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Consumption, Working Hours, and Wealth Determination in a Life Cycle Model∗

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Abstract

This paper presents an empirical analysis of a life cycle model. We incorporate labor supply and family structure into the standard precautionary savings model and estimate structural parameters based on the moment conditions for the life cycle profiles of consumption, working hours, and wealth accumulation. Our empirical analyses with Japanese household data reveal that consideration of both family structure and idiosyncratic shocks are crucial in modeling consumption and working hours profiles simultaneously under plausible parameter values.

Keyword: Life Cycle, Consumption, Asset Accumulation, Labor Supply, Structural Estimation

JEL Classification: D12, E21, C15.

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1 Introduction

Investigation of the mechanisms behind the life cycle profile of consumption has been a central issue in macroeconomics for a long time. A typical household consumption profile over the life cycle is hump-shaped, and closely tracks income profile. Because a very simple life cycle model of consumption under complete markets contradicts observed consumption, to fill the gap between the theory and data, the life cycle model requires a richer structure. So far, two mechanisms have been extensively investigated: a model with incomplete capital markets and a liquidity constraint by Deaton (1991), and a model with demographic change by Attanasio, Banks, Meghir, and Weber (1999). Both models are successful in replicating the observed hump-shaped life cycle consumption profile. However, because both the liquidity constraint and demographics affect the consumption profile in similar ways, it is difficult to identify which mechanism is decisive in the determination of consumption over the life cycle.

Recently, an increasing number of papers use not only consumption but also other information—such as labor-supply or asset accumulation—to investigate the life cycle model of household behavior. Blundell, Browning, and Meghir (1994) and Attanasio and Weber (1995) demonstrate that ignoring labor supply decisions can cause serious biases in the estimation of structural parameters such as the intertemporal marginal rate of substitution. In a comprehensive survey paper, Attanasio (1999) states, “consumption decisions cannot be studied in isolation”.

In this paper, we show that by incorporating the labor-supply decision into the standard life cycle model, it becomes possible to identify the demographic effects and liquidity constraint effects in the life cycle model. More specifically, we build a life cycle model with endogenous labor supply, liquidity constraints, and demographic changes. We then, conduct a structural estimation of the fundamental parameters with moment conditions for consumption, working hours, and asset accumulation. By considering several life cycle profiles simultaneously, we can estimate the parameters that determine the effects of uncertainty and demographics on the life cycle profiles of consumption and working hours.

The basic mechanism of our identification is simple. Although both family structure and liquidity constraints affect the intertemporal allocation of consumption and labor supply, the number of dependent children alters the intratemporal leisure-consumption choice, which enables us to identify the effects of the liquidity constraint and demographics on consumption and labor supply profiles. More precisely, the mechanism can
be explained as follows. Labor supply and consumption at age \( j \) are determined by both the intratemporal leisure-consumption choice at time \( j \) and intertemporal choices represented by the Euler equation. As the number of nonworking family members increases, given the household consumption level, consumption per capita decreases, which implies that the marginal utility from household consumption becomes an increasing function of the number of dependent family members. Therefore, family size affects the Euler equation. As explained by Low (2005) and French (2005), idiosyncratic income risks and the liquidity constraint affect the precautionary saving behavior of households under incomplete markets, which also influences the Euler equation. The difference lies in the effects on the intratemporal choice between leisure and consumption. Because family composition affects the marginal utility from consumption directly, the changes in the number of nonworking family members alters the marginal value of labor supply in terms of consumption. In other words, family composition affects the first-order condition for the intratemporal leisure-consumption choice while the liquidity constraints do not. In this paper, we utilize this information to identify the effects of demography and liquidity constraints.

This work is closely related to previous works on precautionary savings models and labor supply. The mechanism through which liquidity constraints affect the consumption profile in this paper is identical to the one in many precautionary savings models such as Deaton (1991) and Gourinchas and Parker (2002). Following Blundell et al. (1994), Attanasio et al. (1999) and others,\(^1\) we investigate the effects of demographics on consumption, but, following Heckman (1974), we also analyze the effects on labor-supply decisions. Low (2005) investigates the role of labor supply in the precautionary savings model and presents several calibration results. French (2005) also builds a life cycle model with labor supply and health conditions and conducts a structural estimation. This paper can be regarded as an extension of their work by incorporating demographics and conducting structural estimation with several profiles including consumption.\(^2\)

For our structural estimation, we use moment conditions for annual working hours and asset accumulation, as well as consumption profiles based on Japanese household-level panel data that cover more than 10 years.\(^3\) Using Japanese data gives us an

\(^1\)Fernández-Villaverde and Krueger (2004) also point out the importance of demographics for the life cycle of durable and nondurable goods from the Consumer Expenditure Survey.

\(^2\)Although French (2005) uses many moment conditions for his structural estimations, a consumption profile is not included.

\(^3\)In contrast to the Panel Study of Income Dynamics (PSID), the panel data of Japanese households contain detailed information on household expenditure as well as other economic variables such as income
interesting test of the life cycle model under incomplete capital markets. Recent empirical analyses of Japanese data reveal that Japanese households face much smaller idiosyncratic income risks than those in the United States. The estimated variance of permanent income shocks on Japanese households is about one-third of the U.S. level. Life cycle profiles of consumption and working hours, however, exhibit similar patterns to the United States: that is, we can observe hump-shaped consumption and downward-sloping working hours profiles. We would like to investigate whether such small income risks can induce observed patterns of age profiles under plausible parameters.

Our main findings are as follows: (1) the life cycle model with labor-supply and liquidity constraints can replicate the life cycle profiles of consumption and working hours under plausible parameter values, and (2) both demographics and liquidity constraints are important in describing the various profiles simultaneously.

The paper is organized as follows. The next section provides a brief description of the age profiles of consumption, working hours, and asset accumulation of Japanese families. Section 3 builds a dynamic model of households. Section 4 discusses methodology for the structural estimation. Section 5 discusses our empirical results, and the subsequent section provides several simulation analyses. The final section concludes.

2 Data and Features

In this section we describe the characteristics of the age profiles for income, hours worked, consumption and wealth used in our empirical analysis. We use several age profiles obtained by Abe and Inakura (2007b). Abe and Inakura (2007a,b) use the Japanese Panel Surveys of Consumers (JPSC) compiled by the Institute of Household Economy, and create age profiles of these variables in their investigation of the covariance structure of household income and hours worked. Figure 1 shows the age distribution of married and assets.


5To the authors’ best knowledge, the JPSC is the only publicly available panel survey in Japan that spans more than 10 years with detailed information on income and consumption, as well as family structure. See the web site of the Institute of Household Economy for details at http://www.kakeiken.or.jp/research/aboutpanel.html.
household heads in 1993, the first year of the data set. Because the primary target of
the survey is young females in 1993, the data do not contain many elderly households. 
Therefore, we have to concentrate on relatively early stages of life cycle activities.

The definition and characteristics of each profile are summarized below. Figures
2(a)–(d) show the means and the estimated age effects for each variable by age.6

Log of Income  We use before-tax annual labor income of household heads as household
income. The average of income is 5.23 million yen. From Figure 2(a), we can
observe that log income increases until the age of 40 and remains almost constant
after 40. The odd shape around the late 50s is probably caused by a small sample
bias.

Log of Annual Hours of Work  Annual hours worked are used as hours of work.
Overtime working hours for which no salary is paid are excluded. The sample
average of hours worked is 2319 hours. This number implies 8.9 hours a day for a
worker who works 52 weeks in a year with 2 days holiday in a week. We can ob-
serve a downward trend in hours worked. Such a downward trend is also reported
for the United States in French (2005) and Low (2005).

Log of Consumption  The JPSC does not contain data on annual expenditure. The
consumption data used in this paper are expenditure in September. Expenditure is
identified so as not to include savings, life insurance fees, and loan payments. The
average expenditure is 220,000 yen. In Figure 2(c), we can observe a hump-shaped
age profile where expenditure increases until age 50 and decreases after that. A
similar pattern is reported in Gourinchas and Parker (2002) using the CEX of U.S.
household data, and in Abe and Yamada (2005) using Japanese household data.7

Gross Financial Assets  There are a number of ways to define household wealth. In
this paper, we define household wealth as financial assets composed of deposits,
securities, and insurance. Because housing loans are a large part of household debt,
we exclude real estate and housing loans from our definition. Although wealth is

6These tables are Tables 3a, 3b, 3c and 3d from Abe and Inakura (2007b). In each table, the estimated
age effect is labeled “estimated”. Age effects are estimated using OLS. They use family type, number
of dependent children, dummy variable for living with parent who is not retired and year dummy as
explanatory variables in OLS. The aim of these variables is to control for the differences within and
between households. The age dummy should extract age effects after controlling for other factors. See
Abe and Inakura (2007b) for details.

7Abe and Yamada (2005) employ the National Survey of Family Income and Expenditure conducted
by the Japanese government every five years. The survey covers more than 50,000 households for each
surveyed year and contains detailed information on consumption for three months.
defined in gross terms, there is a nonnegligible number of families with zero assets. In our analysis, liquidity constraints play an important role, and rather than taking natural logarithms, we use level values. Average wealth is 6.5 million yen. As can be seen in Figure 2(d), wealth exhibits an upward trend.

Many previous studies of structural estimation of life cycle models use a single moment condition. For example, Gourinchas and Parker (2002) use the age profile of consumption, while Cagetti (2003) uses the wealth profile. The exception is French (2005), who uses both labor and wealth profiles. In this paper, we use the consumption profile as well as the labor and wealth profiles. This is possible because Japanese panel data contain detailed information on consumption as well as financial assets and employment.

3 A Life Cycle Model with Labor–Leisure Choice

The primary purpose of this paper is to investigate whether a standard life cycle model with labor–leisure choice is consistent with the age profiles shown in the previous section. In this section, we build a life cycle model based on the buffer-stock saving model of Carroll (1997), and Hubbard, Skinner and Zeldes (1995). The model is extended to incorporate labor–leisure choice and the effects of dependent children on the consumption decision.

3.1 A Household

We consider a partial-equilibrium finite-horizon life cycle model. Although all households live at most $J$-periods, they face mortality risks, $\{s_j\}_{j=1}^J$. A household of age $j$ elastically supplies labor during their working life and retires at age $j_r$; $1 \leq j \leq j_r < J$. Labor supply $\ell_j$ is endogenously determined by the household’s optimal decision but is bounded by $\ell_j \in [0, \bar{\ell}]$. After retirement, households rely on public pensions and capital income as their only income sources.

3.2 Budget Constraints

At the beginning of each age $j$, a household has wealth $W_{j-1}$ ($W_{j-1} \geq 0, \forall J \geq j \geq 1$) and faces liquidity constraints. The wealth yields interest income, which is fixed through the life cycle. A household can obtain labor income with elastic labor supply. Following Deaton (1991), we define cash on hand at age $j$ as $X_j \equiv (1 + r)W_{j-1} + Y_j$, where $Y_j$ is
labor income. The next period’s wealth $W_j$ equals cash on hand minus consumption $C_j$ such as:

$$W_j = (1 + r)W_{j-1} + Y_j - C_j.$$  \hspace{1cm} (1)

Labor income $Y_j$ at working age is determined by multiplying the hourly wage, $\omega_j$, by labor supply, $\ell_j$, as follows:

$$Y_j = \omega_j \ell_j, \text{ if } j \leq j_r$$

The real hourly wage at age $j$ can be decomposed into household fixed effect, $\omega_0$, permanent income, $\psi_j$, and transitory shock, $\xi_j$, such that:

$$\ln \omega_j = \ln \omega_0 + \ln \psi_j + \ln \xi_j, \ln \xi_j \sim N\left(-\frac{\sigma^2_\xi}{2}, \sigma^2_\xi\right).$$ \hspace{1cm} (2)

The permanent income level, $\psi_j$, is determined by the previous period’s permanent income, $\psi_{j-1}$, permanent income shock, $\phi_j$, and deterministic average income growth rate, $G_j$. Thus, the permanent income level reflects the history of all past permanent shocks\(^{8}\) that is:

$$\ln \psi_j = \ln G_j + \ln \psi_{j-1} + \ln \phi_j, \ln \phi_j \sim N\left(-\frac{\sigma^2_{\phi_j}}{2}, \sigma^2_{\phi_j}\right).$$ \hspace{1cm} (3)

After retirement, namely $j > j_r$, a household receives a public pension, the level of which is determined by the wage rate (the implicitly permanent income level $\psi_{j_r}$) and a fixed parameter $b$.\(^{9}\)

$$Y_j = \omega_j b, \text{ and } \omega_j = \omega_{j_r} \text{ if } j > j_r$$ \hspace{1cm} (4)

\(^{8}\)Generally, the permanent shocks depend upon age $j$. Using a large repeated cross-sectional data set, Ohtake and Saito (1998) find that the age profile of the logarithms of income variances increases as households get older. Moreover, the profile of the variances of income and consumption is convex in Japan. Using the same but more recent data, Abe and Yamada (2006) estimate the stochastic income processes behind the profile, and they also estimate structural parameters using the age-variance profiles. Because Japanese panel data do not contain a lot of observations for each age group, we do not consider the age-dependent variance and assume it to be constant over age.

\(^{9}\)The parameter $b$ is determined from the actual replacement rate in the Japanese public pension system. Because labor supply and household income are endogenous in our model, the replacement rate depends on structural parameters. In particular, $\sigma$, a consumption and leisure share parameter described below, has significant effects on working hours. To avoid this problem, we define the replacement rate parameter as $b = \sigma \ell \bar{b}$. 

7
3.3 Dependent Children

Young and middle-aged households are likely to have children to whom they must dedicate time and extra spending.\(^{10}\) Apparently, if a household has more children, given the household consumption level, the potential consumption of the household head is lower, which leads to higher marginal utility of consumption.\(^{11}\) Blundell, et al. (1994) and Attanasio et al. (1999) also point out the importance of demographics for the life cycle in their nonstructural estimation.

Following Nishiyama and Smetters (2005), we incorporate the effects of family structure into our life cycle model as follows. We define individual consumption in multiplicative form:

\[
\hat{C}_j = \left(1 + \frac{n_j}{2}\right)^{-\xi} C_j,
\]

where \(C_j\) represents total family consumption, \(\hat{C}_j\) is an individual’s consumption, \(n_j\) is the number of dependent children of age \(j\), and \(\xi \geq 0\) is a parameter that adjusts the marginal utility of the household head. Notice that although the consumption that yields utility is \(\hat{C}_j\), the expenditure for consumption that appears in the budget constraint, (1), is \(C_j\).

3.4 Objective Function

The household head has the following objective function:

\[
U(\{\hat{C}_j\}^J_{j=1}, \{\ell_j\}^J_{j=1}) = E \sum_{j=1}^{J} \beta^{j-1} \left[\hat{C}_j^\sigma (\bar{\ell} - \ell_j)^{1-\sigma} \right]^{1-\gamma} S_j, \text{ where } S_j = \prod_{i=1}^{j-1} s_i,
\]

where \(\beta > 0\) is a discount factor, and \(S_j\) is a cumulative survival probability at the beginning of the life cycle.

We assume that the instantaneous utility function is additively nonseparable between consumption and leisure.\(^{12}\) \(\gamma\) is a coefficient that determines relative risk aversion and the intertemporal elasticity of substitution, and \(\sigma\) is the share parameter for consumption

\(^{10}\)There can be other dependent family members such as elderly people. We do not consider the elderly because the JSPC contains relatively young households, and the number of dependent “elderly” is small.

\(^{11}\)Expenditures on children and the family’s common consumption increases with age. On the contrary, the husband and wife’s consumption does not seem to increase so much. For details, see Figures 6, and Section 5.

\(^{12}\)For details of separability of utility functions with leisure, see Browning, Hansen and Heckman (1999).
and leisure. Because the utility function is of the Cobb–Douglas type, the elasticity of consumption and leisure is equal to unity.\(^ {13}\)

### 3.5 Dynamic Programming Problem

Defining the state variables as \((W_j, \psi_j, \xi_j)\), the Bellman equation at age \(j\) can be written as follows.

\[
V_j(W_{j-1}, \psi_j, \xi_j) = \max_{C_j, W_j} \left\{ \left[ \frac{C_j^\sigma (\bar{\ell} - \ell_j)^{1-\sigma}}{1 - \gamma} \right]^{1-\gamma} + s_j \beta E_j V_{j+1}(W_j, \psi_{j+1}, \xi_{j+1}) \right\}
\]  

(5)

subject to \(C_j + W_j = (1 + r)W_{j-1} + Y_j\), (2), (3) and (4),

\[
\ln \omega_1 = \ln \omega_0 + \ln \psi_0 + \ln G_1 + \ln \phi_1, \ G_1 = \phi_1 = 1, \ \ln \omega_0 \sim N\left(-\frac{\sigma^2 \omega_0}{2}, \sigma^2 \omega_0\right)
\]

It is difficult to solve the above Bellman equation directly because the range of the realized permanent income, \(\psi_j\), becomes larger as a household gets older. Thus, following Carroll (1997), we normalize our model by the permanent income level \(\psi_j\).

Because of homogeneity of the objective function, both sides of the Bellman equation can be divided by \(\psi_j^{\sigma(1-\gamma)}\).\(^ {14}\) The normalized Bellman equation of age \(j\) can be written as:

\[
v_j(w_{j-1}, \phi_j, \xi_j) = \max_{\hat{c}_j, \ell_j} \left\{ \left[ \frac{\hat{c}_j^\sigma (\bar{\ell} - \ell_j)^{1-\sigma}}{1 - \gamma} \right]^{1-\gamma} + s_j \beta E_j \Gamma_{j+1}^{\sigma(1-\gamma)} v_{j+1}(w_j, \phi_{j+1}, \xi_{j+1}) \right\}
\]

(6)

subject to

\(c_j + w_j = 1 + \frac{r}{\Gamma_j} w_{j-1} + \xi_j \ell_j\), if \(j \leq j_r\),

(7)

\(c_j + w_j = 1 + \frac{r}{\Gamma_j} w_{j-1} + b\), if \(j > j_r\),

(8)

where \(\Gamma_{j+1} \equiv \phi_{j+1} G_{j+1}\). Note that, even though we have divided the Bellman equation by \(\psi_j\), we cannot reduce the number of state variables. The state vector consists of three elements after the normalization.

\(^{13}\)If the utility function is separable between consumption and leisure, or is of a general CES type, then we cannot normalize our model, which makes it difficult to solve the model numerically. See the appendix for details of the normalization and numerical methods. Low (2005) carefully investigates the working hours profile of his model with a separable utility function.

\(^{14}\)See the appendix for details.
3.6 Labor Supply and Demographics

From equations (6), (7) and (8), intertemporal and intratemporal first-order conditions are as follows.\(^{15}\)

\[
\left(1 + \frac{n_j}{2}\right)^{-\zeta} \frac{c_j^{1-\sigma} (\bar{\ell} - \ell_j)^{1-\sigma}}{\hat{c}_j} \geq \sum_j \beta (1 + r) \left(1 + \frac{n_{j+1}}{2}\right)^{-\zeta} E_j \left\{ \Gamma_{j+1}^{\sigma(1-\gamma)-1} \frac{c_{j+1}^{1-\sigma} (\bar{\ell} - \ell_{j+1})^{1-\sigma}}{\hat{c}_{j+1}} \right\}
\]

\[
(1 - \sigma) \frac{c_j^{1-\sigma} (\bar{\ell} - \ell_j)^{1-\sigma}}{\ell - \ell_j} \geq \sigma \left(1 + \frac{n_j}{2}\right)^{-\zeta} c_j^{1-\sigma} \hat{c}_j \xi_j
\]

Therefore, putting aside corner solutions, we obtain the labor supply function from the intratemporal first-order condition.

\[
\ell_j = \bar{\ell} - \max \left( \frac{1 - \sigma}{\sigma \xi_j \left(1 + n_j/2\right)^{1-\zeta}} \hat{c}_j, 0 \right)
\]

(9)

Though we have no closed-form solution, we can solve the model numerically. Thus, we can empirically test the life cycle model using the Japanese micro data.

From the first-order condition defined above, we can observe that the growth rate of the effect of dependent children, \((1 + n_j/2)^{-\zeta}\), and the intertemporal elasticity of substitution (\(\gamma\)) determine the effective discount factor such as:

\[
\left(1 + \frac{n_j+1}{2}\right)\alpha_j = s_j \beta (1 + r) \left(1 + \frac{n_j+1}{2}\right)^{-\zeta} \hat{c}_j E_j \left\{ \Gamma_{j+1}^{\sigma(1-\gamma)-1} \frac{c_{j+1}^{\sigma(1-\gamma)-1}}{\hat{c}_{j+1}} \right\}
\]

(10)

The equation (10) is a normal Euler equation of family consumption \(c_j\), which equates marginal utility of age \(j\) with that of age \(j + 1\). However, there is a difference in the effective discount factor \(s_j \alpha_j \beta\). The effective discount factor differs across ages because of survival probability and life stage of parental care. Attanasio et al. (1999) have specified the utility function as \(1^{1-\gamma} \exp(\theta_1 x_j + \theta_2 y_j + z_j)\), where \(y_j\) are endogenous factors such as labor supply, \(x_j\) are observable exogenous factors, and \(z_j\) are unobservable exogenous factors. They have estimated preference parameters and concluded that family size

\(^{15}\)For the analytical characterizations of an endogenous labor supply model, see Low (2005). See Blundell and McCurdy (1999) for a survey.
and spouse’s leisure are significant for the utility function. In their model, demographics also has a role for the discount factor. In our model, the number of dependent children changes marginal utility of a household head. Therefore, the effective discount factor depends not only on $\zeta$ but also on $\sigma(1 - \gamma) - 1$, which is the intertemporal elasticity of substitution. We will discuss this point in Section 5, in detail.

4 Estimation Procedures and Calibration

Using the model described in Section 3, we estimate structural parameters by the method of simulated moments. As discussed in the introduction, we use age profiles of consumption, working hours, and financial assets, which are calculated from the JPSC, for structural estimation. Although ideally we should estimate all exogenous components in the model such as the permanent and transitory shocks parameters simultaneously, this is almost impossible to implement. Therefore, following previous research, we adopt a two-stage procedure to estimate the structural parameters. In the first stage, we calibrate some exogenous parameters using the same data. After that, in the second stage, we estimate the structural parameters, which include discount factor $\beta$, relative risk aversion $\gamma$, consumption–leisure share parameter $\sigma$, and an adjustment coefficient for the number of dependent children $\zeta$.

4.1 First-Stage Estimation from the JPSC

4.1.1 Life Cycle and Life Expectancy

We assume that all households enter the economy at age 25 ($j = 1$), that they must retire at age 60, and that they die by age 100, which implies $J = 76, j_r = 36$. Remember that from the assumption on mortality risks, although a household can live for 100 years at the most, most of them die earlier. The survival probability $\{s_j\}$ is taken from the life table in 2000 from the National Institute of Population and Social Security Research (2002). We have calculated the survival probability used in our estimation in Table 1. Notice that after retirement, the subjective discount rate becomes greater than 1% because $s_j$ is below 0.99.

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16By assumption, survival probability from age 100 to 101 is set to zero.
4.1.2 Interest Rate

All households face the same interest rate through the life cycle. Because the Japanese panel data cover from 1993 to 2002, one may think that we should use an average of the interest rate in this period. However, by the end of the 1990s in Japan, the deposit interest rate was almost zero, which is significantly smaller than preceding interest rates. It is highly likely that if we assume that all households face such a low interest rate over their life cycles, the effects of capital income will be underestimated. Thus, we set the real interest rate at 3.44%, which is the average of deflated nominal government bond returns from 1983 to 2001.

4.1.3 Labor Supply

We assume that \( \bar{\ell} = 3.0 \) so that adjusting average working hours becomes approximately one. For example, if a household supplies one-third of its labor endowment \( \bar{\ell} \), they work for 8 hours a day on average. In such a case, average working hours per year are 8-hours \( \times \) 5-days per week \( \times \) 4-weeks per month \( \times \) 12-months. We have adjusted the public pension parameter \( b \) to be the recent replacement rate in Japan, being equal to 40 ~ 50%.

4.2 Income Risks and Age-Income Profile

Using the same micro data from the JPSC, Abe and Inakura (2007a) have estimated permanent and transitory income risks from balanced and unbalanced panels.\(^{17}\) The following values are their estimates of the standard deviations of the permanent and transitory shocks.\(^{18}\)

- \( \sigma_\phi : 0.156 \) (from unbalanced panel as benchmark), 0.091 (from balanced panel)
- \( \sigma_\xi : 0.135 \) (from unbalanced panel), 0.099 (from balanced panel)\(^{19}\)

\(^{17}\)Abe and Inakura (2007a) estimate the income process with the same data set used in this paper. In their paper, Abe and Inakura follow Abowd and Card (1989) and compare several models for describing the Japanese income process. Although most previous studies use balanced data, Abe and Inakura (2007a) report results for both balanced and unbalanced data.

\(^{18}\)These values are much smaller than estimates for the United States. For example, Gourinchas and Parker (2002) use \( \sigma_\phi = 0.146 \) and \( \sigma_\xi = 0.209 \). According to Abe and Inakura (2007a), the variance of income itself in Japan is about one-third of the U.S. level reported in Abowd and Card (1989).

\(^{19}\)As the standard deviation of the fixed effect \( \omega_0 \), we use the standard deviation of the transitory shock. For the average of the initial wealth at age 25, we calculate the fraction of assets at age 25 divided by income at age 25. The average is 0.5739, and the variance is 0.3195.
Unbalanced panel data naturally contain more heterogeneous families, which results in larger income risks than for the balanced panel. Because our age profiles of consumption, working hours, and assets are based on unbalanced panel data, we adopt the income risk estimates based on unbalanced data as our benchmark.\footnote{Another reason to adopt the estimates based on unbalanced panel data is the sample size. Balanced panel data contain 262 families and 2620 observations, while unbalanced panel data contain more than 8500 observations.}

We need the average income profile to compute income growth rates \( \{G_t\} \). Because labor supply in our model is endogenous, we measure average income using the average hourly wage. Figure 3 plots the after-tax average hourly wage profile. We use the smoothed version of the profile when solving the model.

### 4.3 The Number of Dependent Children

The adjustment parameter between consumption and marginal utility, \( \zeta \), is one of the parameters to be estimated. Nishiyama and Smetters (2005) calibrate the parameter \( \zeta \) to be 0.6 as a benchmark of their model. In the benchmark case, a household with two dependent children consumes about 50% more than a household without dependent children, because \( (1 + n_j/2)^\zeta = 2^{0.6} = 1.516 \). From Figure 6, we can confirm that the consumption of the wife and husband changes little by age, but the consumption of their children and family increases as they get older. Because our data set contains only family consumption for September, we cannot observe consumption for annual events such as school admission fees. Moreover, because the JPSC micro data are targeted at young households, the dependent children in the data set are young. Thus, it is possible that we underestimate expenditures for dependent children such as education expenses, which are typically spent in April in Japan.

Table 2 shows the average numbers of dependent children in 5-year age intervals as appearing in Abe and Inakura (2007b). The table also contains the average numbers of dependent children from the Keio Household Panel Survey.\footnote{The Keio Household Panel Survey is a large survey of Japanese households that began in 2004. Sample size is 4005 households.} We also cited the number of dependent children in the U.S. from Nishiyama and Smetters (2005) for comparison. Our definition of a dependent child is as follows.

- **The Number of Dependent Children in Japan from the JPSC and Keio Household Panel Survey:** “preschooler” and “unmarried, nonworking children (include over 18 years old)”
The Number of Dependent Children in the United States: the number of children under 18 years old

Apparently, the number of dependent children over 40 in the JPSC is much larger than in the United States. Because the JPSC is a survey for young and middle-aged women as mentioned above, a household head over 40 with a relatively young wife will overestimate the number of dependent children. To confirm the number of dependent children, we calculate the number using the same definition as the Keio Household Panel Survey, and we find that the number of dependent children in households where the household head is over 50 is much larger in the JPSC. Therefore, we use the number of dependent children from the Keio Household Panel Survey, which is approximated by a 6th-order polynomial as in Figure 4.

As explained in Section 3.6, our specification of adjustment by the number of dependent children affects the effective discount factor as \( \alpha_j = \left( \frac{1 + n_j}{1 + n_j/2} \right)^{-\zeta \sigma (1 - \gamma) - 1} \). Figure 5 plots \( \alpha_j \) for each age and for some parameters.\(^{22}\) Because the family size increases from 25 to 44, the effective discount factor is over one for \( \zeta > 0 \), and after that, a household discounts the future much more. Thus, if \( \zeta \) is estimated to be larger, a household discounts the future more in middle and old age, and the consumption profile becomes strictly hump shaped.\(^{23}\)

4.4 Details of Second-Stage Estimation Procedures

We already have the set of calibrated parameters needed for solving the model, except a set of preference parameters (\( \beta, \gamma, \sigma, \zeta \)): the discount factor, the coefficient of relative risk aversion, the share parameter for consumption, and the consumption adjustment parameter. Therefore, given a set of parameters (\( \beta, \gamma, \sigma, \zeta \)), we can solve the optimal consumption–savings model described in the previous section. Because our model is a finite horizon, we can numerically solve the model by backward induction.\(^{24}\) After computing the optimal policy function, we simulate a sample path of consumption, working hours, and savings for \( L = 10,000 \) families.\(^{25}\)

\(^{22}\)The growth rate of \( (1 + n_j/2)^{-\zeta \sigma (1 - \gamma) - 1} \) seems to be waved, because the number of dependent children is approximated by polynomials.

\(^{23}\)Gourinchas and Parker (2002) and Cagetti (2003) also adjust marginal utility by demographics, although they do not include the parameters in their structural estimation.

\(^{24}\)For details on numerical procedures for solving the model, see the appendix.

\(^{25}\)We exclude households younger than 25 and older than 55 because the data do not contain enough observations for such households.
Given a set of parameters \((\beta, \gamma, \sigma, \zeta)\), let us define the logarithm of consumption (asset, or log of working hours) of the \(i\)-th agent of age \(j\) to be \(\ln Z_{i,j}(w_{i,j-1}; \beta, \gamma, \sigma, \zeta)\). Because a household’s consumption is a function of their assets in our model, we include \(w_{i,j-1}\) in the notation explicitly. Then, average consumption for each age is computed naturally as follows.

\[
\ln \hat{Z}_j(\beta, \gamma, \sigma, \zeta) = \frac{1}{L} \sum_{i=1}^{L} \ln Z_{i,j}(w_{i,j}; \beta, \gamma, \sigma, \zeta)
\]

Following Gourinchas and Parker (2002), we use the simulated average of the profile for estimation, not each household’s profile.

The estimation seeks a set of parameters that generates simulated data that are close to the age-profile data obtained in Section 2. Because of our reliance on micro data, as discussed in Section 2, we omit age profiles over 56 years old. Assume that the prediction errors have a mean of zero. Then, we can use moment conditions and conduct an estimation by a nonlinear least squares estimation method. We define the differences between the actual consumption \(\ln \tilde{Z}_j\) of age \(j\) and the simulated consumption as follows.

\[
g_j(\beta, \gamma, \sigma, \zeta) = \ln \tilde{Z}_j - \ln \hat{Z}_j(\beta, \gamma, \sigma, \zeta)
\]

We can then write an objective function for estimating the structural parameters such as:

\[
\min g'Wg,
\]

where the diagonal of the weighting matrix \(W\) is taken from the inverse of the variance–covariance matrix. The off-diagonal components of \(W\) are set to zero for tractability.\(^{26}\)

We look for parameters that minimize the function. Detailed steps of the estimation are as follows. First, we compute an average of the simulated profile, and calculate the differences between the data and the simulated path as \(\epsilon = \sum_{j=26}^{55} \frac{1}{\sigma_j^2} (\ln \tilde{Z}_j - \ln \hat{Z}_j)^2\). Second, if the parameters do not minimize the adjusted sum of squares, change the parameters \((\beta, \gamma, \sigma, \zeta)\). Repeat these steps until the value converges to the minimum. Variances of the estimator are easily computed as they are the same as variances from standard nonlinear least squares with instrumental variables.

This procedure enables us to conduct estimation using a mixture of each profile, such as working hours and asset profile. Therefore, our estimation proceeds with each single profile and a mixture of those profiles as follows.

\(^{26}\)Although we could use the optimal weight, we do not adopt it because the panel horizon is not long enough to obtain all the covariances. For example, the covariance between consumption at age 30 and age 50 cannot be calculated because the panel covers only 12 years.
Consumption Profile  Consumption in our data covers monthly expenses only, while the model is built based on annual decisions. Multiplying consumption by 12 does not work well probably because of strong seasonality in consumption. For this reason, we normalize both the actual and simulated paths using the average path. These normalized unitless data have sufficient information for estimation because the growth rate of average consumption is determined from the households’ decisions on consumption and savings.

Wealth Profile  Because there are some families with no financial assets, we do not take logarithms of wealth into our estimation. We also normalize the wealth profile using the average profile. One reason for the normalization is the difference between the wealth defined in the model and our data. Ideally, all assets such as durable goods, real estate, and future pensions should be included in the data, which is difficult because of measurement problems. In this paper, we restrict our data to financial assets only, which provides too few observations to match the model’s prediction in levels.

Working Hours Profile  We use the average of logarithms of the working hours profile because our data set contains annual working hours data. The working hours profile is the only profile that fits the data of the model in levels. As stated above, average working hours decrease as households get older. If we normalize the working hours profile using the average, we cannot estimate the share parameter for consumption $\sigma$ because the parameter shifts the level of the working hours profile.

Wealth and Working Hours Profile  French (2005) has estimated structural parameters in the U.S. from profiles of savings levels, working hours, and labor participation rates with good and bad health status respectively (i.e., 6 profiles). Unfortunately, the JPSC data do not contain such detailed health information. The JPSC does not contain many observations for the elderly. Therefore, we could not conduct the same procedure as French (2005). In this paper, we simply conduct estimation of the structural parameters from wealth and the working hours profile.

Consumption and Working Hours Profiles  From equation (9), it is straightforward to see that the consumption profile has a one-to-one correspondence with the working hours profile. In our model, by including the adjustment parameter $\zeta$, our model becomes flexible enough in possible patterns of both profiles to estimate structural parameters from the two profiles.
Consumption and Wealth Profiles Gourinchas and Parker (2002) find that if the fundamental parameters are adjusted for the consumption profile, the corresponding wealth profile does not match the actual data profile. French (2005) also points out the difficulty of matching both profiles simultaneously. We will check this relationship using Japanese data later.

5 Estimated Results

5.1 Estimation Results with Each Single Profile

There have been several papers that conduct structural estimation of life cycle models with age–consumption profiles (Gourinchas and Parker, 2002, Abe and Yamada, 2005), age–wealth profiles (Cagetti, 2003), and age–working hours and age–wealth profiles (French, 2005). Following these previous studies, rather than using several profiles simultaneously, we first estimate our model with a single profile and then examine whether the model is consistent with each profile under plausible parameter values.

Table 4 shows all the estimated results using single profiles. Each column reports our estimates using the corresponding profile with income risks obtained from the unbalanced panel data, which are taken from Abe and Inakura (2007a). The standard errors of each estimator are in parenthesis in Table 4. Seemingly, all estimated parameters are within the plausible range found in the previous literature.

As described in Figure 2(c) in Section 2, the age–consumption profile in Japan has a peak in the late 40s and is hump-shaped. This shape is also observed in other countries; for example, Fernández-Villaverde and Krueger (2004) and Attanasio et al. (1999) have investigated the shape of the consumption profile, and Gourinchas and Parker (2002) use that shape in the United States for structural estimation. They suggest that the hump-shaped profile reflects incomplete capital markets, liquidity constraints and demographics. In a life cycle model under incomplete asset markets and idiosyncratic labor income risks, young households have a precautionary savings motive and face liquidity constraints. As households get older, they accumulate wealth and earn interest income.

French (2005) uses the age profile of smoothed micro data instead of raw data. We have also examined this procedure in our estimation, and find that the results do not change significantly, although the standard errors of the estimator using smoothed data are a little too large. Therefore, we proceed with the estimation using the raw data.
Furthermore, because the elderly have less remaining time alive, they face less uncertainty in lifetime income. Therefore, middle or old households have less precautionary motive because of sufficient wealth accumulation and higher wages than the young. An increase in survival probability decreases consumption after retirement. Therefore, using the precautionary savings model and the age–consumption profile, we can estimate several preference parameters because the shape is determined by the precautionary motive, the intertemporal elasticity of substitution, and the discount factor.

From the MSM estimation of the consumption profile, we have estimated the discount factor $\beta$ to be 0.997. Although it is a little higher than the previous research mentioned above, it does not differ much from previous studies for Japan. Hayashi and Prescott (2002) find that $\beta = 0.978$ using macro data in Japan. The estimated coefficient for relative risk aversion is also in the standard range, $\gamma = 7.069$. For example, French (2005) estimates the same parameter with an endogenous labor supply model, and he finds $\gamma$ to be between 3.19 and 7.69. On the contrary, the estimated family adjustment parameter, $\zeta$, is much larger than the value calibrated by Nishiyama and Smetters (2005), i.e., $\zeta = 0.6$. Furthermore, the share parameter for consumption $\sigma$ is too small compared with previous research and calibration. The under- and overestimation of those parameters are not surprising because $\sigma$ and $\gamma$ affect the shape of the consumption profile quite similarly; note that the relative risk aversion coefficient is $-\frac{\alpha c_u c_u'}{\sigma} = \sigma(\gamma - 1) + 1$. In other words, the shape of the consumption profile only does not have enough power to distinguish $\sigma$ from $\gamma$. As reported in Gourinchas and Parker (2002) and French (2005), the standard errors of $\beta$ are very small, but the standard errors of $\gamma$ are large in our estimation with the consumption and wealth profiles.

Although the definition of the wealth profile in our model differs significantly from the actual age–wealth profile, we obtain reasonable parameters when we use the wealth profile for estimation.\textsuperscript{28} In particular, consumption, leisure share parameter $\sigma$ and adjustment parameter for dependent children $\zeta$ are all much closer to the previous research. Moreover, we find that the estimation works well with the age–working hours profile, provided that the consumption adjustment parameter $\zeta$ is not zero, which implies that the labor supply profile needs to be adjusted through changes to marginal utility by the dependent children. Estimated parameters, $\beta$, $\gamma$, and $\sigma$, are comparable with previous literature such as Gourinchas and Parker (2002) and French (2005). In our specification\textsuperscript{28}

\textsuperscript{28}For a robustness check, we have estimated the parameters with a different wealth profile definition. The wealth profile contains house, land and housing loan values in the definition of asset holdings. Estimation using this wealth profile gives us a very high discount factor, 1.4.
of the utility function, relative risk aversion is represented as $-\frac{cu''}{uc} = \sigma(\gamma - 1) + 1$, thus for example, if $\sigma = 0.33$ and $\gamma = 4$, relative risk aversion is about 2. This implies that, if the utility function does not include leisure and is of CRRA type, then $-\frac{cu''}{uc} = \gamma$, therefore relative risk aversion (intertemporal elasticity of substitution) is higher (lower) than in the standard log-utility function. Our estimation results show that relative risk aversion is greater than unity for a log-utility function. Moreover, consumption and leisure are Frisch substitutes because the cross derivative becomes negative: $u_{cl} = (1 - \gamma)\sigma(1 - \sigma)\hat{\epsilon}_{\gamma(1-\gamma)-1}(\bar{\ell} - \ell)(1-\sigma)(1-\gamma)-1 < 0$.

We have assumed that marginal utility in middle and old age is accommodated through the dependent family’s profile. Therefore, a household has strict preferences toward consuming in middle age. The consumption adjustment parameter is estimated to be 0.866, which is slightly higher than the calibrated value of Nishiyama and Smetters (2005). The estimated result of $\zeta = 0.866$ implies that the effective discount factor is adjusted by $\alpha_j$, which ranges from about $-6\%$ to $6\%$ (see Figure 5). As will be explained in Section 6, the working hours profile tends to be a decreasing function if a household faces high income risks or if marginal utility of the household is adjusted for dependent children. Thus, thanks to high income risks estimated from unbalanced panel data, we succeed in estimating all fundamental parameters with plausible value of $\zeta$. In other words, both high risks and adjustment for dependent children are the keys to explaining age–working hours profiles in Japan. This result may be surprising because the Japanese labor market has been known by its unique customs such as long-term employment.

Figure 7 plots actual data profiles and simulated profiles using the estimated parameters. Our simulated profiles of consumption and working hours exhibit very similar patterns to those of actual profiles: strict hump-shaped consumption and weakly downward sloping working hours.

5.2 Estimation Results with Several Profiles

Next, we conduct estimation with several profiles simultaneously. In the previous subsection, we saw that estimation with a single profile does not work well in some cases. For example, the estimated parameter $\sigma$ is too low and $\zeta$ is extremely high when accommodating the age–consumption profile. One possible reason for this failure is the

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29 See Low (2005) for details.
30 See Aoki, Patrick, and Sheard (1994).
lack of power in each profile to identify all the parameters. By utilizing information contained in several profiles simultaneously, we show that it becomes possible to estimate the parameters. The results are summarized in Table 5.

First, following French (2005), we estimate the parameters using working hours and wealth profiles simultaneously. The results are not convincing for discount factor $\beta$, which is too low. The mechanism for identifying parameters from those profiles is as follows. Both consumption $C_j$ and labor supply $\ell_j$ are functions of wealth at $j$ years old. An increase in wealth will induce a household to increase leisure when consumption and leisure are substitutes. If a household is risk-averse, it prefers to accumulate wealth from precautionary savings in its early life stage. Thus, the relation between working hours and wealth is mainly determined by the degree of relative risk aversion and the Frisch elasticity. French (2005) uses these relationships for his estimation and succeeds in estimating the structural parameters. Measurement errors in our wealth data might be the reason for the failure of our estimation.

When we use consumption and working hours profiles simultaneously, it is necessary to include a consumption adjustment parameter $\zeta$ in the simulation because of the one-to-one relationship that appears in equation (9). In our estimation, the difference in shapes between the consumption and working hours profiles is mainly explained by $\zeta$. Therefore, the parameter $\zeta$ results in a large value, but other estimated parameters are within a plausible range and the standard errors are small. Attanasio et al. (1999) show that the hump-shaped consumption profile is created partially by precautionary saving and also that demographics have a role in determining the peak of the hump-shaped profile. We support their results using a life cycle model with plausible structural parameters.

From estimation of the consumption and wealth profiles, we find that the estimated results are similar to the case of the consumption profile only, $\sigma$ being low and $\beta$ being high. As Gourinchas and Parker (2002) and French (2005) discuss, if we use moment conditions of consumption for the estimation of the structural parameters, the corresponding simulated wealth profile departs significantly from the actual wealth profile. However, compared with the previous estimation using a mixture of the consumption and wealth profiles, this case seems to work well because the discount factor and relative risk aversion estimates are still within plausible ranges. Using all profiles, we obtain moderate and mixed results, which are comparable to all previous estimations although $\zeta$ is a little higher.
5.3 Robustness for Our Estimation

We report estimated results of the same parameters with low income risk estimation, which is taken from balanced panel data of Abe and Inakura (2007a). Table 6 shows all estimated results with low income variances.

The coefficient of relative risk aversion \( \gamma \) has been estimated to be high in almost all cases when assuming small income risk. In particular, an extremely high \( \gamma \) is required for fitting the actual wealth profile. This is consistent because, with a consumption profile of the same shape, if households face lower risk, they need to have high risk aversion to obtain the same level of precautionary savings. Estimation with the working hours profile results in a high discount factor, which also implies a high propensity to save.

Compared with the Family Income and Expenditure Survey used by Abe and Yamada (2005),\(^{31}\) the consumption profile in this paper has a steep slope, implying a strong precautionary motive. We do not consider whether those differences come from our definition of consumption or features of the micro data. Even in the low income risk case, the working hours profile does work well for estimating structural parameters with coefficient \( \zeta \).

6 Simulation Analysis

6.1 Consumption and Working Hours Profiles with Income Risk

So far, we know that each profile can be explained by a life cycle model with plausible parameters, although there are some exceptions. In this section, we discuss the mechanisms through which dependent children and income risks affect profiles. Figures 8 and 9 plot the consumption and working hours profiles for the upper risk case taken from unbalanced panel data (“benchmark”) and the lower risk case from balanced panel (depicted as “low risk”).\(^{32}\) Each figure also plots the cases with and without dependent children (depicted as “no kids”).

Suppose that both the permanent and transitory income risks are very small. Because the average earnings profile is hump-shaped and has a peak at about age 50 as described

\(^{31}\)The Family Income and Expenditure Survey is a monthly national survey of detailed family expenditure and income that covers more than 8,000 households. The survey requests each surveyed family to keep a diary to record precise household accounts so that the data are expected to contain fewer measurement errors.

\(^{32}\)In the simulation analysis, as the benchmark, we set \( \beta = 0.958 \), \( \gamma = 3.18 \), \( \sigma = 0.394 \), and \( \zeta = 1.0 \), which are estimated parameters using the consumption and working hours profiles.
in Figure 3, all households prefer to supply labor at their highest productivity. When a household is young, it can earn less relative to middle age, which implies that the value of leisure is greater compared with consumption. Therefore, the working hours profile traces the average earnings profile, and it is also hump-shaped (see Figure 9 with low risk). Moreover, because a household has a lower precautionary savings motive, it consumes more during early stages of the life cycle. Comparing this with the benchmark case with low risk in Figure 8, the slope of the consumption profile is steeper in the benchmark case.

If households face higher income risks, they supply more labor to accumulate precautionary savings. If households have more wealth compared with the less risky case, they will not work in middle and old age because of the wealth effect. They can also earn capital income from their wealth. Thus, working hours over 50 years old decrease rapidly, and the corresponding age–working hours profile is monotonically decreasing as in the “benchmark” case in Figure 9. From these relationships, structural estimation with both the working hours profile and the savings profile works well in French (2005). The level of income risks also affects the consumption profile through wealth accumulation for the precautionary savings motive. If young workers have a strong precautionary motive, they need to restrain themselves from consuming in their early life stage so that they can enjoy consumption more through the decumulation of wealth and capital income later in life. Thus, the consumption profile becomes steep.

Those mechanisms discussed above are basically the same as those in Low (2005). As described in Section 2, working hours decrease monotonically as households get older, which is the same as in the U.S. On the contrary, a simple life cycle model predicts a hump-shaped labor supply profile, which seems to contradict the data. Low (2005) explains these differences by the level of income uncertainty. However, at least in our estimation of income risks from Japanese panel data, the Japanese income risks are small relative to the United States, and the income risk may not adequately explain the shape.

6.2 Consumption and Working Hours Profiles with/without Dependent Children

In this paper, we have added the effects of dependent children to the framework of Low (2005). Even if households do not face significant income risks, the age–working hours profile decreases if we include dependent children in the life cycle model. From equation (10), the number of dependent children changes the effective discount factor, and a high \( \zeta \) implies a strong hump shape, as depicted in Figure 10.
Although households with high productivity and earnings have strong incentives to supply labor, they need to consume more for dependent children, which reduces their utility levels from consumption. In other words, the life stage of high earnings corresponds to the life stage of high consumption. Therefore, a middle-age household can earn more because of high productivity, but it must increase its expenditures for their dependent children; high productivity is thus offset by the existence of dependent children. A household that expects to have many dependent children tends to save more for spending on food, education, or home renovations. Such households have strong incentives to supply more labor, and they save more during the early stages of their life cycles. Such a life structure of labor income and expenditure causes the age–working hours profile to decrease. If the age–working hours profile accounts for uncertainty only, the working hours decrease too rapidly, as in Figure 9 under “no kids”, and the corresponding consumption profile is too steep. Thus, a mild level of adjustment according to this demographic is required. As stated in Table 4, the age–working hours profile can be explained by the household’s optimal behavior under plausible parameters and relatively high income risks.

6.3 Precautionary Savings

Gourinchas and Parker (2001) measured levels of precautionary savings. The precautionary savings of our model is quite similar to their study, in which they estimate the level of precautionary savings from 1% to 5%. Precautionary savings—defined as differences between saving with and without income shocks—is around 2% over the life cycle, as shown in the left panel of Figure 11. If households cannot adjust labor supply elastically, then the corresponding wealth profile differs significantly, and precautionary savings are much larger than in the model with leisure choice in the right panel of Figure 11.

7 Concluding Remarks

This paper investigates the relationships between consumption, working hours, and asset accumulation over the life cycle. As seen in other countries, Japanese data show hump-shaped consumption and downward-sloping working hours profiles. The main objective of this paper is to explore whether a simple life cycle model can account for the characteristics of several age profiles simultaneously. We built a life cycle model that incorporates precautionary savings, labor supply, and family structure. Using moment
conditions of several age profiles, we estimate the structural parameters of the model. The empirical results show that our model fits the age profiles well even though Japanese households face relatively little income uncertainty.

There are many remaining tasks such as considerations of health risks, bequest, retirement, endogenous determination of family structure, purchase of real estate, etc. Health risks and retirement seem to be important for current Japanese households that are rapidly aging.

References


A Details of the Life Cycle Model

A.1 Normalization of Budget Constraints and State Variables

By normalizing permanent income $\psi_j$, the budget constraint is rewritten as follows.

$$\frac{C_j}{\psi_j} + \frac{W_j}{\psi_j} = (1 + r)\frac{W_{j-1}}{\psi_{j-1}} \psi_j + \frac{Y_j}{\psi_j},$$

$$c_j + w_j = \frac{1 + r}{G_j\phi_j} w_{j-1} + \xi_j \ell_j,$$

where $w_j \geq 0$ \hspace{1cm} (11)

The state variables of age $j$ can be transformed from $(W_{j-1}, \psi_j, \xi_j)$ to $(w_{j-1}, \phi_j, \xi_j)$. Because the new budget constraint (11) includes $(\phi_j, \xi_j)$, we cannot reduce the number of state variables. We will verify the usefulness of such a transformation in the next section.

A.2 Normalized Bellman Equation

Because our model has a finite horizon, we can solve the model by backward induction. Because all households die with certainty at $J$, $V_{J+1} = 0$, they consume all cash on hand $X_J = (1 + r)W_{J-1} + Y_J = C_J$. Note that, because $Y_J$ is a public pension, the cash on hand is deterministic at the beginning of age $J$. From homogeneity of the utility function, we have:

$$V_J(W_{J-1}, \psi_J) = \left[\frac{((1 + n_{J}/2) - \zeta_x X_J)^{\sigma} (\bar{\ell} - \ell_J)^{1-\sigma}}{1 - \gamma}\right]^{1-\gamma},$$

$$= \psi_J^{(1-\gamma)} \left[\frac{((1 + n_{J}/2) - \zeta_x X_J)^{\sigma} (\bar{\ell} - \ell_J)^{1-\sigma}}{1 - \gamma}\right]^{1-\gamma},$$

$$= \psi_J^{(1-\gamma)} v(w_{J-1}),$$

where $w_{j-1} = \frac{W_{j-1}}{\psi_{j-1}}, c_j = \frac{C_j}{\psi_j}, \hat{c}_j = (1 + n_j/2)^{-\zeta c_j}$, and we set the new value function $v_J(w_{J-1})$ with normalized asset $w_{J-1}$ as a new state variable. Because all households have already retired at $J$, they cannot supply labor, and we omit the state variables $(\phi_j, \xi_j)$.

From the above, only the consumption term is normalized by $\psi_j$ without any change to labor supply. Substituting equation (12) into the Bellman equation (5) in period
we obtain:

\[
V_{J-1}(W_{J-2}, \psi_{J-1}) = \max_{C_{J-1}, W_{J-1}} \left\{ \frac{\hat{C}_{J-1}^{\sigma} (\bar{\ell} - \ell_{J-1})^{1-\sigma}}{1-\gamma} + s_{J-1} \beta E_{J-1} V_{J} (W_{J-1}, \psi_{J}) \right\}
\]

\[
= \psi_{J-1}^{\sigma(1-\gamma)} \times \max_{c_{J-1}, w_{J-1}} \left( \frac{\hat{c}_{J-1}^{\sigma} (\bar{\ell} - \ell_{J-1})^{1-\sigma}}{1-\gamma} + s_{J-1} \beta E_{J-1} \Gamma_{J}^{\sigma(1-\gamma)} v_{J} (w_{J-1}) \right)
\]

where \( \psi_{J}^{\sigma(1-\gamma)} = (G_{J} \phi_{J} \psi_{J-1})^{\sigma(1-\gamma)} \) and \( \Gamma_{J} \equiv G_{J} \phi_{J} \). By repeating this step, we can obtain the Bellman equation at age \( j \) years.

B  Numerical Procedures: Endogenous Gridpoints Method

B.1 Policy Function

Solving the model using the standard value function iteration method, we need to extrapolate the approximated value function if the income growth rate and permanent shocks are high. Because the extrapolation usually makes large numerical errors, we employ the “Endogenous Gridpoints Method (hereafter, EGM)” developed by Carroll (2006). The EGM is a safe and relatively rapid method compared with the value function iteration.33

Although we want to employ the EGM, it is difficult to apply directly because the definition of cash on hand contains labor income \( \omega_{J} \ell_{J} \), which is a decision variable.

We define the normalized cash on hand as follows.

\[
\hat{x}_{J} \equiv \frac{1+r}{1+r_{J}} w_{J-1} + \xi_{J} \bar{\ell}, \quad \hat{c}_{J} \equiv \frac{1+r}{1+r_{J}} w_{J-1} + b
\]

We assume that a household always supplies maximum labor, thus if \( (w_{J-1}, \phi_{J}, \xi_{J}) \) is given, the cash on hand \( \hat{x}_{J} \) is uniquely determined. We define policy functions \( \{ g_{c}, g_{r} \} \) over this modified cash on hand.

We redefine the normalized choice variables and budget constraints as:

\[
\begin{cases}
\hat{c}_{J} \equiv c_{J} + \xi_{J} (\bar{\ell} - \ell_{J}), & \text{if } j \leq j_{r}, \\
\hat{c}_{J} \equiv c_{J}, & \text{if } j > j_{r}
\end{cases}
\]

\[\hat{w}_{J} = \hat{x}_{J} - \hat{c}_{J}.\]

33Details of the method used in this paper are based upon the appendix in Krueger and Ludwig (2006).
We do not change the budget constraints equation.

Then, a modified Bellman equation $\tilde{v}_j (\bar{x}_j, \phi_j, \xi_j)$ with new state variables $(\bar{x}_j, \phi_j, \xi_j)$ is:

$$
\tilde{v}_j (\bar{x}_j, \phi_j, \xi_j) = \max_{\tilde{c}_j, \tilde{\ell}_j} \left\{ \left[ \frac{c_j^\sigma (\bar{\ell} - \ell_j)^{1-\sigma}}{1 - \gamma} \right]^{\frac{1}{1-\sigma}} + s_j \beta E_j \Gamma_{j+1}^{\sigma(1-\gamma)} \tilde{v}_{j+1} (\bar{x}_{j+1}, \phi_{j+1}, \xi_{j+1}) \right\},
$$

subject to

$$
\bar{x}_{j+1} = \frac{1 + r}{\Gamma_{j+1}} [\bar{x}_j - \bar{c}_j] + \xi_{j+1} \bar{\ell}, \text{ or } \bar{x}_{j+1} = \frac{1 + r}{\Gamma_{j+1}} [\bar{x}_j - \bar{c}_j] + b,
$$

$$
\Rightarrow \max_{\tilde{c}_j, \tilde{\ell}_j} \left\{ \left[ \frac{c_j^\sigma (\bar{\ell} - \ell_j)^{1-\sigma}}{1 - \gamma} \right]^{\frac{1}{1-\sigma}} + s_j \beta E_j \Gamma_{j+1}^{\sigma(1-\gamma)} \tilde{v}_{j+1} \left( \frac{1 + r}{\Gamma_{j+1}} [\bar{x}_j - \bar{c}_j] + \xi_{j+1} \bar{\ell}, \phi_{j+1}, \xi_{j+1} \right) \right\}.
$$

Define the second term of the Bellman equation as follows:

$$
\Omega_j (\bar{w}_j, \phi_{j+1}, \xi_{j+1}) = s_j \beta E_j \Gamma_{j+1}^{\sigma(1-\gamma)} \tilde{v}_{j+1} \left( \frac{1 + r}{\Gamma_{j+1}} \bar{w}_j + \xi_{j+1} \bar{\ell}, \phi_{j+1}, \xi_{j+1} \right),
$$

$$
\Omega'_{j} (\bar{w}_j, \phi_{j+1}, \xi_{j+1}) = s_j \beta E_j \Gamma_{j+1}^{\sigma(1-\gamma)} \frac{1 + r}{\Gamma_{j+1}} \frac{\partial \tilde{v}_{j+1} (\bar{x}_{j+1}, \phi_{j+1}, \xi_{j+1})}{\partial \bar{w}_j}.
$$

Because the intertemporal first-order condition is:

$$
u'_c (\bar{c}_j, \ell_j) = \Omega' (\bar{w}_j, \phi_{j+1}, \xi_{j+1}), \quad (13)
$$

by calculations of $\Omega'$ on discretized states $(\bar{w}_j, \phi_{j+1}, \xi_{j+1})$, we can obtain consumption $\bar{c}_j$ from the inverse of the marginal utility function. By the Envelope Theorem, we have:

$$
\frac{\partial \tilde{v}_{j+1} (\bar{x}_{j+1}, \phi_{j+1}, \xi_{j+1})}{\partial \bar{w}_j} = (1 + n_{j+1}/2)^{-\frac{1}{\gamma}} \frac{c_j^\sigma (\bar{\ell} - \ell_j)^{1-\sigma}}{\bar{c}_{j+1}}.
$$

Now, suppose that we know next period’s policy functions by backward induction such as:

$$
c_{j+1} = g_{c,j+1}(\bar{x}_{j+1}, \phi_{j+1}, \xi_{j+1}),
$$

$$
\ell_{j+1} = g_{\ell,j+1}(\bar{x}_{j+1}, \phi_{j+1}, \xi_{j+1}), \text{ if } j \leq j_r,
$$

$$
\ell_{j+1} = 0, \text{ if } j > j_r.
$$

---

In our numerical computation, we have used the Gauss–Hermite quadrature to compute expectation operators, and the number of gridpoints of the permanent shocks is set to 9, and that of transitory shocks to 7.
Then, we obtain:

\[ \frac{\partial \tilde{v}_{j+1}}{\partial \tilde{w}_j} = \sigma(1 + n_{j+1}/2)^{-\zeta} \left[ \left( 1 + n_{j+1}/2 \right)^{-\zeta} g_{c,j+1} \left( \tilde{\ell} - g_{c,j+1} \right)^{1-\sigma} \right]^{1-\gamma} \]

Because the cash on hand \( \tilde{x}_{j+1} = \frac{1+r}{1+r} \tilde{w}_j + \xi_{j+1} \tilde{\ell} \) can be calculated by \( \tilde{w}_j \), we can compute \( \Omega' (\tilde{w}_j, \phi_{j+1}, \xi_{j+1}) \) over each grid of \( \{ \tilde{w}_i \}_{i=1}^{n_w} \).

### B.2 Inverse of the Utility Function

From the first-order condition (13), if the marginal utility function \( u'_c(\hat{c}_j, \ell_j) \) is invertible with \( \hat{c}_j \), we can calculate the consumption function. Using the intratemporal Euler equation, we know \( \ell_j = \tilde{\ell} - \frac{1-\sigma}{\sigma \xi_j (1 + n_j / 2)^{-\zeta}} c_j \). Using the equation, we obtain:

\[ u'_c(c_j, \ell_j) = \hat{c}_j^{-\gamma} \sigma \left( 1 + n_j / 2 \right)^{-\zeta} \left( \frac{1 - \sigma}{\sigma \xi_j (1 + n_j / 2)^{-\zeta}} \right)^{(1-\sigma)(1-\gamma)} \]

Because this equation is invertible, we have:

\[ \hat{c}_j = u^{-1} \left( \Omega' (\tilde{w}_j, \phi_{j+1}, \xi_{j+1}) \right), \]

and we also compute \( \ell_j \) from the first-order condition. A crucial point of this method is that we do not need an optimization or nonlinear equation solver, which makes our estimation safe, because our estimation requires several thousand iterations of solving for the policy function.

Because we have a set of \( \{ \hat{c}_j, \ell_j, \tilde{w}_j \} \), we also have cash on hand \( \tilde{x}_j = \tilde{w}_j + \hat{c}_j \), where \( \hat{c}_j \equiv c_j + \xi_j (\tilde{\ell} - \max(\tilde{\ell} - \frac{1-\sigma}{\sigma \xi_j} c_j, 0)) \).

### B.3 Simulation

From the previous step, we already know the optimal policy function. Therefore, given a sequence of realizations of the permanent shocks, transitory shocks, fixed shocks, and initial wealth shocks, we can compute life cycle profiles of consumption, working hours

---

Even if labor supply is zero or exogenously fixed at value \( \tilde{\ell} \), the marginal utility function is invertible as follows.

\[ \sigma \left( 1 + n_j / 2 \right)^{-\zeta} \frac{[\sigma (\tilde{\ell} - \tilde{\ell}_j)^{1-\sigma}]^{1-\gamma}}{\xi_j} = \sigma \left( 1 + n_j / 2 \right)^{-\zeta} \xi_j^{(1-\gamma)-1} (\tilde{\ell} - \tilde{\ell}_j)^{(1-\sigma)(1-\gamma)} \]

\[ = \xi_j^{(1-\gamma)-1} (\sigma (1 + n_j / 2)^{-\zeta} (\tilde{\ell} - \tilde{\ell}_j)^{(1-\sigma)(1-\gamma)}) \]
and wealth for each household. The number of people assumed in the simulation is 10,000. From this sequence, we have estimated the structural parameters by the method of simulated moments.
### C Tables and Figures

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<th>age</th>
<th>surv. prob.</th>
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Table 1: Survival Probability
Table 2: The Number of Dependent Children for Each Age in Japan from JPSC and Keio Panel Data

<table>
<thead>
<tr>
<th>Age</th>
<th>JPSC</th>
<th>Keio</th>
<th>Age</th>
<th>JPSC</th>
<th>Keio</th>
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<tr>
<td>20–24</td>
<td>0.867</td>
<td>0.714</td>
<td>45–49</td>
<td>2.019</td>
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<td>25–29</td>
<td>1.112</td>
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<td>50–54</td>
<td>1.902</td>
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<tr>
<td>30–34</td>
<td>1.566</td>
<td>1.379</td>
<td>55–59</td>
<td>2.167</td>
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<td>35–39</td>
<td>1.866</td>
<td>1.656</td>
<td>60–64</td>
<td>1.400</td>
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</tr>
<tr>
<td>40–44</td>
<td>2.044</td>
<td>1.886</td>
<td>65–</td>
<td>–</td>
<td>0.037</td>
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Table 3: The Number of Dependent Children for Each Age in the U.S. from Nishiyama and Smetters (2005)

<table>
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<tr>
<th>Age</th>
<th>JPSC</th>
<th>Keio</th>
<th>Age</th>
<th>JPSC</th>
<th>Keio</th>
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<td>20–24</td>
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<td>0.847</td>
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<td>25–29</td>
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<td>1.649</td>
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Table 4: MSM Estimation with Each Individual Profile. Standard errors are in parentheses

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<tr>
<th>Consumption</th>
<th>Asset</th>
<th>Labor</th>
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<tr>
<td>$\beta$</td>
<td>0.997</td>
<td>0.936</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.011)</td>
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<tr>
<td>$\gamma$</td>
<td>7.069</td>
<td>7.782</td>
</tr>
<tr>
<td></td>
<td>(1.159)</td>
<td>(1.911)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.084</td>
<td>0.354</td>
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<tr>
<td></td>
<td>(0.015)</td>
<td>(0.044)</td>
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<tr>
<td>$\zeta$</td>
<td>8.014</td>
<td>1.418</td>
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<td>(0.918)</td>
<td>(0.116)</td>
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Table 5: MSM Estimation with a Mixture of Each Profile. Standard errors are in parentheses

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<th>cons. &amp; labor</th>
<th>cons. &amp; asset</th>
<th>all</th>
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<tr>
<td>(\beta)</td>
<td>0.878</td>
<td>0.958</td>
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<tr>
<td></td>
<td>(0.008)</td>
<td>(0.002)</td>
<td>(0.005)</td>
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<td>(\gamma)</td>
<td>8.758</td>
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<td>(\zeta)</td>
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Table 6: MSM Estimation with the Lower Income Risk Case

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<th>labor</th>
<th>asset &amp; labor</th>
<th>cons. &amp; labor</th>
<th>cons. &amp; asset</th>
<th>all</th>
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<td>(0.015)</td>
<td>(0.001)</td>
<td>(0.011)</td>
<td>(0.005)</td>
<td>(0.009)</td>
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<tr>
<td>(\gamma)</td>
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<tr>
<td></td>
<td>(0.051)</td>
<td>(0.035)</td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.003)</td>
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<tr>
<td>(\zeta)</td>
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<td>(0.055)</td>
<td>(0.128)</td>
<td>(0.060)</td>
<td>(0.052)</td>
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Figure 1: Age Distribution of Household Heads in 1993

Figure 2: Each Profile
hourly wage: 10,000 yen

Figure 3: Hourly Wage Profile

dependent children

Figure 4: The Number of Dependent Children (Smoothed)
Figure 5: Growth Rate of Coefficient of Dependent Children

Figure 6: Family Structure and Expenditure for One Month
Figure 7: Simulated Profiles and Data
Figure 8: Simulated Consumption Profile with High and Low Income Risk and with/without Dependent Children

Figure 9: Simulated Working Hours Profile with High and Low Income Risk and with/without Dependent Children

Figure 10: Simulated Consumption Profile with $\zeta = 3.0$
Figure 11: Precautionary Saving with and without Leisure