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A Macro-Finance Analysis of the Term Structure and Monetary Policy in Japan:

Using a Model with Time-Variant Equilibrium Rates of Real Interest and Inflation and with the Zero Lower Bound of Nominal Interest Rates

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A Macro-Finance Analysis of the Term Structure and Monetary Policy in Japan:

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Abstract

We study the term structure of interest rates and monetary policy in Japan empirically, using a macro-finance model. In particular, we investigate whether or not Japan's low long-term interest rates can be explained with economic rationality by taking into account some key features of the economy: possible time-variability of perceived equilibrium rates of real interest and inflation, the effect of the zero lower bound of nominal interest rates, and the effect of the zero interest rate commitment by the Bank of Japan. We are also interested in the estimation of the macroeconomic structure based not only on macroeconomic data but also on market interest rate information.

Specifically, we use a New Keynesian-type macro structural model and an affine diffusion model of the term structure, taking into account the non-linearity related to the zero interest rate constraint. We estimate the models simultaneously using monthly time-series data including the estimated monthly series of GDP. We find that both the perceived equilibrium rates have been time-variant since the end of 1980s, and that the macro-finance model gives us a rational explanation of low interest rates although there are some caveats in interpreting the results. We also carry out a decomposition of the interest rates into various components, and analyze the causes of model errors.

Key words: Macro-finance model; Monetary policy; Term structure of interest rates; Risk premium; Equilibrium real interest rate; Zero interest rate

JEL classification: E43, E52, G12

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1. Introduction

We study the term structure of interest rates and monetary policy in Japan empirically, using a macro-finance model, which combines a macroeconomic model and a term structure model. The motivation of this study stems from the fact that Japan's long-term interest rates have been extremely low in recent years. At the end of 2006, the 10-year risk-free rate in Japan is about 1.6–1.7 percent, which can not be explained away based only on the long-run historical performance of the economy. We investigate whether or not such a low rate can be explained with economic rationality by taking into account some key features of the economy: possible time-variability of perceived equilibrium rates of real interest and inflation, the effect of the zero lower bound of nominal interest rates, and the effect of the Bank of Japan's (BOJ's) zero interest rate commitment since 1999[1]. We are also interested in the estimation of the macroeconomic structure based not only on macroeconomic data but also on market interest rate information.

A macro model and a term structure model had been analyzed separately from each in most studies before. In recent years, however, studies that combine these two models to gain richer information is on the rise, especially in the United States and Europe. This type of macro-finance analysis, as is indicated in the conceptual diagram of Figure 1, utilizes both macroeconomic data and market interest rate data for a simultaneous estimation of the two models. This approach makes it possible to enhance the precision of estimation compared to utilizing both models independently. However, as far as the authors are aware, there have been few studies where Japanese data are applied in earnest to such an analysis. As will be explained below, the main reason for this is likely to be the restrictions on Japanese data. In this paper, the restrictions are overcome with the use of monthly series of data. In particular, we generate Japan's monthly GDP from publicly available quarterly GDP by utilizing other monthly macroeconomic indicators. Using these data, we estimate a New Keynesian-type macro model and an affine diffusion-type term structure model simultaneously. This type of macro-finance analysis

[1] The BOJ set out a policy commitment during the period with zero interest rate policy (ZIRP, from April 1999 through August 2000) and during the period with quantitative monetary easing policy (QMEP, from March 2001 through March 2006), respectively. Although they differed from each other in some points, the two commitments were common in that they stated that the ZIRP or the QMEP would be continued until specific conditions for price development had been satisfied. In this paper, because we focus our attention on the effect of the policy rate being zero, both the commitments are collectively referred to as the "zero interest rate commitment", or simply as the "commitment".

allows for the decomposition of long-term interest rates into expectations components, which correspond to the pure expectations hypothesis, and risk premium components. It also allows for decomposition of the expectations components into more detailed components. Meanwhile, we also deal with the issue of how to analyze an economy under the zero interest rate environments with a linear model, as the policy rate in Japan had been zero since 1999. Regarding this problem, we perform estimations, using Monte-Carlo simulations, to approximately take into account the non-linear effects both from the zero lower bound and from the BOJ's zero interest rate commitment.

We will briefly review previous studies on macro-finance models in the United States and Europe, and point out the specific features of our analysis. A seminal paper on this subject is that by Ang and Piazzesi (2003). They employed a vector autoregression (VAR) model as a macro model and an affine diffusion model as a term structure model[2]. Ever since, a variety of studies have been reported[3]. For example, it has been found that macro-finance analyses are effective in studying how the equilibrium inflation rate and other macroeconomic factors affect the end-point of the term structure (Kozicki and Tinsley (2001), Dewachter and Lyrio (2006)).

Regarding studies with similar motivation or methodology to this paper, we cite Ang, Dong, and Piazzesi (2005), Bekaert, Cho, and Moreno (2003), Rudebusch and Wu (2005), Rudebusch, Swanson, and Wu (2006), and Gurkaynak, Sack, and Swanson (2005). Specifically, use of the New Keynesian-type small-scale structural model, which is effective in examining monetary policy implications, rather than VAR as a macro model by Bekaert, Cho, and Moreno (2003) is common with this paper. Using a Taylor-type policy rule and adopting a model that tolerates the existence of model errors in all of the analyzed interest rates by Ang, Dong, and Piazzesi (2005) is also shared. The framework of model representation and estimation by Rudebusch and Wu (2005) is common with this paper while their use of latent variables is not shared. With respect to the determinant of the perceived equilibrium inflation rate, assuming a mechanism of its

[2] Most previous studies on macro-finance models utilize the affine diffusion model as a term structure model (refer to Section 2.3 for definition of the affine diffusion model). Exceptionally, Piazzesi (2005) utilizes the affine jump-diffusion model, which combines Poisson process with diffusion process, as a stochastic process for the policy rate.

[3] In addition to research in academia, economists at various central banks also have applied macro-finance models in the analysis of monetary policy and term structure. For example, Bernanke, Reinhart, and Sack (2004), and Kim and Wright (2005) of the Federal Reserve Bank, Hördahl, Tristani, and Vestin (2006) of the European Central Bank, Lildholdt, Panigirtzoglou, and Peacock (2007) of the Bank of England, Oda and Ueda (2007), and Ichiue and Ueno (2006) of the Bank of Japan.

dependence on realized inflation rate by Bekaert, Cho, and Moreno (2003) and Gurkaynak, Sack, and Swanson (2005) is also common.

In addition, this paper has two distinctive features: 1) application of a macro-finance model to the Japanese economy with monthly series of GDP data estimated, as mentioned before, and 2) incorporation of the potential for time-variant behavior not only for the equilibrium inflation rate but also for the equilibrium real interest rate and for the underlying potential growth rate. In particular, 2) is a feature that has not been taken up at all in previous studies. This is partly due to the fact that since economies in the United States and Europe in recent years have developed in the vicinity of a linear growth trend, the necessity of assuming non-linearity, or time-variant equilibrium real interest rate, was low. With respect to the Japanese economy, however, the necessity is high because the formation and subsequent collapse of the asset price bubble since late 1980s could cause a significant change in the equilibrium real interest rate.

This paper is organized as follows. Section 2 presents the model in detail. Section 3 explains the methodology and the result of model estimations. Section 4 investigates the development of the equilibrium rates of real interest and inflation based on the estimation results, and then analyzes the development of medium- to long-term interest rates by decomposing them into various components. The effects of the zero interest rate commitment and the impulse responses of medium- to long-term interest rates are also examined. Section 5 conducts regression analysis on model errors to investigate the possible determinant factors of the interest rates that could not be captured by our model. Finally, Section 6 concludes with a summary, caveats to our results, and remaining issues.

2. Model

This section presents the macro-finance model utilized in the following analysis. Figure 2 provides an overview of the model structure. In part of the macro structural model, the dynamics of real GDP, inflation rate and short-term interest rate are described (see Section 2.1 for details). In part of the finance model, or the term structure model, the theoretical values of medium- to long-term interest rates are derived (see Section 2.3 for details). Both parts are mutually related through the short-term interest rate because the short-term rate is determined by monetary policy based on the macroeconomic situation, and the expected series of the short-term rates are important components in shaping medium- to long-term rates. The shadowed variables in Figure 2 are observables to be used as input information for estimations.

The equilibrium level of nominal interest rates is considered theoretically as the expected value of nominal short-term interest rate for the distant future. If decomposed into nominal and real components, it then becomes the equilibrium inflation rate and equilibrium real interest rate. Either of these values cannot be observed directly. We assume a simple learning mechanism that economic agents perceive these equilibrium values based on relevant information (see Section 2.2 for details).

Details for each part of the model are given below.

2.1 Macro Structural Model

Regarding the macroeconomic structure, we adopt a small-scale dynamic stochastic general equilibrium model of the New Keynesian-type, which has been frequently used in monetary policy analysis since the studies by Rotemberg and Woodford (1997) and Woodford (2003), with several modifications. Specifically, IS, AS and MP curves are log-linearly approximated in the vicinity of the steady state and formulated as indicated below.

- Aggregate demand function (hybrid IS curve)

$$x_t = \mu E_t x_{t+1} + (1-\mu) x_{t-1} - \hat{\sigma}(i_t - E_t \pi_{t+1} - \tilde{r}_t^{\,n}) + \varepsilon_t^{IS} . \tag{1}$$

- Aggregate supply function (hybrid AS curve)

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \kappa x_t + \varepsilon_t^{AS} . \tag{2}$$

- Monetary policy rule (Taylor-type MP rule with inertia)

$$i_t = \gamma i_{t-1} + (1-\gamma)[\tilde{r}_t^{\,n} + \tilde{\pi}_t^* + \phi_\pi(\pi_t - \tilde{\pi}_t^*) + \phi_x x_t] + \varepsilon_t^{MP} . \tag{3}$$

In the equations, x_t, π_t, and i_t denote the GDP gap, inflation rate, and short-term nominal interest rate at each month (t), respectively. $\tilde{r}_t^{\,n}$ denotes the equilibrium real interest rate, which is commonly recognized at t both by the private sector and by the central bank as the long-term equilibrium level. $\tilde{\pi}_t^*$ denotes the equilibrium inflation rate recognized at t by the private sector. ε_t^{IS}, ε_t^{AS}, ε_t^{MP} denote demand shock, supply shock, and policy shock, respectively, with each being subject to white noise $N(0,\sigma^{IS})$, $N(0,\sigma^{AS})$, and $N(0,\sigma^{MP})$. Note that all macroeconomic variables are represented as raw values and not as percentage deviations from the steady-state values.

The aggregate demand function, equation (1), is called a hybrid IS curve because it has both forward- and backward-looking properties. Specifically, the part of ratio ϕ ($0 \le \phi \le 1$) for total consumption at the current period is determined based on the relationship of intertemporal substitution with rationally expected consumption at the next period. The remaining part of ratio $1-\phi$ of total consumption is determined as the same consumption as that at the previous period, which is based on Fuhrer's (2000) analysis on the inertial behavior of "rule of thumb" consumers. Amato and Laubach (2003) reported that under this setting the aggregate demand function could be stated as equation (1), of which the structural parameters are defined as:

$$\mu \equiv \frac{1}{2-\phi}, \quad \hat{\sigma} \equiv \frac{\phi}{2-\phi} \cdot \sigma, \tag{4}$$

where σ is elasticity of intertemporal substitution for consumption. Equation (4) leads to the relationship:

$$\hat{\sigma} = (2\mu - 1)\sigma. \tag{5}$$

Consequently, given a value of σ, the greater the "rule of thumb" consumers (that is, the smaller ϕ and the smaller μ), the smaller the responsiveness ($\hat{\sigma}$) of aggregate demand to changes in the real interest rate through monetary policy. Based on equation (5), the aggregate demand function equation (1) can be restated as:

$$x_t = \mu E_t x_{t+1} + (1-\mu)x_{t-1} - (2\mu - 1)\sigma \cdot (i_t - E_t \pi_{t+1} - \tilde{r}_t^{\,n}) + \varepsilon_t^{IS}. \tag{1'}$$

The aggregate supply function, equation (2), is also a hybrid AS curve with both forward- and backward-looking properties, derived by modifying the New Keynesian Phillips curve with Calvo-type nominal rigidities under imperfect competition. The monetary policy rule (MP) in equation (3) is a Taylor-type rule with interest rate inertia[4].

2.2 Learning of the Equilibrium Rates of Inflation and Real Interest

The equilibrium rates of inflation and real interest appeared in equations (1), (2), and (3) cannot be observed directly. We assume the following simple learning mechanism for these variables.

First, we consider the equilibrium inflation rate. As a long-term objective of monetary policy, the central bank sets the equilibrium inflation rate π_t^*, implicitly or explicitly, which is not always constant. It is assumed to be dependent on the deviation between the inflation rate at the previous period and the target value, as in equation (6).

[4] In the standard Taylor rule (Taylor, 1993), the policy rate responds to the deviation of the inflation rate π_t from the equilibrium inflation rate π_t^* that is targeted by the central bank. Instead, in equation (3), the equilibrium inflation rate $\tilde{\pi}_t^*$ that is perceived by the private sector is substituted for π_t^* for convenience of analysis. The possible difference between $\tilde{\pi}_t^*$ and π_t^* is absorbed in ε_t^{MP}.

The same assumption has been employed by many studies including Bekaert, Cho, and Moreno (2003), and Gurkaynak, Sack, and Swanson (2005)[5].

$$\pi_t^* = \pi_{t-1}^* + \theta(\pi_{t-1} - \pi_{t-1}^*) + \varepsilon_t^{\pi^*}, \qquad (6)$$

where θ is the parameter representing the intensity of dependence on realized inflation rate, and $\varepsilon_t^{\pi^*}$ is the shock to the equilibrium inflation rate determined by the central bank.

Our model allows for the asymmetry of information on the equilibrium inflation rate between the central bank and private sector[6]. More specifically, it is assumed that, although the private sector knows the mechanism described by equations (3) and (6), it cannot observe the shock $\varepsilon_t^{\pi^*}$ directly. For this reason, it attempts to indirectly infer $\varepsilon_t^{\pi^*}$ from the development of short-term interest rate i_t which depends on monetary policy. We denote the short-term rate at the next period (*t*) that the private sector anticipates based on information at *t*-1 as $\tilde{i}_t$. The private sector considers that a certain part of ratio ξ of the difference between i_t, the realized rate at *t*, and $\tilde{i}_t$ is attributed to $\varepsilon_t^{\pi^*}$, and it updates its perception ($\tilde{\pi}_t^*$) for π_t^* by reflecting this consideration in equation (6). This type of inference corresponds with expectations via the Kalman filter under specific premises. In this sense, it can be thought of as a learning mechanism with economic rationality[7]. Specifically, this learning can be stated as:

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5 Gurkaynak, Sack, and Swanson (2005) justifies this assumption by indicating a) the possibility that there are some kinds of costs to completely return the inflation rate to π_t^* after the rate deviation from π_t^* due to economic shocks, and b) the possibility that, since perception ($\tilde{\pi}_t^*$) of the private sector on equilibrium inflation rate may gradually respond to the actual inflation rate, the central bank considers that the cost of doing nothing is less than the cost to correct the perception against such fluctuations, as long as the variation is not large.

6 Some central banks have been attempting to share information on the equilibrium inflation rate with the private sector by adopting inflation targeting. However, to share the information completely, it is not enough to simply announce the inflation target. For example, the equilibrium inflation rate needs to be pin-pointed, not indicated as a range, and credibility of the central bank needs to be established based on the record of monetary policy performance. In this sense, it seems appropriate for many countries to conclude the existence of asymmetric information between the central bank and private sector as assumed in this paper.

7 Conditions for simplifying the expectations formation via the Kalman filter into the process of equation (7) include a constant variance of shocks and linearity of the structural model. While the limited-sample estimation in Section 3.4 satisfies the latter condition, the full-sample estimation in Section 3.5 do not satisfy it as a result of taking into account non-linearity due to the zero lower bound. When we do not consider equation (7) and (9) as a rational learning process via the Kalman filter in such cases, we should interpret them as a

$$\tilde{\pi}_t^* = \tilde{\pi}_{t-1}^* + \theta(\pi_{t-1} - \tilde{\pi}_{t-1}^*) - \xi(i_t - \tilde{i}_t), \tag{7}$$

where $\tilde{i}_t$ is the anticipated rate based on information of the equilibrium inflation rate ($\tilde{\pi}_{t-1}^*$) up to period t-1. It can be written as:

$$\tilde{i}_t = \gamma i_{t-1} + (1-\gamma)[\tilde{r}_t^{\,n} + \tilde{\pi}_{t-1}^* + \phi_\pi(\pi_t - \tilde{\pi}_{t-1}^*) + \phi_x x_t]. \tag{8}$$

Substituting equation (8) for equation (7) produces equation (9) appearing below.

$$\tilde{\pi}_t^* = \tilde{\pi}_{t-1}^* + \theta(\pi_{t-1} - \tilde{\pi}_{t-1}^*) - \xi[i_t - \gamma i_{t-1} - (1-\gamma)\{\tilde{r}_t^{\,n} + \tilde{\pi}_{t-1}^* + \phi_\pi(\pi_t - \tilde{\pi}_{t-1}^*) + \phi_x x_t\}]. \tag{9}$$

The private sector's perception for the equilibrium inflation rate is updated every period according to equation (9).

Next, we deal with the update of perceptions for the equilibrium real interest rate. Perceived values based on the information up to period t are denoted as $\tilde{g}_t^n$ and $\tilde{r}_t^{\,n}$, respectively, for the potential growth rate (or the productivity trend) and the equilibrium real interest rate, both of which are in the steady state. These are defined not as variables reflecting short-term shocks but as perceived variables corresponding to long-term equilibrium. They behave as a time-variant because there may occur unexpected and persistent productivity shocks. It is assumed that perception for $\tilde{g}_t^n$ and $\tilde{r}_t^{\,n}$ is shared between the private sector and the central bank.

Given equation (1) for the aggregate demand the relationship between $\tilde{r}_t^{\,n}$ and $\tilde{g}_t^n$ exists as[8]:

$$\tilde{r}_t^{\,n} = \sigma^{-1}\tilde{g}_t^n + \rho, \tag{10}$$

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more simplistic learning process.

[8] Equation (10) is derived as follows. When the equilibrium real interest rate, or the natural interest rate, reflecting short-term demand shocks is denoted as r_t^n, reviewing the process whereby equation (1) is derived from the Euler equation produces the following.

$$\begin{aligned} r_t^n &= \hat{\sigma}^{-1}[\mu E_t y_{t+1}^n + (1-\mu)y_{t-1}^n - y_t^n] + \rho \\ &= \hat{\sigma}^{-1}[\mu(E_t y_{t+1}^n - y_t^n) - (1-\mu)(y_t^n - y_{t-1}^n)] + \rho \\ &= \hat{\sigma}^{-1}[\mu E_t \Delta y_{t+1}^n - (1-\mu)\Delta y_t^n] + \rho. \end{aligned}$$

In order to derive the trend $\tilde{r}_t^{\,n}$ for equilibrium real interest rate r_t^n at t, time-variant potential growth rate $E_t \Delta y_{t+1}^n$ and Δy_t^n should be substituted with trend $\tilde{g}_t^n$ for potential growth rate (which is the perceived trend updated with information through t). Thus,

$$\begin{aligned} \tilde{r}_t^{\,n} &= \hat{\sigma}^{-1}[\mu\tilde{g}_t^n - (1-\mu)\tilde{g}_t^n] + \rho \\ &= \hat{\sigma}^{-1}(2\mu - 1)\tilde{g}_t^n + \rho. \end{aligned}$$

Then, by substituting equation (5) into this equation, equation (10) is derived.

where ρ is a constant representing the subjective discount rate, or the time preference rate. Change in the potential GDP, of which the logarithmic value is denoted as y_t^n, is formulated in conjunction with the potential growth rate shock ε_t^{PG} around the productivity trend $\tilde{g}_t^n$. It follows that

$$\begin{aligned} \Delta y_t^n &\equiv y_t^n - y_{t-1}^n \\ &= \tilde{g}_t^n + \varepsilon_t^{PG}. \end{aligned} \tag{11}$$

Here $\tilde{g}_t^n$ is interpreted as the expected value at t for the future productivity trend. Potential growth shocks ε_t^{PG} follow white noise $N(0,\sigma^{PG})$, interpreted as including demand and supply shocks under flexible prices.

Since productivity trend $\tilde{g}_t^n$ is a potential growth rate in the steady state, it is a "constant" in the sense that it is not affected by business cycles. However, if persistent productivity shocks (ε_t^{PD}) occur, $\tilde{g}_t^n$ changes as shown in equation (12):

$$\tilde{g}_t^n = \tilde{g}_{t-1}^n + \varepsilon_t^{PD}. \tag{12}$$

The dynamics of potential growth rate Δy_t^n can be produced as shown below by taking the difference of equation (11) and rewriting it using equation (12):

$$\begin{aligned} \Delta y_t^n &= \Delta y_{t-1}^n + (\tilde{g}_t^n - \tilde{g}_{t-1}^n) + \varepsilon_t^{PG} - \varepsilon_{t-1}^{PG} \\ &= \Delta y_{t-1}^n + \varepsilon_t^{PD} + \varepsilon_t^{PG} - \varepsilon_{t-1}^{PG} \\ &= \Delta y_{t-1}^n + \varepsilon_t^{\Delta PG} \quad , \end{aligned} \tag{13}$$

where $\varepsilon_t^{\Delta PG} \equiv \varepsilon_t^{PD} + \varepsilon_t^{PG} - \varepsilon_{t-1}^{PG}$, and $\varepsilon_t^{\Delta PG}$ follows $N(0,\sigma^{\Delta PG})$.

Although ε_t^{PD} in equation (12) cannot be observed directly, it is possible to infer it based on equation (11). Specifically, by focusing on the difference between the potential growth rate Δy_t^n realized at t and its expected value $E_{t-1}\Delta y_t^n$ in the previous period, we obtain:

$$\begin{aligned} \Delta y_t^n - E_{t-1}\Delta y_t^n &= \Delta y_t^n - E_{t-1}\tilde{g}_t^n \\ &= \Delta y_t^n - \tilde{g}_{t-1}^n \\ &= \tilde{g}_t^n + \varepsilon_t^{PG} - \tilde{g}_{t-1}^n \\ &= \varepsilon_t^{PG} + \varepsilon_t^{PD} \end{aligned} \tag{14}$$

from equations (11) and (12). Equation (14) indicates that the deviation ($\Delta y_t^n - \tilde{g}_{t-1}^n$) from expectations is the sum of the two types of shocks. Although an exact decomposition of these shocks is not possible, similar to the aforementioned approach for the equilibrium inflation rate, it can be inferred that a certain part of ratio ν of $\Delta y_t^n - \tilde{g}_{t-1}^n$ is attributed to ε_t^{PD}. Then, equation (12) becomes:

$$\begin{aligned}\tilde{g}_t^n &= \tilde{g}_{t-1}^n + \nu(\Delta y_t^n - \tilde{g}_{t-1}^n) \\ &= (1-\nu)\tilde{g}_{t-1}^n + \nu \Delta y_t^n \ .\end{aligned} \tag{15}$$

Rewriting this using equation (10) produces:

$$\tilde{r}_t^n = (1-\nu)\tilde{r}_{t-1}^n + \nu(\sigma^{-1}\Delta y_t^n + \rho) \ . \tag{16}$$

Perception for the equilibrium real interest rate, which means long-term equilibrium level, is updated every period according to equation (16).

In our macro-finance model, this part of the macro structural model is composed of equations (1'), (2), (3), (9), (13), and (16). These are described with state space representation and estimated simultaneously with the finance part. Refer to Section 3 and Appendix 1 for details.

2.3 Term Structure Model of Interest Rates

The finance model in this paper is a term structure model to determine the theoretical values of medium- to long-term interest rates, represented as the compound yields of risk-free bonds. Generally, this kind of model describes a stochastic process of the short-term interest rate, and imposing no arbitrage condition on the model leads to theoretical interest rates with a risk premium taken into account. Many of the models proposed in previous studies have a property that allows the theoretical interest rates to be represented as a linear function of the state variables. For this reason such models are referred to as affine models or affine diffusion models[9]. Even within affine models, there are variations in specifications of the risk price. This paper employs specifications, which have been frequently used in previous macro-finance studies, as indicated below. That is, the risk price vector Λ_t at t is represented as:

$$\Lambda_t = \lambda_0 + \lambda_1 F_t \ , \tag{17}$$

which is a linear function of state vector F_t[10]. Since our model includes four risk factors (ε_t^{IS}, ε_t^{AS}, ε_t^{MP}, and $\varepsilon_t^{\Delta PG}$), Λ_t is a four-element vector where each factor corresponds to each risk price. λ_0 is a 4 × 1 vector and λ_1 is a 4 × 6 matrix. The state vector F_t is defined as:

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9 In term structure models, a diffusion process including a Brownian motion is often employed as the stochastic process representing the development of interest rates. Affine models of this type are known as affine diffusion process.

10 The specification of risk price in equation (17) has been used in numerous macro-finance analyses since addressed by Duffee (2002).

$$F_t = (x_t ,\ \pi_t , i_t,\ \tilde{r}_t^{\,n} ,\ \tilde{\pi}_t^* ,\ \Delta y_t^n)'. \tag{18}$$

We can state a dynamic equation for the state vector based on the aforementioned dynamics for each element of F_t. It is provided below (see Appendix 1 for details).

$$F_t = C^F + \psi F_{t-1} + \Sigma\varepsilon_t , \tag{19}$$

which corresponds to equation (A5) of Appendix 1. State vector F_t at the current period is expressed as the sum of the linear function of state vector F_{t-1} at the previous period and the linear function of economic shock vector ε_t. Refer to Appendix 1 for coefficient vector C^F and coefficient matrices ψ and Σ.

In this type of affine model, the theoretical value of medium- to long-term interest rates is also expressed as the linear function of the state vector as shown below[11]. The observed value of the interest rates at t maturing in j months is stated as the addition of model error ε_t^j to that theoretical value[12]:

$$i_{j,t} = \frac{A_j}{j} + \frac{B_j'}{j} \cdot F_t + \varepsilon_t^j , \tag{20}$$

where A_j and B_j are constants and constant vectors dependent on risk-related parameters λ_0 and λ_1 as well as on structural parameters appearing in equation (19). Model error ε_t^j is the deviation between theoretical rate and market rate due to incompleteness of the model. ε_t^j is assumed to follow the AR (1) process, as shown in equation (21)[13].

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[11] Refer to Constantinides (1992), Dai and Singleton (2000), Duffee (2002), Duffie and Kan (1996) for the derivation of equations (20), (21) and (22), and for the general properties of affine models.

[12] It is known that medium- to long-term interest rates can be expressed as shown in equation (20) when the short-term rate follows an affine model. However, it is necessary to check whether the short-term rate in our model follows an affine model. For this proof, generally, all the state variables need to follow affine processes. In other words, with regard to the variation of state variables, it is necessary that 1) the drift is a linear function of state variables, and 2) all the elements of the variance-covariance matrix are also linear functions. In confirmation of these points, transforming equation (19) into variation form produces:

$$\begin{aligned}\Delta F_t &\equiv F_{t+1} - F_t \\ &= [C^F + (\psi - 1)F_t] + \Sigma\varepsilon_{t+1} .\end{aligned}$$

Thus, it is found that both conditions 1) and 2) above are satisfied.

[13] In previous macro-finance studies using United States data, there are many cases that assume the model errors to be white noise. See Bekaert, Cho, and Moreno (2003) and Rudebusch and Wu (2005), for example. In this paper, we initially analyzed under the white-noise assumption. However, we observed significant inertia in model errors in that case, and thus

$$\varepsilon_t^j = \alpha_j \varepsilon_{t-1}^j + \mu_t^j , \tag{21}$$

where α_j is an autoregressive coefficient and μ_t^j is white noise that follows a normal distribution $N(0,\sigma^j)$. Factors causing model error ε_t^j are examined in Section 5.

A_j and B_j in equation (20) are known to be derived from the recursive equations that follow.

$$A_j - A_{j-1} = B'_{j-1}(C^F - \Sigma \cdot \lambda_0) + \frac{1}{2} B'_{j-1} \Sigma \Sigma' B_{j-1} + A_1 , \quad (j \ge 2) \tag{22}$$

$$B'_j = B'_{j-1}(\psi - \Sigma \cdot \lambda_1) + B'_1 . \quad (j \ge 2) \tag{23}$$

Since $i_t \equiv i_{1,t} = A_1 + B'_1 \cdot F_t$, when $j = 1$, then $A_1 = 0$ and $B'_1 = (0,0,1,0,0,0)$.

Equation (20) shows that, under the affine model in this paper, the theoretical value of medium- to long-term interest rates is determined by perceived equilibrium rates ($\tilde{r}_t^n$, $\tilde{\pi}_t^*$) of real interest and inflation, economic fundamentals (x_t, π_t, i_t, Δy_t^n), and risk prices (λ_0, λ_1).

In some analyses following Section 3, the theoretical value of medium- to long-term interest rates is decomposed into the expectations component and the risk premium component. The expectations component is defined as the hypothetical interest rates in a risk neutral world where no risk premium exists. It is obtained by substituting A_j and B_j, derived from equation (22) and (23) with risk price Λ_t set to be zero, into equation (20). The risk premium component is defined by subtracting the expectations component from the whole theoretical interest rate.

3. Estimations

3.1 Overview of Estimations

The whole model is constructed from equations (1'), (2), (3), (9), (13), (16), (20) and (21). We estimate these equations simultaneously with the maximum likelihood method using monthly data. The estimation period starts from October 1989 because only since then is sufficiently reliable market data for medium- to long-term interest rates available in Japan. The reason for employing the maximum likelihood method, instead of the generalized method of moments (GMM), is that the coefficients A_j and B_j in equation (20) cannot be explicitly expressed. In other words, the recursive property in equations (22) and (23) makes the application of GMM difficult. To derive

concluded that an autoregressive process should be assumed for model errors.

the likelihood function, state space representation is convenient. Refer to Appendix 1 for the details on this point.

The specific procedures of estimation are divided into three steps.

In the first step, three of the 43 parameters in the model are calibrated (see Section 3.3 for details). Specifically, interest rate elasticity of the GDP gap in the aggregate demand curve ($\hat{\sigma}$ in equation (1), or $(2\mu-1)\sigma$ in equation (1')), the slope of the Phillips curve (κ in equation (2)), and the lag coefficient of the short-term interest rate (γ in equation (3)) are not estimated within our macro-finance framework. Rather, these three parameters are locked based on the results of a separate estimation by GMM with a longer estimation period for macro components only. This is an effort to avoid the problem that an inappropriate result can be obtained from the estimation whereby the sample period consists primarily of the asset price bubble collapse period and the period with long-lasting deflation in the 1990s in Japan[14].

In the second step, all other parameters are estimated by the maximum likelihood method with the estimation period limited from October 1989 through December 1998. Because our model is basically linear, it is not easy to deal with non-linear properties such as the zero rate constraint. Since this estimation period predates the realization of zero interest rates and introduction of the zero interest rate commitment, the linear model can be applied appropriately. This is referred to as "limited-sample estimation" hereafter.

In the third step, the estimation period is extended to May 2006, the most recent period at the time of writing, and the model is re-estimated. During this step, the non-linear effects of the zero lower bound and the commitment are approximately taken into account. This is referred to as "full-sample estimation" hereafter.

3.2 Data

The development of the secondary market for medium- to long-term government bonds was delayed in Japan compared to key markets in the United States and Europe.

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[14] We also performed the estimation with these three parameters included. The result indicated that $\hat{\sigma}$ and κ were nearly zero while γ was nearly one. It is considered that this result reflects, along with the non-performing loan problem and the zero interest rate constraint, the long-lasted recessionary and deflationary environment of the 1990s in Japan. If it is assumed that such a special macroeconomic environment will continue in the future, simulations of future economic paths to calculate the theoretical rates tend to generate instable paths such as those leading to a deflationary spiral. Thus, we consider it more appropriate to calibrate the three parameters, based on the result of the longer-period estimation, than to estimate them with data of the special environment in the 1990s.

For this reason, with regard to the term structure of risk-free interest rates, reliable market data can only be obtained following the end of the 1980s. With such a restriction, if we use quarterly data, we lack the information required to estimate the 40 parameters in our model. Therefore, we decided to perform estimations using monthly data.

For this purpose, it is necessary to convert publicly available GDP data on a quarterly base into monthly data. We employ the Bernanke, Gertler, and Watson (1999) method to estimate the monthly data. For each demand component of the GDP, the estimation is performed to decompose the quarterly data into three monthly data by referring to other monthly economic statistics that have a strong relationship to the component. Refer to Appendix 2 for details. Figure 4 (1) shows the monthly series of real GDP data that has been generated with this method.

It is common in theories to define the GDP gap as a percentage deviation of the real GDP from the potential GDP which is the hypothetical real GDP under flexible price. However, since it is difficult to observe a potential GDP of this definition, we alternatively utilize the potential GDP estimated with the production function approach[15], the details of which are explained by Ito *et al.* (2006). Specifically, we calculate the monthly potential GDP by applying the "smooth decomposition method" shown in Appendix 2 (2) to the quarterly potential GDP estimated with the production function approach. The percentage deviation of real GDP, of which the logarithmic value is y_t, from the potential GDP, of which the logarithmic value is y_t^n, is defined as the monthly GDP gap x_t.[16] That is, $x_t \equiv y_t - y_t^n$.

For inflation rate π_t, the year-on-year percentage change in the CPI (excluding perishables, adjusted for consumption tax effects) is utilized, which is shown in Figure 4(2)[17].

With regard to data for market interest rates, we use the average bidding yield of 6-month TB (government discount bond) for the 6-month rate and the yields of long-term government bonds in generic issue for the 3-year, 5-year, and 10-year rate, all

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[15] There are different methods other than the production function approach to estimate the potential GDP. For example, a method is frequently employed whereby the GDP series smoothed with HP-filter is interpreted as the potential GDP series. In the process of our analysis, model estimations and analyses using potential GDP calculated with HP filter were attempted as well. As a result, from a qualitative perspective, a large discrepancy was not observed compared to the production function approach.

[16] In the estimation, we input a value of the GDP gap divided by 12 into the variable x_t in equations, although we do not denote it as $x_t/12$, for ease of understanding of the model and for comparing the estimate with other previous studies.

[17] The base year of the consumer price index utilized in this paper is 2000.

of which are the average values of daily closing rates within a month. The uncollateralized call rate (overnight) is utilized for the short-term rate in equation (3)[18]. Figure 3 shows the development of these interest rates.

3.3 Calibration

As discussed in Section 3.1, for the purpose of calibrating the three parameters ($\hat{\sigma}$, κ, γ), GMM estimation was conducted using quarterly data as presented below.

In order to understand the long-term picture of the economy, instead of that under the special environment in the 1990s, the estimation period runs from 1983Q1 to 1998Q4. The estimation equations are shown in Table 1 along with the estimation results. The estimation equations are composed of the IS curve, AS curve, and policy rule. The basic structure is similar to that of the macro structural part of our macro-finance model. The differences to the macro-finance model is 1) the equilibrium real interest rate is approximated to be a constant rate corresponding to the trend of potential growth rate, 2) the equilibrium inflation rate is fixed at the average inflation rate during the estimation period, and 3) in order to take into account the effect on the economy of asset price bubble formation and collapse, a term dependent upon the stock price index, which we consider as a proxy for various asset prices, is introduced into the IS curve after 1987. According to the estimation results, all the parameters are statistically significant with a correct sign. The results are mostly consistent with those of previous studies on the Japanese economy as well.

For the three parameters to be calibrated, the estimates are $\hat{\sigma}$ = 2・0.5027 - 1 = 0.00538, κ = 0.00606, and γ = 0.767 (Table 1). Since these estimates are the parameters for the quarterly model, it is necessary to convert γ into a parameter for the monthly model for our analysis[19]. The parameters for the monthly model are $\hat{\sigma}$ = 0.00538, κ = 0.00605, and $\gamma = (0.767)^{1/3} = 0.915$.

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[18] Short-term interest rate i_t in our model is the 1-month rate on the side that the model is based on monthly data. It is also the policy rate (overnight call rate) on the other where the rate is the explained variable in the monetary policy rule. Since no significant difference is observed when comparing the developments of the 1-month rate and the overnight call rate in Japan, either could be utilized for estimations. We adopt the uncollateralized call rate, which has been used in many previous studies of monetary policy rule.

[19] In estimations, the input data for the GDP gap is divided by four for quarterly estimations and by 12 for monthly estimations. Therefore, with regard to $\hat{\sigma}$ and κ, results of quarterly estimations can be applied directly to the monthly model. On the other hand, since γ is a lag coefficient for the policy rate, conversion is required as described in the main text.

3.4 Limited-sample Estimation of Macro-finance Model

Next, we proceed to the estimation of the macro-finance model. In this section, "limited-sample estimation" is conducted where the estimation period is limited to the period from October 1989 through December 1998, just before introduction of the zero interest rate policy. For this estimation, we can use the standard maximum likelihood method because the model is linear. Specifically, we calculate the likelihood function, which is derived by describing the model in Section 2 with state space representation, and then determine the parameters which maximize the likelihood. Refer to Appendix 1 for details.

The settings for the estimation are described below.

Regarding the learning for the two equilibrium rates, it is necessary to provide the initial values which the market perceived as the equilibrium inflation rate and the potential growth rate trend in October 1989, the beginning of the estimation period. The initial value for the equilibrium inflation rate is set to be 1.2% per annum, which is the average of year-on-year percentage change in the CPI from April 1983 when the effects of the second oil shock dissipated through September 1989, the point just before the estimation period. That is, $\tilde{\pi}^*_{89/10} = 1.2\%$. The initial value for the trend of potential growth rate is set to be 4.1% per annum, which is the average of the real GDP growth rate from 1971Q1 through 1989Q3, the point just before the estimation period. That is, $\tilde{g}^n_{89/10} = 4.1\%$.

Of all the estimation parameters, there are as many as 20 parameters related to the risk price, meaning that our model has a great amount of flexibility in describing the risk price. The elements included in the risk price vector in equation (17), $\Lambda_t = \lambda_0 + \lambda_1 F_t$, are denoted below[20]:

$$\lambda_0 = \begin{pmatrix} \lambda_0^{IS} & \lambda_0^{AS} & \lambda_0^{MP} & \lambda_0^{\Delta PG} \end{pmatrix}', \tag{24}$$

$$\lambda_1 = \begin{pmatrix} \lambda_1^{IS} & \lambda_1^{AS,IS} & \lambda_1^{MP,IS} & 0 & 0 & \lambda_1^{\Delta PG,IS} \\ \lambda_1^{IS,AS} & \lambda_1^{AS} & \lambda_1^{MP,AS} & 0 & 0 & \lambda_1^{\Delta PG,AS} \\ \lambda_1^{IS,MP} & \lambda_1^{AS,MP} & \lambda_1^{MP} & 0 & 0 & \lambda_1^{\Delta PG,MP} \\ \lambda_1^{IS,\Delta PG}, & \lambda_1^{AS,\Delta PG} & \lambda_1^{MP,\Delta PG} & 0 & 0 & \lambda_1^{\Delta PG} \end{pmatrix}. \tag{25}$$

For limited-sample estimation in this section, non-diagonal elements in equation (25) are set *a priori* to be zero to reduce the computational burden.

[20] Since the equilibrium inflation rate and equilibrium real interest rate do not accompany any independent economic shock in their dynamics, relevant risk price factor do not exist for them. Therefore, the relevant elements in equation (25) are set to be zero.

$$\lambda_1 = \begin{pmatrix} \lambda_1^{IS} & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1^{AS} & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1^{MP} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1^{\Delta PG} \end{pmatrix}. \tag{25'}$$

Note that, when conducting full-sample estimations in Section 3.5, all the parameters in equation (25) are estimated without using diagonal approximation (also see Table 3).

When maximizing the likelihood function (A10) in Appendix 1, we utilize not only local optimization algorithms, such as the Newton-Raphson method, but also the global optimization approach through a grid search of the economically plausible area, because the likelihood function is of high order. In optimization, the following restrictions are imposed: 1) the expectations component and risk premium component for each interest rate (the 6-month, 3-year, 5-year, and 10-year rate) are not permitted to be negative during the full estimation period, 2) the risk premium component for each interest rate is not permitted to exceed the market rate[21], and 3) the subjective discount rate is not permitted to be negative ($\rho \geq 0$).

The estimation results are indicated in the "Limited-sample Estimation" column of Tables 2 and 3. Since the basic properties are not largely different from those of the full-sample estimation in Section 3.5, they are summarized later.

3.5 Full-sample Estimation Taking into Account the Effects of Zero Lower Bound of Nominal Interest Rates and Zero Interest Rate Commitment

In this section, we estimate the macro-finance model with an extended estimation period, i.e. from October 1989 through May 2006. Because this period includes the period when the policy rate was facing zero lower bound and the period when the BOJ's zero interest rate commitment was in effect[22], we should not simply estimate the linear

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[21] This restriction is imposed only at the end of the estimation period, December 1998, instead of during the full period.

[22] On March 19, 2001 the BOJ decided to introduce the so-called "quantitative monetary easing policy (QMEP)." This framework, which includes the BOJ employing the outstanding balance of its current account as an operating target for monetary market operations, provides ample liquidity, and commits to continue the QMEP until the year-on-year percentage change in the CPI (nationwide, excluding perishables) has been stable at above zero percent. In addition, it was also declared that the BOJ was ready to increase the purchase of long-term government bonds if necessary to provide ample liquidity. In this framework, the target was set higher than the required reserve level, and thus the uncollateralized call rate (overnight) was assumed to be stable at almost zero percent under normal conditions. Further, on October

model as in Section 3.4. Below, we present a method to approximately take into account such non-linear effects, and indicate the estimation results.

3.5.1 Treatment of the Effects of Zero Lower Bound of Nominal Interest Rates and Zero Interest Rate Commitment

Equation (20) in Section 2 indicates the theoretical values of medium- to long-term interest rates. However, the equation is correct only when we do not need to take into account the zero lower bound and the commitment. In Japan, since the policy rate faced the zero lower bound after February 1999 and the commitment was introduced by the BOJ thereafter, it is necessary to consider these effects in order to accurately calculate the theoretical interest rates for this period and estimate the model.

Generally, if the zero lower bound is not taken into account, a fan chart indicating the probability distribution for future interest rate paths can include paths with negative policy rates in some periods such as those in deep recessions. However, in the real world where the zero lower bound exists, the policy rate does not become negative but stops at zero in any path. The theoretical value of medium- to long-term interest rates is calculated as the expected value under risk neutral probability of the average of the future policy rate, or short-term interest rate. Thus, for the aforementioned reason, the theoretical value of medium- to long-term interest rates is higher when the zero lower bound is taken into account than that when the bound is not taken into account. As shown in Appendix 3 (1), the specific calculations can be performed with Monte-Carlo simulations.

10, 2003 the BOJ made an additional clarification for the commitment that the current policy would be continued until both the development of actual inflation rate and outlook of inflation rate were above zero percent. The QMEP was exited on March 9, 2006 when the conditions for the exit had been satisfied. Refer to the BOJ's homepage (http://www.boj.or.jp) for details.

Prior to introduction of the QMEP, the BOJ decided to encourage the uncollateralized call rate (overnight), which was then the operating target for money market operations, to be at about zero percent on February 12, 1999. This is the introduction of the so-called "zero interest rate policy (ZIRP)." Thereafter, at the BOJ governor's regular press conference in April 1999, it was declared that the ZIRP would be continued until deflationary concerns had been dispelled. In August 2000, the ZIRP was exited based on the judgment that the conditions had been satisfied.

It is possible to regard both the policy announcements by the BOJ in April 1999 and March 2001 as a commitment on the continuation of the zero interest rate under certain conditions, although there are some differences between the two announcements in terms of transparency of the contents. Thus, both of the commitments are collectively referred to as "zero interest rate commitment" in this paper.

On the other hand, the theoretical value of medium- to long-term interest rates is lower when the zero interest rate commitment exists than that when the commitment does not. This is due to the fact that the policy rate is maintained at zero, even if the policy rule in equation (3) is positive, when the commitment is binding on policy rate with the inflation rate below the specific threshold. For this effect as well, the specific calculations can be performed with simulations as shown in Appendix 3 (2).

To incorporate the commitment into our model, we add a property that it is necessary for the inflation rate at the previous month to exceed the specified threshold rate, $\bar{\pi}$**,** in order to make the current policy rate not zero but positive[23]. This content is not exactly the same as that committed by the BOJ. It has been simplified for ease of incorporation into the model within the scope of preserving the meaning of the content. The policy rate, i_t, in which both effects from the commitment and from the existence of the zero lower bound are taken into account, can be expressed as equation (26). The interest rate $\hat{i}_t$ in the equation is defined as the original policy rule (i.e., the left side of equation (3)) where these effects are not taken into account.

$$\begin{cases} i_t = 0 & \text{if } \hat{i}_t < 0 \text{ or } \pi_{t-1} < \bar{\pi}, \\ i_t = \hat{i}_t & \text{if } \hat{i}_t \geq 0 \text{ and } \pi_{t-1} \geq \bar{\pi}, \end{cases} \tag{26}$$

where $\hat{i}_t = \gamma i_{t-1} + (1-\gamma)[\tilde{r}_t^n + \tilde{\pi}_t^* + \phi_\pi(\pi_t - \tilde{\pi}_t^*) + \phi_x x_t] + \varepsilon_t^{MP}$. (3')

$\bar{\pi}$ denotes the inflation rate threshold that tolerates an exit of the commitment, which is set to be 0.0% for the period with zero interest rate policy (ZIRP; from April 1999 through July 2000) and 0.5% for the period with quantitative monetary easing policy (QMEP; from March 2001 through March 2006)[24]. Note that neither threshold was announced by the BOJ. With regard to the ZIRP period, we interpreted the BOJ's statement to continue the ZIRP "until deflationary concerns had been dispelled" as $\bar{\pi}$ = 0.0%. With regard to the QMEP period, based on the fact that the CPI inflation rate (nationwide, excluding perishables, base year 2000) in the previous month of exiting the QMEP was 0.5%, we assumed that the market had judged that $\bar{\pi}$ = 0.5% in advance.

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[23] The reason for using the inflation rate at the previous month in this context is that the CPI (nationwide) is released with an almost one month time lag in Japan.

[24] Oda and Ueda (2007), through formulation similar to equation (26), analyze policy rule that takes into account the zero lower bound and the commitment. In our paper the threshold inflation rate $\bar{\pi}$ is set *a priori* as mentioned above, whereas Oda and Ueda (2007) assume the value of $\bar{\pi}$ to be time-variant and derive its development from market interest rate information.

Appendix 3 presents a basic methodology to approximately take into account the effects of the zero lower bound and the commitment for both the expectations components and risk premium components when calculating the theoretical values of medium- to long-term interest rates with Monte-Carlo simulations. Based on our experimental simulations, however, we found that simulations for the risk premium components under risk neutral probability might generate unrealistic results with our model. This was due to a partial collapse of the theoretical assumptions for an affine diffusion model where the stochastic process should be a Brownian motion and the model should be linear. Therefore, we decided not to perform simulations under risk neutral probability in the estimations and analyses below. Thus, the effects of the zero lower bound and the commitment are not taken into account for the risk premium components, which is evaluated below based on equation (20) in Section 2[25]. On the other hand, the expectations components are calculated below with Monte-Carlo simulations under subjective probability, where the aforementioned problem does not arise. Consequently, both effects are taken into account for the expectations components. The theoretical values for medium- to long-term interest rates correspond to the sum of both the components calculated in this manner.

3.5.2 Methodology for Full-sample Estimation

As already mentioned, by utilizing the Monte-Carlo simulations presented in Appendix 3, it is possible to derive the theoretical values of medium- to long-term interest rates that approximately take in account the non-linear effects under the given macro-finance model. Consequently, given a set of model parameters, we can calculate the likelihood function, equation (A10), which evaluates not only the likelihood for the macro structural model but also the likelihood for the term structure model. Thus, it is possible to conduct the maximum likelihood estimation, in principle, by searching for the parameters that maximize equation (A10). However, it is difficult to incorporate Monte-Carlo simulations with large computational burdens as in this analysis when solving a global optimization problem with multiple parameters. Therefore, we make an approximation in estimating the model as presented below.

First, the result of the limited-sample estimation in Section 3.4 is assumed to be a

[25] According to Oda and Ueda's (2007) analysis on medium- to long-term interest rates in Japan, the zero interest rate commitment had a significant effect of lowering the expectations component but little effect on the risk premium component. Thus, it does not seem to cause substantial problems not to adopt the Monte-Carlo simulation approach for the risk premium components in this analysis.

good approximation for the full-sample estimation, and is used for extrapolation. Specifically, based on the limited-sample results, medium- to long-term theoretical interest rates after 1999 are calculated using the simulation method in Appendix 3. These calculations take into account the effects of the zero lower bound and the commitment. The differences are calculated from the theoretical rates, based on equation (20), that do not take these effects into account, and are referred to as "adjustment factors." By definition, adding the adjustment factors back to the calculations from equation (20) produces the theoretical rates taking into account both the effects. If the limited-sample estimation is really a good approximation for the full-sample estimation, there should not be any significant difference between the known adjustment factors based on the limited-sample basis and the unknown adjustment factors based on the full-sample basis. Next, we extend the estimation period up to the latest sample, and re-conduct the maximum likelihood estimation in Appendix 1 with the adjustment factors, which is locked as the limited-sample basis, added to equation (20). Other settings for the estimation fundamentally follow those in Section 3.4. Once we obtain the results for this full-sample estimation, we re-calculate the adjustment factors based on the new estimate. We actually found that the adjustment factors based on the full-sample basis virtually corresponded to those based on the limited-sample basis. Consequently, we judged that it was appropriate to apply the adjustment factors based on the limited-sample estimation as an assumption for the re-estimation with full samples[26].

3.5.3 Results of Full-sample Estimation

The estimation results are shown in Tables 2 and 3. Depending on which part of the whole model the parameters belong to, it is possible to classify them into three: 1) macro structural parameters, which are the parameters included in the aggregate demand function, aggregate supply function and monetary policy rule, and those relating to the size of macroeconomic shocks, 2) parameters of learning with regard to the perceived equilibrium rates, or 3) parameters of risk prices. Along with this classification, the

[26] Even if the adjustment factors based on the full-sample re-estimation significantly deviate from the adjustment factors based on the limited-sample estimation, it is possible to gain the final result by repeating the procedures of re-estimation. Specifically, the maximum likelihood estimation should be conducted again and again with the estimation period up to the most recent period using the revised adjustment factors based on the latest re-estimation results, and finally it should be confirmed that the adjustment factors based on the final results match the latest adjustment factors applied in the final estimation. This matching ensures the convergence of the recursive approach.

features of the results for full-sample estimation are described as below.

- For the learning part, the parameter indicating the speed of learning with regard to the equilibrium real interest rate was estimated as $\nu \cong 0.0237$, which was statistically significant and relatively large. On the other hand, the parameter indicating the speed of learning of the equilibrium inflation rate from development of the policy rate was estimated as $\xi \cong 0.00477$, which was small and not statistically significant. As for the parameter indicating the degree of dependence of the perceived equilibrium inflation rate on the realized inflation rate at the previous period, it was statistically significant but relatively small at $\theta \cong 0.00484$. Although there are no comparable previous studies for these results in Japan, there are in the United States. For example, Gurkaynak, Sack, and Swanson (2005) suggest parameters of $\theta \cong 0.02$ and $\xi \cong 0.1$ for the US economy with calibrations based on impulse response analysis. From these results, it can be understood that the perception of the equilibrium inflation rate has been relatively stable in Japan, compared to the United States, in that the change of the perceived equilibrium rate from learning was relatively small. The developments of the estimated equilibrium rates of real interest and inflation are shown in Figures 5 and 6, respectively. These will be investigated in Section 4.1.
- For the macro structural part, the parameters, μ and δ, indicating strength of inertia for the aggregate demand and aggregate supply functions, were estimated as approximately 0.5 for both. For the policy rule, the estimated reaction coefficient for the inflation gap was greater than one, which satisfies the Taylor's Principle. Moreover, all the macro structural parameters are statistically significant.
- For parameters relating to the equilibrium real interest rate, the subjective discount rate ρ, a constant term, was estimated as approximately 0.05% per annum (that is, a monthly rate of 0.0038%). This is extremely small, and is not statistically significant. On the other hand, the coefficient (σ^{-1}) of the potential growth rate trend for the equilibrium real interest rate was estimated as 0.46 (that is, $\sigma = 2.17$), which is comparatively small.
- With regard to risk price parameters, it is difficult to derive economic implications from each estimate. Rather, it can be interpreted that the set of estimated parameters provide appropriate risk premium components as a whole, within the model framework where the total risk price is represented by an affine function.
- Comparing the result of the full-sample estimation with that of the limited-sample estimation, there is no substantial difference for most main parameters excluding risk price parameters. It is likely that the estimations are refined along with

extension of the estimation period.

In addition, we calculate impulse response functions in order to check the basic properties of the macro-finance model based on the full-sample estimation. Figure 7 shows the responses of macroeconomic variables when each of the four economic shocks (demand shock, supply shock, policy shock, and productivity shock) occurs at a size of one standard deviation, respectively, with initial values set at steady state. These prove the occurrence of the typical transmission mechanism observed in the small-scale New Keynesian-type macro structural model for Japan. Moreover, the impulse responses of medium- to long-term interest rates for the same economic shocks were also calculated, which will be investigated in Section 4.4.

4. Investigating the Interest Rates Based on the Estimated Model

4.1 Estimated Equilibrium Rates of Real Interest and Inflation

In Sections 4 and 5, we investigate the medium- to long-term interest rates in Japan based on the result of the full-sample estimation of our macro-finance model.

First, we calculate the development of the perceived equilibrium rates of real interest and inflation, using the estimated equations (16) and (9), respectively. Figures 5 and 6 indicate the results[27].

Looking at the estimated equilibrium real interest rate in Figure 5, the rate was approximately 2% in the later half of 1989, the asset price bubble period, and then it decreased gradually. After the rate recorded the lowest at approximately 0.5% in 2003, the rate started to rise slightly, and was approximately 0.6% in May 2006. This development is considered to roughly correspond to the long-term development of Japan's economic growth while it does not reflect short-term business cycles. The difference between the highest and lowest of the rate during the estimation period is approximately 1.5 percent point, which partly explains the difference of roughly 7 percentage points between the highest and lowest of the 10-year rate in the period.

The development of the estimated equilibrium real interest rate is consistent with the decelerating growth of the Japanese economy in the 1990s. It seems, however, that the estimated level of the rate is lower as a whole than the intuitive level. As has been

[27] Since equations (16) and (9) are of recursive form, it is necessary to set the initial values for both the equilibrium rates at the beginning period. In this paper, the potential growth rate was set at 4.1% and the equilibrium inflation rate at 1.2% in October 1989. Refer to Section 3.4 for the reason.

mentioned, one of the reasons for the low estimated rate is that parameter σ^{-1}, which is a coefficient of the potential growth rate trend for the equilibrium real interest rate, was estimated small at 0.46. The other reason is that the subjective discount rate ρ was also estimated small at approximately 0.05%. When we transform the estimated equilibrium real interest rate into the potential growth rate using equation (10), also shown in Figure 5, the rate was approximately 4% for the later half of 1989, then steadily decreased, recording a low of approximately 1% in 2003, and was estimated as approximately 1.3% in May 2006. It seems that the estimate for this rate is relatively close to the intuitive level, compared to the equilibrium real interest rate.

Incidentally, there is a perspective in macroeconomic theory that parameter σ^{-1} must be equal to one[28]. Moreover, it is often assumed to be one in practical economic analysis. Under these conditions, the development of the potential growth rate corresponds to that of equilibrium real interest rate by excluding the contribution of the subjective discount rate. Contrary to this, we obtain an estimation result, $\sigma^{-1} = 0.46$. This indicates the necessity of assuming a weak relationship between the potential growth rate and the real rate in Japan in order to understand the macroeconomic development and low market rates consistently. It is one of the key findings in our analysis that the estimated equilibrium real interest rate is lower than our intuition based on actual economic development. Note that we have been considering this issue assuming the rational formation of interest rates in the market.

Looking at the estimated equilibrium inflation rate in Figure 6, the rate was approximately 1.2% in the later half of 1989, then increased slightly, and recorded a high at approximately 1.4% in 1993. Then, it decreased very gradually and was estimated at a low of approximately 0.6% in May 2006[29]. The difference between the

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[28] In an economy on a steady growth path, the marginal productivity of labor grows at a constant rate under fixed labor supply. In order to realize this condition, it is necessary for the income effect and substitution effect of the productivity growth on labor hours to offset each other. It is known that the utility function that satisfies this condition is limited to specific functions (King, Plosser, and Rebelo [1988]). Further, for the aggregate demand function to be stated independently of the labor market as in this paper, the utility function is required to be additively separable in consumption and labor. It is commonly known that, it is necessary for the rate σ of intertemporal substitution for consumption to be one, and necessary for the utility function to be expressed in logarithmic functional form for consumption in order to satisfy these conditions.

[29] On March 9, 2006, the BOJ released the statement "The introduction of a New Framework for the Conduct of Monetary Policy." The release indicated that "an understanding of medium- to long-term price stability" of the policy board members then was around 0–2% in terms of year-on-year percentage change in the CPI. It also showed that the midpoint value of

highest and the lowest of the rate during the estimation period is approximately 0.8 percent point. This is roughly half of the difference of approximately 1.5 percentage points between the highest and the lowest of the equilibrium real interest rate. In this sense, the market perception of the equilibrium inflation rate, compared to the equilibrium real interest rate and potential growth rate, has experienced a relatively stable development. Moreover, in explaining the decrease of long-term interest rates during the estimation period, the contribution of changes in the equilibrium real interest rate is greater than that of changes in the equilibrium inflation rate.

These results are, as mentioned in Section 3.5.3, also consistent with the fact that the equilibrium inflation rate in Japan is more stable than that in the United States. Moreover, we pointed out at the outset that it is a distinctive feature of our analysis to take into account not only the variability of the equilibrium inflation rate but also that of the equilibrium real interest rate. It could be said that this strategy has been successful.

4.2 Component Decomposition of Medium- to Long-term Interest Rates

4.2.1 Decomposition into Expectations Component and Risk Premium Component

Next, we investigate the development of medium- to long-term interest rates: 6-month, 3-year, 5-year, and 10-year rate. Figure 8 shows the market rates on a line graph and the estimation results for the expectations components and risk premium components of the theoretical interest rates on a bar graph[30].

With regard to features for each interest rate, first, we see that the 6-month rate has almost no risk premium components during the whole period, and thus, most of its theoretical interest rate is composed of the expectations component. This is due to fact that the path of the short-term interest rate for future 6 months is mostly predictable based on the prevailing information such as the current short-term rate. In other words, uncertainty is quite limited for such a short time horizon. On the other hand, the risk premium components of rates over 3 years are significant at all points of time.

When focusing on the bubble period around 1990, the size of the expectations component is smallest for the 10-year rate and largest for the 6-month rate. This is due

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each member's range was dispersed around 1%. While the major part of the estimation period in our analysis is prior to this release, the result indicates that the development of the estimated equilibrium inflation rate consistently fell within the range indicated by the BOJ's release.

[30] Refer to Section 2.3 for a definition of the expectations components and risk premium components of medium- to long-term interest rates.

to the prevailing perception that high economic growth and high interest rates at that time exceeded the equilibrium growth and equilibrium interest rate. Conversely, when focusing on the QMEP period since 2001, the size of the expectations component is largest for the 10-year rate and smallest for the 6-month rate. This is due to the prevailing perception that low economic growth and low interest rates at that time were below the equilibrium growth and equilibrium interest rate. With regard to risk premium components, the 10-year rate is largest and the 6-month rate is smallest for almost every point. This is due to the perception that the further into the future outlook, the greater the uncertainty of the process whereby the economy finally returns the equilibrium.

Next, we will investigate the development of interest rates during the period, with a special focus on the 10-year rate. In the early 1990s, the expectations component constitutes over half of the theoretical value of the 10-year rate, and the remaining is the risk premium component. Both components peaked at the beginning of 1991, and then decreased gradually through 2002. From a detailed perspective, the pace of decrease for the expectations component is smaller after 1995 than the decrease for the risk premium component. As a result, the portion of risk premium component in the theoretical value decreased to one third around 2002. Based on this development, it seems that the decreasing trend of long-term interest rates from the early 1990s through the early half of 2000s results from the reduction of risk premium in addition to the decrease in the equilibrium interest rate, as shown in Section 4.1. The reduction of risk premium during the period is considered to be due to the perception that the uncertainty from future business cycles and price fluctuations has decreased within a context of continued disinflation and low economic growth. Moreover, the risk premium component of the 10-year rate was approximately 0.5% around 2002, and then unchanged until the latest period. Therefore, the development of long-term interest rates after 2002 mainly reflects the development of the expectations component, increasing gradually along with the economic recovery. The stability of the risk premium at a low level during this period applies not only to the 10-year rate but also to 5-year, 3-year, and 6-month rate. For this reason, we can consider the possibility that the existence of the zero lower bound and the BOJ’s commitment makes volatility of future short-term interest rate very low, and thus, the risk premium has approached the floor.

4.2.2 Further Decomposition of Expectations Component

In this section, the development of the expectations component for medium- to long-term interest rates is analyzed through decomposition of the component. The decomposition results for each interest rate are displayed in Figure 9 (1)-(4), respectively. Regarding the method of decomposition, each term on the right side of equation (20) is considered as a "component", based on the property that the expectations components are represented by equation (20) under a risk neutral world as explained in Section 2.3. Specifically, $x_t B_{1j} / j$ is a GDP gap component, $\pi_t B_{2j} / j$ is an inflation rate component, and $i_t B_{3j} / j$ is a short-term interest rate component. Each component represents the contribution of the macroeconomic variable to the medium- to long-term interest rate at the observation point. $\tilde{r}_t^n B_{4j} / j + \Delta y_t^n B_{6j} / j$ is an equilibrium real interest rate component[31] and $\tilde{\pi}_t^* B_{5j} / j$ is an equilibrium inflation rate component. Each component represents the contribution of the perception for the equilibrium rate to the interest rate at the observation point. A_j / j is a constant component mainly corresponding to the subjective discount rate ρ. Since ρ is estimated as nearly zero, this component is also nearly zero and is omitted in Figure 9. Moreover, Figure 9 also shows a zero lower bound component and a commitment effect component for the period after 1999. These components are not incorporated in equation (20). However, as shown in Section 3.5.1, theoretical interest rates that take into account both effects can be calculated using Monte-Carlo simulations. Specifically, a zero lower bound component is defined as the positive component derived by subtracting the theoretical interest rate in equation (20) from the Monte-Carlo-based theoretical interest rate incorporating the zero lower bound but not the commitment. A commitment effect component is defined as the negative component derived by subtracting the Monte-Carlo-based theoretical interest rate incorporating the zero lower bound but not the commitment from the Monte-Carlo-based theoretical interest rate incorporating both the zero lower bound and the commitment.

First, we investigate the features of the expectations components in Figure 9, focusing on the period before 1999 when the policy rate faced the zero lower bound. For the 6-month rate, contribution of the short-term interest rate component is very large in this period, as mentioned in Section 4.2.1. The GDP gap component, inflation rate component and equilibrium real interest rate component make limited contributions while the equilibrium inflation rate component hardly makes any contribution. On the other hand, for the 10-year rate, contribution of the short-term interest rate component is limited while contributions of the equilibrium real interest rate component and equilibrium inflation rate component are very large. A major part of the expectations component of the 10-year rate can be explained by adding the GDP gap component to

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[31] The potential growth rate affects medium- to long-term interest rates through changes in perceived equilibrium real interest rate. Thus, we deal with the potential growth rate along with the equilibrium real interest rate component here.

the two equilibrium rate components. Especially, the magnitude of fluctuation of the equilibrium real interest rate component is, compared to those of other components, relatively large. It could be said that the development of the theoretical value of the 10-year rate is mostly explained by the development of the equilibrium real interest rate component. The 3-year and 5-year rates exhibit properties that fall between those of the 6-month rate and 10-year rate. For example, as the term of interest rate increases, the contributions of the equilibrium real interest rate and equilibrium inflation rate increase and that of the short-term interest rate decreases.

We will focus on the development of each component during this period. For the GDP gap component, it can be seen that the contributions in the 6-month and 10-year rates are not large being about 0.5% even at its largest, but the component clearly indicates development that reflects short-term business cycles with 2- or 3-year periodicity. For the inflation rate component, although we see a small contribution in the 6-month rate, the contribution decreases as the period of interest rates gets longer. The contribution is very small in the 10-year rate. This is due to the fact that the perception of equilibrium inflation rate is much more important than the current inflation rate in the formation of long-term interest rates.

Next, we will focus on the period after 1999 when the policy rate faced the zero lower bound. For the 6-month rate, the short-term interest rate component disappears when the policy rate at zero. The zero lower bound component makes a positive contribution of about 0.5% at its largest. While the commitment effect component is very small in the early half of the period, it gradually increases as the economy recovers after 2003. The GDP gap component is about 0.5% at its largest, and develops with 2- or 3-year periodicity as in the 1990s. The inflation rate component makes a negative contribution during the deflation period around 2001 and 2002. For the 10-year rate during the period after 1999, features do not markedly differ from that before the period, and the major part of the expectations component can be explained by the equilibrium real interest rate and equilibrium inflation rate components. Contributions of the zero lower bound component and commitment effect component are limited. The 3-year and 5-year rates exhibit properties that fall between those of the 6-month rate and 10-year rate, excluding the fact that contribution of the GDP gap component is comparatively large.

4.3 Effect of Zero Interest Rate Commitment

Figure 10 shows the results of estimation of the extent to which the zero interest rate commitment lowered the medium- to long-term interest rates in the period with ZIRP

(April 1999 through August 2000) and in the period with QMEP (March 2001 through March 2006)[32]. Specifically, the effect of the commitment is defined as the component derived by subtracting the theoretical interest rates which take into account not only the zero lower bound but also the commitment, according to Appendix 3 (2), from the theoretical rates which take into account the zero lower bound, according to Appendix 3 (1), but without the commitment.

We observe the following points in Figure 10.

- With regard to the effects of the commitment for 3-year, 5-year, and 10-year rate, those after the introduction of the QMEP in March 2001 are generally greater than those before. Also refer to Table 4 for this point. This is mainly due to the fact that the threshold inflation rate to exit the commitment is set at a lower level in the ZIRP period than in the QMEP period based on our judgment mentioned in Section 3.5.1[33].
- Comparing the effects of the commitment between 3-year, 5-year, and 10-year rate, the shorter the term of interest rate the greater the effect observed. This is due to the property that, even when the commitment is binding under deflation for example, it will be highly possible for the commitment not to be binding in the distant future as the economy tends towards equilibrium. Thus, the longer the term of interest rate, the more diluted the effects of the commitment.
- Focusing on the 6-month rate, the effect of the commitment increases or decreases phase by phase. It cannot be said that the effect in the period with QMEP is consistently larger, like other interest rates, than that in the period with ZIRP. For example, the effect in the later part of the ZIRP period is larger than that at the beginning of the period. The similar tendency also applies to the QMEP period. In Table 4, the QMEP period is divided into three phases; (1) recession period (March 2001 through June 2003), (2) bottoming out period (July 2003 through December 2004), and (3) recovery period (January 2005 through March 2006), with the average effect of the commitment shown for each period. While

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[32] The analysis in this section assumes that, even after the zero interest rate commitment is exited, the market recognizes a possibility of re-introduction of the same commitment in the future when the economic and price environments worsen to a similar extent as at the time of introduction of the commitment in the past. For this reason, in Figure 10, even in each period after exiting the ZIRP and after exiting the QMEP, the effect of the commitment is observed.

[33] We set the threshold inflation rates *a priori* as explained in Section 3.5.1. Oda and Ueda (2007) show that, based on their macro-finance analysis, the threshold rate is estimated lower in the ZIRP period than in the QMEP period. Thus, their finding is consistent with the assumption adopted in this paper.

the effect in the recession period is very small at 0.03%, as the economy recovers, the effect becomes larger at 0.10% in the bottoming out period and at 0.13% in the recovery period. This is due to the fact that, under severe economic conditions, the rate indicated by the policy rule is zero regardless of presence or absence of the commitment, and therefore the effect is likely to be zero. On the other hand, as the economy recovers, the rate of the policy rule not taking into account the commitment tends to become positive and the difference in the policy rate expands depending on the presence or absence of the commitment. This tendency is also observed somewhat for the 3-year, 5-year and 10-year rate although the extent of the tendency becomes smaller as the term of interest rate increases.

- In 2006, the effect of the commitment for the 6-month rate decreased rapidly. It is likely that, as the inflation rate in this period turned positive, the market widely perceived it as hardly possible for the commitment to produce an effect within six months. On the other hand, for 3-year, 5-year, and 10-year rate, the effects of the commitment did not decrease notably. It seems that the medium- to long-term interest rates, even after deflation is overcome, build in the effects of a possible re-imposition of the commitment in a phase when price conditions revert to deflation in the future.

4.4 Impulse Responses for Medium- to Long-Term Interest Rates

In empirical analyses of macro structural models, the responses of macroeconomic endogenous variables to various macroeconomic shocks are often investigated. Similarly, the response of medium- to long-term interest rates to various macroeconomic shocks can be analyzed using our macro-finance model. Figure 11 shows the impulse responses of the 6-month, 3-year, 5-year, and 10-year rate with an initial value at steady state when the four macroeconomic shocks, of demand shock, supply shock, policy shock, and productivity shock, occur at a magnitude of one standard deviation, respectively[34].

We observe the following points in Figure 11.

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[34] Steady state in this context is defined as the state at which the GDP gap is zero, the inflation rate matches the equilibrium inflation rate, and the policy rate matches the nominal long-term equilibrium interest rate, assuming the market perception for the equilibrium inflation rate and the potential growth rate is 1% and 2%, respectively. In steady state, expectations components of all the medium- to long-term rates are equal to the long-term equilibrium value of the nominal short-term rate. However, since the risk premium components are distinct per term of the rates, the steady-state values of the theoretical interest rates are also distinct per term, as indicated in Figure 11.

- After the demand shock, all the medium- to long-term interest rates rise gradually with lag, and begin to decrease after peaking about a year later. This property is somewhat similar to the impulse response of the short-term interest rate i_t to the demand shock, as observed in Figure 7. The property remains strong in a relatively short-term rate such as the 6-month rate while the magnitude of the response is smaller in a longer-term rate such as the 10-year rate.
- For the supply shock, the magnitudes of the impulse responses of all the medium- to long-term interest rates are comparatively small, reflecting the fact that the magnitude of the impulse response of the short-term rate is small.
- After the policy shock, the short-term rate increases while the GDP gap and inflation rate decrease. Since these responses offset one another in the context of the influence on medium- to long-term interest rates, the magnitudes of impulse responses of the interest rates are relatively small.
- The productivity shock has an effect of permanently raising the potential growth rate. For this reason, it gradually increases the equilibrium real interest rate with lag, and permanently increases the medium- to long-term interest rates.
- The three shocks other than the productivity shock also have the limited effect of permanently raising the medium- to long-term interest rates. This is because, even if the economic shock is temporary, the perceived equilibrium inflation rate is increased as the macroeconomic variables develop. Since the change in the equilibrium inflation rate remains even after the initial shock has disappeared, the effects on the medium- to long-term interest rates remain.

These are the properties of the interest rates under the estimated macro-finance model. It would be interesting to investigate to what extents the derived properties correspond with the developments of actual interest rates. Although we do not systematically check for this point in this paper, we will conduct a simple event study.

The base year for Japan's CPI was revised from 2000 to 2005 on August 25, 2006. In the revision, the so-called CPI bias also changed in a discontinuous manner, and thus the year-on-year percentage change in the CPI (nationwide, excluding perishables) decreased by around 0.5%. Since the change in the CPI that the market had anticipated was about 0.3%, it was pointed out that a surprise of -0.2% in the inflation rate occurred with the release of the revision. In this paper, we interpret the change of -0.2% as being brought about by the unexpected supply shock. Since this corresponds to a supply shock of -2.2 times the standard deviation, it makes the 10-year rate decrease by -0.08% at maximum, based on the results of Figure 11 (2). The actual 10-year rate decreased by -0.17% from 1.80% (closing rate at August 24) on the day before the revision to 1.63%

(closing rate at August 31) a week later when the rate bottomed out. Consequently, this case indicates that a decrease in the long-term interest rate is greater than twice that predicted by the model. It is not possible to determine if this is due to a model error or the market overshooting. However, it can at least be said that the macro-finance model is beneficial in conducting this type of impulse response analysis.

5. Other Determinants of Medium- to Long-Term Interest Rates

Revisiting Figure 8, we see that the model errors, which are defined as the deviations between the theoretical rates based on the model and the market rates, are large in some periods for all the interest rates. For example, all the market rates were higher than the theoretical rates in 1990, in the midst of the bubble period, reflecting monetary tightening. This can be due to the fact that the model does not include a mechanism for monetary policy to react to economic instability lead by asset price fluctuations. The market rates are also higher than the theoretical rates from the later half of 1994 through the beginning of 1995. During this period, upward pressure on interest rates arose not only from supply and demand imbalances for domestic bonds but also from arbitrage with overseas interest rates within a climate of worldwide high interest rates. These mechanisms are also not dealt with in our model. In the era of low interest rates after 1995, the market rates were consistently lower than the theoretical rates, especially in 3-year and 5-year rate. In this period, since there were various recessionary factors that are not taken into account by the model including asset price deflation and financial system instability, the expected interest rates for the future could be considerably lowered. In addition, the monetary policy rule in our model does not include the concept of preemptive easing against the risk of facing the zero lower bound[35]. Supposing that markets recognized the possibility of preemptive easing, market rates would be lower than the theoretical rates based on our model.

In such phases as mentioned above, the causes of model errors can be individually explained. Apart from them, however, model errors are observed more or less in the whole of the estimation period. This is due to the fact that the factors which are not taken into account by the model influence the formation of interest rates. In this section, we assume the following six proxy indices to explain the causes of model errors and

[35] Many papers, including Adam and Billi (2007) and Oda and Nagahata (2007), have pointed out that both from a theoretical and empirical perspective preemptive monetary easing is effective against the risk of facing the zero lower bound.

examine their explanatory power. The time series of model error, as indicated in Figure 12, is defined as the data derived by subtracting the theoretical rate from the market rate. Regression of model errors is conducted for the 10-year rate, using the following proxy indices as explanatory variables[36].

(1) GDP volatility

(2) CPI volatility

(3) US 10-year interest rate

(4) Nikkei stock index volatility

(5) Volatility of interest rate futures

(6) Volatility of 10-year JGB yields

Among these, (1) and (2) are macroeconomic variable volatilities and (4), (5) and (6) are financial asset price volatilities. While the data vary in type, it seems that these function as proxy indices for economic uncertainty. Since the variance of each economic shock is estimated as a constant in our model, the possibility that the degree of uncertainty can change depending on the phase is not taken into account. These indices are adopted with the aim of compensating this constraint. It is anticipated that if economic uncertainty increases, then the risk premium components increase and medium- to long-term rates increase. If this mechanism is significant in the formation of market rates, then the regressed coefficient will be positive. With regard to (3), a rise in the US long-term interest rate can be interpreted as the US economy's expansion and, as a result, an expansion of external demand in Japan, leading to an upward revision of the expected values for Japanese business conditions, price levels and interest rates. In this case, the regressed coefficient will be also positive.

Following Rudebusch, Swanson, and Wu (2006), we conduct simple regressions on each explanatory variable, and a multiple regression on all the explanatory variables[37].

For the result of the simple regressions, indicated in Table 5 (1), there are four explanatory variables of which the coefficients are statistically significant and have a correct sign: CPI volatility, the US 10-year rate, volatility of interest rate futures, and volatility of 10-year JGB yields. For the result of the multiple regression, as indicated in Table 5 (2), there is only one explanatory variable, the US 10-year rate, of which the coefficient is statistically significant and has a correct sign. Although the coefficient of volatility of the 10-year JGB yields has a correct sign, it falls slightly short of statistical

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[36] Refer to the notes in Table 5 for a detailed definition of each proxy index.

[37] This paper conducts single regressions and a multiple regression with a constant term, following Rudebusch, Swanson, and Wu (2006).

significance. Based on these results, it is judged that the model errors in our analysis are correlative with the US long-term interest rate. This result is consistent with the frequently mentioned rule of thumb that there is a significant correlation between the long-term interest rates of the US and other major countries including Japan. To incorporate this feature into the analysis, the part of the macro structural model should be extended to the open-economy macroeconomic model. However, based on the possibility that arbitrage between domestic rates and foreign rates in financial markets is incomplete in the short run, it may be difficult to systematically incorporate this feature into the model. In any case, the question of how to deal with the correlation is a remaining issue.

Meanwhile, Rudebusch, Swanson, and Wu (2006) conducted similar regressions of model errors for the US 10-year rate, using their macro-finance model for the US economy. They found that the implied volatility of long-term interest rate has the highest explanatory power among all explanatory variables[38]. In this paper as well, the coefficient of the volatility of long-term interest rate has a correct sign and a *t*-value of 1.01, which is the second best explanatory power after the US 10-year rate. In this sense, the result is broadly consistent with that of Rudebusch, Swanson, and Wu (2006).

6. Concluding Remarks

We estimated a macro-finance model of Japan's macroeconomic structure and the term structure of interest rates. Then, we analyzed the development of medium- to long-term interest rates as well as the effects of monetary policy in Japan.

The model was composed of a small-scale macro structural model of the New Keynesian-type, an affine diffusion model of the term structure, and a simple learning mechanism for perceived equilibrium rates of inflation and real interest to be updated based on the macroeconomic development. While many previous studies assumed this type of learning only for the equilibrium inflation rate, this paper is unique in assuming a similar process for the equilibrium real interest rate as well. Indeed, we found that the time-variability of the latter equilibrium rate was important for explaining the decline in long-term interest rates in Japan in recent years. Regarding the estimation, we generated original monthly time series of Japan's GDP, and estimated the macro-finance model

[38] Rudebusch, Swanson, and Wu (2006) do not adopt foreign interest rates as explanatory variables for model errors.

with the maximum likelihood method using monthly data. In estimation and analysis, the effects of the zero lower bound of nominal interest rates and the BOJ's zero interest rate commitment after 1999 were approximately taken in account.

With these conditions, we found that the macro-finance model can be plausibly estimated for Japan. Moreover, it was possible to enhance understanding of the development of medium- to long-term interest rates by decomposing them into various components based on the estimated model.

The main results of the analysis are summarized below. In contrast to the United States, change in the perceived equilibrium inflation rate was comparatively gradual in the estimation period, while change in the perceived equilibrium real interest rate was relatively large. With regard to factors that influence long-term interest rates, the effect from the change in the equilibrium real interest rate was the largest while effects were also observed from other factors such as changes in the current GDP gap, inflation rate, and short-term interest rate in accompanying business cycles. In addition, it was confirmed that the existence of the zero lower bound of nominal interest rates and the BOJ's zero interest rate commitment affected medium- to long-term interest rates. The analysis of model errors suggested the possibility that long-term interest rates in Japan were also affected by long-term rates in the United States.

At the beginning of this paper, we raised the question of whether the fact that Japan's long-term interest rates had been extremely low could be explained with economic rationality. It can be said now that the macro-finance model gives us a rational explanation, although we have some caveats to this conclusion. Specifically, estimating a macro-finance model means to identify model parameters that are consistent not only with macroeconomic data but also with market interest rate data. Thus, we need to ascertain the appropriateness of the estimated model from a broad perspective before concluding the model's capability to explain the interest rates. In this context, almost all the estimated parameters were plausible while two of the parameters had an estimate that was smaller than the intuitive value. One of these parameters was the subjective discount rate, and the other was the parameter linking the equilibrium real interest rate to the potential growth rate. This might imply that a potential bias, which was raised in an attempt to explain the low long-term rates, was concentrated in these estimates.

The main results obtained in our analysis have been listed above. Although they do not include a definitive response to the initial question, it can be said that the paper has provided an effective tool to deal with such a question. In addition, a more fundamental contribution of the paper is that of having presented a framework for applying a standard macro-finance analysis to Japan. There are obviously issues that remain. For example, it would be a good challenge to expand the macroeconomic part of the model to an open-economy version in order to incorporate the fact that long-term interest rates in Japan have been significantly affected by those in the United States. It could be expected that further macro-finance analyses lead to more findings in the area of the term structure and monetary policy.

Appendix 1 State Space Representation of the Model and Likelihood Function

In appendix 1, we describe our macro-finance model with state space representation, and derive the likelihood function used for the estimation.

The macro structural part of the model consists of equations (1'), (2), and (3) in Section 2.1, and the learning component for the equilibrium rates of real interest and inflation is composed of equations (9), (13), and (16) in Section 2.2. First, we show their state space representation, and then add the finance part. Specifically, the state space vector Y_t is defined as:

$$Y_t = (x_t \quad \pi_t \quad i_t \quad \tilde{r}_t^n \quad \tilde{\pi}_t^* \quad \Delta y^n{}_t \quad E_t x_{t+1} \quad E_t \pi_{t+1})'. \tag{A1}$$

With this vector the above-mentioned equations are expressed as:

$$\Gamma_0 Y_t = c + \Gamma_1 Y_{t-1} + \Psi \varepsilon_t + \Pi \eta_t, \tag{A2}$$

where the vectors and matrices in equation (A2) are defined as shown below.

$$\varepsilon_t' = (\varepsilon_t^{IS} \quad \varepsilon_t^{AS} \quad \varepsilon_t^{MP} \quad \varepsilon_t^{\Delta PG}) ,$$

$$\eta_t' = (x_t - \mathrm{E}_{t-1} x_t \quad \pi_t - \mathrm{E}_{t-1}\pi_t) ,$$

$$c' = (0 \quad 0 \quad 0 \quad \rho\nu \quad 0 \quad 0 \quad 0 \quad 0) ,$$

$$\varepsilon_t^{IS} \sim N(0, \sigma^{IS}), \ \varepsilon_t^{AS} \sim N(0, \sigma^{AS}), \ \varepsilon_t^{MP} \sim N(0, \sigma^{MP}), \ \varepsilon_t^{\Delta PG} \sim N(0, \sigma^{\Delta PG}),$$

$$\Gamma_0 = \begin{pmatrix} 1 & 0 & (2\mu-1)\sigma & -(2\mu-1)\sigma & 0 & 0 & -\mu & -(2\mu-1)\sigma \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\delta \\ 0 & 0 & 1 & -(1-\gamma) & -(1-\gamma)(1-\phi_\pi) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\sigma^{-1}\nu & 0 & 0 \\ 0 & 0 & \xi & -\xi(1-\gamma) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Psi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma_1 = \begin{pmatrix} 1-\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\delta & 0 & 0 & 0 & 0 & \kappa & 0 \\ 0 & 0 & \gamma & 0 & 0 & 0 & (1-\gamma)\phi_x & (1-\gamma)\phi_\pi \\ 0 & 0 & 0 & 1-\nu & 0 & 0 & 0 & 0 \\ 0 & \theta & \xi\gamma & 0 & 1-\theta+\xi(1-\gamma)(1-\phi_\pi) & 0 & \xi(1-\gamma)\phi_x & \xi(1-\gamma)\phi_\pi \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \Pi = \begin{pmatrix} 0 & 0 \\ \kappa & 0 \\ (1-\gamma)\phi_x & (1-\gamma)\phi_\pi \\ 0 & 0 \\ \xi(1-\gamma)\phi_x & \xi(1-\gamma)\phi_\pi \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

In order to derive the likelihood function, it is necessary to solve for the expectations variables in equation (A2) and obtain a reduced model. Employing the Sims (2002) algorithm, equation (A2) is converted to equation (A3) below.

$$Y_t = C + \Gamma Y_{t-1} + \Omega \varepsilon_t . \tag{A3}$$

Vector C and matrices Γ, Ω in equation (A3) can be numerically calculated, given the coefficient vector and matrices in equation (A2), as long as the convergence conditions are satisfied.

Then, state vector F_t defined in Section 2.1.3 is introduced.

$$F_t \equiv (x_t,\ \pi_t , i_t,\ \tilde{r}_t^{\,n},\ \tilde{\pi}_t^{*},\ \Delta y_t^n)' . \tag{A4}$$

Since F_t is the sub vector for Y_t, the dynamics of F_t can be stated, by extracting part of equation (A3), as:

$$F_t = C^F + \psi F_{t-1} + \Sigma \varepsilon_t , \tag{A5}$$

where coefficient vector C^F and coefficient matrices ψ, Σ in equation (A5) refer to the first six rows of coefficient vector C and matrices Γ, Ω on the right side of equation (A3), respectively.

Next, the finance model part explained in Section 2.3 is incorporated in the state space representation. We define a vector composed of observable variables as:

$$z_t \equiv (x_t, \pi_t, \Delta y_t^n, R_t)' , \tag{A6}$$

where $R_t \equiv (i_t, i_{6,t}, i_{36,t}, i_{60,t}, i_{120,t})'$ expresses data for the term structure of interest rates at t. i_t is the 1-month rate at t and a proxy for policy rate. $i_{j,t}$ is the medium- to long-term interest rate at t for maturing in j months ($j > 1$). Dynamics of z_t can be stated, from combining equation (A5) and equations (20) and (21) in Section 2.3, as:

$$z_t = C^z + \Gamma^z F_{t-1} + \Gamma^\alpha R_{t-1} + \Omega^z \zeta_t , \tag{A7}$$

where ζ_t is the vector defined as:

$$\zeta_t \equiv (\varepsilon_t^{IS}, \varepsilon_t^{AS}, \varepsilon_t^{\Delta PG}, \varepsilon_t^{MP}, \mu_t^6, \mu_t^{36}, \mu_t^{60}, \mu_t^{120})' . \tag{A8}$$

It contains four kinds of economic shocks $\{ \varepsilon_t^k \}$ and random terms $\{ \mu_t^j \}$ of model errors in equation (21), where $\varepsilon_t^k \sim N(0, \sigma^k)$ and $\mu_t^j \sim N(0, \sigma^j)$. Coefficient vector C^z and coefficient matrices Γ^z, Γ^α, Ω^z in equation (A7) are defined respectively as:

$$C^z = \begin{pmatrix} C_1^F \\ C_2^F \\ C_6^F \\ C_3^F \\ (1-\alpha_6)A_6/6 + B_6'/6 \cdot C^F \\ (1-\alpha_{36})A_{36}/36 + B_{36}'/36 \cdot C^F \\ (1-\alpha_{60})A_{60}/60 + B_{60}'/60 \cdot C^F \\ (1-\alpha_{120})A_{120}/120 + B_{120}'/120 \cdot C^F \end{pmatrix}, \qquad \varGamma^z = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_6 \\ \psi_3 \\ B_6'/6 \cdot \psi - \alpha_6 B_6'/6 \\ B_{36}'/36 \cdot \psi - \alpha_{36} B_{36}'/36 \\ B_{60}'/60 \cdot \psi - \alpha_{60} B_{60}'/60 \\ B_{120}'/120 \cdot \psi - \alpha_{120} B_{120}'/120 \end{pmatrix},$$

$$\varGamma^\alpha = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_6 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{36} & 0 & 0 \\ 0 & 0 & 0 & \alpha_{60} & 0 \\ 0 & 0 & 0 & 0 & \alpha_{120} \end{pmatrix}, \qquad \varOmega^z = \begin{pmatrix} \varSigma_1 & 0 & 0 & 0 & 0 \\ \varSigma_2 & 0 & 0 & 0 & 0 \\ \varSigma_6 & 0 & 0 & 0 & 0 \\ \varSigma_3 & 0 & 0 & 0 & 0 \\ B_6'/6 \cdot \varSigma & 1 & 0 & 0 & 0 \\ B_{36}'/36 \cdot \varSigma & 0 & 1 & 0 & 0 \\ B_{60}'/60 \cdot \varSigma & 0 & 0 & 1 & 0 \\ B_{120}'/120 \cdot \varSigma & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Given the coefficient matrices and vector in equation (A7), the log-likelihood function can be calculated. The logarithmic value llh_t of the conditional probability density for observable data z_t, given the information through period t-1, is expressed as:

$$\begin{aligned} llh_t &= \log f_{z_t|z_{t-1},\dots,z_1}(z_t | z_{t-1},\dots,z_1;\theta) \\ &= -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log[\det(\varOmega^z \varOmega^{z'})] - \frac{1}{2}(z_t - C^z - \varGamma^z F_{t-1} - \varGamma^\alpha R_{t-1})'(\varOmega^z \varOmega^{z'})^{-1}(z_t - C^z - \varGamma^z F_{t-1} - \varGamma^\alpha R_{t-1}). \end{aligned} \tag{A9}$$

Utilizing this, the log-likelihood function is derived as equation (A10) below:

$$L_{z_t,z_{t-1},\dots,z_2|z_1}(z_t, z_{t-1},\dots,z_2 | z_1;\theta) = \sum_{\tau=2}^{t} llh_\tau . \tag{A10}$$

Estimations are conducted through numerical search, using the optimization algorithm, for parameter set θ that maximizes the log-likelihood function. Since there are as many as 40 parameters, excluding the three parameters calibrated, to estimate in the model, substantial care is taken. That is, a local solution depending upon initial values is avoided by also conducting a grid search to ensure a globally optimized solution.

Meanwhile, standard errors of the estimates are numerically calculated based on the inverse of the Hessian matrix at the maximum likelihood point.

We can describe estimation results with state space representation by substituting estimated parameters shown in Tables 2 and 3 into the coefficient matrices in equation (A2). Further, the results can also be expressed in terms of the reduced model for equation (A7), where coefficient vector C^z and coefficient matrices Γ^z, Γ^α, Ω^z are shown below.

$$C^z = \begin{pmatrix} 0.014 \\ 0.024 \\ 0.000 \\ 0.004 \\ -0.005 \\ -0.041 \\ -0.129 \\ -0.424 \end{pmatrix} \times 10^{-4}, \quad \Gamma^z = \begin{pmatrix} 0.958 & -0.004 & -0.082 & 0.044 & 0.086 & 0.017 \\ 0.097 & 0.894 & -0.060 & -0.002 & 0.166 & 0.029 \\ -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & 1.000 \\ 0.033 & 0.103 & 0.906 & 0.084 & -0.010 & 0.005 \\ 0.062 & 0.105 & 0.096 & 0.108 & 0.041 & 0.024 \\ 0.034 & 0.014 & -0.039 & 0.031 & 0.071 & 0.031 \\ 0.028 & 0.018 & -0.024 & 0.036 & 0.099 & 0.045 \\ 0.023 & 0.026 & -0.009 & 0.054 & 0.212 & 0.075 \end{pmatrix},$$

$$\Gamma^\alpha = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.756 & 0 & 0 & 0 \\ 0 & 0 & 0.967 & 0 & 0 \\ 0 & 0 & 0 & 0.942 & 0 \\ 0 & 0 & 0 & 0 & 0.887 \end{pmatrix}, \quad \Omega^z = \begin{pmatrix} 0.410 & -0.001 & -0.012 & 0.001 & 0 & 0 & 0 & 0 \\ 0.041 & 0.138 & -0.009 & 0.002 & 0 & 0 & 0 & 0 \\ 0.000 & -0.000 & -0.000 & 0.062 & 0 & 0 & 0 & 0 \\ 0.014 & 0.016 & 0.136 & 0.000 & 0 & 0 & 0 & 0 \\ 0.057 & 0.038 & 0.101 & 0.004 & 1 & 0 & 0 & 0 \\ 0.047 & 0.040 & 0.035 & 0.030 & 0 & 1 & 0 & 0 \\ 0.029 & 0.032 & 0.026 & 0.035 & 0 & 0 & 1 & 0 \\ 0.016 & 0.027 & 0.018 & 0.036 & 0 & 0 & 0 & 1 \end{pmatrix} \times 10^{-3}.$$

Note that these figures are based on representation with all the endogenous variables in equation (A7) being monthly data.

Appendix 2 Estimation of Monthly Series of GDP

(1) Estimation Method of Monthly Series of GDP

To obtain sufficient data for estimating our macro-finance model, we convert publicly available GDP data on a quarterly base into monthly data with the approach shown below[39].

Specifically, for each demand component of the GDP, estimation is performed to decompose the quarterly data into three monthly data[40] by referring to the development of other monthly economic statistics that have a strong relationship to that component. Our methodology follows that in Appendix A of Bernanke, Gertler, and Watson (1997), using a state space model to determine the weights for various reference statistics.

Estimation results are shown in Table 6. Reference statistics for each demand component of the GDP is also shown there. In selecting the reference, we basically follow the approach adopted in the "Quarterly Estimates of GDP" published by Japanese Cabinet Office as well as that in Yamasawa and Fujii (2000) and Iizuka and Kawada (2002). The monthly series of GDP generated based on the estimation of Table 6 is shown in Figure 4.

(2) Method for Decomposing Quarterly Time Series into Smooth Monthly Time Series: Smooth Decomposition Method

This method decomposes a quarterly time series, which is flow data, into a smooth monthly time series by imposing the two conditions shown below. This paper refers to the method as the smooth decomposition method.

(i) The quarterly data (x_t^Q) is equal to the sum of the three monthly values ($x_{3t}^M, x_{3t+1}^M, x_{3t+2}^M$) corresponding to that quarterly data ($t = 0, 1, 2, \ldots$).

(ii) The sum of squares of the curvature, which is defined as the second-order

[39] Apart from the estimation in this paper, the Japan Center for Economic Research has been generating monthly GDP data as well. See, for example, Iizuka and Kawada (2002). However, because this data is limited to the time series after January 1994, the information volume is not sufficient for the estimation in our paper. Therefore, we originally estimate the monthly GDP data.

[40] With regard to the three components among the various demand components of GDP, actual quarterly data are decomposed into monthly data using the smooth decomposition method in Appendix 2(2), because no monthly economic statistics suitable for reference are available. These components are private inventory increase, government final consumption expenditure, and public inventory increase. These three components are not listed in Table 6.

differential of the series, of the monthly time series is minimized to ensure the smoothness of the monthly series.

Specifically, we solve the minimization problem below to calculate the monthly time series ($x_{3t}^M, x_{3t+1}^M, x_{3t+2}^M$) from the quarterly time series (x_t^Q).

$$\min_{\{x_0, x_1, \dots\}} \sum_t [(x_{t+2}^M - x_{t+1}^M) - (x_{t+1}^M - x_t^M)]^2 ,$$

$$\text{subject to} \quad x_{3t}^M + x_{3t+1}^M + x_{3t+2}^M = x_t^Q . \quad (t = 0, 1, 2, \dots)$$

Appendix 3 Calculation of Theoretical Value of Medium- to Long-term Interest Rates during Periods with Zero Interest Rate

A method that calculates the theoretical value of medium- to long-term interest rates is presented in this appendix taking into account 1) the effect of zero lower bound for short-term interest rate, and 2) the effect of the zero interest rate commitment introduced by the BOJ. When incorporating these non-linear effects, the theoretical interest rates given in equation (20) are not correct. However, utilizing the method below makes it possible to approximately calculate the theoretical interest rates based on the macro-finance model with such non-linearity.

(1) Method to Calculate Medium- to Long-term Interest Rates Taking into Account Existence of Zero Lower Bound of Nominal Interest Rates

We consider a method that approximately takes into account the effects of the zero lower bound. Drawing on the basis of the no-arbitrage pricing theory, theoretical interest rates are derived based on calculation of the expected path of future short-term interest rate under risk neutral probability. In this calculation, it is necessary to impose the zero lower bound on the policy rule in equation (3). To this end, we conduct Monte-Carlo simulations to generate paths of future short-term rates because its expected value cannot be analytically solved with the non-linear model. This method is detailed below.

In simulations of future economic paths with the model, if a negative nominal short-term rate is generated, it is assumed that a positive interest rate shock occurs at the same time to produce a short-term rate return of exactly zero[41]. This approach is also used by Reifschneider and Williams (1998). Specifically, a simulation under subjective probability is performed following $F_t = C^F + \psi F_{t-1} + \Sigma\varepsilon_t$, where the third element of

[41] We allowed for an exceptional treatment to an extremely unstable path in the simulation. Specifically, when the nominal short-term interest rate is extremely deep in the negative direction before considering the zero lower bound, we assume that the "large-scale fiscal policy" is mobilized to aid a critical economic environment. The stimulus effects of this fiscal policy are assumed to be the same as the hypothetical monetary easing effects which could be appeared with the short-term rate lowered to the negative value that was then indicated by the policy rule. This assumption is the same as that utilized by Oda and Nagahata (2007). Without this assumption, the economy can fall into a divergent path when facing a critical depression that seldom occurs, and the simulation results may become unrealistic. We set the condition that the fiscal policy is mobilized when the part that is multiplied by $(1-\gamma)$ in equation (3) falls below -5.0% on the annual base. The fiscal policy defined in this manner has the same effect, in terms of the simulation algorithm, as a tentative withdrawal of the zero lower bound.

state vector F_t is short-term interest rate i_t. In the simulation, when $i_t < 0$ as a result of shock $\varepsilon_t = (\varepsilon_t^{IS}, \varepsilon_t^{AS}, \varepsilon_t^{MP}, \varepsilon_t^{PG})'$ at *t*, it is assumed that a positive interest rate shock of $-\min(i_t, 0)/\Sigma_{3,3})$ is added in the same period *t*. In other words, the simulation is performed with a shock of $\varepsilon_t = (\varepsilon_t^{IS}, \varepsilon_t^{AS}, \varepsilon_t^{MP} - \min(i_t, 0)/\Sigma_{3,3}, \varepsilon_t^{PG})'$. The shock stream $\{\varepsilon_t\}$ is stored in every simulation, and the expected value under risk neutral probability is calculated using stored shocks as shown below.

The theoretical value of interest rate $i_{j,t}$ at *t* maturing in *j* months is basically obtained with equation (A11) as the expected value under risk neutral probability of the average of the short-term interest rate from period *t* through future period *t*+*j*-1[42].

$$i_{j,t} = \frac{1}{j} E_t^Q [\sum_{\tau=t}^{t+j-1} i_\tau]. \tag{A11}$$

Since short-term rate i_τ is an element of vector F_t, with equation (A12) below representing the dynamics of F_t under risk neutral probability, it is possible to determine the expected value $E_t^Q[i_\tau]$ ($t < \tau$) by simulating the future paths of F_t.

$$\begin{aligned} F_t &= C^F + \psi F_{t-1} + \Sigma \varepsilon_t^Q \\ &= (C^F - \Sigma \lambda_0) + (\psi - \Sigma \lambda_1) F_{t-1} + \Sigma \varepsilon_t \ . \end{aligned} \tag{A12}$$

Here, the aforementioned shock streams $\{\varepsilon_t\}$, which are modified to prevent a negative interest rate, can be applied to equation (A12).

Meanwhile, expectations component $i_{j,t}^{Exp}$, which excludes the risk premium component of interest rate $i_{j,t}$, can be calculated as the expected value under subjective probability of the average of the short-term interest rate from period *t* through future period *t*+*j*-1.

$$i_{j,t}^{Exp} = \frac{1}{j} E_t [\sum_{\tau=t}^{t+j-1} i_\tau], \tag{A13}$$

where short-term interest rate i_τ on the right side is subject to the dynamics of state vector F_t under subjective probability as:

$$F_t = C^F + \psi F_{t-1} + \Sigma \varepsilon_t . \tag{19, repeated}$$

The risk premium component is defined as the component subtracting the expectations component from the theoretical value of medium- to long-term interest rates.

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[42] Equation (A11) is consistent with the theoretical interest rate, which is the part on the right that excludes model error, of equation (20) when shock vector ε_t completely follows the multivariate normal distribution. This appendix considers equation (A11), instead of equation (20), since it deals with cases where ε_t deviates from the normal distribution through the zero lower bound and the BOJ's commitment.

Note that, as explained in Section 3.5, model estimations in this paper do not take into account the effects of the zero lower bound and the commitment on the risk premium component. For this reason, equation (A13) is utilized, instead of equation (A12), to take these effects into account only for the expectations component. Combining this expectations component with the risk premium component based on the linear framework of equation (20) produces the theoretical value of whole interest rates.

(2) Method of Calculation for Medium- to Long-term Interest Rates Taking into Account the Effects of the Zero Interest Rate Commitment

In this paper, as explained in Section 3.5, the commitment is described as the BOJ's assurance that even if the policy rate in equation (3) is positive, when the inflation rate in the previous month does not exceed the specific threshold, the policy rate in the current month is set to zero. It is assumed that the inflation rate threshold, $\overline{\pi}$, was perceived to be 0.0% during the ZIRP period (from April 1999 through July 2000), and 0.5% during the QMEP period (from March 2001 through March 2006).

The dynamics of policy rate i_t is described as equations (26) and (3'):

$$\begin{cases} i_t = 0 & \text{if } \hat{i}_t < 0 \text{ or } \pi_{t-1} < \overline{\pi}, \\ i_t = \hat{i}_t & \text{if } \hat{i}_t \geq 0 \text{ and } \pi_{t-1} \geq \overline{\pi}, \end{cases} \qquad \text{(26, repeated)}$$

where $\hat{i}_t = \gamma i_{t-1} + (1-\gamma)[\tilde{r}_t^n + \tilde{\pi}_t^* + \phi_\pi(\pi_t - \tilde{\pi}_t^*) + \phi_x x_t] + \varepsilon_t^{MP}$. (3', repeated)

The method employed to incorporate this dynamics into Monte-Carlo simulations is to slightly expand the methodology explained in appendix 3 (1). Specifically, in simulations of the future economic path under subjective probability, when $\hat{i}_t < 0$ or $\pi_{t-1} < \overline{\pi}$, it is assumed that the positive interest rate shock, which returns policy rate i_t exactly to zero, occurs at the same time *t* leading to $i_t = 0 \neq \hat{i}_t$. Shock stream $\{\varepsilon_t\}$, which incorporates the above-mentioned assumption, is stored and the simulation proceeds. After the shock streams $\{\varepsilon_t\}$ are stored sufficiently by repeating the simulation, the theoretical value of medium- to long-term interest rates is approximately derived by calculating the expected value under risk neutral probability. The expectations component of the interest rates is also derived by calculating the expected value under subjective probability with the risk price set at zero. This approach is applied only for the expectations component in estimating the macro-finance model, as explained in appendix 3 (1).

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Table 1 GMM Estimation for Calibration

IS curve	$x_t = \underset{(215.7)}{0.5027} E_t x_{t+1} + (1-0.5027) x_{t-1} - (2 \cdot 0.5027 - 1)\left(i_t - E_t \pi_{t+1} - \tilde{g}_t^n\right)$ $+ (\underset{(10.6)}{0.00241} \cdot S_t - \underset{(-10.6)}{0.00602}) \cdot Dummy_t + \varepsilon_t^{IS}$
AS curve	$\pi_t = \underset{(87.0)}{0.482} E_t \pi_{t+1} + (1-0.482)\pi_{t-1} + \underset{(5.33)}{0.00606} x_t + \varepsilon_t^{AS}$
Monetary policy rule	$i_t = \underset{(125.8)}{0.767} i_{t-1} + (1-0.767)\left[\tilde{g}_t^n + \frac{1.4}{400} + \underset{(32.5)}{1.35}\left(\pi_t - \frac{1.4}{400}\right) + \underset{(15.0)}{0.245} x_t\right] + \varepsilon_t^{MP}$

Notes:

1. The estimation period runs from 1983 Q1 through 1998 Q4.
2. Error terms are corrected for heteroscedasticity and serial correlation with the Newey and West (1987) method. The number of lags for error terms is set to be three. The *p*-value of *J*-test for the estimation result is 0.271, and thus the model is not rejected.
3. Instrumental variables are one- to three-period lags of the explanatory variables as well as other independent variables (the BOJ *Tankan* Output Price DI and the capacity utilization ratio).
4. The *t*-values are shown in parentheses below the estimated parameters.
5. $\tilde{g}_t^n$ denotes the trend for potential growth rate. S_t denotes the logarithmic value of the Nikkei Average Stock Price Index (four-period backward moving average). $Dummy_t$ denotes the dummy variable that is defined to be zero for the period before 1986 Q4 and one for the period after 1987 Q1.
6. The equilibrium inflation rate is set to be the same as the average inflation rate in the estimation period (i.e., 1.4%).

Table 2 Estimation of Macro-Finance Model by Maximum Likelihood Method

		Limited-sample estimation		Full-sample estimation	
Estimation period		Oct 1989 - Dec 1998		Oct 1989 - May 2006	
Aggregate demand function	μ	0.501	(0.0000109)	0.501	(0.00000918)
Equation (1) and (1')	$\hat{\sigma}$	0.00538#	(-)	0.00538#	(-)
	σ	2.32	(-)	2.17	(-)
Aggregate supply function	δ	0.509	(0.0154)	0.513	(0.00378)
Equation (2)	κ	0.00606#	(-)	0.00606#	(-)
Monetary policy rule	γ	0.915#	(-)	0.915#	(-)
Equation (3)	ϕ_{π}	1.36	(0.457)	1.36	(0.0710)
	ϕ_{χ}	0.303	(0.123)	0.268	(0.0233)
Learning of equilibrium real interest rate	ν	0.0234	(0.0336)	0.0237	(0.00963)
Equation (16)	ρ	0.0000653	(0.000128)	0.0000381	(0.0000933)
Learning of equilibrium inflation rate	θ	0.000373	(0.0308)	0.00484	(0.00119)
Equation (9)	ξ	0.00731	(0.0607)	0.00477	(0.0142)
Macroeconomic shocks	σ^{IS}	0.000230	(0.0000255)	0.000214	(0.0000196)
(standard deviations)	σ^{AS}	0.0000709	(0.0000313)	0.0000751	(0.00000682)
	σ^{MP}	0.000168	(0.0000207)	0.000137	(0.0000110)
	$\sigma^{\Delta PG}$	0.0000618	(0.00000955)	0.0000618	(0.00000955)
Model error of interest rates	α_{6M}	0.952	(0.0613)	0.756	(0.0624)
(Autoregressive coefficients in equation (21))	α_{3Y}	0.949	(0.0872)	0.967	(0.0366)
	α_{5Y}	0.908	(0.142)	0.942	(0.0447)
	α_{10Y}	0.883	(0.0960)	0.887	(0.0443)
Model error of interest rates	σ^{6M}	0.000160	(0.0000192)	0.000139	(0.0000109)
(Standard deviations of disturbance terms in equation (21))	σ^{3Y}	0.000194	(0.0000243)	0.000163	(0.0000116)
	σ^{5Y}	0.000212	(0.0000237)	0.000183	(0.0000150)
	σ^{10Y}	0.000208	(0.0000289)	0.000170	(0.0000114)

Notes:

1. Standard errors are shown in parentheses.
2. Parameters of risk prices are also estimated simultaneously with the above parameters. Refer to Table 3 as well.
3. Parameters $\hat{\sigma}$, κ, and γ are calibrated, not estimated, based on the result of GMM estimation indicated in Table 1. These calibrated values are indicated with "#". The value for σ is calculated based on the equation (5).

Table 3 Estimation of Macro-Finance Model by Maximum Likelihood Method: Risk Prices

(1) Limited-sample estimation (Oct 1989 - Dec 1998)

Equation (24) λ_0	Equation (25) λ_1					
-0.0475	708.6	0	0	0	0	0
(0.143)	(183.9)					
0.167	0	823.5	0	0	0	0
(0.151)		(262.1)				
-0.0417	0	0	-415.9	0	0	0
(0.101)			(78.7)			
-0.0274	0	0	0	0	0	-139.6
(0.114)						(178.6)

(2) Full-sample estimation (Oct 1989 - May 2006)

Equation (24) λ_0	Equation (25) λ_1					
0.261	488.2	-24.2	-360.9	0	0	-259.9
(0.0799)	(10.3)	(6.05)	(17.4)			(22.6)
-0.198	38.5	-122.8	77.4	0	0	-48.9
(0.105)	(4.94)	(15.9)	(14.6)			(13.4)
-0.0158	-183.3	50.4	120.6	0	0	17.8
(0.104)	(33.1)	(3.57)	(15.3)			(16.3)
0.205	445.5	197.2	-486.3	0	0	322.5
(0.0530)	(10.7)	(12.5)	(14.2)			(27.9)

Note: Standard errors are shown in parentheses.

Table 4 Effects of Zero Interest Rate Commitment to Lower Medium- to Long-Term Interest Rate

— Average of the effect on interest rates during each period

%

		6M rate	3Y rate	5Y rate	10Y rate
Period with zero interest rate policy (Apr 1999 - Jul 2000)		0.05	0.02	0.03	0.01
Period after exiting the zero interest rate policy (Aug 2000 - Feb 2001)		0.12	0.02	0.03	0.01
Period with quantitative monetary easing policy	Recession period (Mar 2001 - Jun 2003)	0.03	0.08	0.09	0.04
	Bottoming out period (Jul 2003 - Dec 2004)	0.10	0.11	0.11	0.04
	Recovery period (Jan 2005 - Mar 2006)	0.13	0.15	0.13	0.05
Period after exiting the quantitative monetary easing policy (Apr 2006 - May 2006)		0.01	0.20	0.14	0.05

Table 5 Regression Analysis of Model Errors

(1) Simple regression (10-year interest rate)

	Coefficient	*t* -value	Adjusted R^2
GDP volatility	-0.39	-6.24	0.16
CPI volatility	1.00	5.22	0.12
US 10-year rate	0.16	7.99	0.24
Nikkei stock index volatility	-0.09	-1.51	0.01
Volatility of interest rate futures	1.11	2.46	0.02
Volatility of 10-year JGB yields	1.72	3.29	0.05

(2) Multiple regression (10-year interest rate)

	Coefficient	*t* -value
GDP volatility	-0.19	-2.94
CPI volatility	0.11	0.38
US 10-year rate	0.18	6.08
Nikkei stock index volatility	-0.02	-0.43
Volatility of interest rate futures	-2.02	-3.51
Volatility of 10-year JGB yields	0.60	1.01
Adjusted R^2	0.33	

Notes:
1. GDP volatility and CPI volatility are defined as the standard deviation of the percentage rate of changes in the monthly series of real GDP and CPI, respectively, for the previous 24 months.
2. Nikkei stock index volatility, volatility of interest rate futures (specifically, 3-month Euro-yen futures), and volatility of 10-year JGB (Japanese government bond) yields (specifically, 10-year generic issue) are defined as the standard deviation of each of the daily series for the 20 business days within a month.
3. Both the simple regression and the multiple regression are specified with a constant term.

Table 6 Estimation of Japan's Monthly GDP

(1) Private final consumption expenditure

Explanatory variables	Coefficient	Standard deviation
Constant	2.26	0.37
Core household consumption	0.55	0.17
Indices of industrial shipment (consumer goods)	0.45	0.23
Index of tertiary industry activity	0.55	0.18
Export volume index	38.2	25.3
Import volume index	1.33	0.35
Autoregressive coefficient (ρ) in error term	0.86	0.08
Standard deviation (σ) of disturbance term in error term	0.69	0.08

(2) Private housing investment

Explanatory variables	Coefficient	Standard deviation
Constant	-4.15	2.04
Building floor area (dwelling use) [a]	0.85	0.06
Autoregressive coefficient (ρ) in error term	0.97	0.02
Standard deviation (σ) of disturbance term in error term	0.31	0.03

(3) Private non-residential investment

Explanatory variables	Coefficient	Standard deviation
Constant	1.04	0.18
Machinery orders (manufacturing) [b]	0.99	0.11
Machinery orders (nonmanufacturing) [c]	0.75	1.40
Estimated construction cost (nondwelling use) [d]	1.02	0.17
Autoregressive coefficient (ρ) in error term	0.80	0.05
Standard deviation (σ) of disturbance term in error term	0.63	0.06

(4) Public fixed capital formation

Explanatory variables	Coefficient	Standard deviation
Constant	-3.12	1.54
Amount of public construction completed [e]	0.61	0.06
Autoregressive coefficient (ρ) in error term	0.99	0.01
Standard deviation (σ) of disturbance term in error term	0.23	0.02

Table 6 (continued)

(5) Export of goods and services

Explanatory variables	Coefficient	Standard deviation
Constant	0.98	0.16
Trade balance (exports) + service account (receipts) [(f)]	1.00	0.02
Autoregressive coefficient (ρ) in error term	0.77	0.06
Standard deviation (σ) of disturbance term in error term	0.70	0.07

(6) Import of goods and services

Explanatory variables	Coefficient	Standard deviation
Constant	17.42	4.24
Trade balance (imports) + service account (expenses) [(g)]	0.52	0.07
Autoregressive coefficient (ρ) in error term	0.99	0.01
Standard deviation (σ) of disturbance term in error term	0.39	0.03

Notes:

1. Monthly GDPs are estimated based on the model where each of the demand components, (1)-(6) above, of GDPs are represented with a linear combination of the explanatory variables listed in the above tables. Error terms in the model follow the AR(1) process where the autoregressive coefficient is ρ and the disturbance term follows a normal distribution $N(0,\sigma^2)$. Refer to Bernanke, Gertler, and Watson (1997) for details.
2. "Core household consumption" is based on the "Household Expenditure Survey", and is generated by excluding some items, of which the fluctuation due to sample revisions is very large, from original data. "Building floor area" is based on the "estimated construction cost for dwelling use" in "Statistics on Building Construction Starts." Both data are defined in the same way as Iizuka and Kawada (2002).
3. "Machinery orders" is deflated by Corporate Goods Price Index (capital goods), and is generated by taking the six-month moving average for the manufacturing sector and 15-month moving average for the nonmanufacturing sector.
4. "Amount of public construction", "trade balance (exports) + service account (receipts)," and "trade balance (imports) + service account (expenses)" are deflated by Corporate Goods Price Index (capital goods), Export Price Index (total average), and Import Price Index (total average), respectively.

Sources:

(a) Statistics on Building Construction Starts.

(b), (c) Machinery Orders Statistics.

(d) Statistics on Building Construction Starts.

(e) Integrated Statistics on Construction Works.

(f), (g) Balance of Payments.

Figure 1　Concept of Macro-Finance Model

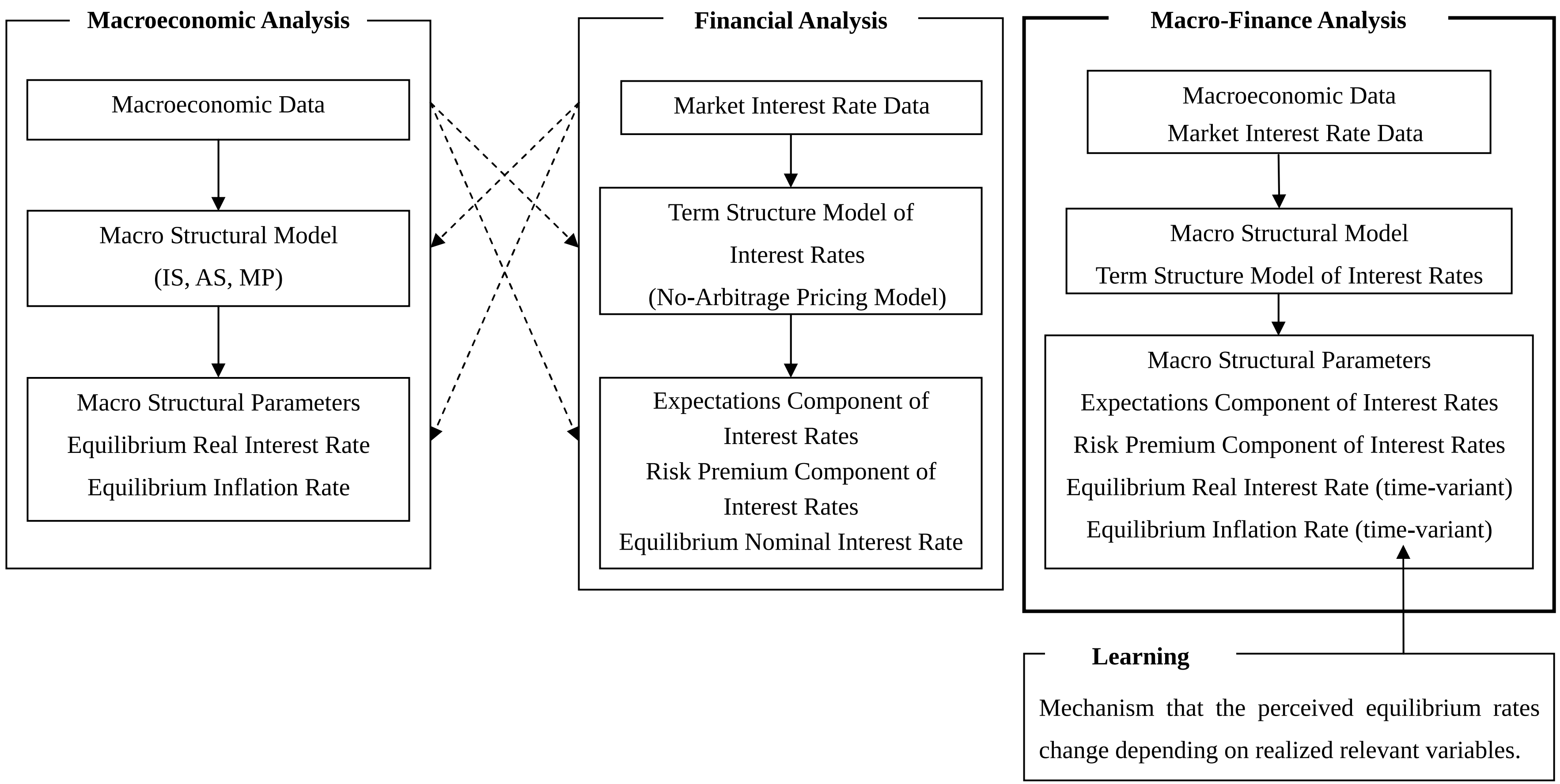


Figure 2 Overview of the Model Structure

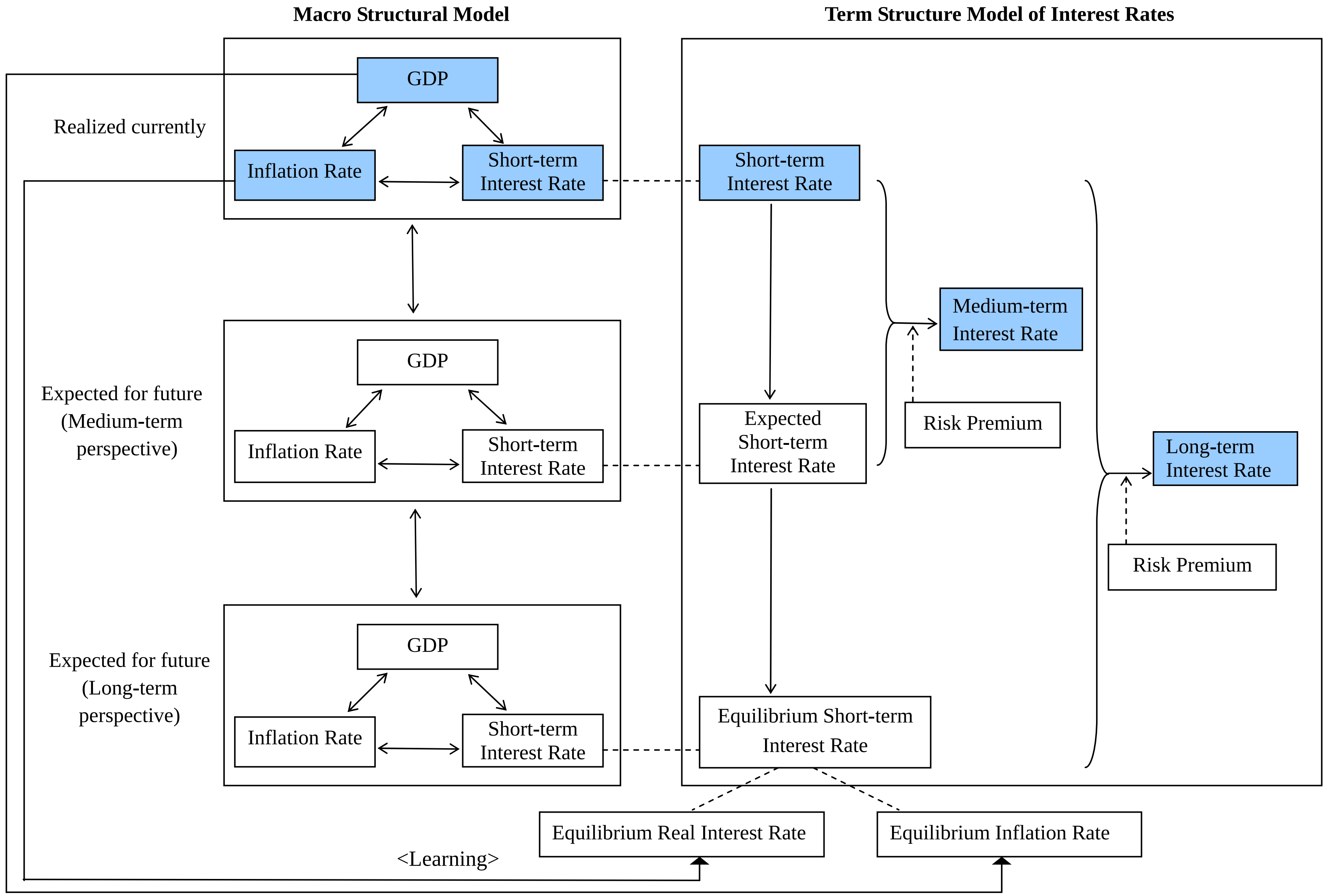
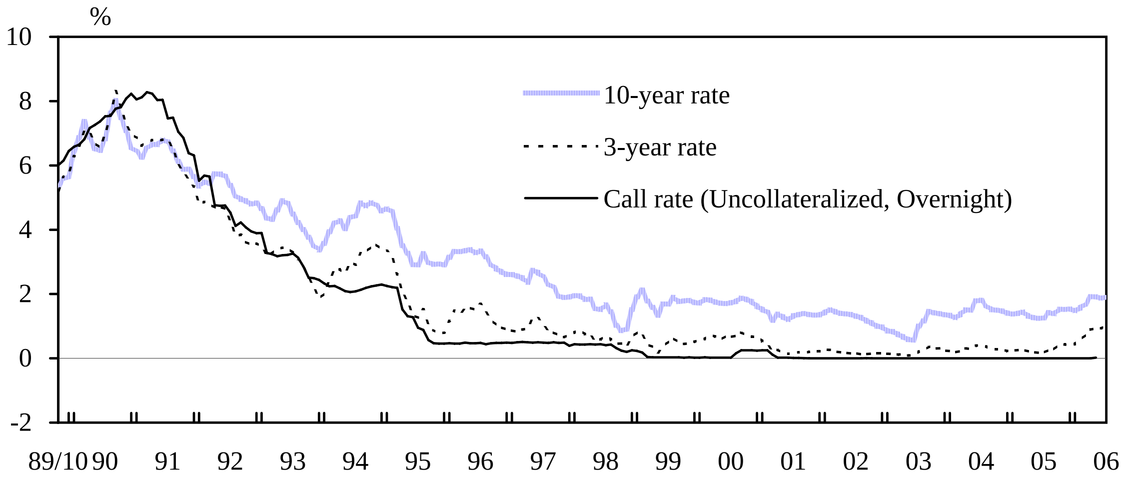


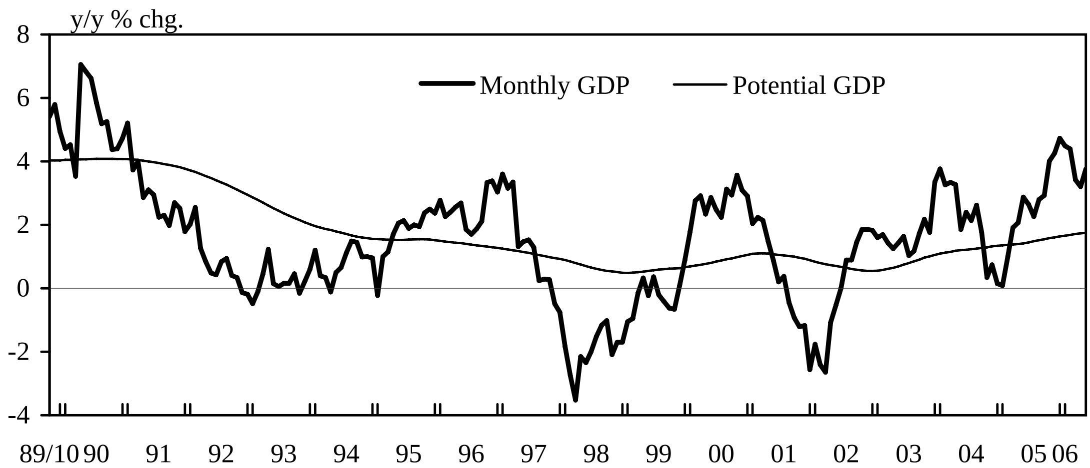
Figure 3 Development of Interest Rates in Japan



Note: Rates are average values of daily closing rates within a month.
Sources: Bank of Japan, Bloomberg.

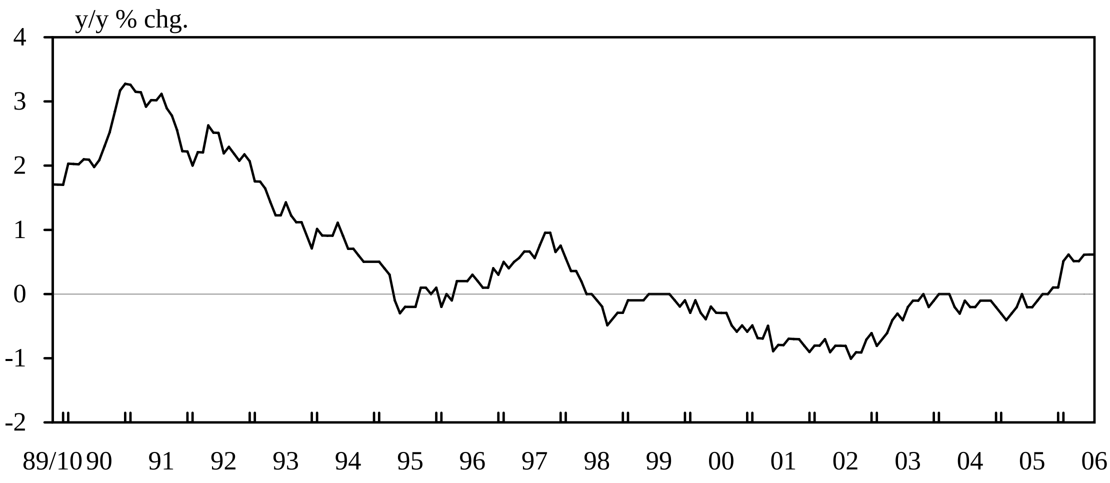
Figure 4 Development of Macroeconomic Data in Japan

(1) Estimated Monthly GDP and Potential GDP



Source: Estimations by authors based on “National Accounts” (Cabinet Office) and other statistics (refer to Table 6 as well).

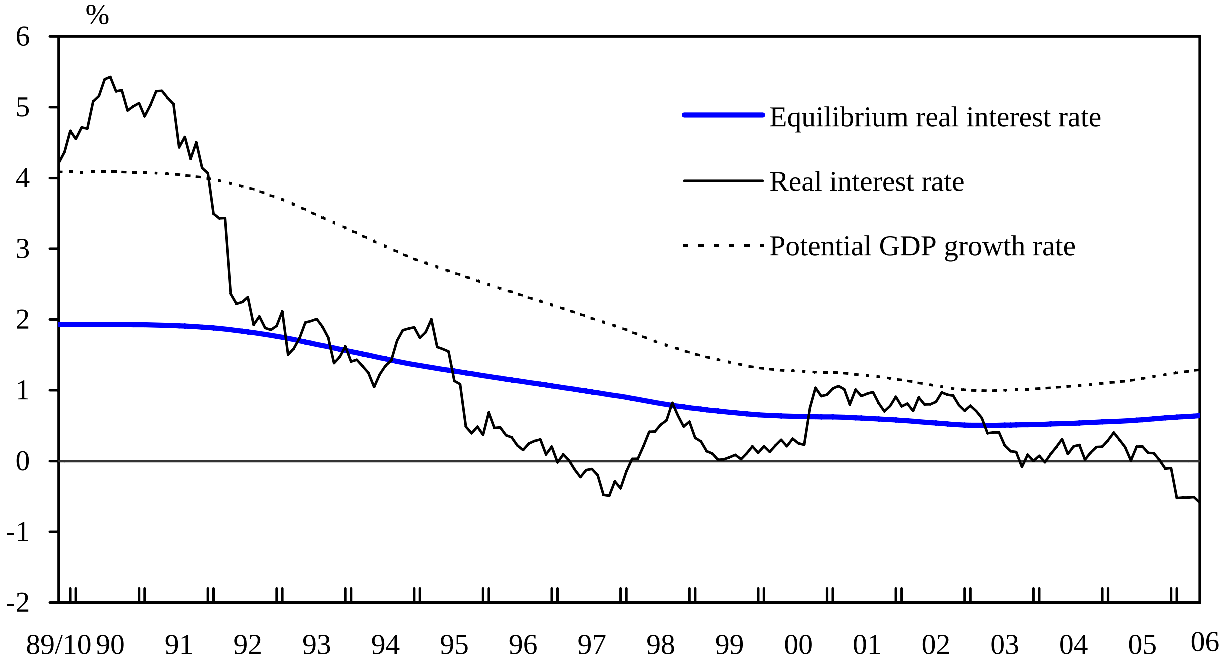
(2) Inflation Rate (nationwide CPI excluding perishables)



Note: Inflation rate is based on the Consumer Price Index (base year 2000) which is adjusted for consumption tax effects.

Source: Ministry of Internal Affairs and Communications

Figure 5 Estimation of Equilibrium Real Interest Rate



Note: The real interest rate is calculated by deflating the call rate (uncollateralized, overnight) with the year-on-year change in CPI (excluding perishables).

Figure 6 Estimation of Equilibrium Inflation Rate

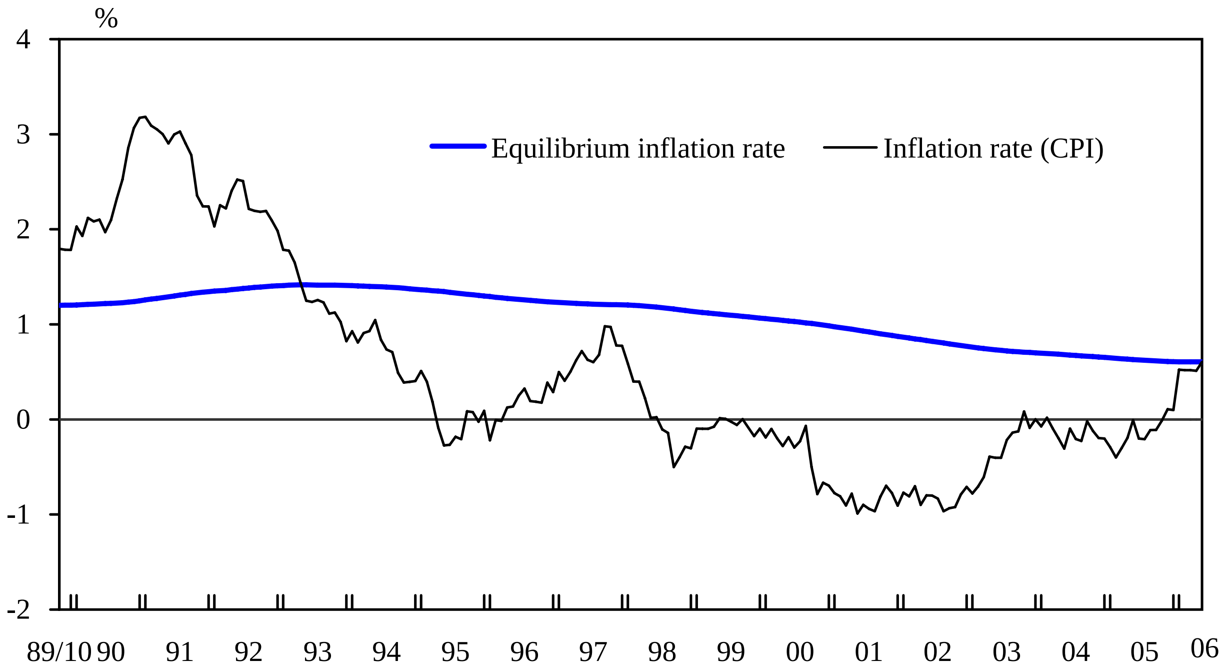


Figure 7 Impulse Responses of Macroeconomic Variable (GDP gap, inflation rate, short-term interest rate, and potential growth rate)

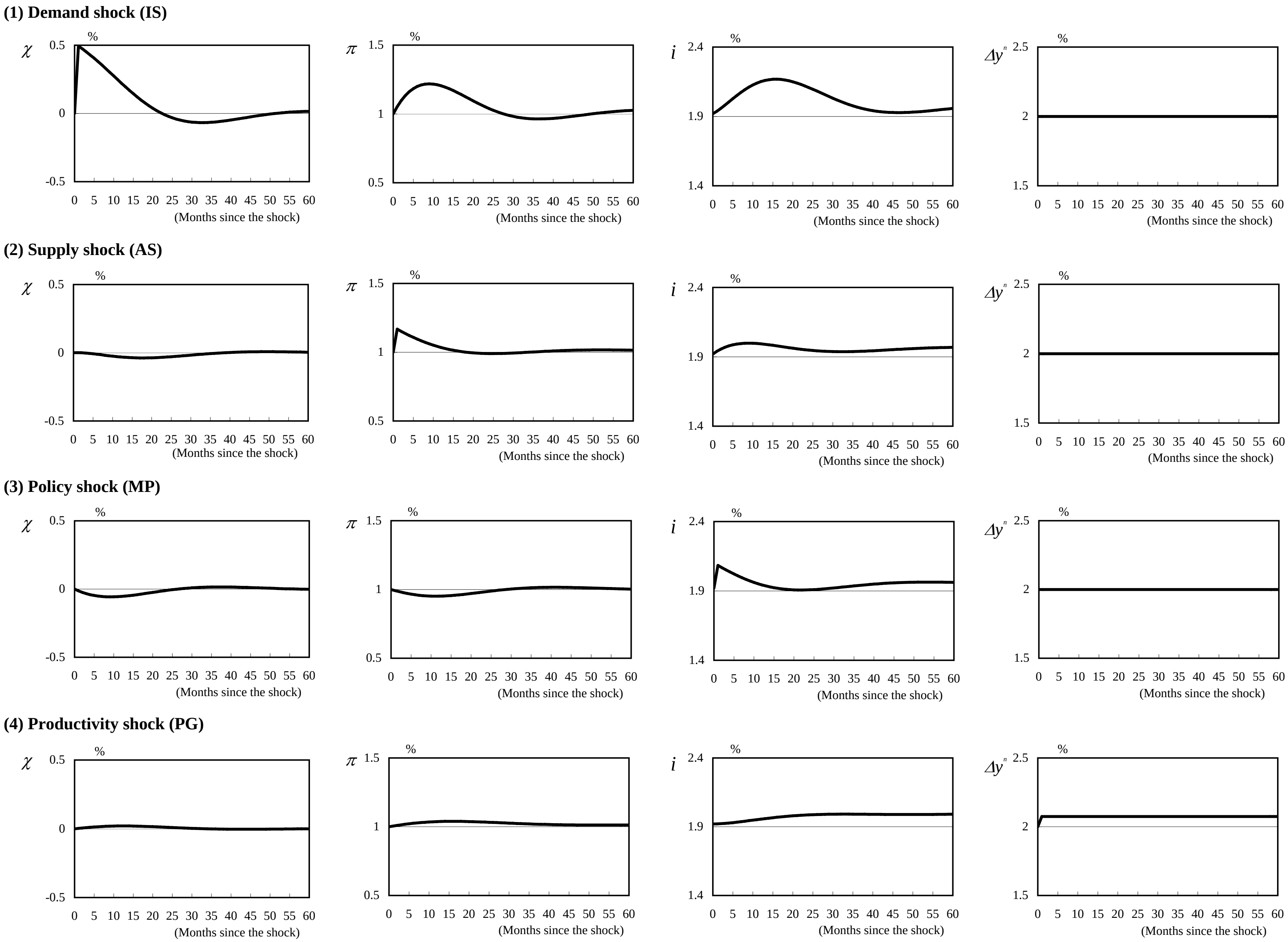
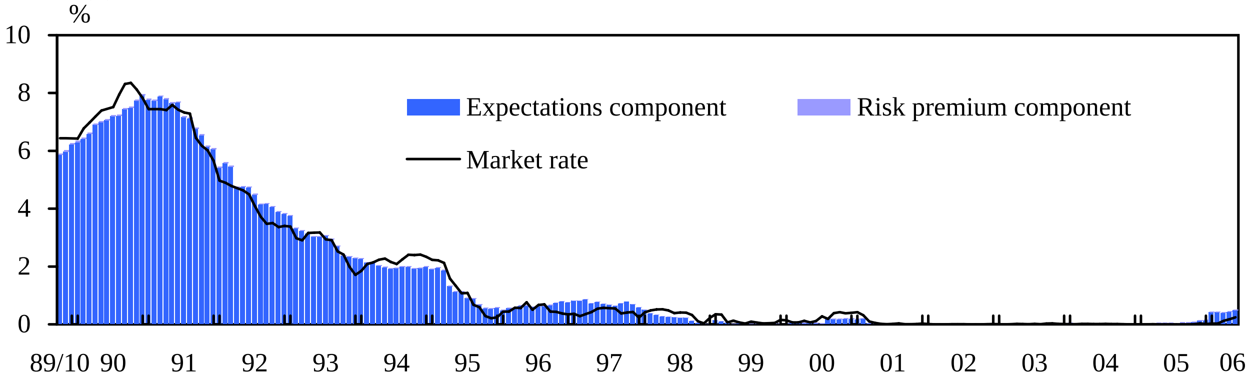
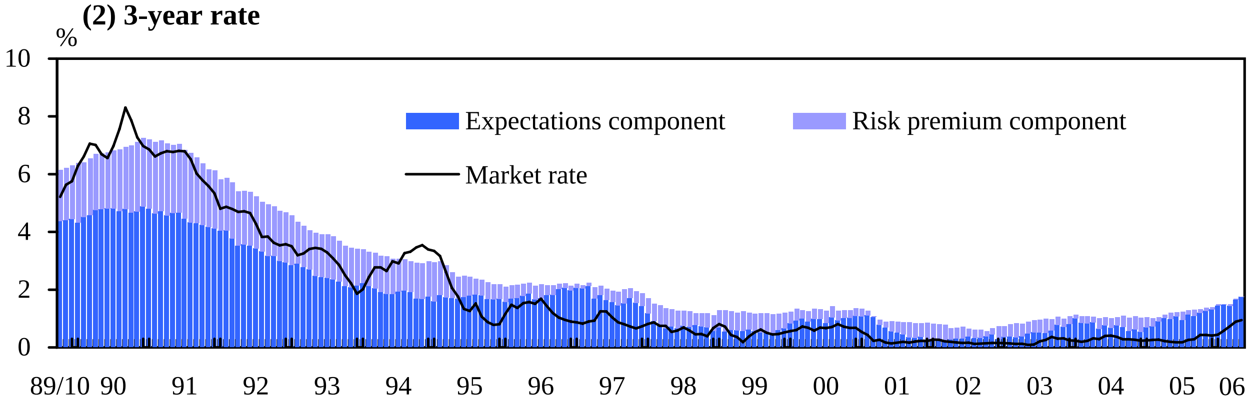


Figure 8 Estimation of Expectations Component and Risk Premium Component of Medium- to Long-Term Interest Rates

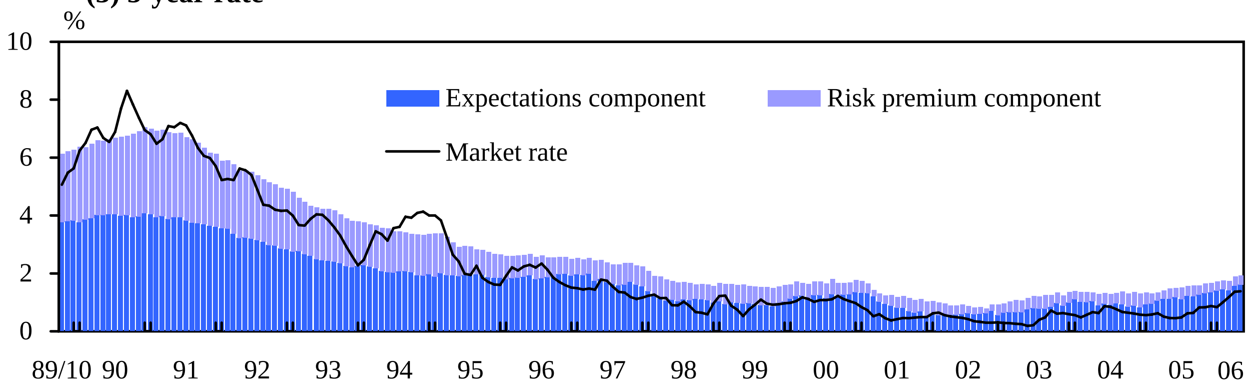
(1) 6-month rate



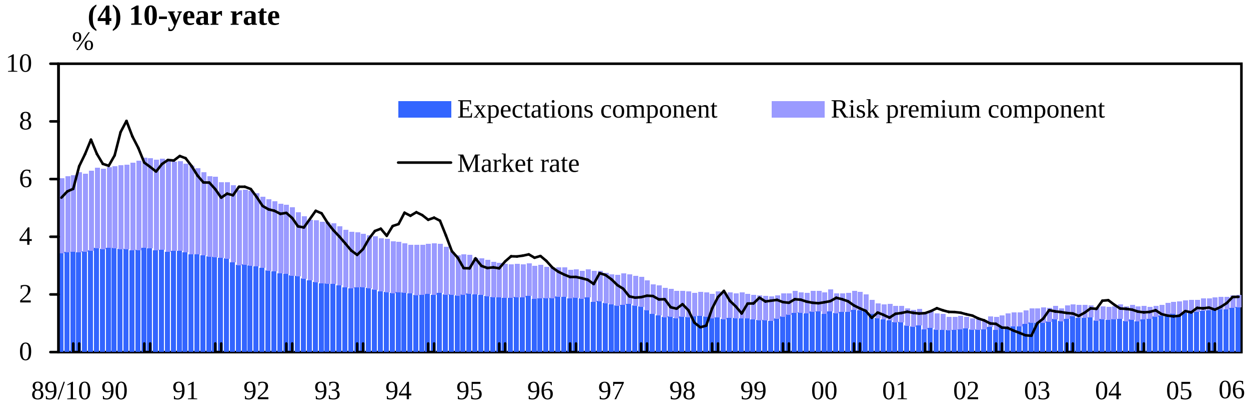
(2) 3-year rate



(3) 5-year rate



(4) 10-year rate



Note: Market rates are average values of the daily closing rates within a month.

Figure 9 Decomposition of Expectations Component of Medium- to Long-Term Interest Rates

(1) Expectations component of 6-month rate

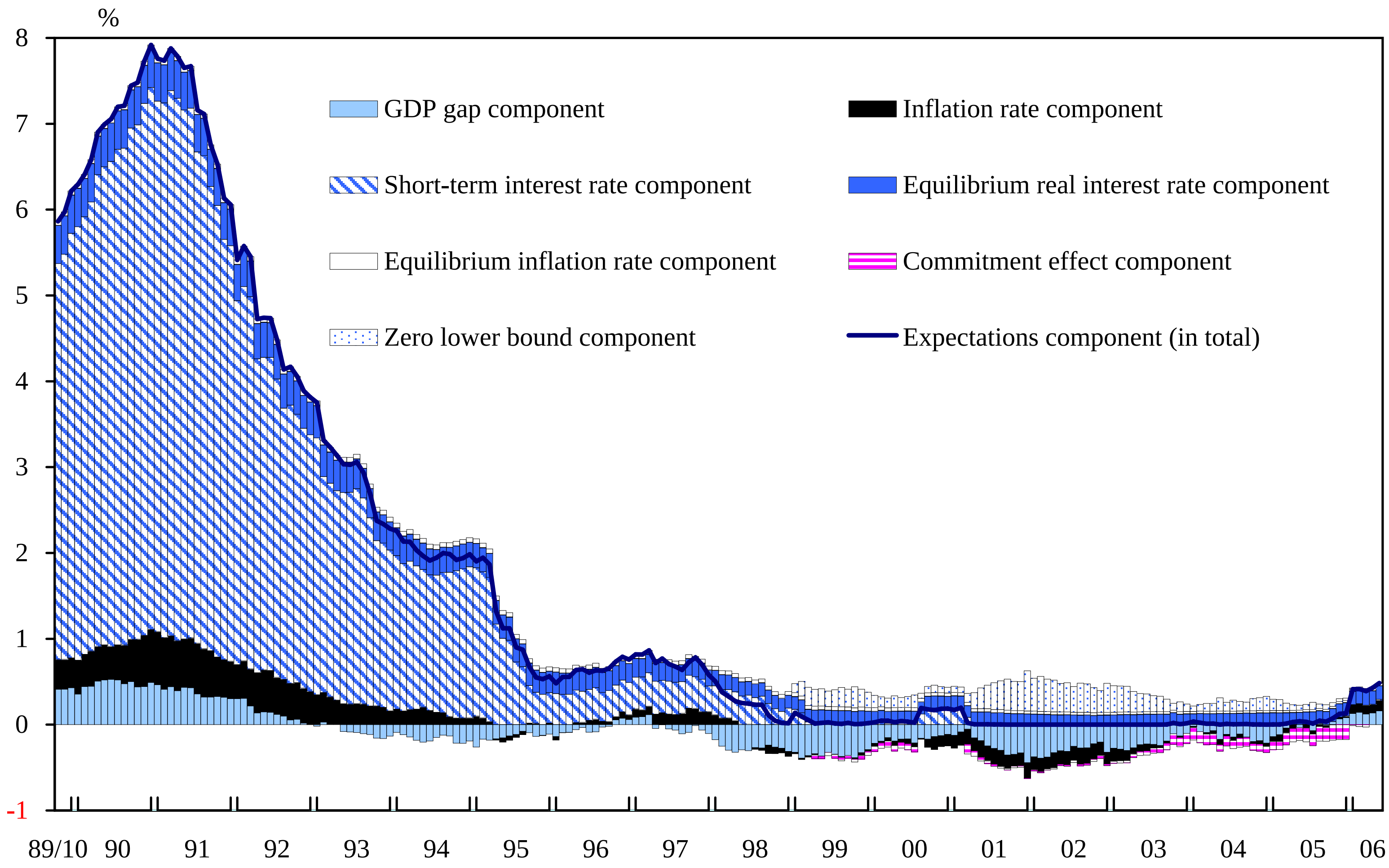


Figure 9 Decomposition of Expectations Component of Medium- to Long-Term Interest Rates

(2) Expectations component of 3-year rate

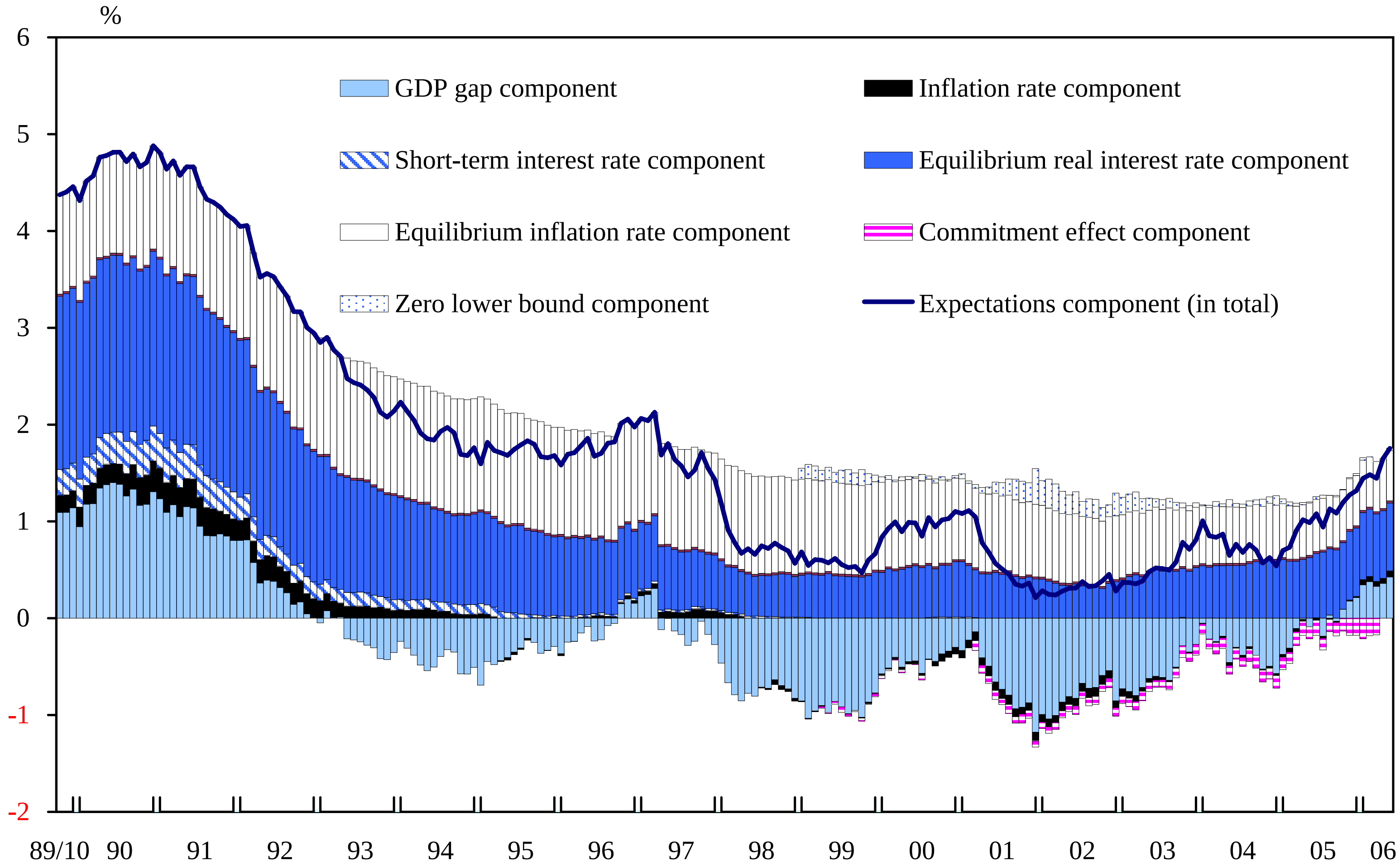


Figure 9 Decomposition of Expectations Component of Medium- to Long-Term Interest Rates

(3) Expectations component of 5-year rate

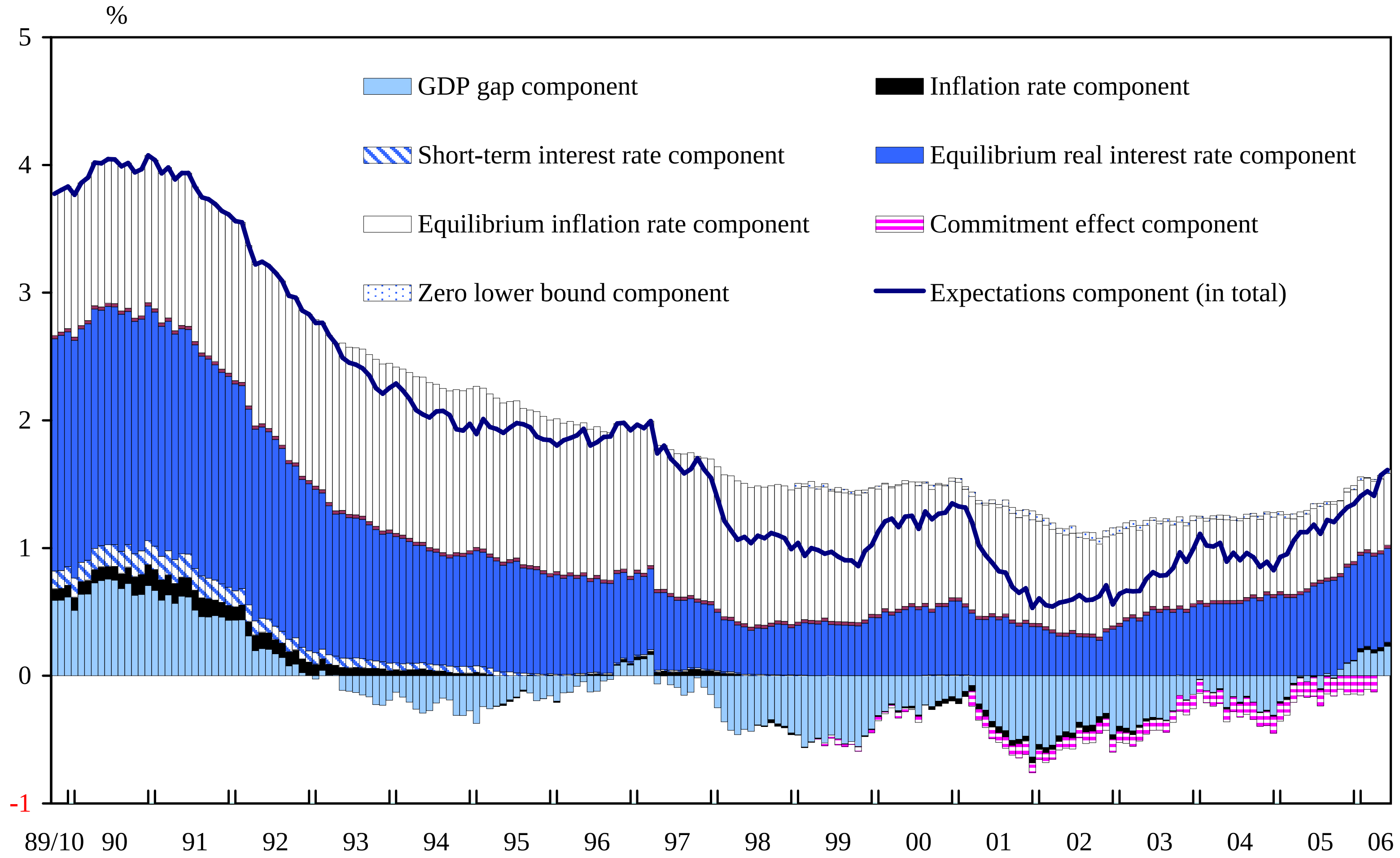


Figure 9 Decomposition of Expectations Component of Medium- to Long-Term Interest Rates

(4) Expectations component of 10-year rate

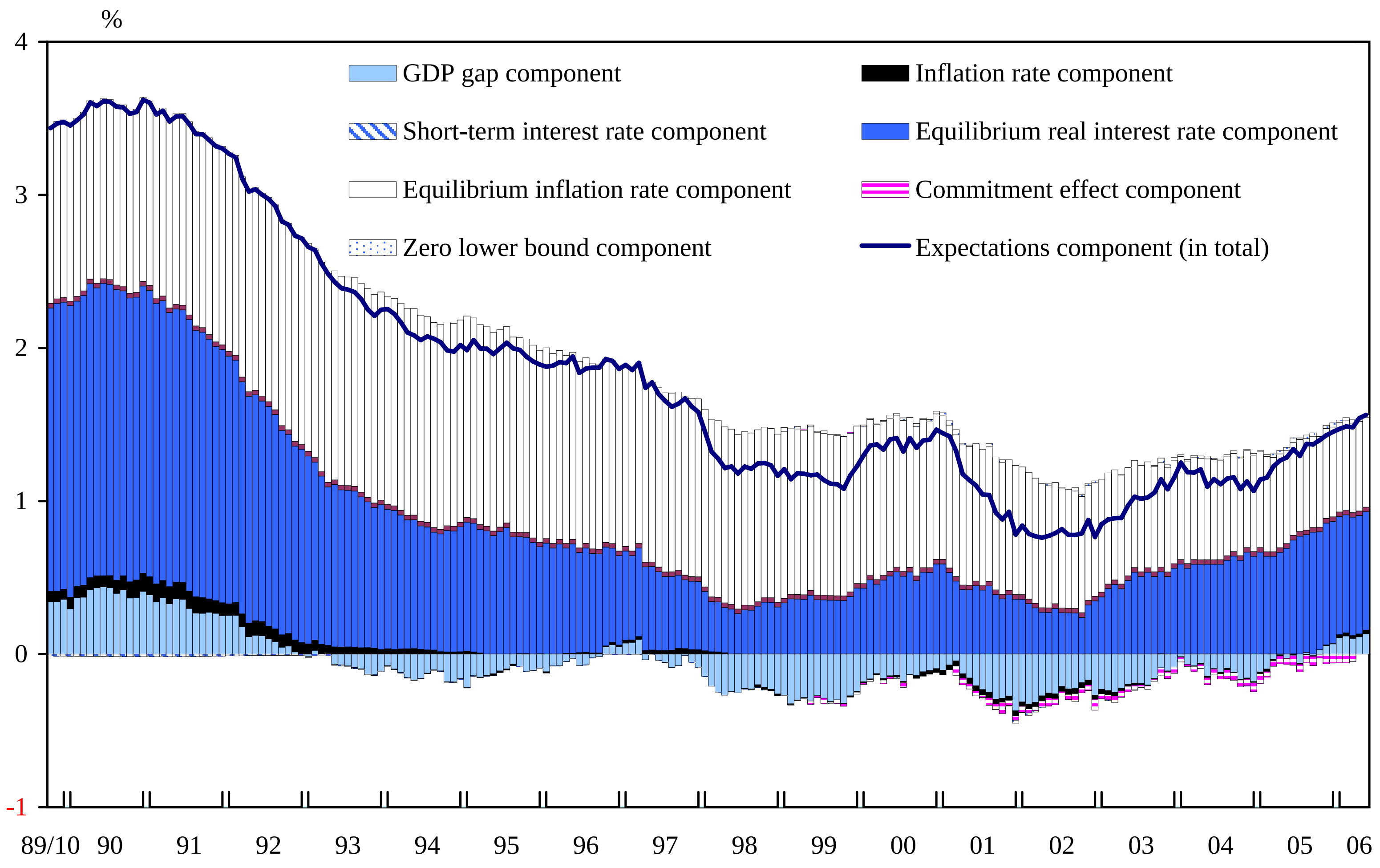
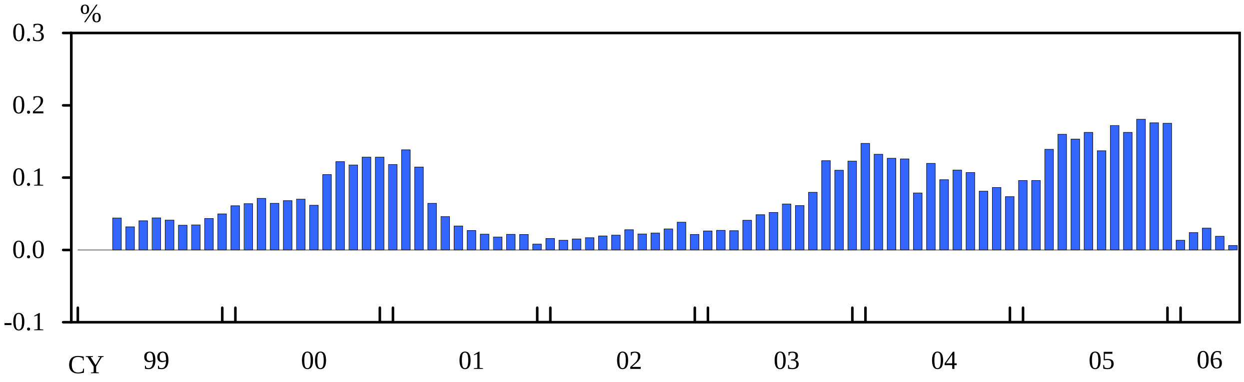
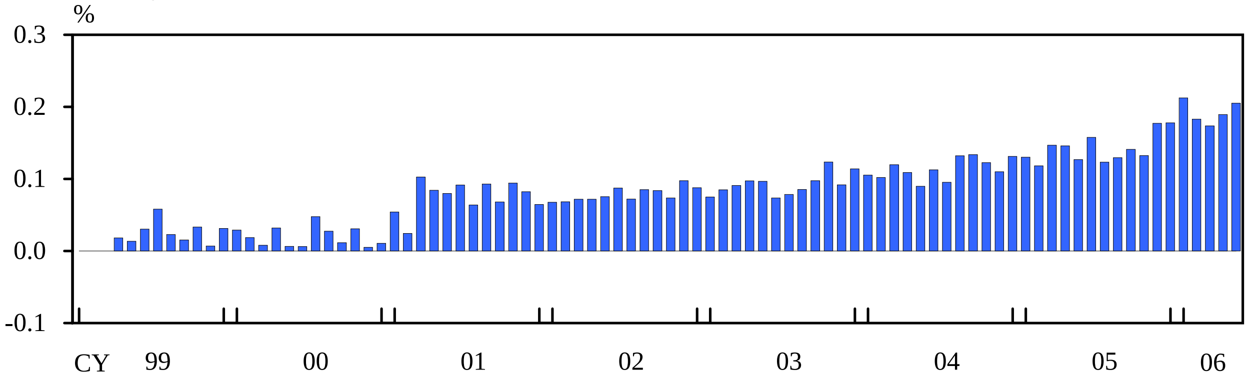


Figure 10 Effects of Zero Interest Rate Commitment to Lower Medium- to Long-Term Interest Rates

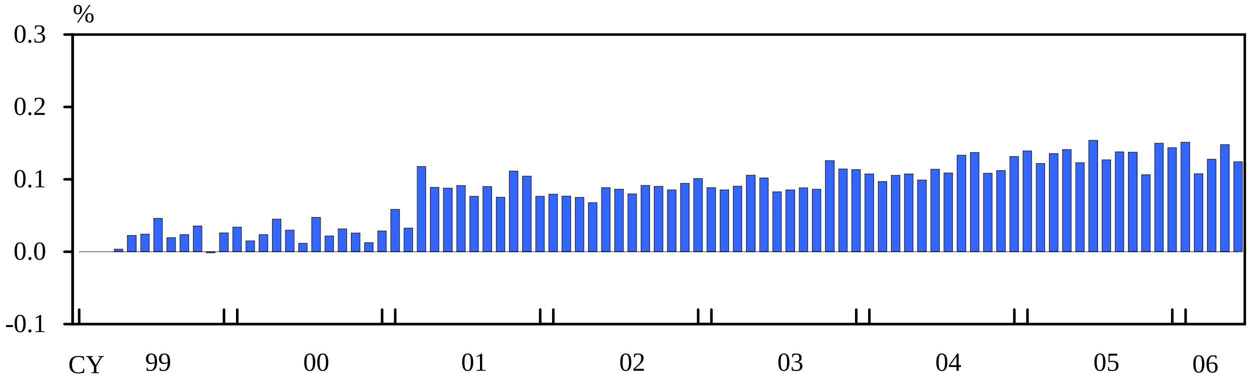
(1) 6-month rate



(2) 3-year rate



(3) 5-year rate



(4) 10-year rate

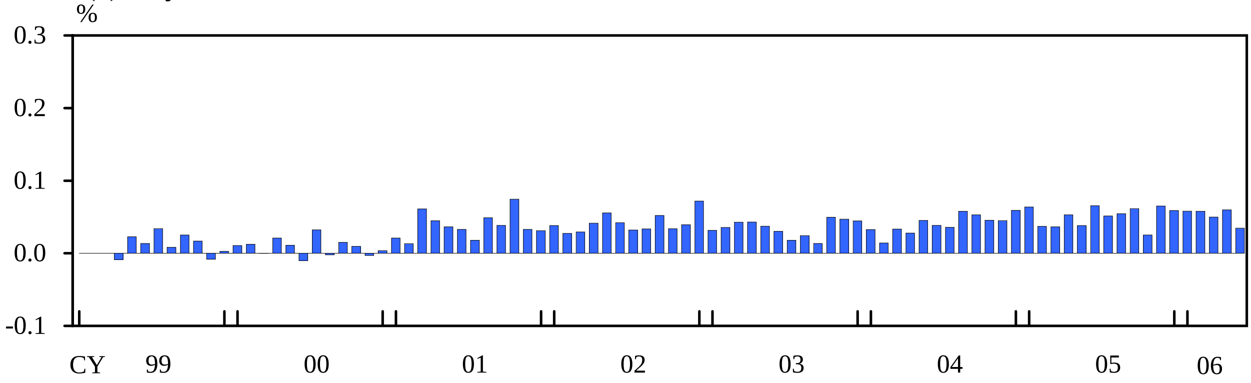
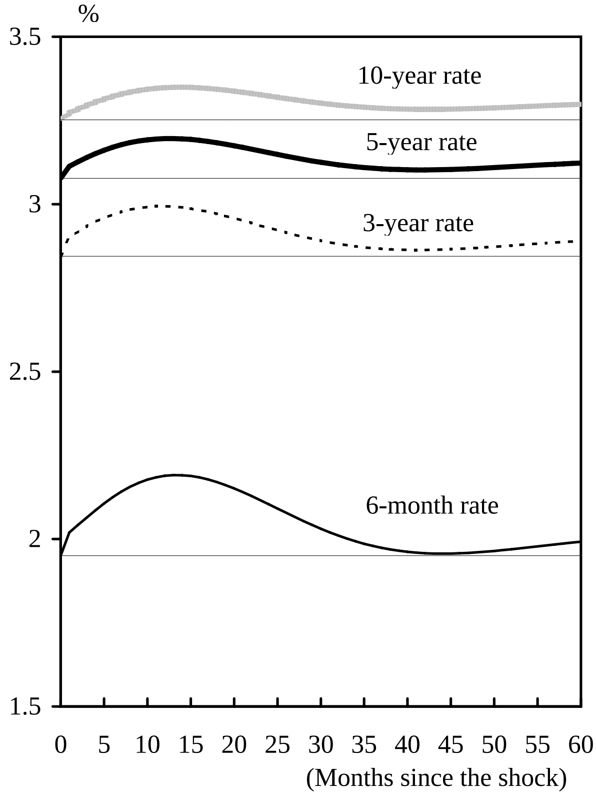
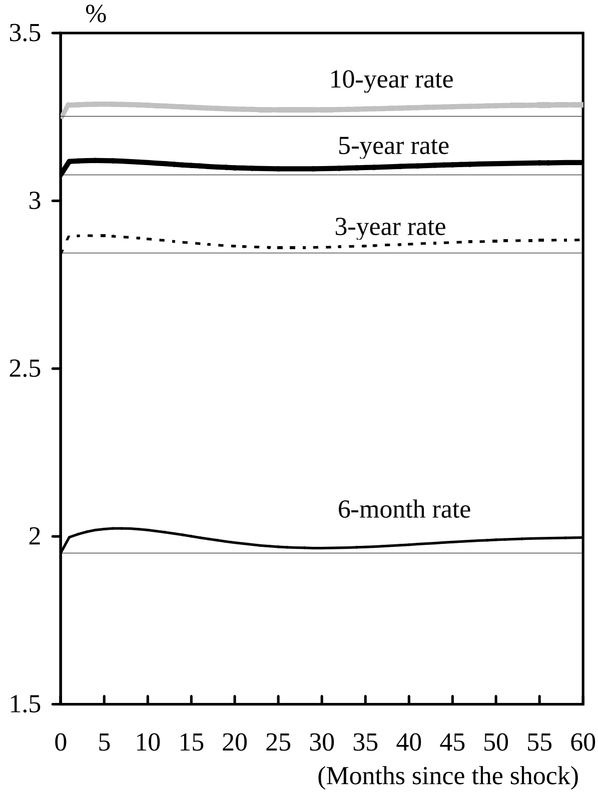


Figure 11 Impulse Responses of Medium- to Long-Term Interest Rates

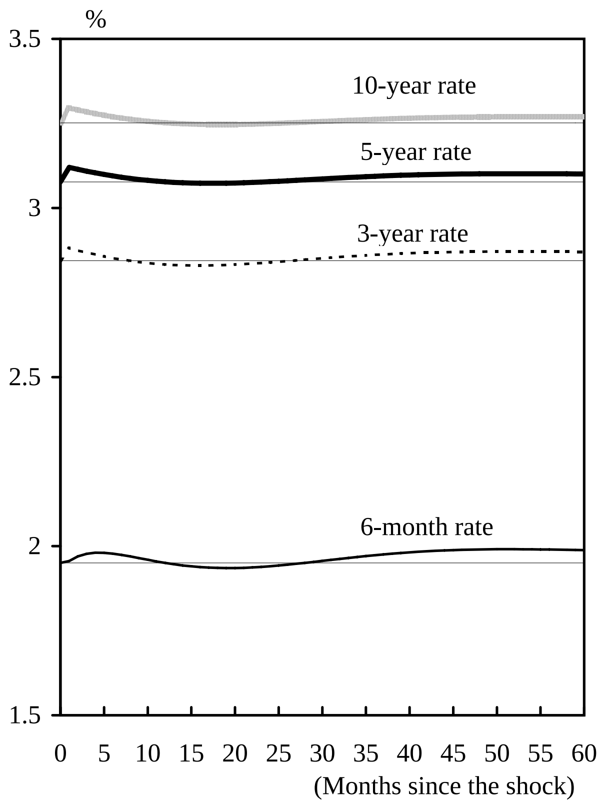
(1) Demand shock (IS)



(2) Supply shock (AS)



(3) Policy shock (MP)



(4) Productivity shock (PG)

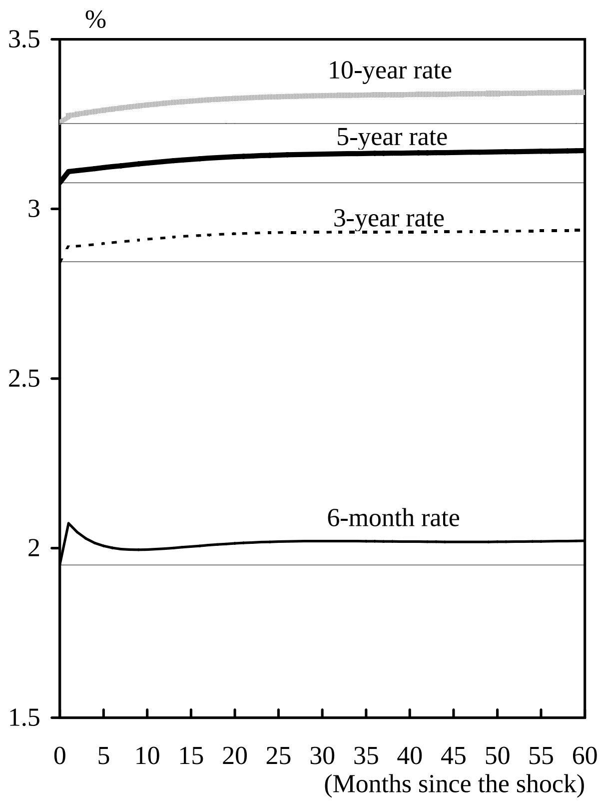


Figure 12 Model Errors for 10-Year Rate