Bank of Japan Working Paper Series

The Effects of Demographic Changes on the Real Interest Rate in Japan

Daisuke Ikeda*
daisuke.ikeda@boj.or.jp

Masashi Saito**
masashi.saitou@boj.or.jp

No.12-E-3
February 2012

Bank of Japan
2-1-1 Nihonbashi-Hongokucho, Chuo-ku, Tokyo 103-0021, Japan

* Monetary Affairs Department
** Monetary Affairs Department

The Effects of Demographic Changes on the Real Interest Rate in Japan*

Daisuke Ikeda† Masashi Saito‡

February 2012

Abstract

What are the effects of demographic changes on the real interest rate in Japan? We present a dynamic general equilibrium model in which demographic changes are captured by exogenous changes in the ratio of workers to the total population. Our model predicts that a decline in this ratio in the process of population aging lowers the real interest rate; and the demographic impact on the real interest rate is amplified by a fall in land prices in the presence of collateral constraints. The model is simulated with the realized and forecasted changes in the working-age population ratio and the TFP growth in Japan. Our results indicate that the TFP growth is the main source of variations in the real interest rate, but the demographic factor is also quantitatively important especially for its long-term movements.

JEL Classifications: E20, E43, J11

Keywords: Demographics; land prices; real interest rate; collateral constraint.

*We thank Kosuke Aoki, Ippei Fujiwara, Hibiki Ichiue, Ryo Kato, Takeshi Kimura, Kazuo Monma, Ichiro Muto, Takushi Kurozumi, Kiyohiko Nishimura, Nao Sudo, and Tsutomu Watanabe for helpful comments. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

†Monetary Affairs Department, Bank of Japan (E-mail: daisuke.ikeda@boj.or.jp)

‡Monetary Affairs Department, Bank of Japan (E-mail: masashi.saitou@boj.or.jp)

1 Introduction

There have been significant changes in the demographic structure in developed countries. The upper panel of Figure 1 shows the evolution of the ratio of the working-age population (persons with age 15-64) to the total population in Japan, where the data after 2010 is the projection of the United Nations. A striking feature that emerges from this figure is that the late 1980s is a turning point for the demographic history of Japan: because of the aging of the population, the working-age population ratio starts to decline in the late 1980s, and a declining trend in this ratio is expected to continue into the future. A similar shift in the demographic trend is observed in other developed countries as shown in the lower panels of Figure 1, although the precise timing of the shift in many countries is later than that in Japan.

Such demographic changes are expected to have a widespread impact on the macroeconomy. Bakshi and Chen (1994) note that the investment behavior of the older age group is different from that of the younger age group, and present an empirical evidence that changes in the age distribution have a significant impact on stock and house prices. Similarly, Mankiw and Weil (1989) argue that the entry of the baby boom generation into its house-buying years caused a U.S. housing boom in the 1970s. Miles (1999) uses an overlapping generations (OLG) model to claim that the aging of the population is an important factor behind the evolutions of the saving rate. The aging of the population is sometimes listed as one of the potential causes of the slowdown in economic growth in Japan since the 1990s, together with a fall in the growth rate of total factor productivity (TFP) and problems in the financial sector.

Among the numerous macroeconomic variables that are expected to be influenced by demographic changes, this paper focuses on the real interest rate. More specifically, we are interested in the movements of the equilibrium real interest rate, or the natural rate of interest. This interest rate is not observable and needs to be estimated, but it provides an essential information when one evaluates the monetary condition and the state of macroeconomic environment: when the natural rate of interest falls below the actual real interest rate, the monetary condition becomes relatively tight and deflationary pressure emerges; such a pressure becomes especially strong if the zero lower bound prevents the policy interest rate from falling. In this paper, we use a model that does not include nominal frictions so that the equilibrium interest rate in our model can be interpreted as the natural rate of interest.[1]

The dynamic general equilibrium model in this paper includes the following three channels through which demographic changes—specifically, changes in the working-age population ratio—affect the real interest rate. The first channel works by increasing the supply of loanable funds by the household which is a lender in the model economy. Our model assumes that the wage income earned by workers is distributed within a household to support the consumption of non-workers. When the ratio of the working-age population to the total population is expected to decline in the process of population aging, the number of wage earners relative to the number of persons who consume is expected to decrease.[2] The household which follows a permanent income hypothesis then consumes less and saves more in order to smooth out the level of per-capita consumption into the future. The increase in the household savings results in an increase in the supply of funds in the loanable funds market, and generates downward pressure on the real interest rate.

The remaining two channels work by reducing the demand for loanable funds by the firm which is a borrower in the model economy. We consider a firm that conducts production using capital stock, labor, and land as inputs. In our model, a decline in the working-age population ratio works like a fall in TFP. This reduces the marginal products of capital and land, and the demand for capital and land by the firm decreases. Since the firm's expenditure on capital is financed partly by borrowing, a decrease in the demand for capital reduces the demand for loanable funds by the firm, placing downward pressure on the real interest rate. This is the second channel.

The third channel operates through a fall in land prices. Our model assumes that the land serves as a collateral in the firm's borrowing and that the amount of the firm's borrowing is constrained by the value of land. Because of the fall in the marginal product of land described above, the firm reduces the demand for land. This leads to a fall in land prices and the collateral value of the firm. The demand for loanable funds by the firm decreases further, and this channel provides an additional downward pressure on the real interest rate, i.e., the effects of demographic changes on the real interest rate are

[1] Of course, the real interest rate in our model is a good approximation for the natural rate of interest only to the extent that the shocks and the structure of the model provide a good characterization of the actual economy.

[2] In contrast, a standard representative agent model assumes that all the household members work. Such a model allows for variations in the total population, but not the variations in the ratio of workers to the total population.

amplified by a change in the balance-sheet conditions of the borrower. A key assumption behind this channel is the presence of collateral constraints that depend on land values. This assumption has an empirical validity because numerous empirical studies including Ogawa and Suzuki (1998) illustrate that the land plays an important role as a collateral in Japanese firms' borrowing, and show that the land value has a significant influence on the investment decisions of credit-constrained firms.

To evaluate the impact of demographic changes on the real interest rate, we simulate the model using as inputs the realized and forecasted variations in the working-age population ratio and variations in the total population. Our results imply that the demographic factor has worked to lower the real interest rate in Japan since the late 1980s, and this factor is expected to keep the interest rate low in the future. Among the two demographic variables, variations in the ratio of the working-age population to the total population are more important than changes in the total population itself; this supports the specification of our model which allows for changes in the ratio of workers to the total population. Additionally, we confirm that the third channel described above amplifies the effects of demographic changes on the real interest rate.

We also conduct a simulation that includes realized variations in the TFP growth in addition to the variations in the working-age population ratio, to evaluate the relative contribution of the demographic factor to the overall movements in the real interest rate. Our quantitative analysis indicates that the TFP growth is the dominant source of fluctuations in the real interest rate, but the demographic factor is also quantitatively important especially for its long-term movements.

The model in this paper builds on the previous theoretical studies that emphasize the role of collateral constraint in amplifying business cycles (Kiyotaki and Moore 1997; Iacoviello 2005; Liu, Wang and Zha 2011). To our knowledge, this paper is the first to demonstrate quantitatively that the negative impact of population aging on the natural rate of interest is amplified by the fall in asset values in the presence of a collateral constraint. Krugman (1998) points out the importance of such mechanism when he discusses the factors behind liquidity trap in Japan, although he does not offer a formal model. Guerrieri and Lorenzoni (2011) present a model in which a tightening of the borrowing constraint decreases the natural rate of interest. In their model, a tightening of the borrowing constraint is given as an exogenous shock–they interpret it as a financial crisis—, while in our model it arises endogenously as a result of a fall in land values induced by population aging or a fall in the TFP growth. Using an OLG framework, Miles (1999), Chen, Imrohoroglu and Imrohoroglu (2007), and Braun, Ikeda and Joines (2009) analyze the impact of demographic changes on macroeconomic variables, including the saving rate and the real interest rate. However, these studies do not consider the collateral constraint.

The rest of the paper is structured as follows. In Section 2, we present the model. Section 3 describes the parameterization of the model and the simulation procedure. We present our results in Section 4. Section 5 concludes.

2 Model

There are two types of agents: households and entrepreneurs. Households supply labor, consume goods, purchase land for housing, and save. The savings of households constitute the supply of funds in the loanable funds market. Within a household there are workers and non-workers, and it is assumed that the ratio of workers to the total population changes exogenously. We interpret that changes in this ratio reflect demographic changes: the aging of the population lowers this ratio. Entrepreneurs produce goods using labor, capital stock, and land as inputs. Part of their expenditures is financed by borrowing. As in Kiyotaki and Moore (1997), they face a collateral constraint that limits the amount of borrowing to the value of collateral. The collateral is the land held by the entrepreneur.

We now describe the optimization problems of households and entrepreneurs, as well as the equilibrium conditions.

2.1 Households

A representative household consists of workers and non-workers. The utility function of workers is defined as

$$u_{y,t} = \frac{\left[c_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma} - 1}{1-\sigma}, \tag{1}$$

where $c_{y,t}$ is consumption, $h_{y,t}$ is the amount of land held by workers, and l_t is hours worked (all of these variables are in per-worker terms).[3] As in Iacoviello (2005), we assume that the household holds land for housing. σ is a positive parameter that represents

[3] Hayashi and Prescott (2002) treat hours worked as an exogenous variable in order to take into account the regulatory changes in Japan in the early 1990s. We treat it as an endogenous variable in order to focus on the effects of demographic changes and the TFP growth.

the inverse of the elasticity of intertemporal substitution; ν is a positive parameter that governs the elasticity of the labor supply; and ϕ and χ are positive parameters. As shown in King, Plosser and Rebelo (2002), this preference specification ensures the presence of a balanced growth path.

The utility function of non-workers is defined as

$$u_{o,t} = \frac{\left[c_{o,t} h_{o,t}^{\phi}\right]^{1-\sigma} - 1}{1-\sigma}, \tag{2}$$

where $c_{o,t}$ is consumption and $h_{o,t}$ is the amount of land owned by non-workers (both variables are in per-non-worker terms).

The total population is N_t, among which the number of workers is N_t^y and the number of non-workers is $N_t - N_t^y$. We assume that the ratio of workers to the total population, N_t^y/N_t, changes exogenously. Thus, the number of workers is determined exogenously, while the hours worked per worker are determined endogenously.

The household is utilitarian. The representative household chooses consumption $(c_{y,t}, c_{o,t})$, land holdings $(h_{y,t}, h_{o,t})$, hours worked per worker (l_t), and the aggregate savings or the aggregate supply of loanable bonds (B_t), in order to maximize the present value of the weighted sum of future utility flows given by,

$$E_0 \sum_{t=0}^{\infty} \beta_h^t \left\{ u_{y,t} N_t^y + u_{o,t} \left(N_t - N_t^y\right) \right\},$$

subject to a flow budget constraint:

$$\begin{aligned} & c_{y,t} N_t^y + c_{o,t} \left(N_t - N_t^y\right) + q_t h_{y,t} N_t^y + q_t h_{o,t} \left(N_t - N_t^y\right) + B_t \\ & \leq w_t l_t N_t^y + q_t h_{y,t-1} N_{t-1}^y + q_t h_{o,t-1} \left(N_{t-1} - N_{t-1}^y\right) + R_{t-1} B_{t-1}, \end{aligned}$$

where q_t is the price of land; w_t is real wages; and $u_{y,t}$ and $u_{o,t}$ are given by (1) and (2) respectively. β_h is the subjective discount factor of households. The loanable bond purchased in period t pays off R_t units of goods in all states of nature in period $t+1$.

The budget constraint above makes clear the first channel through which a decline in the ratio of workers to the total population leads to a fall in the real interest rate, outlined in the introduction. Specifically, consider a situation where the ratio of workers to the total population, N_t^y/N_t, declines, while the total population, N_t, remains unchanged. This reduces the wage income for households, given by $w_t l_t N_t^y$, while the total number of household members who consume goods remains unchanged. Once such changes are expected, the household saves more in the current period in order to smooth out the consumption level of household members into the future. This increases the supply of funds, and places downward pressure on the real interest rate.

By denoting the marginal utility of consumption by λ_t, the optimality conditions of the household are represented by the following equations:

$$\lambda_t = \beta_h E_t \left[\lambda_{t+1} R_t\right], \tag{3}$$

$$\lambda_t = \frac{\left[c_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma}}{c_{y,t}}, \tag{4}$$

$$\frac{\left[c_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma}}{c_{y,t}} = \frac{\left[c_{o,t} h_{o,t}^{\phi}\right]^{1-\sigma}}{c_{o,t}}, \tag{5}$$

$$\lambda_t q_t = \phi \frac{\left[c_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma}}{h_{y,t}} + \beta_h E_t \left[\lambda_{t+1} q_{t+1}\right], \tag{6}$$

$$\frac{\left[c_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma}}{h_{y,t}} = \frac{\left[c_{o,t} h_{o,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma}}{h_{o,t}}, \tag{7}$$

$$\lambda_t w_t = \chi \left(c_{y,t} h_{y,t}^{\phi}\right)^{1-\sigma} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)^{-\sigma} l_t^{1/\nu}. \tag{8}$$

Equation (3) is the optimality condition for household savings, or the supply of loanable funds. It equates the marginal utility of current consumption with the marginal return on savings in terms of utility units. The marginal utility of consumption is given by equation (4). Equation (5) gives us the equilibrium relationship between the consumption of worker and the consumption of non-worker.

Equation (6) is the optimality condition for land holdings. It equates the marginal cost of increasing land holdings with the marginal benefit of doing so. The latter consists of the utility benefit of additional land and the expected benefit from selling land in the next period. Equation (7) gives us the equilibrium relationship between the land holdings of worker and the land holdings of non-worker.

Equation (8) is the labor supply curve, which equates the marginal benefit of supplying extra hours of work with the marginal cost of doing so.

2.2 Entrepreneurs

A representative entrepreneur produces homogenous goods Y_t using labor L_t^d (which is the number of workers times the hours worked per worker), capital stock K_{t-1}, and land $H_{e,t-1}$ as inputs, according to the Cobb-Douglas production function,

$$Y_t = A_t^{1-\alpha_k} K_{t-1}^{\alpha_k} \left(L_t^d\right)^{\alpha_l} H_{e,t-1}^{1-\alpha_k-\alpha_l}, \tag{9}$$

where $A_t^{1-\alpha_k}$ is TFP, and α_k and α_l are the parameters that govern the capital and labor shares of income.

The entrepreneur raises inter-period and intra-period loans. As for the former, the entrepreneur issues loanable bonds B_t in period t that must be repaid in period $t+1$. As for the latter, we assume that a fraction θ of the sum of labor costs and investment in period t must be repaid within the period, as in Bianchi and Mendoza (2011). As in Kiyotaki and Moore (1997), we assume that the land serves as a collateral for loan, and the entrepreneur can borrow up to a fraction κ of the expected present discounted value of the land in the next period: a collateral constraint places an upper bound on the sum of inter-period loans (B_t) and intra-period loans ($\theta\left(w_t L_t^d + I_t\right)$):

$$B_t + \theta\left(w_t L_t^d + I_t\right) \leq \kappa E_t\left[\frac{q_{t+1} H_{e,t}}{R_t}\right]. \tag{10}$$

We include the land value in the collateral because numerous empirical studies on the investment behavior of Japanese firms claim that the land plays an important role as a collateral (Ogawa and Suzuki 1998).

Underlying the collateral constraint above is the following contract enforcement problem. When the entrepreneur defaults on the loan, the lender can seize the land owned by the entrepreneur. To seize and liquidate the land, however, the lender must pay costs proportional to the value of the land, and can recoup only a fraction κ of the land value. Supposing that the lender is willing to lend the amount that can be recovered in the event of default, we obtain constraint (10).

The entrepreneur chooses consumption $c_{e,t}$ (denoted in per-capita terms), labor inputs L_t^d, land holdings $H_{e,t}$, investment I_t, and loanable bonds B_t, to maximize the present value of expected utility flows given by,

$$E_0 \sum_{t=0}^{\infty} \beta_e^t \frac{c_{e,t}^{1-\sigma} - 1}{1-\sigma} N_t,$$

subject to the production function (9), the collateral constraint (10), the budget constraint, and the law of motion for capital, given by,

$$c_{e,t} N_t + q_t H_{e,t} + w_t L_t^d + I_t + R_{t-1} B_{t-1} = Y_t + q_t H_{e,t-1} + B_t, \tag{11}$$

$$K_t = K_{t-1} (1-\delta) + I_t, \tag{12}$$

where δ is the depreciation rate of capital, and β_e is the subjective discount factor of the entrepreneur. The number of entrepreneurs is assumed to be N_t, the same as the total population of households.

We set the value of β_e such that it is lower than the subjective discount factor of households, $\beta_e < \beta_h$. This parameter setting implies that the entrepreneur is impatient relative to the household,[4] and ensures that the collateral constraint binds in the neighborhood of the steady state.[5] As we will illustrate in section 4, a binding collateral constraint serves to amplify the effects of demographic changes on the real interest rate. Specifically, a change in demographics affects the price of land and the value of collateral. This, in turn, influences the entrepreneur's demand for loanable funds, labor inputs, and investment through the collateral constraint (10). This amplifies the original impact of demographic changes on macroeconomic variables.

If we denote by $\tilde{\mu}_t$ the Lagrangian multiplier associated with the collateral constraint (10), the entrepreneur's optimality conditions are represented by the following equations:

$$c_{e,t}^{-\sigma} = \beta_e E_t \left[c_{e,t+1}^{-\sigma} R_t \right] + \tilde{\mu}_t, \tag{13}$$

$$c_{e,t}^{-\sigma} q_t = \beta_e E_t c_{e,t+1}^{-\sigma} \left\{ (1-\alpha_k-\alpha_l) \frac{Y_{t+1}}{H_{e,t}} + q_{t+1} \right\} + \tilde{\mu}_t \kappa E_t \left[\frac{q_{t+1}}{R_t} \right], \tag{14}$$

[4] Iacoviello (2005) cites several empirical studies which document that the discount factor of individuals with small wealth (borrowers) tends to be lower than that of individuals with large wealth (lenders).

[5] If we denote by μ the steady state level of the Lagrangian multiplier for the collateral constraint in the entrepreneur's problem normalized by the marginal utility of consumption, it is given by $\mu = (\beta_h - \beta_e) R$. When $\beta_h > \beta_e$, μ is positive, implying that the collateral constraint is binding.

$$c_{e,t}^{-\sigma} + \tilde{\mu}_t \theta = \beta_e E_t \left\{ c_{e,t+1}^{-\sigma} \left(\alpha_k \frac{Y_{t+1}}{K_t} + 1 - \delta \right) + \tilde{\mu}_{t+1} \theta (1 - \delta) \right\}, \tag{15}$$

$$\alpha_l \frac{Y_t}{L_t^d} = w_t \left(1 + \theta \frac{\tilde{\mu}_t}{c_{e,t}^{-\sigma}} \right). \tag{16}$$

Equation (13) is the optimality condition for the entrepreneur's demand for loanable funds, which equates the marginal utility from consumption with the marginal cost of borrowing. The latter includes the repayments on the loan in the next period (the first term on the right-hand side) and the tightening of the borrowing constraint in the current period (the second term on the right-hand side).

Equation (14) is the optimality condition for land holdings by the entrepreneur. It equates the cost of increasing land holdings with the benefit of doing so. The latter consists of an increase in production, the resale value of land, and the relaxation of the borrowing constraint in the next period.

Equation (15) is the optimality condition for investment in physical capital. It equates the marginal cost of investment with the marginal benefit of investment, both denoted in the unit of utility. The cost of additional investment consists of giving up consumption in the current period and the tightening of the collateral constraint. The benefit includes increase in production and relaxation of the collateral constraint in the next period. The latter arises because there will be less need for investment in the next period.

Equation (16) is the labor demand curve, which equates the marginal product of labor with the marginal cost of hiring. Because of the working capital assumption, the cost consists of both the wage cost and the tightening of the collateral constraint.

The entrepreneur's problem described above makes explicit the second and third channels through which demographic changes affect the real interest rate, outlined in the introduction. In this model, a decline in the ratio of workers to the total population works like a fall in TFP. To see this explicitly, divide both sides of the production function (9) by the total population N, to express the per-capita output as $y = A^{1-\alpha_k} k^{\alpha_k} (lN^y/N)^{\alpha_l} h^{1-\alpha_k-\alpha_l}$ where y, k, and h denote per-capita levels of output, capital stock, and land. This equation implies that a decline in the ratio of workers to the total population, N^y/N, operates like a fall in TFP, $A^{1-\alpha_k}$. The resulting fall in the marginal product of capital reduces the entrepreneur's incentive to invest in capital stock, and decreases the entrepreneur's demand for funds, placing downward pressure on the real interest rate. At the same time, a decrease in the ratio of workers to the total population lowers the marginal product of land. This decreases the entrepreneur's demand for land, and land prices fall. A decline in the collateral values constrains the demand for loanable funds by the entrepreneur, and lowers the real interest rate further.

2.3 Equilibrium

We close the model by imposing market clearing conditions for goods, labor (the number of workers times the hours worked per worker), and land. These are given by the following equations:

$$Y_t = c_{y,t}N_t^y + c_{o,t}(N_t - N_t^y) + c_{e,t}N_t + I_t, \tag{17}$$

$$l_t N_t^y = L_t^d, \tag{18}$$

$$H = h_{y,t}N_t^y + h_{o,t}(N_t - N_t^y) + h_{e,t}N_t. \tag{19}$$

In equation (19), $h_{e,t} \equiv H_{e,t}/N_t$ is land holdings per entrepreneur. We assume that the aggregate supply of land, H, is constant.

The exogenous variables in the model are the TFP growth rate, $z_t \equiv A_t/A_{t-1}$, the ratio of workers to the total population, $n_t^y \equiv N_t^y/N_t$, and the growth rate of the total population, $\gamma_t \equiv N_t/N_{t-1}$.

We normalize the model variables so that they are stationary in the presence of a sustained growth in TFP: (i) we normalize the Lagrangian multiplier for the collateral constraint by the marginal utility of the entrepreneur: $\mu_t \equiv \tilde{\mu}_t/c_{e,t}^{-\sigma}$; (ii) we normalize the per-capita endogenous variables by A_t: $\widehat{y}_t \equiv Y_t/(A_tN_t)$, $\widehat{c}_{y,t} \equiv c_{y,t}/A_t$, $\widehat{c}_{o,t} \equiv c_{o,t}/A_t$, $\widehat{c}_{e,t} \equiv c_{e,t}/A_t$, $\widehat{i}_t \equiv I_t/(A_tN_t)$, $\widehat{k}_t \equiv K_t/(A_tN_t)$, $\widehat{b}_t \equiv B_t/(A_tN_t)$; (iii) we normalize the real wage and land price by the level of TFP: $\widehat{w}_t \equiv w_t/A_t$, $\widehat{q}_t \equiv q_t/A_t$; and (iv) we normalize the marginal utility of household consumption: $\widehat{\lambda}_t \equiv \lambda_t A_t^\sigma$.

Given the process for the exogenous variables $\{z_t, n_t^y, \gamma_t\}$ and the initial conditions for the endogenous state variables ($h_{y,-1}$, $h_{o,-1}$, $h_{e,-1}$, $\widehat{k}_{-1}$, $R_{-1}\widehat{b}_{-1}$), a competitive equilibrium consists of a sequence of prices $\{R_t$, $\widehat{w}_t$, $\widehat{q}_t$, $\mu_t\}$ and a set of allocations $\{\widehat{y}_t$, $\widehat{\lambda}_t$, $\widehat{c}_{y,t}$, $\widehat{c}_{o,t}$, $\widehat{c}_{e,t}$, $\widehat{i}_t$, $\widehat{k}_t$, $\widehat{b}_t$, $h_{y,t}$, $h_{o,t}$, $h_{e,t}$, l_t, $L_t^d\}$ that satisfy the following conditions: (i) given prices, the set of allocations satisfies the optimality conditions (3)-(16); and (ii) all markets clear. The appendix describes the equilibrium conditions expressed in terms of the stationary variables.

The steady state in this model is defined as the situation in which n_t^y, γ_t, and z_t are constant. The steady-state level of the real interest rate in our model is given by $R = z^\sigma/\beta_h$: it is determined by the steady-state TFP growth rate, the subjective discount factor of the household, and the parameter that determines the elasticity of intertemporal substitution.

2.4 A Model without Collateral Constraints

In order to illustrate the role of collateral constraints, we consider an alternative model in which collateral constraints are absent. Specifically in order to consider such a case, we solve the following modified model. First, we remove the collateral constraint (10) from the entrepreneur's optimization problem. Second, we set the subjective discount factor of the entrepreneur equal to the subjective discount factor of the household,[6] i.e., we set β_e equal to β_h. Third, we consider a model similar to a standard real business cycle model in that (i) the household maximizes the weighted average of the utility levels of the household and the entrepreneur where the weights are given by the population share; (ii) capital stock and land are owned by the household and they are lent to the entrepreneur; and (iii) the entrepreneur conducts production to maximize profits. Fourth, we include a constant tax rate on each of the wage income, the return on capital stock, the return on land, and the resale value of land so that the steady-state levels of the endogenous variables in the model without collateral constraints coincide with the steady-state levels of these variables in the model with collateral constraints, except for the consumption shares among workers, non-workers, and entrepreneurs. We do this so that we can properly compare the dynamics of the two models.

Once we solve this alternative model, we obtain equilibrium conditions that consist of the same set of equations as in the baseline model, except that (i) the Lagrangian multiplier for the collateral constraint, $\tilde{\mu}_t$, is constant at zero; (ii) the subjective discount factor of the entrepreneur is the same as that of the household; (iii) the constant tax rates are included; and (iv) the collateral constraint and the household budget constraint are removed from the system of equilibrium conditions (see appendix for details).

[6] In the absence of collateral constraints, we cannot ensure the existence of a balance growth path when β_e is smaller than β_h. We avoid such outcome by setting β_e equal to β_h.

3 Parameterization and Simulation Procedure

3.1 Parameterization

A period in the model is a year. We set the parameter values so that the model captures the main characteristics of the Japanese economy (Table 1).

The subjective discount factor of the household, β_h, is set to 0.995 so that the steady-state real interest rate in the model is equal to the average real interest rate in Japan between 1990 and 2008 (1.92%), where the real interest rate is calculated as the one-year government bond yield minus the inflation rate of the GDP deflator. We follow Hayashi and Prescott (2002) and Braun and Waki (2006) to set the capital income share, α_k, to 0.362 and the depreciation rate of capital, δ, to 0.089. We set the labor income share, α_l, to 0.625 so that the steady-state ratio of land value held by the entrepreneur to output matches the average ratio of land value in private non-financial corporate sector to nominal GDP in Japan for 1990-2008. The data source is the System of National Accounts.

The parameter related to working capital, θ, is set to 0.235 using the method of Bianchi and Mendoza (2011): we choose this parameter so that the steady-state ratio of working capital to output in the model matches the average ratio of the sum of currency and transferable deposits held by non-financial corporations to nominal GDP in Japan for 1990-2008. The data source for the currency and deposits is the Flow of Funds Statistics.

We follow Chen, Imrohoroglu and Imrohoroglu (2007) to set the parameter related to the intertemporal elasticity of substitution, σ, to 1.5. The parameter ν is set to 1.0, which implies that the Frisch elasticity of labor supply is 1. These values are standard in the real business cycle literature. The preference parameter related to the share of non-housing, ϕ, is set to 0.033 so that the model's steady-state ratio of land held by the entrepreneur to that held by the household is equal to the average ratio of land held by private non-financial corporations to that held by households in Japan for 1990-2008. We normalize the steady-state hours worked per worker at unity by setting the preference parameter χ to 0.923. The aggregate supply of land, H, is normalized at unity.

The parameter in the collateral constraint, κ, is set to 0.89, which is the value estimated in Iacoviello (2005). We set the subjective discount factor of entrepreneurs, β_e, at 0.95 so that the steady-state ratio of capital stock to output in the model is equal to the 1990-2008 average ratio of capital stock to output in Japan. As noted previously, β_e must be smaller than β_h for the collateral constraint to bind in the steady state.

3.2 Simulation Procedure

We assume perfect foresight and conduct deterministic simulations. Since we are interested in the Japanese economy before and after the bubble period, we start our simulations in 1975, which is about 10 years before the bubble period. We set the terminal period for our simulations to 2150: as we explain below, we assume that there is no further changes in the exogenous variables after 2100, and we think it is enough to have 50 years of transition for the economy to reach a new steady state.

The model has three exogenous sources of variation: the ratio of the working-age population to the total population (n_t^y); the growth rate of the total population (γ_t); and the growth rate of TFP (z_t). As for the working-age population ratio and the growth rate of the total population, we use the realized values for 1975-2009, and the United Nations' forecast for 2010-2100 (the working-age population ratio for 1975-2040 is shown in Table 1). Since the working-age population ratio is approximately constant after 2070 in the United Nations' forecast,[7] we assume that it is constant for 2100-2150 at the level of 2100. The growth rate of the total population for 2100-2150 is assumed to be 0%: with constant supply of aggregate land H, we need this assumption for a steady state to exist.

As for the growth rate of TFP, we use the realized growth rates for 1975-2009, and assume that it is constant for 2010-2150 at the average growth rate during 1990-2008. We obtain the realized TFP using the method of Hayashi and Prescott (2002), with the following modifications. First, we take into account the fact that our production function includes land, while the production function in Hayashi and Prescott (2002) does not.[8] Second, we use the parameter values for the production function listed in Table 1. Third, we extend the dataset of Hayashi and Prescott (2002) for the period 2001-2009.

The TFP series constructed from the methodology of Hayashi and Prescott (2002) does not adjust for variations in the capital utilization rate. In the sensitivity analysis, we also conduct simulations with the utilization-adjusted TFP series which is contained in the JIP database.[9] Figure 2 presents the growth rate of our baseline TFP series and

[7] In obtaining this forecast, the United Nations assumes that the total fertility rate in Japan rises to a level close to 2.0 around 2080. To the extent that this forecast is too optimistic, the simulated real interest rate in this paper is too high.

[8] When we calculate TFP, we use the model's restriction that the sum of land held by the household and land held by private non-financial corporations (the entrepreneur in the model) is constant over time. The data source for land is the System of National Accounts.

[9] The JIP Database (the Japan Industrial Productivity Database) is compiled by RIETI and Hitotsubashi University, and it is available for 1970-2008. Since the TFP series contained in this database

the utilization-adjusted TFP series. Not surprisingly, the utilization-adjusted series is smoother than the baseline series.

Given the sequence of the exogenous variables $\{n_t^y, \gamma_t, z_t\}$ for 1975-2150 and the initial conditions for the endogenous state variables $(h_{y,1974}, h_{o,1974}, h_{e,1974}, k_{1974}, R_{1974}b_{1974})$, we use a Newton method to solve the system of equilibrium conditions from 1975 to 2150 and simulate the model.[10] We explain how we determine the initial conditions in the next section.

4 Results

This section presents the simulation results. We also conduct sensitivity analysis related to the data and the parameters we use in the simulations.

4.1 Effects of Demographic Changes

We first simulate the model using the variations in the ratio of the working-age population to the total population as the only shocks. We do this to understand the effects of demographics in isolation from the effects of TFP. For the same reason, we set the initial endogenous state variables in 1974 at their steady state levels.[11]

Figure 3 presents the simulation results for the real interest rate, the growth rate of land prices, and the tightness of the collateral constraint (μ_t). The thick line in panel (1) is the real interest rate simulated from the baseline model with collateral constraint. It rises in the late 1980s as the working-age population ratio rises. It is then followed by a sustained decline in the real interest rate between the late 1980s and the recent period, as the working-age population ratio falls. The first baby boomers retire in the latter half of the 2000s, and this is period in which the real interest rate falls most sharply. The real interest rate temporarily rises between the mid 2010s and the mid 2020s, as the pace of the decline in the working-age population ratio slows down. It then falls again in the 2030s as the second baby boomers begin to retire.

is not necessarily based on the same production function as that in this paper, we do not use it as the baseline series.

[10] The Dynare software is used to conduct simulations.

[11] We solve for the steady state in which the working-population ratio is set to the 1974 level, the TFP growth rate is set to the average growth rate for 1990-2008, and the growth rate of the total population is set to 0%.

The thick line in panel (2) of Figure 3 presents the growth rate of land prices simulated from the model with collateral constraint. The trend in land price growth shifts downwards in the late 1980s, roughly coinciding with the timing of the shift in the trend of the working-age population ratio. A decline in the working-age population ratio lowers the land prices through the following mechanism: when the workers decrease, the marginal product of land falls, and the firms reduce the demand for land. Since the supply of land is fixed, the land prices fall.

Note that the simulated growth rate of land prices becomes negative in the mid 1990s, while in the data, this occurs in the beginning of the 1990s. Krugman (1998) makes a related observation: demographic factors in Japan should have contributed to lowering land values beginning in the late 1980s, while the actual timing of the fall in land prices was delayed because of other factors, such as bubbles and expectations that TFP would grow rapidly, that worked to offset the downward pressure on the land prices coming from the demographic factor.

4.2 Amplification through Collateral Constraint

The collateral constraint works to amplify the effects of demographic changes on land prices and the real interest rate. We can see this by comparing the path of the real interest rate simulated from the baseline model with collateral constraint to that simulated from an alternative model that does not include such a constraint. The thin lines in panel (1) and panel (2) of Figure 3 present the real interest rate and the growth rate of land prices obtained from a model without collateral constraint. As we can see from the figure, both of these variables have smaller variations in the absence of collateral constraint.

Underlying this result is the following mechanism. In the presence of collateral constraint, a fall in land values from population aging reduces collateral values and tightens the collateral constraint. This, in turn, works to decrease the firm's demand for loanable funds and land further, and generates larger declines in the interest rate and land prices, relative to the case of no collateral constraint. To illustrate this mechanism more concretely, panel (3) of Figure 3 shows the simulated path of the variable that represents the degree of tightness of the collateral constraint. Specifically, we plot the Lagrangian multiplier attached to the collateral constraint normalized by the marginal utility of the entrepreneur's consumption, μ_t. By construction, an increase in this variable implies a tightening in the collateral constraint. The result suggests that the constraint was relaxed in the 1980s as the growth rate of land prices rose, while it became increasingly tight in the 1990s and 2000s as the growth rate of land prices fell. The case of no collateral constraint corresponds to the case in which μ_t is constant at zero.

4.3 Simulation with Variations in Both Demographics and TFP

What is the contribution of the demographic changes to the overall movements in the real interest rate in Japan? To analyze this issue, we must include other sources of variations. Hayashi and Prescott (2002) claim that a slowdown in the TFP growth is the main factor behind the stagnation of the Japanese economy since the 1990s. Similarly, Chen, Imrohoroglu and Imrohoroglu (2006) argue that changes in the TFP growth is the most important factor for the secular movements in the Japanese saving rate between 1960 and 2000. We now include variations in the TFP growth in our simulations.

As noted earlier, we use the TFP series obtained from the method of Hayashi and Prescott (2002) in our baseline analysis. The realized TFP growth rates are used for 1975-2009, and it is assumed that the TFP growth rate is constant for 2010-2150 at the average growth rate between 1990 and 2008.

In the simulations with both demographic and TFP shocks, we choose the initial conditions for the endogenous state variables in 1974 as follows. First, the initial capital stock is chosen to match the capital stock in the data. Second, since we do not have useful information about the initial condition for the bond holdings in 1974, we choose it so that the percentage deviation in the initial bond holdings from the steady state is the same as the percentage deviation in the initial capital stock from the steady state.[12] Finally, the initial land holdings of the household and the entrepreneur are set at their steady-state levels, because we do not have good information about these variables either.[13]

Before we turn to the simulation result for the real interest rate, we confirm in Figure 4 that the model with both demographic and TFP shocks does a reasonably good job in tracing the actual real GDP per total population in Japan for 1975-2009. The increase in the real GDP in the late 1980 is somewhat smaller in the model, possibly because our model does not take into account the effects of the asset price bubble during this period.

[12] The choice of the initial bond holdings affects the early parts of the simulation results, i.e., if we set it at a lower level than the one described above, the simulated real interest rate would start from a higher level than that reported below.

[13] If the initial land held by the entrepreneur was set at a lower level than that described above, the real interest rate would start from a higher level than that reported below. However, the results for the 1990s and the 2000s are not sensitive to this assumption.

Figure 5 shows the simulation results for the real interest rate, the land price growth, and the tightness of the collateral constraint. The thick lines labeled "Baseline" represent the series simulated from the model with collateral constraint, while the thin lines labeled "No collateral constraint" represent the series simulated from the model without collateral constraint. It is evident that the fluctuations in the real interest rate in Figure 5 are larger than those in the case of demographic shocks only: the real interest rate exceeds 6% in the late 1980s, and it drops below 0% during the period of financial turmoil in Japan (1997-98) as well as in the recession period in the late 2000s. Nonetheless, the overall trend in the real interest rate remains the same as in the case of demographic shocks only: there is an upward trend in the real interest rate until the late 1980s, and there is a declining trend after the early 1990s.[14] This is because until the late 1980s, both the trend in the working-age population ratio and the trend in the TFP growth are high, and after the early 1990s, both of these trends become low. The comparison of Figure 5 with Figure 3 tells us that the TFP is the dominant source of fluctuations in the real interest rate, but the demographic factor is also quantitatively important especially for its low-frequency movements. We obtain a similar result for the land prices. Note that the sharp rise and fall in the real interest rate we observe in Figure 5 are partly because the TFP series used in this simulation does not adjust for variations in the capital utilization rate. In the sensitivity analysis below, we re-do the simulation with a smoother, utilization-adjusted TFP series.

The panel (3) of Figure 5 shows the tightness of the collateral constraint. This also has sharp ups and downs reflecting the volatile movements in the TFP growth. However, its low frequency movements—the credit condition is relatively easy in the late 1980s and it becomes relatively tight after the early 1990s—remain the same as in the case of demographic shocks only. The sharp declines in the real interest rate during 1997-98 and the late 2000s correspond to the strong tightening in the collateral constraint in these periods. As in the case of demographic shocks only, changes in the tightness of the collateral constraint amplify the effects of shocks to the TFP growth rate on the real interest rate and land prices.

Note that the real interest rate is forecasted to remain below the steady-state level after 2010, although we assume that the TFP growth remains constant at the steady-

[14] In Figure 5, the real interest rate starts from a high level in 1975 and then it falls for the first several years, while in the case of demographic shocks only (Figure 3), the real interest rate rises during this period. This difference can be explained by the difference in the assumptions for the initial capital stock and initial bond holdings.

state level after 2010. This is because the working-age population ratio keeps falling during this period.

In the panel (1) of Figure 5, we also plot the actual data on the real interest rate for 1981-2009. The real interest rate here is the nominal one-year government bond yield minus the GDP deflator inflation rate. There is no surprise that our model does not replicate the cyclical movements in this data: the real interest rate in our model is closer in concept to the natural rate of interest, rather than the actual real interest rate in the economy. However, the figure suggests that our model traces the long-term trend in the data reasonably well.

4.4 Sensitivity Analysis

We now conduct sensitivity analysis related to the TFP series and the demographic variable which we use as inputs to simulations, as well as some of the parameters in the model. Table 2 reports the peak-to-bottom difference in the real interest rate between 1985 and 2015 for each of the specifications we consider. We focus on this period because we are interested in the bubble period in Japan and the near future. We also report the results that exclude 2007-09 from the sample period because the drop in TFP during this period is extremely large and we expect the results are sensitive to the way how the TFP series is constructed. The columns labeled "All shocks" report results from the simulations with both TFP and demographic shocks, while the columns labeled "Demographics only" report the results from the simulations with only demographic shocks. All the results in this table are obtained from the model with collateral constraint.

Utilization-adjusted TFP The baseline simulation reported earlier used a TFP series that is not adjusted for the variations in the capital utilization rate. When we instead use the utilization-adjusted TFP series contained in the JIP Database, the movements in the real interest rate become much smoother.[15] Specifically, the peak-to-bottom difference in the real interest rate drops from 1083 basis points in the baseline case to 527 basis points in the case of the utilization-adjusted TFP. Thus, when we use this alternative TFP series, the relative contribution of the demographic factors to the real interest rate becomes larger.

[15] The JIP series is available until 2008. When we use this series in our simulations, we assume that the TFP growth rate is constant for 2009-2150 at the average growth rate of the baseline TFP series during 1990-2008.

Using labor force instead of working-age population The baseline simulation used the data on the working-age population (persons with age 15-64) as a proxy for the number of workers in the model (N_t^y). This approximation is valid as long as (i) most of the persons in this age category work; and (ii) most of the persons in other age categories do not work. Here, we consider labor force as an alternative measure of workers in the model. Using labor force may be justified on the ground that it is more closely related to the concept of workers in the model. The potential drawback of using labor force is that it is more likely than the working-age population to be influenced by non-demographic factors such as the business cycles. According to the result in Table 2, the fluctuations in the real interest rate become larger when we use labor force, if we include only demographic shocks in the simulations. However, once TFP shocks are included in the simulation, we do not find a noticeable difference between the two cases.

Fixing the growth rate of the total population Our baseline simulations allowed for variations in both the working-age population ratio and the growth rate of the total population. In order to understand which of these variables is more important for the movements in the real interest rate, we conduct a couter-factual simulation in which the growth rate of the total population is fixed at 0%. The result shown in Table 2 implies that the fluctuations in the real interest rate become slightly smaller, but the difference from the baseline case is small. This suggests that the main demographic factor that influences the real interest rate is the changes in the working-age population ratio, not the changes in the total population itself. This is for two reasons. First, the realized variations in the total population are relatively small: in the process of population aging, a decrease in the working-age population is partly compensated by an increase in the elderly population. Second, in terms of the structure of the model, a change in the working-age population ratio influences the real interest rate through more diverse channels than a change in the growth rate of the total population. Specifically, the former influences the income per total population of the household, while the latter does not affect this variable.

Alternative parameter values Finally, we discuss the sensitivity of the results to the parameter values. First, lowering the level of the parameter related to the collateral constraint, κ, from our baseline value of 0.98 to 0.7 reduces the fluctuations in the real interest rate, because it reduces the sensitivity of borrowing to land prices. Second, increasing the elasticity of intertemporal substitution by lowering the level of the preference parameter σ from 1.5 to 1.0 reduces the variations in the real interest rate. Third, reducing the elasticity of labor supply by lowering the level of the preference parameter ν from 1.0 to 0.5 increases the volatility of the real interest rate. However, the relative contribution of the demographic factors to the overall movements in the real interest rate remains unchanged in all cases.

5 Concluding Remarks

This paper has developed a dynamic general equilibrium model to analyze the effects of demographic changes on the real interest rate. Our quantitative analysis indicates that a decline in the ratio of the working-age population to the total population in the process of population aging has worked to lower the real interest rate in Japan since the early 1990s, and such a impact has been magnified in the presence of collateral constraints. When we simulate the model using the realized changes in the demographic structure and the TFP growth in Japan, we find that the TFP growth is the dominant source of fluctuations in the real interest rate, but the demographic factor is also quantitatively important especially for its long-term movements.

In order to make our analysis transparent, we have made several simplifying assumptions. One is that the demographic changes and the TFP growth are independent of each other. Potentially, changes in the demographic structure affect TFP through at least two channels. First, population aging may encourage technological progress which helps to mitigate the negative impact of the declining workforce. Second, the aging of the population may lower TFP by increasing mismatches in labor and goods markets. Taking into account such interactions between the two factors is an important avenue for future research.

We have also not considered fiscal and international issues. A worsening fiscal balance through an increase in social security expenditures in the process of population aging may have an important impact on the household savings and the real interest rate. Additionally, if we allow for international mobility in labor and capital, the negative impact of a declining workforce on the real interest rate should become smaller than that reported in this paper.

References

[1] Bakshi, Gurdip S. and Zhiwu Chen, "Baby Boom, Population Aging, and Capital Markets," *Journal of Business 67*(2), 1994, 165-202.

[2] Bianchi, Javier and Enrique G. Mendoza, "Overborrowing, Financial Crises and Macro-Prudential Policy," University of Maryland, 2011.

[3] Braun, Anton R., Daisuke Ikeda and Douglas H. Joines, "The Saving Rate in Japan: Why It Has Fallen and Why It Will Remain Low?" *International Economic Review 50*(1), 2009, 291-321.

[4] Braun, Anton R. and Yuichiro Waki, "Monetary Policy during Japan's Lost Decade," *Japanese Economic Review 57*(2), 2006, 324-344.

[5] Chen, Kaiji, Ayse Imrohoroglu, and Selahattin Imrohoroglu, "The Japanese Saving Rate," *American Economic Review 96*(5), 2006, 1850-1858.

[6] Chen, Kaiji, Ayse Imrohoroglu, and Selahattin Imrohoroglu, "The Japanese Saving Rate between 1960 and 2000: Productivity, Policy Changes, and Demographics," *Economic Theory 32*, 2007, 87-104.

[7] Guerrieri, Veronica and Guido Lorenzoni, "Credit Crises, Precautionary Savings and the Liquidity Trap," MIT, 2011.

[8] Hayashi, Fumio and Edward C. Prescott, "The 1990s in Japan: A Lost Decade," *Review of Economic Dynamics 5*, 2002, 206-235.

[9] Iacoviello, Matteo, "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review 95*(3), 2005, 739–764.

[10] King, Robert G., Charles I. Plosser, and Sergio T. Rebelo, "Production, Growth and Business Cycles: Technical Appendix," *Computational Economics 20*(1-2), 2002, 87-116.

[11] Kiyotaki, Nobuhiro and John Moore, "Credit Cycles," *Journal of Political Economy 105*(2), 1997, 211–248.

[12] Krugman, Paul R., "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 1998, 137-187.

[13] Liu, Zheng, Pengfei Wang and Tao Zha, "Do Credit Constraints Amplify Macroeconomic Fluctuations?" Federal Reserve Bank of Atlanta, 2011.

[14] Mankiw, N. Gregory and David N. Weil, "The Baby Boom, the Baby Bust, and the Housing Market," *Regional Science and Urban Economics 19*, 1989, 235-258.

[15] Miles, David, "Modelling the Impact of Demographic Change upon the Economy," *Economic Journal 109*, 1999, 1-36.

[16] Ogawa, Kazuo and Kazuyuki Suzuki, "Land Value and Corporate Investment: Evidence from Japanese Panel Data," *Journal of the Japanese and International Economies 12*, 1998, 232-249.

Appendix

This appendix presents the equilibrium conditions for the model with collateral constraint and the model without collateral constraint.

1. Equilibrium Conditions for the Model with Collateral Constraints

In terms of the stationary variables which are defined in Section 3, the equilibrium conditions can be written as follows:

$$\widehat{\lambda}_t = \beta_h E_t \left[\widehat{\lambda}_{t+1} z_{t+1}^{-\sigma} R_t \right], \tag{20}$$

$$\widehat{\lambda}_t = \widehat{c}_{y,t}^{-\sigma} \left[h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu} \right) \right]^{1-\sigma}, \tag{21}$$

$$\widehat{c}_{y,t}^{-\sigma} \left[h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu} \right) \right]^{1-\sigma} = \widehat{c}_{o,t}^{-\sigma} \left[h_{o,t}^{\phi} \right]^{1-\sigma}, \tag{22}$$

$$\widehat{\lambda}_t \widehat{q}_t = \phi \frac{\left[\widehat{c}_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu} \right) \right]^{1-\sigma}}{h_{y,t}} + \beta_h E_t \left[\widehat{\lambda}_{t+1} z_{t+1}^{1-\sigma} \widehat{q}_{t+1} \right], \tag{23}$$

$$\frac{\left[\widehat{c}_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu} \right) \right]^{1-\sigma}}{h_{y,t}} = \frac{\left[\widehat{c}_{o,t} h_{o,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu} \right) \right]^{1-\sigma}}{h_{o,t}}, \tag{24}$$

$$\widehat{\lambda}_t \widehat{w}_t = \chi \left(\widehat{c}_{y,t} h_{y,t}^{\phi} \right)^{1-\sigma} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu} \right)^{-\sigma} l_t^{1/\nu}, \tag{25}$$

$$\widehat{q}_t = \beta_e E_t \left\{ \frac{\widehat{c}_{e,t+1}^{-\sigma}}{\widehat{c}_{e,t}^{-\sigma}} z_{t+1}^{-\sigma} \left((1 - \alpha_k - \alpha_l) \frac{\widehat{y}_{t+1}}{h_{e,t}} z_{t+1} \gamma_{n,t+1} + \widehat{q}_{t+1} z_{t+1} \right) \right\} + \mu_t \kappa E_t \left[\frac{\widehat{q}_{t+1} z_{t+1}}{R_t} \right], \tag{26}$$

$$1 = \beta_e E_t \left[\frac{\widehat{c}_{e,t+1}^{-\sigma}}{\widehat{c}_{e,t}^{-\sigma}} z_{t+1}^{-\sigma} R_t \right] + \mu_t, \tag{27}$$

$$1 + \mu_t \theta = \beta_e E_t \left\{ \frac{\widehat{c}_{e,t+1}^{-\sigma}}{\widehat{c}_{e,t}^{-\sigma}} z_{t+1}^{-\sigma} \left(\alpha_k \frac{\widehat{y}_{t+1}}{k_t} z_{t+1} \gamma_{n,t+1} + 1 - \delta + \mu_{t+1} \theta (1 - \delta) \right) \right\}, \tag{28}$$

$$\alpha_l \frac{\widehat{y}_t}{l_t n_t^y} = \widehat{w}_t (1 + \theta \mu_t), \tag{29}$$

$$\widehat{b}_t + \theta \left(\widehat{w}_t l_t n_t^y + \widehat{i}_t \right) = \kappa E_t \left[\frac{\widehat{q}_{t+1} z_{t+1} h_{e,t}}{R_t} \right], \tag{30}$$

$$\widehat{k}_t = (1-\delta) \frac{\widehat{k}_{t-1}}{z_t \gamma_{n,t}} + \widehat{i}_t, \tag{31}$$

$$\widehat{c}_{e,t} + \widehat{q}_t h_{e,t} + \frac{\alpha_l}{1+\theta\mu_t} \widehat{y}_t + \widehat{i}_t + R_{t-1} \frac{\widehat{b}_{t-1}}{z_t \gamma_{n,t}} = \widehat{y}_t + \widehat{q}_t \frac{h_{e,t-1}}{\gamma_{n,t}} + \widehat{b}_t, \tag{32}$$

$$\widehat{y}_t = \widehat{c}_{y,t} n_t^y + \widehat{c}_{o,t}(1-n_t^y) + \widehat{c}_{e,t} + \widehat{i}_t, \tag{33}$$

$$\widehat{y}_t = (l_t n_t^y)^{\alpha_l} \, \widehat{k}_{t-1}^{\alpha_k} h_{e,t-1}^{1-\alpha_k-\alpha_l} \frac{1}{z_t^{\alpha_k} \gamma_{n,t}^{1-\alpha_l}}, \tag{34}$$

$$h_t = h_{y,t} n_t^y + h_{o,t}(1-n_t^y) + h_{e,t}, \tag{35}$$

where $h_t \equiv H/N_t$.

2. Equilibrium Conditions for the Model without Collateral Constraints

The equilibrium conditions in the model without collateral constraints correspond to a special case of the equilibrium conditions in the model with collateral constraints, in which (i) we remove the collateral constraint (30); (ii) we set the Lagrangian multiplier for the collateral constraint, μ_t, to zero in equations (26), (27), (28), (29); (iii) we set the subjective discount factor for the entrepreneur, β_e, at the same value as that for the household, β_h, in equations (27) and (28), (iv) we remove the budget constraint of the household (32); and (v) we include constant tax rates $(\tau_k, \tau_l, \tau_h, \tau_e)$ in equations (23), (25), (26), (28) so that the steady-state levels of the endogenous state variables and output in the model without collateral constraints are equated to the steady-state levels of these variables in the model with collateral constraints. Specifically, the equilibrium conditions are as follows:

$$\widehat{\lambda}_t = \beta_h E_t \left[\widehat{\lambda}_{t+1} z_{t+1}^{-\sigma} R_t \right],$$

$$\widehat{\lambda}_t = \widehat{c}_{y,t}^{-\sigma} \left[h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu} \right) \right]^{1-\sigma},$$

$$\widehat{c}_{y,t}^{-\sigma} \left[h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu} \right) \right]^{1-\sigma} = \widehat{c}_{o,t}^{-\sigma} \left[h_{o,t}^{\phi} \right]^{1-\sigma},$$

$$\widehat{\lambda}_t \widehat{q}_t = \phi \frac{\left[\widehat{c}_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma}}{h_{y,t}} + (1-\tau_h)\beta_h E_t \left[\widehat{\lambda}_{t+1} z_{t+1}^{1-\sigma} \widehat{q}_{t+1}\right],$$

$$\frac{\left[\widehat{c}_{y,t} h_{y,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma}}{h_{y,t}} = \frac{\left[\widehat{c}_{o,t} h_{o,t}^{\phi} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)\right]^{1-\sigma}}{h_{o,t}},$$

$$(1-\tau_l)\widehat{\lambda}_t \widehat{w}_t = \chi \left(\widehat{c}_{y,t} h_{y,t}^{\phi}\right)^{1-\sigma} \left(1 - \chi \frac{l_t^{1+1/\nu}}{1+1/\nu}\right)^{-\sigma} l_t^{1/\nu},$$

$$\widehat{q}_t = \beta_h E_t \left\{ \frac{\widehat{c}_{e,t+1}^{-\sigma}}{\widehat{c}_{e,t}^{-\sigma}} z_{t+1}^{-\sigma} \left((1-\tau_e)(1-\alpha_k-\alpha_l)\frac{\widehat{y}_{t+1}}{h_{e,t}} z_{t+1}\gamma_{n,t+1} + \widehat{q}_{t+1} z_{t+1}\right)\right\},$$

$$1 = \beta_h E_t \left[\frac{\widehat{c}_{e,t+1}^{-\sigma}}{\widehat{c}_{e,t}^{-\sigma}} z_{t+1}^{-\sigma} R_t\right],$$

$$1 = \beta_h E_t \left\{ \frac{\widehat{c}_{e,t+1}^{-\sigma}}{\widehat{c}_{e,t}^{-\sigma}} z_{t+1}^{-\sigma} \left((1-\tau_k)\alpha_k \frac{\widehat{y}_{t+1}}{k_t} z_{t+1}\gamma_{n,t+1} + 1 - \delta\right)\right\},$$

$$\alpha_l \frac{\widehat{y}_t}{l_t n_t^y} = \widehat{w}_t,$$

$$\widehat{k}_t = (1-\delta)\frac{\widehat{k}_{t-1}}{z_t \gamma_{n,t}} + \widehat{i}_t,$$

$$\widehat{y}_t = \widehat{c}_{y,t} n_t^y + \widehat{c}_{o,t}(1-n_t^y) + \widehat{c}_{e,t} + \widehat{i}_t,$$

$$\widehat{y}_t = (l_t n_t^y)^{\alpha_l}\, \widehat{k}_{t-1}^{\alpha_k} h_{e,t-1}^{1-\alpha_k-\alpha_l} \frac{1}{z_t^{\alpha_k}\gamma_{n,t}^{1-\alpha_l}},$$

$$h_t = h_{y,t} n_t^y + h_{o,t}(1-n_t^y) + h_{e,t}.$$

Table 1: Parameter Values

Parameter	Description	Value	Target/Source
β_h	Household discount factor	0.995	Average interest rate
β_e	Entrepreneur discount factor	0.95	Average capital stock/GDP
κ	Collateral constraint	0.89	Iacoviello (2005)
δ	Depreciation rate	0.089	Hayashi and Prescott (2002)
α_k	Capital income share	0.362	Hayashi and Prescott (2002)
α_l	Labor income share	0.625	Average land value/GDP
θ	Working capital	0.235	Average M1 held by firms/GDP
σ	Intertemporal elasticity	1.50	Chen, Imrohoroglu, Imrohoroglu (2007)
ν	Labor supply elasticity	1.00	Standard value in RBC literature
ϕ	Share of non-housing	0.033	Average land holdings by households
χ	Labor supply scale	0.923	Normalization ($l = 1$)

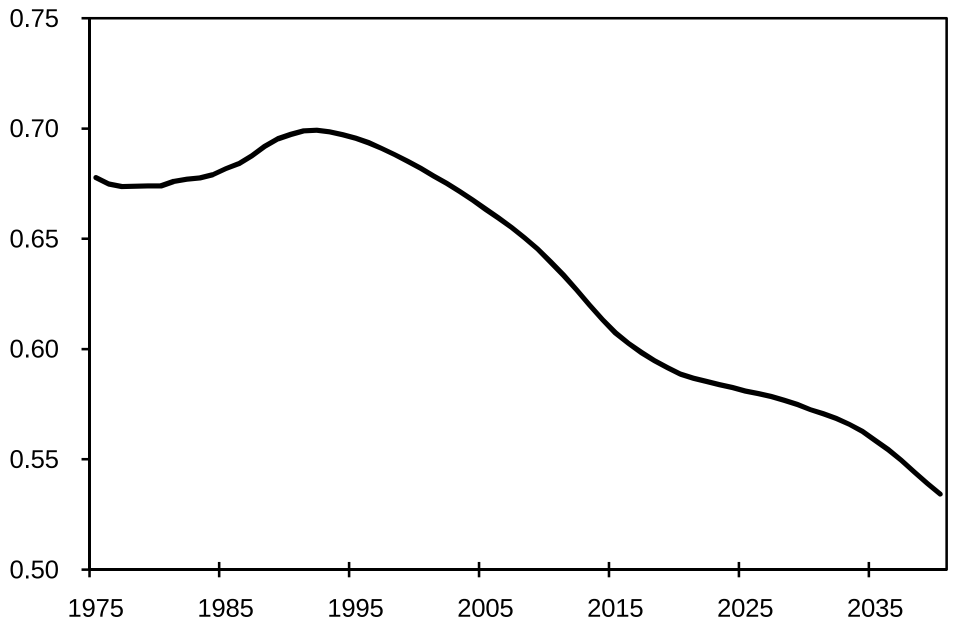
Table 2: Sensitivity Analysis

	All shocks		Demographics only	
	Full sample	Excl. 2007-09	Full sample	Excl. 2007-09
Baseline	1083	706	127	127
Alternative data				
(i) Utilization-adjusted TFP	527	510	127	127
(ii) Labor force	1046	674	150	150
(iii) Constant total population	1001	641	127	127
Alternative parameters				
(i) $\kappa = 0.7$	910	639	125	125
(ii) $\sigma = 1.0$	702	458	75	75
(iii) $\nu = 0.5$	1097	716	149	149

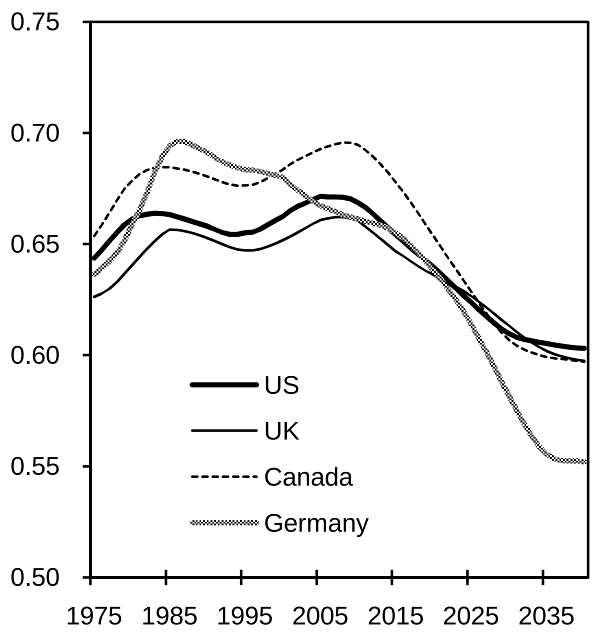
Notes: The table presents the peak-to-bottom difference in the real interest rate in basis points between 1985 and 2015. "All shocks" refers to the simulation that includes changes in both the working-age population ratio and the TFP growth. "Demographic only" refers to the simulation that includes only changes in the working-age population ratio. "Full sample" refers to the results from the whole sample (1985-2015), while "Excl. 2007-09" refers to the results from the sample that excludes 2007-09.

Figure 1: Ratio of Working-age Population to Total Population

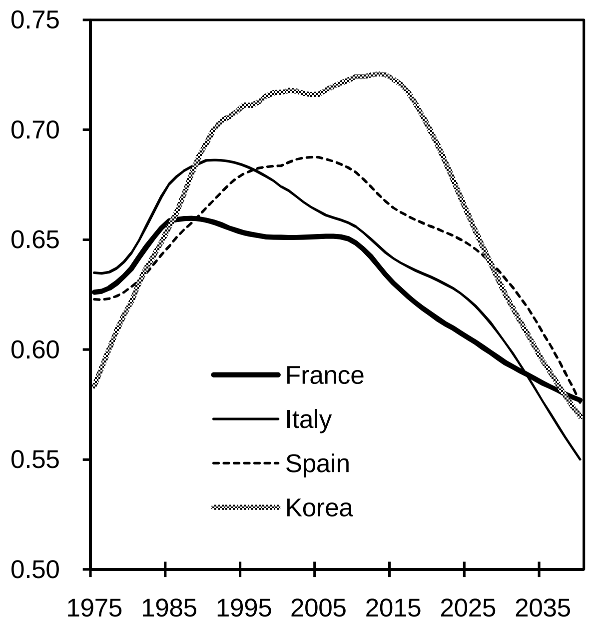
(1) Japan



(2) US, UK, Canada and Germany



(3) France, Italy, Spain and Korea



Note: The data after 2010 is the forecast of the United Nations.

Figure 2: TFP Growth in Japan

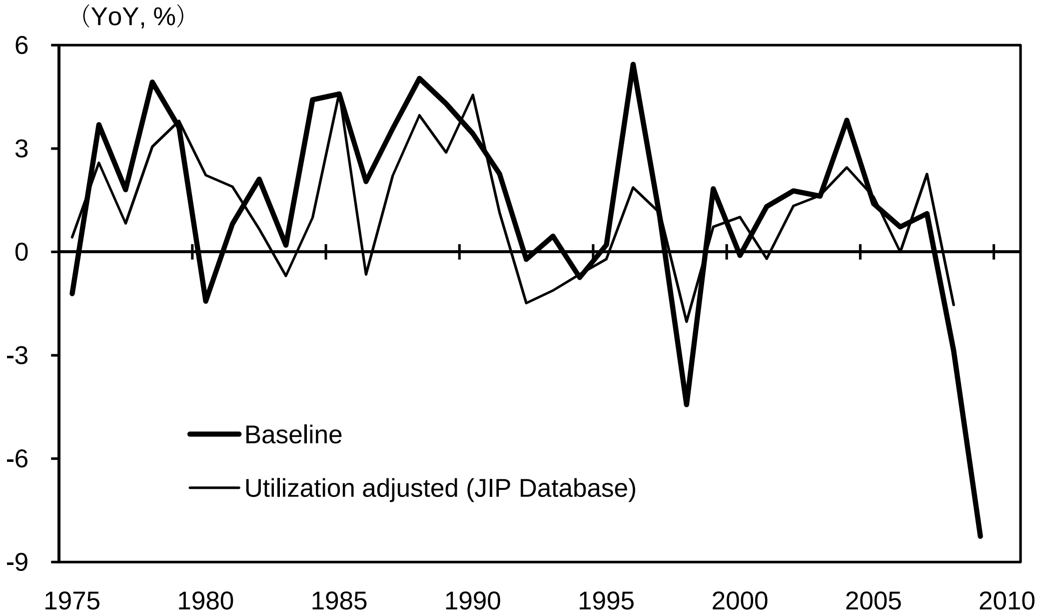
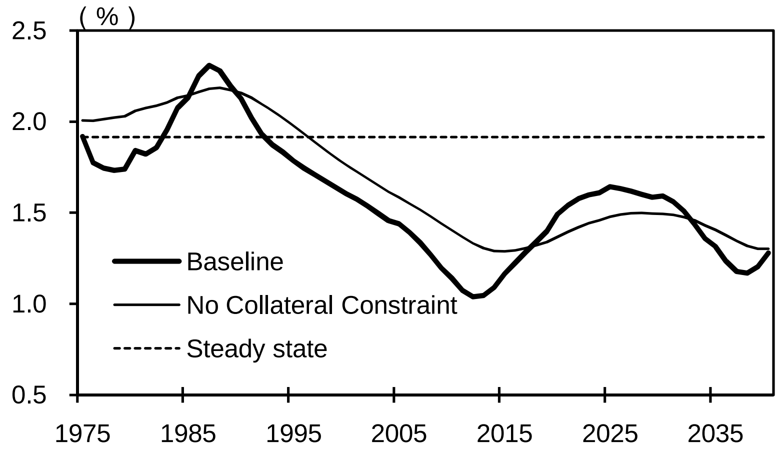
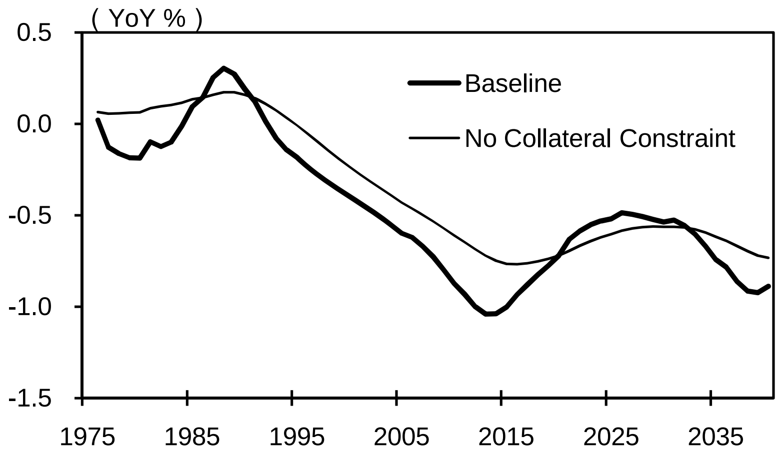


Figure 3: Simulation with Demographic Changes Only

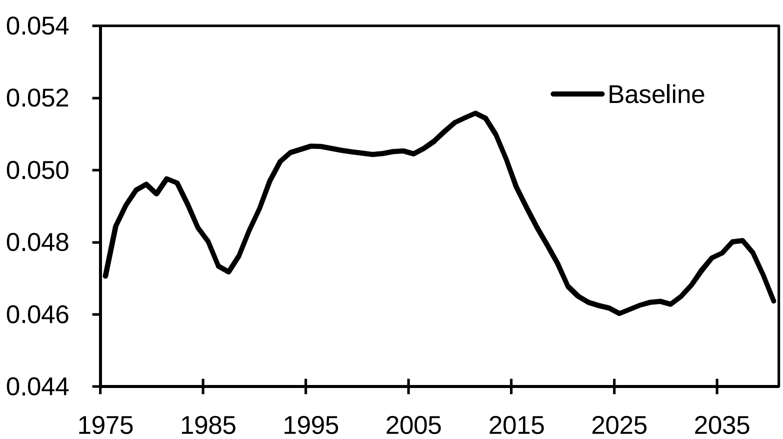
(1) Real Interest Rate



(2) Land Price Growth



(3) Tightness of Collateral Constraint



Notes: The figure plots the real interest rate, the growth rate of land prices, and the tightness of collatetal constraint in Japan for 1975-2040, simulated from the model. "Baseline" refers to the model with collateral constraint. "No Collateral Constraint" refers to the model withtout collateral constraint.

Figure 4: Real GDP per capita

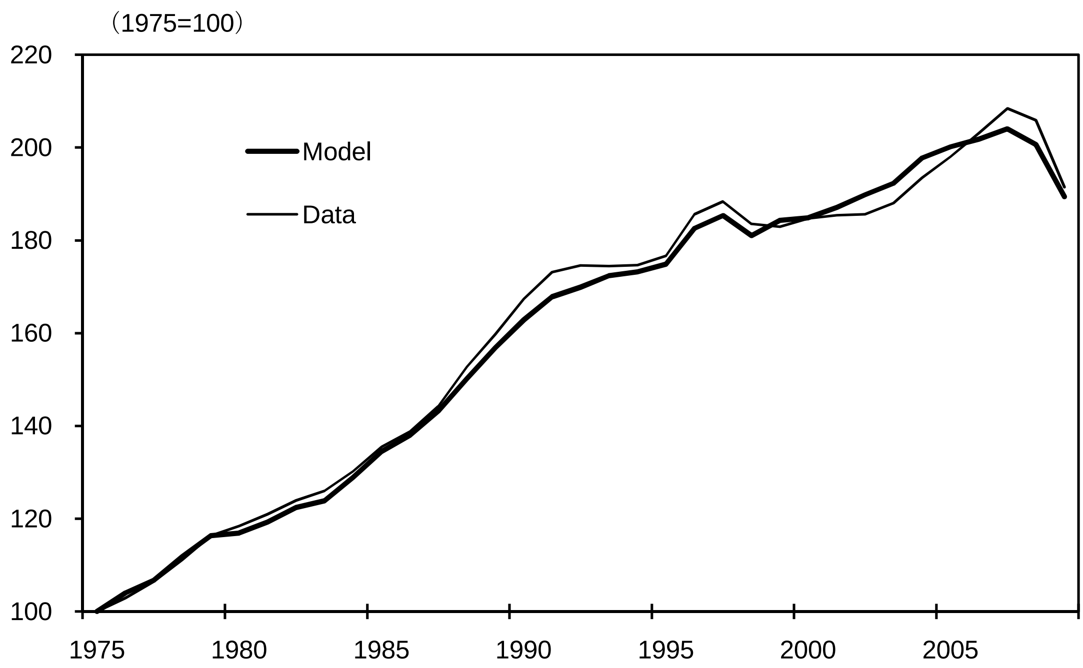
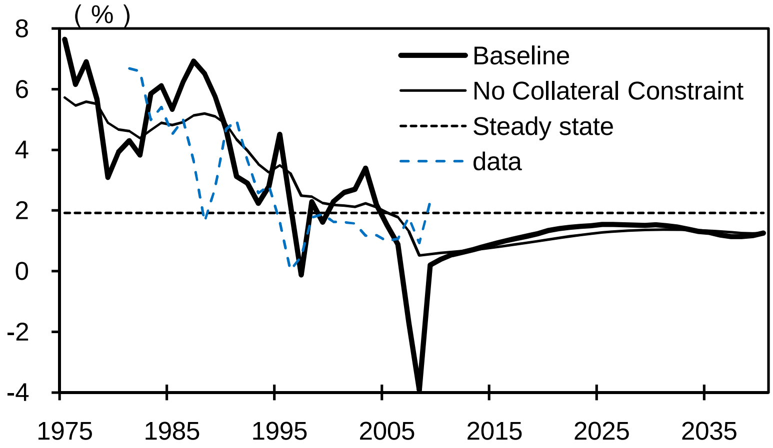
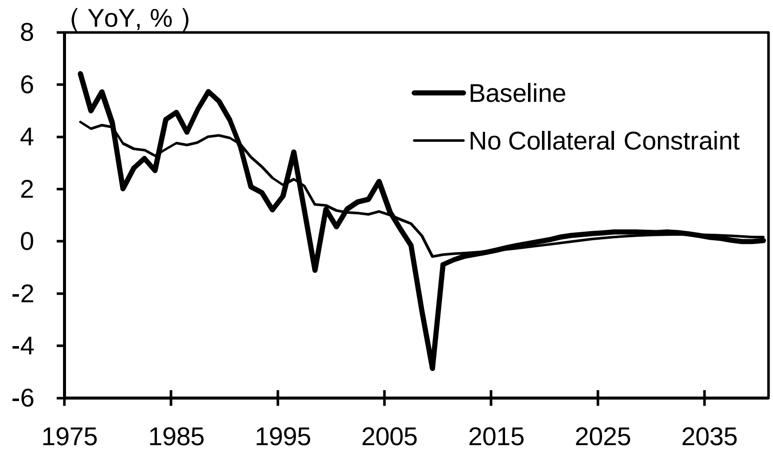


Figure 5: Simulation with Both Demographic Changes and TFP Growth

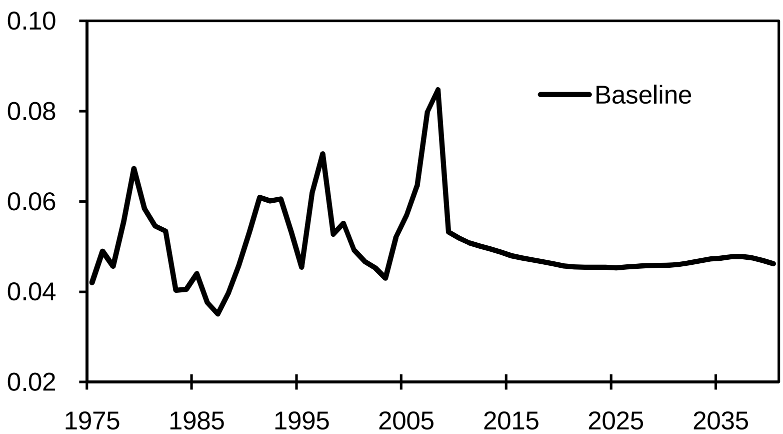
(1) Real Interest Rate



(2) Land Price Growth



(3) Tightness of Collateral Constraint



Notes: The figure plots the real interest rate, the growth rate of land prices, and the tightness of collatetal constraint in Japan for 1975-2040, simulated from the model. "Baseline" refers to the model with collateral constraint. "No Collateral Constraint" refers to the model withtout collateral constraint. "data" in panel (1) is the data for the real interest rate for 1981-2009.