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Abstract

We propose a simple and tractable method to estimate linear DSGE models with the zero lower bound on nominal interest rates. Our method makes use of forward rate curves in order to take into account the effects of the zero lower bound on equilibrium endogenous variables without relying on nonlinear techniques for solving rational expectation equilibrium. Applying the method to Japanese data, we find that the natural interest rate might not have declined to negative values in the late 90s and 2000s. Counterfactual simulations show that the Bank of Japan's zero interest rate policy and quantitative easing policy in those periods had expansionary effects by bull flattening the yield curves.

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1 Introduction

In recent years, the zero lower bound on nominal interest rates has imposed on many central banks in the world serious constraints on stimulating their economies. Analysis of monetary policy under the zero lower bound has been an important topic both from the academic and practical point of view. However, the zero lower bound has been a technical challenge to researchers on monetary policy analysis. It is a common practice to use linearized rational expectations models for analysis of monetary policy, but the presence of the zero lower bound adds nonlinearity into those models. Solving nonlinear models are much more demanding than solving linear models.

Due to this difficulty, researchers have made various types of compromise. On one hand, the existing theoretical research has restricted to a smaller models (e.g. Adam and Billi (2006), Eggertsson and Woodford (2003), Fernandez-Villaverde *et al.* (2012)) to use nonlinear methods to solve rational expectations equilibrium with the zero lower bound. The existing empirical research, on the other hand, has made a shortcut by abstracting from the zero lower bound (e.g. Ireland (2011), Benati (2008)). However, the present paper shows that ignoring the zero lower bound may cause biased estimates of economic shocks.[1] This is because expectations should take into consideration the possibility that the zero lower bound constraint may bind in future periods. Ignoring the zero lower bound may imply that the model allows expectations about the future nominal rates to become negative. Then the effects of the zero lower bound on deflationary pressure through expectations may be undervalued, causing biases in estimated shocks

[1] Alternatively, if one supposes that there is no structural break before and after the zero lower bound becomes binding, it may be possible to estimate parameters accurately by using only the sample period during which the zero lower bound is not binding. For example, Chen *et al.* (2011) and Saito *et al.* (2012) compare the parameter estimates obtained by using the sample that includes zero-interest-rate period and the estimates obtained by using the sample that does not. They report that those parameter estimates remain comparable.

and structural parameters. In this paper, we propose a new method to overcome this problem and to correctly estimate structural shocks and model parameters under the presence of the zero lower bound.

Our method is built on the idea that, in linear rational expectations models, all the endogenous non-policy variables (such as inflation and the output gap) can be expressed in linear form in terms of expectations about future shocks and future nominal interest rates as well as the current nominal interest rate and lagged endogenous variables. We use forward rate curves as a measure of private-sector expectations about future nominal interest rates. Since forward rates contain information on the possibility of hitting the zero lower bound in future periods, we can correctly take into account of the effects of both the current and future zero lower bounds on the current endogenous variables. More specifically, our method consists of the following two steps. Firstly, we identify structural shocks except monetary policy shock by using forward rate curves in a linear framework. Secondly, we can recover monetary policy shock given the distribution of forward rates. The second step builds on the idea of Black (1995) that nominal interest rates (subject to the zero lower bound) can be modeled as options.[2] A great advantage of our approach is that it is not necessary to solve the entire model by nonlinear techniques because, in the linear rational expectations framework, all endogenous variables can be expressed as linear functions of forward rates. Furthermore the zero lower bound affects the endogenous non-policy variables only through its effect on forward rates. This implies that we need nonlinear methods only when we identify monetary policy shock. Thus our methodology can be used to estimate a wider class of linear rational expectations models under the zero bound. Our methodology is also related to Laseen and Svensson (2011)'s algorithm for solving linear rational expectations models with anticipated policy paths.

[2] See, also Ichiue and Ueno (2006) who apply Black (1995) to study the yield curves and monetary policy of the Japanese economy.

They express anticipated policy paths by including a vector of non-zero means of future shocks to a policy rule. It may be possible to apply their method to express the zero lower bound on nominal interest rates in future periods and solve rational expectations equilibrium. For example, Bodenstein *et al.* (2010) apply the algorithm of Laseen and Svensson (2011) to deal with the zero lower bound. However, as is shown later, it is not necessarily the case that model agents expect that the zero lower bound will bind in future periods with probability either one or zero as their method implies. It is more natural to allow agents to expect that the zero lower bound will be binding with some probability (between zero and one) as the economy is always subject to shocks of both directions (positive or negative). Our methodology is capable of handling this problem. A recent independent work by Kim and Pruitt (2012) also propose a method to estimate monetary policy rules under presence of the zero lower bound. While they only focus on estimation of coefficients in a monetary policy rule, our method can estimate entire model parameters as well as structural shocks.[3] Yet another approach to deal with the zero lower bound is to use regime switching models. For example, Hamilton and Wu (2012) and Koeda (2011) use affine term structure models with regime switching to investigate yield curves when the nominal interest rates are subject to the zero lower bound. While their method is also tractable, they estimate reduced-form models. Our work attempts to estimate structural DSGE models.

The next section uses a simple stylized model to illustrate problems of the existing empirical approach that only considers the zero lower bound in the current period but ignores those in subsequent periods. It is shown that use of forward rates can mitigate the problem. Section 3 describes our identification and estimation strategy. Section 4

[3]Since Kim and Pruitt (2012) focus on estimation of monetary policy rules, they do not need to specify the rest of the economy. On the other hand our estimated monetary policy rules depend on specification of the rest of the economy. In that sense, the two approaches are complementary.

applies our method to simulated data of the Eggertsson-Woodford model (Eggertsson and Woodford (2003)) to confirm usefulness of our approach. There we generate artificial data by solving the Eggertsson-Woodford model with nonlinear techniques, and show that the our method can correctly identify shocks while the existing method identifies shocks with bias. Section 5 applies a simple New Keynesian model to the Japanese data and estimate structural parameters and shocks. Section 6 concludes.

2 Simple example

In this section we use a stylized three-period model to illustrate the main idea. Although the model is highly stylized, it has basic features of New Keynesian models. The Phillips curve is given by

$$\pi_t = \begin{cases} \beta E_t[\pi_{t+1}] + (r_t^n - i_t), & t = 1, 2 \\ r_t^n - i_t, & t = 3 \end{cases} \tag{1}$$

where π_t, r_t^n, i_t respectively denote inflation, the natural interest rate, and the nominal interest rate. Equation (1) implies that inflation at time 1 and 2 are purely determined by the current expected values of the interest rate gap—the deviation of the nominal interest rate from the natural rate. In standard New Keynesian models, inflation depends on expectations about the future output gap, which in turn depends on expectations about the future interest rate gap. In order to make the model as simple as possible, we abstract from the IS curve and assume that the interest rate gap directly affects inflation. The monetary policy rule is

$$i_t = \max\left\{r_t^n + e_{i,t},\, 0\right\}. \tag{2}$$

Equation (2) assumes that the central bank chooses the nominal interest rate to keep track of the natural interest rate but that the nominal rate cannot be negative. Here $e_{i,t}$

represents monetary policy shock whose stochastic process is given by

$$e_{i,t} = \rho e_{i,t-1} + \varepsilon_{i,t}, \quad 0 < \rho < 1, \tag{3}$$

and $\varepsilon_{i,t}$ is i.i.d normal with mean zero and variance σ^2. We assume that σ^2 is small enough so that when the natural rate is positive the nominal interest rate will not be negative almost surely.

We make the following assumptions. The natural rate becomes negative at time 1 and 2 ($\underline{r}^n < 0$), and then recovers to a positive value at time 3 ($\overline{r}^n > 0$). Agents and the central bank in the model know the level of the natural rate in each period (perfect foresight). An econometrician at time 1, however, does not directly observe the levels of the natural rate at time 1 and 2. But he knows that the natural rate will remain constant at time 1 and 2, and that it will recover to its normal level ($\overline{r}^n$) at time 3. Endogenous variables π_t and i_t are observable.

Suppose that the econometrician observes at time 1 that $i_1 = 0$ and π_1. His problem is to estimate $\underline{r}^n$ and $e_{i,1}$. The existing approach typically computes the rational expectations equilibrium value of π_1 to express it as a function of $\underline{r}^n$ and $e_{i,1}$. In doing so, in order to keep linearity it abstracts from the zero bound. Firstly, i_1 is assumed to satisfy

$$i_1 = \underline{r}^n + e_{i,1} = 0. \tag{4}$$

Secondly, it is assumed that π_1 is expressed as

$$\begin{aligned} \pi_1 &= (\underline{r}^n - i_1) + \beta(\underline{r}^n - \widetilde{E}_1[i_2]) + \beta^2(\overline{r}^n - \widetilde{E}_1[i_3]) \\ &= \underline{r}^n - \beta\rho(\beta\rho + 1)e_{i,1}, \end{aligned} \tag{5}$$

where $\widetilde{E}_1$ denotes conditional expectations that abstract from the zero lower bound in the future periods. Namely,

$$\widetilde{E}_1[i_2] = \underline{r}^n + \rho e_{i,1}, \quad \widetilde{E}_1[i_3] = \overline{r}^n + \rho^2 e_{i,1}.$$

Equations (4) and (5) jointly determine the estimates of $\underline{r}^n$ and $e_{i,1}$.

However, there are two problems with this approach. Firstly, equation (4) is not necessarily correct when $i_1 = 0$ because the correct specification is

$$i_1 = \max\left[\underline{r}^n + e_{i,1}, 0\right]. \tag{6}$$

For example, suppose that the nominal rate became zero due to $\underline{r}^n < 0$. Use of (4) rather than (6) as one of the observation equations implies that, conditional on observing $i_1 = 0$, the econometrician may incorrectly infer that $e_{i,1}$ is higher than their true values (supposing $\underline{r}^n$ is correctly identified)[4]. Secondly, inflation expectation term $(\beta(\underline{r}^n - \widetilde{E}_1[i_2]) + \beta^2(\overline{r}^n - \widetilde{E}_1[i_3])$ in equation (5)) is not correctly expressed in terms of the underlying shocks $\underline{r}^n$ and $e_{i,1}$ because it abstracts from the zero lower bound. The correct conditional expectations that take the zero lower bound into account is given by[5]

$$E_1\left[i_2\right] = E_1 \max\left[\underline{r}^n + e_{i,2}, 0\right]. \tag{7}$$

[4] See, for example, Ireland (2011). Ireland (2011) estimates a New Keynesian model for the US economy but he does not explicitly takes the zero lower bound into account. He finds that monetary policy shocks of the US economy were positive after 2007 during which the federal funds rate is virtually zero.

[5] Regarding interest rate at period 3, since the zero lower bound will not be binding,

$$E_1\left[i_3\right] = E_1\left[\overline{r}^n + \rho^2 e_{i,1} + \rho\varepsilon_{i,2} + \varepsilon_{i,3}\right] = \overline{r}^n + \rho^2 e_{i,1}.$$

Therefore $\overline{r}^n - \widetilde{E}_1[i_3] = \overline{r}^n - E_1[i_3] = -\rho^2 e_{i,1}$.

Then it is obvious that

$$\widetilde{E}_1[i_2] < E_1\left[i_2\right].$$

Notice that π_1 can be expressed as

$$\pi_1 = (\underline{r}^n - i_1) + \beta(\underline{r}^n - E_1[i_2]) + \beta^2(\overline{r}^n - E_1[i_3]). \tag{8}$$

Comparing (5) and (8), one can notice that inflation expectations are overestimated in equation (5). As a result, ignoring the zero lower bound results in either underestimation of $\underline{r}^n$ or overestimation of $e_{i,1}$. An intuition is as follows. Inflation expectations decline when agents expect that the zero lower bound may bind in the future periods, lowering the current inflation. However, failure to take account of the future zero lower bound makes the econometrician to infer that low inflation in the current period is due to either lower natural rate or higher monetary policy shock.

For those two reasons, ignoring the zero lower bound may result in biased estimates of structural shocks. One solution is to take (7) explicitly, and solve and estimate the nonlinear model.[6] However those techniques often involve heavy computational burden. Here we propose a simpler algorithm that does not require heavy nonlinear techniques to solve rational expectations equilibrium. Suppose that the econometrician has data on forward rates. Unless risk premia are very large, those are good proxies for expectations about future interest rates $E_1\left[i_2\right]$ and $E_1\left[i_3\right]$. Then, by using equation (8) $\underline{r}^n$ can be directly identified without identifying monetary policy shock $e_{i,1}$. How about $e_{i,1}$? Notice that

$$E_1\left[i_2\right] = E_1 \max\left[\underline{r}^n + \rho e_{i,1} + \varepsilon_{i,2}, 0\right],$$

[6] For a recent contribution to this effort, see Fernandez-Villaverde *et al.* (2012). They nonlinearly solve the rational expectations equilibrium of a New Keynesian model subject to the zero lower bound.

and $e_{i,1}$ follows a normal distribution with mean zero and variance σ^2. Since we i_2 follows a truncated normal distribution, $e_{i,1}$ can be identified given the values of $\underline{r}^n$ and $E_1[i_2]$. Namely, since $E_1[i_2]$ is given by

$$E_1[i_2] = (\underline{r}^n + \rho e_{i,1}) \left\{ 1 - \Phi\left(-\frac{\underline{r}^n + \rho e_{i,1}}{\sigma} \right) \right\} + \sigma \phi \left(-\frac{\underline{r}^n + \rho e_{i,1}}{\sigma} \right), \tag{9}$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are respectively the cdf and pdf of the standard normal distribution, one can recover $e_{i,1}$ from (9). The next section builds a general framework. While we assumed that σ^2 is known in this section, in the next section we will estimate jointly σ^2 and monetary policy shock.

As is explained above, an advantage of our approach is that we can identify structural shocks without using nonlinear techniques to solve explicitly rational expectations equilibrium under the zero lower bound. This is possible because all the effects of the zero lower bound on the current endogenous variables are through the current interest rate and expectations about future interest rates, and the relationship between expected future interest rates and the current endogenous variables is linear.

3 Identification of structural shocks under the zero lower bound

In this section we set up the general identification strategy under the zero bound.

3.1 Step 1: Identification of structural shocks except monetary policy shock

Consider a linear rational expectations model:

$$A_0\hat{y}_t = A_1\hat{y}_{t+1|t}+A_2\hat{y}_{t-1}+A_3\hat{i}_t+A_4e_t+A_5, \tag{10}$$

$$e_t = \rho_e e_{t-1} + \varepsilon_{e,t}, \tag{11}$$

for $t = ..., -1, 0, 1, ...$ Here, $\hat{y}_t$ is n_y-vector of observable non-policy endogenous variables in period t; e_t and $\varepsilon_{e,t}$ are respectively n_e-vectors of structural shocks and their innovations in period t; $\hat{\imath}_t$ is the policy rate which is a short term nominal interest rate in the usual case. Variable $\hat{x}$ denotes a percentage deviation of x from its steady state value. Matrices A's consist of structural parameters. The policy rate satisfies[7]

$$\hat{i}_t = \max\{\hat{i}_t^{not}, -\overline{r}_t^n - \overline{\pi}\}, \tag{12}$$

$$\hat{i}_t^{not} = B_1\hat{y}_t+B_2\hat{i}_{t-1}^{not}+B_3e_t+B_4e_{i,t}+B_5 + B_6\varepsilon_{\overline{r},t}, \tag{13}$$

$$e_{i,t} = \rho_i e_{i,t-1} + \varepsilon_{i,t}, \tag{14}$$

where $\hat{i}_t^{not}$ is the notional policy rate which is given by monetary policy rule (13); $\overline{r}_t^n$ is the long-run natural rate of interest in period t; $e_{i,t}$ and $\varepsilon_{i,t}$ are the monetary policy shock and its innovation in period t; $\varepsilon_{\overline{r},t}$ is the innovation to long-run natural rate of interest in period t. While the notional policy rate i_t^{not} is negative policy rate i_t cannot be negative.

If there is no zero lower bound problem, $\hat{i}_t = \hat{i}_t^{not}$ for all periods and the system of

[7] Note that when the lower bound of the nominal interest rate is zero, the lower bound of its percentage deviation from its value in the steady state, $\hat{\imath}_t$, is given by $-\overline{r}_t^n - \overline{\pi}$.

equations (10), (11), (13), and (14) can be solved by linear solution techniques (e.g. Sims (2002)). The solution can be expressed as

$$\hat{y}_t = C_1\hat{y}_{t-1} + C_2\hat{i}_{t-1} + C_3 e_{t-1} + C_4 e_{i,t-1} + C_5\varepsilon_t + C_6. \tag{15}$$

where ε_t is the vector consisting of $\varepsilon_{e,t}, \varepsilon_{i,t}$ and $\varepsilon_{\bar{r},t}$.Under the presence of the zero lower bound, equation (15) is not a solution to rational expectations equilibrium because equation (12) is not taken into account. Consideration of (12) makes the system nonlinear and typically one needs nonlinear solution techniques to solve rational expectations equilibrium.

However, in what follows we argue that it is possible to identify shocks and estimate structural parameters without solving nonlinear rational expectations equilibrium explicitly. We notice that equation (10) always holds in equilibrium regardless of whether the zero lower bound on the nominal interest rate binds or not. Therefore, (10) also holds in j period ahead expectation as follows:

$$A_0\hat{y}_{t+j|t} = A_1\hat{y}_{t+j+1|t} + A_2\hat{y}_{t+j-1|t} + A_3\hat{i}_{t+j|t} + A_4 e_{t+j|t} + A_5. \tag{16}$$

We suppose that the policy rate will not be subject to the zero lower bound from far future date $t+\tau-1$ onwards almost surely even if the policy rate at period t is equal to zero. Choosing a sufficiently large value of τ ensures accuracy of solution.[8] This assumption implies that the zero lower bound problem is modeled as a transitory problem.[9] This assumption is imposed in the previous literature (Eggertsson and Woodford (2003), more

[8] When we estimate the model for the Japanese economy in Section 5, we choose $\tau = 80$ which corresponds the end point of the available forward rate curves.

[9] In that sense, we do not consider an economy that falls into a self-fulfilling liquidity trap analyzed by Benhabib *et al.* (2001). We assume that the zero lower bound problem arises due to temporary but large negative economic shocks.

references) to solve (nonlinear) rational expectations equilibrium under the zero lower bound. Then, expected policy rate at period $t+\tau$ can be expressed as

$$\hat{i}_{t+\tau|t} = B_1\hat{y}_{t+\tau|t}+B_2\hat{i}_{t+\tau-1|t} + B_3 e_{t+\tau|t}+B_4 e_{i,t+\tau|t}+B_5, \tag{17}$$

and the projection of non-policy endogenous variables at time $t+\tau$ $(\hat{y}_{t+\tau|t})$ can be obtained by using equation (15) as

$$\hat{y}_{t+\tau|t}= C_1\hat{y}_{t+\tau-1}+C_2\hat{i}_{t+\tau-1|t}+C_3 e_{t+\tau-1|t}+C_4 e_{i,t+\tau-1|t}+C_5\varepsilon_{t+\tau|t} + C_6. \tag{18}$$

We can stuck equations (10), (16) for $j = 1, ...\ t+\tau-1$, and (18) the following system of equations for $e_t, \hat{y}_{t+1|t}, ...\hat{y}_{t+\tau|t}$:

$$L_0 \begin{bmatrix} e_t \\ \hat{y}_{t+1|t} \\ \hat{y}_{t+2|t} \\ \vdots \\ \hat{y}_{t+\tau|t} \end{bmatrix} = L_1 \begin{bmatrix} \hat{y}_{t-1} \\ \hat{y}_t \\ \hat{\imath}_t \\ \vdots \\ \hat{\imath}_{t+\tau|t} \end{bmatrix} + L_2, \tag{19}$$

where matrices L_0, L_1, and L_2 consist of the structural parameters. In the left hand side of equation (19), e_t is coming from equation (10); $\left\{\hat{y}_{t+1+j|t}\right\}_{j=0}^{\tau-2}$ is from equation (16); and finally $\hat{y}_{t+\tau|t}$ is from (18). Also, we use equations (14) and (17) to eliminate $e_{i,t+\tau-1|t}$ from the right hand side of (19).[10] Equation (19) exploits the fact that all the effects of equation (12) is through the current and expected values of the nominal interest rate, and that, given $\left\{\hat{\imath}_{t+1+j|t}\right\}_{j=0}^{\tau-1}$, the system can be still written in linear form because the behavioral equations (equations (10) and (11)) are all linear. While the variables in

[10]That is why $\hat{\imath}_{t+\tau|t}$ shows up in the right hand side of equation (19)

the left hand side of equation (19) are not observable, those in the right hand side are observable since data on the forward rate curves are available. Equation (19) implies that as far as the economy escapes from the zero lower bound far in the future, structural shocks e_t and the expected values of endogenous variables $\{\hat{y}_{t+j|t}\}_{j=1}^{\tau}$ are linear functions of the observable variables. If $n_y = n_e$, L_0 can be invertible under plausible values of the structural parameters. Then the equation (19) can be written as

$$\begin{bmatrix} e_t \\ \hat{y}_{t+1|t} \\ \hat{y}_{t+2|t} \\ \vdots \\ \hat{y}_{t+\tau|t} \end{bmatrix} = J_1 \begin{bmatrix} \hat{y}_{t-1} \\ \hat{y}_t \\ \hat{\imath}_t \\ \vdots \\ \hat{\imath}_{t+\tau|t} \end{bmatrix} + J_2, \tag{20}$$

where $J_1 = L_0^{-1}L_1$ and $J_2 = L_0^{-1}L_2$. Equation (20) implies that e_t and $\{\hat{y}_{t+j|t}\}_{j=1}^{\tau}$ can be expressed in linear form in terms of observed values of $\hat{y}_t$, $\hat{y}_{t-1}$ and the forward rate curves $\{\hat{\imath}_t, \hat{\imath}_{t+1|t} \cdots, \hat{\imath}_{t+\tau|t}\}$. Therefore we can identify the values of structural shocks e_t by using equation (20).

3.2 Step 2: Identification of monetary policy shock

Step one identifies structural shock e_t without identifying monetary policy shock. In the next step, we identify monetary policy shock by using forward rate curves $\left\{i_t, i_{t+1|t} \cdots, i_{t+\tau|t}\right\}$. A j period ahead short term forward rate at period t can be expressed as expectation of the positive part of the notional policy rate at period $t+j$:

$$i_{t+j|t} \equiv E_t\left[i_{t+j}\right] \tag{21}$$

$$= E_t\left[\max\{i_{t+j}^{not}, 0\}\right]. \tag{22}$$

If the probability distribution of i^{not}_{t+j} can be approximated by normal distribution with mean $i^{not}_{t+j|t}$ and variance σ^2_j, we obtain

$$i_{t+j|t} = i^{not}_{t+j|t}\left(1 - \Phi\left(-\frac{i^{not}_{t+j|t}}{\sigma_j}\right)\right) + \sigma_j \phi\left(-\frac{i^{not}_{t+j|t}}{\sigma_j}\right). \tag{23}$$

where $\Phi(\cdot)$ is the standard normal cumulative density function and $\phi(\cdot)$ is the standard normal probability density function. Here we assume that variance σ^2_j is constant over time. It may not be obvious that it is the case, but in what follows it will be shown that under this assumption it is possible to identify monetary policy shock fairly accurately.

While $i_{t+j|t}$ can be observed from data on forward rate curves, $i^{not}_{t+j|t}$ is not observable. In addition, we have not identified σ_j as a function of structural parameters since we have not solved rational expectations equilibrium explicitly under the zero lower bound. Instead, our strategy is to estimate σ_j from data. Monetary policy rule (13) implies that $i^{not}_{t+j|t}$ consists of $y_{t+j|t}$, $e_{t+j|t}$, $i^{not}_{t+j-1|t}$, and $e_{i,t+j|t}$. In step 1, we have identified the values of $y_{t+j|t}$ and $e_{t+j|t}$. Therefore what remains to be identified is $\left\{e_{i,t+j|t,}\right\}^{\tau}_{j=1}$. Our strategy is to identify $\left\{e_{i,t+j|t,}\right\}^{\tau}_{j=1}$ and $\{\sigma_j\}^{\tau}_{j=1}$ by using forward rate curves $\left\{i_t, i_{t+1|t} \cdots, i_{t+\tau|t}\right\}$. Let t_0 be the first period when the zero lower bound becomes binding, and suppose that it is binding for $N+1$ periods. Then, under the assumption that $e_{i,t,}$ follows an $AR(1)$ process (equation (14)) and $\{\sigma_j\}^{\tau}_{j=1}$ is time invariant, unknown variables are $\{\sigma_j\}^{\tau}_{j=1}$ and $\{e_{i,t,}\}^{t_0+N}_{t=t_0}$ where the zero lower bound is binding from period t_0 until t_0+N. Since the number of unknown variables is $\tau + N + 1$ and the number of observable variables (i.e., forward rate curves) is $\tau \times (N+1)$, we can identify $\{\sigma_j\}^{\tau}_{j=1}$ and $\{e_{i,t,}\}^{t_0+N}_{t=t_0}$.[11] In Appendix A.2, we provide details of how to obtain $\{\sigma_j\}^{\tau}_{j=1}$ and $\{e_{i,t,}\}^{t_0+N}_{t=t_0}$ by using equation (23).

[11] Strictly, we need the condition $\tau N \geq N+1$ for the identification. We choose $\tau = 80$ so that the condition is satisfied.

3.3 Counterfactual simulation under the zero lower bound

It is also possible to conduct counterfactual simulation under the zero bound fairly easily. This solution method relies on $\{\sigma_j\}_{j=1}^{\tau}$ estimated in the previous Section.[12] From monetary policy rule (13), the term structure of the notional policy rate can be expressed as follows:

$$\begin{bmatrix} \hat{i}_t^{not} \\ \hat{i}_{t+1|t}^{not} \\ \hat{i}_{t+2|t}^{not} \\ \vdots \\ \hat{i}_{t+\tau|t}^{not} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -B_2 & 1 & 0 & \cdots & 0 \\ 0 & -B_2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & -B_2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} B_1\hat{y}_t + B_2\hat{i}_{t-1}^{not} + B_3 e_t + B_4 e_{i,t} + B_5 + B_6\varepsilon_{\bar{r},t} \\ B_1\hat{y}_{t+1|t} + B_3 e_{t+1|t} + B_4 e_{i,_{t+1|t}} + B_5 \\ B_1\hat{y}_{t+2|t} + B_3 e_{t+2|t} + B_4 e_{i,_{t+2|t}} + B_5 \\ \vdots \\ B_1\hat{y}_{t+\tau|t} + B_3 e_{t+\tau|t} + B_4 e_{i,_{t+\tau|t}} + B_5 \end{bmatrix} \tag{24}$$

Equation (19) can be rewritten as follows;

$$L_{01}e_t + L_{02}\begin{bmatrix} \hat{y}_{t+1|t} \\ \hat{y}_{t+2|t} \\ \vdots \\ \vdots \\ \hat{y}_{t+\tau|t} \end{bmatrix} = L_{11}\hat{y}_{t-1} + L_{12}\hat{y}_t + L_{13}\begin{bmatrix} \hat{i}_t \\ \hat{i}_{t+1|t} \\ \vdots \\ \vdots \\ \hat{i}_{t+\tau|t} \end{bmatrix} + L_2, \tag{25}$$

[12] Here it is possible to conduct counterfactual simulation only for alternative realization of shocks. Since changing the parameters of monetary policy rule can change $\{\sigma_j\}_{j=1}^{\tau}$, we believe it is not appropriate to conduct such exercises in our framework.

where L_{01} and L_{02} are submatrices of L_0, and L_{11},L_{12}, and L_{13} are submatrices of L_1. Rearranging equation (25), we obtain

$$\begin{bmatrix} \hat{y}_t \\ \hat{y}_{t+1|t} \\ \vdots \\ \vdots \\ \hat{y}_{t+\tau|t} \end{bmatrix} = M_{11}\hat{y}_{t-1} + M_{12}e_t + M_{13} \begin{bmatrix} \hat{i}_t \\ \hat{i}_{t+1|t} \\ \vdots \\ \vdots \\ \hat{i}_{t+\tau|t} \end{bmatrix} + M_2. \tag{26}$$

where $M_{11} = [\ -L_{12} \quad L_{02}\ ]^{-1}L_{11}$, $M_{12} = [\ L_{12} \quad -L_{02}\ ]^{-1}L_{01}$, $M_{13} = [\ -L_{12} \quad L_{02}\ ]^{-1}L_{13}$, and $M_2 = [\ -L_{12} \quad L_{02}\ ]^{-1}L_2$. Using equations (24) and (26), we obtain

$$\begin{bmatrix} \hat{i}^{not}_t \\ \hat{i}^{not}_{t+1|t} \\ \hat{i}^{not}_{t+2|t} \\ \vdots \\ \hat{i}^{not}_{t+\tau|t} \end{bmatrix} = \Psi_1\hat{i}^{not}_{t-1} + \Psi_2\hat{y}_{t-1} + \Psi_3 e_t + \Psi_4 e_{i,t} + \Psi_5 \begin{bmatrix} \max\{\hat{i}^{not}_t, -\overline{r}^n_t - \overline{\pi}\} \\ f\left(\hat{i}^{not}_{t+1|t}; \sigma_1, \overline{r}^n_t, \overline{\pi}\right) \\ f\left(\hat{i}^{not}_{t+2|t}; \sigma_2, \overline{r}^n_t, \overline{\pi}\right) \\ \vdots \\ f\left(\hat{i}^{not}_{t+\tau|t}; \sigma_\tau, \overline{r}^n_t, \overline{\pi}\right) \end{bmatrix} + \Psi_6\varepsilon_{\overline{r},t} + \Psi_7, \tag{27}$$

where $f(\cdot)$ is defined as

$$\begin{aligned} f\left(\hat{i}^{not}_{t+j|t}; \sigma_j, \overline{r}^n_t, \overline{\pi}\right) &\equiv \hat{i}_{t+j|t} \\ &\equiv \left(\hat{i}^{not}_{t+j|t} + \overline{r}^n_t + \overline{\pi}\right)\left[1 - \Phi\left(-\frac{\hat{i}^{not}_{t+j|t} + \overline{r}^n_t + \overline{\pi}}{\sigma_j}\right)\right] \\ &\quad +\sigma_j\phi\left(-\frac{\hat{i}^{not}_{t+j|t} + \overline{r}^n_t + \overline{\pi}}{\sigma_j}\right) - \overline{r}^n_t - \overline{\pi}, \end{aligned} \tag{28}$$

and Ψ_1, Ψ_2, Ψ_3, Ψ_4, Ψ_5, Ψ_6, Ψ_7 are defined in Appendix A.3. The solution can be derived by following three steps. Notice that, in equation (27), $\{\hat{i}^{not}_{t-1},\ \hat{y}_{t-1},\ e_t,\ e_{i,t}\ \varepsilon_{\bar{r},t}\}$ are exogenous or predetermined at time t. Therefore, given $\{\hat{i}^{not}_{t-1},\ \hat{y}_{t-1},\ e_t,\ e_{i,t}\ \varepsilon_{\bar{r},t}\}$, equation (27) represents a nonlinear equation for the term structure of notional policy rate $\{\hat{i}^{not}_t, \hat{i}^{not}_{t+1|t} \cdots, \hat{i}^{not}_{t+\tau|t}\}$ and can be solved numerically. Second, the term structure of policy rate $\{\hat{i}_t, \hat{i}_{t+1|t} \cdots, \hat{i}_{t+\tau|t}\}$ is derived using equation (28). Finally, endogenous variable $\hat{y}_t$ can be obtained by substituting the term structure of policy rate $\{\hat{i}_t, \hat{i}_{t+1|t} \cdots, \hat{i}_{t+\tau|t}\}$ into equation (26).

4 Example: Eggertsson-Woodford (2003) model

Here we use a simulated simple New Keynesian model to illustrate that our method can identify structural shocks better than the existing approaches that abstract from the zero lower bound. As in the previous sections, all the variables are expressed in terms of percentage deviations from their values at the steady state with zero inflation rate. The model is the one considered by Eggertsson and Woodford (2003). The Phillips curve is given by

$$\hat{\pi}_t = \beta E_t\left[\hat{\pi}_{t+1}\right] + \kappa \hat{x}_t + e_{\pi,t},$$

where $\hat{\pi}_t$, $\hat{x}_t$ and $e_{\pi,t}$ respectively denote inflation, the output gap and mark-up shock. The mark-up shock follows i.i.d. normal with mean zero and variance σ^2_π. The expectational IS curve is given by

$$\hat{x}_t = E_t\left[\hat{x}_{t+1}\right] - \frac{1}{\sigma}\left(\hat{i}_t - E_t\left[\hat{\pi}_{t+1}\right] - \hat{r}^n_t\right),$$

where $\hat{i}_t$ denotes nominal interest rate and $\hat{r}_t^n$ is the natural interest rate. Finally, monetary policy follows a Taylor-type policy rule subject to the zero lower bound

$$\hat{i}_t = \max\left[\hat{r}_t^n + \phi\hat{\pi}_t + e_{i,t}, -\overline{r}^n\right],$$

and monetary policy shock e_t follows

$$e_{i,t} = \rho e_{i,t-1} + \varepsilon_{i,t},$$

where $\varepsilon_{i,t}$ follows i.i.d. normal distribution with mean zero and variance σ_i^2. Finally, $\overline{r}^n$ is the level of the natural interest rate at the steady state. In this section we assume that the steady state value of inflation is zero ($\overline{\pi} = 0$). The evolution of the level of the natural interest rate is as follows. At time 1, it drops to -2%, and in subsequent periods, it recovers to 2% with probability α. With probability $1-\alpha$, it remains at -2%. Finally, at period 400, it recovers to 2% with probability 1. We choose the variance of monetary policy shock (σ_i^2) small enough so that the nominal interest rate is almost always equal to zero when $r_t^n = -2\%$. Therefore, in the simulated path, $i_t = 0$ whenever r_t^n is negative.

Eggertsson and Woodford (2003) show how to solve the rational expectations equilibrium of this model. We use their solution method to solve the model and generate artificial data. In order to focus on the biases of estimates of monetary policy shock and the natural rate, we set $e_{\pi,t} = 0$ at all times when generating artificial data. A typical simulated path is given in Figure 1.

Figure 1 here

In Figure 1, when the natural interest rate is negative the nominal interest rate hits the zero lower bound, and both the output gap and inflation are negative.[13] After the natural rate recovers to 2%, the nominal interest rate becomes positive. Figure 2 shows the forward rate curve (rational expectations equilibrium of $E_t\left[i_{t+j}\right]$).

Figure 2 here

Figure 2 shows that while the zero lower bound is binding ($t = 5, 20$) the forward rate curve is upward sloping. This is because the probability (conditional on time t) that the zero lower bound becomes unbinding at time $t + j$ is higher when j is larger. Once the zero lower bound indeed becomes unbinding ($t = 24$), the forward rate curve becomes flatter.

Figure 3 shows the identified structural shocks.

Figure 3 here

In Figure 3, the solid lines represent artificial true shocks. The circles represent estimated shocks. For comparison, we also plot by dashed lines the identified shocks when we use the traditional approach that abstracts from the zero lower bound. Our method identifies the natural rate, monetary policy shock, and mark-up shock almost accurately. However, abstracting from the zero lower bound results in a downward bias in the natural rate, an upward bias in monetary policy shock, or a downward bias in mark-up shock. The direction of those biases are in line with those shown in the simple example of Section 2. The reason for the downward bias of the mark-up shock is also similar.

[13]Since monetary policy shock is a persistent process, it affects inflation even if the current nominal interest rate is zero. This is because the nominal interest rate can become positive after the natural interest rate recovers to 2% some time in the future, and the agents expect that the level of the interest rate at that period is affected by the current level of monetary policy shock.

Possibility of binding zero lower bound lowers inflation expectations in rational expectations equilibrium, lowering the current inflation. However, ignoring the zero lower bound in future periods means that a decline in the current inflation is mistakenly be explained by negative structural shocks, such as a decline in mark-up shock or the natural rate shock, or a positive monetary policy shock.

5 Application to Japanese data

In the previous section we verified usefulness of our method by using artificial data. In this section we apply it to Japanese data. In order to capture properly the movements of the end point of the forward rate curve in each period that are observed in data, we model the level of the natural interest rate by the following stochastic process:

$$r_t^n = \overline{r}_t^n + \eta_t,$$

where

$$\overline{r}_t^n = \overline{r}_{t-1}^n + \varepsilon_{r,t},$$

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_{\eta,t}.$$

Here $\overline{r}_t^n$ is the long-run natural rate at time t, and η_t is the transitory deviation from it. We assume that the end point of the forward rate curve in each period converges to the long run value of the natural rate $\overline{r}_t^n$ plus the average inflation rate $\overline{\pi}$.

The model is given by

$$\hat{\pi}_t = \frac{\beta}{1+\alpha\beta} E_t\left[\hat{\pi}_{t+1}\right] + \frac{\alpha}{1+\alpha\beta} \hat{\pi}_{t-1} + \kappa \hat{x}_t + e_{\pi,t}, \tag{29}$$

$$\hat{x}_t = \gamma E_t\left[\hat{x}_{t+1}\right] + (1-\gamma)\hat{x}_{t-1} + \frac{1}{\sigma}\left(\hat{r}_t^n + E_t\left[\hat{\pi}_{t+1}\right] - \hat{i}_t\right), \tag{30}$$

$$\hat{i}_t = \max\left[\hat{i}_t^{not}, -\overline{\pi} - \overline{r}_t^n\right], \tag{31}$$

$$\hat{i}_t^{not} = \rho\hat{i}_{t-1}^{not} + (1-\rho)\left[\hat{r}_t^n + \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t\right] + e_{i,t} - \rho\varepsilon_{r,t}, \tag{32}$$

where the nominal interest rate, the natural rate and inflation are normalized as

$$\hat{i}_t \equiv i_t - \overline{\pi} - \overline{r}_t^n, \quad \hat{r}_t^n \equiv r_t^n - \overline{r}_t^n = \eta_t, \quad \hat{\pi}_t = \pi_t - \overline{\pi}_t.$$

Now the Phillips curve (29) and expectational IS curve have backward looking terms ($\hat{\pi}_{t-1}$ and $\hat{x}_{t-1}$). Also, the notional interest rate $\hat{i}_t^{not}$ is given by a Taylor-type interest rate with smoothing (equation (32)). Here $\overline{\pi}$ denotes long-run inflation rate, which we calibrate by the average inflation rate from 1981 to 2011. The structural shocks are markup shocks ($e_{\pi,t}$), the natural rate ($\hat{r}_t^n$), and monetary policy shock ($e_{i.t}$). The other shocks, $e_{\pi,t}$ and $e_{i,t}$, follow AR(1) processes

$$e_{\pi,t} = \rho_u e_{\pi,t-1} + \varepsilon_{\pi,t},$$

$$e_{i,t} = \rho e_{i,t-1} + \varepsilon_{e,t}.$$

We assumed that all the innovations follow i.i.d. normal distribution.

The data used in estimation are the output gap, inflation rate, collateralized overnight call rate and the forward rate curves. The output gap is calculated by applying the Hodrick-Prescott filter to real GDP.[14] Inflation rate is quarterly changes in the consumer price index less fresh food, which is adjusted so as to exclude the effects of the changes

[14] Our GDP measure is Real GDP in "National Accounts" published by the Cabinet Office.

in the consumption tax rate and the subsidy for high school tuition.[15] The forward rate curves for the Japanese Government Bond are calculated from the yield to maturity for the Japanese Government Bond published by the Ministry of Finance. We compute the forward rate curves by using the P-spline approach proposed by Jarrow *et al.* (2004). Details of the calculation of the forward rate curves are provided in Appendix A.1. In estimation, we use one quarter to 20 year ahead instantaneous forward rates. Due to data limitation, when identifying the structural shocks in the 1980s we use the forward rate curves only up to 9 years. The long-run natural rate, $\overline{r}_t^n$, is assumed to be equal to the end point of the forward rate curve minus the average inflation rate $\overline{\pi}$.

In addition to identifying structural shocks, we also estimate structural parameters by Bayesian methods as in Cogley *et al.* (2010) to compute the posterior distribution of the model's structural parameters. While the method described in Section 3 works very well for artificial data generated by Eggertsson and Woodford (2003)'s model, we need an adjustment to apply it to actual data. As is explained in Section 3, we identify structural shocks by two steps. Step 1 identifies structural shocks other than monetary policy shocks by using forward rate curves and step 2 identifies monetary policy shocks. Since the forward rates in reality include risk premium, identified shock e_t can be contaminated by risk premium. In our setting, risk premium can be interpreted as the deviation of the forward rate in data from the right hand side of equation (23). Therefore, we need to compute the forward rates that are consistent with equation (23) in order to obtain model-consistent e_t. To accomplish this objective, we add one more step in the above identification procedure. The third step is to compute updated values of the forward rates using equation (23) given identified shocks e_t and $e_{i,t}$. After deriving new forward rates based on the equation (23), we go back and follow the first and second steps and

[15] Consumer Price Index (CPI) is published by the Ministry of Internal Affairs and Communications.

obtain new identified shocks e_t and $e_{i,t}$ by using the new forward rate curves, which are not necessarily equal to the forward rate curves observed in data. Starting from the observed forward rate curves, we update the identified shocks and the forward rate curves without risk premium until these values are mutually consistent. When the identified shocks converge by iterating these three steps, they are consistent with the forward rate curves without risk premium defined by equation (23). Indeed we find that convergence is fairly quick. In this procedure, the observed forward rate curves are used as a good initial guess since those forward rate curves enable us to find the initial guesses of $\{\sigma_j\}_{j=1}^{\tau}$ and $\left\{i_{t+j|t}^{not}\right\}_{j=1}^{\tau}$.

5.1 Identified shocks and deflation in Japan

Table 1 reports estimation results.

Table 1 here

We set prior distributions following Benati and Surico (2009) who estimate a simple New Keynesian model similar model to ours. Regarding the parameters on monetary policy rule (32), we follow Cogley *et al.* (2010). The variance of the long-run natural rate shock, $\sigma_{\bar{r}}^2$, is calibrated to the variance of changes in the end point of the forward rate curve. Table 1 compares the estimation results obtained by our method (labeled "W/ZLB") and those obtained by the existing approach that abstracts from the zero lower bound (labeled "W/O ZLB"). Comparing those two, median estimates of the parameters are fairly similar except two persistence parameters α and ρ_i. Rather, the two approaches differ in identified shocks, which are shown in Figure 4(1) - 4(3). Figure 4(1) shows estimated paths of the natural rate.

Figure 4 here

For comparison, we also show with a thin line the estimated natural rate when we abstract from the zero lower bound. Comparing those two estimates, they are quite similar before 1992 during which the nominal interest rates were above some 3%, but differed from each other when the nominal interest rates became extremely low afterwards. Our estimation result suggests that the natural rate has declined since the early 1990s, but the decline has not been as large as the previous literature finds. The estimated natural rate has remained at some 2% in most periods since the 1990s. The direction of the bias caused by ignoring the zero lower bound is in line with the numerical example in Section 4. The Japanese economy was characterized by prolong deflation after the late 1990s. One of the common explanations has been that the natural rate has become negative. Oda and Muranaga (2003) is one of the earliest attempts to estimate the natural interest rate for the Japanese economy by using the method of Laubach and Williams (2003). They report that the natural rate declined substantially after 1990 and became negative for most periods after 1997. Iwamura *et al.* (2005) also estimate the natural interest rate by applying a Kalman Filter to a small structural model similar to ours but do not take into account explicitly the zero lower bound. They report a similar result to Oda and Muranaga. However, our finding suggests that the magnitude of the decline may have been overemphasized. Kamada (2009) compared various methods of estimating the natural interest rates their results. He concludes that although the natural rate indeed became negative in 1997 and 1998, the level was at most -1% and its duration was not very long.

Figure 4(2) shows estimated markup shocks. Markup shocks have been positive in most sample periods. In contrast, the identified markup shocks when we abstract from the zero lower bound are negative after 1994. It has been argued that negative markup shocks have been contributing to deflation in the Japanese economy since the late 1990s (See, for example, Saito *et al.* (2012) and Leigh (2009)).[16] Our result suggests that, once the zero lower bound is properly treated in estimation, the deflationary effects of markup shocks are found to be not very large. Again, the direction of the bias is in line with the result of Section 4.

Figure 4(3) shows estimated monetary policy shocks. The estimated shocks are negative in most periods after 1991. Notice that the nominal interest rates have been zero or near zero in since the mid 1990s. Even though monetary policy shocks may not change the observed levels of the nominal interest rate when the zero lower bound is binding, they can be interpreted as representing monetary policy stance which is not captured by the observed levels of the policy rate (which is zero). As long as monetary policy shock is a persistent process, its current innovation affects future policy rates even after the economy escapes from the zero lower bound. Therefore, a negative and persistent monetary policy shock in our model can represent a commitment to expansionary monetary policy even after the zero lower bound stops binding. Also, the effects of other unconventional monetary policy measures such as quantitative easing that flatten the yield curve can be captured in our framework by persistent monetary policy shocks in the notional interest rate rule when the zero lower bound is binding. Flattening of forward rate curves lowers long term interest rates, and lower long rates have expansionary effects on the current economic activity. Therefore, identified negative monetary policy shocks in Figure 4(3) indicate that the policy of the Bank of Japan during those periods have had expansionary

[16] On the other hand, Saito *et al.* (2012) also report that declines in the natural interest rate were not very significant and remained positive.

effects. This is in contrast to Ireland (2011). Ireland (2011) estimates a New Keynesian model for the US economy but he does not explicitly takes the zero lower bound into account when he estimates the model and identifies structural shocks. He finds that monetary policy shocks of the US economy were positive after 2007 when the Federal Funds Rate hits its lower bound. He interprets this result as representing contractionary effects of the zero lower bound. However, monetary policy shocks he identifies does not necessarily represents policy stance of the Fed who used various types of unconventional policies to stimulate aggregate demand. Figure 4(3) also shows by thin line the identified monetary policy shocks when the zero lower bound is ignored. It is shown that the identified shocks in that case are positive after 1996, which is in line with the finding of Ireland (2011). This indicates that his results can be subject to bias that is caused by ignoring the zero lower bound in estimation.

5.2 Asymmetric effects of shocks under the zero lower bound

In linear rational expectations models, impulse responses to a positive and negative shock of the same kind are symmetric and are not dependent on the initial state of the model economy. However, when the economy is subject to the zero lower bound on the nominal interest rate, it may respond asymmetrically to positive and negative shocks.

Figure 5 shows impulse response functions to the three shocks. Due to the zero lower bound the economy's responses to those shocks become nonlinear, and hence the impulse responses depend on the initial state of the economy. Here we chose the initial values of the state variables equal to those of 2003Q2 when the notional interest rate was the lowest. We chose those initial values because it is of interest to see the effects of the zero lower bound on the propagation of the shocks when monetary policy is severely constrained by the zero lower bound.

Black thick lines represent the responses of the economy to positive shocks while grey lines represent the responses to negative shocks.[17]

Figure 5(1) here

For comparison, black thin lines represent impulse response functions in absence of the zero lower bound. Because the model without the zero lower bound is linear, responses to positive and negative shocks are symmetric. Figure 5(1) shows that the zero lower bound makes the economy's responses asymmetric. In the presence of the zero lower bound, shocks affect the endogenous variables through two channels. The first channel is the standard mean effect, which is shown by the black thin lines (the responses without the zero lower bound). The second channel is shocks' effects on the probability of hitting the zero lower bound in the current and subsequent periods. This is because the zero lower bound prevents monetary policy from responding sufficiently to those negative shocks. This implies that the zero lower bound itself can generate downward pressure on the output gap and inflation. Because of the second channel, the economy responds by more to negative shocks than positive shocks. In a different context, Benhabib *et al.* (2001) argue that the zero lower bound creates possibility of self-fulfilling deflationary equilibria. This also stems from inability of the central bank to respond sufficiently to deflationary pressure when the zero lower bound is binding. In the present paper, we do not focus on deflationary equilibria that are driven by self-fulfilling deflationary expectations. Instead, our impulse responses show that due to the presence of the zero lower bound contractionary effects of negative shocks are stronger than expansionary

[17] When negative shocks happen, inflation, the output gap and yield curves become lower, i.e., take negative values. In order to make comparison clearer, in Figure 5 we plot the absolute values of those responses.

effects of positive shocks, creating deflationary pressure.

Figure 5(1) also shows that the economy responds to positive shocks by more in the presence of the zero lower bound than in the absence of it. This is again because of the second channel mentioned above. A positive shock reduces the probability of hitting the zero lower bound in subsequent periods and this fact removes deflationary pressure. As a result, inflation and output gap expands by more.

To see this channel further, Figure 5(2) shows the impulse responses with the initial values of the state variables equal to those of 1992Q2.

Figure 5(2) here

At that time, the nominal interest rates were much higher than those in 2003. Compared with Figure 5(1), Figure 5(2) shows that the differences between the responses to positive shocks and those to negative shocks are smaller.[18] This is because when the nominal interest rates are higher the possibility of hitting the zero bound in subsequent periods are smaller. This implies that the second channel is weaker when the nominal rates are higher.

Finally, the impulse responses of the nominal forward rate curves shown in Figure 5(1) and 5(2) are in line with some empirical findings. For example, Gurkaynak *et al.* (2005) report that the peak response of forward rates to economic shocks occurs at about three years ahead, and that the peak response to monetary policy surprises occurs at one year ahead.[19] Since their estimation period is not subject to the zero lower bound, we compare their results with Figure 5(2) where the zero lower bound was not a serious

[18] By construction, the responses without zero lower bound (black thin lines) are identical between Figure 5(1) and 5(2).

[19] See Table 1 and Figure 2 Gurkaynak *et al.* (2005).

constraint to the Japanese economy. Figure 5(2) shows that the peak response of the forward rate curves to natural rate shocks occur at about three years ahead and the peak response to monetary policy shock occurs at one year ahead. Our findings are in line with their finding.[20] How about the effects of the zero lower bound? A recent paper by Swanson and Williams (2012) analyze how the responses of nominal yield curves of the US economy to economic news are affected by the zero lower bound. They report that the responses of the yields less than six months to economic news became smaller due to the presence of the zero lower bound, and that the responses of longer yields are less affected. This finding is in line with our finding. In Figure 5(1), while responses of the forward rates at shorter horizon (less than two years ahead) become smaller under the presence of the zero lower bound, the responses at longer horizon became larger. This means that the responses of the yield curves of our model economy (implied by the responses of the forward rate curves in Figure 5(1)) is in line with the finding of Swanson and Williams (2012).

5.3 Counterfactual simulation: how severe was the zero lower bound constraint?

One of the common hypotheses for the Japanese deflation in the 1990s and 2000s was that the natural interest rate and markup shocks declined to negative values. However, the previous section implies that those shocks were not as weak as the previous literature found. Then what caused deflation? In this section we conduct several counterfactual simulation to answer to this question. The results are shown in Figures 6 (inflation) and 7 (the output gap). In each figure, we conduct counterfactual simulations in which (i)

[20] Note that Gurkaynak *et al.* (2005) studies the US economy. A more extensive empirical study on the Japanese forward rate curves is left for future research.

no markup shock; (ii) no natural rate shock; (iii) no monetary policy shock; and (iv) no zero lower bound.

Figure 6 here

Figure 7 here

Figures 6 shows that the natural rate shock and the zero lower bound are the two main sources of deflationary pressure. Deflationary effects of declines in the natural interest rate are shown to be very strong for the entire sample period except around 2006-2007. Figure 6 also shows that, without the zero lower bound, inflation would have had remained positive despite those declines in the natural interest rate.[21] Figure 7 shows that those two shocks are also the main drivers of fluctuations in the output gap. In Section 5.2, we note that the deflationary effects of negative shocks are larger than inflationary effects of positive shocks under the presence of the zero lower bound. Therefore, we conclude that the decline in the natural interest rate, even though its level remained positive, contributed to deflation because of the presence of the zero lower bound.

Compared with the natural rate shock, markup shock does not contribute much to deflation. Figure 6 and 7 also show that monetary policy shocks had expansionary effects. Without monetary policy shocks, the economy would have had resulted in severer deflation and negative output gap. We discussed in Section 5.1 that negative monetary policy shocks under the binding zero lower bound can be interpreted as expansionary effects of various unconventional monetary policy measures such as a commitment to the

[21] The effect of the zero lower bound on inflation is found to be large even in the early 1990s during which the nominal interest rates still remained high. This is because the negative natural rate shocks in the early 1990s were found to be very large. As is explained above, the responses of the model economy subject to the zero lower bound to negative shocks are larger than those without the zero lower bound. And this difference is larger when shocks are larger even when the nominal interest rate remains positive.

zero rate in the future and quantitative easing. Figure 6 shows that those policy measures had indeed expansionary effects. Those expansionary effects were through flattening of the yield curve.

6 Conclusion

This paper develops a tractable method to estimate forward-looking monetary models with the zero lower bound on the nominal interest rate. The method builds on the idea that all nonlinearity due to the presence of the zero lower bound can be summarized by expectations about the future policy rates (forward rate curves). Since it is not necessary to solve entire models nonlinearly, our method can be applied to a wider variety of medium size DSGE models.

By using a simple New Keynesian model, we show that ignoring the zero lower bound may cause biases in estimated shocks. Applying to the Japanese data, we show that the natural interest rate might not have declined to negative values in the 1990s and 2000s, in contrast to the previous literature. However, the decline in the natural interest rate indeed contributed to the Japanese deflation during those periods due to the presence of the zero lower bound on the nominal interest rate. However, because of its simplicity, the model may not necessarily capture all the propagation mechanism to identify shocks with sufficient accuracy. Estimation of a larger and more realistic DSGE model for the Japanese economy under the zero lower bound is left for future research.

A Appendix

A.1 Construction of the forward rate curves

We construct the forward rate curves from the data on the yield to maturity of the Japanese Government Bonds. Those are daily data published by the Ministry of Finance. The available maturities are 1 to 10 years and 15, 20, 25, 30, and 40 years. We use maturities up to 20 years because those over 20 years are available only after the 2000s. In addition, the maturities over 9 years are not available in the early 1980s. For this period, we estimate the forward rate curves up to 9 years. We also use the collateralized overnight call rates as starting points of the forward rate curves.

To estimate the forward rate curves, we calculate the coupon bond prices by using yield to maturity and coupon rates, and then fit cubic P-spline as in Jarrow *et al.* (2004). Specifically, we solve the following problem to find the parameters of cubic P-spline, δ , in each period;

$$\min_{\delta}\left[\frac{1}{N}\sum\nolimits_{i=1}^{N}\left\{P_i-\sum_{j=1}^{z_i}C\left(t_{i,j}\right)\exp\left\{-\delta' B^I\left(t_{i,j}\right)\right\}\right\}^2+\lambda\delta' G\delta\right], \tag{A.1}$$

where P_i is the price of the Japanese Government Bonds with maturity T_i; z_i is the number of coupon payments; $C\left(t_{i,j}\right)$ is coupon payment at $t_{i,j}$; and $B^I$is given by

$$B^I\left(t_{i,j}\right)\equiv\int_0^{t_{i,j}}B\left(s\right)ds, \tag{A.2}$$

where $B\left(s\right)$ is truncated power basis function as in Jarrow *et al.* (2004). Finally, G is defined as

$$G\equiv\int_0^{T_N}B'\left(s\right)B'\left(s\right)' ds. \tag{A.3}$$

The second term in equation (A.1) is a penalty function for smoothness. Parameter λ controls the shape of the spline. There are several methods proposed in literature to determine λ (e.g. generalized cross validation in Fisher *et al.* (1995), and empirical bias bandwidth selection in Jarrow *et al.* (2004)). Following Lukas *et al.* (2011), we use the robust generalized cross validation method to determine λ. We set robustness parameter γ equal to 0.3 since this value is recommended in Lukas *et al.* (2011). Following the robust generalized cross validation criteria, the value of λ is determined. The selected forward rate curves are shown in Figure 8.

Figure 8 here

A.2 Identification of monetary policy shock

Here we provide details of our identification procedure of monetary policy shock. In order to compute $\{\sigma_j\}_{j=1}^{\tau}$ and $\{e_{i,t,}\}_{t=t_0}^{t_0+N}$, we use equation (23). Given a certain guess of $\{\widetilde{\sigma}_j\}_{j=1}^{\tau}$, we take a first order approximation of equation (23) around $\{\widetilde{i}_{t+j|t}^{not}, \widetilde{\sigma}_j\}$ as

$$i_{t+j|t} \approxeq \widetilde{i}_{t+j|t}^{not}\left[1-\Phi\left(-\frac{\widetilde{i}_{t+j|t}^{not}}{\widetilde{\sigma}_j}\right)\right]+\left[1-\Phi\left(-\frac{\widetilde{i}_{t+j|t}^{not}}{\widetilde{\sigma}_j}\right)\right]\left(i_{t+j|t}^{not}-\widetilde{i}_{t+j|t}^{not}\right)+\sigma_j\phi\left(-\frac{\widetilde{i}_{t+j|t}^{not}}{\widetilde{\sigma}_j}\right)+u_{j,t}, \tag{A.4}$$

where $\widetilde{i}_{t+j|t}^{not}$ is the value of $i_{t+j|t}^{not}$ when innovation in monetary policy shock ($\varepsilon_{i,t}$) is set equal to zero.[22] Then $i_{t+j|t}^{not} - \widetilde{i}_{t+j|t}^{not}$ can be expressed as a function of $\{\varepsilon_{i,t,}\}_{t=t_0}^{t_0+N}$. Finally,

[22] Regarding $\{\widetilde{\sigma}_j\}_{j=1}^{\tau}$, we use 1% as initial guess.

$u_{j,t}$ is an approximation error. In matrix form, equation (A.4) can be expressed as

$$\begin{bmatrix} i_{t_0+1|t_0} & \cdots & i_{t_0+\tau|t_0} \\ \vdots & \ddots & \vdots \\ i_{t_0+N+1|t_0+N} & \cdots & i_{t_0+N+\tau|t_0+N} \end{bmatrix} = \Lambda_0 + \sum_{j=1}^{N+1} \Lambda_{1j}\varepsilon_{i,t_0+j-1} + \sum_{s=1}^{\tau} \Lambda_{2s}\sigma_s + u, \tag{A.5}$$

where Λ_0, Λ_{1j}, and Λ_{2s} are given by

$$\Lambda_0 = \begin{bmatrix} \widetilde{i}_{t_0+1|t_0}\left(1-\Phi\left(-\frac{\widetilde{i}^{not}_{t_0+1|t_0}}{\widetilde{\sigma_1}}\right)\right) & \cdots & \widetilde{i}_{t_0+\tau|t_0}\left(1-\Phi\left(-\frac{\widetilde{i}^{not}_{t_0+\tau|t_0}}{\widetilde{\sigma_\tau}}\right)\right) \\ \vdots & \ddots & \vdots \\ \widetilde{i}_{t_0+N+1|t_0+N}\left(1-\Phi\left(-\frac{\widetilde{i}^{not}_{t_0+N+1|t_0+N}}{\widetilde{\sigma_1}}\right)\right) & \cdots & \widetilde{i}_{t_0+N+\tau|t_0+N}\left(1-\Phi\left(-\frac{\widetilde{i}^{not}_{t_0+N+\tau|t_0+N}}{\widetilde{\sigma_\tau}}\right)\right) \end{bmatrix}, \tag{A.6}$$

$$\Lambda_{1j} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ \left(1-\Phi\left(-\frac{\widetilde{i}^{not}_{t_0+1|t_0}}{\widetilde{\sigma_1}}\right)\right)\frac{\rho^2-\rho_i^2}{\rho-\rho_i} & \cdots & \left(1-\Phi\left(-\frac{\widetilde{i}^{not}_{t_0+1|t_0}}{\widetilde{\sigma_\tau}}\right)\right)\frac{\rho^{\tau+1}-\rho_i^{\tau+1}}{\rho-\rho_i} \\ \vdots & \ddots & \vdots \\ \left(1-\Phi\left(-\frac{\widetilde{i}^{not}_{t_0+N+1|t_0+N}}{\widetilde{\sigma_1}}\right)\right)\frac{\rho^{N-j+2}-\rho_i^{N-j+2}}{\rho-\rho_i} & \cdots & \left(1-\Phi\left(-\frac{\widetilde{i}^{not}_{t_0+N+\tau|t_0+N}}{\widetilde{\sigma_\tau}}\right)\right)\frac{\rho^{N-j+\tau+1}-\rho_i^{N-j+\tau+1}}{\rho-\rho_i} \end{bmatrix}, \tag{A.7}$$

$$\Lambda_{2s} = \begin{bmatrix} 0 & \cdots & 0 & \phi\left(-\frac{\widetilde{i}^{not}_{t_0+s|t_0}}{\widetilde{\sigma}_s}\right) & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \phi\left(-\frac{\widetilde{i}^{not}_{t_0+N+s|t_0+N}}{\widetilde{\sigma}_s}\right) & 0 & \cdots & 0 \end{bmatrix}. \tag{A.8}$$

Finally, u is a vector of approximation errors. Λ_{1j}'s columns from the first to $(j-1)$th are zero vectors since ε_{i,t_0+j-1} doesn't have any influence on the past forward rate curves.

Given an initial guess $\{\widetilde{\sigma}_j\}_{j=1}^{\tau}$ (which is in Λ_{0j}, Λ_{1j}, Λ_{2j}), we compute an update of

$\{\sigma_j\}_{j=1}^{\tau}$ by regressing the following equation

$$\begin{bmatrix} i_{t_0+1|t_0} & \cdots & i_{t_0+\tau|t_0} \\ \vdots & \ddots & \vdots \\ i_{t_0+N+1|t_0+N} & \cdots & i_{t_0+N+\tau|t_0+N} \end{bmatrix} = \Lambda_0 + \sum_{s=1}^{\tau} \Lambda_{2s}\sigma_s + \widetilde{u}, \tag{A.9}$$

where $\widetilde{u}$ includes the second term $(\sum_{j=1}^{N+1} \Lambda_{1j}\varepsilon_{i,t_0+j-1})$ in the right hand side of equation (A.5). We use equation (A.9) to obtain updates of $\{\sigma_j\}_{j=1}^{\tau}$ in order to avoid the over-fitting problem.[23] Starting from initial guess $\{\widetilde{\sigma}_j\}_{j=1}^{\tau}$, we obtain estimate of $\{\sigma_j\}_{j=1}^{\tau}$ by repeating the ordinary least squares estimation of (A.9) until those estimates converge. After obtaining $\{\sigma_j\}_{j=1}^{\tau}$, we identify $e_{i,t+j}$ by using the following equation

$$\begin{bmatrix} i_{t_0+j+1|t_0+j} \\ \vdots \\ i_{t_0+j+\tau|t_0+j} \end{bmatrix} = \begin{bmatrix} \widetilde{\widetilde{i}}_{t_0+j+1|t_0+j}\left(1-\Phi\left(-\frac{\widetilde{\widetilde{i}}^{not}_{t_0+j+1|t_0+j}}{\sigma_1}\right)\right) \\ \vdots \\ \widetilde{\widetilde{i}}_{t_0+j+\tau|t_0+j}\left(1-\Phi\left(-\frac{\widetilde{\widetilde{i}}^{not}_{t_0+j+\tau|t_0+j}}{\sigma_\tau}\right)\right) \end{bmatrix} + \begin{bmatrix} \sigma_1\phi\left(-\frac{\widetilde{\widetilde{i}}^{not}_{t_0+j+1|t_0+j}}{\sigma_1}\right) \\ \vdots \\ \sigma_\tau\phi\left(-\frac{\widetilde{\widetilde{i}}^{not}_{t_0+j+\tau|t_0+j}}{\sigma_\tau}\right) \end{bmatrix}$$
$$+ \begin{bmatrix} \left(1-\Phi\left(-\frac{\widetilde{\widetilde{i}}^{not}_{t_0+j+1|t_0+j}}{\sigma_1}\right)\right)\frac{\rho^2-\rho_i^2}{\rho-\rho_i} \\ \vdots \\ \left(1-\Phi\left(-\frac{\widetilde{\widetilde{i}}^{not}_{t_0+j+\tau|t_0+j}}{\sigma_\tau}\right)\right)\frac{\rho^{\tau+1}-\rho_i^{\tau+1}}{\rho-\rho_i} \end{bmatrix} \varepsilon_{i,t_0+j} + \widetilde{\widetilde{u}}, \tag{A.10}$$

where $\widetilde{\widetilde{i}}_{t_0+j+s|t_0+j}$ is the value of $i^{not}_{t_0+j+s|t_0+j}$ when monetary policy shock ε_{i,t_0+j} is set equal to zero and $\widetilde{\widetilde{u}}$ is an approximation error.

[23] At some trials we experience that the estimate of $\{\sigma_j\}_{j=1}^{\tau}$ tend to be negative if we estimate $\{\sigma_j\}_{j=1}^{\tau}$ and $\{\varepsilon_{i,t,}\}_{t=t_0}^{t_0+N}$ simultaneously. Using (A.9) can avoid this problem.

A.3 Matrices used in Section 3.3

Define Ξ by

$$\Xi \equiv \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -B_2 & 1 & 0 & \cdots & 0 \\ 0 & -B_2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & -B_2 & 1 \end{bmatrix}^{-1} . \tag{A.11}$$

Then, Ψ_1, Ψ_2, Ψ_3, Ψ_4, Ψ_5, Ψ_6, Ψ_7 used in equation (27) are defined by

$$\Psi_1 \equiv \Xi \begin{bmatrix} B_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{A.12}$$

$$\Psi_2 \equiv \Xi \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & B_1 \end{bmatrix} M_{11}, \tag{A.13}$$

$$\Psi_3 \equiv \Xi \begin{bmatrix} B_3 & 0 & \cdots & 0 \\ 0 & B_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & B_3 \end{bmatrix} \begin{bmatrix} I \\ \rho_e \\ \vdots \\ \rho_e^{\tau} \end{bmatrix} + \Xi \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & B_1 \end{bmatrix} M_{12}, \tag{A.14}$$

$$\Psi_4 \equiv \Xi \begin{bmatrix} B_4 & 0 & \cdots & 0 \\ 0 & B_4 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & B_4 \end{bmatrix} \begin{bmatrix} 1 \\ \rho_i \\ \vdots \\ \rho_i^{\tau} \end{bmatrix}, \tag{A.15}$$

$$\Psi_5 \equiv \Xi \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & B_1 \end{bmatrix} M_{13}, \tag{A.16}$$

$$\Psi_6 \equiv \Xi \begin{bmatrix} B_6 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{A.17}$$

$$\Psi_7 \equiv \Xi \begin{bmatrix} B_5 \\ B_5 \\ \vdots \\ B_5 \end{bmatrix} + \Xi \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & B_1 \end{bmatrix} M_2. \tag{A.18}$$

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Table 1 – Bayesian Estimates of the Structural Parameters

Parameter	Density	Prior distribution		Posterior distribution					
				W/O ZLB			W/ ZLB		
		Mode	Standard deviation	Median	25^{th} pct	75^{th} pct	Median	25^{th} pct	75^{th} pct
β	Calibrated	0.99	—		—			—	
$\sigma_{\bar{r}}^2$	Calibrated	0.39	—		—			—	
σ_π^2	Inverse Gamma	0.50	0.50	0.098	0.090	0.107	0.099	0.091	0.108
σ_g^2	Inverse Gamma	5.00	0.25	4.135	3.924	4.363	4.116	3.910	4.346
σ_i^2	Inverse Gamma	0.50	0.50	0.117	0.106	0.130	0.117	0.107	0.129
κ	Gamma	0.05	0.01	0.042	0.036	0.048	0.042	0.037	0.048
σ	Gamma	8.00	1.00	7.548	6.936	8.208	7.867	7.239	8.538
α	Beta	0.75	0.20	0.277	0.192	0.370	0.431	0.313	0.546
γ	Beta	0.25	0.20	0.803	0.751	0.855	0.825	0.771	0.876
ρ	Beta	0.75	0.20	0.982	0.976	0.987	0.978	0.971	0.982
ϕ_π	Normal	1.70	0.30	1.900	1.736	2.073	1.983	1.849	2.115
ϕ_g	Gamma	0.30	0.20	0.188	0.124	0.269	0.112	0.070	0.178
ρ_i	Beta	0.25	0.20	0.175	0.108	0.239	0.458	0.410	0.517
ρ_π	Beta	0.25	0.20	0.948	0.928	0.964	0.962	0.937	0.980
ρ_g	Beta	0.25	0.20	0.998	0.996	0.998	0.998	0.997	0.999
Fraction of accepted draws					0.241			0.254	

Figure 1 : a typical simulated path

	β	κ	σ	φ	ρ	α
parameter value	0.99	0.02	5	1.5	0.7	0.1

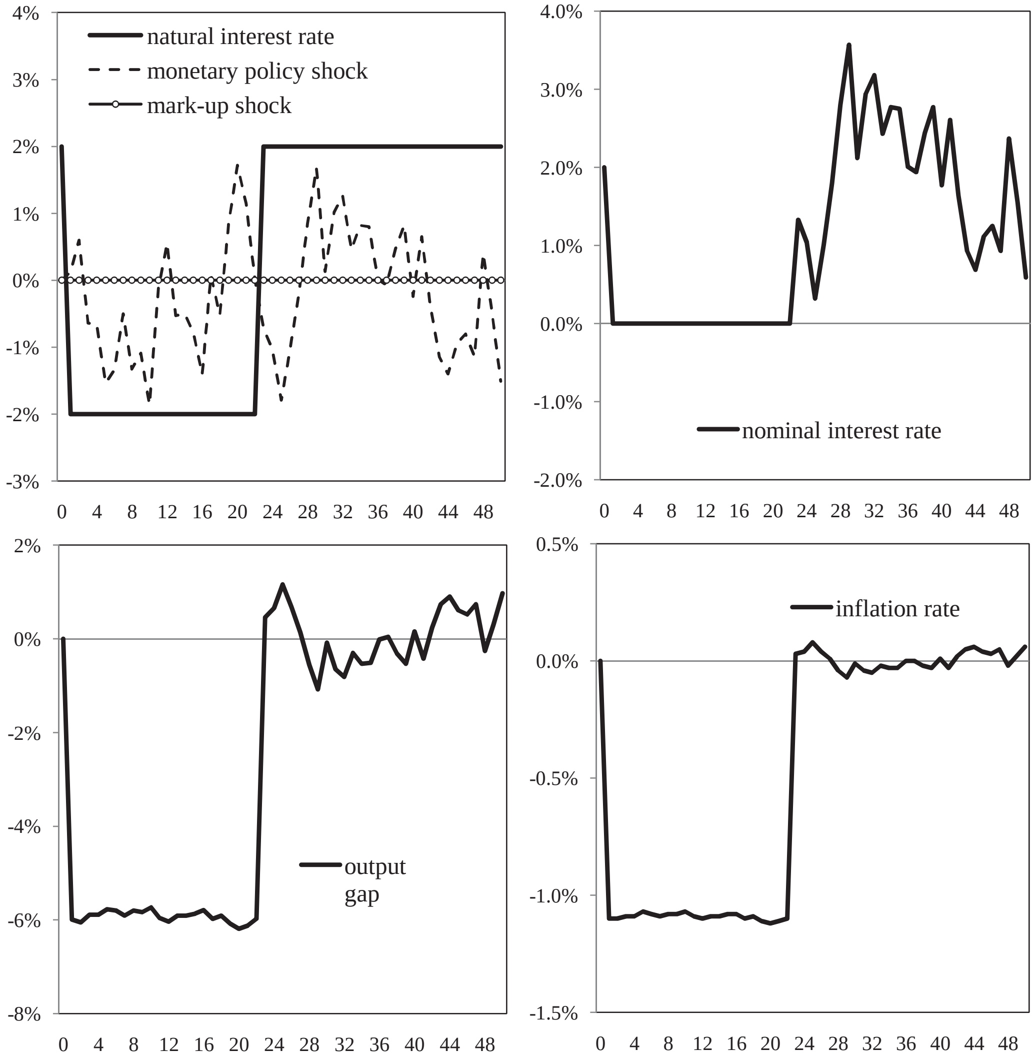


Figure 2 : forward rate curves

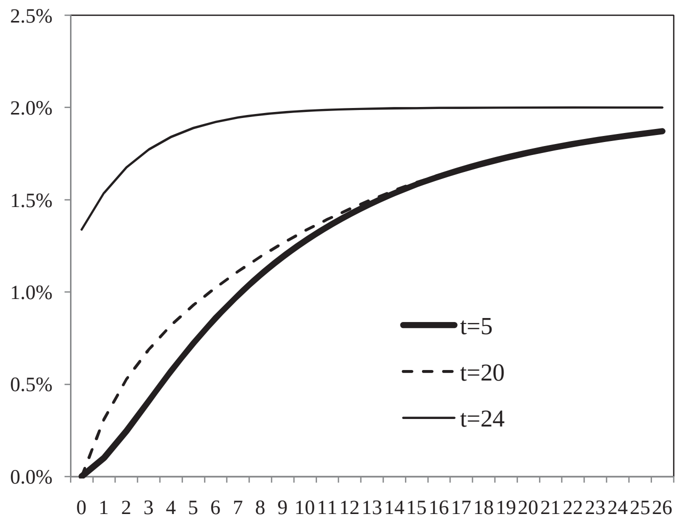


Figure 3 : identified structural shocks

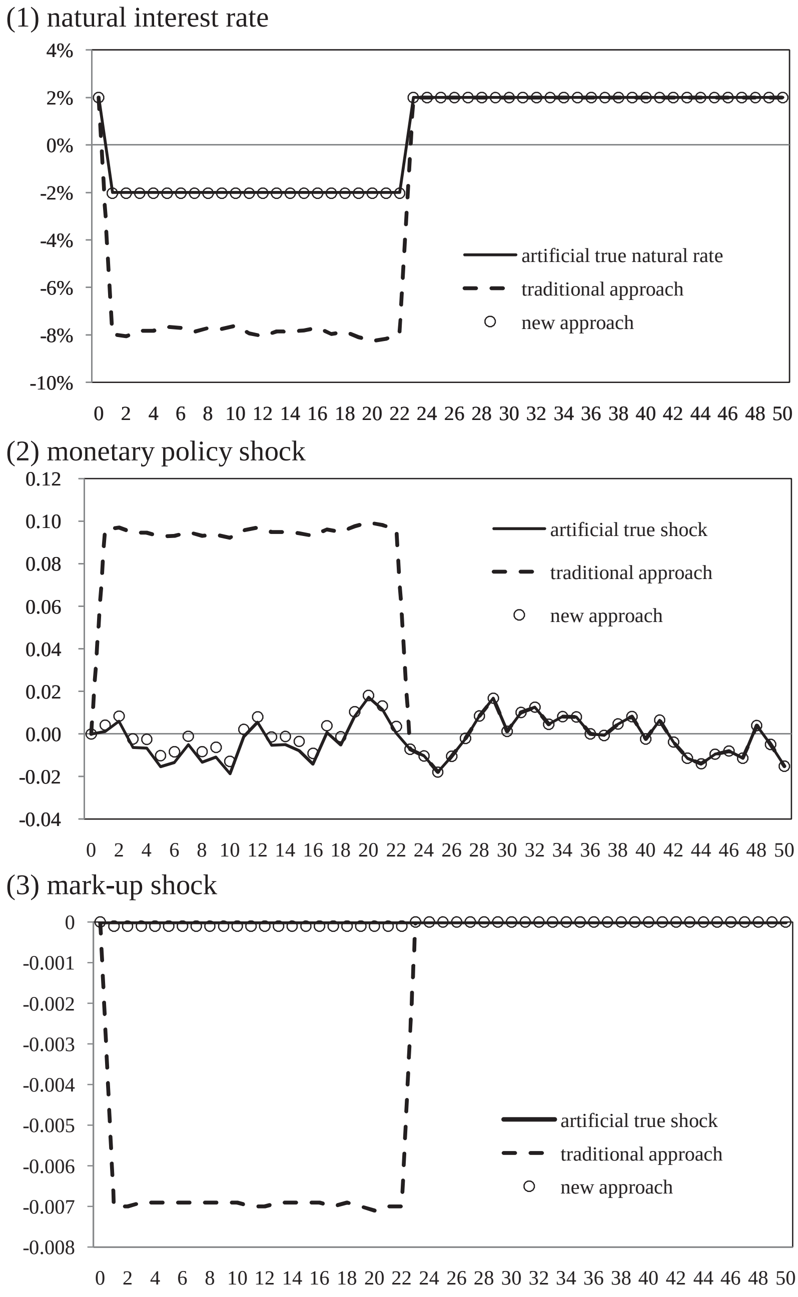


Figure 4 (1) : Identified structural shocks - natural rate -

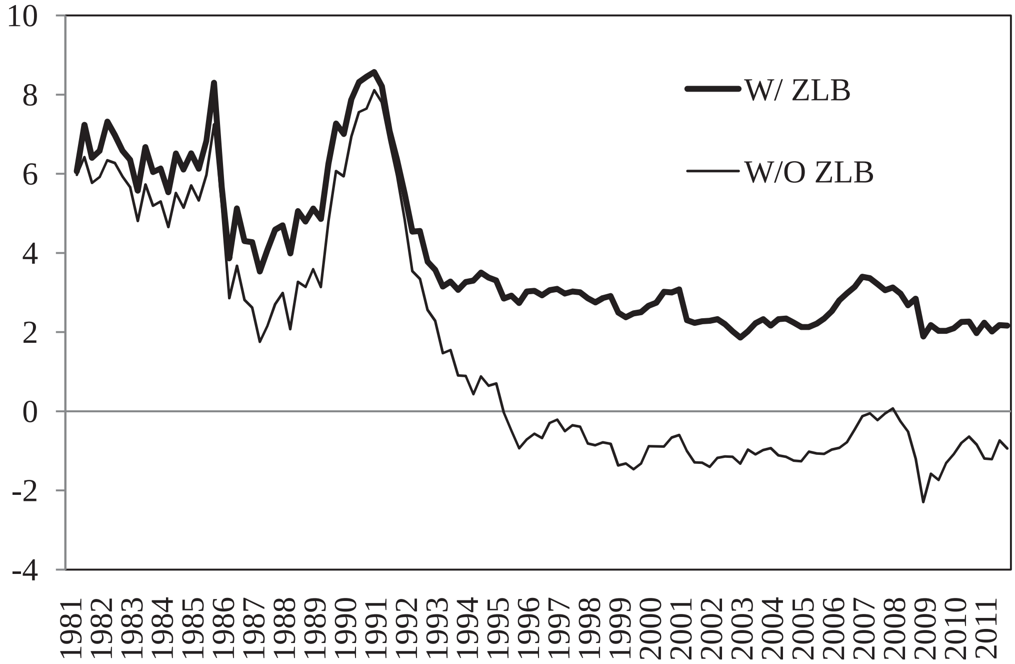


Figure 4 (2) : Identified structural shocks - mark-up shock -

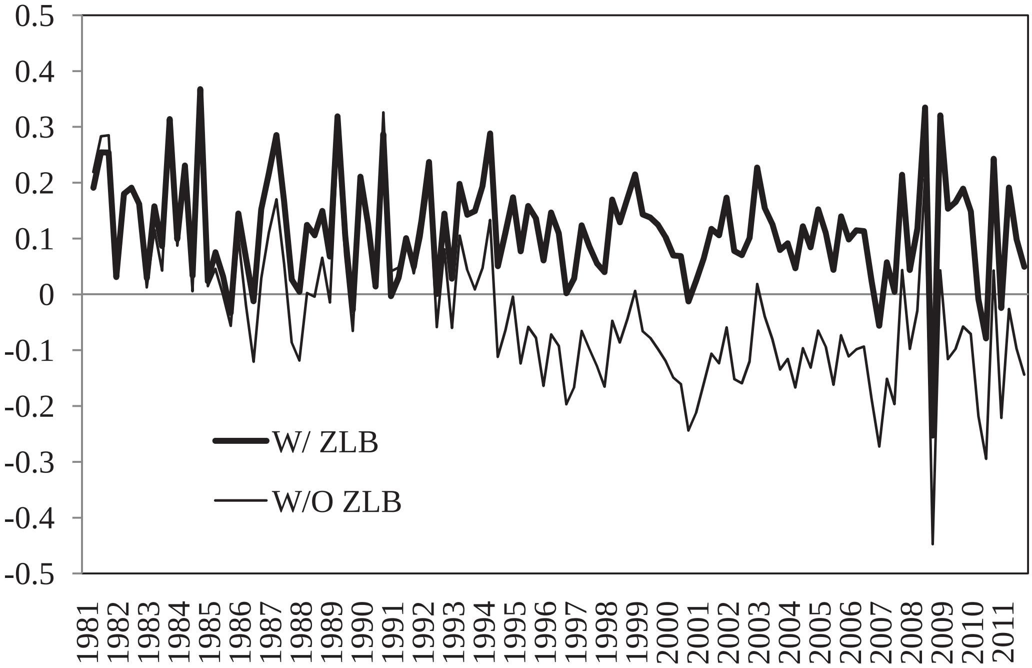


Figure 4 (3) : Identified structural shocks - monetary policy shock -

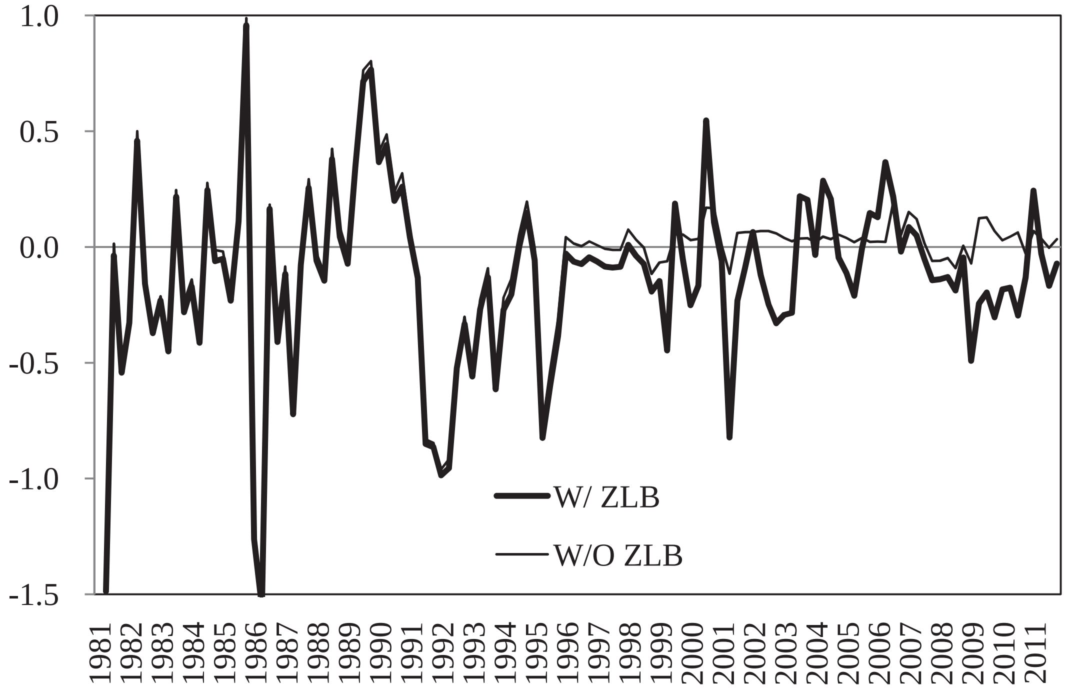
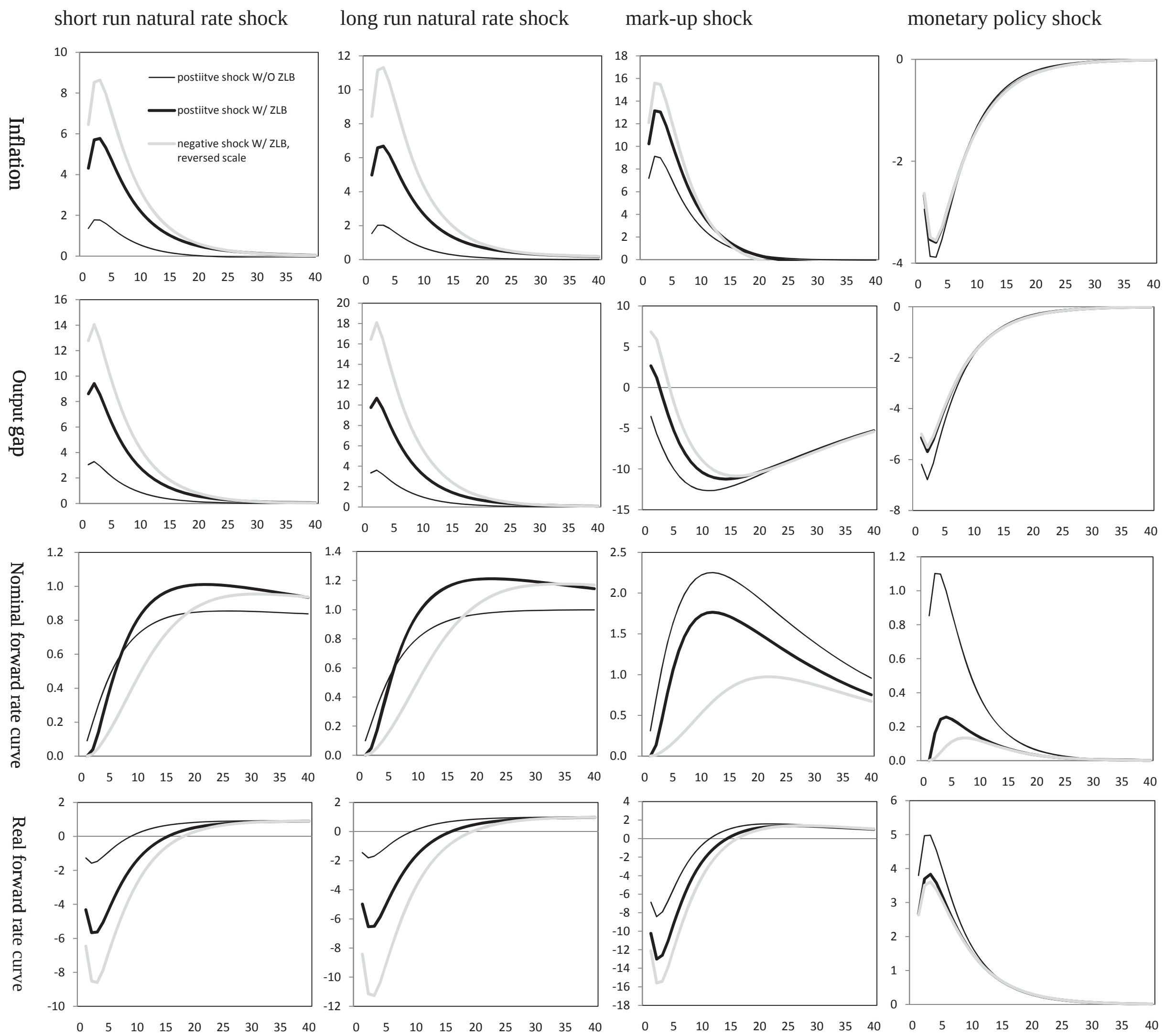
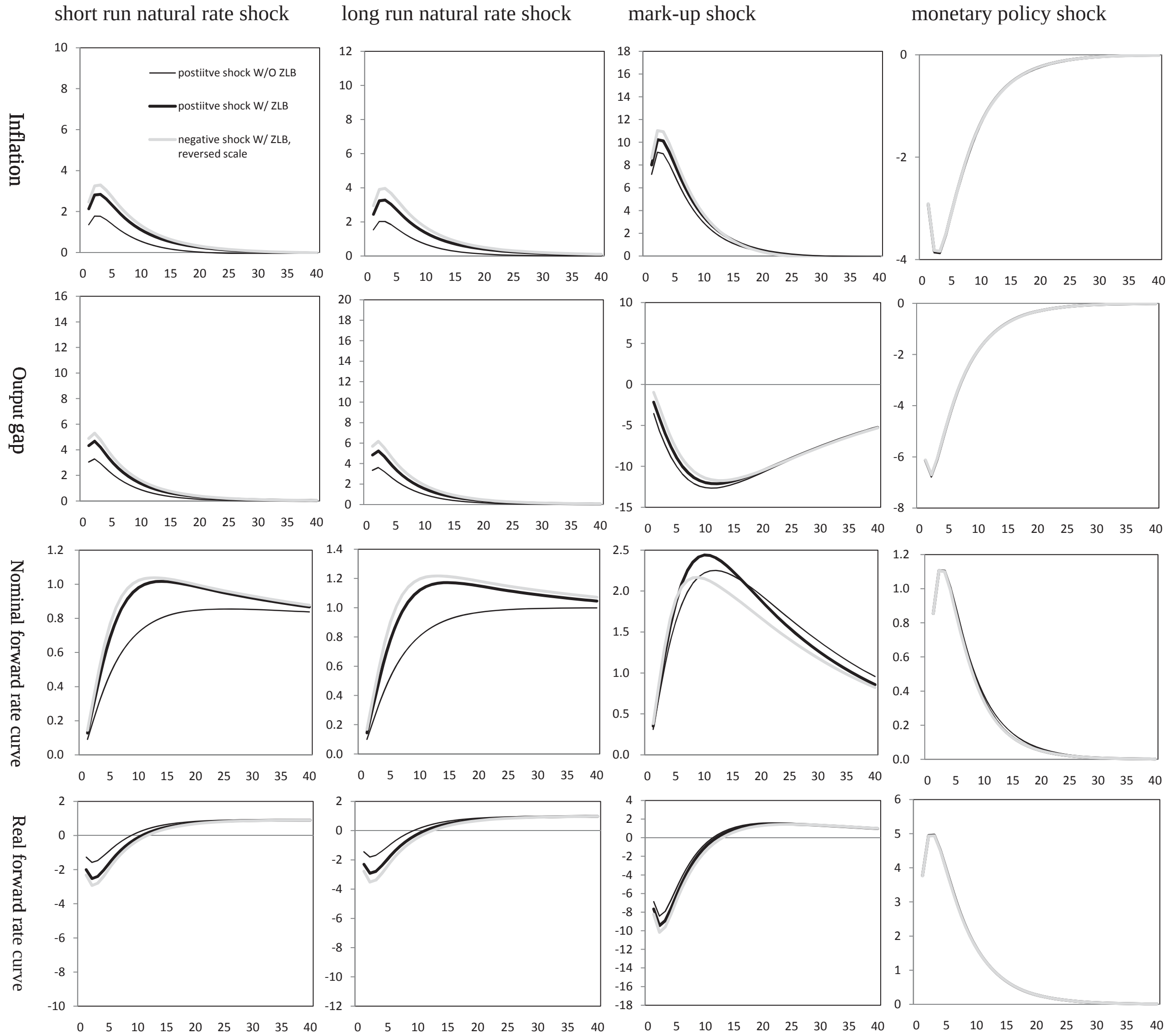


Figure 5 (1) : Impulse-Response Functions



Notes: 1. In each impulse response, the initial values of the state variables in the model are set equal to those of 2003Q2.
2. One percent shock is added in each case.

Figure 5 (2) : Impulse-Response Functions



Notes: 1. In each impulse response, the initial values of the state variables in the model are set equal to those of 1995Q2.
2. One percent shock is added in each case.

Figure 6 : Counter Factual Simulation - Inflation -

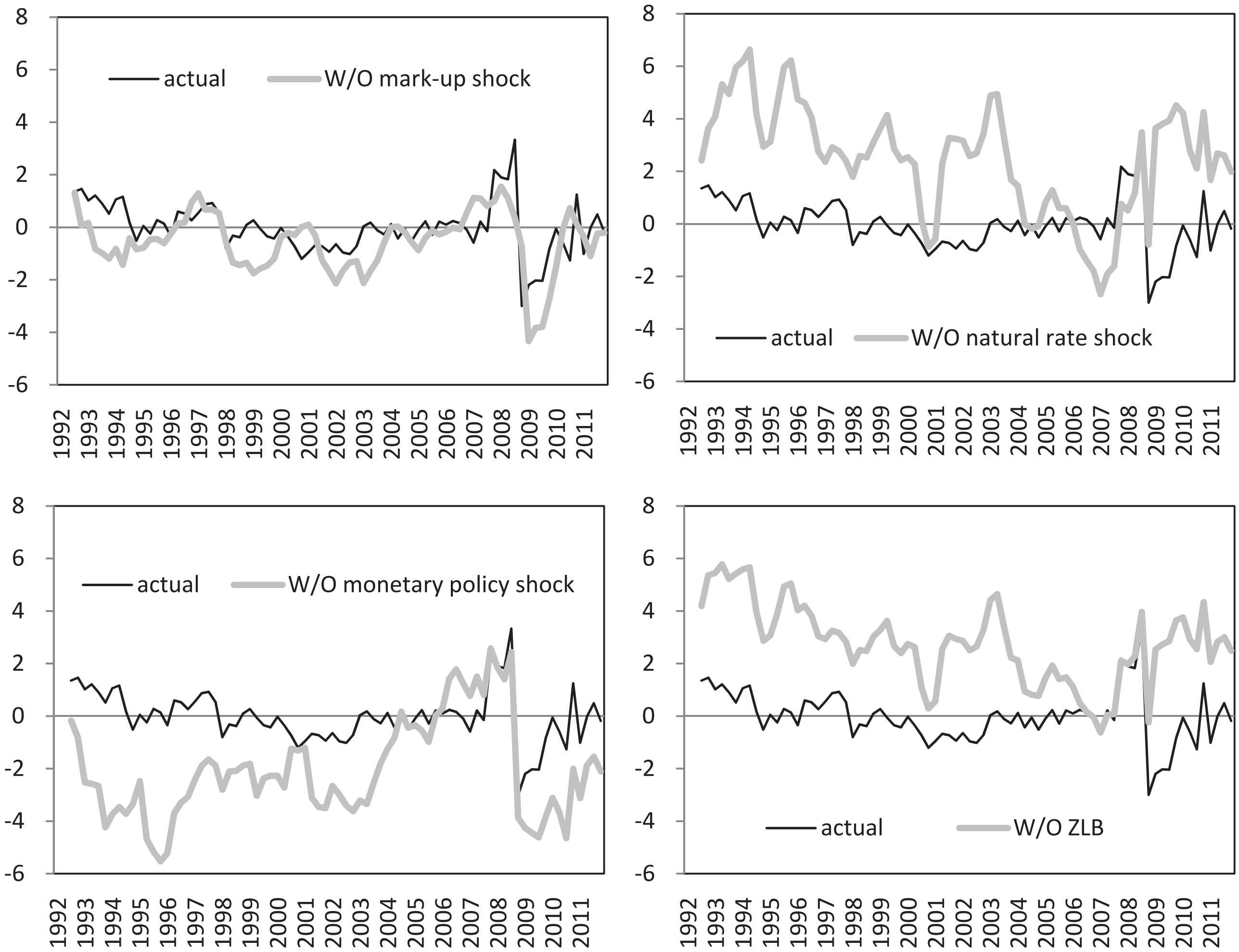


Figure 7 : Counter Factual Simulation - Output gap -

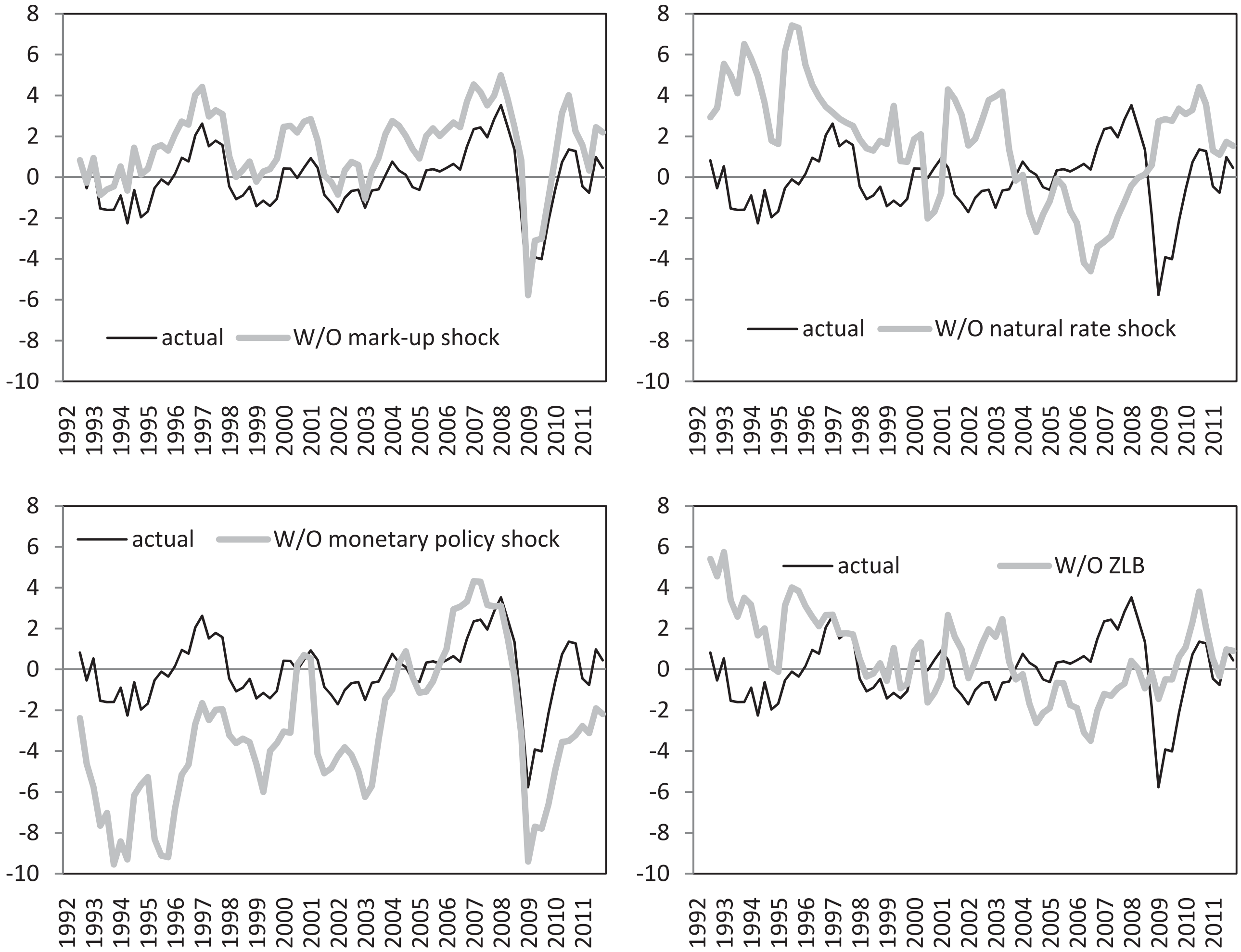


Figure 8 : selected yield curve