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Measuring Potential Growth with an Estimated DSGE Model of Japan's Economy*

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Abstract

In this paper, we calculate the potential output and the output gap using a Bayesian-estimated DSGE model of Japan's economy. The model is a two-sector growth model that takes into account growth rate shocks including investment-goods sector-specific technological progress. For bridging the gap with conventional measures, we define our measure of potential output as a component of the efficient output generated only by growth rate shocks. Our potential growth displays a high degree of smoothness and moves closely with conventional measures. Moreover, the output gap from our measure of potential output has forecasting power for inflation. We analyze the sensitivity of our measure to the specifications of monetary policy rules, labor supply shocks, price and wage markup shocks, and technology shocks as well as the robustness with respect to data revisions and updates.

Keywords: Potential growth; Output gap; Dynamic stochastic general equilibrium model; Bayesian estimation; Real-time data
JEL classification: E32; E37; O41; O47

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1 Introduction

It has been widely acknowledged that estimated dynamic stochastic general equilibrium (DSGE) models are able to fit the data as well as do reduced-form vector autoregression (VAR) models, as shown by Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003), and Levin et al. (2005). A recent trend in developing DSGE models is to pursue their ability to "tell stories" in a policymaking context (Edge, Kiley, and Laforte, 2008). For monetary and fiscal policy discussions, empirically plausible and theoretically coherent explanations for model-based estimates of potential output and output gap would be invaluable and essential.

Despite their conceptual importance in a policymaking context, measures of potential output and output gap from DSGE models are controversial.[1] In general, model-based measures of potential output, which estimate an efficient level of output without pressure for inflation to either accelerate or decelerate, tend to be more volatile than conventional measures based on the production function approach or on statistical smoothing methods (e.g., the Hodrick-Prescott (HP) filter) which try to capture medium-term growth trends of output. This tendency reflects a significant difference in views between modelers and policymakers on which types of shocks drive the short-run macroeconomic fluctuations. While DSGE models attribute a substantial fraction of the fluctuations to fundamental shocks such as temporary technology shocks, policymakers' traditional views implicitly assume that "animal spirit" expenditure shocks play a central role in the short-run fluctuations and that an efficient level of output is driven mainly by permanent technology shocks. Many policymakers accordingly disagree with the DSGE view that a substantial fraction of the short-run fluctuations is efficient and does not require policy responses.[2]

[1]For instance, Mishkin (2007) and Basu and Fernald (2009) discuss the characteristics of several measures and concepts of potential output, and Kiley (2010) discusses those of output gap based on DSGE models.

[2]For instance, Bean (2005) objects to Hall (2005)'s argument that even short-run macroeconomic fluctuations are the natural consequences of a well-functioning economy responding to shocks to productivity and real spending.

The aim of this paper is to bridge the gap between the conventional and model-based measures of potential output using a Bayesian-estimated DSGE model of Japan's economy.[3] Our model shares many similar features with recent New Keynesian DSGE models in the literature and those practically used in central banks. A key feature of our model is that it takes into account growth rate shocks, so that we can estimate directly the growth trend of output without detrending the data.[4] Based on this model, we define our measure of potential output as a component of the efficient level of output *generated only by growth rate shocks*. Then our potential growth, the year-on-year growth rate of our measure of potential output, displays a high degree of smoothness and moves closely with the conventional measures of potential growth. Meanwhile, the efficient output itself, which is defined in our model as the level of output in an environment without nominal rigidities in goods and labor markets and without shocks to price and wage markups,[5] moves closely with the actual output and thus is more volatile than our measure and conventional measures of potential output. While the model views a substantial fraction of short-run fluctuations in the actual output as efficient, we take only the medium- and longer-term growth component of the efficient output as our measure of potential output, for telling stories consistent with the policymakers' traditional views.

Another key feature of our model is a two-sector production structure that reflects the trends in relative prices and categories of real expenditure apparent in the Japanese data. Our model explicitly divides the final goods into the consumption goods produced by the slow-growing sector and the investment goods produced by the fast-growing sector. This two-sector pro-

[3]The model is a variant of the Medium-scale Japanese Economic Model (M-JEM), which has been developed at Research and Statistics Department, Bank of Japan.

[4]Most DSGE models of Japan's economy are estimated or calibrated using detrended data. For instance, Sugo and Ueda (2008) use the data detrended with kinked linear trends, and Ichiue et al. (2008) use those detrended by potential output based on the production function approach. Hirose and Kurozumi (2010) consider technology growth rate shocks, but use the output gap based on the production function approach in the estimation.

[5]A slightly different DSGE-based measure of potential output is the "natural output," which is defined as the level of output in an environment without nominal rigidities in goods and labor markets but with shocks to price and wage markups.

duction structure is useful not only because it generates empirically plausible comovement between consumption and investment in response to investment-specific technology shocks,[6] but also because it allows us to tell stories about our measure of potential growth. Since we consider the economy-wide and investment-specific technology growth rate shocks in our model, our measure of potential growth can be decomposed into those two types of technology growth rate shocks in addition to an exogenous population growth. While the investment-specific technology growth rate shock has constantly raised the potential growth during the sample period since the 1980s, the economy-wide technology growth rate shock has reduced the potential growth since the 1990s.

Based on our measure and the conventional measures of potential output as well as the efficient output (a standard DSGE model-based measure of potential output), we can calculate the several corresponding measures of output gap, which is defined as the deviation of the actual output from a measure of potential output. We compare the predictability of inflation across those measures of output gaps by testing Granger causality and by estimating bivariate models of output gap and inflation. Although the gap from the efficient output is theoretically the most relevant measure that indicates inflationary or deflationary pressure, it does not necessarily show better forecast performance than the other measures of output gap. Meanwhile, the gap from our measure of potential output has some forecasting power for inflation and can be used practically as an indicator of inflationary or deflationary pressure.[7]

Since some aspects of the model structure and the identification of shocks

[6] Under certain conditions, as shown by Greenwood, Hercowitz, and Krussell (1997), a technology shock to investment goods in a one-sector model is equivalent to a technology shock to an investment-goods-producing sector in a two-sector model. Guerrieri, Henderson, and Kim (2010), however, argue that the above conditions are highly restrictive and that a one-sector model in which the investment-specific technology shocks play a prominent role often generates the counterfactually negative correlation between consumption and investment.

[7] Kiley (2010) shows that the deviation of output from its long-run stochastic trend (Beveridge-Nelson cycle) estimated using a DSGE model of the U.S. economy used at the Federal Reserve Board (the EDO model) is similar to the output gap based on the production function approach and has forecasting power for inflation.

are controversial among researchers and the sensitivity to the details of models is a great concern for users of model-based measures, it is important to check the robustness of those measures.[8] We find that our measures of potential growth and output gap are robust with respect to the specifications of monetary policy rules and identifications of labor supply shocks, price and wage markup shocks, and measurement errors in prices and wages. Meanwhile, our measure of potential growth naturally depends on the specifications of technology level shocks and technology growth rate shocks.

We also check the robustness of our results with respect to data revisions and updates. In general, the real-time estimates move closely with the latest estimate, but they sometimes deviate from each other, especially near the end of each sample period of the real-time estimates. For both conventional and model-based measures, the real-time estimates of potential growth and output gap tend to be revised to a large extent, which is practically a serious problem in policymakers' use of those measures.

The remainder of the paper is organized as follows. Section 2 describes our model. Section 3 explains the estimation procedures and reports the estimation results. In Section 4, we calculate our measure of potential growth and compare it with alternative measures. In Section 5, we calculate the several corresponding measures of output gap and compare the predictability of inflation across those measures. Section 6 discusses the robustness check with respect to the model structure and the identification of shocks. Section 7 discusses the robustness check with respect to data revisions and updates. Section 8 concludes. Appendices A to D provide the details of our model, the data, and the lists of variables and parameters.

[8] Some recent studies including Coenen, Smets, and Vetlov (2008), Justiniano and Primiceri (2008), and Sala, Söderström, and Trigari (2010) investigate the sensitivity of model-based measures of potential output and output gap to the details of DSGE models.

2 The Model

In this section, we provide an overview and a brief description of our model. More details, including the equilibrium conditions, stationary equilibrium conditions, and log-linearized system, are provided in Appendix A.

2.1 Overview

Our model is a two-sector growth model that takes into account growth rate shocks including investment-specific technological progress.[9] There are two final goods in the model: the consumption goods produced by the slow-growing sector and the investment goods produced by the fast-growing sector. We assume that the former goods are purchased by households and the government and that the latter goods are purchased by capital owners and foreign countries (net exports). The two-sector production structure with differential rates of technological progress across sectors induce different trends in categories of real expenditure and secular relative price differentials, which are both apparent in the Japanese data.[10]

Meanwhile, our model shares many similar features with recent New Keynesian DSGE models in the literature, such as monopolistic competition, sticky prices and wages, adjustment costs, habit persistence, etc. The goods are produced in two stages by intermediate- and then final-goods-producing firms in each sector. The final-goods-producing firms aggregate differentiated sector-specific intermediate goods. The intermediate-goods-producing firms combine the aggregate labor inputs with utilized capital and set prices of their differentiated output. The capital owners rent their capital to the intermediate-goods-producing firms in both sectors. Households supply differentiated labor forces to the intermediate-goods-producing firms in both

[9] Our model closely follows the Federal Reserve Board's Estimated, Dynamic, Optimization-based (EDO) model (Edge, Kiley, and Laforte, 2007; Chung, Kiley, and Laforte, 2010). The two-sector representation of the investment-specific technological progress is also described by Whelan (2003), Ireland and Schuh (2008), and others.

[10] In our dataset from 1981 to 2009 (explained in Section 3), the average annual growth rate of the real value added of the slow-growing sector is 1.86%, and that of the fast-growing sector is 2.81%, while the price of the investment goods relative to the consumption goods has declined at 1.75% per year on average.

sectors. In what follows, we describe the decisions made by each of the agents in our economy.

2.2 Final goods producers

Final goods producers in the slow-growing sector (sector c) produce the consumption goods X_t^c, and those in the fast-growing sector (sector k) produce the investment goods X_t^k. They face competitive markets and produce the final goods, X_t^s, $s \in \{c, k\}$, by combining a continuum of s sector-specific intermediate goods, $X_t^s(j)$, $j \in [0, 1]$, according to the following Dixit-Stiglitz type technology.

$$X_t^s = \left(\int_0^1 X_t^s(j)^{\frac{\Theta_t^{x,s}-1}{\Theta_t^{x,s}}} dj \right)^{\frac{\Theta_t^{x,s}}{\Theta_t^{x,s}-1}}, \quad s = \{c, k\} \tag{1}$$

where $\Theta_t^{x,s}$ is the elasticity of substitution between the differentiated intermediate goods input. Letting $\theta_t^{x,s}$ be the log-deviation from its steady-state value, we assume that $\theta_t^{x,s}$ follows an ARMA(1,1) process.[11]

$$\theta_t^{x,s} = \rho^{\theta_{x,s}} \theta_{t-1}^{x,s} + \epsilon_t^{\theta,x,s} - \rho^{\theta_{x,s},ma} \epsilon_{t-1}^{\theta,x,s} \tag{2}$$

where $\epsilon_t^{\theta,x,s}$ is an i.i.d. shock process. This stochastic elasticity of substitution introduces transitory markup shocks into the pricing decisions of intermediate goods producers. Subject to the above aggregation technology, a final goods producer in each sector chooses the optimal level of each intermediate goods to minimize the cost of purchasing them, taking their prices as given.

2.3 Intermediate goods producers

Intermediate goods producers in both sectors face the monopolistically competitive market and produce the sector-specific intermediate goods $X_t^s(j)$, $s \in \{c, k\}$ with the following production function.

[11] Smets and Wouters (2007) assume that the price and wage markup shocks follow ARMA(1,1) processes to capture the high-frequency fluctuations in price and wage inflations.

$$X_t^s(j) = [K_t^{u,s}(j)]^{\alpha} \left[AZ_t^m AZ_t^s L_t^s(j)\right]^{1-\alpha} \tag{3}$$

where $K_t^{u,s}(j)$ and $L_t^s(j)$ are the effective capital input and the labor input of a firm j, respectively. Letting $U_t^s(j)$ be the capital utilization rate in sector s, the effective capital input is written as $K_t^{u,s}(j) \equiv K_t^s(j) \times U_t^s(j)$. Further, the labor input of a firm j is the continuum of the differentiated labor input, $L_t^s(j) = [\int_0^1 L_t^s(i,j)^{(\Theta_t^l-1)/\Theta_t^l} di]^{\Theta_t^l/(\Theta_t^l-1)}$, where Θ_t^l is the elasticity of substitution, and its log-deviation θ_t^l follows an ARMA(1,1) process. This stochastic elasticity of substitution introduces transitory wage markup shocks into households' labor supply decisions.

AZ_t^m is the economy-wide technology shock and AZ_t^k is the fast-growing (investment-goods-producing) sector-specific technology shock. In order to reduce the number of shocks in the model, we presume that the slow-growing (consumption-goods-producing) sector does not have the sector-specific shock ($AZ_t^c \equiv Z_t^c = 1$). We assume that each of the technology shocks contains two separate stochastic components: one (A_t^n) is stationary in levels and the other (Z_t^n) is stationary in growth rates, where $n \in \{m, k\}$.

$$\ln AZ_t^n = \ln A_t^n + \ln Z_t^n \tag{4}$$

$$\ln A_t^n = \ln A_*^n + \epsilon_t^{a,n} \tag{5}$$

$$\ln Z_t^n - \ln Z_{t-1}^n = \ln \Gamma_t^{z,n} = \ln(\Gamma_*^{z,n} \times \exp[\gamma_t^{z,n}]) = \ln \Gamma_*^{z,n} + \gamma_t^{z,n} \tag{6}$$

$$\gamma_t^{z,n} = \rho^{z,n}\gamma_{t-1}^{z,n} + \epsilon_t^{z,n} \tag{7}$$

where $\epsilon_t^{a,n}$ and $\epsilon_t^{z,n}$ are i.i.d. shock processes, and A_*^n and $\Gamma_*^{z,n}$ are the constant technology level and growth rate, respectively. (Hereafter, variables with subscript $*$ represent the variables at steady state.)

An intermediate goods producer j in sector $s \in \{c, k\}$ maximizes the discounted future profit,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^c}{P_t^c} \Bigg\{ P_t^s(j) X_t^s(j) - MC_t^s(j) X_t^s(j) \tag{8}$$

$$- \frac{100 \cdot \chi^p}{2} \left(\frac{P_t^s(j)}{P_{t-1}^s(j)} - \eta^p \Pi_{t-1}^{p,s} - (1-\eta^p)\Pi_*^{p,s} \right)^2 P_t^s X_t^s \Bigg\}$$

subject to the final goods producers' demand schedule,

$$X_t^s(j) = \left(\frac{P_t^s(j)}{P_t^s}\right)^{-\Theta_t^{x,s}} X_t^s \tag{9}$$

taking as given the marginal cost of production, $MC_t^s(j)$, the aggregate price level for its sector, $P_t^s = \{\int_0^1 [P_t^s(j)]^{(\Theta_t^s-1)/\Theta_t^s} dj\}^{\Theta_t^s/(\Theta_t^s-1)}$, and households' valuation of a unit nominal income in each period, Λ_t^c/P_t^c where Λ_t^c is the marginal utility of consumption. The second line in (8) represents the quadratic price adjustment cost as in Rotemberg (1982), where $\Pi_t^{p,s} = P_t^s/P_{t-1}^s$ and $\Pi_*^{p,s}$ is the time-invariant trend inflation. Since the cost is imposed on the deviation of the optimum price inflation from the past inflation, the equilibrium inflation as well as the price response to the marginal cost becomes sticky.

2.4 Capital stock owners

Capital stock owners provide the capital service to the intermediate goods producers in both sectors, receive the rental cost of capital in exchange, and accumulate the investment goods. Each capital stock owner k chooses investment expenditure, $I_t(k)$, the amount and utilization of capital in both sectors, $K_t^c(k)$, $U_t^c(k)$, $K_t^k(k)$, and $U_t^k(k)$, to maximize its discounted profit,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^c}{P_t^c} \left[R_t^c U_t^c(k) K_t^c(k) + R_t^k U_t^k(k) K_t^k(k) - P_t^k I_t(k)\right] \tag{10}$$

subject to the capital evolution process with quadratic investment adjustment cost and the costs from higher utilization rates,

$$\begin{aligned} K_{t+1}(k) &= (1-\delta)K_t(k) + I_t(k) \\ &\quad - \frac{100 \cdot \chi}{2} \left[\frac{I_t(k)A_t^{\varphi} - I_{t-1}(k)\Gamma_t^{z,m}\Gamma_t^{z,k}}{K_t}\right]^2 K_t \\ &\quad - \sum_{s=c,k} \kappa \left[\frac{\left(Z_t^U U_t^s(k)\right)^{1+\psi} - 1}{1+\psi}\right] K_t^s \end{aligned} \tag{11}$$

where $K_t(k) = K_t^c(k) + K_t^k(k)$. The third term (in the second line) of the right hand side is the quadratic investment adjustment cost, where A_t^{φ} is a stochastic variation in the adjustment cost that is assumed to be unity at the steady state and to follow an AR(1) process. The last term of the right hand side is the utilization cost, which presumes that high capital utilization leads to faster capital depreciation, as in Greenwood, Hercowitz, and Huffman (1988). Z_t^U is a stochastic variation in the utilization cost that is assumed to be common in both sectors and to follow an AR(1) process. We set κ so that the utilization rate is unity at the steady state.

2.5 Households

Each household i chooses its purchase of consumption goods, $C_t(i)$, its holdings of bonds, $B_t(i)$, its wages for both sectors, $W_t^c(i)$ and $W_t^k(i)$, and supply of labor consistent with each wage, $L_t^c(i)$ and $L_t^k(i)$, given the demand schedule for the differentiated labor supply, $L_t^c(i) = (W_t^c(i)/W_t^c)^{-\Theta_t^l} L_t^c$ and $L_t^k(i) = (W_t^k(i)/W_t^k)^{-\Theta_t^l} L_t^k$, to maximize the utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \Xi_t^{\beta} \left\{ \varsigma^c \ln \left(C_t(i) - hC_{t-1}(i) \right) - \varsigma^l \frac{\left[\left(L_t^c(i) + L_t^k(i) \right) / \Xi_t^l \right]^{1+\nu}}{1+\nu} \right\} \quad (12)$$

subject to its budget constraint,

$$\begin{aligned} \frac{1}{R_t} B_{t+1}(i) &= B_t(i) + \sum_{s=c,k} W_t^s(i) L_t^s(i) + \Omega_t(i) - P_t^c C_t(i) \\ &\quad - \sum_{s=c,k} \frac{100 \cdot \chi^w}{2} \left\{ \frac{W_t^s(i)}{W_{t-1}^s(i)} - \eta^w \Pi_{t-1}^{w,s} - (1-\eta^w) \Pi_*^{w,s} \right\}^2 W_t^s L_t^s \\ &\quad - \frac{100 \cdot \chi^l}{2} \left(\frac{L_*^c}{L_*^c + L_*^k} W_t^c + \frac{L_*^k}{L_*^c + L_*^k} W_t^k \right) \\ &\quad \times \left(\frac{L_t^c(i)}{L_t^k(i)} - \eta^l \frac{L_{t-1}^c}{L_{t-1}^k} - (1-\eta^l) \frac{L_*^c}{L_*^k} \right)^2 \frac{L_t^k}{L_t^c} L_t. \end{aligned} \quad (13)$$

In the utility function, Ξ_t^{β} is the *inter*temporal preference shock, Ξ_t^{l} is the labor supply shock (*intra*temporal preference shock), ς^c and ς^l are scale parameters that determine the ratio between the household's consumption and leisure, and h is the degree of the habit persistence of the household. We assume that the log-deviation of the intertemporal preference shock follows an AR(1) process and that the labor supply shock is non-stochastic ($\Xi_t^l = 1$) to properly identify the wage markup shock. We consider the case of stochastic labor supply shock in Section 5.1. In the budget constraint, R_t is the nominal interest rate on the bonds and $\Omega_t(i)$ is the household's capital and profits income. The fifth term (in the second line) of the right hand side is the quadratic wage adjustment cost imposed on the deviation of the optimum wage growth from the past wage inflation, $\Pi_{t-1}^{w,s}$, and from the trend wage inflation, $\Pi_*^{w,s}$. With this formulation, the wage inflation as well as the wage level becomes sticky. The last term of the right hand side is the labor reallocation cost, which helps to generate realistic sectoral comovement of labor inputs during business cycles.

2.6 Real GDP growth and GDP deflator inflation

Since the trend growth rate is different in each sector, we aggregate the real GDP as a divisia index, following Whelan (2003) and Edge, Kiley, and Laforte (2007). This divisia-index aggregation allows us to avoid the base-year bias of the deflator and to mimic the SNA data compiled with the chain-index aggregation. The growth rate of the real GDP is calculated as

$$H_t^{gdp} = \left[\left(\frac{X_t^c}{X_{t-1}^c} \right)^{P_*^c X_*^c} \left(\frac{X_t^k}{X_{t-1}^k} \right)^{P_*^k X_*^k} \right]^{\frac{1}{P_*^c X_*^c + P_*^k X_*^k}} . \tag{14}$$

The inflation rate of the GDP deflator, $\Pi_t^{p,gdp}$, is implicitly defined by

$$\Pi_t^{p,gdp} H_t^{gdp} = \frac{P_t^{gdp} X_t^{gdp}}{P_{t-1}^{gdp} X_{t-1}^{gdp}} = \frac{P_t^c X_t^c + P_t^k X_t^k}{P_{t-1}^c X_{t-1}^c + P_{t-1}^k X_{t-1}^k}. \tag{15}$$

2.7 Monetary authority

Following Chung, Kiley, and Laforte (2010), we assume that the monetary authority sets the short-term nominal interest rate following a Taylor-type feedback rule with interest rate smoothing.

$$R_t = (R_{t-1})^{\phi^r} (\bar{R}_t)^{1-\phi^r} \exp(\epsilon_t^r) \tag{16}$$

$$\bar{R}_t = R_* \left(\tilde{X}_t\right)^{\phi^{h,gdp}} \left(\frac{\tilde{X}_t}{\tilde{X}_{t-1}}\right)^{\phi^{\Delta h,gdp}} \left(\frac{\Pi_t^{p,gdp}}{\Pi_*^{p,gdp}}\right)^{\phi^{\pi,gdp}} \tag{17}$$

where ϕ^r is the degree of interest rate smoothing, ϵ_t^r is the interest rate shock, and $\phi^{h,gdp}$, $\phi^{\Delta h,gdp}$, and $\phi^{\pi,gdp}$ are the degrees of responsiveness in the policy rule. $\tilde{X}_t$ is the output gap, which is defined as the deviation of real GDP from its efficient level (to be precise, the deviation from the unconditional efficient output defined in Section 5).[12] As will be shown in Section 6, we examine an alternative policy rule that responds to the gap from our measure of potential output instead of the efficient output, but it makes little difference in the estimation results of the potential growth and output gap.

2.8 Market clearing

Before closing the model, we assume that government expenditure, G_t, and net exports, F_t, are produced by the slow-growing sector and fast-growing sector, respectively. Both factors are stochastic, and obey AR(1) processes as follows.

$$\ln G_t - \ln\{Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}\} = \ln \tilde{G}_t = \rho^g \ln \tilde{G}_{t-1} + \epsilon_t^g \tag{18}$$

$$\ln F_t - \ln\{Z_t^m Z_t^k\} = \ln \tilde{F}_t = \rho^f \ln \tilde{F}_{t-1} + \epsilon_t^f \tag{19}$$

[12]The level of real GDP is calculated as a divisia index, in similar to its growth rate (14).

At the symmetric equilibrium, each market clears.

$$\begin{aligned}
X_t^c &= \int_0^1 C_t(i)di + G_t + \frac{100\cdot\chi^w}{2}\left[\Pi_t^{w,c} - \eta^w\Pi_{t-1}^{w,c} - (1-\eta^w)\Pi_*^w\right]^2 W_t^c L_t^c \\
&+\frac{100\cdot\chi^p}{2}\left[\Pi_t^{p,c} - \eta^p\Pi_{t-1}^{p,c} - (1-\eta^p)\Pi_*^{p,c}\right]^2 P_t^c X_t^c \\
&+\frac{100\cdot\chi^l}{2}\left(\frac{L_*^c}{L_*^c+L_*^k}W_t^c + \frac{L_*^k}{L_*^c+L_*^k}W_t^k\right) \\
&\times\left\{\frac{L_t^c}{L_t^k} - \eta^l\frac{L_{t-1}^c}{L_{t-1}^k} - (1-\eta^l)\frac{L_*^c}{L_*^k}\right\}^2 \frac{L_t^k}{L_t^c}L_t
\end{aligned} \tag{20}$$

$$\begin{aligned}
X_t^k &= \int_0^1 I_t(k)dk + F_t + \frac{100\cdot\chi^w}{2}\left[\Pi_t^{w,k} - \eta^w\Pi_{t-1}^{w,k} - (1-\eta^w)\Pi_*^w\right]^2 W_t^k L_t^k \\
&+\frac{100\cdot\chi^p}{2}\left[\Pi_t^{p,k} - \eta^p\Pi_{t-1}^{p,k} - (1-\eta^p)\Pi_*^{p,k}\right]^2 P_t^k X_t^k
\end{aligned} \tag{21}$$

$$L_t^s(i) = \int_0^1 L_t^s(i,j)dj, \quad \forall i\in[0,1],\ s\in\{c,k\} \tag{22}$$

$$\int_0^1 U_t^s(k)K_t^s(k)dk = \int_0^1 K_t^{u,s}(j)dj, \quad s\in\{c,k\} \tag{23}$$

3 Model Estimation

3.1 Estimation procedures

We solve the model and estimate its structural parameters using Bayesian methods. Since growth rate shocks in the system make some of the endogenous variables non-stationary, we divide the non-stationary variables by the corresponding I(1) trends and stationarize the model. We then log-linearize the set of equilibrium conditions, solve the linear rational expectation system, and obtain the transition dynamics of the whole system.

$$\hat{\zeta}_t = G(\vartheta)\hat{\zeta}_{t-1} + H(\vartheta)\hat{\varepsilon}_t \tag{24}$$

where $\hat{\zeta}_t$ is a properly defined $k\times 1$ vector of stationarized and log-linearized endogenous variables, $\hat{\varepsilon}_t$ is the $n\times 1$ vector of exogenous i.i.d. disturbances and ϑ is the $p \times 1$ vector of structural unknown coefficients. $G(\vartheta)$ and $H(\vartheta)$ are the conformable matrices of coefficients that depend on the structural parameters ϑ.

To estimate the model, we specify the observation equation,

$$x_t = J\hat{\zeta}_t + cons \tag{25}$$

where x_t is the observed data described in the next subsection and $cons$ is the vector of constant terms. Since the variables in our model $\hat{\zeta}_t$ include growth rate shocks, we do not detrend or demean any data series, while some of the data are transformed into log-differences. As a benchmark estimation, we do not incorporate measurement errors into the observation equation. In Section 6.3, we instead estimate alternative models with measurement errors.

Letting x^T be a set of observable data, the likelihood function $L(\vartheta \mid x^T)$ is evaluated by applying the Kalman filter. We combine the likelihood function $L(\vartheta \mid x^T)$ with priors for the parameters to be estimated, $p(\vartheta)$, to obtain the posterior distribution: $L(\vartheta \mid x^T)p(\vartheta)$. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods using Dynare. Draws from the posterior distribution are generated with the Metropolis-Hastings algorithm.[13] We obtain the posterior median estimates and posterior intervals of unobservable model variables, including the efficient output, by applying the Kalman smoother.

3.2 Data

The model is estimated using 10 key macroeconomic quarterly Japan's time series from 1981:Q1 to 2009:Q4 as observed data:[14] nominal value added of the slow-growing sector, nominal value added of the fast-growing sector, nominal household consumption, nominal business investment, deflator of the slow-growing sector, deflator of the fast-growing sector, compensation of

[13] A sample of 800,000 draws was created (neglecting the first half of these draws). Our selected step size for the jumping distribution in the Metropolis-Hastings algorithm results in an acceptance ratio of 0.39. The resulting sample properties are not sensitive to the step size. To test the stability of the sample, we use the convergence diagnostic based on Brooks and Gelman (1998).

[14] The GDP data are the second preliminary quarterly estimates released in March 2010.

employees, total hours worked, short-term nominal interest rate (call rate),[15] and capital utilization rate (operating ratio). All the variables, except for the last two, are transformed into log differences. None of them, however, are detrended or demeaned.

The nominal value added of the slow-growing sector is the sum of nominal private consumption, nominal private residential investment, and nominal government expenditure. The nominal value added of the fast-growing sector is the sum of nominal private business investment, nominal private inventory investment, and nominal net exports. Those GDP components are transformed into a per-capita base (divided by the population over 15 years old).

Our model has two price indices for the slow-growing and the fast-growing sectors. In order to match the SNA data with our theoretical model, we construct the chain index of the real value added of each sector and calculate the implicit deflator.

More details of the data are summarized in Appendix B.

3.3 Estimation results

The model's calibrated parameters are presented in Table 1, and the estimated parameters are reported in Table 2. Referring to previous studies, we set 6 structural parameter values such as households' subjective discount rate, capital share, capital depreciation, and elasticity of substitution. We also set the steady-state values based on historical averages of the data.[16] Meanwhile, we estimate 14 structural parameters as well as the parameters that characterize 13 shock processes. Prior and posterior distributions of the model parameters are shown in Figure 1. In general, most of our posterior

[15] The sample period includes the period after the short-term nominal interest rate effectively hit the zero lower bound. However, the estimation results using the data up to 1998:Q4, just before the Bank of Japan started the zero interest rate policy, are not much different from the baseline results using the full sample data.

[16] Although the GDP growth rate has declined during the sample period, we set the steady-state growth rates to its full sample average. The estimation results using the data from 1991:Q1, when Japan's "lost decade" started, are not much different from the baseline results using the full sample data.

estimates of the structural parameters are consistent with previous studies.

Next we report the variance decompositions in Table 3. It shows the posterior mean estimates of forecast error variance decompositions of output (real GDP) growth, consumption growth, investment growth, and GDP deflator inflation at forecast horizon $T = 1$, 4, 10, and 100. The output fluctuations, both in the short and in the long run, are mainly caused by the technology shocks (economy-wide and investment-specific), the investment adjustment cost shocks, and the intertemporal preference shocks. The contributions of the investment-specific technology shocks are smaller than those of the economy-wide technology shocks, as in Hirose and Kurozumi (2010). The investment adjustment cost shocks contribute substantially to the investment fluctuations, and the intertemporal preference shocks contribute substantially to the consumption fluctuations. Meanwhile, the inflation fluctuations are mainly caused by the consumption-goods price mark-up shocks. The investment-specific technology shocks and the intertemporal preference shocks also have large contributions to the long-run fluctuations in inflation.

Finally, we report the impulse responses of key variables to the economy-wide and investment-specific technology shocks and the monetary policy shock in Figure 2. The technology *growth rate* shocks, either economy-wide or investment-specific, increase labor input and accelerates inflation, while the technology *level* shocks decrease labor input and decelerate inflation.[17] This certainly relates to the following result that our measure of potential output driven by technology *growth rate* shocks moves procyclically. Meanwhile, the economy-wide and investment-specific technology shocks, either in growth rate or in level, increase both consumption and investment.[18] The notable result here is that the investment-specific technology shocks increase consumption as well as investment, which is hard to obtain in an one-sector model. An important advantage of our two-sector model is that it generates empirically plausible comovement between consumption and investment.[19]

[17] Christiano, Trabandt, and Walentin (2010) discuss this point in detail.

[18] The responses of consumption and investment are generally consistent with those in the VAR results of Braun and Shioji (2007).

[19] Guerrieri, Henderson, and Kim (2010) discuss this point in detail. (See footnote 6.)

As for the responses to the monetary policy shock, their signs and the hump-shaped patterns are generally consistent with empirical studies, although the inflation response seems quicker than VAR results in the literature.[20]

4 Potential Growth

In this section, we report our measure of potential growth calculated from the estimated benchmark model and compare it with alternative measures.

4.1 Efficient output

First, we estimate the efficient output, which is usually considered as a DSGE model-based measure of potential output. It is defined as the level of output in an environment without nominal rigidities in goods and labor markets and without shocks to price and wage markups. Following the recent literature such as Adolfson, Laséen, Linde, and Svensson (2008) and Sala, Söderström, and Trigari (2010), we estimate both the "unconditional" efficient output calculated using the state variables in the counterfactually efficient allocation from the past to the future, and the "conditional" efficient output calculated using the actual values of the state variables and assuming that the allocation becomes unexpectedly efficient (prices and wages become flexible) today and is expected to remain efficient in the future.

Figure 3 shows the level (not per-capita but total; the same hereafter) and the year-on-year growth rate of the estimated unconditional and conditional efficient outputs together with the actual output (real GDP).[21] The efficient output, either unconditional or conditional, moves closely with the actual output. This implies that nominal rigidities and markup shocks have only limited effects on the actual economic fluctuations and thus a substantial

[20]Some VAR studies using Japanese data, including Shioji (2002) and Nakashima (2006), show that the response of price level to a monetary policy shock is slower than that of output.

[21]The estimated efficient outputs are the Kalman-smoothed posterior median. The total efficient outputs shown in the figure are obtained by multiplying the estimated per-capita efficient outputs by the population over 15 years old. The year-on-year growth rate is calculated as the sum of the quarter-on-quarter growth rates for a year.

fraction of the fluctuations is viewed as efficient in our model. Many policy-makers disagree with this view and use conventional measures of potential output that move much smoother than the model-based efficient output.

4.2 Potential growth

As discussed in the introduction, we try to bridge the gap between model-based measures and conventional measures of potential output. We define our measure of potential output as a component of the (unconditional) efficient output *generated only by growth rate shocks*.[22] Figure 4-1 shows the level and the year-on-year growth rate of our measure of potential output together with the unconditional and conditional efficient outputs. Compared with the growth rate of the efficient outputs, our measure of potential growth displays a higher degree of smoothness. In Figure 4-2, we compare the level and the year-on-year growth rate of our measure of potential output with the HP-filtered output and the potential output based on the production-function approach (PFA) by Hara et al. (2006). Our measure of potential growth moves closely with those conventional measures of potential growth.

In Figure 5 we decompose our measure of potential growth into component parts generated by each source. Since the growth rate shocks we consider in our model are the economy-wide and investment-specific technology shocks, our measure of potential growth can be decomposed into those two types of technology growth rate shocks in addition to the exogenous population growth. While the investment-specific technology growth rate shock has constantly raised the potential growth during the sample period,[23] the economy-wide technology growth rate shock has reduced the potential growth since the 1990s, except in the early 2000s when information technology (IT) propagated through the economy.[24] This widening of the difference

[22] We can also define a measure of potential output similarly based on the conditional rather than unconditional efficient output, but it makes qualitatively little difference.

[23] Braun and Shioji (2007) show that the investment-specific technological progress sustained the potential growth of Japan's economy, even in the 1990s, by calibrating a neoclassical growth model.

[24] Fueki and Kawamoto (2009) suggest the possibility that Japan experienced IT-driven pickup in productivity growth in the 2000s, which occurred not only in the investment-

in the pace of technological progress between the two sectors could result in sluggish reallocation or misallocation of resources in the labor and financial markets, which in turn could lead to further decline in the economy-wide technology growth. This decomposition makes different but somewhat related stories from those in the PFA-based "growth accounting," in which the capital inputs and the total factor productivity have raised the potential growth while the labor inputs have reduced it.[25]

5 Output Gap

Based on several measures of potential output discussed in the previous section, we can calculate the several corresponding measures of output gap, which is defined as the deviation of the actual output from a measure of potential output. In this section, we report and compare several measures of output gap.

5.1 Several measures of output gap

The upper panel of Figure 6 shows the output gap from our measure of potential output and the gaps from the unconditional and conditional efficient outputs. The former is more volatile than the latter, because our measure of potential output is smoother than the efficient output, as shown in Figure 4-1. Apart from the difference in volatility, these model-based output gaps move in parallel with each other and procyclically in accordance with the expansion and recession dates determined by the Economic and Social

goods-producing sector but also in the consumption-goods-producing sector.

[25]Hayashi and Prescott (2002) show that Japan's "lost decade" in the 1990s can be explained by the fall in the growth rate of total factor productivity and by the reduction of the workweek length, using the growth accounting and a one-sector neoclassical growth model. They conjecture that the low productivity growth was the result of policy-induced misallocation in which inefficient firms and declining industries were subsidized. Caballero, Hoshi, and Kashyap (2008) discuss the possibility that Japanese banks' lending to otherwise insolvent firms ('zombies') had distorting effects on healthy firms and played an important role in the productivity slowdown in the lost decade.

Research Institute (ESRI).[26]

The lower panel of Figure 6 shows the output gap from our measure of potential output together with the gaps from the HP-filtered output and the PFA-based potential output. The gaps from these conventional measures of potential outputs are as volatile as the gap from our measure of potential output, and they move closely with each other.

5.2 Predictability of inflation

Among the several measures of output gap discussed above, the gap from the efficient output is theoretically the most relevant measure that indicates inflationary or deflationary pressure. Practically, however, the conventional measures of output gap have been widely used for forecasting future inflation. The gap from our measure of potential output, which moves closely with the conventional measures, could also be useful for forecasting inflation. We compare the predictability of inflation across those several measures of output gap.

First, we check whether the above measures of output gap Granger cause inflation, following Kiley (2010). Table 4-1 presents the F-statistics and the p-values associated with tests of Granger causality from the several measures of output gap to the quarter-to-quarter changes in GDP deflator and consumption-goods deflator (deflator of the slow-growing sector), over the period from 1985:Q1 to 2009:Q4 with the lags selected by the Schwartz information criteria. We can see that all the measures of output gap Granger cause inflation of both deflators.[27] According to the F-statistics, the gap from the conditional efficient output has the strongest causality on the GDP-deflator inflation, and the gap from our measure of potential output has the strongest causality on the consumption-goods inflation. Meanwhile, the gap from the HP-filtered output has the weakest causality on both deflators.

[26]The fact that our model-based output gaps move procyclically is in contrast to those based on simple sticky-prices DSGE models in which the output gap is proportional to labor's share that tends to move countercyclically in Japan.

[27]We also test the Granger causality from output gaps to unemployment, following Kiley (2010), and confirm that all the measures of output gap Granger cause unemployment.

We also evaluate the predictability of inflation by comparing bivariate models of output gap and inflation with an univariate autoregressive model of inflation, following Coenen, Smets, and Vetlov (2009). The general specification of the bivariate models is as follows.

$$\pi_{t+h}^{h} = a + b(L)\pi_t + c(L)x_t + \epsilon_{t+h}^{h} \tag{26}$$

where π_{t+h}^{h} is the annualized h-period change in GDP or consumption-goods deflator, π_t is the annualized one-period inflation ($= \pi_{t+1}^{1}$), x_t is each measure of output gap, and $b(L)$ and $c(L)$ are finite polynomials of order p and q selected by the Schwartz information criteria. Parameters are estimated by ordinary least squares on rolling samples from 1985:Q1-1999:Q1 through 1985:Q1-2009:Q4. We then calculate the mean squared forecast errors (MSFE) of the bivariate models (26) and an univariate autoregressive model of inflation at forecast horizons (h) of 1, 4, and 8 quarters ahead. The results are summarized in Table 4-2. Judging from the MSFE of the bivariate models relative to that of the univariate model, all the measures of output gap, except some measures for the 8-quarters ahead forecast, have forecasting power for inflation when they are included in the bivariate models in addition to the lagged inflation. The comparison of the forecasting power across the several measures of output gap shows quite a different picture from that of Granger causality. For the GDP-deflator inflation, the gap from the conditional efficient output gives the best performance at the one-quarter horizon, and the gap from the HP-filtered output gives the best performance beyond the four-quarter horizon. For the consumption-goods inflation, the gap from our measure of potential output gives the best performance at the one-quarter horizon, and the gap from the HP-filtered output gives the best performance at the eight-quarter horizon. Meanwhile, the gap from the unconditional efficient output shows consistently poorer performance than the gap from our measure of potential output, and shows the worst performance for the forecast of the consumption-goods inflation.

To sum up, we have seen that the comparison of the predictability of inflation across several measures of output gap is highly dependent on the evaluation methods and the forecast horizons. It is fair to say that our results are inconclusive about which measure of output gap has better predictability of inflation. At least we can point out that the gap from the efficient output does not necessarily show the best forecast performance, while the gap from our measure of potential output has some forecasting power and can be used practically as an indicator of inflationary or deflationary pressure.

6 Alternative Models

In this section, we check the robustness of the above results from our benchmark model with respect to the model structure and the identification of shocks. In particular, we analyze the sensitivity to the specifications of monetary policy rules, labor supply shocks, prices and wage markup shocks, and technology shocks.

6.1 Monetary policy rules

First we check the robustness of the results from the benchmark model with respect to the specification of monetary policy rules. The benchmark model assumes that the monetary authority sets the short-term nominal interest rate in response to the output gap from the unconditional efficient output, following many New Keynesian models in the literature. Here we examine an alternative monetary policy rule that responds to the gap from our measure of potential output.

Figure 7 shows our measure of potential growth and the output gap from our measure of potential output calculated from the benchmark model and the alternative model in which the monetary policy rule responds to the gap from our measure of potential output instead of the gap from the unconditional efficient output. We can see that the potential growths calculated from these models move very closely together, and so do the output gaps calculated from these models. These results are related to the fact that the output gap from the unconditional efficient output and that from our measure of potential output move in parallel with each other, as shown in the upper panel of Figure 6. While the gap from our measure of potential output is more volatile than the gap from the unconditional efficient output, the estimated parameter of the responsiveness in the monetary policy rule in the alternative model (response to the gap from our measure of potential output) is smaller than that in the benchmark model. That is why the results of the calculated potential growths and output gaps are not so different between the two models. These results imply that the monetary policy response to the gap from our measure of potential output, with adjusting the degree of responsiveness, could obtain similar outcomes to the policy response to the gap from the efficient output.

6.2 Labor supply shocks

In the rest of this section, we analyze the sensitivity of our results to the specifications and identifications of shocks.

The benchmark model assumes that the labor supply shock is non-stochastic. This is because the temporary labor supply shock is observationally equivalent to the wage markup shock considered in the benchmark model, as shown in Smets and Wouters (2003) among others. These two shocks, however, have different implications for efficient output: the labor supply shock affects efficient output, but the wage markup shock does not.

To consider the differences in their implications for our measure of potential output, we replace the wage markup shock with the temporary labor supply shock, following Sala, Söderström, and Trigari (2010). We now assume that the labor supply shock, Ξ_t^l, in the households' utility function follows an AR(1) process. Figure 8 shows our measure of potential growth and the output gap from our measure of potential output calculated from the benchmark model and the alternative model with the temporary labor supply shock. We can see that the potential growths and the output gaps calculated from these models move very closely together. This is because our measure of potential output is not affected by temporary shocks.

Then we also check the robustness of our measure of potential growth with respect to a permanent labor supply shock. Following Chang, Doh, and

Schorfheide (2007), we introduce a labor supply growth rate shock, which influences our measure of potential growth, into the above model without the wage markup shock. As does the technology shock, the labor supply shock contains two separate stochastic components: the temporary labor supply shock is an i.i.d. process and the permanent labor supply shock follows an AR(1) process, as follows.[28]

$$\begin{aligned}
\ln \Xi_t^l &= \ln B_t^l + \ln Z_t^l \\
\ln B_t^l &= \ln B_*^l + \epsilon_t^{\Xi,l} \\
\ln Z_t^l - \ln Z_{t-1}^l &= \ln \Gamma_t^{z,l} = \ln(\Gamma_*^{z,l} \times \exp[\xi_t^l]) = \ln \Gamma_*^{z,l} + \xi_t^l \\
\xi_t^l &= \rho^{\xi,l} \xi_{t-1}^l + \epsilon_t^{\xi,l}
\end{aligned}$$

where $\epsilon_t^{\Xi,l}$ and $\epsilon_t^{\xi,l}$ are i.i.d. shock processes, and B_*^l and $\Gamma_*^{z,l}$ are the constant level and growth rate of the labor supply shock, respectively. Figure 9 shows our measure of potential growths and output gaps calculated from the benchmark model and the alternative model with the permanent labor supply shock. The potential growth from this alternative model still moves closely with that from the benchmark model.[29] These results imply that our measure of potential growth is robust with respect to the specifications and identifications of the labor supply shocks.

6.3 Markup shocks and measurement errors

The price markup shocks as well as the wage markup shocks have identification problems, and their structural interpretations are often questioned. As shown by de Walque, Smets, and Wouters (2006), the price markup shocks are observationally equivalent to relative price shocks.[30] Moreover, Justiniano and Primiceri (2008) argue that markup shocks may only capture measurement errors in prices and wages.

[28] The estimated posterior mean of $\rho^{\xi,l}$ is 0.98.

[29] The permanent labor supply shock may capture the decrease in the number of statutory workdays per week ('jitan') which gradually proceeded in the 1990s.

[30] Chari, Kehoe, and McGrattan (2009) criticize New Keynesian DSGE models by referring to these points.

Following Justiniano and Primiceri (2008), we replace price and wage markup shocks with measurement errors in price and wage inflation. Figure 10 shows our measure of potential growth and the output gap from our measure of potential output calculated from the benchmark model and the following alternative models: one without either price or wage markup shocks and with measurement errors in both price and wage (Alternative Model 1),[31] one without price markup shock (but with wage markup shock) and with measurement error only in price (Alternative Model 2), one without wage markup shock (but with price markup shock) and with measurement error only in wage (Alternative Model 3), and one with both price and wage markup shocks as well as measurement errors in both price and wage (Alternative Model 4). In general, the potential growths calculated from the benchmark model and the four alternative models move in parallel with each other, and so do the output gaps from these models. These results, however, do not necessarily mean that the markup shocks capture the measurement errors as argued by Justiniano and Primiceri (2008). In particular, the potential growth calculated from Alternative Model 4 deviates from those calculated from both the benchmark model and Alternative Model 1 from the middle of the 1990s to the middle of the 2000s, which implies that some structural shocks to price and wage markups that could not be explained by measurement errors might actually occur in that period.

6.4 Technology shocks

Our measure of potential output naturally depends on the specifications of technology level shocks and technology growth rate shocks. The benchmark model assumes that the level shock is an i.i.d. process while the growth rate shock follows an AR(1) process. Here we examine an alternative model that assumes that both the level shock and the growth rate shock obey AR(1) processes, following Ireland and Schuh (2008). In this alternative model,

[31] In Fueki et al. (2010), the potential growth calculated from Alternative Model 1 is treated as the "benchmark case" of the DSGE approach.

Equation (5) in Section 2.3 is replaced by[32]

$$\ln A_t^n \quad = \quad (1-\rho^{a,n}) \ln A_*^n + \rho^{a,n} \ln A_{t-1}^n + \epsilon_t^{a,n}.$$

Figure 11-1 shows our measure of potential growth and the output gap from our measure of potential output calculated from the benchmark model and this alternative model. The potential growths from these models move quite differently: they sometimes move in the opposite directions, especially in the 2000s. The output gaps calculated from these models generally move in parallel, but the gap from the alternative model is much more volatile than that from the benchmark model.

We can explain these differences by looking at how differently shocks are identified in these models. Figure 11-2 shows the identified technology level shocks in the benchmark and the alternative models. While the economy-wide technology level shock in the benchmark model is identified as small temporary variations under the assumption of i.i.d. process, that in the alternative model is identified as large persistent movements under the assumption of AR(1) process. Note that these movements in technology *level* shocks are excluded in our measure of potential output, which is defined as a component of the efficient output generated by *growth rate* shocks. Meanwhile, the persistent movements in the identified economy-wide technology level shock in the alternative model are counted into the movements in the output gap, which is why the output gap in the alternative model is more volatile than that in the benchmark model, as shown in the lower panel of Figure 11-1. Therefore, the differences between the results calculated from the two models are attributed to the differences in views about whether persistent technology level shocks should be considered as movements in the potential output or those in the output gap.[33]

[32] The estimated posterior mean of $\rho^{a,n}$ is 0.98.

[33] Some types of shocks other than technology shocks are also identified differently in the benchmark and in the alternative models. For instance, the positive and persistent investment adjustment cost shocks are identified throughout the 2000s in the benchmark model, but not in the alternative model. This might imply that the persistent investment adjustment cost shocks, which might correspond to financial shocks, are captured as negative economy-wide technology growth rate shocks and counted into our measure of

7 Real-time Data

Finally, we check the robustness of the results from the benchmark model with respect to data revisions and updates using a real-time database.

Figure 12 shows the real-time sequential estimates of our measure of potential growth and the output gap from our measure of potential output calculated from the benchmark model, with the start date of the sample period fixed and the end date moved from 2001:Q1 to 2009:Q1.[34] In general, the real-time estimates moved closely with the latest estimate. However, they sometimes deviate from each other, especially near the end of each sample period of the real-time estimates.

In Figure 12, we also show the posterior intervals of the latest estimates of potential growth and output gap. By comparing the real-time estimates with those posterior intervals, we could consider the uncertainty in data revisions and updates as generally comparable to the uncertainty in estimation. However, the real-time estimates of the output gap in the middle of the 2000s deviate from the corresponding posterior interval of the latest estimate.

Figure 13 shows the HP-filtered measures of potential growth and output gap using the same real-time GDP data. We can see that the real-time estimates generally move closely with the latest estimate, but they sometimes deviate from each other, as our model-based measures do. For both conventional and model-based measures, the real-time estimates of potential growth and output gap tend to be revised to a large extent. That is practically a serious problem in policymakers' use of those measures.

8 Concluding Remarks

In this paper, we have calculated the potential output and the output gap using a Bayesian-estimated DSGE model of Japan's economy. For bridging the gap with conventional measures, we define our measure of potential output

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potential output in the alternative model.

[34] We utilize the real-time dataset collected by Hara and Ichiue (2010), who analyze the real-time measures of Japan's labor productivity.

as a component of the efficient output generated only by growth rate shocks. Our potential growth displays a high degree of smoothness and moves closely with conventional measures. Moreover, the output gap from our measure of potential output has forecasting power for inflation.

Behind the gap between the DSGE model-based measures and the conventional measures of potential output, there has been a significant difference in views on which types of shocks drive the short-run macroeconomic fluctuations, as discussed in the introduction. The efficient output calculated from our model is volatile and moves very closely with the actual output because nominal rigidities and markup shocks have only limited effects on the actual economic fluctuations and thus a substantial fraction of the fluctuations is viewed as efficient in our model. Some recent DSGE models, however, consider various kinds of real frictions in the financial market, the labor market, and the open economy so that the models can generate substantially inefficient fluctuations. Developing those kinds of models would be another way for bridging the gap with conventional measures of potential output. That will be an important future task.

A The model

This appendix describes the details of the model.

A.1 The equilibrium conditions

The first order conditions of the intermediate-goods-producing firms' cost minimization problem are given as follows.

$$L_t^s = (1-\alpha) X_t^s \frac{MC_t^s}{W_t^s} \quad \text{for } s \in \{c, k\},$$
$$K_t^{u,s} = \alpha X_t^s \frac{MC_t^s}{R_t^s} \quad \text{for } s \in \{c, k\},$$
$$X_t^s = \left(AZ_t^m AZ_t^s L_t^s\right)^{1-\alpha} \left(K_t^{u,s}\right)^{\alpha} \quad \text{for } s \in \{c, k\}.$$

The inflation Euler equation of the intermediate-goods-producing firms is given as follows.

$$\begin{aligned}
\Theta_t^{x,s} MC_t^s X_t^s = {} & \left(\Theta_t^{x,s} - 1\right) P_t^s X_t^s \\
& + 100 \cdot \chi^p \left[\Pi_t^{p,s} - \eta^p \Pi_{t-1}^{p,s} - (1-\eta^p) \Pi_*^{p,s}\right] \Pi_t^{p,s} P_t^s X_t^s \\
& - \beta E_t \left\{ \frac{\Lambda_{t+1}^c / P_{t+1}^c}{\Lambda_t^c / P_t^c} \cdot 100 \cdot \chi^p \left[\Pi_{t+1}^{p,s} - \eta^p \Pi_t^{p,s} - (1-\eta^p) \Pi_*^{p,s}\right] \Pi_{t+1}^{p,s} P_{t+1}^s X_{t+1}^s \right\}.
\end{aligned}$$

The first order conditions of the capital owners are given as follows.

$$Q_t = \beta E_t \frac{\Lambda_{t+1}^c / P_{t+1}^c}{\Lambda_t^c / P_t^c} \left[R_{t+1} + (1-\delta) Q_{t+1}\right],$$
$$R_t^s = \frac{R_t}{U_t^s} \quad \text{for } s \in \{c, k\},$$
$$U_t^s = \frac{1}{Z_t^U} \left(\frac{R_t^s}{\kappa Z_t^U Q_t}\right)^{\frac{1}{\psi}},$$

$$\begin{aligned}
P_t^k = {} & Q_t \left[1 - 100 \cdot \chi \cdot A_t^{\varphi} \left(\frac{I_t \cdot A_t^{\varphi} - I_{t-1} \Gamma_t^{z,m} \Gamma_t^{z,k}}{K_t}\right)\right] \\
& + \beta E_t \left[\frac{\Lambda_{t+1}^c / P_{t+1}^c}{\Lambda_t^c / P_t^c} Q_{t+1} \cdot 100 \cdot \chi \cdot \Gamma_{t+1}^{z,m} \Gamma_{t+1}^{z,k} \left(\frac{I_{t+1} \cdot A_{t+1}^{\varphi} - I_t \Gamma_{t+1}^{z,m} \Gamma_{t+1}^{z,k}}{K_{t+1}}\right)\right],
\end{aligned}$$

where Q_t denotes Tobin's q.

The process of the capital accumulation and the economy-wide capital stock are given as follows.

$$
\begin{aligned}
K_{t+1} =& (1-\delta)K_t + I_t - \frac{100 \cdot \chi}{2} \left(\frac{I_t \cdot A_t^{\varphi} - I_{t-1}\Gamma_t^{z,m}\Gamma_t^{z,k}}{K_t} \right)^2 K_t \\
& - \sum_{s=c,k} \kappa \frac{\left(Z_t^U U_t^s\right)^{1+\psi} - 1}{1+\psi} K_t^s,
\end{aligned}
$$

$$
K_t^c + K_t^k = K_t.
$$

The first order conditions of the households utility maximization problem are given as follows.

$$
\begin{aligned}
&\frac{\Lambda_t^c}{P_t^c} = \beta R_t E_t \left[\frac{\Lambda_{t+1}^c}{P_t^c} \right], \\
&\Lambda_t^c = \varsigma^c \cdot \frac{\Xi_t^{\beta}}{C_t - hC_{t-1}} - \beta \varsigma^c E_t \left[\frac{h\Xi_{t+1}^{\beta}}{C_{t+1} - hC_t} \right],
\end{aligned}
$$

$$
\begin{aligned}
&\Theta_t^l \frac{\Lambda_t^{l,c}}{\Lambda_t^c} P_t^c \cdot L_t^c \\
&= \left(\Theta_t^l - 1\right) W_t^c L_t^c \\
&- \Theta_t^l \cdot 100 \cdot \chi^l \left(\frac{L_*^c}{L_*^c + L_*^k} \cdot W_t^c + \frac{L_*^k}{L_*^c + L_*^k} \cdot W_t^k \right) \left(\frac{L_t^c}{L_t^k} - \eta^l \frac{L_{t-1}^c}{L_{t-1}^k} - (1-\eta^l) \frac{L_*^c}{L_*^k} \right) L_t \\
&+ 100 \cdot \chi^w \left(\Pi_t^{w,c} - \eta^w \Pi_{t-1}^{w,c} - (1-\eta^w) \Pi_*^{w,c} \right) \Pi_t^{w,c} W_t^c L_t^c \\
&- \beta E_t \left\{ \frac{\Lambda_{t+1}^c / P_{t+1}^c}{\Lambda_t^c / P_t^c} 100 \cdot \chi^w \left(\Pi_{t+1}^{w,c} - \eta^w \Pi_t^{w,c} - (1-\eta^w) \Pi_*^{w,c} \right) \Pi_{t+1}^{w,c} W_{t+1}^c L_{t+1}^c \right\},
\end{aligned}
$$

$$
\begin{aligned}
&\Theta_t^l \frac{\Lambda_t^{l,k}}{\Lambda_t^c} P_t^c \cdot L_t^k \\
&= \left(\Theta_t^l - 1\right) W_t^k L_t^k \\
&+ \Theta_t^l \cdot 100 \cdot \chi^l \left(\frac{L_*^c}{L_*^c + L_*^k} \cdot W_t^c + \frac{L_*^k}{L_*^c + L_*^k} \cdot W_t^k \right) \left(\frac{L_t^c}{L_t^k} - \eta^l \frac{L_{t-1}^c}{L_{t-1}^k} - (1-\eta^l) \frac{L_*^c}{L_*^k} \right) L_t \\
&+ 100 \cdot \chi^w \left(\Pi_t^{w,k} - \eta^w \Pi_{t-1}^{w,k} - (1-\eta^w) \Pi_*^{w,k} \right) \Pi_t^{w,k} W_t^k L_t^k \\
&- \beta E_t \left\{ \frac{\Lambda_{t+1}^c / P_{t+1}^c}{\Lambda_t^c / P_t^c} 100 \cdot \chi^w \left(\Pi_{t+1}^{w,k} - \eta^w \Pi_t^{w,k} - (1-\eta^w) \Pi_*^{w,k} \right) \Pi_{t+1}^{w,k} W_{t+1}^k L_{t+1}^k \right\},
\end{aligned}
$$

where $\Lambda_t^{l,c} = \Lambda_t^{l,k} = \varsigma^l (L_t^c + L_t^k)^{\nu} \Xi_t^{\beta}$.

A.2 Definitions of stationary model variables

To render all the variables stationary, we redefine the non-stationary variables and transform them to stationary ones.

$$\tilde{X}_t^c = \frac{X_t^c}{Z_t^m \left(Z_t^k\right)^\alpha \left(Z_t^c\right)^{1-\alpha}}$$
$$\tilde{X}_t^k = \frac{X_t^k}{Z_t^m Z_t^k}$$
$$\tilde{C}_t = \frac{C_t}{Z_t^m \left(Z_t^k\right)^\alpha \left(Z_t^c\right)^{1-\alpha}}$$
$$\tilde{I}_t = \frac{I_t}{Z_t^m Z_t^k}$$
$$\tilde{\Lambda}_t^c = \Lambda_t^c Z_t^m \left(Z_t^k\right)^\alpha \left(Z_t^c\right)^{1-\alpha}$$

$$\tilde{K}_{t+1}^{u,c} = \frac{K_{t+1}^{u,c}}{Z_t^m Z_t^k}$$
$$\tilde{K}_{t+1}^{u,k} = \frac{K_{t+1}^{u,k}}{Z_t^m Z_t^k}$$
$$\tilde{K}_{t+1}^{c} = \frac{K_{t+1}^{c}}{Z_t^m Z_t^k}$$
$$\tilde{K}_{t+1}^{k} = \frac{K_{t+1}^{k}}{Z_t^m Z_t^k}$$
$$\tilde{K}_{t+1} = \frac{K_{t+1}}{Z_t^m Z_t^k}$$

$$\tilde{P}_t^k = \frac{P_t^k}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}$$
$$\tilde{W}_t^c = \frac{W_t^c}{P_t^c}\frac{1}{Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}}$$
$$\tilde{W}_t^k = \frac{W_t^k}{P_t^c}\frac{1}{Z_t^m (Z_t^k)^\alpha (Z_t^c)^{1-\alpha}}$$

$$\tilde{R}_t^c = \frac{R_t^c}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}$$
$$\tilde{R}_t^k = \frac{R_t^k}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}$$
$$\tilde{R}_t = \frac{R_t}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}$$

$$\tilde{MC}_t^c = \frac{MC_t^c}{P_t^c}$$
$$\tilde{MC}_t^k = \frac{MC_t^k}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}$$
$$\tilde{Q}_t = \frac{Q_t}{P_t^c}\left(\frac{Z_t^k}{Z_t^c}\right)^{1-\alpha}$$

$$\tilde{G}_t = \frac{G_t}{Z_t^m\left(Z_t^k\right)^{\alpha}\left(Z_t^c\right)^{1-\alpha}}$$
$$\tilde{F}_t = \frac{F_t}{Z_t^m Z_t^k}$$

A.3 Stationary equilibrium conditions

Rewriting the equilibrium conditions with the transformed variables, we obtain the following stationary equilibrium conditions.

$$L_t^s = (1-\alpha)\tilde{X}_t^s\frac{\tilde{MC}_t^s}{\tilde{W}_t^s} \quad \text{for } s \in \{c,k\}$$
$$\frac{\tilde{K}_t^{u,s}}{\Gamma_t^{x,k}} = \alpha\tilde{X}_t^s\frac{\tilde{MC}_t^s}{\tilde{R}_t^s} \quad \text{for } s \in \{c,k\}$$
$$\tilde{X}_t^s = (A_t^m A_t^s L_t^s)^{1-\alpha}\left(\frac{\tilde{K}_t^{u,s}}{\Gamma_t^{x,k}}\right)^{\alpha} \quad \text{for } s \in \{c,k\}.$$

$$\begin{aligned}\Theta_t^{x,c}\tilde{MC}_t^c\tilde{X}_t^c = {} & \left(\Theta_t^{x,c}-1\right)\tilde{X}_t^c \\ & + 100\cdot\chi^p\left[\Pi_t^{p,c} - \eta^p\Pi_{t-1}^{p,c} - (1-\eta^p)\Pi_*^{p,c}\right]\Pi_t^{p,c}\tilde{X}_t^c \\ & - \beta E_t\left\{\frac{\tilde{\Lambda}_{t+1}^c}{\tilde{\Lambda}_t^c}\cdot 100\cdot\chi^p\left[\Pi_{t+1}^{p,c} - \eta^p\Pi_t^{p,c} - (1-\eta^p)\Pi_*^{p,c}\right]\Pi_{t+1}^{p,c}\tilde{X}_{t+1}^c\right\}\end{aligned}$$

$$
\begin{aligned}
\Theta_t^{x,k}\tilde{MC}_t^k\tilde{X}_t^k = & \left(\Theta_t^{x,k}-1\right)\tilde{P}_t^k\tilde{X}_t^k \\
& + 100\cdot\chi^p\left[\Pi_t^{p,k}-\eta^p\Pi_{t-1}^{p,k}-(1-\eta^p)\Pi_*^{p,k}\right]\Pi_t^{p,k}\tilde{P}_t^k\tilde{X}_t^k \\
& - \beta E_t\left\{\frac{\tilde{\Lambda}_{t+1}^c}{\tilde{\Lambda}_t^c}\cdot 100\cdot\chi^p\left[\Pi_{t+1}^{p,k}-\eta^p\Pi_t^{p,k}-(1-\eta^p)\Pi_*^{p,k}\right]\Pi_{t+1}^{p,k}\tilde{P}_{t+1}^k\tilde{X}_{t+1}^k\right\}
\end{aligned}
$$

$$
\tilde{Q}_t = \beta E_t\frac{\tilde{\Lambda}_{t+1}^c}{\tilde{\Lambda}_t^c}\frac{1}{\Gamma_{t+1}^{x,k}}\left[\tilde{R}_{t+1}+(1-\delta)\,\tilde{Q}_{t+1}\right]
$$

$$
\tilde{R}_t^s = \frac{\tilde{R}_t}{U_t^s} \quad \text{for } s\in\{c,k\}
$$

$$
U_t^s = \frac{1}{Z_t^U}\left(\frac{\tilde{R}_t^s}{\kappa Z_t^U\tilde{Q}_t}\right)^{\frac{1}{\psi}}
$$

$$
\begin{aligned}
\tilde{P}_t^k = & \tilde{Q}_t\left[1-100\cdot\chi\cdot A_t^\varphi\left(\frac{\tilde{I}_t\cdot A_t^\varphi-\tilde{I}_{t-1}}{\tilde{K}_t}\Gamma_t^{x,k}\right)\right] \\
& + \beta E_t\left[\frac{\tilde{\Lambda}_{t+1}^c}{\tilde{\Lambda}_t^c}\tilde{Q}_{t+1}\cdot 100\cdot\chi\left(\frac{\tilde{I}_{t+1}\cdot A_{t+1}^\varphi-\tilde{I}_t}{\tilde{K}_{t+1}}\Gamma_{t+1}^{x,k}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\tilde{K}_{t+1} = & (1-\delta)\frac{\tilde{K}_t}{\Gamma_t^{x,k}} \\
& + \tilde{I}_t - \frac{100\cdot\chi}{2}\left(\frac{\tilde{I}_t\cdot A_t^\varphi-\tilde{I}_{t-1}}{\tilde{K}_t}\Gamma_t^{x,k}\right)^2\frac{\tilde{K}_t}{\Gamma_t^{x,k}} \\
& - \sum_{s=c,k}\kappa\frac{\left(Z_t^U U_{t+1}^s\right)^{1+\psi}-1}{1+\psi}\frac{\tilde{K}_t^s}{\Gamma_t^{x,k}}
\end{aligned}
$$

$$
\tilde{K}_t^c + \tilde{K}_t^k = \tilde{K}_t
$$

$$
\tilde{\Lambda}_t^c = \beta R_t E_t\left[\tilde{\Lambda}_{t+1}^c\frac{1}{\Pi_{t+1}^c\Gamma_{t+1}^{x,c}}\right]
$$

$$
\tilde{\Lambda}_t^c = \varsigma^c\cdot\frac{\Xi_t^\beta}{\tilde{C}_t-\left[h/\Gamma_t^{x,c}\right]\tilde{C}_{t-1}} - \beta\varsigma^c E_t\left\{\frac{\left[h/\Gamma_{t+1}^{x,c}\right]\Xi_{t+1}^\beta}{\tilde{C}_{t+1}-\left[h/\Gamma_{t+1}^{x,c}\right]\tilde{C}_t}\right\}
$$

$$
\begin{aligned}
&\Theta_t^l \frac{\tilde{\Lambda}_t^{l,c}}{\tilde{\Lambda}_t^c} \cdot L_t^c \\
&= \left(\Theta_t^l - 1\right) \tilde{W}_t^c L_t^c \\
&- \Theta_t^l \cdot 100 \cdot \chi^l \left(\frac{L_*^c}{L_*^c + L_*^k} \cdot \tilde{W}_t^c + \frac{L_*^k}{L_*^c + L_*^k} \cdot \tilde{W}_t^k \right) \left(\frac{L_t^c}{L_t^k} - \eta^l \frac{L_{t-1}^c}{L_{t-1}^k} - (1-\eta^l) \frac{L_*^c}{L_*^k} \right) L_t \\
&+ 100 \cdot \chi^w \left(\Pi_t^{w,c} - \eta^w \Pi_{t-1}^{w,c} - (1-\eta^w) \Pi_*^{w,c} \right) \Pi_t^{w,c} \tilde{W}_t^c L_t^c \\
&- \beta E_t \left\{ \frac{\tilde{\Lambda}_{t+1}^c}{\tilde{\Lambda}_t^c} 100 \cdot \chi^w \left(\Pi_{t+1}^{w,c} - \eta^w \Pi_t^{w,c} - (1-\eta^w) \Pi_*^{w,c} \right) \Pi_{t+1}^{w,c} \tilde{W}_{t+1}^c L_{t+1}^c \right\}
\end{aligned}
$$

$$
\begin{aligned}
&\Theta_t^l \frac{\tilde{\Lambda}_t^{l,k}}{\tilde{\Lambda}_t^c} \cdot L_t^k \\
&= \left(\Theta_t^l - 1\right) \tilde{W}_t^k L_t^k \\
&+ \Theta_t^l \cdot 100 \cdot \chi^l \left(\frac{L_*^c}{L_*^c + L_*^k} \cdot \tilde{W}_t^c + \frac{L_*^k}{L_*^c + L_*^k} \cdot \tilde{W}_t^k \right) \left(\frac{L_t^c}{L_t^k} - \eta^l \frac{L_{t-1}^c}{L_{t-1}^k} - (1-\eta^l) \frac{L_*^c}{L_*^k} \right) L_t \\
&+ 100 \cdot \chi^w \left(\Pi_t^{w,k} - \eta^w \Pi_{t-1}^{w,k} - (1-\eta^w) \Pi_*^{w,k} \right) \Pi_t^{w,k} \tilde{W}_t^k L_t^k \\
&- \beta E_t \left\{ \frac{\tilde{\Lambda}_{t+1}^c}{\tilde{\Lambda}_t^c} 100 \cdot \chi^w \left(\Pi_{t+1}^{w,k} - \eta^w \Pi_t^{w,k} - (1-\eta^w) \Pi_*^{w,k} \right) \Pi_{t+1}^{w,k} \tilde{W}_{t+1}^k L_{t+1}^k \right\}
\end{aligned}
$$

where $\tilde{\Lambda}_t^{l,c} = \tilde{\Lambda}_t^{l,k} = \varsigma^l \left(L_t^c + L_t^k\right)^\nu \Xi_t^\beta$, $\Gamma_t^{x,k} = \Gamma_t^{z,m} \cdot \Gamma_t^{z,k}$, and $\Gamma_t^{x,c} = \Gamma_t^{z,m} \cdot (\Gamma_t^{z,k})^\alpha$.

A.4 Log-linearized system

We log-linearize the above stationary equilibrium conditions as follows. Hereafter, each of log-linearized variables is expressed as a small letter.

A.4.1 Labor share, marginal cost, and production function

$$
\begin{aligned}
&\hat{l}_t^s = \hat{x}_t^s + \hat{mc}_t^s - \hat{w}_t^s \quad \text{for } s \in \{c, k\} \\
&\hat{k}_t^{u,s} - \hat{\gamma}_t^{x,k} = \hat{x}_t^s + \hat{mc}_t^s - \hat{r}_t^s \quad \text{for } s \in \{c, k\} \\
&\hat{x}_t^s = (1-\alpha) \left(\hat{a}_t^m + \hat{a}_t^s + \hat{l}_t^s \right) + \alpha \left(\hat{k}_t^{u,s} - \hat{\gamma}_t^{x,k} \right) \quad \text{for } s \in \{c, k\}
\end{aligned}
$$

A.4.2 Inflation Euler equations

$$\hat{\pi}_t^{p,c} = \frac{\beta}{1+\beta\eta_p} E_t \hat{\pi}_{t+1}^{p,c} + \frac{\eta_p}{1+\beta\eta_p} \hat{\pi}_{t-1}^{p,c}$$
$$+ \frac{\Theta_*^{x,c} \tilde{MC}_*^c}{100\chi_p \left(\pi_*^{p,c}\right)^2 (1+\beta\eta_p)} \left(\hat{mc}_t^c + \frac{\tilde{MC}_*^c - 1}{\tilde{MC}_*^c} \hat{\theta}_t^{x,c} \right)$$

$$\hat{\pi}_t^{p,k} = \frac{\beta}{1+\beta\eta_p} E_t \hat{\pi}_{t+1}^{p,k} + \frac{\eta_p}{1+\beta\eta_p} \hat{\pi}_{t-1}^{p,k}$$
$$+ \frac{\Theta_*^{x,k} \tilde{MC}_*^k / \tilde{P}_*^k}{100\chi_p \left(\pi_*^{p,k}\right)^2 (1+\beta\eta_p)} \left(\hat{mc}_t^k - \hat{p}_t^k + \frac{\tilde{MC}_*^k - \tilde{P}_*^k}{\tilde{MC}_*^k} \hat{\theta}_t^{x,k} \right)$$

A.4.3 Tobin's q, effective interest rate, capital utilization, investment, capital accumulation, and economy-wide capital stock

$$\hat{q}_t = E_t \hat{\lambda}_{t+1}^c - \hat{\lambda}_t^c - E_t \hat{\gamma}_{t+1}^{x,k} + \frac{\tilde{R}_*}{\tilde{R}_* + (1-\delta)\tilde{Q}_*} E_t \hat{r}_{t+1} + \frac{(1-\delta)\tilde{Q}_*}{\tilde{R}_* + (1-\delta)\tilde{Q}_*} E_t \hat{q}_{t+1}$$
$$\hat{r}_t^s = \hat{r}_t - \hat{u}_t^s \quad \text{for } s \in \{c,k\}$$
$$\hat{u}_t^s = \frac{1}{\psi} \left(\hat{r}_t^s - \hat{q}_t - \hat{z}_t^U \right) - \hat{z}_t^U \quad \text{for } s \in \{c,k\}$$
$$\hat{p}_t^k = \frac{\tilde{Q}_*}{\tilde{P}_*^k} \hat{q}_t + 100\chi \frac{\tilde{Q}_*}{\tilde{P}_*^k} \frac{\tilde{I}_*}{\tilde{K}_*} \Gamma_*^{x,k} \left[\beta E_t \hat{i}_{t+1} - (1+\beta)\hat{i}_t + \hat{i}_{t-1} + \beta E_t \hat{\varphi}_{t+1} - \hat{\varphi}_t \right]$$
$$\hat{k}_t = \frac{1-\delta}{\Gamma_*^{x,k}} \left(\hat{k}_{t-1} - \hat{\gamma}_t^{x,k} \right) + \frac{\tilde{I}_*}{\tilde{K}_*} \hat{i}_t - \sum_{s\in\{c,k\}} \frac{\kappa}{\Gamma_*^{x,k}} \frac{\tilde{K}_*^s}{\tilde{K}_*} (\hat{u}_t^s + \hat{z}_t^U)$$
$$\hat{k}_t = \frac{\tilde{K}_*^c}{\tilde{K}_*} \hat{k}_t^c + \frac{\tilde{K}_*^k}{\tilde{K}_*} \hat{k}_t^k$$

A.4.4 Consumption Euler equations

$$\hat{\lambda}_t^c = \hat{R}_t + E_t \hat{\lambda}_{t+1}^c - E_t \hat{\pi}_{t+1}^c - E_t \hat{\gamma}_{t+1}^{x,c}$$

$$\hat{\lambda}_t^c = \tfrac{1}{1-\beta h^c/\Gamma_*^{x,c}} \Big\{ \hat{\xi}_t^\beta - \beta \tfrac{h^c}{\Gamma_*^{x,c}} E_t \hat{\xi}_{t+1}^\beta + \tfrac{h^c/\Gamma_*^{x,c}}{1-h^c/\Gamma_*^{x,c}} \left(\hat{c}_{t-1} + \beta E_t \hat{c}_{t+1} \right) - \tfrac{1+\beta\left(\frac{h^c}{\Gamma_*^{x,c}}\right)^2}{1-h^c/\Gamma_*^{x,c}} \hat{c}_t$$
$$- \left(\tfrac{h^c/\Gamma_*^{x,c}}{1-h^c/\Gamma_*^{x,c}} \right) \left(\hat{\gamma}_t^{x,c} \right) + \beta \left(\tfrac{h^c/\Gamma_*^{x,c}}{1-h^c/\Gamma_*^{x,c}} \right) \left(E_t \hat{\gamma}_{t+1}^{x,c} \right) \Big\}$$

A.4.5 Labor Euler equations

$$\hat{\lambda}_t^{l,c} - \hat{\lambda}_t^c - \hat{w}_t^c - \tfrac{1}{\Theta_*^l - 1}\hat{\theta}_t^l = -\tfrac{\Theta_*^l}{\Theta_*^l - 1}\tfrac{100\chi^l}{L_*^k}\left[\left(\hat{l}_t^c - \hat{l}_t^k\right) - \eta^l\left(\hat{l}_{t-1}^c - \hat{l}_{t-1}^k\right)\right]$$
$$+\tfrac{100\chi^w\left(\pi_*^{w,c}\right)^2}{\Theta_*^l - 1}\left[\left(\hat{\pi}_t^{w,c} - \eta^w\hat{\pi}_{t-1}^{w,c}\right) - \beta\left(E_t\hat{\pi}_{t+1}^{w,c} - \eta^w\hat{\pi}_t^{w,c}\right)\right]$$

$$\hat{\lambda}_t^{l,k} - \hat{\lambda}_t^c - \hat{w}_t^k - \tfrac{1}{\Theta_*^l - 1}\hat{\theta}_t^l = \tfrac{\Theta_*^l}{\Theta_*^l - 1}\tfrac{100\chi^l L_*^c}{\left(L_*^k\right)^2}\left[\left(\hat{l}_t^c - \hat{l}_t^k\right) - \eta^l\left(\hat{l}_{t-1}^c - \hat{l}_{t-1}^k\right)\right]$$
$$+\tfrac{100\chi^w\left(\pi_*^{w,k}\right)^2}{\Theta_*^l - 1}\left[\left(\hat{\pi}_t^{w,k} - \eta^w\hat{\pi}_{t-1}^{w,k}\right) - \beta\left(E_t\hat{\pi}_{t+1}^{w,k} - \eta^w\hat{\pi}_t^{w,k}\right)\right]$$

$$\hat{\lambda}_t^{l,c} = \hat{\lambda}_t^{l,k} = \nu\left(\frac{L_*^c}{L_*^c + L_*^k}\hat{l}_t^c + \frac{L_*^k}{L_*^c + L_*^k}\hat{l}_t^k\right) + \hat{\xi}_t^\beta$$

A.4.6 Identities

$$\hat{x}_t^c = \frac{C_*}{X_*^c}\hat{c}_t + \frac{G_*}{X_*^c}\hat{g}_t$$
$$\hat{x}_t^k = \frac{I_*}{X_*^k}\hat{i}_t + \frac{F_*}{X_*^k}\hat{f}_t$$

$$\hat{k}_t^{u,c} = \hat{u}_t^c + \hat{k}_t^c$$

$$\hat{k}_t^{u,k} = \hat{u}_t^k + \hat{k}_t^k$$

$$\hat{p}_t^k = \hat{\pi}_t^{p,k} - \hat{\pi}_t^{p,c} + \hat{\gamma}_t^{x,k} - \hat{\gamma}_t^{x,c} + \hat{p}_{t-1}^k$$

$$\hat{w}_t^c = \hat{\pi}_t^{w,c} - \hat{\pi}_t^{p,c} - \hat{\gamma}_t^{x,c} + \hat{w}_{t-1}^c$$

$$\hat{w}_t^k = \hat{\pi}_t^{w,k} - \hat{\pi}_t^{p,c} - \hat{\gamma}_t^{x,c} + \hat{w}_{t-1}^k$$

$$\hat{\gamma}_t^{x,k} = \hat{\gamma}_t^{z,m} + \hat{\gamma}_t^{z,k}$$

$$\hat{\gamma}_t^{x,c} = \hat{\gamma}_t^{z,m} + \alpha\cdot\hat{\gamma}_t^{z,k}$$

A.4.7 GDP and GDP deflator

$$\hat{H}_t^{gdp} = \frac{1}{P_*^c\tilde{X}_*^c + P_*^k\tilde{X}_*^k}$$
$$\times\left[P_*^c\tilde{X}_*^c\Gamma_*^{x,c}\left(\hat{x}_t^c - \hat{x}_{t-1}^c + \hat{\gamma}_t^{x,c}\right) + P_*^k\tilde{X}_*^k\Gamma_*^{x,k}\left(\hat{x}_t^k - \hat{x}_{t-1}^k + \hat{\gamma}_t^{x,k}\right)\right]$$

$$\hat{\pi}_t^{pgdp} + \hat{H}_t^{gdp} = \hat{\pi}_t^{p,c} + \hat{\gamma}_t^{x,c}$$
$$+\frac{1}{\tilde{X}_*^c + \tilde{P}_*^k\tilde{X}_*^k}\left[\tilde{X}_*^c\left(\hat{x}_t^c - \hat{x}_{t-1}^c\right) + \tilde{P}_*^k\tilde{X}_*^k\left(\hat{p}_t^k - \hat{p}_{t-1}^k + \hat{x}_t^k - \hat{x}_{t-1}^k\right)\right]$$

A.4.8 Monetary policy

$$\hat{R}_t = \phi^r\hat{R}_{t-1} + (1-\phi^r)\hat{\bar{R}}_t + \epsilon_t^r$$
$$\hat{\bar{R}}_t = \phi^{\pi,gdp}(\hat{\pi}_t^{pgdp}) + \phi^{h,gdp}(\hat{x}_t) + \phi^{\Delta h,gdp}(\hat{x}_t - \hat{x}_{t-1})$$

where $\hat{R}_t$ and $\hat{\bar{R}}_t$ denote log-linearized variables.

A.4.9 Exogenous shock process

$$\hat{g}_t = \rho^g\hat{g}_{t-1} + \epsilon_t^g$$

$$\hat{f}_t = \rho^f\hat{f}_{t-1} + \epsilon_t^f$$

$$\hat{\gamma}_t^{z,m} = \rho^{z,m}\hat{\gamma}_{t-1}^{z,m} + \epsilon_t^{z,m}$$

$$\hat{\gamma}_t^{z,k} = \rho^{z,k}\hat{\gamma}_{t-1}^{z,k} + \epsilon_t^{z,k}$$

$$\hat{a}_t^m = \epsilon_t^{a,m}$$

$$\hat{a}_t^k = \epsilon_t^{a,k}$$

$$\hat{\theta}_t^{x,c} = \rho^{\theta_{x,c}}\hat{\theta}_{t-1}^{x,c} + \epsilon_t^{\theta_{x,c}} - \rho^{\theta_{x,c},ma}\epsilon_{t-1}^{\theta_{x,c}}$$

$$\hat{\theta}_t^{x,k} = \rho^{\theta_{x,k}}\hat{\theta}_{t-1}^{x,k} + \epsilon_t^{\theta_{x,k}} - \rho^{\theta_{x,k},ma}\epsilon_{t-1}^{\theta_{x,k}}$$

$$\hat{\theta}_t^l = \rho^{\theta_l}\hat{\theta}_{t-1}^l + \epsilon_t^{\theta,l} - \rho^{\theta_l,ma}\epsilon_{t-1}^{\theta,l}$$

$$\hat{z}_t^U = \rho^U\hat{z}_{t-1}^U + \epsilon_t^U$$

$$\hat{\varphi}_t = \rho^\varphi\hat{\varphi}_{t-1} + \epsilon_t^\varphi$$

$$\hat{\xi}_t^\beta = \rho^{\xi,\beta}\hat{\xi}_{t-1} + \epsilon_t^{\xi,\beta}$$

B Data

This appendix summarizes the data used for estimation.

1. Nominal consumption expenditures + nominal residential investment expenditure + nominal government expenditure, as output of the slow-growing sector (divided by the population over 15 years old).

2. Nominal consumption expenditures + nominal residential investment expenditure, as the households consumption (divided by the population over 15 years old).

3. Nominal business investment expenditure + nominal inventory expenditure + net exports, as output of the fast-growing sector (divided by the population over 15 years old).

4. Nominal business investment expenditure + nominal inventory expenditure, as the capital owners investment (divided by the population over 15 years old).

5. The rate of inflation for the deflater of #1.

6. The rate of inflation for the deflater of #3.

7. Compensation of employees.

8. Total hours worked (Monthly Labour Survey) multiplied by the number of employees over 15 years old (Labour Force Survey).

9. Uncollateralized overnight call rate.

10. Index of operating ratio (manufacturing).

Since the corresponding deflaters of the c sector and the k sector do not exist, we create the chain index from the SNA data, following Whelan (2003). Assuming that the real expenditure of X_t consists of $X_t^s,\ s \in S$, and letting $P_t^{s,a}$ be the corresponding deflater of the previous year in each component, we define the chain indexed real variable, X_t, as

$$\frac{\Delta X_t}{X_{t-1}} = \frac{\sum_{s \in S} P_t^s \Delta X_t^s}{\sum_{s \in S} P_{t-1}^{s,a} X_{t-1}^s}.$$

C Endogenous and exogenous variables

This appendix lists the endogenous and exogenous variables in the benchmark model.

X_t^c = Production in the slow-growing sector ("consumption" goods sector).

X_t^k = Production in the fast-growing sector ("investment" goods sector).

$X_t^c(j)$ = Slow-growing sector-specific intermediate goods produced by firm j.

$X_t^k(j)$ = Fast-growing sector-specific intermediate goods produced by firm j.

$K_t^{u,c}(j)$ = *Effective* capital input of firm j producing the slow-growing sector-specific intermediate goods.

$K_t^{u,k}(j)$ = *Effective* capital input of firm j producing the fast-growing sector-specific intermediate goods.

$L_t^c(j)$ = Labor input of firm j producing the slow-growing sector-specific intermediate goods.

$L_t^k(j)$ = Labor input of firm j producing the fast-growing sector-specific intermediate goods.

$U_t^c(j)$ = Capital utilization rate of firm j producing the slow-growing sector-specific intermediate goods.

$U_t^k(j)$ = Capital utilization rate of firm j producing the fast-growing sector-specific intermediate goods.

$K_t^c(j)$ = *Physical* capital input of firm j producing the slow-growing sector-specific intermediate goods.

$K_t^k(j)$ = *Physical* capital input of firm j producing the fast-growing sector-specific intermediate goods.

$P_t^c(j)$ = The price level set by firm j producing the slow-growing sector-specific intermediate goods.

$P_t^k(j)$ = The price level set by firm j producing the fast-growing sector-specific intermediate goods.

$MC_t^c(j)$ = Marginal cost of firm j producing the slow-growing sector-specific intermediate goods.

$MC_t^k(j)$ = Marginal cost of firm j producing the fast-growing sector-specific intermediate goods.

P_t^c = Aggregate price level in the slow-growing sector-specific intermediate goods.

P_t^k = Aggregate price level in the fast-growing sector-specific intermediate goods.

Λ_t^c = Marginal utility of consumption.

$\Pi_t^{p,c}$ = Inflation rate for prices in the slow-growing sector.

$\Pi_t^{p,k}$ = Inflation rate for prices in the fast-growing sector.

$\Pi_*^{p,c}$ = Time-invariant trend inflation rate for prices in the slow-growing sector.

$\Pi_*^{p,k}$ = Time-invariant trend inflation rate for prices in the fast-growing sector.

R_t^c = Nominal interest rate on the physical capital for the slow-growing sector.

R_t^k = Nominal interest rate on the physical capital for the fast-growing sector.

$U_t^c(k)$ = Capital utilization rate of capital owner k for the slow-growing sector.

$U_t^k(k)$ = Capital utilization rate of capital owner k for the fast-growing sector.

$K_t^c(k)$ = *Physical* capital of capital owner k for the slow-growing sector.

$K_t^k(k)$ = *Physical* capital of capital owner k for the fast-growing sector.

$K_t(k)$ = Aggregate *physical* capital of capital owner k.

$I_t(k)$ = The amount of investment expenditure of capital owner k.

$C_t(i)$ = Purchase of consumption goods of household i.

$B_t(i)$ = Bonds holdings of household i.

$W_t^c(i)$ = Wages for the slow-growing sector set by household i.

$W_t^k(i)$ = Wages for the fast-growing sector set by household i.

$L_t^c(i)$ = Supply of labor for the slow-growing sector by household i.

$L_t^k(i)$ = Supply of labor for the fast-growing sector by household i.

L_t = Aggregate labor supply.

W_t^c = Aggregate wages for the slow-growing sector.

W_t^k = Aggregate wages for the fast-growing sector.

L_t^c = Supply of labor for the slow-growing sector.

L_t^k = Supply of labor for the fast-growing sector.

$\Pi_t^{w,c}$ = Wage inflation rate in the slow-growing sector.

$\Pi_t^{w,k}$ = Wage inflation rate in the fast-growing sector.

$\Pi_*^{w,c}$ = Time-invariant trend wage inflation rate in the slow-growing sector.

$\Pi_*^{w,k}$ = Time-invariant trend wage inflation rate in the fast-growing sector.

$\Omega_t(i)$ = Capital and profits income of household i.

H_t^{gdp} = Growth rate of real (chain-weighted) GDP.

$\Pi_t^{p,gdp}$ = Inflation rate of the GDP deflator.

R_t = Nominal interest rate.

$\tilde{X}_t$ = Output gap (the deviation from the unconditional efficient output defined in Section 5).

$\Pi_*^{p,gdp}$ = Time-invariant trend inflation rate of the GDP deflator.

G_t = Government expenditure.

F_t = Net exports.

$\Gamma_t^{z,k}$ = Growth rate of the fast-growing sector-specific technology.

$\Gamma_t^{z,m}$ = Growth rate of the economy-wide technology.

A_t^k = Level of the fast-growing sector-specific technology.

A_t^m = Level of the economy-wide technology.

$\Theta_t^{x,c}$ = Elasticity of substitution between the differentiated intermediate inputs in the slow-growing sector.

$\Theta_t^{x,k}$ = Elasticity of substitution between the differentiated intermediate inputs in the fast-growing sector.

Θ_t^l = Elasticity of substitution between the differentiated labor inputs into production.

A_t^φ = Investment adjustment cost shock.

Ξ_t^β = Intertemporal preference shock.

Z_t^U = Capital utilization adjustment cost shock.

D Model parameters

This appendix lists the model parameters.

h = Degree of habit-persistence for consumption.

α = Elasticity of output with respect to capital.

β = Discount factor of household.

δ = Depreciation rate of capital.

χ^p = Size of adjustment costs in resetting prices.

χ^w = Size of adjustment costs in resetting wages.

η^p = Relative importance of lagged price inflation in the adjustment cost function.

η^w = Relative importance of lagged wage inflation in the adjustment cost function.

κ = Variable capacity utilization scaling parameter.

ν = Inverse of labor supply elasticity.

ς^c = Coefficient on the consumption goods component of the utility function.

ς^l = Coefficient on the labor supply component of the utility function.

χ^l = Parameter representing the size of adjustment costs in the labor sectoral adjustment cost function.

η^l = Parameter representing the relative importance of lagged labor supply in the labor sectoral adjustment cost function.

χ = Investment adjustment costs in the capital accumulation.

ψ = Elasticity of utilization costs.

ϕ^r = Coefficient on lagged nominal interest rates in the Taylor type feedback rule.

$\phi^{h,gdp}$ = Coefficient on the output gap in the Taylor type feedback rule.

$\phi^{\Delta h,gdp}$ = Coefficient on change of the output gap in the Taylor type feedback rule.

$\phi^{\pi,gdp}$ = Coefficient on GDP price inflation in the Taylor type feedback rule.

$\rho^{z,k}$ = Persistence parameter in the AR(1) process describing the evolution of $\Gamma_t^{z,k}$.

$\rho^{z,m}$ = Persistence parameter in the AR(1) process describing the evolution of $\Gamma_t^{z,m}$.

$\rho^{\theta_{x,c}}$ = Persistence parameter in the ARMA(1,1) process describing the evolution of $\Theta_t^{x,c}$.

$\rho^{\theta_{x,k}}$ = Persistence parameter in the ARMA(1,1) process describing the evolution of $\Theta_t^{x,k}$.

ρ^{θ_l} = Persistence parameter in the ARMA(1,1) process describing the evolution of Θ_t^l.

$\rho^{\theta_{x,c},ma}$ = Coefficient on moving average term in the ARMA(1,1) process describing the evolution of $\Theta_t^{x,c}$.

$\rho^{\theta_{x,k},ma}$ = Coefficient on moving average term in the ARMA(1,1) process describing the evolution of $\Theta_t^{x,k}$.

$\rho^{\theta_l,ma}$ = Coefficient on moving average term in the ARMA(1,1) process describing the evolution of Θ_t^l.

ρ^{φ} = Persistence parameter in the AR(1) process describing the evolution of A_t^{φ}.

ρ^g = Persistence parameter in the AR(1) process describing the evolution of G_t.

ρ^f = Persistence parameter in the AR(1) process describing the evolution of F_t.

$\rho^{\xi,\beta}$ = Persistence parameter in the AR(1) process describing the evolution of Ξ_t^{β}.

ρ^U = Persistence parameter in the AR(1) process describing the evolution of Z_t^U.

σ^r = Standard deviation of interest rate shock.

$\sigma^{z,k}$ = Standard deviation of the shock to $\Gamma_t^{z,k}$.

$\sigma^{z,m}$ = Standard deviation of the shock to $\Gamma_t^{z,m}$.

$\sigma^{\theta_{x,c}}$ = Standard deviation of the shock to $\Theta_t^{x,c}$.

$\sigma^{\theta_{x,k}}$ = Standard deviation of the shock to $\Theta_t^{x,k}$.

$\sigma^{\theta,l}$ = Standard deviation of the shock to Θ_t^l.

$\sigma^{a,k}$ = Standard deviation of the shock to A_t^k.

$\sigma^{a,m}$ = Standard deviation of the shock to A_t^m.

σ^{φ} = Standard deviation of the shock to A_t^{φ}.

σ^g = Standard deviation of the shock to G_t.

σ^f = Standard deviation of the shock to F_t.

$\sigma^{\xi,\beta}$ = Standard deviation of the shock to Ξ_t^{β}.

σ^U = Standard deviation of the shock to Z_t^U.

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Table 1 : Calibrated Parameter Values

α	β	δ	$\Theta_*^{x,c}$	$\Theta_*^{x,k}$	$\Theta_*^{x,l}$	$\Gamma_*^{z,m}$	$\Gamma_*^{z,k}$
0.30	0.99	0.02	6.00	6.00	6.00	1.002	1.004

Table 2 : Estimated Parameter Values (Prior and Posterior Distributions)

Param.	Prior Distribution	Prior Mean	Prior S.D.	Posterior Distribution		
				Mean	5th Percentiles	95th Percentiles
h	beta	0.6	0.15	0.49	0.39	0.59
ν	gamm	2	1	0.31	0.06	0.55
χ^p	gamm	4	2	13.85	8.21	19.26
η^p	beta	0.5	0.15	0.16	0.05	0.26
χ^w	gamm	4	2	11.49	7.56	15.37
η^w	beta	0.5	0.15	0.16	0.05	0.26
χ^l	gamm	2	1	1.90	0.40	3.37
η^l	beta	0.5	0.15	0.51	0.26	0.75
χ	gamm	2	1	1.24	0.47	1.99
ϕ^r	beta	0.7	0.15	0.92	0.90	0.94
$\phi^{\pi,gdp}$	norm	1.5	0.5	1.16	0.76	1.57
$\phi^{h,gdp}$	norm	0.5	0.5	0.16	0.08	0.23
$\phi^{\Delta h,gdp}$	norm	0	0.5	1.13	0.67	1.59
ψ	norm	1	1	3.14	2.09	4.19
$\rho^{z,k}$	norm	0.98	0.01	0.97	0.96	0.99
$\rho^{z,m}$	norm	0.98	0.01	0.97	0.95	0.98
$\rho^{\theta_{x,c}}$	beta	0.5	0.15	0.83	0.75	0.91
$\rho^{\theta_{x,k}}$	beta	0.5	0.15	0.44	0.25	0.63
ρ^{θ_l}	beta	0.5	0.15	0.26	0.08	0.42
ρ^{φ}	beta	0.7	0.15	0.65	0.47	0.82
ρ^g	beta	0.5	0.15	0.93	0.90	0.97
ρ^f	beta	0.5	0.15	0.87	0.82	0.92
$\rho^{\xi,\beta}$	beta	0.8	0.15	0.96	0.93	0.98
ρ^U	beta	0.7	0.15	0.91	0.86	0.97
$\rho^{\theta_{x,c},ma}$	beta	0.5	0.15	0.70	0.55	0.86
$\rho^{\theta_{x,k},ma}$	beta	0.5	0.15	0.50	0.33	0.67
$\rho^{\theta_l,ma}$	beta	0.4	0.15	0.39	0.27	0.51
σ^r	invg	0.1	2	0.10	0.09	0.11
$\sigma^{z,k}$	invg	0.5	2	0.23	0.12	0.34
$\sigma^{z,m}$	invg	0.5	2	0.16	0.10	0.22
$\sigma^{\theta_{x,c}}$	invg	0.5	5	0.42	0.33	0.50
$\sigma^{\theta_{x,k}}$	invg	1.5	5	2.77	2.35	3.18
$\sigma^{\theta,l}$	invg	0.5	5	3.10	1.90	4.28
$\sigma^{a,k}$	invg	2	5	1.12	0.53	1.71
$\sigma^{a,m}$	invg	5	5	1.08	0.94	1.21
σ^{φ}	invg	3	5	9.88	3.47	16.09
σ^g	invg	1	5	1.52	1.35	1.69
σ^f	invg	0.5	5	0.24	0.22	0.27
$\sigma^{\xi,\beta}$	invg	5	5	4.67	3.19	6.08
σ^U	invg	1	2	1.56	1.38	1.74

Table 3: Variance Decomposition

T = 1

	Output	Consumption	Investment	Inflation
Monetary Policy Shock	3.15	2.6	1.24	0.07
Economy-Wide Technology Shock	27.66	30.48	3.34	0.04
Investment-Specific Technology Shock	5.51	5.77	0.68	0.59
Price Mark-up Shock (Consumption Goods)	2.57	6.82	0.01	95.92
Price Mark-up Shock (Investment Goods)	4.52	2.63	17.21	0.64
Wage Mark-up Shock	0.05	0.04	0.21	1.09
Investment Adjustment Cost Shock	39.62	0.11	76.38	0.1
Government Expenditure Shock	0.14	0.11	0.09	0
Net Export Shock	2.46	0.18	0.25	0
Intertemporal Preference Shock	13.34	49.82	0.44	1.32
Capital Utilization Adjustment Cost Shock	0.97	1.45	0.14	0.21

T = 4

	Output	Consumption	Investment	Inflation
Monetary Policy Shock	2.35	2.07	1.22	0.12
Economy-Wide Technology Shock	35.43	35.75	5.66	0.08
Investment-Specific Technology Shock	7.48	5.29	2.53	1.28
Price Mark-up Shock (Consumption Goods)	3.74	11.48	0.01	92.49
Price Mark-up Shock (Investment Goods)	4.78	2.27	16.29	0.96
Wage Mark-up Shock	0.04	0.07	0.19	1.73
Investment Adjustment Cost Shock	32.26	1.83	72.9	0.21
Government Expenditure Shock	0.23	0.1	0.12	0.01
Net Export Shock	3.46	0.15	0.5	0
Intertemporal Preference Shock	9.4	39.75	0.39	2.81
Capital Utilization Adjustment Cost Shock	0.83	1.25	0.19	0.3

T = 10

	Output	Consumption	Investment	Inflation
Monetary Policy Shock	2.11	2.13	0.91	0.16
Economy-Wide Technology Shock	31.65	35.75	3.73	0.13
Investment-Specific Technology Shock	8.21	4.31	3.69	2.84
Price Mark-up Shock (Consumption Goods)	2.66	9.71	0.02	87.15
Price Mark-up Shock (Investment Goods)	3.95	3.38	12.14	0.85
Wage Mark-up Shock	0.08	0.06	0.17	2.39
Investment Adjustment Cost Shock	40.41	3.07	78.38	0.38
Government Expenditure Shock	0.31	0.09	0.07	0.01
Net Export Shock	2.65	0.16	0.29	0
Intertemporal Preference Shock	7.38	40.26	0.48	5.73
Capital Utilization Adjustment Cost Shock	0.59	1.08	0.11	0.36

T = 100

	Output	Consumption	Investment	Inflation
Monetary Policy Shock	1.89	1.85	1	0.16
Economy-Wide Technology Shock	36.24	41.44	3.92	0.26
Investment-Specific Technology Shock	14.15	6.32	7.03	10.93
Price Mark-up Shock (Consumption Goods)	2.55	9.2	0.04	74.4
Price Mark-up Shock (Investment Goods)	3.65	2.77	13.08	0.96
Wage Mark-up Shock	0.07	0.05	0.18	2.33
Investment Adjustment Cost Shock	31.99	2.6	73.63	0.6
Government Expenditure Shock	0.36	0.08	0.08	0.01
Net Export Shock	2.02	0.12	0.38	0
Intertemporal Preference Shock	6.53	34.61	0.53	10.04
Capital Utilization Adjustment Cost Shock	0.56	0.95	0.13	0.3

Note : Each table shows variance decompositions of the output growth rate, the investment growth rate, the consumption growth rate, and the inflation rate.

Table 4-1: Granger Causality from Gaps to Inflation

(1) GDP deflator

Null Hypothesis	F-Statistic	Prob.
GAP from the PFA-based output does not Granger Cause Inflation_q	19.38	0.00
GAP from the HP-filterd output does not Granger Cause Inflation_q	12.95	0.00
GAP from the unconditional efficient output does not Granger Cause Inflation_q	65.26	0.00
GAP from the conditional efficient output does not Granger Cause Inflation_q	71.97	0.00
GAP from our measure of potential output does not Granger Cause Inflation_q	48.29	0.00

(2) Consumption goods deflator

Null Hypothesis	F-Statistic	Prob.
GAP from the PFA-based output does not Granger Cause Inflation_q	16.26	0.00
GAP from the HP-filterd output does not Granger Cause Inflation_q	4.28	0.02
GAP from the unconditional efficient output does not Granger Cause Inflation_q	34.52	0.00
GAP from the conditional efficient output does not Granger Cause Inflation_q	46.28	0.00
GAP from our measure of potential output does not Granger Cause Inflation_q	52.85	0.00

Note : Inflation_q and cons_inflation_q represent the percentage change from previous quarter.

Table 4-2: Analysis of Forecast Accuracy

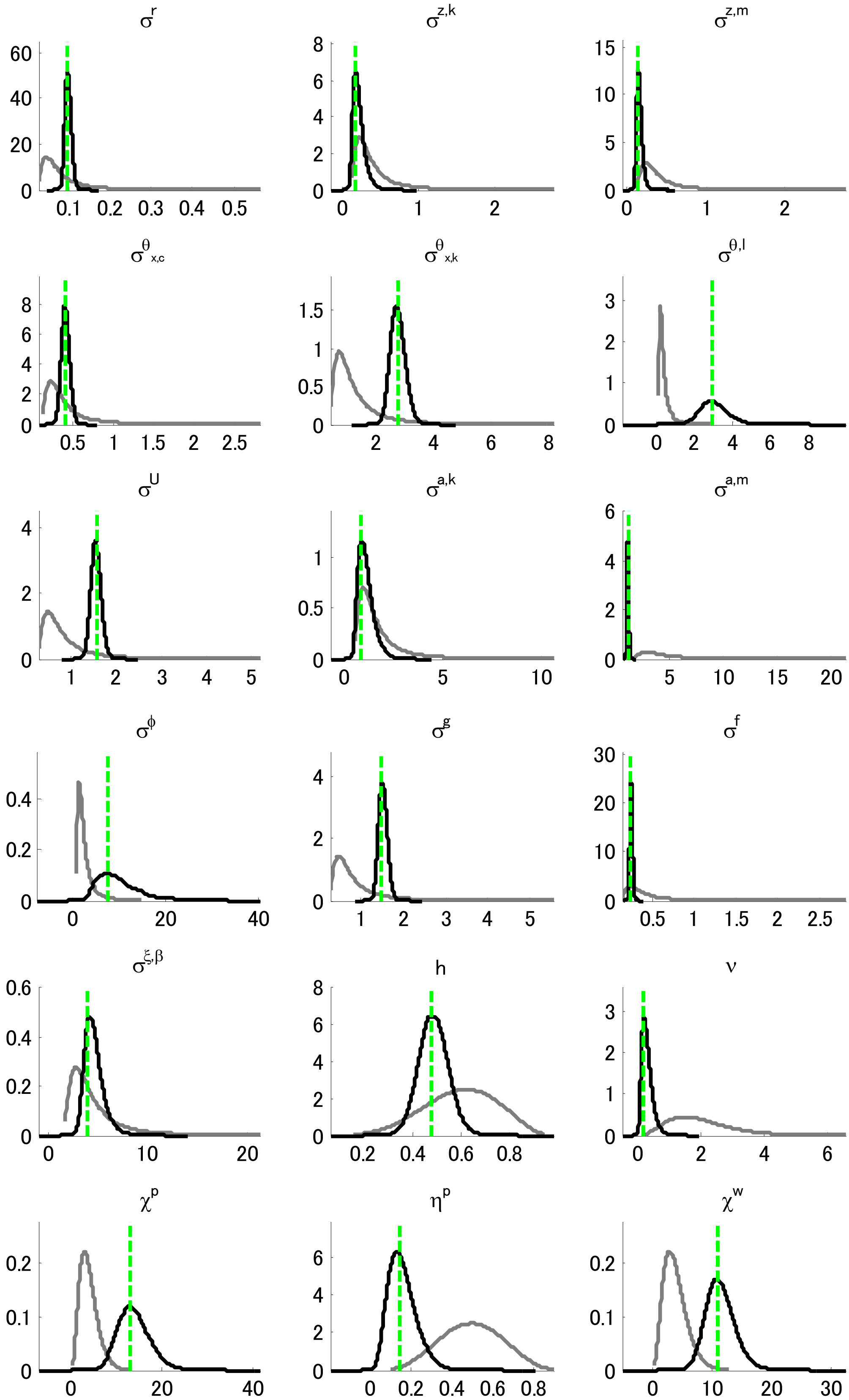
(1) GDP deflator

	MSFE			Relative to AR		
	horizon 1q	horizon 4q	horizon 8q	horizon 1q	horizon 4q	horizon 8q
GAP from the PFA-based output	2.98	0.74	0.65	0.78	0.96	1.40
GAP from the HP-filterd output	3.33	0.38	0.18	0.87	0.49	0.39
GAP from the unconditional efficient output	2.73	0.69	0.41	0.72	0.89	0.88
GAP from the conditional efficient output	2.12	0.51	0.34	0.56	0.66	0.72
GAP from our measure of potential output	2.63	0.41	0.35	0.69	0.53	0.76
AR	3.81	0.77	0.47	1.00	1.00	1.00

(2) Consumption goods deflator

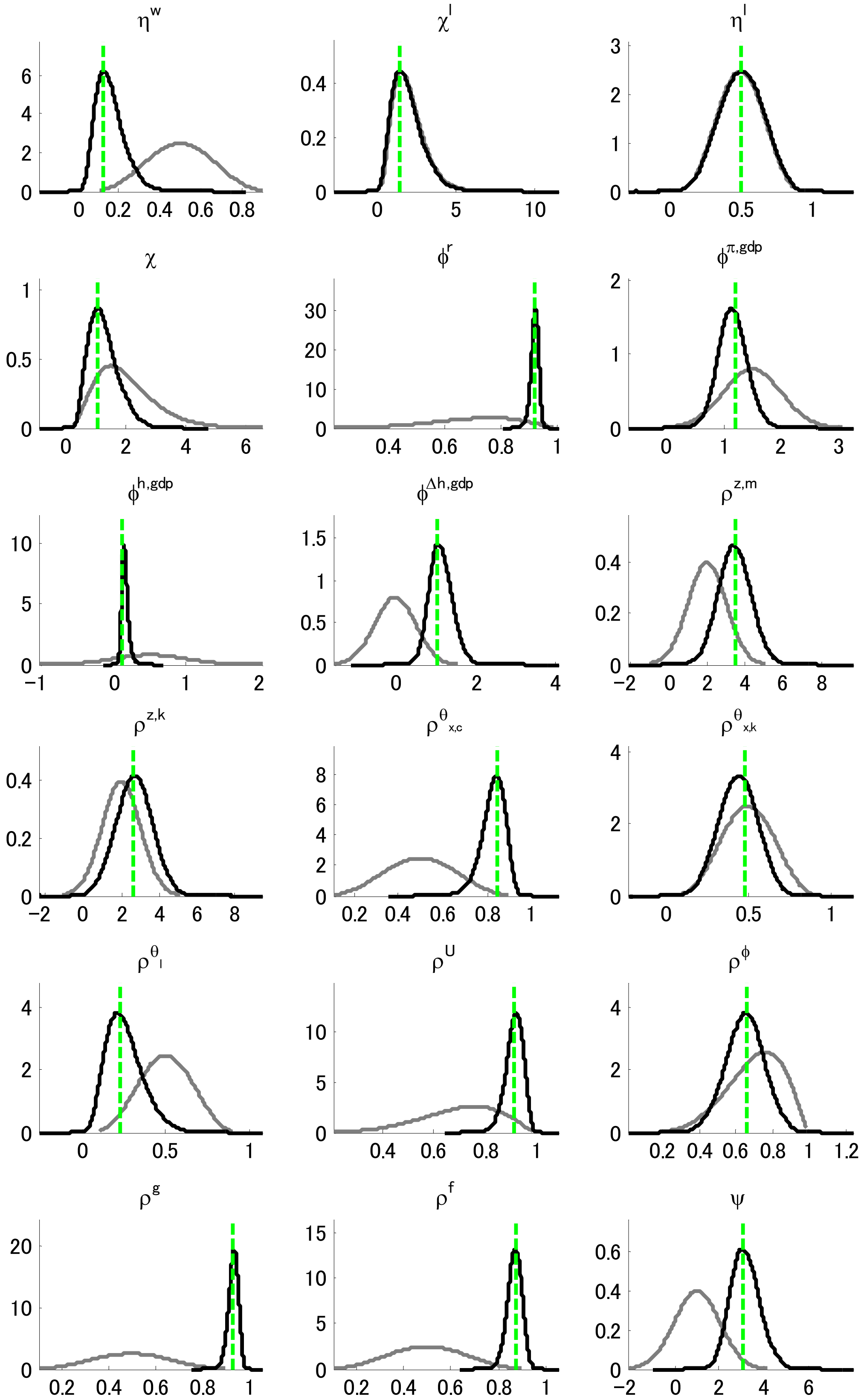
	MSFE			Relative to AR		
	horizon 1q	horizon 4q	horizon 8q	horizon 1q	horizon 4q	horizon 8q
GAP from the PFA-based output	1.74	0.76	0.55	0.72	0.64	1.20
GAP from the HP-filterd output	2.31	1.00	0.39	0.96	0.84	0.85
GAP from the unconditional efficient output	2.51	1.27	0.54	1.04	1.07	1.16
GAP from the conditional efficient output	1.82	1.06	0.44	0.76	0.89	0.95
GAP from our measure of potential output	1.60	0.87	0.53	0.67	0.73	1.14
AR	2.41	1.19	0.46	1.00	1.00	1.00

Figure 1-1: Estimated parameter values (prior and posterior distributions)



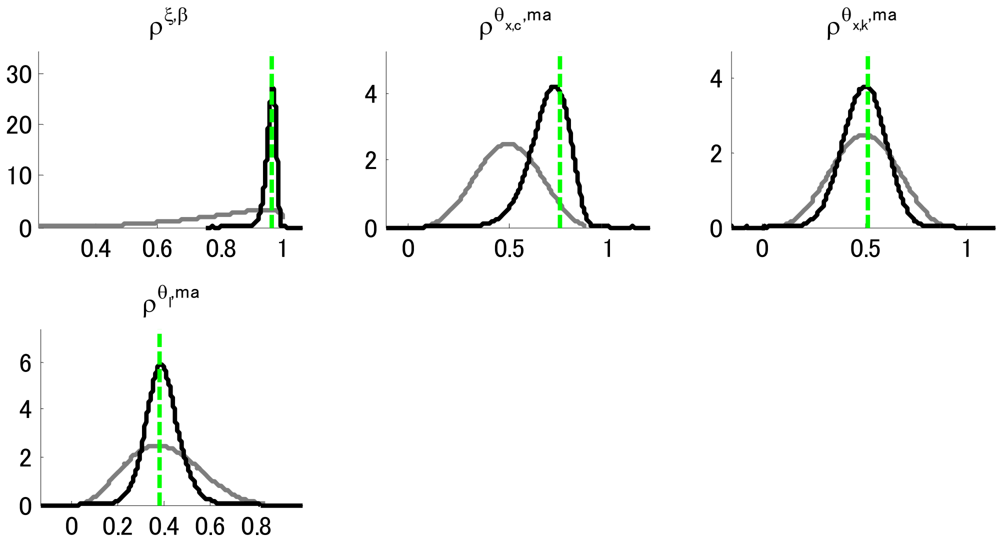
Note: Black and gray curves indicate the posterior and prior distribution of parameters respectively. Vertical lines represent their modes of posterior distributions.

Figure 1-2: Estimated parameter values (prior and posterior distributions)



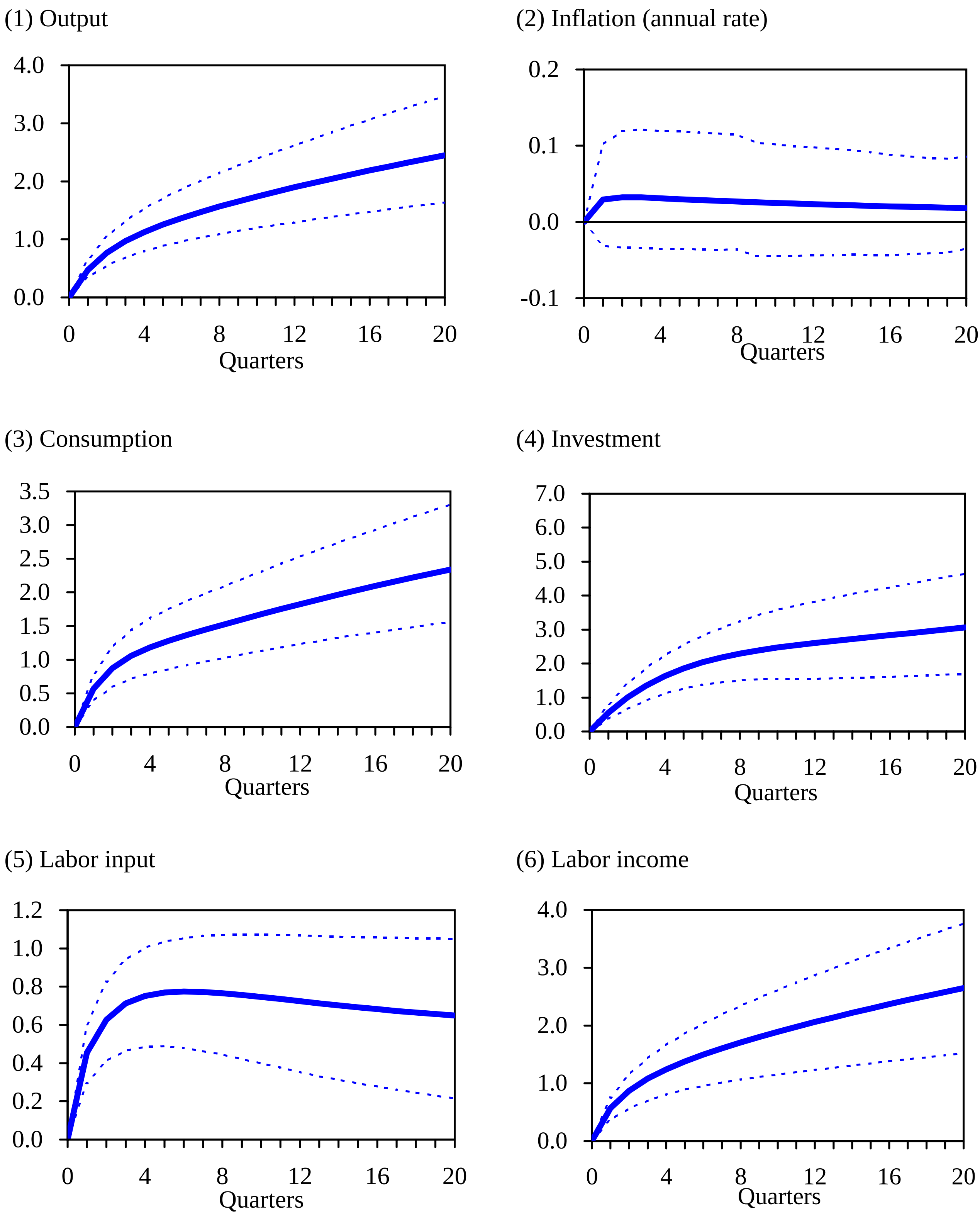
Note: Black and gray curves indicate the posterior and prior distribution of parameters respectively. Vertical lines represent their modes of posterior distributions.

Figure 1-3: Estimated parameter values (prior and posterior distributions)



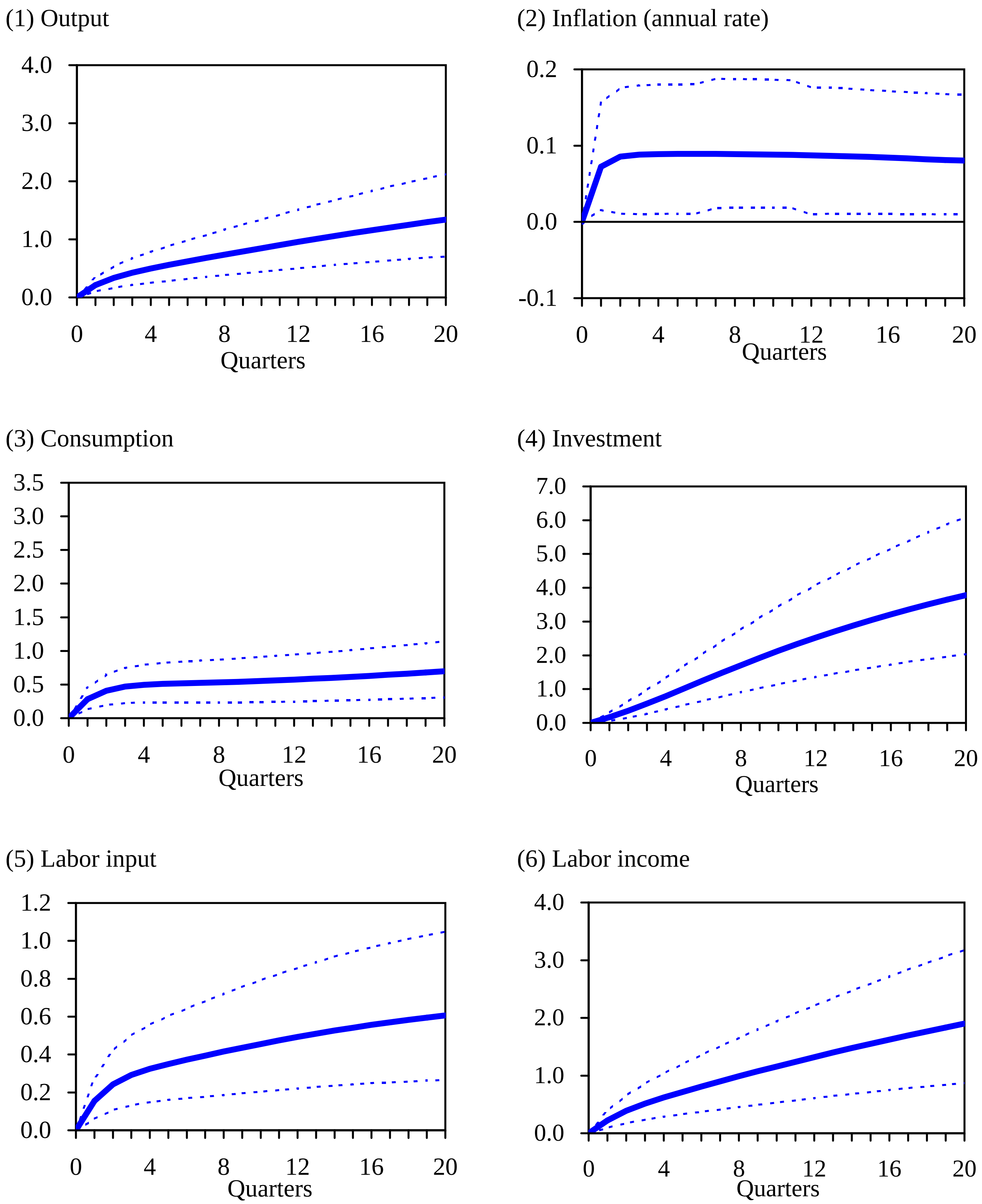
Note: Black and gray curves indicate the posterior and prior distribution of parameters respectively. Vertical lines represent their modes of posterior distributions.

Figure 2-1: Responses to the Economy-Wide Technology Shock (growth rate shock)



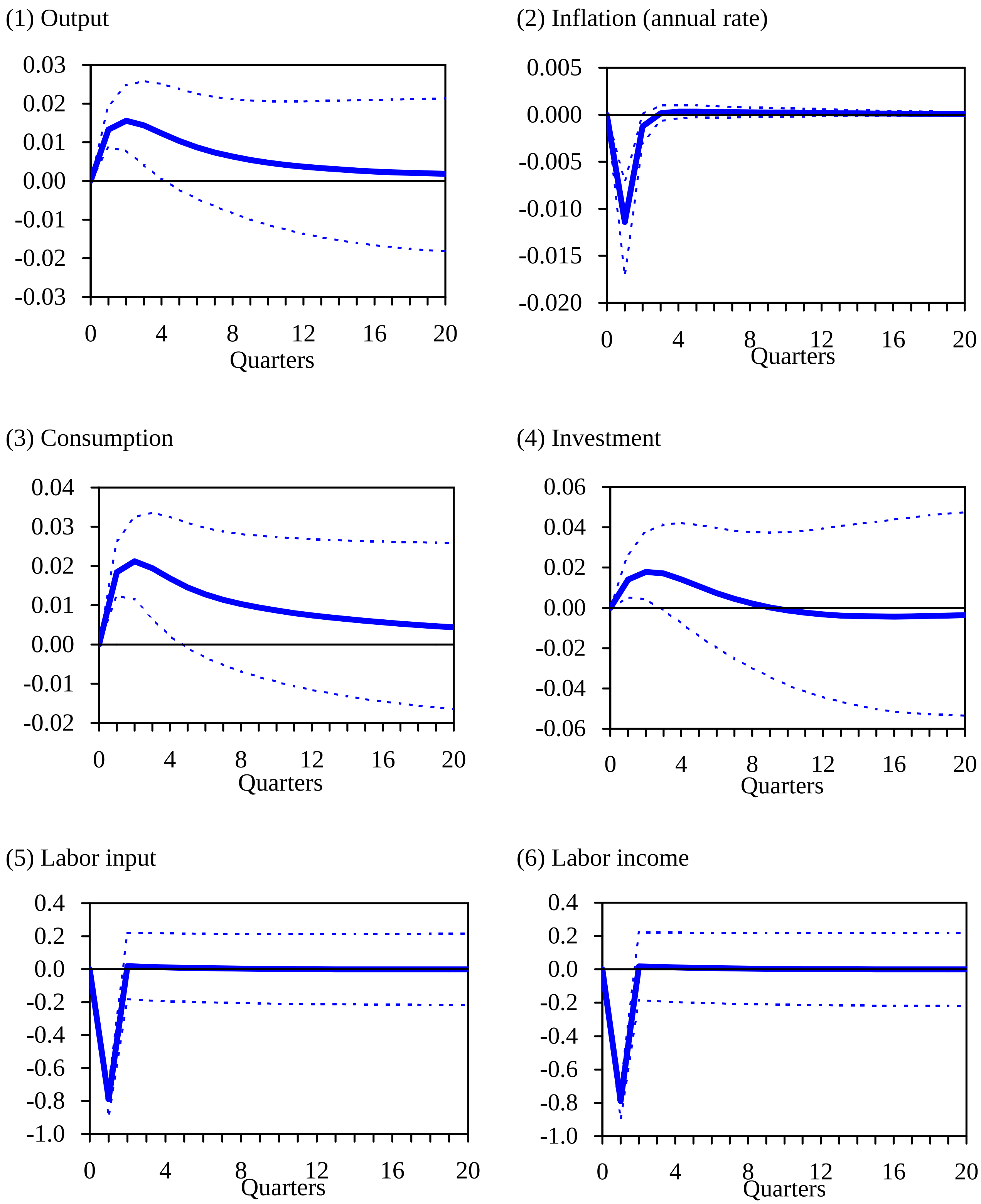
Note : Each figure shows the impulse responses to a shock equal to one standard deviation. All impulse responses are reported as percentage deviations from non-stochastic steady state. The dotted lines are the 90% posterior intervals.

Figure 2-2: Responses to the Investment-Specific Technology Shock (growth rate shock)



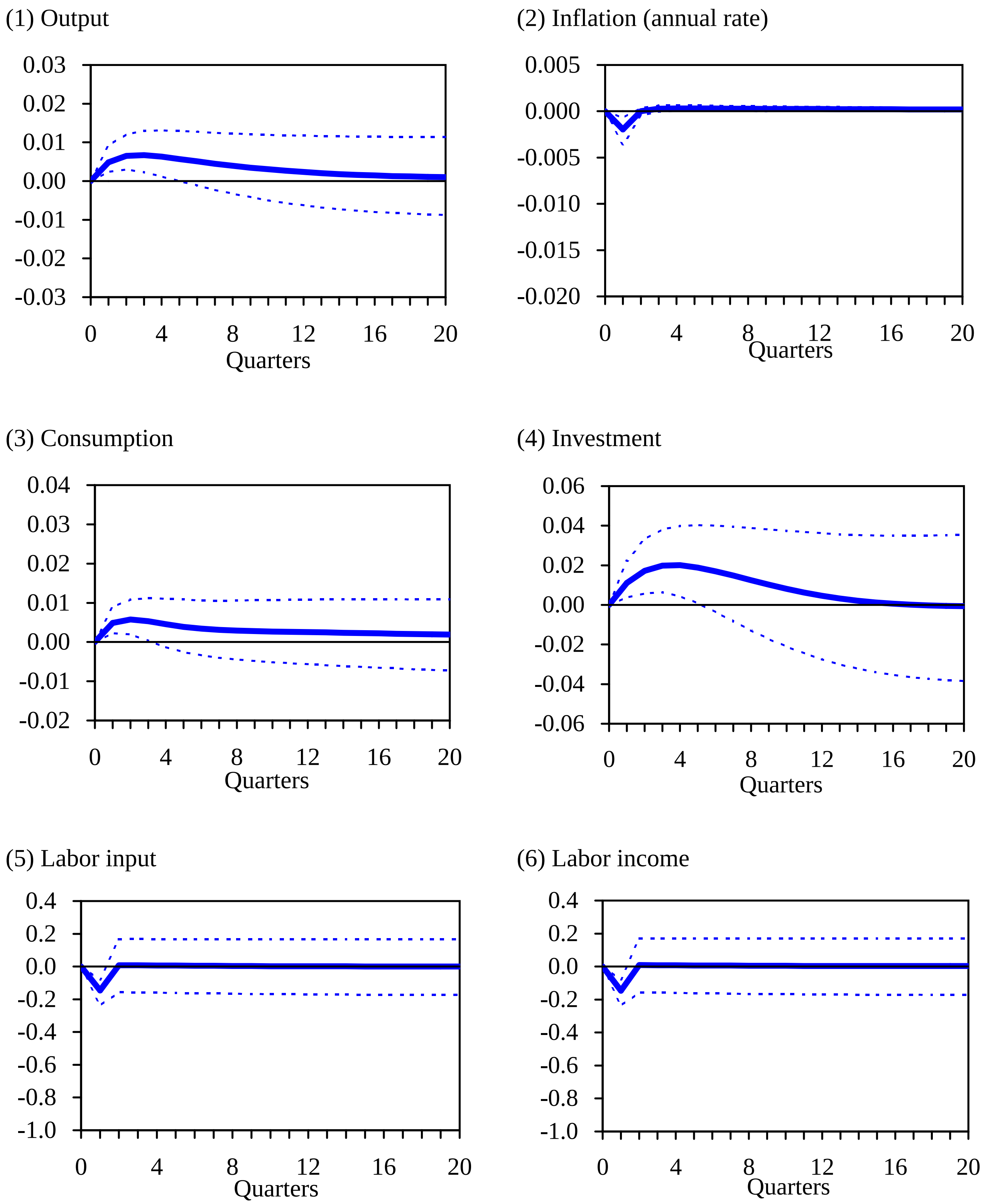
Note : Each figure shows the impulse responses to a shock equal to one standard deviation. All impulse responses are reported as percentage deviations from non-stochastic steady state. The dotted lines are the 90% posterior intervals.

Figure 2-3: Responses to the Economy-Wide Technology Shock (in level)



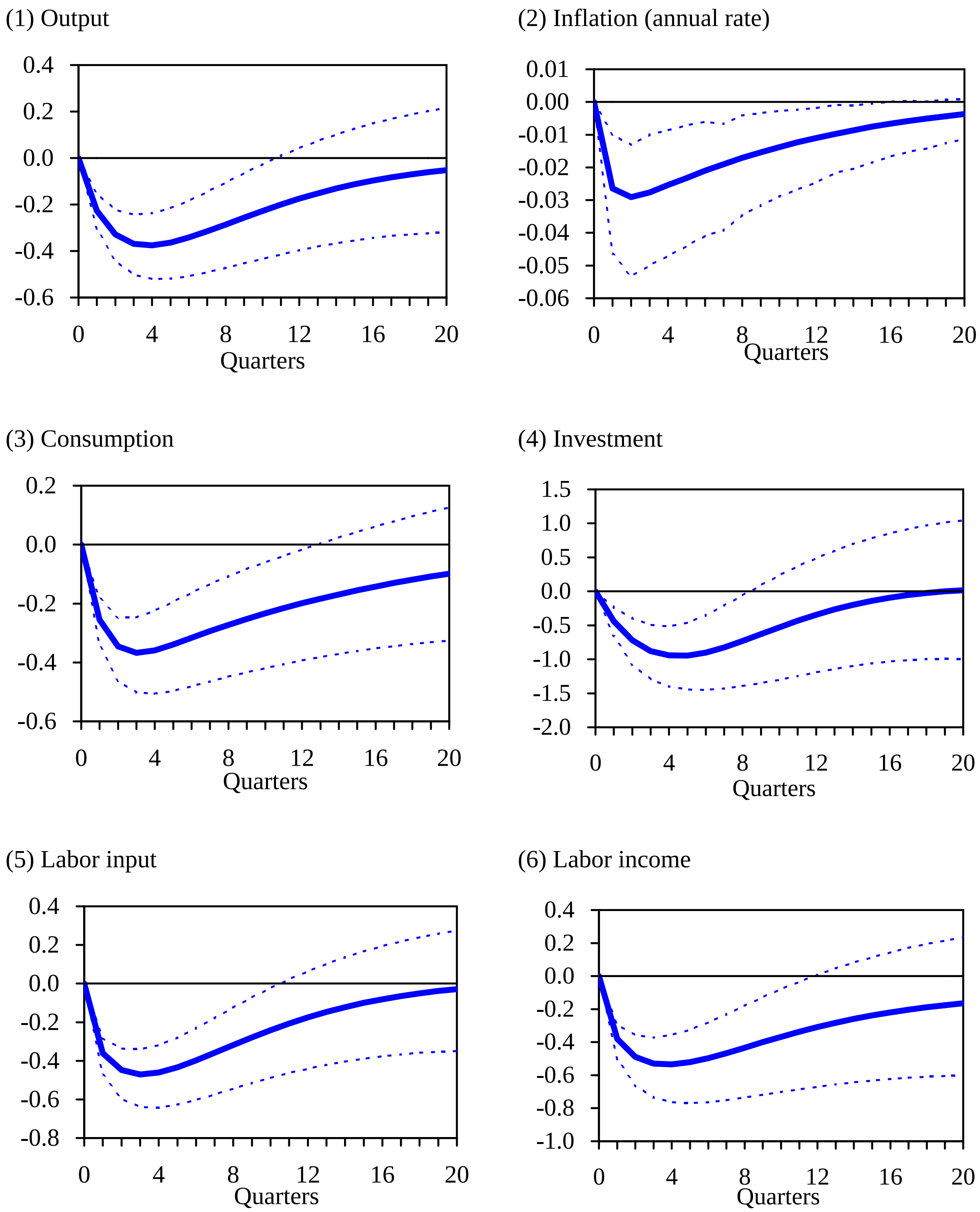
Note : Each figure shows the impulse responses to a shock equal to one standard deviation. All impulse responses are reported as percentage deviations from non-stochastic steady state. The dotted lines are the 90% posterior intervals.

Figure 2-4: Responses to the Investment-Specific Technology Shock (in level)



Note : Each figure shows the impulse responses to a shock equal to one standard deviation. All impulse responses are reported as percentage deviations from non-stochastic steady state. The dotted lines are the 90% posterior intervals.

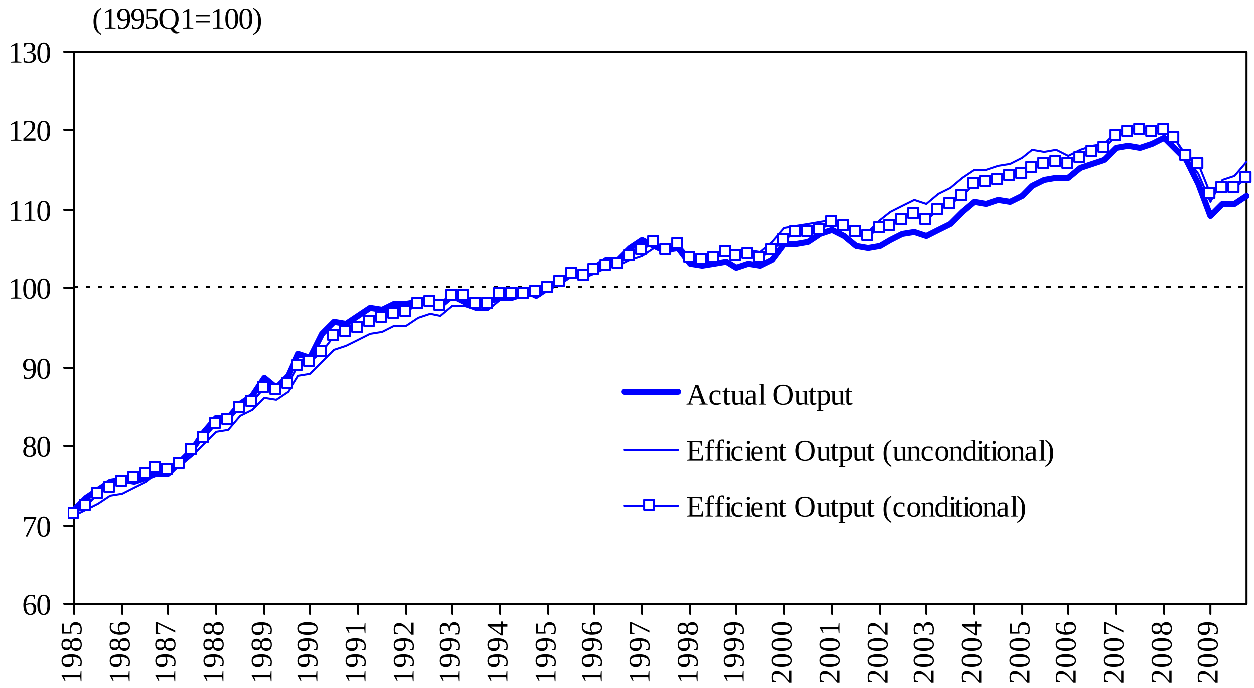
Figure 2-5: Responses to the Monetary Policy Shock



Note : Each figure shows the impulse responses to a shock equal to one standard deviation. All impulse responses are reported as percentage deviations from non-stochastic steady state. The dotted lines are the 90% posterior intervals.

Figure 3: Efficient Output (level & year-on-year growth rate)

Actual and Efficient Output (Level)



Actual and Efficient Output (Year-on-Year Growth Rate)

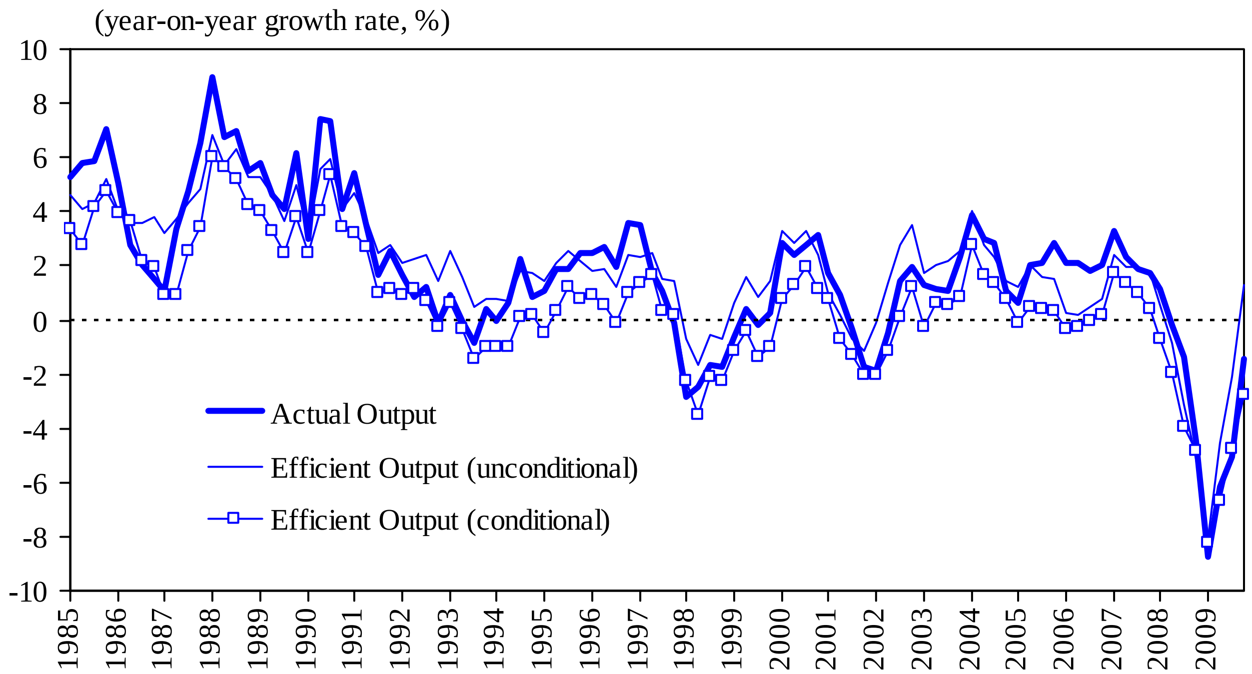
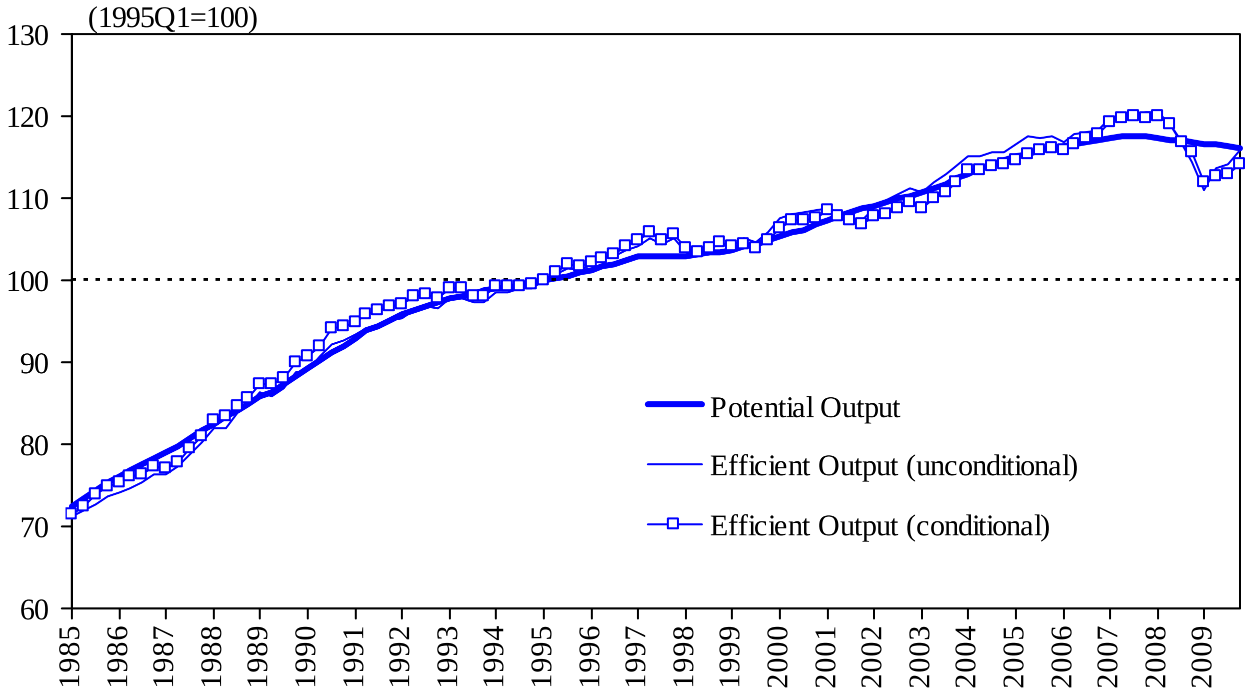


Figure 4-1: Potential Output (level & year-on-year growth rate)

Efficient and Potential Output (Level)



Efficient and Potential Output (Year-on-Year Growth Rate)

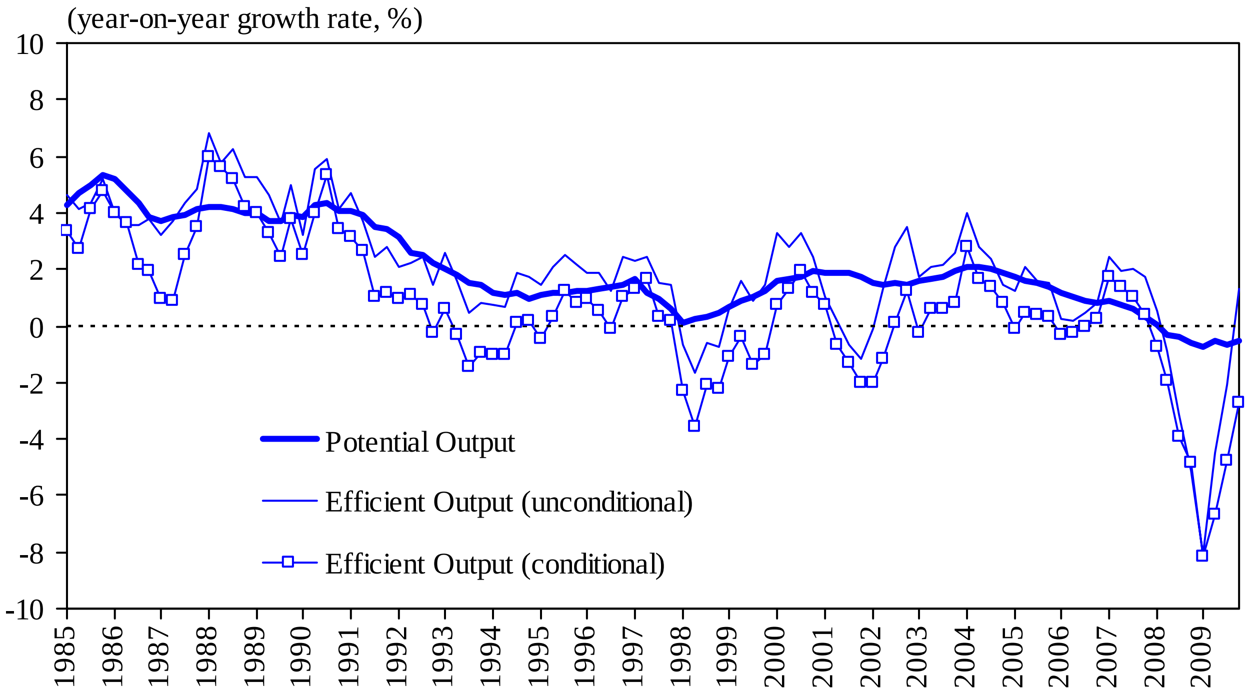
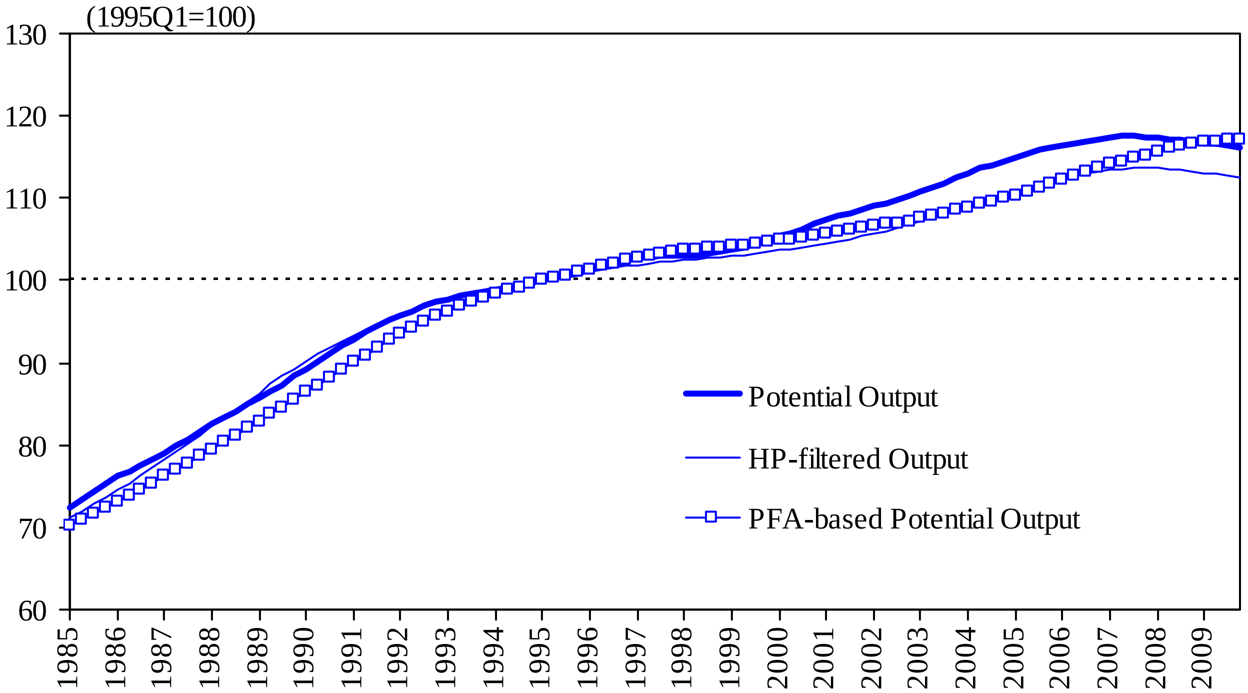
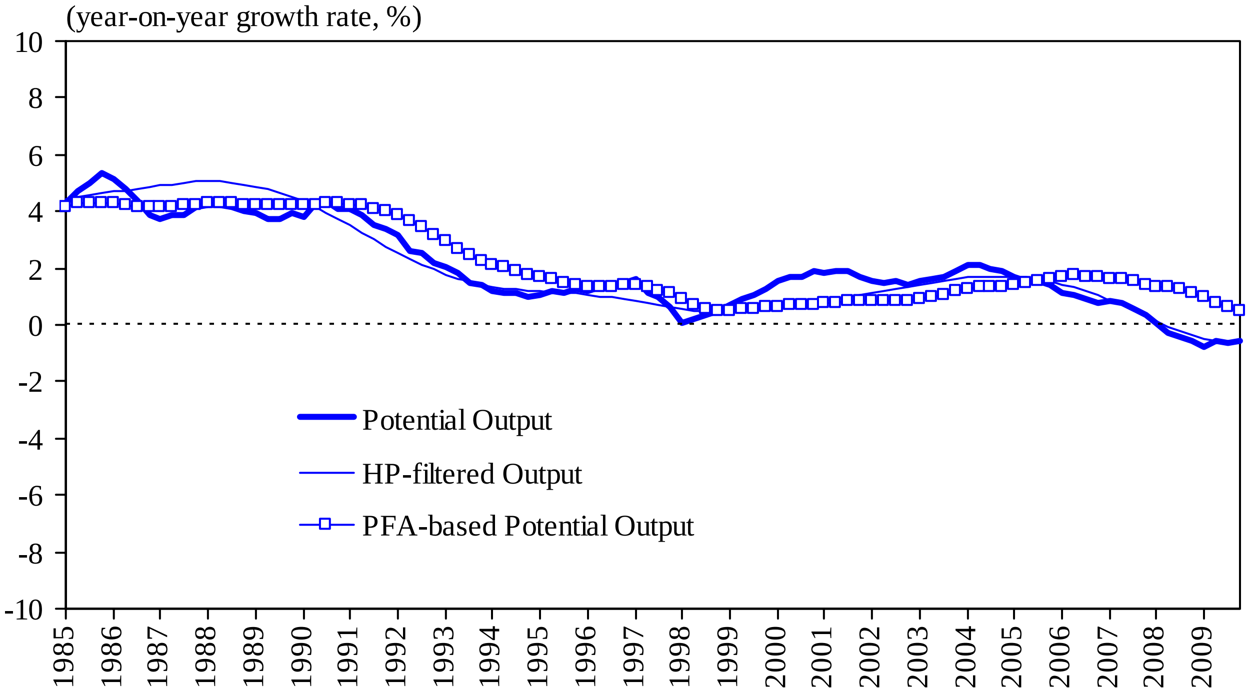


Figure 4-2: Potential Output (level & year-on-year growth rate)

Potential Output (Level)



Potential Output (Year-on-Year Growth Rate)



Note: A full write-up of the production-function approach (PFA) can be found in Hara et al. (2006).

Figure 5: Decomposition of Potential Growth

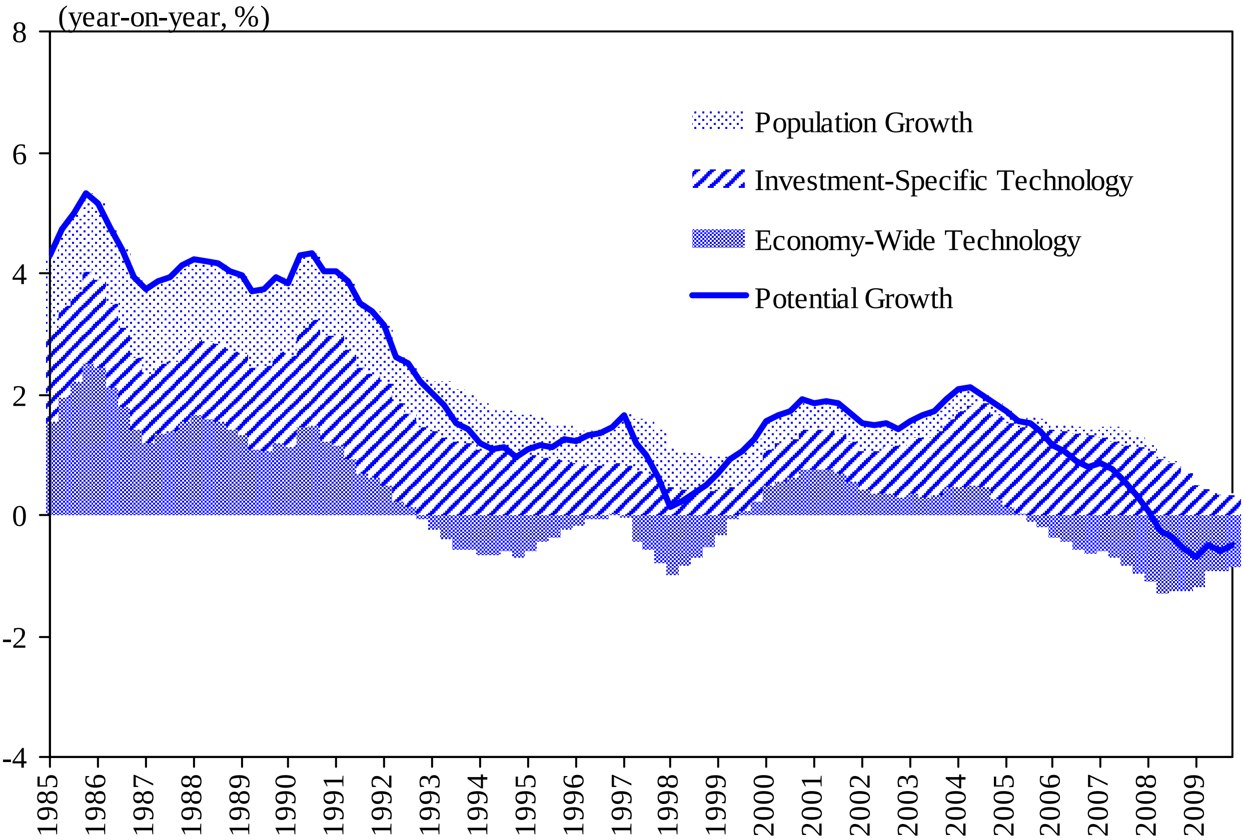
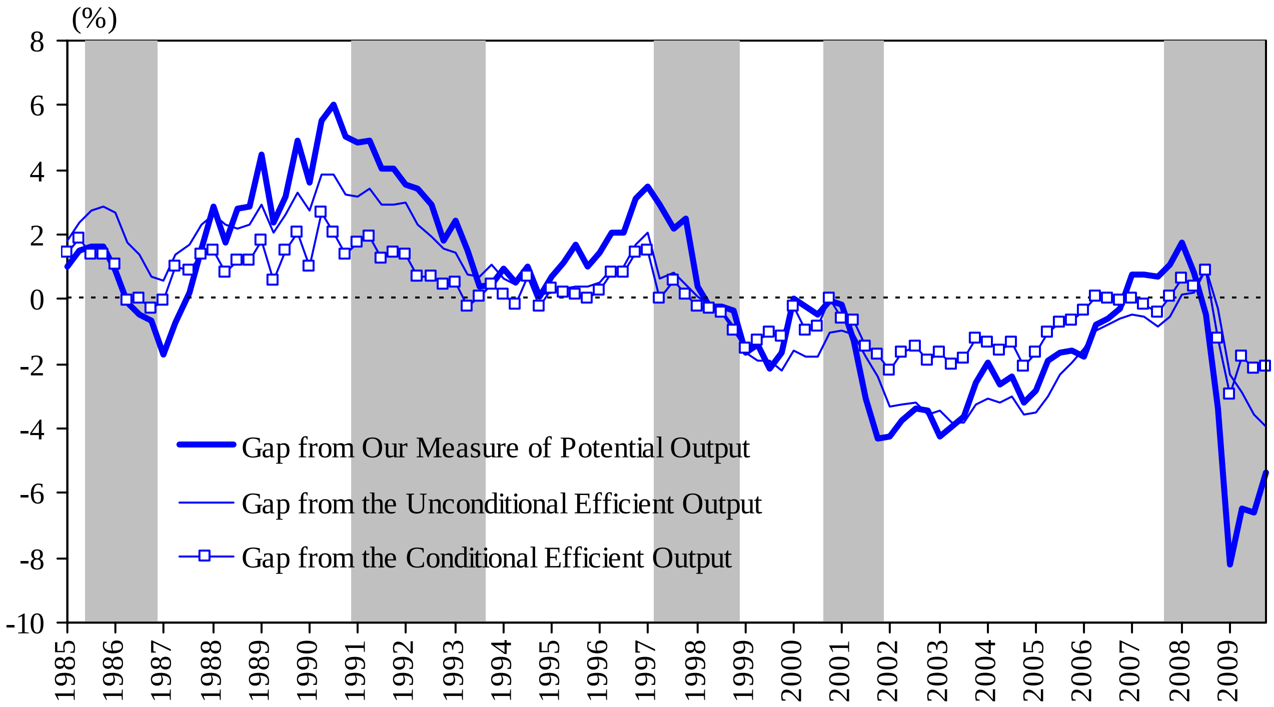
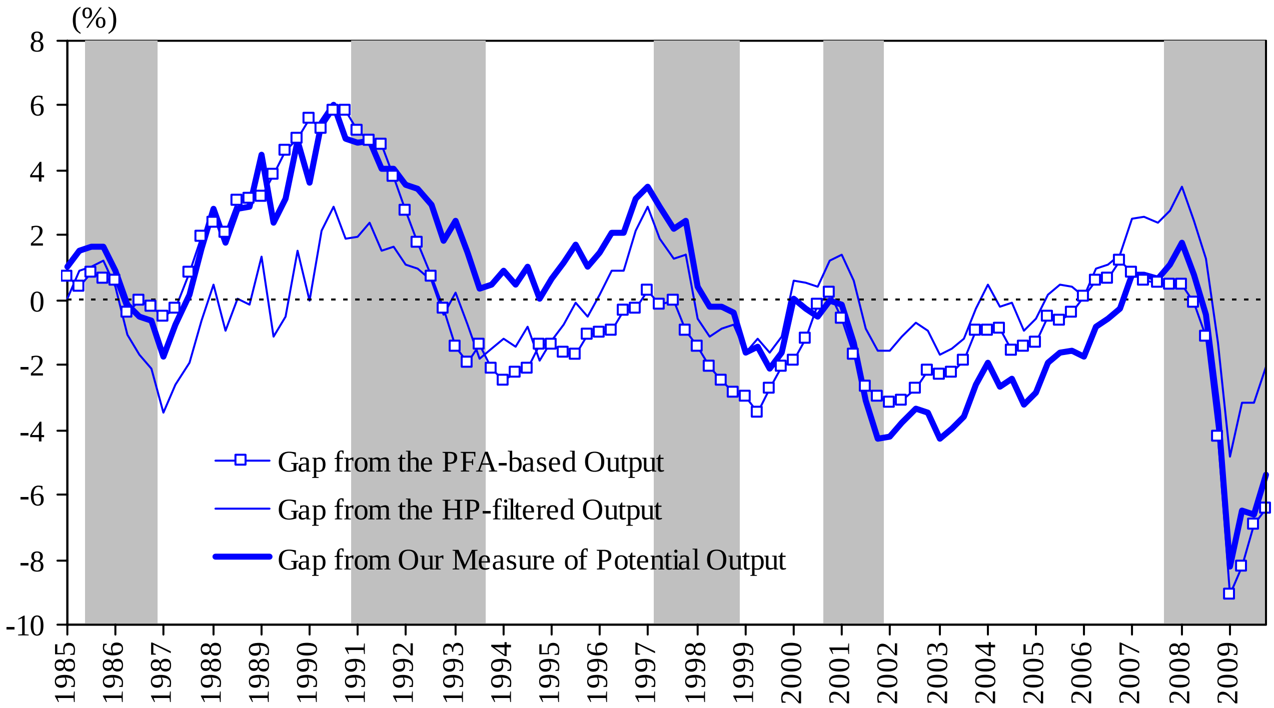


Figure 6: Output Gap

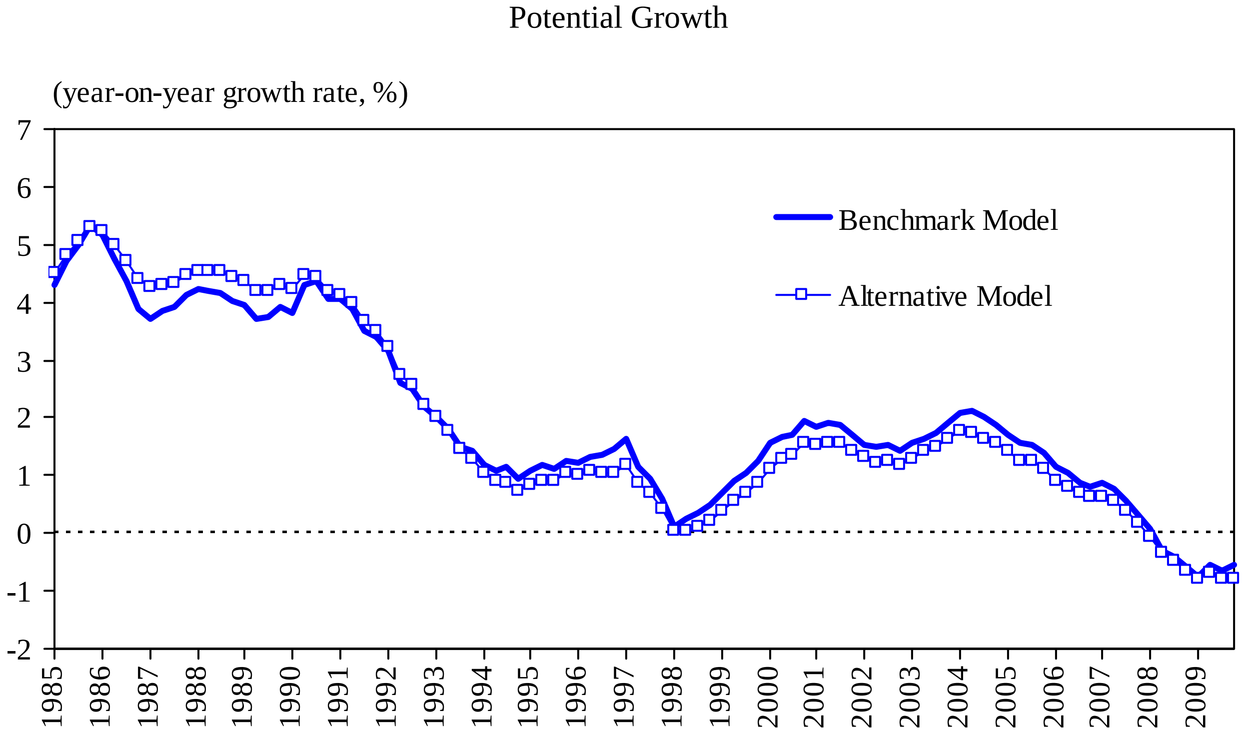


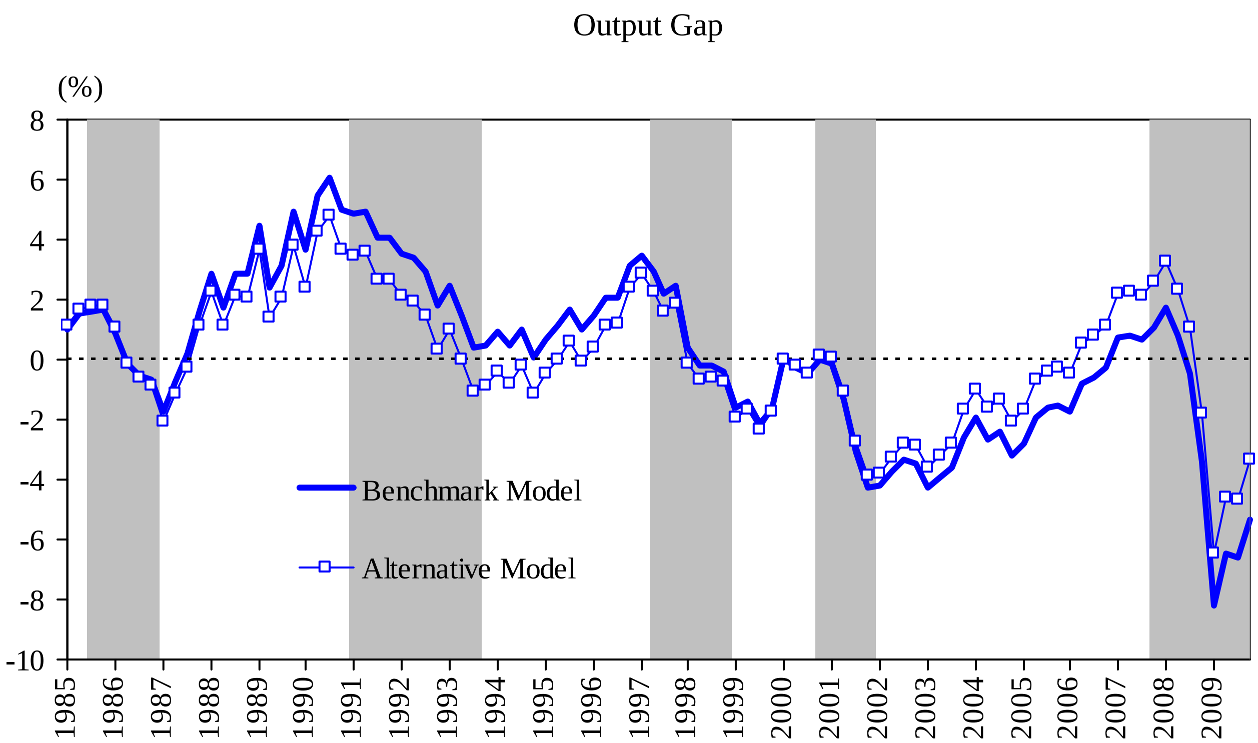


Note: Shaded regions show the ESRI recession dates.

Figure 7: Potential Growth and Output Gap

(alternative Taylor Rule)





Note: Alternative model corresponds to a model replacing the gaps from unconditional efficient outputs with those from potential outputs in a Taylor type feedback rule. Shaded regions show the ESRI recession dates.

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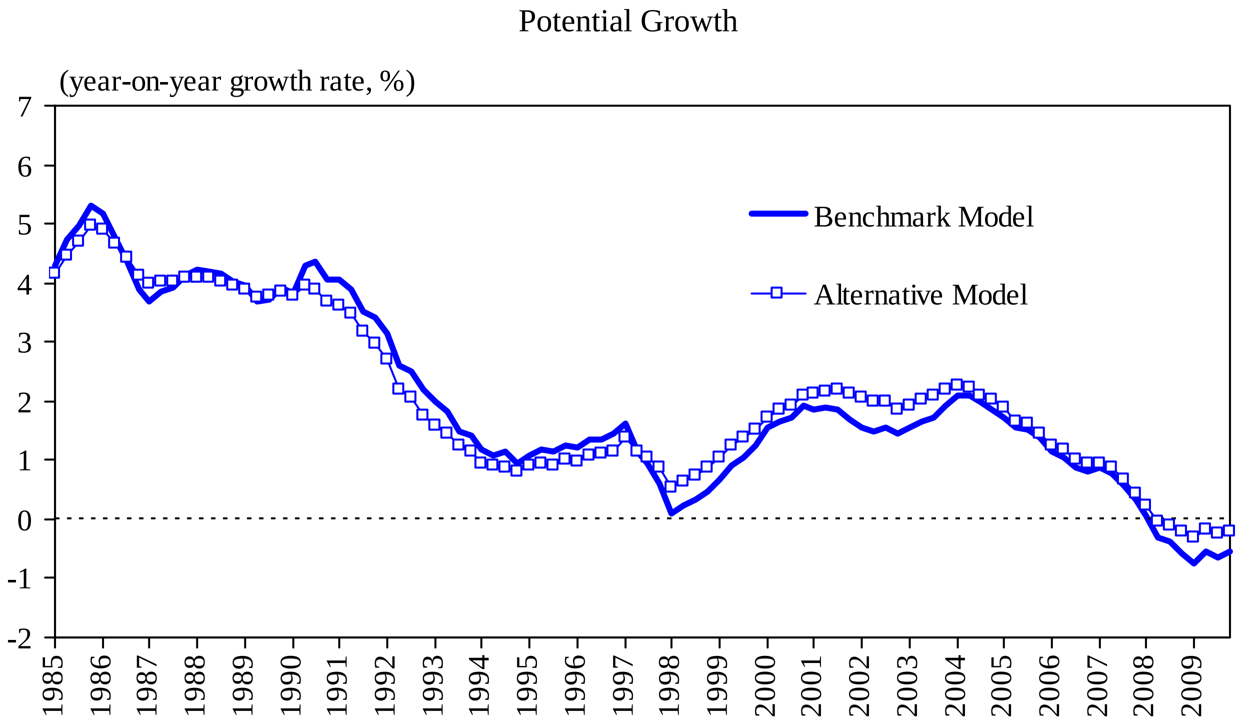
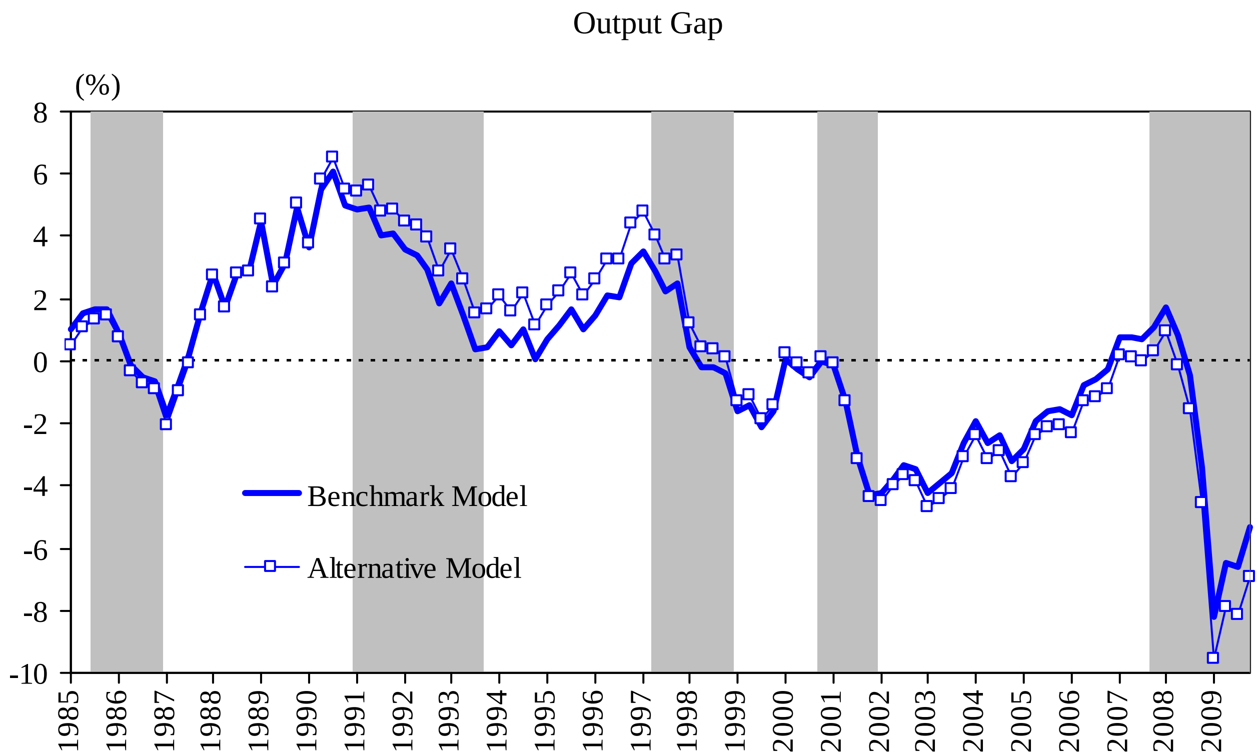
Figure 8: Potential Growth and Output Gap

(alternative model with the temporary labor supply shock)

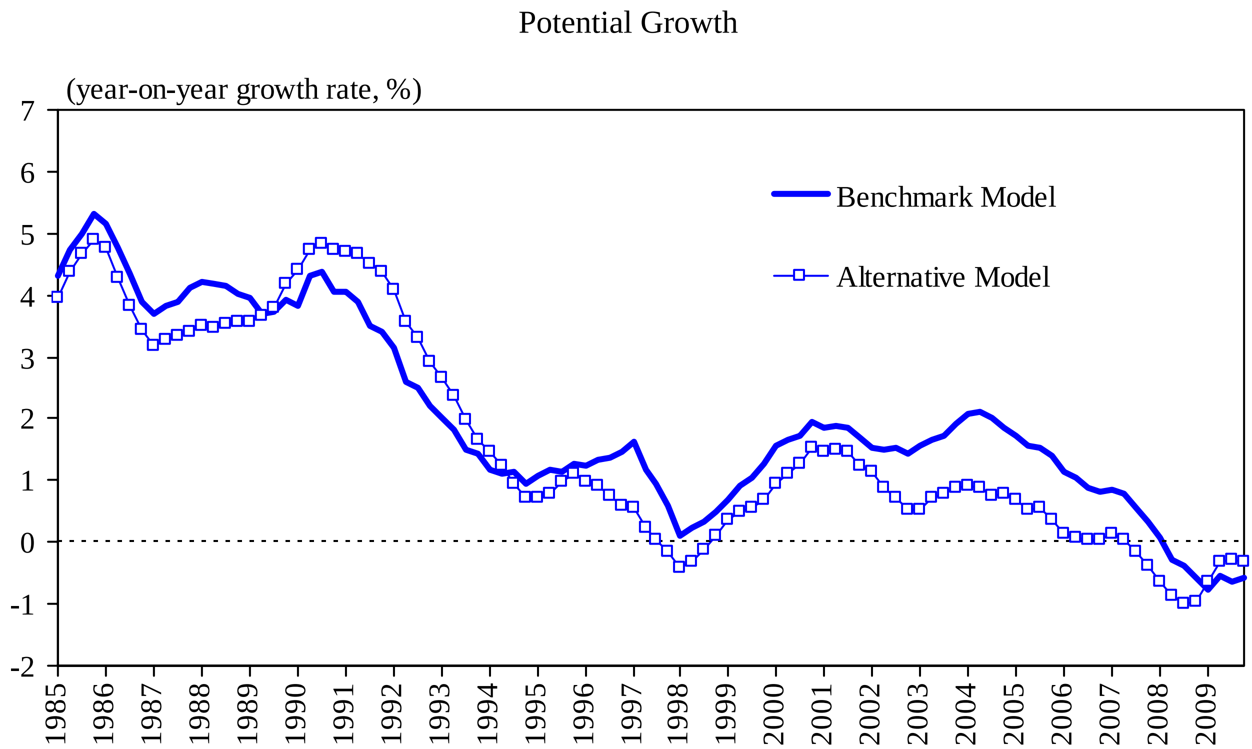


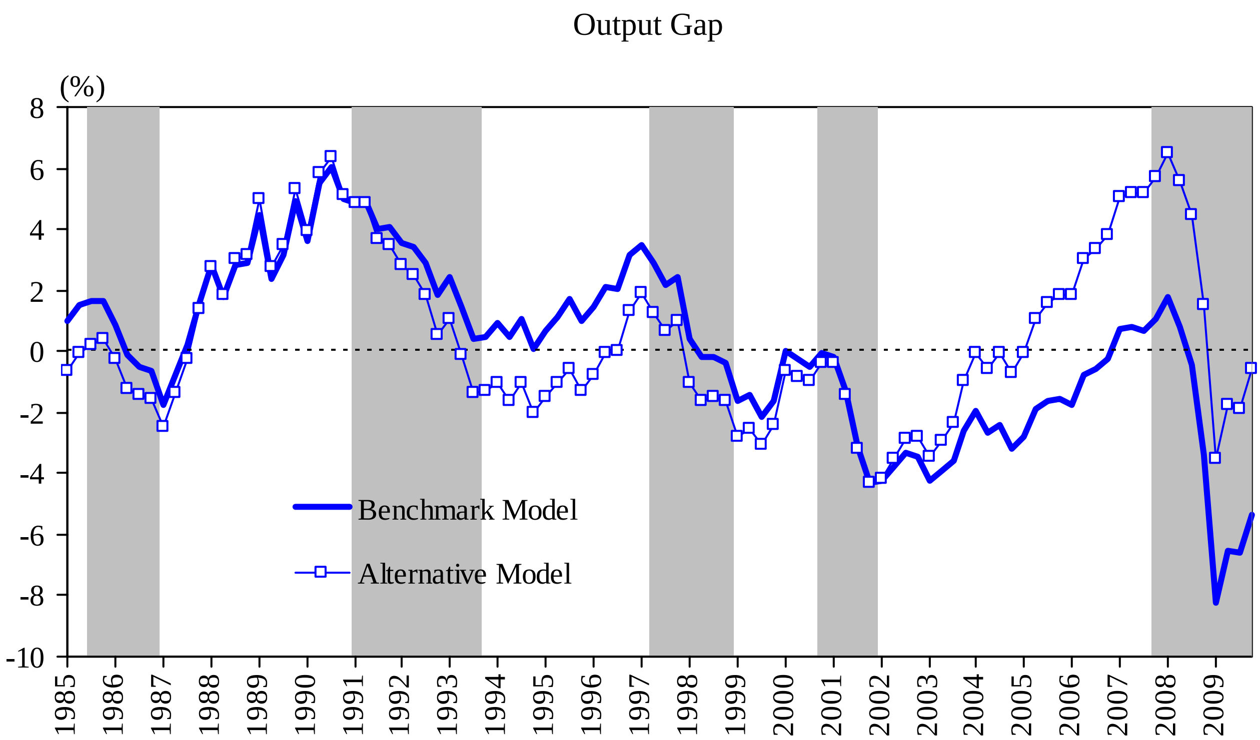


Note: Alternative model corresponds to a model replacing the wage markup shock with the temporary labor supply shock. Shaded regions show the ESRI recession dates.

Figure 9: Potential Growth and Output Gap

(alternative model with the permanent and temporary labor supply shocks)



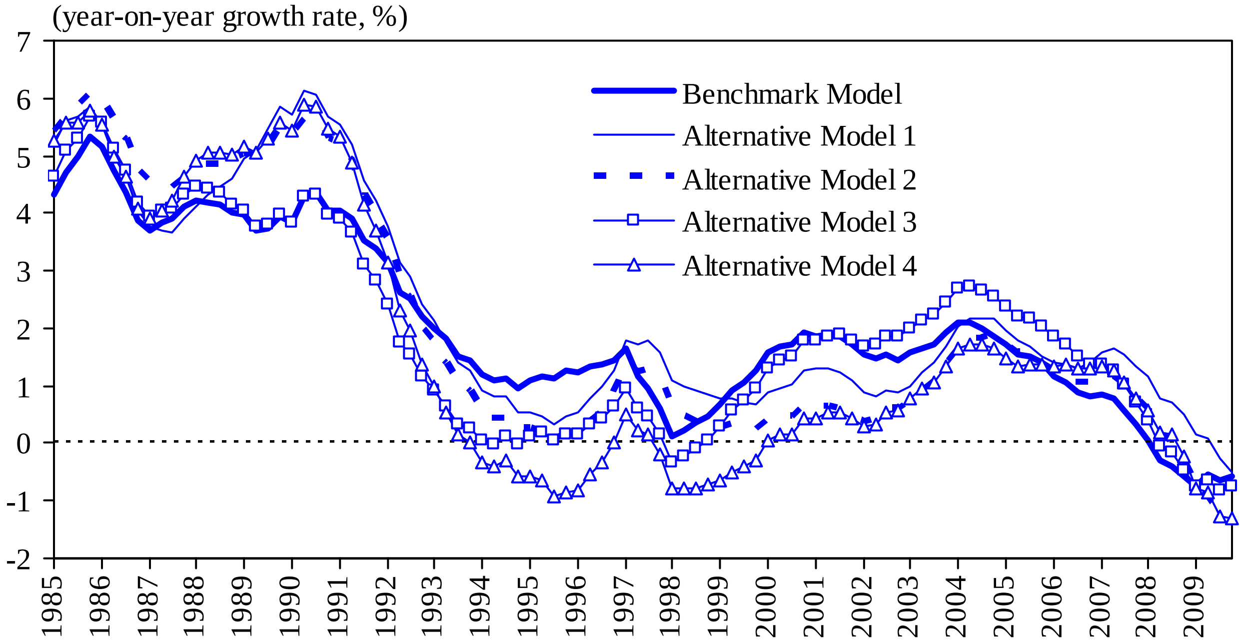


Note: Alternative model corresponds to a model with permanent and temporary labor supply shocks. Shaded regions show the ESRI recession dates.

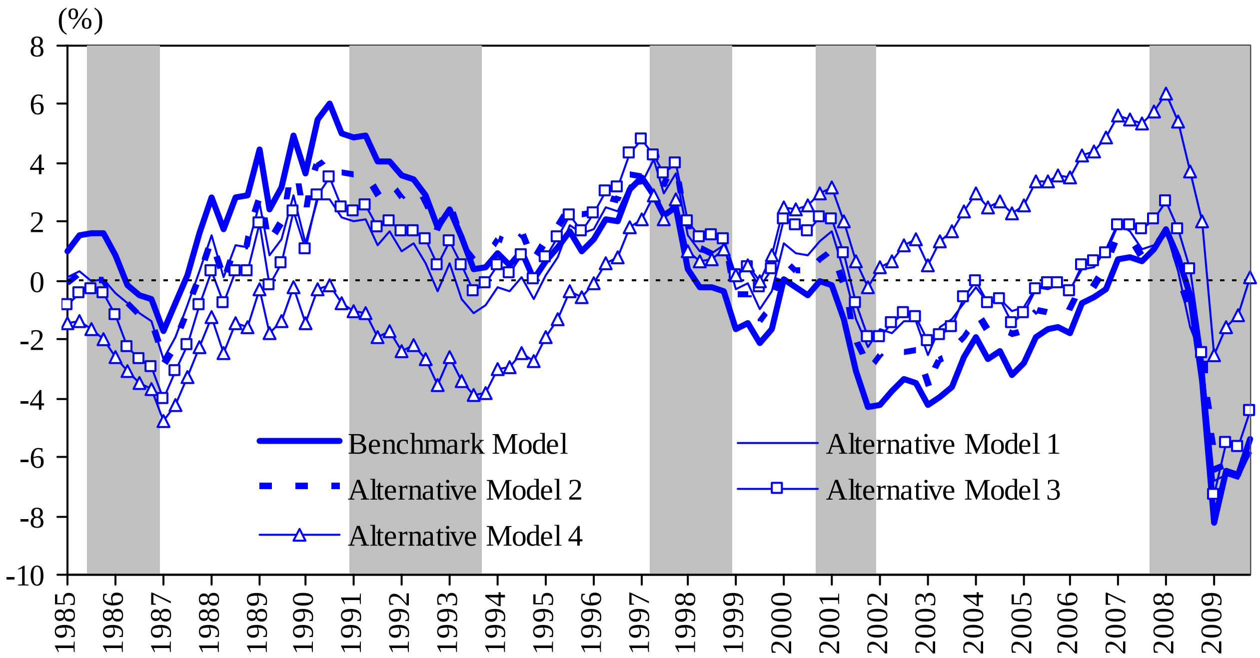
Figure 10: Potential Growth and Output Gap

(alternative models with measurement errors)

Potential Growth



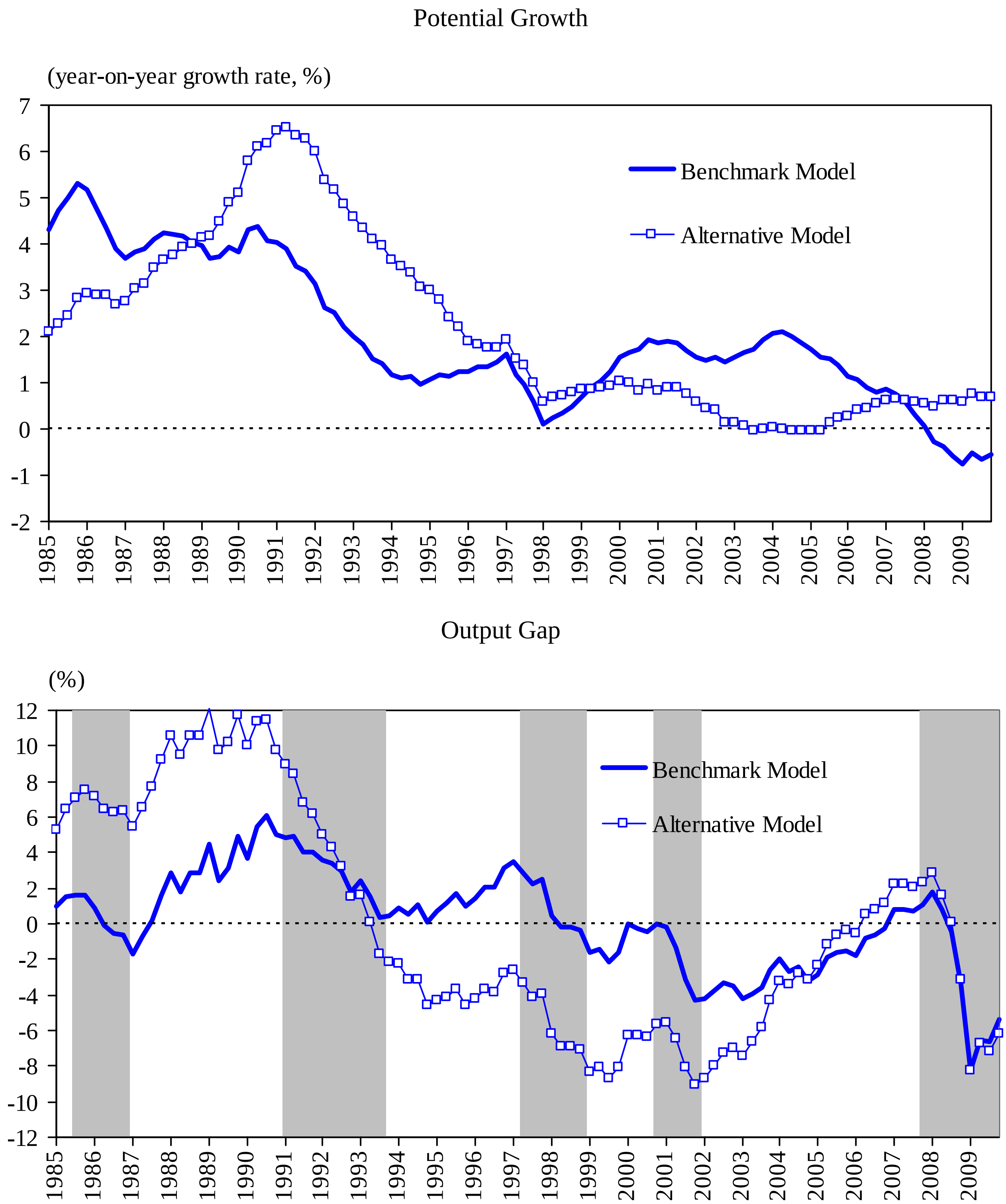
Output Gap



Note:
Alternative Model 1: with measurement errors in both prices and wage instead of markup shocks
Alternative Model 2: with measurement errors only in price instead of markup shock
Alternative Model 3: with measurement errors only in wage instead of markup shock
Alternative Model 4: with measurement errors in both price and wage in addition to markup shocks
Shaded regions show the ESRI recession dates.

Figure 11-1: Potential Growth and Output Gap

(alternative model with AR(1) technology process in level)

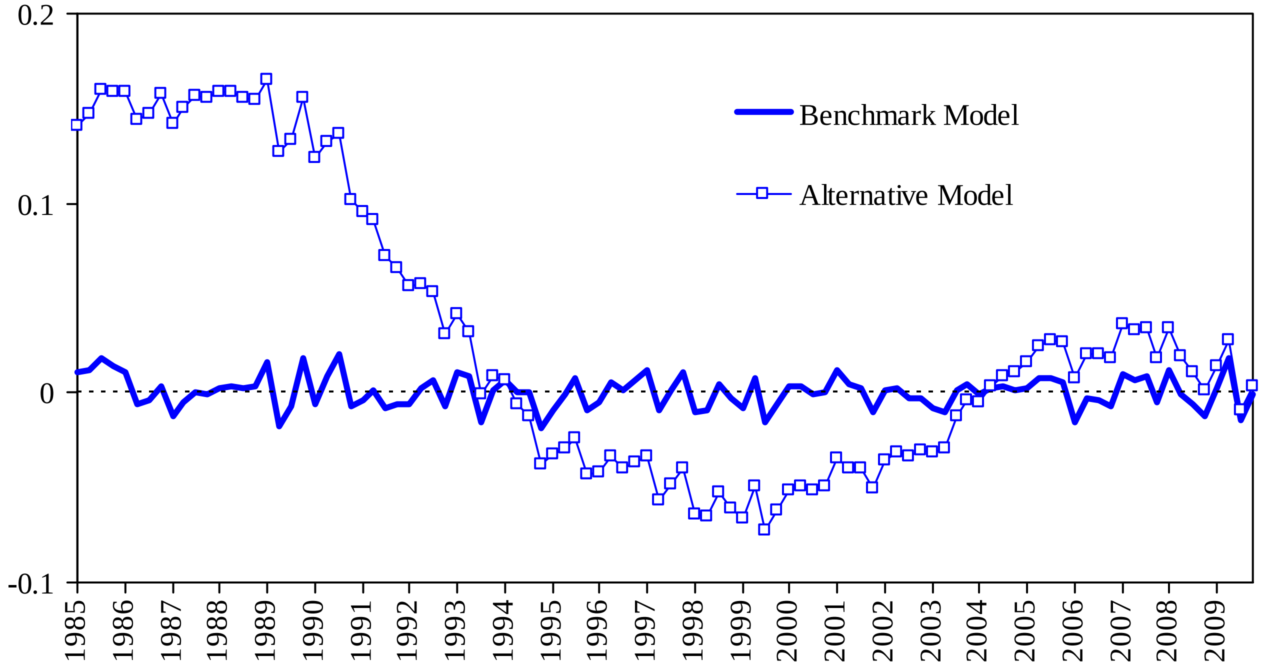


Note: Alternative model corresponds to a model in which we assume that both the economy-wide technology shock and the investment-specific technology shock in level follow a first-order autoregressive process. Shaded regions show the ESRI recession dates.

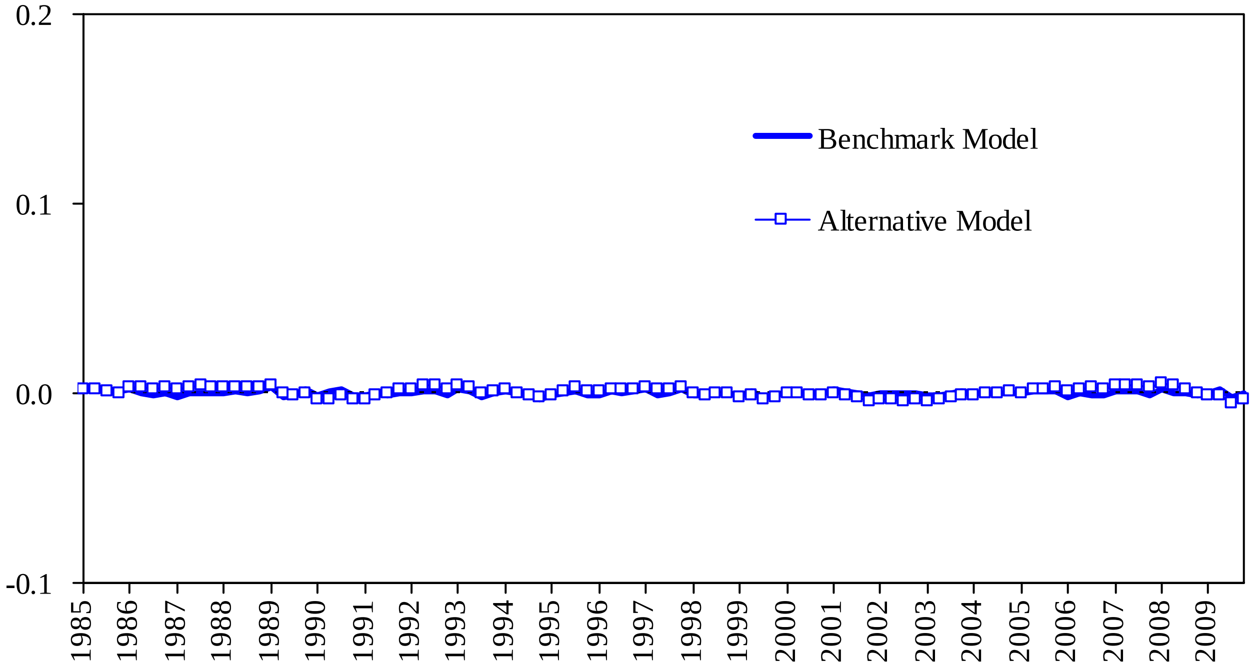
Figure 11-2: Estimated Shocks

(alternative model with AR(1) technology process in level)

Economy-Wide Technology Shock (in level)



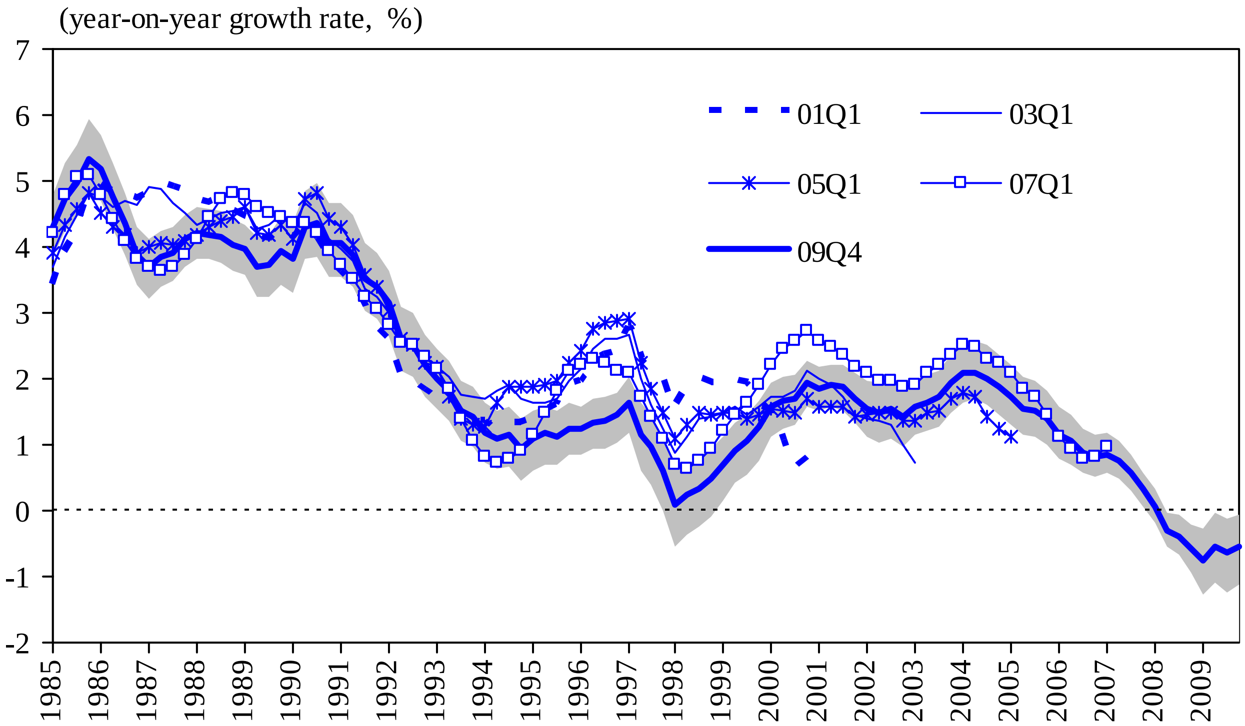
Investment-Specific Technology Shock (in level)



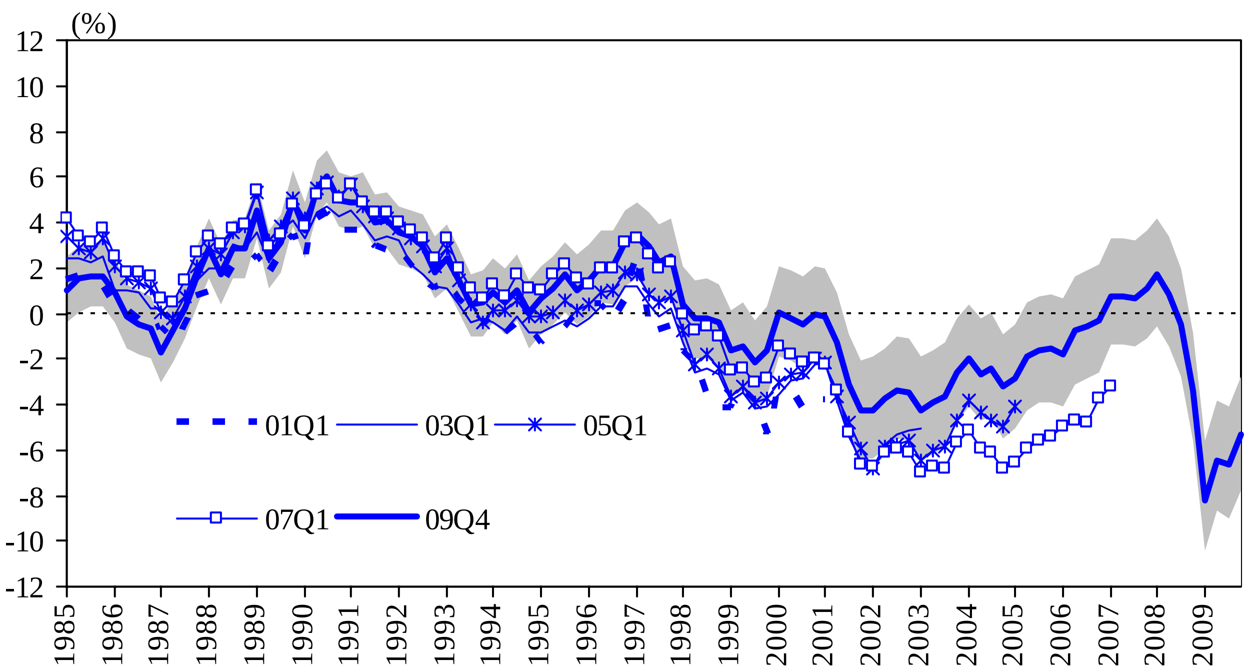
Note: Alternative model corresponds to a model in which we assume that both the economy-wide technology shock and the investment-specific technology shock in level follow a first-order autoregressive process.

Figure 12: Potential Growth and Output Gap: Real Time Estimations of Our Measures

Potential Growth



Output Gap



Note: Shaded regions are the 90% posterior intervals of the latest estimates of potential output growth and output gap.

Figure 13: Potential Growth and Output Gap: Real Time Estimations by HP-filter