Bank of Japan Working Paper Series

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Ko Munakata [*]
kou.munakata@boj.or.jp

Koji Nakamura [**]
kouji.nakamura@boj.or.jp

Yuki Teranishi [**]
yuuki.teranishi@boj.or.jp

No.13-E-1
March 2013

Bank of Japan
2-1-1 Nihonbashi-Hongokucho, Chuo-ku, Tokyo 103-0021, Japan

[*] Oita Branch

[**] Financial System and Bank Examination Department

Welfare Analysis of Policy Measures for Financial Stability*

Ko Munakata[†], Koji Nakamura[‡], and Yuki Teranishi[§]

Abstract

We introduce the financial market friction through the search and matching in the loan market into a dynamic stochastic general equilibrium (DSGE) model. We reveal that the second order approximation of social welfare includes the terms relating credit, such as credit market tightness, the volume of credit, and a loan separation rate, in addition to the inflation rate and the output gap under the financial market friction. Our analytical result justifies the reason why the optimal policy should take the credit variation into account. We introduce a monetary policy and other policy measures for the financial stability into the model. The optimal outcome is achieved through the monetary and other policy measures by taking into account not only price stability but also financial stability.

JEL Classification: E44; E52; E61

Keywords: Optimal macroprudential policy; optimal monetary policy; financial market friction

*We thank Kosuke Aoki for valuable suggestions and comments. Views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

†Oita Branch, Bank of Japan. E-mail: `kou.munakata@boj.or.jp`

‡Financial System and Bank Examination Department, Bank of Japan. E-mail: `kouji.nakamura@boj.or.jp`

§Financial System and Bank Examination Department, Bank of Japan. E-mail: `yuuki.teranishi@boj.or.jp`

1 Introduction

Financial crises reveal critical roles of financial and credit markets by inducing economic disruptions. Under such a situation, accompanied with a lack of room of the monetary policy, policy makers shed light on the policy measures including macroprudential policy for financial stability as a new frontier of the policy.

Some empirical and theoretical studies clarify the need of policy measures for financial stability. Borio (2011) empirically shows a difference between financial cycles and business cycles and then justifies necessity of coexistence of the monetary policy and the macroprudential policy.[1] Theoretical studies for optimal policy under models with financial frictions have started to be conducted. For example, Quint and Rabanal (2011) study the optimal combination of the monetary and macroprudential policies in the DSGE model. They show that the macroprudential policies become quantitatively important if the credit term is in the policy maker's objective function. Suh (2012) shows that the macroprudential policy should respond to credit to improve welfare apart from the monetary policy responses to the output gap and the inflation rate. Kannan, Rabanal, and Scott (2012) show that the monetary policy reaction to credit growth can help macroeconomic stabilization. These studies, however, do not provide the answers for why counting the credit into the policy can improve welfare.

In this paper, we introduce the financial market friction through the search and matching in the loan market into a DSGE model, following Wasmer and Weil (2000) and Den Haan, Ramey, and Watson (2003).[2][3][4] We then reveal that this policy objective function

[1] Drehmann, Borio, and Tsatsaronis (2012) also show such empirical results.

[2] Blanchflower and Oswald (1998) and Peterson and Rajan (2002) empirically show that search and matching frictions play an important role in the loan market.

[3] This type of search and matching friction is also assumed in the labor market as in Mortensen and Pissarides (1994) and Rogerson, Shimer, and Wright (2005). Ravenna and Walsh (2011) derive the quadratic welfare function in the model with the search and matching friction in the labor market.

[4] Different types of financial frictions are assumed in other former studies. Bernanke, Gertler, and Gilchrist (1999, henceforth BGG), which is the first to stress that credit market imperfections have a significant influence on business cycle dynamics. In the BGG model, this financial market wedge is determined by time-varying leverage, in that endogenous mechanisms in credit markets work to amplify and propagate shocks to the economy. In Bianchi (2010), the financial market friction is introduced by externalities in which private agents undervalue the dynamics of net worth since the agents fail to internalize spillover effect among them.

includes a term closely related to the volume of credit. In addition, we introduce a monetary policy and policy measures for the financial market friction into the model. The policy measures for the financial market friction aim to achieve financial stability. We show that optimal policy measures change according to the type of policies. When the degree of competition between a firm and a bank in a loan interest rate setting through Nash bargaining is endogenized and the policy measure, which is interpreted as a regulation policy, intervenes in such a bargaining, accompanied with the optimal monetary policy, the optimal policy measure perfectly stabilizes the inflation rate. This is because the policy measure for financial stability leads to price stability through the cost channel. Another policy measure that is interpreted as a macroprudential policy intervenes in a endogenous credit separation. In this case, a welfare function additionally includes terms for credit separation. Thus, this optimal macroprudential policy adjusts the separation rate by taking into account trade-off among the separation rate, credit market tightness, the inflation rate, and consumption. In any case, the policy measures for financial stability have a close relation with price stability and consequently with the monetary policy.

The rest of the paper is organized as follows. The following section sets up a model. In Section 3, we derive the second order approximation from the consumer's utility function and comment on its properties. In Section 4, we derive an optimal monetary policy and show the properties of this optimal monetary policy. In Section 5, we derive optimal policy measures for financial stability under various settings and reveal the interaction between the optimal policy measures for financial stability and the optimal monetary policy. In Section 6, we conclude the paper.

2 The Model

2.1 Household

At the beginning of time t, a representative household with money M_t deposits D_t in its bank account, receives Π_t as a lump-sum profit from firms and a bank in real terms, and then enters the goods market to consume C_t in real term with price P_t. The cash-in-advance (CIA) constraint of the household is

$$C_t \leqq \frac{M_t}{P_t} + \Pi_t - \frac{D_t}{P_t}.$$

The household gets the deposit back with the nominal interest rate R_t^D at the end of time t, leading to the budget constraint

$$\frac{M_{t+1}}{P_t} = \frac{M_t}{P_t} + \frac{(R_t^D - 1)D_t}{P_t} + \Pi_t - C_t.$$

Assuming that the utility of the household depends only on the consumption C_t, the CIA constraint is always binding if $R_t^D > 1$, and the household's problem is expressed as

$$\max_{C,D} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i}), \tag{1}$$

subject to

$$C_t = \Pi_t + \frac{R_{t-1}^D D_{t-1} - D_t}{P_t}.$$

We assume the following utility function.

$$u(C_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma},$$

where σ is a coefficient of relative risk aversion of the household.

This optimization problem leads to

$$\lambda_t = C_t^{-\sigma},$$

$$1 = \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} R_t^D, \tag{2}$$

where λ_t is the Lagrangian multiplier in the household's optimization problem.

It is assumed that the consumption C_t consists of goods labeled by $j \in [0,1]$. The consumption of each good $c_t(j)$ is related to the total consumption C_t by

$$C_t = \left[\int_0^1 c_t(j)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dj \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$$

where $\varepsilon_t \in (0,1)$ is an exogenous stochastic variable related to the elasticity of substitution. The households choose the level of consumption of each good $c_t(j)$ in order to minimize the cost $\int_0^1 p_t(j)c_t(j)dj$, given the level of total consumption C_t and the price of each good $p_t(j)$. This minimization yields

$$c_t(j) = \left[\frac{p_t(j)}{P_t}\right]^{-\varepsilon_t} C_t \ ,$$

where

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\varepsilon_t} dj\right]^{\frac{1}{1-\varepsilon_t}} . \tag{3}$$

2.2 Wholesale Firms

Each firm seeks for a credit line in the credit market at time t if the firm is a credit seeker. The number of credit seekers is u_t, the total number of credit lines is L_t, and the amount of each credit line in real terms is a. If a firm has a credit contract at time $t-1$ but fails to receive an update on the credit line with the probability ρ at time t, the firm becomes a credit seeker. On the other hand, if a firm does not have a credit contract at time $t-1$, it automatically becomes a credit seeker at time t. Under this setting, the number of credit seekers at time t is obtained by

$$u_t = 1 - (1-\rho)L_{t-1}. \tag{4}$$

Credit seeking firms participate in the bargaining of credit lines with a representative bank. As a result of the matching process, L_t (the number of credit lines at time t) is related to L_{t-1} by

$$L_t = (1-\rho)L_{t-1} + p_t^F u_t,$$

where p_t^F is the probability of getting a credit line for each credit seeking firm.

When a firm has a credit line, the firm uses the amount of credit a to buy retail goods $a(j)$, which are then used as an input or a sweat cost to produce wholesale goods with productivity Z. A firm's gain when it has a credit line is then

$$f_t^1 = \frac{Z}{\mu_t} - aR_t^L + \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[(1-\rho)f_{t+1}^1 + \rho(p_{t+1}^F f_{t+1}^1 + (1-p_{t+1}^F)f_{t+1}^0)\right],$$

where $R_t^L - 1$ is the real interest rate for the credit, $\mu_t \equiv \frac{P_t}{P_t^w}$ is the price markup by retail firms, and P_t^w is the price of a wholesale good. The first two terms show the net profit from production at time t and the third term is the discounted present value of the future profit. On the other hand, a firm's gain when it fails to receive a credit line is

$$f_t^0 = \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[p_{t+1}^F f_{t+1}^1 + (1 - p_{t+1}^F) f_{t+1}^0 \right].$$

Since the firm cannot obtain a credit line, it does not produce at time t and it only has the discounted future values. These equations imply that the gain from a credit line for a credit seeking firm is

$$f_t \equiv f_t^1 - f_t^0 = \frac{Z}{\mu_t} - aR_t^L + \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t} (1-\rho)(1-p_{t+1}^F) f_{t+1}. \tag{5}$$

2.3 Bank

A representative bank decides on the number of credit vacancies V_t to post in order to maximize its profit B_t with respect to L_t

$$B_t = (R_t^L - 1)aL_t - \frac{(R_t^D - 1)D_t}{P_t} - \kappa V_t + \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t} B_{t+1},$$

subject to

$$L_t = (1-\rho)L_{t-1} + q_t^B V_t,$$

where κ is a cost for posting each credit vacancy and q_t^B is the vacancy filling rate. In reality, κ includes the cost of collecting borrowers' information and examining the loan applications. This maximization problem yields

$$\frac{\kappa}{q_t^B} = (R_t^L - 1)a + \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t} (1-\rho) \frac{\kappa}{q_{t+1}^B}. \tag{6}$$

The bank's expected gain from one credit line should be, with free entry, the same as the cost of a vacancy post. This condition implies that the bank's gain for a credit line is

$$J_t = \frac{\kappa}{q_t^B}. \tag{7}$$

We assume that the cost of posting credit vacancies κV_t is again paid by consumption of retail goods $v_t(j)$, in a similar way as the household consumption C_t and the firms' input for production a. Therefore, the demand for good j and the total demand are given by

$$y_t^d(j) \equiv c_t(j) + a(j)L_t + \kappa v_t(j),$$

and

$$Y_t^d \equiv C_t + aL_t + \kappa V_t,$$

respectively, where

$$y_t^d(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon_t} Y_t^d. \tag{8}$$

2.4 Retail Firms

A retail firm producing goods j faces the demand given by the equation (8), and it has to pay P_t^w to buy wholesale goods as an input to produce the final goods. In order to introduce the price stickiness, it is assumed that a firm can adjust its price each period with probability $1-\omega$ as in the model by Calvo (1983). The profit maximization problem of a retail firm when it has a chance to adjust its price P_t^* becomes

$$\max_{P_t^*} \sum_{i=0}^{\infty} (\omega\beta)^i \mathrm{E}_t \left[\left(\frac{\lambda_{t+i}}{\lambda_t}\right) \left(\frac{(1+\tau)P_t^* - P_{t+i}^w}{P_{t+i}}\right) \left(\frac{P_t^*}{P_{t+i}}\right)^{-\varepsilon_{t+i}} Y_{t+i}^d \right],$$

where the demand equation (8) is used. We here assume that the subsidy for firms τ is set to ensure that the price flexibility is achieved at the efficient steady-state equilibrium discussed below. Note that the average price level P_t is given by

$$P_t^{1-\varepsilon_t} = (1-\omega)\left(P_t^*\right)^{1-\varepsilon_t} + \omega P_{t-1}^{1-\varepsilon_t}.$$

2.5 Loan Market Matching

The number of new credit matches is

$$p_t^F u_t = q_t^B V_t = \chi u_t^{1-\alpha} V_t^{\alpha}$$

in a Cob-Douglas form where α is a substitution for u_t and V_t, and χ is a constant parameter for matching. To make the calculation clearer, we use the credit market tightness

$$\theta_t = \frac{V_t}{u_t}, \tag{9}$$

so that

$$V_t = \theta_t u_t,$$

$$p_t^F = \chi \theta_t^{\alpha}, \tag{10}$$

$$q_t^B = \chi \theta_t^{\alpha-1}, \tag{11}$$

$$L_t = (1-\rho)L_{t-1} + \chi \theta_t^{\alpha} u_t. \tag{12}$$

In the loan market, the total gain from loan extension is the sum of credit seeking firms' gain f_t and the bank's gain J_t as follows

$$\max_{R^L} f_t^b J_t^{1-b},$$

where b is the bargaining power of credit seeking firms. This is an asymmetric Nash bargaining condition. By taking the first order condition with respect to R_t^L,

$$(1-b)f_t = bJ_t. \tag{13}$$

For convenience, we simplify the equations (5) and (6) by using the equations (10), (11), (13), and (7) to eliminate p_t^F, q_t^B, f_t, and J_t, yielding

$$\frac{b}{1-b}\frac{\kappa}{\chi}\theta_t^{1-\alpha} = \frac{Z}{\mu_t} - aR_t^L + \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\left(1-\chi\theta_{t+1}^{\alpha}\right)\frac{b}{1-b}\frac{\kappa}{\chi}\theta_{t+1}^{1-\alpha}$$

and

$$\frac{\kappa}{\chi}\theta_t^{1-\alpha} = (R_t^L - 1)a + \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\frac{\kappa}{\chi}\theta_{t+1}^{1-\alpha},$$

respectively. If we further eliminate R_t^L from these equations, we obtain the following condition that relates to the markup μ_t to the credit market tightness θ_t:

$$\begin{aligned}\frac{Z}{\mu_t} &= a + \frac{1}{1-b}\frac{\kappa}{\chi}\theta_t^{1-\alpha} \\ &\quad - \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\frac{1}{1-b}\left(\frac{\kappa}{\chi}\theta_{t+1}^{1-\alpha} - b\kappa\theta_{t+1}\right).\end{aligned} \tag{14}$$

2.6 Market Clearing Condition

Since one unit of the wholesale goods is needed as an input to produce one unit of each retail goods j, the market clearing condition for the wholesale goods is

$$ZL_t = \int_0^1 y_t^d(j)dj.$$

Together with the demand equation for the retail goods (8), the following market clearing condition is obtained:

$$\frac{ZL_t}{Q_t} = C_t + aL_t + \kappa V_t. \tag{15}$$

where

$$Q_t \equiv \int_0^1 \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon_t} dj \tag{16}$$

represents the dispersion of the prices of retail goods due to the price stickiness for retail firms.

3 Welfare Criteria

3.1 Efficient Steady-State Equilibrium

The second order expansion of the household's utility function is set around the efficient steady-state equilibrium, which is the equilibrium designed by a social planner without credit matching inefficiency or price dispersion. Such a situation can only be achieved (1) when the Hosios condition $b = 1 - \alpha$ for the bargaining power b and the matching share α is satisfied and (2) when the subsidy for retail firms τ is chosen to make sure $\overline{\mu} = \frac{\varepsilon}{(\varepsilon-1)(1+\tau)} = 1$. Under these assumptions, the optimization problem for the social planner is expressed as

$$\max_{C,L,u,\theta} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \{ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \lambda_{t+i} \left[ZL_{t+i} - aL_{t+i} - \kappa\theta_{t+i}u_{t+i} - C_{t+i}\right]$$
$$+ \psi_{t+i} \left[(1-\rho)L_{t+i-1} + \chi\theta_{t+i}^{\alpha}u_{t+i} - L_{t+i}\right] + s_{t+i} \left[u_{t+i} - 1 + (1-\rho)L_{t+i-1}\right]\},$$

where λ_t, ψ_t, and s_t are Lagrangian multipliers for the constraints.

The solution to this optimization problem yields the following condition for the efficient steady-state equilibrium:

$$Z - a - \frac{\kappa\overline{v}}{\alpha\rho\overline{L}} = -\beta(1-\rho)\frac{\kappa\overline{v}}{\alpha\rho\overline{L}}\left(1-(1-\alpha)\frac{\rho\overline{L}}{\overline{u}}\right),$$

which is simplified for convenience as

$$\delta_1 = -\beta\delta_2. \tag{17}$$

Note that the bar above each variable (e.g., $\overline{v}$) implies the efficient steady-state value of the variable (v_t).

3.2 Policy Objective Function

The second order expansion of the household's utility function around the efficient steady state yields

$$\begin{aligned} u(C_t) &= u(\overline{C}) + u_c\overline{C}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) + \frac{1}{2}u_{cc}\overline{C}^2\widehat{c}_t^2 \\ &= u(\overline{C}) + u_c\overline{C}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{1}{2}\sigma u_c\overline{C}\widehat{c}_t^2, \end{aligned}$$

where $\widehat{c}_t$ is the log-deviation of C_t from the efficient steady-state value $\overline{C}$. The goal of the following calculation in this subsection is to express this utility $u(C_t)$ by only $\widehat{l}_t$, $\widehat{l}_{t-1}$, and the inflation rate $\pi_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$. This calculation closely follows the derivation of a policy objective function by Ravenna and Walsh (2011) for the search and matching in the labor market. By using the market clearing condition of the equation (15),

$$\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2 = -\frac{Z\overline{L}}{\overline{C}}\widehat{q}_t + \frac{(Z-a)\overline{L}}{\overline{C}}\left(\widehat{l}_t + \frac{1}{2}\widehat{l}_t^2\right) - \frac{\kappa\overline{v}}{\overline{C}}\left(\widehat{v}_t + \frac{1}{2}\widehat{v}_t^2\right). \tag{18}$$

Note that the efficient steady-state value of the price dispersion term Q_t is $\overline{Q} = 1$, and the log-deviation of this term $\widehat{q}_t$ is already in the second order, as is shown below.

Using the equation (9)

$$\widehat{v}_t = \widehat{u}_t + \widehat{\theta}_t,$$

and the equation (4)

$$\widehat{u}_t + \frac{1}{2}\widehat{u}_t^2 = -\eta\left(\widehat{l}_{t-1} + \frac{1}{2}\widehat{l}_{t-1}^2\right),$$

where

$$\eta \equiv (1-\rho)\frac{\overline{L}}{\overline{u}}.$$

Then, we obtain up to the second order

$$\widehat{v}_t + \frac{1}{2}\widehat{v}_t^2 = \left(\widehat{\theta}_t + \frac{1}{2}\widehat{\theta}_t^2\right) - \eta\left(\widehat{l}_{t-1} + \frac{1}{2}\widehat{l}_{t-1}^2\right) - \eta\widehat{\theta}_t\widehat{l}_{t-1}. \tag{19}$$

By using this equation and the equation (12), we can eliminate $\widehat{\theta}_t$ and get

$$\begin{aligned} u(C_t) &= u(\overline{C}) - u_c Z\overline{L}\widehat{q}_t \\ &+ u_c\overline{C}\left[\frac{\overline{L}}{\overline{C}}\left(\delta_1\widehat{l}_t + \delta_2\widehat{l}_{t-1}\right) + \frac{\overline{L}}{2\overline{C}}\left(\delta_1\widehat{l}_t^2 + \delta_2\widehat{l}_{t-1}^2\right) - \frac{\kappa\overline{v}}{\overline{C}}\frac{1-\alpha}{2(\alpha\rho)^2}\left(\widehat{l}_t - \rho_u\widehat{l}_{t-1}\right)^2\right] \\ &- \frac{1}{2}\sigma u_c\overline{C}\left(\frac{\overline{L}}{\overline{C}}\right)^2\left(\delta_1\widehat{l}_t + \delta_2\widehat{l}_{t-1}\right)^2, \end{aligned}$$

where

$$\rho_u \equiv 1 - \rho - \rho\eta.$$

Thus, the utility of the equation (1) is rewritten as

$$\begin{aligned} \sum_{i=0}^{\infty}\beta^i u(C_{t+i}) &= \frac{u(\overline{C})}{1-\beta} - u_c Z\overline{L}\sum_{i=0}^{\infty}\beta^i\widehat{q}_{t+i} \\ &+ u_c\overline{L}\sum_{i=0}^{\infty}\beta^i\left(\delta_1\widehat{l}_{t+i} + \delta_2\widehat{l}_{t+i-1}\right) \\ &+ \frac{1}{2}u_c\overline{L}\sum_{i=0}^{\infty}\beta^i\left(\delta_1\widehat{l}_{t+i}^2 + \delta_2\widehat{l}_{t+i-1}^2\right) \\ &- u_c\kappa\overline{v}\frac{1-\alpha}{2(\alpha\rho)^2}\sum_{i=0}^{\infty}\beta^i\left(\widehat{l}_{t+i} - \rho_u\widehat{l}_{t+i-1}\right)^2 \\ &- \frac{1}{2}\sigma u_c\overline{C}\left(\frac{\overline{L}}{\overline{C}}\right)^2\sum_{i=0}^{\infty}\beta^i\left(\delta_1\widehat{l}_{t+i} + \delta_2\widehat{l}_{t+i-1}\right)^2. \end{aligned}$$

By using the efficient steady-state condition given by the equation (17), the two terms on the right-hand side of the equation above are shown to depend only on $\widehat{l}_{t-1}$, and the utility becomes

$$\begin{aligned} \sum_{i=0}^{\infty}\beta^i u(C_{t+i}) &= -u_c Z\overline{L}\sum_{i=0}^{\infty}\beta^i\widehat{q}_{t+i} \\ &- u_c\kappa\overline{v}\frac{1-\alpha}{2(\alpha\rho)^2}\sum_{i=0}^{\infty}\beta^i\left(\widehat{l}_{t+i} - \rho_u\widehat{l}_{t+i-1}\right)^2 \\ &- \frac{1}{2}\sigma u_c\overline{C}\left(\frac{\overline{L}}{\overline{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right)^2 \\ &+ t.i.p., \end{aligned} \tag{20}$$

where $t.i.p.$ denotes "terms independent of policy."

Next, we consider the sum over the price dispersion terms $\widehat{q}_{t+i}$. By the definition of the equation (16),

$$\begin{aligned}\widehat{q}_t &= \int_0^1 dj \exp\left[-\varepsilon_t\left(\widehat{p}_t(j) - \widehat{p}_t\right)\right] - 1 \\ &\simeq -\varepsilon(\Delta_t^E - \widehat{p}_t)(1 + \widehat{\varepsilon}_t) + \frac{1}{2}\varepsilon^2\left[\Delta_t^V + \left(\Delta_t^E - \widehat{p}_t\right)^2\right],\end{aligned}$$

where $\Delta_t^E \equiv E_j\widehat{p}_t(j) = \int_0^1 \widehat{p}_t(j)dj$ and $\Delta_t^V \equiv Var_j\widehat{p}_t(j) = E_j\widehat{p}_t(j)^2 - (E_j\widehat{p}_t(j))^2$. Since the definition of the aggregate price P_t given by the equation (3) can be used to show that

$$\Delta_t^E - \widehat{p}_t \simeq -\frac{1}{2}(1-\varepsilon)\Delta_t^V.$$

Up to the second order in $\widehat{p}_t$, we can thus rewrite $\widehat{q}_t$ as

$$\widehat{q}_t \simeq \frac{1}{2}\varepsilon\Delta_t^V,$$

leading to

$$\sum_{i=0}^{\infty}\beta^i\widehat{q}_{t+i} = \frac{1}{2}\varepsilon\sum_{i=0}^{\infty}\beta^i\Delta_{t+i}^V. \tag{21}$$

On the other hand, the equation to obtain Δ_t^V is written as

$$\begin{aligned}\Delta_t^V &= Var_j\widehat{p}_t(j) \\ &= Var_j\left(\widehat{p}_t(j) - \Delta_{t-1}^E\right) \\ &= E_j(\widehat{p}_t(j) - \Delta_{t-1}^E)^2 - (\Delta_t^E - \Delta_{t-1}^E)^2.\end{aligned}$$

Here, we remember that only the fraction $1-\omega$ of all firms adjust their prices to P_t^*, while other firms do not change their prices $p_{t-1}(j)$. $\widehat{p}_t^*$, the log-deviation of P_t^*, can in turn be expressed by $\widehat{p}_t$ and $\widehat{p}_{t-1}$. Hence,

$$\begin{aligned}\Delta_t^V &\simeq \omega E_j(\widehat{p}_{t-1}(j) - \Delta_{t-1}^E)^2 + (1-\omega)E_j\left(\widehat{p}_t^* - \Delta_{t-1}^E\right)^2 - \left(\Delta_t^E - \Delta_{t-1}^E\right)^2 \\ &\simeq \omega\Delta_{t-1}^V + (1-\omega)E_j\left(\frac{1}{1-\omega}\widehat{p}_t - \frac{\omega}{1-\omega}\widehat{p}_{t-1} - \Delta_{t-1}^E\right)^2 - (\Delta_t^E - \Delta_{t-1}^E)^2 \\ &\simeq \omega\Delta_{t-1}^V + (1-\omega)E_j\left(\frac{1}{1-\omega}\widehat{p}_t - \frac{\omega}{1-\omega}\widehat{p}_{t-1} - \widehat{p}_{t-1}\right)^2 - \left(\widehat{p}_t - \widehat{p}_{t-1}\right)^2,\end{aligned}$$

up to the second order again in $\widehat{p}_t$. Using the inflation rate $\pi_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$, we thus have

$$\Delta_t^V \simeq \omega \Delta_{t-1}^V + \frac{\omega}{1-\omega}\pi_t^2.$$

The sum of Δ_t^V becomes

$$\begin{aligned}
\sum_{i=0}^{\infty} \beta^i \Delta_{t+i}^V &= \Delta_t^V + \sum_{i=1}^{\infty} \beta^i \Delta_{t+i}^V \\
&= \Delta_t^V + \beta \sum_{i=0}^{\infty} \beta^i \Delta_{t+i+1}^V \\
&= \Delta_t^V + \beta \sum_{i=0}^{\infty} \beta^i \left(\omega \Delta_{t+i}^V + \frac{\omega}{1-\omega}\pi_{t+i}^2 \right) \\
&= \Delta_t^V + \omega\beta \sum_{i=0}^{\infty} \beta^i \Delta_{t+i}^V + \frac{\omega\beta}{1-\omega} \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2.
\end{aligned}$$

We therefore obtain

$$\sum_{i=0}^{\infty} \beta^i \Delta_{t+i}^V = \frac{\omega\beta}{(1-\omega)(1-\omega\beta)} \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 + t.i.p..$$

Combining this equation with the equations (20) and (21),

$$\begin{aligned}
\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) = &-\frac{1}{2} u_c Z \overline{L} \frac{\varepsilon}{\delta} \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 \\
&- u_c \kappa \overline{v} \frac{1-\alpha}{2(\alpha\rho)^2} \sum_{i=0}^{\infty} \beta^i \left(\widehat{l}_{t+i} - \rho_u \widehat{l}_{t+i-1} \right)^2 \\
&- \frac{1}{2} \sigma u_c \overline{C} \left(\frac{\overline{L}}{\overline{C}} \delta_2 \right)^2 \sum_{i=0}^{\infty} \beta^i \left(-\beta \widehat{l}_{t+i} + \widehat{l}_{t+i-1} \right)^2 \\
&+ t.i.p.,
\end{aligned}$$

where

$$\delta \equiv \frac{(1-\omega)(1-\omega\beta)}{\omega}.$$

We also use the approximations up to the second order

$$\widehat{\theta}_{t+i}^2 = \frac{1}{(\alpha\rho)^2} \left(\widehat{l}_{t+i} - \rho_u \widehat{l}_{t+i-1} \right)^2 = \frac{1}{(\alpha\rho\eta)^2} \left(\widehat{u}_{t+i} - \rho_u \widehat{u}_{t+i-1} \right)^2,$$

and

$$\widehat{c}^2_{t+i} = \left(\frac{\overline{L}}{\overline{C}}\delta_2\right)^2 \left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right)^2,$$

and finally, we have the following second order expansion of the household's utility function

$$\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) = -\frac{1}{2}\sum_{i=0}^{\infty} \beta^i \left(\lambda_\pi \pi^2_{t+i} + \lambda_\theta \widehat{\theta}^2_{t+i} + \lambda_c \widehat{c}^2_{t+i}\right) + t.i.p.,$$

where $\lambda_\pi \equiv u_c Z \overline{L} \frac{\varepsilon}{\delta}$, $\lambda_\theta \equiv u_c \kappa \overline{v}(1-\alpha)$ and $\lambda_c \equiv \sigma u_c \overline{C}$.

We note that, even if we introduce exogenous productivity shock Z_t in the model, we can derive a mathematically-similar formula for the utility by taking the difference from the efficient stochastic state. This point is discussed in the appendix.

The optimal policy faces the trade-off among the inflation rate, consumption, and credit market tightness.[5] The credit market tightness is a new feature for the optimal policy under the loan market friction. Moreover, the approximated welfare function can be transformed as

$$\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) = -\sum_{i=0}^{\infty} \beta^i \frac{1}{2}\left[\lambda_\pi \pi^2_{t+i} + \frac{\lambda_\theta}{(\alpha\rho)^2}\left(\widehat{l}_{t+i} - \rho_u \widehat{l}_{t+i-1}\right)^2 + \lambda_c \widehat{c}^2_{t+i}\right].$$

Thus, the optimal policy should respond to the changes in credit. This result supports the papers of Quint and Rabanal (2011), Suh (2012), and Kannan, Rabanal, and Scott (2012), claiming that the policy should respond to the changes in credit, in addition to the inflation rate and the output gap.

3.3 Linearization

We show the closed linearized system of the economy around the efficient equilibrium. For general stochastic non-efficient state, the Calvo-type stickiness introduced in the retail sector leads to the standard Phillips curve with a cost-push shock $\widehat{\varepsilon}_t$ as

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \delta\left(\frac{1}{\epsilon - 1}\widehat{\varepsilon}_t + \widehat{\mu}_t\right). \tag{22}$$

[5] Cúrdia and Woodford (2009) show that an approximated welfare function includes the credit spread term under the model where households face a financial market friction. Also, Teranishi (2008) shows that under the staggered cost channel model an approximated welfare function includes the growth of the loan interest rate.

The retail price markup term $\widehat{\mu}_t$ in this equation can be obtained by the linearized equation (14),

$$\begin{aligned} Z\widehat{\mu}_t = &-(1-\alpha)\frac{\kappa\overline{v}}{\alpha\rho\overline{L}}\left(\widehat{\theta}_t - \beta\rho_u \mathrm{E}_t\widehat{\theta}_{t+1}\right) \\ &- \beta\delta_2\sigma\left(\mathrm{E}_t\widehat{c}_{t+1} - \widehat{c}_t\right). \end{aligned} \tag{23}$$

The IS relation is given by

$$\widehat{c}_t = \mathrm{E}_t\widehat{c}_{t+1} - \frac{1}{\sigma}\left(\widehat{r}_t^D - \mathrm{E}_t\pi_{t+1}\right). \tag{24}$$

We call $\widehat{c}_t$ as the output gap.

The credit market tightness is given by

$$\widehat{\theta}_t = \frac{\left(\widehat{l}_t - \rho_u\widehat{l}_{t-1}\right)}{\alpha\rho}. \tag{25}$$

The relation between consumption and credit is given by

$$\widehat{c}_t = \frac{\overline{L}\delta_2}{\overline{C}}\left(-\beta\widehat{l}_t + \widehat{l}_{t-1}\right). \tag{26}$$

It is also noteworthy that the log-linearized deviation of the loan interest rate $\widehat{r}_t^L$ is related to the credit market tightness $\widehat{\theta}_t$ and the deposit interest rate $\widehat{r}_t^D$ by the following equation:

$$\begin{aligned} a\overline{R}^L\widehat{r}_t^L = &(1-\alpha)\frac{\kappa}{\chi}\overline{\theta}^{1-\alpha}\widehat{\theta}_t \\ &- (1-\rho)\beta(1-\alpha)\frac{\kappa}{\chi}\overline{\theta}^{1-\alpha}\mathrm{E}_t\widehat{\theta}_{t+1} \\ &+ (1-\rho)\beta\frac{\kappa}{\chi}\overline{\theta}^{1-\alpha}\left(\widehat{r}_t^D - \mathrm{E}_t\pi_{t+1}\right). \end{aligned}$$

The credit market tightness and the real deposit rate determine the loan interest rate. This equation can be transformed as

$$\begin{aligned} a\overline{R}^L\widehat{r}_t^L = &-\beta(1-\rho)\frac{\kappa\overline{\theta}^{1-\alpha}}{\chi}\left[\frac{(1-\alpha)}{\alpha\rho} + \beta\sigma\frac{\overline{L}\delta_2}{\overline{C}}\right]\mathrm{E}_t\widehat{l}_{t+1} \\ &+ \frac{\kappa\overline{\theta}^{1-\alpha}}{\chi}\left[\frac{(1-\alpha)}{\alpha\rho} + \frac{(1-\rho)\beta(1-\alpha)\rho_u}{\alpha\rho} + \frac{(1-\rho)\beta\sigma\overline{L}\delta_2(1+\beta)}{\overline{C}}\right]\widehat{l}_t \\ &- \frac{\kappa\overline{\theta}^{1-\alpha}}{\chi}\left[\frac{(1-\alpha)\rho_u}{\alpha\rho} + \frac{(1-\rho)\beta\sigma\overline{L}\delta_2}{\overline{C}}\right]\widehat{l}_{t-1}. \end{aligned} \tag{27}$$

Thus, the loan interest rate and credit have a close relation.[6]

4 Monetary Policy

We assume that a central bank control the nominal interest rate on deposits to maximize the social welfare, following Woodford (2003). The optimal precommitment policy rule for the central bank under the timeless perspective should therefore be obtained by solving the following minimization problem:

$$\min_{\pi,c,\theta,r^D,l} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \frac{1}{2} \left(\lambda_\pi \pi_{t+i}^2 + \lambda_\theta \widehat{\theta}_{t+i}^2 + \lambda_c \widehat{c}_{t+i}^2 \right)$$

subject to the Phillips curve of equation (22), the markup of equation (23), the IS relation of equation (24), and the expressions for the credit market tightness of equation (25), and the consumption of equation (26).

Then, the optimal monetary policy, which is the nominal interest rate, is given by the first order conditions for π_t, $\widehat{\theta}_t$, $\widehat{c}_t$, $\widehat{l}_t$, and $\widehat{r}_t^D$, respectively as:

$$\lambda_\pi \pi_t + \varphi_{1t} - \varphi_{1t-1} - \beta^{-1}\sigma^{-1}\varphi_{2t-1} = 0, \tag{28}$$

$$\lambda_\theta \widehat{\theta}_t - \delta(1-\alpha)\frac{\kappa \overline{v}}{Z\alpha\rho\overline{L}}(\varphi_{1t} - \rho_u \varphi_{1t-1}) + \varphi_{3t} = 0, \tag{29}$$

$$\lambda_c \widehat{c}_t - \frac{\delta\delta_2\sigma}{Z}(\varphi_{1t-1} - \beta\varphi_{1t}) + \varphi_{2t} - \beta^{-1}\varphi_{2t-1} + \varphi_{4t} = 0, \tag{30}$$

$$-\frac{1}{\alpha\rho}\varphi_{3t} + \frac{\beta\rho_u}{\alpha\rho}\mathrm{E}_t\varphi_{3t+1} + \frac{\overline{L}\delta_2\beta}{\overline{C}}\varphi_{4t} - \frac{\overline{L}\delta_2\beta}{\overline{C}}\mathrm{E}_t\varphi_{4t+1} = 0, \tag{31}$$

$$\varphi_{2t} = 0, \tag{32}$$

where φ_{1t}, φ_{2t}, φ_{3t}, and φ_{4t} are Lagrangian multipliers for the Phillips curve of the equation (22), the IS relation of the equation (24), and the expressions for the credit market tightness of the equation (25), and the consumption of the equation (26), respectively.

[6] By using the equation (27), it is possible to include the loan rate term in the approximated welfare function. This result is consistent with the one in Teranishi (2008).

5 Policy Measures for Financial Stability

5.1 Intervention for Nash Bargaining

We introduce a policy measure that intervenes in the setting of the loan interest rate between a firm and a bank through Nash bargaining. The monetary authority control the parameter b of Nash bargaining as

$$\max_{R^L} f_t^{b_t} J_t^{1-b_t},$$

where b_t is the policy variable of the monetary authority. In reality, the monetary authority controls the degree of competition in the loan market by changing regulations. In this case, the retail price markup term $\widehat{\mu}_t$ in the equation (23) changes as

$$\begin{aligned} Z\widehat{\mu}_t = &-(1-\alpha)\frac{\kappa\overline{v}}{\alpha\rho\overline{L}}(\widehat{\theta}_t - \beta\rho_u \mathrm{E}_t\widehat{\theta}_{t+1}) \\ &- \beta\delta_2\sigma\left(\mathrm{E}_t\widehat{c}_{t+1} - \widehat{c}_t\right) \\ &- \frac{b}{(1-b)^2}\frac{\kappa\overline{v}}{\rho\overline{L}}\left(\widehat{b}_t - \beta\rho_u \mathrm{E}_t\widehat{b}_{t+1}\right). \end{aligned}$$

The welfare function and other parts of the model, however, do not change. Then, the optimal policy measure for financial stability is given by

$$\varphi_{1t} - \rho_u\varphi_{1t-1} = 0.$$

Accompanied with the optimal monetary policy, under the optimal policy measure for financial stability, the inflation rate is fixed to zero, and other variables are also zero against the cost-push shock. This is because financial stability leads to price stability via the cost channel. This case shows that the policy measure for financial stability can hold a close relation with price stability and ultimately with the monetary policy.

5.2 Intervention for Credit Separation

Another way to intervene in the credit market could be limiting or controlling the credit separation rate ρ.[7] In this policy measure, the monetary authority controls the volume of credit, and this can be interpreted as the macroprudential policy. If the change in ρ

[7] We can also make the credit vacancies κ or the matching parameter χ as policy variables.

affects no other exogenous parameters, then it can be shown in our model that the utility is maximized by always setting ρ to zero. In the real economy, however, constraining natural separation of credit between banks and firms may result in a deterioration of productivity in a long run, and a similar feature for the separation between workers and firms is endogenously captured by the matching model by Mortensen and Pissarides (1994). In addition to allowing the policy makers to control the separation rate ρ_t, we therefore introduce the following phenomenological relation between the average productivity of wholesale firms Z_t and the separation rate ρ_t:

$$Z_t = f(\rho_t),$$

where f is a monotonically-increasing concave function. We then assume that the expansion of the above relation up to the second order from a steady-state value can be expressed as

$$Z\left(\widehat{z}_t + \frac{1}{2}\widehat{z}_t^2\right) = \overline{\rho}\left[f_1\widehat{\rho}_t + \frac{1}{2}\left(f_1 + f_2\overline{\rho}\right)\widehat{\rho}_t^2\right], \tag{33}$$

where the first and the second derivative of the function f under the efficient steady-state equilibrium satisfy $f_1 > 0$ and $f_2 \leq 0$, respectively.

In addition to the equation (17), the efficient steady-state condition becomes,

$$(1-\overline{\rho})f_1 = \delta_2, \tag{34}$$

where $\overline{\rho}$ can be nonzero. In this case, the welfare function becomes

$$\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = -\frac{1}{2}\sum_{i=0}^{\infty}\beta^i\left(\lambda_\pi\pi_{t+i}^2 + \lambda_\theta\widehat{\theta}_{t+i}^2 + \lambda_c\widehat{c}_{t+i}^2 + \lambda_\rho\widehat{\rho}_{t+i}^2\right) \tag{35}$$
$$+\sum_{i=0}^{\infty}\beta^i\psi_\rho\widehat{\rho}_{t+i}\left(\widehat{l}_{t+i} - \widehat{l}_{t+i-1}\right) + t.i.p.,$$

where $\lambda_\rho \equiv u_c\overline{L}\overline{\rho}^2\left|f_2\right|$ and $\psi_\rho \equiv u_c\overline{L}\overline{\rho}f_1$. See the appendix for the details of the calculation. The implication of this equation is clear. The fact that the last term is linear in $\widehat{\rho}$ suggests that, when the number of credit increases, the society is better off by setting the separation rate higher than the efficient equilibrium value. The second-order term $\lambda_\rho\widehat{\rho}_{t+i}^2$ is then the cost incurred by excessive control of $\widehat{\rho}$ and reflect the concavity of the function f.

The time-variation in the separation rate modifies the relation between the credit tightness and the number of credit as

$$\widehat{\theta}_t = \frac{1}{\alpha\overline{\rho}}\left[\widehat{l}_t - \rho_u \widehat{l}_{t-1} + \overline{\rho}\left(1 - \frac{\overline{\rho}\overline{L}}{\overline{u}}\right)\widehat{\rho}_t\right]. \tag{36}$$

In addition, the markup equation is modified as

$$\begin{aligned} Z\widehat{\mu}_t = &-(1-\alpha)\frac{\kappa\overline{v}}{\alpha\overline{\rho}\overline{L}}\left(\widehat{\theta}_t - \beta\rho_u \mathrm{E}_t\widehat{\theta}_{t+1}\right) \\ &- \beta\delta_2\sigma\left(\mathrm{E}_t\widehat{c}_{t+1} - \widehat{c}_t\right) \\ &+ \delta_2\frac{\overline{\rho}}{1-\overline{\rho}}\left(\widehat{\rho}_t - \beta\mathrm{E}_t\widehat{\rho}_{t+1}\right). \end{aligned} \tag{37}$$

On the other hand, the efficient steady-state condition (34) ensures that the equation (26) between consumption and credit is virtually unchanged up to the first order.

The optimal policy can thus be obtained by maximizing the above utility subject to the Phillips curve of the equation (22), the modified markup of the equation (37), the IS relation of the equation (24), and the expressions for the credit market tightness of the equation (36), and the consumption of the equation (26). This maximization replaces one of the first order conditions (31) by

$$-\psi_\rho(\widehat{\rho}_t - \beta\mathrm{E}_t\widehat{\rho}_{t+1}) - \frac{1}{\alpha\rho}\varphi_{3t} + \frac{\beta\rho_u}{\alpha\rho}\mathrm{E}_t\varphi_{3t+1} + \frac{\overline{L}\delta_2\beta}{\overline{C}}\varphi_{4t} - \frac{\overline{L}\delta_2\beta}{\overline{C}}\mathrm{E}_t\varphi_{4t+1} = 0. \tag{38}$$

In addition, the following first order condition for $\widehat{\rho}_t$ is obtained:

$$\lambda_\rho\widehat{\rho}_t - \psi_\rho(\widehat{l}_t - \widehat{l}_{t-1}) + \frac{\delta}{Z}\delta_2\frac{\overline{\rho}}{1-\overline{\rho}}(\varphi_{1t} - \varphi_{1t-1}) - \frac{\rho_u}{\alpha(1-\overline{\rho})}\varphi_{3t} = 0, \tag{39}$$

or by using the equations (28) and (32),

$$\varphi_{3t} = \frac{\alpha\overline{\rho}}{\rho_u}u_c\overline{L}\left[\overline{\rho}\left(1-\overline{\rho}\right)|f_2|\widehat{\rho}_t - \delta_2\left(\varepsilon\pi_t + \widehat{l}_t - \widehat{l}_{t-1}\right)\right]. \tag{40}$$

The optimal macroprudential policy adjusts the separation rate by taking account of the trade-off among the separation rate, credit market tightness, consumption, and the inflation rate, while all of the separation rate, credit market tightness, and consumption are closely related to the amount of credit $\widehat{l}_t$ by the equations (26) and (36). In fact, we can explicitly derive the following policy by substituting the equations (30) and (40) into the equation (38) to eliminate φ_{3t} and φ_{4t}:

$$A_1\widehat{\rho}_t - \beta A_2\mathrm{E}_t\widehat{\rho}_{t+1} = A_3\left(\varepsilon\pi_t + \widehat{l}_t - \widehat{l}_{t-1}\right) - \beta\delta_2 A_4\mathrm{E}_t\left(\varepsilon\pi_{t+1} + \widehat{l}_{t+1} - \widehat{l}_t\right), \tag{41}$$

where

$$A_1 \equiv \frac{\overline{\rho}}{1-\overline{\rho}}\delta_2 + \frac{\overline{\rho}(1-\overline{\rho})|f_2|}{\rho_u},$$

$$A_2 \equiv \frac{\overline{\rho}}{1-\overline{\rho}}\delta_2 + \overline{\rho}(1-\overline{\rho})|f_2|,$$

$$A_3 \equiv \frac{\overline{L}\delta_2\beta\sigma}{\overline{C}} + \frac{1}{\rho_u},$$

$$A_4 \equiv \frac{\overline{L}\delta_2\beta\sigma}{\overline{C}} + 1.$$

Note that the equation (26) is used to eliminate $\widehat{c}_t$. The form of the equation (41) clearly suggests that the optimal choice of the separation rate is determined by the inflation rate and the increase in the credit $\varepsilon\pi_t + \widehat{l}_t - \widehat{l}_{t-1}$.

6 Concluding Remarks

We introduce the search and matching type of friction into the loan market in a DSGE model. In this model, the second order approximation of social welfare includes the term relating to credit, such as credit market tightness, the volume of credit, and a loan separation rate, in addition to the inflation rate and the output gap. This is a new finding in a field of the optimal policy. The outcome of the optimal policy measure changes in accordance with the type of policy measures for financial stability.

For the future research, the following points could be of interest. The model considered in this paper is restricted to the case where the central bank and monetary authority coordinate their policy choices so as to maximize social welfare. An alternative assumption would be that two policy makers set their respective policies in a non-cooperative way. Moreover, we can assume a situation where the macroprudential policy can have impact on disturbances, even though the monetary policy cannot affect them. They can be disturbances from credit spreads on the policy interest rate. Another issue is a question for what kinds of policies are simple and implementable to replicate the optimal policy measure for financial stability, is as in the case with the Taylor rule in the optimal monetary policy analysis. In such an analysis, it would be interesting to quantitatively evaluate priority among stabilization of credit, the inflation rate, and the output gap.

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Appendix

A Policy Objective Function with Productivity Shock

The models in the main text do not consider the effect of the productivity shock Z_t. This is because, even if Z_t is taken into account as a shock, the mathematical forms of the model would remain the same by taking the difference from an efficient stochastic state equilibrium. In this subsection, we show that the productivity shock alters neither the policy objective function nor any of the linearized structural equations.

When we have a stochastic exogenous productivity Z_t, the second order expansion of the household's utility function around the efficient steady-state equilibrium becomes

$$\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) = -\frac{1}{2} \sum_{i=0}^{\infty} \beta^i \left(\lambda_\pi \pi_{t+i}^2 + \lambda_\theta \widehat{\theta}_{t+i}^2 + \lambda_c \widehat{c}_{t+i}^2 \right)$$
$$+ u_c Z \overline{L} \sum_{i=0}^{\infty} \beta^i \left(\widehat{z}_{t+i} + \frac{1}{2} \widehat{z}_{t+i}^2 \right) + u_c Z \overline{L} \sum_{i=0}^{\infty} \beta^i \widehat{z}_{t+i} \widehat{l}_{t+i} + t.i.p..$$

Here, although the term

$$u_c Z \overline{L} \sum_{i=0}^{\infty} \beta^i \left(\widehat{z}_{t+i} + \frac{1}{2} \widehat{z}_{t+i}^2 \right)$$

is clearly $t.i.p.$, the cross term between $\widehat{z}_{t+i}$ and $\widehat{l}_{t+i}$ seems relevant. In order to eliminate this term, we consider the log-linearized deviation from the dynamics of efficient stochastic state. This efficient stochastic state is obtained by imposing the Hosios condition $b = 1 - \alpha$ and no price markup $\overline{\mu} = 1$, although allowing the productivity shock Z_t to move. We write the log-linearized value of a variable X_t at the efficient stochastic state as x_t^e and the deviation from the efficient stochastic state as $\widetilde{x}_t \equiv \widehat{x}_t - x_t^e$. At the efficient stochastic state, the consumption up to the first order is

$$c_t^e = \frac{\overline{L}}{\overline{C}} \left(Z \widehat{z}_t - \beta \delta_2 l_t^e + \delta_2 l_{t-1}^e \right). \tag{42}$$

On the other hand, at the efficient stochastic equilibrium, the Euler equation (2) becomes

$$c_t^e = \mathrm{E}_t c_{t+1}^e - \frac{1}{\sigma} r_t^e,$$

where r_t^e is the real interest rate. By substituting the equation (42) into this equation, we obtain

$$\widehat{z}_t = \mathrm{E}_t \widehat{z}_{t+1} + \frac{\delta_2}{Z}(-\beta \mathrm{E}_t l^e_{t+1} + l^e_t + \beta l^e_t - l^e_{t-1}) - \frac{\overline{C}}{\sigma Z \overline{L}} r^e_t. \tag{43}$$

The policy objective function can be divided as

$$\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) = A_t + B_t + t.i.p., \tag{44}$$

where

$$A_t = u_c Z \overline{L} \sum_{i=0}^{\infty} \beta^i \widehat{z}_{t+i} \widehat{l}_{t+i} - \sigma u_c Z \overline{C} \delta_2 \left(\frac{\overline{L}}{\overline{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left(\widehat{z}_{t+i} \widehat{l}_{t+i-1} - \beta \widehat{z}_{t+i} \widehat{l}_{t+i} \right)$$

is the collection of the terms that include $\widehat{z}_t$, and

$$B_t = -\frac{1}{2} \lambda_\pi \sum_{i=0}^{\infty} \beta^i \pi^2_{t+i} - u_c \kappa \overline{v} \frac{1-\alpha}{2} \sum_{i=0}^{\infty} \beta^i \widehat{\theta}^2_{t+i} - \frac{1}{2} \sigma u_c \overline{C} \left(\frac{\overline{L}}{\overline{C}} \delta_2 \right)^2 \sum_{i=0}^{\infty} \beta^i \left(-\beta \widehat{l}_{t+i} + \widehat{l}_{t+i-1} \right)^2$$

represents the other terms. By using the equation (43) only for the last term in A_t,

$$\begin{aligned}
A_t &= u_c Z \overline{L} \sum_{i=0}^{\infty} \beta^i \widehat{z}_{t+i} \widehat{l}_{t+i} - \sigma u_c Z \overline{C} \delta_2 \left(\frac{\overline{L}}{\overline{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left(\widehat{z}_{t+i} \widehat{l}_{t+i-1} - \beta \widehat{z}_{t+i+1} \widehat{l}_{t+i} \right) \\
&\quad + \beta \sigma u_c Z \overline{C} \delta_2 \left(\frac{\overline{L}}{\overline{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left[\frac{\delta_2}{Z} \left(-\beta l^e_{t+i+1} + l^e_{t+i} + \beta l^e_{t+i} - l^e_{t+i-1} \right) - \frac{\overline{C}}{\sigma Z \overline{L}} r^e_{t+i} \right] \widehat{l}_{t+i} \\
&= u_c Z \overline{L} \sum_{i=0}^{\infty} \beta^i \widehat{z}_{t+i} \widehat{l}_{t+i} + t.i.p. \\
&\quad + \beta \sigma u_c Z \overline{C} \delta_2 \left(\frac{\overline{L}}{\overline{C}} \right)^2 \sum_{i=0}^{\infty} \beta^i \left[\frac{\delta_2}{Z} \left(-\beta l^e_{t+i+1} + l^e_{t+i} + \beta l^e_{t+i} - l^e_{t+i-1} \right) - \frac{\overline{C}}{\sigma Z \overline{L}} r^e_{t+i} \right] \widehat{l}_{t+i}.
\end{aligned}$$

On the other hand, the equation (14) implies that, at the efficient stochastic equilibrium, we have

$$Z \widehat{z}_t = (1-\alpha) \frac{\kappa \overline{v}}{\alpha \rho \overline{L}} \left(\theta^e_t - \beta \rho_u \mathrm{E}_t \theta^e_{t+1} \right) + \beta \delta_2 r^e_t.$$

By substituting this, A_t can be further simplified as

$$
\begin{aligned}
A_t &= u_c(1-\alpha)\frac{\kappa\overline{v}}{\alpha\rho}\sum_{i=0}^{\infty}\beta^i\left(\theta^e_{t+i}\widehat{l}_{t+i} - \beta\rho_u\theta^e_{t+i+1}\widehat{l}_{t+i}\right) \\
&+ \beta\sigma u_c\overline{C}\left(\delta_2\frac{\overline{L}}{\overline{C}}\right)^2\sum_{i=0}^{\infty}\beta^i\left(-\beta l^e_{t+i+1} + l^e_{t+i} + \beta l^e_{t+i} - l^e_{t+i-1}\right)\widehat{l}_{t+i} + t.i.p. \\
&= u_c(1-\alpha)\frac{\kappa\overline{v}}{\alpha\rho}\sum_{i=0}^{\infty}\beta^i\left(\rho_u\theta^e_{t+i}\widehat{l}_{t+i-1} - \beta\rho_u\theta^e_{t+i+1}\widehat{l}_{t+i}\right) \\
&+ u_c(1-\alpha)\kappa\overline{v}\sum_{i=0}^{\infty}\beta^i\theta^e_{t+i}\widehat{\theta}_{t+i} \\
&+ \beta\sigma u_c\overline{C}\left(\delta_2\frac{\overline{L}}{\overline{C}}\right)^2\sum_{i=0}^{\infty}\beta^i(-\beta l^e_{t+i+1} + l^e_{t+i} + \beta l^e_{t+i} - l^e_{t+i-1})\widehat{l}_{t+i} + t.i.p.,
\end{aligned}
$$

where $\widehat{\theta}_t = \frac{1}{\alpha\rho}(\widehat{l}_t - \rho_u\widehat{l}_{t-1})$ is used. The two terms in the first summation on the right-hand side are again $t.i.p.$, and the last summation can be rewritten as

$$
\begin{aligned}
&\sum_{i=0}^{\infty}\beta^i(-\beta l^e_{t+i+1} + l^e_{t+i})\widehat{l}_{t+i} + \sum_{i=0}^{\infty}\beta^i(\beta l^e_{t+i} - l^e_{t+i-1})\widehat{l}_{t+i} \\
&= \beta^{-1}\sum_{i=0}^{\infty}\beta^i(-\beta l^e_{t+i} + l^e_{t+i-1})\widehat{l}_{t+i-1} - \beta^{-1}(-\beta l^e_t + l^e_{t-1})\widehat{l}_{t-1} + \sum_{i=0}^{\infty}\beta^i(\beta l^e_{t+i} - l^e_{t+i-1})\widehat{l}_{t+i} \\
&= \beta^{-1}\sum_{i=0}^{\infty}\beta^i(-\beta l^e_{t+i} + l^e_{t+i-1})\left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right) + t.i.p..
\end{aligned}
$$

We thus obtain

$$
\begin{aligned}
A_t = \beta\sigma u_c\overline{C}\left(\frac{\overline{L}}{\overline{C}}\delta_2\right)^2 &\sum_{i=0}^{\infty}\beta^i(-\beta l^e_{t+i} + l^e_{t+i-1})\left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right) \\
&+ u_c(1-\rho)\kappa\overline{v}\sum_{i=0}^{\infty}\beta^i\theta^e_{t+i}\widehat{\theta}_{t+i} + t.i.p..
\end{aligned}
$$

This expression for A_t is substituted into the policy objective function of equation (44), yielding

$$
\begin{aligned}
\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) &= \beta\sigma u_c \overline{C}\left(\frac{\overline{L}}{\overline{C}}\delta_2\right)^2 \sum_{i=0}^{\infty}\beta^i(-\beta l^e_{t+i} + l^e_{t+i-1})\left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right) \\
&+ u_c(1-\alpha)\kappa\overline{v}\sum_{i=0}^{\infty}\beta^i\theta^e_{t+i}\widehat{\theta}_{t+i} \\
&- \frac{1}{2}\lambda_\pi\sum_{i=0}^{\infty}\beta^i\pi^2_{t+i} - u_c\kappa\overline{v}\frac{1-\alpha}{2}\sum_{i=0}^{\infty}\beta^i\widehat{\theta}^2_{t+i} \\
&- \frac{1}{2}\sigma u_c\overline{C}\left(\frac{\overline{L}}{\overline{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right)^2 + t.i.p. \\
&= -\frac{1}{2}\lambda_\pi\sum_{i=0}^{\infty}\beta^i\pi^2_{t+i} \\
&- \frac{1}{2}\beta\sigma u_c\overline{C}\left(\frac{\overline{L}}{\overline{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left[\left(-\beta\widehat{l}_{t+i} + \widehat{l}_{t+i-1}\right) - \left(-\beta l^e_{t+i} + l^e_{t+i-1}\right)\right]^2 \\
&- u_c\kappa\overline{v}\frac{1-\alpha}{2}\sum_{i=0}^{\infty}\beta^i\left(\widehat{\theta}_{t+i} - \theta^e_{t+i}\right)^2 + t.i.p. \\
&= -\frac{1}{2}\lambda_\pi\sum_{i=0}^{\infty}\beta^i\pi^2_{t+i} - \frac{1}{2}\beta\sigma u_c\overline{C}\left(\frac{\overline{L}}{\overline{C}}\delta_2\right)^2\sum_{i=0}^{\infty}\beta^i\left(-\beta\widetilde{l}_{t+i} + \widetilde{l}_{t+i-1}\right)^2 \\
&- u_c\kappa\overline{v}\frac{1-\alpha}{2}\sum_{i=0}^{\infty}\beta^i\widetilde{\theta}^2_{t+i} + t.i.p.,
\end{aligned}
$$

which can be simplified as:

$$
\sum_{i=0}^{\infty}\beta^i u(C_{t+i}) = -\sum_{i=0}^{\infty}\beta^i\frac{1}{2}\left(\lambda_\pi\pi^2_{t+i} + \lambda_\theta\widetilde{\theta}^2_{t+i} + \lambda_c\widetilde{c}^2_{t+i}\right).
$$

We therefore confirm that the form of the utility-based policy objective function remains the same even when we introduce the productivity shock. In addition, we can easily see that all the relevant structural equations (22, 23, 24, 25, and 26) can be written identically if we replace $\widehat{x}$ by $\widetilde{x}$ for all the variables X_t.

B Policy Objective Function with Time-Dependent Separation Rate

In this section, we show the derivation of the equations (34) and (35).

The efficient steady-state condition is obtained by the following maximization problem:

$$\max_{C,L,u,\theta,\rho} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \{ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \lambda_{t+i}[f(\rho_{t+i})L_{t+i} - aL_{t+i} - \kappa\theta_{t+i}u_{t+i} - C_{t+i}]$$
$$+ \psi_{t+i}[(1-\rho_{t+i})L_{t+i-1} + \chi\theta_{t+i}^{\alpha}u_{t+i} - L_{t+i}] + s_{t+i}[u_{t+i} - 1 + (1-\rho_{t+i})L_{t+i-1}]\}.$$

By taking the first-order conditions for the five variables and rearranging the equations, we obtain

$$f(\rho_t) - a - \frac{\kappa\theta_t^{1-\alpha}}{\alpha\chi} = -\beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t}(1-\rho_{t+1}) \left(\frac{\kappa\theta_{t+1}^{1-\alpha}}{\alpha\chi} + \frac{\alpha-1}{\alpha}\kappa\theta_{t+1} \right),$$

and

$$f'(\rho_t)\frac{L_t}{L_{t-1}} = \frac{\kappa\theta_t^{1-\alpha}}{\alpha\chi} + \frac{\alpha-1}{\alpha}\kappa\theta_t.$$

At the efficient steady-state equilibrium, the former condition becomes identical to the equation (17), while the latter can be rearranged to the condition (34).

We note that the latter equation, when linearized around the efficient steady-state equilibrium, is written as

$$f_2\overline{\rho}\widehat{\rho}_t + f_1\left(\widehat{l}_t - \widehat{l}_{t-1}\right) = \frac{1-\alpha}{\alpha}\frac{\kappa\overline{\theta}^{1-\alpha}}{\chi}\left(1 - \chi\overline{\theta}^{\alpha}\right),$$

and could be obtained from the optimal policy in the main text, if there were no inflation. This point can be easily confirmed by substituting the equation (29) into the equation (39) to eliminate φ_{3t} and by remembering that φ_{1t} is zero without inflation from the equation (28).

As for the second-order expansion of the utility, both the time-dependence of the separation rate ρ_t and the productivity Z_t make the calculation slightly complicated, although the basic procedure of the derivation is straightforward. For example, the productivity enters the expansion of consumption, and the equation corresponding to the equation (18) becomes:

$$\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2 = \frac{Z\overline{L}}{\overline{C}}(\widehat{z}_t + \frac{1}{2}\widehat{z}_t^2 + \widehat{z}_t\widehat{l}_t - \widehat{q}_t) + \frac{(Z-a)\overline{L}}{\overline{C}}\left(\widehat{l}_t + \frac{1}{2}\widehat{l}_t^2\right) - \frac{\kappa\overline{v}}{\overline{C}}\left(\widehat{v}_t + \frac{1}{2}\widehat{v}_t^2\right).$$

On the other hand, the expansion of the credit vacancy (19) is modified by the time-dependence of the separation rate as:

$$
\begin{aligned}
\widehat{v}_t + \frac{1}{2}\widehat{v}_t^2 = &\left(\widehat{\theta}_t + \frac{1}{2}\widehat{\theta}_t^2\right) - \eta\left(\widehat{l}_{t-1} + \frac{1}{2}\widehat{l}_{t-1}^2\right) - \eta\widehat{\theta}_t\widehat{l}_{t-1} \\
&+ \frac{\overline{\rho}\overline{L}}{\overline{u}}\left(\widehat{\rho}_t + \frac{1}{2}\widehat{\rho}_t^2 + \widehat{\rho}_t\widehat{l}_{t-1} + \widehat{\rho}_t\widehat{\theta}_t\right).
\end{aligned}
$$

After eliminating the credit market tightness $\widehat{\theta}_t$ by using

$$
L_t = (1-\rho_t)L_{t-1} + \chi\theta_t^{\alpha}u_t,
$$

we obtain the following expansion of the utility:

$$
\begin{aligned}
u(C_t) = u(\overline{C}) &+ u_c Z\overline{L}\left(\widehat{z}_t + \frac{1}{2}\widehat{z}_t^2 + \widehat{z}_t\widehat{l}_t - \widehat{q}_t\right) \\
&- u_c\frac{\kappa\overline{v}}{\alpha}\left(1-(1-\alpha)\frac{\overline{\rho}\overline{L}}{\overline{u}}\right)\left(\widehat{\rho}_t + \frac{1}{2}\widehat{\rho}_t^2 + \widehat{\rho}_t\widehat{l}_{t-1}\right) \\
&+ u_c\overline{C}\left[\frac{\overline{L}}{\overline{C}}\left(\delta_1\widehat{l}_t + \delta_2\widehat{l}_{t-1}\right) + \frac{\overline{L}}{2\overline{C}}\left(\delta_1\widehat{l}_t^2 + \delta_2\widehat{l}_{t-1}^2\right) - \frac{\kappa\overline{v}}{\overline{C}}\frac{1-\alpha}{2(\alpha\rho)^2}\left(\widehat{l}_t - \rho_u\widehat{l}_{t-1}\right)^2\right] \\
&- \frac{1}{2}\sigma u_c\overline{C}\left(\frac{\overline{L}}{\overline{C}}\right)^2\left[Z\widehat{z}_t + \delta_1\widehat{l}_t + \delta_2\widehat{l}_{t-1} - \frac{\kappa\overline{v}}{\alpha\overline{L}}\left(1-(1-\alpha)\frac{\overline{\rho}\overline{L}}{\overline{u}}\right)\widehat{\rho}_t\right]^2.
\end{aligned}
$$

At this stage, by using the equations (33) and (34), we observe that the first-order difference between $\widehat{z}_t$ and $\widehat{\rho}_t$ is cancelled, and we finally obtain the equation (35).