Bank of Japan Working Paper Series

The Quarterly Japanese Economic Model (Q-JEM): 2011 Version

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No.11-E-11
November 2011

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The Quarterly Japanese Economic Model (Q-JEM): 2011 Version*

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November 2011

Abstract

This paper provides a brief explanation and a detailed documentation of the current version of the *Quarterly Japanese Economic Model* (Q-JEM), which has been developed and constantly updated since the mid-2000s at Research and Statistics Department, Bank of Japan. Q-JEM is a large-scale hybrid-type macroeconomic model that pursues both long-run theoretical coherence and short-run empirical validity, as is the Federal Reserve Board's FRB/US model. The model captures various aspects of Japan's economy, including inflation dynamics, the zero lower bound on nominal interest rates, linkages with overseas economies, and the effects of population aging and declining potential economic growth.

Keywords: Macroeconomic model; Japan's economy; Inflation; Monetary policy; Error-correction mechanism
JEL classification: E17; E37

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*This paper is a revised version of "A Hybrid-type Model of Japan's Economy: the Quarterly Japanese Economic Model (Q-JEM)" authored by Hibiki Ichiue, Tomiyuki Kitamura, Satoko Kojima, Toyoichiro Shirota, Koji Nakamura, and Naoko Hara, which was released in July 2009 (in Japanese only) as Bank of Japan Working Paper Series 09–J–6. Takemasa Oda, Masashi Saito and Ikuo Takei have also contributed to the updates of Q-JEM. Saki Inoue and Naoko Ozaki provided excellent assistance with manuscript and data preparation. The authors are grateful to John Roberts (Federal Reserve Board) for his kind support and advice since the early stages of the development of Q-JEM. They would also like to acknowledge helpful comments from Kosuke Aoki, Etsuro Shioji, and the staff at the Bank of Japan. Views expressed in this paper are those of the authors and do not reflect those of the Bank of Japan.

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1 Introduction

Macroeconomic models have been increasingly used in many central banks because they are efficient and transparent tools for economic projection, risk assessment, and policy simulations. In recent years, the Bank of Japan has constructed various types of macroeconomic models to refine its "suite of models." This suite covers a broad range of models including rigorously theory-based dynamic stochastic general equilibrium (DSGE) models and statistical models. Among them, this paper provides a brief explanation and a detailed documentation of the current version of the *Quarterly Japanese Economic Model* (Q-JEM), which has been developed and constantly updated since the mid-2000s at Research and Statistics Department.[1] Q-JEM is a large-scale macroeconomic model that pursues both long-run theoretical coherence and short-run empirical validity. The model is mainly used for short- to medium-term projections, macroeconomic risk assessment, and various scenario analyses. The output from Q-JEM is reported to the members of the policy board and some analyses are published in the *Outlook for Economic Activity and Prices* (Outlook Report).[2]

Traditional (Keynesian-type) large-scale macroeconomic models, including those previously used at the Bank of Japan,[3] have been criticized for their heavy dependence on observed relationships among economic variables in the data without theoretical micro-foundations. This is problematic especially when the model is used for policy analyses, because policy changes may affect the observed relationships themselves through private agents' behavior and expectations (the Lucas critique). In response to this criticism, the development of macroeconomic models has shifted toward more theory-based

[1]Earlier versions of Q-JEM were explained in Hara et al. (2009) and Ichiue et al. (2009).

[2]In the Outlook Report, the policy board members' judgmental forecasts and risk balance charts for real GDP and inflation rates are presented. The report also provides extensive background explanations on the outlook, including analyses using macroeconomic models. (For example, in the April 2011 report, Chart 47(3) was based on Q-JEM.) An explanation of how macroeconomic models are used in the preparation of the Outlook Report is provided by Hara et al. (2009).

[3]An early developed macroeconomic model at the Bank of Japan was mentioned in its *Monthly Research Bulletin* of September 1972 (in Japanese only).

models that explicitly consider private agents' behavior and expectations. Nowadays, many central banks are developing and practically using DSGE models in which short-run dynamics as well as long-run relationships are rigorously formulated based on microeconomic theory such as households' and firms' optimization problems. In the Bank of Japan, a large-scale calibrated DSGE model named the *Japanese Economic Model* (JEM) was developed in 2004 and then several medium-scale estimated DSGE models, including the *Medium-scale Japanese Economic Model* (M-JEM), have been developed and used for various kinds of analysis.[4]

Although these DSGE models are useful and promising for future development, there are some drawbacks at this stage to their practical use. First, it is not easy to expand these models to take into account a large number of practically important variables such as foreign variables and detailed breakdown of spending, income, and prices, in a rigorous manner that is consistent with economic theory and empirical methodology. Second, some features that are particularly relevant to Japan's economy, such as the zero lower bound on nominal interest rates and financial frictions, cannot necessarily be dealt with easily in contemporary DSGE models. Third, related to the above two drawbacks, the fitness to the data and forecasting accuracy of these models, especially in the short run, are not necessarily satisfactory for practical use.

We developed Q-JEM and added it to our suite of models in order to overcome the above drawbacks of contemporary DSGE models as well as the problems in traditional large-scale macroeconomic models. Such a hybrid-type macroeconomic model that pursues both theoretical coherence and empirical validity has been used at the Federal Reserve Board since the late 1990s: the FRB/US model,[5] which has been a key reference for the development of Q-JEM.

[4]Fujiwara et al. (2005) provide the documentation of JEM. Fueki et al. (2010) provide the documentation of a recent version of M-JEM and shows the estimation results of potential growth and output gap based on the model. Other medium-scale estimated DSGE models developed by the staff of the Bank of Japan include Sugo and Ueda (2008) and Ichiue, Kurozumi, and Sunakawa (2008).

[5]The basic references for the FRB/US model are Brayton and Tinsley (1996) and Brayton et al. (1997).

Q-JEM has the following three major features. First, it is a large-scale model that contains a large number of equations estimated using quarterly data to capture key aspects of short- to medium-term macroeconomic developments in Japan. Including many variables in a model helps to make it not only more realistic but also more useful for policymakers, because they actually use various types of information for making their own judgmental projections, deciding on policies, and explaining their decisions.[6] The number of variables and equations in Q-JEM can be flexibly increased or decreased, since most of the equations are estimated individually.

The second main feature of Q-JEM is that it explicitly models private agents' expectations of future macroeconomic conditions. Following the FRB/US model, Q-JEM assumes that private agents share a condensed description of the aggregate economy represented by a vector auto-regression (VAR) model of a small number of key variables, estimated using historical and survey data, from which private agents derive their expectations (VAR expectations).[7] Although those VAR expectations are not necessarily consistent with the whole structure of Q-JEM, they can be viewed as bounded-rational expectations under a limited scope of information available to private agents.[8] Based on the VAR expectations, several expected variables are calculated and incorporated into many equations in Q-JEM, including the equations on financial variables, inflation dynamics, and private agents' behavior. As for the expected policy interest rate and long-term interest rates, Q-JEM explicitly considers the term structure, the term premium, the credit spread, and the zero lower bound, under the assumption that private agents correctly understand the specification of the monetary policy rule. Therefore, the model captures the effects of monetary policy

[6]On the other hand, Q-JEM does not explicitly use some variables that may be important in the long run but are not necessarily tractable in the short run due to measurement problems (such as capital stock) or structural changes (such as money stock).

[7]The VAR model in the system of VAR expectations, which is assumed to represent an average-history summary of the dynamic behavior of the economy, is sometimes called Historical VAR (H-VAR), though our VAR model is estimated using survey data on inflation forecasts as well as historical data.

[8]In the FRB/US model, the assumption of "full-model expectations" is also available as an alternative option for simulation purposes.

through changes in private agents' expectations in a more systematic way than traditional large-scale macroeconomic models.

Lastly, many equations in Q-JEM take the form of error-correction specifications, in which long-run equilibrium relationships ("core" equations) and short-run dynamics ("non-core" equations) are specified. In equations representing private agents' behavior, such as consumption and investment, the long-run equilibrium determines the theoretically desired level of spending, while the short-run dynamics capture the adjustment process represented by the error-correction terms and may also contain various explanatory variables for improving the model's fit to the data. It is such an error-correction specification that gives Q-JEM its "hybrid" feature pursuing both theoretical coherence and empirical validity. This characteristic is in contrast to DSGE models that derive the whole structure, including short-run dynamics, explicitly from the optimization problems. Moreover, the long-run equilibrium in the error-correction specification also captures the long-run statistical properties of Japanese data. In particular, the equations for the GDP components in Q-JEM are specified so that their shares in nominal GDP are stable, or follow stable trends that reflect population aging, in the long run. This enhances the stability and plausibility of the model's empirical properties and medium- to long-term forecast accuracy.

The remainder of the paper is organized as follows. Section 2 provides a brief overview of the overall structure of Q-JEM. Section 3 shows the estimated dynamic properties of core inflation, focusing on the role of inflation expectations and a comparison of inflation dynamics between Japan and the U.S. Section 4 describes the monetary policy rule and the whole system of VAR expectations in Q-JEM, focusing on the term structure and the zero lower bound on nominal interest rates. Section 5 explains the error-correction mechanisms formulated in Q-JEM, using as examples the private consumption and non-residential investment functions. Section 6 concludes. In each section from 3 to 5, we conduct some simulations of the impulse response in order to examine quantitatively the estimated dynamics in Q-JEM. In Appendix I, we explain the details of the estimation procedure of inflation dynamics. Appendix II provides a detailed documentation of all estimated equations of Q-JEM together with a comprehensive list of variables.

2 Overview of Q-JEM

Q-JEM is a large-scale model with some 200 equations, among which about 70 are estimated.[9] The actual equations and identities are shown in Appendix II. In this section, we provide a brief overview of the model structure, a graphical representation of which is provided in Figures 1 and 2.

As in modern New Keynesian DSGE models, three key variables play a central role in Q-JEM: inflation, the output gap (GDP gap), and the policy interest rate (shown at the top of Figure 1). Inflation dynamics are based on the New Keynesian Phillips curve, in which the GDP gap affects inflation. The policy interest rate follows a rule responding to inflation and the GDP gap.

Private agents' expectations of future values of these key variables are crucial for their decisions on current spending such as consumption and investment as well as for the inflation dynamics themselves. As mentioned in Section 1, Q-JEM assumes that private agents form their expectations from a small VAR model of the three key variables. The VAR model assumes that all three variables converge to the corresponding long-run variables: trend inflation, the long-run GDP gap (which is assumed to be zero), and the equilibrium interest rate. Based on the VAR expectations, financial variables that affect real economic activity, such as (real) long-term interest rates and exchange rates, are calculated. Expectations of some non-financial variables such as households' long-term (permanent) income and the future unemployment rate are also calculated based on the VAR expectations of the three key variables.

Q-JEM contains estimated equations for the main GDP components and their deflators, including private consumption, residential and non-residential investment, and public spending (bottom center of Figure 1). Although the equations for consumption and investment spending are not explicitly

[9] When those used for computational and miscellaneous purposes are added to the count, there are approximately 1,000 equations in Q-JEM.

derived from the optimization problems of households and firms, they are represented by error-correction specifications in which long-run equilibrium relationships or "core" equations determine the desired level of spending based on standard economic theory. To explain private spending, Q-JEM also contains estimated equations for various components of households' income and corporate profits. Meanwhile, public spending is assumed to follow a simple fiscal policy rule responding to the GDP gap. Real GDP in Q-JEM is calculated as a divisia index that approximates the chain-weighted aggregate of these GDP components.

Supply-side variables, such as potential GDP, are given exogenously (right side of Figure 1). Potential GDP is estimated using the production function approach outside Q-JEM together with the potential labor input.[10] In the model, potential GDP growth affects consumption and investment demand through the long-run expectations of income, stock prices, and interest rates. Q-JEM also takes account of demographic factors that affect household spending and saving behavior as well as the potential labor input. The GDP gap is then calculated as the deviation of real GDP from potential GDP.

Foreign variables and import prices are also important factors that affect domestic activity and prices (left side of Figure 1). Q-JEM divides overseas economies into the U.S. economy and the "non-U.S. economy" (shown in Figure 2). The model contains estimated equations for the three key variables for each economy. Japan's economy is treated as a small open economy in the sense that it does not affect either the U.S. or the non-U.S. economy. The two economies are aggregated into the "overseas economy" and weighted by Japan's exports to each economy. The model also contains estimated equations for the oil price, Japan's import prices, and the exchange rate between the Japanese yen and the U.S. dollar.

[10] An explanation of the estimation of potential GDP using the production function approach is provided by Hara et al. (2006).

3 Inflation Dynamics

This section explains the estimated dynamic properties of core inflation, defined as the change in the consumer price index (CPI) excluding food and energy,[11] in Q-JEM. The estimated equations for other components of the CPI and other price indices are shown in Appendix II. (The equations that we explain in the text below are also shown in Appendix II, although the equation numbers and the names of variables here are different from those in Appendix II.[12])

3.1 Specification and estimation

Core inflation depends on the GDP gap as well as on lagged and expected inflation, following the "hybrid" New Keynesian Philips curve:

$$\pi_t = \gamma y_t + \rho \pi_{t-1} + (1-\rho)\pi^e_{t,1} + w_t, \quad 0 < \rho < 1, \tag{1}$$

where π_t is the rate of core inflation in period t, y_t is the GDP gap in t, and $\pi^e_{t,n}$ is the n-period ahead expectation of core inflation in t. (The latter may be rewritten as $E_t\pi_{t+n}$ where E_t is the expectation operator conditional on information as of t.) γ and ρ are parameters to be estimated and w_t is the error term that is assumed to follow an i.i.d. normal process.

The above core inflation equation is estimated simultaneously with the expected and trend inflation rates using a Kalman filter algorithm with maximum likelihood estimation. Inflation expectations are formed in accordance with the actual inflation dynamics as follows:

$$\pi^e_{t,n} = \alpha y^e_{t,n} + \beta \pi^e_{t,n-1} + (1-\beta)\overline{\pi}_t, \quad 0 < \beta < 1, \tag{2}$$

where $y^e_{t,n}$ is the n-period ahead expectation of the GDP gap in t and $\overline{\pi}_t$ is the time-varying trend inflation rate in t.[13] Q-JEM assumes that the expected

[11] "High-school fees" (for which substantial subsidies were introduced in 2010) and the effects of changes in the consumption tax are also excluded. Alcoholic beverages are included while food is excluded.

[12] Equations (1), (2), (3), and (4) below correspond to equations (J.36), (A.2.1), (A.2.3), and (A.2.2), respectively, in Appendix II.

[13] Cogley and Sbordone (2008), among others, derive and estimate the New Keynesian Philips curve that incorporates a time-varying trend inflation.

GDP gap is generated by the following AR(2) process:

$$y^e_{t,n} = \phi_1 y^e_{t,n-1} + \phi_2 y^e_{t,n-2}, \tag{3}$$

and that trend inflation follows a random walk process:

$$\overline{\pi}_t = \overline{\pi}_{t-1} + v_t, \tag{4}$$

where v_t is an i.i.d. normal error term. Equations (1), (2), and (4) are estimated simultaneously using survey data on inflation forecasts as well as historical data on inflation and the GDP gap. The survey data we use are economists' average forecasts with horizons of one, two, three, four, five, and the average of six to ten years.[14] We estimate the above equations so that the expected inflation rates for the corresponding horizons calculated from the model can be matched with those from the survey data. Details of the estimation procedures are provided in Appendix I.

3.2 Trend inflation and expected inflation

The estimated trend inflation rate and the expected inflation rates for one, two, and five years ahead, together with the actual core inflation rate (year-on-year) are shown in the upper panel of Figure 3. While actual inflation has been negative since the late 1990s, trend inflation has remained around one percent since the mid-1990s. One-year ahead expected inflation has moved almost in parallel with, but slightly above, the actual inflation, and five-year ahead expected inflation has moved closely with trend inflation. For any horizon, expected inflation has been consistently above actual inflation since the mid-1990s.[15] These "anchored" inflation expectations have prevented actual inflation from spiraling downward in the presence of the persistent negative GDP gap shown in the lower panel of Figure 3.

[14] The survey data we use are published by Consensus Economics Inc. An explanation of the data is provided at the company's website (www.consensuseconomics.com/what_are_consensus_forecasts.htm).

[15] This is largely due to the use of the survey data in the estimation. Without the survey data included in the estimation, expected inflation moves closely with actual inflation. The long-term expected inflation rate implied by market data (10-year break-even inflation rate) has also been lower than the corresponding survey data.

Q-JEM contains exactly the same specification of the dynamics of core inflation consisting of equations (1) through (4) for the U.S. economy as for Japan's economy.[16] Figure 4 shows the estimated trend inflation rate and the expected inflation rates for the U.S. economy, corresponding to Figure 3 for Japan's economy. The trend inflation rate has steadily declined along with actual inflation, and the expected inflation rates, even for one-year ahead, have moved closely with trend inflation, except after 2008 when the GDP gap fell into deep negative territory. We can see the differences in inflation dynamics between the two countries, especially with regards to the role of inflation expectations, by comparing the estimated parameter values.[17] While the estimated ρ's in the actual inflation equation (1) are similar for the two countries (0.355 for Japan and 0.403 for the U.S.), the estimated β in the expected inflation equation (2) for the U.S. (0.106) is far smaller than that for Japan (0.724). This means that the inflation expectations in the U.S. converge more quickly to trend inflation than in Japan.[18]

3.3 Simulations

To examine quantitatively the estimated inflation dynamics in Q-JEM, we conduct simulations of the impulse response. Here, we particularly focus on the role of inflation expectations and compare inflation dynamics between Japan and the U.S.

Figure 5 shows the response of core inflation and the GDP gap in Japan to a 10-percent negative shock to the U.S. GDP gap.[19] All responses are ex-

[16]For the non-U.S. economy, trend inflation is calculated as a stochastic trend because survey data on inflation forecasts are not available.

[17]Equations (1) and (2) for the U.S. economy correspond to equations (D.16) and (D.22.1), respectively, in Appendix II.

[18]Fuhrer, Olivei, and Tootell (2010) discuss the difference between Japan and the U.S. inflation dynamics, focusing on the role of inflation expectations. Cogley, Primiceri, and Sargent (2010) show that the persistence of the "inflation gap," defined as the difference between actual and trend inflation, decreased after the Volcker disinflation and that non-monetary shocks (markup shocks) have also contributed to the decrease in the persistence of the inflation gap in the U.S.

[19]For illustrative purposes, we assume in these simulations that the domestic fiscal policy rule, the non-U.S. monetary policy rule, and the nominal exchange rate between the Japanese yen and currencies other than the U.S. dollar are fixed. (The same applies

pressed as percentage deviations from the baseline. Japan's GDP gap (lower panel) falls more than 3 percent below the baseline around one year (4 quarters) after the shock, mainly due to the fall in exports, and then returns to the baseline around 10 quarters after the shock. Core inflation (upper panel) falls more gradually and persistently until around two years (8 quarters) after the shock. With the estimated benchmark parameter values, the peak deviation from the baseline is around -0.8 percent, as shown by the bold line. For comparison, we also examine the case when the parameter values on actual and expected inflation (equations (1) and (2)) for Japan are replaced by those estimated for the U.S. while keeping all other parameters unchanged. The peak deviation under this alternative parameterization, as shown by the thin line, is almost half of that under the benchmark parameterization. This result shows that the difference in the dynamics of inflation expectations significantly affects actual inflation dynamics. Meanwhile, the persistence in the response of actual inflation does not differ greatly between the benchmark and the alternative case, since the estimated ρ's in equation (1) do not differ greatly between the two countries.

4 Monetary Policy Rule

This section explains the monetary policy rule and the whole system of VAR expectations in Q-JEM. The model explicitly takes into account the zero lower bound and the term structure of nominal interest rates.

4.1 Specification and estimation

The monetary policy rule is specified as the following Taylor rule with interest-rate smoothing:[20]

$$i_t = \max\{\theta i_{t-1} + (1-\theta)[\,\bar{i}_t + 0.5 y_t + 1.5(\pi_t - \overline{\pi}_t)\,],\, 0\}, \qquad (5)$$

to the simulations in Section 4.3).

[20] Equations (5), (6), and (7) below correspond to equations (A.1.4), (A.1.1), and (A.1.2), respectively, in Appendix II.

where i_t is the policy interest rate (the overnight call rate) and $\bar{i}_t$ is its "equilibrium" value. The smoothing parameter θ is estimated using the data up to 1995/Q4, since which period the policy interest rate has been 0.5 percent or less and assumed to be affected by the zero lower bound. The parameter values on the reaction to the GDP gap and the inflation gap are set to 0.5 and 1.5, respectively, following Taylor (1993).[21] The max function is used to take into account the zero lower bound.

The equilibrium policy interest rate is estimated as follows. First the *real* policy interest rate deflated by core inflation is regressed on potential GDP growth:

$$i_t - \pi_t = \lambda_1 + \lambda_2 \log(\overline{Y}_t/\overline{Y}_{t-4}) + e_t, \tag{6}$$

where $\overline{Y}_t$ is potential GDP and e_t is an i.i.d. normal error term. Using the estimated parameters λ_1 and λ_2 and replacing actual inflation with trend inflation, we define the *equilibrium* policy interest rate, $\bar{i}_t$, as follows:

$$\bar{i}_t = \lambda_1 + \lambda_2 \log(\overline{Y}_t/\overline{Y}_{t-4}) + \overline{\pi}_t. \tag{7}$$

Figure 6 shows the actual and equilibrium policy interest rates for Japan, the U.S., and the non-U.S. economy.[22] The equilibrium rate for Japan declined gradually during the 1990s, together with the estimated trend inflation rate (shown in Figure 3) and potential GDP growth, but has remained positive and consistently above the actual policy interest rate since the 1990s. The equilibrium policy rates for the U.S. and the non-U.S. economy have also been above the actual policy rates in recent years, implying that the monetary policy stances have been accommodative.

4.2 VAR expectations

As mentioned in Section 1, Q-JEM assumes that private agents share a condensed description of the aggregate economy represented by a vector auto-

[21] We calibrate these parameters because the estimated results vary depending on the sample period.

[22] Equations (5) and (7) for the U.S. economy correspond to equations (D.21.4) and (D.21.2), while those for the non-U.S. economy correspond to equations (D.24.4) and (D.24.2), respectively, in Appendix II.

regression (VAR) model of the three key variables: (core) inflation, the GDP gap, and the policy interest rate. Expected inflation and the expected GDP gap are specified in equations (2) and (3), respectively. The expected policy interest rate is formed in accordance with the actual policy interest rate, following equation (5):[23]

$$i^e_{t,n} = \max\{\theta i^e_{t,n-1} + (1-\theta)[\,\bar{i}_t + 0.5 y^e_{t,n} + 1.5(\pi^e_{t,n} - \overline{\pi}_t)\,],\, 0\}, \tag{8}$$

where $i^e_{t,n}$ is the n-period ahead expectation of the policy interest rate in period t. This specification of the expected policy interest rate implies that private agents correctly understand the specification of the monetary policy rule. By summarizing equations (2), (3), and (8), the whole system of VAR expectations can be expressed as follows:[24]

$$\begin{pmatrix} 1 & -1.5(1-\theta) & -0.5(1-\theta) & 0 \\ 0 & 1 & -\alpha & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} i^e_{t,n} - \bar{i}_t \\ \pi^e_{t,n} - \overline{\pi}_t \\ y^e_{t,n} \\ y^e_{t,n-1} \end{pmatrix} = \begin{pmatrix} \theta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} i^e_{t,n-1} - \bar{i}_t \\ \pi^e_{t,n-1} - \overline{\pi}_t \\ y^e_{t,n-1} \\ y^e_{t,n-2} \end{pmatrix}$$

$$\begin{pmatrix} i^e_{t,n} - \bar{i}_t \\ \pi^e_{t,n} - \overline{\pi}_t \\ y^e_{t,n} \\ y^e_{t,n-1} \end{pmatrix} = A \begin{pmatrix} i^e_{t,n-1} - \bar{i}_t \\ \pi^e_{t,n-1} - \overline{\pi}_t \\ y^e_{t,n-1} \\ y^e_{t,n-2} \end{pmatrix} \tag{9}$$

[23] Equations (8), (9), (10), and (11) below correspond to equations (A.2.4), (A.2.6), (A.3.1), and (A.3.2), respectively, in Appendix II.

[24] While the system of VAR expectations in Q-JEM incorporates some structures in accordance with the actual dynamics, the system of VAR expectations in the FRB/US model is represented by a reduced-form VAR.

where

$$A \equiv \begin{pmatrix} \theta & 1.5(1-\theta)\beta & 1.5(1-\theta)\alpha\phi_1 + 0.5(1-\theta)\phi_1 & 1.5(1-\theta)\alpha\phi_2 + 0.5(1-\theta)\phi_2 \\ 0 & \beta & \alpha\phi_1 & \alpha\phi_2 \\ 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

When $n \leq 0$, $i^e_{t,n} = i_{t+n}$, $\pi^e_{t,n} = \pi_{t+n}$, and $y^e_{t,n} = y_{t+n}$. Beginning with the realized values i_t, π_t, y_t, and y_{t-1}, we can calculate the n-period ahead expectations of the three key variables by iterating equation (9). Private agents expect that the future policy interest rate will converge to the current equilibrium policy interest rate, $\bar{i}_t$, that future inflation will converge to the current trend inflation rate, $\bar{\pi}_t$, and that the future GDP gap will converge to zero.

Using this system of VAR expectations, based on expectations theory of the term structure, we can calculate the long-term (10-year) expected policy interest rate, i^z_t, as follows:

$$i^z_t = \frac{1}{40}\sum_{n=0}^{39} \max\left[\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} A^n \begin{pmatrix} i^e_{t,n} - \bar{i}_t \\ \pi^e_{t,n} - \bar{\pi}_t \\ y^e_{t,n} \\ y^e_{t,n-1} \end{pmatrix} + \bar{i}_t,\, 0 \right] \tag{10}$$

Similarly, long-term expected core inflation, π^z_t, is defined as follows:

$$\pi^z_t = \frac{1}{40}\sum_{n=0}^{39} \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} A^n \begin{pmatrix} i^e_{t,n} - \bar{i}_t \\ \pi^e_{t,n} - \bar{\pi}_t \\ y^e_{t,n} \\ y^e_{t,n-1} \end{pmatrix} + \bar{\pi}_t \tag{11}$$

Using these, together with the estimated term premium and the credit spread, we calculate real long-term interest rates, which then affect consumption and investment, as will be explained in Section 5.

4.3 Simulations

We now continue the impulse-response simulations started in Section 3.3, to consider the effects of the zero lower bound (ZLB) on nominal interest rates.

In Figure 7, just as in Figure 5, the upper panel shows the response of core inflation in Japan to a 10-percent negative shock to the U.S. GDP gap. In the benchmark case shown by the bold line, which is exactly the same as the benchmark case in Figure 5, we assume that the ZLB is never binding even after the negative shock. Here, we examine two alternative cases taking account of the ZLB. The thin line corresponds to the case in which Japan's policy interest rate is zero when the shock occurs (Alternative Case 1).[25] Since the monetary authority cannot cut the policy rate in response to the negative shock, the decrease in the long-term interest rate (lower panel) is smaller than in the benchmark case and, accordingly, the GDP gap (not shown in the figure) and inflation decline more than in the benchmark case. Moreover, the broken line corresponds to the case in which the policy rates are zero both in the U.S. as well as in Japan when the shock occurs (Alternative Case 2). Since the U.S. GDP gap falls more than in the other cases, Japan's GDP gap and inflation decrease further than in Alternative Case 1. Meanwhile, the long-term interest rate is lower than in Alternative Case 1 because the exit from the ZLB and the following increase in the policy interest rate are expected to be slower. These results are consistent with previous studies showing that the propagation of foreign shocks is larger than usual in the case of the ZLB.[26]

5 Consumption and Investment

This section explains the error-correction mechanisms formulated in Q-JEM, using the private consumption and non-residential investment functions as examples. Although these functions are not explicitly derived from the optimization problems of households and firms, they are assumed to converge to long-run equilibria or "core" equations that determine the desired level of spending based on standard economic theory, such as the permanent income

[25]The simulation assumes that the shock occurs in 2011/Q2. In Alternative Case 1, however, the U.S. policy interest rate is assumed to be never binding the ZLB.

[26]For instance, Bodenstein, Erceg, and Guerrieri (2009) show the above result using a multi-region DSGE model.

and life-cycle theories of consumption and the user-cost theory of investment. Meanwhile, the short-run dynamics or "non-core" equations capture the adjustment process represented by the error-correction terms and may also contain various explanatory variables to improve the model's fit to the data.

The core equations in Q-JEM also capture the long-run properties of Japanese data. In particular, the model assumes that the shares of the main GDP components in *nominal* GDP are stable, or follow stable trends that reflect population aging, in the long run. Figure 8 shows the nominal shares of private consumption, residential investment, and non-residential investment in GDP. Reflecting the steady trend of population aging, the share of consumption has trended upward since 1990, while the share of residential investment has gradually trended downward. Meanwhile, the share of the sum of these two components seems to be stationary. As for private non-residential investment (lower panel), its nominal share also seems stationary, except during the "bubble" period in the late 1980s. While these nominal shares are stationary, the non-residential investment deflator shows a downward trend relative to the consumption deflator, potentially reflecting investment-specific technological progress.[27] Under the assumption of the stationary nominal shares, the model can capture such a trend in relative prices and a difference in technological progress among different sectors.

5.1 Private consumption

The basic idea underlying the desired level of private consumption in Q-JEM is the permanent income and life-cycle theories.

A simple optimization problem of infinitely-lived households is formulated as follows:

$$\max_{\{C_{t+j}\}} \sum_{j=0}^{\infty} \beta^j u(C_{t+j}) \quad \text{s.t. } W_t = (1+r_t)(W_{t-1} - C_t) + YW_t,$$

[27] Many recent DSGE models consider the investment-specific technological change, following Greenwood, Hercowitz, and Krusell (1997).

where $u(C_t)$ is the one-period utility function of consumption C_t, β represents the discount factor, W_t stands for the net financial assets at the end of period t, r_t is the real interest rate, and YW_t represents real labor income in t. Under certain conditions, the optimal consumption in t can be expressed as follows:[28]

$$C_t = (1-\beta)\left(\frac{YW_t}{r-g} + W_{t-1}\right),$$

where r is the constant real interest rate and g is the constant growth rate of real labor income. A few additional conditions ensure that the ratio of labor income to net financial assets is stationary in the long run.

We then incorporate life-cycle features into the above infinite-horizon framework. First, we assume that the ratio of the elderly population affects the propensity to consume from labor income and from net financial assets. The higher the ratio of the elderly is, the more consumption depends on net financial assets and the less it depends on labor income. This assumption captures the effects of population aging on household spending and saving behavior.

Second, the labor income that explains consumption is the finite long-term (10 years) expected labor income defined as follows:

$$YW_t^z = \frac{1}{40}\sum_{n=0}^{39}\frac{YW_{t,n}^e}{(1+i_t^L-\pi_t^z)^{n+1}},$$

where YW_t^z is the real long-term expected labor income, i_t^L is the long-term interest rate which includes a term premium in addition to the long-term expected policy interest rate calculated from equation (10), π_t^Z is the long-term expected inflation rate calculated from equation (11), and $YW_{t,n}^e$ is the n-period ahead expectation of real labor income ("compensation of employee" divided by the GDP deflator) in t. The latter is assumed to be formed as

[28]When $u(C_t) \equiv \log C_t$, the optimal consumption in each period is a constant fraction of wealth.

follows:

$$\log(YW^e_{t,n}/YW^e_{t,n-4}) = \log(\overline{Y}_t/\overline{Y}_{t-4}) + \omega\, y^e_{t,n}, \tag{12}$$

where $\overline{Y}_t$ is potential GDP, $y^e_{t,n}$ is the expected GDP gap calculated from equation (3) as a part of the VAR expectations, and ω is the parameter to be estimated.

Third, we assume that consumption to some extent is also explained by temporary income, defined as disposable income minus long-term expected labor income.

Thus, the estimated core equation of private consumption in Q-JEM is specified as follows:[29]

$$\begin{aligned}\log(P^C_t C^Q_t) \quad = \quad & a_0 + (1 - a_1\, EP_t) \log\left\{ \frac{1}{2} \sum_{s=0}^{1} (1 - YT_{t-s}) P_{t-s} YW^z_{t-s} \right\} \\ & + a_1\, EP_t \log(FA_{t-1} - FL_{t-1}) \\ & + a_2 \frac{1}{2} \sum_{s=0}^{1} (YD_{t-s} - YW^z_{t-s})/(YD - YW^z), \end{aligned} \tag{13}$$

where P^C_t is the consumption deflator, C^Q_t is the desired level of real consumption, EP_t is the ratio of the elderly to the working-age population (five-year backward moving average), YT_t is the income tax rate, P_t is the GDP deflator, FA_t represents household financial assets, FL_t stands for household loans, YD_t is the disposable income, $YD - YW^z$ is the historical average of temporary income, and a_0 to a_2 are parameters to be estimated. The restriction that the coefficients on the second and third terms on the right-hand side sum to unity ensures stationarity of the propensity to consume. The two-period backward moving averages of after-tax long-term expected labor income and temporary income are used as explanatory variables because some part of consumption, especially of consumer durables, may depend on past as well as current income.

[29] Equations (13), (14), and (15) below correspond to equations (I.1.1), (I.1.2), and (I.3), while equations (12) and (16) correspond to equations (A.4.1) and (A.5) respectively, in Appendix II.

Meanwhile, the non-core equation for the short-run dynamics of private consumption contains the consumer confidence index, a lagged dependent variable, dummy variables for before and after the consumption tax hikes in 1989 and 1997, and the error-correction term, as follows:

$$\begin{aligned}\log(C_t/C_{t-1}) &= b_0 + b_1 \log(C_{t-1}/C^Q_{t-1}) \\ &\quad + b_2\, CM_t + b_3 \sum_{s=2}^{3} \log(C_{t-s}/C_{t-s-1}) \\ &\quad + b_4\, D891_t + b_5\, D892_t + b_6\, D971_t + b_7\, D972_t, \qquad (14)\end{aligned}$$

where C_t is real consumption, CM_t is the consumer confidence index, $D{*}{*}{*}_t$ are dummy variables, and b_0 to b_7 are parameters to be estimated.

The consumer confidence index is assumed to depend on a lagged dependent variable, the government debt, the expected unemployment rate gap (the deviation of the actual unemployment rate from the "structural" unemployment rate), stock prices, and the dummy variable for the effects of policy measures taken between 2009 and 2010,[30] as follows:

$$\begin{aligned}CM_t &= c_0 + c_1\, CM_{t-1} + c_2 \{(FA_t - GB_t) - (FA_{t-1} - GB_{t-1})\}/P_tY_t \\ &\quad + c_3\, (u^e_{t,2} - u_t) + c_4 \{\log(SP_t/SP_{t-4}) - \log(P_t/P_{t-4})\} \\ &\quad + c_5\, D0910_t, \qquad (15)\end{aligned}$$

where GB_t is the level of government debt, Y_t is real GDP, SP_t is the TOPIX, u_t is the unemployment rate gap, and $u^e_{t,n}$ is the n-period ahead expectation of the unemployment rate gap that is assumed to be formed as follows:

$$u^e_{t,n} = \upsilon_1\, u^e_{t,n-1} + \upsilon_2\, y^e_{t,n}, \qquad (16)$$

where $y^e_{t,n}$ is the expected GDP gap. c_0 to c_5 and υ_1 and υ_2 are parameters to be estimated.

[30] Various policy measures to stimulate durable goods consumption were introduced in 2009 including the "eco-point system" promoting the purchase of household electrical appliances with higher energy efficiency.

5.2 Private non-residential investment

The desired level of private non-residential investment is the level that would be realized without adjustment costs and any other real and financial frictions. In Q-JEM, it is determined based on the neoclassical user-cost theory.

The user cost of capital consists of the price of capital goods, the interest cost, and the depreciation of capital. At the desired level of capital, the marginal user cost must be equal to the marginal product of capital, as follows:

$$\alpha \frac{Y_t}{K_t} = \frac{P_t^K}{P_t} \left(i_t^{LL} - \frac{P_{t+1}^K - P_t^K}{P_t^K} + \delta_t \right), \tag{17}$$

where K_t is the capital stock, α is the capital share (so the marginal product of capital is $\alpha\frac{Y_t}{K_t}$), P_t^K is the price of capital, i_t^{LL} is the long-term loan interest rate, and δ_t is the depreciation rate.

Meanwhile, the capital accumulation takes the following form:

$$K_{t+1} = K_t(1 - \delta_t) + I_t,$$

where I_t is real investment spending. Letting the growth rate of potential GDP be g_t and the growth rate of the ratio of capital to potential GDP be γ_t, the above equation can be rewritten as

$$\frac{I_t}{K_t} = \gamma_t + g_t + \delta_t. \tag{18}$$

Combining equations (17) and (18), we have

$$\frac{P_t^K I_t}{P_t Y_t} = \frac{\alpha(\gamma_t + g_t + \delta_t)}{i_t^{LL} - \frac{P_{t+1}^K - P_t^K}{P_t^K} + \delta_t}.$$

Since K_t is eliminated, the above equation can be estimated without using capital stock data. Assuming that γ_t is constant and that the difference between the capital-goods inflation rate and the core inflation rate is constant, we specify the estimated core equation of private non-residential investment in Q-JEM as follows:[31]

$$\log\left(\frac{P_t^I I_t^Q}{P_t Y_t}\right) = d_0 + d_1\{\log(\overline{Y}_t/\overline{Y}_{t-4}) + \delta_t\} + d_2(i_t^{LL} - \pi_t^z + \delta_t), \quad (19)$$

where P_t^I is the non-residential investment deflator, I_t^Q is the desired level of real non-residential investment, and d_0 to d_2 are parameters to be estimated.

The non-core equation for the short-run dynamics of private non-residential investment, which considers financial frictions and foreign demand shocks as well as the error-correction mechanism, is specified as follows.

$$\begin{aligned}\log(I_t/I_{t-1}) &= e_0 + e_1 \log(I_{t-1}/I_{t-1}^Q) \\ &+ e_2 \log(EX_{t-1}/EX_{t-2}) + e_3\, LA_{t-1} \\ &+ e_4 \{\log(CF_t/CF_{t-1}) - \log(P_t^I/P_{t-1}^I)\} \quad (20)\end{aligned}$$

where EX_t is exports, LA_t is the lending attitude of financial institutions for small non-manufacturing firms (the Tankan diffusion index), CF_t is the corporate (after-tax) cash flow, and e_0 to e_4 are parameters to be estimated.

5.3 Simulations

Lastly, we conduct two more simulations of the impulse response to examine quantitatively the estimated dynamics of private consumption and non-residential investment in Q-JEM.

Figure 9 shows the responses to a tightening shock to the monetary policy rule, in which Japan's policy interest rate increases by 0.5 percent (as shown in panel (1)). The long-term interest rate (panel (2)) increases almost in parallel with the policy interest rate. Since long-term expected labor income is discounted more heavily, the desired level of private consumption is reduced. Moreover, the high long-term interest rate lowers stock prices, which dampens consumer confidence and thereby affects the short-run dynamics of

[31]Equations (19) and (20) below correspond to equations (F.1.1) and (F.1.2), respectively, in Appendix II.

private consumption as well. As a result, private consumption (panel (3)) decreases gradually until around two years (8 quarters) after the shock. The high long-term interest rate also affects both the desired level and the short-run dynamics of private non-residential investment. The heightened user cost of capital reduces the desired level of investment, and lower stock prices lead to a tightening of the lending attitude of financial institutions and affect the short-run dynamics of investment with a one-quarter lag. As a result, private non-residential investment (panel (4)) falls in the quarter after the shock and decreases for about one-and-a-half years. Owing mainly to these decreases in private consumption and non-residential investment, real GDP (panel (5)) decreases gradually until around two years (8 quarters) after the shock.[32] The response of core inflation (panel (6)) is slower than that of real GDP, which starts to fall about half a year (2 quarters) after the shock and then declines for more than two years. The magnitudes of the peak responses of real GDP and inflation to the 0.5 percent rate increase are small: around -0.07 percent and -0.02 percent, respectively.

Next, we examine the effects of a more powerful but not necessarily unrealistic shock. Figure 10 shows the responses to a temporary fall in private agents' *expectations* of Japan's potential economic growth. As shown in panel (1), we assume that the expected potential growth rate falls by one percent and then gradually returns to the baseline three years (12 quarters) after the shock. We incorporate this shock into Q-JEM by replacing potential growth in the equations for private agents' behavior and for stock prices with *expected* potential growth. Meanwhile, *actual* potential GDP growth and other potential variables, such as the equilibrium interest rate, the structural unemployment rate, etc., are not affected by this shock.[33] Since the GDP gap relative to baseline potential GDP, which remains unaffected, falls, core inflation (panel (6)) also falls, leading the monetary authority to cut the policy interest rate. Since the equilibrium interest rate has also remained

[32]The fall in exports, caused by the real appreciation of the yen, also contributes to the decrease in real GDP to a limited extent.

[33]In this sense, the shock we are considering here is similar to a "news shock" that affects the expectation of future technology but does not influence current technology.

unaffected, the long-term interest rate (panel (2)) decreases. Despite this monetary easing, private consumption (panel (3)) and non-residential investment (panel (4)) decrease persistently. Since potential growth in the equation of expected labor income (12) and the core equation of non-residential investment (19) is replaced by *expected* potential growth,[34] the desired levels of consumption and non-residential investment are reduced by the shock. Moreover, the fall in stock prices due to the shock also affect the short-run dynamics of consumption and non-residential investment through its impact on consumer confidence and financial institutions' lending attitude. Owing mainly to these decreases in private consumption and non-residential investment, real GDP (panel (5)) decreases gradually until around two years (8 quarters) after the shock, leading to a delayed and persistent decline in core inflation (panel (6)). The magnitudes of the peak responses of real GDP and inflation to this shock are far larger than those to the monetary policy shock in Figure 9: around -1.5 percent and -0.4 percent, respectively.[35]

6 Concluding Remarks

In this paper, we showed how Q-JEM captures various aspects of Japan's economy, including inflation dynamics, the zero lower bound on nominal interest rates, linkages with overseas economies, and the effects of population aging and declining potential economic growth. As a hybrid-type macroeconomic model for practical use, Q-JEM pursues both long-run theoretical coherence and short-run empirical validity. It also explicitly models private agents' expectations of future macroeconomic conditions.

One important advantage of such a hybrid-type model over rigorously theory-based models (DSGE models) is flexibility. By utilizing this advantage, we will continue to develop and update Q-JEM, reflecting various prac-

[34]Potential growth is also replaced by expected potential growth in the core equation of residential investment.

[35]In this simulation, we assume that the zero lower bound on nominal interest rates is never binding despite the negative shock. If it is binding, real GDP and inflation decrease further because the long-term interest rate does not decrease as much as in the case without the zero lower bound.

tical needs, developments in macroeconomic research, and structural changes in Japan's economy.[36] For instance, it has been increasingly important, especially for simulation purposes, to elaborately model financial frictions, fiscal policy, and linkages with overseas economies. Moreover, there is still much room for strengthening theoretical coherence, in particular the formulation of expectations and links to the supply side, which would further enhance the plausibility of the model's long-run properties.

Meanwhile, we have to keep in mind that there is no single model that can perfectly describe reality. It is important to utilize several different types of models for different purposes, while understanding the advantages and limitations of each model. We will use Q-JEM as one tool in our suite of models together with other types of models.

[36] According to Tetlow and Ironside (2007), a large number of equations of the FRB/US model changed frequently, especially in the years immediately after its inception.

Appendix I: Estimation of core inflation dynamics

The purpose of this appendix is to provide details on the procedure used to estimate core inflation dynamics and on the data used.

As explained in Section 3 in the text, the core inflation dynamics in Q-JEM consist of “hybrid” New Keynesian Philips curves for actual and expected inflation, and time varying trend inflation, which is assumed to be an unobservable state variable and follow a random walk process. The equations in this system are simultaneously estimated using a Kalman filter algorithm with maximum likelihood estimation.

Data used for the estimation include economists’ average forecasts (the consensus forecasts) with horizons of one, two, three, four, five, and the average of six to ten years as well as historical data. The consensus forecasts are used as proxies for expected inflation rates. In the estimation procedure, expected inflation rates are assumed to be observable. We need to adjust for the following discrepancies between the two sets of data in order to use them for the estimation procedure.

- The consensus forecasts consist of a survey of forecasts on the change in the headline consumer price index. On the other hand, inflation expectations in Q-JEM are modeled as the expectations of the change in the consumer price index excluding food and energy.
- The consensus forecasts are on a semi-annual basis, which differs from the frequency of other historical data published quarterly.
- The expected inflation rates in Q-JEM are modeled as quarter-on-quarter changes. On the other hand, the consensus forecasts consist of forecasts of the average rate of inflation for a calendar year.

This appendix is organized as follows. Subsection A presents the specification of core inflation dynamics in Q-JEM. Next, Subsection B shows the observation equations in the state space representation. Subsection C then presents the whole state space representation, while Subsection D provides some details on the estimation procedure.

A. Specification of core inflation dynamics in Q-JEM

We begin by presenting the specification of core inflation dynamics in Q-JEM. First, we restate equations (1) through (4) in the text, which specify actual and expected core CPI inflation, the expected GDP gap, and trend inflation, respectively:

$$\pi_t = \gamma y_t + \rho\pi_{t-1} + (1-\rho)\pi^e_{t,1} + w_t, \tag{Ap.1}$$

$$y^e_{t,n} = \phi_1 y^e_{t,n-1} + \phi_2 y^e_{t,n-2}, \tag{Ap.2}$$

$$\pi^e_{t,n} = \alpha y^e_{t,n} + \beta\pi^e_{t,n-1} + (1-\beta)\bar{\pi}_t, \tag{Ap.3}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + v_t, \tag{Ap.4}$$

where $\pi^e_{t,n} \equiv \pi_{t+n}$ and $y^e_{t,n} \equiv y_{t+n}$ if $n \le 0$. γ, ρ, ϕ_1, ϕ_2, α and β are parameters to be estimated. ϕ_1 and ϕ_2 are estimated in advance using ordinary least squares.

As noted above, the consensus forecasts consist of a survey of forecasts of the headline rate of inflation, while expected inflation in Q-JEM focuses on the core inflation rate. Therefore, it is necessary to adjust for this discrepancy in the estimation procedure. To do so, we assume that the actual dynamics of headline inflation, $\tilde{\pi}_t$, take the same form with the same parameter values as those of core inflation (Ap.1):

$$\tilde{\pi}_t = \gamma y_t + \rho\tilde{\pi}_{t-1} + (1-\rho)\tilde{\pi}^e_{t,1} + \tilde{w}_t. \tag{Ap.5}$$

Moreover, we assume that headline inflation expectations can be described by equation (Ap.3) with the same parameter values:

$$\tilde{\pi}^e_{t,n} = \alpha y^e_{t,n} + \beta\tilde{\pi}^e_{t,n-1} + (1-\beta)\bar{\pi}_t. \tag{Ap.6}$$

These equations are auxiliary equations for the estimation of this system and not a part of Q-JEM. The reason we use these equations is to avoid poor forecasts of the actual inflation rate arising from laying too heavy weight on the survey data.[1]

Since short-term forecasts of headline inflation within several quarters depend on movements of food and energy prices, we do not use economists' consensus forecasts of the immediate future inflation rate for estimation, which is the average rate of inflation for the

[1] In Graeve, Emiris and Wouters (2009), a balanced set of macro-aggregates and bond yields is used to avoid poor macroeconomic forecasts.

current year.

B. Observation equations in the state space representation

There are two types of observation equations in this estimation. The first is the equation on actual inflation, and the other are the equations on the expectations of the headline inflation.

B-1. Derivation of the observation equation for actual inflation

Combining equations (Ap.1) through (Ap.3), the observation equation for actual core inflation can be written as follows:

$$\pi_t = \mathbf{A}_\pi \mathbf{x}_t + \frac{(1-\rho)(1-\beta)}{1-\beta(1-\rho)}\bar{\pi}_t + \frac{1}{1-\beta(1-\rho)} w_t. \qquad \text{(Ap.7)}$$

where $\mathbf{x}_t \equiv [\pi_{t-1}, \tilde{\pi}_t, \tilde{\pi}_{t-1}, y_t, y_{t-1}]'$ and the coefficient vector $\mathbf{A}_\pi$ is

$$\mathbf{A}_\pi \equiv \begin{pmatrix} \frac{\rho}{1-\beta(1-\rho)} & 0 & 0 & \frac{\gamma+(1-\rho)\alpha\phi_1}{1-\beta(1-\rho)} & \frac{\alpha\phi_2(1-\rho)}{1-\beta(1-\rho)} \end{pmatrix}. \qquad \text{(Ap.8)}$$

Using equations (Ap.2) and (Ap.3), one-period ahead expected inflation can be represented as follows:

$$\pi^e_{t,1} = \alpha(\phi_1 y_t + \phi_2 y_{t-1}) + \beta\pi_t + (1-\beta)\bar{\pi}_t.$$

The observation equation for actual headline inflation is similar to the one for actual core inflation and is given by

$$\tilde{\pi}_t = \mathbf{A}_{\tilde{\pi}} \mathbf{x}_t + \frac{(1-\rho)(1-\beta)}{1-\beta(1-\rho)}\bar{\pi}_t + \frac{1}{1-\beta(1-\rho)} \tilde{w}_t. \qquad \text{(Ap.9)}$$

where coefficient vector $\mathbf{A}_{\tilde{\pi}}$ is

$$\mathbf{A}_{\tilde{\pi}} \equiv \begin{pmatrix} 0 & 0 & \frac{\rho}{1-\beta(1-\rho)} & \frac{\gamma+(1-\rho)\alpha\phi_1}{1-\beta(1-\rho)} & \frac{\alpha\phi_2(1-\rho)}{1-\beta(1-\rho)} \end{pmatrix}. \qquad \text{(Ap.10)}$$

B-2. Derivation of the observation equations for expected inflation

For later convenience, we show how the expected rates of inflation are calculated by "VAR expectations".[2]

[2] As explained in Section 4 in the text, the VAR expectation system in Q-JEM contains the monetary

The VAR expectations here consist of equations (Ap.2), (Ap.4), and (Ap.6), which can be written as the following VAR(1) process:

$$\tilde{\xi}_t = \mathbf{Z}\tilde{\xi}_{t-1} + \varepsilon_{z,t} \tag{Ap.11}$$

where $\tilde{\xi}_t \equiv [\tilde{\pi}_t, \tilde{\pi}_{t-1}, y_t, y_{t-1}, \bar{\pi}_t]'$ and

$$\mathbf{Z} \equiv \begin{pmatrix} \beta & 0 & \alpha\phi_1 & \alpha\phi_2 & 1-\beta \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{Ap.12}$$

By iterating equation (Ap.11) *n* times, the *n*-period ahead expectation of headline inflation in the VAR expectations $\tilde{\pi}^e_{t,n}$ can be represented by

$$\tilde{\pi}^e_{t,n} = \mathbf{e}_1 \mathbf{Z}^n \tilde{\xi}_t \tag{Ap.13}$$

where $\mathbf{e}_i$ is the five dimension selector vector, which has one in the *i*-element and zero in the other elements.

In the observation equations for expected inflation, the VAR expectations of headline inflation represented by equation (Ap.13) should be matched to the survey data. As noted above, the survey data are available only in the first and third quarter of each year. To deal with these issues, we employ these observation equations for estimation only for periods for which the survey data are available.[3]

Apart from adjusting for differences in data availability, we need to transform the VAR expectations of inflation to match them to the survey data, since the VAR expectations are for quarter-on-quarter inflation while the survey data are for annual inflation on a calendar year basis. Specifically, we need to calculate the annual, calendar-year-based rate of inflation from the model. Equation (Ap.14) shows the formula to calculate the average annual calendar-year-based rate of inflation and an approximation:[4]

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policy rule. However, since the monetary policy rule does not have an impact on the expectation of core inflation in Q-JEM, we do not include it here.

[3] Specifically,following Harvey (1989) and Kim and Orphanides (2005) we use a selector matrix to select available data. Ichiue and Ueno (2007) use a similar method.

[4] Equation (Ap.14) is derived by log-approximating the year-on-year rate of inflation and using the geometric mean instead of the arithmetic mean.

$$\left[\left\{\left(\tilde{P}_{T+7}+\tilde{P}_{T+6}+\tilde{P}_{T+5}+\tilde{P}_{T+4}\right)/4\right\}/\left\{\left(\tilde{P}_{T+3}+\tilde{P}_{T+2}+\tilde{P}_{T+1}+\tilde{P}_{T}\right)/4\right\}-1\right]\times 100$$
$$\approx\left(\tilde{\pi}_{T+7}+2\tilde{\pi}_{T+6}+3\tilde{\pi}_{T+5}+4\tilde{\pi}_{T+4}+3\tilde{\pi}_{T+3}+2\tilde{\pi}_{T+2}+\tilde{\pi}_{T+1}\right)/16 \quad \text{(Ap.14)}$$

where $\tilde{P}_t$ is the headline price level at time t and $\tilde{\pi}_t \equiv 400\cdot\log\left(\tilde{P}_t/\tilde{P}_{t-1}\right)$ is the annualized headline rate of inflation. We calculate the model-based average calendar-year rate of inflation using this approximation method.

Employing equations (Ap.13) and (Ap.14), we can transform quarter-on-quarter inflation expectations to average annual inflation expectations on a calendar year basis. Since the number of remaining quarters in a calendar year differs depending on what quarter it is, the way the transformation is conducted depends on whether the end period of the historical data is the first or the third quarter.

When the end period of the historical data is the first quarter, the average rate of inflation in the next calendar year can be written as

$$\left(\tilde{\pi}^e_{t,7}+2\tilde{\pi}^e_{t,6}+3\tilde{\pi}^e_{t,5}+4\tilde{\pi}^e_{t,4}+3\tilde{\pi}^e_{t,3}+2\tilde{\pi}^e_{t,2}+\tilde{\pi}^e_{t,1}\right)/16=\mathbf{Z}_{1Q,1}\tilde{\xi}_t$$

where

$$\mathbf{Z}_{1Q,1}\equiv\mathbf{e}_1\left(\mathbf{Z}^7+2\mathbf{Z}^6+3\mathbf{Z}^5+4\mathbf{Z}^4+3\mathbf{Z}^3+2\mathbf{Z}^2+\mathbf{Z}\right)/16.$$

Generally, the average rate of inflation in N calendar years ahead is given by $\mathbf{Z}_{1Q,N}\tilde{\xi}_t$ where

$$\mathbf{Z}_{1Q,N}\equiv\mathbf{e}_1\left(\mathbf{Z}^7+2\mathbf{Z}^6+3\mathbf{Z}^5+4\mathbf{Z}^4+3\mathbf{Z}^3+2\mathbf{Z}^2+\mathbf{Z}\right)\mathbf{Z}^{4(N-1)}/16. \quad \text{(Ap.15)}$$

On the other hand, when the end period of the historical data is the third quarter, the inflation rates for the second and third quarters, ($\tilde{\pi}_{T+1}$ and $\tilde{\pi}_{T+2}$ in equation (Ap.14)) are available and are given by $\mathbf{e}_1\tilde{\xi}_t$ and $\mathbf{e}_2\tilde{\xi}_t$. Therefore, average inflation in the next calendar year can be written as

$$\left(\tilde{\pi}^e_{t,5}+2\tilde{\pi}^e_{t,4}+3\tilde{\pi}^e_{t,3}+4\tilde{\pi}^e_{t,2}+3\tilde{\pi}^e_{t,1}+2\tilde{\pi}_t+\tilde{\pi}_{t-1}\right)/16=\mathbf{Z}_{3Q,1}\tilde{\xi}_t$$

where

$$\mathbf{Z}_{3Q,1}\equiv\left\{\mathbf{e}_1\left(\mathbf{Z}^5+2\mathbf{Z}^4+3\mathbf{Z}^3+4\mathbf{Z}^2+3\mathbf{Z}\right)+2\mathbf{e}_2+\mathbf{e}_1\right\}/16. \quad \text{(Ap.16)}$$

Similarly, the average inflation rate in in N calendar years ahead is given by $\mathbf{Z}_{3Q,N}\tilde{\xi}_t$ where

$$\mathbf{Z}_{3Q,N} \equiv \mathbf{e}_1\left(\mathbf{Z}^7 + 2\mathbf{Z}^6 + 3\mathbf{Z}^5 + 4\mathbf{Z}^4 + 3\mathbf{Z}^3 + 2\mathbf{Z}^2 + \mathbf{Z}\right)\mathbf{Z}^{4(N-1)-2}/16. \tag{Ap.17}$$

Using the above equations and matching the expected rates of inflation in Q-JEM to the survey data, we obtain the observations equations for expected inflation:

$$s_t = \mathbf{Z}_{i_tQ}\tilde{\xi}_t + \mathbf{w}_t \tag{Ap.18}$$

where $s_t = [s_{1t}, s_{2t}, s_{3t}, s_{4t}, s_{5t}, s_{6t}]'$ is the vector of inflation forecasts in the consensus forecasts at time t, which are forecasts for the annual inflation rate in one, two, three, four, five, and the average of six to ten years ahead, and $\mathbf{w}_t = [\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t}, \varepsilon_{5t}, \varepsilon_{6t}]'$ is the vector of observation errors. The coefficient matrix $\mathbf{Z}_{i_tQ}$ is given by

$$\mathbf{Z}_{i_tQ} \equiv \begin{pmatrix} \mathbf{Z}_{i_tQ,1} \\ \mathbf{Z}_{i_tQ,2} \\ \mathbf{Z}_{i_tQ,3} \\ \mathbf{Z}_{i_tQ,4} \\ \mathbf{Z}_{i_tQ,5} \\ 1/5\cdot\left(\mathbf{Z}_{i_tQ,6} + \mathbf{Z}_{i_tQ,7} + \mathbf{Z}_{i_tQ,8} + \mathbf{Z}_{i_tQ,9} + \mathbf{Z}_{i_tQ,10}\right) \end{pmatrix} \tag{Ap.19}$$

where i_t is a variable which specifies the current quarter. For example, if the current period is the third quarter, $i_t = 3$. For later convenience, we define

$$\mathbf{Z}_{2Q} \equiv \mathbf{0}, \quad \mathbf{Z}_{4Q} \equiv \mathbf{0} \tag{Ap.20}$$

and partition the matrix $\mathbf{Z}_{i_tQ}$ as follows:

$$\mathbf{Z}_{i_tQ} = \begin{pmatrix} \underset{6\times 4}{\mathbf{Z}^{\bar{x}}_{i_tQ}} & \underset{6\times 1}{\mathbf{Z}^{\bar{\pi}}_{i_tQ}} \end{pmatrix}. \tag{Ap.21}$$

C. State space representation

Using equations (Ap.4), (Ap.7), (Ap.9), and (Ap.18), core inflation dynamics can be denoted by the following state space representation:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + v_t \quad \text{(transition equation)}$$

$$\underset{8\times1}{\mathbf{S}_t \mathbf{y}_t} = \mathbf{S}_t \left(\underset{8\times5}{\mathbf{A}_t} \underset{5\times1}{\mathbf{x}_t} + \underset{8\times1}{\mathbf{H}_t} \bar{\pi}_t + \underset{8\times1}{\mathbf{w}_t} \right) \quad \text{(observation equation)}$$

$$E\left(v_t^2\right) = \sigma^2, \quad E\left(\mathbf{w}_t \mathbf{w}_\tau^{'}\right) = \begin{cases} \underset{8\times8}{\mathbf{R}} & t = \tau \\ \mathbf{0} & t \neq \tau \end{cases}$$

where $\mathbf{y}_t = [\pi_t, \tilde{\pi}_t, s_{1t}, s_{2t}, s_{3t}, s_{4t}, s_{5t}, s_{6t}]^{'}$ is the vector of observable variables, σ^2 is the variance of the shock to trend inflation, and $\mathbf{R}$ is the variance-covariance matrix, which is assumed to be diagonal. The coefficient matrix depends on time as follows:

$$\mathbf{S}_t = \begin{cases} \underset{8\times8}{\mathbf{I}} & \text{if } t \text{ is Q1 or Q3} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \text{otherwise} \end{cases}$$

$$\underset{8\times5}{\mathbf{A}_t} = \begin{pmatrix} \underset{1\times5}{\mathbf{A}_\pi} \\ \underset{1\times5}{\mathbf{A}_{\tilde{\pi}}} \\ \underset{6\times1}{\mathbf{0}} \quad \underset{6\times4}{\mathbf{Z}^{\bar{x}}_{i_tQ}} \end{pmatrix}, \quad \underset{8\times1}{\mathbf{H}_t} = \begin{pmatrix} \dfrac{(1-\rho)(1-\beta)}{1-\beta(1-\rho)} \\ \dfrac{(1-\rho)(1-\beta)}{1-\beta(1-\rho)} \\ \underset{6\times1}{\mathbf{Z}^{\bar{\pi}}_{i_tQ}} \end{pmatrix}$$

where $\mathbf{Z}^{\bar{x}}_{i_tQ}$ and $\mathbf{Z}^{\bar{\pi}}_{i_tQ}$ depend on α, β, γ and ρ, which are defined in equations (Ap.8), (Ap.10), (Ap.12), (Ap.15) to (Ap.17), and (Ap.19) to (Ap.21).

D. Some details on the maximum likelihood estimation

The number of estimated parameters is thirteen, including α, β, γ, ρ, σ^2 and the elements of diagonal matrix, $\mathbf{R}$. Each parameter is determined to maximize the likelihood via the Kalman filter algorithm. The estimated trend inflation is the smoothed estimator.

The starting value of trend inflation for the Kalman filter is given by the trend component in the second quarter of 1980 of the annualized percentage change in the consumer price index calculated by the Hodrick-Prescott (HP) filter ($\lambda = 1600$). The unconditional variance of the shock to trend inflation is set to be quite large (100%), as proposed by Harvey (1989), since the transition equation is non-stationary.

Appendix II: Documentation of Q-JEM Equations

In this appendix, we provide a detailed documentation of all estimated equations of Q-JEM, preceded by a comprehensive list of variables used.

The list includes all variables that appear in the documentation except the long-run equilibrium level (expressed as "-Q" in the documentation). Note that the names of variables differ from those used in the text.

The following conventions are used in the documentation of equations:

- Values in angle brackets following estimated parameter values indicate the t-value.
- $\mathrm{d}\log(x_t) \equiv \log(x_t / x_{t-1})$ approximates the rate of quarter-on-quarter change in x_t, and $\log(x_t / x_{t-4})$ approximates the rate of year-on-year change in x_t.
- The p-value of the Lagrange Multiplier (LM) test shown under some equations indicates how likely it is that the error term is serially uncorrelated. If the p-value is less than 0.05, we estimate the equation assuming that the error term follows an AR(1) process.

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Variable		Unit	Source, calculation, etc.	Equation
BPNEXI	Nominal income balance	100 million yen	Ministry of Finance, Bank of Japan, "Balance of Payments"	(E.7)
BPNTR	Nominal transfer balance	100 million yen	Ministry of Finance, Bank of Japan, "Balance of Payments"	(E.8)
C	Real private consumption	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(I.1.2)
CA	Current account	100 million yen	Ministry of Finance, Bank of Japan, "Balance of Payments"	(E.9)
CALL	Policy interest rate	% per annum	Uncollateralized overnight call rate (up to 1985/Q2, collateralized overnight call rate)	(A.1.4)
CALLQ	Equilibrium policy interest rate	% per annum	Calculated by equation (A.1.1)	(A.1.2)
CG	Real government consumption	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(C.2)
CGN	Nominal government consumption	Billion yen	Cabinet Office, "National Accounts"	(C.5)
CGPD	Domestic corporate goods price index (all commodities)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.6)
CGPDEV	Domestic corporate goods price index (electric & electronic products, excluding consumption tax)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.14)
CGPDEW	Weight of domestic corporate goods price index (electric & electronic products)	Per 1000	Bank of Japan, "Corporate Goods Price Index"	
CGPDGASOLINE	Domestic corporate goods price index (gasoline)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.10)
CGPDGASOLINEW	Weight of domestic corporate goods price index (gasoline)	Per 1000	Bank of Japan, "Corporate Goods Price Index"	
CGPDOIL	Domestic corporate goods price index (petroleum & coal products)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.8)
CGPDOILV	Domestic corporate goods price index (petroleum & coal products, excluding consumption tax)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.9)
CGPDOILW	Weight of domestic corporate price index (petroleum & coal products)	Per 1000	Bank of Japan, "Corporate Goods Price Index"	
CGPDOILXG	Domestic corporate goods price index (petroleum & coal products, excluding gasoline)	CY2005 average = 100	Calculated by equation (J.8)	(J.11)
CGPDV	Domestic corporate goods price index (all commodities, excluding consumption tax)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.7)
CGPDXO	Domestic corporate goods price index (excluding petroleum & coal products)	CY2005 average = 100	Calculated by equation (J.6)	(J.12)

Variable		Unit	Source, calculation, etc.	Equation
CGPDXOESV	Domestic corporate goods price index (excluding electric & electronic products, petroleum & coal products, consumption tax, and extra charges for summer electricity)	CY2005 average = 100	"Petroleum & coal products excluding consumption tax" and "electric & electronic products excluding consumption tax" are excluded from "all commodities, excluding consumption tax and extra charges for summer electricity."	(J.15)
CGPDXOV	Domestic corporate goods price index (excluding petroleum & coal products and consumption tax)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.13)
CMIND	Consumer confidence index (households more than two-persons, S.A.)	-	Cabinet Office, "Monthly Consumer Confidence Survey"	(I.3)
CN	Nominal private consumption	Billion yen	Cabinet Office, "National Accounts"	(I.2)
CPI	Consumer price index (general, S.A.)	CY2005 average = 100	Seasonally adjusted by X-12-ARIMA	(J.20)
CPIENOR	Consumer price index (energy)	CY2005 average = 100	Calculated using CPIXFOR, CPIFDXFAOR and CPIXFDENOR data	(J.29)
CPIENW	Weight of consumer price index (energy)	-	Ministry of Internal Affairs and Communications, "Consumer Price Index"	
CPIENXGASOLINEOR	Consumer price index (energy, excluding gasoline)	CY2005 average = 100	Calculated by equation (J.29)	(J.32)
CPIENXGASOLINEV	Consumer price index (energy, excluding gasoline and consumption tax, S.A.)	CY2005 average = 100	Seasonally adjusted by X-12-ARIMA (consumption tax adjusted simultaneously)	(J.33)
CPIFDXFAOR	Consumer price index (food, excluding fresh food and alcoholic beverages)	CY2005 average = 100	"Alcoholic beverages" are excluded from "food, excluding fresh food."	(J.27)
CPIFDXFAV	Consumer price index (food, excluding fresh food, alcoholic beverages and consumption tax, S.A.)	CY2005 average = 100	Seasonally adjusted by X-12-ARIMA (consumption tax adjusted simultaneously)	(J.28)
CPIFDXFAW	Weight of consumer price index (food, excluding fresh food, alcoholic beverages)	-	Ministry of Internal Affairs and Communications, "Consumer Price Index"	
CPIFF	Consumer price index (fresh food, S.A.)	CY2005 average = 100	Seasonally adjusted by X-12-ARIMA	(J.23)
CPIFFOR	Consumer price index (fresh food)	CY2005 average = 100	Ministry of Internal Affairs and Communications, "Consumer Price Index"	(J.22)
CPIFFW	Weight of consumer price index (fresh food)	-	Ministry of Internal Affairs and Communications, "Consumer Price Index"	
CPIGASOLINEOR	Consumer price index (gasoline)	CY2005 average = 100	Ministry of Internal Affairs and Communications, "Consumer Price Index"	(J.30)
CPIGASOLINEV	Consumer price index (gasoline, excluding consumption tax, S.A.)	CY2005 average = 100	Seasonally adjusted by X-12-ARIMA (consumption tax adjusted simultaneously)	(J.31)

Variable		Unit	Source, calculation, etc.	Equation
CPIGASOLINEW	Weight of consumer price index (gasoline)	-	Ministry of Internal Affairs and Communications, "Consumer Price Index"	
CPIHSW	Weight of consumer price index (high school fees)	-	Ministry of Internal Affairs and Communications, "Consumer Price Index"	
CPIOR	Consumer price index (general)	CY2005 average = 100	Ministry of Internal Affairs and Communications, "Consumer Price Index"	(J.19)
CPIV	Consumer price index (general, excluding consumption tax, S.A.)	CY2005 average = 100	Seasonally adjusted by X-12-ARIMA (consumption tax adjusted simultaneously)	(J.21)
CPIXF_AD	Consumer price index (general, excluding fresh food and high school fees, S.A.)	CY2005 average = 100	CPIXFOR_AD/SFCPIXF*100	(J.26)
CPIXFDENOR	Consumer price index (general, excluding food (but including alcoholic beverages) and energy)	CY2005 average = 100	Ministry of Internal Affairs and Communications, "Consumer Price Index"	(J.34)
CPIXFDENOR_AD	Consumer price index (general, excluding food (but including alcoholic beverages), energy and high school fees, S.A.)	CY2005 average = 100	"High school fees" are excluded from "general, excluding food (excluding alcoholic beverages) and energy."	(J.35)
CPIXFDENV_AD	Consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees and consumption tax, S.A.)	CY2005 average = 100	CPIXFDEN_AD/(1+VATCPIXFDEN)	(J.36)
CPIXFDENW	Weight of consumer price index (general, excluding food (but including alcoholic beverages) and energy)	-	Ministry of Internal Affairs and Communications, "Consumer Price Index"	
CPIXFOR	Consumer price index (general, excluding fresh food)	CY2005 average = 100	Ministry of Internal Affairs and Communications, "Consumer Price Index"	(J.24)
CPIXFOR_AD	Consumer price index (general, excluding fresh food and high school fees)	CY2005 average = 100	Calculated by equation (J.25)	(J.25)
CPIXFW	Weight of consumer price index (general, excluding fresh food)	-	Ministry of Internal Affairs and Communications, "Consumer Price Index"	
CTAX	Current taxes on income, wealth, etc., receivable (general government), excluding those from households	Billion yen	GYTAX-YTAX	(C.11)
CUMFB	Stocks of liabilities (general government)	Billion yen	Converted from yearly to quarterly data using a cubic spline (FY data: Cabinet Office, "National Accounts")	(C.20)
D	Consumption of fixed capital (total economy)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(L.5)
DDF	Real domestic final demand	Billions of chained (2000) yen	Calculated by equation (L.8.1)	(L.8.1)
DDFN	Nominal domestic final demand	Billion yen	CN+IHN+INVN+CGN+IGN	(L.9)
DFT	Depreciation rate of corporate capital stock	%	Research Institute of Economy, Trade & Industry, IAA, "JIP Database 2010"	
Dyyq	Dummy variable	-	yyQq = 1, otherwise = 0	

Variable		Unit	Source, calculation, etc.	Equation
Dyyqhhk	Dummy variable for temporary level shift	-	yyQq to hhQk = 1, otherwise = 0	
DyyqZ	Dummy variable for parmanent level shift	-	Through yyQq = 0, after yyQq = 1	
EPI	Export price index (all commodities, yen basis)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.1.2)
EX	Real exports of goods and services	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(E.1.2)
EXN	Nominal exports of goods and services	Billion yen	Cabinet Office, "National Accounts"	(E.2)
EXUSWEIGHT	Weight of exports to the United States in total exports	%	Ministry of Finance, "Trade Statistics"	
FA	Financial assets (households and private nonprofit institutions serving households)	Billion yen	FASAFE+FARISK	(H.19)
FARISK	Risky financial assets (households and private nonprofit institutions serving households)	Billion yen	Foreign currency deposits, investment trust beneficiary certificates, trust beneficiary rights, shares, insurance and pension reserves and outward investment in securities (Bank of Japan, "Flow of Funds"). Some data are converted from yearly to quarterly data using a cubic spline.	(H.21)
FASAFE	Safe financial assets (households and private nonprofit institutions serving households)	Billion yen	Currency, transferable deposits, time and savings deposits, central government securities, local government securities, public corporation securities, bank debentures and industrial securities (Bank of Japan, "Flow of Funds"). Some data are converted from yearly to quarterly data using a cubic spline.	(H.20)
FCPI	Consumer price index of overseas economy (S.A.)	CY2005 average = 100	Seasonally adjusted by X-12-ARIMA	(D.13)
FCPIOR	Consumer price index of overseas economy	CY2005 average = 100	CPIOR*FXN/FXR	(D.14.1)
FCPIUSWEIGHT	Weight of U.S. dollar in effective exchange rate	%	Bank for International Settlements, "Effective Exchange Rates"	
FD	Real final demand	Billions of chained (2000) yen	Calculated by equation (L.6.1)	(L.6.1)
FDN	Nominal final demand	Billion yen	CN+IHN+INVN+CGN+IGN+EXN	(L.7)
FGAP	GDP gap of overseas economy	%	log(FGDP/FGDPQ)*100	(D.3)
FGDP	Real gross domestic product of overseas economy	1960/Q1 =100	The overseas GDP is calculated as the weighted average of individual countries' GDP, where countries' shares in Japanese exports are used as weights.	(D.1)

Variable		Unit	Source, calculation, etc.	Equation
FGDPQ	Potential output of overseas economy	-	Trend using the Hodrick-Prescott filter (lambda=1600); data up to 2030/Q1 are used, with overseas GDP extrapolated using its steady-state rate of growth	
FPPI	Producer price index of overseas economy	-	The overseas producer price index (PPI) is calculated as the weighted average of individual countries' PPI, where countries' shares in Japanese exports are used as weights.	(D.10)
FXN	Nominal effective exchange rate (broad basis)	CY2005 average = 100	Bank for International Settlements, "Effective Exchange Rates" (data up to 1993/M12 are on a narrow basis)	(B.6)
FXR	Real effective exchange rate (broad basis)	CY 2005 average = 100	Bank for International Settlements, "Effective Exchange Rates" (data up to 1993/M12 are on a narrow basis)	(B.7)
FXYEN	Nominal exchange rate (yen per U.S. dollar)	Yen per U.S. dollar	Closing price, monthly average (Tokyo Foreign Exchange Matket)	(B.5.2)
G	Real public demand	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(C.1)
GAP	GDP gap	%	Estimated following the method of Hara et al. (2006)	(L.12)
GASOLINEMP	Gasoline price	Yen per liter	The Oil Information Center, "Price Information" (regular-gasoline, cash sale price, including consumption tax)	(J.18)
GASOLINET	Gasoline tax	Yen per liter	53.8 yen (temporarily dropped to 28.7 yen in April 2008 due to expiration of the provisional tax on gasoline)	
GCTRP	Capital transfers, payable (general government)	Billion yen	Converted from yearly to quarterly data using a cubic spline. (FY data: Cabinet Office, "National Accounts")	
GCTRR	Capital transfers, receivable (general government)	Billion yen	Converted from yearly to quarterly data using a cubic spline. (FY data: Cabinet Office, "National Accounts")	
GD	Real consumption of fixed capital (general government)	Billion yen	GDN/(PGIG/100)	(C.17)
GDN	Nominal consumption of fixed capital (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(C.16)
GDP	Real gross domestic product	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(L.1.1)
GDPN	Nominal gross domestic product	Billion yen	Cabinet Office, "National Accounts"	(L.2)
GDPNADJ	Adjusted nominal gross domestic product	Billion yen	GDPN-YHOUSEGR-YMIXGR-NINDTAX	(L.3)
GDPQ	Real potential gross domestic product	Billion yen	Estimated following the method of Hara et al. (2006)	(L.10)
GDPQNONHP	Real potential gross domestic product (non-filtering)	Billion yen	Estimated following the method of Hara et al. (2006)	(L.11)
GIG	Real gross fixed capital formation (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(C.15)

Variable		Unit	Source, calculation, etc.	Equation
GIGN	Nominal gross fixed capital formation (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(C.14)
GIIGN	Nominal changes in inventories (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	
GIP	Interest, payable (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(C.21)
GIR	Interest, receivable (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	
GK	Social capital stock	Billion yen	Accumulation of GIG-GD based on general government fixed assets at the end of CY2000	(C.18)
GL	Purchases of land, net (general government)	Billion yen	Converted from yearly to quarterly data using a cubic spline. (FY data: Cabinet Office, "National Accounts")	(C.19)
GN	Nominal public demand	Billion yen	Cabinet Office, "National Accounts"	(C.8)
GSP	Social benefits other than social transfer in kind, payable (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(C.13)
GSR	Social contributions, receivable (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(C.12)
GYPXI	Property income, excluding interest, net (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	
GYTAX	Current taxes on income, wealth, etc., receivable (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(C.10)
GYTRP	Other current transfers, excluding current transfers within general government, payable (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	
GYTRR	Other current transfers, excluding current transfers within general government, receivable (general government)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	
H	Total hours worked (industries covered, S.A.)	Hours per capita	Translate index into hours per capita [original data: Ministry of Health, Labour and Welfare, "Monthly Labour Survey" (from 1990/Q1, data are for establishments with 5 or more employees)]	(G.9)
HGAP	Gap of total hours worked	-	log(H/12/HQ)	(G.10)
HQ	Potential total hours worked	Hours per capita and per month	Estimated following the method of Hara et al. (2006)	
HQFGDP_SS	Steady-state rate of real overseas GDP growth	-	Historical average of dlog(FGDP)	
HQIPIOIL_SS	Steady-state value of import price index (petroleum)	-	Historical average of dlog(IPIOIL)	
HQIPIOILD_SS	Steady-state value of import price index (petroleum, U.S. dollar basis)	-	Historical average of dlog(IPIOILD)	

Variable		Unit	Source, calculation, etc.	Equation
HQNUSCPIQ	Trend inflation rate of the non-U.S. consumer price index	% per annum	Trend using the Hodrick-Prescott filter (lambda=1600); data up to 2030/Q1 are used, with inflation rate of the non-U.S. CPI extrapolated using its past average	
HQNUSGDP_SS	Steady-state rate of the real non-U.S. GDP growth	-	Historical average of dlog(NUSGDP)	
HQUSCPIQ	Trend inflation rate of the U.S. consumer price index	% per annum	Estimated based on the method described in Appendix I.	
HQUSGDP_SS	Steady-state rate of the real U.S. GDP growth	-	Historical average of dlog(USGDP)	
HYGDPQ	Potential GDP growth rate	-	log(GDPQ/GDPQ(-4))	
IG	Real public investment	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(C.3)
IGN	Nominal public investment	Billion yen	Cabinet Office, "National Accounts"	(C.6)
IH	Real private residential investment	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(I.4.2)
IHN	Nominal private residential investment	Billion yen	Cabinet Office, "National Accounts"	(I.5)
IIG	Real public inventory	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(C.4)
IIGN	Nominal public inventory	Billion yen	Cabinet Office, "National Accounts"	(C.7)
IIGN_SS	Steady-state value of nominal public inventory	Billion yen	Historical moving average (last 4 quarter) of IIGN	
IIPIV	Producer inventories (mining and manufacturing)	CY2005 average = 100	Ministry of Economy, Trade and Industry, "Indices of Industrial Production" (data are at the end of period)	(F.5.2)
IIPP	Industrial production (mining and manufacturing)	CY2005 average = 100	Ministry of Economy, Trade and Industry, "Indices of Industrial Production"	(F.4.2)
IIPS	Producer shipments (mining and manufacturing)	CY2005 average = 100	Ministry of Economy, Trade and Industry, "Indices of Industrial Production"	(F.3)
IM	Real imports of goods and services	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(E.6)
IMN	Nominal imports of goods and services	Billion yen	Cabinet Office, "National Accounts"	(E.5)
IMOILN	Nominal imports (petroleum)	Billion yen	Seasonally adjusted by X11; data are calculated as nominal goods imports ("Balance of Payments") times the ratio of petroleum imports to overall goods imports ("Trade Statistics")	(E.3)
IMXON	Nominal imports (excluding petroleum)	Billion yen	IMN-IMOILN*4/10	(E.4.2)
INV	Real private non-residential investment	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(F.1.2)
INVN	Nominal private non-residential investment	Billion yen	Cabinet Office, "National Accounts"	(F.2)
IPI	Import price index (all commodities, yen basis)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.2)

Variable		Unit	Source, calculation, etc.	Equation
IPIOIL	Import price index (petroleum, yen basis)	CY2005 average = 100	Bank of Japan, "Corporate Goods Price Index"	(J.3)
IPIOILD	Import price index (petroleum, U.S. dollar basis)	CY2005 average = 100	IPIOIL/FXYEN, indexed with the base year CY2005	(J.4)
IPIOILW	Weight of petroleum price in import price index	Per 1000	Bank of Japan, "Corporate Goods Price Index"	
IPIXO	Import price index (excluding petroleum, yen basis)	CY2005 average = 100	(IPI-IPIOIL*IPIOILW/1000)/(1-IPIOILW/1000)	(J.5.2)
IRL	Long-term interest rate (Japanese government bond yields, 10-year)	% per annum	Yield on newly issued government bonds (10 years); data up to 1998/Q4 are yields to subscribers of government bonds (10 years)	(B.1.2)
IRLOAN	Long-term interest rate on loans	% per annum	Average contracted interest rates on new long-term loans and discounts; data up to 1991/Q2 are the prime lending rate on long-term loans	(B.2.2)
KIV	Real private inventory stock	Billion yen	Accumulation of SNAIV based on KIV at the end of CY2000	(F.6.2)
L	Employed persons	10,000 persons	Misintry od Internal Affairs and Communications, "Labour Force Survey"	(G.1)
LF	Labour force	10,000 persons	Misintry od Internal Affairs and Communications, "Labour Force Survey"	(G.5)
LFGAP	Gap of labour force	-	log(LF/LFQ)	(G.6)
LFQ	Potential labour force	10,000 persons	Estimated following the method of Hara et al. (2006)	
LGAP	Gap of employed persons	-	log(L/LQ)	(G.2)
LOAN	Loans (households and private nonprofit institutions serving households)	Billion yen	Private and public financial institutions (Bank of Japan, "Flow of Funds"). Some data are converted from yearly to quarterly data using a cubic spline.	(H.22)
LOANDI	Lending attitude of financial institutions D.I. (small enterprises, nonmanufacturing)	% points	Bank of Japan, "TANKAN"	(B.3)
LOANH	Housing loans (households and private nonprofit institutions serving households)	Billion yen	Housing loans by private and public financial institutions (Bank of Japan, "Flow of Funds"). Some data are converted from yearly to quarterly data using a cubic spline.	(H.23)
LOANXH	Loans, excluding housing loans (households and private nonprofit institutions serving households)	Billion yen	LOAN-LOANH	(H.24)
LQ	Potential employed persons	10,000 persons	Estimated following the method of Hara et al. (2006)	
LSELF	Self-employed workers and family workers	10,000 persons	L-LW	(G.4)
LW	Employees	10,000 persons	Misintry od Internal Affairs and Communications, "Labour Force Survey"	(G.3)

Variable		Unit	Source, calculation, etc.	Equation
NINDTAX	Taxes on production and imports, excluding subsidies	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(L.4)
NUSCALL	Policy interest rate of the non-U.S. economy	% per annum	Weighted average of non-U.S. countries' short-term interest rates, where countries shares in Japanese exports are used as weights	(D.24.4)
NUSCALLQ	Equilibrium policy interest rate of the non-U.S. economy	% per annum	Calculated by equation (D.24.1)	(D.24.2)
NUSCPI	Consumer price index of the non-U.S. economy	CY2005 average = 100	Calculated by equation (D.13)	(D.18)
NUSGAP	GDP gap of the non-U.S. economy	%	log(NUSGDP/NUSGDPQ)*100	(D.9)
NUSGDP	Real gross domestic product of the non-U.S. economy	1960/Q1 =100	Calculated by equation (D.1)	(D.7)
NUSGDPQ	Potential real gross domestic product of the non-U.S. economy	-	Calculated by equation (D.2)	(D.8)
NUSPIX	Inflation rate of the non-U.S. consumer price index	% per annum	dlog(NUSCPI)*400	(D.19)
NUSPPI	Producer price index of the non-U.S. economy	CY2005 average = 100	Calculated by equation (D.10)	(D.12)
NUSZCALL	Expected policy interest rate of the non-U.S. economy	% per annum	Calculated by equation (D.25.4)	(D.25.4)
NUSZCALL10	Long-term expected policy interest rate of the non-U.S. economy (10 year average)	% per annum	Calculated by equation (D.26.1)	(D.26.1)
NUSZGAP	Expected GDP gap of the non-U.S. economy	%	Calculated by equation (D.25.3)	(D.25.3)
NUSZGAP10	Long-term expected GDP gap of the non-U.S. economy (10 year average)	%	Calculated by equation (D.26.3)	(D.26.3)
NUSZPI	Expected inflation rate of the non-U.S. consumer price index	% per annum	Calculated by equation (D.25.1)	(D.25.1)
NUSZPI10	Long-term expected inflation rate of the non-U.S. consumer price index (10 year average)	% per annum	Calculated by equation (D.26.2)	(D.26.2)
PB	Primary fiscal balance	Billion yen	Calculated by equation (C.9)	(C.9)
PC	Private consumption deflator	-	Cabinet Office, "National Accounts"	(K.1)
PCG	Government consumption deflator	-	Cabinet Office, "National Accounts"	(K.5)
PEX	Exports of goods & services deflator	-	Cabinet Office, "National Accounts"	(K.9)
PGDP	GDP deflator	-	Cabinet Office, "National Accounts"	(K.11)
PGIG	Gross fixed capital formation deflator (general government)	-	GIGN/GIG*100	(K.7)
PICGPDVL	Trend inflation rate of the domestic corporate price index (all commodities, excluding consumption tax)	% per annum	Updated with equation (A.6.3); the base period is 1974/Q1.	(A.6.3)
PIG	Public investment deflator	-	Cabinet Office, "National Accounts"	(K.6)
PIH	Private residential investment deflator	-	Cabinet Office, "National Accounts"	(K.2)
PIIG	Public inventory deflator	-	Cabinet Office, "National Accounts"	(K.8)
PIM	Imports of goods & services deflator	-	Cabinet Office, "National Accounts"	(K.10)
PINV	Private non-residential investment deflator	-	Cabinet Office, "National Accounts"	(K.3)

Variable		Unit	Source, calculation, etc.	Equation
PIQ	Trend inflation rate of the consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees and consumption tax, S.A.)	% per annum	Estimated based on the method described in Appendix I.	(A.2.2)
PIUSPPIL	Trend inflation rate of the U.S. producer price index	% per annum	Updated with equation (A.6.6); the base period is 1974/Q1.	(A.6.6)
PIX	Inflation rate of the consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees and consumption tax, S.A.)	% per annum	dlog(CPIXFDENV_AD)*400	(J.39)
POP15	Population of 15 years old or more (S.A.)	10,000 persons	Ministry od Internal Affairs and Communications, "Labour Force Survey"	
POP60	Population of 60 years old or more (S.A.)	10,000 persons	Ministry od Internal Affairs and Communications, "Labour Force Survey"	
PROF	Ordinary profits (large enterprises, S.A.)		Ministry of Finance, "Financial statements statistics of corporations by industry"	(F.9)
PSNAIV	Private inventory deflator	-	SNAIVN/SNAIV*100	(K.4)
RSDPROF	Operating surplus as the residual	Billion yen	Calculated by equation (F.10)	(F.10)
SNAIV	Real private inventory	Billions of chained (2000) yen	Cabinet Office, "National Accounts"	(F.7)
SNAIVN	Nominal private inventory	Billion yen	Cabinet Office, "National Accounts"	(F.8)
SUMMERCGPDXOE	Extra charges for summer electricity in the domestic corporate price index (excluding electric & electronic products, petroleum & coal products)	-	"Including extra charges for summer electricity" / "Excluding extra charges for summer electricity" - 1	(J.17)
T801	Time trend	1980/Q1=1	Increase 1 per quarter	
TMPINCOME_SS	Steady-state value of temporary income (households including private unincorporated enterprises)	Billion yen	Historical average of YDN-ZYW10*PGDP	
TOPIX	Tokyo Stock Price Index	Jan.4, 1968=100	Tokyo Stock Exchange	(B.4)
U	Unemployment rate (S.A.)	%	Misintry od Internal Affairs and Communications, "Labour Force Survey"	(G.7)
UGAP	Unemployment rate gap	%	U-UQ	(G.8)
UQ	Structural unemployment rate	%	Estimated following the method of Hara et al. (2006)	
USCALL	Policy interest rate of the United States	% per annum	Federal funds rate (monthly average)	(D.21.4)
USCALLQ	Equilibrium policy interest rate of the United States	% per annum	Calculated by equation (D.21.1)	(D.21.2)
USCPI	Consumer price index of the United States (for all urban consumers, all items)	1982-1984=100	Department of Labor Bureau of Labor Statistics	(D.15)
USCPIXFDEN	Consumer price index of the United States (for all urban consumers, all items less food & energy)	1982-1984=100	Department of Labor Bureau of Labor Statistics	(D.16)
USGAP	GDP gap of the United States	%	log(USGDP/USGDPQ)*100	(D.6)
USGDP	Real gross domestic product of the United States	1960/Q1 =100	Department of Labor Bureau of Labor Statistics	(D.4)

Variable		Unit	Source, calculation, etc.	Equation
USGDPQ	Potential real gross domestic product of the United States	-	Estimated by Congressional Budget Office	
USIRL	Long-term interest rate of the United States	% per annum	Yield on treasury securities (10 years, monthly average)	(D.20)
USPIX	Inflation rate of the U.S. consumer price index (for all urban consumers, all items less food & energy)	% per annum	dlog(USCPIXFDEN)*400	(D.17)
USPPI	Producer price index of the United States	1982=100	Department of Labor Bureau of Labor Statistics	(D.11)
USZCALL	Expected policy interest rate of the United States	% per annum	Calculated by equation (D.22.4)	(D.22.4)
USZCALL10	Long-term expected policy interest rate of the United States (10 year average)	% per annum	Calculated by equation (D.23.1)	(D.23.1)
USZGAP	Expected GDP gap of the United States	%	Calculated by equation (D.22.3)	(D.22.3)
USZGAP10	Long-term expected GDP gap of the United States (10 year average)	%	Calculated by equation (D.23.3)	(D.23.3)
USZPI	Expected inflation rate of the U.S. consumer price index (for all urban consumers, all items less food & energy)	% per annum	Calculated by equation (D.22.1)	(D.22.1)
USZPI10	Long-term expected inflation rate of the U.S. consumer price index (for all urban consumers, all items less food & energy) (10 year average)	% per annum	Calculated by equation (D.23.2)	(D.23.2)
WHN	Nominal wage (per hour)	100,000 yen per capita and per hour	YWN/LW/H	(G.11.2)
YDN	Nominal disposable income, net (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.1)
YHOUSE	Operating surplus (imputed service of owner-occupied dwellings), net (households including private unincorporated enterprises)	Billion yen	YOSMI-YMIX	(H.11)
YHOUSE_GDPN_HP	Trend ratio of operating surplus (imputed service of owner-occupied dwellings, net) to nominal gross domestic product	-	Trend using the Hodrick-Prescott filter (lambda=1600) on YHOUSE/GDPN	
YHOUSEGR	Operating surplus (imputed service of owner-occupied dwellings), gross (households including private unincorporated enterprises)	Billion yen	YOSMIGR-YMIXGR	(H.10)
YHOUSEGR_GDPN_HP	Trend ratio of operating surplus (imputed service of owner-occupied dwellings, gross) to nominal gross domestic product	-	Trend using the Hodrick-Prescott filter (lambda=1600) on YHOUSEGR/GDPN	
YMIX	Mixed income, net (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.13.2)
YMIXGR	Mixed income, gross (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.12.2)
YOSMI	Operating surplus and mixed income, net (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.9)

Variable		Unit	Source, calculation, etc.	Equation
YOSMIGR	Operating surplus and mixed income, gross (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.8)
YPP	Property income, payable (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.7)
YPR	Property income, receivable (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.4)
YPRDIV	Property income (dividends), receivable (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.5)
YPRXDIV	Property income (excluding dividends), receivable (households including private unincorporated enterprises)	Billion yen	YPR-YPRDIV	(H.6)
YTAX	Current taxes on income, wealth, etc., payable (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.17)
YTAXRAT	Effective income tax rate	-	YTAX/(YWN+YMIXGR+YTR)	(H.18)
YTR	Social contributions and other current transfers, excluding social transfer in kind , net (households including private unincorporated enterprises)	Billion yen	YDN-(YWN+YP+YHOUSE+YMIX-YTAX)	(H.14)
YTRP	Social contributions and other current transfers, payable (households including private unincorporated enterprises)	Billion yen	YTRR-YTR	(H.16)
YTRR	Social benefits other than social transfers in kind and other current transfer, receivable (households including private unincorporated enterprises)	Billion yen	Seasonally adjusted by X-12-ARIMA (original data: Cabinet Office, "National Accounts")	(H.15)
YW	Real compensation of employees	Billion yen	YWN/PGDP*100	(H.3)
YWN	Nominal compensation of employees	Billion yen	Cabinet Office, "National Accounts"	(H.2)
ZCALL	Expected policy interest rate	% per annum	Calculated by equation (A.2.4)	(A.2.4)
ZCALL10	Long-term expected policy interest rate (10 year average)	% per annum	Calculated by equation (A.3.1)	(A.3.1)
ZGAP	Expected GDP gap	%	Calculated by equation (A.2.3)	(A.2.3)
ZGAP10	Long-term expected GDP gap (10 year average)	%	Calculated by equation (A.3.3)	(A.3.3)
ZPI	Expected inflation rate of the consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees and consumption tax, S.A.)	% per annum	Calculated by equation (A.2.1)	(A.2.1)
ZPI10	Long-term expected inflation rate of the consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees and consumption tax, S.A.) (10 year average)	% per annum	Calculated by equation (A.3.2)	(A.3.2)
ZPICGPDV	Expected inflation rate of the domestic corporate price index (all commodities, excluding consumption tax)	% per annum	Calculated by equation (A.6.1)	(A.6.2)
ZPIUSPPI	Expected inflation rate of the U.S. producer price index	% per annum	Calculated by equation (A.6.4)	(A.6.5)

Variable		Unit	Source, calculation, etc.	Equation
ZUGAP	Expected unemployment rate gap	%	Calculated by equation (A.5)	(A.5)
ZYW	Expected real compensation of employees	Billion yen	Calculated by equation (A.4.1)	(A.4.1)
ZYW10	Long-term expected real compensation of employees (10 year average)	Billion yen	Calculated by equation (A.4.2)	(A.4.2)

A. Monetary Policy Rule and Expectational Variables

A.1 **CALL**: Policy interest rate (See Section 4.1 in the text.)

The real policy interest rate is regressed on potential GDP growth:

$$CALL_t - PIX_t = -0.406\langle -2.68\rangle + 0.970\langle 17.5\rangle \cdot HYGDPQ_t \cdot 100 \qquad \text{(A.1.1)}$$

Sample period: 1981/Q1 - 2010/Q4, Adjusted R^2: 0.900

The equilibrium policy interest rate is defined as the real policy interest rate estimated using equation (A.1.1) plus the trend inflation rate:

$$CALLQ_t = -0.406 + 0.970 \cdot HYGDPQ_t \cdot 100 + PIQ_t \qquad \text{(A.1.2)}$$

The monetary policy rule is the Taylor Rule with interest rate smoothing; only the smoothing parameter is estimated in this rule:

$$\begin{aligned} CALL_t = {} & 0.838\langle 21.3\rangle \cdot CALL_{t-1} \\ & + (1-0.838)\cdot\left[CALLQ_t + 0.5\cdot GAP_t + 1.5\cdot\left(PIX_t - PIQ_t\right)\right] \end{aligned} \qquad \text{(A.1.3)}$$

Sample period: 1980/Q3 - 1995/Q4, Adjusted R^2: 0.942

The following equation taking into account the zero lower bound is used in forecasts and simulations:

$$CALL_t = \max\left\{ \begin{array}{l} 0.838\cdot CALL_{t-1} + (1-0.838)\cdot\left[\begin{array}{l} CALLQ_t + 0.5\cdot GAP_t \\ +1.5\cdot\left(PIX_t - PIQ_t\right) \end{array}\right], \\ 0 \end{array} \right\} \qquad \text{(A.1.4)}$$

A.2 **ZPI**: Expected inflation rate of the consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees, and consumption tax, S.A.); **PIQ**: Trend inflation rate of the consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees, and consumption tax, S.A.); **ZGAP**: Expected GDP gap; **ZCALL**: Expected policy interest rate

The *n*-period ahead expectation of core CPI inflation formed in period t, $ZPI_{t,n}$, the trend inflation, *PIQ* (A.2.2), and the core CPI, *CPIXFDENV_AD* (J.36) are estimated simultaneously using a Kalman filter algorithm with maximum likelihood estimation (see Section 3.1 in the text and Appendix. I):

$$ZPI_{t,n} = 0.724\langle 19.1\rangle \cdot ZPI_{t,n-1} + (1-0.724)\cdot PIQ_t + 0.0384\langle 2.90\rangle \cdot ZGAP_{t,n} \qquad \text{(A.2.1)}$$

Sample period: 1980/Q3 - 2010/Q4

Trend inflation is assumed to follow a random walk process:

$$PIQ_t = PIQ_{t-1} \qquad \text{(A.2.2)}$$

The expected GDP gap is assumed to be generated by an AR(2) process:

$$ZGAP_{t,n} = 1.39\langle 17.2\rangle \cdot ZGAP_{t,n-1} - 0.460\langle -5.66\rangle \cdot ZGAP_{t,n-2} \qquad \text{(A.2.3)}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.926

The expected policy interest rate is formed in accordance with the monetary policy rule (A.1.3) (see Section 4.2 in the text.):

$$\begin{aligned} ZCALL_{t,n} &= 0.838\cdot ZCALL_{t,n-1} \\ &\quad + (1-0.838)\cdot\left[CALLQ_t + 1.5\cdot\left(ZPI_{t,n} - PIQ_t\right) + 0.5\cdot ZGAP_{t,n}\right] \end{aligned} \qquad \text{(A.2.4)}$$

By summarizing equations (A.2.1), (A.2.3), and (A.2.4), the whole system of VAR expectations can be expressed as follows (when $n \le 0$, $ZCALL_{t,n}$ = $CALL_{t+,n}$, $ZPI_{t,n}$ = PIX_{t+n}, $ZGAP_{t,n}$ = GAP_{t+n}):

$$\begin{pmatrix} 1 & -0.244 & -0.0812 & 0 \\ 0 & 1 & -0.0384 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ZCALL_{t,n} - CALLQ_t \\ ZPI_{t,n} - PIQ_t \\ ZGAP_{t,n} \\ ZGAP_{t,n-1} \end{pmatrix} = \begin{pmatrix} 0.838 & 0 & 0 & 0 \\ 0 & 0.724 & 0 & 0 \\ 0 & 0 & 1.39 & -0.460 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} ZCALL_{t,n-1} - CALLQ_t \\ ZPI_{t,n-1} - PIQ_t \\ ZGAP_{t,n-1} \\ ZGAP_{t,n-2} \end{pmatrix} \qquad \text{(A.2.5)}$$

$$
\begin{pmatrix} ZCALL_{t,n} - CALLQ_t \\ ZPI_{t,n} - PIQ_t \\ ZGAP_{t,n} \\ ZGAP_{t,n-1} \end{pmatrix}
= \begin{pmatrix} 0.838 & 0.177 & 0.126 & -0.0417 \\ 0 & 0.724 & 0.0534 & -0.0177 \\ 0 & 0 & 1.39 & -0.460 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} ZCALL_{t,n-1} - CALLQ_t \\ ZPI_{t,n-1} - PIQ_t \\ ZGAP_{t,n-1} \\ ZGAP_{t,n-2} \end{pmatrix}
= A \begin{pmatrix} ZCALL_{t,n-1} - CALLQ_t \\ ZPI_{t,n-1} - PIQ_t \\ ZGAP_{t,n-1} \\ ZGAP_{t,n-2} \end{pmatrix} \quad \text{(A.2.6)}
$$

A.3 **ZCALL10**: Long-term expected policy interest rate (10 year average); **ZPI10**: Long-term expected inflation rate of the consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees, and consumption tax, S.A.) (10 year average); **ZGAP10**: Long-term expected GDP gap (10 year average)

$$
ZCALL10_t = \frac{1}{40} \sum_{n=0}^{39} \max \left[\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} A^n \begin{pmatrix} ZCALL_{t,n} - CALLQ_t \\ ZPI_{t,n} - PIQ_t \\ ZGAP_{t,n} \\ ZGAP_{t,n-1} \end{pmatrix} + CALLQ_t,\ 0 \right] \quad \text{(A.3.1)}
$$

$$
ZPI10_t = \frac{1}{40} \sum_{j=0}^{39} \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} A^j \begin{pmatrix} ZCALL_{t,j} - CALLQ_t \\ ZPI_{t,j} - PIQ_t \\ ZGAP_{t,j} \\ ZGAP_{t,j-1} \end{pmatrix} + PIQ_t \quad \text{(A.3.2)}
$$

$$
ZGAP10_t = \frac{1}{40} \sum_{j=0}^{39} \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} A^j \begin{pmatrix} ZCALL_{t,j} - CALLQ_t \\ ZPI_{t,j} - PIQ_t \\ ZGAP_{t,j} \\ ZGAP_{t,j-1} \end{pmatrix} \quad \text{(A.3.3)}
$$

A.4 **ZYW**: Expected real compensation of employees; **ZYW10**: Long-term expected real compensation of employees (10 year average) (See Section 5.1 in the text)

$$\log\left(ZYW_{t,n}/ZYW_{t,n-4}\right) = HYGDPQ_t + 0.00274\langle 5.53\rangle \cdot ZGAP_{t,n} \quad \text{(A.4.1)}$$

Sample period: 1981/Q2 - 2010/Q4, Adjusted R^2: 0.815

$$\begin{aligned} ZYW10_t = & \frac{1}{40} \cdot \frac{1}{\left(1 + IRL_t/400 - ZPI10_t/400\right)} \\ & \cdot \sum_{n=0}^{39} \left\{ ZYW_{t,n} \Big/ \left(1 + IRL_t/400 - ZPI10_t/400\right)^n \right\} \end{aligned} \quad \text{(A.4.2)}$$

A.5 **ZUGAP**: Expected unemployment rate gap (See Section 5.1 in the text)

$$\begin{aligned} ZUGAP_{t,n} = & 0.968\langle 71.6\rangle \cdot ZUGAP_{t,n-1} \\ & -0.0130\langle -4.70\rangle \cdot ZGAP_{t,n} \end{aligned} \quad \text{(A.5)}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.969

A.6 **ZPICGPDV**: Expected inflation rate of the domestic corporate goods price index (all commodities, excluding consumption tax); **ZPIUSPPI**: Expected inflation rate of the U.S. producer price index

The expectation of the domestic corporate goods price index excluding consumption tax, *CGPDV*, is formulated in terms of the deviation from its trend rate, *PICGPDVL*, which is updated each period by 5 percent of realized inflation:

$$\begin{aligned} & \mathrm{dlog}\left(CGPDV_t\right) - PICGPDVL_{t-1} \\ & \quad = 0.601\langle 7.84\rangle \cdot \left(\mathrm{dlog}\left(CGPDV_{t-1}\right) - PICGPDVL_{t-1}\right) \end{aligned} \quad \text{(A.6.1)}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.316

$$\begin{aligned} ZPICGPDV_t = & PICGPDVL_{t-1} \cdot 400 \\ & + 1/40 \left[\frac{\left(1 - 0.601^{40}\right)}{1 - 0.601} \cdot \left(\mathrm{dlog}\left(CGPDV_t\right) - PICGPDVL_{t-1}\right) \cdot 400 \right] \end{aligned} \quad \text{(A.6.2)}$$

$$PICGPDVL_t = 0.95 \cdot PICGPDVL_{t-1} + 0.05 \cdot \text{dlog}\left(CGPDV_t\right) \qquad \text{(A.6.3)}$$

Similarly, the expectation of the U.S. producer price index, *USPPI*, is formulated in terms of the deviation from its trend rate, *PIUSPPIL*:

$$\begin{aligned} &\text{dlog}\left(USPPI_t\right) - PIUSPPIL_{t-1} \\ &\quad = 0.398\langle 4.53\rangle \cdot \left(\text{dlog}\left(USPPI_{t-1}\right) - PIUSPPIL_{t-1}\right) \end{aligned} \qquad \text{(A.6.4)}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.130

$$\begin{aligned} ZPIUSPPI_t &= PIUSPPIL_{t-1} \cdot 400 \\ &+ 1/40\left[\frac{\left(1-0.398^{40}\right)}{1-0.398} \cdot \left(\text{dlog}\left(USPPI_t\right) - PIUSPPIL_{t-1}\right) \cdot 400\right] \end{aligned} \qquad \text{(A.6.5)}$$

$$PIUSPPIL_t = 0.95 \cdot PIUSPPIL_{t-1} + 0.05 \cdot \text{dlog}\left(USPPI_t\right) \qquad \text{(A.6.6)}$$

B. Financial Variables

B.1 **IRL**: Long-term interest rate (Japanese government bond yields, 10-year)

$$IRL_t - ZCALL10_t = 0.0388\langle 1.17\rangle + 0.891\langle 23.6\rangle \cdot \left(IRL_{t-1} - ZCALL10_{t-1}\right) \qquad \text{(B.1.1)}$$

Sample period: 1983/Q3 - 2010/Q4, Adjusted R^2: 0.836, LM Test (p-value): 0.90

The following equation taking into account the zero lower bound is used in forecasts and simulations:

$$IRL_t = \max\left\{0.0388 + ZCALL10_t + 0.891 \cdot \left(IRL_{t-1} - ZCALL10_{t-1}\right),\ 0\right\} \qquad \text{(B.1.2)}$$

B.2 **IRLOAN**: Long-term interest rate on loans

$$IRLOAN_t = 1.12\langle 81.2\rangle \cdot IRL_t - 0.00828\langle -2.52\rangle \cdot LOANDI_t \qquad \text{(B.2.1)}$$

Sample period: 1983/Q3 - 2010/Q4, Adjusted R^2: 0.970, LM Test (p-value): 0.00

The following equation taking into account the zero lower bound is used in forecasts and simulations:

$$IRLOAN_t = \max\left\{1.12 \cdot IRL_t - 0.00828 \cdot LOANDI_t,\ 0\right\} \qquad \text{(B.2.2)}$$

B.3 **LOANDI**: Lending attitude of financial institutions D.I. (small enterprises, nonmanufacturing)

$$\begin{aligned} LOANDI_t &= 0.215\langle 0.878\rangle + 0.950\langle 65.1\rangle \cdot LOANDI_{t-1} \\ &\quad + 16.1\langle 6.56\rangle \cdot \mathrm{d}\log\left(TOPIX_t / PGDP_t\right) - 18.2\langle -7.83\rangle \cdot D981_t \end{aligned} \qquad \text{(B.3)}$$

Sample period: 1983/Q3 - 2010/Q4, Adjusted R^2: 0.978, LM Test (p-value): 0.07

B.4 **TOPIX**: Tokyo Stock Price Index

$$\begin{aligned} \log\left(TOPIX_t / PROF_t\right) &= -7.80\langle -247\rangle \\ &\quad - 62.7\langle -5.56\rangle \cdot \left(IRLOAN_t - ZPI10_t - HYGDPQ_t\right)/400 \\ &\quad - 0.801\langle -14.5\rangle \cdot D023Z_t \end{aligned} \qquad \text{(B.4)}$$

Sample period: 1981/Q1 - 2010/Q4, Adjusted R^2: 0.639, LM Test (p-value): 0.00

B.5 **FXYEN**: Nominal exchange rate (yen per U.S. dollar)

$$\begin{aligned} \log\left(FXYENQ_t\right) &= 5.03\langle 276\rangle + \log\left(CGPDV_t / USPPI_t\right) \\ &\quad + \left[USIRL_t - IRL_t - \left(ZPIUSPPI_t - ZPICGPDV_t\right)\right]/100 \cdot 10 \end{aligned} \qquad \text{(B.5.1)}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.566

$$\log\left(FXYEN_t / FXYENQ_t\right) = 0.954\langle 42.3\rangle \cdot \log\left(FXYEN_{t-1} / FXYENQ_{t-1}\right) \qquad \text{(B.5.2)}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.936, LM Test (p-value): 0.05

B.6 **FXN**: Nominal effective exchange rate (broad basis)

$$\begin{aligned}\mathrm{dlog}\left(FXN_t\right) = & -0.000175\langle -0.0975\rangle - 0.859\langle -24.5\rangle \cdot \mathrm{d}\log\left(FXYEN_t\right) \\ & +0.0595\langle 3.37\rangle \cdot D981_t\end{aligned} \tag{B.6}$$

Sample period: 1986/Q1 - 2010/Q4, Adjusted R^2: 0.859, LM Test (p-value): 0.00

B.7 **FXR**: Real effective exchange rate (broad basis)

$$FXR_t = FXN_t \cdot CPIOR_t / FCPIOR_t \tag{B.7}$$

C. Government

C.1 **G**: Real public demand

$$\log\left(G_t / G_{t-1}\right) = \log\left(GDPQ_t / GDPQ_{t-1}\right) \tag{C.1}$$

C.2 **CG**: Real government consumption

$$CG_t / CG_{t-1} = G_t / G_{t-1} \tag{C.2}$$

C.3 **IG**: Real public investment

$$IG_t / IG_{t-1} = G_t / G_{t-1} \tag{C.3}$$

C.4 **IIG**: Real public inventory

$$IIG_t / IIG_{t-1} = G_t / G_{t-1} \tag{C.4}$$

C.5 **CGN**: Nominal government consumption

$$CGN_t = CG_t \cdot PCG_t / 100 \tag{C.5}$$

C.6 **IGN**: Nominal public investment

$$IGN_t = IG_t \cdot PIG_t / 100 \tag{C.6}$$

C.7 **IIGN**: Nominal public inventory

$$IIGN_t = IIGN\_SS_t \tag{C.7}$$

C.8 **GN**: Nominal public demand

$$GN_t = CGN_t + IGN_t + IIGN_t \tag{C.8}$$

C.9 **PB**: Primary fiscal balance

$$\begin{aligned} PB_t = {} & NINDTAX_t + GYTAX_t + GYPXI_t + \left(GSR_t - GSP_t\right) \\ & + \left(GYTRR_t - GYTRP_t\right) + \left(GCTRR_t - GCTRP_t\right) \\ & - CGN_t - \left(GIGN_t - GDN_t\right) - GIIGN_t - GL_t \end{aligned} \tag{C.9}$$

C.10 **GYTAX**: Current taxes on income, wealth, etc., receivable (general government)

$$GYTAX_t = CTAX_t + YTAX_t \tag{C.10}$$

C.11 **CTAX**: Current taxes on income, wealth, etc., receivable (general government), excluding those from households

$$\begin{aligned} \log\left(CTAX_t / RSDPROF_t\right) = {} & -0.666\langle -6.42\rangle \\ & + 0.601\langle 9.71\rangle \cdot \log\left(CTAX_{t-1} / RSDPROF_{t-1}\right) \\ & + 2.00\langle 5.60\rangle \cdot GAP_t / 100 \end{aligned} \tag{C.11}$$

Sample period: 1980/Q2 - 2010/Q1, Adjusted R^2: 0.722, LM Test (p-value): 0.14

C.12 **GSR**: Social contributions, receivable (general government)

$$\begin{aligned}\log\left(GSR_t/YWN_t\right) &= -0.0463\langle -1.56\rangle \\ &+0.889\langle 18.2\rangle\cdot\log\left(GSR_{t-1}/YWN_{t-1}\right) \\ &+0.0780\langle 2.20\rangle\cdot\frac{1}{20}\sum_{s=1}^{20}\log\left(GSP_{t-s}/YWN_{t-s}\right)\end{aligned} \tag{C.12}$$

Sample period: 1985/Q1 - 2010/Q1, Adjusted R^2: 0.980, LM Test (p-value): 0.00

C.13 **GSP**: Social benefits other than social transfer in kind, payable (general government)

$$\begin{aligned}\log\left(GSP_t/GDPN_t\right) &= -0.640\langle -3.94\rangle \\ &+0.715\langle 10.8\rangle\cdot\log\left(GSP_{t-1}/GDPN_{t-1}\right) \\ &+0.823\langle 4.03\rangle\cdot\frac{1}{20}\sum_{s=0}^{19}\frac{POP60_{t-s}}{POP15_{t-s}} \\ &+0.0696\langle 3.02\rangle\cdot\log\left(U_t/100\right)\end{aligned} \tag{C.13}$$

Sample period: 1980/Q2 - 2010/Q1, Adjusted R^2: 0.981, LM Test (p-value): 0.00

C.14 **GIGN**: Nominal gross fixed capital formation (general government)

$$\text{dlog}\left(GIGN_t\right) = 0.00185\langle 1.34\rangle + 1.09\langle 32.0\rangle\cdot\text{dlog}\left(IGN_t\right) \tag{C.14}$$

Sample period: 1980/Q2 - 2010/Q1, Adjusted R^2: 0.896, LM Test (p-value): 0.84

C.15 **GIG**: Real gross fixed capital formation (general government)

$$GIG_t = GIGN_t / PGIG_t \cdot 100 \qquad \text{(C.15)}$$

C.16 **GDN**: Nominal consumption of fixed capital (general government)

$$GDN_t = PGIG_t / 100 \cdot GD_t \qquad \text{(C.16)}$$

C.17 **GD**: Real consumption of fixed capital (general government)

For the depreciation rate (general government), a constant value, namely the value at the end of FY2009, 4.737, is used:

$$GD_t = 4.737/100 \cdot GK_{t-1} \qquad \text{(C.17)}$$

C.18 **GK**: Social capital stock

$$GK_t = GK_{t-1} + \left(GIG_t - GD_t\right)/4 \qquad \text{(C.18)}$$

C.19 **GL**: Purchases of land, net (general government)

$$\begin{aligned} \log\left(GL_t / GIGN_t\right) = & -0.0553\langle -1.19\rangle \\ & +0.971\langle 38.8\rangle \cdot \log\left(GL_{t-1} / GIGN_{t-1}\right) \end{aligned} \qquad \text{(C.19)}$$

Sample period: 1980/Q3 - 2010/Q1, Adjusted R^2: 0.927, LM Test (p-value): 0.48

C.20 **CUMFB**: Stocks of liabilities (general government)

$$CUMFB_t = CUMFB_{t-1} + \left(PB_t + GIR_t - GIP_t\right)/4 \qquad \text{(C.20)}$$

C.21 **GIP**: Interest, payable (general government)

The coefficients on lagged long-term interest rates reflect the maturity structure of the outstanding amount of Japanese government bonds (JGBs) at the end of FY2009:

$$GIP_t / (-CUMFB_{t-1}) = -0.00659\langle -11.7 \rangle$$
$$+0.882\langle 73.2 \rangle \cdot \left(\begin{array}{l} 0.178 \cdot \frac{1}{4}\sum_{s=0}^{3} IRL_{t-s}/100 + 0.130 \cdot \frac{1}{4}\sum_{s=4}^{7} IRL_{t-s}/100 \\ +0.085 \cdot \frac{1}{4}\sum_{s=8}^{11} IRL_{t-s}/100 + 0.084 \cdot \frac{1}{4}\sum_{s=12}^{15} IRL_{t-s}/100 \\ +0.088 \cdot \frac{1}{4}\sum_{s=16}^{19} IRL_{t-s}/100 + 0.052 \cdot \frac{1}{4}\sum_{s=20}^{23} IRL_{t-s}/100 \\ +0.049 \cdot \frac{1}{4}\sum_{s=24}^{27} IRL_{t-s}/100 + 0.055 \cdot \frac{1}{4}\sum_{s=28}^{31} IRL_{t-s}/100 \\ +0.052 \cdot \frac{1}{4}\sum_{s=32}^{35} IRL_{t-s}/100 + 0.047 \cdot \frac{1}{4}\sum_{s=36}^{40} IRL_{t-s}/100 \\ +0.180 \cdot IRL_{t-60}/100 \end{array} \right) \tag{C.21}$$

Sample period: 1987/Q1 - 2010/Q1, Adjusted R^2: 0.983, LM Test (p-value): 0.00

D. Overseas Economy

D.1 **FGDP**: Real gross domestic product of overseas economy

$$\begin{aligned} \text{dlog}(FGDP_t) = & (1 - EXUSWEIGHT_t/100) \cdot \text{dlog}(NUSGDP_t) \\ & + EXUSWEIGHT_t/100 \cdot \text{dlog}(USGDP_t) \end{aligned} \tag{D.1}$$

D.2 **FGDPQ**: Potential output of overseas economy

$$\begin{aligned} \text{dlog}(FGDPQ_t) = & (1 - EXUSWEIGHT_t/100) \cdot \text{dlog}(NUSGDPQ_t) \\ & + EXUSWEIGHT_t/100 \cdot \text{dlog}(USGDPQ_t) \end{aligned} \tag{D.2}$$

D.3 **FGAP**: GDP gap of overseas economy

$$FGAP_t = \log\left(FGDP_t / FGDPQ_t\right) \cdot 100 \tag{D.3}$$

D.4 **USGDP**: Real gross domestic product of the United States

$$\log\left(USGDP_t\right) = \log\left(USGDPQ_t\right) + USGAP_t / 100 \tag{D.4}$$

D.5 **USGDPQ**: Potential real gross domestic product of the United States

$$\mathrm{dlog}\left(USGDPQ_t\right) = HQUSGDP\_SS_t \tag{D.5}$$

D.6 **USGAP**: GDP gap of the United States

$$\begin{aligned} USGAP_t = & -0.0216\langle -0.338\rangle \\ & + 0.905\langle 36.1\rangle \cdot USGAP_{t-1} + 0.269\langle 2.70\rangle \cdot \mathrm{d}\left(NUSGAP_t\right) \\ & - 0.0900\langle -2.73\rangle \cdot \frac{1}{4}\sum_{s=0}^{3}\begin{pmatrix} USIRL_{t-s} - USZPI10_{t-s} \\ -USCALLQ_{t-s} + HQUSCPIQ_{t-s} \end{pmatrix} \\ & + 0.233\langle 2.68\rangle \cdot \mathrm{d}\left(USGAP_{t-1}\right) + 0.346\langle 4.06\rangle \cdot \mathrm{d}\left(USGAP_{t-2}\right) \end{aligned} \tag{D.6}$$

Sample period: 1981/Q2 - 2010/Q4, Adjusted R^2: 0.948, LM Test (p-value): 0.31

D.7 **NUSGDP**: Real gross domestic product of the non-U.S. economy

$$\log\left(NUSGDP_t\right) = \log\left(NUSGDPQ_t\right) + NUSGAP_t / 100 \tag{D.7}$$

D.8 **NUSGDPQ**: Potential real gross domestic product of the non-U.S. economy

$$\log\left(NUSGDPQ_t\right) = HQNUSGDP\_SS_t \tag{D.8}$$

D.9 **NUSGAP**: GDP gap of the non-U.S. economy

$$\begin{aligned} & NUSGAP_t \\ & \quad = 0.898\langle 22.5\rangle \cdot NUSGAP_{t-1} + 0.270\langle 4.10\rangle \cdot \mathrm{d}\left(USGAP_{t-1}\right) \\ & \quad - 0.101\langle -2.37\rangle \cdot \frac{1}{4}\sum_{s=0}^{3}\begin{pmatrix} NUSZCALL10_{t-s} - NUSZPI10_{t-s} \\ -NUSCALLQ_{t-s} + HQNUSCPIQ_{t-s} \end{pmatrix} \end{aligned} \tag{D.9}$$

Sample period: 1981/Q2 - 2010/Q4, Adjusted R^2: 0.809, LM Test (p-value): 0.00

D.10 **FPPI**: Producer price index of overseas economy

$$\begin{aligned} \mathrm{dlog}\left(FPPI_t\right) = & \left(1 - EXUSWEIGHT_t/100\right) \cdot \mathrm{dlog}\left(NUSPPI_t\right) \\ & + EXUSWEIGHT_t/100 \cdot \mathrm{dlog}\left(USPPI_t\right) \end{aligned} \tag{D.10}$$

D.11 **USPPI**: Producer price index of the United States

$$\begin{aligned} \mathrm{dlog}\left(USPPI_t\right) \cdot 400 = & \ 3.26\langle 8.13\rangle \\ & +2.85\langle 10.3\rangle \cdot \left(\mathrm{dlog}\left(USCPI_t\right) \cdot 400 - HQUSCPIQ_t\right) \\ & +0.0196\langle 2.23\rangle \cdot \mathrm{dlog}\left(IPIOILD_t\right) \cdot 400 \end{aligned} \tag{D.11}$$

Sample period: 1984/Q1 - 2010/Q4, Adjusted R^2: 0.733, LM Test (p-value): 0.01

D.12 **NUSPPI**: Producer price index of the non-U.S. economy

$$\begin{aligned} \mathrm{dlog}\left(NUSPPI_t\right) = & \ 0.00485\langle 5.96\rangle + 0.00244\langle 3.54\rangle \cdot FGAP_t \\ & +0.219\langle 2.83\rangle \cdot \mathrm{dlog}\left(NUSPPI_{t-1}\right) \\ & +0.0294\langle 6.89\rangle \cdot \mathrm{dlog}\left(IPIOILD_t\right) \end{aligned} \tag{D.12}$$

Sample period: 1984/Q1 - 2010/Q4, Adjusted R^2: 0.494, LM Test (p-value): 0.01

D.13 **FCPI**: Consumer price index of overseas economy (S.A.)

$$\begin{aligned}\text{dlog}\left(FCPI_t\right) = & \left(1 - FCPIUSWEIGHT_t/100\right) \cdot \text{dlog}\left(NUSCPI_t\right) \\ & + FCPIUSWEIGHT_t/100 \cdot \text{dlog}\left(USCPI_t\right)\end{aligned} \quad \text{(D.13)}$$

D.14 **FCPIOR**: Consumer price index of overseas economy

$$FCPIOR_t = FCPI_t \cdot \left(SFFCPI_t/100\right) \quad \text{(D.14.1)}$$

$$SFFCPI_t = SFFCPI_{t-4} + \left(SFFCPI_{t-4} - SFFCPI_{t-8}\right)/2 \quad \text{(D.14.2)}$$

D.15 **USCPI**: Consumer price index of the United States (for all urban consumers, all items)

$$\begin{aligned}\text{dlog}\left(USCPI_t\right) \cdot 400 = & \; 0.949\langle 26.2\rangle \cdot \text{dlog}\left(USCPIXFDEN_t\right) \cdot 400 \\ & + 0.0213\langle 11.6\rangle \cdot \text{dlog}\left(IPIOILD_t\right) \cdot 400\end{aligned} \quad \text{(D.15)}$$

Sample period: 1984/Q1 - 2010/Q4, Adjusted R^2: 0.658, LM Test (p-value): 0.28

D.16 **USCPIXFDEN**: Consumer price index of the United States (for all urban consumers, all items less food and energy)

$$\begin{aligned}USPIX_t = & \; 0.403\langle 35.8\rangle \cdot USPIX_{t-1} + \left(1 - 0.403\right) \cdot USZPI_{t,1} \\ & + 0.0526\langle 2.40\rangle \cdot USGAP_t\end{aligned} \quad \text{(D.16)}$$

Sample period: 1980/Q3 - 2010/Q4

D.17 **USPIX**: Inflation rate of the U.S. consumer price index (for all urban consumers, all items less food and energy)

$$USPIX_t = \text{dlog}\left(USCPIXFDEN_t\right) \cdot 400 \quad \text{(D.17)}$$

D.18 **NUSCPI**: Consumer price index of the non-U.S. economy

$$\begin{aligned} NUSPIX_t &= 0.240\langle 1.86\rangle \cdot FGAP_t + 0.640\langle 5.46\rangle \cdot NUSZPI_{t-1,1} \\ &+\left(1-0.640\right)\cdot HQNUSCPIQ_t \\ &+0.00508\langle 2.59\rangle \cdot \left(\text{dlog}\left(IPIOILD_t\right) - HQIPIOILD\_SS_t\right) \\ &+5.49\langle 4.45\rangle \cdot D942_t \end{aligned} \quad \text{(D.18)}$$

Sample period: 1984/Q1 - 2010/Q4, Adjusted R^2: 0.656, LM Test (p-value): 1.00

D.19 **NUSPIX**: Inflation rate of the non-U.S. consumer price index

$$NUSPIX_t = \text{dlog}\left(NUSCPI_t\right)\cdot 400 \quad \text{(D.19)}$$

D.20 **USIRL**: Long-term interest rate of the United States

$$\begin{aligned} USIRL_t - USZCALL10_t &= 0.0467\langle 0.844\rangle \\ &+0.955\langle 38.0\rangle \cdot \left(USIRL_{t-1} - USZCALL10_{t-1}\right) \end{aligned} \quad \text{(D.20)}$$

Sample period: 1980/Q4 - 2010/Q4, Adjusted R^2: 0.923, LM Test (p-value): 0.01

D.21 **USCALL**: Policy interest rate of the United States

The real policy interest rate is regressed on potential GDP growth:

$$\begin{aligned} USCALL_t - USPIX_t &= -2.57\langle -2.09\rangle \\ &+1.73\langle 4.05\rangle \cdot \log\left(USGDPQ_t / USGDPQ_{t-4}\right)\cdot 100 \end{aligned} \quad \text{(D.21.1)}$$

Sample period: 1981/Q1 - 2010/Q4, Adjusted R^2: 0.650

The equilibrium policy interest rate is defined as the real policy interest rate estimated using equation (D.21.1) plus the trend inflation rate:

$$\begin{aligned} USCALLQ_t &= -2.57 + 1.73\cdot \log\left(USGDPQ_t / USGDPQ_{t-4}\right)\cdot 100 \\ &+HQUSCPIQ_t \end{aligned} \quad \text{(D.21.2)}$$

The monetary policy rule is the Taylor Rule with interest rate smoothing; only the smoothing parameter is estimated in this rule:

$$\begin{aligned} USCALL_t = 0.857\langle 30.7\rangle \cdot USCALL_{t-1} \\ +(1-0.857)\cdot \begin{bmatrix} USCALLQ_t + 0.5 \cdot USGAP_t \\ +1.5 \cdot (USPIX_t - HQUSCPIQ_t) \end{bmatrix} \end{aligned} \quad \text{(D.21.3)}$$

Sample period: 1980/Q3 - 2007/Q2, Adjusted R^2: 0.937

The following equation taking into account the zero lower bound is used in forecasts and simulations:

$$USCALL_t = \max \left\{ \begin{array}{l} 0.857 \cdot USCALL_{t-1} \\ \quad +(1-0.857)\cdot \begin{bmatrix} USCALLQ_t + 0.5 \cdot USGAP_t \\ +1.5 \cdot (USPIX_t - HQUSCPIQ_t) \end{bmatrix}, \\ 0 \end{array} \right\} \quad \text{(D.21.4)}$$

D.22 **USZPI**: Expected inflation rate of the U.S. consumer price index (for all urban consumers, all items less food and energy); **HQUSCPIQ**: Trend inflation rate of the U.S. consumer price index; **USZGAP**: Expected GDP gap of the United States; **USZCALL**: Expected policy interest rate of the United States

The *n*-period ahead expectation of core CPI inflation formed in period t, $USZPI_{t,n}$, the trend inflation, *HQUSCPIQ* (D.22.2), and the core CPI, *USCPIXFDEN* (D.16) are estimated simultaneously using a Kalman filter algorithm with maximum likelihood estimation:

$$\begin{aligned} USZPI_{t,n} = 0.106\langle 9.54\rangle \cdot USZPI_{t,n-1} + (1-0.106)\cdot HQUSCPIQ_t \\ +0.0595\langle 10.2\rangle \cdot USZGAP_{t,n} \end{aligned} \quad \text{(D.22.1)}$$

Sample period: 1980/Q3 - 2010/Q4

Trend inflation is assumed to follow a random walk process:

$$HQUSCPIQ_t = HQUSCPIQ_{t-1} \quad \text{(D.22.2)}$$

The expected GDP gap is assumed to be generated by an AR(2) process:

$$USZGAP_{t,n} = 1.37\langle 17.6\rangle \cdot USZGAP_{t,n-1} - 0.418\langle -5.26\rangle \cdot USZGAP_{t,n-2} \qquad \text{(D.22.3)}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.933

The expected policy interest rate is formed in accordance with the monetary policy rule (D.21.3):

$$\begin{aligned} USZCALL_{t,n} &= 0.857 \cdot USZCALL_{t,n-1} \\ &+ (1-0.857) \cdot \begin{bmatrix} USCALLQ_t + 1.5 \cdot \left(USZPI_{t,n} - HQUSCPIQ_t\right) \\ +0.5 \cdot USZGAP_{t,n} \end{bmatrix} \end{aligned} \qquad \text{(D.22.4)}$$

By summarizing equations (D.22.1), (D.22.3), and (D.22.4), the whole system of VAR expectations can be expressed as follows (when $n \leq 0$, $USZCALL_{t,n}$ = $USCALL_{t+,n}$, $USZPI_{t,n}$ = $USPIX_{t+n}$, $USZGAP_{t,n}$ = $USGAP_{t+n}$):

$$\begin{aligned} &\begin{pmatrix} 1 & -0.214 & -0.0713 & 0 \\ 0 & 1 & -0.0595 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} USZCALL_{t,n} - USCALLQ_t \\ USZPI_{t,n} - HQUSCPIQ_t \\ USZGAP_{t,n} \\ USZGAP_{t,n-1} \end{pmatrix} \\ &= \begin{pmatrix} 0.857 & 0 & 0 & 0 \\ 0 & 0.106 & 0 & 0 \\ 0 & 0 & 1.37 & -0.418 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} USZCALL_{t,n-1} - USCALLQ_t \\ USZPI_{t,n-1} - HQUSCPIQ_t \\ USZGAP_{t,n-1} \\ USZGAP_{t,n-2} \end{pmatrix} \end{aligned} \qquad \text{(D.22.5)}$$

$$\begin{pmatrix} USZCALL_{t,n} - USCALLQ_t \\ USZPI_{t,n} - HQUSCPIQ_t \\ USZGAP_{t,n} \\ USZGAP_{t,n-1} \end{pmatrix}$$

$$= \begin{pmatrix} 0.857 & 0.0227 & 0.272 & -0.0830 \\ 0 & 0.106 & 0.0815 & -0.0249 \\ 0 & 0 & 1.37 & -0.418 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} USZCALL_{t,n-1} - USCALLQ_t \\ USZPI_{t,n-1} - HQUSCPIQ_t \\ USZGAP_{t,n-1} \\ USZGAP_{t,n-2} \end{pmatrix} \qquad \text{(D.22.6)}$$

$$= B \begin{pmatrix} USZCALL_{t,n-1} - USCALLQ_t \\ USZPI_{t,n-1} - HQUSCPIQ_t \\ USZGAP_{t,n-1} \\ USZGAP_{t,n-2} \end{pmatrix}$$

D.23 **USZCALL10**: Long-term expected policy interest rate of the United States (10 year average); **USZPI10**: Long-term expected inflation rate of the U.S. consumer price index (for all urban consumers, all items less food and energy) (10 year average); **USZGAP10**: Long-term expected GDP gap of the United States (10 year average)

$$USZCALL10_t$$

$$= \frac{1}{40} \sum_{n=0}^{39} \max \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} B^n \begin{pmatrix} USZCALL_{t,n} - USCALLQ_t \\ USZPI_{t,n} - HQUSCPIQ_t \\ USZGAP_{t,n} \\ USZGAP_{t,n-1} \end{pmatrix} + USCALLQ_t, \\ 0 \end{bmatrix} \qquad \text{(D.23.1)}$$

$$USZPI10_t = \frac{1}{40} \sum_{j=0}^{39} \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} B^j \begin{pmatrix} USZCALL_{t,j} - USCALLQ_t \\ USZPI_{t,j} - HQUSCPIQ_t \\ USZGAP_{t,j} \\ USZGAP_{t,j-1} \end{pmatrix} + HQUSCPIQ_t \qquad \text{(D.23.2)}$$

$$USZGAP10_t = \frac{1}{40} \sum_{j=0}^{39} \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} B^j \begin{pmatrix} USZCALL_{t,j} - USCALLQ_t \\ USZPI_{t,j} - HQUSCPIQ_t \\ USZGAP_{t,j} \\ USZGAP_{t,j-1} \end{pmatrix} \qquad \text{(D.23.3)}$$

D.24 **NUSCALL**: Policy interest rate of the non-U.S. economy

The real policy interest rate is regressed on potential GDP growth:

$$\begin{aligned} & NUSCALL_t - NUSPIX_t \\ & \quad = 0.707\langle 12.4\rangle \cdot \log\left(NUSGDPQ_t / NUSGDPQ_{t-4}\right) \cdot 100 \end{aligned} \tag{D.24.1}$$

Sample period: 1981/Q1 - 2010/Q4, Adjusted R^2: 0.211

The equilibrium policy interest rate is defined as the real policy interest rate estimated using equation (D.24.1) plus the trend inflation rate:

$$\begin{aligned} NUSCALLQ_t = & \ 0.707 \cdot \log\left(NUSGDPQ_t / NUSGDPQ_{t-4}\right) \cdot 100 \\ & + HQNUSCPIQ_t \end{aligned} \tag{D.24.2}$$

The monetary policy rule is the Taylor Rule with interest rate smoothing:

$$\begin{aligned} NUSCALL_t = & \ 0.962\langle 47.5\rangle \cdot NUSCALL_{t-1} \\ & + (1 - 0.962) \cdot NUSCALLQ_t \\ & + 0.141\langle 3.36\rangle \cdot NUSGAP_t \\ & + 0.0852\langle 2.85\rangle \cdot \left(NUSPIX_t - HQNUSCPIQ_t\right) \end{aligned} \tag{D.24.3}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.974

The following equation taking into account the zero lower bound is used in forecasts and simulations:

$$\begin{aligned} & NUSCALL_t \\ & \quad = \max\left\{ \begin{array}{l} 0.962 \cdot NUSCALL_{t-1} + (1 - 0.962) \cdot NUSCALLQ_t \\ + 0.141 \cdot NUSGAP_t + 0.0852 \cdot \left(NUSPIX_t - HQNUSCPIQ_t\right), \\ 0 \end{array} \right\} \end{aligned} \tag{D.24.4}$$

D.25 **NUSZPI**: Expected inflation rate of the non-U.S. consumer price index; **HQNUSCPIQ**: Trend inflation rate of the non-U.S. consumer price index; **NUSZGAP**: Expected GDP gap of the non-U.S. economy; **NUSZCALL**: Expected policy interest rate of the non-U.S. economy

The *n*-period ahead expectation of CPI inflation formed in period t, $NUSZPI_{t,n}$, is formulated as follows:

$$\begin{aligned} NUSZPI_{t,n} &= 0.441\langle 5.90\rangle \cdot NUSZPI_{t,n-1} + (1-0.441)\cdot HQNUSCPIQ_t \\ &+0.293\langle 2.80\rangle \cdot NUSZGAP_{t,n} + 5.31\langle 4.11\rangle \cdot D942_t \end{aligned} \tag{D.25.1}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.760

Trend inflation is assumed to follow a random walk process:

$$HQNUSCPIQ_t = HQNUSCPIQ_{t-1} \tag{D.25.2}$$

The expected GDP gap is assumed to be generated by an AR(2) process:

$$NUSZGAP_{t,n} = 1.39\langle 18.4\rangle \cdot NUSZGAP_{t,n-1} - 0.566\langle -7.51\rangle \cdot NUSZGAP_{t,n-2} \tag{D.25.3}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.848

The expected policy interest rate is formed in accordance with the monetary policy rule (D.24.3):

$$\begin{aligned} NUSZCALL_{t,n} &= 0.962 \cdot NUSZCALL_{t,n-1} \\ &+ (1-0.962)\cdot NUSCALLQ_t \\ &+0.141\cdot NUSZGAP_{t,n} \\ &+0.0852\cdot\left(NUSZPI_{t,n} - HQNUSCPIQ_t\right) \end{aligned} \tag{D.25.4}$$

By summarizing equations (D.25.1), (D.25.3), and (D.25.4), the whole system of VAR expectations can be expressed as follows (when $n \leq 0$, $NUSZCALL_{t,n} = NUSCALL_{t+,n}$, $NUSZPI_{t,n} = NUSPIX_{t+n}$, $NUSZGAP_{t,n} = NUSGAP_{t+n}$):

$$\begin{pmatrix} 1 & -0.0852 & -0.141 & 0 \\ 0 & 1 & -0.293 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} NUSZCALL_{t,n} - NUSCALLQ_t \\ NUSZPI_{t,n} - HQNUSCPIQ_t \\ NUSZGAP_{t,n} \\ NUSZGAP_{t,n-1} \end{pmatrix}$$

$$= \begin{pmatrix} 0.962 & 0 & 0 & 0 \\ 0 & 0.441 & 0 & 0 \\ 0 & 0 & 1.39 & -0.566 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} NUSZCALL_{t,n-1} - NUSCALLQ_t \\ NUSZPI_{t,n-1} - HQUSNCPIQ_t \\ NUSZGAP_{t,n-1} \\ NUSZGAP_{t,n-2} \end{pmatrix} \tag{D.25.5}$$

$$\begin{pmatrix} NUSZCALL_{t,n} - NUSCALLQ_t \\ NUSZPI_{t,n} - HQNUSCPIQ_t \\ NUSZGAP_{t,n} \\ NUSZGAP_{t,n-1} \end{pmatrix}$$

$$= \begin{pmatrix} 0.962 & 0.0376 & 0.231 & -0.0940 \\ 0 & 0.441 & 0.407 & -0.166 \\ 0 & 0 & 1.39 & -0.566 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} NUSZCALL_{t,n-1} - NUSCALLQ_t \\ NUSZPI_{t,n-1} - HQNUSCPIQ_t \\ NUSZGAP_{t,n-1} \\ NUSZGAP_{t,n-2} \end{pmatrix} \tag{D.25.6}$$

$$= C \begin{pmatrix} NUSZCALL_{t,n-1} - NUSCALLQ_t \\ NUSZPI_{t,n-1} - HQNUSCPIQ_t \\ NUSZGAP_{t,n-1} \\ NUSZGAP_{t,n-2} \end{pmatrix}$$

D.26 **NUSZCALL10**: Long-term expected policy interest rate of the non-U.S. economy (10 year average); **NUSZPI10**: Long-term expected inflation rate of the non-U.S. consumer price index (10 year average); **NUSZGAP10**: Long-term expected GDP gap of the non-U.S. economy (10 year average)

$$NUSZCALL10_t$$

$$= \frac{1}{40} \sum_{n=0}^{39} \max \left[\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} C^n \begin{pmatrix} NUSZCALL_{t,n} - NUSCALLQ_t \\ NUSZPI_{t,n} - HQNUSCPIQ_t \\ NUSZGAP_{t,n} \\ NUSZGAP_{t,n-1} \end{pmatrix} + NUSCALLQ_t, \right. \tag{D.26.1}$$

$$
\begin{aligned}
&NUSZPI10_t \\
&= \frac{1}{40}\sum_{j=0}^{39}\begin{pmatrix}0 & 1 & 0 & 0\end{pmatrix} C^j \begin{pmatrix} NUSZCALL_{t,j} - NUSCALLQ_t \\ NUSZPI_{t,j} - HQNUSCPIQ_t \\ NUSZGAP_{t,j} \\ NUSZGAP_{t,j-1} \end{pmatrix} + HQNUSCPIQ_t
\end{aligned} \qquad \text{(D.26.2)}
$$

$$
NUSZGAP10_t = \frac{1}{40}\sum_{j=0}^{39}\begin{pmatrix}0 & 0 & 1 & 0\end{pmatrix} C^j \begin{pmatrix} NUSZCALL_{t,j} - NUSCALLQ_t \\ NUSZPI_{t,j} - HQNUSCPIQ_t \\ NUSZGAP_{t,j} \\ NUSZGAP_{t,j-1} \end{pmatrix} \qquad \text{(D.26.3)}
$$

E. Balance of Payments

E.1 **EX**: Real exports of goods and services

$$
\begin{aligned}
\log\left(EXQ_t\right) = {} & 4.55\langle 35.2\rangle + 1.16\langle 102\rangle \cdot \log\left(FGDP_t\right) \\
& +2.80\langle 7.19\rangle \cdot FGAP_t/100 \\
& +6.14\langle 5.25\rangle \cdot \log\left(FGDPQ_t/FGDPQ_{t-4}\right) \\
& -0.269\langle -9.73\rangle \cdot \log\left(FXR_t\right)
\end{aligned} \qquad \text{(E.1.1)}
$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.989

$$
\begin{aligned}
\text{dlog}\left(EX_t\right) = {} & -0.0253\langle -4.80\rangle - 0.321\langle -5.40\rangle \cdot \log\left(EX_{t-1}/EXQ_{t-1}\right) \\
& +0.230\langle 3.49\rangle \cdot \text{dlog}\left(EX_{t-1}\right) + 3.53\langle 7.20\rangle \cdot \text{dlog}\left(FGDP_t\right) \\
& -0.157\langle -2.93\rangle \cdot \text{dlog}\left(FXR_t\right)
\end{aligned} \qquad \text{(E.1.2)}
$$

Sample period: 1980/Q4 - 2010/Q4, Adjusted R^2: 0.541, LM Test (p-Value): 0.20

E.2 **EXN**: Nominal exports of goods and services

$$
EXN_t = EX_t \cdot PEX_t/100 \qquad \text{(E.2)}
$$

E.3 **IMOILN**: Nominal imports (petroleum)

$$\begin{aligned}&\text{dlog}\left(IMOILN_t \cdot 4/10/FDN_t\right)\\&\quad = -0.00741\langle -1.04\rangle + \text{dlog}\left(IPIOIL_t/CGPD_t\right)\end{aligned} \tag{E.3}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.772, LM Test (p-Value): 0.01

E.4 **IMXON**: Nominal imports (excluding petroleum)

$$\begin{aligned}&\log\left(IMXONQ_t\right) = -4.88\langle -377\rangle + \log\left(EXN_t\right)\\&\quad + \log\left(FXR_t\right) + \log\left(IPIXO_t/EPI_t\right)\end{aligned} \tag{E.4.1}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.817

$$\begin{aligned}\text{dlog}\left(IMXON_t\right) &= 0.00174\langle 0.565\rangle\\&\quad -0.0562\langle -2.89\rangle \cdot \log\left(IMXON_{t-1}/IMXONQ_{t-1}\right)\\&\quad +0.552\langle 7.22\rangle \cdot \text{dlog}\left(EX_t\right) + 0.552\langle 1.81\rangle \cdot \text{dlog}\left(DDF_t\right)\\&\quad +0.658\langle 9.43\rangle \cdot \text{dlog}\left(IPIXO_t\right)\end{aligned} \tag{E.4.2}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.681, LM Test (p-value): 0.96

E.5 **IMN**: Nominal imports of goods and services

$$IMN_t = IMOILN_t \cdot 4/10 + IMXON_t \tag{E.5}$$

E.6 **IM**: Real imports of goods and services

$$IM_t = IMN_t/\left(PIM_t/100\right) \tag{E.6}$$

E.7 **BPNEXI**: Nominal income balance

$$\frac{BPNEXI_t \cdot 4/10}{GDPN_t} = -0.0120\langle -17.6\rangle + 26.0\langle 42.1\rangle \cdot \frac{FGDP_t \cdot FCPI_t}{GDP_t \cdot CPI_t / 100 \cdot FXN_t} \quad \text{(E.7)}$$

Sample period: 1988/Q1 - 2010/Q4, Adjusted R^2: 0.951, LM Test (p-value): 0.00

E.8 **BPNTR**: Nominal transfer balance

$$\frac{BPNTR_t \cdot 4/10}{GDPN_t} = -0.00122\langle -21.3\rangle - 0.00512\langle -11.2\rangle \cdot D911_t - 0.000911\langle -11.1\rangle \cdot D961Z_t \quad \text{(E.8)}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.649, LM Test (p-value): 0.00

E.9 **CA**: Current account

$$\begin{aligned} CA_t / GDPN_t = & -0.00105\langle -3.55\rangle \\ & + \left(\left(EXN_t - IMN_t \right) \cdot 10/4 + BPNEXI_t + BPNTR_t \right) / GDPN_t \\ & + 0.00535\langle 9.17\rangle \cdot D031Z_t \end{aligned} \quad \text{(E.9)}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.991, LM Test (p-value): 0.06

F. Business Investment Expenditure and Production

F.1 **INV**: Real private non-residential investment (See Section 5.2 in the text.)

$$\begin{aligned} \log\left(\left(INVQ_t \cdot PINV_t \right) / \left(GDP_t \cdot PGDP_t \right) \right) = & -2.76\langle -55.5\rangle \\ & + 0.113\langle 12.9\rangle \cdot \left(HYGDPQ_t \cdot 100 + DFT_t \right) \\ & - 0.0208\langle -3.33\rangle \cdot \left(IRLOAN_t - ZPI10_t + DFT_t \right) \end{aligned} \quad \text{(F.1.1)}$$

Sample period: 1981/Q2 - 2010/Q1, Adjusted R^2: 0.741

$$\begin{aligned}\text{dlog}\left(INV_t\right) = &-0.00191\langle -0.612\rangle - 0.124\langle -2.55\rangle \cdot \log\left(INV_{t-1}/INVQ_{t-1}\right)\\ &+ 0.183\langle 2.80\rangle \cdot \text{dlog}\left(EX_{t-1}\right) + 0.000631\langle 3.44\rangle \cdot LOANDI_{t-1}\\ &+ 0.221\langle 2.59\rangle \cdot \text{dlog}\left(\left(RSDPROF_t - CTAX_t + D_t - GDN_t\right)/PINV_t\right)\end{aligned} \qquad \text{(F.1.2)}$$

Sample period: 1983/Q3 - 2010/Q1, Adjusted R^2: 0.306, LM Test (p-value): 0.45

F.2 **INVN**: Nominal private non-residential investment

$$INVN_t = INV_t \cdot PINV_t/100 \qquad \text{(F.2)}$$

F.3 **IIPS**: Producer shipments (mining and manufacturing)

$$\begin{aligned}\text{dlog}\left(IIPS_t\right) = &-0.00308\langle -2.48\rangle\\ &+1.80\langle 2.59\rangle \cdot IHN_{t-1}/GDPN_{t-1} \cdot \text{dlog}\left(IH_t\right)\\ &+0.867\langle 3.41\rangle \cdot INVN_{t-1}/GDPN_{t-1} \cdot \text{dlog}\left(INV_t\right)\\ &+4.21\langle 19.0\rangle \cdot EXN_{t-1}/GDPN_{t-1} \cdot \text{dlog}\left(EX_t\right)\end{aligned} \qquad \text{(F.3)}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.792, LM Test (p-value): 0.64

F.4 **IIPP**: Industrial production (mining and manufacturing)

$$\begin{aligned}\log\left(IIPPQ_t/IIPS_t\right) = &\ 0.0650\langle 41.7\rangle - 0.000628\langle -36.1\rangle \cdot T801_t\\ &+0.0224\langle 17.9\rangle \cdot D834981_t\end{aligned} \qquad \text{(F.4.1)}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.959

$$\begin{aligned}\text{dlog}\left(IIPP_t\right) = & -0.000465\langle -1.11\rangle \\ & -0.345\langle -4.75\rangle \cdot \log\left(IIPP_{t-1}/IIPPQ_{t-1}\right) \\ & +0.944\langle 31.4\rangle \cdot \text{dlog}\left(IIPS_t\right) \\ & -0.0610\langle -2.71\rangle \cdot \text{dlog}\left(IIPIV_{t-1}/IIPS_t\right)\end{aligned} \tag{F.4.2}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.977, LM Test (p-value): 0.65

F.5 **IIPIV**: Producer inventories (mining and manufacturing)

$$\log\left(IIPIVQ_t/IIPS_t\right) = 0.362\langle 26.8\rangle - 1.15\langle -15.3\rangle \cdot T801_t/400 \tag{F.5.1}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.654

$$\begin{aligned}\text{dlog}\left(IIPIV_t\right) = & \ 0.000908\langle 0.701\rangle \\ & -0.0783\langle -3.93\rangle \cdot \log\left(IIPIV_{t-1}/IIPIVQ_{t-1}\right) \\ & +0.439\langle 5.50\rangle \cdot \text{dlog}\left(IIPIV_{t-1}\right) + 1.962\langle 7.05\rangle \cdot \text{dlog}\left(IIPP_t\right) \\ & -1.86\langle -6.68\rangle \cdot \text{dlog}\left(IIPS_t\right)\end{aligned} \tag{F.5.2}$$

Sample period: 1980/Q2 - 2010/Q4: Adjusted R^2: 0.509, LM Test (p-value): 0.02

F.6 **KIV**: Real private inventory stock

$$\begin{aligned}\log\left(KIVQ_t/FD_t\right) = & -2.01\langle -130\rangle \\ & +8.17\langle 3.92\rangle / \left(T801_t + 30.4\langle 4.14\rangle\right)\end{aligned} \tag{F.6.1}$$

Sample period: 1980/Q1 - 2008/Q4, Adjusted R^2: 0.783

$$\begin{aligned}\text{dlog}\left(KIV_t\right) = & \ 0.00379\langle 5.65\rangle \\ & -0.0614\langle -3.06\rangle \cdot \log\left(KIV_{t-1}/KIVQ_{t-1}\right) \\ & +0.241\langle 2.48\rangle \cdot \text{dlog}\left(KIV_{t-1}\right) \\ & +0.0726\langle 2.20\rangle \cdot \text{dlog}\left(IIPIV_{t-1}\right)\end{aligned} \tag{F.6.2}$$

Sample period: 1980/Q2 - 2008/Q4, Adjusted R^2: 0.266, LM Test (p-value): 0.02

F.7 **SNAIV**: Real private inventory

$$SNAIV_t = \left(KIV_t - KIV_{t-1}\right) \cdot 4 \tag{F.7}$$

F.8 **SNAIVN**: Nominal private inventory

$$SNAIVN_t = SNAIV_t \cdot PSNAIV_t / 100 \tag{F.8}$$

F.9 **PROF**: Ordinary profits (large enterprises, S.A.)

$$\begin{aligned} \log\left(PROF_t / GDPNADJ_t\right) = & 1.55\langle 4.82\rangle \\ & -5.76\langle -9.25\rangle \cdot \log\left(YWN_t / GDPNADJ_t\right) \\ & +0.688\langle 7.33\rangle \cdot \log\left(EXN_t / GDPNADJ_t\right) \end{aligned} \tag{F.9}$$

Sample period: 1980/Q1 - 2010/Q1, Adjusted R^2: 0.575, LM Test (p-value): 0.00

F.10 **RSDPROF**: Operating surplus as the residual

$$RSDPROF_t = GDPN_t - \left(YWN_t + D_t + NINDTAX_t\right) \tag{F.10}$$

G. Labor Market

G.1 **L**: Employed persons

$$\log\left(L_t\right) = \log\left(LQ_t\right) + LGAP_t \tag{G.1}$$

G.2 **LGAP**: Gap of employed persons

$$LGAP_t = 0.879\langle 27.1\rangle \cdot LGAP_{t-1} + 0.0397\langle 4.02\rangle \cdot GAP_{t-1}/100 \qquad \text{(G.2)}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.879. LM Test (p-value): 1.00

G.3 **LW**: Employees

$$LW_t = L_t - LSELF_t \qquad \text{(G.3)}$$

G.4 **LSELF**: Self-employed workers and family workers

$$\begin{aligned}\log\left(LSELF_t/LW_t\right) = &-0.186\langle -2.57\rangle \\ &+0.948\langle 45.2\rangle \cdot \log\left(LSELF_{t-1}/LW_{t-1}\right) \\ &+7.24\langle 2.41\rangle \cdot \frac{1}{20}\sum_{s=0}^{19}\left(L_{t-s}/GDP_{t-s}\right)\end{aligned} \qquad \text{(G.4)}$$

Sample period: 1984/Q4 - 2010/Q4, Adjusted R^2: 0.997, LM Test (p-value): 0.14

G.5 **LF**: Labour force

$$\log\left(LF_t\right) = \log\left(LFQ_t\right) + LFGAP_t \qquad \text{(G.5)}$$

G.6 **LFGAP**: Gap of labour force

$$LFGAP_t = 0.866\langle 20.0\rangle \cdot LFGAP_{t-1} + 0.0182\langle 2.20\rangle \cdot GAP_{t-1}/100 \qquad \text{(G.6)}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.776, LM Test (p-value): 0.07

G.7 **U**: Unemployment rate (S.A.)

$$U_t = \left(1 - L_t / LF_t\right) \cdot 100 \tag{G.7}$$

G.8 **UGAP**: Unemployment rate gap

$$UGAP_t = U_t - UQ_t \tag{G.8}$$

G.9 **H**: Total hours worked (industries covered, S.A.)

$$\log\left(H_t/12\right) = \log\left(HQ_t\right) + HGAP_t \tag{G.9}$$

G.10 **HGAP**: Gap of total hours worked

$$\begin{aligned} HGAP_t + LGAP_t = 0.932\langle 31.8\rangle \cdot \left(HGAP_{t-1} + LGAP_{t-1}\right) \\ -0.0544\langle -2.64\rangle \cdot \log\left(WHN_{t-1}/WHNQ_{t-1}\right) \end{aligned} \tag{G.10}$$

Sample period: 1981/Q2 - 2010/Q1, Adjusted R^2: 0.909, LM Test (p-value): 0.00

G.11 **WHN**: Nominal wage (per hour)

$$\begin{aligned} &\log\left(\left(WHNQ_t \cdot LW_t \cdot H_t\right)/GDPNADJ_t\right) \\ &\quad = -0.351\langle -68.6\rangle \\ &\quad - 0.0324\langle -5.73\rangle \cdot D851Z_t - 0.0365\langle -7.94\rangle \cdot D033Z_t \end{aligned} \tag{G.11.1}$$

Sample period: 1981/Q1 - 2010/Q1, Adjusted R^2: 0.507

$$\begin{aligned} &\mathrm{dlog}\left(WHN_t\right) = \mathrm{dlog}\left(GDPQ_t/\left(LQ_t \cdot HQ_t \cdot 12\right)\right) \\ &-0.126\langle -2.71\rangle \cdot \log\left(WHN_{t-1}/WHNQ_{t-1}\right) \\ &+0.348\langle 4.06\rangle \cdot \left(\mathrm{dlog}\left(WHN_{t-4}\right) - \mathrm{dlog}\left(GDPQ_{t-4}/\left(LQ_{t-4} \cdot HQ_{t-4} \cdot 12\right)\right)\right) \\ &+\left(1 - 0.348\right) \cdot ZPI_{t,1}/400 \end{aligned} \tag{G.11.2}$$

Sample period: 1981/Q2 - 2010/Q1, Adjusted R^2: 0.230, LM Test (p-value): 1.00

H. Household Income

H.1 **YDN**: Nominal disposable income, net (households including private unincorporated enterprises)

$$YDN_t = YWN_t + \left(YPR_t - YPP_t\right) + YOSMI_t + YTR_t - YTAX_t \qquad \text{(H.1)}$$

H.2 **YWN**: Nominal compensation of employees

$$YWN_t = WHN_t \cdot LW_t \cdot H_t \qquad \text{(H.2)}$$

H.3 **YW**: Real compensation of employees

$$YW_t = YWN_t / PGDP_t \cdot 100 \qquad \text{(H.3)}$$

H.4 **YPR**: Property income, receivable (households including private unincorporated enterprises)

$$YPR_t = YPRDIV_t + YPRXDIV_t \qquad \text{(H.4)}$$

H.5 **YPRDIV**: Property income (dividends), receivable (households including private unincorporated enterprises)

$$\begin{aligned} YPRDIV_t/4/FA_{t-1} = {} & 0.000152\langle 2.58\rangle + 0.485\langle 6.02\rangle \cdot YPRDIV_{t-1}/4/FA_{t-2} \\ & +0.000187\langle 3.17\rangle \cdot PROF_t/4/FA_{t-1} \\ & +0.0000943\langle 2.54\rangle \cdot D044Z_t \cdot PROF_t/4/FA_{t-1} \end{aligned} \qquad \text{(H.5)}$$

Sample period: 1981/Q1 - 2010/Q1, Adjusted R^2: 0.559, LM Test (p-value): 0.00

H.6 **YPRXDIV**: Property income (excluding dividends), receivable (households including private unincorporated enterprises)

$$\begin{aligned} YPRXDIV_t/4/FASAFE_{t-1} = & -0.0000123\langle -0.0825\rangle \\ & +0.920\langle 45.9\rangle \cdot YPRXDIV_{t-1}/4/FASAFE_{t-2} \\ & +0.114\langle 4.39\rangle \cdot IRL_t/400 \end{aligned} \quad \text{(H.6)}$$

Sample period: 1980/Q3 - 2010/Q1, Adjusted R^2: 0.992, LM Test (p-value): 0.42

H.7 **YPP**: Property income, payable (households including private unincorporated enterprises)

$$\begin{aligned} YPP_t/4/LOAN_{t-1} = & \ 0.00165\langle 9.03\rangle + 0.784\langle 36.57\rangle \cdot YPP_{t-1}/4/LOAN_{t-2} \\ & +0.157\langle 10.2\rangle \cdot IRLOAN_t/400 \end{aligned} \quad \text{(H.7)}$$

Sample period: 1981/Q1 - 2010/Q1, Adjusted R^2: 0.998, LM Test (p-value): 0.10

H.8 **YOSMIGR**: Operating surplus and mixed income, gross (households including private unincorporated enterprises)

$$YOSMIGR_t = YHOUSEGR_t + YMIXGR_t \quad \text{(H.8)}$$

H.9 **YOSMI**: Operating surplus and mixed income, net (households including private unincorporated enterprises)

$$YOSMI_t = YHOUSE_t + YMIX_t \quad \text{(H.9)}$$

H.10 **YHOUSEGR**: Operating surplus (imputed service of owner-occupied dwellings), gross (households including private unincorporated enterprises)

$$\log\left(\frac{YHOUSEGR_t / GDPN_t}{YHOUSEGR\_GDPN\_HP_t}\right) = 0.618\langle 8.58\rangle \cdot \log\left(\frac{YHOUSEGR_{t-1} / GDPN_{t-1}}{YHOUSEGR\_GDPN\_HP_{t-1}}\right) \quad \text{(H.10)}$$

Sample period: 1980/Q2 - 2010/Q1, Adjusted R^2: 0.382, LM Test (p-value): 0.25

H.11 **YHOUSE**: Operating surplus (imputed service of owner-occupied dwellings), net (households including private unincorporated enterprises)

$$\log\left(YHOUSE_t / GDPN_t / YHOUSE\_GDPN\_HP_t\right) = 0.708\langle 11.0\rangle \cdot \log\left(YHOUSE_{t-1} / GDPN_{t-1} / YHOUSE\_GDPN\_HP_{t-1}\right) \quad \text{(H.11)}$$

Sample period: 1980/Q2 - 2010/Q1, Adjusted R^2: 0.503, LM Test (p-value): 0.58

H.12 **YMIXGR**: Mixed income, gross (households including private unincorporated enterprises)

$$\log\left(YMIXGRQ_t / GDPNADJ_t\right) - \log\left(LSELF_t / LW_t\right) = -0.0317\langle -0.522\rangle + 0.333\langle 15.0\rangle \cdot \log\left(IGN_t / GDPN_t\right) \quad \text{(H.12.1)}$$

Sample period: 1980/Q1 - 2010/Q1, Adjusted R^2: 0.650

$$\text{dlog}\left(YMIXGR_t\right) = -0.00473\langle -1.18\rangle - 0.397\langle -5.93\rangle \cdot \log\left(YMIXGR_{t-1} / YMIXGRQ_{t-1}\right) \quad \text{(H.12.2)}$$

Sample period: 1980/Q2 - 2010/Q1, Adjusted R^2: 0.223, LM Test (p-value): 0.17

H.13 **YMIX**: Mixed income, net (households including private unincorporated enterprises)

$$\begin{aligned}&\log\left(YMIXQ_t/GDPNADJ_t\right)-\log\left(LSELF_t/LW_t\right)\\&\quad=-0.206\langle-2.99\rangle+0.356\langle 14.1\rangle\cdot\log\left(IGN_t/GDPN_t\right)\end{aligned} \tag{H.13.1}$$

Sample period: 1980/Q1 - 2010/Q1, Adjusted R^2: 0.622

$$\begin{aligned}\mathrm{dlog}\left(YMIX_t\right)=&-0.00550\langle-1.10\rangle\\&-0.463\langle-6.28\rangle\cdot\log\left(YMIX_{t-1}/YMIXQ_{t-1}\right)\end{aligned} \tag{H.13.2}$$

Sample period: 1980/Q2 - 2010/Q1, Adjusted R^2: 0.244, LM Test (p-value): 0.25

H.14 **YTR**: Social contributions and other current transfers, excluding social transfer in kind, net (households including private unincorporated enterprises)

$$YTR_t = YTRR_t - YTRP_t \tag{H.14}$$

H.15 **YTRR**: Social benefits other than social transfers in kind and other current transfer, receivable (households including private unincorporated enterprises)

$$\begin{aligned}\log\left(YTRR_t/GDPN_t\right)=&-0.663\langle-5.11\rangle\\&+0.575\langle 7.54\rangle\cdot\log\left(YTRR_{t-1}/GDPN_{t-1}\right)\\&+0.675\langle 4.87\rangle\cdot\frac{1}{20}\sum_{s=0}^{19}\frac{POP60_{t-s}}{POP15_{t-s}}\\&+0.0842\langle 4.32\rangle\cdot\log\left(U_t/100\right)\end{aligned} \tag{H.15}$$

Sample period: 1980/Q3 - 2010/Q1, Adjusted R^2: 0.974, LM Test (p-value): 0.51

H.16 **YTRP**: Social contributions and other current transfers, payable (households including private unincorporated enterprises)

$$\begin{aligned}\log\left(YTRP_t/YWN_t\right) = &-0.102\langle -2.96\rangle \\ &+0.387\langle 4.68\rangle \cdot \log\left(YTRP_{t-1}/YWN_{t-1}\right) \\ &+0.440\langle 5.61\rangle \cdot \log\left(YTRP_{t-2}/YWN_{t-2}\right) \\ &+0.0769\langle 2.39\rangle \cdot \log\left(YTRR_t/YWN_t\right)\end{aligned} \quad \text{(H.16)}$$

Sample period: 1980/Q3 - 2010/Q1, Adjusted R^2: 0.912, LM Test (p-value): 0.70

H.17 **YTAX**: Current taxes on income, wealth, etc., payable (households including private unincorporated enterprises)

$$YTAX_t = YTAXRAT_t \cdot \left(YWN_t + YMIXGR_t + YTR_t\right) \quad \text{(H.17)}$$

H.18 **YTAXRAT**: Effective income tax rate

$$\begin{aligned}\log\left(YTAXRAT_t\right) = &-0.280\langle -3.54\rangle \\ &+0.528\langle 6.10\rangle \cdot \log\left(YTAXRAT_{t-1}\right) \\ &+0.348\langle 4.19\rangle \cdot \log\left(YTAXRAT_{t-2}\right) \\ &+0.836\langle 4.53\rangle \cdot GAP_t/100\end{aligned} \quad \text{(H.18)}$$

Sample period: 1980/Q3 - 2010/Q1, Adjusted R^2: 0.902, LM Test (p-value): 0.63

H.19 **FA**: Financial assets (households and private nonprofit institutions serving households)

$$FA_t = FASAFE_t + FARISK_t \quad \text{(H.19)}$$

H.20 **FASAFE**: Safe financial assets (households and private nonprofit institutions serving households)

$$FASAFE_t / FASAFE_{t-1} - 1 = 0.00161\langle 3.49\rangle + 0.905\langle 27.9\rangle \cdot \left(YDN_t - CN_t\right) / 4 / FASAFE_{t-1} \quad \text{(H.20)}$$

Sample period: 1980/Q2 - 2010/Q1, Adjusted R^2: 0.868, LM Test (p-value): 0.00

H.21 **FARISK**: Risky financial assets (households and private nonprofit institutions serving households)

$$\text{dlog}\left(FARISK_t\right) = 0.00391\langle 1.66\rangle + 0.148\langle 5.98\rangle \cdot \text{dlog}\left(TOPIX_t\right) \quad \text{(H.21)}$$

Sample period: 1998/Q2 - 2010/Q4, Adjusted R^2: 0.410, LM Test (p-value): 0.41

H.22 **LOAN**: Loans (households and private nonprofit institutions serving households)

$$LOAN_t = LOANH_t + LOANXH_t \quad \text{(H.22)}$$

H.23 **LOANH**: Housing loans (households and private nonprofit institutions serving households)

$$\begin{aligned} &LOANH_t / LOANH_{t-1} - 1 \\ &= -0.0126\langle -12.1\rangle + 0.598\langle 23.9\rangle \cdot IHN_t / 4 / LOANH_{t-1} \\ &- 0.0776\langle -2.96\rangle \cdot \left(YDN_t - CN_t - 4 \cdot \text{d}\left(FASAFE_t\right)\right) / 4 / LOANH_{t-1} \end{aligned} \quad \text{(H.23)}$$

Sample period: 1986/Q1 - 2010/Q1, Adjusted R^2: 0.867, LM Test (p-value): 0.00

H.24 **LOANXH**: Loans, excluding housing loans (households and private nonprofit institutions serving households)

$$\begin{aligned}\text{dlog}\left(LOANXH_t/GDPN_t\right) = &-0.0179\langle -7.21\rangle + 0.738\langle 7.95\rangle \cdot HYGDPQ_t \\ &-0.184\langle -3.44\rangle \cdot GAP_t/100\end{aligned} \quad \text{(H.24)}$$

Sample period: 1981/Q1 - 2010/Q4, Adjusted R^2: 0.367, LM Test (p-value): 0.86

I. Household Expenditures

I.1 **C**: Real private consumption (See Section 5.1 in the text)

$$\begin{aligned}\log\left(CQ_t \cdot PC_t/100\right) = &\ 0.0469\langle 3.74\rangle \\ &+ 0.0784\langle 9.09\rangle \cdot \frac{1}{2}\sum_{s=0}^{1}\frac{\left(YDN_{t-s} - ZYW10_{t-s}\cdot PGDP_{t-s}/100\right)}{TMPINCOME\_SS_t} \\ &+\left(1 - 0.116\langle 5.32\rangle \cdot \frac{1}{20}\sum_{s=0}^{19}\frac{POP60_{t-s}}{POP15_{t-s}}\right) \\ &\quad \cdot \log\left(\frac{1}{2}\sum_{s=0}^{1}\left(1 - YTAXRAT_{t-s}\right)\cdot ZYW10_{t-s}\cdot PGDP_{t-s}/100\right) \\ &+ 0.116 \cdot \log\left(FA_{t-1} - LOAN_{t-1}\right)\cdot \frac{1}{20}\sum_{s=0}^{19}\frac{POP60_{t-s}}{POP15_{t-s}}\end{aligned} \quad \text{(I.1.1)}$$

Sample period: 1983/Q2 - 2010/Q1, Adjusted R^2: 0.983

$$\begin{aligned}\text{dlog}\left(C_t\right) = &-0.0195\langle -2.75\rangle - 0.0539\langle -1.76\rangle \cdot \log\left(C_{t-1}/CQ_{t-1}\right) \\ &+0.000540\langle 3.14\rangle \cdot CMIND_t + 0.301\langle 2.29\rangle \cdot \frac{1}{2}\sum_{s=2}^{3}\text{dlog}(C_{t-s}) \\ &+0.0194\langle 2.57\rangle \cdot D891_t - 0.0264\langle -3.51\rangle \cdot D892_t \\ &+0.0168\langle 2.23\rangle \cdot D971_t - 0.0418\langle -5.56\rangle \cdot D972_t\end{aligned} \quad \text{(I.1.2)}$$

Sample period: 1983/Q3 - 2010/Q1, Adjusted R^2: 0.434, LM Test (p-value): 0.20

I.2 **CN**: Nominal private consumption

$$CN_t = C_t \cdot PC_t / 100 \qquad \text{(I.2)}$$

I.3 **CMIND**: Consumer confidence index (households more than two-persons, S.A.)

$$\begin{aligned} CMIND_t = {} & 10.2\langle 4.42\rangle + 0.748\langle 13.5\rangle \cdot CMIND_{t-1} \\ & +27.3\langle 3.75\rangle \cdot \mathrm{d}\left(FA_t + CUMFB_t\right) / GDPN_t \\ & -12.9\langle -2.34\rangle \cdot \left(ZUGAP_{t,2} - UGAP_t\right) \\ & +0.0333\langle 4.06\rangle \cdot \log\left(\frac{TOPIX_t / PGDP_t}{TOPIX_{t-4} / PGDP_{t-4}}\right) \cdot 100 \\ & +2.95\langle 3.23\rangle \cdot D091102_t \end{aligned} \qquad \text{(I.3)}$$

Sample period: 1982/Q3 - 2010/Q1, Adjusted R^2: 0.876, LM Test (p-value): 0.02

I.4 **IH**: Real private residential investment

$$\begin{aligned} \log\left(\frac{IHQ_t \cdot \left(PIH_t / 100\right)}{CN_t}\right) = {} & -1.35\langle -24.3\rangle \\ & -34.2\langle -6.32\rangle \cdot \left((IRLOAN_t - ZPI10_t)/400 - HYGDPQ_t / 4\right) \\ & -4.93\langle -22.4\rangle \cdot \frac{1}{20} \sum_{s=0}^{19} \frac{POP60_{t-s}}{POP15_{t-s}} \end{aligned} \qquad \text{(I.4.1)}$$

Sample period: 1983/Q2 - 2010/Q4, Adjusted R^2: 0.825

$$\begin{aligned} \mathrm{dlog}\left(IH_t\right) = {} & -0.0000918\langle -0.0312\rangle \\ & -0.122\langle -4.06\rangle \cdot \log\left(IH_{t-1} / IHQ_{t-1}\right) \\ & +0.430\langle 5.70\rangle \cdot \mathrm{dlog}\left(IH_{t-1}\right) - 0.0748\langle -2.39\rangle \cdot D972_t \end{aligned} \qquad \text{(I.4.2)}$$

Sample period: 1983/Q3 - 2010/Q4, Adjusted R^2: 0.344, LM Test (p-value): 0.23

I.5 **IHN**: Nominal private residential investment

$$IHN_t = IH_t \cdot PIH_t / 100 \quad \text{(I.5)}$$

J. Prices

J.1 **EPI**: Export price index (all commodities, yen basis)

$$\begin{aligned}\log(EPIQ_t) = &\ 1.44\langle 4.07\rangle + 0.345\langle 16.3\rangle \cdot \log(FPPI_t / FXN_t) \\ &+0.305\langle 3.36\rangle \cdot \log(CGPDXOV_t) \\ &+2.19\langle 22.3\rangle \cdot USGDPQ_t / FGDPQ_t\end{aligned} \quad \text{(J.1.1)}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.977

$$\begin{aligned}\mathrm{dlog}(EPI_t) = &-0.00338\langle -3.67\rangle \\ &-0.0912\langle -2.91\rangle \cdot \log(EPI_{t-1} / EPIQ_{t-1}) \\ &+0.571\langle 29.4\rangle \cdot \mathrm{dlog}(FPPI_t / FXN_t) \\ &+(1-0.571) \cdot \mathrm{dlog}(CGPDXOV_t)\end{aligned} \quad \text{(J.1.2)}$$

Sample period: 1980/Q4 - 2010/Q4, Adjusted R^2: 0.887, LM Test (p-value): 0.00

J.2 **IPI**: Import price index (all commodities, yen basis)

$$IPI_t = IPIOILW_t / 1000 \cdot IPIOIL_t + (1 - IPIOILW_t / 1000) \cdot IPIXO_t \quad \text{(J.2)}$$

J.3 **IPIOIL**: Import price index (Petroleum, yen basis)

$$IPIOIL_t = IPIOILD_t \cdot FXYEN_t \quad \text{(J.3)}$$

J.4 **IPIOILD**: Import price index (petroleum, U.S. dollar basis)

$$\begin{aligned}
&\text{dlog}\left(IPIOILD_t\right) - HQIPIOILD_SS_t \\
&\quad = 0.230\langle 2.74\rangle \cdot \left(\text{dlog}\left(IPIOILD_{t-1}\right) - HQIPIOILD_SS_t\right) \\
&\quad + 7.84\langle 3.44\rangle \cdot \left(\text{dlog}\left(FGDP_t\right) - HQFGDP_SS_t\right)
\end{aligned} \tag{J.4}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.133, LM Test (p-value): 0.09

J.5 **IPIXO**: Import price index (excluding petroleum, yen basis)

$$\begin{aligned}
&IPIXOQ_t = IPIQ_t \\
&\log\left(IPIQ_t\right) = 3.84\langle 272\rangle + \log\left(FPPI_t / FXN_t\right) \\
&\qquad -0.300\langle -19.5\rangle \cdot D854Z_t
\end{aligned} \tag{J.5.1}$$

Sample period: 1981/Q1 - 2010/Q4, Adjusted R^2: 0.946

$$\begin{aligned}
\text{dlog}\left(IPIXO_t\right) = &-0.00243\langle -1.39\rangle \\
&-0.0752\langle -2.59\rangle \cdot \log\left(IPI_{t-1} / IPIXOQ_{t-1}\right) \\
&+0.727\langle 20.5\rangle \cdot \text{dlog}\left(FPPI_t / FXN_t\right) \\
&+\left(1-0.727\right) \cdot \text{dlog}\left(CGPDXOV_{t-1}\right) \\
&-0.117\langle -6.07\rangle \cdot D901_t
\end{aligned} \tag{J.5.2}$$

Sample period: 1981/Q2 - 2010/Q4, Adjusted R^2: 0.796, LM Test (p-value): 0.00

J.6 **CGPD**: Domestic corporate goods price index (all commodities)

$$\begin{aligned}
CGPD_t = &CGPDOILW_t / 1000 \cdot CGPDOIL_t \\
&+\left(1 - CGPDOILW_t / 1000\right) \cdot CGPDXO_t
\end{aligned} \tag{J.6}$$

J.7 **CGPDV**: Domestic corporate goods price index (all commodities, excluding consumption tax)

$$CGPDV_t = CGPD_t / (1 + VATCGPD_t) \qquad \text{(J.7)}$$

J.8 **CGPDOIL**: Domestic corporate goods price index (petroleum and coal products)

$$\begin{aligned} CGPDOIL_t &= CGPDGASOLINEW_t / CGPDOILW_t \cdot CGPDGASOLINE_t \\ &+ (1 - CGPDGASOLINEW_t / CGPDOILW_t) \cdot CGPDOILXG_t \end{aligned} \qquad \text{(J.8)}$$

J.9 **CGPDOILV**: Domestic corporate goods price index (petroleum and coal products, excluding consumption tax)

$$CGPDOILV_t = CGPDOIL_t / (1 + VATCGPDOIL_t) \qquad \text{(J.9)}$$

J.10 **CGPDGASOLINE**: Domestic corporate goods price index (gasoline)

The percentage change (q/q) of gasoline in the corporate goods price index is equal to the growth rate of its wholesale price deducting the constant margin (11.6 yen) from the retail price of gasoline (*GASOLINEMP*).

$$\text{dlog}(CGPDGASOLINE_t) = \text{dlog}(GASOLINEMP_t - 11.6) \qquad \text{(J.10)}$$

J.11 **CGPDOILXG**: Domestic corporate goods price index (petroleum and coal products, excluding gasoline)

$$\begin{aligned} \text{dlog}(CGPDOILXG_t) = 0.00425 &\langle 1.01 \rangle \\ &+ 0.356 \langle 12.0 \rangle \cdot \text{dlog}(IPIOIL_t) \\ &+ 0.159 \langle 5.35 \rangle \cdot \text{dlog}(IPIOIL_{t-1}) \\ &+ 0.0649 \langle 2.18 \rangle \cdot \text{dlog}(IPIOIL_{t-2}) \end{aligned} \qquad \text{(J.11)}$$

Sample period: 1980/Q4 - 2010/Q4, Adjusted R^2: 0.667, LM Test (p-value): 0.61

J.12 **CGPDXO**: Domestic corporate goods price index (excluding petroleum and coal products)

$$CGPDXO_t = \frac{CGPDEW_t}{1000 - CGPDOILW_t} \cdot CGPDEV_t \cdot \left(1 + VATCGPDE_t\right) + \frac{1000 - CGPDOILW_t - CGPDEW_t}{1000 - CGPDOILW_t} \cdot CGPDXOESV_t \cdot \left(1 + VATCGPDXOE_t\right) \cdot \left(1 + SUMMERCGPDXOE_t\right) \quad \text{(J.12)}$$

J.13 **CGPDXOV**: Domestic corporate goods price index (excluding petroleum and coal products and consumption tax)

$$CGPDXOV_t = CGPDXO_t \big/ \left(1 + VATCGPDXO_t\right) \quad \text{(J.13)}$$

J.14 **CGPDEV**: Domestic corporate goods price index (electric and electronic products, excluding consumption tax)

$$\begin{aligned} \mathrm{dlog}\left(CGPDEV_t\right) = & -0.00699\langle -6.42\rangle \\ & +0.293\langle 3.22\rangle\, \mathrm{dlog}\left(CGPDEV_{t-1}\right) \\ & +0.0536\langle 2.66\rangle \cdot GAP_{t-1}/100 \\ & -0.0101\langle -3.90\rangle \cdot D003014_t \end{aligned} \quad \text{(J.14)}$$

Sample period: 1986/Q1 - 2010/Q4: Adjusted R^2: 0.379, LM Test (p-value): 0.34

J.15 **CGPDXOESV**: Domestic corporate goods price index (excluding electric and electronic products, petroleum and coal products, consumption tax, and extra charges for summer electricity)

$$
\begin{aligned}
&\text{dlog}\left(CGPDXOESV_t\right) = \\
&\quad 0.00181\langle 3.90\rangle + 0.0894\langle 5.43\rangle \cdot GAP_{t-1}/100 \\
&\quad + 0.0159\langle 2.48\rangle \cdot \frac{CGPDOILV_{t-1}}{CGPDXOESV_{t-1}} \cdot \text{dlog}\left(CGPDOILV_t\right) \\
&\quad + 0.0173\langle 3.49\rangle \cdot \frac{CGPDOILV_{t-2}}{CGPDXOESV_{t-2}} \cdot \text{dlog}\left(CGPDOILV_{t-1}\right) \\
&\quad + 0.0585\langle 3.97\rangle \cdot \text{dlog}\left(IPIXO_t\right)
\end{aligned} \tag{J.15}
$$

Sample period: 1990/Q3 - 2010/Q4, Adjusted R^2: 0.605, LM Test (p-value): 0.00

J.16 **VATCGPDs** (VATCGPD, VATCGPDOIL, VATCGPDE, VATCGPDXO, VATCGPDXOE): Effects of consumption tax on the domestic corporate goods price index

$$VAT_t = VAT_{t-1} \tag{J.16}$$

J.17 **SUMMERCGPDXOE**: Extra charges for summer electricity in the domestic corporate price index (excluding electric and electronic products, petroleum and coal products)

$$SUMMERCGPDXOE_t = SUMMERCGPDXOE_{t-4} \tag{J.17}$$

J.18 **GASOLINEMP**: Gasoline price

$$
\begin{aligned}
&\text{dlog}\left(GASOLINEMP_t - GASOLINET_t\right) \\
&\quad = 0.218\langle 3.66\rangle \cdot \text{dlog}\left(GASOLINEMP_{t-1} - GASOLINET_{t-1}\right) \\
&\quad + \left(1 - 0.218\right) \cdot PIQ_t/400 \\
&\quad + 0.397\langle 13.1\rangle \cdot \left(\text{dlog}\left(IPIOIL_t\right) - HQIPIOIL\_SS_t\right)
\end{aligned} \tag{J.18}
$$

Sample period: 1987/Q4 - 2010/Q4, Adjusted R^2: 0.673, LM Test (p-value): 1.00

J.19 **CPIOR**: Consumer price index (general)

$$CPIOR_t = \frac{CPIFFW_t}{CPIFFW_t + CPIXFW_t} \cdot CPIFFOR_t + \frac{CPIXFW_t}{CPIFFW_t + CPIXFW_t} \cdot CPIXFOR_t \quad \text{(J.19)}$$

J.20 **CPI**: Consumer price index (general, S.A.)

$$CPI_t = CPIOR_t / \left(SFCPI_t / 100 \right) \quad \text{(J.20)}$$

J.21 **CPIV**: Consumer price index (general, excluding consumption tax, S.A.)

$$CPIV_t = CPI_t / \left(1 + VATCPI_t \right) \quad \text{(J.21)}$$

J.22 **CPIFFOR**: Consumer price index (fresh food)

$$\begin{aligned} &\log\left(CPIFFOR_t / CPIFFOR_{t-4} \right) \\ &= 0.374 \langle 4.32 \rangle \cdot \log\left(CPIFFOR_{t-1} / CPIFFOR_{t-5} \right) \\ &+ \left(1 - 0.374 \right) \cdot PIQ_t / 100 + 0.401 \langle 2.00 \rangle \cdot GAP_t / 100 \end{aligned} \quad \text{(J.22)}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.206, LM Test (p-value): 1.00

J.23 **CPIFF**: Consumer price index (fresh food, S.A.)

$$CPIFF_t = CPIFFOR_t / \left(SFCPIFF_t / 100 \right) \quad \text{(J.23)}$$

J.24 **CPIXFOR**: Consumer price index (general, excluding fresh food)

$$\begin{aligned} CPIXFOR_t = & \frac{CPIXFDENW_t}{CPIXFW_t} \cdot CPIXFDENOR_t \\ & + \frac{CPIFDXFAW_t}{CPIXFW_t} \cdot CPIFDXFAOR_t + \frac{CPIENW_t}{CPIXFW_t} \cdot CPIENOR_t \end{aligned} \quad (J.24)$$

J.25 **CPIXFOR_AD**: Consumer price index (general, excluding fresh food and high school fees)

$$\begin{aligned} CPIXFOR\_AD_t = & \frac{CPIXFDENW_t - CPIHSW_t}{CPIXFW_t - CPIHSW_t} \cdot CPIXFDENOR\_AD_t \\ & + \frac{CPIFDXFAW_t}{CPIXFW_t - CPIHSW_t} \cdot CPIFDXFAOR_t \\ & + \frac{CPIENW_t}{CPIXFW_t - CPIHSW_t} \cdot CPIENOR_t \end{aligned} \quad (J.25)$$

J.26 **CPIXF_AD**: Consumer price index (general, excluding fresh food and high school fees, S.A.)

$$CPIXF\_AD_t = CPIXFOR\_AD_t \Big/ \left(SFCPIXF_t / 100 \right) \quad (J.26)$$

J.27 **CPIFDXFAOR**: Consumer price index (food, excluding fresh food and alcoholic beverages)

$$\begin{aligned} & CPIFDXFAOR_t \\ & = CPIFDXFAV_t \cdot \left(1 + VATCPIFDXFA_t\right) \cdot \left(SFCPIFDXFA_t / 100\right) \end{aligned} \quad (J.27)$$

J.28 **CPIFDXFAV**: Consumer price index (food, excluding fresh food, alcoholic beverages and consumption tax, S.A.)

$$\begin{aligned} & \mathrm{dlog}\left(CPIFDXFAV_t\right) \\ & \quad = 0.709 \langle 13.6 \rangle \cdot \mathrm{dlog}\left(CPIFDXFAV_{t-1}\right) \\ & \quad + \left(1 - 0.709\right) \cdot PIQ_t / 400 + 0.0301 \langle 3.38 \rangle \cdot GAP_t / 100 \end{aligned} \quad (J.28)$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.688, LM Test (p-value): 1.00

J.29 **CPIENOR**: Consumer price index (energy)

$$CPIENOR_t = \frac{CPIGASOLINEW_t}{CPIENW_t} \cdot CPIGASOLINEOR_t + \left(1 - \frac{CPIGASOLINEW_t}{CPIENW_t}\right) \cdot CPIENXGASOLINEOR_t \quad \text{(J.29)}$$

J.30 **CPIGASOLINEOR**: Consumer price index (gasoline)

$$CPIGASOLINEOR_t = CPIGASOLINEV_t \cdot \left(1 + VATCPIGASOLINE_t\right) \cdot \left(SFCPIGASOLINE_t / 100\right) \quad \text{(J.30)}$$

J.31 **CPIGASOLINEV**: Consumer price index (gasoline, excluding consumption tax, S.A.)

$$\text{dlog}\left(CPIGASOLINEV_t\right) = \text{dlog}\left(GASOLINEMP_t\right) \quad \text{(J.31)}$$

J.32 **CPIENXGASOLINEOR**: Consumer price index (energy, excluding gasoline)

$$CPIENXGASOLINEOR_t = CPIENXGASOLINEV_t \cdot \left(1 + VATCPIENXGASOLINE_t\right) \cdot \left(SFCPIENXGASOLINE_t / 100\right) \quad \text{(J.32)}$$

J.33 **CPIENXGASOLINEV**: Consumer price index (energy, excluding gasoline and consumption tax, S.A.)

$$\begin{aligned}&\text{dlog}\left(CPIENXGASOLINEV_t\right)\\&\quad=0.0995\langle 6.99\rangle\cdot\frac{1}{4}\sum_{s=0}^{3}\left(\text{dlog}\left(IPIOIL_{t-s}\right)-HQIPIOIL\_SS_t\right)\\&\quad+PIQ_t/400\end{aligned}\tag{J.33}$$

Sample period: 1983/Q2 - 2010/Q4, Adjusted R^2: 0.282, LM Test (p-value): 1.00

J.34 **CPIXFDENOR**: Consumer price index (general, excluding food (but including alcoholic beverages) and energy)

$$\begin{aligned}CPIXFDENOR_t&=-0.734\cdot D102Z_t\\&+CPIXFDENOR\_AD_t\cdot\left(1-\frac{CPIHSW_t}{CPIXFDENW_t}\right)\end{aligned}\tag{J.34}$$

J.35 **CPIXFDENOR_AD**: Consumer price index (general, excluding food (but including alcoholic beverages), energy and high school fees, S.A.)

$$\begin{aligned}CPIXFDENOR\_AD_t&=CPIXFDENV\_AD_t\cdot\left(1+VATCPIXFDEN_t\right)\\&\cdot\left(SFCPIXFDEN_t/100\right)\end{aligned}\tag{J.35}$$

J.36 **CPIXFDENV_AD**: Consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees and consumption tax, S.A.) (See Section 3.1 in the text.)

$$\begin{aligned}&\text{dlog}\left(CPIXFDENV\_AD_t\right)\\&\quad=0.355\langle 6.70\rangle\cdot\text{dlog}\left(CPIXFDENV\_AD_{t-1}\right)\\&\quad+\left(1-0.355\right)\cdot ZPI_{t,1}/400+0.0313\langle 2.50\rangle\cdot GAP_t/400\end{aligned}\tag{J.36}$$

Sample period: 1980/Q3 - 2010/Q4

J.37 **SFCPIs** (SFCPI, SFCPIXF, SFCPIFF, SFCPIFDXFA, SFCPIXFDEN, SFCPIEN, SFCPIGASOLINE, SFCPIENXGASOLINE): Seasonal factors of the consumer price index

$$SF_t = SF_{t-4} + \left(SF_{t-4} - SF_{t-8}\right)/2 \qquad \text{(J.37)}$$

J.38 **VATCPIs** (VATCPIXF, VATCPIFDXFA, VATCPIXFDEN, VATCPIGASOLINE, VATCPIENXGASOLINE): Effects of consumption tax on the consumer price index

$$VAT_t = VAT_{t-1} \qquad \text{(J.38)}$$

J.39 **PIX**: Inflation rate of consumer price index (general, excluding food (but including alcoholic beverages), energy, high school fees and consumption tax, S.A.)

$$PIX_t = \mathrm{dlog}\left(CPIXFDENV\_AD_t\right) \cdot 400 \qquad \text{(J.39)}$$

K. Deflators

K.1 **PC**: Private consumption deflator

$$\begin{aligned} \mathrm{dlog}\left(PC_t\right) &= -0.00118\langle -6.80\rangle \\ &+ 0.844\langle 19.0\rangle \cdot \mathrm{dlog}\left(CPIXF\_AD_t\right) + 0.0243\langle 6.83\rangle \cdot \mathrm{dlog}\left(CPIFF_t\right) \\ &+ 0.0393\langle 2.71\rangle \cdot \mathrm{dlog}\left(WHN_{t-1}\right) + 0.0504\langle 3.44\rangle \cdot \mathrm{dlog}\left(WHN_{t-2}\right) \end{aligned} \qquad \text{(K.1)}$$

Sample period: 1980/Q4 - 2010/Q4, Adjusted R^2: 0.871, LM Test (p-value): 0.04

K.2 **PIH**: Private residential investment deflator

$$\begin{aligned}\text{dlog}\left(PIH_t\right) = {} & 0.00140\langle 2.36\rangle + 0.279\langle 3.67\rangle \cdot \text{dlog}\left(PIH_{t-1}\right) \\ & +0.440\langle 3.99\rangle \cdot \text{dlog}\left(CGPDXOV_t\right) \\ & +0.0448\langle 3.24\rangle \cdot \text{dlog}\left(IPIXO_t\right) \\ & +0.235\langle 3.04\rangle \cdot \frac{1}{4}\sum_{s=0}^{3}\text{dlog}\left(WHN_{t-s}\right)\end{aligned} \quad \text{(K.2)}$$

Sample period: 1981/Q1 - 2010/Q4, Adjusted R^2: 0.510, LM Test (p-value): 0.77

K.3 **PINV**: Private non-residential investment deflator

$$\begin{aligned}\text{dlog}\left(PINV_t\right) = {} & -0.00215\langle -5.73\rangle \\ & +0.449\langle 8.46\rangle \cdot \text{dlog}\left(CGPDXOV_t\right) \\ & +0.112\langle 4.12\rangle \cdot \text{dlog}\left(WHN_t\right) \\ & +0.125\langle 4.68\rangle \cdot \text{dlog}\left(WHN_{t-1}\right)\end{aligned} \quad \text{(K.3)}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.479, LM Test (p-value): 0.03

K.4 **PSNAIV**: Private inventory deflator

$$PSNAIV_t = 36.4\langle 0.70\rangle - 1940\langle -3.36\rangle \cdot D034_t \quad \text{(K.4)}$$

Sample period: 1980/Q1 - 2010/Q4, Adjusted R^2: 0.0771, LM Test (p-value): 0.66

K.5 **PCG**: Government consumption deflator

$$\begin{aligned}\text{dlog}\left(PCG_t\right) = {} & 0.000485\langle 0.509\rangle - 0.383\langle -4.67\rangle \cdot \text{dlog}\left(PCG_{t-1}\right) \\ & +0.259\langle 3.31\rangle \cdot \text{dlog}\left(WHN_t\right) + 0.384\langle 2.93\rangle \cdot \text{dlog}\left(PIG_t\right)\end{aligned} \quad \text{(K.5)}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.224, LM Test (p-value): 0.01

K.6 **PIG**: Public investment deflator

$$\text{dlog}\left(PIG_t\right) = 0.00181\langle 3.62\rangle + 0.737\langle 11.9\rangle \cdot \text{dlog}\left(CGPDXOV_t\right) + 0.0816\langle 2.07\rangle \cdot \text{dlog}\left(WHN_t\right) \quad \text{(K.6)}$$

Sample period: 1980/Q2 - 2010/Q4: Adjusted R^2: 0.557, LM Test (p-value): 0.02

K.7 **PGIG**: Gross fixed capital formation deflator (general government)

$$\text{dlog}\left(PGIG_t\right) = \text{dlog}\left(PIG_t\right) \quad \text{(K.7)}$$

K.8 **PIIG**: Public inventory deflator

$$PIIG_t = IIGN_t / IIG_t \cdot 100 \quad \text{(K.8)}$$

K.9 **PEX**: Exports of goods and services deflator

$$\text{dlog}\left(PEX_t\right) = -0.00148\langle -1.53\rangle + 0.670\langle 21.3\rangle \cdot \text{dlog}\left(EPI_t\right) \quad \text{(K.9)}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.788, LM Test (p-value): 0.03

K.10 **PIM**: Imports of goods and services deflator

$$\text{dlog}\left(PIM_t\right) = -0.000650\langle -0.634\rangle + 0.807\langle 43.3\rangle \cdot \text{dlog}\left(IPI_t\right) \quad \text{(K.10)}$$

Sample period: 1980/Q3 - 2010/Q4, Adjusted R^2: 0.939, LM Test (p-value): 0.03

K.11 **PGDP**: GDP deflator

$$PGDP_t = GDPN_t / GDP_t \cdot 100 \qquad \text{(K.11)}$$

L. Aggregate Output Identities

L.1 **GDP**: Real gross domestic product

$$\begin{aligned}\frac{GDP_t}{GDP_{t-1}} = & \frac{CN_t^*}{GDPN_t^*}\frac{C_t}{C_{t-1}} + \frac{IHN_t^*}{GDPN_t^*}\frac{IH_t}{IH_{t-1}} + \frac{INVN_t^*}{GDPN_t^*}\frac{INV_t}{INV_{t-1}} \\ & + \frac{SNAIVN_t^*}{GDPN_t^*}\frac{SNAIV_t}{SNAIV_{t-1}} + \frac{CGN_t^*}{GDPN_t^*}\frac{CG_t}{CG_{t-1}} + \frac{IGN_t^*}{GDPN_t^*}\frac{IG_t}{IG_{t-1}} \\ & + \frac{IIGN_t^*}{GDPN_t^*}\frac{IIG_t}{IIG_{t-1}} + \frac{EXN_t^*}{GDPN_t^*}\frac{EX_t}{EX_{t-1}} - \frac{IMN_t^*}{GDPN_t^*}\frac{IM_t}{IM_{t-1}}\end{aligned} \qquad \text{(L.1.1)}$$

$$\begin{aligned}GDPN_t^* = & CN_t^* + IHN_t^* + INVN_t^* + SNAIVN_t^* \\ & + CGN_t^* + IGN_t^* + IIGN_t^* + EXN_t^* - IMN_t^*\end{aligned} \qquad \text{(L.1.2)}$$

L.2 **GDPN**: Nominal gross domestic product

$$\begin{aligned}GDPN_t = & CN_t + IHN_t + INVN_t + SNAIVN_t \\ & + CGN_t + IGN_t + IIGN_t + EXN_t - IMN_t\end{aligned} \qquad \text{(L.2)}$$

L.3 **GDPNADJ**: Adjusted nominal gross domestic product

$$GDPNADJ_t = GDPN_t - YOSMIGR_t - NINDTAX_t \qquad \text{(L.3)}$$

L.4 **NINDTAX**: Taxes on production and imports, excluding subsidies

$$NINDTAX_t / GDPN_t = 0.0626\langle 98.7\rangle + 0.00632\langle 6.78\rangle \cdot D892Z_t + 0.00783 \cdot D972Z_t \qquad \text{(L.4)}$$

Sample period: 1980/Q1 - 2010/Q1, Adjusted R^2: 0.711, LM Test (p-value): 0.00

L.5 **D**: Consumption of fixed capital (total economy)

$$\begin{aligned}\log\left(D_t / GDPN_t\right) = &-0.0165\langle -0.974\rangle \\ &+0.643\langle 7.37\rangle \cdot \log\left(D_{t-1}/GDPN_{t-1}\right) \\ &+0.344\langle 3.97\rangle \cdot \log\left(D_{t-2}/GDPN_{t-2}\right)\end{aligned} \qquad \text{(L.5)}$$

Sample period: 1980/Q3 - 2010/Q1, Adjusted R^2: 0.989, LM Test (p-value): 0.66

L.6 **FD**: Real final demand

$$\begin{aligned}\frac{FD_t}{FD_{t-1}} = &\frac{CN_t^*}{FDN_t^*}\frac{C_t}{C_{t-1}} + \frac{IHN_t^*}{FDN_t^*}\frac{IH_t}{IH_{t-1}} + \frac{INVN_t^*}{FDN_t^*}\frac{INV_t}{INV_{t-1}} \\ &+ \frac{CGN_t^*}{FDN_t^*}\frac{CG_t}{CG_{t-1}} + \frac{IGN_t^*}{FDN_t^*}\frac{IG_t}{IG_{t-1}} + \frac{EXN_t^*}{FDN_t^*}\frac{EX_t}{EX_{t-1}}\end{aligned} \qquad \text{(L.6.1)}$$

$$FDN_t^* = CN_t^* + IHN_t^* + INVN_t^* + CGN_t^* + IGN_t^* + EXN_t^* \qquad \text{(L.6.2)}$$

L.7 **FDN**: Nominal final demand

$$FDN_t = CN_t + IHN_t + INVN_t + CGN_t + IGN_t + EXN_t \qquad \text{(L.7)}$$

L.8 **DDF**: Real domestic final demand

$$\frac{DDF_t}{DDF_{t-1}} = \frac{CN_t^*}{DDFN_t^*}\frac{C_t}{C_{t-1}} + \frac{IHN_t^*}{DDFN_t^*}\frac{IH_t}{IH_{t-1}} + \frac{INVN_t^*}{DDFN_t^*}\frac{INV_t}{INV_{t-1}} + \frac{CGN_t^*}{DDFN_t^*}\frac{CG_t}{CG_{t-1}} + \frac{IGN_t^*}{DDFN_t^*}\frac{IG_t}{IG_{t-1}} \quad \text{(L.8.1)}$$

$$DDFN_t^* = CN_t^* + IHN_t^* + INVN_t^* + CGN_t^* + IGN_t^* \quad \text{(L.8.2)}$$

L.9 **DDFN**: Nominal domestic final demand

$$DDFN_t = CN_t + IHN_t + INVN_t + CGN_t + IGN_t \quad \text{(L.9)}$$

L.10 **GDPQ**: Real potential gross domestic product

$$GDPQ_t = \left(1 + HYGDPQ_t\right)^{0.25} \cdot GDPQ_{t-1} \quad \text{(L.10)}$$

L.11 **GDPQNONHP**: Real potential gross domestic product (non-filtering)

$$\log\left(GDPQNONHP_t / GDPQ_t\right) = 0.342\langle 4.07\rangle \cdot \log\left(GDPQNONHP_{t-1} / GDPQ_{t-1}\right) \quad \text{(L.11)}$$

Sample period: 1980/Q2 - 2010/Q4, Adjusted R^2: 0.119, LM Test (p-value): 0.42

L.12 **GAP**: GDP gap

$$GAP_t = GDP_t / GDPQNONHP_t \cdot 100 - 100 \quad \text{(L.12)}$$

L.13 **HYGDPQ**: Potential GDP growth rate

$$HYGDPQ_t = \log\left(GDPQ_t / GDPQ_{t-4}\right) \quad \text{(L.13)}$$

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Figure 1: Overview of Q-JEM

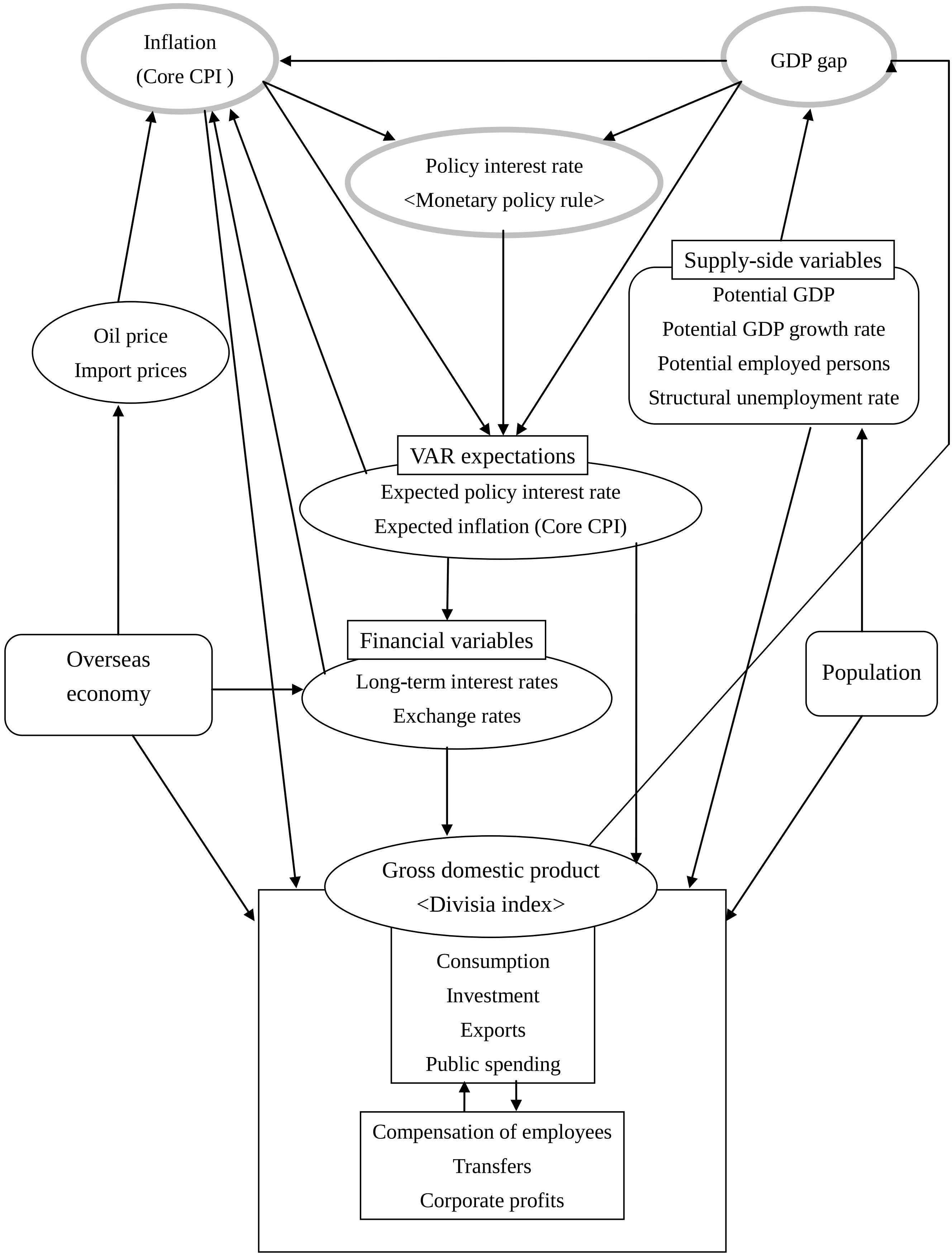


Figure 2: Overseas Economy in Q-JEM

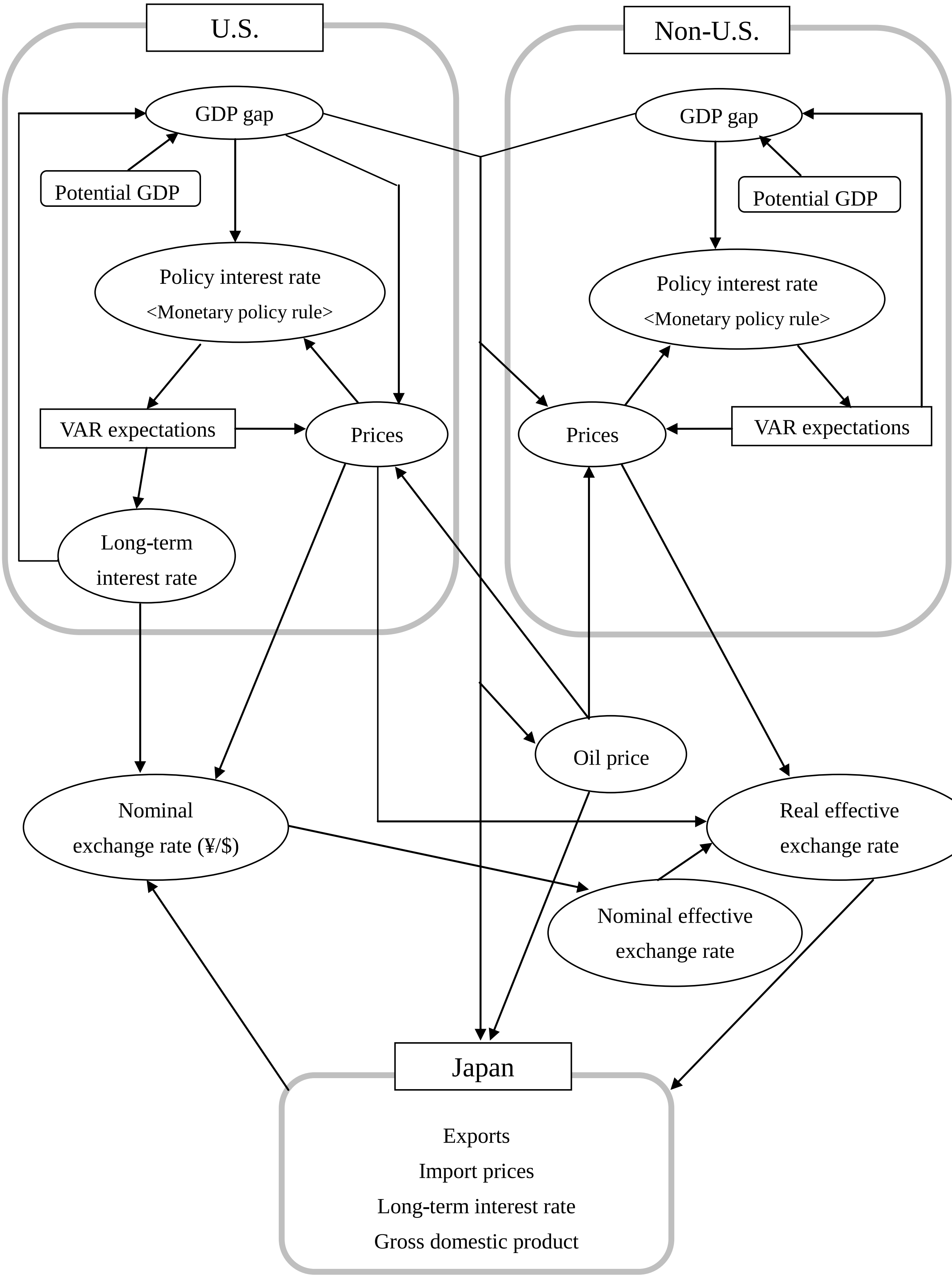
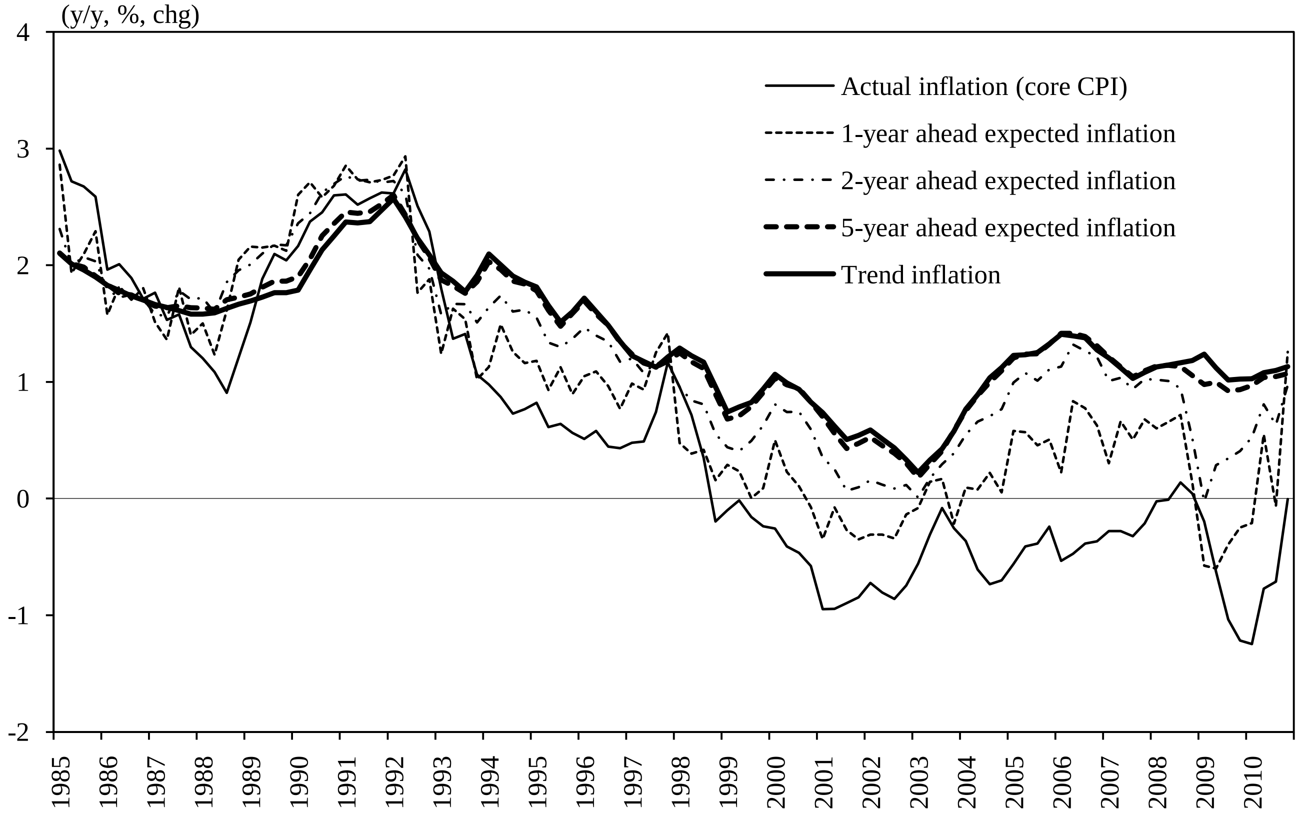


Figure 3: Trend Inflation and Expected Inflation (Japan)

Actual, Expected, and Trend Inflation



GDP Gap

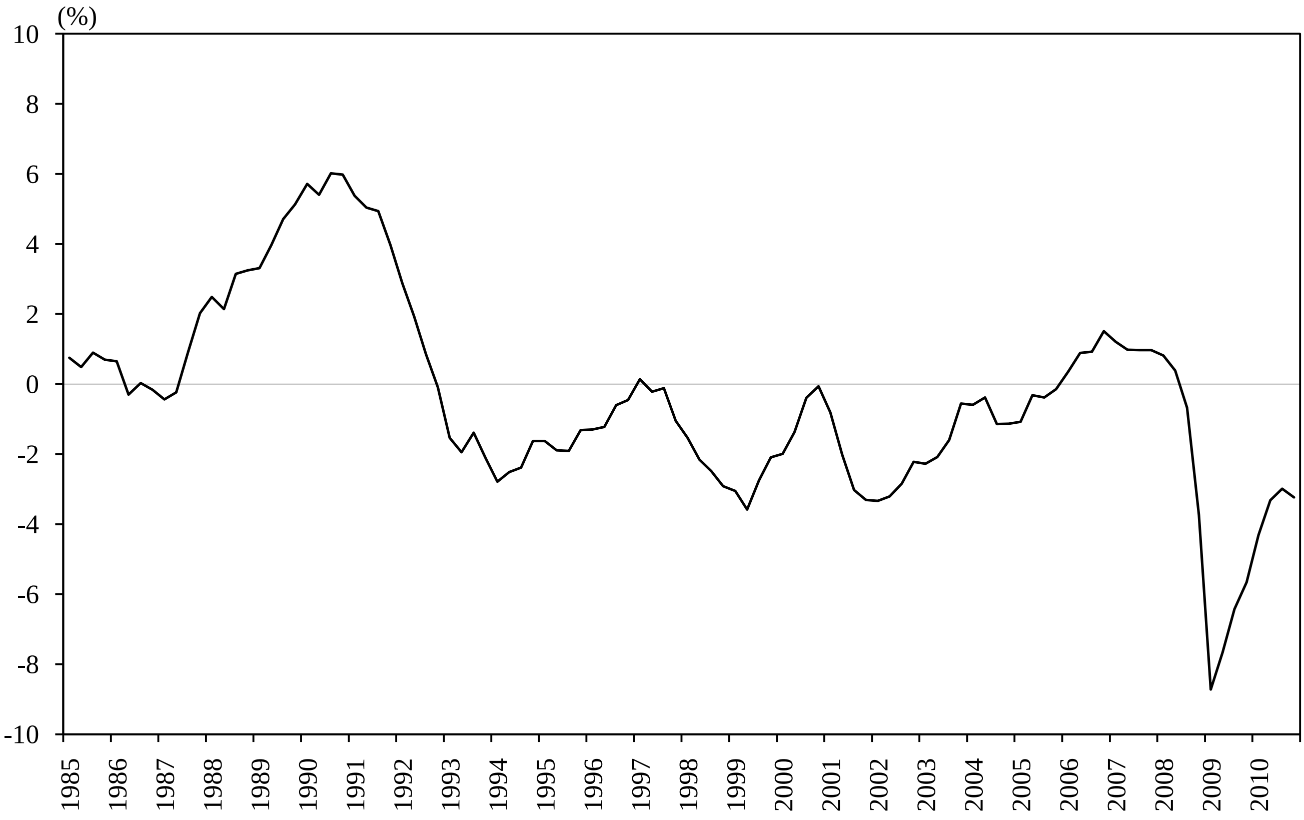
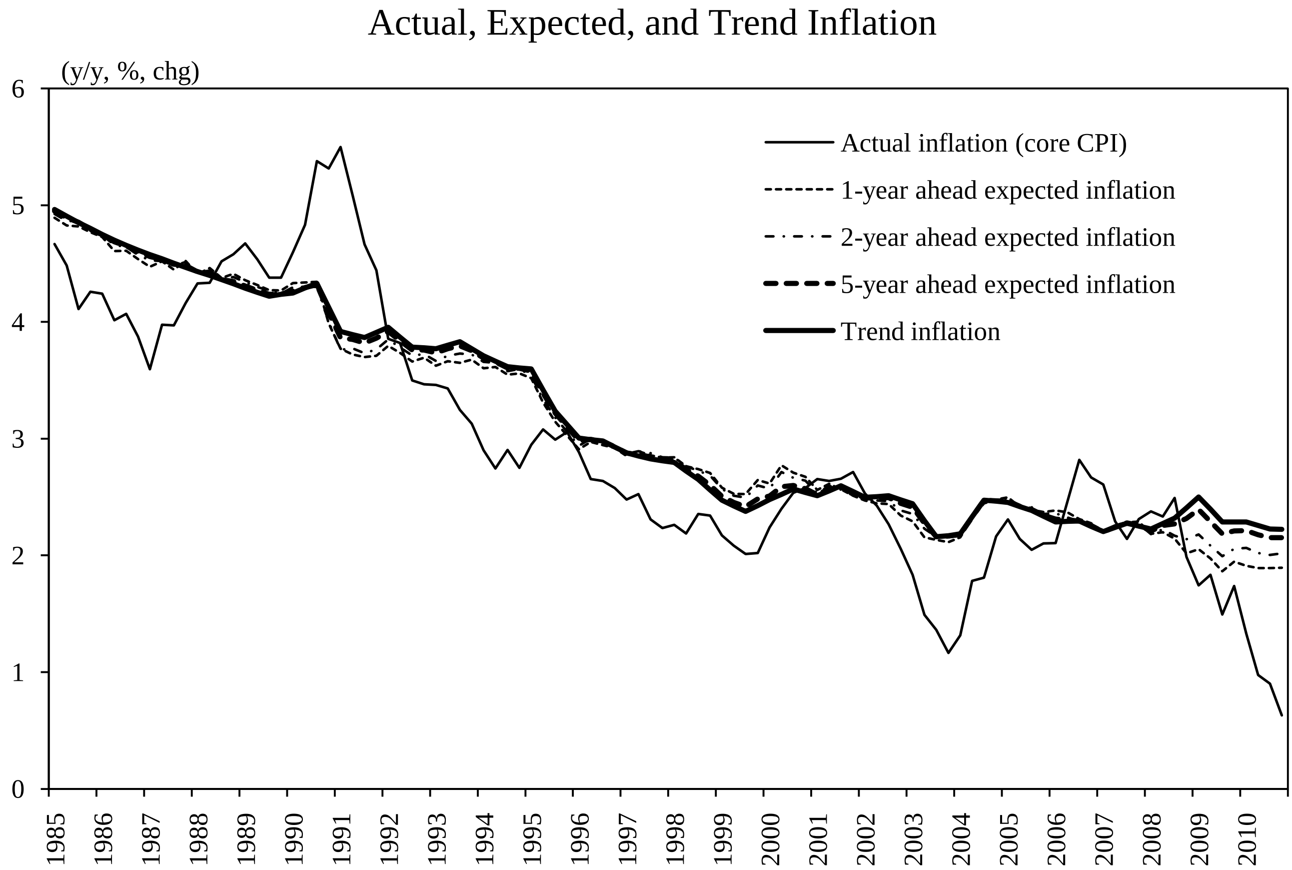


Figure 4: Trend Inflation and Expected Inflation (U.S.)



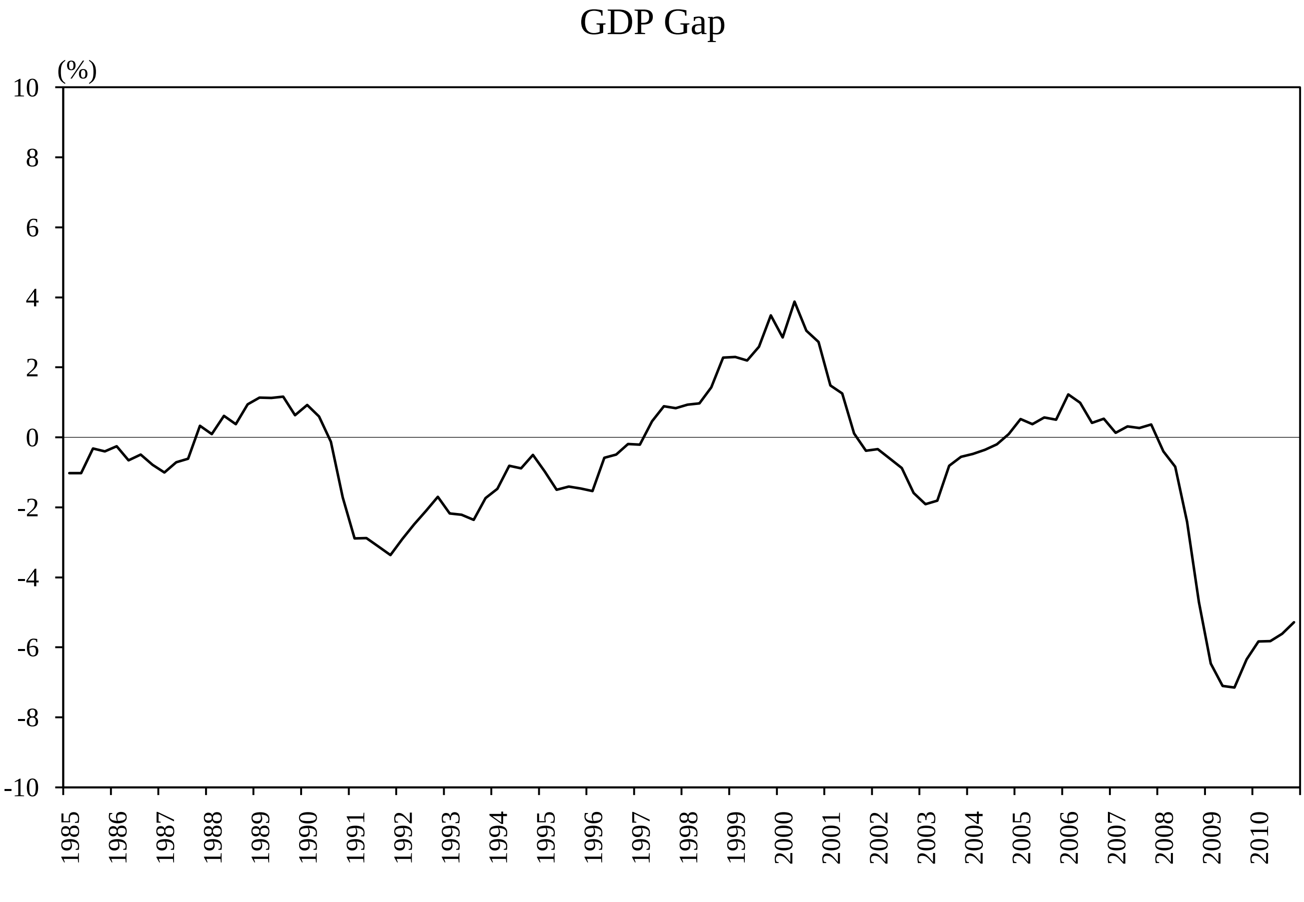
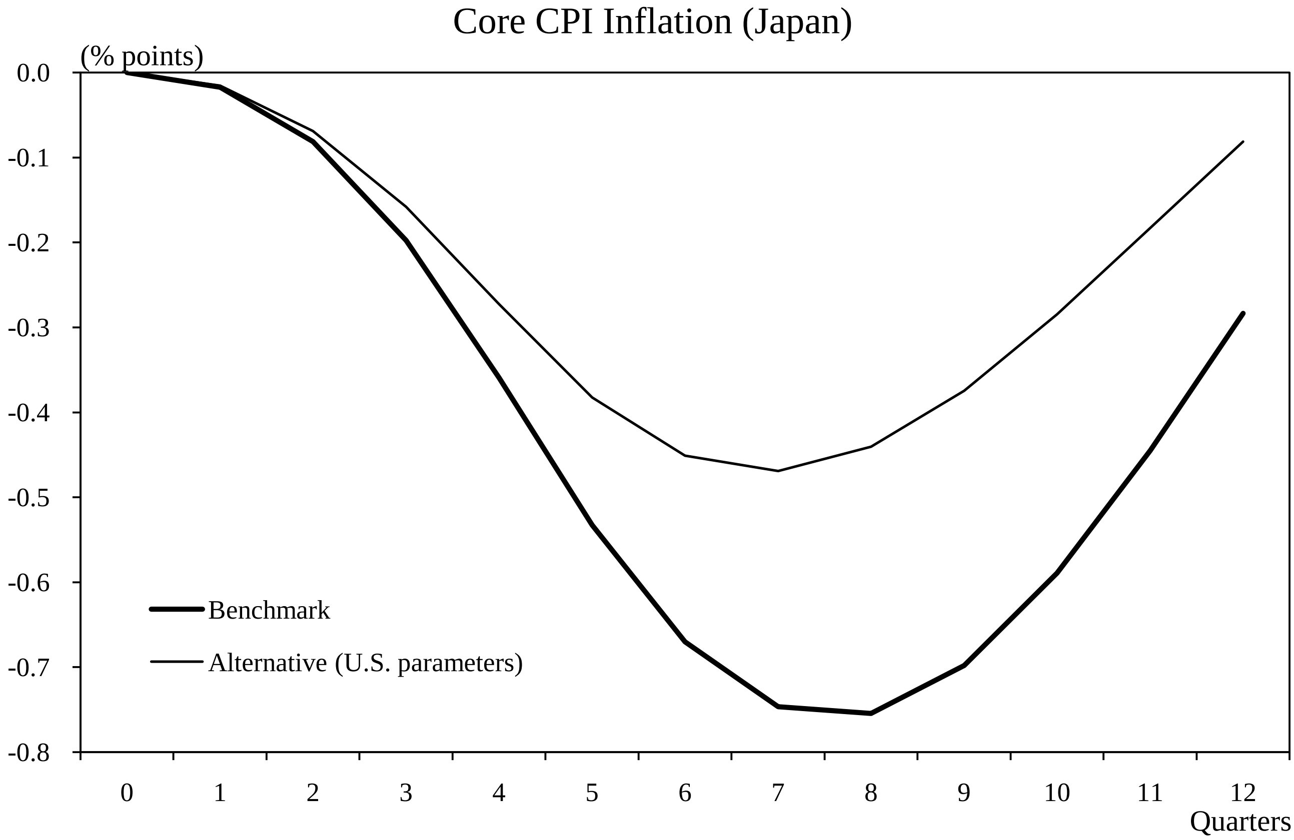
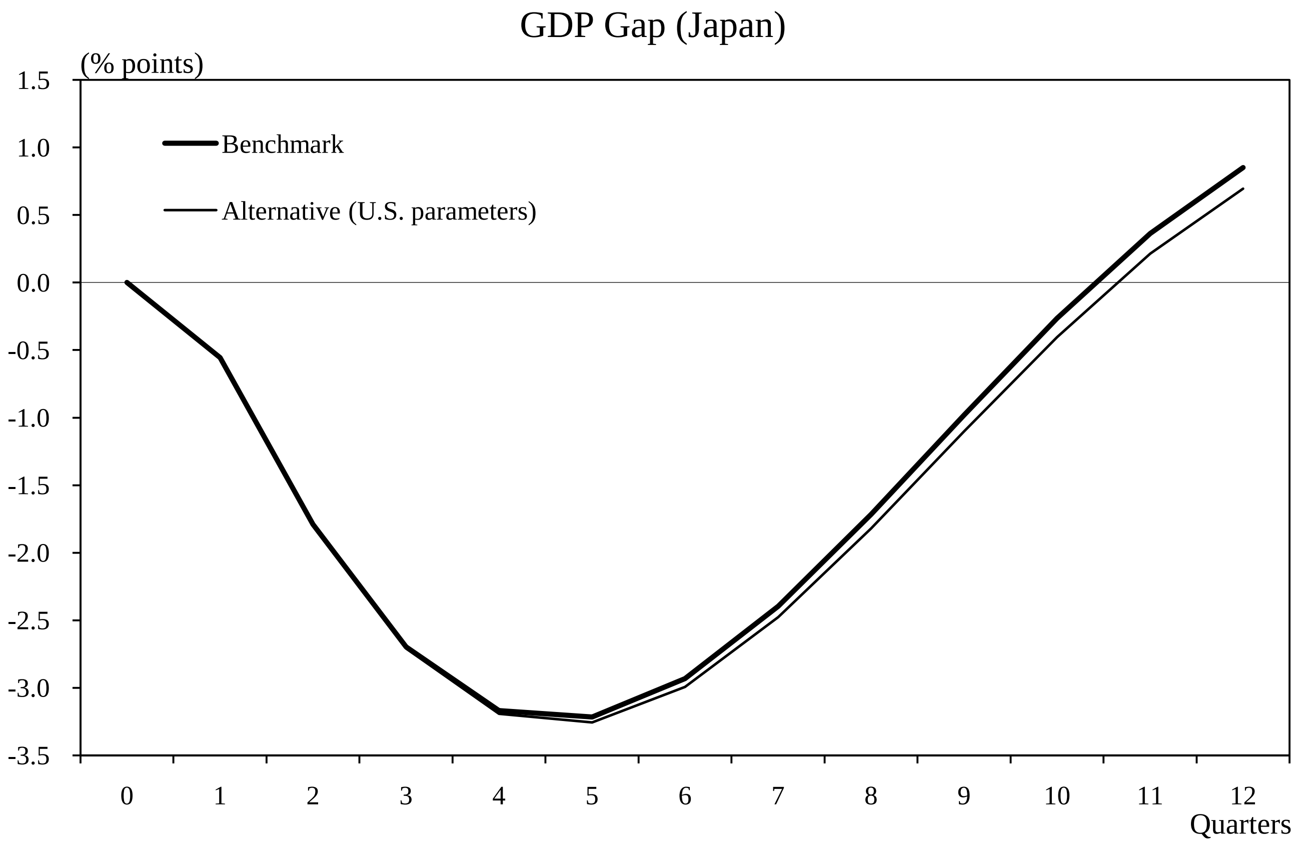


Figure 5: Response to an Adverse Shock to the U.S. Economy

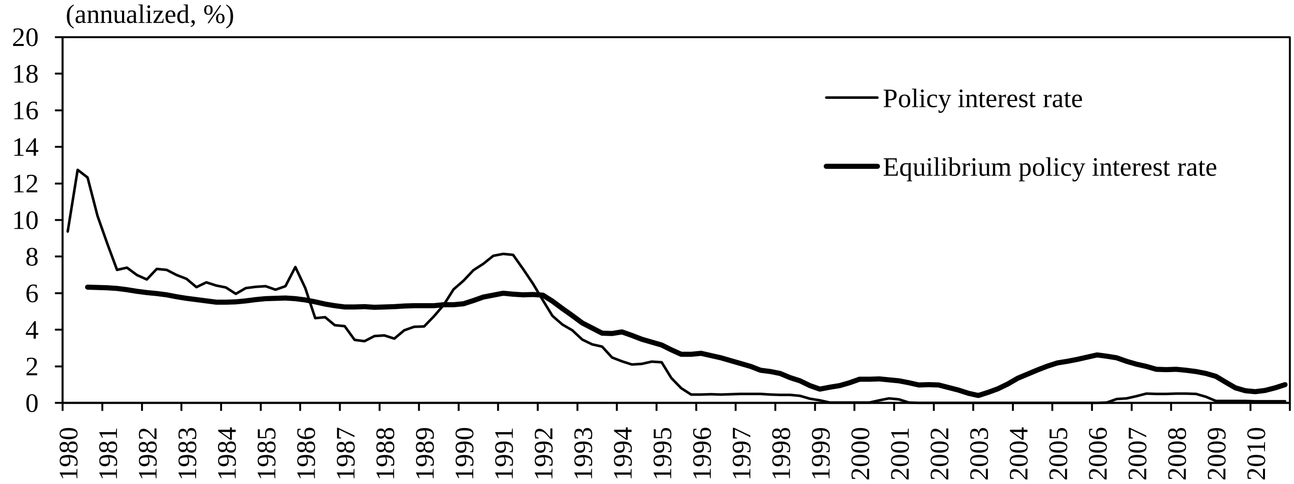




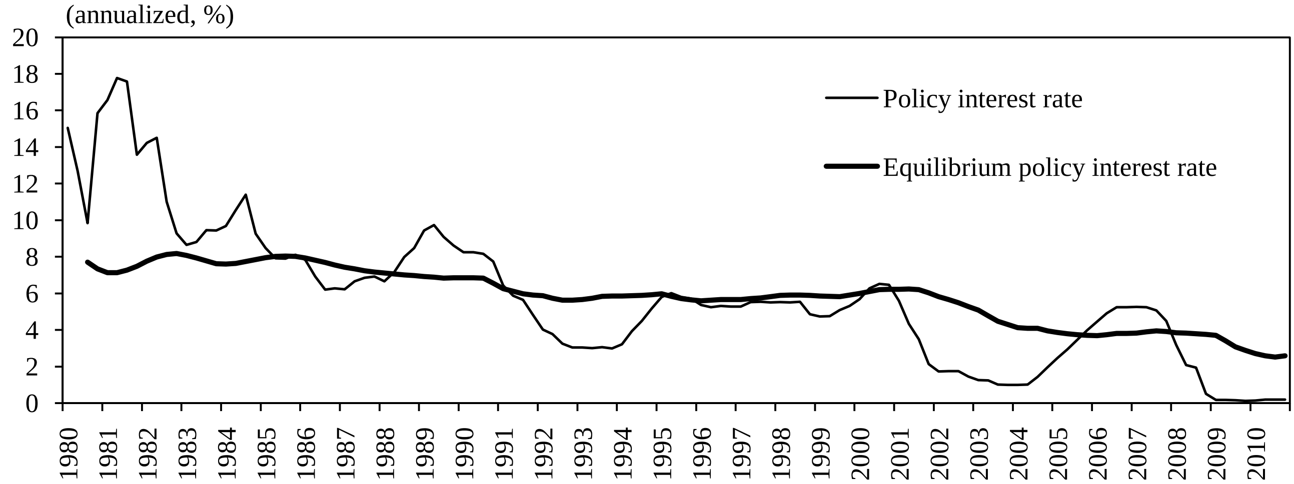
Note : All impulse responses are expressed as percentage deviations from the baseline.

Figure 6: Policy Interest Rate

Japan



United States



Non-U.S.

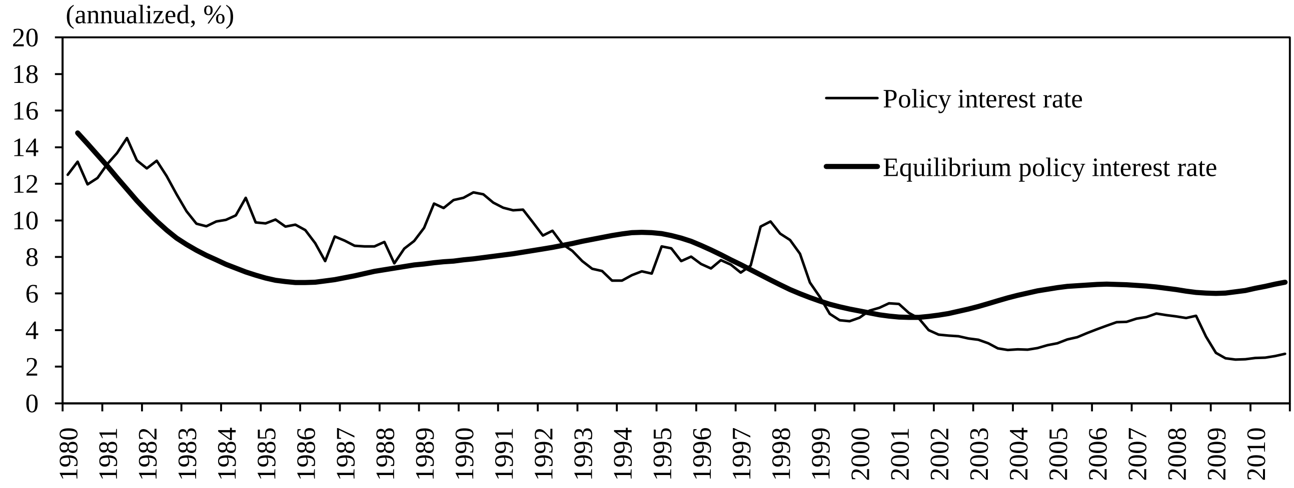
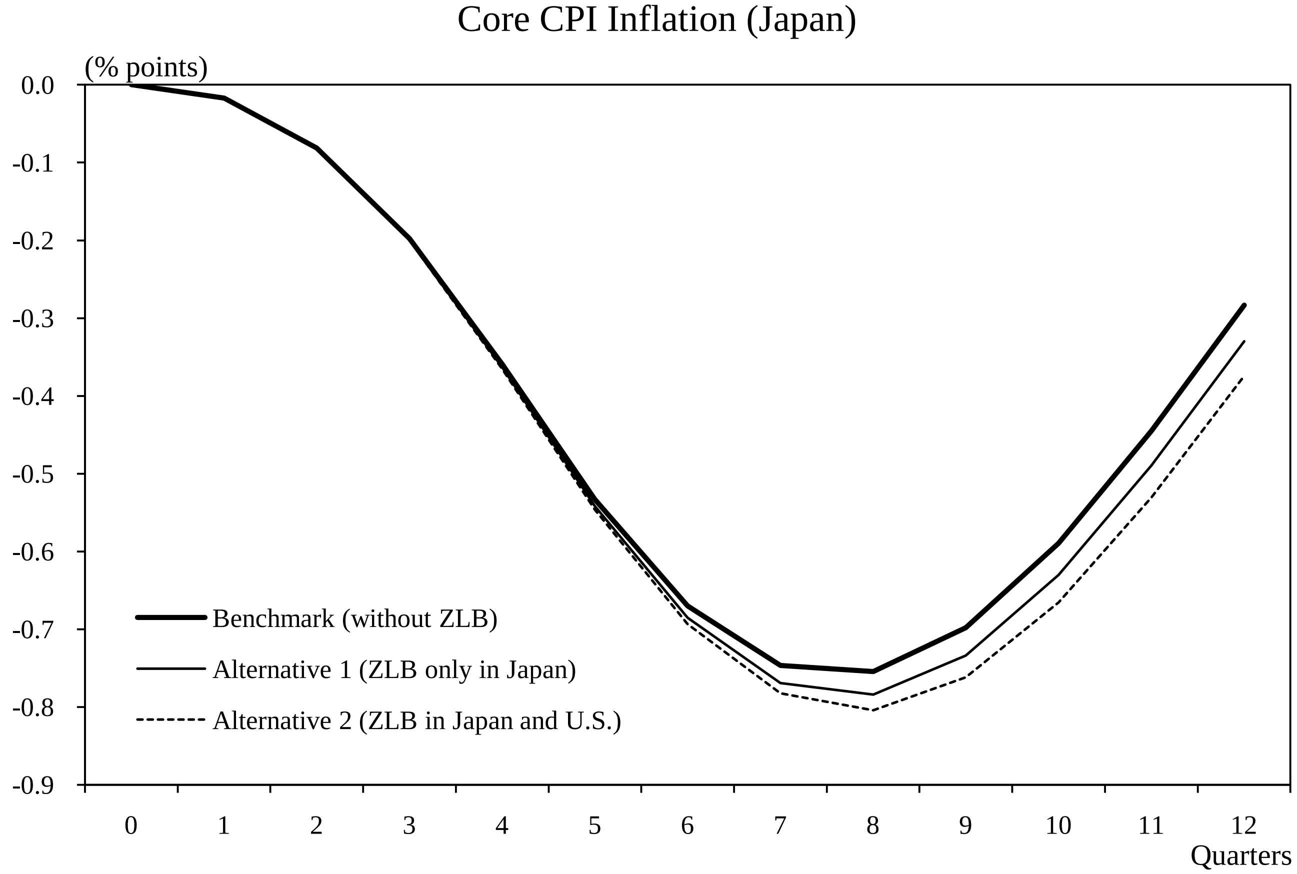
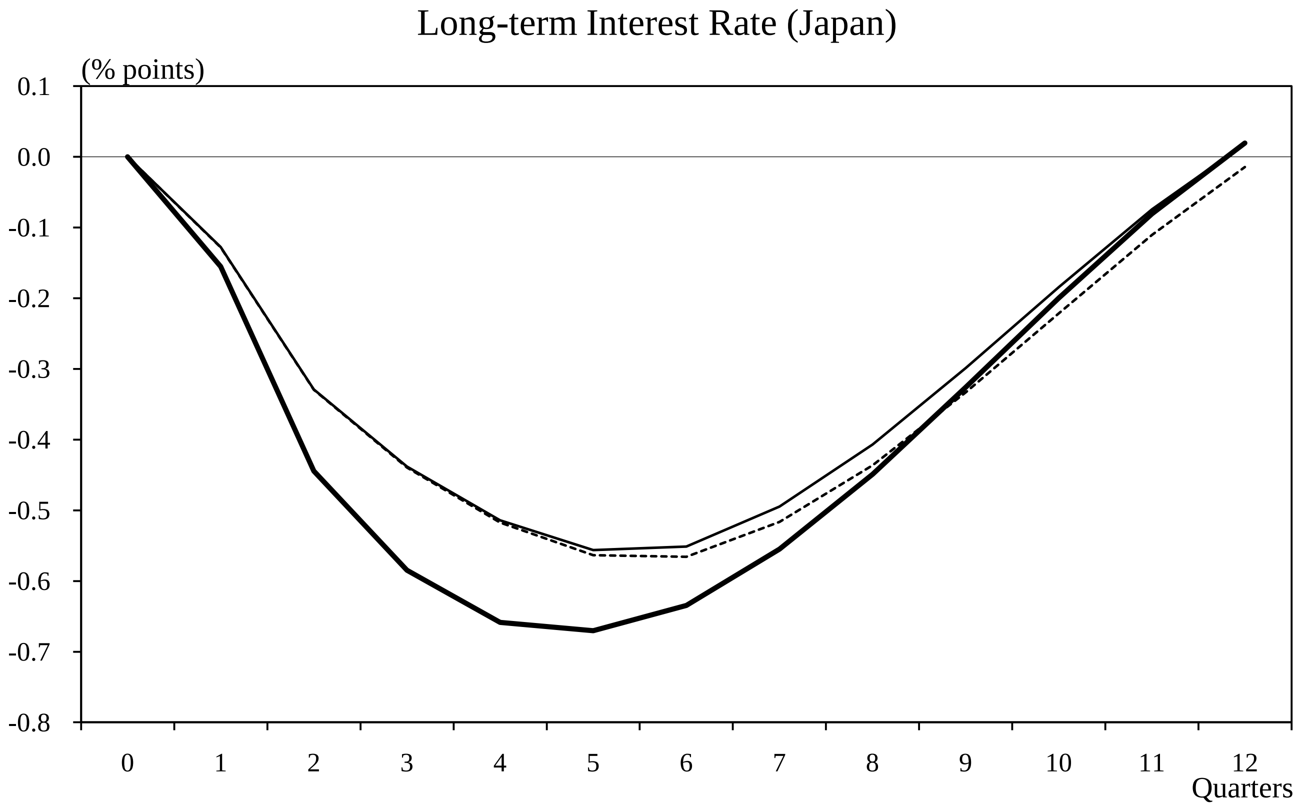


Figure 7: Response to an Adverse Shock to the U.S. Economy (With and Without a Zero Lower Bound on the Policy Interest Rate)

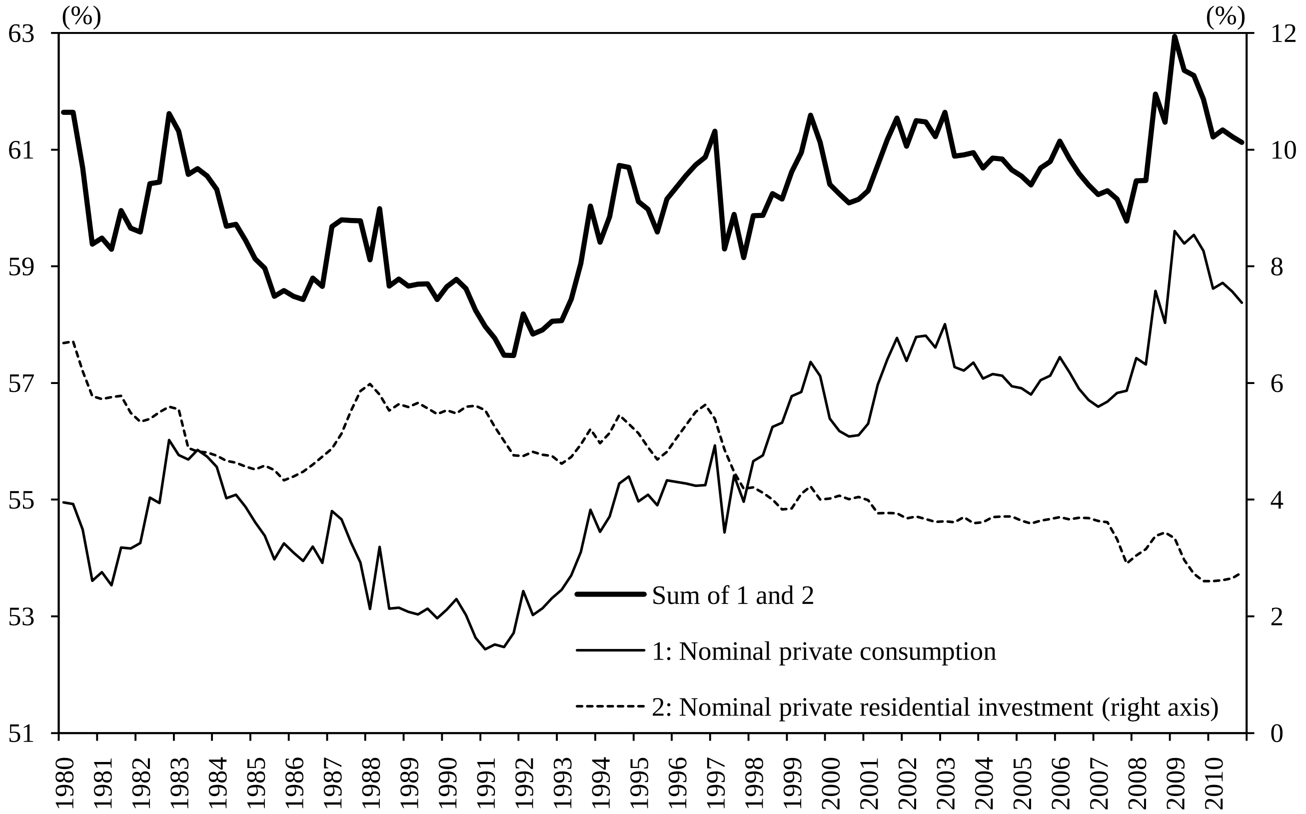




Note : All impulse responses are expressed as percentage deviations from the baseline.

Figure 8: Shares in Nominal GDP

Nominal Private Consumption and Residential Investment



Nominal Private Non-residential Investment

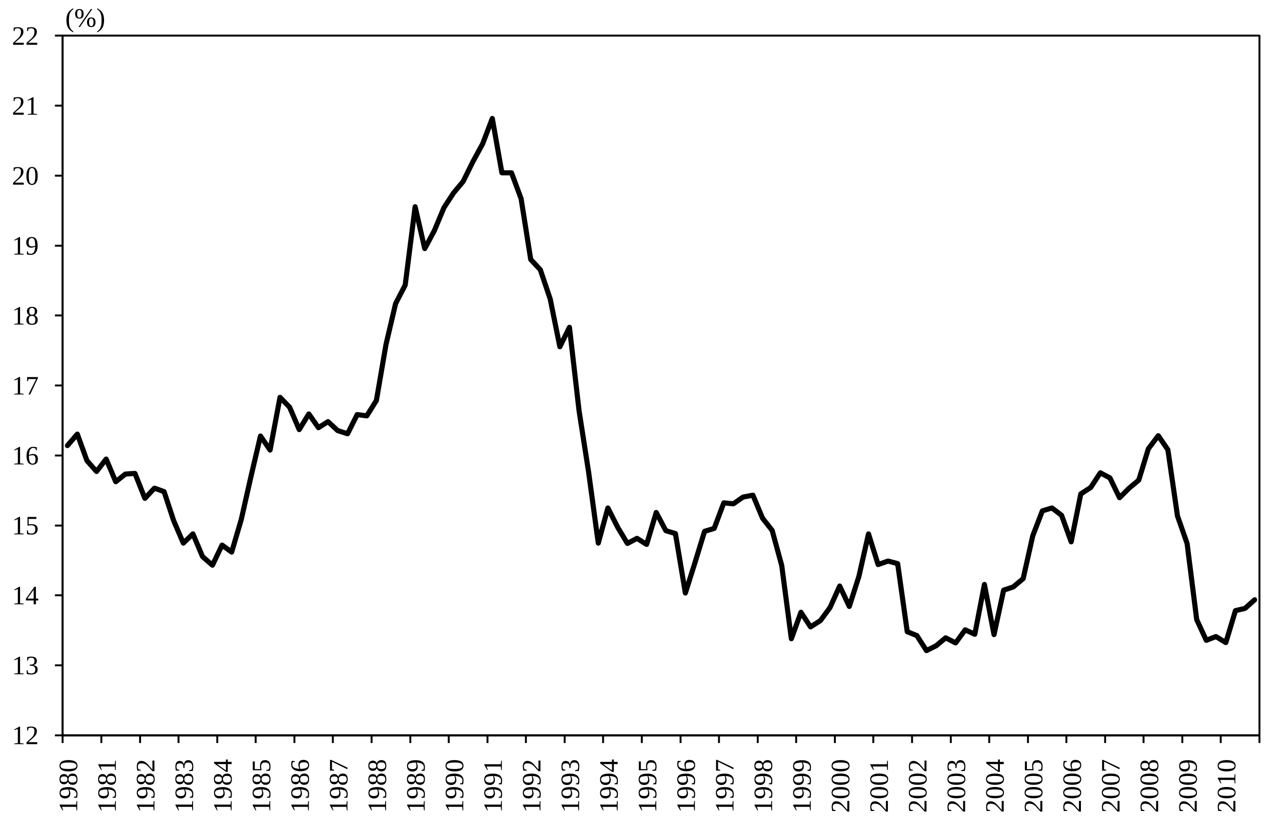
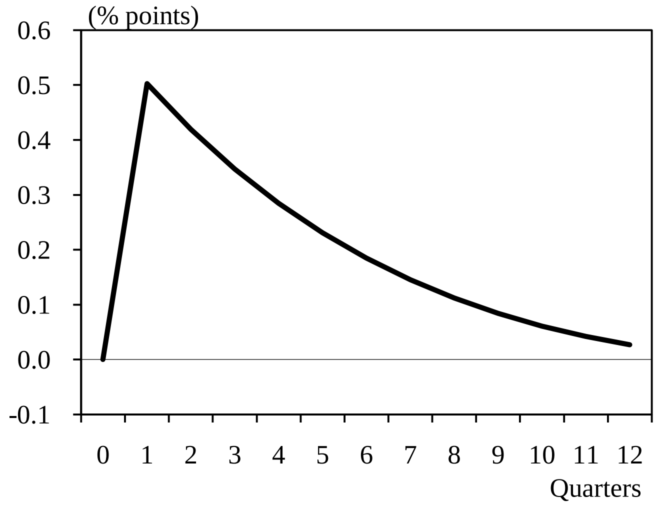
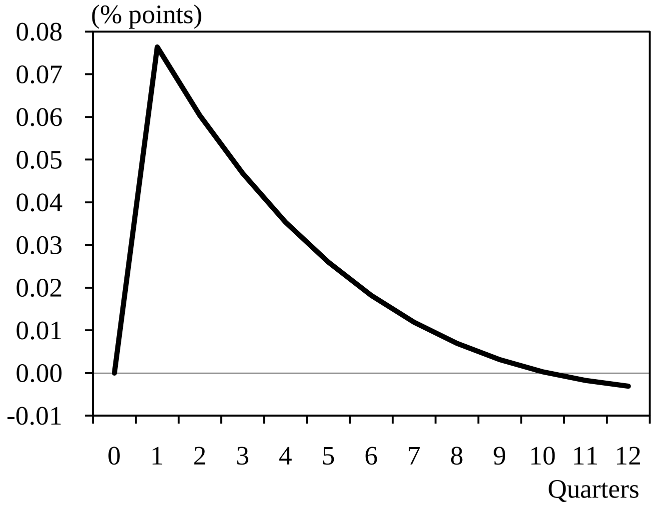


Figure 9: Responses to a Tightening Shock to the Policy Interest Rate

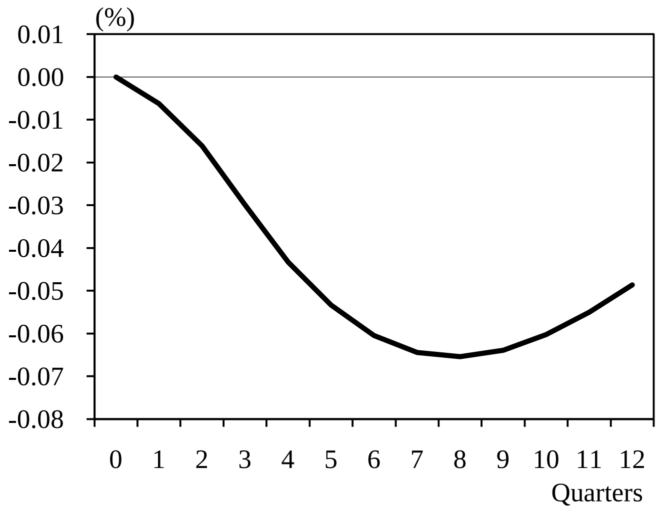
(1) Policy Interest Rate



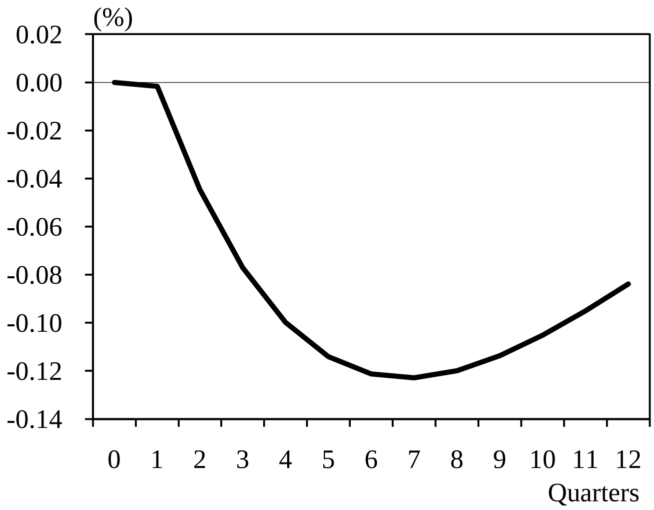
(2) Long-term Interest Rate



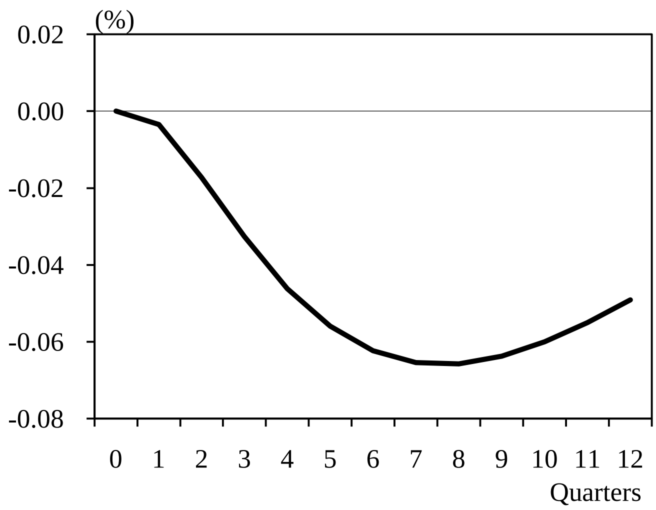
(3) Real Private Consumption



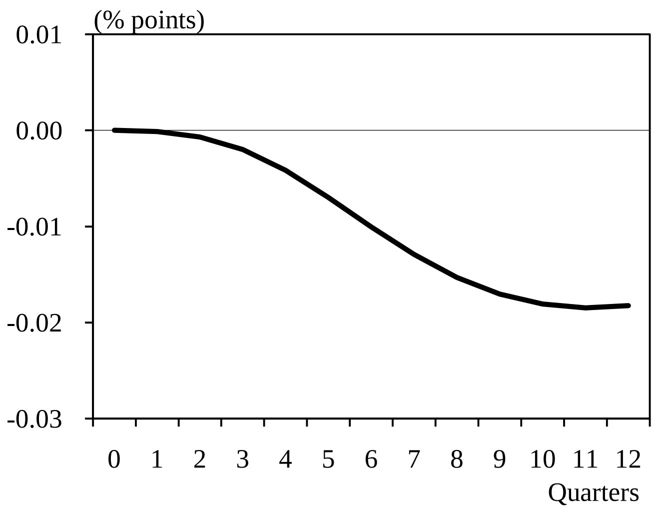
(4) Real Private Non-residential Investment



(5) Real GDP



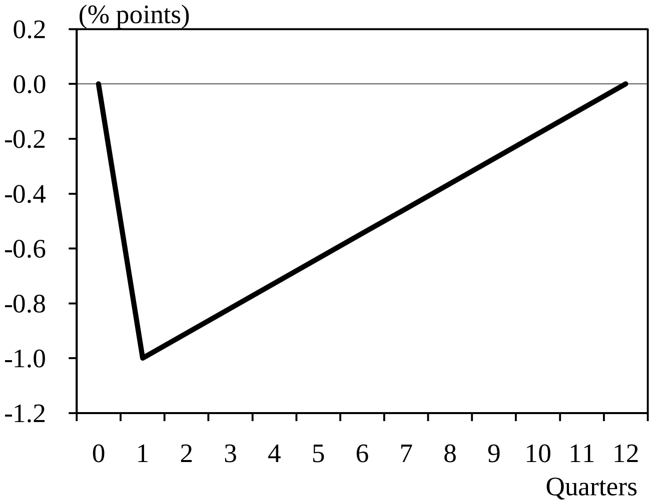
(6) Core CPI Inflation



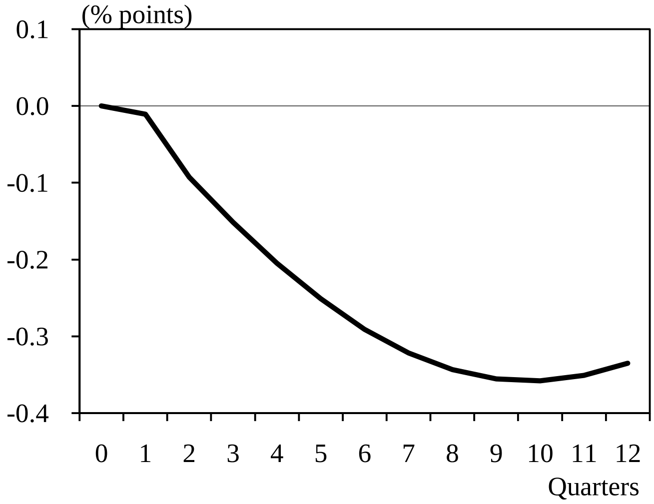
Note : All impulse responses are expressed as percentage deviations from the baseline.

Figure 10: Responses to a Negative Shock to Expected Potential Growth

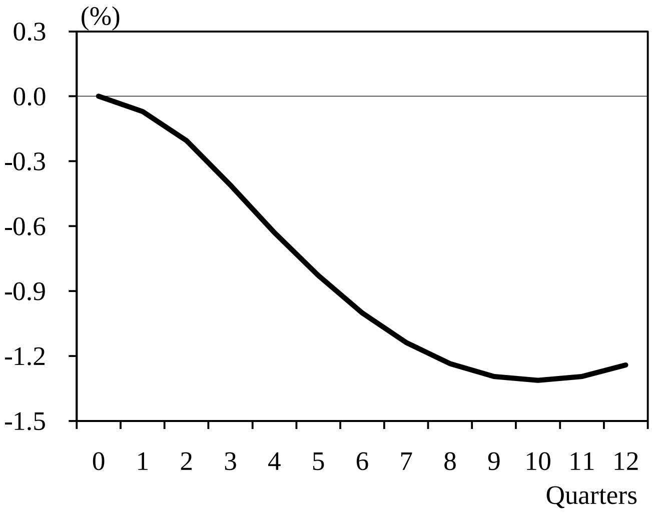
(1) Expected Potential Growth Rate



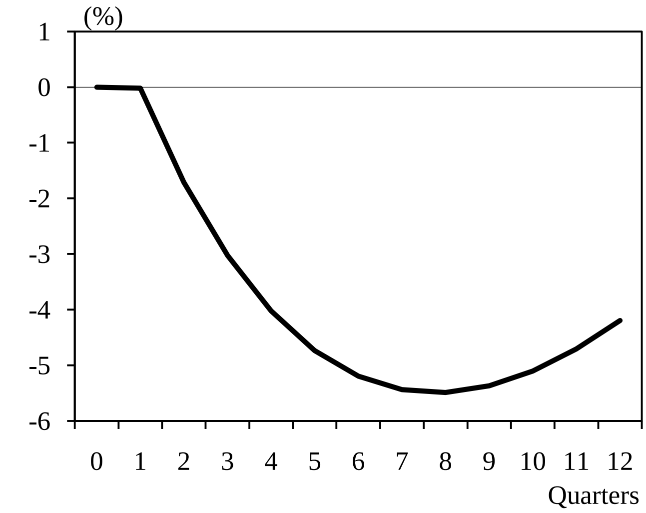
(2) Long-term Interest Rate



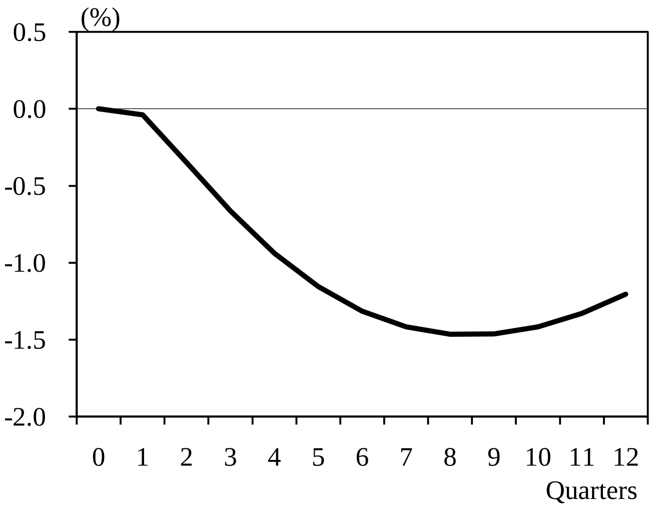
(3) Real Private Consumption



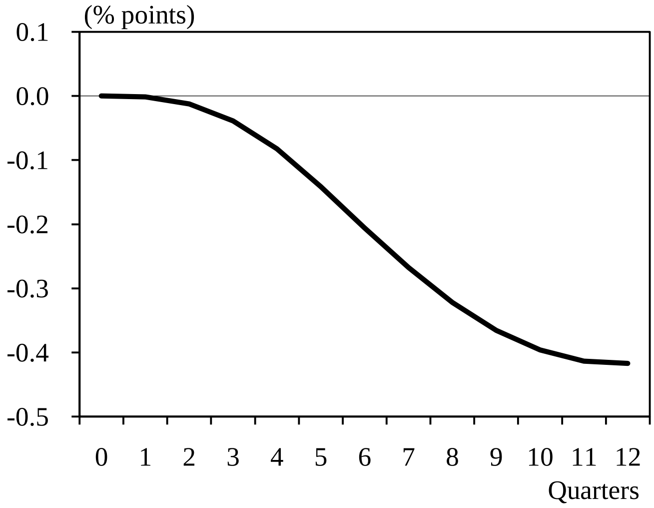
(4) Real Private Non-residential Investment



(5) Real GDP



(6) Core CPI Inflation



Note : All impulse responses are expressed as percentage deviations from the baseline.