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An empirical study of the dynamic correlation of Japanese stock returns

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Abstract

We focus on the pairwise correlations of Japanese stock returns to study their correlation dynamics empirically. Two types of reduced size sample portfolios are created to observe the changes in conditional correlation: a set of individual stock portfolios created by using a network-based clustering algorithm and a single portfolio created from the mean return indexes of the individual sample portfolios. A multivariate GARCH model with dynamic conditional correlation (DCC) is then fitted to the return data of these sample portfolios independently. The estimation results show that the correlation matrices change over time in a way that depends on the sample portfolios; further, the DCC parameters are significantly different between them. Then, the time series of the maximum eigenvalues of the correlation matrices are calculated to observe the changes in correlation intensity. A higher level of correlation intensity is observed during crisis periods, namely after both the Lehman shock and the Great East Japan Earthquake. We also examine the impact of correlation changes on the risk of sample portfolios by using a numerical simulation, with the results showing non-negligible positive impacts. The comparative VaR backtesting simulation also suggests that DCC performs better than CCC.

***Keywords*:** Stock returns, dynamic correlation, DCC–GARCH, clustering, portfolio risk

Contents

1 Introduction

The correlation of asset returns is one of the key issues for quantitative risk measurement and portfolio investment control. Among the many heavily debated issues related to this topic, dynamic changes in correlation remain particularly controversial. Intuitively, the risk of financial asset portfolios measured by a certain risk measure such as value at risk (VaR) comprises two components: variance and covariance. Correlation hence plays a key role in calculating portfolio risk since it directly affects the covariance part. Financial asset returns including stock returns exhibit fat-tailed properties in that large losses occur more frequently with much higher probabilities than those expected by the normal distribution (Fama [1965], Mandelbrot [1963], Mantegna and Stanley [2000]). Another well-known feature of financial returns is volatility clustering: large fluctuations of returns tend to cluster together, resulting in the persistence of volatilities (Cont [2007], Mandelbrot [1963]). These features can significantly affect the estimation of the correlation between asset returns.

Many measures capture the correlation (or comovement) of returns. The most frequently used of these measures is the Pearson product moment correlation; however, this type of linear correlation is significantly distorted for fat-tailed returns, showing a much higher degree of interdependence than actually exists, especially in crisis periods when many assets tend to have larger volatilities. The key issue here is how to overcome volatility fluctuations, which are seemingly a major source of the fat-tailedness of return distributions. In this regard, conditional volatility models such as the generalized autoregressive conditional heteroskedasticity (GARCH) model provide a useful method with which to address the conditional volatility of fat-tailed asset returns.

Another concern is the possible dynamic changes in correlation between asset returns. The choice of dynamic or static correlation is rather an empirical issue that depends on the actual return data. A more complicated dynamic correlation model can involve a static model as a special case; however, such a complicated model requires more resources for model fitting. The choice should thus be carefully made by considering the empirical features of the return data as well as the computational burden of the parameter estimation. Estimating the correlation of fat-tailed asset returns is already a difficult task that only becomes more challenging when the number of assets is large.

In this study, we empirically examine how the correlation between individual Japanese stock returns changes over time. A correlation structure exists between different asset classes such as stocks and bonds as well as between the individual assets that belong to the same asset class. We focus on the latter case to cope with a higher level of dimension, although our research can be extended easily to the former case. We aim to gather useful information regarding the correlation between individual stocks through empirical analyses of stock returns by using a multivariate GARCH (MGARCH) model with dynamic conditional correlation (DCC–GARCH hereafter). Specifically, we propose using a dimension reduction method to build a set of sample portfolios to observe the conditional correlation of returns. Our approach employs clustering techniques of time series data to form homogeneous groups of stocks. The analysis includes assessing the dynamic changes in within-group and between-group correlations to infer the underlying correlation dynamics in the stock market. We also quantitatively evaluate the impact of changes in correlation intensity on the risk of sample portfolios.

Our three research questions are as follows:

- Does the correlation of stock returns change over time?
- More specifically, does correlation intensity change significantly during crisis periods?
    - We focus on the collapse of Lehman Brothers in 2008 (the Lehman shock hereafter) and the Great East Japan Earthquake in 2011 (the Great Earthquake hereafter) as the crisis events.
- Do correlation changes significantly affect portfolio risk?

The remainder of this paper is organized as follows. Section 2 focuses on the development of MGARCH models with conditional correlation in the current body of knowledge on this topic. In Section 3, we discuss the difficulties of modeling a high-dimensional correlation matrix of fat-tailed returns; then, our modeling approach based on DCC–GARCH and the method used to build a sample portfolio that comprises selected individual stocks are described. In Section 4, we fit the model to the return data of the sample portfolios presented in Section 3. The estimation results of the DCC–GARCH models are also summarized. In Section 5, the dynamic changes in correlation intensity are examined for every sample portfolio, focusing on correlation changes during crisis periods. Further, the impact of correlation changes on the risk amount of the sample portfolios in terms of VaR and Expected Shortfall (ES) is analyzed by using numerical simulations. In Section 6, several technical discussion points are listed after the summary of our analysis. Section 7 concludes and offers possible directions of future work.

2 Literature review

Bollerslev [1986] is the seminal paper on the univariate GARCH model, which generalized the ARCH process introduced in Engle [1982] to allow for past conditional variances in the current conditional variance equation. The numerous variants of the GARCH model, such as IGARCH (Engle and Bollerslev [1986]) and EGARCH (Nelson [1991]), can accurately replicate time-varying volatility and volatility clustering to describe the dynamics of the dependency of conditional volatility.

In addition to volatility modeling, the comovement of financial returns is of great practical importance. When extending univariate GARCH to MGARCH, the key issue is how to model the conditional covariances of asset returns. Importantly, the conditional covariance matrix has to be positive definite at any time. Because such a theoretical restriction complicates the estimation of the model, building a flexible but parsimonious model is therefore crucial.

The first generation of MGARCH is the VECH–GARCH model proposed by Bollerslev et al. [1988], which is a natural multivariate extension of univariate GARCH. VECH–GARCH enables flexible multivariate modeling, but the number of parameters increases rapidly as the number of assets increases. The BEKK–GARCH model proposed by Engle and Kroner [1995] ensures positive definiteness; however, it still suffers from the high dimensionality problem. FACTOR–ARCH-type models (Engle et al. [1990], Van der Weide [2002], and Lanne and Saikkonen [2007]) assume common factors that generate the conditional covariances for dimension reduction. The second type of MGARCH model decomposes the conditional covariance matrix of returns into two parts: the conditional volatility and the conditional correlation of the residuals. Bollerslev [1990] first introduced a class of constant conditional correlation (CCC) models, in which conditional correlation is assumed to be constant over time, with only conditional volatility time-varying.

Engle [2002] generalized the CCC model to make the conditional correlation matrix time-varying as in the DCC one. In particular, the author introduced a proxy variable with a GARCH-type structure to establish the positive definiteness of the correlation matrix, whereas VC–GARCH (Tse and Tsui [2002]) formulates the correlation matrix as a weighted sum of past correlations. The advantage of DCC–GARCH is that the dynamics of the correlation matrix are described by a small number of parameters, assuming the same correlation dynamics for all assets. Hence, DCC–GARCH may be applied to large portfolios.

This benefit of DCC, however, becomes too restrictive when the assumption of the same correlation dynamics for all assets does not hold true and thus many variants of DCC have been proposed. For instance, BDCC (Block DCC, Billio et al. [2006]) has a block-diagonal structure to DCC assuming different correlation dynamics; AG–DCC (asymmetric generalized DCC, Cappiello et al. [2006]) incorporates the asymmetry of the dynamics of the proxy variable; STCC (Silvennoinen and Teräsvirta [2005, 2009]) and regime-switching DCC (Pelletier [2006]) introduce smooth change and regime-switching mechanisms to DCC dynamics, respectively; and CDCC (consistent DCC, Aielli [2011]) introduces a corrective step in the proxy variable dynamics to overcome the estimation bias problem of DCC. Moreover, Aielli and Caporin [2013, 2014] proposed a clustering method to reduce the complexity of large-scale DCC–GARCH models, in which the GARCH model parameter matrices depend on the clustering of individual assets.

Finally, other extensions of DCC–GARCH offer a flexible choice of non-Gaussian distributions as the residual distribution. As mentioned in Lee and Long [2009], the copula-based method can be applied to many MGARCH models including DCC and CCC to link the marginals. The research presented by Jondeau and Rockinger [2006], Patton [2006], and Lee and Long [2009] are just some of the many theoretical and empirical studies in the literature.

3 Modeling the correlation of stock returns

3.1 Fat-tailedness and the correlation of stock returns

This study examines the correlation of individual asset returns within the same asset class, namely Japanese stock returns. In particular, we aim to observe how the correlation between individual stock returns changes over time, as mentioned in Section 1. More than 3,000 stocks are listed on the Tokyo Stock Exchange, with at least 1,700 of these listed on the First Section that includes larger stocks (blue chips). Here, we are only interested in the whole market portfolio that covers every stock listed. However, because handling a correlation matrix of returns at such a large scale is challenging, some dimension reduction operation is required before carrying out the empirical analysis.

When modeling the correlation structure of asset returns, a factor model approach is frequently employed in which one or multiple factors are defined according to external or internal information on the financial asset. Dimension reduction can be easily accomplished in factor models as the pairwise correlations between individual assets are attributed to their responses to the common factor(s). However, the most pressing problem of this approach is identifying the factor. CAPM or Fama and French n-factor models are good examples of the factor model approach for stock price modeling. The market factor here can be defined as the mean return of all stocks or a stock price index (e.g., the Nikkei225). Nevertheless, those factors may not be sufficiently reliable to approximate the correlation of asset returns.

A more direct approach to observe the correlation structure is thus to calculate a sample linear correlation matrix of asset returns during a defined observation period. Although the sample linear correlation matrix approach may suffer from insufficient positive definiteness, especially in large portfolios, several methods have been proposed to recover positive definiteness including a shrinkage estimator (Ledoit and Wolf [2003]). Nonetheless, the sample linear correlation matrix can still be significantly distorted by the fat-tailedness of returns. An MGARCH model is useful to work around this problem; here, fat-tailedness is removed, or significantly reduced, by controlling volatility fluctuation. In the context of the CCC– or DCC–GARCH models, the correlation matrix is defined as the correlation of standardized residuals, which is independently and identically distributed (i.i.d.). Thus, we use DCC–GARCH to model the conditional correlation matrix of Japanese stock returns.

3.2 DCC–GARCH

Let $(\Omega,\ \boldsymbol{\mathcal{F}},\ \{\boldsymbol{\mathcal{F}}_t\},\ \mathbb{P})$ be a filtered probability space equipped with the filtration $\{\boldsymbol{\mathcal{F}}_t\}$ of its σ-field $\boldsymbol{\mathcal{F}}$ on a set Ω and probability measure $\mathbb{P}$ on $(\Omega,\ \boldsymbol{\mathcal{F}})$. Consider multiple asset returns as a stochastic vector process $\boldsymbol{r}_t$ that is assumed to be described as

$$\boldsymbol{r}_t = \mathrm{E}\left(\boldsymbol{r}_t | \boldsymbol{\mathcal{F}}_{t-1}\right) + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\mathrm{E}\left(\cdot|\cdot\right)$ denotes a conditional expectation operator with respect to the measure $\mathbb{P}$, $\boldsymbol{\mathcal{F}}_{t-1}$ is the filtration (information set) at time $t-1$, generated by the observed series $\boldsymbol{r}_t$ up to and including $t-1$, and $\boldsymbol{\varepsilon}_t$ is a vector of unpredictable residuals.

Assuming the predictable conditional (time-varying) mean and volatility of $\boldsymbol{r}_t$, equation (1), is written as[1]

$$\begin{aligned} &\boldsymbol{r}_t = \boldsymbol{\mu}_t + \boldsymbol{H}_t^{1/2} \boldsymbol{z}_t, \\ &\boldsymbol{\mu}_t = \mathrm{E}\left(\boldsymbol{r}_t | \boldsymbol{\mathcal{F}}_{\boldsymbol{t-1}}\right),\ \mathrm{E}\left(\boldsymbol{z_t}\right) = \boldsymbol{0},\ \mathrm{Var}\left(\boldsymbol{z_t}\right) = \boldsymbol{I}_N \end{aligned} \tag{2}$$

where $\boldsymbol{\mu}_t$ is a vector of conditional means at time t, $\boldsymbol{H}_t$ is an $N \times N$ (N is the number of returns) symmetric positive definite matrix, which is a conditional variance–covariance

[1] The description of MGARCH models follows Bollerslev [1990] and Ghalanos [2014] with some modifications.

matrix of $\boldsymbol{r}_t$, $\mathrm{Var}(\cdot)$ is a variance operator, and $\boldsymbol{z}_t$ is a vector of i.i.d. standardized residuals, the mean and variance of which are $\mathbf{0}$ and $\boldsymbol{I}_N$: an identity matrix of order N, respectively.[2] Further, $\boldsymbol{z}_t$ follows a multivariate distribution, although this distribution is only specified when estimating the model.

As for the matrix process $\boldsymbol{H}_t$, there are generally two approaches, namely modeling the conditional covariance matrix $\boldsymbol{H}_t$ directly (e.g., VECH or BEKK models) and modeling the conditional correlation matrix indirectly by using a correlation matrix (e.g., CCC and DCC models). We adopt the latter approach in which only the variance part of $\boldsymbol{H}_t$ is modeled explicitly.

Three factors must be considered when running a multivariate model: the interactions of the individual mean processes and volatility processes as well as the correlation structure of the standardized residuals. For the interactions mentioned above, we follow the standard simplified settings frequently used to reduce the computational burden of the parameter estimation. The two sub-models are then implemented as the mean and volatility models.

Mean model

The conditional mean process is modeled separately for each stock return to allow us to estimate each autoregressive moving average (ARMA) model independently as

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \sum_{i=1}^{P} \boldsymbol{A}_i \boldsymbol{r}_{t-i} + \sum_{j=1}^{Q} \boldsymbol{B}_j \boldsymbol{\varepsilon}_{t-j} + \boldsymbol{\varepsilon}_t \tag{3}$$

where $\boldsymbol{A}_i$ and $\boldsymbol{B}_j$ are diagonal matrices.[3]

Variance model

The equation of the volatility dynamics comprises a simple vector form of the univariate GARCH(p, q) model as

$$\boldsymbol{h}_t = \boldsymbol{\omega} + \sum_{i=1}^{q} \boldsymbol{S}_i \boldsymbol{\varepsilon}_{t-i} \odot \boldsymbol{\varepsilon}_{t-i} + \sum_{j=1}^{p} \boldsymbol{T}_j \boldsymbol{h}_{t-j} \tag{4}$$

[2] The variance of $\boldsymbol{r}_t$ is confirmed to be $\boldsymbol{H}_t$ as

$$\mathrm{Var}\left(\boldsymbol{r}_t | \mathcal{F}_{t-1}\right) = \mathrm{Var}_{t-1}\left(\boldsymbol{r}_t\right) = \boldsymbol{H}_t^{1/2} \mathrm{Var}_{t-1}\left(\boldsymbol{z}_t\right) \left(\boldsymbol{H}_t^{1/2}\right)' = \boldsymbol{H}_t$$

where $\mathrm{Var}(\cdot|\cdot)$ is a conditional variance operator. Note that $\boldsymbol{H}_t$ is assumed to be deterministic in the context of the GARCH model. The correlation of $\boldsymbol{r}_t$ is equivalent to that of $\boldsymbol{z}_t$, since $\boldsymbol{H}_t^{1/2}$ does not affect the correlation.

[3] The degree $(P,\ Q)$ can take different values for every stock return, while the values and diagonal elements of $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are determined empirically.

where $\odot$ denotes the Hadamard operator (the entry-wise product), $\boldsymbol{h}_t$ is the diagonalized matrix of $\boldsymbol{H}_t$, and both $\boldsymbol{S}_i$ and $\boldsymbol{T}_j$ are diagonal matrices.[4] Note that equation (4) only models the variance of $\boldsymbol{r}_t$ as $\boldsymbol{h}_t$; the covariance of $\boldsymbol{r}_t$ is not modeled. Note also that equation (4) with the diagonal coefficient matrices means that there are no inter-temporal volatility spillover effects between stock returns.[5] While this assumption enables us to estimate the univariate GARCH model separately, such a simplification may be too restrictive and can lead to a biased estimation result. This point is the major drawback of this modeling approach.

Further, it is possible to adopt a more flexible GARCH structure; however, we adopt the simple linear GARCH model to reduce the model fitting burden for a large number of stock returns.

The above-mentioned mean and variance models can be estimated by fitting the univariate ARMA–GARCH model to historical data on individual stock returns.

Correlation structure (CCC and DCC)

The third part to be implemented is the correlation of the residuals $\boldsymbol{z}_t$, which is the same as the correlation of returns $\boldsymbol{r}_t$, as mentioned earlier. The CCC of Bollerslev [1990] is a typical unconditional correlation model, in which an $N \times N$ positive definite constant correlation matrix $\boldsymbol{R}$ is defined as

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R} \boldsymbol{D}_t = \left[\rho_{kl}\sqrt{h_{kk\cdot t} h_{ll\cdot t}}\right]_{k,\ l=1,\ \ldots,\ N} \tag{5}$$

where $\boldsymbol{D}_t$ is a diagonal matrix with the elements of $\boldsymbol{H}_t$ as $\left(\sqrt{h_{11\cdot t}},\ \ldots,\ \sqrt{h_{NN\cdot t}}\right)$ and ρ_{kl} is the unconditional correlation of the returns between stock k and l.

The DCC–GARCH model proposed by Engle [2002] replaces $\boldsymbol{R}$ in the CCC with dynamic correlation $\boldsymbol{R}_t$:

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t = \left[\rho_{kl\cdot t}\sqrt{h_{kk\cdot t} h_{ll\cdot t}}\right]_{k,\ l=1,\ \ldots,\ N} \tag{6}$$

where $\rho_{kl\cdot t}$ is the conditional correlation of returns between stock k and l at time t. DCC has been widely used to implement dynamic correlation in MGARCH.

4 The degree $(p,\ q)$ can take different values for every stock return, while the values and diagonal elements of $\boldsymbol{S}_i$ and $\boldsymbol{T}_i$ are determined empirically.

5 We have not defined $\boldsymbol{H}_t^{1/2}$ in equation (2). The decomposition from $\boldsymbol{H}_t$ to $\boldsymbol{H}_t^{1/2}$ is apparent for the diagonal matrix $\boldsymbol{h}_t$. $\sqrt{\boldsymbol{h}_t}$ is defined as the volatility of $\boldsymbol{r}_t$.

Correlation dynamics

DCC is more flexible than CCC; however, the number of parameters in $\boldsymbol{R}_t$ increases significantly when the number of stocks becomes large. Moreover, because the correlation matrix can change depending on time t, ensuring that every correlation matrix satisfies the positive definite condition throughout the entire period is a challenge. Engle [2002] satisfied this constraint by modeling a dynamic correlation process with the proxy variable $\boldsymbol{Q_t}$.[6] The proxy variable $\boldsymbol{Q}_t$ is modeled as

$$\begin{aligned}\boldsymbol{Q}_t &= \bar{\boldsymbol{Q}} + \sum_{i=1}^{m} a_i \left(\boldsymbol{z}_{t-i}\boldsymbol{z}_{t-i}^{'} - \bar{\boldsymbol{Q}}\right) + \sum_{j=1}^{n} b_j \left(\boldsymbol{Q}_{t-i} - \bar{\boldsymbol{Q}}\right) \\ &= \left(1 - \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j\right) \bar{\boldsymbol{Q}} + \sum_{i=1}^{m} a_i \boldsymbol{z}_{t-i}\boldsymbol{z}_{t-i}^{'} + \sum_{j=1}^{n} b_j \boldsymbol{Q}_{t-j}\end{aligned} \tag{7}$$

where a_i and b_j are non-negative scalars and $\bar{\boldsymbol{Q}}_t$ is the unconditional matrix of the standardized residual z_t. The DCC model with time lags in conditional correlation is described as DCC $(m,\ n)$. The parameter a_i shows the sensitivity of $\boldsymbol{Q}_t$ to previous shocks, while the parameter b_j represents the persistence of correlation in previous periods. The concept of dynamic modeling is similar to volatility process modeling in the GARCH model. The correlation matrix $\boldsymbol{R}_t$ is then obtained by rescaling $\boldsymbol{Q}_t$ such that,

$$\boldsymbol{R}_t = \mathrm{diag}\left(\boldsymbol{Q}_t\right)^{-\frac{1}{2}} \boldsymbol{Q}_t \mathrm{diag}\left(\boldsymbol{Q}_t\right)^{-\frac{1}{2}}. \tag{8}$$

The positive definiteness of $\boldsymbol{Q}_t$ as well as $\boldsymbol{R}_t$ is ensured by the following conditions:

$$a_i \geq 0, \quad b_j \geq 0, \quad \sum_{i=1}^{m} a_i + \sum_{j=1}^{n} b_j < 1. \tag{9}$$

For more details on the DCC–GARCH, see Engle and Sheppard [2001] and Engle [2002].

An inconsistency problem exists when estimating $\bar{\boldsymbol{Q}}$ in equation (7) with variance targeting. Aielli [2011] pointed out that $\bar{\boldsymbol{Q}}$ is not the unconditional covariance matrix of $\boldsymbol{z}_t$, as $\mathrm{E}\left[\boldsymbol{z}_t\boldsymbol{z}_t^{'}\right] = \mathrm{E}\left[\boldsymbol{R}_t\right] \neq \mathrm{E}\left[\bar{\boldsymbol{Q}}\right]$. Instead, the author proposed CDCC, which includes a corrective term for bias adjustment.[7]

While DCC–GARCH models have many technical limitations,[8] the parsimonious parameterization of the dynamic correlation is helpful for our empirical study. We hence choose unrestricted scalar DCC–GARCH to model the conditional correlation, even though improved estimation performance can be expected by applying more complicated models.

[6] We follow the notation of the DCC–GARCH of Engle [2002] with some modifications.

[7] Some studies have already addressed this issue, including Engle and Kelly [2012].

[8] The limitations of DCC–GARCH models are discussed in more detail in Caporin and McAleer [2013]

4 Model fitting and estimation results

4.1 Building reduced size sample portfolios

Data on stock returns

Before discussing our approach for fitting the DCC–GARCH model, we first identify the stock return data and define the whole universe of stocks. The data frequency is daily; the period runs from the beginning of January 2008 to the end of December 2013 to include the two major financial shocks examined herein: the Lehman shock (2008) and the Great Earthquake (2011). The whole universe of stocks comprises those listed on the First Section of the Tokyo Stock Exchange that have complete daily price data (at close) for the given period. These selection criteria have been introduced to avoid any inconsistency in the time series when calculating the correlations. The total number of stocks in the universe is 1,354 in 33 sectors. Price data are converted into daily log-returns.

Dimension reduction of the correlation matrix

To achieve unbiased observations, we need a data set that covers all the stocks in the universe such as a market portfolio rather than one that focuses on specific stocks. However, using such a large portfolio complicates the correlation analysis, since a 1,300 $\times$ 1,300 correlation matrix is too large to fit a single DCC–GARCH model. We hence propose creating sample portfolios to reduce the magnitude of the data. Specifically, we overcome the complexity of a large-scale correlation structure not by building a more generalized and complicated model but by reducing the data structure to apply a simple but robust model. The main issue here is the selection of individual stocks to be included in the sample portfolios. Some stock index portfolios such as the Nikkei225 are options, but more flexible choices with wider coverage would be preferred.

To create reduced size sample portfolios, one approach would be to divide the whole universe into several homogeneous groups. This approach is similar to common factor modeling, but without the need to identify those factors. If such a grouping were available, observing changes in within-group and between-group correlations in a reduced dimension could be possible. Further, the groups would not only be homogeneous but also be balanced in size to avoid bias and concentration problems.

The standard sector classification that comprises 33 sectors is frequently used to categorize stocks. Sample portfolios can be created by selecting representatives from these 33 sectors based on certain criteria. Such a sector classification approach, however, has a fundamental problem in that the distribution of group size is significantly unbalanced. Moreover, it is not necessarily consistent with the comovement of stock returns, since the classification is based on the definitions of the business sectors. The use of sector classification to select sample portfolios may thus cause bias to arise.

Against this background, a more data-oriented grouping of stock returns was studied in Isogai [2014]. Fourteen homogeneous and balanced groups of stocks were identified by applying correlation clustering based on complex networks theory as shown in Table 1. Homogeneity means that the stocks grouped together show a higher level of correlation than those that belong to different groups. The correlation matrix is calculated by fitting the CCC–GARCH model to avoid the distortion effect caused by the fat-tailedness of returns. Two major categories of groups—cyclical and defensive—are also identified and adopted to create the sample portfolios. For more detailed information on the clustering algorithm, see Isogai [2014].[9] Two types of sample portfolios are created based on this grouping.

[9] The total number of stocks is slightly smaller in this study than in Isogai [2014] because some stocks had been delisted from the Tokyo Stock Exchange. The data period has also been updated.

Table 1: Stock group and over-expressing sectors

	Group name	(GID)	Number of stocks	Mean correlation	Mean Topix Beta		Typical sectors
Cyclical	G_A	(25)	88	0.39	1.05		Electric Appliances, Transportation Equipment, Precision Instruments
	G_B	(11)	143	0.36	0.96		Iron and Steel, Nonferrous Metals, Marine Transportation, Securities
	G_C	(26)	108	0.32	0.82	0.77	Transportation Equipment, Machinery, Rubber Products
	G_D	(30)	107	0.25	0.67		Other Financing business
	G_E	(29)	97	0.23	0.56		-
	G_F	(13)	168	0.19	0.58		Construction, Textiles and Apparels, Real Estate
Defensive	G_G	(22)	56	0.39	0.65		Banks
	G_H	(15)	67	0.26	0.63		Construction
	G_I	(21)	56	0.26	0.58		Information and Communication, Land Transportation
	G_J	(16)	120	0.25	0.47	0.48	-
	G_K	(17)	91	0.24	0.40		-
	G_L	(19)	72	0.22	0.46		Electric Power and Gas, Pharmaceutical, Foods, Land Transportation
	G_M	(18)	99	0.21	0.31		Information and Communication
	G_N	(20)	82	0.18	0.37		Retail Trade, Foods
		Total	1,354				

Note: GID is the group ID number that was originally used to identify the 14 groups in Isogai [2014]. Topix Beta is calculated as the sensitivity of the rates of return on an individual stock compared with the rates of return of the TOPIX index. The two categories cyclical and defensive correspond to the two largest groups identified in the first round of recursive clustering, which reflect the level of mean TOPIX Beta. Typical sector is determined statistically by using the hypergeometric test as the sector that characterizes the corresponding group significantly. For more details on the definitions and methods used, see Isogai [2014].

Group portfolios

The first type of sample portfolio is a set of partial portfolios, which covers only specific groups identified by the correlation clustering mentioned above. The large-scale single correlation matrix is separated into 14 diagonal blocks. The 14 sample portfolios are then created based on the grouping to observe within-group dynamic correlation. The number of stocks in these 14 groups is around 100 on average, which is still large for estimating the dynamic correlation of returns when using the DCC–GARCH model. Thus, a second round of dimension reduction is required to specify the individual stocks to be included in each sample portfolio.

To identify and select the stocks in each group, we adopt the eigenvector centrality measure, which is frequently used in network analyses. Network centrality is one of the structural characteristics of a node in a network; an individual with a higher centrality measure is often more likely to be a leading individual according to network theory.[10] The eigenvector centrality of a node is defined as an element of the eigenvector of a network adjacency matrix with the maximum eigenvalue. Here, a node corresponds to a stock, while a network corresponds to the group to which the stock belongs. The eigenvector centrality measure is designed to provide a higher score to a node that has more links to a node with many links. In the context of stock returns, the eigenvector centrality of a stock is higher when it is correlated more with a stock that is highly correlated with other stocks.

Technically, the centrality measure is generally assumed to take a positive value.[11] Our network adjacency matrix is designed to be a non-negative regular matrix; therefore, we can safely define the eigenvector centrality measure. For more detailed information on the eigenvector centrality measure, see Newman [2008].

Finally, the stocks that have the 20 largest eigenvector centrality values are selected to create a sample portfolio for each group. The coverage of the total number of stocks selected is about 20% of the total stocks.[12] The 14 individual group models are built on the selected 20 stocks. We define these sample portfolios as group portfolios, the correlation matrices of which are all of equal size. Note that the selection of the stocks depends on

[10] Typical centrality measures include degree centrality, closeness centrality, and betweenness centrality.

[11] The Perron–Frobenius theorem ensures that the eigenvector centrality measure takes a positive number. This theorem ensures that there is a unique eigenvector of matrix $\boldsymbol{A}$ with the largest positive eigenvalue; further, the eigenvector is positive and any non-negative eigenvector of $\boldsymbol{A}$ is a positive multiple of the vector, on condition that $\boldsymbol{A}$ is a non-negative regular matrix.

[12] The 20 largest values were used by balancing the coverage of stocks in the universe and the complexity of the parameter estimation and evaluation.

the centrality measure used; therefore, other centrality measures may suggest a different set of stocks.

Market portfolio

The second type of sample portfolio covers the entire market. To reduce the dimensions of the correlation matrix of the whole universe, an equally weighted stock portfolio is first created for each group. Note that each group portfolio includes all of the stocks that belong to the group at this stage. Then, the return index of each portfolio is calculated as the mean of the individual stock returns in each group. The 14 return indexes can be regarded as the underlying factors of the development of the stock market, since any stock could belong to one of these 14 groups. Lastly, a single sample portfolio is created as an equally weighted portfolio of the 14 return indexes to observe market-wide or between-group (between-factor) dynamic correlation.

Non-constant correlation test

As mentioned in Section 1, the choice of dynamic or static correlation is rather an empirical issue that depends on the actual return data. Before delving into the details of the DCC–GARCH model estimation, it is informative to examine if the static correlation is statistically acceptable for our data.

In that context, we perform the non-constant correlation test proposed by Engle and Sheppard [2001] for the market portfolio and group portfolios. The GARCH(1, 1) model, which is uniformly assumed to be a typical GARCH model, is first fitted to the individual return data on every portfolio to calculate standardized residuals. The constant correlation is then calculated from the standardized residuals. The null hypothesis (H_0) is $R_t = \bar{R}$. The test is based on an artificial regression of the outer products of the residuals on a constant and lagged outer products to explore if there is any time dependency between R_t and $R_{t-1}, \cdots, R_{t-i}$. The numbers of lags are set to 5 and 10.

Table 2 shows the test results. In many cases, we can safely reject the null hypothesis in favor of the dynamic correlation model rather than the static one. This result provides strong motivation to estimate the DCC–GARCH model with a more detailed specification, although the test assumes a simple univariate GARCH(1, 1) model and has some technical limitations.[13]

[13] Engle and Sheppard [2001] discussed the technical difficulties associated with testing the null of constant correlation against an alternative of dynamic correlation. More recently, McCloud and Hong [2011] proposed a specification test for the constant and dynamic structures of conditional correlations, which is based on a generalized spectrum approach. Other testing approaches and their technical limitations

Table 2: Constant correlation test

		5 lags		10 lags	
		Stat	P-value	Stat	P-value
Market		14.800	0.022	18.567	0.069
Cyclical	G_A	22.618	0.001	24.566	0.011
	G_B	39.777	0.000	65.617	0.000
	G_C	11.613	0.071	30.414	0.001
	G_D	35.870	0.000	44.261	0.000
	G_E	46.986	0.000	52.558	0.000
	G_F	60.859	0.000	64.939	0.000
Defensive	G_G	29.062	0.000	32.831	0.001
	G_H	12.087	0.060	22.925	0.018
	G_I	18.305	0.006	27.586	0.004
	G_J	22.597	0.001	30.010	0.002
	G_K	59.057	0.000	64.219	0.000
	G_L	84.126	0.000	87.503	0.000
	G_M	93.684	0.000	105.109	0.000
	G_N	28.176	0.000	39.241	0.000

Note: "Stat" is the test statistic of the non-constant correlation test proposed by Engle and Sheppard [2001], which is asymptotically distributed as a chi-squared distribution. P-value is calculated for the null hypothesis (H_0): $R_t = \bar{R}$. For more details of the test, see Engle and Sheppard [2001].

4.2 Modeling the dependency of returns using the copula function

To estimate the parameters of the DCC–GARCH model by using MLE, the likelihood function needs to be specified. Two approaches can be used to build the conditional joint distribution of return $\boldsymbol{r}_t$ in equation (2). The first approach assumes a multivariate distribution (e.g., the multivariate normal) to specify the density function to maximize the log-likelihood with respect to the model parameters. In the case of the normal distribution, the maximization process can be simplified by separating the first-stage estimation of the individual GARCH models from the second-stage DCC parameter estimation. However, because the assumption of a normal distribution might not apply in every case, we select an alternative approach based on the copula function to model the dependency structure of the residuals.

The concept of the copula of an arbitrary distribution is a function to connect the marginal distributions to a joint distribution. The joint distribution function $F(x_1, ..., x_n)$ of a vector of variables $\boldsymbol{X} = (X_1, \ldots, X_n)$ with marginal distribution functions

are also summarized there.

$F_1(x_1), \ldots, F_n(x_n)$ can be represented by the copula function $C(\cdot)$ as

$$F(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)) \tag{10}$$

under absolutely continuous margins (Sklar's Theorem, Sklar [1959]). Considering that $x_1, ..., x_n = F_1^{-1}(u_1), ..., F_n^{-1}(u_n)$, the copula is obtained uniquely as

$$C(u_1, ..., u_n) = F\left(F_1^{-1}(u_1), ..., F_n^{-1}(u_n)\right) \tag{11}$$

where $F_i^{-1}(\cdot)$ is the quantile function of the i-th marginal distribution. Consequently, the joint density function $f(x)$ of $\boldsymbol{X}$ can be described as

$$f(x_1, ..., x_n) = c(F_1(x_1), ..., F_n(x_n)) \prod_{i=1}^{n} f_i(x_i) \tag{12}$$

where $f_i(x_i)$ is the marginal distribution of x_i and $c(\cdot)$ is the density function of the copula. The joint density of returns $\boldsymbol{r}_t$ is defined as a combination of the copula density and the density of the i.i.d. residual $\boldsymbol{z}_t$, as described by equation (12).

As for the marginal distribution of the individual residuals $\boldsymbol{z}_t$, we assume one of the (standardized) normal, Student t, and skew t distributions.[14] The parameter set to be estimated for the i-th return includes the ARMA–GARCH parameters as θ_i^{AG} and distributional parameters of z_i as θ_i. The parameters in θ_i depend on the distribution type: θ_i includes ξ_i and ν_i for the skew t, ν_i for the Student t, and none for the normal, where ν_i and ξ_i are the shape and skew parameters, respectively. As such, the use of the copula enables the flexible modeling of the marginal distributions. Further, the separation of the fat-tailedness of residuals and tail dependency between them enables a more precise parameter estimation. On the contrary, the multivariate distribution approach assumes the same marginal distribution for all stocks.

The dependence structure of the marginals is modeled by using a copula; specifically, we select the Student t-copula, since we assume possible tail dependency between the residuals. The Student t-copula can handle tail dependency, whereas the Gaussian copula cannot. The Student t-copula is defined as

$$C^{S_t}(\boldsymbol{u}|\nu, \boldsymbol{R}) = \boldsymbol{t}_{\nu \cdot \boldsymbol{R}}\left(t_\nu^{-1}(u_1), \ \ldots, \ t_\nu^{-1}(u_n)\right) \tag{13}$$

where $\boldsymbol{R}$ is a correlation matrix, ν is a shape parameter, $t_\nu(\ )$ is the cdf of the univariate Student t-distribution, and $\boldsymbol{t}_{\nu \cdot \boldsymbol{R}}$ is the cdf of the multivariate Student t-distribution. The

[14] We use the skew t-distribution defined by Fernández and Steel [1998].

density function of the Student t-copula is defined as

$$c^{S_t}\left(\boldsymbol{u}|\nu, \boldsymbol{R}\right) = \frac{\Gamma\left(\frac{\nu+n}{2}\right)\left(\Gamma\left(\frac{\nu}{2}\right)\right)^n \left(1+\nu^{-1}\boldsymbol{q'}\boldsymbol{R}^{-1}\boldsymbol{q}\right)^{-(\nu+n)/2}}{\sqrt{|\boldsymbol{R}|}\left(\Gamma\left(\frac{\nu+n}{2}\right)\right)^n \Gamma\left(\frac{\nu}{2}\right)\prod_{i=1}^{n}\left(1+\frac{q_i^2}{\nu}\right)^{-(\nu+1)/2}} \tag{14}$$

where $\boldsymbol{q} = (q_1, ..., q_n)$ is defined such that $q_i = t_\nu^{-1}(u_i)$ for $i = 1, \ldots, n$. For more details on the Student t-copula, see Demarta and McNeil [2005].

Then, the conditional joint density of returns $\boldsymbol{r}_t$ can be defined as a combination of the copula density and density of the i-th residual $z_{i\cdot t}$ based on equation (12), substituting N (the number of stocks) for n:

$$f\left(\boldsymbol{r}_t|\boldsymbol{\mu}_t,\ \sqrt{\boldsymbol{h}_t},\ \boldsymbol{R}_t,\ \bar{\nu}\right) = c^{S_t}\left(u_{1\cdot t},\ \ldots,\ u_{N\cdot t}|\boldsymbol{R}_t,\ \bar{\nu}\right)\prod_{i=1}^{N}\frac{1}{\sqrt{h_{i\cdot t}}}f_{i\cdot t}\left(z_{i\cdot t}|\theta_i\right) \tag{15}$$

where $u_{i\cdot t} = F_i(r_{i\cdot t}|\mu_{i\cdot t}, \sqrt{h_{i\cdot t}}, \theta_i)$, $c^{S_t}(\cdot)$ is the Student t-copula density defined in equation (14), and $\bar{\nu}$ is the shape parameter of the Student t-copula.[15]

The log-likelihood function $LL\left(\boldsymbol{\theta}|\boldsymbol{r}_t\right)$ is given by the density function (15) as

$$\begin{aligned} LL\left(\boldsymbol{\theta}|\boldsymbol{r}_t\right) &= LL_R\left(\boldsymbol{R}_t,\ \bar{\nu}\right) \\ &\quad +LL_V\left(\left(\theta_1,\ \mu_{1\cdot t},\ \sqrt{h_{1\cdot t}}\right), \ldots, \left(\theta_N,\ \mu_{N\cdot t},\ \sqrt{h_{N\cdot t}}\right)\right) \\ &= LL_R\left(\boldsymbol{a}_1,\ \ldots,\ \boldsymbol{a}_N,\ \boldsymbol{b}_1,\ \ldots,\ \boldsymbol{b}_N,\ \bar{\nu}\right) \\ &\quad +LL_{V_1}\left(\theta_1,\ \theta_1^{AG}\right)+,\ldots,+LL_{V_N}\left(\theta_N,\ \theta_N^{AG}\right) \end{aligned} \tag{16}$$

where $\boldsymbol{\theta}$ is the whole parameter set, $LL_R(\cdot)$ is the Copula–DCC part with the DCC parameters ($\boldsymbol{a}$, $\boldsymbol{b}$) as in equation (7), and $LL_{V_i}(\cdot)$ is the univariate ARMA–GARCH part with a set of parameters θ_i^{AG} for stock i $(i = 1,\ \ldots,\ N)$.

As such, the log-likelihood can easily be separated into two parts when maximizing $\sum_{t=1}^{p} LL(\cdot)$, where p is the length of the time series data: the joint Copula–DCC part and the individual univariate GARCH part. The two parts of the log-likelihood function can be safely maximized independently without any shared parameters between them. Thus, the individual ARMA–GARCH parameters as well as their distributional parameters are estimated first for the individual stocks by maximizing LL_{Vi}; then, the Copula–DCC parameters are estimated by maximizing LL_R.

4.3 Estimation results of DCC–GARCH

The DCC–GARCH model is simply fitted to the market portfolio and 14 group portfolios, independently. When estimating the DCC–GARCH model for the market portfolio, the

[15] $\frac{1}{\sqrt{h_{i\cdot t}}}$ in equation (15) is the Jacobian of the variable transformation between r_t and z_t.

univariate ARMA–GARCH models are first fitted to the individual group return indexes defined in Section 4.1 based on the two-step estimation approach described in Section 4.2. The ARMA–GARCH lags and residual distribution should be determined to identify the model (model selection). The multiple models with different lag patterns and choices of residual distribution are then estimated by using MLE, and the model with the highest AIC is selected for every return index. In the second step, the DCC lags are determined similarly by selecting the model with the highest AIC from the alternatives. Specifically, the Copula–DCC model is fitted to the standardized residuals to estimate the DCC model parameters by using MLE. The whole likelihood maximization process shown in equation (16) is thus completed. Similar to the market portfolio, the univariate ARMA–GARCH model is first fitted to the individual stock returns when estimating the DCC–GARCH model for the group portfolios. The remaining estimation process is the same as that for the market portfolio.

Table 3 shows the estimation results for the DCC parameters.[16] The results of the univariate ARMA–GARCH model for the market portfolio are summarized in Table 4 (for the cyclical groups) and Table 5 (for the defensive groups). The estimation results of the univariate ARMA–GARCH model for the group portfolios are omitted because of space limitations.

[16] We used the R (http://cran.r-project.org/) package "rmgarch" (Ghalanos [2014]) for the parameter estimation.

Table 3: DCC estimation results

Group ID	m, n	$a1$	Std error	$b1$	Std error	$b2$	Std error	$b1 + b2$	Shape	Std error
Market	1, 1	0.018	(0.004)	0.939	(0.021)		-	0.939	14.2	(0.959)
Cyclical										
G_A	1, 2	0.009	(0.002)	0.293	(0.130)	0.638	(0.131)	0.931	18.8	(0.945)
G_B	1, 1	0.010	(0.001)	0.930	(0.015)		-	0.930	20.9	(1.257)
G_C	1, 1	0.008	(0.002)	0.881	(0.039)		-	0.881	22.0	(1.279)
G_D	1, 2	0.012	(0.002)	0.301	(0.114)	0.610	(0.112)	0.911	28.9	(2.282)
G_E	1, 2	0.013	(0.003)	0.424	(0.092)	0.368	(0.132)	0.792	27.1	(2.063)
G_F	1, 1	0.015	(0.003)	0.806	(0.064)			0.806	22.3	(1.997)
Defensive										
G_G	1, 2	0.015	(0.002)	0.331	(0.071)	0.561	(0.072)	0.892	13.9	(0.779)
G_H	1, 1	0.009	(0.002)	0.852	(0.049)		-	0.852	22.5	(1.904)
G_I	1, 1	0.006	(0.001)	0.943	(0.020)		-	0.943	22.9	(1.607)
G_J	1, 1	0.009	(0.001)	0.889	(0.024)		-	0.889	26.2	(1.860)
G_K	1, 2	0.011	(0.002)	0.483	(0.144)	0.331	(0.160)	0.814	23.5	(1.807)
G_L	1, 2	0.012	(0.002)	0.363	(0.141)	0.563	(0.140)	0.926	16.0	(0.873)
G_M	1, 2	0.017	(0.002)	0.352	(0.143)	0.389	(0.131)	0.740	28.5	(2.683)
G_N	1, 1	0.006	(0.001)	0.908	(0.031)		-	0.908	29.2	(2.554)

Note: m and n are the DCC order as in equation (7). $a1$, $b1$, and $b2$ are the DCC parameters in equation (7). “Shape” is the shape parameter of the Student t-copula.

The DCC order (m, n) in equation (7) is almost (1, 1) or (1, 2) as shown in Table 3. The lag order m for a_i in equation (7) is 1 in all cases. The parameter a_i indicates the degree of responses of $\boldsymbol{Q_t}$ to the past covariances of shocks in equation (7). The result that the order $m = 1$ means that the effect of past shocks on $\boldsymbol{Q_t}$, and hence the correlation $\boldsymbol{R_t}$, do not last longer. The lag order n for b_j is 1 for the market portfolio and 1 or 2 for both the cyclical and the defensive group portfolios. The parameter b_j indicates the degree of persistence of $\boldsymbol{Q_t}$ as well as $\boldsymbol{R_t}$. The order $n = 1$ (or 2) corresponds to the DCC parameter $b1$ (and $b2$) in Table 3.

The DCC parameters $a1$ are all non-zero positive numbers with enough significance, but are very small numbers (< 0.02) compared with $b1$ and $b2$. Both $b1$ and $b2$ take relatively large numbers. We calculate $b1 + b2$ to compare the relative persistence of the sample portfolios; $b1 + b2$ is higher than 0.9 for some of them including the market portfolio.[17] Hence, we can say that DCC is more realistic for the sample portfolios than CCC is, which assumes that $a_i = b_j = 0$ in equation (7). These findings are similar to those of previous studies that have estimated DCC models.

The DCC parameter estimates, especially $b1$ and $b2$, vary widely between the groups, implying that the correlation dynamics may differ across them. Indeed, the parameter estimates vary even within the cyclical and defensive groups. We explore the pattern of correlation changes in every group more in detail in Section 5.2. The shape parameters of the Student t-copula range between about 14 and 29. These relatively high values mean that the tail dependency of the standardized residuals seems to be limited, if any.

Tables 4 and 5 summarize the estimation results of the univariate ARMA–GARCH model for the market portfolio. The parameter set depends on the individual ARMA–GARCH lag degrees and distribution types of standardized residuals. The distribution is selected to be the skew t in most instances with the Student t in one group based on the AIC. The estimates of the shape parameters of the skew t and Student t show values below 10 in many of the defensive groups, but higher values in many of the cyclical groups. A lower shape value means that the standardized residuals still exhibit fat-tailedness even after the fat-tailedness of stock returns is reduced by adjusting the volatility by using GARCH. An important advantage of the copula approach is that it can handle such heterogeneities in marginal distributions very well.

[17] The values of $a1+b1+b2$ are all below 1, which indicates that the condition of equation (9) is satisfied.

Table 4: Univariate ARMA–GARCH: Six cyclical groups (portfolio return indexes)

Group ID	$(P,\ Q)$ $(p,\ q)$	Parameter	Estimate	Std error	Cdist
G_A	(0,1) (1,2)	ma1	0.056	(0.026)	sstd
		garch1	0.830	(0.036)	
		arch1	0.079	(0.030)	
		arch2	0.060	(0.038)	
		skew	1.080	(0.043)	
		shape	26.46	(19.986)	
G_B	(1,0) (1,2)	ar1	0.053	(0.026)	sstd
		garch1	0.773	(0.040)	
		arch1	0.095	(0.036)	
		arch2	0.081	(0.043)	
		skew	1.140	(0.051)	
		shape	31.789	(26.417)	
G_C	(0,1) (1,1)	ma1	0.066	(0.028)	sstd
		garch1	0.828	(0.026)	
		arch1	0.134	(0.021)	
		skew	1.142	(0.058)	
		shape	17.587	(8.342)	

Group ID	$(P,\ Q)$ $(p,\ q)$	Parameter	Estimate	Std error	Cdist
G_D	(1,0) (1,1)	ar1	0.072	(0.027)	sstd
		garch1	0.767	(0.032)	
		arch1	0.168	(0.027)	
		skew	1.254	(0.063)	
		shape	14.849	(6.582)	
G_E	(2,0) (1,1)	ar1	0.102	(0.027)	sstd
		ar2	0.048	(0.027)	
		garch1	0.763	(0.030)	
		arch1	0.161	(0.025)	
		skew	1.197	(0.051)	
		shape	10.844	(3.632)	
G_F	(1,1) (1,1)	ar1	0.561	(0.240)	sstd
		ma1	-0.452	(0.256)	
		garch1	0.732	(0.033)	
		arch1	0.192	(0.028)	
		skew	1.207	(0.055)	
		shape	11.002	(3.473)	

Note: $(P,\ Q)$ $(p,\ q)$ is (AR order, MA order) (GARCH order, ARCH order) as in equations (3) and (4). ar1, 2 are the parameter estimates for the AR part; ma1, 2 for the MA part in equation (3). garch1, 2 are the parameter estimates for the GARCH part; arch1, 2 for the ARCH part in equation (4). "Cdist" is the conditional distribution of the standardized residuals: "std" represents Student t, "sstd" skew t, and "norm" normal. The best model is selected from the multiple alternatives by using an AIC-type criterion.

Table 5: Univariate ARMA–GARCH: Eight defensive groups (portfolio return indexes)

Group ID	(P, Q) (p, q)	Parameter	Estimate	Std error	Cdist
G_G	(1,1) (1,1)	ar1	-0.870	(0.069)	std
		ma1	0.833	(0.076)	
		garch1	0.838	(0.052)	
		arch1	0.117	(0.036)	
		shape	8.956	(2.344)	
G_H	(2,2) (1,1)	ar1	-1.963	(0.001)	sstd
		ar2	-0.985	(0.001)	
		ma1	1.964	(0.000)	
		ma2	0.987	(0.000)	
		garch1	0.798	(0.141)	
		arch1	0.154	(0.094)	
		skew	1.129	(0.048)	
		shape	12.031	(3.985)	
G_I	(0,0) (1,1)	mu	0.000	(0.000)	sstd
		garch1	0.810	(0.050)	
		arch1	0.140	(0.035)	
		skew	1.133	(0.048)	
		shape	12.310	(4.084)	
G_J	(0,0) (1,1)	mu	-0.001	(0.000)	sstd
		garch1	0.788	(0.033)	
		arch1	0.159	(0.028)	
		skew	1.142	(0.042)	
		shape	11.516	(3.725)	

Group ID	(P, Q) (p, q)	parameter	Estimate	Std error	Cdist
G_K	(2,2) (2,1)	ar1	-0.553	(0.005)	sstd
		ar2	-0.989	(0.003)	
		ma1	0.543	(0.003)	
		ma2	0.990	(0.000)	
		garch1	0.473	(0.182)	
		garch2	0.257	(0.155)	
		arch1	0.188	(0.037)	
		skew	1.122	(0.040)	
		shape	9.095	(2.502)	
G_L	(1,0) (2,1)	ar1	-0.065	(0.027)	sstd
		garch1	0.325	(0.173)	
		garch2	0.444	(0.172)	
		arch1	0.191	(0.050)	
		skew	1.073	(0.044)	
		shape	7.130	(1.211)	
G_M	(2,0) (1,1)	ar1	0.054	(0.028)	sstd
		ar2	0.085	(0.027)	
		garch1	0.747	(0.035)	
		arch1	0.173	(0.029)	
		skew	1.141	(0.044)	
		shape	8.481	(2.516)	
G_N	(0,0) (1,1)	mu	-0.001	(0.000)	sstd
		garch1	0.787	(0.041)	
		arch1	0.171	(0.035)	
		skew	1.086	(0.040)	
		shape	7.998	(1.813)	

Note: (P, Q) (p, q) is (AR order, MA order) (GARCH order, ARCH order) as in equations (3) and (4). ar1, 2 are the parameter estimates for the AR part; ma1, 2 for the MA part in equation (3). garch1, 2 are the parameter estimates for the GARCH part; arch1, 2 for the ARCH part in equation (4). "Cdist" is the conditional distribution of the standardized residuals: "std" represents Student t, "sstd" skew t, and "norm" normal. The best model is selected from the multiple alternatives by using an AIC-type criterion.

We conducted goodness of fit tests to ensure that the model assumptions are satisfied. Specifically, the selection of the distribution type (shown as “Cdist” in Table 4) of the standardized residuals should be confirmed. The absence of the serial correlation of the standardized residuals should also be ensured, since the i.i.d. condition is assumed in equation (2). We performed the Anderson–Darling test for the goodness of fit of the selected distribution and the Ljung–Box test for the auto-correlation. These two tests are portmanteau tests in which only the null hypothesis is well specified.

Table 6 shows the test results for the market portfolio. In every case, the Anderson–Darling test results with high p-values show that the null hypothesis cannot be rejected at the 10% significance level (or much higher significance level in most cases). We can say that there is no significant misspecification with regard to the distribution of the standardized residuals. As for the Ljung–Box test results, the null hypothesis of no serial correlation cannot be rejected at the 10% significance level in most cases (excluding $\mathrm{G_F}$). These test results suggest that the model assumptions are generally well satisfied.

We also conducted the same tests for the group portfolios. No significant misspecification or serial correlation problem was detected. The test results are omitted owing to space limitations.

Further, to confirm the stability of the estimation result of the DCC–GARCH model, we fit the same model to two sub-period data sets of the market portfolio that have almost the equal numbers of trading days. We find that the parameter estimates differ little between the whole period and sub-period cases. The same check is then performed for the group portfolios and the results are similar.

5 Dynamic changes in correlation intensity

5.1 A measure of correlation intensity

The parameters of the DCC–GARCH were estimated for the market portfolio and group portfolios presented in Section 4. In this section, we calculate DCC $\boldsymbol{R}_t$ in equation (8). Because one instance of $\boldsymbol{R}_t$ exists at a time, the total number of correlation matrices is the same as the length of the return series (i.e., larger than 1,300). The dimension of $\boldsymbol{R}_t$ is 20×20 for every group portfolio and 14×14 for the market portfolio. It is difficult to observe the time series development of $\boldsymbol{R}_t$ as it is in matrix form. We hence need a further dimension reduction of $\boldsymbol{R}_t$.

The eigenvalues of the correlation matrix can be used as a vector of proxies for the

Table 6: Goodness of fit test (Market portfolio)

		Anderson–Darling test			Ljung–Box test	
		Cdist	AD	P-value	LB	P-value
Cyclical	G_A	sstd	0.356	0.891	5.873	0.555
	G_B	sstd	0.510	0.737	8.472	0.293
	G_C	sstd	0.769	0.504	8.175	0.417
	G_D	sstd	0.728	0.536	11.429	0.179
	G_E	sstd	0.345	0.901	8.278	0.407
	G_F	sstd	0.578	0.669	17.617	0.024
Defensive	G_G	std	0.499	0.748	3.341	0.911
	G_H	sstd	0.348	0.898	6.034	0.643
	G_I	sstd	1.096	0.311	6.087	0.638
	G_J	sstd	1.828	0.114	10.560	0.228
	G_K	sstd	0.248	0.971	9.999	0.189
	G_L	sstd	0.465	0.783	10.131	0.181
	G_M	sstd	0.601	0.647	8.373	0.398
	G_N	sstd	1.296	0.234	10.269	0.247

Note: "Cdist" is the conditional distribution of standardized residual z_t in equation (2); "std" and "sstd" stand for the Student t and skew t distribution, respectively. AD is the test statistics for the Anderson–Darling test for the null hypothesis (H_0) that assumes the conditional distribution as the one specified by "cdist." A higher p-value for AD means a lower risk of the misspecification of the conditional distribution. LB is the test statistics for the Ljung–Box test applied to the standardized residuals. The null hypothesis is that the data are independently distributed without any observed correlations. The number of lags tested is 10 and the degree of freedom is adjusted appropriately considering the number of parameters in the model.

correlation intensities on the corresponding axes. A larger eigenvalue indicates a stronger correlation. The positive maximum eigenvalue of $\boldsymbol{R}_t$ is the proxy for the correlation intensity on the first axis with the largest variance.[18] If the maximum eigenvalue is large enough, other eigenvalues may have limited influence on the correlation intensity of $\boldsymbol{R}_t$. In that case, the time series of the maximum eigenvalues approximate well the development of the correlation intensity between stock returns.

To answer the first and second research questions, we focus on the time series of the maximum eigenvalues of $\boldsymbol{R}_t$.[19] We first calculate a series of $\boldsymbol{R}_t$ by using the estimated DCC–GARCH model for the market portfolio and group portfolios. Then, the time series of the maximum eigenvalue of $\boldsymbol{R}_t$ are calculated for every sample portfolio.

[18] This indicates "the maximum amount of the variance of the variables which can be accounted for with a linear model by a single underlying factor" (Friedman and Weisberg [1981]).

[19] The changes in a correlation matrix have two components: correlation intensity (eigenvalues) and direction (eigenvectors). We focus on correlation intensity to observe any dynamic changes, assuming that intensity has a larger influence on portfolio risk. When simulating the quantitative impact of correlation changes in Section 5.3, changes in both intensity and direction are considered with different $\boldsymbol{R}_t$.

Eigenvalues and random matrix theory

Table 7 summarizes the eigenvalues of $\boldsymbol{R}_t$ and unconditional correlation matrix $\overline{\boldsymbol{R}}$. The three largest eigenvalues (EV1, EV2, and EV3) are listed from the whole set. Note that the length of the corresponding eigenvector is normalized to one for all eigenvalues.[20] "Min" and "max" represent the minimum and maximum values of the time series of the eigenvalues of $\boldsymbol{R}_t$, respectively. "Uncon" represents the eigenvalue of $\overline{\boldsymbol{R}}$. The maximum eigenvalue (EV1) is much larger than the second and third eigenvalues (EV2 and EV3) for the market portfolio and for all individual group portfolios, suggesting that EV1 mostly determines correlation intensity. If so, we can now focus on the time series development of the maximum eigenvalue as a proxy for correlation intensity.

Random matrix theory provides a reliable measure for distinguishing informative eigenvalues from uninformative ones. The Marčenko–Pastur distribution is a good approximation to the density of the eigenvalues of the correlation matrix of randomized returns.[21] We are, however, interested in which of the eigenvalues are meaningful by examining the largest eigenvalue of the correlation matrix of randomized returns. Importantly, we must know the threshold value that the maximum eigenvalue of the correlation matrix of randomized returns can take. If an eigenvalue of a correlation matrix is larger than the threshold value, we can safely say that it is meaningful.

To determine the threshold, we need to know the limiting distribution of the maximum eigenvalue of the randomized return correlation matrix with the same size as the sample correlation matrix. Johnstone [2001] showed that the asymptotic distribution of the properly rescaled largest eigenvalue of the white Wishart population covariance matrix is the Tracy–Widom distribution, which provides the limiting distribution of the maximum eigenvalue, while the Marčenko–Pastur distribution suggests the boundary of the distribution of eigenvalues. For more mathematical details on eigenvalues and the Tracy–Widom distribution, see Johnstone [2001] and Tracy and Widom [2009, 1996, 1994].

The distribution function of the Tracy–Widom distribution $F_\beta(\cdot)$ has three types of definitions depending on the value of β (1, 2, and 4).[22] We set β as 1, which provides the most conservative (largest) quantile value (to be used as a threshold) compared with the

[20] A symmetrical and positive definite matrix $\boldsymbol{R}_t$ has orthonormal eigenvectors.

[21] The largest and smallest eigenvalues of a Wishart matrix almost surely converge to the respective boundaries of the support of the Marčenko–Pastur distribution when the true covariance matrix is an identity matrix (Marčenko and Pastur [1967], Johnstone [2001]).

[22] The value of β depends on the assumption of the correlation matrix structure: $\beta = 1$ for the Gaussian orthogonal ensemble, $\beta = 2$ for the Gaussian unitary ensemble, and $\beta = 4$ for the Gaussian symplectic ensemble.

other settings. The distribution function $F_1(\cdot)$ is defined as

$$F_1(x) = \exp\left(-\frac{1}{2}\int_x^\infty q(y)\,dy\right)(F_2(x))^{\frac{1}{2}} \tag{17}$$

$$F_2(x) = \exp\left(-\int_x^\infty (y-x)\,q^2(y)\,dy\right) \tag{18}$$

where q is the unique solution to the ordinary differential equation called the *Painlevé* (type II) equation. For more exact and complete definitions, see Tracy and Widom [1996]. We calculate the 99th percentile of the Tracy–Widom distribution ($\beta = 1$) to identify the non-random eigenvalues that are beyond this value (Table 7). The 99th percentile value of the Marčenko–Pastur distribution is also calculated for reference.[23]

Table 7 shows that the minimum value of the maximum eigenvalues (EV1) of $\boldsymbol{R}_t$ during the data period is larger than the 99th percentile of the Tracy–Widom distribution in all sample portfolios as indicated by "$*$" in the EV1 column. This finding means that these maximum eigenvalues are all meaningful enough. Next, we find that the minimum value of EV2 is larger than the 99th percentile of the Tracy–Widom distribution only in one group portfolio, while the maximum value of EV2 is larger than the threshold only in four sample portfolios (the market portfolio and three group portfolios). This finding means that EV2 is only meaningful at certain points of time during the period.[24] Finally, the maximum value of EV3 is larger than the threshold only in one group portfolio. Hence, EV3 does not convey meaningful information.

[23] We use R package "RMTstat" to calculate the density and quantiles of the Tracy–Widom and Marčenko–Pastur distributions.

[24] EV2 is only meaningful in $\mathrm{G_L}$ (one of the defensive groups), including Electric Power and Gas. $\mathrm{G_G}$ (one of the defensive groups), including Banks, has the largest maximum of EV1, which implies very strong correlations in regional banks.

Table 7: Eigenvalues of the correlation matrix

Group ID	EV1 Min	EV1 Uncon	EV1 Max		EV2 Min	EV2 Uncon	EV2 Max		EV3 Min	EV3 Uncon	EV3 Max		TW(99)	MP(99)
Market	10.890	11.619	12.520	*	0.487	0.849	1.645	†	0.221	0.389	0.685		1.239	1.190
Cyclical														
G_A	12.947	13.377	14.126	*	0.757	0.872	1.047		0.549	0.602	0.730			
G_B	12.197	13.018	14.099	*	0.727	0.842	1.166		0.582	0.675	0.832			
G_C	10.687	11.130	12.367	*	0.654	0.775	1.011		0.611	0.683	0.805			
G_D	9.954	10.521	12.286	*	0.783	0.960	1.298	†	0.552	0.681	0.877		1.280	1.229
G_E	9.365	9.728	11.625	*	0.665	0.795	1.104		0.548	0.657	0.872			
G_F	8.878	9.329	11.144	*	0.832	1.014	2.327	†	0.617	0.733	1.024			
Defensive														
G_G	14.287	14.832	16.425	*	0.440	0.638	0.798		0.320	0.400	0.578			
G_H	9.749	10.659	11.664	*	1.149	1.282	2.927	†	0.717	0.852	1.016			
G_I	9.999	10.360	11.527	*	0.808	0.900	1.050		0.730	0.850	0.954			
G_J	10.040	10.368	11.863	*	0.708	0.869	1.063		0.628	0.674	0.827			
G_K	9.980	10.263	11.730	*	0.740	0.885	1.128		0.568	0.645	0.825		1.280	1.229
G_L	9.748	10.484	12.433	*	2.018	2.709	3.573	*	0.757	0.910	1.418	†		
G_M	8.416	8.686	11.180	*	0.653	0.927	1.216		0.606	0.776	0.882			
G_N	8.113	8.543	9.871	*	0.905	0.964	1.109		0.809	0.924	1.002			

Note: EV1, EV2, and EV3 are the maximum eigenvalue, second largest eigenvalue, and third largest eigenvalue, respectively. "Min" of EV1, -2, and -3 stands for the minimum value of the time series of the corresponding eigenvalue, "uncon" represents the eigenvalue of the unconditional correlation, and "max" the maximum value of the time series of the corresponding eigenvalue. TW(99) and MP(99) are the 99th percentiles of the Tracy–Widom and Marčenko–Pastur distributions, respectively. "∗" shows the case that the minimum eigenvalue during the period is higher than TW(99). "†" shows the case that the maximum eigenvalue is larger than TW(99).

5.2 Dynamic changes in maximum eigenvalues

The time series of the maximum eigenvalue of a conditional correlation matrix $\boldsymbol{R}_t$ reveals that the correlation intensity changes dynamically in the market portfolio and group portfolios. This means that both the between-group and the within-group correlations of stock returns change over time. We next describe the changes in the cyclical and defensive group portfolios as well as the market portfolio more in detail.

Cyclical group portfolios

Figure 1 depicts how the correlation intensities of the cyclical groups change over time. There are two charts for every group in Figure 1. The top chart shows the time series development of the maximum eigenvalues of $\boldsymbol{R}_t$ as a proxy measure for correlation intensity. The bottom chart shows the mean volatilities, calculated as the mean of the conditional volatilities of individual stock returns estimated by using the univariate GARCH model. The two dotted vertical lines indicate the trading date closest to the Lehman shock and Great Earthquake in that order.

Overall, correlation intensity changes dynamically in every group. Sharp increases in within-group correlation intensity are observed after the Lehman shock and Great Earthquake, with sharp increases in volatility observed as well. The differences between the two events, however, vary by group. While the persistence of increased correlation intensity as well as mean volatility is observed in many groups, the degrees of persistence differ.

In groups $\mathrm{G_A}$, $\mathrm{G_B}$, and $\mathrm{G_C}$, for example, mean volatility is much higher after the Lehman shock than it is after the Great Earthquake. These groups include stocks in Electric Appliances and Transportation Equipment, both of which are more export-oriented sectors.[25] The larger increases in volatility suggest that the stock returns in these groups were affected more by the overseas shock. Further, the maximum eigenvalues increased significantly after both events; however, their peak levels are not necessarily higher after the Lehman shock compared with after the Great Earthquake. In $\mathrm{G_C}$, the maximum eigenvalue is the highest after the Great Earthquake, whereas the peak levels in $\mathrm{G_A}$ and $\mathrm{G_B}$ are similar for the two events.

By contrast, in $\mathrm{G_D}$, $\mathrm{G_E}$, and $\mathrm{G_F}$, which are relatively less export-oriented, mean volatility increased markedly after both events. The maximum eigenvalue also increased after

[25] For more details on the correspondence between the groups and business sectors, see Table 4 of Isogai [2014].

both events in G_D and G_E, while the degree of increase after the Great Earthquake was limited in G_F.[26] In summary, the pattern of changes in correlation intensity seems to be significantly different by group.

Defensive group portfolios

Figures 2 and 3 show how the correlation intensity of the defensive groups changes over time. The maximum eigenvalue of $\boldsymbol{R}_t$ changes dynamically in every group as observed in the cyclical groups. Sharp increases in within-group correlation intensity are observed after both events; mean volatility also increased significantly after both. Comparing the changes after the two events, the peak levels of the maximum eigenvalues are higher after the Great Earthquake than they are after the Lehman shock in many groups. This trend seems to be more evident in the defensive groups, which are less export-oriented, than in the cyclical groups.

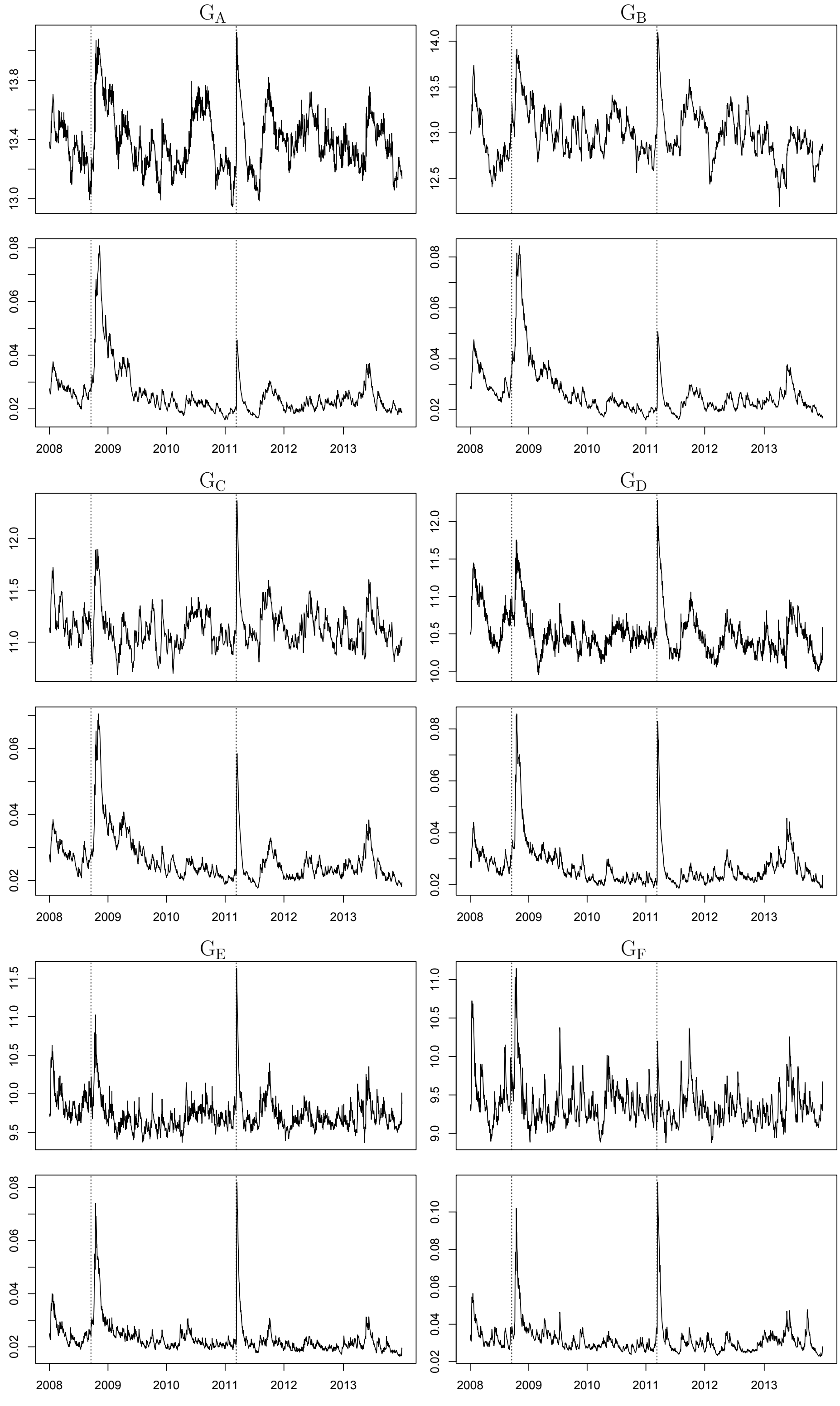
The persistence of increased correlation intensity and mean volatility is observed in many groups to different degrees. In G_H, correlation intensity increased after the Lehman shock, whereas the maximum eigenvalue decreased significantly after the Great Earthquake. G_H includes many construction companies like G_F in the cyclical category; hence, the same type of temporal correlation breakdown with a greater impact occurred at that time. In G_L, a typical defensive group, which includes Electric Power and Gas, Pharmaceutical, and Foods, a sharp increase in correlation intensity is observed after the Great Earthquake. In G_I (Information and Communication; Land Transportation) and G_N (Retail Trade; Foods), sharp increases are also observed after the Great Earthquake. Note that the mean volatilities of these groups have similar peak levels after both events, excluding G_G (Regional banks) and G_H.

Hence, the combination of the observations from the cyclical and defensive groups confirms that within-group correlation intensity changes over time, with a significant increase in crisis periods accompanied by a sharp rise in volatility. Further, we find significant differences in the changes in correlation intensity as well as volatility across the groups.

Market portfolio

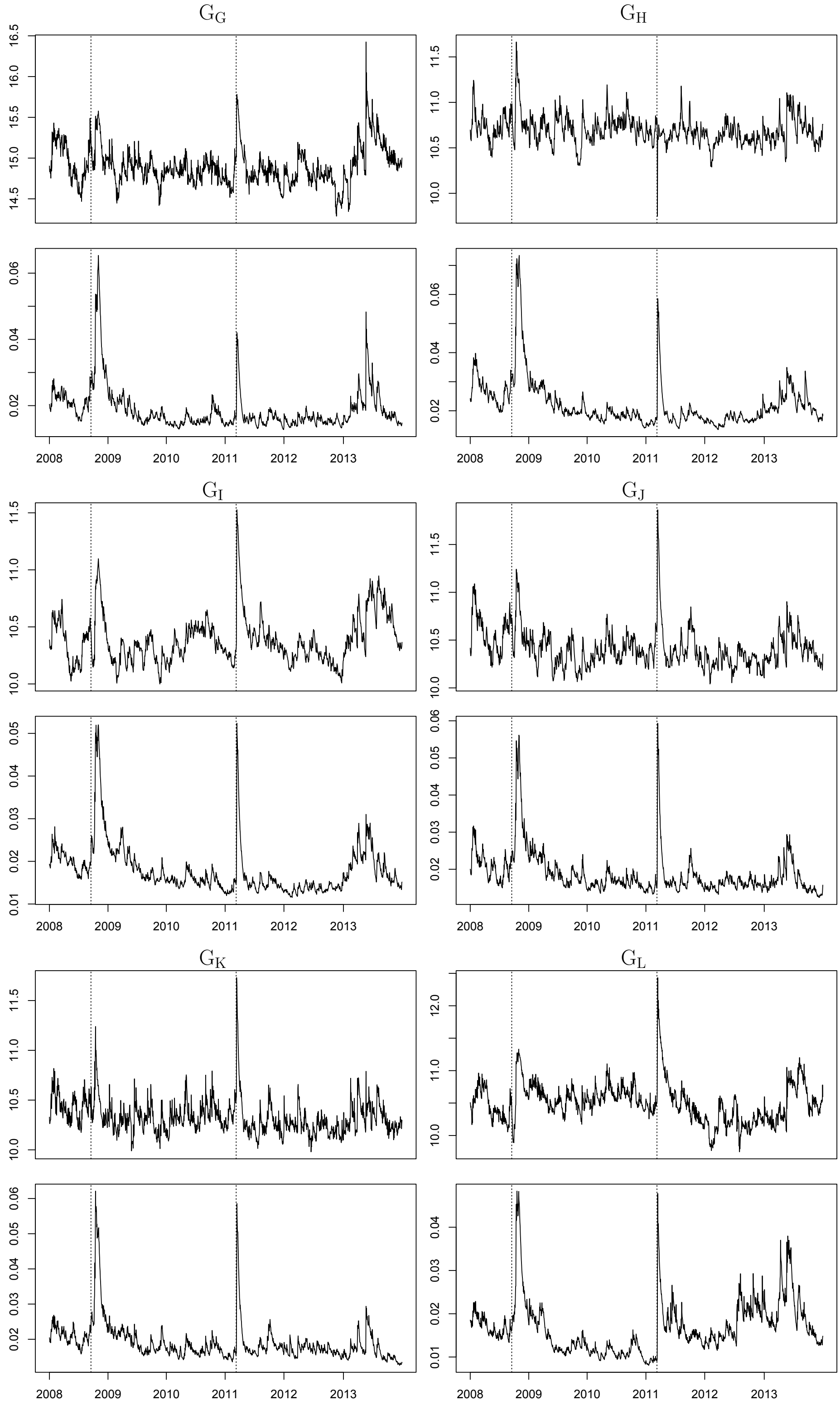
Figure 4 shows the time series development of the maximum eigenvalue of $\boldsymbol{R}_t$ of the market portfolio as well as mean conditional volatility. Recall that the market portfolio comprises

[26] The stock prices of some construction companies in G_E showed an unusual pattern after the Great Earthquake; sharp increases were partially observed, which seemingly contributed to the lower correlation intensity at that time.



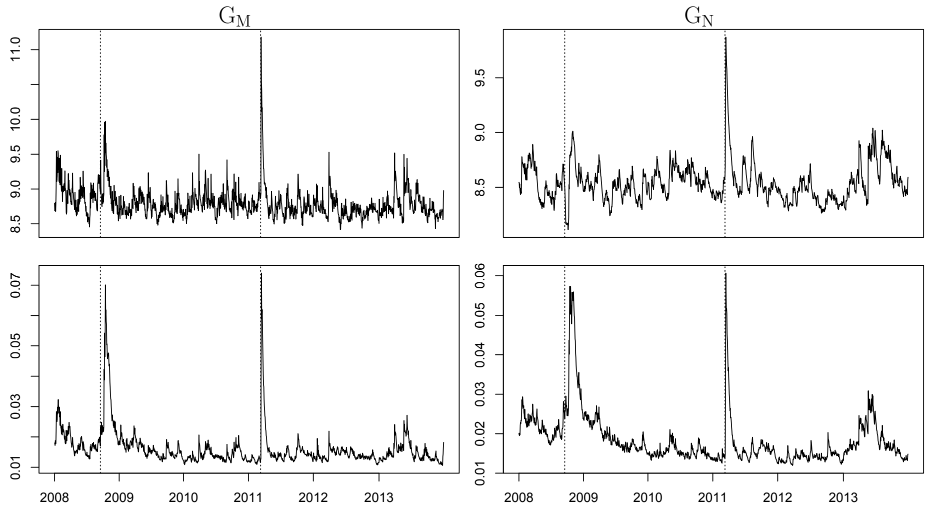
Note: The maximum eigenvalue of the correlation matrix (top) and mean volatility (bottom).

Figure 1: Maximum eigenvalue of the correlation matrix: Cyclical



Note: The maximum eigenvalue of the correlation matrix (top) and mean volatility (bottom).

Figure 2: Maximum eigenvalue of the correlation matrix: Defensive 1



Note: The maximum eigenvalue of the correlation matrix (top) and mean volatility (bottom).

Figure 3: Maximum eigenvalue of the correlation matrix: Defensive 2

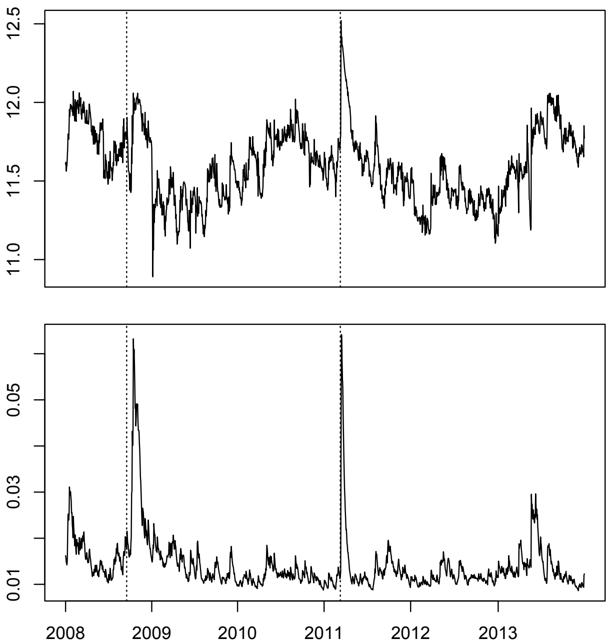
14 equally weighted index returns calculated from the 14 individual group portfolios, as mentioned in Section 4.1. The correlation matrix $\boldsymbol{R}_t$ shows the between-group or market-wide factor correlation of the market portfolio. We see that mean volatility increased significantly after both events, whereas the peak levels were similar.

Figure 4 clearly shows that between-group correlation changes over time as observed in the within-group correlations. The market-wide factor correlation also intensified during crisis periods as in many of the cyclical and defensive groups. Moreover, the maximum eigenvalue peaked after the Great Earthquake, while the persistence of increased correlation intensity was also observed.

The trends of the maximum eigenvalue after the Lehman shock are complicated. This value increased considerably after the shock and remained at relatively high levels before dropping sharply. Nonetheless, the peak level after the Lehman shock is lower than that after the Great Earthquake. Further, an upward trend is observed after the sharp decrease, which lasted for around two years, although a lack of information prevents us from clarifying the background of these movements.

5.3 Impact of correlation changes on portfolio risk

Having confirmed that both within-group and between-group correlation intensities change over time, we are now interested in answering the third research question. To evaluate the influence on the risk of the sample portfolios, we conduct a numerical simulation analysis.



Note: The maximum eigenvalue of the correlation matrix (top) and mean volatility (bottom).

Figure 4: Maximum eigenvalue of the correlation matrix: Market

The simulation focuses on the changes in correlation intensities and their influence on the portfolio risk measures: VaR and ES.

Relative changes in correlation intensity and volatility

Table 8 summarizes the maximum eigenvalues of the conditional correlation matrix $\boldsymbol{R}_t$ and mean volatilities of both the market and the group portfolios. This table also compares the relative changes in the maximum eigenvalue and mean volatility. The maximum eigenvalues are calculated as the mean values during the 20 trading days after the Lehman shock and Great Earthquake to smooth fluctuations. Moreover, the maximum eigenvalue of the unconditional correlation matrix $\overline{\boldsymbol{R}}$ is used as the benchmark for the relative comparison.

We can make two main observations here. First, the changes in the maximum eigenvalues from the unconditional one are relatively small compared with those of mean volatility, while the fluctuation in correlation intensity is also much smaller than that of mean volatility for the sample portfolios.[27] Second, the changes in the maximum eigenvalues are significantly different across the sample portfolios, while the changes in mean volatilities also differ across the sample portfolios, but not to a significant degree. At an event level,

[27] Because the maximum eigenvalue and mean volatility are measured by using different scales, the same changes in the two factors may influence the risk amount of the sample portfolio to a different degree.

Table 8: Maximum eigenvalues and mean conditional volatilities

Group ID	Maximum eigenvalue					Mean volatility				
	Uncon (a)	L (b)	E (c)	$\frac{b-a}{a}$ (%)	$\frac{c-a}{a}$ (%)	Uncon (a)	L (b)	E (c)	$\frac{b-a}{a}$ (%)	$\frac{c-a}{a}$ (%)
Market	11.61	11.67	12.21	0.54	5.16	0.014	0.027	0.035	86.78	143.20
Cyclical										
G_A	13.38	13.33	13.75	-0.37	2.77	0.025	0.034	0.033	36.00	32.00
G_B	12.99	13.26	13.66	2.08	5.19	0.027	0.046	0.038	72.15	42.60
G_C	11.14	11.17	11.71	0.26	5.15	0.026	0.036	0.041	34.72	55.44
G_D	10.50	10.93	11.53	4.11	9.82	0.027	0.044	0.053	63.22	97.11
G_E	9.74	10.13	10.46	4.01	7.37	0.023	0.038	0.049	63.24	111.05
G_F	9.37	10.05	9.52	7.27	1.56	0.032	0.050	0.072	56.64	126.78
Defensive										
G_G	14.90	15.05	15.45	0.98	3.69	0.019	0.030	0.030	61.51	60.34
G_H	10.70	10.84	10.62	1.32	-0.73	0.022	0.037	0.040	67.22	78.88
G_I	10.39	10.35	11.09	-0.40	6.74	0.018	0.028	0.034	56.35	87.27
G_J	10.42	10.60	11.15	1.77	7.02	0.019	0.029	0.037	52.19	97.12
G_K	10.33	10.52	10.93	1.87	5.82	0.019	0.031	0.037	64.40	94.09
G_L	10.51	10.28	11.56	-2.14	10.02	0.017	0.023	0.029	39.05	74.50
G_M	8.80	9.15	9.53	4.01	8.26	0.016	0.032	0.039	97.81	138.21
G_N	8.54	8.32	9.23	-2.53	8.03	0.018	0.032	0.037	71.50	101.19

Note: "Uncon" of the maximum eigenvalue column denotes the maximum eigenvalue of the unconditional correlation matrix $\overline{\boldsymbol{R}}$. "Uncon" of the mean volatility column is the mean of the volatilities estimated by CCC–GARCH. L and E denote the Lehman shock and Great Earthquake, respectively. The maximum eigenvalues of L and E are calculated as the means of the maximum eigenvalues of the conditional correlation matrix $\boldsymbol{R}_t$ during the 20 trading days after the two events. The mean volatilities of L and E are calculated as the means of conditional volatility during the 20 trading days after the two events.

the changes are much larger after the Great Earthquake in terms of both the maximum eigenvalue and mean volatility.[28]

Impact study of correlation changes: A numerical simulation

We next present the results of a numerical simulation conducted to compare quantitatively the impact of correlation changes on the risk amount of the sample portfolios: the market portfolio and the 14 group portfolios. Three factors are required when calculating portfolio risk: volatility, correlation, and the distribution of the probabilistic variable. The timing of the evaluation and confidence level should also be specified.

The timing here is set as two trading days after the Lehman shock and Great Earthquake.[29] The estimated conditional volatility and correlation on these two trading days

[28] Please note that because the sharp rise in volatility was slightly delayed after the Lehman shock, whereas it occurred immediately after the Great Earthquake, this lag might overemphasize the changes after the latter event.

[29] The selected trading days are October 15, 2008 and March 16, 2011, when the maximum eigenvalue peaked for most of the sample portfolios.

are specified in the simulation. To randomly sample the residuals, we use the Student t-copula, which has the same parameters, namely the conditional correlation matrix and shape parameter, as those estimated by the DCC–GARCH model presented in Section 4.3. A sample data set is generated by randomly sampling 100,000 draws from the Student t-copula for every sample portfolio. These data are then converted into individual returns by the quantile functions of the marginal distributions; finally, the portfolio returns are calculated by applying the conditional volatilities.

Note that the initial portfolio value is normalized to 1; therefore, the portfolio value after the one-day holding period is calculated as the 1 + the sum of individual simulated returns. The portfolio risk measures (VaR and ES) are calculated by using the historical simulation method:

$$\begin{aligned} \mathrm{VaR}_p[X] &= -\inf\left\{x|\Pr[X \le x] > 1-p\right\}, \quad 0<p<1 \\ \mathrm{ES}_p[X] &= \mathrm{E}\left[-X| -X \ge \mathrm{VaR}_p[X]\right], \quad 0<p<1 \end{aligned} \tag{19}$$

where X is portfolio returns and p is the confidence level of these risk measures (set at 99%).

We calculate the risk measures for the sample portfolios with both unconditional and conditional correlations to observe the differences between these two cases. The case with the unconditional correlation is regarded as the benchmark. For comparison, VaR and ES are also calculated with the Gaussian copula and unconditional correlation, the combination of which is the most naive assumption when measuring risk.

Simulation results

Tables 9 and 10 summarize the simulation results for VaR and ES, respectively. These results show that the VaR and ES of the sample portfolios increased in many cases when calculated using conditional correlation. This finding means that changes in correlation intensity can have a non-negligible positive impact on portfolio risk.

In Table 9, the VaR values of the market portfolio with the conditional correlation is about 1.2% and 3.1% (b→a and e→d, respectively) larger than those with the unconditional correlation after the Lehman shock and Great Earthquake, respectively. The ES of the market portfolio is also about 0.3% and 3.7% larger, as shown in Table 10. The impact is larger after the Great Earthquake than after the Lehman shock, as shown by the larger maximum eigenvalue, implying that the larger maximum eigenvalues after the Great Earthquake contribute to the larger impact.

In the case of the cyclical and defensive group portfolios, a similar tendency is observed as that in the market portfolio, although the impact differs considerably by group because of the different correlation intensities. The VaR and ES values with the conditional correlation are larger than those with the unconditional correlation in most groups. The VaR value with the conditional correlation is larger by about 11% (G_M, after the Great Earthquake) at the maximum but is similar (G_F and G_H, both after the Great Earthquake) at the minimum. The ES value is also larger by about 13% (G_M, after the Great Earthquake) at the maximum and similar (G_F and G_H, both after the Great Earthquake) at the minimum. These higher levels of the maximum eigenvalues seemingly contribute to the larger risk amount compared with the unconditional case, although the impacts differ significantly in both cyclical and defensive groups.[30]

When we use the Gaussian copula, the differences (between VaR and VaR* in Table 9; ES and ES* in Table 10) become larger, since this copula underestimates the risk without considering the tail dependency of the residuals unlike the Student *t*-copula. The existence of the tail dependency of returns is confirmed; however, the differences from the unconditional cases are still not large in many of the sample portfolios. The tail dependency is rather limited for the GARCH-filtered standardized residuals, as shown by the higher level of the shape parameters listed in Table 3.

[30] In some cyclical groups (G_B, G_D, and G_E), the order of the maximum eigenvalues is not consistent with that of the risk amount, although the differences in VaR and ES changes are not large. This inconsistency is probably because of the differences in the random sample sets as well as the smaller eigenvalues and direction of the eigenvector of the conditional correlation contributing to the risk differently.

Table 9: Impact of correlation changes: VaR

	Unconditional	Lehman shock						Great Earthquake					
		Conditional		Unconditional		Changes		Conditional		Unconditional		Changes	
Group ID	Max eigv	Max eigv	VaR (a)	VaR (b)	VaR^* (c)	(b→a) %	(c→a) %	Max eigv	VaR (d)	VaR (e)	VaR^* (f)	(e→d) %	(f→d) %
Market	11.61	12.06	0.124	0.123	0.121	1.21	2.61	12.37	0.128	0.124	0.122	3.12	4.66
Cyclical													
G_A	13.38	13.98	0.121	0.118	0.117	1.95	3.11	13.86	0.081	0.080	0.080	1.05	1.86
G_B	12.99	13.98	0.140	0.134	0.133	4.97	5.14	14.12	0.089	0.086	0.085	3.57	3.86
G_C	11.14	11.88	0.109	0.105	0.103	4.30	5.67	12.37	0.098	0.093	0.092	6.19	7.20
G_D	10.50	11.76	0.146	0.138	0.137	5.27	6.51	11.77	0.140	0.134	0.132	4.40	6.11
G_E	9.74	11.02	0.125	0.117	0.115	7.65	8.71	11.37	0.137	0.128	0.126	7.60	8.80
G_F	9.37	11.24	0.164	0.150	0.146	9.50	12.40	10.13	0.167	0.167	0.163	-0.43	2.31
Defensive													
G_G	14.90	15.51	0.110	0.107	0.107	2.81	2.67	15.78	0.088	0.085	0.085	3.37	3.28
G_H	10.70	11.66	0.124	0.116	0.116	6.34	6.23	10.63	0.097	0.098	0.098	-0.29	-0.60
G_I	10.39	10.86	0.087	0.084	0.084	3.49	3.38	11.53	0.093	0.087	0.087	6.68	7.38
G_J	10.42	11.24	0.096	0.093	0.090	2.66	5.80	11.86	0.105	0.100	0.098	4.90	7.78
G_K	10.33	11.24	0.110	0.105	0.103	5.21	7.10	11.65	0.104	0.099	0.098	5.57	6.70
G_L	10.51	11.10	0.080	0.078	0.077	2.33	4.56	12.43	0.088	0.083	0.081	6.09	8.39
G_M	8.80	9.97	0.115	0.107	0.107	7.52	7.12	10.54	0.124	0.112	0.112	11.00	10.95
G_N	8.54	8.80	0.089	0.087	0.086	1.81	2.69	9.87	0.098	0.091	0.091	6.83	7.92

Note: "Unconditional" and "conditional" represent the correlation matrix type. "Max eigv" stands for the maximum eigenvalue of the correlation matrix. The initial portfolio value is set to 1 for each portfolio. VaR is calculated at the 99% confidence level. VaR^* is calculated from the estimation result of the DCC–GARCH model with the Gaussian copula.

Table 10: Impact of correlation changes: ES

	Unconditional	Lehman shock						Great Earthquake					
		Conditional		Unconditional		Changes		Conditional		Unconditional		Changes	
Group ID	Max eigv	Max eigv	ES (a)	ES (b)	ES^* (c)	(b→a) %	(c→a) %	Max eigv	ES (d)	ES (e)	ES^* (f)	(e→d) %	(f→d) %
Market	11.61	12.06	0.145	0.145	0.142	0.28	2.22	12.37	0.150	0.146	0.144	3.68	4.40
Cyclical													
G_A	13.38	13.98	0.143	0.139	0.136	2.35	5.02	13.86	0.096	0.095	0.092	1.72	4.33
G_B	12.99	13.98	0.165	0.158	0.156	4.58	5.99	14.12	0.106	0.102	0.100	3.69	5.24
G_C	11.14	11.88	0.131	0.126	0.123	3.96	6.44	12.37	0.120	0.112	0.109	7.69	10.28
G_D	10.50	11.76	0.175	0.166	0.162	5.60	8.04	11.77	0.169	0.161	0.157	4.82	7.29
G_E	9.74	11.02	0.153	0.141	0.138	8.65	11.31	11.37	0.166	0.155	0.151	7.67	10.48
G_F	9.37	11.24	0.201	0.182	0.175	9.99	14.76	10.13	0.202	0.202	0.194	-0.25	3.99
Defensive													
G_G	14.90	15.51	0.135	0.130	0.129	3.46	4.75	15.78	0.108	0.104	0.103	3.82	5.13
G_H	10.70	11.66	0.149	0.142	0.138	5.34	7.99	10.63	0.118	0.119	0.116	-0.40	1.85
G_I	10.39	10.86	0.106	0.102	0.100	3.58	5.70	11.53	0.113	0.106	0.104	6.06	8.05
G_J	10.42	11.24	0.116	0.112	0.108	3.60	7.71	11.86	0.129	0.121	0.117	6.38	10.40
G_K	10.33	11.24	0.136	0.127	0.124	6.63	9.79	11.65	0.129	0.121	0.117	6.75	9.90
G_L	10.51	11.10	0.100	0.097	0.093	3.08	7.50	12.43	0.110	0.103	0.099	6.86	10.66
G_M	8.80	9.97	0.143	0.131	0.129	9.19	11.38	10.54	0.155	0.138	0.135	12.81	15.03
G_N	8.54	8.80	0.109	0.106	0.103	2.79	5.64	9.87	0.120	0.111	0.108	8.14	11.26

Note: "Unconditional" and "conditional" represent the correlation matrix type. "Max eigv" stands for the maximum eigenvalue of the correlation matrix. The initial portfolio value is set to 1 for each portfolio. ES is calculated at the 99% confidence level. ES^* is calculated from the estimation result of the DCC–GARCH model with the Gaussian copula.

VaR backtesting

In addition to the numerical impact simulation, we conducted another test to evaluate the model performance of DCC–GARCH in comparison with CCC–GARCH. We assume that DCC–GARCH shows better VaR backtesting performance compared with CCC–GARCH if DCC captures the correlation changes that affect VaR values, while CCC, by definition, does not.[31]

The VaR value is calculated over the whole in-sample period at the 99% confidence level by using the DCC– and CCC–GARCH models with the parameters estimated in Section 4.3. The holding period is one trading day. The conditional volatilities are updated daily by the univariate GARCH using the previous time series of return data. The number of VaR exceedances is counted for every sample portfolio to compare the theoretically expected number of exceedances. The two types of VaR backtests proposed by Kupiec [1995] and Christoffersen and Diebold [2006] are conducted to evaluate the frequency of exceedances statistically.

Table 11 shows the results of the VaR backtesting. The p-values are calculated for the null hypothesis (H_0): the VaR model is correctly specified. The null hypothesis cannot be rejected in six groups (G_C, G_D, G_G, G_H, G_J, and G_N) for DCC and in five groups (G_C, G_D, G_G, G_J, and G_N) for CCC at the 5% significance level in both tests.[32] Even for the groups in which the null hypothesis is rejected, the exceedance counts are closer to the expected level (14) in four groups (G_B, G_F, G_I, and G_K) in DCC than in CCC, while they diverge in two groups (G_A and G_L). These results suggest that DCC performs better than CCC in terms of VaR backtesting.

[31] VaR backtesting is normally conducted as an out-of-sample test with a rolling model parameter estimation. We simplified the test, since our VaR backtesting is conducted only to compare the relative performance of DCC and CCC. As for ES backtesting, technical issues that are related to the elicitability concept have been much debated since Gneiting [2011]. Recently, Acerbi and Szekely [2014] proposed a new backtesting method for ES that uses a MonteCarlo hypothesis test. We do not cover the ES backtesting here, since the issue is beyond our scope.

[32] The VaR backtesting results may seem to be unsatisfactory for formulating risk management strategies. As described in Sections 3.2 and 4.2, we assume a simple GARCH model for univariate returns and a scalar DCC model with the Student *t*-copula for their dependency structure. For improved backtesting performance, this modeling framework needs to be reexamined; however, this investigation is beyond the scope of the present study.

Table 11: VaR (99%) backtest

	DCC						CCC					
	Exceedance	(ratio, %)	LR_K	P-value	LR_C	P-value	Exceedance	(ratio, %)	LR_K	P-value	LR_C	P-value
Market	2	(0.14)	17.462	0.000	-	-	2	(0.14)	17.4622	0.000	-	-
Cyclical												
G_A	24	(1.64)	5.040	0.024	5.726	0.057	23	(1.57)	4.085	0.043	4.881	0.087
G_B	23	(1.57)	4.085	0.043	4.881	0.087	25	(1.71)	6.081	0.013	6.666	0.036
G_C	13	(0.89)	0.197	0.656	2.872	0.237	14	(0.96)	0.030	0.861	2.432	0.296
G_D	14	(0.96)	0.030	0.861	2.432	0.296	14	(0.96)	0.030	0.861	2.432	0.296
G_E	22	(1.50)	3.217	0.072	11.807	0.002	22	(1.50)	3.217	0.072	11.807	0.003
G_F	23	(1.57)	4.085	0.043	7.899	0.019	28	(1.91)	9.679	0.001	15.553	0.000
Defensive												
G_G	11	(0.75)	1.010	0.314	4.322	0.115	11	(0.89)	1.010	0.314	4.322	0.115
G_H	18	(1.23)	0.716	0.397	2.246	0.325	18	(1.23)	0.716	0.397	6.338	0.042
G_I	14	(0.96)	0.030	0.861	7.646	0.021	13	(2.12)	0.197	0.656	8.422	0.015
G_J	12	(0.82)	0.519	0.471	3.497	0.174	14	(1.16)	0.030	0.861	2.432	0.296
G_K	16	(1.09)	0.120	0.728	12.608	0.001	17	(0.55)	0.358	0.549	12.081	0.002
G_L	32	(2.18)	15.487	0.000	32.610	0.000	31	(0.89)	13.934	0.000	37.280	0.000
G_M	8	(0.55)	3.659	0.055	10.156	0.006	8	(0.75)	3.659	0.055	8.249	0.016
G_N	13	(0.89)	0.197	0.656	2.872	0.237	13	(0.89)	0.197	0.656	2.872	0.238

Note: VaR values are calculated over the whole in-sample period at a 99% confidence level. The expected frequency of VaR exceedances is 14 for each group at the given confidence level. The DCC and other parameters used are those summarized in Tables 3, 4, and 5. LR_K is the likelihood ratio statistic of the unconditional coverage test for VaR exceedances proposed by Kupiec [1995]. P-value is shown for the null hypothesis (H_0): the VaR model is correctly specified. A higher p-value indicates that the null hypothesis cannot be rejected, meaning that the VaR exceedances are consistent with the expected level. LR_C is the likelihood ratio statistic of the conditional coverage test for VaR exceedances proposed by Christoffersen and Diebold [2006]. The test considers the potential violation of the assumption of the independence of the number of exceedances, which is not considered in the Kupiec test. LR_C is not available for the market portfolio because of the very low level of VaR exceedances.

6 Discussion

The correlation of asset returns is a key issue for quantitative risk measurement and portfolio investment control. In this empirical study, we propose a data-driven approach to observe the dynamic changes in the correlation matrices of Japanese stock returns by using an MGARCH model, namely DCC–GARCH. While it is difficult to fit any multivariate model with conditional correlation to the whole universe of stock returns (about 1,400 stocks according to our definition), we overcome the high dimensionality problem by fitting a reduced size MGARCH model with DCC to two types of sample portfolios: the market portfolio and group portfolios.

When building these sample portfolios, we apply the clustering method originally developed in complex networks theory. The unconditional correlation matrix of the whole universe is first estimated by using the CCC–GARCH model. The universe is then divided into 14 sub-groups with two large categories: cyclical and defensive. The group portfolios are next built with 20 representative stocks selected by using a network centrality measure. These portfolios cover each segment of the universe as homogeneous groups. The market portfolio is finally built as a set of the 14 mean return indexes of the individual groups.

We fit the scalar DCC–GARCH model to the return data of the market and group portfolios to observe the between-group and within-group correlation dynamics, respectively. The likelihood function of DCC–GARCH is built by using the Student t-copula considering the tail dependency of returns. The parameter estimation results show that DCC–GARCH is more realistic than CCC–GARCH, confirming that both within-group and between-group correlations change over time and that the dynamics are significantly different for the sample portfolios.

The conditional correlation matrices are calculated from the DCC–GARCH estimation results for all sample portfolios. Then, the time series of the maximum eigenvalues of the conditional correlation matrices are calculated to observe the dynamic changes in correlation intensity. The findings confirm that both the between-group and the within-group correlations intensified after the Lehman shock and Great Earthquake; however, the patterns of changes are significantly different across the sample portfolios.

We also explore the impact of correlation changes on the risk of sample portfolios by using a numerical simulation. The VaR and ES values of the market and group portfolios with the conditional correlation are compared with those with the unconditional correlation, assuming the highest level of conditional volatilities after the Lehman shock and

Great Earthquake. The results show that the dynamic change in the correlation matrix has non-negligible positive influences on the risk of the stock portfolio. The comparative VaR backtesting results also suggest that DCC performs better than CCC.

These empirical findings suggest a number of discussion points. The first point relates to technical limitations when modeling correlations. Although DCC–GARCH can model dynamic correlation and the copula-based two-step estimation procedure can also incorporate the heterogeneity of individual return distributions efficiently, it remains difficult to evaluate the changes in the estimated correlation matrix. We adopted the maximum eigenvalue of a correlation matrix as the proxy measure for correlation intensity. The maximum eigenvalue reveals the dynamic changes in correlation intensity as a scalar indicator, which helps us follow the pattern of these changes. Nevertheless, the maximum eigenvalue is still not directly linked to the calculation of portfolio risk. More strictly, the changes in the eigenvector of a correlation matrix can also influence portfolio risk, demanding a simulation analysis of the quantitative impact of correlation changes on portfolio risk. As for the impact study, the simulation results depend on the modeling assumptions of DCC–GARCH: no volatility spillover is considered. Further, the correlation dynamics can be described differently by other more structural dynamic correlation models. To quantify the effect of volatility spillovers on dynamic correlation, we must estimate conditional correlation or covariance by using other types of multivariate models including the BEKK model.

The second discussion point is the practical aspects of DCC–GARCH. Since DCC–GARCH provides a consistent framework with which to combine conditional mean, volatility, and correlation to measure portfolio risk, it is thus flexible to capture any time-varying changes in those three factors. Despite the technical limitations of DCC–GARCH, its compact and flexible modeling framework with parsimonious parameters and easy parameter estimation procedure are valuable from a practical viewpoint of risk control. For example, the conditional approach is beneficial for stress testing portfolio risk by using possible combinations of extreme volatilities and correlation matrices. In brief, we need extreme but plausible scenarios for meaningful stress testing. The estimated historical time series of the conditional correlation matrix as well as conditional volatility can further provide a set of realistic combinations of volatility and correlation, which may not be available with a static correlation matrix, in order to set the stress level. Moreover, the eigenvalues may be used to adjust correlation intensity when building the scenarios, although further study is required to clarify their quantitative impact.

Another discussion point is the high dimensionality of the correlation matrix of stock returns. We adopt the reduced size of sample portfolios to monitor the whole stock market, since we are interested in a more general portfolio with wider coverage rather than a specifically targeted portfolio. The clustering algorithm is based on the correlation matrix of the whole universe of stock return data, which is calculated by assuming the use of CCC–GARCH. The volatility interaction between stocks is thus not considered. Using other methods to estimate the correlation matrix may lead to different group samples. This point is an important caveat to this study. Nevertheless, our dimension reduction method works well, even for a very large number of assets with fat-tailed returns, while group size as well as the size of sample portfolios can be modified flexibly. The method of selecting representatives from a group based on a network centrality measure can also be easily applied to other groupings.

7 Conclusion

In this study, the dynamic correlation of Japanese stock returns is modeled by using DCC–GARCH, which is fitted to reduced size sample portfolios. It is confirmed that the correlation matrix changes over time in both the market portfolio and the group portfolios. Significant differences in the patterns of the changes between the sample portfolios are also identified, with sharp increases in correlation intensity observed during crisis periods.

The presented findings suggest two possible directions for future research. First, our empirical findings depend on the assumption of no volatility spillovers. A more generalized multivariate model could thus be applied to similar types of sample portfolios if the size was appropriately reduced. Second, it would be meaningful to test if the same result would be obtained when other clustering methods are used to create a set of reduced size sample portfolios. A higher level of coverage of stocks in a sample portfolio is another issue to be considered. We used a simple scalar DCC–GARCH model; however, more generalized and complicated DCC models including BDCC and AG–DCC would improve estimation performance. Moreover, with regard to issues related to correlation intensity, a more detailed study of the relationship between the eigenvalues and portfolio risk would be an interesting topic. Finally, a practical application of the estimation of dynamic correlation change to portfolio risk management needs to be explored.

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References

Acerbi, C. and Szekely, B. (2014). Back-testing expected shortfall. *Risk*, 27(11).

Aielli, G. P. (2011). Dynamic conditional correlation: On properties and estimation. *SSRN Electronic Journal, http://www.ssrn.com/abstract=1507743.*

Aielli, G. P. and Caporin, M. (2013). Fast clustering of GARCH processes via Gaussian mixture models. *Mathematics and Computers in Simulation*, 94:205–222.

Aielli, G. P. and Caporin, M. (2014). Variance clustering improved dynamic conditional correlation MGARCH estimators. *Computational Statistics & Data Analysis*, 76:556–576.

Billio, M., Caporin, M., and Gobbo, M. (2006). Flexible dynamic conditional correlation multivariate GARCH models for asset allocation. *Applied Financial Economics Letters*, 2(02):123–130.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307–327.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *The Review of Economics and Statistics*, 72(3):498–505.

Bollerslev, T., Engle, R. F., and Wooldridge, J. M. (1988). A capital asset pricing model with time-varying covariances. *Journal of Political Economy*, 96(1):116–131.

Caporin, M. and McAleer, M. (2013). Ten things you should know about the dynamic conditional correlation representation. *Econometrics*, 1(1):115–126.

Cappiello, L., Engle, R. F., and Sheppard, K. (2006). Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics*, 4(4):537–572.

Christoffersen, P. F. and Diebold, F. X. (2006). Financial asset returns, direction-of-change forecasting, and volatility dynamics. *Management Science*, 52(8):1273–1287.

Cont, R. (2007). Volatility clustering in financial markets: Empirical facts and agent-based models. In *Long Memory in Economics*, pages 289–309. Springer, Berlin, Heidelberg.

Demarta, S. and McNeil, A. J. (2005). The t copula and related copulas. *International Statistical Review*, 73(1):111–129.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3):339–350.

Engle, R. and Kelly, B. (2012). Dynamic equicorrelation. *Journal of Business & Economic Statistics*, 30(2):212–228.

Engle, R. and Sheppard, K. (2001). Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. *National Bureau of Economic Research*, w8554.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4):987–1007.

Engle, R. F. and Bollerslev, T. (1986). Modelling the persistence of conditional variances. *Econometric Reviews*, 5(1):1–50.

Engle, R. F. and Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. *Econometric Theory*, 11(01):122–150.

Engle, R. F., Ng, V. K., and Rothschild, M. (1990). Asset pricing with a factor-ARCH covariance structure. *Journal of Econometrics*, 45(1-2):213–237.

Fama, E. F. (1965). The behavior of stock-market prices. *The Journal of Business*, pages 34–105.

Fernández, C. and Steel, M. F. J. (1998). On bayesian modeling of fat tails and skewness. *Journal of the American Statistical Association*, 93(441):359–371.

Friedman, S. and Weisberg, H. F. (1981). Interpreting the first eigenvalue of a correlation matrix. *Educational and Psychological Measurement*, 41(1):11–21.

Ghalanos, A. (2014). *rmgarch: Multivariate GARCH Models.* R package version 1.2-8 (`http://cran.r-project.org/web/packages/rmgarch/index.html`).

Gneiting, T. (2011). Making and evaluating point forecasts. *Journal of the American Statistical Association*, 106(494):746–762.

Isogai, T. (2014). Clustering of Japanese stock returns by recursive modularity optimization for efficient portfolio diversification. *Journal of Complex Networks*, 2(4):557–584.

Johnstone, I. M. (2001). On the distribution of the largest eigenvalue in principal components analysis. *Annals of Statistics*, 29(2):295–327.

Jondeau, E. and Rockinger, M. (2006). The copula-GARCH model of conditional dependencies: An international stock market application. *Journal of International Money and Finance*, 25(5):827–853.

Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives*, 3(2):73–84.

Lanne, M. and Saikkonen, P. (2007). A multivariate generalized orthogonal factor GARCH model. *Journal of Business & Economic Statistics*, 25(1):61–75.

Ledoit, O. and Wolf, M. (2003). Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of Empirical Finance*, 10(5):603–621.

Lee, T. H. and Long, X. (2009). Copula-based multivariate GARCH model with uncorrelated dependent errors. *Journal of Econometrics*, 150(2):207–218.

Mandelbrot, B. B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4):394–419.

Mantegna, R. N. and Stanley, H. E. (2000). *An Introduction to Econophysics: Correlations and Complexity in Finance.* Cambridge University Press, Cambridge.

Marčenko, V. A. and Pastur, L. A. (1967). Distribution of eigenvalues for some sets of random matrices. *Sbornik: Mathematics*, 1(4):457–483.

McCloud, N. and Hong, Y. (2011). Testing the structure of conditional correlations in multivariate garch models: A generalized cross-spectrum approach. *International Economic Review*, 52(4):991–1037.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, 59(2):347–370.

Newman, M. E. J. (2008). Mathematics of networks. In Durlauf, S. N. and Blume, L. E., editors, *The New Palgrave Encyclopedia of Economics*. Palgrave Macmillan, Basingstoke, 2nd edition.

Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. *International Economic Review*, 47(2):527–556.

Pelletier, D. (2006). Regime switching for dynamic correlations. *Journal of Econometrics*, 131(1):445–473.

Silvennoinen, A. and Teräsvirta, T. (2005). Multivariate autoregressive conditional heteroskedasticity with smooth transitions in conditional correlations. Technical report, SSE/EFI Working Paper Series in Economics and Finance.

Silvennoinen, A. and Teräsvirta, T. (2009). Modeling multivariate autoregressive conditional heteroskedasticity with the double smooth transition conditional correlation GARCH model. *Journal of Financial Econometrics*, 7(4):373–411.

Sklar, M. (1959). Fonctions de répartition à n dimensions et leurs marges. In *Publ. Inst. Stat. 8*, pages 229–231. Université Paris, Paris.

Tracy, C. A. and Widom, H. (1994). Level-spacing distributions and the airy kernel. *Communications in Mathematical Physics*, 159(1):151–174.

Tracy, C. A. and Widom, H. (1996). On orthogonal and symplectic matrix ensembles. *Communications in Mathematical Physics*, 177(3):727–754.

Tracy, C. A. and Widom, H. (2009). The distributions of random matrix theory and their applications. In *New Trends in Mathematical Physics*, pages 753–765. Springer, Netherlands.

Tse, Y. K. and Tsui, A. K. C. (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business & Economic Statistics*, 20(3):351–362.

Van der Weide, R. (2002). GO-GARCH: A multivariate generalized orthogonal GARCH model. *Journal of Applied Econometrics*, 17(5):549–564.