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Abstract

Many central banks implement forward guidance according to an implicit or explicit policy rule in practice, and thus it is expected to influence the economy by changing expectations formation of private agents. In this paper, I investigate the effects of forward guidance particularly via expectations formation by formulating forward guidance as a monetary policy rule in a non-linear new Keynesian model. A quantitative analysis using the U.S. and Japanese data implies that a rule-based forward guidance significantly mitigates a decline in inflation and output growth in a crisis period via changing expectations formation.

Keywords: Forward Guidance; Expectations Formation; Effective Lower Bound; Particle Filter

JEL Classification: E31; E32; E42; E52

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1 Introduction

Recently, a number of central banks have implemented monetary easing by keeping interest rates at zero longer than expected to mitigate the adverse effects of the effective lower bound (ELB) of nominal interest rates. For example, the Federal Reserve kept the interest rates at zero after the Great Recession even though the estimated contemporaneous Taylor rule – a popular policy rule for nominal interest rates – has already suggested positive values (Figure 1).[1] Such a commitment to keep interest rates low and longer than expected is referred to as a type of forward guidance (FG). The basic mechanism behind its effect after reaching the ELB has been theoretically established by, for example, Reifschneider and Williams (2000) and Eggertsson and Woodford (2003) among others.[2]

A quantitative evaluation of FG in a rational expectations model is, however, not so simple task because FG is usually implemented by an implicit or explicit policy rule and consequently it potentially influences the economy via changing expectations of private agents. That is, since the private agents recognize that FG is endogenously introduced and removed (more generally, strengthened and weakened) in response to economic situations, FG must change the way of forming expectations among private agents not only at the ELB but also before reaching the ELB. Therefore, to correctly and precisely evaluate the effects of FG in a rational expectations model, FG needs to be formulated as a monetary policy rule rather than exogenous policy actions.

In this paper, I quantitatively investigate the effects of FG by modeling FG as a monetary policy rule in a non-linear new Keynesian model. The approach to model

[1]For computing the interest rates suggested by the Taylor rule, I first assume a contemporaneous Taylor rule without interest rate inertia responding inflation and output growth, and set the reaction parameter to inflation rates and output growth to 2.45 and 0.39, respectively, based on the estimated rule in the later section.

[2]While FG is recently defined as broader policy measures, especially a clarification of policy rules, than a commitment policy, this paper defines FG as the traditional commitment policy. Campbell, Evans, Fisher, and Justiniano (2012) categorize FG depending on whether or not it entails a commitment to future actions. They call these two types of FG Odyssean and Delphic, respectively.

FG as a monetary policy rule is in contrast to the previous literature in the sense that the previous literature usually models FG as an exogenous extension of zero interest rate periods in a perfect foresight model or as an exogenous news shock to nominal interest rates. Therefore, the previous literature just investigates the effects of FG after it is actually implemented at the ELB, and tells nothing about the effects of FG via expectations formation at the ELB as well as before reaching the ELB. To describe FG as a policy rule, the central bank in the model is assumed to conduct the monetary policy not only by setting nominal interest rates according to the Taylor rule with ELB, but also by committing to zero interest rates à la Reifschneider and Williams (2000). A rule-based FG following the Reifschneider-Williams rule suggests that when the nominal interest rate by the Taylor rule is constrained by the ELB, the central bank commits to an extended period of zero interest rate policy or a lower interest rate policy according to the cumulative deviation of nominal interest rates from the ELB. Therefore, the event of reaching ELB induces the expectation for future monetary easing and consequently leads to an increase in inflation expectations. The Reifschneider-Williams rule is of course a reduced form of a FG rule, but since it appropriately captures the endogenous and historically dependent implementation of the central bank's FG in response to inflation and output growth as observed in real economy, the quantitative analysis based on the Reifschneider-Williams rule should provide a useful implication for the effects of a rule-based FG in general.

The quantitative analysis using policy functions and impulse responses indicates that while the economy faces a significant negative pressure on inflation and output growth at the ELB as emphasized by the previous literature, a rule-based FG can significantly mitigate those adverse effects at the ELB. Furthermore, the quantitative analysis shows that a rule-based FG has effects on inflation and output growth even at non-ELB (but close to ELB) states via changing expectation for the central bank's policy at the ELB, and as a result reduces the probability for reaching ELB. That is, it implies that the central bank can positively influence inflation and output growth even at non-ELB states *by clarifying in advance that they would commit to low interest rates according to the FG rule if they are constrained by the ELB.* This policy implication sounds relatively new in the literature of FG, but Davig and Leeper (2006) call such policy effects "expectations formation effects" and show that those effects are quantitatively important when investigating the effects of a non-linear policy rule.

To apply the model to the U.S. economy and quantify the effects of FG, I estimate the model parameters and identify the structural shocks by a Bayesian method. The model is non-linear with respect to: (i) the nominal interest rates are constrained by the ELB, and (ii) the economy is faced with a potential large decline in output growth (i.e., the crisis shock). The second non-linearity, which is supposed to capture the large decline in output growth during the Great Recession, is incorporated as in Barro (2006). Since the non-linear structure of the model makes it practically difficult to estimate all parameters by computing exact solutions, I adopt the following two-step approach. First, most parameters are estimated by a linearized model *without* ELB and the crisis shock using the data up to 2007/4Q. The implicit assumption in the first-step estimation is that the economy had not been faced with the crisis shocks as well as the ELB up to the Great Recession. The parameters of the fully non-linear model are set to the estimated values, and, as the second-step estimation, the degree of commitment to FG as well as the structural shock sequences are estimated by a non-linear model *with* ELB and the crisis shock using the particle filter and the data up to 2015/4Q.

The estimated FG rule strongly implies that they respond to economic changes even at the ELB by strengthening and weakening its commitment to FG. To understand the role taken by the rule-based FG in more detail, the counterfactual simulations for the U.S. economy are conducted. The counterfactual experiments indicate that the U.S. economy would possibly experience deflation and a sharper decline in output growth during and after the Great Recession if the Federal Reserve adopted a weaker commitment to FG. Furthermore, they imply that the exogenous extensions of zero interest rate periods, which are identified as negative monetary policy shocks at the ELB, contributed to support inflation and output growth during the Great Recession. Put together, the results of the policy experiments suggest that the monetary policy both as a rule and as an exogenous extension of the zero interest rate periods at the ELB played a key role accounting for the "missing deflation" during the Great Recession.

Finally, a quantitative analysis on the Japanese economy is carried out. Since the Japanese economy experienced long-lasting low inflation periods in contrast to the U.S. economy, it is expected that the quantitative effects of FG are less evident than in the case of the U.S. economy. The counterfactual simulation examining the case of (i) a stronger commitment to FG, and (ii) a higher target inflation rate, implies that the Japanese economy would mitigate the sharp drop in inflation and output growth during the Great Recession under those alternative policy choices. Interestingly, however, it also shows that, even under a stronger commitment to FG, the Japanese economy would still experience the long-lasting deflation that occurred in the late 1990s and early 2000s. This result implies that the efficacy of FG crucially depends on the assumption that recession or deflation lasts only for a few periods, and that the commitment to FG works for mitigating a sharp drop in inflation, particularly during crisis periods, but its effects for supporting inflation could be reduced if the economy faces long-lasting deflationary pressures. This is because the FG, by design, commits to a future lower interest rate policy after recovering from the ELB. If the deflationary pressures are expected to continue for many periods and the recovery from the ELB is not expected in a short period of time, the expectations formation effects of FG on current inflation rates would not be very powerful.

Literature Review

In terms of literature, this paper is related to the literature on the quantitative investigation of non-linear policy rules. Davig and Leeper (2006) incorporate the endogenous monetary policy shift into a forward-looking model. While they do not mention FG at all, FG in my paper is in effect by the same mechanism via expectations formation as in their model. Adam and Billi (2006) and Hara, Kimura, and Okina (2008) analyze the effect of a non-linear monetary policy rule by the model with ELB, and emphasize the expectations formation effects near ELB. Nakov (2008) is a good summary of this literature.

This paper is also related to the recent studies that solve a non-linear rational expectations model with the ELB (e.g., Aruoba, Cuba-Borda, and Schorfheide (2013), Fernandez-Villaverde, Gordon, Guerron-Quintana, and Rubio-Ramirez (2015), Nakata (2013), Hirose and Sunakawa (2015) and Hills, Nakata, and Schmidt (2016)). These studies, however, may overestimate the adverse effects caused by the ELB because they do not consider the effects of FG at the ELB.

Another strand of studies related to this paper is the quantitative evaluation to FG. In a spirit of Eggertsson and Woodford (2003), a number of studies quantitatively investigate the effect of FG by DSGE models. These studies, however, do not examine the effect of FG as a policy rule but as an exogenous news shock to nominal interest rates (e.g., Hirose and Kurozumi (2011), Campbell, Evans, Fisher, and Justiniano (2012), Del Negro, Giannoni, and Patterson (2012), and Gavin, Keen, Richter, and Throckmorton (2013)) or an exogenous extension of zero interest rate periods in a perfect foresight model (e.g., McKay, Nakamura, and Steinsson (2015)). Reifschneider and Williams (2000), on the other hand, consider FG as a policy rule. While they use a backward-looking model and consequently do not mention the effects via expectations formation, their idea of a rule-based FG is used in my paper to investigate the effects of FG including those via expectations formation. In a similar vein, Harrison (2015) points out the importance of systematic components of FG, and Harrison, Korber, and Waldron (2015) consider FG as an endogenous threshold-based policy and investigate on the optimal policy rule.

The paper proceeds as follows. In Section 2, I construct a non-linear new Keynesian model to analyze policy effects and transmission mechanism of FG. Section 3 estimates the model parameters using the U.S. data and quantitatively investigates the effects of FG by policy functions and impulse responses, and conducts counterfactual policy simulations with respect to the degree of commitment to FG and the monetary policy shocks. Section 4 applies the model to the Japanese economy and quantitatively analyzes the role of FG there. Concluding remarks are given in Section 5.

2 Model

The model follows a standard small-scale new Keynesian model with the ELB of nominal interest rates as Aruoba, Cuba-Borda, and Schorfheide (2013) and Nakata (2013). The private sector of the economy consists of a representative household, consumption-good firms, and intermediate-good firms. The central bank conducts a monetary policy by using nominal interest rates as a policy tool and follows the Taylor rule with ELB. In contrast to the conventional literature, however, the central bank can commit to future monetary easing through FG in a spirit of Reifschneider and Williams (2000). Each agent's behavior is described in turn.

2.1 Household

The representative household supplies labor force to obtain wage income, $W_t L_t$, where W_t denotes real wages and L_t denotes hours worked. In addition, because the household owns all firms in the economy as a stockholder, it also obtains dividend, D_t, as another source of income. The household allocates its income to consumption, C_t, and savings as a form of nominal one-period bonds, B_t. The household faces the budget constraint,

$$P_t C_t + B_t = R_{t-1} B_{t-1} + P_t W_t L_t + P_t D_t \tag{1}$$

where P_t is the price level and R_t is the nominal interest rate.

The household maximizes its lifetime utility by choosing consumption and labor supplies,

$$\max_{C_t, L_t} \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t/A_t)^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\nu}}{1+\nu} \right]$$

subject to the budget constraint (1). $\beta \in (0,1)$ is a constant discount factor and A_t is a non-stationary part of aggregate productivity. The first order conditions for C_t, B_t and L_t give the Euler equation,

$$c_t^{-\sigma} = \beta R_t E_t \left[\frac{c_{t+1}^{-\sigma}}{a_{t+1} \pi_{t+1}} \right] \tag{2}$$

as well as the labor supply function,

$$w_t c_t^{-\sigma} = \psi L_t^{\nu}, \tag{3}$$

where the detrended consumption and real wages are denoted by lower-case letters as $c_t = C_t/A_t$ and $w_t = W_t/A_t$. Also, the growth rate of non-stationary part of aggregate productivity is denoted by $a_t = A_t/A_{t-1}$, which follows an exogenous stochastic process specified below.

2.2 Firm

The representative consumption-good firm produces the final good, Y_t, by aggregating the intermediate goods, Y_{it}, using the following CES aggregator, $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of substitution. Let $p_{i,t}$ be the price of each intermediate good. The price index, P_t, is then defined as $P_t = \left(\int_0^1 p_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}}$, and the demand for each intermediate good is derived as a result of profit maximization of the representative consumption good firm,

$$Y_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{-\theta} Y_t. \tag{4}$$

The detrended consumption goods and intermediate goods production are defined by dividing them by A_t, and are denoted by lower-case letters as $y_t = Y_t/A_t$ and $y_{i,t} = Y_{i,t}/A_t$.

A continuum of intermediate-good firms produces differentiated intermediate goods using labor, $L_{i,t}$, according to the following linear technology,

$$Y_{i,t} = A_t z_t L_{i,t}, \tag{5}$$

where A_t and z_t is a non-stationary and stationary part of the aggregate productivity at period t, respectively.

Under monopolistic competition, intermediate-good firm i maximizes its discounted profits by setting the price of its differentiated products under the cost for price changes, γ. Hence, the intermediate-good firm i chooses its prices so as to maximize,

$$\max_{\{p_{i,t+k}\}_{k=0}^{\infty}} E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{c_{t+k}}{c_t} \right)^{-\sigma} \left[\frac{p_{i,t+k}}{P_t} y_{i,t+k} - w_{t+k} L_{i,t+k} - \frac{\gamma}{2} \left(\frac{p_{i,t+k}}{p_{i,t+k-1}} - \pi^* \right)^2 y_{t+k} \right]$$

subject to (4) and (5). Here, π^* represents the steady-state inflation rate. As a result of the intermediate-good firms' optimization, inflation dynamics can be described by the following new Keynesian Phillips curve,

$$1 - \theta + \theta \frac{w_t}{z_t} = \gamma \left(\pi_t - \pi^* \right) \pi_t - \beta \gamma E_t \left(\pi_{t+1} - \pi^* \right) \pi_{t+1} \left(\frac{y_{t+1}}{y_t} \right)^{1-\sigma}. \tag{6}$$

where w_t/z_t represents the real marginal cost.

2.3 Central Bank

The central bank sets the nominal interest rate according to the Taylor rule with the ELB and the forward guidance term (FG term),

$$R_t = \max\left[R_t^{tlr} - m_{t-1}, 1\right]. \tag{7}$$

where R_t^{tlr} is the nominal interest rate suggested by the Taylor rule, and m_t is the FG term, which is specified below. The max function indicates that the central bank would set the nominal interest rate to one if the nominal interest rate based on the Taylor rule subtracted by the FG term, $R_t^{tlr} - m_{t-1}$, is lower than one. The nominal interest rate based on the policy rule, R_t^{tlr}, is specified by the following a type of Taylor rule responding to inflation rates and output growth rates,

$$R_t^{tlr} = R_{ss}\left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi}\left(\frac{y_t}{y_{t-1}}\frac{a_t}{a^*}\right)^{\phi_y} i_t \tag{8}$$

where $R_{ss} = \pi^* a^*/\beta$ is the nominal interest rate at the steady-state. i_t is the monetary policy shock which is determined by the following stochastic process,

$$i_t = i_{t-1}^{\rho i}\exp(\varepsilon_{i,t}) \tag{9}$$

where $\varepsilon_{i,t}$ follows $N(0, \sigma_i)$.

The FG term, m_t, is determined by the following law of motion à la Reifschneider and Williams (2000),

$$m_t = \begin{cases} m_{t-1} + \zeta\left(1 - R_t^{tlr}\right) & \text{if } R_t^{tlr} < 1 \\ m_{t-1} - \left(R_t^{tlr} - R_t\right) & \text{otherwise} \end{cases} \tag{10}$$

where $\zeta \geq 0$. This FG rule indicates that when the nominal interest rate based on (7) is lower than one (i.e., $R_t^{tlr} < 1$), the central bank commits to future monetary easing by accumulating m_t, which is proportional to the negative deviation from the rule, $\zeta\left(1 - R_t^{tlr}\right)$. That is, the increase in m_t induces the expectation for future monetary easing and consequently leads to an increase in inflation expectations, because the accumulated m_t will be subtracted from nominal interest rates as specified in (7) after the economy leaves the ELB. Since the parameter ζ determines how much the negative deviation is accumulated as m_t, it is interpreted as the strength of commitment to the FG.[3] After the economy leaves the ELB, the above law of motion (10) indicates that the accumulated m_t is reduced by $R_t^{tlr} - R_t$, which is the same amount of m_{t-1} subtracted from the nominal interest rate in period t.

Figure 2 shows a simple example for describing how the Reifschneider-Williams rule works for committing to future monetary easing in response to deflation. In this example, the nominal interest rate indicated by the Taylor rule R_t^{tlr} is assumed to decline to a negative value in period 2, and gradually return to the previous level (the green line in the top figure). Given the path of R_t^{tlr}, the interest rate R_t and the FG term m_t are determined according to (7) and (10), respectively (the middle and bottom figures). The dashed red lines represent the interest rate R_t and the FG term m_t for the case of $\zeta = 0$. This case corresponds to the policy rule without FG because the FG term m_t is always equal to zero. As a result, the interest rate for $\zeta = 0$ (the red dashed line in the middle figure) is the same as the Taylor rule with ELB, i.e., $R_t = \max\left[R_t^{tlr}, 1\right]$. The blue solid lines, on the other hand, represent the interest rate R_t and the FG term m_t for the case of $\zeta = 1.5$. In this case, the negative deviation from the rule $\zeta\left(1 - R_t^{tlr}\right)$ is accumulated as the FG term m_t (the blue solid line in the bottom figure). Therefore, as indicated by (7), the gross interest rate R_t continues to be one (i.e., the net interest rate $R_t - 1$ is zero) as long as $m_t > 0$, and it lifts off from the ELB after using up the accumulated FG term. Since such a commitment to future monetary easing is expected by private agents right after the economy is hit by a deflationary shock in period 2, the commitment to FG as a policy rule (i.e., a positive ζ) works as supporting current inflation rates via the expectations formation.[4] There are some presumptions for the policy effects via expectations formation. First, it is assumed that the value of ζ is common knowledge among private agents. Therefore, if private agents have only imperfect information for

[3] Note that if $\zeta = 0$, the FG term m_t is always equal to zero, and as a result the monetary policy rule (7) just becomes the Taylor rule with ELB as is used in the previous literature.

[4] The path of nominal interest rate looks similar to the Taylor rule with interest rate smoothing. The Reifschneider-Williams rule, however, does not require a smoothed decline of interest rate in the face of negative shocks, and such an asymmetric response is a key difference to the interest rate smoothing.

the value of ζ, the effects of FG via expectations formation would be limited, and would depend on how precisely the private agents know about the true value of ζ. Second, it is assumed that the central bank can commit to FG perfectly. While some papers cast doubt on the feasibility of FG in terms of the time consistency problem, Nakata (2014) provides a positive result about the ability of commitment to FG by taking into account the central bank's reputation, thus supporting the assumption here.

The Reifschneider-Williams rule is of course a reduced form of FG rule, and it looks different from the actual policies, including, for example, the date-dependent FG or the FG that is directly contingent on inflation rates and unemployment rates. Nevertheless, the FG based on the Reifschneider-Williams rule responds to inflation and output growth through changes in R_t^{tlr}, as observed in the real economy. Therefore, this rule appropriately captures the endogenous and historically dependent responses of the central bank's FG to inflation and output growth in principle. Consequently, the quantitative analysis based on the rule provides useful implications for the effects of a rule-based FG in general.

2.4 Productivity Process

The economic dynamics are driven by the non-stationary and stationary part of productivity, A_t and z_t. The growth rate of the non-stationary part of productivity, $a_t \equiv A_t/A_{t-1}$, as well as the stationary part of productivity, z_t, follow a standard AR(1) process,

$$a_t = a_{t-1}^{\rho_a} a^{*1-\rho_a} \exp(\varepsilon_{a,t}) \tag{11}$$

and

$$z_t = z_{t-1}^{\rho_z} \exp(\varepsilon_{z,t}), \tag{12}$$

where $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ are stochastic shocks to productivity growth and level, respectively. a^* is the growth rate of productivity at the steady-state. While both the growth rate of productivity, a_t, and the productivity level, z_t, entail exogenous changes in productivity of intermediate-good firms as shown in (5), they have different effects on aggregate output and inflation: The positive shock to productivity growth $\varepsilon_{a,t} > 0$ increases both output and inflation on impact while the positive shock to productivity level $\varepsilon_{z,t} > 0$ increases output but decreases inflation on impact. The positive shock to a_t increases inflation because it generates the expectation for the increase in future income in addition to current income. Consequently, it raises the current consumption more than its supply through the wealth effect, thus leading to higher inflation rates. The difference in responses of output and inflation means that the shock to productivity growth $\varepsilon_{a,t}$ acts as a "demand shock" while the shock to productivity level $\varepsilon_{z,t}$ acts as a "supply shock" according to the classification in, for example, Brave, Campbell, Fisher, and Justiniano (2012). In the estimation of the model, given the fact that all economic variables are driven only by those two shocks in the model, the shock to productivity growth $\varepsilon_{a,t}$ is supposed to capture all demand shocks, including the discount rate shock, while the shock to productivity level $\varepsilon_{z,t}$ is supposed to capture all supply shocks, including the mark-up shock, in a very parsimonious way.

As for the probability distribution of the structural shocks, $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$, I assume that there are two regimes: the non-crisis regime and crisis regime. In the non-crisis regime, $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ follow $N(0, \sigma_a)$ and $N(0, \sigma_z)$ as in a standard DSGE model. In the crisis regime, however, $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ follow the uniform distribution $U[-6\%, -3\%]$ and $U[-10\%, -3\%]$, respectively.[5] That is, the economy in the crisis regime is faced with the risk for very large negative shocks to both non-stationary and stationary decline in aggregate productivity. The economy is assumed to be hit by the "crisis shock" and go into the crisis regime with probability 1% in an i.i.d manner. This specification follows the setting in the disaster shock literature such as Barro (2006). Also, from a practical perspective, the crisis shock is necessary for likelihood evaluation of a non-linear model, particularly for the period of large fluctuations such as the Great Recession, because the probability for a large fluctuation in the normal distribution is so small that it is

[5] The lower bound of distribution is set to -16% based on Barro (2006). Then, since Nakamura, Steinsson, Barro, and Ursua (2013) argues that a temporary shock is more dominant than a permanent shock in a crisis, I set -6% and -10% for the lower bound of distribution for the permanent and temporary productivity shock, respectively. The upper bound of distribution, on the other hand, is set to a value high enough to capture only the Great Recession as a disaster in the estimation.

impossible to describe such an event by a simulation-based method like the particle filter. This point is discussed in more detail in the estimation part.

3 Quantitative Analysis

This section quantitatively investigates the effects of a rule-based FG, particularly focusing on those in the U.S. economy around the Great Recession. Given the highly non-linear structure of the model, I adopt the following two-step approach for estimation of parameters as similarly in Aruoba, Cuba-Borda, and Schorfheide (2013): First, most parameters are estimated by a linearized model *without* ELB and the crisis shock using the data up to 2007/4Q. Next, as the second step estimation, the strength of commitment to FG, ζ, is estimated by a non-linear model *with* ELB and the crisis shock using the particle filter and the data up to 2015/4Q.

With the estimated values of parameters, the effects of a rule-based FG are investigated by examining policy functions and impulse responses. As a part of the investigation on the effect of FG, I try to give an interpretation of "forward guidance puzzle" in this model briefly. Finally, to measure the influence of the strength of commitment, the counterfactual simulations for the U.S. economy are conducted, with a special focus on the following two counterfactual cases during the Great Recession: (i) a weak commitment to FG, and (ii) no exogenous extensions of zero interest rate periods.

3.1 Estimation up to 2007/4Q

Since the model described in the previous section is highly non-linear due to the ELB and the crisis shock, the computational burden is too heavy to estimate all parameters by computing the exact solutions. Therefore, the parameters except for the strength of commitment to FG, ζ, are estimated by a linearized model without ELB and the crisis shock using the data only from 1983/1Q to 2007/4Q as in Aruoba, Cuba-Borda, and Schorfheide (2013). The implicit assumption for this estimation strategy is that the economy had not been faced with the crisis shocks as well as the ELB of nominal interest rates up to the Great Recession, and that the estimated parameters are structural and not changed during and after the Great Recession. The linearized model consists of eight state variables $(y_t, w_t, \pi_t, R_t, R_t^{tlr}, a_t, z_t, i_t)$, and its dynamics can be described by a linearized version of eight equations: (2), (3), (6), (8), (9), (11), (12), and $R_t = R_t^{tlr}$. The parameters are estimated by a Bayesian method using the following four data sequences: the annualized quarter-to-quarter real GDP growth rate (GDPR), the annualized quarter-to-quarter core PCE deflator (INF), the annualized FF rate (SR), and the annualized 5-year government bond yield (5R). Specifically, the measurement equations are formulated as,

$$GDPR_t = \left(\frac{y_t}{y_{t-1}} a_t - 1\right) \times 400$$
$$INF_t = (\pi_t - 1) \times 400$$
$$SR_t = (R_t - 1) \times 400$$
$$5R_t = \left(\eta_t R_t^{20} - 1\right) \times 400,$$

where R_t^j is the long-term interest rate with maturity j quarter and η_t is a measurement error associated with the long-term interest rate.

The long-term interest rate with maturity j is computed by the following recursive structure,

$$R_t^j = E_t \left[R_{t+1}^{j-1}\right]^{j-1} R_t \tag{13}$$

where $R_t^1 = R_t$ and $R_t^0 = 1$. In the estimation, the long-term interest rate is supposed to identify the effect through the accumulation of m_t at the ELB because it reflects the decline in expected short-term interest rates by FG.[6] A measurement error associated with the long-term interest rate η_t follows the AR(1) process,

$$\eta_t = \eta_{t-1}^{\rho_\eta} \eta^{*1-\rho_\eta} \exp(\varepsilon_{\eta,t})$$

where $\varepsilon_{\eta,t}$ is generated from $N(0, \sigma_\eta)$ and η^* is the measurement error at the steady

[6] Long-term interest rates have been used for identifying the effects of FG in the previous literature. For example, Hirose and Kurozumi (2011) model FG as a news shock and argue that the long-term interest rate provides important information to identify its effects.

state. Since η_t is interpreted as including term-premia, its steady state value η^* is not necessarily equal to zero but usually takes a positive value.

Before running the estimation, some parameters are calibrated. The discount factor β is chosen so that the annual discount rate is 0.5%. The elasticity of labor supply ν and the elasticity of substitution θ are set to 2.0 and 6.0, both of which are conventional values. The disutility of labor supply ψ is set to 4.0 just as normalization. Finally, the steady-state inflation rate π^* is set to 1.005, which means that the Federal Reserve set the target inflation rate at 2%. The rest of the parameters are estimated by a Bayesian technique using the U.S. quarterly data from 1983/1Q to 2007/4Q. Table 1 summarizes the estimation results with the prior distribution used for estimation. The table indicates that the estimated parameters are close to the previous studies such as Del Negro, Eusepi, Giannoni, Sbordone, Tambalotti, Cocci, Hasegawa, and Linder (2013). For the monetary policy rule, the response to inflation, ϕ_π (output growth, ϕ_y), is a little bit larger (smaller) than the conventional value probably because the growth-rate rule rather than the output-gap rule is adopted and because the interest rate inertia is not incorporated. The steady-state value of measurement error for the long-term interest rate η^* is 1.0032, which means that the 5-year interest rate contains about 1.4% term-premium on average. For the productivity processes, the growth of non-stationary productivity a_t follows a persistent process with a small shock variance while the stationary productivity z_t and the monetary policy shock i_t follow a relatively less persistent process.[7]

3.2 Effects of a Rule-Based Forward Guidance

This subsection quantitatively investigates the effects of a rule-based FG by examining policy functions as well as impulse responses with respect to the productivity growth a_t. To compute policy functions and impulse responses, the parameters of the non-linear

[7] A slightly tight prior distribution is assumed for a_t because the shock to a_t is supposed to capture the persistent potential growth and consequently work as a demand shock in the model, as is discussed in Section 2.4.

model *with* ELB and the crisis shock are set to the estimated values by the linearized model summarized in Table 1, and it is solved by the Coleman's policy function iteration method. A more detailed description for the non-linear model and its solution method is provided in Appendix A.

3.2.1 Policy Functions and Impulse Responses

Figure 3 shows the policy functions of the inflation rate (π_t), output growth rate ($a_t y_t / y_{t-1}$), the short-term interest rate (R_t), and the interest rate indicated by the Taylor rule (R_t^{tlr}). The horizontal axis represents the log-deviation of productivity growth from its steady state, $\log(a_t/a^*)$. The solid blue lines show the policy functions with no accumulated FG term (i.e., $m_{t-1} = 0$) while the dashed red lines show those with positive accumulated FG term (i.e., $m_{t-1} > 0$). For both cases, the strength of commitment to FG ζ is set to 0.5, which is the lower bound value for solving the model.[8] First of all, the figure shows that both inflation and output growth positively respond to a_t. That is, the shock to productivity growth ε_t^a works as a demand shock, as explained in Section 2.4. The figure also points out that there is strong non-linearity in the policy functions for inflation and output growth. Specifically, as the policy function of interest rates suggested by the Taylor-rule R_t^{tlr} (the right bottom figure) implies, the non-linear responses of inflation and output growth to productivity growth are observed around the point where the ELB binds. For instance, the policy function of output growth becomes clearly steeper than a 45 degree line (the dotted black line) only at the ELB. Such non-linearities at the ELB, which are not observed in a standard linearized model but commonly observed in a non-linear model with the ELB, point out the possibility that inflation and output growth face significant negative pressure at the ELB because the central bank cannot lower the nominal interest rate in response to the negative productivity growth. Finally, the figure shows that the dashed red lines are always higher than the bold blue lines in the figures for inflation and output growth, implying that the accumulation of FG term,

[8] If ζ is lower than 0.5, the policy functions in this model would be difficult to compute because the policy function iteration would not converge.

m_t, works for increasing both inflation and output growth. Behind the effects of FG on inflation and output growth, note that the central bank keeps zero interest rates even at the states where the Taylor rule suggests a positive level of interest rates, as indicated by the policy function of nominal interest rates R_t for $m_{t-1} > 0$ (the dashed red line in left bottom figure).

Figure 4 shows the policy functions under different values of ζ. Since ζ represents the degree of commitment to FG as a monetary policy rule in (10), the differences in policy functions under the different values of ζ can be interpreted as the policy effects of a rule-based FG. The figures show the policy functions of inflation (π_t), output growth ($a_t y_t / y_{t-1}$), the short-term interest rate (R_t), and the 5-year interest rate (R_t^{20}) with respect to the log-deviation of productivity growth from its steady state, $\log(a_t/a^*)$. The solid blue lines show the policy functions with a weak commitment to FG ($\zeta = 0.5$) while the dashed red lines show those with a tight commitment to FG ($\zeta = 4$). The figures indicate that, with a tight commitment to FG as a rule, the adverse effects on inflation and output growth at the ELB are significantly mitigated, and consequently their non-linear responses to negative productivity growth are merely observed. Furthermore, the policy functions point out that commitment to FG has effects on inflation and output growth even at non-ELB (but close to ELB) states, which implies that a rule-based FG influences the economy not only through the actual accumulation of the FG term $m_t > 0$ but also through *the expectation for* such commitment at the ELB. As the policy function for interest rates R_t (the left bottom figure in Figure 4) shows, such policy effects of FG at non-ELB states significantly shrink the region of a_t in which the ELB binds, and consequently mitigates the adverse effects due to the ELB by reducing the probability for reaching the ELB.

Those policy functions therefore imply that central banks can positively influence inflation and output growth even at non-ELB (but close to ELB) states and reduce the probability for reaching ELB *by clarifying in advance that they would commit to low interest rates following the FG rule if they face the ELB.* This policy implication at a non-ELB state is new in the literature of FG, but such effects of a rule-based policy through the expectations for future policy responses have already been discussed in the literature of monetary policy analyses. For example, Davig and Leeper (2006) emphasize that a non-linear Taylor rule for nominal interest rates has effects on inflation not only through the actual non-linear responses by central banks but also through *the expectation for* the non-linear responses. They call such policy effects "expectations formation effects" and show that those effects are quantitatively important.

To quantify the expectations formation effects of FG, I compute (1) the impulse responses to the same size of productivity growth shock under the different values of ζ, and (2) the forecast paths of inflation. Figure 5 shows the impulse responses of inflation, output growth and interest rates to the productivity growth shock $\varepsilon_{a,t}$ of -3%. The sequence of productivity growth rates a_t after the shock realization is shown in the top left figure. The solid blue lines represent the impulse responses with a weak commitment to FG ($\zeta = 0.5$) while the dashed red lines represent those with a tight commitment to FG ($\zeta = 4$). The figures imply that, even with the same size of negative productivity growth shock $\varepsilon_{a,t}$, the responses of inflation and output growth can be significantly different depending on the degree of commitment to FG. Specifically, the figure implies that the economy would be faced with deflation, large fluctuations in output growth and a long-lasting ELB experience if the central bank does not firmly commit to FG as a policy rule, but those negative consequences including a long lasting ELB experience could be avoided by clarifying the commitment to FG as a policy rule in advance. Figure 6 shows unconditional forecast paths of inflation rates with the 99%, 95%, 90% and 80% confidence intervals under different values of ζ. The figure indicates that while the forecasting path of inflation is negatively skewed under a weak commitment to FG ($\zeta = 0.5$) due to the adverse effects of ELB, the negative skew almost disappears under a tight commitment to FG ($\zeta = 4$). For example, while the 90% confidence interval includes negative inflation rates under $\zeta = 0.5$, the 95% confidence interval slightly touches on negative inflation rates under $\zeta = 4$. Those impulse responses and forecasting paths imply that if FG is announced as a monetary policy rule, it would have quantitatively relevant effects on inflation via expectations formation. Therefore, FG is more powerful if it is implemented as a systematic component of monetary policy, and, in such cases, FG should be modeled as a policy rule rather than neither an exogenous news shock to nominal interest rates nor an exogenous extension of zero interest rate periods.[9] In other words, if FG is not implemented as a policy rule, the effects of FG via the expectations formation effects would completely disappear, and consequently FG would have effects on inflation and output growth only after it is actually implemented at the ELB.

3.2.2 Target Inflation Rate and Forward Guidance

The effectiveness of FG as a policy rule heavily depends on the possibility of being constrained by the ELB. As is often argued, with a low target inflation rate, the central bank would be easily constrained by the ELB and consequently lose a room for lowering nominal interest rates in the face of deflation and/or recessions. In such cases, FG might be a useful tool to avoid being trapped by the ELB. Actually, the quantitative results in the previous subsection imply that the adverse effects due to the ELB could be mitigated by implementing a rule-based FG appropriately. On the other hand, with a high target inflation rate and a low probability of being trapped by the ELB, the effectiveness of FG might not be as evident as an economy with a lower target inflation rate.

To examine such arguments in this model, Figure 7 computes the policy functions of inflation and output growth with respect to the log-deviation of productivity growth from its steady state, $\log(a_t/a^*)$, under different levels of target inflation π^*. The solid blue lines represent the policy functions with a 2% inflation target ($\pi^* = 1.005$) while the dashed red lines represent those with a 1% inflation target ($\pi^* = 1.0025$). In both cases, the degree of commitment to FG ζ is set to 0.5. Those policy functions point out the advantage for setting a higher inflation target, as argued above. As productivity growth rates decline, inflation and output growth decline much more non-linearly for the 1% target case than for the 2% target case, because there is smaller room for lowering nominal interest rates in the former case. In particular, note that the steady-state inflation rate (i.e., inflation at $a_t = 0$) is around 2% for the 2% target case while it is

[9] Recently, Guerrieri and Iacoviello (2015) build a non-linear model in which the ZLB constraint binds endogenously and occasionally, and propose an easier way of solving it. As emphasized by themselves, however, their approach also cannot take into account the expectations formation effects at non-ZLB states by construction.

clearly less than 1% and close to 0% for the 1% target case. While steady-state inflation is usually lower than target inflation in a model with ELB, the figures indicate that such an adverse effect of ELB on steady-state inflation is not quantitatively negligible and more severe in the economy with a lower inflation targets.[10]

Figure 8 shows the policy functions under a tight commitment to FG (i.e., $\zeta = 4$) and different levels of inflation target. They show that, with a tight commitment to FG, the adverse effects due to a lower inflation target are significantly mitigated: the policy functions of inflation in the 1% target case and the 2% target case are almost parallel with each other and those of output growth are almost on the same line. Also, the steady-state inflation for the 1% target case is now at around 1%. Given the fact that the ELB constraint does not bind at the steady-state, the effect of FG on the steady-state inflation is induced by the expectations formation effects. That is, the expectation that the central bank would commit to FG at the ELB mitigate the adverse effects on inflation at the steady state, thus making it easier for the central bank to realize the target inflation at the steady state than under a weak commitment.

Figures 7 and 8 imply that a strong commitment to FG (i.e., a high ζ) and a high target inflation rate (i.e., a high π^*) are partly substitutable as a policy tool for mitigating the adverse effects due to ELB. In particular, a strong commitment might be more preferable than the higher inflation target if the central bank can have private agents trust the FG rule, because a high steady-state inflation may entail non-negligible welfare losses, as pointed out by Ascari and Sbordone (2014).

3.2.3 Relation to the "Forward Guidance Puzzle"

As some previous studies argue, FG modeled as an exogenous extension of zero interest rate periods has (unrealistically) large effects on inflation and output, which is called the "forward guidance puzzle" (e.g., Del Negro, Giannoni, and Patterson (2012) and McKay, Nakamura, and Steinsson (2015)). In this model, such an exogenous extension of zero

[10] Hills, Nakata, and Schmidt (2016) argues that steady-state inflation can be somewhat lower than target inflation in a non-linear model with ELB because steady-state inflation is determined by taking into account the risk for reaching the ELB.

interest rate periods can be described as negative monetary policy shocks $\varepsilon_{i,t} < 0$ at the ELB. That is, the negative monetary policy shocks at the ELB increase the FG term m_t by about $\zeta \times R_{ss}$ as indicated by (10), and lead to the expectation for longer zero interest rate periods. Therefore, the model in this paper is a good laboratory to examine how the "forward guidance puzzle" can be interpreted in the model with a rule-based FG.

Figure 9 shows the impulse responses of inflation and interest rates to a negative productivity growth shock $\varepsilon_{a,t} < 0$ *and* a negative monetary policy shock $\varepsilon_{i,t} < 0$. That is, the figure describes the situation that the central bank exogenously extends the period of zero interest rates at the same time as the economy is hit by a negative productivity growth shock in $t = 2$. The thick blue lines represent the case for no monetary policy shock (i.e., $\varepsilon_{a,t} < 0$ and $\varepsilon_{i,t} = 0$) for comparison, and indicate that the economy would be faced with deflation and as a result constrained by the ELB for five periods. The thin red lines in the same figure represent the case that the central bank tries to extend the period of zero interest rates by adding $\varepsilon_{i,t} = -4\%$ in $t = 2$. The -4% shock of $\varepsilon_{i,t}$ in $t = 2$ seems large enough to extend the periods of zero interest rates by one period, because the interest rate level right after leaving the ELB for the case without any monetary policy shocks (i.e., the thick blue line in the period 6 in the right figure) is less than 1%. The thin red lines indicate, however, that the interest rate in $t = 6$ would be slightly lower than that for the case of $\varepsilon_{i,t} = 0$ (the blue line), but the periods of zero interest rates would not be extended at all; there would still be five . The reason why the interest rate path would not significantly change even with the large negative monetary policy shock in $t = 2$ is that the negative monetary policy shock works for increasing inflation, as shown in the left figure, and consequently making the liftoff from the ELB earlier. That is, the increases in inflation due to the negative monetary policy shock in $t = 2$ would raise the interest rate indicated by the Taylor rule R_t^{tlr} and reduce the FG term m_t as indicated by (10), thus offsetting the increase in m_t induced by the negative productivity shock. The dotted green lines represent the case for $\varepsilon_{a,t} < 0$ and $\varepsilon_{i,t} = -8\%$, which imply that the periods of zero interest rates would be extended by one period only if the central bank adds such an unrealistically large monetary policy shock in $t = 2$.

In summary, the quantitative exercise in this subsection provides the following interpretation of the forward guidance puzzle. When the central bank announces to exogenously extend the periods of zero interest rates as a form of FG, inflation rates would increase in response to the announcement. Such an increase in inflation, however, makes the liftoff from the ELB earlier because the rule-based FG endogenously responds to the increased inflation rates, thus partially offsetting the central bank's initial intention for extending the zero interest rates. Therefore, considering the central bank's endogenous response to the increased inflation, an unrealistically large monetary policy shock needs to be added for extending the zero interest rate periods even just by one period. Since such an unrealistically large monetary policy shock has significant positive effects on inflation and output, a tiny exogenous extension of the zero interest rate periods and puzzlingly large policy effects on inflation and output growth are simultaneously observed in the model. In other words, the exercise implies that since such an unrealistically large monetary policy shock needs to be added, it is merely puzzling to observe the large policy effects on inflation and output growth.

3.3 Estimation with Full Sample

This subsection estimates the degree of commitment to FG, ζ, and identifies the structural shock sequences by a fully non-linear model using the U.S. data up to 2015/4Q. Since model parameters except for ζ have been already estimated by a linearized model using the U.S. data up to the Great Recession (i.e., up to 2007/4Q) in Section 3.1, the parameters of the non-linear model are set to the estimated posterior mean in Table 1. The model is then solved by the Coleman policy function iteration method under various values of ζ. Given the non-linear structure of the model due to the ELB and the crisis shock, the likelihood under each ζ cannot be calculated by the Kalman filter; it must be computed by simulation based methods like the particle filter. This subsection uses the particle filter with 1,000,000 particles and assumes measurement errors for output growth and inflation in data.[11] When applying the particle filter to the data with large

[11] This paper uses an algorithm for the particle filter as described in An and Schorfheide (2007). See Herbst and Schorfheide (2015) for other algorithms. Also, note that the number of particles needs to

economic fluctuations such as the U.S. data around the Great Recession, the crisis shock takes an important role for computing the likelihood in practice. That is, since the normal distribution has very small probability for a tail event, the particle filter only with normally distributed shocks cannot cover large economic fluctuations like the Great Recession even with a huge number of particles. Such a problem is not materialized when using the Kalman filter because the Kalman filter can cover any event, as long as it occurs with a positive probability. This problem, however, casts doubt on the use of a linearized model only with normally distributed shocks and the estimation of the model using the Kalman filter for describing crisis periods such as the Great Recession.

The degree of commitment to FG, ζ, is estimated as follows. First, the likelihood under a particular value of ζ, which is denoted by $p(Y|\zeta)$ where Y represents the data sequences, is computed by the particle filter. Then, the posterior mode based on the posterior density, $p(\zeta|Y)$, can be found using the joint density, $p(\zeta, Y)$, which is computed by multiplying the likelihood with the prior density, $p(\zeta)$, because the joint density is proportional to the posterior density, i.e., $p(\zeta|Y) \propto p(\zeta)p(Y|\zeta)$. Since ζ is the only parameter to be estimated, it is not needed to search the posterior mode by sampling posterior draws. Instead, the posterior mode is searched by calculating the joint probability $p(\zeta, Y)$ for each ζ according to $p(\zeta) \times p(Y|\zeta)$ one by one. Figure 10 shows the likelihood (dotted green line), prior density (gray line), and posterior density (blue line) with respect to ζ. The figure shows that the likelihood function for ζ is clearly increasing up to around $\zeta = 4$, thus strongly implying that the Federal Reserve has adopted FG as a policy rule. The figure also shows, however, that the likelihood function becomes almost flat for $\zeta > 4$, thus implying that the degree of commitment to FG ζ is weakly identified. This flat likelihood indicates that the only one ELB episode after the Great Recession in the U.S. is not enough to identify between the expected FG by a large ζ and the unexpected FG by the monetary policy shock i_t. Therefore, to identify a particular baseline value of ζ, a slightly downward sloping function Beta[1,2500] is used for the prior density, which implies that a higher ζ has slightly small probability than a lower ζ. The

be much more than usual cases in order to accurately describe the crisis event.

figure indicates that, given this downward sloping prior density, the posterior density is hump-shaped and maximized around $\zeta = 4$ (dashed black line) while the likelihood is almost flat for $\zeta > 4$. Hereafter, the value of $\zeta = 4$ is used as the baseline value.

Given the degree of commitment to FG in the baseline case, $\zeta = 4$, the sequences of structural shocks as well as the FG term m_t can be identified by the particle filter based on the model. The set of exogenous variables which drive the economy in this model consists of (i) the productivity growth, a_t, (ii) the productivity level, z_t, and (iii) the monetary policy shock, i_t. Figure 11 shows the identified sequences of exogenous variables as well as the FG term in the U.S. economy from 1983/1Q to 2015/4Q. The dotted red lines show 95% confidence intervals. The sequence of productivity growth (the left top figure) indicates that the U.S. productivity growth dropped dramatically in the Great Recession, but it has recovered and has now returned to around the steady state level. The productivity level (the right top figure), on the other hand, dropped somewhat during the Great Recession but it too has recovered strongly and remained at a higher level than the steady state. Because the productivity shocks are supposed to represent all supply shocks in the economy, the current high level of productivity in the figure implies that the weak inflation rates after the Great Recession are not driven by negative demand shocks but mainly by positive supply shocks, including lower commodity prices.[12] The sequences of monetary policy shock and the FG term (the bottom two figures) imply that the Federal Reserve aggressively carries out monetary easing or tightening as a form of monetary policy shocks and FG.[13] In particular, both positive FG term and negative monetary policy shocks are observed during and after the Great Recession, implying that the Federal Reserve kept zero interest rates longer than suggested by the Taylor rule both via expected rule-based FG and via exogenous extensions of zero interest rate periods. The FG term significantly increased right after the Great Recession, but it has gradually decreased and reached zero in 2015/4Q reflecting the fact that the Federal Reserve left

[12] This result is consistent with an argument by policy makers. See, for example, Yellen (2015).

[13] In addition to FG, the U.S. central bank adopted the Quantitative Easing (QE) as a monetary policy measure at the ELB. Since the model in this paper does not take into account the QE at all, the effects of QE, if any, are mostly identified as a demand shock in the estimation.

the ELB then. In the following subsections, counterfactual policy experiments are carried out through comparative statics under the identified sequences of exogenous variables.

3.4 Policy Experiment

This subsection conducts counterfactual policy experiments to investigate how much a rule-based FG affects inflation and output growth, particularly during and after the Great Recession. In particular, the policy experiments intend to answer the following "missing deflation" puzzle: Why did the U.S. economy not experience deflation, even with the huge decline in output growth during the Great Recession? While several studies, including Del Negro, Giannoni, and Schorfheide (2015), investigate the missing deflation problem using a linearized DSGE model, the policy experiments in this subsection try to add another but possibly compatible interpretation for this problem using a non-linear model. Also, the policy experiments that examine the inflation and output dynamics under various monetary policy choices should be helpful to consider the policy prescription for deflationary pressure observed in developed countries other than the U.S. For instance, since many European countries continue to suffer from deflationary pressure after the financial crisis, the results of the policy experiments may give some suggestive implications for their future policy decisions.

The counterfactual policy experiments via comparative statics are conducted by examining what would happen in the U.S. economy if the Federal Reserve adopted different monetary policies, given the structural shocks identified in the previous subsection remain unchanged. As for the counterfactual policy choices, I examine different monetary policy measures at the ELB in the following two aspects: (i) A weaker commitment to FG (i.e., a lower ζ), (ii) No exogenous extension of the zero interest rate periods (i.e., zero monetary policy shocks, i_t). Since the identified monetary policy shocks i_t in Figure 11 indicate that the Federal Reserve extended the zero interest rate periods via exogenous extensions in addition to via expected rule-based FG, these two counterfactual policy choices intend to describe the situation where the Federal Reserve did not expectedly and/or unexpectedly extend zero interest rate periods longer than suggested by the estimated Taylor rule.

Figure 12 shows the counterfactual paths of inflation and output growth under a weaker commitment to FG ($\zeta : 4 \rightarrow 0.5$). The figure indicates that inflation and output growth would not been changed for most periods even under the weaker commitment to FG, a reflection of the fact that the U.S. economy had not been faced with the ELB of nominal interest rates up to the late 2000s. During and after the Great Recession, however, the figure suggests that the U.S. economy would possibly experience deflation and a sharper drop in output growth if the Federal Reserve adopted the weaker commitment to FG. Furthermore, the effect of FG on inflation is also observed around the periods after the dot-com bubble in the early 2000s, even though the economy was not constrained by the ELB at that time. This result therefore implies the possibility that the commitment to FG has effects on inflation through not only the accumulation of FG term m_t at ELB but also through the expectations formation effects of a rule-based FG at non-ELB states.[14]

Next, Figure 13 shows the counterfactual paths of inflation and output growth with zero monetary policy shocks ($i_t = 1$ for all t) under a weaker commitment to FG ($\zeta : 4 \rightarrow 0.5$). This counterfactual scenario intends to describe the situation where the Federal Reserve did not deviate from the Taylor rule both expectedly via the FG rule and unexpectedly via the monetary policy shock, and as a result did not carry out any monetary easing via FG. The figure shows that, in this case, the U.S. economy not only would suffer from deflation during the Great Recession but also would experience relatively long-lasting low inflation rates in the early 2000s and after the Great Recession. Therefore, while the monetary easing that deviated from the Taylor rule during and after the recession periods is criticized for potentially having caused the economic bubble, the policy experiment indicates that these negative deviations from the Taylor rule played a key role to avoiding deflation.

[14] A caveat to this result is that the analysis here assumes that the value of ζ is constant throughout the sample period, and thus does not take into account the possibility that a commitment to a rule-based FG in the early 2000s might not be as strong as that in the late 2000s.

4 Application to the Japanese Economy

Finally, a quantitative analysis on the Japanese economy is carried out. Compared to the U.S. economy, there are some notable features in the Japanese economy. First, the Japanese economy was constrained by the ELB for most of the time since the late 1990s, and the Bank of Japan introduced FG to deal with the ineffectiveness of monetary policy at the ELB.[15] Although it is difficult to estimate the central bank's monetary policy behavior at the ELB, the estimation results are expected to give a more clear-cut conclusion than the U.S. case because the Japanese experience of the ELB should provide enough information for the estimation. Second, the Japanese economy had faced prolonged deflationary pressure. As I mentioned earlier, the efficacy of FG could crucially depend on the assumption that recession or deflation lasts only for a few periods, and thus an application of the model to the Japanese experiences is an interesting policy analysis to examine such an argument.

For the estimation of parameters, the same two-step approach is adopted as in the analysis on the U.S. economy. After estimating the parameters and identifying the shock sequences, the counterfactual policy simulations are conducted, with a special focus on the following two counterfactual cases: (i) a stronger commitment to FG, and (ii) a higher inflation target.

4.1 Estimation Result

As in the analysis on the U.S. economy, the parameters except for the strength of commitment to FG, ζ, are estimated by a linearized model without ELB and the crisis shock using the data only from 1983/1Q to 1997/4Q. The measurement equations for the Bayesian estimation are exactly the same as those for the analysis on the U.S. economy. As for calibrated parameters, only the steady-state inflation rate π^* is different from the U.S. case and set to 1.0025, which means that the Bank of Japan had set the

[15] In the 1990s, the Bank of Japan committed to a zero interest rate policy to deal with deflationary pressure at the ELB, even though it was not called FG at that time. See Ueda (2000) for more detail about the commitment policy in the 1990s in Japan.

target inflation rate at 1% up to 2013/1Q. The estimation results summarized in Table 1 indicate that the estimated parameters are close to the U.S. case.

Next, the degree of commitment to FG, ζ, as well as the structural shock sequences are estimated by a non-linear model with ELB and the crisis shock using the particle filter and the data up to 2013/1Q. The parameters of the fully non-linear model are set to the estimated values by the linearized model in Table 1. Figure 14 shows the likelihood with respect to ζ. In contrast to the U.S. case, the value of ζ is identified by the likelihood that it has a peak around $\zeta = 0.9$. Such a difference between the U.S. case and the Japanese case probably stems from the fact that the Japanese economy has reached the ELB three times in the sample periods and thus provides enough information to identify the value of ζ. The estimated $\zeta = 0.9$ implies that the Bank of Japan has adopted FG as a policy rule but its degree of commitment is much weaker than the U.S. case, reflecting the fact that, for example, the Bank of Japan raised interest rates in the early 2000s even in the low inflation environment.

Given the degree of commitment to FG in the baseline case, $\zeta = 0.9$, the sequences of structural shocks as well as the FG term m_t can be identified by the particle filter based on the model as in the U.S. case. Figure 15 shows the identified sequences of exogenous variables as well as the FG term in the Japanese economy from 1983/1Q to 2013/1Q. As in the U.S. economy, it indicates that the productivity growth declined around the Great Recession but has already returned to the steady state level, and that the high productivity level causes deflationary pressure. In contrast to the U.S. case, however, the FG term is still at a high level, even in 2013/1Q, thus implying that the Bank of Japan is expected to continue the zero interest rate for a while in the face of low inflation. Furthermore, the monetary policy shocks are identified as almost zero during all the zero interest rate periods. This result is also in contrast to the U.S. economy, where significant negative monetary policy shocks are identified after the Great Recession, and implies that the Bank of Japan committed to FG as expected by a rule but did not exogenously extend the zero interest rate periods more than (or less than) expected in the all ELB episodes.

When interpreting the quantitative result for a rule-based FG, however, we should be careful about the following caveats. First, the value of ζ is assumed to be constant throughout the sample period. This assumption may be a little bit strong, particularly for the Japanese economy, because the Bank of Japan has introduced several different versions of commitment on the future monetary policy in the course of its struggle with long-lasting deflation. Furthermore, considering the fact that FG has been treated as a rule-based policy only recently, the value of ζ for recent periods might be estimated with a downward bias. Second, the zero interest rate policy in Japan implemented in the late 2000s was quite different from what the U.S. had introduced in 2008. In particular, while the former intended to deal with the long-lasting deflationary pressure, the latter was introduced mainly as a crisis management tool. Third, the estimation presumes that the value of ζ is precisely perceived among private agents. In practice, however, even in the case that the central bank tried to strongly commit to a rule-based FG, private agents might not perceive such a commitment of the central bank perfectly. If this is the case, the estimated ζ would not necessarily correspond to a degree of commitment by the central bank, but would correspond to pervasiveness of FG as a policy rule. Since only the Japanese economy experienced the ELB in the early 2000s, the small value of ζ for the Japanese economy may reflect the fact that FG was not as pervasive as it was in the late 2000s or as it is in the U.S.

4.2 Counterfactual Policy Experiment

This subsection conducts counterfactual policy experiments to investigate how much a rule-based FG affects inflation and output growth in the Japanese economy. As for the counterfactual policy choices, I examine different monetary policy measures at the ELB in the following two aspects: (i) A stronger commitment to FG (i.e., a higher ζ), and (ii) a higher target inflation (i.e., a higher π^*). Since the Bank of Japan raised the target inflation in 2013/1Q, the second counterfactual case is supposed to provide some suggestions for the effect of the recent policy change.[16]

[16] On January 22, 2013, the Bank of Japan was recognized as increasing the target inflation from 1% to 2% when it announced that "...the Bank sets the 'price stability target' at 2 percent in terms of the

Figure 16 shows the counterfactual paths of inflation and output growth when the Bank of Japan adopts as strong commitment to FG as in the U.S. case ($\zeta : 0.9 \rightarrow 4$). The figure indicates that inflation and output growth would not been changed even under the stronger commitment to FG when the Japanese economy was not constrained by the ELB (i.e., periods up to the late 1990s and 2007-2008). During and after the Great Recession, however, the figure suggests that the Japanese economy would avoid the sharp drop in inflation and output growth if the Bank of Japan was believed to adopt a stronger commitment to FG. Interestingly, however, the figure implies that, even under a stronger commitment to FG, the Japanese economy would have experienced the long-lasting deflation in the late 1990s and the early 2000s, thus implying that the commitment to FG works for mitigating a sharp drop in inflation, particularly during a crisis period, but does not work for supporting inflation in the face of long-lasting deflationary pressure.

Next, Figure 17 shows the counterfactual paths of inflation and output growth with a higher target inflation ($\pi^* : 1.0025 \rightarrow 1.005$) with the baseline value of ζ. As pointed out by Figure 7 and 8, with a higher target inflation rate, the central bank has more room for lowering nominal interest rates, thus dealing with a decline in inflation or output growth by lowering nominal interest rates. Figure 17 indicates that the counterfactual path of inflation is higher than the actual data by more than 1%, particularly during the Great Recession, thus implying that a high target inflation rate works as a buffer for lowering interest rates and supports inflation rates.

A caveat to the counterfactual policy experiments for the Japanese economy is that the trend inflation is assumed to be anchored at the target inflation rate π^* throughout the sample periods. This assumption is controversial in the sense that some studies, including Aruoba, Cuba-Borda, and Schorfheide (2013) and Hirose (2014), consider the possibility that the trend inflation in Japan was not anchored at the target inflation in the 1990s, and argue that Japanese economy remained around the deflationary steady state, which is proposed by Benhabib, Schmitt-Grohe, and Uribe (2002). If the trend

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year-on-year rate of change in the consumer price index (CPI) – a main price index. Previously, the 'price stability goal in the medium to long term' was in a positive range of 2 percent or lower in terms of the year-on-year rate of change in the CPI and the Bank set a goal at 1 percent for the time being."

inflation was not anchored at the target inflation rate but remained at the lower level, the effects of FG via the expectations formation effect would be much smaller because its efficacy significantly relies on the assumption that inflation rates will eventually return to around the target inflation rate. In that sense, it may still be an open question that the FG can work for getting the economy out of deflation even after the economy runs into chronic deflation, as observed in Japan.

5 Concluding Remarks

In this paper, I quantitatively investigate the effects of FG by modeling it as a monetary policy rule rather than an exogenous policy shock in a non-linear new Keynesian model. By doing so, I investigate the effects of FG via changing expectations formation of private agents at the ELB as well as before reaching the ELB. A quantitative analysis points out significant policy effects of a rule-based FG via expectations formation, and provides a new interpretation of "forward guidance puzzle." The counterfactual policy experiments using the U.S. and Japanese data imply that a rule-based FG significantly mitigates a decline in inflation and output growth in a crisis period like the Great Recession via expectations formation. However, FG has, by design, smaller effects on inflation if the economy faces a long-lasting deflationary pressure, as observed in the 1990s in Japan.

There are some directions for future works. First, while this paper just focuses on the U.S. and Japanese economy, the same method can be applied to other developed countries, including European countries. Since deflation is a main concern for central banks in most developed countries, the methodologies developed in this paper would be helpful to consider the policy prescriptions for those economies. Second, the effect of FG in a model with the deflationary steady state in the sense of Benhabib, Schmitt-Grohe, and Uribe (2002) is an open question. While the possibility of the deflationary steady state is excluded by assumption even in face of long-lasting deflation, this assumption should be taken away when analyzing how to escape from the equilibrium dynamics around the deflationary steady state, particularly when investigating the Japanese economy. Third, the investigation on the optimal level of ζ is another important step. While those topics are important from theoretical and practical perspectives, I leave them for future research.

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Appendix A

In Appendix A, I describe how to solve a non-linear new Keynesian model in 2 by the Coleman's policy function iteration with a discretized state space. The procedure is basically the same as in Nakata (2013), which solves the New Keynesian model with the ELB by the same method.

First, the system to be solved is formulated. By plugging (3) and (5) into (6), the labor supply L_t and the real wage rate w_t are deleted from the equations and the following output-based new Keynesian Phillips curve (NKPC) is obtained,

$$1 - \theta + \theta\psi \frac{y_t^{\nu+\sigma}}{z_t^{\nu+1}} = \gamma\left(\pi_t - \pi^*\right)\pi_t - \beta\gamma E_t\left(\pi_{t+1} - \pi^*\right)\pi_{t+1}\left(\frac{y_{t+1}}{y_t}\right)^{1-\sigma}. \tag{14}$$

There are five state variables in this system: $(y_{t-1}, m_{t-1}, a_t, z_t, i_t)$. The next step is making a discretized state space for shocks (a_t, z_t and i_t), and approximating the AR(1) processes (11), (12), and (9) as a first-order Markov chains by the Tauchen's method. Also, the discretized state spaces for y_{t-1} and m_{t-1} are made in the space $[0.9y^*, 1.1y^*]$ and $[0.0, 0.4]$.

Then, the policy function iteration starts with the guess for the policy function $y^0(y_{-1}, m_{-1}, a, z, i)$ and $\pi^0(y_{-1}, m_{-1}, a, z, i)$, which are defined on the discretized state space for $(y_{t-1}, m_{t-1}, a_t, z_t, i_t)$. With this guess, the k-th interation ($k = 1, ...$) proceeds as follows.

1. For $k = 1, ...,$ the function for the interest rate based on the Taylor rule R^{tlr} and the observed interest rate R are defined as:

$$R^{tlr}(\tilde{y}, \tilde{\pi}, y_{-1}, a, i) = R_{ss} \left(\frac{\tilde{\pi}}{\pi^*} \right)^{\phi_\pi} \left(\frac{\tilde{y}}{y_{-1}} \frac{a}{a^*} \right)^{\phi_y} i$$

and

$$R(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, i) = \max \left[R^{tlr}(\tilde{y}, \tilde{\pi}, y_{-1}, a, i) - m_{-1}, 1 \right].$$

where the variables with tilde ($\tilde{y}$ and $\tilde{\pi}$) represent variables other than the state variable. Given the function for R^{tlr} and R, define the law of motion for the forward guidance term m as:

$$\begin{aligned} & m(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, i) \\ &= \begin{cases} m_{-1} + \zeta \left(1 - R^{tlr}(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, i)\right) & \text{if } R^{tlr} < 1 \\ m_{-1} - \left(R^{tlr}(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, i) - R(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, i)\right) & \text{otherwise} \end{cases} \end{aligned}$$

2. Define and compute functions Γ and Φ for $(\tilde{y}, \tilde{\pi})$ on the discretized state space as follows:

$$\begin{aligned} \Gamma(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, z, i) = 0.99 \times E^N_{a_{+1}|a} E^N_{z_{+1}|z} E^N_{i_{+1}|i} \left[\frac{[y^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1})]^{-\sigma}}{a_{+1}\pi^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1})} \right] \\ + 0.01 \times E^C_{a_{+1}|a} E^C_{z_{+1}|z} E^C_{i_{+1}|i} \left[\frac{[y^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1})]^{-\sigma}}{a_{+1}\pi^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1})} \right] \end{aligned}$$

and

$$\begin{aligned} & \Phi(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, z, i) \\ &= 0.99 \times E^N_{a_{+1}|a} E^N_{z_{+1}|z} E^N_{i_{+1}|i} \left[\begin{array}{l} \left(\pi^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1}) - \pi^*\right) \\ \times \pi^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1}) \left(\frac{y^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1})}{\tilde{y}} \right)^{1-\sigma} \end{array} \right] \\ &+ 0.01 \times E^C_{a_{+1}|a} E^C_{z_{+1}|z} E^C_{i_{+1}|i} \left[\begin{array}{l} \left(\pi^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1}) - \pi^*\right) \\ \times \pi^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1}) \left(\frac{y^{k-1}(\tilde{y}, \tilde{m}, a_{+1}, z_{+1}, i_{+1})}{\tilde{y}} \right)^{1-\sigma} \end{array} \right] \end{aligned}$$

where $\tilde{m} = m(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, i)$. Also, $E^N_{x_{+1}|x}$ and $E^C_{x_{+1}|x}$ represent the one-period forward expectation operator of the variable $x = a, z, i$ in the non-crisis and crisis regime, respectively. In the computation, those expectations are computed by the transition matrix of Markov chain for a_t, z_t and i_t.

3. Given the functions Γ and Φ, the two residual functions V_1 and V_2 based on the Euler equation (2) and the NKPC (14) are defined as follows:

$$V_1(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, z, i) = \tilde{y}^{-\sigma} - \beta R(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, i)\Gamma(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, z, i)$$

and

$$V_2(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, z, i) = \gamma\left(\tilde{\pi} - \pi^*\right)\tilde{\pi} - \beta\gamma\Phi(\tilde{y}, \tilde{\pi}, y_{-1}, m_{-1}, a, z, i) - \left[1 - \theta + \theta\psi\frac{\tilde{y}^{\nu+\sigma}}{z^{\nu+1}}\right].$$

Since there are two unknowns, $\tilde{y}$ and $\tilde{\pi}$, and two residual functions, V_1 and V_2, the values of $\tilde{y}$ and $\tilde{\pi}$ can be pinned down by solving

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

using a non-linear equation solver. Let the solution for $\tilde{y}$ and $\tilde{\pi}$ be $y^k(y_{-1}, m_{-1}, a, z, i)$ and $\pi^k(y_{-1}, m_{-1}, a, z, i)$, respectively, for the state vector $(y_{t-1}, m_{t-1}, a_t, z_t, i_t)$. Repeat Step 1 - 3 for every state vector $(y_{t-1}, m_{t-1}, a_t, z_t, i_t)$.

4. If $\left\|y^k - y^{k-1}\right\| + \left\|\pi^k - \pi^{k-1}\right\| < \bar{\varepsilon}$, then stop and use $y^k(y_{-1}, m_{-1}, a, z, i)$ and $\pi^k(y_{-1}, m_{-1}, a, z, i)$ as the policy functions in the non-linear new Keynesian model. Otherwise, $k \rightarrow k+1$ and go back to Step 1.

5. Once the policy functions $y(y_{-1}, m_{-1}, a, z, i)$ and $\pi(y_{-1}, m_{-1}, a, z, i)$ are obtained, the policy function for the interest rate based on the Taylor rule, the observed interest rate and the forward guidance term, $R^{tlr}(y_{-1}, m_{-1}, a, z, i)$, $R(y_{-1}, m_{-1}, a, z, i)$ and $m(y_{-1}, m_{-1}, a, z, i)$, are computed by

$$R^{tlr}(y_{-1}, m_{-1}, a, z, i) = R_{ss}\left(\frac{\pi(y_{-1}, m_{-1}, a, z, i)}{\pi^*}\right)^{\phi_\pi}\left(\frac{y(y_{-1}, m_{-1}, a, z, i)}{y_{-1}}\frac{a}{a^*}\right)^{\phi_y} i,$$

$$R(y_{-1}, m_{-1}, a, z, i) = \max\left[R^{tlr}(y_{-1}, m_{-1}, a, z, i) - m_{-1}, 1\right],$$

and

$$m(y_{-1}, m_{-1}, a, z, i) = \begin{cases} m_{-1} + \zeta\left(1 - R^{tlr}(y_{-1}, m_{-1}, a, z, i)\right) & \text{if } R^{tlr} < 1 \\ m_{-1} - \left(R^{tlr}(y_{-1}, m_{-1}, a, z, i) - R(y_{-1}, m_{-1}, a, z, i)\right) & \text{otherwise} \end{cases}.$$

Given those policy functions, recursively compute the policy function for the long-term interest rate $R^j(y_{-1}, m_{-1}, a, z, i)$ according to

$$R^j(y_{-1}, m_{-1}, a, z, i)$$
$$= \left\{ \begin{array}{l} 0.99 \times E^N_{a_{+1}|a} E^N_{z_{+1}|z} E^N_{i_{+1}|i} \left[R^{j-1}(y(y_{-1}, m_{-1}, a, z, i), m(y_{-1}, m_{-1}, a, z, i), a_{+1}, z_{+1}, i_{+1}) \right] \\ +0.01 \times E^C_{a_{+1}|a} E^C_{z_{+1}|z} E^C_{i_{+1}|i} \left[R^{j-1}(y(y_{-1}, m_{-1}, a, z, i), m(y_{-1}, m_{-1}, a, z, i), a_{+1}, z_{+1}, i_{+1}) \right] \end{array} \right\}^{j-1}$$
$$\times R(y_{-1}, m_{-1}, a, z, i)$$

where $j = 1, 2, ...$. Note that $R^0(y_{-1}, m_{-1}, a, z, i) = 1$ and $R^1(y_{-1}, m_{-1}, a, z, i) = R(y_{-1}, m_{-1}, a, z, i)$ by definition.

The obtained policy functions for y, π, m, R^{tlr}, R and R^{20} with respect to the growth shock a are shown, for example, in Figure 3. Those policy functions are used for computing the likelihood and identifying the structural shocks in the particle filter.

Table 1: Parameter values for the U.S. economy

parameter	posterior mean (the U.S.)	posterior mean (Japan)	prior dist.	prior mean	prior stdev
σ	2.1395	1.3589	Gamma	2.0	0.4
γ	51.5968	50.9266	Gamma	58.0	10.0
$\phi_\pi - 1$	1.4259	1.4452	Gamma	0.5	0.2
ϕ_y	0.2713	0.3890	Gamma	0.5	0.2
a^*	1.0048	1.0036	Gamma	1.005	0.001
η^*	1.0032	1.0027	Gamma	1.00	0.01
ρ_a	0.9177	0.8997	Beta	0.9	0.05
ρ_z	0.8742	0.8625	Beta	0.5	0.25
ρ_i	0.5153	0.3686	Beta	0.5	0.25
ρ_η	0.9050	0.8227	Beta	0.7	0.15
σ_a	0.0018	0.0019	Inv. Gamma	0.010	inf.
σ_z	0.0075	0.0102	Inv. Gamma	0.015	inf.
σ_i	0.0050	0.0053	Inv. Gamma	0.015	inf.
σ_η	0.0015	0.0015	Inv. Gamma	0.005	inf.

Figure 1: The U.S. output, inflation and interest rates

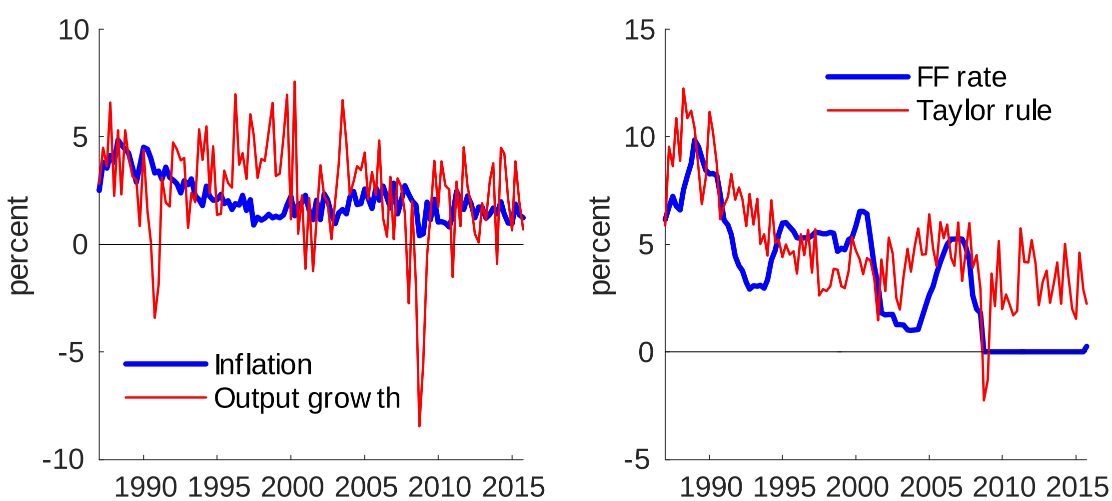

Figure 2: Forward guidance: a simple example

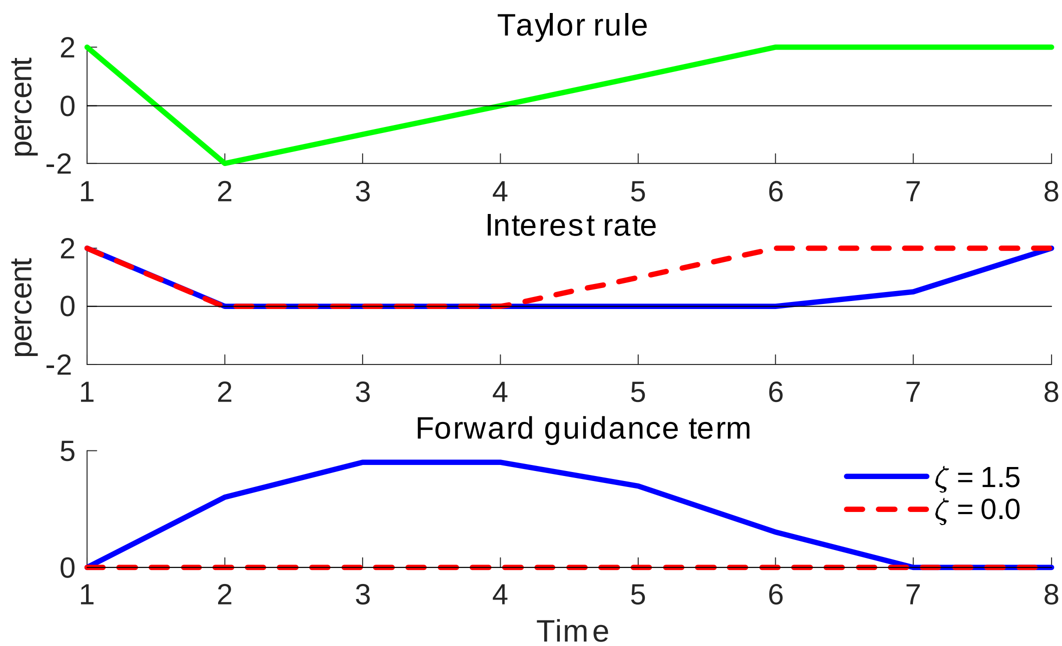

Figure 3: Policy functions by FG term, m_{t-1}

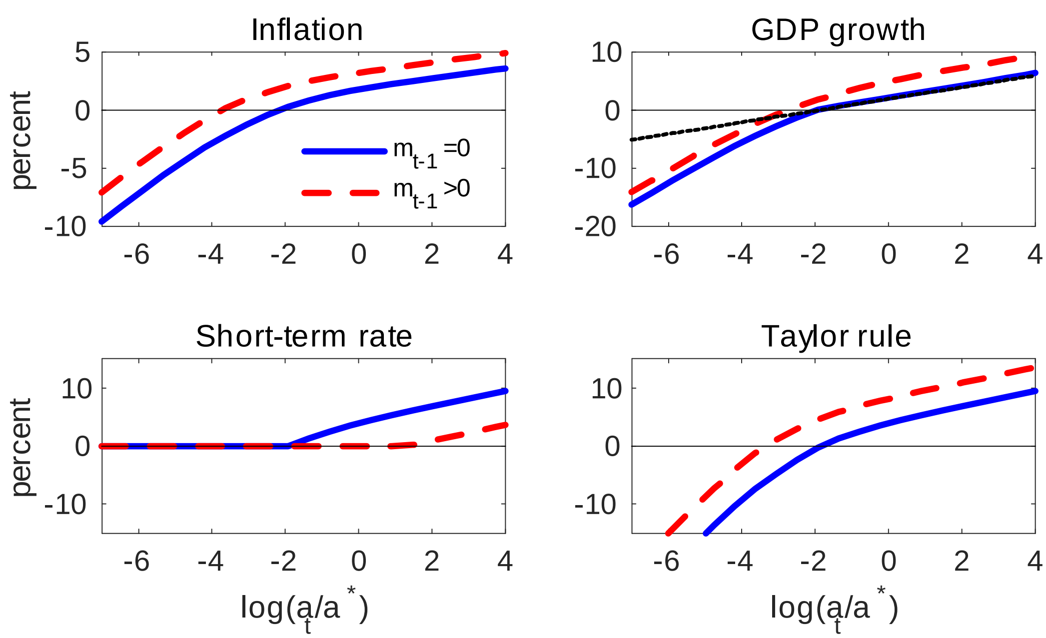

Figure 4: Policy functions by FG rule, ζ

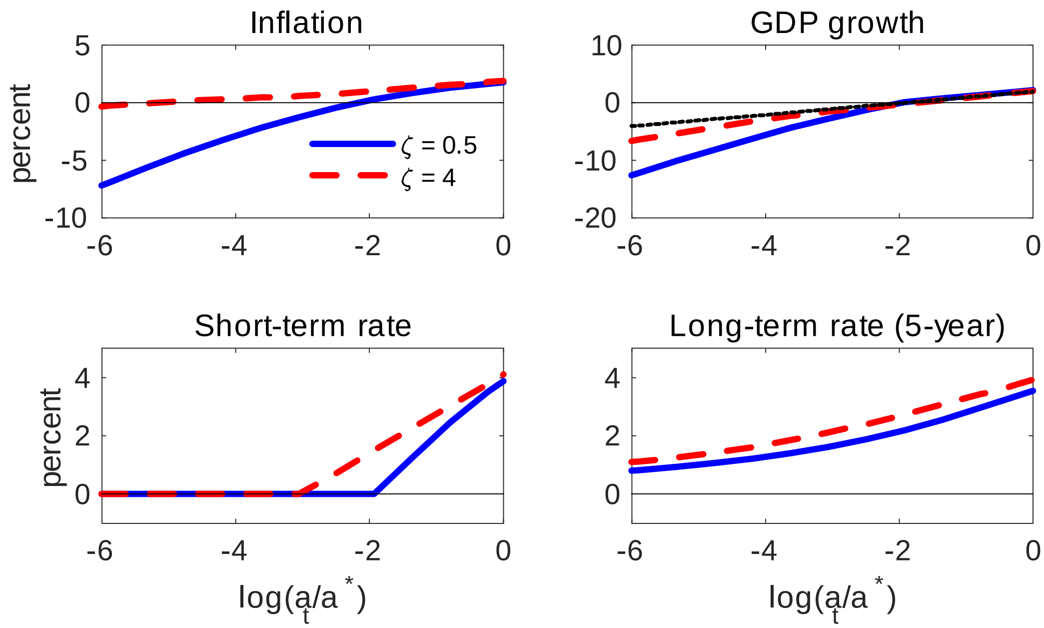

Figure 5: Impulse responses to a growth shock

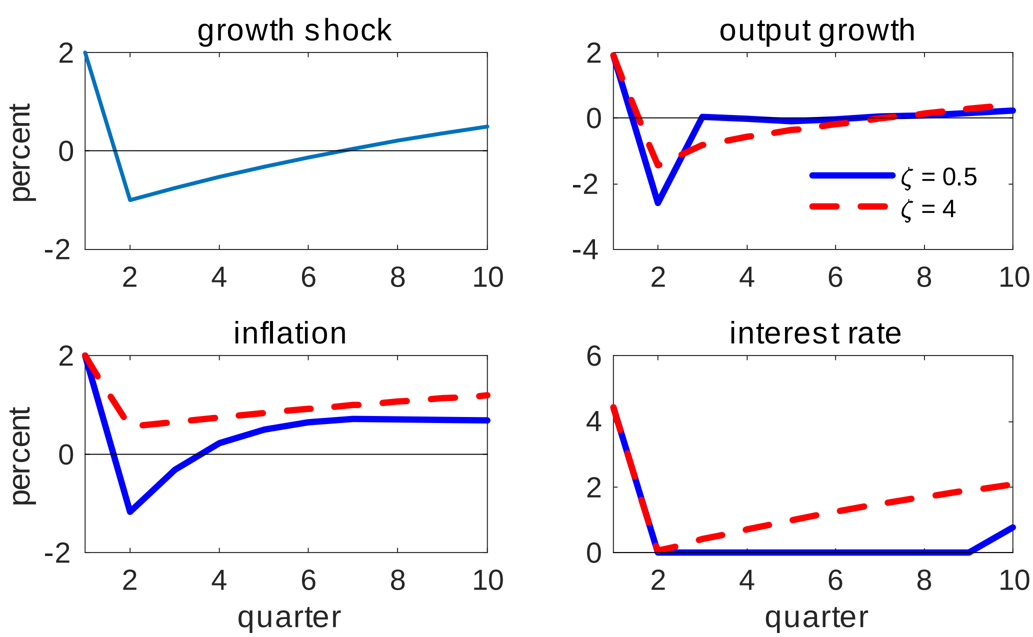

Figure 6: Forecasting Path of Inflation Rates

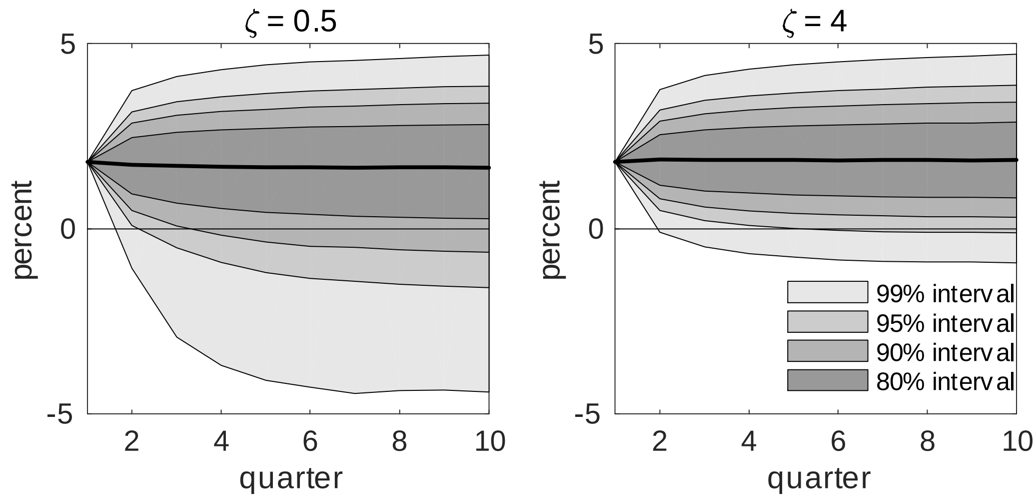

Figure 7: Policy function by target inflation, π^* (a weak commitment to FG case, $\zeta = 0.5$)

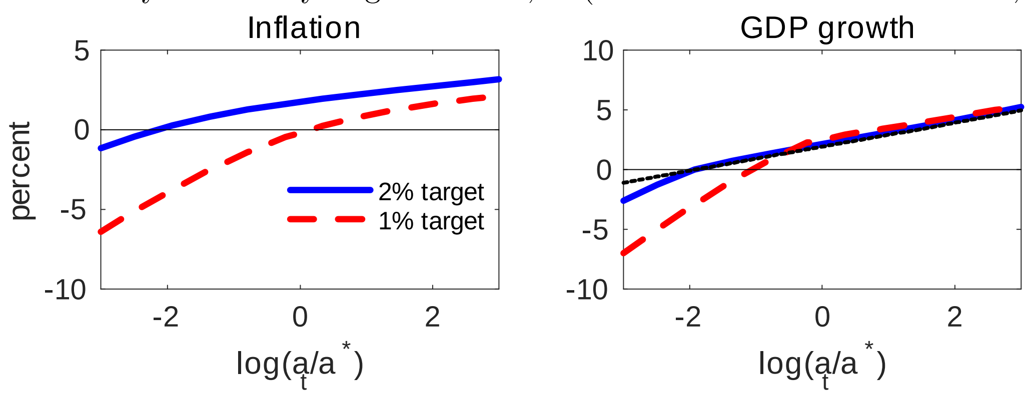

Figure 8: Policy function by target inflation, π^* (a strong commitment case, $\zeta = 4$)

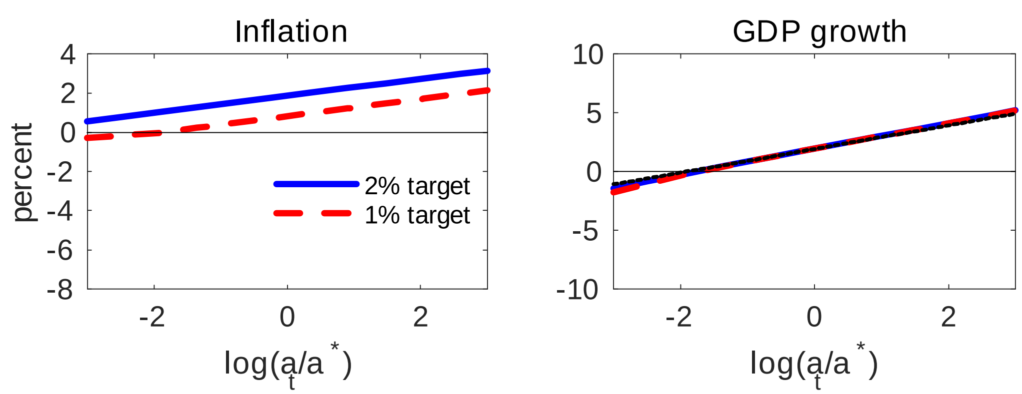

Figure 9: Responses to a growth shock with a negative monetary policy shock

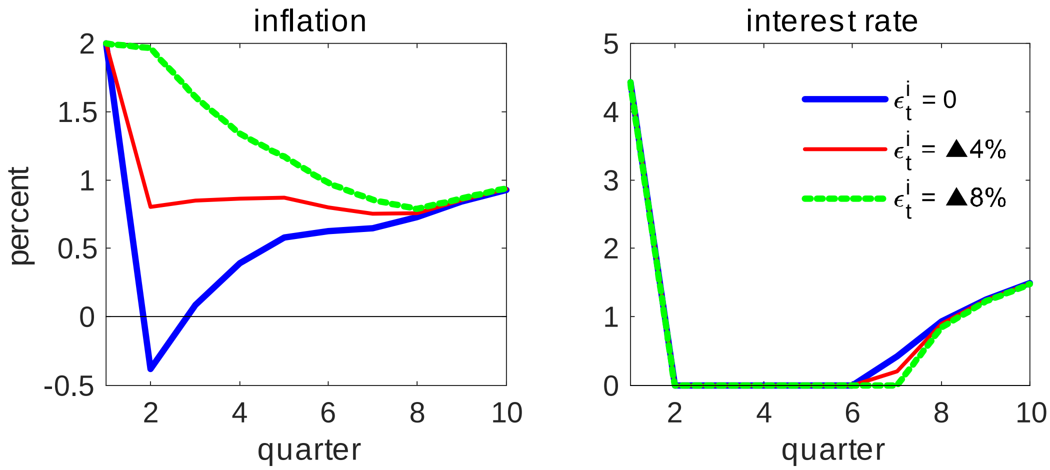

Figure 10: Likelihood and posterior distribution for ζ (the U.S.)

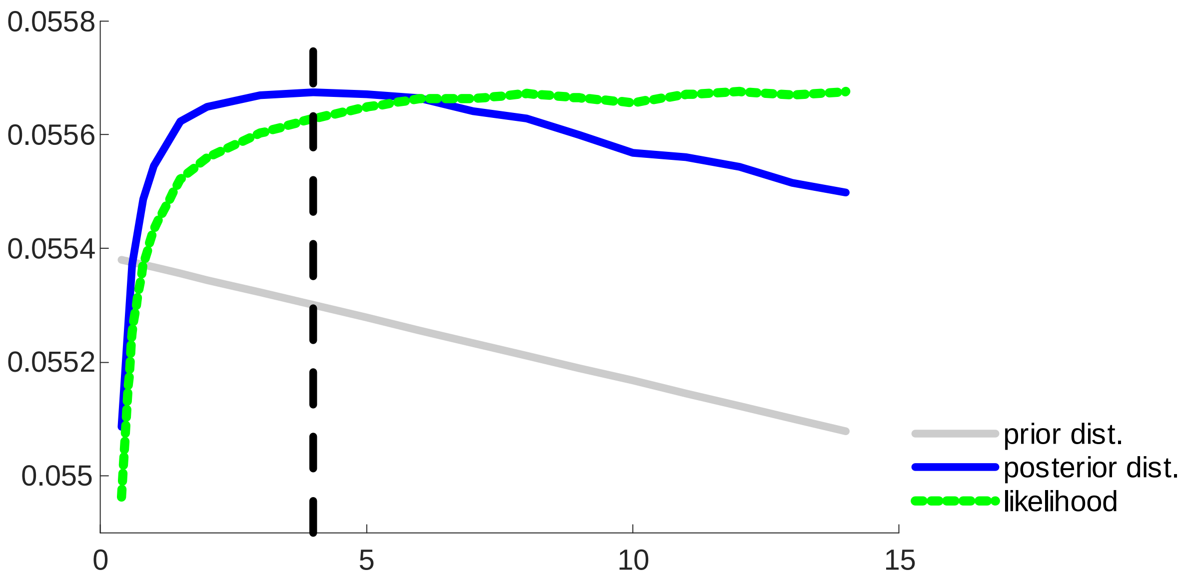

Figure 11: Identified shocks and FG term

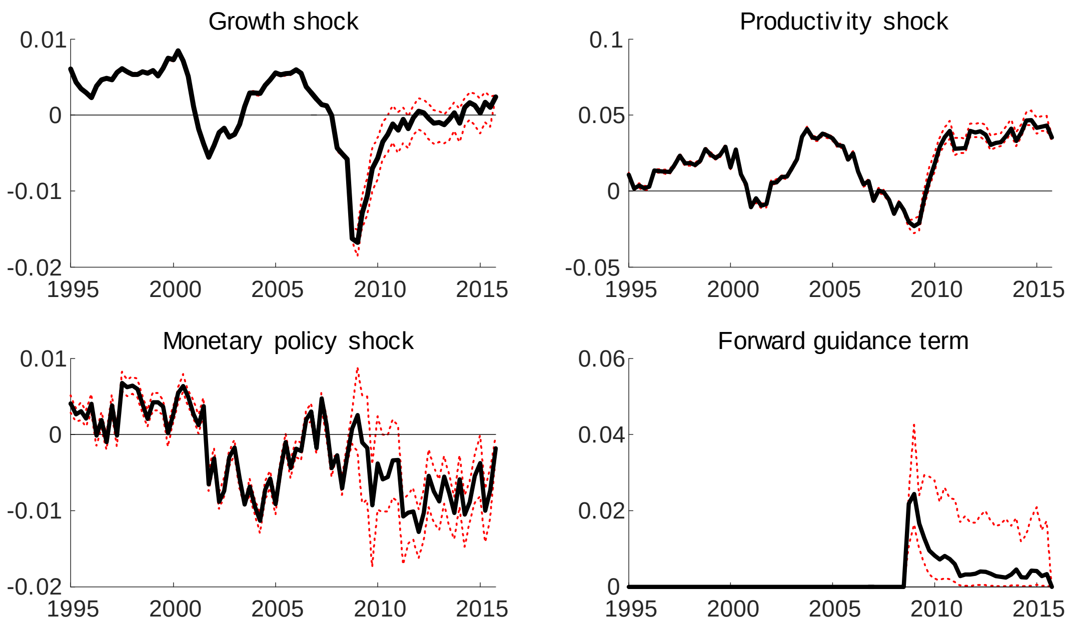

Figure 12: Counterfactual simulation ($\zeta : 4 \to 0.5$)

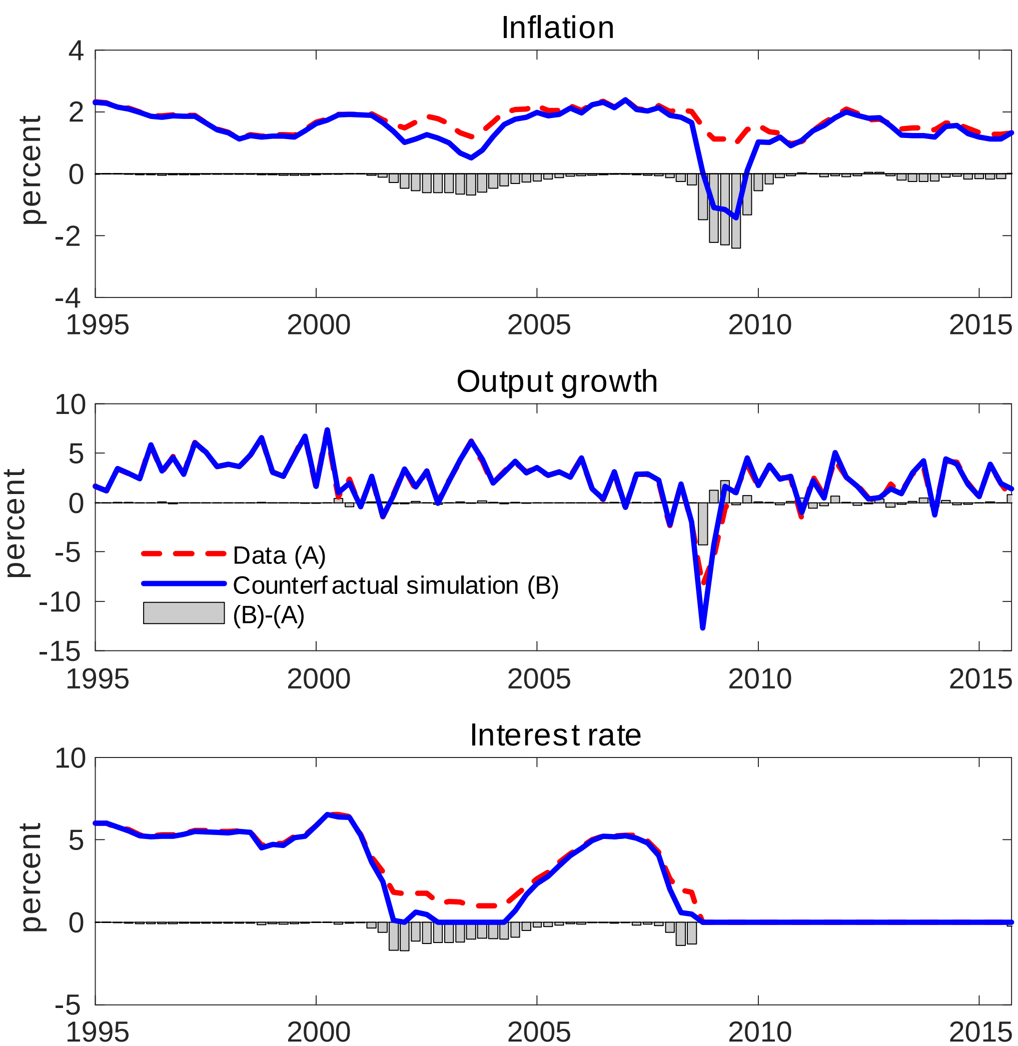

Figure 13: Counterfactual simulation ($\zeta : 4 \rightarrow 0.5$ and $i_t = 1$ for all t)

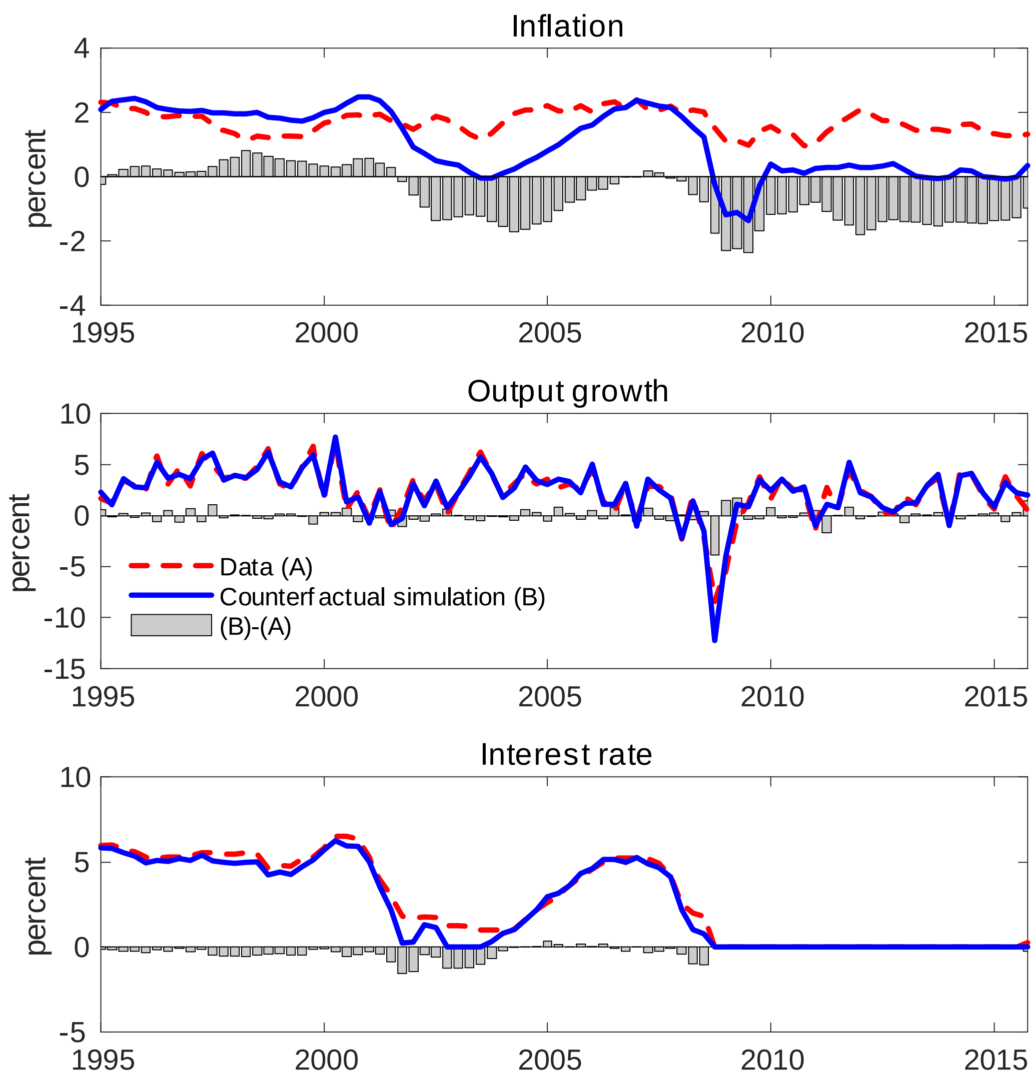

Figure 14: Likelihood for ζ (Japan)

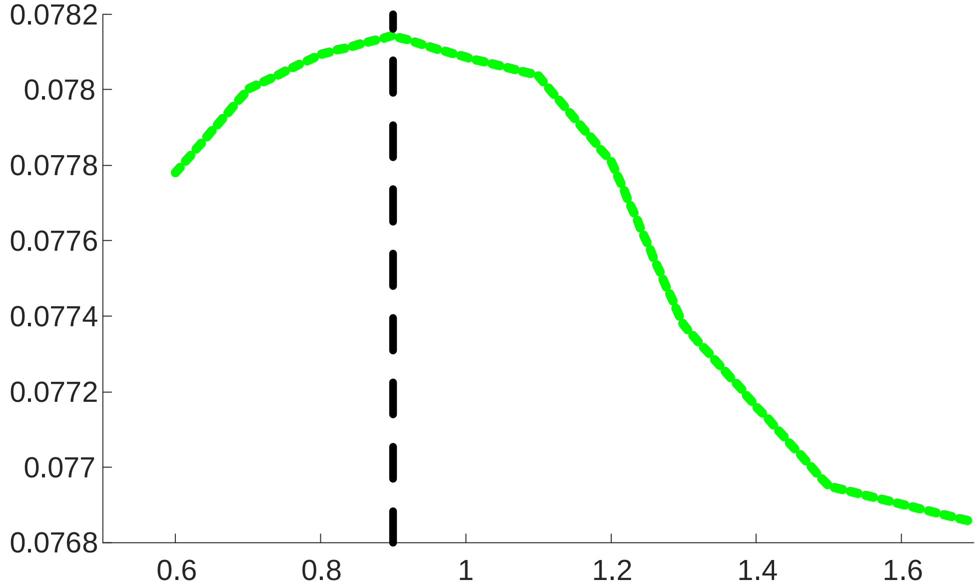

Figure 15: Identified shocks and FG term (Japan)

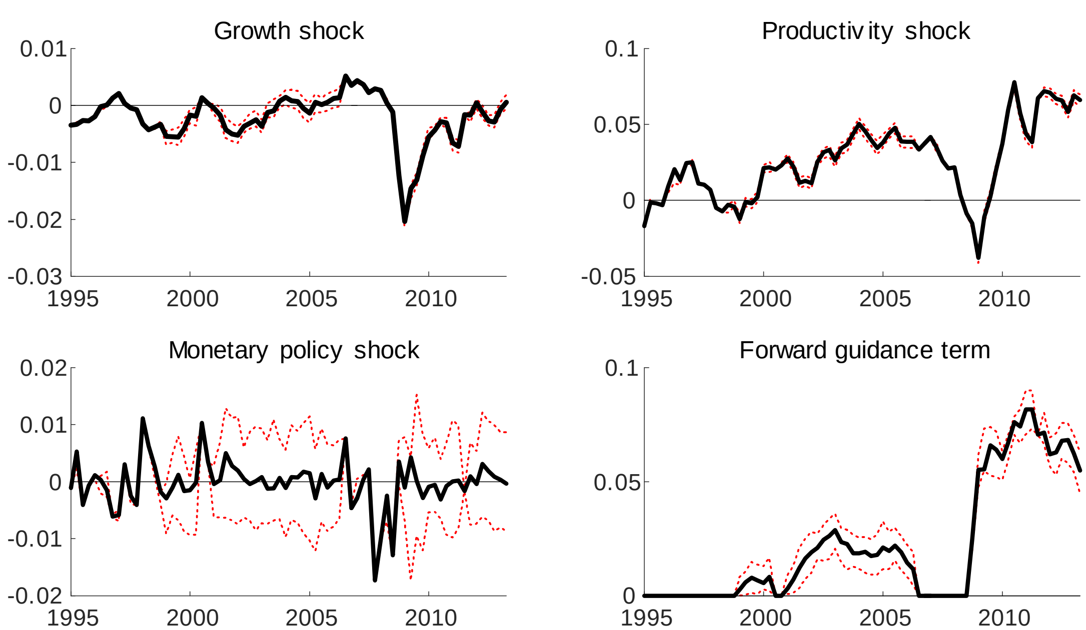

Figure 16: Counterfactual simulation ($\zeta : 0.9 \to 4$)

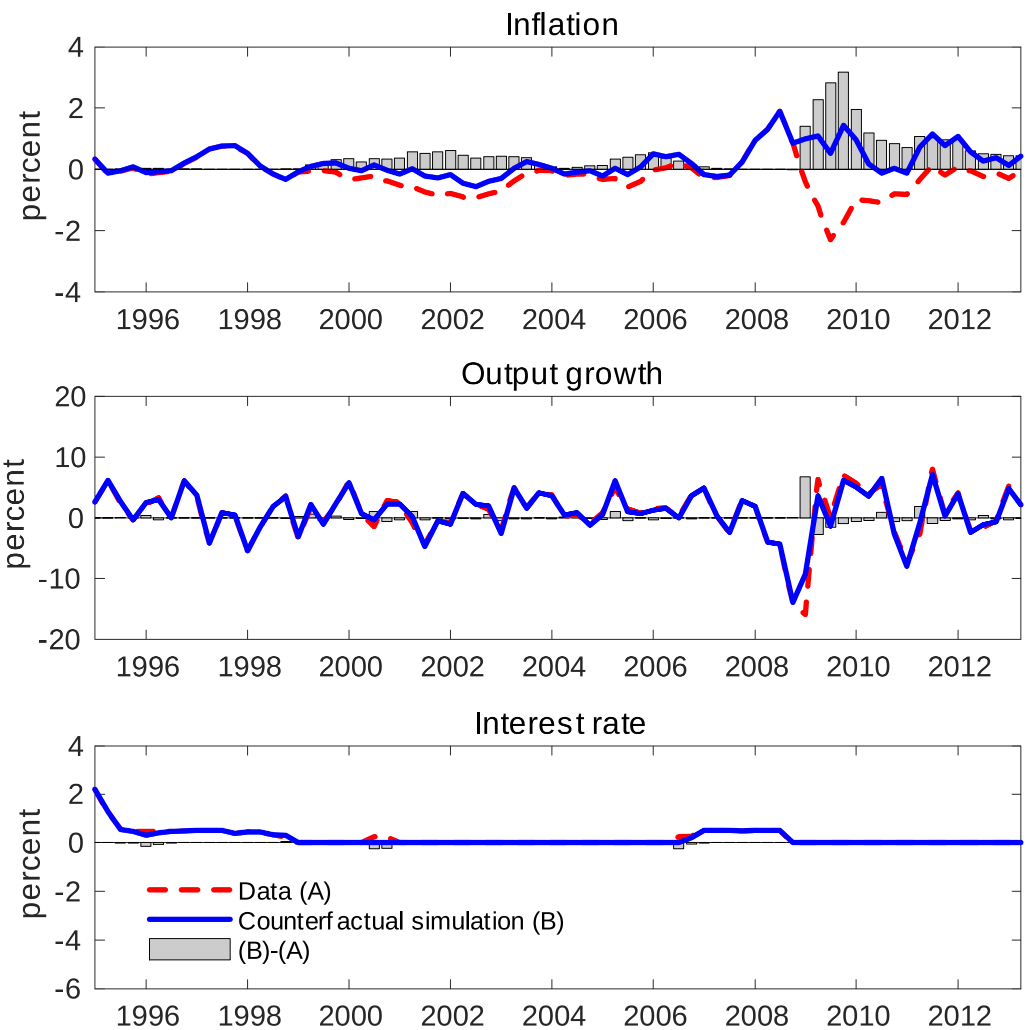

Figure 17: Counterfactual simulation ($\pi^* : 1.0025 \rightarrow 1.005$)