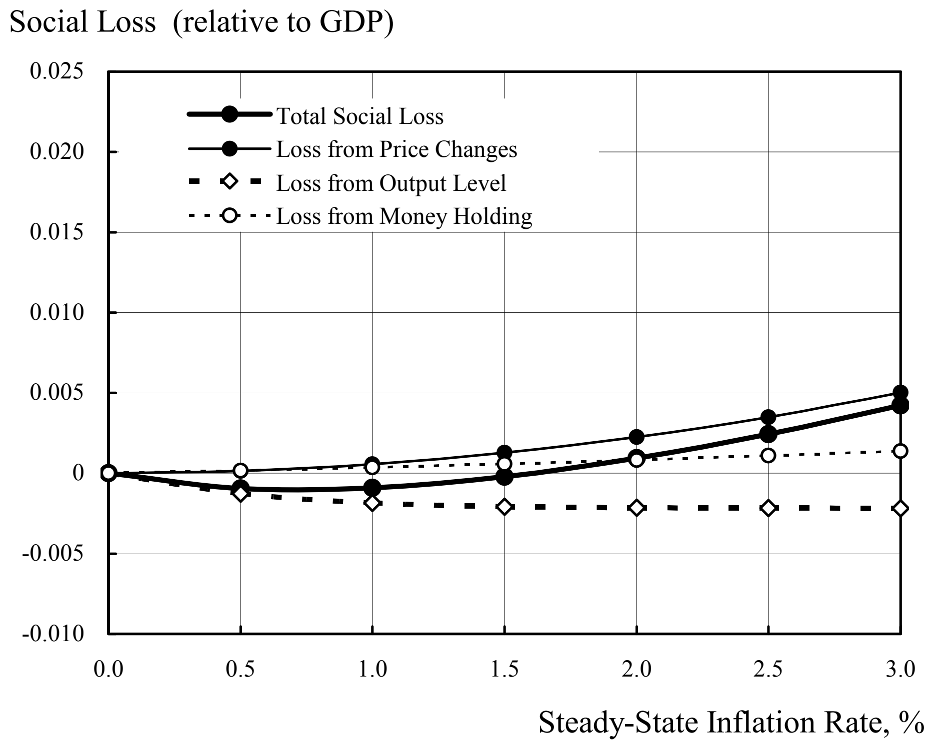
(5) Evaluation of Social Loss in the Steady State

Figure 6 shows the results of calculations of $\overline{SL}$ for the social loss in Equation (27) using the parameters set above and considering various steady states with the steady-state inflation rate set in increments of 0.5% for the range 0% to 3%. The scale of the social loss on the vertical axis corresponds to the logarithmic deviation of GDP; 0.01 means constant generation of social losses equivalent to 1% of GDP in each year.

Looking at the figure, for example, when the inflation rate is 3%, a social loss corresponding to a 0.4% reduction of GDP every year will be generated, compared to the zero-inflation steady state. In line with the definitions in Section 3 (6), the breakdown of the social loss is as follows: (i) a loss from price changes equivalent to a 0.5% reduction of GDP, (ii) a loss from the output level of between -0.2 and -0.3% reduction of GDP (in other words, a benefit from an increase of GDP), and (iii) a loss from money holding of a 0.1-0.2% reduction of GDP. The “optimal rate of inflation” that minimizes the social loss is estimated at about 0.5% or 1.0%. We should emphasize, however, that the curve of the social loss function is quite flat. For example, the deviation of the social loss for a deviation in the inflation rate of ±0.5% seems less than

0.1% of GDP. Given the uncertainty involved in the statistical data, the modeling, and parameter settings of this paper, one can interpret the difference in the social loss as being relatively small.

Figure 6: Social Loss in the Steady State



Note that when the long-term tradeoff relationship between wages and unemployment is disallowed and the NAIRU-type relationship is posited, the loss from the output level is unrelated to the long-run inflation rate. Therefore, the loss of output is eliminated from calculations of the social loss. In the circumstances, the assumption of an inflation rate above zero as shown in Figure 6 will lead to the conclusion that the social loss is minimized exactly at zero inflation. If deflation is allowed, the social loss will be minimized in the deflation domain.

5. Evaluation of Social Loss Accounting for Deviation from the Steady State

(1) Concept of social loss generated by deviation from the steady state

The analysis in the previous section focused on the social loss in the steady state. Section 5 expands on that analysis to include considerations of a social loss generated by deviation from the steady state. We will account for the nonlinearity of the zero lower bound by conducting a stochastic simulation under the non-negativity constraint of interest rates while maintaining the assumption that the economic structure is within the range of linear approximation.

(2) Specifications of monetary policy rule and method for solving the model

First, we will specify a monetary policy rule to conduct a stochastic simulation that accounts for the zero lower bound on interest rates. One method would be to analyze a theoretically optimal policy rule as shown in Adam and Billi [2006, 2007]. This paper, however, focuses on using a policy rule with more real-life feasibility. Specifically, we work from a Taylor-type simple policy response function to express the policy rate i_t under the zero lower bound on interest rates as:

$$i_t^f = r + \bar{\pi} + \phi_\pi(\hat{\pi}_t - \bar{\pi}) + \phi_x(\hat{x}_t - \bar{x}) - (i_{t-1} - i_{t-1}^f), \text{ and}$$

$$i_t = \max(i_t^f, 0),$$

where i_t^f expresses the policy rate when there is no zero lower bound; ϕ_π expresses policy response to the inflation rate; and ϕ_x expresses policy response to the output gap. The second equation above shows that the policy rate i_t is equal to the rate i_t^f derived from the first equation when i_t^f is non-negative, but that the policy rate i_t is set at zero when i_t^f is negative. The final term in the first equation above is to incorporate a kind of "policy duration effects" in the same way as Reifschneider and Williams [2000]. It has the effect of supplementing the current shortfall in monetary easing at the zero lower bound by using commitment to maintain easy monetary policy in the future. In other words, the model takes into account the possibility of obtaining an advance on future monetary easing to increase the current effect of monetary policy.[19]

Parameters ϕ_π and ϕ_x are set based on actual policy response. They are estimated as:

$$i_t = \phi_\pi \pi_t + \phi_x \hat{x}_t + constant.$$

The sample period is from 1981 until 1998, after which the Japanese economy hit the zero lower bound. The ordinary least squares (OLS) method is adopted for estimation.

[19] Many empirical studies report that the quantitative monetary easing adopted in Japan between 2001 and 2006 generated such a policy duration effect through a policy with commitment to CPI developments. For a survey of these studies, see Ugai [2007].

Parameters are estimated as 1.36 for ϕ_π with standard error of 0.18, and as 1.06 for ϕ_x with standard error of 0.42. Based on the result, we set $\phi_\pi = 1.4$ and $\phi_x = 1.1$.

We need to solve our model because it includes forward-looking variables. The model, however, cannot be solved analytically since it has non-linearity stemming from the zero lower bound on nominal interest rates. Thus, we conduct numerical calculations using the stacked-time algorithm (Hollinger [1996]) to solve the model.

Regarding the expected value of the interest rate in the future, one would ideally impose the non-negativity constraint on individual forecasted values of the interest rate and average them to arrive at the expected interest rate:

$$E_t i_s = E_t \max\left(i_s^f, 0\right) \quad for \forall s > t.$$

However, in our numerical calculations, the non-negativity constraint is imposed on the expected interest rate to reduce calculation burdens as:

$$E_t i_s = \max\left(E_t i_s^f, 0\right) \quad for \forall s > t.$$

With this treatment, we partly take into account the feature that economic agents form their expectation taking into consideration the potential of hitting the zero lower bound on nominal interest rates in the future.

(3) Method of stochastic simulation

To conduct the stochastic simulation, we must also specify stochastic processes for exogenous variables. We assume the trend growth rate γ_t to be linear and $\hat{\gamma}_t = 0$. In addition, productivity change shocks, demand shocks and price shocks are assumed to follow the first-order autoregressive processes:

$$\Delta\hat{a}_t = \rho_a \Delta\hat{a}_{t-1} + \varepsilon_{a,t}, \text{ where } \varepsilon_{a,t} \sim N\left(0, \sigma_a^2\right),$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t}, \text{ where } \varepsilon_{g,t} \sim N\left(0, \sigma_g^2\right),$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \varepsilon_{\mu,t}, \text{ where } \varepsilon_{\mu,t} \sim N\left(0, \sigma_\mu^2\right).$$

The autoregressive process of productivity change shocks can be directly estimated from the data. The autoregressive parameter ρ_a is set from the estimated value so that $\rho_a = 0.25$; the standard deviation σ_a is set from the standard error of the regression so that $\sigma_a = 0.014$.

For demand shocks, we substitute data for the variables in the IS curve into Equation

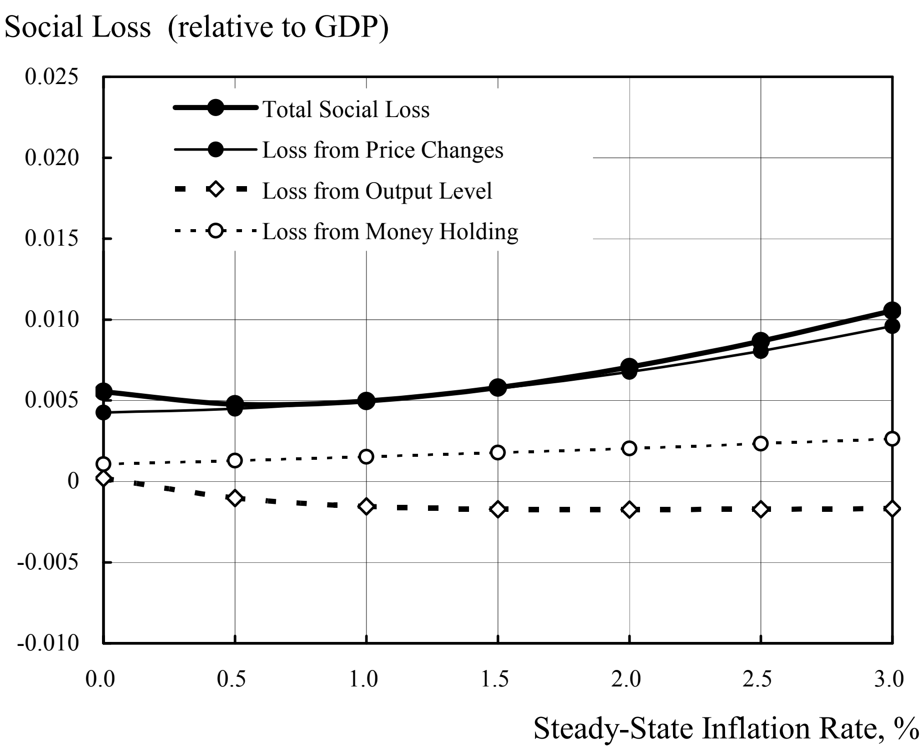
(18) and calculate the residuals in the same way as Rotemberg and Woodford [1998] and other studies. Among the variables in Equation (18), the historical data is used for the realized values, and the forecast data from the three-variable (υ_t, π_t, i_t) unconditional VAR for the expected values. Price shocks are handled similarly as the residuals from the Phillips curve in Equation (20). Having done this, the next step is to estimate the autoregressive processes. Based on the shocks obtained, we estimated the autoregressive parameters and the standard deviations of autoregressions, and set ρ_g = 0.71 and σ_g = 0.0056 for demand shocks and ρ_g = 0.36 and σ_g = 0.0065 for price shocks.

We generate 1,000 series of the three shocks to evaluate the social loss in a simulation. The social loss is calculated not as the discounted present value TSL_0 in Equation (25) but as the average of a social loss for each term SL_t in Equation (26) over ten periods (i.e., ten years).

(4) Result of evaluating social loss

The simulation results are shown in Figure 7.

Figure 7: Social Loss Accounting for Deviation from the Steady State



The results in Figure 7 indicate that, in comparison to the steady-state social loss in the previous section (Figure 6), there is little change in the loss from the output level and the loss from money holding, but the curve expressing the loss from price changes has generally shifted upwards. This is because there are additional losses generated due to economic fluctuations around the steady state.

There are no other significant changes in the shape of the total social loss, compared to the steady-state analysis, other than the shift upwards. The conclusion, therefore, is that the steady-state inflation rate that minimizes the social loss is estimated to be in the same range as for the steady-state analysis, 0.5-1.0%. This means that the additional "safety margin" in the steady-state inflation rate to mitigate the risk of destabilizing the economy due to the zero lower bound on interest rates is not particularly large for this case.

The size of the desired additional safety margin will depend on the degree to which the policy duration effect can be utilized in monetary policy. The above results partly reflect the fact that the capability of monetary policy to obtain an advance on future monetary easing through the policy duration effect reduces the latent social loss from the zero lower bound on interest rates. For contrast, Section 6 (1) analyzes a scenario in which monetary policy has no duration effect.

6. Evaluation of Social Loss with Alternative Model Settings

The model used in Sections 4 and 5 attempts to match the characteristics of the Japanese economy as closely as possible. There is room for debate, however, on the possibility of better model settings. In this section, we change some of the assumptions underlying the previous simulations and analyze several alternative cases to determine the extent to which the simulation results change. Four specific cases will be considered: (1) no policy duration effect in monetary policy, (2) a stronger tradeoff between wages and unemployment, (3) no long-term tradeoff between wages and unemployment and (4) a lower productivity growth rate.

(1) Social loss in the economy with no policy duration effect in monetary policy

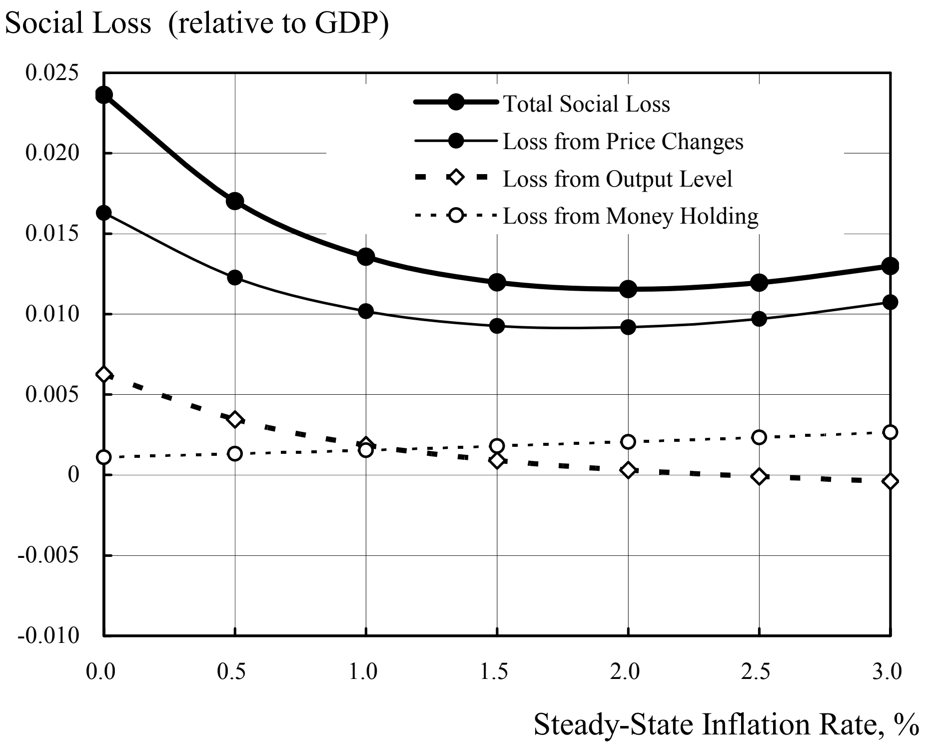
For monetary policy, we assume a policy rule that is just a Taylor-type rule with a zero lower bound on interest rates. In other words, we change the rule of the policy rate i_t to:

$$i_t^f = r + \overline{\pi} + \phi_\pi\left(\hat{\pi}_t - \overline{\pi}\right) + \phi_x\left(\hat{x}_t - \overline{x}\right),$$

$$i_t = \max\left(i_t^f, 0\right).$$

This means that even if the zero lower bound is hit, the policy duration effect cannot be used to obtain an advance on future monetary easing. Figure 8 shows the results of the simulation to evaluate the social loss.

Figure 8: Social Loss in the Economy with No Policy Duration Effect in Monetary Policy



The curves, in Figure 8, expressing the loss from price changes and the loss from the output level are shifted about 1.5% to the right compared to Figure 7. With the model in the previous section taking account of a policy duration effect of monetary policy, even if the zero lower bound is hit, the inflation rate tends to make a comparatively smooth return to positive territory, and thus there is no large deviation between the steady-state inflation rate that is the long-run target and the average rate of inflation that is achieved. On the other hand, with the model in this subsection taking account of no policy duration effect, a relatively long period of time is required to break out once the zero lower bound is hit, and thus the average rate of inflation actually achieved is lower than the targeted steady-state inflation rate. For example, when the targeted steady-state inflation rate is set at 3%, the average rate of inflation achieved in the simulation is only about 1.5%.

The size of the loss from price changes and the loss from the output level depend on the level of the average inflation rate. Therefore, in a figure with the horizontal axis expressing the steady-state inflation rate, the lack of a policy duration effect lowers the average inflation rate and shifts the curve to the right. This feature means that the loss from hitting the zero lower bound is larger when there is no policy duration effect.

In Figure 8, the steady-state inflation rate that minimizes the social loss is around 2%, which is somewhat higher than seen in Figure 7. Compared to Figure 7, in which the policy duration effect was assumed, the case illustrated in Figure 8 indicates that larger safety margins are desired against the zero lower bound because the larger social loss arises when the zero lower bound is hit.

(2) Social loss in the economy with a stronger tradeoff between wages and unemployment

In the preceding sections, we set model parameters based on the estimation with a sample period starting in 1981. Here we reset the parameters for a tradeoff between wages and unemployment based on the estimation with a sample period beginning in 1970. The purpose of this analysis is to take account of the possibility that the characteristics observed from recent Japanese economic data are likely to reflect the extraordinary circumstances resulting from the generation and collapse of the asset price bubbles that began in the late 1980s. It may be more appropriate, therefore, to use a longer-term estimation in order to make a more general evaluation. It should be noted, however, that the tradeoff between wages and unemployment were much stronger in the Japanese labor market of the 1970s.

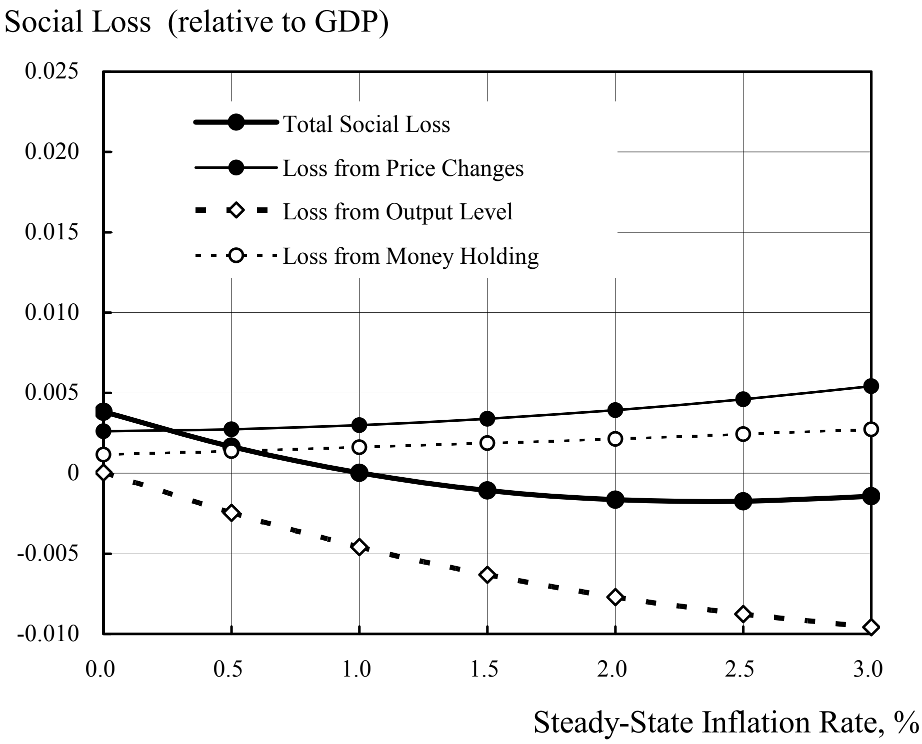
The table below contains the parameter estimations and settings, which actually indicate a stronger tradeoff than the parameters adopted in Sections 4 and 5.

Table 2: Estimation Result (1970-2004) of the Tradeoff between Wages and Unemployment and Parameter Settings

	Estimated value	(Standard error)	Parameter set
υ^{upper}	0.0656	(0.0029)	0.066
υ^{lower}	0.0178	(0.0015)	0.018
σ_{ω}	0.0280	(0.0044)	0.028

For price stickiness as well, we reset the parameters based on the estimation with a sample period starting in 1970, resulting in an estimated value of 0.510 for α with standard error of 0.108. Based on this, the slope λ of the price Phillips curve changes from the 0.27 in the preceding sections to 0.56, which means that the price Phillips curve is steeper. Figure 9 shows the results of the simulation.

Figure 9: Social Loss in the Economy with a Stronger Tradeoff between Wages and Unemployment



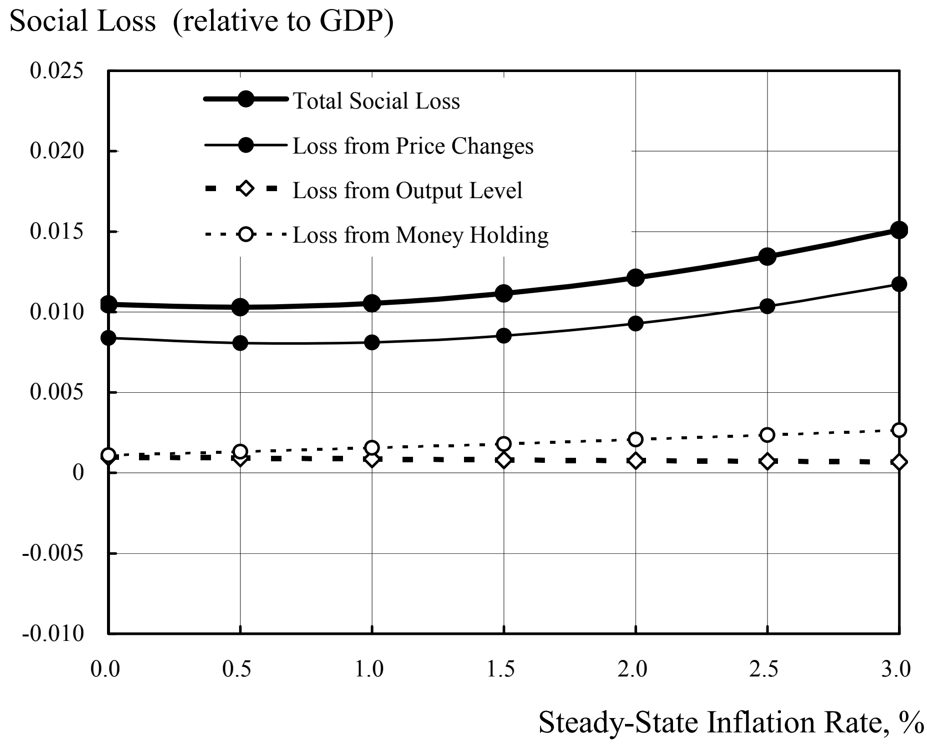
The curve, in Figure 9, expressing the loss from the output level shifts downwards compared to Figure 7 because this case places more value on the benefit in which a higher inflation rate will result in a higher wage growth rate and lower unemployment rate. In addition, the curve expressing the loss from price changes shifts downwards in Figure 9 compared to Figure 7 because, in this case, price stickiness is lower and thus unevenness between individual goods prices tends to be smaller with a given inflation rate.

Because of this, the benefits of inflation are relatively large and the costs relatively small. The steady-state inflation rate that minimizes the social loss is in the low 2% range, which is higher than found in Section 5.

(3) Social loss in the economy with no long-term tradeoff between wages and unemployment

In place of Equation (21) that assumes a long-term tradeoff between wages and unemployment, in this subsection we use the NAIRU-type wage Phillips curve in Equation (22), which is based on the natural rate hypothesis and assumes only a short-term tradeoff. Figure 10 shows the results of the simulation.

Figure 10: Social Loss in the Economy with No Long-Term Tradeoff between Wages and Unemployment



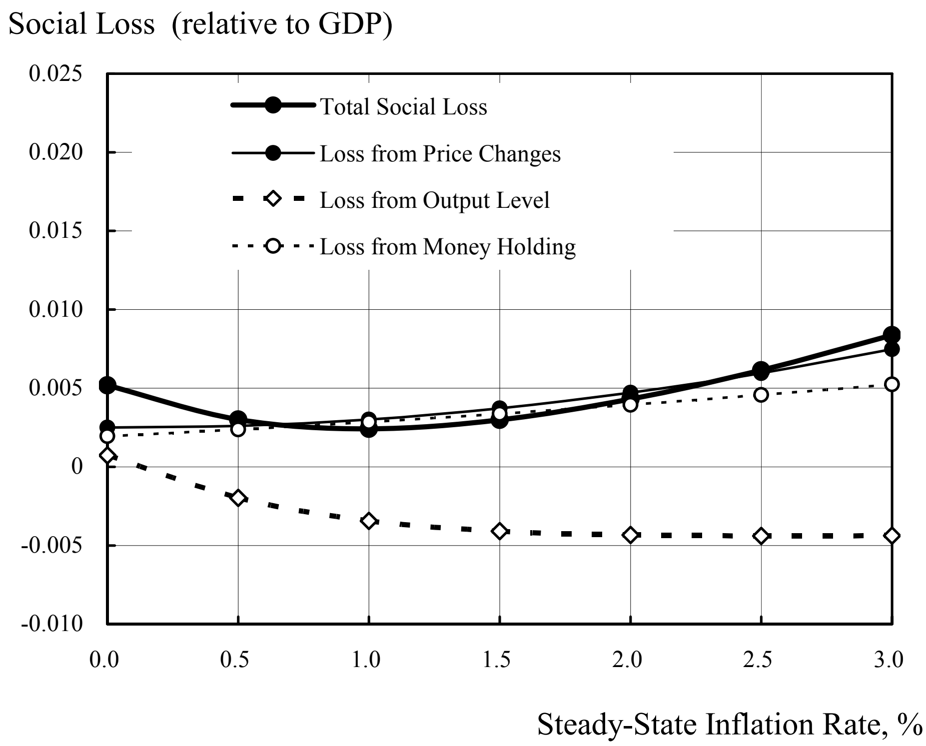
Compared to Figure 7 in Section 5, the curve expressing the loss from the output level shifts upwards in Figure 10 because the "grease effect," in which unemployment is reduced with a higher steady-state inflation rate, is eliminated with the NAIRU-type Phillips curve. The steady-state inflation rate that minimizes the social loss is at around 0.5% in Figure 10, which is somewhat lower than found in Figure 7, since the benefits from inflation are recognized to be relatively small in this case.

(4) Social loss in the economy with a lower productivity growth rate

The analysis so far has assumed that the economy continues to grow along the log-linear trend beginning in 1981, and specifically posited a trend productivity growth rate of 1.8%. In contrast, we below consider the potential of a decline in the Japanese productivity growth rate in the 1990s.

More specifically, we assume a variable trend for productivity growth, deeming the most recent trend growth rate to be the future trend and assuming that productivity grows in that vicinity. We calculate the variable trend using the HP filter with a smoothing parameter of 100. The most recent value for the trend growth rate is 1.2% from 2004, and we assume that this trend growth rate will continue into the future. The trend growth rate used in Sections 5 and 6 was 1.8%, so this scenario envisions a growth rate of only two-thirds of that level. Figure 11 shows the results.

Figure 11: Social Loss in the Economy with a Lower Productivity Growth Rate



All other factors being constant, a lower trend for productivity growth in the future means lower real interest rates and wage growth rates. In such a scenario, raising the steady-state inflation rate has a relatively strong impact on improving employment and increasing output, resulting in relatively larger benefits from inflation compared to the case in Section 5. Real interest rates are also lower than they were in the case in Section 5, which increases the risk of hitting the zero lower bound. This fact enlarges the benefits of inflation in that a higher steady-state inflation rate mitigates the risk of hitting the zero lower bound. The result shows a slightly higher steady-state inflation rate that minimizes the social loss, which is about 1.0% in Figure 11.

(5) Summary of evaluations

Let us briefly summarize the results of our evaluations thus far. The case in Section 5 finds that the steady-state inflation rate that minimizes the social loss is between 0.5% and 1.0%; in the case in Section 6 (1) where there is no policy duration effect in monetary policy, it is around 2.0%; in Section 6 (2) where there is a stronger tradeoff between wages and unemployment, it is around 2.0-2.5%; in Section 6 (3) where there are no long-term tradeoff between wages and unemployment, it is around 0.5%; and in Section 6 (4) where there is a lower productivity growth rate, it is around 1.0%. It can be seen from the results that the steady-state inflation rate that minimizes the social loss will vary to some extent according to analytical assumptions, but the analyses in this paper find it generally to be in the range between 0.5% and 2.0%.

It should also be noted that all of the analytical results indicate that the shape of the social loss function is quite flat with a steady-state inflation rate in the range of 0-3%. More specifically, a divergence of about 1% in the steady-state inflation rate from the level that minimizes the social loss will increase the social loss by no more than about 0.2-0.3% of GDP.

7. Conclusions

This paper has examined the level of steady-state inflation rate desirable for the Japanese economy. We began by setting out points concerning the costs and benefits that accompany inflation. Then we built a model of the Japanese economy capable of evaluating the effects, on social welfare, of several of these points: the opportunity cost of holding money, the zero lower bound on nominal interest rates, price stickiness and the downward wage rigidity. Building on this, we quantitatively evaluated the social loss by conducting a stochastic simulation in which we take into account the zero lower bound on nominal interest rates. We also analyzed the range of changes in the steady-state inflation rate that minimizes the social loss when we change the model settings. Our analysis indicated that the steady-state inflation rate that minimizes the social loss is generally between 0.5% and 2.0%. We also found that a divergence of around 1% in the steady-state inflation from the level that minimizes the social loss will increase the social loss by no more than about 0.2-0.3% of GDP. We should note that the analytical findings in this paper assume a specific model and specific parameters and that there is the potential for significantly different results if these assumptions are changed.

We should list some reservations regarding our analysis.

First, the steady-state inflation rate analyzed in this paper is conceptually an open-ended, long-run inflation rate and is not the rate of change in prices actually achieved in any individual term. In the context of inflation targeting, one often discusses the "medium-term" inflation rate that should be targeted by a central bank, but the "target inflation rate" in this sense is not necessarily the same as the steady-state inflation rate analyzed in this paper.

Turning to technical issues, there is the problem of measuring inflation rates. The price index analyzed in this paper is the GDP deflator, but the yardstick currently used by many countries in conducting actual monetary policy is the consumer price index. If there are differences in the rate of technological progress among the goods that comprise the GDP and the goods that comprise the consumer price index, there will potentially be differences in the steady-state inflation rates calculated from the GDP deflator and the consumer price index. For the sake of simplicity, this paper assumes that technological progress is the same among goods and uses the GDP deflator as the price index in order to maintain consistency with the real GDP growth rate. However, the GDP deflator may not necessarily be an ideal price index for monetary policy. Handling consistently both the real GDP growth rate and an inflation rate measured with the consumer price index in a single model would require expansion to a multi-sector growth model that allows for different rates of technological progress for different goods.

There is also the problem of price index bias. In particular, the social loss incurred from price stickiness is directly influenced by price index bias.[20] To account for this impact, it would be necessary to construct a price stickiness model that incorporates price index bias.

There may be room to deepen the discussion about the framework of monetary policy. As we saw in this analysis, the desired safety margin against the zero interest bound should differ depending on whether there is a policy duration effect. This means that the desired safety margin depends on the monetary policy framework. Though not covered in our evaluation, some studies advocate a stronger-than-normal degree of monetary easing in the vicinity of zero interest rates as "preemptive easing" to avoid hitting the zero lower bound. Adopting such a nonlinear policy response would reduce the required safety margin. The safety margin requirement would also be smaller if it were possible

---

[20] Except for price stickiness, all of the other points covered in this paper are unrelated to the problem of price index bias. The social losses from the opportunity cost of holding money and from the zero interest bound are losses related to nominal interest rates, and the social loss from the output level is related to the downward wage rigidity. Therefore, none of them is directly influenced by the price index bias.

to achieve a commitment to the theoretically optimal monetary policy.

Finally, there may be problems resulting from the simplification of the model. The analysis in this paper could be extended to incorporate habit formation in consumption, endogenous investments and credit constraints in imperfect financial markets, for example. The analysis could also more realistically account for the behavior of economic agents, for instance, the potential for behavior to be based on “rules of thumb” rather than dynamic optimization and the potential for the formation of irrational expectations.

Further studies on the costs and benefits of the steady-state inflation rate taking these remaining points into account are desired.

Appendix A Derivation of a Tradeoff between Wages and Unemployment

Appendix A explains details of the producer model, which were omitted from Section 3 (3), and theoretically derives a tradeoff between wages and unemployment.

As already noted, we rely on the efficiency wage model, which is considered the standard theory regarding unemployment, and assume that workers will reduce labor efficiency as a result of declines in their nominal wages. It is on this basis that we model the downward rigidity of wages.

Prior to incorporating the downward rigidity of wages, we first observe a case in which there is no downward rigidity. Denoting a producer index as $f \in [0,1]$, we define the production function of producer *f* as:

$$Y_t(f) = A_t e_t l_t(f), \tag{29}$$

where $Y_t(f)$ is the output of producer *f* and $l_t(f)$ is the number of employees, each normalized in per capita terms, and $e_t(f)$ is labor efficiency, which depends on wages and other factors. Aggregating $l_t(f)$ for all producers allows us to find the employment rate l_t:

$$l_t \equiv \int_0^1 l_t(f)df .$$

The unemployment rate is defined as $\upsilon_t \equiv -\log l_t$.

Individual producers purchase labor in the labor market for nominal wages $W_t(f)$ and sell products (intermediate goods) to retailers at prices P_t^m in a competitive wholesale market. The profits from this are expressed as:

$$P_t^m Y_t(f) - W_t(f) l_t(f). \tag{30}$$

Producers are assumed to maximize the profits under the constraints of the production function in Equation (29). The key here is the feature of the labor efficiency $e_t(f)$. We begin by assuming the following labor efficiency function:

$$e_t(f) = e\left(\frac{W_t(f)}{W_t l_t}\right), \tag{31}$$

where W_t is an average wage index defined by the formula:

$$\log W_t \equiv \int_0^1 \log W_t(f)df \ .$$

The numerator of the variable in the right side of Equation (31) represents the wages of the relevant producer, and the denominator is the product of average wages and the employment rate. The denominator can be considered to express the average wage income expected if a worker leaves his job.

The producer controls employment $l_t(f)$ and wages $W_t(f)$ to maximize profits. This results in the equilibrium, at which the first-order conditions are satisfied, where the elasticity of the labor efficiency function is 1. In other words, when the elasticity of the labor efficiency function is defined as:

$$\hat{e}\left(\frac{W_t(f)}{W_t l_t}\right) \equiv \frac{e'\left(\frac{W_t(f)}{W_t l_t}\right)}{e\left(\frac{W_t(f)}{W_t l_t}\right)} \frac{W_t(f)}{W_t l_t},$$

the equilibrium condition is expressed as:

$$\hat{e}\left(\frac{W_t(f)}{W_t l_t}\right) = 1.$$

With the reverse function it can also be expressed as:

$$\frac{W_t(f)}{W_t l_t} = \hat{e}^{-1}(1).$$

Wages of individual producers depend only on the macroeconomic variables of average wages and employment rate. Among producers, wages are all at the same level in the equilibrium. Integrating the wages of all producers and expressing them as an aggregate produces the following formula for determining the unemployment rate:

$$\upsilon_t = \log \hat{e}^{-1}(1). \tag{32}$$

This equation implies that the unemployment rate is always constant. The unemployment rate is unrelated to the wage growth rate and there is therefore no tradeoff between wages and unemployment. Note that wages are determined in the following equation from the first-order condition:

$$W_t(f) = A_t e\left(\hat{e}^{-1}(1)\right) P_t^m .$$

Next, we consider a contrastive case in which there is full downward rigidity of wages. We assume the following equation to be added to the constraint equations for the problem of producer profit maximization:

$$W_t(f) \geq W_{t-1}(f).$$

Based on the Kuhn-Tucker condition, we obtain the following relationships:[21]

$$\frac{W_t(f)}{W_t l_t} = \hat{e}^{-1}(1) \ \text{ in case of } \ W_t(f) > W_{t-1}(f), \text{ and}$$
$$\frac{W_t(f)}{W_t l_t} > \hat{e}^{-1}(1) \ \text{ in case of } \ W_t(f) = W_{t-1}(f).$$

Because wages are the same for all producers, aggregating the above relationships leads to:

$$\upsilon_t = \log \hat{e}^{-1}(1) \ \text{ in case of } \ W_t > W_{t-1}, \text{ and}$$

$$\upsilon_t < \log \hat{e}^{-1}(1) \ \text{ in case of } \ W_t = W_{t-1}.$$

In this case, the tradeoff relationship between wages and unemployment has the overall shape of an "L," as Figure 3 in Section 3 (3) illustrates.

However, a model like this, with full downward wage rigidity, is not suited to the Japanese economy, which experienced wage declines in the 1990s. Therefore, rather than assume full downward rigidity, we assume a labor efficiency function in which the

---

21 The results here depend on the fact that the profit function in Equation (30), which producers maximize, represents only the current profits, not the discounted present value of future profits. This assumption implies that producers, when they set their current wages, do not take account of the potential impossibility of reducing wages in future terms. Elsby [2006] argues that when firms are cognizant of such a potential impossibility in the future they will tend not to raise wages to begin with, and therefore the impact on the economy of the downward wage rigidity should be small.

labor efficiency is lower when wages are in decline. It allows us to incorporate the downward rigidity of wages. Specifically, Equation (31) for the labor efficiency function is modified to:

$$e_t(f) = e\left(\frac{W_t(f)}{W_t l_t}\nu_t(f)\right), \tag{33}$$

where $\nu_t(f)$ is the variable defined as $\nu_t(f) = 1$ when $W_t(f) \geq W_{t-1}(f)$ and $\nu_t(f) = \nu < 1$ when $W_t(f) < W_{t-1}(f)$. ν is a constant that expresses the degree of decline in labor efficiency when wages decline. At an equilibrium, wages are the same for all producers, and so is the value of function $\nu_t(f)$, which is expressed as ν_t. Then, the unemployment rate υ_t is expressed as:

$$\upsilon_t = \log \hat{e}^{-1}(1) - \log \nu_t .$$

As illustrated in Figure 4 in Section 3 (3), the tradeoff relationship between wages and unemployment is vertical for both wage rises and wage declines.

Next, we modify the production function, as below, to create a distribution of wage growth rates among individual producers, on the assumption of uneven wage growth rates resulting from idiosyncratic shocks to individual producers:

$$Y_t(f) = A_t \xi_t(f) e_t(f) l_t(f), \tag{34}$$

where $\xi_t(f)$ is productivity, which is uneven among individual producers and follows a log-normal distribution. In other words, $\log \xi_t(f)$ follows a normal distribution with an average of zero and a variance of σ_ξ^2:

$$\log \xi_t(f) \sim N\left(0, \sigma_\xi^2\right).$$

Labor efficiency is assumed to be given by[22]:

[22] This labor efficiency function, although it looks relatively complex, has an advantage of causing the productivity growth rate distribution and wage growth rate distribution to match, as well as to make the wage growth rate follow a normal distribution. If this is not the case, the wage growth rate distribution is very complex.

$$e_t(f) = e\left(\frac{W_t(f)\nu_t(f)}{W_t l_t^a(f)\xi_t(f)}\right).$$

In the background of this labor efficiency function, it is assumed that workers adjust their labor efficiency according to the productivity $\xi_t(f)$ of the producers for whom they work. The labor market is considered to be divided between producers for whom wages are falling and producers for whom they are not. $l_t^a(f)$ is the employment rate for the labor market to which producer *f* belongs.

As done before, the reverse function of labor efficiency elasticity can be used for the first-order condition to arrive at:

$$\frac{W_t(f)\nu_t(f)}{W_t l_t^a(f)\xi_t(f)} = \hat{e}^{-1}(1).$$

The wages of individual producers and average wages are:

$$W_t(f) = e\left(\hat{e}^{-1}(1)\right)P_t^m A_t \xi_t(f), \text{ and } W_t = e\left(\hat{e}^{-1}(1)\right)P_t^m A_t, \tag{35}$$

respectively, which results in:

$$\frac{W_t(f)}{W_t} = \xi_t(f). \tag{36}$$

It can be seen that the productivity distribution and wage distribution of individual producers follow the same log-normal distribution. This can be inserted into Equation (34) and simplified to arrive at:

$$l_t(f) = \frac{\nu_t(f)}{\hat{e}^{-1}(1)}.$$

This is then totaled for the entire labor market as:

$$l_t = \frac{1}{\hat{e}^{-1}(1)}\int_0^1 \nu_t(f)df,$$

where $\nu_t(f)$ is defined as $\nu_t(f) = 1$ for producers for whom wages do not decline and $\nu_t(f) = \nu$ for producers for whom wages do decline. Therefore, if we denote the percentage of producers for whom wages do not decline as s_t^w, then:

$$l_t = \frac{1}{\hat{e}^{-1}(1)} \left\{ s_t^W + \left(1 - s_t^W\right) \nu \right\}. \tag{37}$$

The percentage of producers for whom wages do not decline can be found as follows. First, using the logarithm of Equation (36) it can be seen that the logarithmic value of wages follows a normal distribution:

$$\log W_t(f) - \log W_t = \log \xi_t(f) \sim N\left(0, \sigma_\xi^2\right).$$

Because a linear combination of normally distributed variables also follows a normal distribution, a change of a normally distributed variable between periods t and t-1 follows the normal distribution of which the standard deviation is twice that of the original distribution:

$$\Delta \log W_t(f) - \Delta \log W_t \sim N\left(0_t, 2\sigma_\xi^2\right).$$

We define the logarithmic change of average wages between current and previous periods and the variance of its distribution as:

$$\varpi_t \equiv \Delta \log W_t \text{ and } \sigma_\varpi^2 \equiv 2\sigma_\xi^2 .$$

Then, the wage growth rate of individual producers follows the following normal distribution:

$$\Delta \log W_t(f) \sim N\left(\varpi_t, \sigma_\varpi^2\right).$$

s_t^W can be expressed as follows using a cumulative standard normal distribution function *CN* because it is the percentage of producers with non-negative wage growth rates $\Delta \log W_t(f)$:

$$s_t^W = CN\left(\frac{\varpi_t}{\sigma_\varpi}\right).$$

This can be substituted into Equation (37) to derive:

$$l_t = \frac{1}{\hat{e}^{-1}(1)}\left\{(1-\nu)CN\left(\frac{\varpi_t}{\sigma_\varpi}\right)+\nu\right\}.$$

With the log-linear approximation of the left side in the vicinity of $l_t = 1$ and the conversion of the employment rate to the unemployment rate, we arrive at:

$$\upsilon_t = -\left(\upsilon^{upper} - \upsilon^{lower}\right)CN\left(\frac{\varpi_t}{\sigma_\varpi}\right) + \upsilon^{upper}, \tag{38}$$

where the upper and lower bounds for the unemployment rate are defined as:

$$\upsilon^{upper} \equiv 1 - \frac{\nu}{\hat{e}^{-1}(1)}, \quad \upsilon^{lower} = 1 - \frac{1}{\hat{e}^{-1}(1)}.$$

As illustrated in Figure 5 in Section 3 (3), the tradeoff between average wages and unemployment is expressed by a smooth curve that follows a cumulative normal distribution function.

On the other hand, the wage Phillips curve, in which there is no long-term tradeoff, can be derived as follows. We begin by noting that in Equation (32) there is no tradeoff relationship in either the long or the short term. We will modify the labor efficiency function in Equation (31), which resulted in Equation (32), so that the labor efficiency is influenced by short-term changes. Specifically, we assume that it is impossible to observe the denominator of current average wages in the same term and replace it with workers' forecasts W_t^b:

$$e_t(f) = e\left(\frac{W_t(f)}{W_t^b l_t}\right).$$

In the same way as before, we obtain:

$$W_t = \hat{e}^{-1}(1)W_t^b l_t,$$

which can be expressed, by taking logarithms of both sides, as:

$$\log W_t = \log W_t^b + \log l_t + \log \hat{e}^{-1}(1).$$

We assume that workers forecast the current wage growth rate based on the previous-term growth rate as:

$$\log W_t^b - \log W_{t-1} = \varpi_{t-1}.$$

This results in:

$$\varpi_t = \varpi_{t-1} - \upsilon_t + \log \hat{e}^{-1}(1).$$

The steady state for the above equation indicates that:

$$\upsilon = \log \hat{e}^{-1}(1),$$

where v is constant regardless of the inflation rate, —in other words, it is the non-accelerating inflation rate of unemployment (NAIRU). The equation can be expressed in terms of deviation from the zero-inflation steady state as:

$$\hat{\varpi}_t = \hat{\varpi}_{t-1} - \hat{\upsilon}_t.$$

Thus far, we have derived the tradeoff between wages and unemployment. We can also derive other formulas, from this model of producer behavior, for the intermediate goods price, the unit labor cost, the output gap and the wage growth rate. We begin with the intermediate goods price as below, which is derived from the production function and the first-order condition for wages:

$$\frac{P_t^m}{P_t} = \frac{W_t(f)l_t(f)}{P_t Y_t(f)}.$$

The right side expresses the real unit labor cost, or the labor's share of income. It can be seen that the relative price of intermediate goods matches the real unit labor cost. Inasmuch as the relative price of intermediate goods is common among producers, the real unit labor costs will also be common, which can be defined as:

$$s_t \equiv \frac{W_t l_t}{P_t Y_t}.$$

This results in a match between the relative price of intermediate goods and real unit labor cost as:

$$p^m = s \quad \text{and} \quad \hat{p}_t^m = \hat{s}_t . \tag{39}$$

Moreover, from the production function we can see that:

$$y_t = a_t l_t , \quad y = al \text{, and } \hat{y}_t = \hat{a}_t + \hat{l}_t .$$

We define the "output gap" to be the logarithmic deviation of actual output from output capacity, as $\hat{x}_t \equiv \hat{y}_t - \hat{a}_t$, leading to the following identity relationship:

$$\hat{x}_t = \hat{l}_t = -\hat{\upsilon}_t .$$

Finally, in the steady state, the real wage growth rate must be equivalent to the trend growth rate $\log\gamma$. In the zero-inflation steady state, the wage growth rate ϖ is equal to the real wage growth rate, resulting in the relationship:

$$\varpi = \log\gamma .$$

Appendix B Derivation of the Social Loss Function with a Second-order Approximation

Appendix B derives the social loss function with a second-order approximation, which was omitted from Section 3 (6).

We begin with calculating a second-order approximation around the zero-inflation steady state to the consumption utility function found in Equation (1), so that:

$$u(c_t,\xi_t)=u_c(c,\xi)(c_t-c)+\frac{1}{2}u_{cc}(c,\xi)(c_t-c)^2+u_{c\xi}(c,\xi)(c_t-c)(\xi_t-\xi)+t.i.p.+o(3), \quad (40)$$

where "*t.i.p.*" stands for "terms that are independent of policy," so that it is comprised of constants and shock terms only. *o* (3) is an abbreviation for the third-order terms and beyond.

We can rewrite the function using variables defined as a logarithmic deviation from the steady state as:

$$u(c_t,\xi_t)=u_c(c,\xi)c\left\{\hat{c}_t-\frac{1}{2}(\sigma^{-1}-1)\hat{c}_t^{\,2}+\hat{c}_t\frac{u_{c\xi}(c,\xi)\xi}{u_c(c,\xi)}\hat{\xi}_t\right\}+t.i.p.+o(3),$$

where c_t is the Dixit-Stiglitz index of individual goods consumption as defined in Section 3 (1), not an average index of consumption. If we define the average index as:

$$c_t^a\equiv E_z c_t(z)\left(\equiv\int_0^1 c_t(z)dz\right),$$

the following approximation holds true between the Dixit-Stiglitz index and the average index:

$$\hat{c}_t=\hat{c}_t^a-\frac{1}{2}\theta^{-1}\operatorname{var}_z\log c_t(z)+o(3). \quad (41)$$

Meanwhile, we obtain the following equation by calculating the variance of the log of the demand function in Equation (5):

$$\operatorname{var}_z\log c_t(z)=\theta^2\operatorname{var}_z\log p_t(z). \quad (42)$$

In addition, the average index of consumption and that of output are equivalent:

$$\hat{c}_t^a = \hat{y}_t . \tag{43}$$

By substituting Equations (42) and (43) into Equation (41) we obtain:

$$\hat{c}_t = \hat{y}_t - \frac{1}{2}\theta \operatorname{var}_z \log p_t(z) + o(3).$$

The second term on the right side can be interpreted as distortion arising from unevenness in individual goods prices. This can be substituted into Equation (40) to derive:

$$u(c_t, \xi_t) = u_c(y, \xi) y \left\{ -\frac{1}{2} \operatorname{var}_z \log p_t(z) + \hat{y}_t - \frac{1}{2}\left(\sigma^{-1} - 1\right)\hat{y}_t^{\,2} + \hat{y}_t \frac{u_{c\xi}(y,\xi)\xi}{u_c(y,\xi)} \hat{\xi}_t \right\} + t.i.p. + o(3).$$

The equation can be simplified as follows by replacing output $\hat{y}_t$ with $\hat{x}_t + \hat{a}_t$, abstracting the cross term of shock and $\hat{x}_t$, and omitting the $t.i.p.$ and $o(3)$ terms:

$$u(c_t, \xi_t) = u_c(y, \xi) y \left\{ -\frac{1}{2} \operatorname{var}_z \log p_t(z) + \hat{x}_t - \frac{1}{2}\left(\sigma^{-1} - 1\right)\hat{x}_t^{\,2} \right\}.$$

It can be seen that the consumption utility function can be broken down into three terms: dispersion of individual goods prices, the linear term (level term) of the output gap, and the quadratic term (fluctuation term) of the output gap. Based on this, the expectation of the discounted present value of the consumption utility function for each term is derived as:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \xi_t) = E_0 \sum_{t=0}^{\infty} \beta^t u_c(y, \xi) y \left\{ -\frac{1}{2} \operatorname{var}_z \log p_t(z) + \hat{x}_t - \frac{1}{2}\left(\sigma^{-1} - 1\right)\hat{x}_t^{\,2} \right\}. \tag{44}$$

In this equation, the first term on the right side can be interpreted as the loss arising from the dispersion of individual goods prices at the same point in time. As shown in Woodford [2003], the dispersion of individual goods prices follows the transition equation:

$$\mathrm{var}_z \log p_t(z) = \alpha\, \mathrm{var}_z \log p_{t-1}(z) + \frac{\alpha}{1-\alpha}\pi_t^2 + o(3).$$

From this we can derive the following formula for the discounted value of the dispersion of individual goods prices:

$$\sum_{t=0}^{\infty} \beta^t\, \mathrm{var}_z \log p_t(z) = \sum_{t=0}^{\infty} \beta^t \frac{1}{\lambda}\pi_t^2 + o(3).$$

By substituting this into Equation (44) we obtain:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \xi_t) = E_0 \sum_{t=0}^{\infty} \beta^t u_c(y, \xi) y \left\{ -\frac{1}{2}\frac{\theta}{\lambda}\pi_t^2 + \hat{x}_t - \frac{1}{2}\left(\sigma^{-1} - 1\right)\hat{x}_t^{\,2} \right\}, \qquad (45)$$

where the third-order terms and beyond are omitted. From this result, it can be seen that the consumption utility function includes the quadratic term (fluctuation term) of the inflation rate, the linear term (level term) of the output gap, and the quadratic term (fluctuation term) of the output gap.

Next, we calculate a second-order approximation around the zero-inflation steady state to the money-holding utility function. We can express it, using variables defined as a logarithmic deviation from the zero-inflation steady state, as:

$$\begin{aligned} v(m_t, \xi_t) &= v_m(m)(m_t - m) + \frac{1}{2} v_{mm}(m)(m_t - m)^2 + t.i.p. + o(3) \\ &= v_m(m) m \left\{ \hat{m}_t - \frac{1}{2}\left(\chi^{-1} - 1\right)\hat{m}_t^2 \right\} + t.i.p. + o(3) \\ &= v_m(m) m \left\{ -\frac{\delta}{1-\delta}\chi \hat{i}_t - \frac{1}{2}\left(\chi^{-1} - 1\right)\left(\frac{\delta}{1-\delta}\chi \hat{i}_t\right)^2 \right\} + t.i.p. + o(3). \end{aligned}$$

We derived the last equation by replacing the real money balance with the nominal interest rate based on Equation (9). By defining $k \equiv m / y$ and using Equation (8), we obtain:

$$v(m_t, \xi_t) = u_c y (1-\delta) k \left\{ -\frac{\delta}{1-\delta}\chi \hat{i}_t - \frac{1}{2}\left(\chi^{-1} - 1\right)\left(\frac{\delta}{1-\delta}\chi \hat{i}_t\right)^2 \right\}, \qquad (46)$$

where the $t.i.p.$ and $o(3)$ terms have been omitted.

We can specifically write the utility function U_0 in Equation (1) based on a combination of Equation (45) and (46). By defining the total social loss function TSL_0 as $TSL_0 \equiv -u_c y U_0$, we obtain Equations (25) and (26) in Section 3 (6).

References

Adam, K., and R. M. Billi, "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 38, 2006, pp. 1877-1906.

Adam, K., and R. M. Billi, "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates," *Journal of Monetary Economics*, 2007, forthcoming.

Akerlof, G. A., W. T. Dickens, and G. L. Perry, "The Macroeconomics of Low Inflation," *Brookings Papers on Economic Activity*, 2, 1996, pp. 1-76.

Barro, R. J., Inflation and Economic *Growth, Bank of England Quarterly Bulletin*, 1995, pp. 166-176.

Barro, R. J., *Determinants of Economic Growth: A Cross-country Empirical Study*, Cambridge: MIT Press, 1997.

Benigno, P., and M. Woodford, "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," *Journal of the European Economics Association*, 3, 2005, pp. 1185-1236.

Blanchard, O. J., "Debt, Deficits and Finite Lives," *Journal of Political Economy*, 93, 1985, pp. 223−247.

Blanchard, O. J., and L. H. Summers, "Hysteresis and the European Unemployment Problem," *NBER Macroeconomics Annual*, MIT Press, 1986.

Blanchard, O. J., and L. H. Summers, "Hysteresis in Unemployment," *European Economic Review*, 31, 1987

Brock, W. A., "Money and Growth: The Case of Long Run Perfect Foresight," *International Economic Review*, 15 (3), 1974, pp. 750-777.

Bruno, M., and W. R. Easterly, "Inflation and Growth: In Search of a Stable Relationship," *Federal Reserve Bank of St. Louis Review*, 78 (3), 1996, pp. 139–146.

Bruno, M., and W. R. Easterly, "Inflation Crises and Long-Run Growth," *Journal of Monetary Economics*, 41, 1998, pp. 3-26.

Bullard, J. B., and J. W. Keating, "The Long-run Relationship between Inflation and Output in Postwar Economies," *Journal of Monetary Economics*, 36 (3), 1995, pp. 477-496.

Calvo, G., "Staggered Prices in a Utility-maximizing Framework," *Journal of Monetary Economics*, 12, 1983, pp. 383-398.

Chang, W., and C. Lai, "Anticipated Inflation in a Monetary Economy with Endogenous Growth," *Economica*, 67, 2000, pp. 399-417.

Cole, H. L., and N. Kocherlakota, "Zero Nominal Interest Rates: Why They're Good and How to Get Them," *Federal Reserve Bank of Minneapolis Quarterly Review*, 22 (2), 1998, pp. 2-10.

De Gregrio, J. "The Effects of Inflation on Economic Growth," *European Economic Review*, 36, 1992, pp. 417-425,

De Gregrio, J., "Inflation, Taxation and Long-Run Growth," *Journal of Monetary Economics*, 31, 1993, pp. 271-298.

Drazen, A., "Inflation and Capital Accumulation under a Finite Horizon," *Journal of Monetary Economics*, 8, 1981, pp. 247-260.

Elsby, M. W., "Evaluating the Economic Significance of Downward Nominal Wage Rigidity," NBER Working Paper No. 12611, 2006.

Erceg, C. J., D. Henderson, and A. T. Levin, "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, 2000, pp. 281-313.

Fuhrer, J., and B. Madigan, "Monetary Policy When Interest Rates Are Bounded at Zero," *Review of Economics and Statistics*, 79, 1997, pp. 573-585.

Feldstein, M., ed., *The Costs and Benefits of Price Stability*, Chicago: University of Chicago Press, 1999.

Fischer, S. "The Role of Macroeconomic Factors in Growth," *Journal of Monetary Economics*, 32, 1993, pp. 485-515.

Fisher, I. "The Debt-Deflation Theory of Great Depressions," *Econometrica*, 1 (4), 1933, pp. 337-357.

Friedman, M., "The Role of Monetary Policy," *American Economic Review Papers and Proceedings* 58, 1968, pp. 1-17.

Friedman, M., "The Optimum Quantity of Money," in *The Optimum Quantity of Money and Other Essays*, Chicago: Aldine, 1969.

Friedman, M., "Novel Lecture: Inflation and Unemployment," *The Journal of Political Economics*, 85, 1977, pp. 451-471.

Galí, J., and M. Gertler, "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 44 (2), 1999, pp. 195-222.

Greenspan, A., "Opening Remarks," in *Achieving Price Stability: A Symposium Sponsored by the Federal Reserve Bank of Kansas City*, 1996.

Groshen, E. L., and M. E. Schweitzer, "Identifying Inflation's Grease and Sand Effects in the Labor Market," in *The Costs and Benefits of Price Stability*, Martin Feldstein ed., The University of Chicago Press, 1999, pp. 273-308.

Issing, O., "Panelists' Remarks", in Concluding Panel Discussion: Sustained Economic Growth and Central Banking, *Monetary and Economic Studies (Special Edition)*, Institute for Monetary and Economic Studies, Bank of Japan, 2004, pp. 233-237.

Hollinger, P., "The Stacked-Time Simulator in Troll: A Robust Algorithm for Solving Forward-Looking Models," Presented at the Computing in Economics and Finance 1996, Society for Computational Economics, 1996.

Hunt, B., and D. Laxton, "The Zero Interest Rate Floor (ZIF) and its Implications for Monetary Policy in Japan," in T. Callen and J.D. Ostroy eds., *Japan's Lost Decade: Policies for Economic Revival,* pp. 179-205, Washington, D.C.: IMF.

Judson, R., and A. Orphanides, "Inflation, Volatility, and Growth," *International Finance*, 2 (1), 1999, pp. 117-138.

Jung, T., Y. Teranishi, and T. Watanabe, "Optimal Monetary Policy at the Zero-Interest-Rate Bound," *Journal of Money, Credit, and Banking*, 37, 2005, pp. 813-835.

Kimura, T. and T. Kurozumi, "Effectiveness of History-Dependent Monetary Policy," *Journal of the Japanese and International Economies*, 18, 2004, pp. 330-361.

Kimura, T., and K. Ueda, "Downward Nominal Wage Rigidity in Japan," *Journal of the Japanese and International Economies*, 15, 2001, pp. 50-67.

King, R. G., and A. Wolman, "Inflation Targeting in a St. Louis Model of the 21st Century," *Federal Reserve Bank of St. Louis Review*, 78, 1996, pp. 83-107.

Krugman, P., "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 2, 1998, pp. 137-205.

Kuroda, S. and I. Yamamoto, "Are Japanese Nominal Wages Downwardly Rigid? (Part I): Examinations of Nominal Wage Change Distributions," *Monetary and Economic Studies*, 21 (2), Institute for Monetary and Economic Studies, Bank of Japan, 2003a, pp. 1-29.

Kuroda, S. and I. Yamamoto, "Are Japanese Nominal Wages Downwardly Rigid? (Part II): Examinations using a Friction Model", *Monetary and Economic Studies*, 21 (2), Institute for Monetary and Economic Studies, Bank of Japan, 2003b, pp. 31-68.

Kuroda, S. and I. Yamamoto, "Wage Fluctuations in Japan after the Burst of the Bubble: Downward Nominal Wage Rigidity, Payroll, and the Unemployment Rate," *Monetary and Economic Studies*, 23 (2), Institute for Monetary and Economic Studies, Bank of Japan, 2005, pp. 1-30.

Kuroda, S. and I. Yamamoto, "Why are Nominal Wages Downwardly Rigid, but Less So in Japan?: An Explanation based on Behavioral Economics and Labor Market/Macroeconomic Differences," IMES Discussion Paper Series, Institute for Monetary and Economic Studies, Bank of Japan, E-28, 2006.

Leeper, E., "Equilibria under Active and Passive Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27, 1991.

Levin, A., V. Wieland, and J. Williams, "Robustness of Simple Monetary Policy Rules under Model Uncertainty," in John Taylor ed., *Monetary Policy Rules*, University of Chicago Press, 1999, pp. 263-318.

Lucas, R. E. Jr., “Inflation and Welfare”, *Econometrica*, 68, 2000, pp. 247-274.

Nishiyama, S., “Inflation Target as a Buffer against Liquidity Trap,” IMES Discussion Paper Series, Institute for Monetary and Economic Studies, Bank of Japan, E-8, 2003.

Obstfeld, M., and K. Rogoff, “Exchange Rate Dynamics Redux,” *Journal of Political Economy*, 103 (3), 1995, pp. 624-660.

Obstfeld, M., and K. Rogoff, *Foundations of International Macroeconomics*, MIT Press, 1996.

Orphanides, A., and V. Wieland, “Price Stability and Monetary Policy Effectiveness When Nominal Interest Rates Are Bounded at Zero,” Finance and Economics Discussion Series, 98-35, Board of Governors of the Federal Reserve System, 1998.

Reifschneider, D., and J. C. Williams, “Three Lessons for Monetary Policy in a Low-Inflation Era,” *Journal of Money, Credit and Banking*, 32 (4), 2000, pp. 936-966.

Rotemberg, J. J., and M. Woodford, “An Optimization-based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version,” NBER Technical Working Paper, No. T233, 1998.

Stockman, A. C., “Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy,” *Journal of Monetary Economics*, 8, 1981, pp. 534-44.

Shiratsuka, S., “Is There a Desirable Rate of Inflation? A Theoretical and Empirical Survey,” *Monetary and Economic Studies*, 19 (2), Institute for Monetary and Economic Studies, Bank of Japan, 2001, pp. 49-84.

Sidrauski, M., “Rational Choice and Patterns of Growth in a Monetary Economy,” *American Economic Review Papers and Proceedings*, 57, 1967, pp. 534–544.

Summers, L., “How Should Long-Term Monetary Policy Be Determined?” *Journal of Money, Credit and Banking*, 23 (3), 1991, pp. 625-631.

Taylor, J. B., “Discretion versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39, 1993, pp. 195-214.

Teranishi, Y., “Zero Bound on Nominal Interest Rates and Ex Ante Positive Inflation: A Cost Analysis,” Bank of Japan Working Paper Series, No.03-E-8, 2003.

Tobin, J., “Money and Economic Growth,” *Econometrica*, 33 (4), 1965, pp. 671-684.

Tobin, J., “Inflation and Unemployment,” *American Economic Review*, 62, 1972, pp. 1-18.

Ueda, K., “Costs of Inflation in Japan: Tax and Resource Allocation,” Working Paper, 01-10, Research and Statistics Department, Bank of Japan, 2001.

Ugai, H., “Effects of the Quantitative Easing Policy: A Survey of Empirical Analyses,” *Monetary and Economic Studies,* 25 (1), Institute for Monetary and Economic Studies, Bank of Japan, 2007, pp. 1-47.

Van der Pleog, F., and G. S. Alogoskoufis, “Money and Endogenous Growth,” *Journal of Money, Credit, and Banking*, 26, 1994, pp. 771-91.

Wang, P. and C. K. Yip, “Alternative Approaches to Money and Growth,” *Journal of Money, Credit, and Banking*, 24, 1992, pp. 553-62.

Weiss, L., “The Effects of Money Supply on Economic Welfare in the Steady State,” *Econometrica*, 48, pp. 565-576.

Weil, P., “Overlapping Families of Infinitely-Lived Agents,” *Journal of Public Economics*, 20, 1987, pp. 183-198.

Weil, P., “Is Money Net Wealth?” *International Economic Review*, 32, 1991, pp. 37-53.

Woodford, M, “The Optimum Quantity of Money,” *Handbook of Monetary Economics*, Volume II, edited by B. M. Friedman and F. H. Hahn, 1990, pp. 1067-1152.

Woodford, M. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.