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Macro Stress-Testing on the Loan Portfolio of Japanese Banks

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Abstract

In recent years, an increasing number of central banks use macro stress-testing as a main tool to assess the robustness of the financial system against severe stresses to the economy, such as deep recessions and sharp rises in interest rates. This paper describes a framework for macro stress-testing on credit risk currently used at the Bank of Japan (BOJ). That framework takes account of changes in borrowers' creditworthiness over the business cycle, thereby enabling us to examine the robustness of loan portfolios for major banks and regional banks against a severe economic downturn. The simulation results, taken from the September 2008 issue of the BOJ's *Financial System Report*, show that the framework successfully replicates the asymmetric responses of credit risk between deep recession and subsequent economic recovery by using the combination of borrowers' transition between rating classes and different sensitivity of transition probabilities to economic fluctuations across rating classes.

Key Words: Macro Stress-Testing, Credit Risk, Transition Matrix, Robustness of Financial System

JEL Classification Code: C51, E32, E37, G21

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Table of Contents

I. Introduction

This paper describes a framework for macro stress-testing on credit risk used in the Bank of Japan from the September 2007 issue of the *Financial System Report*. The framework takes account of changes in borrowers' creditworthiness over the business cycle, thereby enabling us to examine the robustness of loan portfolios for major banks and regional banks against a severe economic downturn.[1]

In recent years, an increasing number of central banks have been regularly publishing reports on the financial system from macro-prudential viewpoints, with titles such as "Financial Stability Report/Review" or "Financial System Report/Review" (hereafter called "FSR"). In these reports, central banks present their analyses and assessments of the stability and functioning of the financial system. They use macro stress-testing as a main tool to assess the robustness of the financial system against severe stresses to the economy, such as deep recessions and sharp rises in interest rates.

Macro stress-testing generally involves: (i) analyzing the sensitivity of variables expressing the quality of banks' loan portfolios to the business cycle, (ii) setting up a stress scenario, and (iii) calculating the credit risk and loan loss provisions under the stress scenario. However, details of the frameworks used for macro stress-testing vary from country to country, depending on the availability of data for central banks. For example, a central bank with limited access to detailed data for individual banks will tend to focus on aggregate data, such as using loan loss provisions in the banking sector and the rate of default of the corporate sector as proxies for the quality of banks' loan portfolios.

Our framework takes advantage of our access to data on the transition matrix of borrower classifications at individual banks, to construct macro stress-testing for credit risk.[2, 3] Based on Japan's experience from the late 1990s to the early 2000s, such data

1 This paper uses the total outstanding loans of the 12 major banks and 109 regional banks, as of the end of March 2008. The 12 major banks include Mizuho Bank, The Bank of Tokyo-Mitsubishi UFJ, Sumitomo Mitsui Banking Corporation, Resona Bank, Mizuho Corporate Bank, Saitama Resona Bank, Mitsubishi UFJ Trust and Banking Corporation, Mizuho Trust and Banking Company, The Chuo Mitsui Trust and Banking Company, The Sumitomo Trust and Banking Company, Shinsei Bank, and Aozora Bank. The 109 regional banks comprise the 64 member banks of the Regional Banks Association of Japan and the 45 member banks of the Second Association of Regional Banks.

2 All Japanese banks report the data on the transition matrix on borrower classifications to the Bank of Japan since 2002.

3 Some central banks implement macro stress-testing by using transition matrix data for

provides an important basis for analyzing the dynamic nature of credit risk. In fact, many Japanese banks were confronted with deterioration in borrowers' creditworthiness and further downgrading of their borrower classifications, resulting in drastic increases in the amounts of credit risk. Given these experiences, many banks currently employ their internal transition matrix of the loan portfolio as a main tool for managing credit risk, including stress-testing to investigate the dynamic effects of an economic downturn through increases in downgradings and default rates.

To this end, we construct a long-term time-series of the transition matrix of borrower classifications by linking two datasets: the Japanese banks' aggregated transition matrix data for borrower classifications compiled by the Bank of Japan (BOJ) since 2002 and credit score data for Japanese firms provided by *Teikoku Data Bank*, a large private rating agency in Japan, since 1985.[4] Using these data, we estimate the range of credit scores which corresponds to borrower classifications in the BOJ's transition matrix, thereby constructing a quasi-transition matrix of Japanese banks' loan portfolios since 1985. We then examine the relationship between changes in the transition matrix and macroeconomic variables.

As for a stress scenario, we estimate a vector autoregression (VAR) model with five variables: gross domestic product (GDP), consumer price index (CPI), the amount outstanding of bank lending, the nominal exchange rate, and the overnight call rate. We then make two types of projections regarding the future path of macroeconomic variables: a baseline scenario that assumes no external shock occurs during the projection period and a stress scenario that assumes an adverse shock to GDP occurs at the beginning of the projection period of a size that is likely to occur with a probability of 1 percent on an annualized basis, and that the shock subsides by half in three quarters.

In assessing the robustness of the banking sector against credit risk, we employ two indicators, "excess credit risk" and "excess credit cost." We define excess credit

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corporate bond ratings provided by private rating agencies. Such transition matrix data, however, does not necessarily reflect the credit condition of the loan portfolio for the banking sector as a whole, since the data covers only large- and medium-sized firms that are able to issue corporate bonds.

4 *Teikoku Data Bank* provides information on the migration of credit scores of around one million Japanese firms including many small-sized enterprises. Credit scores take a value from zero to 100, with a higher value meaning higher creditworthiness of the firm.

risk as the difference between the ratio of maximum losses to Tier I capital for both the stress and baseline scenarios. We also define excess credit cost as the difference between the ratio of expected losses for both the stress and baseline scenarios.

The maximum loss in the baseline scenario is expected to be covered, with an expected loss (EL) portion by loan loss provisions and an unexpected loss (UL) portion by allocated risk capital. Therefore, the difference of the ratio of maximum loss to Tier I capital between the baseline scenario and the stress scenario can be interpreted as an additional burden of credit risk on Tier I capital under a stressed condition. Similarly, excess credit cost can be regarded as an additional loan loss provision under a stressed condition, indicating the impact of a financial stress on the current profit level through increased credit costs.

The simulation results show that our framework for macro stress-testing successfully replicates the asymmetric responses of credit risk between deep recession and subsequent recovery. That is, by considering the transition of borrowers between rating classes, we can use a linear model that has different sensitivity of transition probabilities across rating classes to produce a kind of non-linearity in the fluctuations of credit risk over the business cycle.

This paper is structured as follows. In Section II, we review the frameworks for macro stress-testing employed by central banks around the world and examine their constraints and limitations. We then propose our new framework that incorporates the transition of borrower creditworthiness in the macro stress-testing. In Section III, we construct a long-term time-series of the transition matrix on borrower classifications by linking two datasets: the transition matrix data for a short observation period available at the BOJ and the migration data on credit scores provided by *Teikoku Data Bank*. In Section IV, we analyze the transition dynamics of borrower creditworthiness. To do this, we first introduce the credit portfolio view model (CPV model) proposed by Wilson (1997c) and the multi-factor model (MF model) proposed by Wei (2003). We then empirically analyze the transition dynamics of borrower creditworthiness with modified versions of the two models, and choose a model for the macro stress-testing based on the goodness-of-fit comparison both in in-sample and out-of-sample tests. In Section V, we carry out a macro stress-testing exercise to simulate the future loss from the loan portfolio under the stress scenario by Monte Carlo simulation. Finally in Section VI, we summarize the empirical results and discuss the possibility of extending our analytical framework.

II. Frameworks for Macro Stress-Testing

In this section, we first summarize the recent practice of macro stress-testing at central banks around the world. We then propose a new framework to incorporate the transition of borrower creditworthiness over the business cycle, using available data in Japan.

A. Recent Practice at Central Banks around the World

In recent years, an increasing number of central banks have been regularly publishing FSRs, in which they present their analyses and assessments of the stability and functioning of the financial system. They use macro stress-testing as a main tool to assess the robustness of the financial system against severe stresses to the economy, such as deep recessions and sharp rises in interest rates.

Macro stress-testing generally consists of three steps. In the first step, the sensitivity of a variable expressing the quality of banks' loan portfolios to the business cycle is analyzed. In the second step, extreme but plausible stress scenarios for the economic and financial environment are chosen. In the third and final step, maximum loss and/or loan loss provisions are estimated under the stress scenarios and the robustness is assessed based on the estimation results.

Table 1 summarizes the frameworks adopted in the FSRs of the selected central banks or FSAP (Financial Sector Assessment Program[5]). Details of the macro stress-testing framework vary from central bank to central bank, depending on the available data, stress scenario setting and so on. Regarding the analytical methods, all central banks focus on aggregated variables such as profits or loan loss provisions in the whole banking sector, the default rate of the corporate sector, and the delinquency rate of the household sector. These variables are analyzed in terms of macroeconomic variables including GDP growth rate.

As for the stress scenarios, many central banks assume a severe economic downturn in the domestic economy and a drastic depreciation of the US dollar (in other words, a drastic appreciation of the home currency). Meanwhile, several central banks

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[5] *FSAP* is a joint IMF-World Bank program that provides member countries that request participation with a comprehensive assessment of their financial systems. For the details of stress testing at the IMF, see Moretti, Stolz, and Swinburne (2008).

suppose a decline in property prices or an increase in oil prices, depending on the risks faced by each country. Regarding the future path of macroeconomic variables under the stress scenarios, some central banks employ projections from their own macroeconomic models, while others employ a simpler approach of univariate or multivariate time series models, such as an autoregressive model and a vector autoregressive model. As for the time horizon for analyses, most central banks examine the impact of the stress event on the banking sector for one to three years ahead.

B. A Framework to Incorporate the Transition of Borrower Creditworthiness

In this paper, we develop a new framework for macro stress-testing on credit risk, taking into consideration changes in borrowers' creditworthiness over the business cycle.

In the late 1990s and early 2000s, the creditworthiness of Japanese banks' borrowers deteriorated dramatically and the banks were forced to downgrade the ratings of many of their borrowers under mild deflation. This increase in downgrades immediately pushed up credit costs and also increased future credit risks.[6] To investigate such dynamic nature of credit risk, it is necessary to incorporate the sensitivity of firms' creditworthiness and default rates against macroeconomic fluctuations into the macro stress-testing framework. It is thus beneficial to employ transition matrices of borrower classifications, since these contain information on firms' ratings and default rates over time.[7]

The framework for macro stress-testing is comprised of four steps, as shown in

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[6] An increase in downgrades leads to a decrease in the amount outstanding of bank lending in higher ratings and an increase in the amount outstanding of bank lending in lower ratings in the next period, and the probability of default is higher in the lower ratings. Since the credit risk is calculated from data on the outstanding amount of loans in each rating and its probability of default, an increase in downgrades increases credit risk in the future.

[7] The French Banking Commission conducts macro stress-testing explicitly incorporating transition matrices. It analyzes the sensitivity of the difference between probability of upgrades and that of downgrades to GDP and interest rate, and then forecasts banks' capital adequacy ratio under the pre-assumed stress scenarios. Note that it does not use all information contained in the transition matrix, that is, changes in the probability of a firm remaining at the same rating.

Figure 1. In the first step, transition matrices for borrower classifications from 1985 onward are constructed by linking two datasets: borrower classification transition matrices for the overall banking sector compiled by the Bank of Japan since 2002, and credit score data for Japanese firms since 1985 provided by *Teikoku Data Bank*, a large private rating agency in Japan.

In the second step, using the multifactor model developed by Wei (2003), the common factor component of changes in the transition probabilities for each borrower classification is extracted. Then, the relationship between the common factor component and macroeconomic variables, including the GDP growth rate, is examined.

In the third step, a vector autoregression (VAR) model using five variables—real GDP, the CPI (excluding fresh food), the amount outstanding of bank lending, the nominal effective exchange rate, and the overnight call rate—is estimated. The VAR model is employed to project the future path of the real GDP growth rate under the assumption of an adverse shock to GDP of a size that is likely to occur with a probability of 1 percent.

In the fourth step, the future path of the GDP growth rate is inserted into the above estimated equation and the changes in transition matrices during an economic downturn are estimated. Using these results, the ratio of credit risk to Tier I capital is then computed to assess the robustness of Japan's banking system against credit risk.

III. Transition Matrix of Borrower Classifications

In this section, we present the data for the transition matrix of borrower classifications. We construct time-series data for borrower classifications since 1985 by linking two datasets: the Japanese banks' aggregated matrix data compiled by the Bank of Japan (BOJ) since 2002 and credit score data for Japanese firms provided by *Teikoku Data Bank*, a larges private rating agency in Japan, since 1985.

A. Data on Transition Matrix Available to the BOJ

The BOJ collects data from financial institutions regarding the number of borrowers of banks' loan portfolios based on five borrower classifications, "normal," "need attention," "special attention," "in danger of bankruptcy," and "*de facto* bankrupt" or "bankrupt,"[8]

[8] The definitions of borrower classifications are as follows. "Normal" borrowers are debtors

and total outstanding amount of loans and uncovered amount in each borrower classification. The BOJ collects data on around 5.5 million borrowers. Note that the definition of borrower classifications was changed in the early 2000s and thus these data are available from 2002.

The BOJ currently compiles the data used to calculate transition matrices based on the cohort approach. In the cohort approach, the element $p_{ij,t}$ in a transition matrix is constructed as follows:

$$p_{ij,t} = \frac{N_{ij,t}}{\sum_{j=1}^{5} N_{ij,t}},$$

where $N_{ij,t}$ is the number of borrowers at the i-th rating at time t-1 and at the j-th rating at time t. Note that the borrower classifications are divided into five and thus $\sum_{j=1}^{5} N_{ij,t}$ is the total number of firms rated at the i-th rating at t-1.[9]

Given that transition matrix data for a short observation period is readily available at the BOJ, we construct a long-term time-series of the transition matrix on borrower classifications by making an extrapolation using the migration data on credit scores provided by *Teikoku Data Bank*. As mentioned above, *Teikoku Data Bank* publishes the migration of credit scores of around one million Japanese firms since 1985. We explain below the method used to estimate the range of credit scores that corresponds to each borrower classification and calculate the quasi-transition matrices from the credit scores.

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with strong business conditions and no particular problems in their financial positions. "Need attention" borrowers are those who have problems with lending conditions (i.e., reduction or exemption of interest), have problems with making debt repayments (i.e., fall into arrears on principal or interest payments), or have poor or unstable business conditions. "Special attention" borrowers are debtors all or some of whose debts are classified as "special attention claims" (i.e., fall into arrears on principal or interest payments over three months). "In danger of bankruptcy" borrowers represents those who are not bankrupt at this stage but face business difficulties and are highly likely to go bankrupt in the future. "*De facto* bankrupt" represents debtors who are not yet legally and formally bankrupt but who are in serious business difficulties and without any possibility of recovery, and "bankrupt" represents those who are legally and formally bankrupt. These borrower classifications are defined by the Japanese Financial Services Agency and all the banks in Japan use such definitions.

[9] The value in the transition matrix compiled by the Bank of Japan fluctuates over time. This indicates that borrowers' classifications are not rigorous absolute classifications in terms of the probabilities of upgrades and downgrades.

B. Estimation of Transition Matrix

The range of credit scores corresponding to each borrower classification can be defined to minimize the Kullback-Leibler divergences between the transition matrices compiled by the BOJ and that calculated from credit scores.[10] We therefore solve the following minimization problem to obtain the exact ranges of credit scores:

$$\min \sum_{i,j,t} p_{ij,t} \log \frac{p_{ij,t}}{q_{ij,t}},$$

where $p_{ij,t}$ and $q_{ij,t}$ are element (i, j) in the transition matrix at time t compiled by the BOJ and estimates from credit scores.

Note that since data on the transition matrix compiled by the BOJ is only available from 2002, we estimate the range from the data on the transition matrix and credit scores from 2002 to 2005. Table 2 depicts the estimated range corresponding to each borrower classification.[11]

We then compute the quasi-transition matrices based on borrower classifications from credit scores provided by *Teikoku Data Bank*. Table 3 shows the average of the transition matrices estimated from credit scores and that compiled by the BOJ. The table demonstrates that the estimated quasi-transition matrices for borrower classifications are sufficiently precise to be used in macro stress-testing. Transition probabilities in "normal" and "need attention" in the transition matrices, around 97 percent of all borrowers, estimated from credit scores are almost the same as that compiled by the BOJ. Note, however, that transition probabilities in "special attention" and "in danger of bankruptcy" in the former matrices do not well replicate those in the latter matrices, because the coverage of lower classifications differs significantly in two datasets: the share of "special attention" and "in danger of bankruptcy" borrowers in credit scores is smaller than that used in the transition matrices compiled by the BOJ.

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10 Kullback-Leibler divergence is a widely used concept in information theory and measures the distance between density functions.

11 Many Japanese banks use credit scores to examine the accuracy of their own internal rating of borrowers and estimate the range of credit scores which corresponds to each internal rating. Based on information obtained through on-site examinations and off-site monitoring of the banks, our estimates seem to be generally consistent with their estimates.

IV. Transition Dynamics of Borrower Creditworthiness

In this section, we attempt to apply two strands of models for the transition dynamics of borrower creditworthiness to the Japanese data and compare their performances: the credit portfolio view (CPV) model of Wilson (1997c), and the multi-factor (MF) model of Wei (2003).

A. Modeling Transition Dynamics

1. Modified CPV model

The CPV model, which was proposed by Wilson (1997c), expresses a conditional transition matrix as an unconditional transition matrix multiplied by a shifting matrix as follows:

$$P_{cond} = (I + \tau S) \times P_{uncond} ,$$

where P_{cond}, P_{uncond}, I and S denote conditional transition matrix, unconditional transition matrix, unit matrix, and unknown shift matrix, respectively. τ is the parameter for sensitivity of unconditional transition probability to the business cycle. Wilson (1997c) suggests deriving τ from the ratio of conditional default rates to unconditional ones. Wilson (1997c) also argues the form of the shift matrix S, but does not explain how to choose it, which is a major drawback of the model from an analytical perspective (Trück [2008]).

Following Wilson (1997c), we suppose that the transition matrix at time t can be reproduced as the average of long-term data on the transition matrix multiplied by a matrix Q as follows:

$$P_t = \overline{P} \times Q_t , \tag{1}$$

where P_t and $\overline{P}$ denote transition matrix at time t and the average of long-term data on the transition matrix, respectively.

To specify a form of matrix Q_t, we compare two forms Q_{1t} and Q_{2t} below, referring to the Toshiba-CRAFT model:

$$Q_{1t}=\begin{pmatrix}1+\kappa_{1t} & -\kappa_{1t} & 0 & 0 & 0\\ 0 & 1+\kappa_{1t} & -\kappa_{1t} & 0 & 0\\ 0 & 0 & 1+\kappa_{1t} & -\kappa_{1t} & 0\\ 0 & 0 & 0 & 1+\kappa_{1t} & -\kappa_{1t}\\ 0 & 0 & 0 & 0 & 1\end{pmatrix}.$$

$$Q_{2t}=\begin{pmatrix}1 & 0 & 0 & 0 & 0\\ \kappa_{2t} & 1-\kappa_{2t} & 0 & 0 & 0\\ 0 & \kappa_{2t} & 1-\kappa_{2t} & 0 & 0\\ 0 & 0 & \kappa_{2t} & 1-\kappa_{2t} & 0\\ 0 & 0 & 0 & \kappa_{2t} & 1-\kappa_{2t}\end{pmatrix}$$

Q_{1t} and Q_{2t} need only one parameter, κ_{1t} and κ_{2t} respectively, to produce fluctuations in the transition probabilities. An increase in the value of κ_{1t} and κ_{2t} implies that the $\overline{P}\times Q_t$ matrix has higher upgrade probabilities than $\overline{P}$ and vice versa.

We estimate κ_1 and κ_2 so as to minimize the difference between the actual transition probabilities and reproduced ones by eq. (1) in each rating at time t, as follows:[12]

$$\min\sum_{ij}(p_{ij,t}-\hat{p}_{ij,t})^2 \quad s.t. \quad 0\le\forall\hat{p}_{ij}\le 1, \tag{2}$$

where $p_{ij,t}$ and $\hat{p}_{ij,t}$ denote actual and reproduced transition probabilities by eq. (1) from the i-th rating to the j-th rating, respectively. Subscript t represents time. Note that we impose a non-negativity constraint in each transition probability in P_t in estimating κ_{1t} and κ_{2t} to avoid transition probabilities becoming negative.

From these two matrix forms, we choose Q_{1t} considering its higher correlation of estimated κ_{1t} with GDP growth rate. Figure 2 depicts estimated κ_{1t} and GDP growth rate. Figure 2 demonstrates that κ_{1t} increased as the economy expanded in the late 1980s and the 2000s, and decreased during the economic stagnation of the 1990s.

2. Modified MF model

The MF model, which was proposed by Wei (2003), explains the fluctuations of

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12 Wilson (1997c) suggests that the sensitivity parameter be estimated only from default rates. Since we are interested in the sensitivity of changes in the transition matrix to the business cycle, we use all the information in the transition matrix.

transition probability by several factors, which are common to all rating classes as well as specific to each rating class.[13] More precisely, the transition probability from the i-th rating is transformed by the inverse of the cumulative standard normal distribution function as follows:[14]

$$z_{ij,t} = \Phi^{-1}\left(\sum_{k=1}^{j} p_{ik,t}\right).$$

Wei (2003) defined this transformed transition probability as the z-score (see Figure 3 for the relation between transition probability and corresponding z-score). Then, the deviation of the z-score at time t from the long-term average is considered to be determined as follows:

$$y_{ij,t} = \alpha(x_t + x_i) + \sqrt{1-2\alpha^2}\,\varepsilon_{ij,t}\,, \tag{3}$$

where y_{ij} denotes the deviation of the z-score of element (i, j). x, x_i, and ε_{ij} represent common factors, specific factors to the i-th rating, and idiosyncratic factor following i.i.d. standard normal distributions, respectively. α is the sensitivity of deviations of z-scores to macro and specific factors.

Regarding empirical analysis, it is often pointed out that the changes in default rates are larger in lower rating classes than in higher rating classes (Figure 4). This difference in default rates among rating classes implies that the sensitivity of the changes in transition probability varies from rating class to rating class. At the same time, it is difficult to specify an appropriate indicator for x_i in practice. These observations lead us to modify eq. (3) to eq. (4).

$$y_{ij,t} = \alpha_i x_t + \sqrt{1-\alpha_i^{\,2}}\,\varepsilon_{ij,t} \tag{4}$$

That is, although the deviation of z-scores depends on common and idiosyncratic factors, their sensitivity to the common factor varies among ratings.

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[13] In contrast, the one-factor model, which was proposed by several studies including Kim (1999), explains fluctuations in z-scores only by a common factor.

[14] Since the distribution in each row differs, it may be better to use another functional form. In this paper, however, we use the standard normal distribution for computational simplicity.

In the next step, we estimate deviations of z-scores from their average and extract the part of them that can be explained by a common factor, i.e., $\alpha_i x$. As for the derivation of $\alpha_i x$, we follow Wei's procedure (Wei [2003]). Assuming that z_{ij} can be expressed by the sum of its historical average ($\bar{z}_{ij}$) and $\alpha_i x$, we minimize the sum of squares of difference between the observed probability ($p_{ij,t} = \Phi(z_{ij,t}) - \Phi(z_{ij-1,t})$) and the fitted probability ($\hat{p}_{ij,t} = \Phi(\bar{z}_{ij,t} + \alpha_i x_t) - \Phi(\bar{z}_{ij-1,t} + \alpha_i x_t)$) over the i-th rating at time t as the following equation:

$$\min_{\alpha_i x} \sum_j (p_{ij,t} - \hat{p}_{ij,t})^2 \,. \tag{5}$$

Figure 5 shows $\alpha_i x$ for four borrower classifications. $\alpha_i x$ increased during economic expansion in the late 1990s and 2000s, and decreased during economic stagnation in the 1990s in all classifications. Moreover, the changes in $\alpha_i x$ are larger in higher classifications than in lower classifications.

B. Relationship between Transition Matrix and Macroeconomic Variables

We next analyze the relationship between the parameter indicating changes in transition matrix, estimated from the modified CPV model and the modified MF model, and macroeconomic factors.

As for the estimates from the modified CPV model, we estimate the following eq. (6) to investigate their relation with the business cycle:

$$\kappa_{1t} = c + \beta \times GDP_t + \gamma \times Debt_t \tag{6}$$

where *GDP* and *Debt* denote GDP growth rate and the ratio of interest-bearing liability to cash flow, respectively. Banks usually regard profit and liability conditions as important factors in judging borrowers' creditworthiness, so we employ GDP growth rate and the ratio of interest-bearing liability to cash flow as proxies of profit and liability conditions. The expected signs of β and γ are positive and negative respectively, because an increase in profit raises the probability of upgrade and an increase in debt ratio lowers it.

Table 4 shows the estimation results for the period from 1985 to 2005. The estimates for β and γ are both significantly positive, while we expect γ to be negative, and the adjusted R^2 is not so high.

We further estimate the following eq. (7) to investigate the relation between the shifting factors in each borrower classification, estimates from the modified MF model, and macroeconomic variables:

$$\alpha_i x_t = c_i + \beta_i \times GDP_t + \gamma_i \times Debt_t \text{,} \qquad (7)$$

The expected signs of β and γ are positive and negative respectively, as the case in the eq. (6). Note that the above equation is estimated by seemingly unrelated regression (SUR) taking account of possible correlations between error terms in each rating category. The estimation period ranges from 1985 to 2005.

Table 5 shows the estimation results. First, the effect of *GDP* is positive in all rating categories as expected, and its effect is larger in higher rating classifications and is not significant in the lowest rating. Second, the effect of *Debt* is significantly negative in the lowest two rating classifications and the negative effect is larger in "in danger of bankruptcy" than in "special attention." On the contrary, its effect is positive in the highest two rating classifications. The positive effects are significant in "normal" and insignificant in "need attention." The estimated positive effect may indicate that the rise in *Debt* caused by aggressive investment may be considered to increase a firm's value for higher rating borrowers, especially in "normal."

As for the goodness-of-fit in each rating category, the adjusted R^2s are high at around 0.7 in "normal" and "need attention" but not so high at around 0.5 in "special attention" and "in danger of bankruptcy". This observation suggests a possibility that the idiosyncratic factors exert larger effects than macroeconomic factors in lower rating categories.

C. Model Selection: In-sample and Out-of-sample Tests

In the previous sub-section, we showed that both the modified CPV model and MF model trace the relationship between transition matrices of borrower classifications and macroeconomic fluctuations. We next select a model to be used in our macro stress-testing, based on in-sample and out-of-sample tests to compare the performance of the two models and the naïve approach of assuming that the transition matrix remains unchanged from the previous year.

In the in-sample test, we construct a series of the transition matrix from 1985 to 2005 from the fitted values of κ_1 and $\alpha_i x$ based on the results estimated from eq. (6) and eq. (7) and those from the naïve approach. We then compare the differences between the actual and reproduced transition matrices among the modified CPV model, the modified MF model, and the naïve approach. To compare the models' performance, we employ two sets of criteria, which focus on the goodness-of-fit of all the transition probabilities in transition matrices and that of only the default rates of all ratings, respectively.[15] The goodness-of-fit can be expressed by the distance between actual and estimated transition probabilities or default rates. We use the absolute value and square of difference between them as the distance. The goodness-of-fit criteria on all the transition probabilities, L_1 and L_2, and those on default rates, L_1' and L_2', are defined as follows:

$$L_1 = \sum_{i,j} w_i \left| \hat{p}_{ij,t} - p_{ij,t} \right|, \quad L_2 = \sum_{i,j} w_i (\hat{p}_{ij,t} - p_{ij,t})^2 \ ,$$

$$L_1' = \sum_{i} w_i \left| \hat{p}_{i5,t} - p_{i5,t} \right|, \quad L_2' = \sum_{i} w_i (\hat{p}_{i5,t} - p_{i5,t})^2$$

where $\hat{p}_{ij}$ and p_{ij} denote estimated and actual transition probability from the i-th rating in the base year to the j-th rating, respectively. $\hat{p}_{i5}$ and p_{i5} express estimated and actual default rates in the i-th rating. w_i represents the share of firms rated as the i-th rating in the base year.

Table 6 shows the sum of each distance from 1985 to 2005. In Table 6, the in-sample test exhibits mixed results. That is, the naïve approach is the best for L_1 and L_2 and the modified MF model provides the best performance for L_1' and L_2'. The modified CPV model's performance is, however, the worst in all criteria.

Next, we carry out an out-of-sample test. In the first step of the out-of-sample test, we estimate eq. (6) and eq. (7) using the data from 1985 to 2002 and then forecast the series of κ_1 and $\alpha_i x$ from 2003 to 2005, based on the estimated parameters and actual macroeconomic variables. In the second step, we construct data on transition matrices from 2003 to 2005 using forecasted values. In the third step, we use the same criteria with the in-sample test, L_1, L_2, L_1' and L_2', to compare the forecasting performance among the three approaches. Note that in the naïve approach, the transition matrix in

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[15] The reason why we estimate the distance in not only transition probabilities but also in rates of default is that the rate of default is the most important factor when calculating credit risk.

2002 is assumed to be constant from 2003 to 2005. Table 7 presents the results. As for L_1, L_1' and L_2', the modified MF model performs best, while the naïve approach exhibits the best forecasting performance for L_2. The modified CPV model provides the worst performance in all criteria.

Although the performance of the modified MF model is almost the same as the naïve approach in the in-sample test, the MF model provides superior forecasting performance to the naïve approach. Since the stress-testing requires the best forecasting performance in the model, we employ the modified MF model in our macro stress-testing.

V. A Macro Stress-Testing Exercise

In this section, we set out an example of macro stress-testing exercises, taken from the September 2008 issue of the BOJ's *Financial System Report*.

A. Assessment of Banking Sector Robustness

In assessing the robustness of the banking sector against credit risk, we employ two indicators: the "excess credit risk" and the "excess credit cost" (see Figure 6).

We define the excess credit risk as the differences between the ratios of maximum losses to Tier I capital for both the stress and baseline scenarios. The maximum loss in the baseline scenario is expected to be covered, with an expected loss (EL) portion by loan loss provisions and an unexpected loss (UL) portion at the 99 percent level by allocated risk capital. Therefore, the difference of the ratio of maximum loss to Tier I capital between the baseline scenario and the stress scenario can be interpreted as an additional burden of credit risk on Tier I capital under a stressed condition.

We also define the excess credit cost as the difference between the ELs for both the stress and baseline scenarios. As noted above, the EL under the baseline scenario is expected to be covered by loan loss provisions. Thus, excess credit cost can be regarded as an additional loan loss provision under a stressed condition, indicating the impact of a financial stress on the current profit level through increased credit costs.

B. Baseline and Stress Scenarios

As for the future path of the real GDP growth rate, we construct a VAR model comprised of five variables: the nominal effective exchange rate, GDP, CPI, the amount outstanding of bank lending, and the overnight call rate. We take the logarithm of effective exchange rate, GDP, CPI, and the amount outstanding of bank lending. We take four-period lags based on AIC (Akaike information criterion). We employ a recursive restriction with the ordering of variables mentioned above for identifying the innovations in the VAR model.[16] We use quarterly seasonally adjusted series for the period from 1978/I to 2008/I.

Figure 7 depicts the estimated impulse responses with confidence intervals of one standard deviations, which are generally consistent with those of previous studies.[17] The first column in the figure shows the responses of five macroeconomic variables to an exchange rate shock. A positive exchange rate shock, i.e. appreciation of the yen, induces a decline in the CPI due to a decrease in import prices. In addition, since a positive exchange rate shock produces a deflationary impact on the economy, the overnight call rate is also reduced to offset such a deflationary impact from an exchange rate shock. In the second column, a positive GDP shock raises the CPI in the medium term, and it also raises the amount outstanding of bank loans in a real term and the overnight call rate in the short term. In the third column, a positive CPI shock leads to a rise in the overnight call rate in the short term. In the fourth and fifth columns, a positive shock of real bank loans outstanding and the overnight call rate produces no significant impacts. In this regard, a positive shock to the overnight call rate results in an increase in CPI, the so-called price puzzle, but its impact is not statistically significant.[18]

Based on the estimated VAR model, we compute the baseline scenario by assuming no external shock after the second quarter of 2008 to compute the path of real GDP growth rate. We also compute the stress scenario by assuming an adverse shock

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16 We check the robustness of the ordering of variables and estimate the impulse responses by using different ordering of variables. The estimated impulse responses are, however, almost the same, irrespective of different ordering conditions.

17 See Miyao (2006) for empirical studies based on VAR models using Japanese data.

18 This result is known as the "price puzzle" (see Sims [1992]) in many empirical studies based on VAR models.

to GDP in the second quarter of 2008 of a size that is likely to occur with a probability of 1 percent on an annualized basis, and that the shock will subside by half in three quarters. The real GDP growth rate in the stress scenario is lower than that in the baseline scenario by about 4 percentage points in fiscal 2008, and subsequently recovers to the baseline in about two to three years (Figure 8). However, it should be noted that the stress scenario is set to crystallize the risks in Japan's banking sector, and does not necessarily mean that they are likely to manifest themselves.

C. Simulation Results for Macro-Stress Testing

We simulate the credit risk of Japanese banks' loan portfolios by the following two steps. In the first step, we employ the modified MF model to forecast transition matrices for the future five years under the stress scenario. In the second step, we use the aggregated loan portfolios of the major banks and the regional banks at the end of fiscal 2007 (end-March 2008) to compute excess credit risk by Monte Carlo simulation and then assess their robustness against credit risk. We use the data of outstanding amount of uncovered loans in each borrower classification and assume the LGD (loss given default) to be 100 percent.[19, 20]

Figure 9 shows the amounts of excess credit risk for the major banks and the regional banks using data on loan portfolios at the end of March 2008. According to the estimate, the increase in excess credit risk induced by the significant decline in real GDP growth rate peaks in fiscal 2008, then subsides as the GDP growth rate recovers and approaches the baseline level. The maximum excess credit risk, however, reaches about 30 percent for the regional banks and 25 percent for the major banks.

Figure 10 shows the ratios of excess ELs to total loans outstanding. This figure shows that additional credit cost under the stress scenario reach 180 bps for the major banks and 190 bps for the regional banks in fiscal 2009. In this regard, even though interest margins for Japanese banks are narrow, almost half of the excess credit cost is

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[19] Therefore, we carry out the simulation on the assumption that when the borrowers go bankrupt, the entire uncovered loans turn out to be losses. Note that we assume that the total outstanding amount of bank loans remains constant for the future 5 years.

[20] We define the state of default as classified below "special attention" in Basel II and estimate the loss distribution by performing the simulation 100,000 times.

expected to be covered by the operating profits, since break-even credit cost ratios, defined as the points where operating profits from core banking business equal credit cost, are about 90 bps on average.

In the meantime, although excess credit risk increases sharply after a severe economic downturn, the risk declines rather gradually after reaching its peak, compared with the tempo of economic recovery. This is because "normal" borrowers are more sensitive to the business cycle than lower rating borrowers as is shown in Table 5, mentioned earlier, and many borrowers rated as "normal" are thus downgraded due to severe recession. Once downgraded, however, they take longer to be upgraded during economic recovery, since lower rating borrowers are not so sensitive to the business cycle.

The above observations suggest that our framework for macro stress-testing successfully replicates the asymmetric responses of credit risk between deep recession and subsequent recovery. That is, by considering the transition of borrowers between rating classes, we can use a linear model that has different sensitivity of transition probabilities across rating classes to produce a kind of non-linearity in the fluctuations of credit risk over the business cycle.

VI. Concluding Remarks

This paper described the framework for macro stress-testing on credit risk currently used at the BOJ. The framework takes account of changes in borrowers' creditworthiness over the business cycle, thereby enabling us to examine the robustness of loan portfolios for major banks and regional banks against a severe economic downturn.

The simulation results, taken from the September 2008 issue of the BOJ's *Financial System Report*, show that the framework successfully replicates the asymmetric responses of credit risk between deep recession and subsequent economic recovery. That is, by considering the transition of borrowers between rating classes, we can use a linear model that has different sensitivity of transition probabilities across rating classes against business conditions to produce a kind of non-linearity in the fluctuations of credit risk over the business cycle.

The framework is applicable to micro-stress testing at an individual bank level as well. Note that the macro stress-testing exercises employ aggregated transition matrices on borrower classifications but each bank has its own transition matrix on internal ratings for its credit risk management. Even though a bank has relatively short-term data on its transition matrix of its internal ratings, as shown in the paper, it is possible to extrapolate transition matrix data by using the migration data on credit scores provided by a rating agency. Also, each bank has detailed data including industrial transition matrices, and so can construct its own framework for stress-testing by incorporating the detailed characteristics of its borrowers.

In addition, from the standpoint of macro-prudential policy, it would be helpful to encourage private banks to establish frameworks for micro stress-testing and to discuss the state of robustness in the banking sector with them. For example, it would be possible to examine the robustness of the results of macro stress-testing through cross-checking with those from micro stress-testing for each bank's data.

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Table 1 Frameworks for Macro Stress-Testing

	U.K. (FSR)	France (FSAP)	Germany (FSR)	Italy (FSAP)	Netherlands (FSR)	Spain (FSAP)	Sweden (FSR)	Swiss (FSR)
Source of Stress	interest rate, exchange rate, global economy	global economy, oil price, exchange rate, interest rate	domestic economy, exchange rate, oil price	oil price, exchange rate, credit spread	domestic economy, exchange rate, house prices	oil price, exchange rate, asset prices	domestic economy	domestic economy, interest rate, general prices
Time Horizon	1 to 3 years	2 years	2 years	2 years	1 to 3 years	1 to 3 years	1 to 3 years	1 year
Analytical Framework								
Purpose	banks' profit	banks' profit, risk weighted assets	loan loss provisions	credit risk, banks' profit	banks' profit, capital adequacy ratio	banks' profit, capital adequacy ratio	credit risk, banks’ profit	banks’ profit, loan loss provisions
Methods	PD equation, profit equation, household delinquency equation	PD equation, profit equation,	loan loss provision equation, bank lending equation	PD equation, profit equation,	profit equation	N.A.	PD equation	profit equation, loan loss provision equation
Projection method for Macroeconomic Variables	large scaled macro model	N.A.	N.A.	N.A.	large scaled macro model	N.A.	N.A.	standard deviation of variables from 1985 to 2005

Table 2 Range of Credit Scores Corresponding to Each Borrower Classification

	Range of Credit Scores
"Normal"	46 to 100
"Need attention"	42 to 45
"Special attention"	41
"In danger of bankruptcy"	38 to 40
"*De facto* bankrupt" or "bankrupt"	below 37

Note: Ranges are estimated by minimization of relative entropies between transition matrix compiled by the BOJ and that from credit scores.

Table 3 Average of Transition Matrix from 2002 to 2005

A. Transition Matrix compiled by the BOJ (%)

t \ t+1	Normal	Need attention	Special attention	In danger of bankruptcy	*De facto* bankrupt or bankrupt	Firms (share %)
Normal	98.0	1.5	0.1	0.2	0.2	5,401,215 (92.1)
Need attention	14.0	75.1	3.9	4.7	2.3	320,885 (5.5)
Special attention	4.7	21.8	53.9	12.7	6.9	41,654 (0.7)
In danger of bankruptcy	2.9	6.9	2.0	71.2	17.0	56,327 (1.0)
De facto bankrupt or bankrupt	0.0	0.0	0.0	0.0	100.0	43,292 (0.7)

B. Transition Matrix from Credit Scores (%)

	Normal	Need attention	Special attention	In danger of bankruptcy	*De facto* bankrupt or bankrupt	Firms (share %)
Normal	97.5	1.8	0.0	0.0	0.7	699,896 (89.9)
Need attention	12.5	84.5	0.5	0.3	2.1	66,752 (8.6)
Special attention	4.8	16.9	72.3	1.9	4.1	1,974 (0.3)
In danger of bankruptcy	4.3	15.6	2.0	72.6	5.6	1,757 (0.2)
De facto bankrupt or bankrupt	0.0	0.0	0.0	0.0	100.0	7,817 (1.0)

Note: Borrowers classified as "*De facto* bankrupt" or "bankrupt" in time t are assumed not to upgrade in time t+1.

Table 4 Estimation Results on κ_1

	Constant term	*GDP*	*Debt*	*Adj* R^2
κ_1	-0.051^{***} (-3.83)	0.006^{***} (6.36)	0.002^{***} (2.99)	0.62

Note: The estimation period is from 1985 to 2005. *** represents statistical significance at the 1 percent level. Figures in parentheses are *t*-statistics.

Table 5 Estimation Results on Shifting Factor from the Modified MF Model

	Constant term	*GDP*	*Debt*	*Adj* R^2
"Normal" borrowers	-0.47^{***} (-2.97)	0.09^{***} (7.28)	0.02^{**} (2.15)	0.69
Borrowers that "need attention"	-0.15^{*} (-1.89)	0.05^{***} (7.54)	0.00 (0.64)	0.73
Borrowers requiring "special attention"	0.08 (1.05)	0.02^{***} (3.99)	-0.01^{*} (-1.92)	0.56
Borrowers "in danger of bankruptcy"	0.31^{***} (3.46)	0.01 (1.05)	-0.02^{***} (-4.02)	0.50

Note: The estimation period is from 1985 to 2005. ***, **, and * represent statistical significance at the 1, 5, and 10 percent level, respectively. Figures in parentheses are *t*-statistics.

Table 6 In-Sample Test: Goodness-of-Fit

	Modified CPV model	Modified MF model	Naïve approach
L_1	6.379 (3)	4.340 (2)	2.724 (1)
L_2	0.190 (3)	0.106 (2)	0.064 (1)
L_1’	1.046 (3)	0.264 (1)	0.299 (2)
L_2'	0.028 (3)	0.002 (1)	0.004 (2)

Note: Figures in parentheses denote the ranking of the goodness-in-fit.

Table 7 Out-of-Sample Test: Goodness-of-Fit

	Modified CPV model	Modified MF model	Naïve approach
L_1	0.545 (3)	0.428 (1)	0.430 (2)
L_2	0.022 (3)	0.015 (2)	0.014 (1)
L_1'	0.094 (3)	0.042 (1)	0.078 (2)
L_2'	0.004 (3)	0.001 (1)	0.002 (2)

Note: Figures in parentheses denote the ranking of the goodness-in-fit.

Figure 1 Framework of Macro Stress-testing

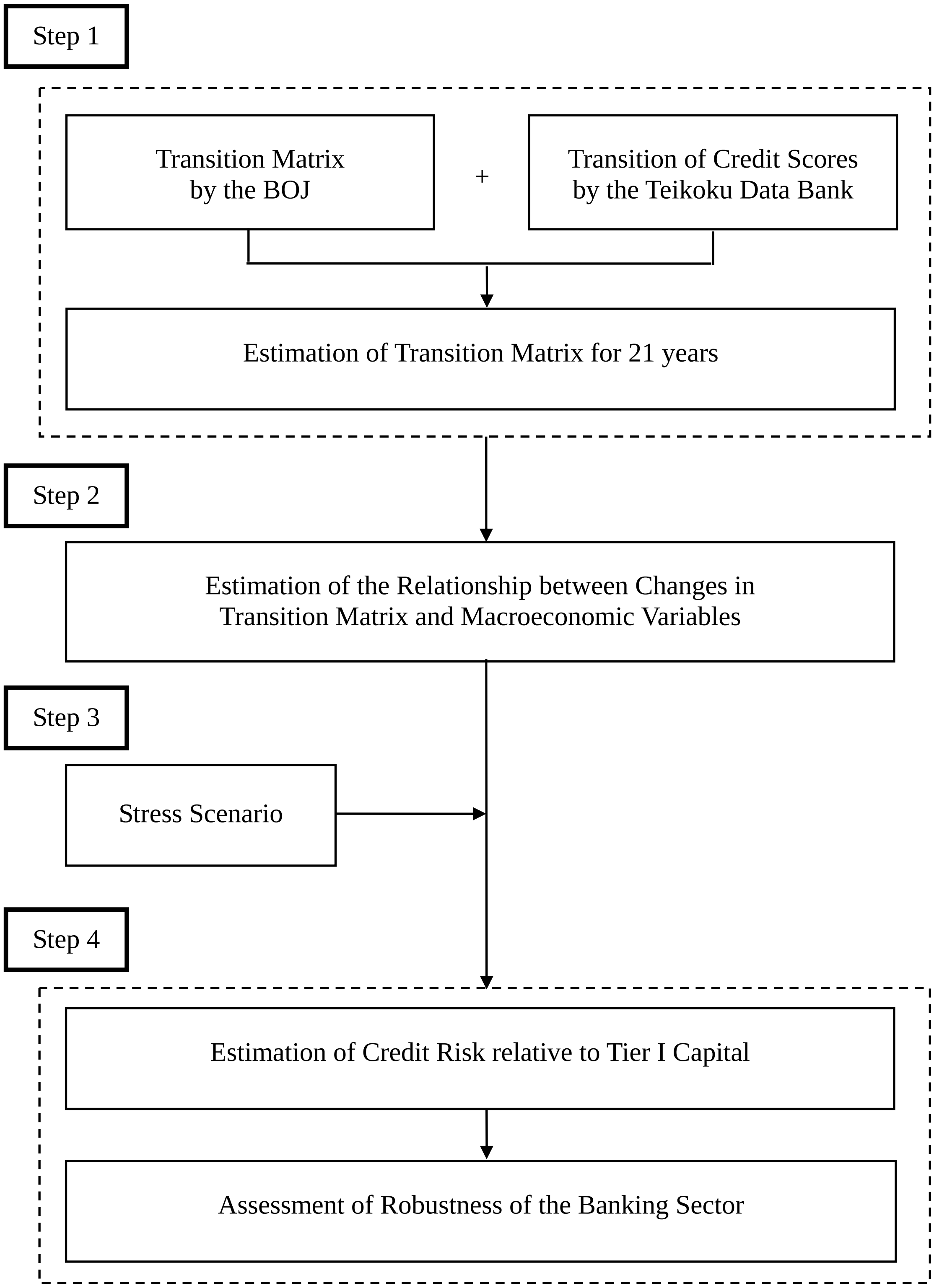


Figure 2 κ_1 estimated from the Modified CPV Model

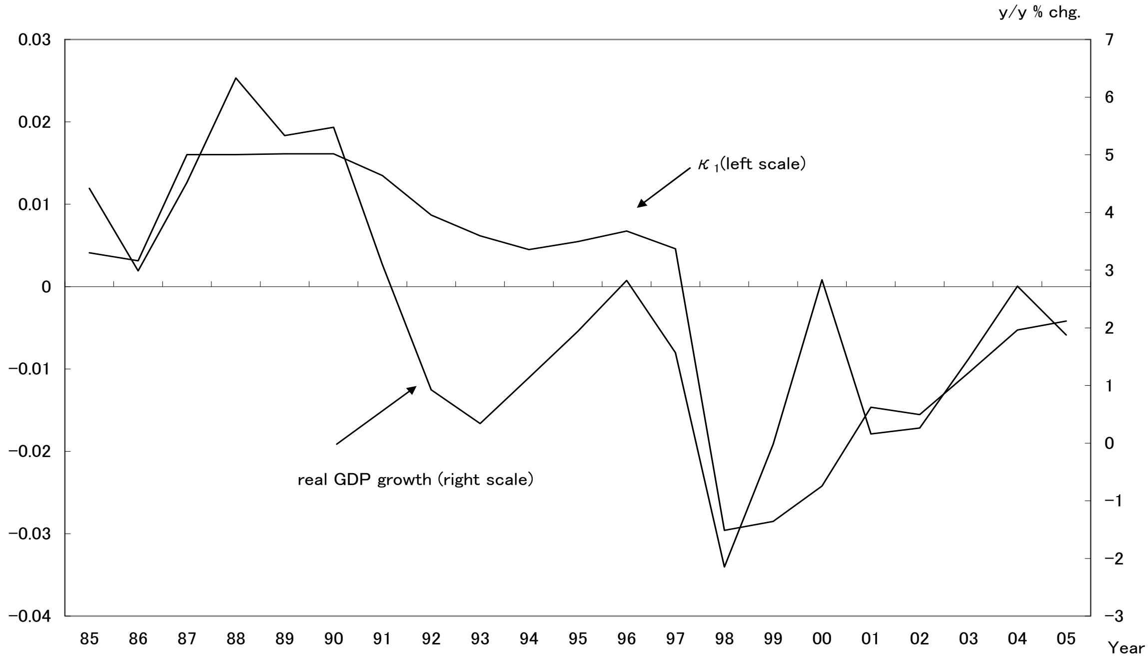
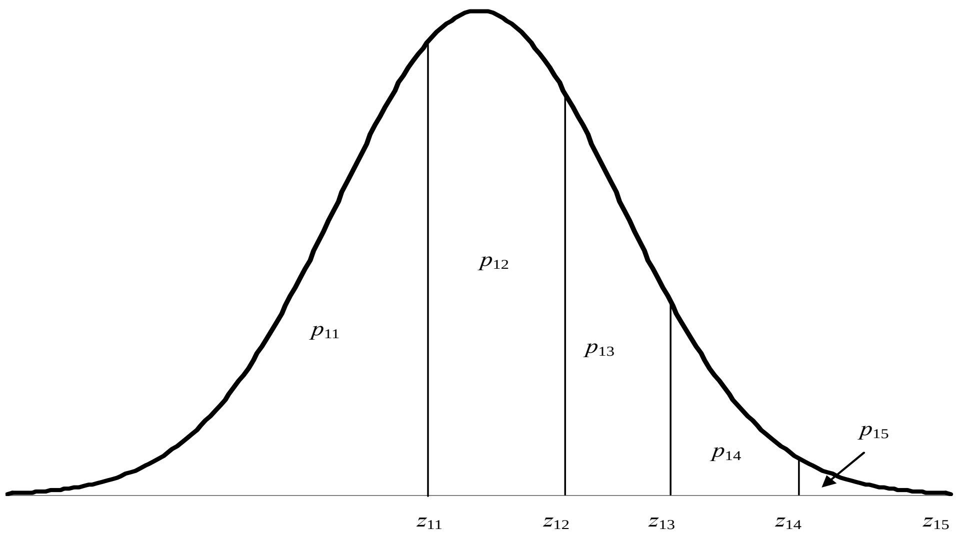


Figure 3 Relation between Transition Probabilities and z-Scores: Case of "Normal" Borrowers



Note: p_{11}, p_{12}, p_{13}, p_{14}, and p_{15} denote the transition probabilities from "normal" to "normal", to "need attention", to "special attention", to "in danger of bankruptcy", and "*de facto* bankruptcy" or "bankrupt". z_{11}, z_{12}, z_{13}, z_{14}, and z_{15} are corresponding z-scores to p_{11}, p_{12}, p_{13}, p_{14}, and $p_{15,}$ respectively.

Figure 4 Default Rates in Each Borrower Classification

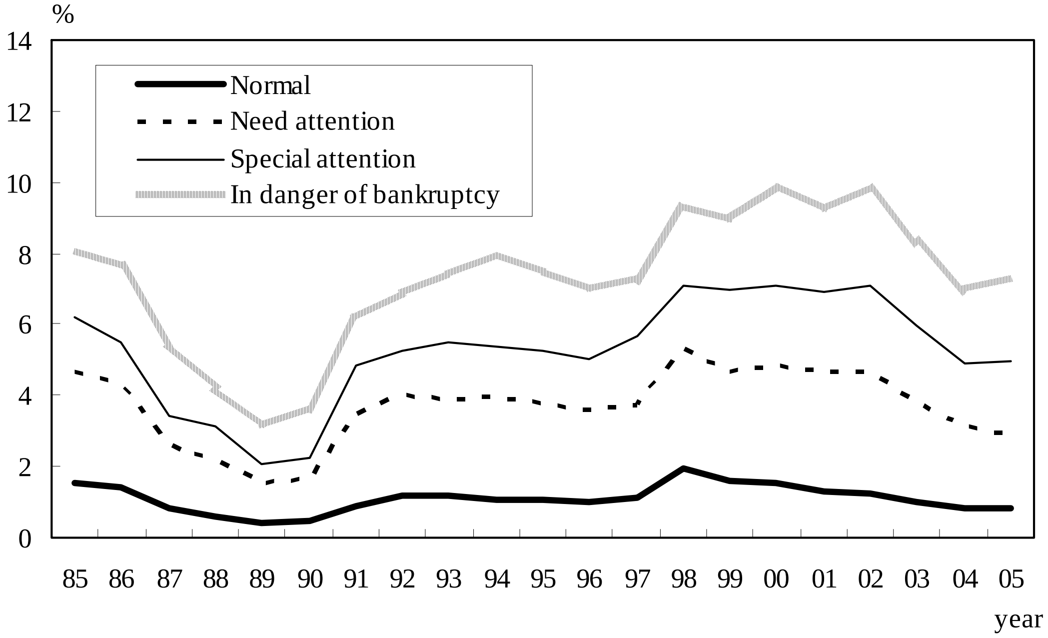


Figure 5 $\alpha_i x$ estimated from the Modified MF Model

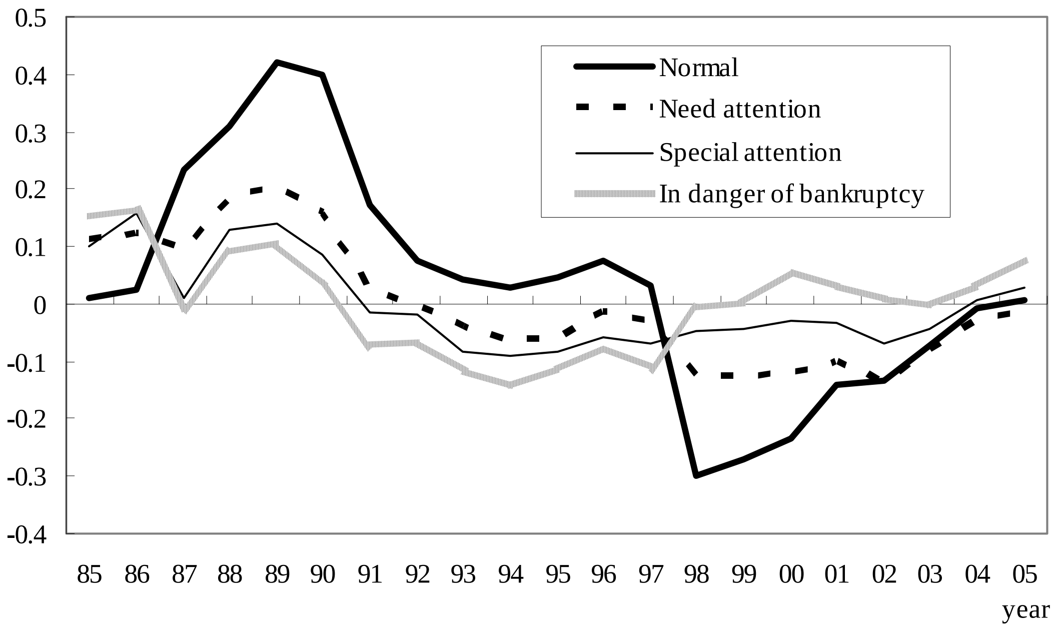


Figure 6 Excess Credit Risk and Excess EL: Illustration

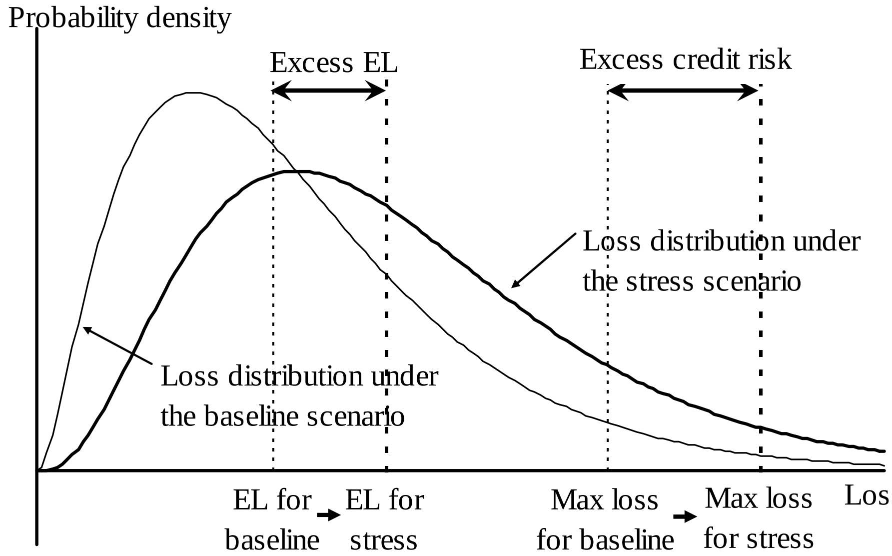
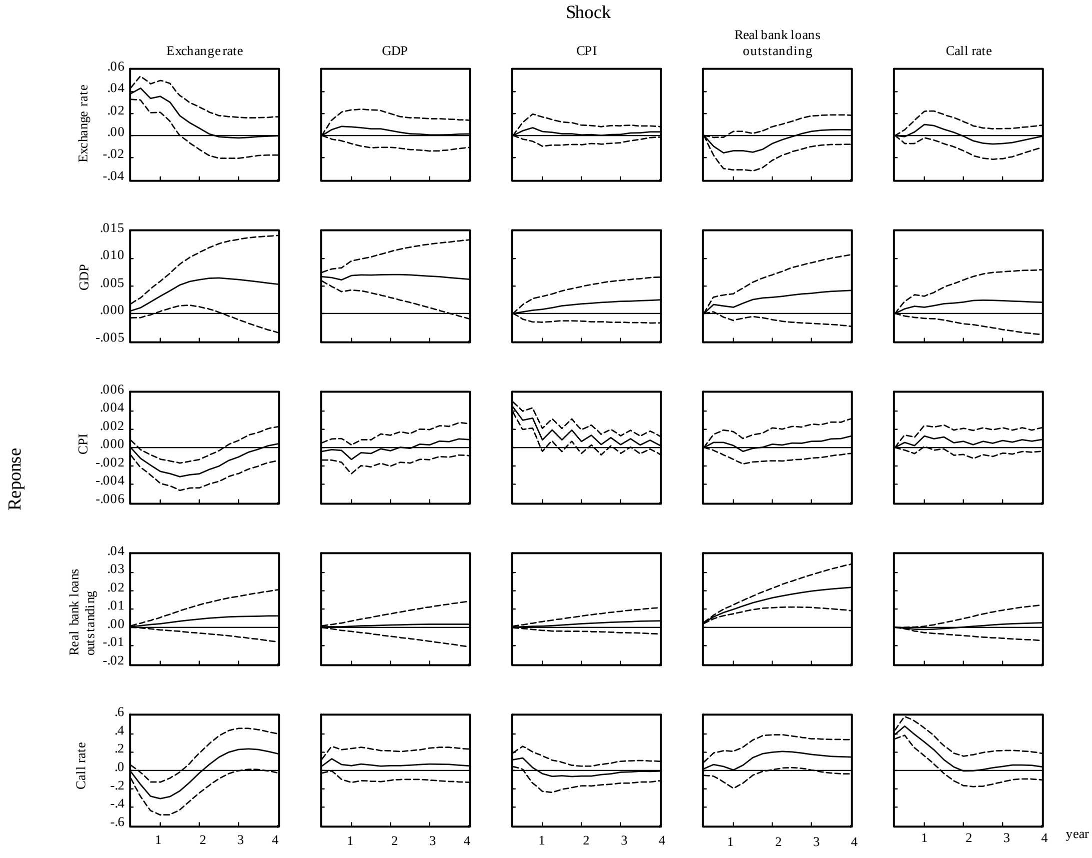


Figure 7 Impulse Response of VAR Model



Note: Dotted lines are one standard error bands.

Figure 8. GDP Growth Rate under the Stress Scenario:
Deviation from the Baseline Scenario

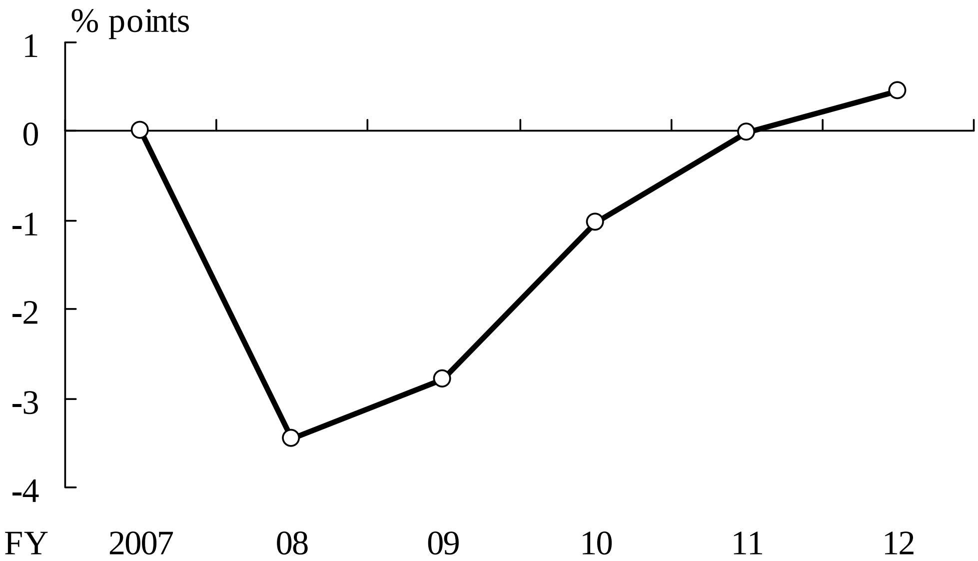


Figure 9. Simulated Excess Credit Risk

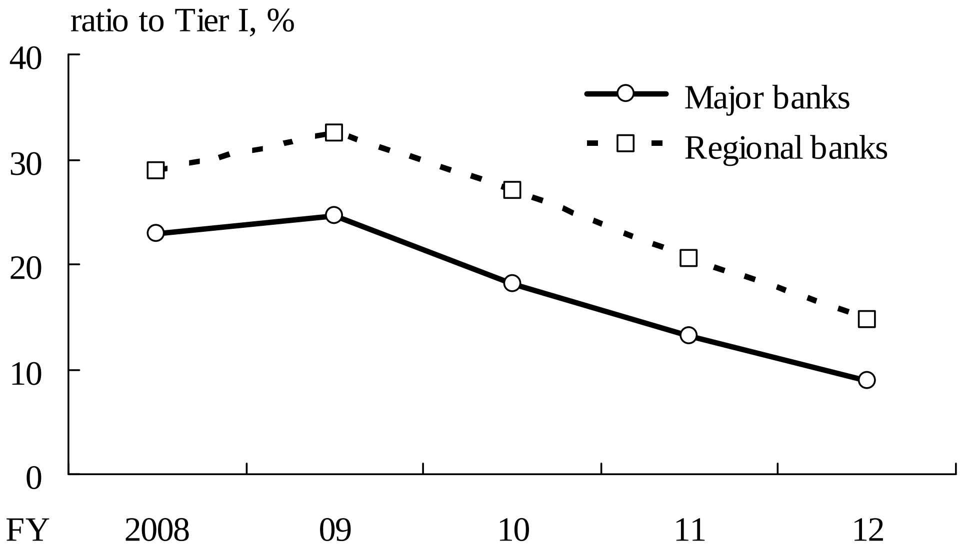
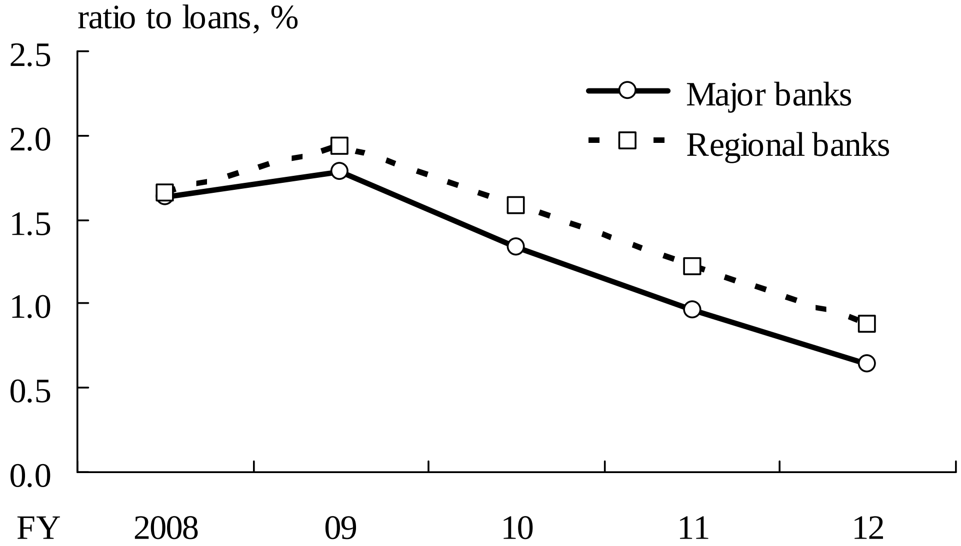


Figure 10.　Simulated Excess Credit Cost



Note: Figures are ratio to total loan outstanding.