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Asset Portfolio Choice of Banks and Inflation Dynamics

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Abstract

Since the mid-1990s, the asset portfolios of Japanese banks have continuously tilted toward government bonds, while lending to firms has declined. In this paper, we investigate the causes and consequences of such changes in banks' behavior by introducing banks' asset portfolio decision into an otherwise standard New Keynesian dynamic stochastic general equilibrium model. In our model, banks construct their portfolios under the value at risk constraint, which requires banks to repay their debt regardless of the return on their assets or whether the maximum loss on their assets materialized. As a result, the maximum loss on assets and banks' net worth affect banks' balance sheet and asset portfolio allocation by changing their risk taking capacity. For instance, an increase in downside risks or a deterioration in banks' net worth reduces their risk taking capacity, and results in a contraction of their balance sheets as well as rebalancing of their portfolios toward government bonds, thus dampening output and inflation. We estimate the model by Bayesian estimation and find that such portfolio decisions played an important role in the accumulation of government bonds and deflation in Japan since the latter half of the 1990s.

Keywords: Value at Risk Constraint; Banks' Asset Allocation; Deflation; Lost Decade.

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1 Introduction

Japan's long-lasting economic stagnation since the beginning of the 1990s, or what is increasingly coming to be called the two lost decades, is, in general, attributed to adverse environmental changes in the real side of the economy, such as the permanent slowdown of total factor productivity in the early 1990s, or the introduction of the mandatory reduction of working hour in the late 1980s.[1] The pioneering study by Hayashi and Prescott (2002), based on a simple growth model, shows that Japan's economic downturn in the 1990s can be explained by an exogenous decline in total factor productivity growth. Another strand of studies, such as Bayoumi (2001), Hoshi and Kashyap (2004, 2010), Caballero, Hoshi, and Kashyap (2008), and Hirose and Kurozumi (2010), by contrast, suggests that malfunction of the banking sector plays a large role in explaining Japan's two lost decades. Bayoumi (2001), for instance, using vector autoregression, argues that disruption in financial intermediation due to the deterioration of banks' balance sheets and the need to meet capital adequacy standards is the major explanation for the slowdown of the economy.

In fact, the banking sector has been subject to constant changes in the economic environment over the past two decades. From the beginning of the 1990s onward the banks needed to rebuild their balance sheets and reduce lending to firms in the face of mounting bad loans originating from the burst of the asset bubble and the full-scale enforcement of Basel capital adequacy requirements. Before the economy, including the banking sector, was able to embark on a recovery path, Japan suffered a banking crisis triggered by the collapse of Sanyo Securities and Yamaichi Securities, leading to a deterioration in banks' profit structures and balance sheets.[2] As a result, the disruption of financial intermediation worsened further.

The aim of this study is to examine the second explanation of Japan's disappointing performance focusing on malfunction in the financial sector further by scrutinizing the reasons for, and consequences of, economic activities in the banking sector. To this end, we shed light on one other peculiar change in the economic environment during the two lost decades of the 1990s and 2000s. That is, we focus on the secular increase in government bond issuance, particularly from the latter half of the 1990s. Figure 1 displays the time path of government bonds outstanding relative to GDP and the aggregate capital stock.[3] The figure clearly shows that the amount of government bonds outstanding has grown more quickly than GDP, and the aggregate asset portfolio is

[1] Also see Otsu (2011) for the role played by the wedge associated with the labor supply decision in explaining the Japanese business cycle.

[2] Kaihatsu and Kurozumi (2010) develop a model incorporating both frictions associated with the non-financial part of the economy and with the financial sector and quantitatively examine their relative importance in explaining Japanese business cycles.

[3] Unless otherwise noted, government bonds include treasury discount bills, central government securities and FILP bonds, local government securities, and public corporation securities.

tilted toward government bonds.[4] This acceleration of government debt accumulation is closely related to banks' adjustments of their balance sheets and of the composition of their asset portfolios. Figure 2 shows changes over the past three decades in banks' asset allocation between government bond purchases and loans to firms, as well as the ratio of the amount of government bonds outstanding held by banks over total government debt. As can be clearly seen, banks' purchases of government bonds started to rise in the mid-1990s, while banks' loan claims began to decline at around the same time. Consequently, as the bottom panel in Figure 2 indicates, the bulk of the increase in government bonds outstanding during the two lost decades has been absorbed by the banking sector. Banks' asset portfolio allocation thus has played a key role in the steady demand for government debt as the issuance of government bonds increased over time.

How then do the changes in banks' economic activities affect fluctuations in the macroeconomy. Figure 3 shows the path of GDP growth and inflation over time. GDP growth remained strong up until the beginning of the 1990s, but declined immediately after the burst of the bubble. Following a moderate recovery in the middle of the 1990s, GDP growth declined further after the banking crisis and never reverted back to the rates seen in the 1980s. Meanwhile, inflation was positive during the 1980s, began to weaken after the burst of the bubble in February 1991,[5] and has been negative since November 1997, the start of the banking crisis that gripped Japan for a number of years.[6][7]

In this paper, we explore the linkage between the macroeconomic variables and banks' adjustments of their balance sheets and allocation of assets, both from a theoretical and an empirical perspective. To this end, we develop a model that incorporates banks and government bonds into an otherwise standard New Keynesian dynamic stochastic general equilibrium (DSGE) model. In the model, banks collect deposits from households, and invest the deposits and their own net worth in two assets: loans to firms, which are equivalent to investment in productive capital, and government bonds. Banks decide the size of their balance sheets and their asset portfolio allocation between the two assets, so as not to violate the value at risk (VaR) constraint. Under the VaR constraint, banks have to repay all of their debt to households, regardless of the ex-post returns on the two types of assets. Since the ex-post returns from holding the assets are uncertain at the timing of the adjustment of asset portfolios, and can be lower than the deposit

[4] There is a growing literature on the accumulation of government bonds in the Japanese economy from the perspective of government debt sustainability, including Doi, Hoshi, and Okimoto (2011) and Imrohologlu and Sudo (2011). See Enomoto and Iwamoto (2008) for the welfare implications of fiscal policy undertaken during the two lost decades.

[5] There are several different views about when the bubble burst. February 1991 is the peak of the economic expansion that started from November 1986, dated by the cabinet office.

[6] Sugo and Ueda (2008), estimating a dynamic stochastic general equilibrium (DSGE) model à la Smets and Wouters (2003) based on Japanese data, report that most of the variation in long-run inflation is accounted for by variation in the target in the monetary policy rule.

[7] Hayakawa and Maeda (2000) and Sudo (2011) argue that the banking crisis hampered financial intermediation, encouraged households to engage in precautionary saving, and lowered the velocity of circulation of money and the price level.

rate, banks construct their asset portfolios such that they remain solvent even if the maximum losses on each type of asset is realized. Our VaR constraint is similar to the VaR constraint analyzed by Adrian and Shin (2011). In their study, banks invest external funds and their own net worth only in capital goods, and the VaR constraint works as a source of fluctuations in the size of banks' balance sheets. By contrast, in our model, there are two assets in which to invest, and the VaR constraint also works as a source of compositional changes in banks' asset portfolio allocation between government bonds and loans to firms.

The central purpose of introducing the VaR constraint is that it allows us to incorporate banks' risk taking capacity into the model.[8] When there is no VaR constraint in the economy, banks' optimal asset portfolio decision requires that the expected returns from the two assets are equalized in equilibrium. However, in the presence of a VaR constraint, banks' asset portfolio depends not only on the expected returns of the two assets, but also on the maximum loss on the two types of assets and banks' net worth. For instance, when the maximum loss of holding a loan claim increases, banks rearrange the size of their balance sheets and the composition of asset portfolios so as to avert bankruptcy. Because such downside risk reduces banks' risk taking capacity, banks remain solvent in the worst-case scenario by shrinking their balance sheets and investing more in assets with a smaller maximum loss. Changes in the institutional environment, such as the strengthening of capital requirements, may affect the economy in the similar manner.[9] Such changes in the institutional environment may lead banks to rearrange their balance sheets and asset portfolios by directly affecting their risk taking capacity. Banks' net worth also plays a significant role in their asset portfolio decision. When their net worth deteriorates, banks' repayment capacity in the worst-case scenario becomes smaller than would otherwise be the case. In such a situation, banks, as shown by Adrian and Shin (2011), avert bankruptcy by reducing their balance sheets and reallocating their asset portfolio from assets with a larger maximum loss to assets with a smaller maximum loss.

Next, we discuss the implications of banks' investment decisions under the VaR constraint for the dynamics of output and inflation. Suppose that uncertainty regarding the ex-post capital return rises and hence the maximum loss on loan claim holdings increases. In this case, banks facing a VaR constraint will reduce their investment in loan claims, as they adjust the size of their balance sheets and the composition of their asset portfolio. As a result, the supply of capital to goods producing firms will fall, reducing

[8] In the present paper, we focus on an economy where banks' risk taking capacity is limited because of the VaR constraint. Consequently, capital investment by banks is lower when compared with an economy where such a constraint is absent. By contrast, recent studies, including Korinek (2011) and Kato and Tsuruga (2011), investigate an economy in which excessive investment by an individual bank leads to negative externalities, such as a fall in asset prices driven by the fire-sale of assets.

[9] Gerali et al. (2009), employing a model in which the interest rate at which banks lend to firms decreases with banks' net worth, show that a deterioration in banks' net worth or a strengthening of capital requirements may increase the interest rate at which banks lend to firms and hence dampen lending and output.

output. As output declines, aggregate demand weakens and inflation declines. The initial effect, originating from the change in the maximum loss, brings about second-round effects on the macroeconomy through endogenous developments in the banks' net worth. When the initial effect leads to a decline in banks' net worth, it also dampens output and inflation through changes in the risk taking capacity originating from insufficiencies in banks' net worth.

The implications of our model are consistent with Japan's experience since 1990. As we saw above, Japanese banks experienced an increase in capital requirements, an accumulation of bad loans triggered by the burst of the asset bubble, and a deterioration in their balance sheets stemming from the banking crisis that started in 1997. In our model, all of these events may lead to a shrinkage of banks' balance sheets and a reallocation of banks' asset portfolios toward government bonds, generating downward pressure on economic activity and inflation. To see how well our model accounts for developments in Japanese economy, both quantitatively and qualitatively, we make use of Bayesian techniques and estimate the parameters of the model and underlying shocks based on data from 1980 to 2007. We confirm that the VaR constraint plays an important role in explaining macroeconomic fluctuations through its effect on banks' behavior. In particular, we find that shocks to banks' net worth contribute significantly to the persistent deflation since the outbreak of the banking crisis in 1997. Shocks to the maximum loss on loan claims also play a sizable role. In addition, we find that the presence of the VaR constraint increases the volatility of business cycles through the endogenous development in banks' net worth. Under the VaR constraint, the macroeconomic outcomes of the exogenous shocks are amplified by affecting banks' balance sheets and asset portfolio allocation.

Our analysis is closely related to Braun and Nakajima (2011), which examines the impact of accumulated government debt on the price level, focusing on banks' asset allocation. Banks in their model hold government bonds as collateral to finance their asset purchases. As long as a certain share of banks are optimistic about future bond prices, these banks purchase government bonds by raising funds from other agents, using government bonds as collateral. As a result, an accumulation of government bonds and deflation coexist in the economy. Our paper also highlights the effects of banks' asset allocation. The economic mechanism that leads banks to hold government bonds, however, differs from the one discussed in Braun and Nakajima (2011). In our paper, the key determinant of banks' asset portfolio is the severity of the VaR constraint. Whenever the constraint tightens, banks become more inclined to reallocate their portfolio from risky to less risky assets.

Another study with which the present paper is related is that by Brunnermeier and Sanikov (2011). In their model, there are market imperfections in financial intermediation activity. Whenever the economy is hit by an adverse shock, agents reallocate their asset portfolio toward safe asset, resulting in deflation. Because safe asset is nominal assets, higher demand for such assets raises their value, leading to a fall in the price level. Although a similar mechanism is present in our model, how banks' asset portfolio allocation affects inflation dynamics differs between the two models. In our model, banks' flight to safe assets impedes capital accumulation in the economy, and dampens output. Inflation falls because aggregate demand declines.

Regarding the role played by uncertainty, our analysis is also related to the work by Fernandez-Villaderde et al. (2011). Using structural vector autoregression, they empirically show that higher volatility in productivity lowers the price level and output, and provide a theoretical framework for analyzing the relationship between uncertainty and households' asset allocation. Further, using the inventory model of money demand, they show that households facing greater uncertainty prefer safer and more liquid assets, such as money, to riskier assets.

The rest of the paper is organized as follows. Section 2 presents our model with banks that endogenously choose their asset portfolio under the VaR constraint. In addition, we explore the qualitative properties of our model using a simplified setting. Section 3 considers the quantitative implications of our model based on parameters estimated using Japanese data from 1981Q1 to 2007Q4. Section 4 concludes the analysis and discusses future extensions of our analysis.

2 The Model

This section describes the structure of our model. The economy that we model consists of seven types of agents: a representative household, banks, intermediate goods producers, wholesale goods producers, final goods producers, the government and the central bank. See Figure 4 for a graphic representation of the model.

The representative household supplies labor to intermediate goods producers, receives wages, makes deposits at the banks, and receives the repayment of the deposits in turn. The household has no means of accessing the financial market directly and cannot own the financial assets other than bank deposits. Banks collect deposits from the household, and invest their own net worth and deposits in two types of assets: loans for capital goods used by intermediate goods producers and government bonds. Banks choose their asset portfolio allocation such that they do not violate the VaR constraint. Intermediate goods producers hire labor and capital goods from the household and banks, respectively, to produce final goods. Wholesale goods producers produce differentiated wholesale goods from intermediate goods. They are monopolistic suppliers of wholesale goods, and set their prices so as to maximize profits. Final goods producers convert the differentiated wholesale goods into final goods. The government collects a lump-sum tax from the household and issues government bonds to finance government debt and government expenditure. The central bank controls inflation by adjusting the nominal interest rate, according to a Taylor rule.

2.1 Household

The infinitely-lived representative household makes decisions on consumption and deposit holdings. It is barred from the financial market and thus possesses no real capital stock or government bonds, and holds all of its savings in the form of bank deposits.

The household's preferences with regard to consumption goods $c(s^t)$ and work effort $l(s^t)$ are presented in the following expected utility function, (1)

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c(s^t), l(s^t)) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c(s^t) + \eta \log \left(1 - l(s^t)\right)\right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and η is the weight assigned to leisure.

The budget constraint of the household is given by the following equation:

$$c(s^t) + d(s^t) = r_d(s^{t-1}) d(s^{t-1}) + \frac{W(s^t)}{P(s^t)} l(s^t) + \Pi(s^t) - \tau(s^t) \tag{2}$$

where $d(s^t)$ represents the household's deposits, $r_d(s^{t-1})$ is the real deposit rate paid by banks for deposits made in period $t-1$, $W(h, s^t)$ is the nominal wage rate, $P(s^t)$ is the price index, $\Pi(s^t)$ is the sum of intermediate goods producers' and banks' real profits that are returned to the household as dividends, and $\tau(s^t)$ is the lump-sum real tax collected by the government. We assume that deposits are a risk-free asset and the real deposit rate is the real risk-free rate.

The first-order condition associated with the household's intertemporal consumption decision is given by

$$U_c(c(s^t), l(s^t)) = \beta r_d(s^t) \mathrm{E}_t U_c(c(s^{t+1}), l(s^{t+1})),$$

where $U_c(c(s^t), l(s^t))$ denotes the marginal utility with respect to consumption in period t. Because the household's only financial assets are its bank deposits, its consumption growth depends on the risk-free rate.

The first-order condition associated with the household's intra-temporal consumption-leisure decision is given by

$$\frac{U_l(c(s^t), l(s^t))}{U_c(c(s^t), l(s^t))} = \frac{W(s^t)}{P(s^t)},$$

where $U_l(c(s^t), l(s^t))$ denotes the marginal utility with respect to leisure in period t.

2.2 Banks

Outline of banks' choice

There is a continuum of risk-neutral banks, indexed by $i \in (0,1)$. Each bank i collects deposits $d(i, s^t)$ from the household and purchases loan claims, i.e., capital stock

$k(i,s^t)$, and real government bonds $b(i,s^t) \equiv \frac{B(i,s^t)}{P(s^t)}$, from final goods producers and the government, respectively. Banks finance these purchases using the deposits $d(i,s^t)$ they have collected and their own real net worth $n(i,s^t)$. Bank i's balance sheet each period is therefore given by

$$k\left(i,s^t\right)+\frac{B\left(i,s^t\right)}{P\left(s^t\right)}=n\left(i,s^t\right)+d\left(i,s^t\right). \tag{3}$$

Bank i receives returns on the two types of assets it invested in in the previous period, repays the deposits to the household, and retains the rest of the earnings as part of its own net worth. Consequently, the bank's net worth evolves according to the following law of motion

$$n\left(i,s^{t+1}\right)=r_k\left(s^{t+1}\right)k\left(i,s^t\right)+r_b\left(s^{t+1}\right)b\left(i,s^t\right)-r_d\left(s^t\right)d_t\left(i,s^t\right), \tag{4}$$

where $r_k(s^{t+1})$ and $r_b(s^{t+1})$ are the ex-post real return on loan claims and government bonds, respectively. Note that the real return on government bonds is given by the policy rate $R_B(s^t)$ set by the central bank, divided by the inflation rate $\pi(s^{t+1})$, that is:

$$r_b\left(s^{t+1}\right)=\frac{R_B\left(s^t\right)}{\pi\left(s^{t+1}\right)}.$$

Bank i accumulates net worth up until the period when it exits from the economy.[10] We assume that bank i's exit probability each period is exogenously given by $1-\gamma(s^t)$. The continuation value of bank i is then given by

$$V\left(n\left(i,s^t\right)\right)=\beta \mathrm{E}_t\Lambda_{t,t+1}\left[\gamma\left(s^t\right)V\left(n\left(i,s^{t+1}\right)\right)+\left(1-\gamma\left(s^t\right)\right)n\left(i,s^{t+1}\right)\right], \tag{5}$$

where $n(i,s^t)$ is the net worth of bank i and $\Lambda_{t,t+1}$ denotes the household's stochastic discount factor from period t to period $t+1$.

In choosing its portfolio allocation between the two types of assets, bank i considers a VaR constraint similar to the one discussed in Adrian and Shin (2011), together with the expected average returns of the two types of assets. Namely, bank i adjusts its balance sheet in period t, so that it is able to repay all of its debt to the household, even if the two types of assets yield the maximum loss in period $t+1$. Denoting the maximum loss from holding the two types of assets by $\mathrm{E}_t\underline{r}_k(s^{t+1})$ and $\mathrm{E}_t\underline{r}_b(s^{t+1})$, respectively, the VaR constraint is given by

$$E_t\underline{r}_k\left(s^{t+1}\right)k\left(i,s^t\right)+E_t\underline{r}_b\left(s^{t+1}\right)b\left(i,s^t\right)-r_d\left(s^t\right)d\left(i,s^t\right)\geq 0. \tag{6}$$

[10] Following Gertler and Karadi (2011), we assume that a bank transfers all of its accumulated net worth to the household when it exits from the economy.

Here, we assume that loan claim holdings have a larger risk associated with them than government bond holdings, so that $\mathrm{E}_t \underline{r}_k\left(s^{t+1}\right) < \mathrm{E}_t \underline{r}_b\left(s^{t+1}\right)$.[11] There are two possible interpretations why the maximum loss associated with a particular type of asset might vary over time. The first interpretation is that it might reflect shocks to the economic environment, such as an increase in downside risks of the ex-post return of an asset or an increase in uncertainty regarding the macroeconomic outlook. The second interpretation is that it might reflect institutional changes, such as a strengthening of capital requirements. Such institutional changes directly limit banks' risk taking capacity, generating similar consequences to those under the first interpretation.[12]

Banks' maximization problem

In Adrian and Shin's (2011) model, where there is only one type of asset, the VaR constraint matters only for the size of a bank's leverage. By contrast, in our model with two types assets, the VaR constraint influences the asset portfolio allocation as well as the size of leverage. Bank i's optimization problem is formulated as the maximization of its net worth in the last period prior to exiting from the economy, which is shown by equation (5), subject to bank i's balance sheet equation (3), the law of motion for bank i's net wort accumulation (5), and the VaR constraint (3). Because banks are risk-neutral, we first guess that the value function of the bank i is given by

$$V\left(n\left(i, s^t\right)\right) = \phi\left(s^t\right) n\left(i, s^t\right).$$

Equation (5) then reduces to

$$\begin{aligned} \max V\left(n\left(i, s^t\right)\right) = \beta \mathrm{E}_t \Lambda_{t,t+1} \Bigg[& \gamma\left(s^t\right) \phi\left(s^{t+1}\right) \begin{pmatrix} q_k\left(s^{t+1}\right) k\left(i, s^t\right) \\ +q_b\left(s^{t+1}\right) b\left(i, s^t\right) \\ +r_d\left(s^{t+1}\right) n\left(i, s^t\right) \end{pmatrix} \\ & + \left(1-\gamma\left(s^t\right)\right)\left(q_k\left(s^{t+1}\right) k\left(i, s^t\right) + q_b\left(s^{t+1}\right) b\left(i, s^t\right) + r_d\left(s^t\right) n\left(i, s^t\right)\right) \Bigg]. \end{aligned}$$

[11] In the present paper, we concentrate our analysis on an equilibrium where banks hold both types of risky assets, and the worst-case returns of the two types of assets are smaller than the risk-free rate, so that following the two equations hold:

$$\begin{aligned} \underline{r}_k\left(s^{t+1}|s^t\right) - r_d\left(s^t\right) &< 0, \\ \underline{r}_b\left(s^{t+1}|s^t\right) - r_d\left(s^t\right) &< 0. \end{aligned}$$

[12] Admittedly, the maximum loss on loan claims or government bonds may be endogenously affected by the current economic environment, such as the amount of capital stock available in the economy or the type of government policy being pursued. Here, however, we assume that the law of motion for the maximum loss is given and concentrate our analysis on how variations in the maximum loss affect the economy.

The corresponding first order condition yields

$$\mathrm{E}_t\left[\frac{\left(\gamma\phi\left(s^{t+1}\right)+1-\gamma\left(s^t\right)\right)\Lambda_{t,t+1}q_k\left(s^{t+1}\right)}{\underline{q}_k\left(s^{t+1}\right)}\right]=\mathrm{E}_t\left[\frac{\left(\gamma\phi\left(s^{t+1}\right)+1-\gamma\left(s^t\right)\right)\Lambda_{t,t+1}q_b\left(s^{t+1}\right)}{\underline{q}_b\left(s^{t+1}\right)}\right]. \tag{7}$$

Here $q_k\left(s^{t+1}\right)\equiv r_k\left(s^{t+1}\right)-r_d\left(s^t\right)$ and $q_b\left(s^{t+1}\right)\equiv r_b\left(s^{t+1}\right)-r_d\left(s^t\right)$ respectively denote the excess return to loan claim holdings and government bond holdings relative to deposits, respectively. Similarly, $\underline{q}_k\left(s^{t+1}\right)\equiv\underline{r}_k\left(s^{t+1}\right)-r_d\left(s^t\right)$ and $\underline{q}_b\left(s^{t+1}\right)\equiv\underline{r}_b\left(s^{t+1}\right)-r_d\left(s^t\right)$ denote the excess return to the two types of risky assets when the worst possible return is realized.

Equation (7) provides the fundamental principle based on which bank i allocates its assets to loan claims and government bonds. When the VaR constraint is effective, it is not necessary that the expected excess returns of the two types of assets are equalized in equilibrium. Instead, bank i's asset portfolio is constructed so that the expected excess returns weighted by the maximum loss for each type of asset are equalized. Under the premise that loan claims are riskier than government bonds, i.e., $\underline{r}_k\left(s^{t+1}\right)<\underline{r}_b\left(s^{t+1}\right)$, the expected excess return on loan claims needs to exceed that on government bonds, i.e., $\mathrm{E}_t r_k\left(s^{t+1}\right)>\mathrm{E}_t r_b\left(s^{t+1}\right)$, to compensate for this.

From equations (6) and (7), we obtain the expression for $\phi\left(s^t\right)$.

$$\phi\left(s^t\right)=\beta\mathrm{E}_t\left[\Lambda_{t,t+1}\left\{\gamma\left(s^t\right)\phi\left(s^{t+1}\right)+\left(1-\gamma\left(s^t\right)\right)\right\}r_d\left(s^t\right)\left(1-q_k\left(s^{t+1}\right)/\underline{q}_k\left(s^{t+1}\right)\right)\right]. \tag{8}$$

Aggregation

Banks exit from the economy with probability $1-\gamma\left(s^t\right)$ each period, and banks' aggregate net worth, i.e., the sum of all banks' net worth, evolves according to the following law of motion:

$$n\left(s^t\right)=\gamma\left(s^t\right)\left[r_k\left(s^t\right)k\left(s^{t-1}\right)+r_b\left(s^t\right)b\left(s^{t-1}\right)-r_d\left(s^{t-1}\right)d\left(s^{t-1}\right)\right],$$

where $n\left(s^t\right)$ is aggregate banks' aggregate net worth. An increase in the exit probability reduces banks' net worth. As shown in equation (6), the reduced net worth leads to a tightening of banks' VaR constraint, affecting the size of banks' balance sheets and the composition of their asset portfolios in the subsequent period. In addition to the fundamental earnings from investment in loan claims and government bonds, the accumulation of aggregate net worth is affected by exogenous shocks to the exit probability $\gamma\left(s^t\right)$. Such exogenous shocks include phenomena such as asset bubble, irrational exuberance, or an innovation in the efficiency of banks' investment.[13][14]

2.3 Intermediate Goods Producers

Intermediate goods producers produce intermediate goods $y\left(s^t\right)$, and sell them to wholesale goods producers at price $P_y\left(s^t\right)$. They hire labor input $l\left(s^t\right)$ from the household and borrow effective capital $v\left(s^t\right)K\left(s^{t-1}\right)$ from banks. Both the input and output markets of intermediate goods producers are competitive. The maximization problem for intermediate goods producer is given by

$$\max_{y(s^t),v(s^t)k(s^{t-1}),l(s^t)} \frac{P_y\left(s^t\right)y\left(s^t\right)}{P\left(s^t\right)} - \tilde{r}\left(s^t\right)v\left(s^t\right)k\left(s^{t-1}\right) - W\left(s^t\right)l\left(s^t\right),$$

subject to

$$y\left(s^t\right) = \left(v\left(s^t\right)k\left(s^{t-1}\right)\right)^{\alpha}\left(A\left(s^t\right)Z\left(s^t\right)l\left(s^t\right)\right)^{1-\alpha}, \tag{9}$$

where $v\left(s^t\right)$ is the capital utilization rate, $k\left(s^{t-1}\right)$ is the capital stock, $\tilde{r}\left(s^t\right)$ is the real return to the use of effective capital, $A\left(s^t\right)$ is the stationary component of the technology level, $Z\left(s^t\right)$ is the non-stationary component of the technology level, and $\alpha \in [0,1]$ is the capital share. The first order conditions for intermediate goods producers yield the following equalities.

$$\begin{aligned}
\tilde{r}_k\left(s^t\right) &= \alpha\frac{P_y\left(s^t\right)}{P\left(s^t\right)}\left(v\left(s^t\right)k\left(s^{t-1}\right)\right)^{\alpha-1}\left(A\left(s^t\right)Z\left(s^t\right)l\left(s^t\right)\right)^{1-\alpha}, \\
\frac{W\left(s^t\right)}{P\left(s^t\right)} &= \frac{P_y\left(s^t\right)}{P\left(s^t\right)}(1-\alpha)\left(v\left(s^t\right)k\left(s^{t-1}\right)\right)^{\alpha}\left(A\left(s^t\right)Z\left(s^t\right)\right)^{1-\alpha}\left(l\left(s^t\right)\right)^{-\alpha}.
\end{aligned}$$

The capital utilization rate is determined by the banks. Assuming that choosing capital utilization $v\left(s^t\right)$, together with the capital stock $k\left(s^{t-1}\right)$, banks incur the real cost of

$$\frac{\kappa_v k\left(s^{t-1}\right)\left(v\left(s^t\right)\right)^{\phi+1}-1}{\phi+1},$$

[13]Based on the financial accelerator model developed by Bernanke, Gertler, and Gilchrist (1999), Gilchrist and Leahy (2002) and Nolan and Thoenissen (2009) examine the consequences of an exogenous deterioration in entrepreneurial net worth, which is similar to the exogenous change in $\gamma\left(s^t\right)$ in our model.

[14]There are alternative ways to incorporate exogenous shocks to banks' net worth into the model. In Gertler and Karadi's (2011) model, for example, the existing capital stock becomes out of date, resulting in a deterioration in the value of banks' loan claims and net worth. On the other hand, in Aoki and Nikolov's (2011) study, which analyzes the consequence of banks' investment on the unproductive bubble, the collapse of the bubble leads to a deterioration in banks' net worth.

so that banks' optimal capital utilization rate is expressed by

$$\tilde{r}_k\left(s^t\right) = \left(\phi+1\right)\kappa_v v_t^{\phi},$$

where κ_v and ϕ are parameters that govern the capital utilization rate. Consequently, banks' net return to investment on productive capital $r_k\left(s^t\right)$ is given by

$$r_k\left(s^t\right)k\left(s^t\right) = \tilde{r}_k\left(s^t\right)k\left(s^t\right) - \kappa_v k\left(s^{t-1}\right)\left(v\left(s^t\right)\right)^{\phi+1} + \left(1-\delta\right)k\left(s^t\right),$$

where $\delta \in [0,1]$ is the depreciation rate of the capital stock. Similarly, the real wage paid to the household is expressed by

$$\frac{W\left(s^t\right)}{P\left(s^t\right)} = s\left(s^t\right)\left(1-\alpha\right)\left(v\left(s^t\right)k\left(s^{t-1}\right)\right)^{\alpha}\left(A\left(s^t\right)\right)^{1-\alpha}\left(Z\left(s^t\right)\right)^{1-\alpha}\left(l\left(s^t\right)\right)^{-\alpha}.$$

2.4 Wholesale and Final Goods Producers

Optimization problem of wholesale and final goods producers

The wholesale goods sector contains a continuum of firms, each producing differentiated products, as indexed by $z \in [0,1]$, from intermediate goods employing a linear production technology;

$$x(z,s^t) = y(z,s^t).$$

Here, $x(z,s^t)$ denotes the differentiated wholesale goods made by wholesale goods producer z and $y(z,s^t)$ is the intermediate goods used as inputs by producer z.

Final goods producers purchase these differentiated goods in a competitive market, producing the final goods from wholesale goods employing the following constant elasticity of substitution (CES) aggregate technology:

$$x\left(s^t\right) = \left[\int_0^1 x(s^t,z)^{\frac{\varepsilon\left(s^t\right)-1}{\varepsilon\left(s^t\right)}}dz\right]^{\frac{\varepsilon\left(s^t\right)}{1-\varepsilon\left(s^t\right)}}, \quad \varepsilon\left(s^t\right) > 1$$

where $\varepsilon\left(s^t\right) \in (1,\infty)$ denotes the time-varying elasticity of substitution between differentiated wholesale goods. Given this CES technology for final goods, the demand for each differentiated wholesale good $x\left(z,s^t\right)$ is given by a function of its price $p(z,s^t)$, the aggregate price index $P\left(s^t\right)$, and the aggregate demand for final goods $x\left(s^t\right)$:

$$x(z,s^t) = \left(\frac{p(z,s^t)}{P\left(s^t\right)}\right)^{-\varepsilon\left(s^t\right)} x\left(s^t\right).$$

Each wholesale goods producer z maximizes its profit by choosing the optimal product price. The maximization problem of each differentiated wholesale goods producer is given by

$$\max_{p(z,s^{t+j})} \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \Lambda_{j-1,j} \left[\begin{array}{l} \left(\frac{p(z,s^{t+j})}{P(s^{t+j})}\right)^{1-\varepsilon(s^t)} x\left(s^{t+j}\right) \\ -\left(\frac{P_y(s^{t+j})}{P(s^{t+j})}\right)\left(\frac{p(z,s^{t+j})}{P(s^{t+j})}\right)^{\varepsilon(s^t)} x\left(s^{t+j}\right) \\ -\frac{\kappa}{2}\left(\frac{p(z,s^{t+j})}{p(z,s^{t+j-1})} - \frac{p\left(s^{t+j-1}\right)}{p(s^{t+j-2})}\right)^2 \left(\frac{p(z,s^{t+j})}{P(s^{t+j})}\right)^{-\varepsilon(s^t)} x\left(s^{t+j}\right) \end{array} \right],$$

where the third term denotes the adjustment cost that the wholesale goods producer pays for changing its product price $p(z, s^t)$, and κ is the parameter that governs the size of the adjustment cost.

Because in the symmetric equilibrium all differentiated goods prices $p(z, s^t)$ set by wholesale goods producers are identical, we can obtain the Phillips curve of the economy from the first order condition of the firms' maximization problem

$$\begin{aligned} -\varepsilon\left(s^t\right)\left(1 - \frac{P_y\left(s^t\right)}{P\left(s^t\right)} - 0.5\left(\pi\left(s^t\right)-1\right)^2\right) + 1 - \kappa\left(\pi\left(s^t\right)-1\right)\pi\left(s^t\right) \\ + \beta\kappa\left(\pi\left(s^{t+1}\right)-1\right)\pi\left(s^{t+1}\right)\frac{x\left(s^{t+1}\right)}{x\left(s^t\right)} = 0. \quad (10) \end{aligned}$$

Market clearing condition

The market clearing conditions for intermediate goods and wholesale goods are given by

$$\begin{aligned} \int_0^1 x\left(s^t, z\right) dz &= y\left(s^t\right), \\ x\left(s^t\right) &= \int_0^1 x\left(s^t, z\right) dz \end{aligned}$$

Final goods serve for household consumption, investment in productive capital, and government expenditure. The market clearing condition for final goods is given by

$$c\left(s^t\right) + k\left(s^t\right) - (1-\delta)\,k\left(s^{t-1}\right) + G\left(s^t\right) = x\left(s^t\right) - \frac{\kappa}{2}\left(\pi\left(s^t\right)-1\right)^2 x\left(s^t\right) - \kappa_v k\left(s^{t-1}\right) v\left(s^t\right)^{\phi+1}$$

2.5 The government and the central bank

The government collects a lump-sum tax $P\left(s^t\right)\tau\left(s^t\right)$ from the household and issues government bonds $B\left(s^t\right)$ to finance its repayment $R_B\left(s^{t-1}\right) B\left(s^{t-1}\right)$ to banks as well as government expenditure $P(s^t) G(s^t)$. We assume that a balanced budget is maintained in each period t, that is:

$$R_B\left(s^{t-1}\right) B\left(s^{t-1}\right) + P\left(s^t\right) G\left(s^t\right) = P\left(s^t\right) \tau\left(s^t\right) + B\left(s^t\right). \tag{11}$$

The government lump-sum tax is an increasing function of the amount of outstanding government bonds and is specified as follows:

$$\tau\left(s^t\right) = b\left(s^{t-1}\right)\left(\frac{b\left(s^{t-1}\right)}{x\left(s^t\right)}\right)^{\psi} T, \tag{12}$$

where $\psi \in (1, \infty]$ is the elasticity of the lump-sum tax with respect to the amount of outstanding government debt, with an increase in debt leading to an increase in the lump-sum tax and T is a constant parameter.

Next, we turn to the central bank. The central bank sets the nominal interest rate according to a simple Taylor rule given by

$$\ln R_B\left(s^t\right) = (1-\rho_M)\ln R + \rho_M \ln R_B\left(s^{t-1}\right) + (1-\rho_M)\phi \ln \pi\left(s^t\right) + \epsilon_r\left(s^t\right), \tag{13}$$

where R is constant, $\rho_M \in [0,1]$ is the autoregressive coefficient of the policy rate, and $\phi > 1$ is the policy weight attached to the inflation rate and $\epsilon_r(s^t)$ is an i.i.d. shock to the monetary policy rule.[15]

2.6 Shock Process

The exogenous shocks in our economy, a shock to the markup-related elasticity $\varepsilon(s^t)$, to banks' net worth $\gamma(s^t)$, to the maximum loss on capital assets $\underline{r}_k(s^t)$, to the maximum loss on government bonds $\underline{r}_b(s^t)$, to the stationary component of technology $A(s^t)$, to the non-stationary component of technology $Z(s^t)$, and to government expenditure

[15] In our model, the policy parameters ψ and ϕ are both greater than unity, implying that our economy is in the Ricardian regime with regard to both fiscal and monetary policy. A matter that we do not consider in the present paper is the case of government default. In non-Ricardian regimes with a government default, the inflation rate is only uniquely pinned down when the central bank responds to inflation aggressively. See, for example, Kocherlakota (2012).

$G\left(s^t\right)$, evolve according to the equations below:

$$\begin{aligned}
\ln\varepsilon\left(s^t\right) &= \ln\varepsilon+\epsilon_\varepsilon\left(s^t\right), && (14)\\
\ln\gamma\left(s^t\right) &= \left(1-\rho_\gamma\right)\ln\gamma+\rho_\gamma\ln\gamma\left(s^{t-1}\right)+\epsilon_\gamma\left(s^t\right), && (15)\\
\ln\underline{r}_k\left(s^t\right) &= \left(1-\rho_{\underline{r}_k}\right)\ln\underline{r}_k+\rho_{\underline{r}_k}\ln\underline{r}_k\left(s^{t-1}\right)+\epsilon_{\underline{r}_k}\left(s^t\right), && (16)\\
\ln\underline{r}_b\left(s^t\right) &= \left(1-\rho_{\underline{r}_b}\right)\ln\underline{r}_b+\rho_{\underline{r}_b}\ln\underline{r}_b\left(s^{t-1}\right)+\epsilon_{\underline{r}_b}\left(s^t\right), && (17)\\
\ln A\left(s^t\right) &= \left(1-\rho_A\right)\ln A+\rho_A\ln A\left(s^{t-1}\right)+\epsilon_A\left(s^t\right), && (18)\\
\ln Z\left(s^t\right) &= \ln Z\left(s^{t-1}\right)+u_Z\left(s^t\right), && (19)\\
u_Z\left(s^t\right) &= \rho_Z u_Z\left(s^{t-1}\right)+\epsilon_Z\left(s^t\right), && (20)\\
\ln G\left(s^t\right) &= \left(1-\rho_G\right)\ln G+\rho_G\ln G\left(s^{t-1}\right)+\epsilon_G\left(s^t\right), && (21)
\end{aligned}$$

where ρ_ε, ρ_γ, $\rho_{\underline{r}_k}$, $\rho_{\underline{r}_b}$, ρ_A, ρ_Z, and $\rho_G \in (0,1)$ are the autoregressive root of the corresponding shocks, and $\epsilon_\varepsilon\left(s^t\right), \epsilon_\gamma\left(s^t\right), \epsilon_{\underline{r}_k}\left(s^t\right), \epsilon_{\underline{r}_b}\left(s^t\right), \epsilon_A\left(s^t\right), \epsilon_Z\left(s^t\right)$ and $\epsilon_G\left(s^t\right)$ are the exogenous i.i.d. shocks that are normally distributed with mean zero.

2.7 Equilibrium Conditions

An equilibrium consists of a set of prices, $\{W\left(s^t\right), P\left(s^t\right), P_y\left(s^t\right), r_k\left(s^t\right), \tilde{r}_k\left(s^t\right), r_d\left(s^t\right), r_b\left(s^t\right), R_B\left(s^t\right)\}_{t=0}^{\infty}$, and the allocations $\{c\left(s^t\right), l\left(s^t\right), d\left(s^t\right), \Pi\left(s^t\right), k\left(s^t\right), v\left(s^t\right), x\left(s^t\right), y\left(s^t\right)\}\}_{t=0}^{\infty}$, for a given government policy $\{G\left(s^t\right),\ \tau\left(s^t\right)\}_{t=0}^{\infty}$, realization of exogenous variables $\{\epsilon_\varepsilon\left(s^t\right), \epsilon_\gamma\left(s^t\right), \epsilon_{\underline{r}_k}\left(s^t\right), \epsilon_{\underline{r}_b}\left(s^t\right), \epsilon_A\left(s^t\right), \epsilon_Z\left(s^t\right), \epsilon_G\left(s^t\right), \epsilon_r\left(s^t\right)\}_{t=0}^{\infty}$, expected worst-case returns $\{\underline{r}_k\left(s^t\right), \underline{r}_b\left(s^t\right)\}_{t=0}^{\infty}$, and initial conditions $\{B_{-1}\}$, $\{d_{-1}\}$, $\{k_{-1}\}$ such that for all t, i, and z :

(i) the household maximizes its utility given prices;
(ii) banks maximize their profits given prices and expected worst-case returns;
(iii) intermediate goods producers maximize their profits given prices;
(iv) wholesale goods producers maximize their profits given prices;
(v) final goods producers maximize their profits given prices;
(vi) the government budget constraint holds;
(vii) the central bank sets the policy rate following the Taylor rule; and
$(viii)$ markets clear.

2.8 Steady State Analysis

Before investigating the dynamics of the model, we explore its mechanism in the steady state to show the determinants of banks' balance sheets and asset portfolio allocations. In particular, we focus on how the expected returns from holding the two types of risky assets in the steady state, which we denote by r_b and r_k, are affected by banks' VaR constraint, and how banks' decision with regard to their portfolio allocation between government bonds b and loan claims k is made.[16] For illustrative purpose, we make two simplifying assumptions in this subsection: (1) the household supplies labor inelastically, i.e., $l = 1$, and (2) banks' capital utilization cost is zero, i.e., $\phi = 0$.[17]

Evaluating the variables appearing in the portfolio choice equation, the VaR constraint equation, and the law of motion of banks' net worth at the steady state values, we have

$$\frac{r_k - r_d}{r_d - \underline{r}_k} = \frac{r_b - r_d}{r_d - \underline{r}_b}, \tag{22}$$

$$\left(\underline{r}_k - r_d\right) k + \left(\underline{r}_b - r_d\right) b = -r_d n, \tag{23}$$

$$n = \frac{\gamma}{1 - \gamma r_d} \left[\left(r_k - r_d\right) k + \left(r_b - r_d\right) b\right]. \tag{24}$$

Note that the household's Euler equation in the steady state implies that

$$r_d = \frac{1}{\beta}.$$

The three equations above yield the excess return from holding the two types of risky assets, and the spread of the two types of risky assets:

$$r_b - r_d = \frac{1 - \gamma r_d}{\gamma r_d} \left(r_d - \underline{r}_b\right), \tag{25}$$

$$r_k - r_d = \frac{1 - \gamma r_d}{\gamma r_d} \left(r_d - \underline{r}_k\right). \tag{26}$$

$$r_k - r_b = \frac{1 - \gamma r_d}{\gamma r_d} \left(\underline{r}_b - \underline{r}_k\right). \tag{27}$$

According to equations (25) and (26), the excess return from holding the two types of risky assets and the spread between them are expressed by the expected maximum loss from holding the two types of risky assets $\underline{r}_b$ and $\underline{r}_k$, together with banks' survival probability γ.

For instance, when the maximum loss of holding loan claims $\underline{r}_k$ rises, which implies the value of $\underline{r}_k$ falls, banks' VaR constraint becomes tighter, reducing banks' risk taking

[16] The definition of the steady state in our economy needs to be carefully stated. Suppose that we define the steady state as a situation in which there are no exogenous shocks and every endogenous variable grows at a constant rate. Banks' asset allocation then becomes indeterminate because their portfolio choice depends on the riskiness of the assets. In the present analysis, we define the steady state following Devereux and Sutherland's (2010, 2011) approach, in which banks take the possibility that the worst-case scenario with regard to asset returns is realized into consideration. Consequently, the risks of holding an asset affect banks' portfolio in the steady state.

[17] This assumption implies that capacity utilization is unity.

capacity. If banks maintain the same amount of loan claim holdings, they require a higher spread vis-à-vis government bonds to hold loan claims than would otherwise be the case. The government bond yield is unaffected by the change in $\underline{r}_k$. The similar mechanism works if the maximum loss of holding the government bonds $\underline{r}_b$ increases.

By contrast, a reduction in survival probability γ leads to a rise in the excess returns associated with the two types of assets. As indicated by equation (24), the smaller survival probability prevents banks from increasing their net worth. Because a low level of net worth tightens the VaR constraint by increasing the risk of default, banks shrink their balance sheet. Since banks' demand for both types of assets falls, the excess returns on them need to rise to clear the asset market.

Next, we discuss how banks allocate their asset between loan claims k and the government bonds b. Because the return from holding loan claims r_k equals the return to the capital stock in the economy, we have

$$r_k = \alpha A Z k^{\alpha-1} + (1-\delta).$$

Similarly, because in equilibrium the amount of loan claims is equivalent to the total capital stock, we have

$$k = \left[\frac{r_k - (1-\delta)}{\alpha A Z}\right]^{\frac{1}{\alpha-1}}. \tag{28}$$

Taking into the consideration that $\alpha - 1 < 0$, a higher return on loan claims implies a smaller amount of loan claims and thus a lower amount of investment in the economy. Based on the considerations above, an increase in the maximum loss on loan claim holdings or a decline in banks' survival probability reduces their loans to firm through a rise in the return on the capital stock, r_k.

Banks' decision regarding the holding of government bonds is affected by the government's policy regarding taxes and budget balance. From equations (11) and (12), we have

$$\begin{aligned} r_b b &= T b^{\psi} + b, \\ b &= \left[\frac{r_b - 1}{T}\right]^{\frac{1}{\psi}} x = \left[\frac{r_b - 1}{T}\right]^{\frac{1}{\psi}} A Z k^{\alpha}. \end{aligned} \tag{29}$$

Here, we assume that the inflation rate is unity in the steady state. These equations suggest that, for a $\psi > 1$, banks tilt toward holding government bonds as the return on them increases. Under the tax policy described by the equations above, an increase in government interest rate payments is met by an equivalent increase in government bond issuance, leading to higher government bond holdings by banks. Similar to the mechanism that determines the amount of loan claims k, an increase in the maximum loss of holding government bonds, expressed by a decline in $\underline{r}_b$, or a deterioration in banks' net worth, expressed by a decline in γ, leads to an increase in banks' government bond holdings through a rise in government bond yields.

Lastly, we discuss how banks allocate their assets between government bond holdings and loans to firms. From equations (28) and (29), the ratio of government bond holdings relative to loan claims is given by

$$\frac{b}{k} = \left[\frac{r_b - 1}{T}\right]^{\frac{1}{\psi}} \left[\frac{r_k - (1-\delta)}{\alpha}\right].$$

According to the above equation, any changes in the economic environment that increase the returns on the two types of risky assets, r_b and r_k, including an increase in the maximum loss on loans to firms or of holding government bonds, or a deterioration of banks' net worth, leads banks to purchase more government bonds relative to loan claims.

2.9 VaR Constraint and Inflation Dynamics

Log-linearizing equations (7) and (13) around the non-stochastic steady state, we obtain

$$\mathrm{E}_t\left(\frac{r_k}{r_k - r_d}\hat{r}_k\left(s^{t+1}\right)\right) + \mathrm{E}_t\left(\frac{\underline{r}_k}{r_d - \underline{r}_k}\hat{\underline{r}}_k\left(s^{t+1}\right)\right) - \mathrm{E}_t\left(\frac{r_b}{r_b - r_d}\hat{r}_b\left(s^{t+1}\right)\right) - \mathrm{E}_t\left(\frac{\underline{r}_b}{r_d - \underline{r}_b}\hat{\underline{r}}_{b,}\left(s^{t+1}\right)\right) \tag{30}$$

$$= \left(\frac{r_d}{r_k - r_d} - \frac{r_d}{r_b - r_d} + \frac{r_d}{r_d - \underline{r}_k} - \frac{r_d}{r_d - \underline{r}_b}\right)\hat{r}_d\left(s^t\right).$$

$$\hat{R}_b\left(s^t\right) = \phi\hat{\pi}\left(s^t\right). \tag{31}$$

Here $\hat{\lambda}(s^t)$ denotes the log deviation of variable $\lambda(s^t)$ from its non-stochastic steady state value. Taking the following relationship

$$\hat{r}_b\left(s^{t+1}\right) = \hat{R}_b\left(s^t\right) - \hat{\pi}\left(s^{t+1}\right)$$

into consideration, we have

$$\hat{\pi}\left(s^t\right) = \phi^{-1}\mathrm{E}_t\left[\hat{\pi}\left(s^{t+1}\right) + a_1\hat{r}_k\left(s^{t+1}\right) + a_2\hat{\underline{r}}_k\left(s^{t+1}\right) - a_3\hat{\underline{r}}_b\left(s^{t+1}\right) + a_4\hat{r}_d\left(s^t\right)\right]. \tag{32}$$

Here, a_1, a_2, a_3, and a_4 are all positive values that are denoted by

$$a_1 = \frac{r_b - r_d}{r_d}\frac{r_k}{r_k - r_d}, \quad a_2 = \frac{r_b - r_d}{r_b}\frac{\underline{r}_k}{r_d - \underline{r}_k}, \quad a_3 = \frac{r_b - r_d}{r_d}\frac{\underline{r}_b}{r_d - \underline{r}_b}$$

$$a_4 = \frac{r_b - r_d}{r_b}\left[\left(\frac{r_d}{r_b - r_d} - \frac{r_d}{r_k - r_d}\right) + \left(\frac{r_d}{r_d - \underline{r}_b} - \frac{r_d}{r_d - \underline{r}_k}\right)\right].$$

Equation (32) shows the qualitative linkage between banks' asset allocation and the inflation rate in the economy. Other things, including inflation expectations $\mathrm{E}_t\left[\hat{\pi}\left(s^{t+1}\right)\right]$, being equal, current inflation is determined by the four variables $\mathrm{E}_t\hat{r}_k\left(s^{t+1}\right)$, $\mathrm{E}_t\hat{\underline{r}}_k\left(s^{t+1}\right)$, $\mathrm{E}_t\hat{\underline{r}}_b\left(s^{t+1}\right)$, and $\hat{r}_d\left(s^t\right)$. Suppose the expected return from holding loan claims, $\mathrm{E}_t\hat{r}_k\left(s^{t+1}\right)$, increases, banks invest more in loan claims and a larger amount of capital is accumulated in the economy, strengthening the economy and generating inflation. A decrease in the maximum loss of holding loan claims $\mathrm{E}_t\hat{\underline{r}}_k\left(s^{t+1}\right)$ or an increase in the maximum loss of holding government bonds $\mathrm{E}_t\hat{\underline{r}}_b\left(s^{t+1}\right)$ cause inflation through a similar mechanism. Similarly, a rise in the deposit rate $\hat{r}_d\left(s^t\right)$ would also generate inflationary pressures in the economy. While a higher deposit rate reduces banks' retained earnings and hampers their accumulation of net worth, it also encourages them to invest more in loan claims than government bonds, increasing the capital supply in the economy. The underlying mechanism is that, under the premise that

$$\mathrm{E}\underline{r}_k < \mathrm{E}\underline{r}_b < r_d < \mathrm{E}r_b < \mathrm{E}r_k,$$

the unconditional expected excess return on government bonds is more sensitive to a change in the deposit rate than that on loan claims. Consequently, banks' asset allocation tilts toward loan to firms, leading to inflation in the economy.

3 Quantitative Analysis

In this section, we investigate the quantitative implications of our model, paying attention to the role played by banks' VaR constraint. Based on data for Japan, we first estimate the model's parameters and eight structural shocks: a markup shock, $\epsilon_\varepsilon\left(s^t\right)$; a shock to banks' net worth, $\epsilon_\gamma\left(s^t\right)$; a shock to the maximum loss on loan claims, $\epsilon_{\underline{r}_k}\left(s^t\right)$; a shock to the maximum loss on government bonds, $\epsilon_{\underline{r}_b}\left(s^t\right)$; a temporary technology shock, $\epsilon_A\left(s^t\right)$; a permanent technology shock, $\epsilon_Z\left(s^t\right)$; a monetary policy shock, $\epsilon_r\left(s^t\right)$; and a government expenditure shock, $\epsilon_G\left(s^t\right)$, using Bayesian techniques. We then explore the model's equilibrium response to these exogenous shocks. In particular, we examine how the VaR constraint affects the dynamics of the model in response to these shocks. Lastly, we evaluate the quantitative contribution of each shock in explaining variations in macroeconomic variables observed in Japan during the past three decades.

3.1 Data

Our benchmark dataset includes eight time series for the Japanese economy from 1980Q1 to 2007Q4: (1) labor input[18], which corresponds to $l\left(s^t\right)$ in the model; (2) real private investment, which is taken from the National Accounts and corresponds to $k\left(s^t\right) -$

[18]To construct the labor input series, we follow the methodology adopted in Hayashi and Prescott (2002).

$(1-\delta)\, k\left(s^{t-1}\right)$ in the model; (3) the sum of treasury discount bills, central government securities and FILP bonds, local government securities, and public corporation securities held by domestically licensed banks, deflated by the GDP deflator; the series is constructed from the Flow of Funds Accounts and the National Accounts and corresponds to $b\left(s^t\right)$ in the model; (4) the stock price index of banks, deflated by the GDP deflator, which is constructed from Tokyo Stock Exchange data and the National Accounts and corresponds to $n\left(s^t\right)$ in the model; (5) the capacity utilization rate of manufacturing industry, which is taken from the Indices of Industrial Production and corresponds to $v\left(s^t\right)$ in the model; (6) the GDP deflator, taken from the National Accounts, which corresponds to $P\left(s^t\right)$ in the model; (7) the call rate set by the Bank of Japan, which corresponds to $R_B\left(s^t\right)$ in the model; and (8) GDP, taken from the National Accounts, which corresponds to $x\left(s^t\right)$ in the model. All of the series, other than series (5) and (7), are first differenced. Series (5) and (7) are used in levels.

3.2 Prior and Posterior Distribution of Parameters

The parameter values used for our quantitative analysis are reported in Table 1. The parameter values are quarterly unless stated otherwise. Since our model is a standard New Keynesian model except that it incorporates banks' asset portfolio choice, we set some of the parameters, that are not related to banks' portfolio decision to the conventional values. These parameters are reported in Table 1(b).

Other parameters are estimated using Bayesian techniques, because they are specific to the current model. The third to fifth columns of Table 1(a) report the prior distribution of the estimated parameters, while the last three columns display the posterior means and the confidence intervals of the model parameters.

3.3 Impulse Responses

In this subsection, we investigate the economy's dynamic response to structural shocks. Figures 5 to 12 display the economy's impulse response function to a negative shock to the technology growth rate, $\epsilon_Z\left(s^t\right)$; a negative shock to the stationary component of technology, $\epsilon_A\left(s^t\right)$; a negative shock to banks' survival probability (a deterioration in banks' net worth), $\epsilon_\gamma\left(s^t\right)$; a negative shock to the maximum loss on loan claims, $\epsilon_{\underline{r}_k}\left(s^t\right)$; a negative shock to the maximum loss on government bonds, $\epsilon_{\underline{r}_b}\left(s^t\right)$; a positive shock to the monetary policy rule, $\epsilon_r\left(s^t\right)$; a positive shock to the markup, $\epsilon_\varepsilon\left(s^t\right)$; and a negative shock to government expenditure, $\epsilon_G\left(s^t\right)$, respectively. All of the equilibrium paths after the shock are approximated by log-linearization around the non-stochastic steady state.

In order to highlight the quantitative role played by the VaR constraint in the economic dynamics, we plot the equilibrium response to a shock in an alternative economy in which the VaR constraint is absent (labeled "No VaR" and denoted by the lines with black circles) in addition to the equilibrium response to a shock in an economy where the VaR constraint is present (labeled "Benchmark" and denoted by the lines with white circles). The settings of the economy with "No VaR" are equivalent to those of our "Benchmark" economy, except that there is no constraint equation (6). In this case, because banks no longer consider the maximum loss on assets and their own net worth in their decision making, banks' leverage and asset portfolio allocation are both independent from these factors. Consequently, banks invest more in those assets that offer a higher expected return, and in equilibrium, the expected returns on the two types of assets are equalized.

As shown in Figure 5, a permanent downward shift in technology dampens long-run economic activity, including investment and banks' net worth accumulation, and results in downward pressure on current inflation. It is noteworthy that this shock has two opposing effects on inflation dynamics. First, it lowers the productivity of goods production, which has a positive effect on inflation. Second, it reduces the economy's output and weakens the household's demand permanently, which has a negative effect on inflation. In our setting, the latter effect dominates the former. Compared with the economy in which the VaR constraint is absent, in the benchmark economy the shock generates quantitatively larger macroeconomic effects because of the endogenous development in banks' net worth. When the technology slowdown reduces banks' net worth, banks shrink their balance sheet so as to avert default. Consequently, less capital is accumulated in the economy, leading to a further decline in output.

Next, Figure 6 shows the economic response when a negative temporary technology shock hits the economy. This shock leads to a short-run economic downturn and a deterioration in banks' net worth. As for price dynamics, since the first channel discussed above dominates the second channel, the temporary technology shock leads to higher marginal costs and inflation. In the economy with the VaR constraint, since net worth responds to the shock cyclically, the downturns in output, the increase in inflation, and the extent of balance sheet adjustments all become larger than in the economy without the VaR constraint.

Turning to Figure 7, this depicts the macroeconomic consequence of a deterioration in banks' net worth. Note that in the economy where the VaR constraint is absent, banks' net worth cannot be a source of economic fluctuations, as banks' investment decisions are unaffected by their net worth. In the VaR model, the deterioration in net worth brings about a recession and deflation in the economy. There are two distinct channels through which net worth affects banks' investment decisions. The first channel is the balance sheet effect. As pointed out by Adrian and Shin (2011), with a smaller net worth, banks shrink their balance sheet, reducing the total amount of investment. Consequently, other things being equal, both loan claims and government bond purchases fall. The second channel is the asset allocation effect. When their net worth is low, banks reduce lending more than they reduce government bond purchases, since the maximum loss on loan claims is larger than that on government bonds. Banks' asset allocation then tilts toward relatively safer assets and their losses in the worst-case scenario become smaller.

In case of a negative shock to banks' net worth, both channels contribute to the reduction in investment in real capital, dampening output and suppressing inflation.[19]

Figure 8 shows the equilibrium response to an increase in the maximum loss associated with holding loan claims. As indicated by equation (7) , since loan claims become riskier as a result, banks tilt their asset portfolios toward government bonds, whose expected return weighted by the maximum loss is now higher. In addition, because of the risk associated with investment in loan claims, the VaR constraint tightens and banks shrink their balance sheet. As a result, the supply of productive capital to the economy falls, leading to a decline in output and inflation in the period that the shock occurs.

Figure 9 depicts the equilibrium response to an increase in the maximum loss associated with holding government bonds. In this case, since government bond holdings become relatively risky, banks shift their asset portfolio toward loan claims. They reduce their holdings of government bonds and increase lending, following equation (7) , resulting in an expansion of output and an increase in inflation.[20]

The equilibrium responses of our model economy to a positive markup shock, a contractionary monetary policy shock and a contractionary government expenditure shock, which are shown in Figure 10, 11, and 12, are, in general, qualitatively the same as those in the standard New Keynesian model. That is, a positive markup shock dampens output and raises inflation, reflecting an exogenous increase in goods prices, while the two contractionary policy shocks lower both output and inflation by dampening aggregate demand. Similar to the macroeconomic outcome of the other five shocks discussed above, the endogenous evolution of banks' net worth plays an important role in the dynamics of the economy. The markup and government expenditure shocks cause an endogenous deterioration in banks' net worth. As a result, the economic downturn becomes larger than would otherwise be the case through the adjustment in banks' balance sheets and asset portfolios. By contrast, a contractionary monetary policy shock increases government debt repayments to banks, which helps banks to accumulate net worth. Consequently, the adverse effect of the shock is partially offset by the countercyclical net worth dynamics.

3.4 The Role of the VaR Constraint

To summarize the effect of incorporating the VaR constraint into the model, we report the steady state values and the theoretical moments of the macroeconomic variables

[19] When banks' net worth endogenously deteriorates, for instance due to a negative temporary technology shock, both the balance sheet effect and the asset allocation effect play a role in the downturn in output and inflation.

[20] The mechanism underlying the expansionary effect of a negative shock to the maximum loss on government bonds depends on our assumption that we are dealing with a closed economy. In a closed economy, other things being equal, a reduction in government bond purchases implies an increase in loans to firms. If we assume that there is room for domestic banks to invest overseas as well, then capital may be invested overseas, reducing domestic capital accumulation.

around the steady state for the benchmark and the no VaR model in Table 2. Shared parameters in the two models are set to equal values for a fair comparison. Because three of the structural shocks, that is, shocks to banks' net worth, $\varepsilon_{\gamma}(s^t)$, shocks to the maximum loss on loan claims, $\varepsilon_{\underline{r}_k}(s^t)$, and shocks to the maximum loss on government bonds, $\varepsilon_{\underline{r}_b}(s^t)$, are absent in the no VaR model, we report the standard deviations of macroeconomic variables in the benchmark economy for two different settings, namely when all of the eight shocks are present in the economy and when only five out of the eight shocks are present in the economy, for comparison.

Starting with the steady state values of the macroeconomic variables, which are shown in Table 2(a), output, capital accumulation, and banks' investment are smaller, and government bond accumulation is larger in the presence of the VaR constraint. Because the VaR constraint reduces banks' risk taking capacity, banks' balance sheets are smaller and they tilt their asset allocation more toward government bond holdings than loan claims when compared with the economy where such a constraint is absent. Consequently, capital and output at the steady state remain at a lower level.

Next, Table 2(b) reports the standard deviation of the growth rates of macroeconomic variables. As can be seen, the VaR constraint amplifies the macroeconomic effects of exogenous shocks hitting the economy. In the VaR model, in addition to the direct effects of shocks, the endogenous change in banks' net worth leads to further variations in banks' leverage and asset portfolio allocation, raising the volatility of growth rates.

3.5 Contribution of Structural Shocks During the Two Lost Decades

Next, using the estimated model parameters and shocks, we investigate the quantitative role that each of the structural shocks plays in explaining macroeconomic variations in Japan during the two lost decades. Table 3 shows the results of decomposing variations in GDP, inflation, and government bond purchases into the eight structural shocks. To examine how economic circumstances over the period differed, we do so not only for the observation period as a whole, i.e., from 1981Q1 to 2007Q4, but also for two sub-periods, namely the period up to burst of bubble economy, i.e., 1981Q1 to 1990Q4, which we label period I, and the period during and after the Japanese banking crisis, i.e., 1997Q1 to 2007Q4, which we label period II. The rows in the table show the contribution of each of the structural shocks in explaining the variations in the corresponding macroeconomic variable. The first, second, and fourth column of the table display the decomposition results for period I, period II, and the whole observation period, respectively. The third column shows the difference in the contribution of each shock between periods I and II. For instance, Δ indicates that the shock moves the corresponding macroeconomic variable in a more positive direction during period II relative to period I.

In explaining GDP variations, temporary shocks to technology play the quantitatively dominant role, regardless of the observation period. While the GDP growth rate drops from 0.4% to 0.1% from period I to period II, most of the shocks, other than shocks to government expenditure, contribute to lowering the GDP growth rate in the latter period.

In explaining inflation variations, it is the quantitative role played by permanent technology shocks rather than temporary technology shocks that is crucial. One reason for this result likely is that the inflation rate is a forward-looking variable and is therefore less affected by temporary changes in technology. Turning to other factors, the second-most important shocks during period I were monetary policy shocks. One reason is that the variation in the monetary policy rate is relatively large in period I when compared with period II, so that there was ample potential for inflation to respond to monetary policy shocks. During period II, shocks to banks' net worth made the greatest contribution among the shocks. As discussed in detail in Hoshi and Kashyap (2011), balance sheet problems at Japanese banks were most acute during the period from 1997 to 1999. Banks incurred loan losses amounting to about 10 trillion yen annually, eroding their net worth. Our result indicates that the deterioration in banks' net worth at the time substantially affected inflation dynamics then. The permanent technology shocks and shocks to bank's net worth contribute to the reduction in the inflation rate from 0.3 % during period I to -0.3% during period II. This is in line with the results of our impulse response exercise that a slowdown in technology growth and a deterioration in banks' net worth hamper the supply of capital to goods producers by directly reducing banks' leverage and leading to a change in banks' asset portfolio allocation toward government bonds. Consequently, inflation falls, reflecting the weakened demand.

The growth rate of government bond purchases slightly increased from -0.2% during period I to 0.4% during period II. For the observation period as a whole, the most important factor underlying variations in government bond purchases was permanent technology shocks. The result suggests that the main reason for the increase in government bond purchases in period II is a decline vis-a-vis period I in the expected return on capital due to the slowdown in technology growth. Next, turning to other factors, similar to the case of inflation, the contribution of monetary policy shocks is considerable for period I, but then becomes negligible for period II. Conversely, shocks to banks' net worth and the maximum loss on loan claims play an important role only during period II. Specifically, the deterioration in banks' net worth lowers the growth in government bond purchases by reducing banks' leverage, while the increase in the maximum loss on loan claims contributes to the growth in government bond purchases by leading banks to shift their asset portfolios away from loan claims. The contribution of shocks to the maximum loss on government bonds is negligible. One possible reason is that the risk of default by the Japanese government has not materialized and that government bonds are regarded as a safe asset.

In sum, our historical decomposition analysis indicates shocks to banks' net worth and the maximum loss on either type of assets considered here are not the dominant force behind the variations in output. This result, however, does not imply that the VaR constraint is not important in explaining economic fluctuations. First, these two shocks play a sizable role in explaining variations in inflation particularly during the period of the banking crisis. Second, as discussion in Section 3.4, the VaR constraint amplifies the macroeconomic consequences of some of the exogenous shocks, including technology shocks, through the endogenous development in the banks' net worth.

4 Conclusion

During the so-called two lost decades, the Japanese economy witnessed an unprecedentedly rapid accumulation of government debt, together with long-lasting economic stagnation and deflation. While a considerable number of studies have investigated the reasons behind the economic stagnation and deflation, issues regarding government debt, such as the reasons for the rapid accumulation of government debt and how this affects economic activity, including inflation dynamics, have not received much attention in the literature so far.

The purpose of our paper was to address these questions by shedding light on the role played by banks' asset portfolio choice in generating demand for government bonds. Banks' asset allocation substantially shifted from lending to private firms to the purchase of government bonds, particularly from the mid-1990s onward. Consequently, a sizable amount of government bonds is now held by banks. Taking these facts into consideration, we developed a New Keynesian model that incorporates banks' endogenous decisions regarding their asset allocation. In the model, banks acquire two types of assets: loan claims, which are expected to yield a higher return but have a larger maximum loss, and government bonds, which are expected to yield a lower return but carry a smaller maximum loss. The key feature of the model is that it considers the effect of the Value at Risk (VaR) constraint in banks' asset portfolio decisions. Specifically, it is assumed that banks allocate their assets such that they are able to repay all of their debts even when the maximum loss on both types of assets materializes.

Our model suggests that, changes in economic circumstances, such as a deterioration in banks' balance sheets, a slowdown in technology growth, or an increase in the maximum loss on the two types of assets, leads to a change in banks' allocation of assets between loan claims and government bonds, as these changes affect the tightness of the VaR constraint. If banks' asset allocation tilts toward government bond holdings due to a rebalancing of banks' asset allocation, the supply of capital to goods producers declines. As a result, output decreases and deflation emerges.

Next, using Japanese data from 1981 to 2007, we estimated the fundamental sources of developments in output, deflation, and government bond purchases applying Bayesian techniques and examined the quantitative role of banks' asset allocation. Our results indicate that although most of the variation in output is explained by technology shocks, shocks to banks' net worth do play an important role in variations in inflation, particularly during the period after Japan's banking crisis erupted in 1997. The deterioration in banks' balance sheets stemming from bad loans, and the implementation of the Basel Accords, which took place at that time, imposed constraints on banks' risk taking capacity, as a result of which banks tilted their asset allocation toward government bonds, leading to a decline in output and inflation.

Our results imply that government policies aiming to encourage private investment in productive capital by reducing the associated risk may be a useful tool for boosting economic activity when the economy is weakened by a decline in risk taking behaviors. Suppose that macroeconomic risks, including the maximum loss on capital investment, may be successfully mitigated by policy, then such policy would lead to a favorable shift in asset allocation toward loan claims, helping the economy back onto a recovery path. What such a policy might look like, and what effects it could have, are issues left for the future research.

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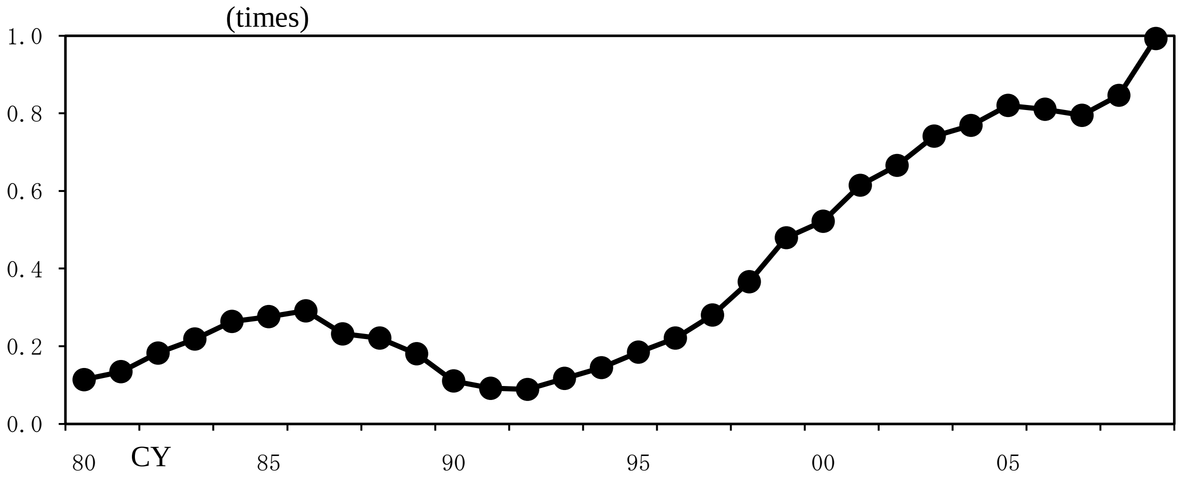
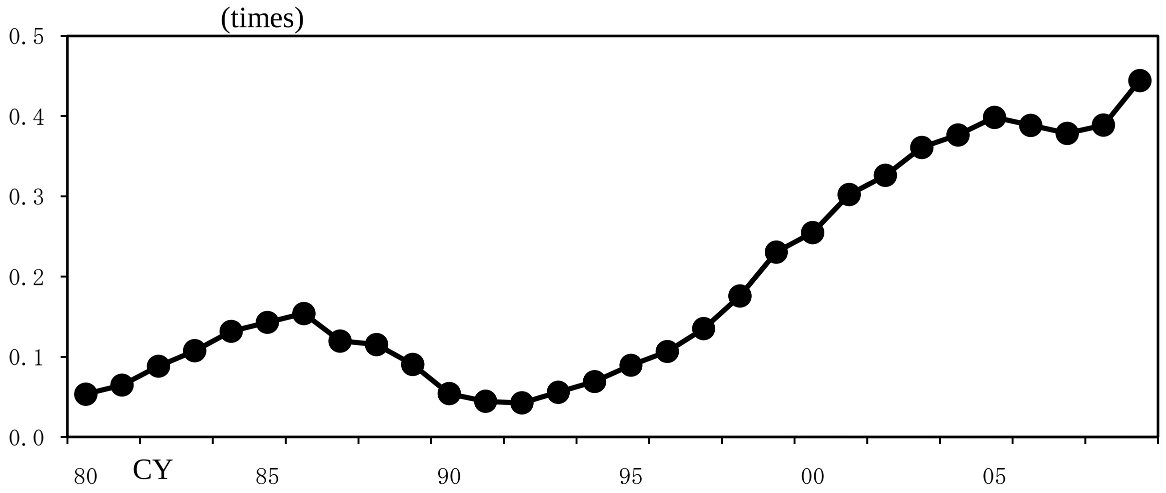
Figure 1. Accumulation of Government Debt

(1) Government Debt over GDP



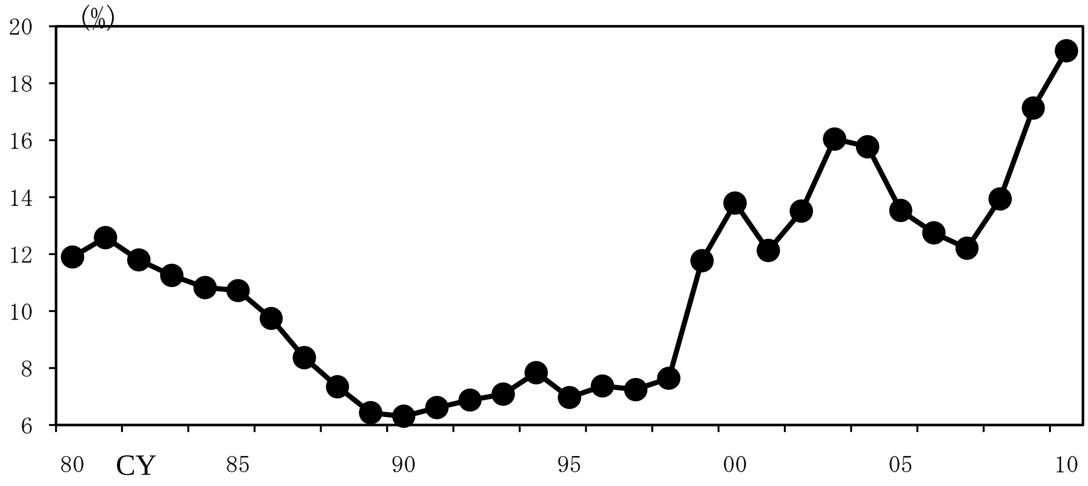
(2) Government Debt over Capital



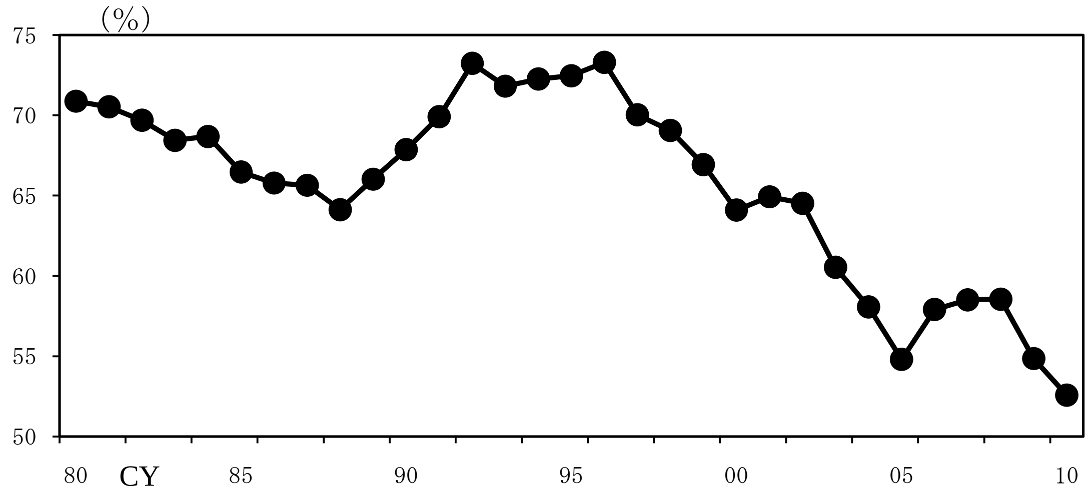
Source: Annual Report on National Accounts, Cabinet Office.

Figure 2. Banks' Asset Allocation

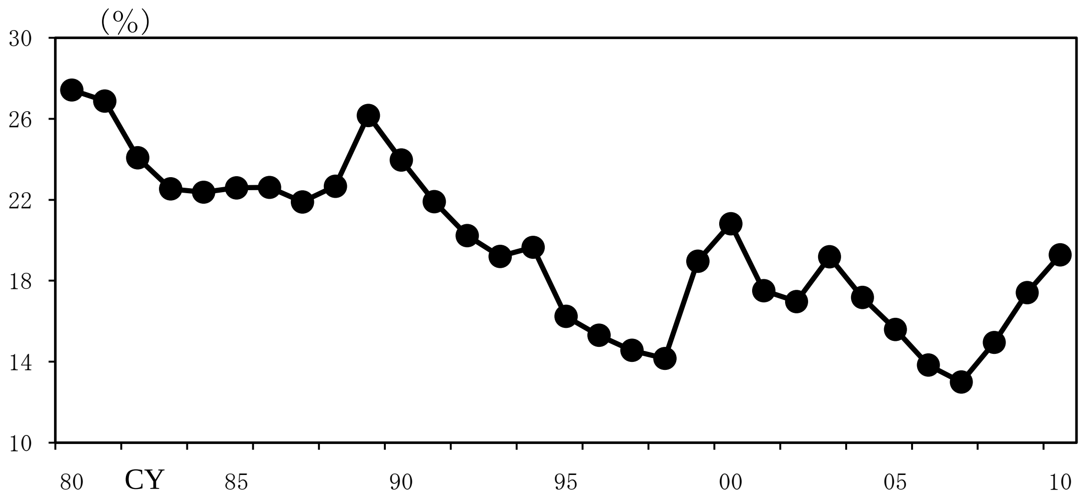
(1) Share of Government Bonds in Banks' Assets



(2) Share of Loan Claim in Banks' Assets



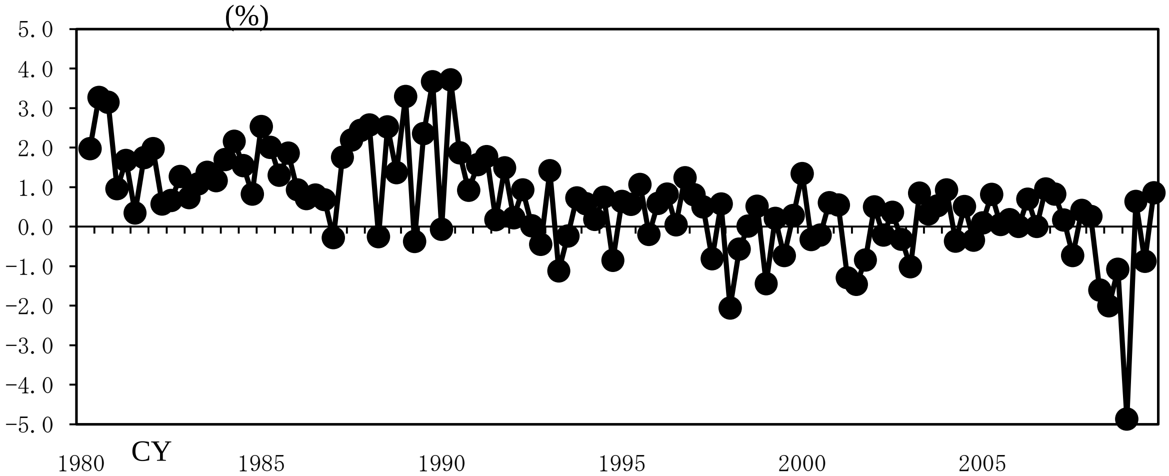
(3) Government Bonds Held by Japanese Banks



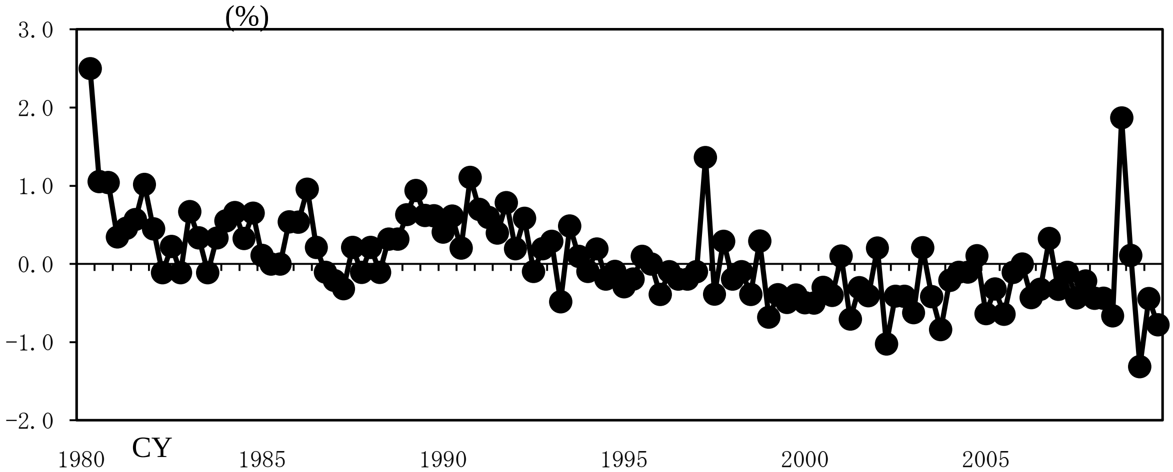
Source: Flow of Funds Accounts, Bank of Japan.

Figure 3. Macroeconomic Variables

(1) GDP

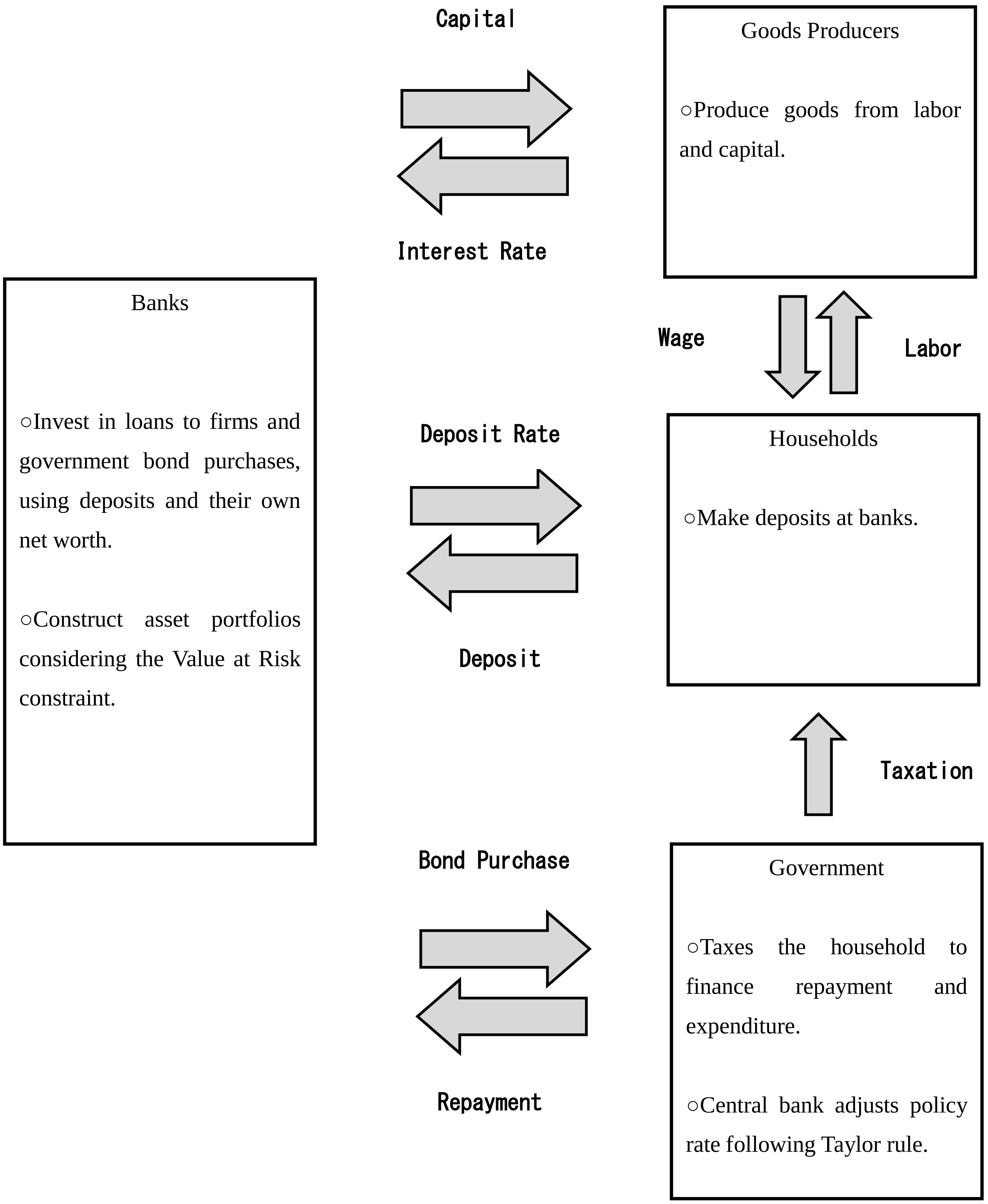


(2) GDP Deflator



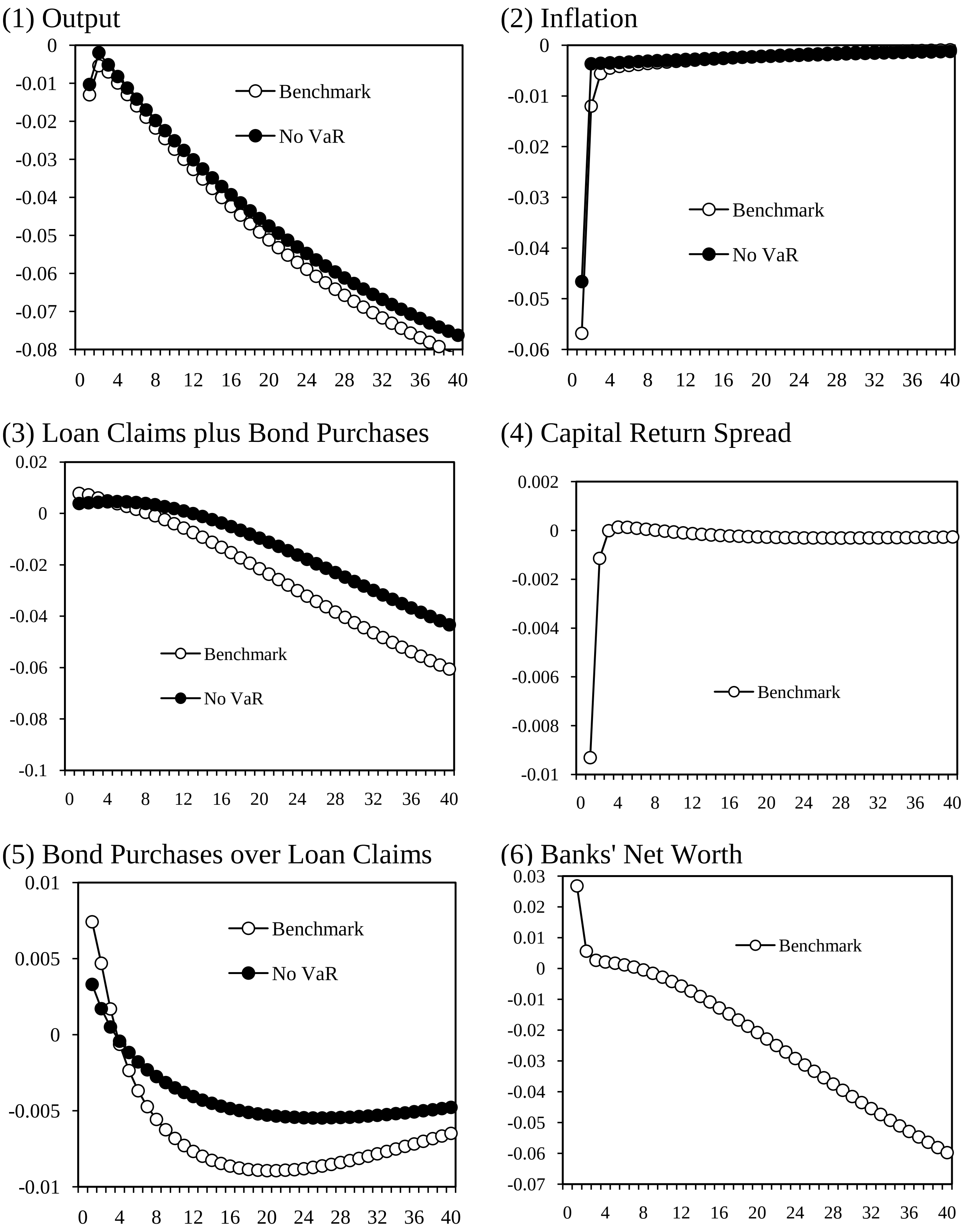
Source: Annual Report on National Accounts, Cabinet Office.

Figure 4



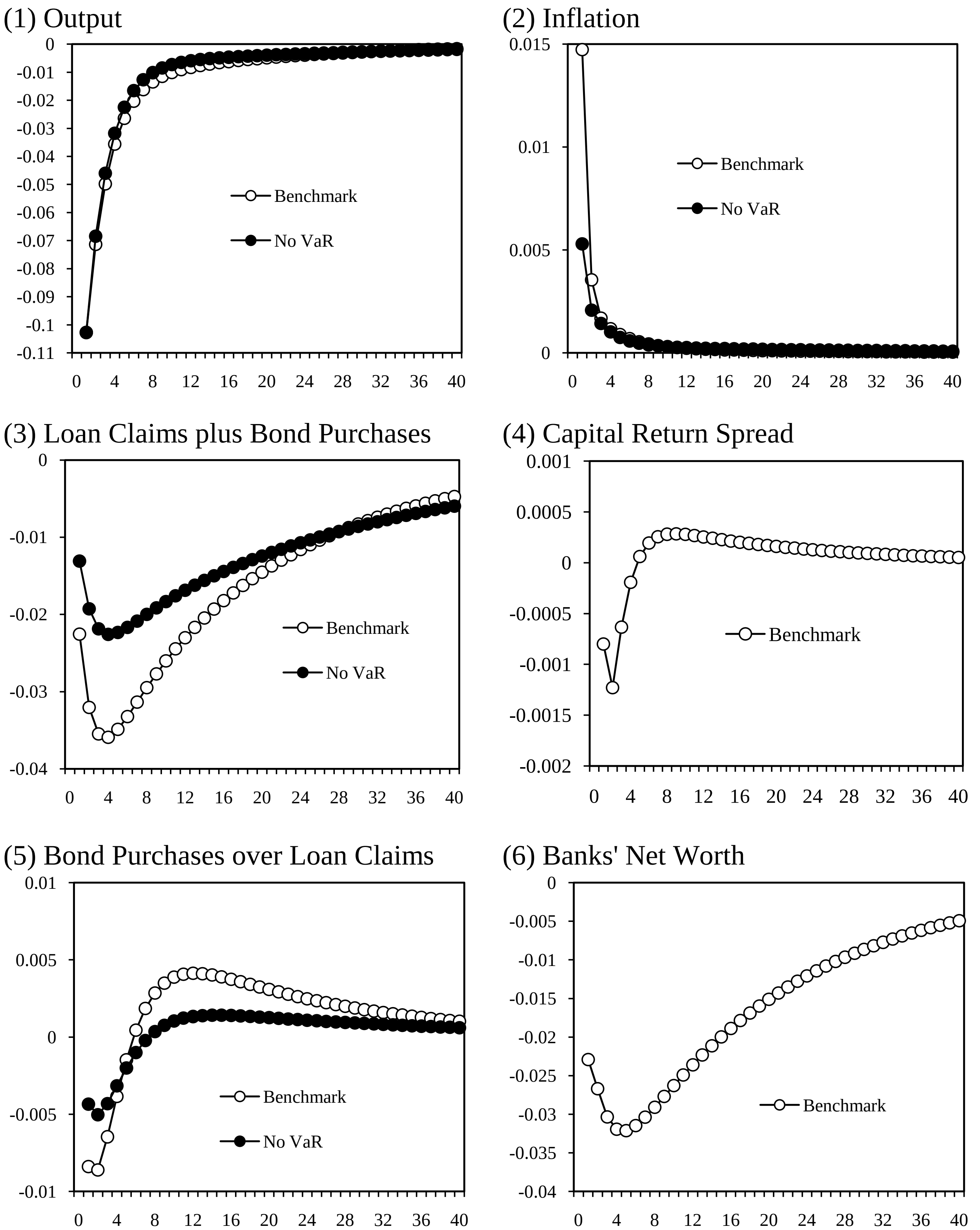
Capital
Goods Producers
○Produce goods from labor and capital.
Interest Rate
Banks
○Invest in loans to firms and government bond purchases, using deposits and their own net worth.
○Construct asset portfolios considering the Value at Risk constraint.
Wage
Labor
Deposit Rate
Households
○Make deposits at banks.
Deposit
Taxation
Bond Purchase
Government
○Taxes the household to finance repayment and expenditure.
○Central bank adjusts policy rate following Taylor rule.
Repayment

Figure 5. Economic Response to Permanent Technology Shock



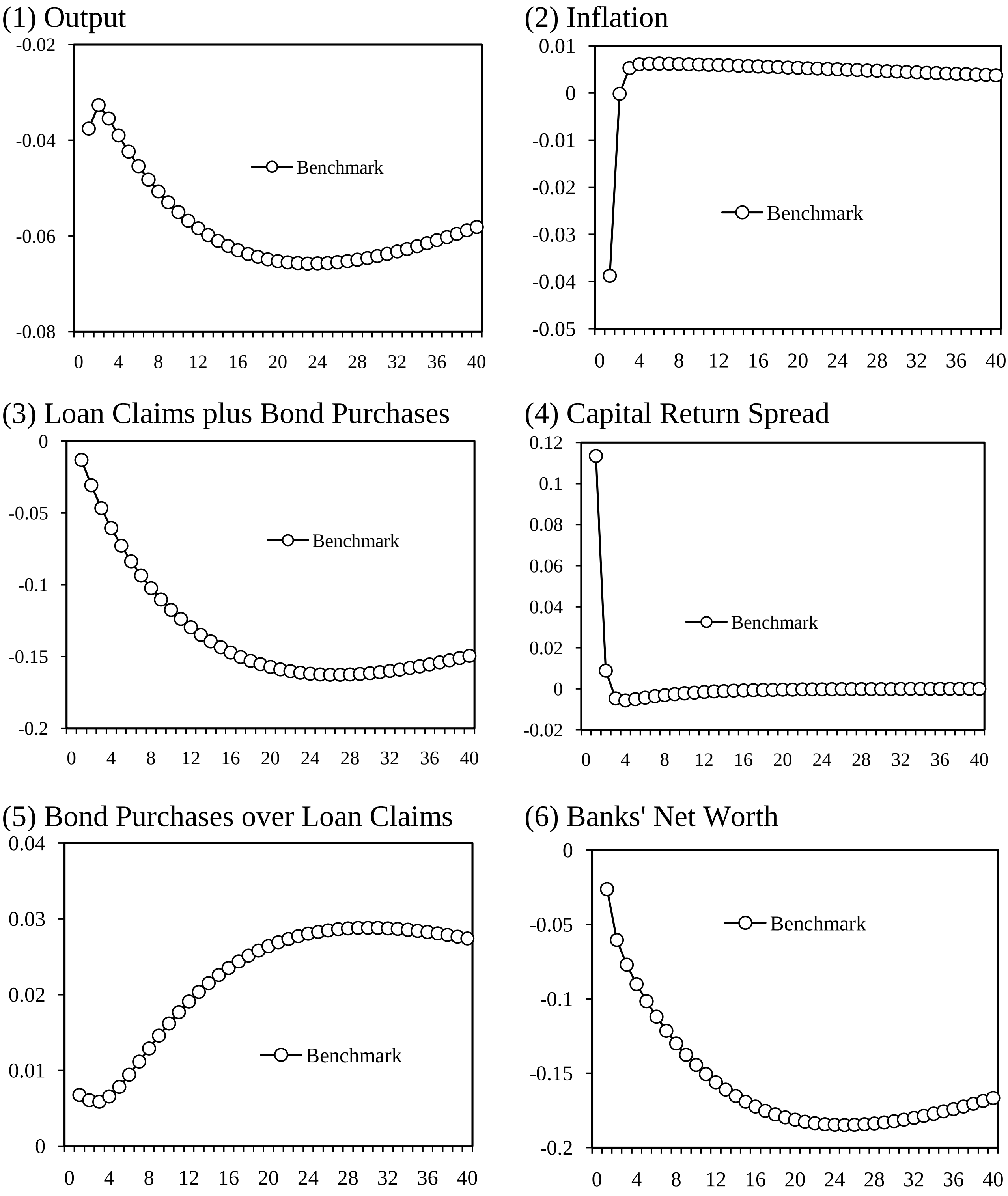
Note: The vertical axes denote the deviation from the steady state, while horizontal axes denote the number of quarters after the shock.

Figure 6. Economic Response to Temporary Technology Shock



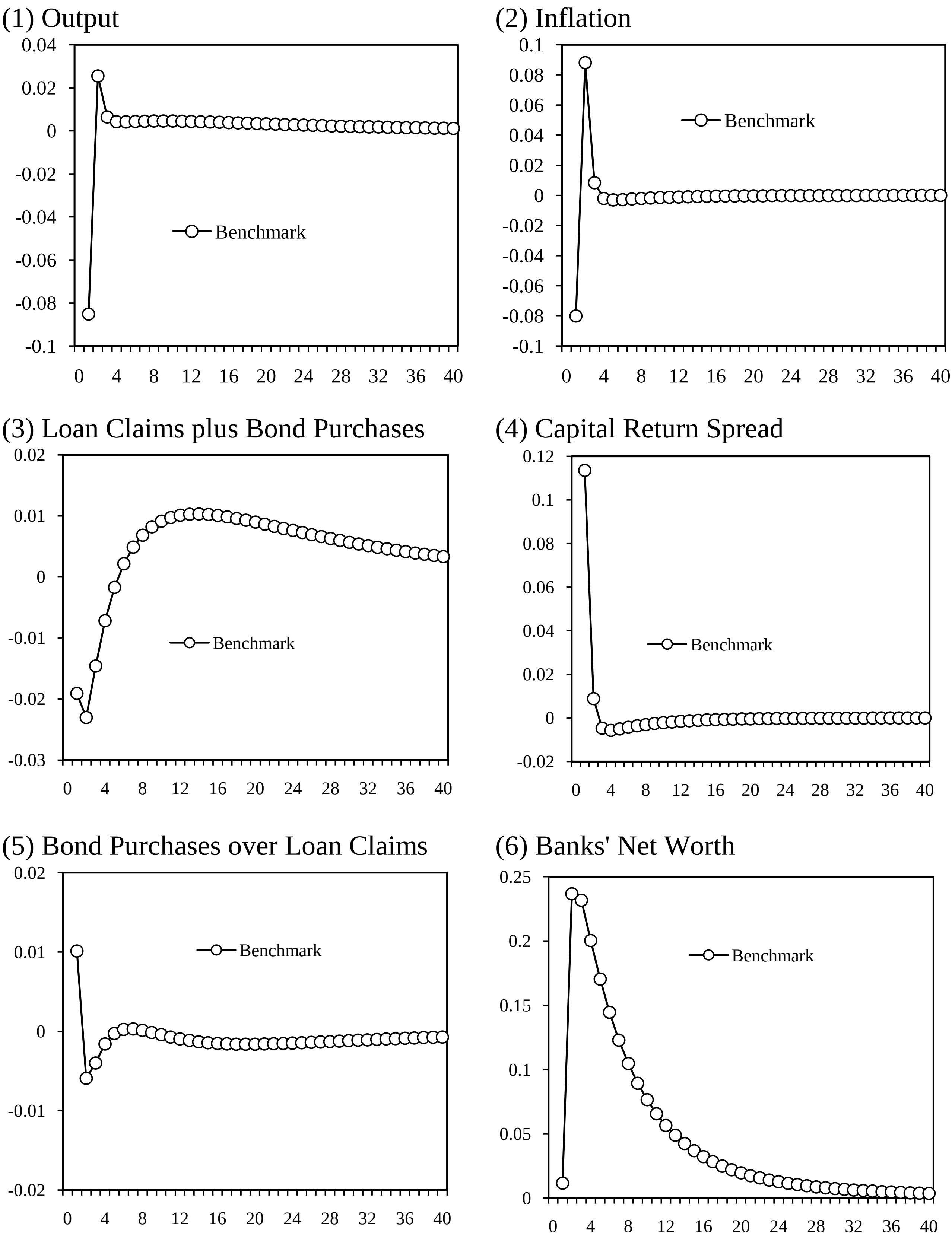
Note: The vertical axes denote the deviation from the steady state, while horizontal axes denote the number of quarters after the shock.

Figure 7. Economic Response to Shocks to Banks' Net Worth



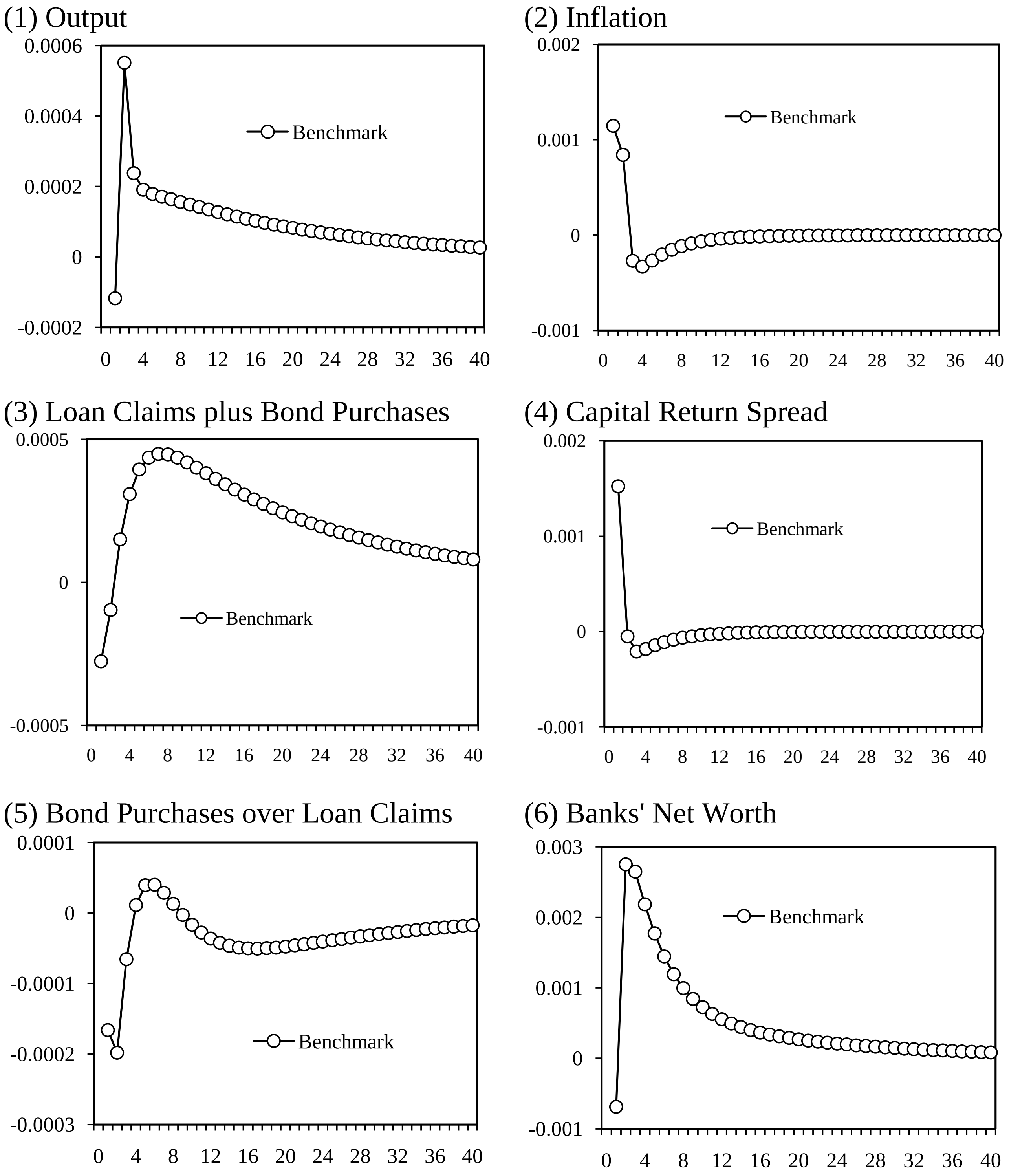
Note: The vertical axes denote the deviation from the steady state, while horizontal axes denote the number of quarters after the shock.

Figure 8. Economic Response to Shock to Maximum Loss on Loan Claims



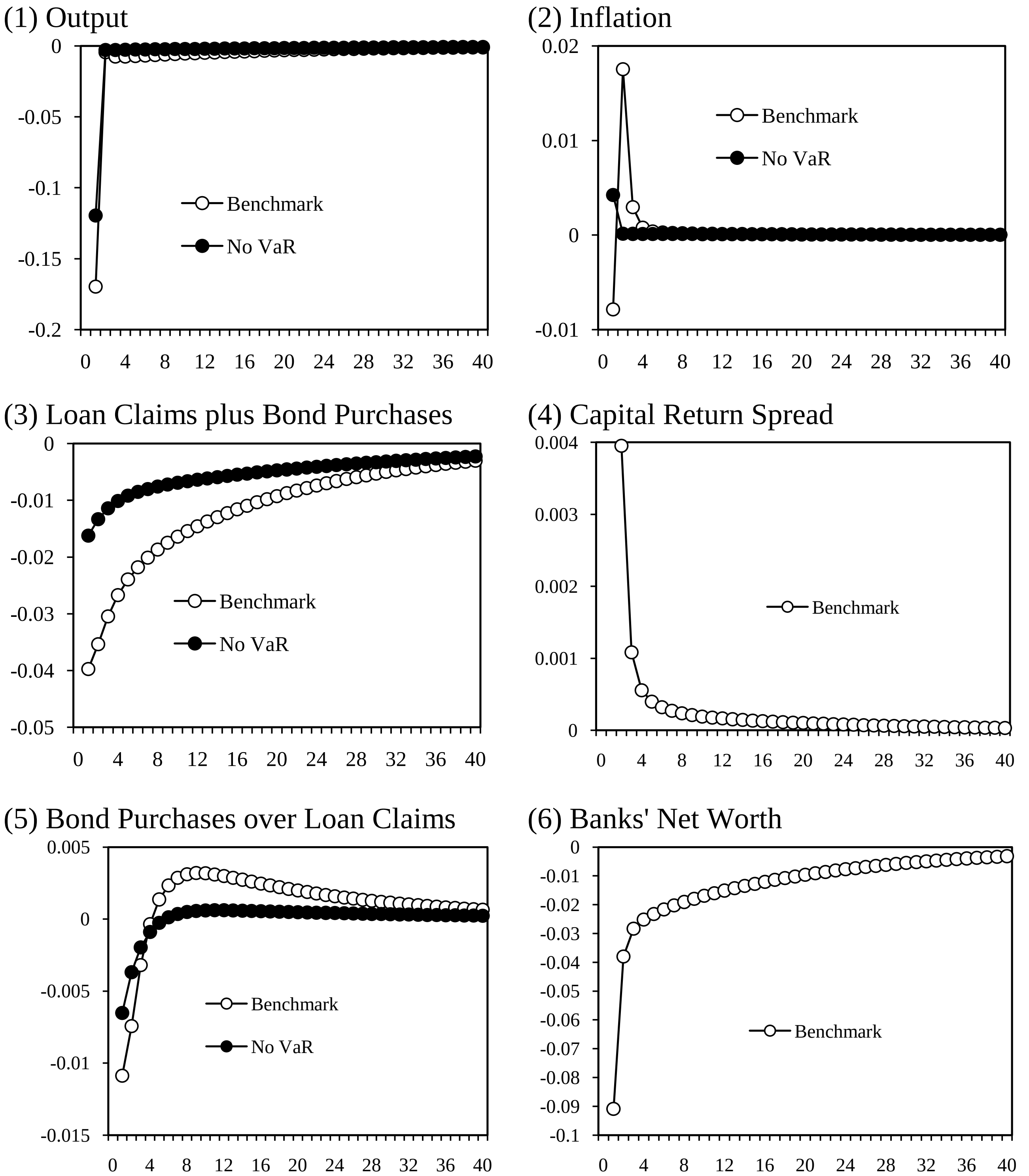
Note: The vertical axes denote the deviation from the steady state, while horizontal axes denote the number of quarters after the shock.

Figure 9. Economic Response to Shock to Maximum Loss on Government Bonds



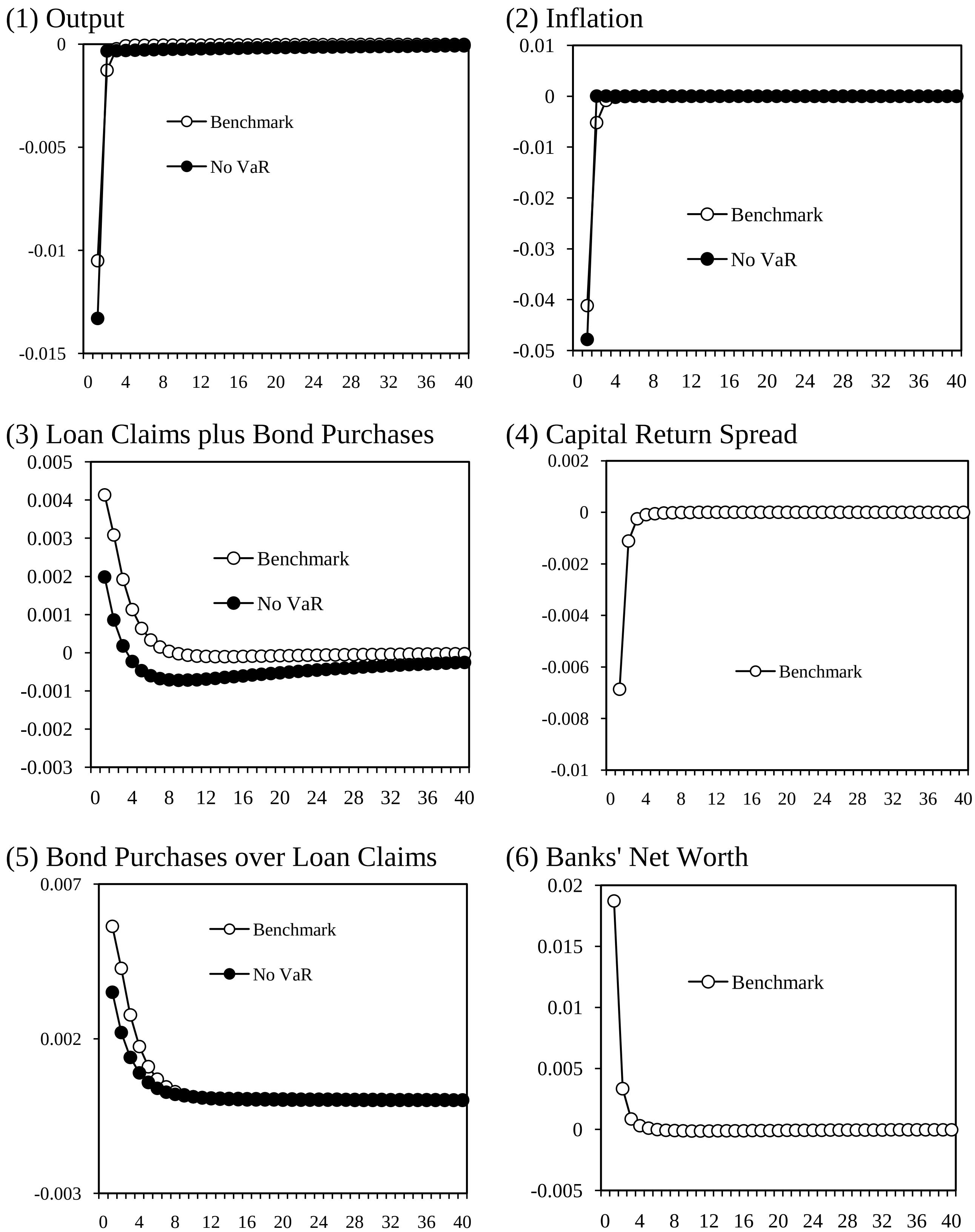
Note: The vertical axes denote the deviation from the steady state, while horizontal axes denote the number of quarters after the shock.

Figure 10. Economic Response to Markup Shock



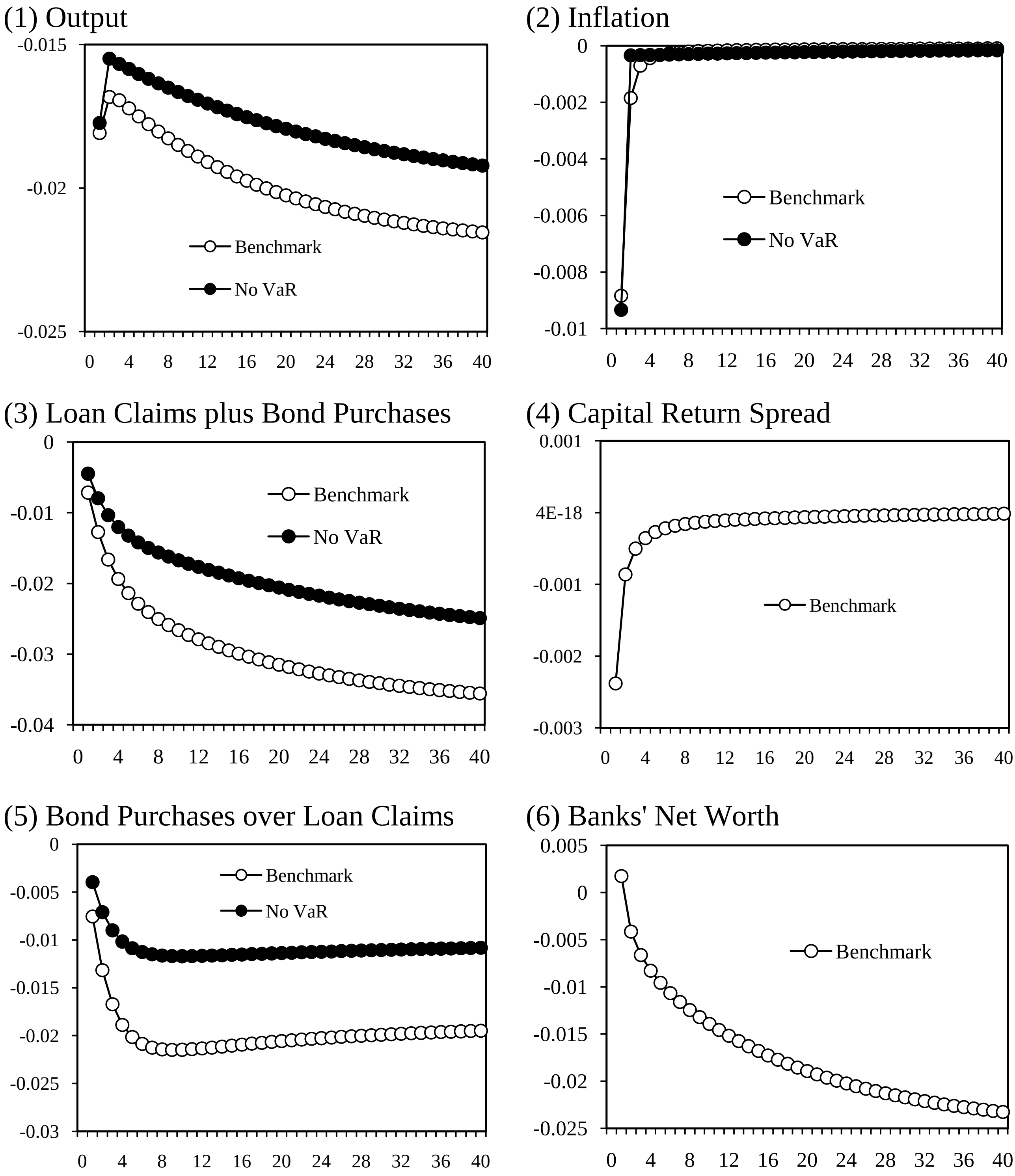
Note: The vertical axes denote the deviation from the steady state, while horizontal axes denote the number of quarters after the shock.

Figure 11. Economic Response to Monetary Policy Shock



Note: The vertical axes denote the deviation from the steady state, while horizontal axes denote the number of quarters after the shock.

Figure 12. Economic Response to Government Expenditure Shock



Note: The vertical axes denote the deviation from the steady state, while horizontal axes denote the number of quarters after the shock.

Table 1. Parameters of the Model

(a) Estimated Parameters

Parameter		Prior Dist.	Prior Mean	Prior Std.	Posterior		
					mean	5%	95%
κ_P	Adjustment Cost of Prices	norm	4	1.5	6.09	6.08	6.11
ϕ	Monetary Policy Rule (coefficient for inflation)	norm	1.5	0.125	1.19	1.19	1.19
ρ_A	Monetary Policy Rule AR	norm	0.75	0.125	0.93	0.93	0.93
γ	Survival Probability of Banks	norm	0.9	0.01	0.90	0.90	0.90
$\underline{r_k}$	Maximum Loss on Loan Claims	norm	0.7	0.01	0.70	0.70	0.70
$\underline{r_b}$	Maximum Loss on Government Bonds	norm	0.95	0.01	0.95	0.95	0.95
T	Fiscal Policy Rule	norm	0.01	0.01	0.003	0.002	0.004
$\underline{\psi}$	Fiscal Policy Rule (coefficient for debt)	norm	2	0.125	1.97	1.96	1.97
G/X	Government Expenditure Share	norm	0.17	0.1	0.17	0.16	0.17
ε	Preference with regard to Goods Variety	norm	7	0.125	6.97	6.97	7.00
ρ_Z	Permanent Technology Shock AR	beta	0.75	0.22	0.98	0.98	0.98
ρ_A	Temporary Technology Shock AR	beta	0.129	0.22	0.64	0.63	0.64
ρ_γ	Shock to Banks' Net Worth AR	beta	0.75	0.22	0.98	0.98	0.98
ρ_{r_k}	Shock to Maximum Loss on Loan Claims AR	beta	0.75	0.22	0.83	0.83	0.83
ρ_{r_b}	Shock to Maximum Loss on Government Bonds AR	beta	0.129	0.22	0.74	0.74	0.75
ρ_G	Government Expenditure Shock AR	beta	0.129	0.22	0.99	0.99	0.99
σ_r	Monetary Policy Shock SD	invg	0.009	Inf	0.005	0.002	0.009
σ_Z	Permanent Technology Shock SD	invg	0.035	Inf	0.005	0.005	0.005
σ_A	Temporary Technology Shock SD	invg	0.009	Inf	0.083	0.079	0.086
σ_ε	Price Markup Shock SD	invg	0.009	Inf	2.646	2.612	2.617
σ_γ	Shock to Banks' Net Worth SD	invg	0.035	Inf	0.039	0.034	0.042
σ_{r_k}	Shock to Maximum Loss on Loan Claims SD	invg	0.009	Inf	0.154	0.134	0.174
σ_{r_b}	Shock to Maximum Loss on Government Bonds SD	invg	0.035	Inf	0.005	0.002	0.009
σ_G	Government Expenditure Shock SD	invg	0.035	Inf	0.280	0.260	0.320

(b)Calibrated Parameter

α	Capital Share in Final Goods Production	0.35
β	Household's Discount Factor	0.99
η	Household's Preference over Leisure	0.5
δ	Depreciation Rate	0.025

Table 2. Effect of the VaR Constraint on the Economy

(a) Comparison of the Steady State Values of the Macroeconomic Variables

		Economy with VaR	Economy without VaR
y	Output at Steady State	1.5	2.3
k	Capital at Steady State	6.6	19.3
b	Government Bonds at Steady State	1.5	0.9
b/k	Ratio of Government Bond Holdings to Loan Claim Holding	0.2	0.1

(b) Variation of Macroeconomic Variables around the Steady State

		Economy with VaR		Economy without VaR
$std(\Delta y)$	Standard Deviation of Output Growth	0.30	[0.26]	0.20
$std(\Delta k)$	Standard Deviation of Capital Growth	0.07	[0.04]	0.02
$std(\Delta b)$	Standard Deviation of Growth of Government Bond Purchases	0.15	[0.12]	0.11
$std(\Delta(b+k))$	Standard Deviation of Leverage	0.07	[0.05]	0.03
$std(\pi)$	Standard Deviation of Inflation Rate	0.15	[0.08]	0.07

Note: Square brackets denote the standard deviation of variable when there are no shocks to banks' net worth and the maximum loss on assets in the economy.

Table 3. Contribution of Structural Shocks to Macroeconomic Variations

(1) Output Growth (Quarterly)

	(a) Before 1991 (1981Q1-1990Q4)	(b) After Banking Crisis (1997Q4-2007Q4)	(b)-(a)	(c) Full Sample (1981Q1-2007Q4)
Average Growth Rate (%)	0.4	0.1		
Permanent Technology Shocks	5.6	2.1	(▼)	6.7
Temporary Technology Shocks	68.8	82.0	(▼)	72.0
Shocks to Banks' Net Worth	3.8	3.8	(▼)	4.3
Shocks to Maximum Loss on Loan Claims	2.7	2.5	(▼)	2.7
Shocks to Maximum Loss on Gov. Bonds	0.0	0.0	(▼)	0.0
Markup Shocks	14.8	6.7	(▼)	10.5
Monetary Policy Shocks	1.3	0.0	(▼)	0.8
Government Expenditure Shocks	3.1	2.9	(△)	3.0

(2) Inflation (Quarterly)

	(a) Before 1991 (1981Q1-1990Q4)	(b) After Banking Crisis (1997Q4-2007Q4)	(b)-(a)	(c) Full Sample (1981Q1-2007Q4)
Average Growth Rate (%)	0.3	-0.3		
Permanent Technology Shocks	49.7	30.0	(▼)	41.9
Temporary Technology Shocks	5.0	3.2	(△)	6.2
Shocks to Banks' Net Worth	2.3	42.1	(▼)	13.0
Shocks to Maximum Loss on Loan Claims	4.1	14.3	(△)	6.0
Shocks to Maximum Loss on Gov. Bonds	0.0	0.0	(▼)	0.0
Markup Shocks	11.4	5.8	(▼)	9.3
Monetary Policy Shocks	26.6	3.6	(△)	22.8
Government Expenditure Shocks	0.9	1.0	(△)	0.9

(3) Growth in Bond Purchases (Quarterly)

	(a) Before 1991 (1981Q1-1990Q4)	(b) After Banking Crisis (1997Q4-2007Q4)	(b)-(a)	(c) Full Sample (1981Q1-2007Q4)
Average Growth Rate (%)	-0.2	0.4		
Permanent Technology Shocks	23.1	32.5	(△)	35.0
Temporary Technology Shocks	9.7	16.8	(▼)	13.2
Shocks to Banks' Net Worth	3.4	20.4	(▼)	5.9
Shocks to Maximum Loss on Loan Claims	5.7	19.9	(△)	5.8
Shocks to Maximum Loss on Gov. Bonds	0.0	0.0	(▼)	0.0
Markup Shocks	17.2	4.2	(▼)	9.5
Monetary Policy Shocks	31.6	1.9	(▼)	23.0
Government Expenditure Shocks	9.2	4.2	(△)	7.5

Notes:

1. △ (▼) indicates that the corresponding shock makes a positive (negative) contribution during period (b) relative to period (a).
2. Markup Shocks includes the contribution of initial values as well as the contribution of markup shocks.

The contribution of the former component is, however, negligible compared with the latter.