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Monetary Policy and Sunspot Fluctuation in the U.S. and the Euro Area

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Monetary Policy and Sunspot Fluctuation in the U.S. and the Euro Area*

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Abstract

We estimate a two-country open economy version of the New Keynesian DSGE model for the U.S. and the Euro area, using Bayesian techniques that allow for both determinacy and indeterminacy of the equilibrium. Our empirical analysis shows that the worldwide equilibrium is indeterminate due to a passive monetary policy in the Euro area, even if U.S. policy is aggressive enough. We demonstrate that the impulse responses under indeterminacy exhibit different dynamics than those under determinacy and that sunspot shocks affect the Euro economy to a substantial degree, while the propagation of sunspots to the U.S. is limited.

Keywords: Monetary Policy, Indeterminacy, Sunspot Shock, Open Economy Model, Bayesian Analysis

JEL Classification: C11, C62, E52, F41

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1 Introduction

Macroeconomic stability is a primary concern for central banks. In this respect, researchers have emphasized the possibility that certain monetary policy rules lead to indeterminacy of rational expectations equilibrium, which is considered an unfavorable outcome, since the economic system might be unexpectedly volatile. It is well known that in the prototypical New Keynesian dynamic stochastic general equilibrium (DSGE) model, indeterminacy can occur if the monetary authority does not raise the interest rate aggressively enough in response to an increase in inflation, which is called the Taylor principle. In a two-country open economy setting, Bullard and Schaling (2006) discuss from a theoretical perspective that the worldwide equilibrium can be indeterminate when one country satisfies the Taylor principle and the other does not.[1]

The primary purpose of this paper is to examine whether the worldwide equilibrium is determinate or not based on a two-country open economy version of the New Keynesian DSGE model for the U.S. and the Euro area. Although the previous studies such as Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004) have shown that U.S. monetary policy post-1982 is consistent with determinacy in a closed economy framework, the equilibrium might be indeterminate in a two-country open economy framework if the monetary policy in the Euro area is not aggressive enough. In the empirical literature on European monetary policy, while Clarida, Galí, and Gertler (1998) and Gerdesmeier and Roffia (2004) among others report that the reaction functions for the Bundesbank or the ECB respond to inflation aggressively enough, Faust, Rogers, and Wright (2001) and Gerlach (2007) point out that the ECB's policy might put a higher weight on the output gap than on inflation. The findings in the latter studies imply the possibility of equilibrium indeterminacy in our present context, although they do not explicitly discuss it.

To investigate the possibility of worldwide indeterminacy, we estimate the model over a parameter space in which the equilibrium can be both determinate and indeterminate, using the system-based Bayesian methods developed by Lubik and Schorfheide (2004).[2] The

[1] A number of papers have investigated conditions for determinacy in small open economy settings. For instance, Carlstrom and Fuerst (1999) report that determinacy conditions in a closed economy framework carry over to a small open economy, whereas De Fiore and Liu (2005) point out the importance of trade openness in generating a determinate equilibrium.

[2] While a growing number of papers have estimated open economy DSGE models using Bayesian techniques, most of the studies estimate small open economy models. See Adolfson, Laséen, Lindé, and Villani (2007), Lubik and Schorfheide (2007), and Justiniano and Preston (2008) among others. On the other

full-information likelihood-based approach has advantages over single-equation GMM estimation in that we can implicitly generate an optimal set of instruments for the coefficients of the equations and provide an overall measure of the time series fit of the model. Based on this measure, we can construct posterior weights for the determinacy and indeterminacy regions of the parameter space. Our empirical analysis suggests that the equilibrium is indeterminate due to the passive monetary policy in the Euro area. This finding casts doubt on the common practice that calibrates a European monetary policy similar to that of the U.S. or estimates policy rules in the parameter space leading only to determinacy.

Another contribution of this paper is to investigate the propagation of shocks and the dynamic behavior under indeterminacy in the two-country model. DSGE models under indeterminacy exhibit two remarkable features: First, the propagation of fundamental shocks is ambiguous, since agents are unable to coordinate on a particular equilibrium among multiple equilibria. Second, sunspot shocks, which are non-fundamental disturbances, affect the equilibrium dynamics. Taking into account these features, we demonstrate that impulse responses to fundamental shocks under indeterminacy are in stark contrast to those under determinacy, and that while sunspot shocks affect the Euro economy to a substantial degree, the transmission of sunspots to the U.S. is limited.

Additionally, we examine whether sunspot shocks are helpful in explaining the exchange rate dynamics. Lubik and Schorfheide (2006) estimate a richer two-country model, and conclude that their model as well as those of other previous studies is still far from explaining the exchange rate movements. Since the model under indeterminacy generally exhibits more persistent dynamics, it might be a good candidate for explaining persistence in the observed dynamics. However, our results are subject to the same difficulties as the previous studies, even if we allow for sunspots, which suggests that self-fulfilling expectations are not the main source of the exchange rate dynamics.

The most closely related paper is Bullard and Singh (2007). They consider a multi-country version of Clarida, Galí, and Gertler's (2002) model and numerically characterize determinacy conditions that depend on monetary policy parameters. They then estimate monetary policy rules for the U.S., Germany (the Euro area in a recent period) and Japan using GMM, and point out the possibility of worldwide indeterminacy and transmission of

hand, empirical studies of two-country settings are scarce. Lubik and Schorfheide (2006) is one of the first attempts to estimate a two-country model. More recently, Adjemian, Pariès, and Smets (2008) estimate a richer two-country model that incorporates the wide range of nominal and real frictions. These papers consider only determinacy regions for parameter estimation.

sunspots across borders even when the U.S. monetary policy is consistent with determinacy. Our approach is methodologically different in that we apply the system-based Bayesian estimation techniques that help to estimate DSGE models with cross-equation restrictions, coping well with misspecification and identification problems. While our main findings about worldwide indeterminacy are the same as those of Bullard and Singh (2007), our empirical results with regard to the low degree of sunspot transmission across countries contrast with their simulation results.

This paper is organized as follows. The next section presents the two-country DSGE model that is used for our analysis. In Section 3, we review a full set of rational expectations solutions, which allows for both determinacy and indeterminacy, and discuss the sources of indeterminacy in the two-country model. Section 4 explains the Bayesian estimation strategy. In Section 5, our benchmark estimation results are presented and we explore whether the data suggests equilibrium indeterminacy and to what extent sunspot shocks affect the equilibrium dynamics in the U.S. and the Euro area. Section 6 is to check the robustness of our findings under alternative dynamic structures and monetary policy rules. Section 7 is the conclusion.

2 The Model

The model is a two-country extension of the standard New Keynesian monetary DSGE model, and is considered to be a two-country version of Galí and Monacelli (2005) or a simplified version of Lubik and Schorfheide (2006). The world economy consists of the U.S. (the domestic or home country) and the Euro area (the foreign country), which are assumed to be of the same size.[3] In each country, the representative household gains utility from aggregate consumption composed of home and foreign goods, and trades state contingent assets in complete international asset markets. Monopolistically competitive firms produce differentiated goods, and are subject to Calvo-type staggered price-setting. Monetary authorities adjust the nominal interest rates in response to inflation and output growth. While we assume symmetric consumer preferences, the two regions differ in price-setting, monetary policy and fundamental shocks. The assumptions with regard to preferences, technology and complete financial markets give us a highly tractable framework for the open economy so that it can be comparable to the simplest New Keynesian closed economy.

[3] Indeed, the two regions are roughly the same size and have a similar per capita income.

2.1 Households

The infinitely lived household in the home country maximizes the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t/A_{W,t})^{1-\tau}}{1-\tau} - N_t \right],$$

where β is a discount factor and $\tau^{-1} > 0$ is the elasticity of intertemporal substitution for consumption. $A_{W,t}$ is a non-stationary world-wide technology component that follows a random walk with drift:

$$\ln A_{W,t} = \gamma + \ln A_{W,t-1} + \widetilde{z}_t,$$

where $\widetilde{z}_t$ is a shock to world-wide technology growth. N_t denotes the labor supply. C_t is a composite consumption index defined by

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $C_{H,t}$ is consumption of domestic goods, and $C_{F,t}$ is imported goods from a foreign country. $0 \leq \alpha \leq 1$ represents the degree of openness, which is inversely related to home bias in the preference. $\eta > 0$ denotes the elasticity of substitution between domestic and foreign goods.

The maximization problem above is subject to the budget constraint:

$$P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + E_t \left[\Delta_{t,t+1} D_{t+1} \right] \leq D_t + W_t N_t + T_t,$$

where $P_{H,t}$ is the price of domestic goods and $P_{F,t}$ is the price of imported goods in terms of domestic currency. D_{t+1} is the nominal payoff from a portfolio of assets, $\Delta_{t,t+1}$ is the stochastic discount factor, W_t is the nominal wage, and T_t is lump-sum taxes of transfers.

The consumer price index (CPI) is defined as the minimum expenditure required to buy one unit of the composite goods C_t, given the prices of home and foreign goods:

$$P_t \equiv \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{1}$$

Then, the optimal allocation of any given expenditure between domestic and imported goods yields:

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \tag{2}$$

and

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t. \tag{3}$$

Accordingly, the first order conditions for the households' maximization problem are derived in aggregate terms:

$$\left(\frac{C_t}{A_{W,t}}\right)^{-\tau} = A_{W,t}\lambda_t \tag{4}$$

$$\frac{1}{\lambda_t} = \frac{W_t}{P_t}, \tag{5}$$

and

$$\Delta_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}, \tag{6}$$

where λ_t is the Lagrange multiplier interpreted as the marginal utility of income. Let R_t denote the gross return on a nominal one-period discount bond. Then, from (6), we obtain the following Euler equation:

$$R_t^{-1} = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\right], \tag{7}$$

where $R_t^{-1} = E_t\Delta_{t,t+1}$ is the price of the discount bond.

2.2 Price Setting Firms and Market Clearing

In the home country, each firm, indexed by j, produces differentiated goods $Y_{H,t}(j)$ using the following technology:

$$Y_{H,t}(j) = A_{W,t}A_tN_t(j),$$

where A_t is a stationary and country-specific technology shock.

The monopolistically competitive firms are assumed to set their prices with probability $1-\theta_H$, as in Calvo (1983). Then, θ_H represents a degree of price stickiness. In resetting a new price in period t, each firm j maximizes:

$$E_t\sum_{k=0}^{\infty}\theta_H^k\Delta_{t,t+k}Y_{H,t+k}(j)\left[\overline{P}_{H,t}(j) - P_{H,t+k}MC_{H,t+k}\right],$$

subject to:[4]

$$Y_{H,t+k}(j) = \left(\frac{\overline{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\mu}Y_{H,t+k},$$

where $\overline{P}_{H,t}(j)$ is the price adjusted by the firm j at period t, $MC_{H,t} = \frac{W_t}{P_{H,t}A_{W,t}A_t}$ denotes the real marginal cost, $Y_{H,t}$ is the aggregate output in the home country.

[4]The demand curve that each firm faces is derived from the definition of aggregate demand as the composite of individual goods and the corresponding prices in the framework of Dixit and Stiglitz (1977): $Y_{H,t} = \left[\int_0^1 Y_{H,t}(j)^{\frac{\mu-1}{\mu}}dj\right]^{\frac{\mu}{\mu-1}}$ and $P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\mu}dj\right]^{\frac{1}{1-\mu}}$.

Then, $\overline{P}_{H,t}(j)$ must satisfy the first order condition:

$$\sum_{k=0}^{\infty}\theta_H^k E_t\left[\Delta_{t,t+k}Y_{H,t+k}\left(\overline{P}_{H,t}-\frac{\mu}{\mu-1}P_{H,t+k}MC_{H,t+k}\right)\right]=0. \tag{8}$$

where the j subscript is dropped since all firms resetting prices in any given period will choose the same price. Under the present price-setting structure, the domestic price index is given by:

$$P_{H,t}=\left[\theta_H P_{H,t-1}^{1-\mu}+(1-\theta_H)\,\overline{P}_{H,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}. \tag{9}$$

2.3 Monetary Policy

As in Lubik and Schorfheide (2006), the central bank adjusts the nominal interest rate R_t in response to deviation of inflation $\pi_t=\log(P_t/P_{t-1})$ and output growth $\log(Y_{H,t}/Y_{H,t-1})$ from each steady state value. The monetary policy rule is of the following form:

$$\log\left(\frac{R_t}{R}\right)=\rho_r\log\left(\frac{R_{t-1}}{R}\right)+(1-\rho_r)\left[\psi_\pi\log\left(\frac{\pi_t}{\pi}\right)+\psi_y\log\left(\frac{Y_{H,t}/Y_{H,t-1}}{\gamma}\right)\right]+\varepsilon_{R,t}, \tag{10}$$

with $0\le\rho_r<1$, $\psi_\pi>0$ and $\psi_y>0$. $\varepsilon_{R,t}\sim N(0,\sigma_R^2)$ is a serially uncorrelated exogenous policy shock that can be interpreted as an unsystematic component of the monetary policy.

2.4 The Foreign Economy and International Dependence

We assume that the foreign country is characterized by the same functional forms and household preferences as the home country, but can differ in terms of price setting behavior and monetary policy. The equations that describe the foreign economy are then obtained in a straightforward way by substituting the relevant variables with subscript F and superscript $*$ into the expressions above.

The terms of trade in domestic currency are defined as:

$$q_t=\frac{P_{H,t}}{P_{F,t}}, \tag{11}$$

while the foreign terms of trade are:

$$q_t^*=\frac{P_{F,t}^*}{P_{H,t}^*}. \tag{12}$$

Under the assumption that the law of one price holds, then:

$$P_{F,t}=e_tP_{F,t}^*,$$

and

$$P_{H,t}=e_tP_{H,t}^*,$$

where e_t is the nominal exchange rate (the price of the foreign currency in terms of the home currency). Combining this purchasing parity (PPP) condition with the definition of the terms of trade, we have:

$$q_t = \frac{1}{q_t^*}. \tag{13}$$

We define the real exchange rate expressed in home currency as:

$$s_t = \frac{e_t P_t^*}{P_t}, \tag{14}$$

and the foreign real exchange rate as:

$$s_t^* = \frac{P_t/e_t}{P_t^*}, \tag{15}$$

so that $s_t^* = s_t^{-1}$.

Given the international tradability of state contingent assets, the stochastic discount factors in the two countries have to be equalized in equilibrium:

$$\Delta_{t,t+1} = \beta \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right] = \beta \left[\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{P_t^*}{P_{t+1}^*} \frac{e_t}{e_{t+1}} \right]. \tag{16}$$

Then, assuming complete international asset markets, the risk-sharing condition between households in the two countries is:

$$\lambda_t = \lambda_t^* s_t^*. \tag{17}$$

Goods market clearing in each country requires that:

$$Y_{H,t} = C_{H,t} + C_{H,t}^* + G_{H,t}, \tag{18}$$

and

$$Y_{F,t}^* = C_{F,t}^* + C_{F,t} + G_{F,t}^*, \tag{19}$$

where government expenditures $G_{H,t}$ and $G_{F,t}^*$ are exogenous and work as a demand shock to each economy.

2.5 Detrending and Linearization

Since the model economy described above contains a non-stationary world-wide technology shock $A_{W,t}$, we detrend the associated real variables by the level of $A_{W,t}$ to induce stationarity. The detrended model then has a non-stochastic steady state. We proceed by log-linearizing the first-order conditions and market clearing conditions together with definitional relationships. All the variables below are expressed as log-deviation from their respective steady states such that $\widetilde{x}_t = \log x_t - \log \overline{x}$.

Detrending and log-linearizing the first order conditions (4) and (7) for the home consumer problem, we obtain the marginal utility of consumption and the Euler equation:

$$-\widetilde{\lambda}_t = \tau\widetilde{c}_t,$$

and

$$-\widetilde{\lambda}_t = -E_t\widetilde{\lambda}_{t+1} - (\widetilde{R}_t - E_t\widetilde{\pi}_{t+1}) + E_t\widetilde{z}_{t+1}.$$

Recall that $\widetilde{z}_t = \Delta\widetilde{A}_{W,t}$. CPI inflation is derived from the definition (1):

$$\widetilde{\pi}_t = \alpha\widetilde{\pi}_{F,t} + (1-\alpha)\widetilde{\pi}_{H,t}$$

The firms' optimal price setting (8) together with (9) can be approximated by the following Phillips curve relationship:

$$\widetilde{\pi}_{H,t} = \beta E_t\widetilde{\pi}_{H,t+1} + \kappa_H\widetilde{mc}_t,$$

with $\kappa_H = \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}$. Using the condition (5), the real marginal cost for domestic firm can be written as:

$$\widetilde{mc}_t = -\widetilde{\lambda}_t - \alpha\widetilde{q}_t - \widetilde{A}_t.$$

A linearized version of the monetary policy rule is:

$$\widetilde{R}_t = \rho_R\widetilde{R}_{t-1} + (1-\rho_R)\left[\psi_\pi\widetilde{\pi}_t + \psi_y(\Delta\widetilde{y}_{H,t} + \widetilde{z}_t)\right] + \varepsilon_{R,t}, \tag{20}$$

where $\varepsilon_{R,t} \sim N(0,\sigma_R^2)$.

Using equations (2) – (3) and the definitions of the real exchange rate (14) and terms of trade (11), the condition for home goods market clearing can be given by:

$$\widetilde{y}_{H,t} = \widetilde{c}_t + \widetilde{g}_t - \alpha\eta\widetilde{q}_t + \alpha\left(\eta - \frac{1}{\tau}\right)\widetilde{s}_t.$$

An analogous set of equations holds for the foreign country:

$$-\widetilde{\lambda}^*_t = \tau\widetilde{c}^*_t,$$

$$\widetilde{\pi}^*_{F,t} = \beta E_t\widetilde{\pi}^*_{F,t+1} + \kappa^*_F\widetilde{mc}^*_t,$$

with $\kappa^*_F = \frac{(1-\theta^*_F)(1-\beta\theta^*_F)}{\theta^*_F}$,

$$\widetilde{mc}^*_t = -\widetilde{\lambda}^*_t - \alpha\widetilde{q}^*_t - \widetilde{A}^*_t,$$

$$\widetilde{\pi}^*_t = \alpha\widetilde{\pi}^*_{H,t} + (1-\alpha)\widetilde{\pi}^*_{F,t},$$

$$\widetilde{R}^*_t = \rho_{R^*}\widetilde{R}^*_{t-1} + (1-\rho_{R^*})\left[\psi^*_\pi\widetilde{\pi}^*_t + \psi^*_y(\Delta\widetilde{y}^*_{F,t} + \widetilde{z}_t)\right] + \varepsilon_{R^*,t}, \tag{21}$$

where $\varepsilon_{R^*,t} \sim N(0, \sigma^2_{R^*})$, and

$$\widetilde{y}^*_{F,t} = \widetilde{c}^*_t + \widetilde{g}^*_t - \alpha\eta\widetilde{q}^*_t - \alpha\left(\eta - \frac{1}{\tau}\right)\widetilde{s}_t.$$

Combining the definitions of the real exchange rate (14) and terms of trade (11) with that of CPI (1), we obtain a relationship between the real exchange rate and terms of trade:

$$\widetilde{s}_t = -(1-\alpha)\,\widetilde{q}_t.$$

Terms of trade evolve according to:

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \widetilde{\pi}_{H,t} - \widetilde{\pi}_{F,t},$$

$$\widetilde{q}^*_t = \widetilde{q}^*_{t-1} + \widetilde{\pi}^*_{F,t} - \widetilde{\pi}^*_{H,t},$$

with

$$\widetilde{q}_t = -\widetilde{q}^*_t.$$

The definition of the real exchange rate also gives nominal exchange rate dynamics:

$$\Delta\widetilde{e}_t = \widetilde{\pi}_t - \widetilde{\pi}^*_t + \widetilde{s}_t - \widetilde{s}_{t-1} + \varepsilon_{E,t} \tag{22}$$

where $\varepsilon_{E,t} \sim N(0, \sigma^2_E)$ is the exogenous PPP shock that is introduced to account for the empirical invalidity of the PPP relationship. Note that in the present model, the PPP shocks are estimated as a residual of the equation (22), so that the shocks are not transmitted to the other variables except for the observed movements of the nominal exchange rate.

A linearized version of the international asset pricing equation for nominal bonds (17) implies the uncovered interest parity (UIP) condition:

$$\widetilde{R}_t - \widetilde{R}^*_t = E_t\widetilde{\pi}_{t+1} - E_t\widetilde{\pi}^*_{t+1} + E_t\widetilde{s}_{t+1} - \widetilde{s}_t, \tag{23}$$

whereas the risk-sharing condition is given by:

$$\widetilde{\lambda}_t = \widetilde{\lambda}^*_t - \widetilde{s}_t.$$

The exogenous shocks are assumed to follow the following autoregressive process:

$$\widetilde{z}_t = \rho_z\widetilde{z}_{t-1} + \varepsilon_{z,t},$$

$$\widetilde{A}_t = \rho_A\widetilde{A}_{t-1} + \varepsilon_{A,t},$$

$$\widetilde{g}_t = \rho_G\widetilde{g}_{t-1} + \varepsilon_{G,t},$$

$$\widetilde{A}^*_t = \rho_{A^*}\widetilde{A}^*_{t-1} + \varepsilon_{A^*,t},$$

$$\widetilde{g}^*_t = \rho_{G^*}\widetilde{g}^*_{t-1} + \varepsilon_{G^*,t},$$

where $0 \leq \rho_x < 1$ and $\varepsilon_{x,t} \sim N(0, \sigma^2_x)$ for $x \in \{z, A, G, A^*, G^*\}$.

3 Sunspot Solution and Sources of Indeterminacy

A distinctive feature of our analysis is that the two-country model is analyzed over the parameter space where the equilibrium can be indeterminate, and hence, sunspot shocks can affect the equilibrium dynamics. In this section, a full set of sunspot solutions of the linear rational expectations system is first presented. Next, in our framework, we address the possibility that indeterminacy occurs due to a passive monetary policy in one country in spite of the other country responding aggressively enough to inflation.

3.1 Sunspot Solution

In solving a rational expectations system, we follow the approach of Lubik and Schorfheide (2003), which provides a full set of non-unique solutions in linear rational expectations models by extending the solution algorithm developed by Sims (2002).[5] In their approach, the log-linearized system can be written in the following canonical form:

$$\Gamma_0(\theta)\, x_t = \Gamma_1(\theta)\, x_{t-1} + \Psi_0(\theta)\, \varepsilon_t + \Pi_0(\theta)\, \eta_t, \tag{24}$$

where Γ_0, Γ_1, Ψ_0 and Π_0 are the conformable matrices of coefficients that depend on the structural parameters θ, x_t is a stacked vector of endogenous variables and those with expectations at t, and ε_t is a vector of fundamental shocks. η_t is a vector of endogenous forecast errors, defined as:

$$\eta_t = \widehat{x}_t - E_{t-1}\widehat{x}_t,$$

where $\widehat{x}_t$ is a subvector of x_t that contains expectational variables. In the present model, $\widehat{x}_t$ consists of $\pi_{H,t}$, π_t, $\pi^*_{F,t}$, π^*_t, s_t, λ_t and z_t.

Assuming that $\Gamma_0(\theta)$ is non-singular,[6] the conditions for determinacy depend on the number of unstable eigenvalues in $\Gamma_0^{-1}(\theta)\,\Gamma_1(\theta)$ and the number of endogenous forecast errors. According to Lubik and Schorfheide (2003), the full set of sunspot solutions under indeterminacy is:

$$x_t = \Gamma(\theta)\, x_{t-1} + \Psi\left(\theta, \widetilde{M}\right) \varepsilon_t + \Pi(\theta, M_\zeta)\, \zeta_t, \tag{25}$$

where $\widetilde{M}$ and M_ζ are arbitrary matrices, and ζ_t is a vector of sunspot shocks, which are non-fundamental stochastic disturbances. If the equilibrium is determinate, the solution

[5] Sims' solution method generalizes the technique in Blanchard and Kahn (1980) and characterizes one particular solution in the case of indeterminacy.

[6] If Γ_0 is singular, the canonical system can be transformed through a generalized Schur decomposition.

(25) is reduced to:

$$x_t = \Gamma^D(\theta)\, x_{t-1} + \Psi^D(\theta)\, \varepsilon_t. \tag{26}$$

The solution (25) has two important features under indeterminacy. First, business cycle fluctuations are generated not only by fundamental shocks but also by sunspot shocks. Second, the equilibrium representation cannot be unique due to the arbitrary matrices $\widetilde{M}$ and M_ζ, i.e., the model has multiple solutions, and different solutions may exhibit different propagation of shocks. Therefore, in order to apply the sunspot solution to an analysis of economic dynamics, we need to specify $\widetilde{M}$ and M_ζ and to select a particular equilibrium path from an infinite number of equilibria; otherwise, any path can be considered as the equilibrium solution. In this paper, following Lubik and Schorfheide (2004), the arbitrary matrices are estimated using Bayesian methods.

3.2 Sources of Indeterminacy

The model employed in this paper is an extension of the closed New Keynesian model to the two-country open economy. It is well known that determinacy of the equilibrium in a prototypical closed New Keynesian model depends on the monetary policy rules.[7] In particular, when the monetary policy follows a current inflation targeting rule, the equilibrium is determinate if the monetary authority raises the nominal interest rate more than one percent in response to a one percent increase in the inflation rate, which is called the Taylor principle. It is also known, in the case of Taylor rule that responds to both inflation and the output gap, that the coefficient on the output gap does not substantially affect the condition for determinacy under plausible parameter settings.

For a two-country model as in Clarida, Galí, and Gertler (2002), which is a simple variant of our model, Bullard and Schaling (2006) show that determinacy of the worldwide equilibrium is attainable if both countries obey the Taylor principle in terms of domestic producer price inflation; that is, if one country satisfies the Taylor principle and the other does not, the worldwide equilibrium will be indeterminate. Bullard and Schaling (2006) also consider the CPI-based targeting rules, as is the case with our model, and point out the possibility that the equilibrium can be determinate even when one of the countries chooses a passive policy, depending on the size of each country. However, if both countries are of equal size, determinacy is maintained by both countries satisfying the Taylor principle.

In our present setting, we have confirmed by numerical calculation that the findings

[7] See, for instance, Bullard and Mitra (2002) or Woodford (2003).

in Bullard and Schaling (2006) carry over to our model, and that the CPI-based Taylor principle for both countries meets the condition for determinacy, since we presume an equal size for each country. Also, while our model exploits the monetary policy rules that respond to output growth as well as inflation, as specified in (20) and (21), we have checked that the effect of the coefficients on output growth to the condition for determinacy is quite marginal.[8] Therefore, in the empirical analysis that follows, we focus on the policy parameters on inflation as a source of indeterminacy.

4 Estimation Strategy

For our empirical investigation, the distributions of the structural parameters are estimated using Bayesian techniques. One of the important features of our analysis is that the parameters are estimated over the parameter space in which the equilibrium can be both determinate and indeterminate. Following Lubik and Schorfheide (2004), we begin with a review of how the inferences are made in such a parameter region. Next, we describe the data used for estimation and explain the prior distributions of the parameters.

4.1 Bayesian Estimation Methodology

First, consider the case in which the parameter space Θ^D is restricted so that the equilibrium is determinate. Let y_t be a vector of observables and $Y^T = \{y_1, ..., y_T\}$. The parameters of the structural model are collected in vector θ. Then, the data and the model can be related in the following state-space form:

$$y_t = A + Bx_t,$$

$$x_t = C(\theta)x_{t-1} + D(\theta)\varepsilon_t,$$

The first equation is the measurement equation, where the matrices A and B select and scale the relevant model variables x_t to link them with observed data y_t. The second equation is the state transition equation for x_t which corresponds to the solution (26) for the linear rational expectations model. Assuming that all the shocks are normally

[8]In addition, we have found that the policy smoothing parameters do not affect the conditions for determinacy.

distributed and uncorrelated over time, we obtain the likelihood function:

$$\begin{aligned}\mathcal{L}^D\left(\theta|Y^T\right) &= p\left(Y^T|\theta\right) \\ &= \prod_{t=1}^{T} p\left(y_t|Y^{t-1},\theta\right),\end{aligned}$$

which can be evaluated using the Kalman filter based on the state-space form above.

Next, consider the parameter space Θ^I in which the model exhibits equilibrium indeterminacy. In such a case, the law of motion for state variables is affected by sunspot shocks, and the solution multiplicity problem arises due to the arbitrary matrices $\widetilde{M}$ and M_ζ, as we can see from the solution (25). Since there is no relative relationship between these two matrices,[9] we impose normalization such that $M_\zeta = 1$ with the dimension of the sunspot shock vector being unity, following Lubik and Schorfheide (2004). The resulting sunspot shock is considered to be a reduced-form sunspot shock, as defined in Lubik and Schorfheide (2003), in the sense that it contains beliefs associated with all the expectational variables. Then, the state transition equation is modified as follows:

$$x_t = F(\theta)x_{t-1} + G(\theta,\widetilde{M})\varepsilon_t + H(\theta)\zeta_t.$$

The corresponding likelihood function is:

$$\mathcal{L}^I\left(\theta,\widetilde{M}|Y^T\right) = \prod_{t=1}^{T} p\left(y_t|Y^{t-1},\theta,\widetilde{M}\right).$$

In the subsequent analysis, the parameters are estimated over the parameter space which allows for both determinacy and indeterminacy. Hence, the overall likelihood function is evaluated by:

$$\mathcal{L}\left(\theta,\widetilde{M}|Y^T\right) = \left\{\theta\in\Theta^D\right\}\mathcal{L}^D\left(\theta|Y^T\right) + \left\{\theta\in\Theta^I\right\}\mathcal{L}^I\left(\theta,\widetilde{M}|Y^T\right),$$

where $\left\{\theta\in\Theta^i\right\}$ for $i\in\{D,I\}$ is the indicator function that is one if $\theta\in\Theta^i$ and zero otherwise. According to Bayes' theorem with a prior distribution $p\left(\theta,\widetilde{M}\right)$, the posterior distribution of θ is expressed as:

$$\begin{aligned}p\left(\theta,\widetilde{M}|Y^T\right) &= \frac{\mathcal{L}\left(\theta,\widetilde{M}|Y^T\right)p\left(\theta,\widetilde{M}\right)}{p\left(Y^T\right)} \qquad (27)\\ &= \frac{\mathcal{L}\left(\theta,\widetilde{M}|Y^T\right)p\left(\theta,\widetilde{M}\right)}{\int\mathcal{L}\left(\theta,\widetilde{M}|Y^T\right)p\left(\theta,\widetilde{M}\right)d\theta d\widetilde{M}}.\end{aligned}$$

[9] $\widetilde{M}$ and M_ζ can be related to the correlation between fundamental shocks and sunspot shocks. For details, see Lubik and Schorfheide (2003).

Markov Chain Monte Carlo methods are used to generate draws from the posterior distribution.[10] Based on the posterior draws, we can make inferences about structural parameters, impulse responses, and variance decompositions.

4.2 Data and Priors

The data used for estimation are identical to those of Lubik and Schorfheide (2006), who estimate a similar model to ours for the U.S. and the Euro area, so that we can compare the estimation results in terms of factors other than the data. The model is fitted to the data on output growth, inflation, the nominal interest rate series for the U.S. and the Euro area, and the nominal exchange rate. All the data are at quarterly frequencies from 1983:I to 2002:IV. For a detailed description of the data, see Lubik and Schorfheide (2006).

The prior distributions of the structural parameters are reported in Table 1. Most of the priors are by and large in line with Lubik and Schorfheide (2006), who set them based on a pre-sample analysis of the observations and relevant micro-econometric studies. The priors for the parameters that are specific to our analysis are set as follows. While Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004) have shown that U.S. monetary policy satisfied the Taylor principle during the sample period, whether the policy in the Euro area did is an open question in our context. Thus, we allow the monetary policy coefficient on inflation for the Euro area to take a wide range of values that leads to both determinacy and indeterminacy; that is, ψ_π^* is distributed around 1.1 with a wide confidence interval ranging from 0.3 to 1.8 so that the fraction of the parameter space that leads to indeterminacy is about half. On the other hand, we set relatively tight priors for the U.S. monetary policy. The standard deviation of the sunspot shock σ_ζ has the same distribution as that of the demand shock for each country. The priors for the components of the arbitrary matrix $\widetilde{M}$ are normally distributed around zero, based on the fact that most of the previous studies typically ignore the solution multiplicity under indeterminacy by setting $\widetilde{M}$ as zero.[11] Since we have no references for these parameters, we set wide confidence intervals for them.

[10]For our subsequent analysis, 500,000 draws are generated with a random-walk Metropolis Algorithm, and the first 50,000 draws are discarded.

[11]The equilibrium representation with $\widetilde{M} = 0$ is a particular solution that Sims (2002) characterizes under indeterminacy.

5 Estimation Results

In this section, posterior distributions of the structural parameters, impulse responses, and variance decompositions are presented. Based on these estimates, we examine the possibility of equilibrium indeterminacy and whether and to what extent sunspot shocks can affect the equilibrium dynamics in the U.S and the Euro area.

5.1 Posterior Distribution of the Structural Parameters

The posterior distributions of the structural parameters are reported in the last two columns of Table 1. The monetary policy coefficients in the U.S. are almost the same as their corresponding priors since we impose relatively tight priors on them. Our primary interest is whether the monetary policy in the Euro area is aggressive enough to lead to equilibrium determinacy. The estimation results show that the policy coefficient on inflation rate ψ_π^* is below unity and does not satisfy the Taylor principle, implying the equilibrium indeterminacy. On the other hand, the policy coefficient on the output growth ψ_y^* is higher than the prior. These results are consistent with the findings in Gerlach (2007), who estimates the monetary policy rule of the European Central Bank (ECB) with careful examination of its official statements and concludes that the ECB has adjusted the nominal interest rate in response to real economic activity rather than inflation. The estimated policy smoothing parameter ρ_{R^*} indicates the gradual adjustment of the interest rate.

While the data suggest higher value for the inverse of the intertemporal substitution elasticity τ, the import share α and the parameter η that relates to the elasticity of labor supply are smaller than the prior. The mean value of the so-called Calvo parameter θ_H for the U.S. firms is almost the same as the prior mean, whereas θ_F for the Euro area is far above the prior. These results are similar to the estimates in Lubik and Schorfheide (2006).

As for the arbitrary matrix $\widetilde{M}$ that takes effect under indeterminacy, some of the components are different from zero, implying that the propagation of the fundamental shocks can be altered compared with the determinacy case. The other parameters are in line with the priors, and are consistent with the findings in Lubik and Schorfheide (2006).

5.2 Determinacy vs. Indeterminacy

Our estimation procedure takes into account both the possibility of determinacy and indeterminacy. According to the prior distributions, the prior probability of indeterminacy is 0.48 as noted in Table 1. In what follows, we investigate the extent to which the data favors indeterminacy against determinacy by computing posterior probabilities for each parameter space.

The posterior probabilities for the determinacy and indeterminacy region of the parameter space are computed from the following marginal data densities:

$$p^i\left(Y^T\right) = \int \left\{\theta \in \Theta^i\right\} \mathcal{L}\left(\theta, \widetilde{M}|Y^T\right) p\left(\theta, \widetilde{M}\right) d\theta d\widetilde{M},$$

for $i \in \{D, I\}$. Then, the posterior probability of indeterminacy is given by

$$\pi^I = \frac{p^I\left(Y^T\right)}{p^I\left(Y^T\right) + p^D\left(Y^T\right)}.$$

The resulting log-data densities[12] are $\ln p^D\left(Y^T\right) = -779.60$ and $\ln p^I\left(Y^T\right) = -768.03$. Thus, the posterior probability of indeterminacy is higher than that of determinacy. Therefore, the data prefers equilibrium indeterminacy and suggests that sunspot shocks affect the macroeconomic dynamics during the period.

The marginal data densities also illustrate an advantage of our estimation procedure that allows for indeterminacy since they measure the overall fit of the model under each parameter region. If we restricted the parameter space that leads to determinacy, the empirical performance of the model would be substantially worse.

Our finding about indeterminacy due to the passive monetary policy against inflation in the Euro area is in contrast to the univariate GMM estimates reported in the previous studies such as Clarida, Galí, and Gertler (1998) and Gerdesmeier and Roffia (2004). However, our full-information system-based estimators are more efficient and are less subject to identification problems than instrumental variable estimators based on single equations, as discussed in Ruge-Murcia (2007). Our approach involves the solution for the rational expectations system with cross-equation restrictions both under determinacy and indeterminacy, so that we can estimate the parameters based on the measure of the overall time series fit, assessing the importance of the sunspot shocks and the propagation of the fundamental shocks. On the other hand, a weakness of our approach is that the results might be sensitive to model misspecification. On this issue, we investigate the robustness of our estimation results under alternative model specifications in Section 6.

[12]The log marginal data densities are approximated using the harmonic mean estimator proposed by Geweke (1999).

5.3 Impulse Responses

The dynamic property of the estimated model is examined by impulse response analysis. Figure 1 and 2 depict the posterior means (solid lines) and 90-percent posterior probability intervals (dashed lines) for impulse responses of output growth, inflation, the nominal interest rate, and depreciation of the nominal exchange rate in each country to the one-standard deviation shocks: home and foreign policy, home and foreign technology, home and foreign demand, world-wide technology, and sunspot shocks.[13] Since the model is based on a two-country version of a New Keynesian monetary DSGE framework, most of the impulse responses are as expected. However, some of the responses may be non-standard since we allow for equilibrium indeterminacy. Under indeterminacy, as demonstrated in Lubik and Schorfheide (2003, 2004), both the difference in structural parameters and the existence of the arbitrary matrix $\widetilde{M}$ can significantly alter the propagation of fundamental shocks.

The positive monetary policy shock in the U.S. has contractionary effects on domestic output and inflation. These negative impacts are transmitted to the Euro area. As is consistent with the nominal exchange rate dynamics (22) and the UIP condition (23), the U.S. currency is appreciated in the first period and depreciated in the next period. Judging from the responses, the dynamic behavior in the U.S. is similar to the one under determinacy that is demonstrated in Lubik and Schorfheide (2006). On the other hand, the monetary policy shock in the Euro area generates dramatically different dynamics. A contractionary monetary policy is supposed to lower output and have a negative impact on inflation. Since the monetary policy in the Euro area does not respond more than one-to-one to inflation, the real interest rate remains positive, which furthermore lowers output and hence inflation. However, such a spiral trajectory is explosive and must be eliminated for the equilibrium to be stationary. Therefore, to ensure stationarity, inflation must be positive to some extent so that the real interest rate eventually leads output toward its steady state. As a result, the positive inflation in the Euro area is transmitted to U.S. inflation.

The U.S. technology shock has a negative effect on marginal cost and hence on inflation. Responding to negative inflation, the monetary authority lowers the interest rate, which

[13] The responses of the PPP shock are excluded from the figures since the PPP shock is treated as a residual of the equation (22) and only affects the movement of the nominal exchange rate depreciation, as noted in Section 2.

results in depreciating the dollar. U.S. output is boosted because of high productivity, the low interest rate, and improvement of the terms of trade. Transmission from the U.S. to the Euro area is negative, since production shifts to the country with the higher productivity. The effects of the Euro technology shock are quite marginal because of high price stickiness, and the direction of the impulse responses are ambiguous due to the uncertainty of the arbitrary component $\widetilde{M}_{A^*}$.

The demand shock (specifically, the government expenditure shock) in the U.S. increases domestic output, but has a negative crowding-out effect on domestic and foreign consumption goods. Resulting changes in the relative prices lead to depreciation in the dollar. On the other hand, symmetric effects are not observed in the case of the demand shock in the Euro area. This is because the posterior estimate of $\widetilde{M}_{G^*}$ takes high value, implying that the effects of the associated shock can be substantially different from those under determinacy. The larger drop in inflation lowers the nominal interest rate in the Euro area and depreciates the dollar, which in turn give rise to a positive effect on U.S. inflation.

The world technology shock has expansionary effects on output and lowers inflation for both the U.S. and the Euro area. Different degrees of price stickiness and monetary policy reactions generate the differential in inflation and the interest rate, which results in the U.S. dollar being depreciated.

The sunspot shock is specific to our analysis. Based on the estimated sunspot shocks, sunspots have substantial negative impacts on the Euro variables and depreciate the dollar. While the depreciation of the dollar has a positive effect on U.S. inflation, the overall effects on the U.S. are marginal.

5.4 Variance Decompositions

In order to evaluate the relative importance of the individual shocks, we decompose the endogenous volatilities based on the posterior distributions of the parameters. Table 2 reports variance decompositions of output, inflation, the nominal interest rate in each country and the nominal exchange rate into the disturbances in the model.

For the U.S., output fluctuations are mainly driven by domestic demand shocks and world productivity shocks, while volatilities of inflation and the interest rate are explained for the most part by U.S. technology shocks. The effects of the disturbances in the Euro area on the U.S. variables are relatively marginal.

For the Euro area, as for the U.S., output movements are largely explained by domestic demand shocks and world productivity shocks. Changes in inflation and the interest rate are mainly attributed to domestic policy and demand shocks. A remarkable finding here is that output and inflation are driven by sunspot shocks to a substantial degree. This result suggests that sunspots might cause the economic system in the Euro area to be unexpectedly volatile due to the monetary policy that leads to indeterminacy.

One of our primary interests is whether the sunspots ascribed to the monetary policy in the Euro area affect the volatilities of U.S variables. The result shows that the effect of sunspot shocks on the U.S. is negligible, implying that the main source of the sunspots is self-fulfilling prophecies with respect to Euro variables, and that the transmission of sunspots to the U.S. is limited. This finding stands in contrast to the result in Bullard and Singh (2007). They simulate sunspot fluctuations in a three-country version of Clarida, Galí, and Gertler's (2002) model that consists of U.S., Germany (the Euro area in a recent period) and Japan under plausible calibrations, and report that sunspots that originate in the country with a passive monetary policy are transmitted to a substantial degree to the country with an aggressive policy. This contrast can be explained by the difference in the parameters; that is, our estimates for the degree of openness α and the elasticity of substitution between domestic and foreign goods η are both lower than the calibrated values in Bullard and Singh (2007).[14] Another possible explanation is the difference in the way the sunspot variables are introduced. While Bullard and Singh (2007) assume that the sunspot shock originates with inflation in countries with a passive monetary policy, we employ a reduced-form sunspot shock so that the dynamics are considered to be driven by beliefs in all the expectational variables. Thus, in our estimates, various beliefs in different directions might alleviate the total endogenous volatilities.

In the present model, following Lubik and Schorfheide (2006), a PPP shock is added in the form of an error term to the equation (22) that defines the nominal depreciation rate. The PPP shock provides a measure of the extent to which the exchange rate data is explained by specific features of the model. Lubik and Schorfheide (2006) conclude that their model as well as those in other previous studies has difficulty explaining the exchange rate dynamics. In our approach, we can investigate whether sunspot shocks are helpful in explaining it. Our result shows that sunspots explain only 4 percent of the exchange rate volatility. Therefore, the model is still very far from providing an explanation for exchange rate movements even if we allow for sunspots under indeterminacy.

[14]The composite consumption index in Bullard and Singh (2007) is defined by the Cobb-Douglas aggregator, which corresponds to $\eta = 1$ in our CES aggregator.

6 Robustness Analysis

Our empirical results have shown that the two-country economy exhibits indeterminacy of equilibrium. In this section, we investigate the robustness of our findings to alternative dynamic structures that are relevant to identification issues discussed in Beyer and Farmer (2007). Also, we consider another specification for monetary policy rules and examine whether or not our findings regarding indeterminacy are altered.

6.1 Alternative Dynamic Structures

Our experiment suggests that the data favors indeterminacy of the equilibrium ascribed to monetary policy in the Euro area. Beyer and Farmer (2007), however, point out that determinacy and indeterminacy can be observationally equivalent depending on lag structures of a model. They show that an indeterminate model possibly exhibits the same law of motion as a determinate one with additional lags, and argue that a model with simple dynamic structure could lead to a false finding of indeterminacy. In response to their critique, Lubik and Schorfheide (2007) address the fact that Lubik and Schorfheide (2004) have checked the robustness using a model with additional lag structures, which does not alter their findings about indeterminacy. In this section, we conduct the same exercise as Lubik and Schorfheide (2004); that is, we incorporate consumption habit formation and backward-looking price setting into the baseline model to ensure that our results are robust to alternative dynamic structures.

First, we introduce internal habit formation as specified in Lubik and Schorfheide (2006). C_t in the period utility function for the home country is replaced with the following effective consumption under habit formation:

$$\mathcal{C}_t = C_t - h\gamma C_{t-1},$$

where $0 \leq h \leq 1$ is the habit persistence parameter. Then, the loglinearized version of the first order condition with respect to consumption is modified as:

$$-\widetilde{\lambda}_t = \frac{\tau}{1-h\beta}\widetilde{\mathcal{C}}_t - \frac{h\beta}{1-h\beta}\left(\tau E_t\widetilde{\mathcal{C}}_{t+1} + E_t\widetilde{z}_{t+1}\right),$$

with

$$(1-h)\,\widetilde{\mathcal{C}}_t = \widetilde{c}_t - h\widetilde{c}_{t-1} + \widetilde{z}_t.$$

Similarly, the equation for the foreign country is:

$$-\widetilde{\lambda^*}_t = \frac{\tau}{1-h\beta}\widetilde{\mathcal{C}^*}_t - \frac{h\beta}{1-h\beta}\left(\tau E_t\widetilde{\mathcal{C}^*}_{t+1} + E_t\widetilde{z}_{t+1}\right),$$

with

$$(1-h)\widetilde{\mathcal{C}^*}_t = \widetilde{c^*}_t - h\widetilde{c^*}_{t-1} + \widetilde{z}_t.$$

Next, the equations for price-setting behavior are replaced with the hybrid New Keynesian Phillips curves proposed by Galí and Gertler (1999). Let ω and ω^* denote, respectively, a fraction of the domestic and foreign firms that set their prices using a simple rule of thumb based on the previous aggregate prices. Then, the hybrid New Keynesian Phillips curve for domestic firms is of the following form:

$$\widetilde{\pi}_{H,t} = \frac{1}{1+\beta\omega}\left[\omega\widetilde{\pi}_{H,t-1} + \beta E_t\widetilde{\pi}_{H,t+1} + \kappa_H\left(-\widetilde{\lambda}_t - \alpha\widetilde{q}_t - \widetilde{A}_t\right)\right],$$

and that for foreign firms is:

$$\widetilde{\pi}^*_{F,t} = \frac{1}{1+\beta\omega^*}\left[\omega^*\widetilde{\pi}^*_{F,t-1} + \beta E_t\widetilde{\pi}^*_{F,t+1} + \kappa_F\left(-\widetilde{\lambda}^*_t - \alpha\widetilde{q}^*_t - \widetilde{A}^*_t\right)\right].$$

With these equations, we re-estimate the model and compare the marginal data densities under determinacy and indeterminacy regions. The prior for h is distributed according to a Beta distribution with mean 0.5 and standard deviation 0.1, as is set in Lubik and Schorfheide (2006). The prior for ω has a Beta distribution with mean 0.25 and standard deviation 0.05, which is in line with the GMM estimates for U.S. data reported in Galí and Gertler (1999). The other priors are the same as the baseline case. According to the posterior distributions reported in Table 3, h is centered around 0.54, which suggests high persistency in consumption. The posterior estimates for ω and ω^* are distributed around 0.15 and 0.20, which are lower than their priors. The other parameters are by and large in line with our baseline estimates.

The primary issue here is whether the inclusion of the additional lags in the model can change our result suggesting indeterminacy. Based on the posterior estimates, the marginal data densities for the determinacy and indeterminacy region are $\ln p^D\left(Y^T\right) = -782.46$ and $\ln p^I\left(Y^T\right) = -762.87$, respectively. Therefore, the posterior probabilities imply that the data still favors indeterminacy, and our findings about indeterminacy are not overturned.

6.2 Alternative Monetary Policy Rules

In our baseline estimation, we have considered monetary policies that react to current variables as specified in (20) and (21) to maintain comparability to the analysis of Lubik and Schorfheide (2004). On the other hand, Clarida, Galí, and Gertler (1998, 2000) and Bullard and Singh (2007) among others have estimated the monetary policy rules that respond to expected values of inflation and output. We now examine the robustness of our findings by replacing the baseline policy specification with the forward-looking policy rules.

We consider the following monetary policies reacting to expected inflation and expected output growth between periods t and $t+1$:

$$\widetilde{R}_t = \rho_R \widetilde{R}_{t-1} + (1-\rho_R)\left[\psi_\pi E_t \widetilde{\pi}_{t+1} + \psi_y (E_t \Delta \widetilde{y}_{H,t+1} + E_t \widetilde{z}_{t+1})\right] + \varepsilon_{R,t},$$

and

$$\widetilde{R}^*_t = \rho_{R^*} \widetilde{R}^*_{t-1} + (1-\rho_{R^*})\left[\psi^*_\pi E_t \widetilde{\pi}^*_{t+1} + \psi^*_y (E_t \Delta \widetilde{y}^*_{F,t+1} + E_t \widetilde{z}_{t+1})\right] + \varepsilon_{R^*,t}.$$

In general, alternative monetary policy rules can change the conditions for determinacy in New Keynesian monetary DSGE models. However, Bullard and Mitra (2002) show that, in a closed economy model with the forward-looking policy targeting expected inflation and expected output, the Taylor principle still applies unless the coefficients on these variables are too large. Bullard and Singh (2007) also characterize similar conditions in their open economy framework. In our present model, we have confirmed that the modification above gives the same prior probability of indeterminacy, 0.48, as in the baseline specification.

The last two columns in Table 3 report the posterior distributions of the parameters based on the same priors as the baseline. The components in the arbitrary matrix $\widetilde{M}$ are remarkably different from the baseline estimates, implying that the alternative policy specification can alter the propagation of the fundamental shocks. The other parameters are in line with our baseline estimates.

In the same way as the previous subsection, we compute the marginal data densities for the determinacy and indeterminacy regions, which are $\ln p^D\left(Y^T\right) = -810.60$ and $\ln p^I\left(Y^T\right) = -796.22$, respectively. Therefore, the data still suggests indeterminacy under the forward-looking policy rules. Also, we can see that the data densities here are both lower than those in the baseline estimates reported in Section 5.2. The last finding empirically validates our baseline specification and estimates in the previous section.

7 Conclusion

We have estimated a two-country monetary DSGE model for the U.S. and the Euro area over a parameter space where the equilibrium can be both determinate and indeterminate using Bayesian methods. Our estimation results show that the data favors indeterminacy due to passive monetary policy in the Euro area. We have demonstrated that the estimated impulse responses are different from those under determinacy. While sunspot shocks substantially affect the endogenous volatilities in the Euro area, the sunspot effects transmitted to the U.S. are negligible. These findings are novel in the estimated open-economy DSGE literature, since the previous studies consider the parameter space that leads only to determinacy.

While the full-information likelihood-based approach employed in this paper delivers more efficient estimates than univariate GMM estimates, one drawback is that this approach is potentially sensitive to model misspecification. Although we have conducted a robustness analysis for the model with the additional lag structures and the alternative monetary policy rules, a variety of other specifications could be considered. Incorporating other types of frictions and policy rules than those examined in this paper might lead to distinct conditions for determinacy, and hence, might give rise to different results.

References

Adjemian, S., M. D. Pariès, and F. Smets (2008): "A Quantitative Perspective on Optimal Monetary Policy Cooperation between the US and the Euro Area," Working Paper Series 884, European Central Bank.

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2007): "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," *Journal of International Economics*, 72(2), 481–511.

Beyer, A., and R. E. A. Farmer (2007): "Testing for Indeterminacy: An Application to U.S. Monetary Policy: Comment," *American Economic Review*, 97(1), 524–529.

Blanchard, O. J., and C. M. Kahn (1980): "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 48, 1305–1311.

Bullard, J., and K. Mitra (2002): "Learning About Monetary Policy Rules," *Journal of Monetary Economics*, 49(6), 1105–1129.

Bullard, J., and E. Schaling (2006): "Monetary Policy, Determinancy, and Learnability in the Open Economy," Working Paper Series 611, European Central Bank.

Bullard, J., and A. Singh (2007): "Worldwide Macroeconomic Stability and Monetary Policy Rules," Journal of Monetary Economics, forthcoming.

Calvo, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.

Carlstrom, C. T., and T. S. Fuerst (1999): "Optimal Monetary Policy in a Small Open Economy: A General Equilibrium Analysis," Working Paper 9911, Federal Reserve Bank of Cleveland.

Clarida, R., J. Galí, and M. Gertler (1998): "Monetary Policy Rules in Practice: Some International Evidence," *European Economic Review*, 42(6), 1033–1067.

——— (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115(1), 147–180.

——— (2002): "A Simple Framework for International Monetary Policy Analysis," *Journal of Monetary Economics*, 49(5), 879–904.

De Fiore, F., and Z. Liu (2005): "Does Trade Openness Matter for Aggregate Instability?," *Journal of Economic Dynamics and Control*, 29(7), 1165–1192.

Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67(3), 297–308.

Faust, J., J. H. Rogers, and J. H. Wright (2001): "An Empirical Comparison of Bundesbank and ECB Monetary Policy Rules," International Finance Discussion Papers 705, Board of Governors of the Federal Reserve System.

Galí, J., and M. Gertler (1999): "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 44(2), 195–222.

Galí, J., and T. Monacelli (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72(3), 707–734.

Gerdesmeier, D., and B. Roffia (2004): "Empirical Estimates of Reaction Functions for the Euro Area," *Swiss Journal of Economics and Statistics*, 140(1), 37–66.

Gerlach, S. (2007): "Interest Rate Setting by the ECB, 1999-2006: Words and Deeds," *International Journal of Central Banking*, 3(3), 1–46.

Geweke, J. F. (1999): "Using Simulation Methods for Bayesian Econometric Models: Inference, Development and Communication," *Econometric Reviews*, 18(1), 1–73.

Justiniano, A., and B. Preston (2008): "Monetary Policy and Uncertainty in an Empirical Small Open Economy Model," *Journal of Applied Econometrics*, forthcoming.

Lubik, T. A., and F. Schorfheide (2003): "Computing Sunspot Equilibria in Linear Rational Expectations Models," *Journal of Economic Dynamics and Control*, 28(2), 273–285.

——— (2004): "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, 94(1), 190–217.

——— (2006): "A Bayesian Look at New Open Economy Macroeconomics," in *NBER Macroeconomics Annual 2005*, ed. by M. Gertler, and K. Rogoff. Cambridge: MIT Press.

——— (2007): "Testing for Indeterminacy: An Application to U.S. Monetary Policy: Reply," *American Economic Review*, 97(1), 530–533.

Ruge-Murcia, F. J. (2007): "Methods to Estimate Dynamic Stochastic General Equilibrium Models," *Journal of Economic Dynamics and Control*, 31(8), 2599–2636.

Sims, C. A. (2002): "Solving Linear Rational Expectations Models," *Computational Economics*, 20(1-2), 1–20.

Woodford, M. (2003): *Interest and Prices.* Princeton: Princeton University Press.

Table 1: Prior and Posterior Distributions

		Prior distributions			Posterior distributions	
Name	Range	Density	Mean	90% interval	Mean	90% interval
ψ_π	$\Re^+$	Gamma	1.50	[1.38, 1.62]	1.53	[1.41, 1.65]
ψ_y	$\Re^+$	Gamma	0.50	[0.34, 0.66]	0.58	[0.43, 0.72]
ψ_π^*	$\Re^+$	Gamma	1.10	[0.32, 1.84]	0.54	[0.27, 0.83]
ψ_y^*	$\Re^+$	Gamma	0.50	[0.07, 0.93]	0.77	[0.20, 1.31]
τ	$\Re^+$	Gamma	2.00	[1.19, 2.80]	3.15	[2.29, 3.94]
α	$[0, 1)$	Beta	0.12	[0.08, 0.16]	0.10	[0.08, 0.13]
η	$\Re^+$	Gamma	1.00	[0.22, 1.71]	0.24	[0.07, 0.40]
θ_H	$[0, 1)$	Beta	0.50	[0.37, 0.62]	0.46	[0.37, 0.55]
θ_F^*	$[0, 1)$	Beta	0.75	[0.63, 0.87]	0.94	[0.91, 0.97]
ρ_A	$[0, 1)$	Beta	0.80	[0.65, 0.96]	0.85	[0.79, 0.91]
ρ_R	$[0, 1)$	Beta	0.50	[0.17, 0.83]	0.70	[0.64, 0.76]
ρ_G	$[0, 1)$	Beta	0.80	[0.65, 0.96]	0.91	[0.86, 0.97]
ρ_{A^*}	$[0, 1)$	Beta	0.60	[0.28, 0.93]	0.6	[0.29, 0.94]
ρ_{R^*}	$[0, 1)$	Beta	0.50	[0.17, 0.83]	0.94	[0.91, 0.96]
ρ_{G^*}	$[0, 1)$	Beta	0.80	[0.65, 0.96]	0.76	[0.63, 0.90]
ρ_z	$[0, 1)$	Beta	0.66	[0.42, 0.91]	0.58	[0.41, 0.76]
r	$\Re^+$	Gamma	0.85	[0.53, 1.17]	0.78	[0.52, 1.04]
γ	$\Re$	Normal	0.40	[0.06, 0.72]	0.43	[0.31, 0.55]
π	$\Re^+$	Gamma	3.00	[1.39, 4.55]	2.97	[2.37, 3.56]
σ_A	$\Re^+$	InvGamma	1.25	[0.53, 1.98]	1.18	[0.90, 1.47]
σ_G	$\Re^+$	InvGamma	1.25	[0.53, 1.98]	0.55	[0.45, 0.64]
σ_R	$\Re^+$	InvGamma	0.50	[0.21, 0.79]	0.20	[0.16, 0.23]
σ_{A^*}	$\Re^+$	InvGamma	0.50	[0.22, 0.80]	0.44	[0.21, 0.68]
σ_{G^*}	$\Re^+$	InvGamma	1.25	[0.53, 1.98]	0.74	[0.56, 0.91]
σ_{R^*}	$\Re^+$	InvGamma	0.25	[0.11, 0.40]	0.13	[0.11, 0.14]
σ_z	$\Re^+$	InvGamma	0.63	[0.27, 0.99]	0.28	[0.21, 0.36]
σ_E	$\Re^+$	InvGamma	4.39	[1.82, 6.90]	4.27	[3.70, 4.82]
σ_ζ	$\Re^+$	InvGamma	1.25	[0.53, 1.98]	0.73	[0.53, 0.93]
M_A	$\Re$	Normal	0.00	[-2.47, 2.47]	0.71	[0.45, 0.96]
M_G	$\Re$	Normal	0.00	[-2.47, 2.47]	0.42	[-0.13, 0.96]
M_R	$\Re$	Normal	0.00	[-2.47, 2.47]	0.13	[-0.96, 1.23]
M_{A^*}	$\Re$	Normal	0.00	[-2.47, 2.47]	-0.08	[-1.29, 1.18]
M_{G^*}	$\Re$	Normal	0.00	[-2.47, 2.47]	1.53	[1.06, 1.97]
M_{R^*}	$\Re$	Normal	0.00	[-2.47, 2.47]	1.21	[-0.48, 2.82]
M_z	$\Re$	Normal	0.00	[-2.47, 2.47]	0.20	[-1.04, 1.50]

Note: According to the prior distributions, the fraction of the parameter space which leads to indeterminacy is 48 percent.

Table 2: Variance Decompositions

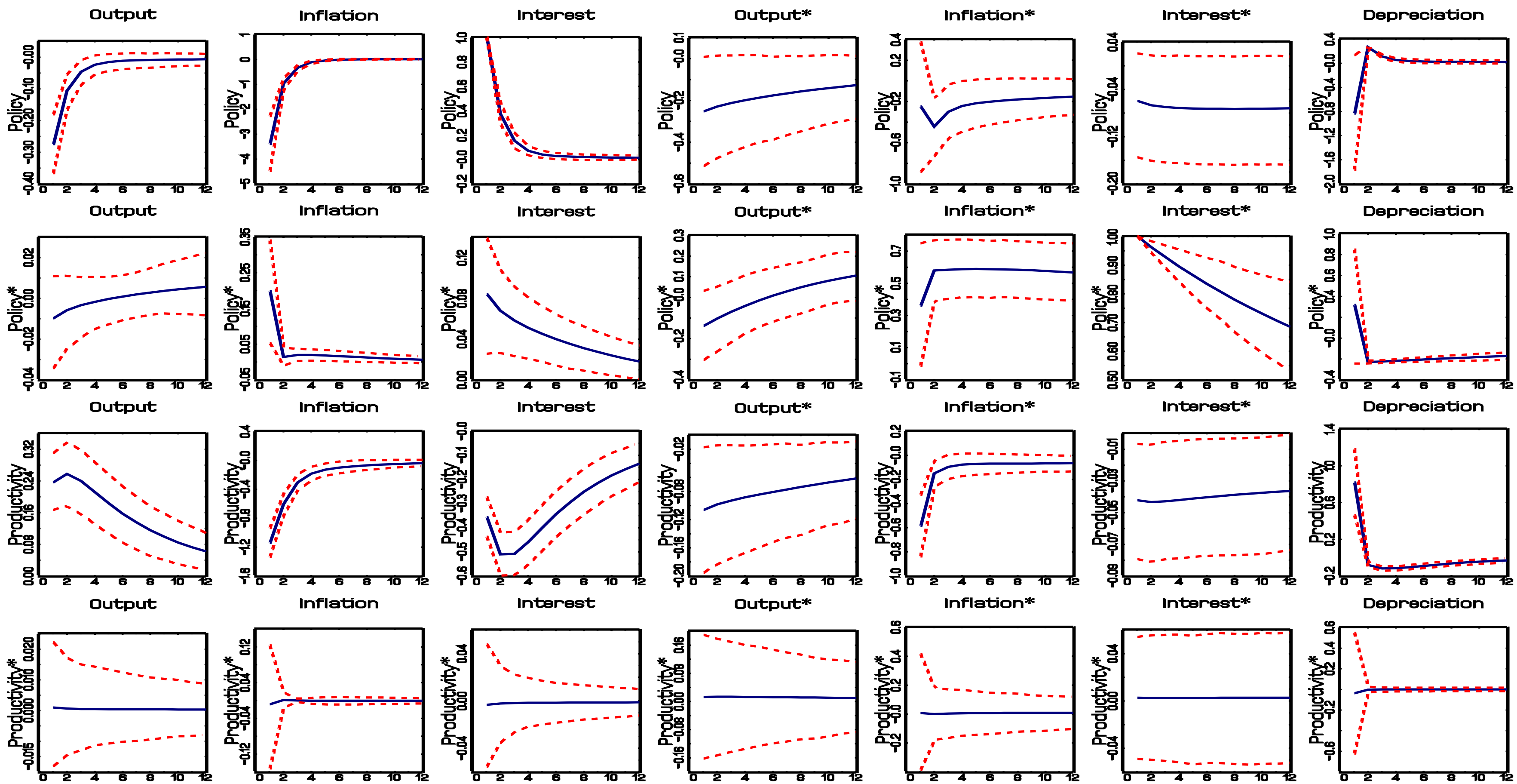
Shock	Mean	90% interval	Shock	Mean	90% interval
US output			Euro output		
Policy	0.02	[0.01, 0.03]	Policy	0.02	[0.00, 0.05]
Policy*	0.00	[0.00, 0.00]	Policy*	0.02	[0.00, 0.05]
Productivity	0.13	[0.07, 0.20]	Productivity	0.04	[0.00, 0.08]
Productivity*	0.00	[0.00, 0.00]	Productivity*	0.03	[0.00, 0.08]
Demand	0.57	[0.44, 0.69]	Demand	0.04	[0.00, 0.09]
Demand*	0.01	[0.00, 0.01]	Demand*	0.36	[0.22, 0.52]
World productivity	0.27	[0.16, 0.37]	World productivity	0.28	[0.10, 0.44]
PPP	0.00	[0.00, 0.00]	PPP	0.00	[0.00, 0.00]
Sunspot	0.00	[0.00, 0.01]	Sunspot	0.21	[0.09, 0.34]
US inflation			Euro inflation		
Policy	0.31	[0.21, 0.42]	Policy	0.01	[0.00, 0.03]
Policy*	0.00	[0.00, 0.01]	Policy*	0.34	[0.14, 0.54]
Productivity	0.57	[0.45, 0.68]	Productivity	0.06	[0.01, 0.12]
Productivity*	0.00	[0.00, 0.01]	Productivity*	0.02	[0.00, 0.06]
Demand	0.04	[0.01, 0.06]	Demand	0.03	[0.00, 0.07]
Demand*	0.04	[0.02, 0.06]	Demand*	0.34	[0.15, 0.52]
World productivity	0.02	[0.01, 0.04]	World productivity	0.02	[0.00, 0.05]
PPP	0.00	[0.00, 0.00]	PPP	0.00	[0.00, 0.00]
Sunspot	0.02	[0.01, 0.03]	Sunspot	0.16	[0.06, 0.25]
US interest rate			Euro interest rate		
Policy	0.05	[0.03, 0.08]	Policy	0.01	[0.00, 0.01]
Policy*	0.00	[0.00, 0.01]	Policy*	0.75	[0.54, 0.98]
Productivity	0.73	[0.62, 0.85]	Productivity	0.01	[0.00, 0.03]
Productivity*	0.00	[0.00, 0.00]	Productivity*	0.01	[0.00, 0.02]
Demand	0.10	[0.04, 0.16]	Demand	0.01	[0.00, 0.03]
Demand*	0.02	[0.01, 0.04]	Demand*	0.12	[0.01, 0.24]
World productivity	0.08	[0.01, 0.16]	World productivity	0.04	[0.00, 0.10]
PPP	0.00	[0.00, 0.00]	PPP	0.00	[0.00, 0.00]
Sunspot	0.01	[0.00, 0.02]	Sunspot	0.05	[0.00, 0.11]
Exchange rate					
Policy	0.00	[0.00, 0.01]			
Policy*	0.02	[0.01, 0.03]			
Productivity	0.04	[0.01, 0.06]			
Productivity*	0.01	[0.00, 0.02]			
Demand	0.01	[0.00, 0.01]			
Demand*	0.09	[0.03, 0.15]			
World productivity	0.00	[0.00, 0.01]			
PPP	0.80	[0.73, 0.87]			
Sunspot	0.04	[0.02, 0.06]			

Note: ‘*’ indicates the shocks to the Euro area.

Table 3: Posterior Distributions under Additional Lag Structure and Alternative Policy Rules

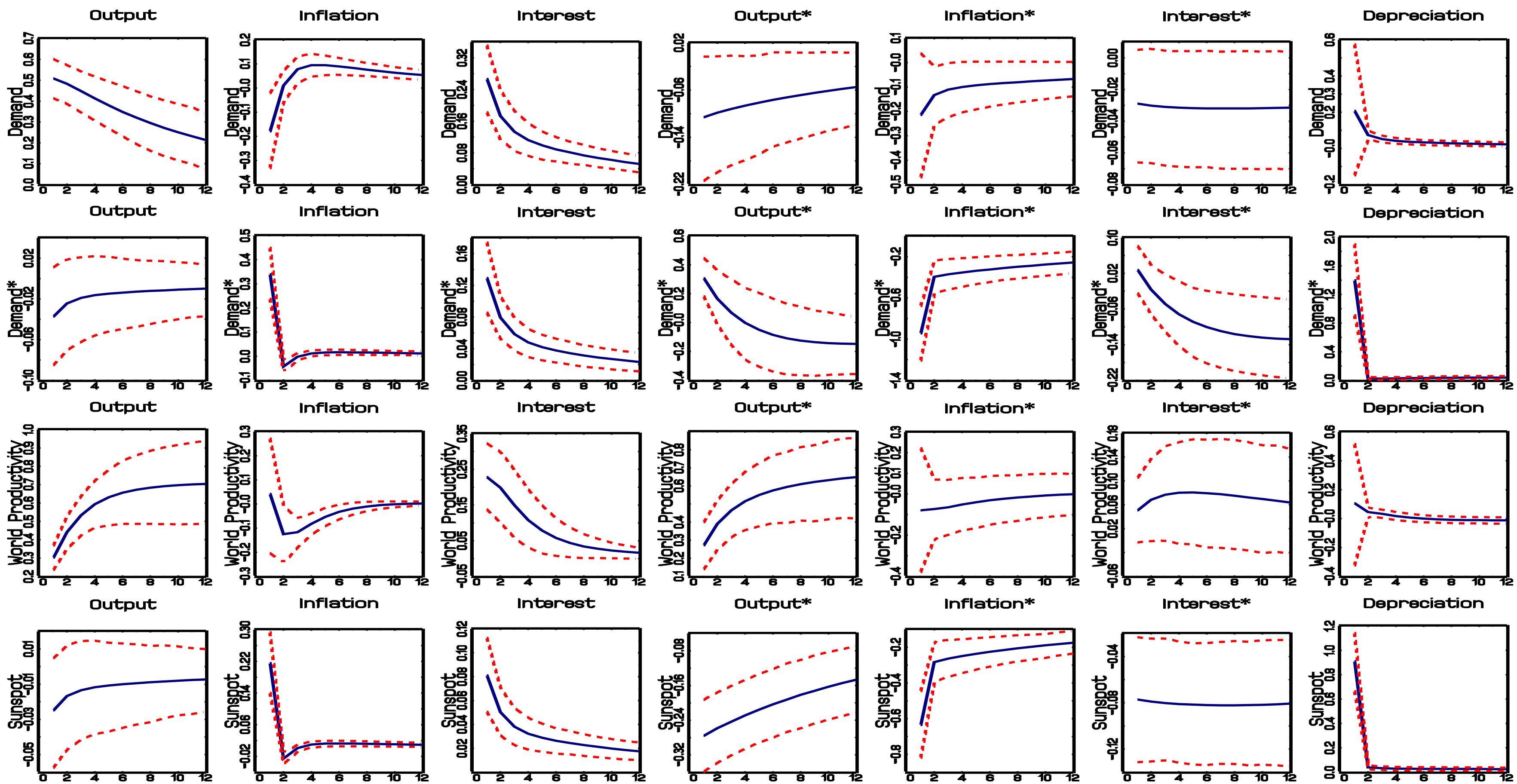
	Baseline		Additional lags		Alternative policies	
Name	Mean	90% interval	Mean	90% interval	Mean	90% interval
h			0.54	[0.37, 0.71]		
ω			0.15	[0.04, 0.25]		
ω^*			0.20	[0.06, 0.33]		
ψ_π	1.53	[1.41, 1.65]	1.54	[1.42, 1.66]	1.54	[1.42, 1.67]
ψ_y	0.58	[0.43, 0.72]	0.50	[0.36, 0.63]	0.51	[0.35, 0.67]
ψ_π^*	0.54	[0.27, 0.83]	0.55	[0.27, 0.83]	0.55	[0.25, 0.84]
ψ_y^*	0.77	[0.20, 1.31]	0.69	[0.18, 1.16]	0.45	[0.06, 0.84]
τ	3.15	[2.29, 3.94]	2.45	[1.66, 3.22]	3.89	[2.93, 4.81]
α	0.10	[0.08, 0.13]	0.12	[0.09, 0.14]	0.09	[0.06, 0.12]
η	0.24	[0.07, 0.40]	0.22	[0.06, 0.36]	0.25	[0.05, 0.44]
θ_H	0.46	[0.37, 0.55]	0.41	[0.32, 0.50]	0.58	[0.50, 0.67]
θ_F^*	0.94	[0.91, 0.97]	0.94	[0.90, 0.97]	0.90	[0.85, 0.96]
ρ_A	0.85	[0.79, 0.91]	0.86	[0.81, 0.92]	0.86	[0.81, 0.91]
ρ_R	0.70	[0.64, 0.76]	0.70	[0.65, 0.76]	0.68	[0.60, 0.76]
ρ_G	0.91	[0.86, 0.97]	0.91	[0.86, 0.97]	0.95	[0.92, 0.99]
ρ_{A^*}	0.60	[0.29, 0.94]	0.60	[0.29, 0.93]	0.60	[0.29, 0.94]
ρ_{R^*}	0.94	[0.91, 0.96]	0.93	[0.91, 0.96]	0.94	[0.91, 0.97]
ρ_{G^*}	0.76	[0.63, 0.90]	0.87	[0.78, 0.96]	0.80	[0.68, 0.93]
ρ_z	0.58	[0.41, 0.76]	0.52	[0.33, 0.73]	0.46	[0.26, 0.67]
r	0.78	[0.52, 1.04]	0.79	[0.53, 1.06]	0.86	[0.56, 1.16]
γ	0.43	[0.31, 0.55]	0.44	[0.32, 0.56]	0.45	[0.36, 0.54]
π	2.97	[2.37, 3.56]	3.08	[2.43, 3.70]	2.98	[2.07, 3.87]
σ_A	1.18	[0.90, 1.47]	1.20	[0.92, 1.47]	1.16	[0.82, 1.48]
σ_G	0.55	[0.45, 0.64]	0.52	[0.44, 0.60]	0.55	[0.45, 0.64]
σ_R	0.20	[0.16, 0.23]	0.19	[0.16, 0.23]	0.18	[0.15, 0.20]
σ_{A^*}	0.44	[0.21, 0.68]	0.47	[0.21, 0.74]	0.43	[0.21, 0.65]
σ_{G^*}	0.74	[0.56, 0.91]	0.56	[0.46, 0.66]	0.69	[0.53, 0.84]
σ_{R^*}	0.13	[0.11, 0.14]	0.13	[0.11, 0.14]	0.13	[0.11, 0.14]
σ_z	0.28	[0.21, 0.36]	0.35	[0.23, 0.47]	0.29	[0.21, 0.36]
σ_E	4.27	[3.70, 4.82]	4.29	[3.72, 4.85]	4.26	[3.69, 4.81]
σ_ζ	0.73	[0.53, 0.93]	1.00	[0.65, 1.33]	0.77	[0.55, 0.98]
M_A	0.71	[0.45, 0.96]	0.86	[0.51, 1.20]	-0.65	[-0.93, -0.37]
M_G	0.42	[-0.13, 0.96]	0.54	[-0.16, 1.25]	-0.36	[-0.92, 0.21]
M_R	0.13	[-0.96, 1.23]	0.82	[-0.54, 2.22]	-0.91	[-2.43, 0.61]
M_{A^*}	-0.08	[-1.29, 1.18]	-0.16	[-2.17, 1.80]	0.10	[-1.38, 1.45]
M_{G^*}	1.53	[1.06, 1.97]	1.33	[0.63, 2.03]	-1.05	[-1.62, -0.48]
M_{R^*}	1.21	[-0.48, 2.82]	1.65	[-0.11, 3.48]	-0.72	[-2.28, 0.85]
M_z	0.20	[-1.04, 1.50]	-0.28	[-1.90, 1.43]	0.26	[-1.03, 1.50]

Figure 1: Impulse Responses



Notes: The Figure depicts posterior means (solid lines) and pointwise 90-percent posterior probability intervals (dashed lines) for the impulse responses to one-standard deviation shocks. ‘*’ indicates the variables and shocks for the Euro area.

Figure 2: Impulse Responses (Continued)



Notes: The Figure depicts posterior means (solid lines) and pointwise 90-percent posterior probability intervals (dashed lines) for the impulse responses to one-standard deviation shocks. '*' indicates the variables and shocks for the Euro area.