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Measuring Underlying Inflation Using Dynamic Model Averaging

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Abstract

This paper presents a new framework for measuring underlying inflation with multiple core indicators for Japan's consumer price index (CPI). Specifically, a combined core indicator is constructed by applying an econometric method based on dynamic model averaging as a weighted average of individual core indicators. The combined core indicator has time-varying combination weights reflecting changes in the predictive performance of each individual core indicator on a real time basis. Thus, the combined core indicator has the potential to adapt to changes in the nature and sources of price movements. Empirical evidence indicates that the combined core indicator firmly outperforms the individual core indicators over time. In addition, the combination weights for the exclusion-based indicators (e.g., the CPI excluding fresh food) tend to be high when aggregate shocks drive the overall inflation. In contrast, combination weights for the distribution-based indicators (e.g., trimmed mean) tend to be high when idiosyncratic shocks are dominant.

Keywords: Consumer price; Core inflation measure; Dynamic model averaging

JEL classification: C52, C53, E31

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1. Introduction

Accurately tracking underlying inflation is crucial to central banks when conducting monetary policy to achieve price stability. Therefore, the Bank of Japan monitors several core indicators for the consumer price index (CPI), which are designed to remove transitory price movements.[1]

Core CPI indicators can be classified into two groups (Figure 1). The first group includes indicators that exclude the fixed set of items with high short-term volatility; for example, the “CPI excluding fresh food,” “CPI excluding fresh food and energy,” and “CPI excluding food and energy.” We refer to these types of indicators as the “exclusion-based indicators” hereafter. The second group includes indicators that exclude the outlying portion of the cross-sectional price change distribution. For example, the “trimmed mean,” “weighted median,” and “mode” belong to this group.[2] We refer to these types of indicators as “distribution-based indicators.” The Bank of Japan comprehensively monitors these various indicators to identify underlying inflation more accurately. This is because, as indicated by Shiratsuka (2015), the performance of each indicator can vary over time with changes in the nature of inflationary developments. However, an issue arises when different core indicators imply different underlying inflation: To what extent can we trust each core indicator?

To address the aforementioned question, a combined core indicator is constructed by applying an econometric method based on dynamic model averaging (DMA) developed by Raftery *et al*. (2010). The combined core indicator has time-varying combination weights, reflecting changes in the predictive performance of each

[1] The research and statistics department of the Bank of Japan releases several measures of underlying inflation, e.g., “the CPI (excluding fresh food and energy),” “trimmed mean,” “weighted median,” and “mode” in line with the monthly release of the official CPI for Japan. See, e.g., Shiratsuka (1997, 2006, 2015), Mio and Higo (1999), and Hogen *et al*. (2015) for the characteristics of such indicators.

[2] The “trimmed mean” excludes items located in the upper and lower tails of the weighted price change distribution. The “weighted median” is the weighted average of price changes of the item at approximately the 50 percentile point of the weighted price change distribution. The “mode” corresponds to the highest frequency in the distribution.

individual core indicators on a real time basis. Thus, the combined core indicator has the potential to adapt to the change in the nature of inflationary development.

The idea of a “combined core indicator” dates back at least to Cogley (2002). He constructed a weighted average of individual core indicators by regressing the future overall inflation on several individual core indicators. Using the CPI data for Brazil, Figueiredo and Staub (2002) calculated a combined core indicator in two ways. One is a simple arithmetic average and the other is a weighted average whose weights are defined using the inverse of the variability of individual core indicators. However, both studies indicate that their combined core indicators do not improve the capacity of capturing the underlying inflation compared with the case of using only the best individual core indicator.

Their failures are attributed mainly to two problems. First, the relation between the performance of individual core indicators and combination weights is unclear. Second, the weights do not necessarily reflect time-varying nature of the predictive performance of individual core indicators. These problems are serious, especially in the study by Figueiredo and Staub (2002), in the sense that they make an ad hoc assumption on the combination weights. In contrast, our approach with DMA has the advantage that weights are allowed to change over time in response to the change in the predictive performance of individual core indicators.[3]

The empirical evidence reveals that the combined core indicator outperforms the individual core indicators in tracking and forecasting the underlying inflation rate. To the best of our knowledge, this study is the first to succeed in constructing a combined core indicator that performs better than any individual core indicator. In addition, this is one of the few studies that present the usefulness of DMA in its application to economic data.[4]

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[3] The combined core indicator is a “real-time” measure of underlying inflation in the sense that it only uses information available at the current period, i.e., information arriving after the current period has no effect on the current weights.

[4] Koop and Korobilis (2012) and Nicoletti and Passaro (2012) are other examples of the application of DMA to economic data.

In addition, this study reveals that when aggregate shocks are dominant, the group of exclusion-based indicators tends to perform well. When idiosyncratic shocks are dominant, the group of distribution-based indicators tend to outperform the group of exclusion-based indicators.

The remainder of the paper is organized as follows. Section 2 outlines the method to construct the combined core indicator. Section 3 applies the method to the core indicators of Japan's CPI. Section 4 examines whether the resulting indicator outperforms the individual core indicators. Section 5 presents the conclusions.

2. Methodology for Constructing the Combined Core Indicator

This section outlines our methodology for constructing the combined core indicator with DMA in three steps. First, the criteria for evaluating each core indicator is specified since the combination weights are calculated on the basis of the performance of each individual core indicator. Second, time-varying combination weights are estimated using DMA. Third, the combined core indicator is defined as point forecasts based on the combination weights from DMA.

2.1. Criteria for evaluating individual core indicators

According to Cogley (2002), the well-accepted definition of the underlying inflation by Bryan and Cecchetti (1994)—"the component of price changes that is expected to persist over medium-run horizon of several years"—can be formally written as

$$\pi_t^{core} = \mathrm{E}_t\left(\pi_{t+h}^{all}\right) \tag{1}$$

where π_t^{core} denotes the year-on-year change of the core indicator at time t (month), and π_{t+h}^{all} denotes that of the overall CPI at time $t+h$, respectively. This formulation implies that the core indicator should be an unbiased predictor of the h-month-ahead inflation rate of the overall CPI.

Based on the definition above, how can we evaluate the individual core indicators? We assume that the deviation between π_{t+h}^{all} and π_t^{core} follows the normal distribution with mean 0. Then, the criteria for evaluating the individual core indicator $k \in \{1,2,\ldots K\}$ can be written as

$$\pi_{t+h}^{all} = \pi_t^{core,k} + \varepsilon_{t,k} \ \ \varepsilon_{t,k} \sim \mathrm{N}(0, \sigma_{t,k}^2). \tag{2}$$

Due to the assumption for the error term, the expected value of the dependent variable conditional on the information up to time t equals the value of the independent variable. The performance of each core indicator corresponds to its goodness-of-fit to Equation (2).

The analysis below considers the case where core indicators are predictors of one-year-ahead inflation rates ($h = 12$). As a robustness check, we estimate the combined core indicators in the cases where $h = 15, 18$ and confirm that the estimates do not change substantially when we set different values of h.

2.2. Estimation of time-varying combination weights using DMA

We now describe how time-varying combination weights are estimated in our framework. The procedure is decomposed into the prediction step and updating step, as in the standard state-space model (e.g., the Kalman filter). In the prediction step, weights are estimated on the basis of the information up to the previous period. In contrast, in the updating step, the weights are updated using the newly available information in the current period. The key feature of DMA is that it weighs the most recent information more heavily in the prediction step. This allows the combined core indicator to adapt to sudden changes in the performance of the individual core indicators.

Before describing the detailed procedure for the estimation, we explain the definition of the weights in DMA. In general, the weights for combining different models are based on the posterior probabilities within a Bayesian framework. In our analysis, the weight for individual core indicator k corresponds to the posterior probability that core indicator k is the best core indicator. Let $\omega_{t|t-1,k}$ denote the weight for the core indicator k in the prediction step and $\omega_{t|t,k}$ denote that in the updating step. Then, they are defined as follows:

$$\text{Prediction step}: \omega_{t|t-1,k} \equiv P(L_t = k | \pi_{1:t+11}^{all}) \tag{3}$$

$$\text{Updating step}: \omega_{t|t,k} \equiv P(L_t = k | \pi_{1:t+12}^{all}) \tag{4}$$

where $L_t = k$ indicates that the core indicator k is the best predictor; $P(L_t = k| \cdot)$

denotes the conditional probability that such an event happens; and $\pi^{all}_{1:t+12}$ denotes the set of inflation rates from the beginning to the period at $t+12$.

In the prediction step, DMA produces estimates of the weights for the current period, given information up to the previous period. Raftery *et al.* (2010) assumed that the model governing the system evolves according to a $K \times K$ transition matrix $Q = (q_{kl})$, where $q_{kl} = P(L_t = k | L_{t-1} = l)$; but, instead of specifying Q, they propose the approximation below:

$$\omega_{t|t-1,k} = \frac{\omega^{\alpha}_{t-1|t-1,k}}{\sum_{k=1}^{K} \omega^{\alpha}_{t-1|t-1,k}}. \tag{5}$$

$\alpha \in [0,1]$ is a calibrated parameter and is called the *forgetting factor* since it determines to what extent the weights incorporate past information. For example, when $\alpha = 0$, $\omega_{t|t-1,k}$ always equals $1/K$. That is, past information does not affect the prediction of the current weights at all. On the other hand, when $\alpha = 1$, $\omega_{t|t-1,k} = \omega_{t-1|t-1,k}$ holds. This implies that the case with $\alpha = 1$ corresponds to the estimation with an expanding window.

Once information at time t arrives, the weights obtained in the prediction step ($\omega_{t|t-1,k}$) are updated. Let $P(\pi^{all}_{t+12} | L_t = k)$ denote the conditional probability that π^{all}_{t+12} is observed, given $L_t = k$. Equation (6) is obtained by applying the Bayes' theorem straightforwardly.

$$\omega_{t|t,k} = \frac{\omega_{t|t-1,k}\, P(\pi^{all}_{t+12} | L_t = k)}{\sum_{k=1}^{K} \omega_{t|t-1,k}\, P(\pi^{all}_{t+12} | L_t = k)}. \tag{6}$$

From the assumption on the error term in Equation (2), π^{all}_{t+12} follows the normal distribution with mean $\pi^{core,k}_t$ and variance $\sigma^2_{t,k}$ ($\mathrm{N}(\pi^{core,k}_t, \sigma^2_{t,k})$). Using this, $P(\pi^{all}_{t+12} | L_t = k)$ is calculated as follows.[5]

$$P(\pi^{all}_{t+12} | L_t = k) = \frac{1}{\sqrt{2\pi\sigma^2_{t,k}}} \exp\left(-\frac{\left(\pi^{all}_{t+12} - \pi^{core,k}_t\right)^2}{2\sigma^2_{t,k}}\right). \tag{7}$$

[5] $\sigma^2_{t,k}$ is estimated by exponentially weighted moving average (EWMA). For details, see Koop and Korobilis (2012).

By iterating these steps, we calculate the combination weights at each time period.

2.3. Construction of the combined core indicator

Using the rule of the conditional probability, it is simple to derive the following:

$$\mathrm{E}\left(\pi_{t+12}^{all} \middle| \pi_{1:t}\right) = \sum_{k=1}^{K} \mathrm{E}\left(\pi_{t+12}^{all} \middle| \pi_{1:t}, L_t = k\right) P(L_t = k | \pi_{1:t}) \,. \tag{8}$$

From Equation (2), $\mathrm{E}\left(\pi_{t+12}^{all} \middle| \pi_{1:t}, L_t = k\right)$ in the right-hand side of Equation (8) is given by $\pi_t^{core,k}$. On the other hand, $P(L_t = k|\pi_{1:t})$ indicates the probability that the core indicator k is the highest performance indicator at time t. Then, it can be replaced by $\omega_{t-11|t-12,k}$. Therefore, Equation (8) can be rewritten as

$$\mathrm{E}\left(\pi_{t+12}^{all} | \pi_{1:t}\right) = \sum_{k=1}^{K} \omega_{t-11|t-12,k} \, \pi_t^{core,k}. \tag{9}$$

This equation gives the point estimates of the overall inflation rate at $t+12$ by DMA, given information up to the current period. We call this point estimate *the combined core indicator*.[6] It should be noted that the weights that are used to calculate the combined core indicator at time t are not revised even after the new information arrives.

3. Estimation of the Combined Core Indicator for Japan's CPI

In this section, we estimate the combined core indicator for Japan's CPI and analyze the factors that affect combination weights for individual core indicators. The evaluation of the performance of the combined core indicator is discussed in the next section.

3.1. Data and calibration

In our baseline estimation, we use eight indicators, starting from 1981: M1 to 2015:

[6] Our attempt can be interpreted as a forecast combination if we interpret each individual core indicator as a predictor. The idea of combining forecasts was first proposed by Bates and Granger (1969), and much effort has been devoted to improving the technique over the past decades. For the survey, see Timmermann (2006). Ohyama (2001) and Kitamura and Koike (2003) are representative examples that apply forecast combination methods to Japanese data.

M11.[7] It comprises five exclusion-based indicators (the CPI excluding fresh food; CPI excluding fresh food and energy; CPI excluding food and energy; CPI excluding fresh food, energy, and imputed rent; and CPI excluding food, energy, and imputed rent) and two distribution-based indicators (the trimmed mean and weighted median).[8] The remainder is the overall index. We exclude the effect of changes in the consumption tax rate and also adjust the series so that they are real-time data without any revisions. In addition to the baseline estimation, we estimate and examine the combined core indicators, which use only a subset of individual indicators in the baseline estimation to explore the effects of alternative combinations of the individual core indicators.

Following the suggestion by Koop and Korobilis (2012), we set the forgetting parameter α to maximize the predictive power of the estimated core indicator ($\alpha = 0.7$).[9] The values of weight are initialized by the noninformative prior; that is, $\omega_{0|0,k} = 1/K$.

3.2. Estimated combined core indicator and combination weights

Inflation rates of the combined core indicator

Figure 2 illustrates the inflation rates of the combined core indicator and individual core indicators. Most of the time, the combined core indicator coincides with the CPI excluding fresh food. However, the inflation rates tend to deviate from each other when the commodity prices greatly fluctuate (e.g., the period from 2008 to 2009 and the period after 2013). The deviations are especially large in 2015, and the development of

[7] The overall index is included to consider the possibility that, in some period, the overall index outperforms the core indicators.

[8] We exclude "mode" from our estimation because the combined core indicator which includes "mode," performs worse, especially in the 1980s. The deterioration in the performance can be attributed to the fact that "mode" does not satisfy the assumption for the error term in Equation (2). More concretely, when we conduct ADF tests on the residuals of Equation (2) with each individual indicator, the constant term is significant at the 10% level only for "mode." This implies that $\hat{\sigma}^2_{t,k}$ suffers from an upward bias since it is calculated based on the assumption that the mean is zero. Then, the upward bias $\hat{\sigma}^2_{t,k}$ leads to the upward bias in the estimation of the combination weights. Thus, it is inappropriate to include "mode" in estimating the combined core indicators.

[9] The grid-search method is applied.

the combined core indicator is akin to that of the CPI excluding fresh food and energy rather than the CPI excluding fresh food. Compared with the distribution-based indicators, the inflation rate of the combined core indicator is similar to that of the trimmed mean in almost all periods.

Time variation of combination weights for the individual core indicators

While the total sum of weights of the exclusion-based indicators tends to be greater than that of the distribution-based indicators until the early 2000s, the magnitude of the relation reverses thereafter (Figure 3). However, it should be noted that the differences are small on the whole. This result is consistent with that by Shiratsuka (2015) in that monitoring a wide range of indicators is important in assessing the underlying inflation properly. Incidentally, the weight on the overall index has been almost zero, which implies that we should exclude at least one item.

Figure 4 illustrates the estimated combination weights for the individual core indicators. Among the exclusion-based indicators, the weight on the CPI excluding fresh food is the highest in most periods. However, the weights on the indicators that include energy have been declining recently, and the CPI excluding fresh food and energy outperforms the other exclusion-based indicators in 2015. On the other hand, among the distribution-based indicators, the weight for trimmed mean tends to be higher than that for weighted median until the early 2000s, whereas the magnitude of the relation has reversed since the mid-2000s.

Factors behind the evolution of the combination weights

What factors affect the changes in the weights over time? Considering the methodologies of constructing the individual core indicators, the following hypotheses are obtained:

1. When persistent aggregate shocks, which are common to all items, change the trend inflation and dominate idiosyncratic shocks,[10] the performance of distribution-based indicators worsens because those indicators tend to exclude informative items responsive to the aggregate shocks.

[10] Regarding the estimation of the trend inflation for Japan, see, e.g., Kaihatsu and Nakajima (2015).

2. When idiosyncratic shocks dominate aggregate shocks, distribution-based indicators outperform the exclusion-based indicators because the former indicators are supposed to exclude items with large but transitory fluctuations, which are caused by the idiosyncratic shocks.

To examine these hypotheses quantitatively, we calculate the partial correlation between the total sum of weights for the distribution-based indicators and the variation width of the trend inflation. Here, the variation width of the trend inflation can be interpreted as the proxy for aggregate shocks. In addition, we calculate the partial correlation between the weights and the variation of the price-change distribution, which can be interpreted as the proxy for idiosyncratic shocks. The results shown in Figure 5 (2) reveal that the sum of weights have a negative correlation with the former and a positive correlation with the latter.[11] Figures 5 (3) and 5 (4) imply that when the performance of the distribution-based indicators is relatively low (e.g., in the 1990s and early 2000s), the variation width of the trend inflation is large, whereas the variation of the price-change distribution is relatively small. In other words, the performance of the distribution-based indicators is poor when the aggregate shock is large. This fact supports the above hypothesis.

Similarly, we can explain the performance order among the distribution-based indicators. The weight for the trimmed mean has a positive correlation with the variation width of the trend inflation. By construction, weighted median excludes more items than trimmed mean. If the items excluded only from weighted median contain any forecasting power for future inflation, then the predictive performance of weighted median is expected to be below that of trimmed mean. In addition, we observe a positive correlation between the kurtosis of the price-change distribution and the weight

[11] This result does not contradict the view of Shiratsuka (2015), who indicated the superiority of the distribution-based indicators for tracking underlying inflation. The scatter plot between the deviation from the underlying trend and the change in trend for the individual core indicators is depicted in the reference chart. The distribution-based indicators show a smaller deviation from trend on average; however, as the change in trend becomes larger, the deviations get larger. The former observation corresponds with the point by Shiratsuka (2015). On the other hand, the latter observation corresponds to the analysis in this paper.

for the trimmed mean. This is consistent with the findings of Bryan et al. (1997), who indicated that the rise in kurtosis increases the optimal rate of trimming.

Finally, we discuss the factors that affect the performance order among the exclusion-based indicators. The performance of the exclusion-based indicators depends on whether the excluded items contain any information about the future fluctuations of inflation. Using the estimated weights for the exclusion-based indicators, we calculate the probabilities of excluding each item, such as fresh food or energy (Figure 7).[12] There are three points to note. First, the exclusion probability for fresh food is very high in almost all periods. This result is consistent with that by Mio and Higo (1999), who argue that fresh food does not contain any information about future inflation. Second, the probability of excluding energy is high in the periods when the trend of oil prices suddenly changes. For example, when oil prices fell sharply after the financial crisis, the probability of excluding energy became high. On the other hand, the probability decreased in the late 1980s when, as Mio and Higo (1999) argued, the yen appreciated sharply versus the U.S. dollar and oil prices declined. Third, the probability of excluding food and imputed rent is consistently below 0.5. This implies that these items contain information about future inflation rates.

3.3. Selection of individual core indicators

How does the inclusion of a specific individual core indicator affect the estimations of the combination weights? Is there any specific core indicator that biases the estimations of the combination weights? To answer these questions, we examine how excluding variables affect the results.[13]

Let $\omega'_{t-11|t-12,k}$ be a weight for the core indicator $k(\neq l)$ of the combined core indicator, which excludes the core indicator l. When the noninformative prior on the

[12] The calculation of the probabilities proceeds as follows. First, the sum of the weights for the exclusion-based indicators and the overall index is normalized to 1. Then, the probability of excluding a specific item is calculated by summing up the normalized weights for the indicators that exclude the item.

[13] As discussed in footnote 8, the weight for the indicator that does not satisfy the assumption for the error terms tends to be overestimated. Therefore, these indicators should be excluded from the outset. The discussion in this section presupposes the nonexistence of such indicators.

individual core indicators is used, Equations (5) and (6) imply:[14]

$$\omega'_{t-11|t-12,k} = \frac{\omega_{t-11|t-12,k}}{1-\omega_{t-11|t-12,l}}. \quad (10)$$

Thus, the weight for the indicator k in the baseline case is distributed proportionally.

Equation (10) implies two points. First, the relative weights among the individual core indicators do not depend on whether the indicator l is included in the estimation or not. For example, the relative weights among the exclusion-based indicators in the baseline case are unchanged, even when other indicators are included in the estimation. That is, the CPI excluding fresh food and energy outperforms the other exclusion-based indicators in 2015 even if new core indicators are added.

Second, we can infer from Equation (10) that including low-performance indicators does not bias the estimation of the combination weights. Suppose that the weight for the indicator l is almost equal to 0 ($\omega_{t-11|t-12,l} \approx 0$). This is equivalent to $1-\omega_{t-11|t-12,l} \approx 1$. Then, $\omega'_{t-11|t-12,k} \approx \omega_{t-11|t-12,k}$ holds. This means that the inflation rates of the combined core indicator in the baseline case are almost the same as those in the alternative cases.

In Figure 8, these points are confirmed empirically. For comparison, we estimate two alternative combined core indicators. One alternative excludes two low-performance indicators (the CPI excluding fresh food, energy, and imputed rent and the CPI excluding food, energy, and imputed rent), whereas the other alternative excludes the distribution-based indicators. We call these alternatives the "combined core indicator (6 indicators)" and "combined core indicator (only exclusion-based indicators)," respectively. First, the inflation rate is the same between the baseline and the combined core indicator (6 indicators) cases, while the combined core indicator (only exclusion-based indicators) frequently deviates from the baseline case. Second, the relative combination weights are almost the same in all three cases.

These findings justify the claim that the combined core indicator should broadly include various individual core indicators as long as they are unbiased predictors.

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[14] For a derivation, see the Appendix.

4. The Performance of the Combined Core Indicator

In this section, we evaluate the performance of the combined core indicator. Various tests are suggested in the related literature. Following Shiratsuka (2015), we evaluate the combined core indicator from two points of view: one is the ability to track the current underlying trend of inflation and the other is to evaluate its predictive power on the future underlying trend.

4.1. The ability to track the current underlying trend

A good core indicator should track the long-run inflation rate of the overall index. There are two ways to evaluate the performance of the core indicators in this respect.

Deviation from the HP trend

One way is to examine the extent to which the combined core indicator tracks the trend of inflation in real time. The trend of inflation is constructed by applying the HP filter to the overall index. To gauge its accuracy, we compute the root mean squared error (RMSE) repeatedly using subsamples of 120 months. Looking at the estimation results in Figure 9 (1), the RMSE of the combined core indicator is generally as small as the trimmed mean, whose RMSE is the smallest among the individual indicators.

Deviation from the overall CPI

We examine whether the deviation between the year-on-year change in the combined core indicator and that in the overall CPI is equal to zero over the medium to long term. If the deviation is not equal to zero, it implies that the combined core indicator converges to different values from the overall index.

Specifically, the equation below is estimated repeatedly using a subsample of 120 months and tested whether a constant term is significantly different from zero at a certain level:

$$(\pi_t^{all} - \pi_t^{core}) = c + \varepsilon_{t,} \tag{11}$$

where c denotes a constant term, and ε_t is the error term.

Figure 9 (2) shows that the confidence interval of the constant terms includes zero. Compared with the individual core indicators, the performance of the combined core indicator is in no way inferior to any other indicator.

In sum, these two types of tests show that the combined core indicator is comparable with the best individual indicator; that is, the trimmed mean.

4.2. Predictive power for the future underlying trend

In this section, we statistically evaluate the predictive power of the combined core indicator for the future underlying trend.

Unbiasedness

First, the unbiasedness of the combined core indicator is evaluated by repeatedly estimating the equation below using subsamples of 120 months:

$$(\pi_{t+12}^{all} - \pi_t^{all}) = \beta_0 + \beta_1(\pi_t^{core} - \pi_t^{all}) + \varepsilon_t \ , \qquad (12)$$

where β_0 and β_1 are the estimated coefficients. In order for the combined core indicator to be an unbiased predictor, it is necessary that the joint hypothesis of $\beta_0 = 0$ and $\beta_1 = 1$ is not rejected. Unless both $\beta_0 = 0$ and $\beta_1 = 1$, $\mathrm{E}_t\left(\pi_{t+12}^{all}\right) = \pi_t^{core}$ does not hold.

Looking at the results in Figure 10 (1), although the joint hypothesis is rejected during the early 2000s, the combined core indicator exhibits a highly stable performance compared with the individual core indicators.

Nonconvergence to the overall index

The rejection of the joint hypothesis $\beta_0 = 0$ and $\beta_1 = 1$ in Equation (12) does not necessarily imply that the overall index converges to the combined core indicator. Therefore, we need to examine whether the combined core indicator converges to the overall index. Therefore, we employ a similar regression as follows:

$$(\pi_{t+12}^{core} - \pi_t^{core}) = \gamma_0 + \gamma_1(\pi_t^{core} - \pi_t^{all}) + \varepsilon_t \qquad (13)$$

where γ_0 and γ_1 are the estimated coefficients. If the slope of γ_1 is significantly different from zero at a certain significance level, then the deviation of the overall index from the core indicator is partly eliminated by the conversion of the core indicator to the overall index.

Figure 10 (2) shows that the coefficient of the slope during the whole sample period is insignificant at the 5% level. Thus, the combined core indicator outperforms the individual core indicators.

In sum, the results of these two regressions show that the combined core indicator is a better predictor of the future underlying trend than any individual core indicator.

5. Conclusion

This study proposed a new framework for measuring underlying inflation using multiple core indicators. More specifically, a combined core indicator is constructed by applying DMA as a weighted average of individual core indicators.

Empirical evidence indicates that the combined core indicator adapts to the change in the nature and sources of price movements. Accordingly, it firmly outperforms the individual core indicators over time in terms of both tracking the current underlying trend of inflation and forecasting the future underlying trend.

In addition, we analyze the factors that affect the evolution of time-varying combination weights. First, we observe that the weights for the exclusion-based indicators (e.g., the CPI excluding fresh food) tend to be high when aggregate shocks are dominant. The weights for the distribution-based indicators tend to be high when idiosyncratic shocks are dominant. Furthermore, the probabilities of excluding specific items, which are calculated from the estimated weights, indicate that (i) the probability of excluding fresh food stays high for almost entire period; (ii) the probability of excluding energy becomes higher when the trend of oil prices changes greatly; and (iii) the probability of excluding food and imputed rent is definitely low.

Thus, the combined core indicator is useful for capturing the time-varying performance of the individual core indicators. Its weights are calculated on the basis of past performances without solid theoretical foundations. Although a simple analysis is conducted to examine the factors behind the changes in the weights, the theoretical background remains unclear. Thus, accumulating the empirical evidence and constructing theoretical frameworks to better understand the circumstances behind the evolution of the combination weights are important.

Appendix: Derivation of Equation (10)

Here, we show the detailed derivation of Equation (10), which indicates the relation between $\omega'_{t-11|t-12,k}$ and $\omega_{t-11|t-12,k}$.

Since the noninformative prior is assumed ($\omega_{0|0,k} = 1/K$), we can obtain Equation (A1) using the characteristic that the total sum of weights is equal to 1.

$$\omega'_{0|0,k} = \frac{\omega_{0|0,k}}{1 - \omega_{0|0,l}} \tag{A1}$$

Following Equation (5) in the main text, the weight in the prediction step, $\omega'_{1|0,k}$, is calculated by the equation below:

$$\omega'_{1|0,k} = \frac{\omega'^{\alpha}_{0|0,k}}{\sum_{k \in S'} \omega'^{\alpha}_{0|0,k}}, \tag{A2}$$

where S' indicates the set of individual core indicators other than the core indicator l. Substituting (A1) into (A2), we have

$$\omega'_{1|0,k} = \frac{\omega^{\alpha}_{0|0,k}}{\sum_{k \in S'} \omega^{\alpha}_{0|0,k}}.$$

Since $\sum_{k \in S'} \omega^{\alpha}_{0|0,k} = \sum_{k \in S} \omega^{\alpha}_{0|0,k} - \omega^{\alpha}_{0|0,l}$ holds, this equation can be rewritten as

$$\omega'_{1|0,k} = \frac{\omega_{1|0,k}}{1 - \omega_{1|0,l}} \tag{A3}$$

Next, using Equation (6) in the main text, the weight in the updating step is given by

$$\omega'_{1|1,k} = \frac{\omega'_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k)}{\sum_{k \in S'} \omega'_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k)} \tag{A4}$$

Then, we substitute Equation (A3) into Equation (A4) to obtain

$$\omega'_{1|1,k} = \frac{\omega_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k)}{\sum_{k \in S'} \omega_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k)}.$$

Since $\sum_{k \in S'} \omega_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k) = \sum_{k \in S} \omega_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k) - \omega_{1|0,l} P(\pi^{all}_{1+12} | L_1 = l)$ holds, this equation can be rewritten as

$$\omega'_{1|1,k} = \frac{\frac{\omega_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k)}{\sum_{k \in S} \omega_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k)}}{1 - \frac{\omega_{1|0,l} P(\pi^{all}_{1+12} | L_1 = l)}{\sum_{k \in S} \omega_{1|0,k} P(\pi^{all}_{1+12} | L_1 = k)}}.$$

From Equation (6) in the main text, the denominator of this equation is equal to $\omega_{1|1,k}$ and the second term in the numerator is equal to $\omega_{1|1,l}$. Thus,

$$\omega'_{1|1,k} = \frac{\omega_{1,1,k}}{1 - \omega_{1,1,l}}. \quad \text{(A5)}$$

By iterating these calculations sequentially ($t = 2,3, \ldots$), we can confirm that Equation (10) holds.

References

Bates, J. M. and C. W. J. Granger. (1969) "The Combination of Forecasts," *Operational Research Quarterly*, 20(4), pp.451-468.

Bryan, Michael F. and Stephen G. Cecchetti. (1994) "Measuring Core Inflation," in Monetary Policy, edited by N. Gregory Mankiw, pp.195-215.

Bryan, Michael F., Stephen G. Cecchetti, and Rodney L.Wiggins II. (1997) "Efficient Inflation Estimation," NBER Working Paper, No. 6183.

Cogley, Timothy. (2002) "A Simple Adaptive Measure of Core Inflation," *Journal of Money, Credit and Banking*, 34(1), pp. 94-113.

Figueiredo, Francisco Marcos Rodrigues and Roberta Blass Staub. (2002) "Evaluation and Combination of Core Inflation Measures for Brazil," *Money Affairs*, 15(1), pp.1-20.

Hogen, Yoshihiko, Takuji Kawamoto, and Moe Nakahama (2015) "Core Inflation and the Business Cycle," *Bank of Japan Review*, 2015-E-6

Kaihatsu, Sohei and Jouchi Nakajima (2015) "Has Trend Inflation shifted? : An Empirical Analysis with a Regime-Switching model," Bank of Japan Working Paper Series, No.15-E-3

Kitamura, Tomoyuki and Ryoji Koike (2003) "The Effectiveness of Forecasting Methods Using Multiple Information Variables," *Monetary and Economic Studies*, 21(1) pp.105-143

Koop, Gary and Dimitris Korobilis. (2012) "Forecasting Inflation Using Dynamic Model Averaging," *International Economic Review*, 53(3), pp.867-886.

Mio, Hitoshi and Masahiro Higo (1999) "Underlying Inflation and the Distribution of Price Change: Evidence from the Japanese Trimmed-Mean," *Monetary and Economic Studies*, 17(1), pp.103-132

Nicoletti, Giulio and Raffaele Passaro. (2012) "Sometimes It Helps the Evolving Predictive Power of Spreads on GDP," European Central Bank Working Paper Series, No.1447.

Ohyama, Shinsuke (2001) "Forecasting Japanese GDP Using Forecast Combination," Bank of Japan Working Paper Series, 01-3 (in Japanese)

Raftery, Adrian E., Miroslav Kárný, and Pavel Ettler. (2010) "Online Prediction Under

Model Uncertainty via Dynamic Model Averaging: Application to a Cold Rolling Mill," *Technometrics*, 52(1), pp.52–66.

Shiratsuka, Shigenori (1997) "Inflation Measures for Monetary Policy : Measuring the Underlying Inflation Trend and Its Implication for Monetary Policy Implementation," *Monetary and Economic Studies*, 15(2), pp.1-26

———— (2006) "Core Indicators of Japan's Consumer Price Index," *Bank of Japan Review*, 2006-E-7

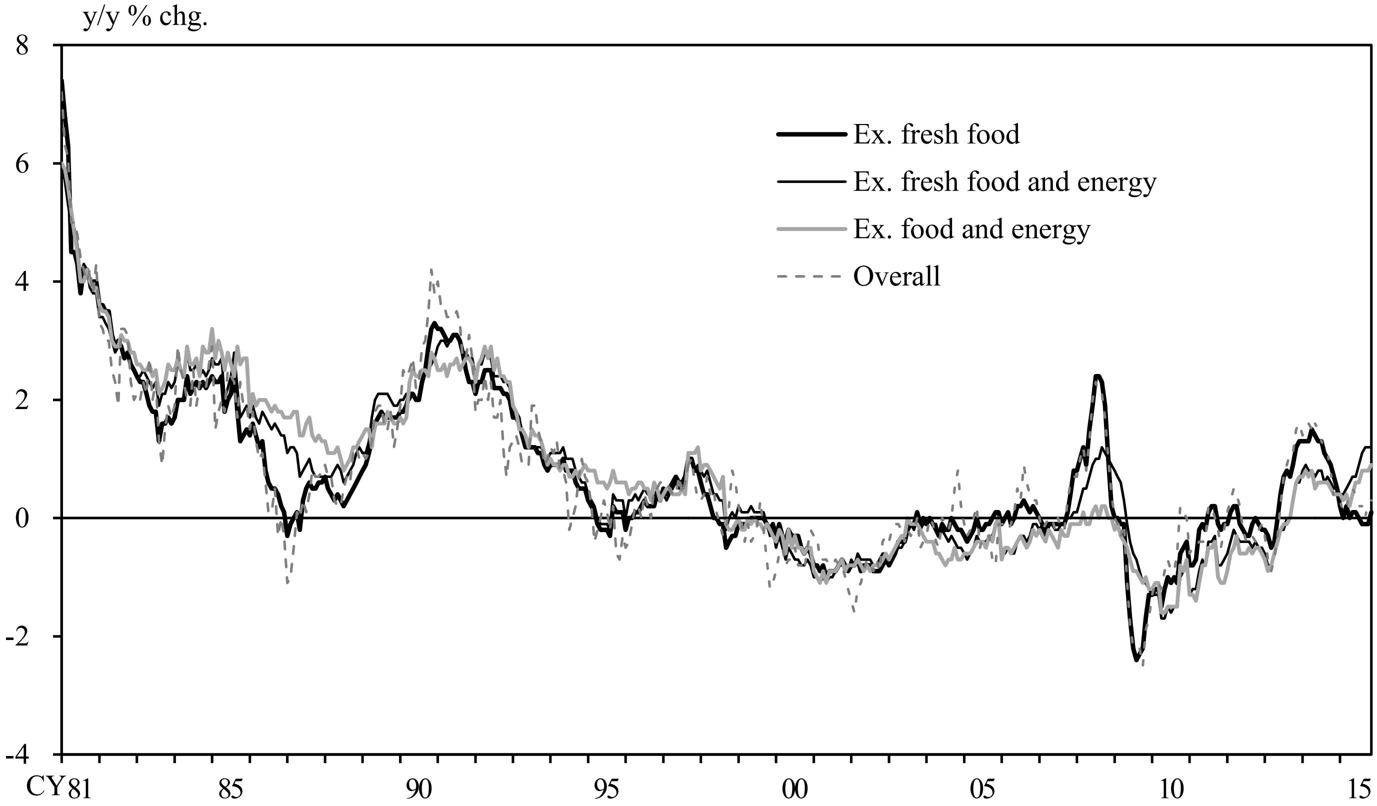
———— (2015) "Performance of Core Indicators of Japan's Consumer Price Index," *Bank of Japan Review*, 2015-E-7

Timmermann, Allan. (2006) "Forecast Combination," in *Handbook of Economic Forecasting*, edited by Graham Elliot, Clive W. J. Granger, and Allan Timmermann, pp.135-196.

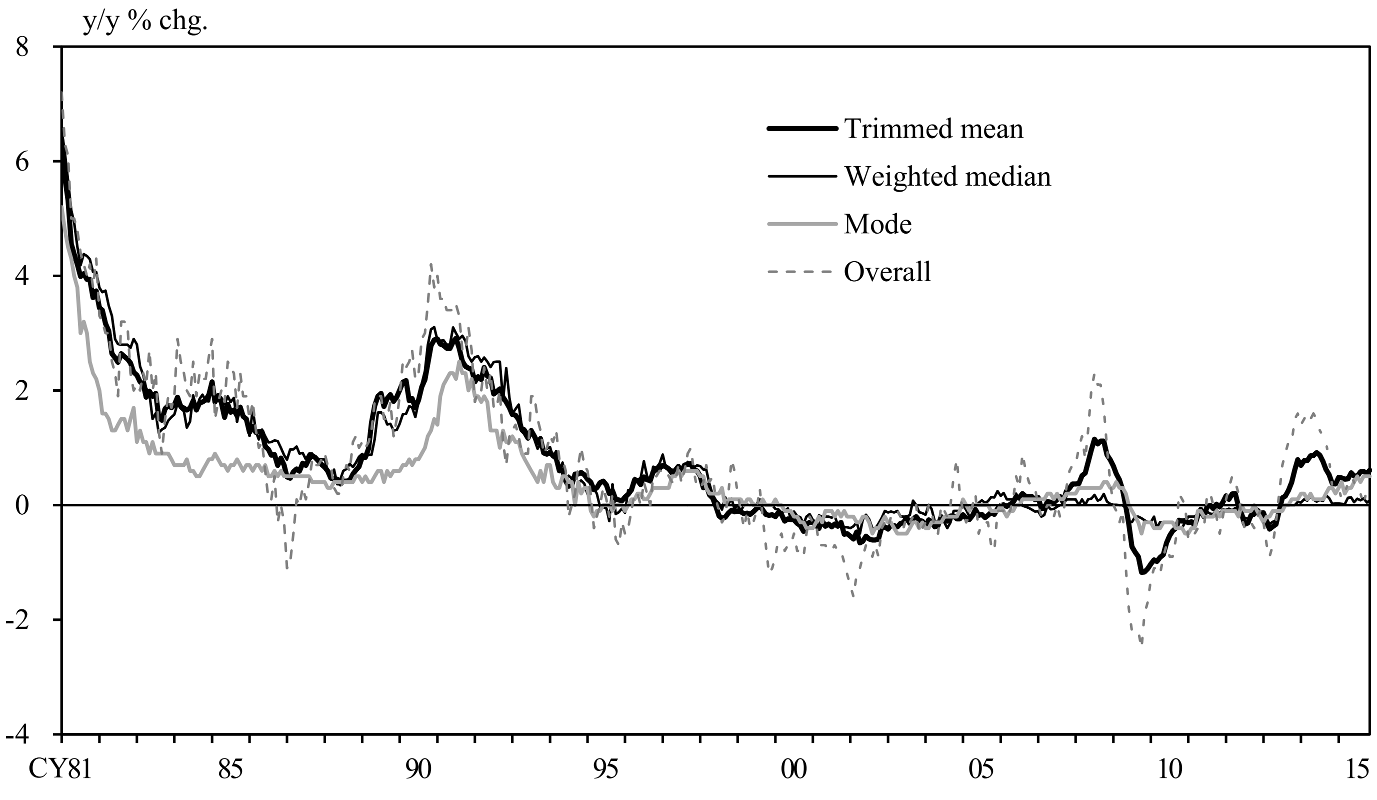
Figure 1

CPI core indicators

(1) Exclusion-based indicators



(2) Distribution-based indicators



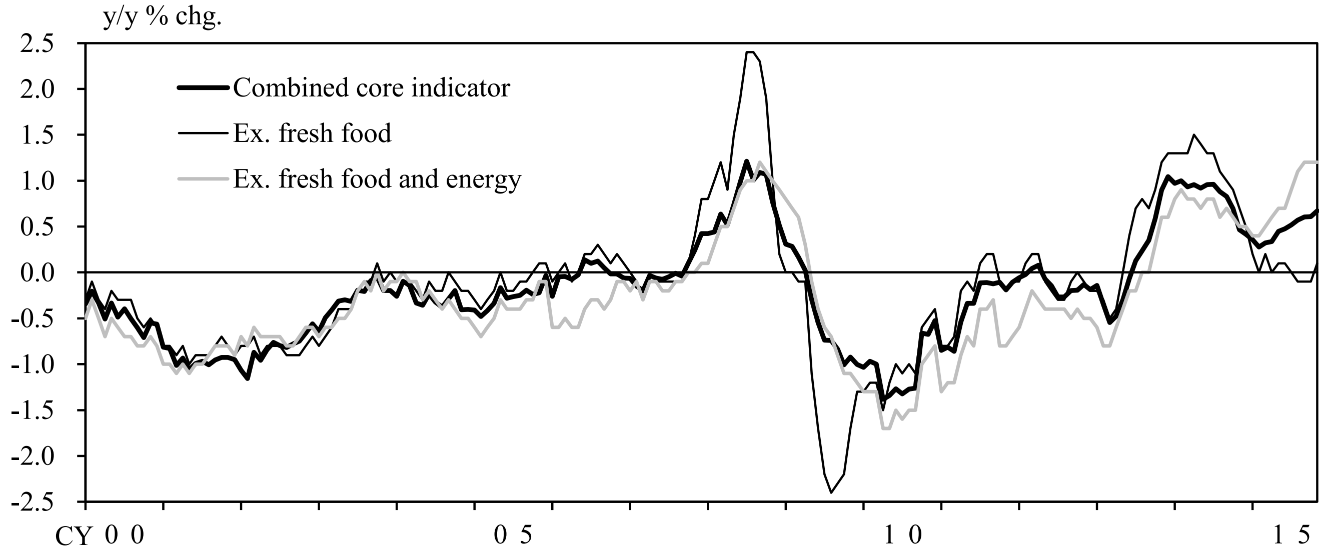
Note: CPI excluding fresh food and energy, trimmed mean, weighted median, and mode are calculated by the Research and Statistics Department, Bank of Japan. Figures are adjusted to exclude the effect of changes in the consumption tax rate.

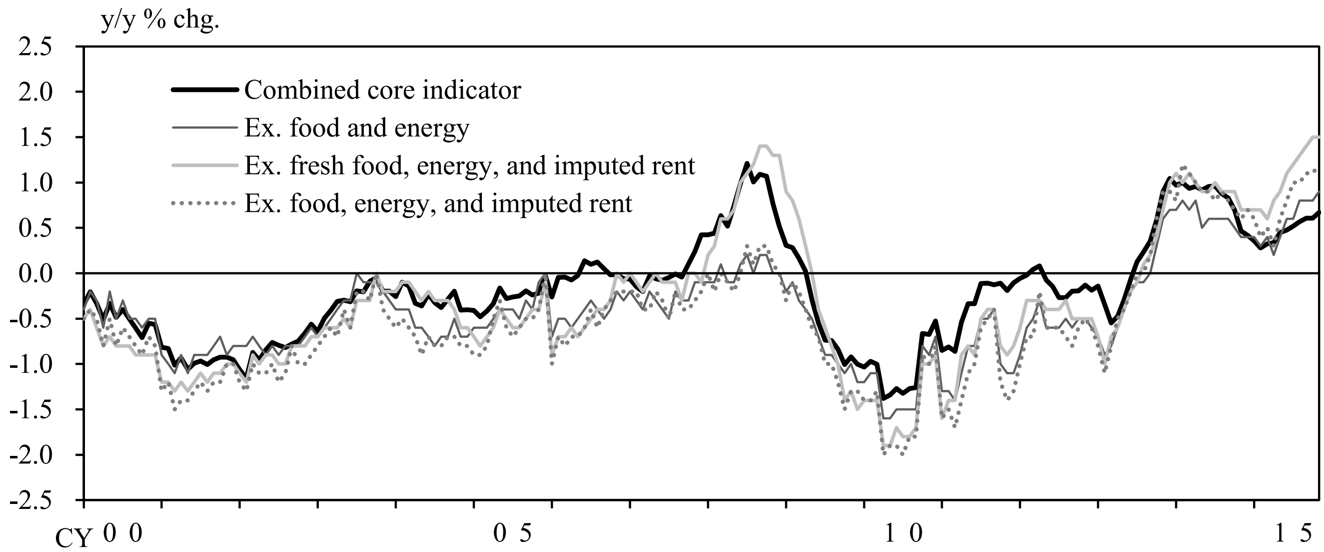
Source: Ministry of Internal Affairs and Communications.

Figure 2

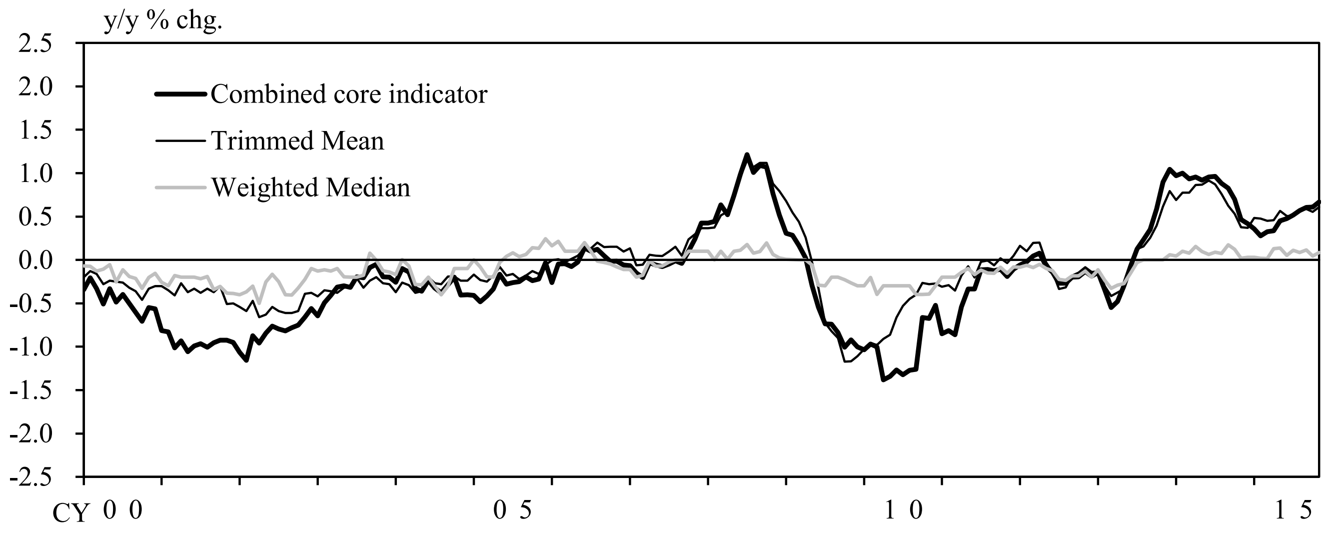
Combined core indicator and individual core indicators

(1) Comparison with exclusion-based indicators





(2) Comparison with distribution-based indicators

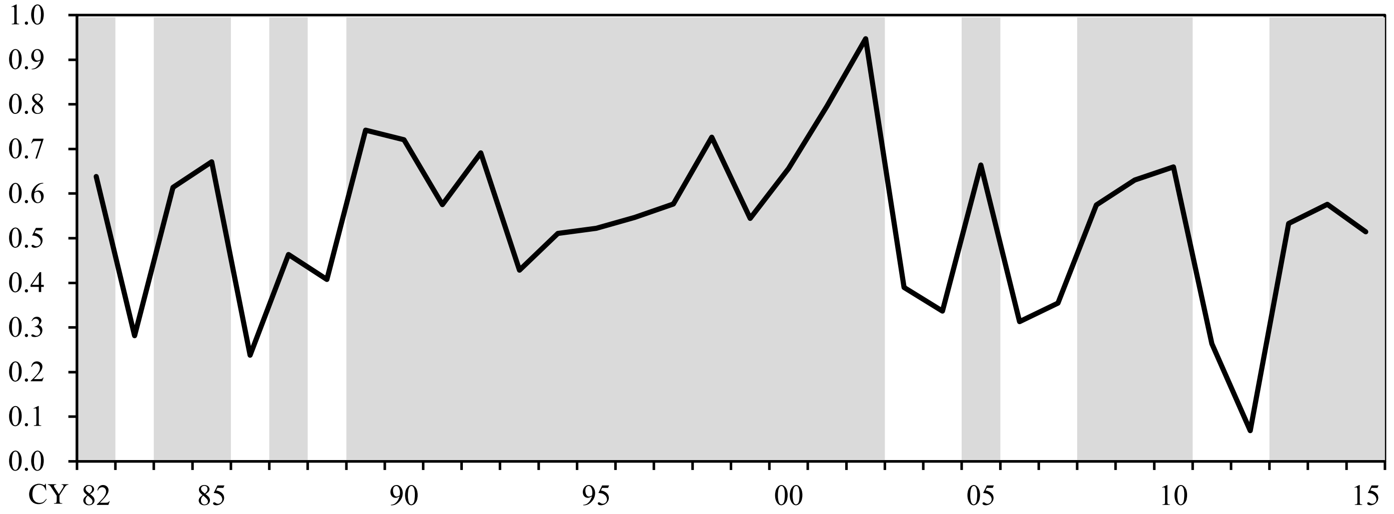


Source: Ministry of Internal Affairs and Communications.

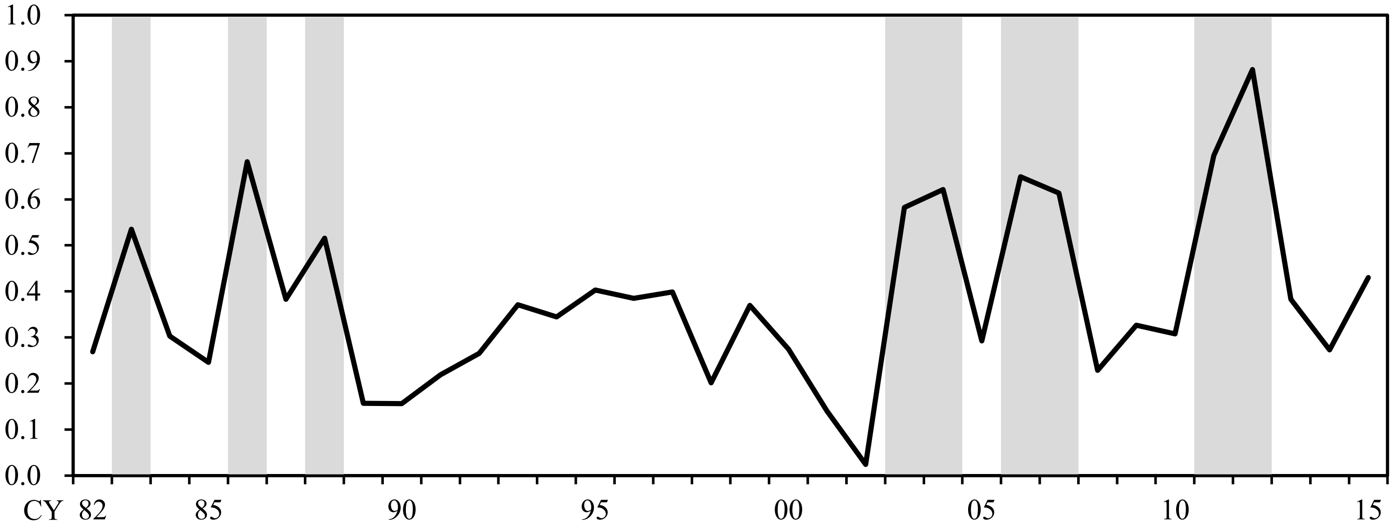
Figure 3

Time-varying combination weights (1)

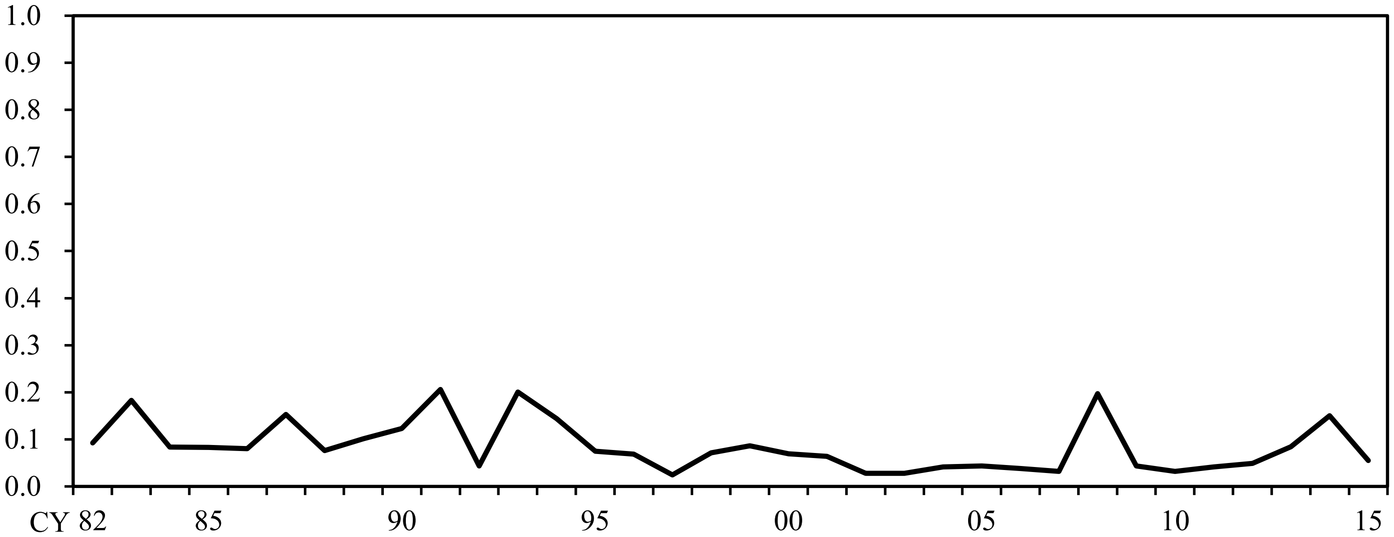
(1) Total sum of weights for exclusion-based indicators



(2) Total sum of weights for distribution-based indicators



(3) Weight for the overall index



Notes: 1. Yearly average.

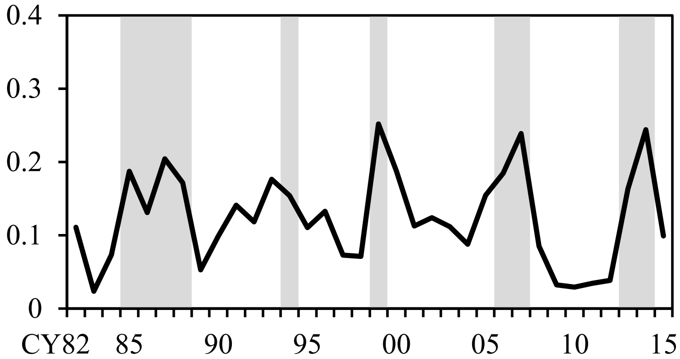
2. The shaded areas indicate the highest weight in each year.

Figure 4

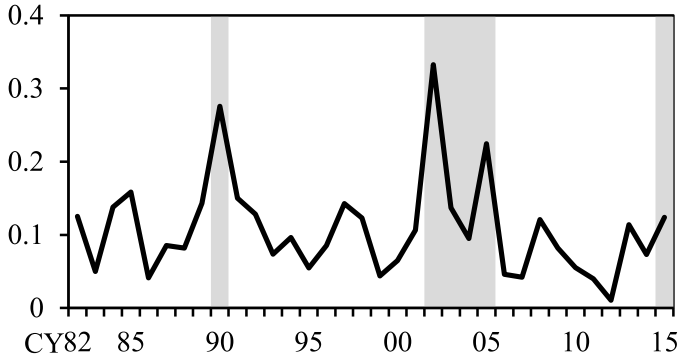
Time-varying combination weights (2)

(1) Exclusion-based indicators

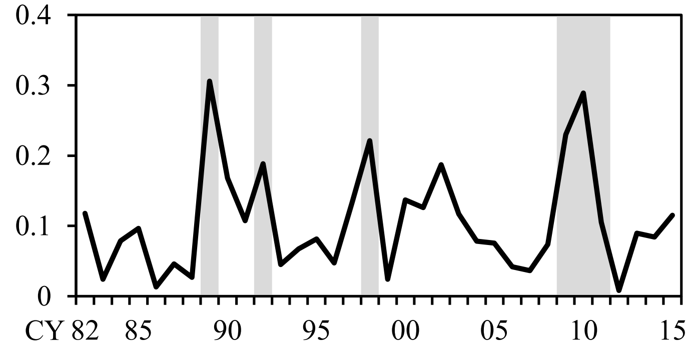
(i) Ex. fresh food



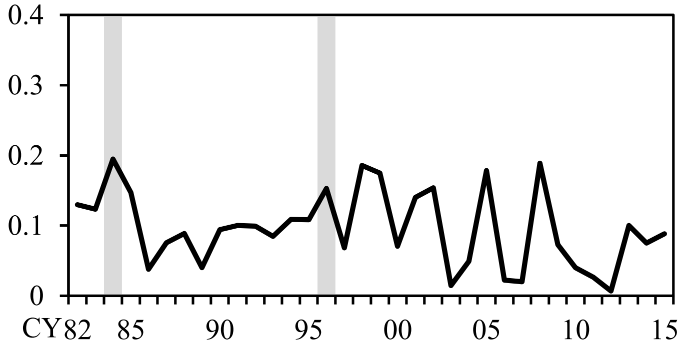
(ii) Ex. fresh food and energy



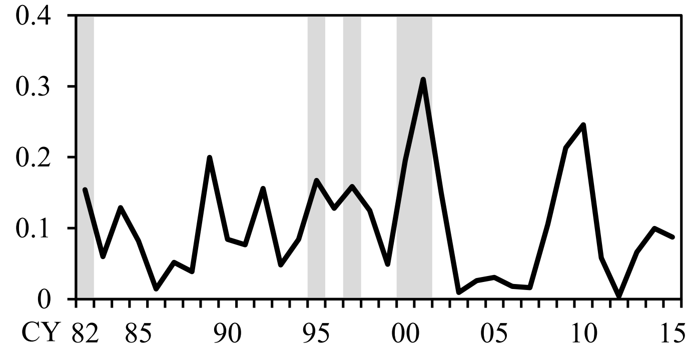
(iii) Ex. food and energy



(iv) Ex. fresh food, energy, and imputed rent

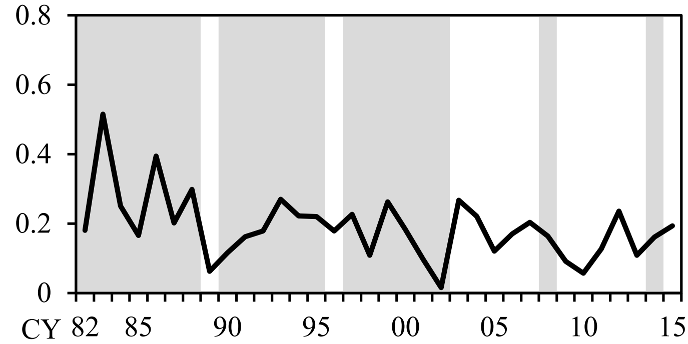


(v) Ex. food, energy, and imputed rent

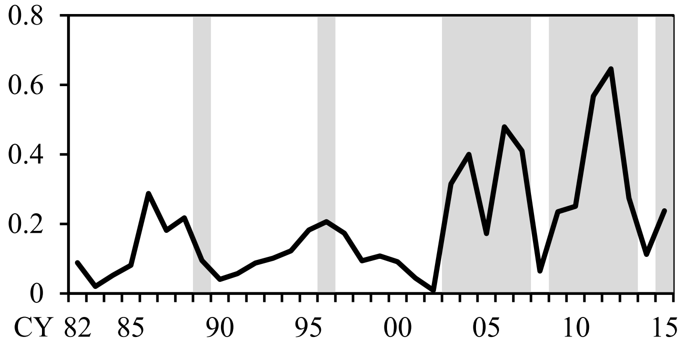


(2) Distribution-based indicators

(i) Trimmed mean



(ii) Weighted median

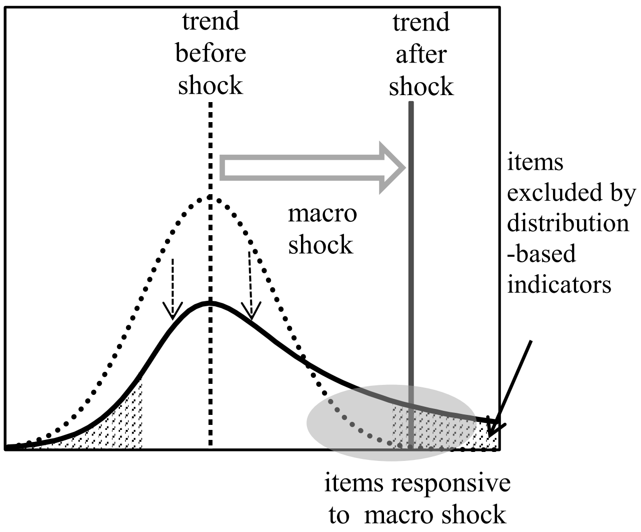


Notes: 1.Yearly average.
2.The shaded areas in (1) and (2) indicate the highest weight among exclusion-based indicators and distribution-based indicators, respectively.

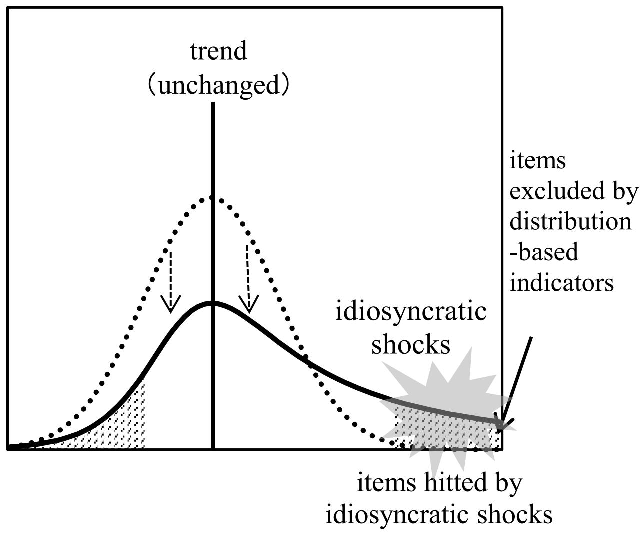
Hypothesis about the evolution of combination weights

(1) Conceptual analysis

(i) Response to a macro shock



(ii) Response to idiosyncratic shocks



(2) Simple test of the hypothesis

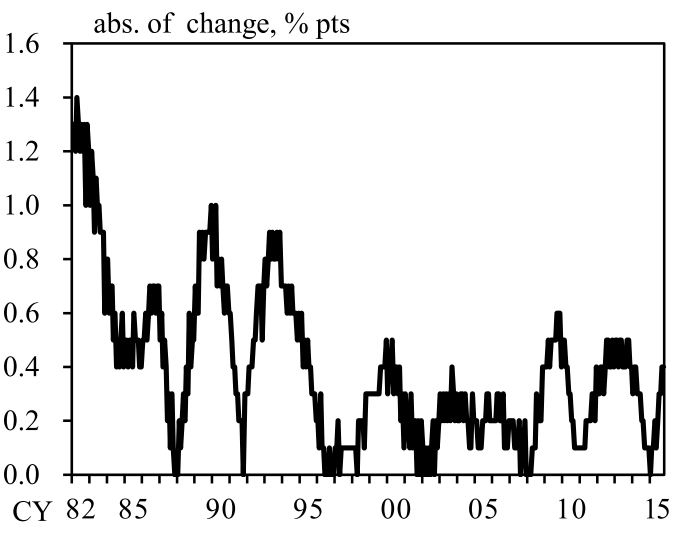
	Variation width of the trend inflation	Variation	Skewness	Kurtosis
Partial correlation	-0.2661 ***	0.3603 ***	-0.0638	0.0692

Notes: 1. "***", "**", "*" indicate 1%, 5%, 10% level significance, respectively.

2. “Variation” is the absolute value of the coefficient of variation, which is calculated by dividing weighted standard deviation by weighted average. “Skewness” and “Kurtosis” are weighted skewness and weighted kurtosis that we calculate using CPI weights. “Variation width of the trend inflation” is the absolute value of the year-on-year difference of HP filtered trend (λ=14,400).

3. We remove outliers that fall outside three interquartile range below the first quartile or above the third quartile.

(3) Variation width of the trend inflation



(4) Variation

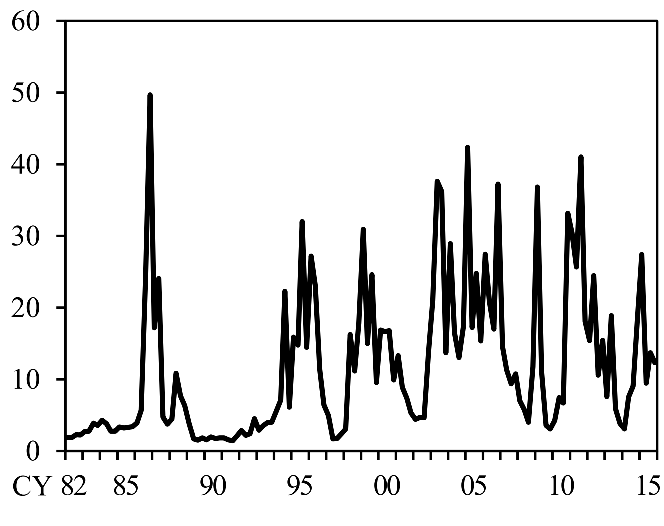
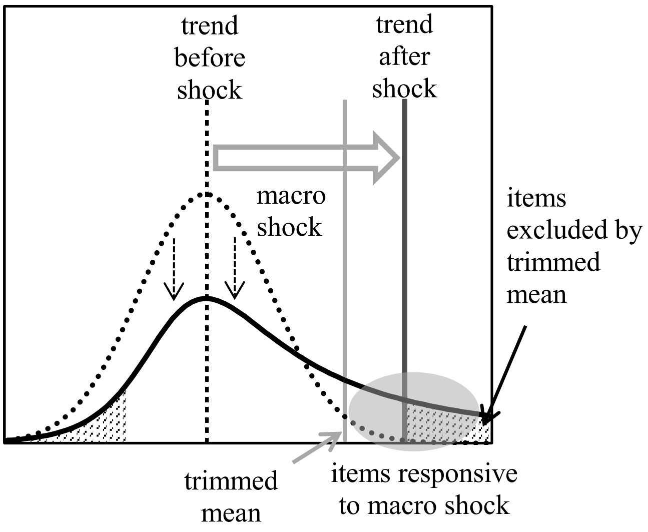


Figure 6

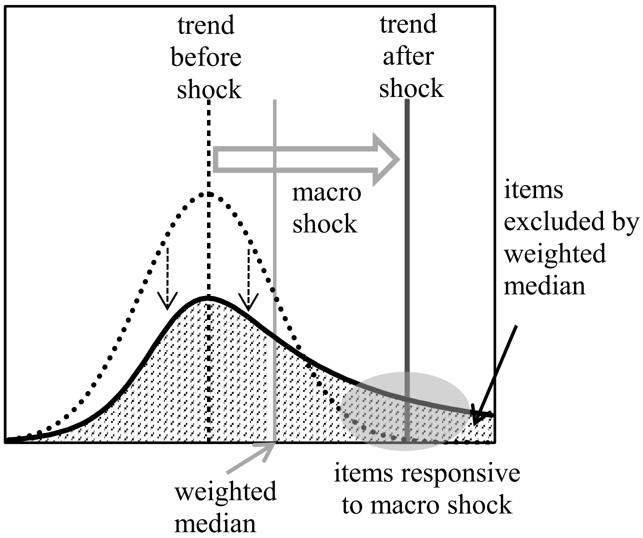
Hypothesis about the evolution of weights for distribution-based indicators

(1) Conceptual analysis

(i) Trimmed mean



(ii) Weighted median

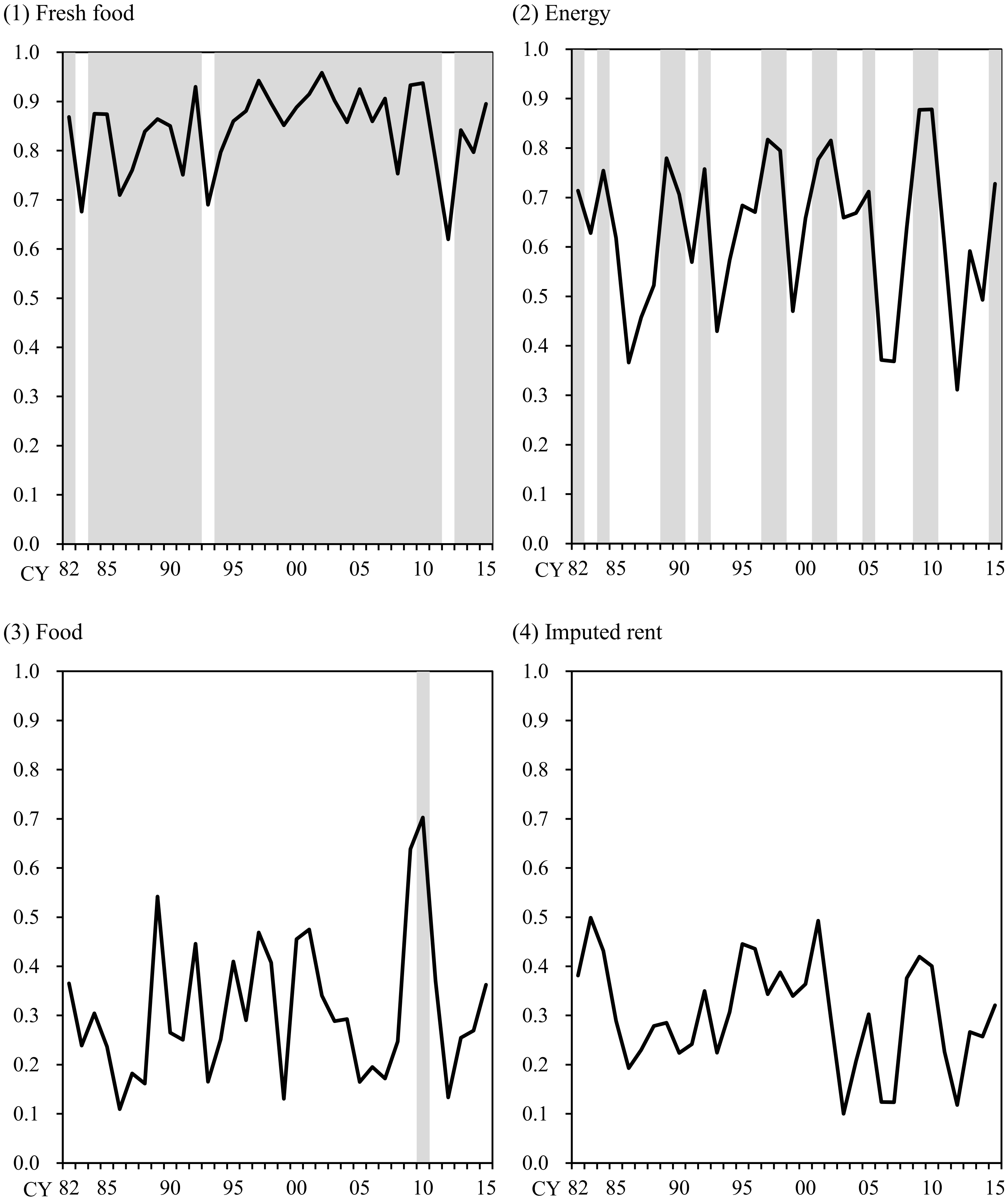


(2) Simple test of the hypothesis

	Variation width of the trend inflation	Variation	Skewness	Kurtosis
Partial correlation	0.2603 ***	-0.0572	0.099	-0.1011 *

Notes: 1. "***", "**", "*" indicate 1%, 5%, 10% level significance, respectively.
2. "Variation" is the absolute value of the coefficient of variation, which is calculated by dividing weighted standard deviation by weighted average. "Skewness" and "Kurtosis" are weighted skewness and weighted kurtosis that we calculate using CPI weights. "Variation width of the trend inflation" is the absolute value of the year-on-year difference of HP filtered trend (λ=14,400).
3. We remove outliers that fall outside three interquartile range below the first quartile or above the third quartile.

Exclusion probabilities of individual items

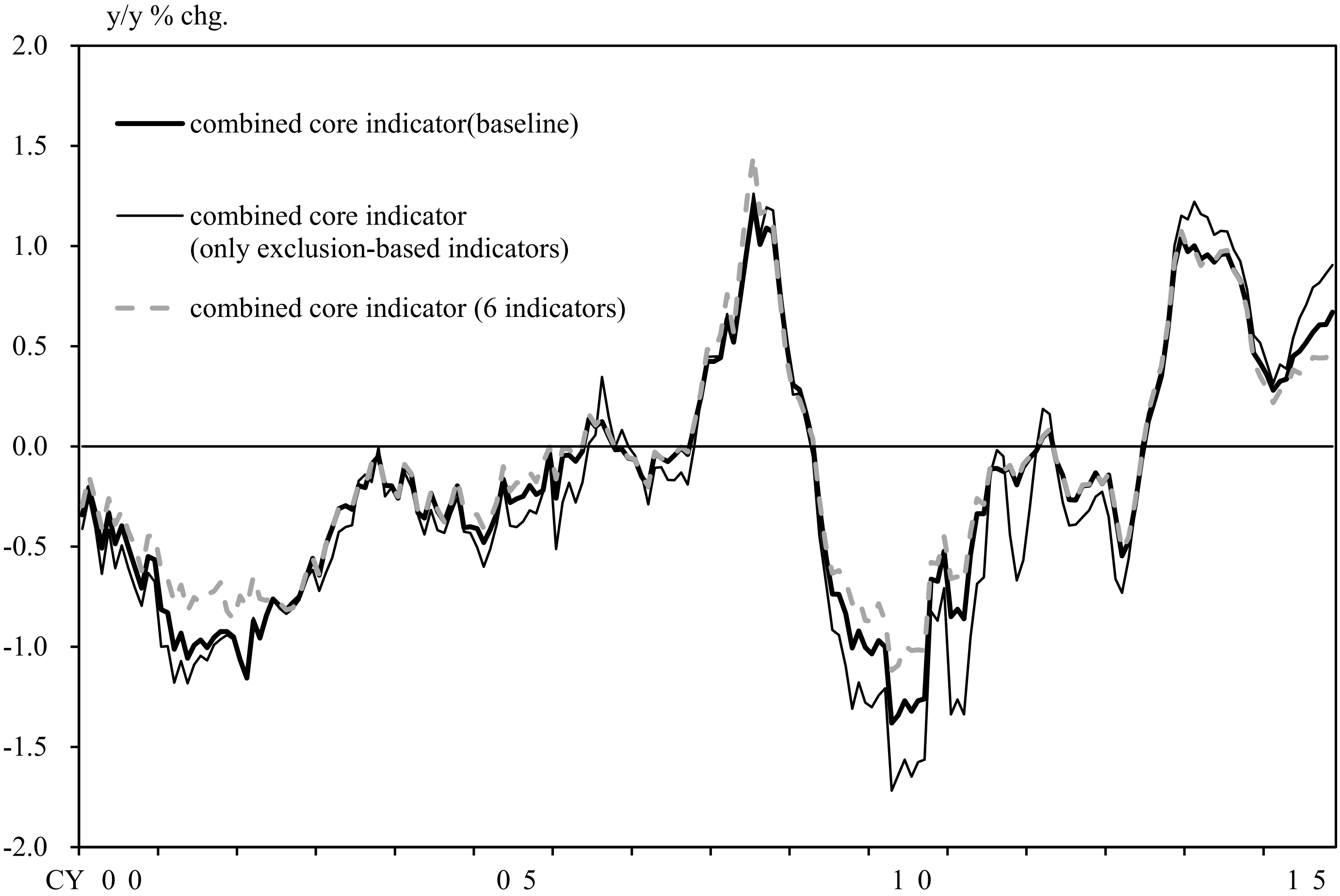


Notes 1. Yearly average.

2. The shaded areas indicate that probabilities are above 0.7.

Comparison with alternative specifications

(1) Inflation rates



(2) Relative weights

(i) Ex.fresh food (ii) Ex.fresh food and energy (iii) Ex.food and energy

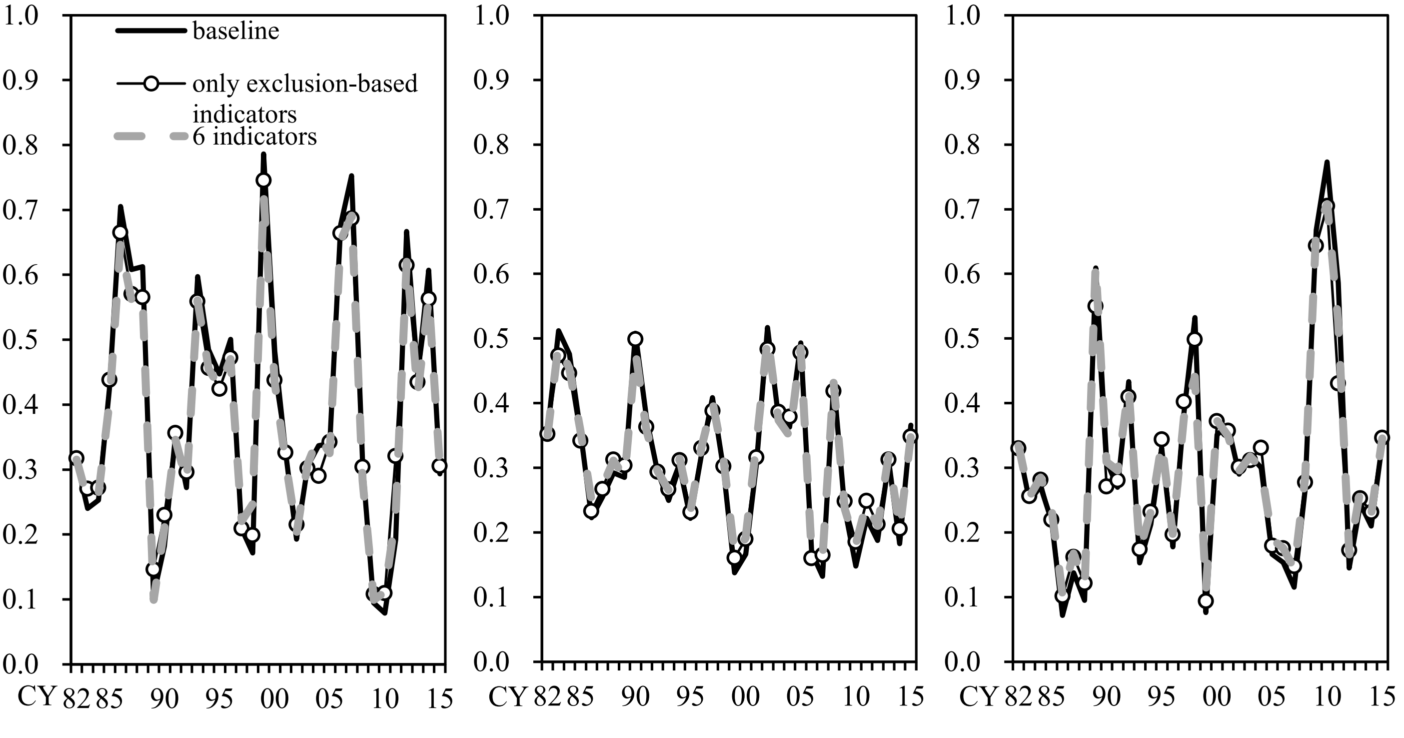
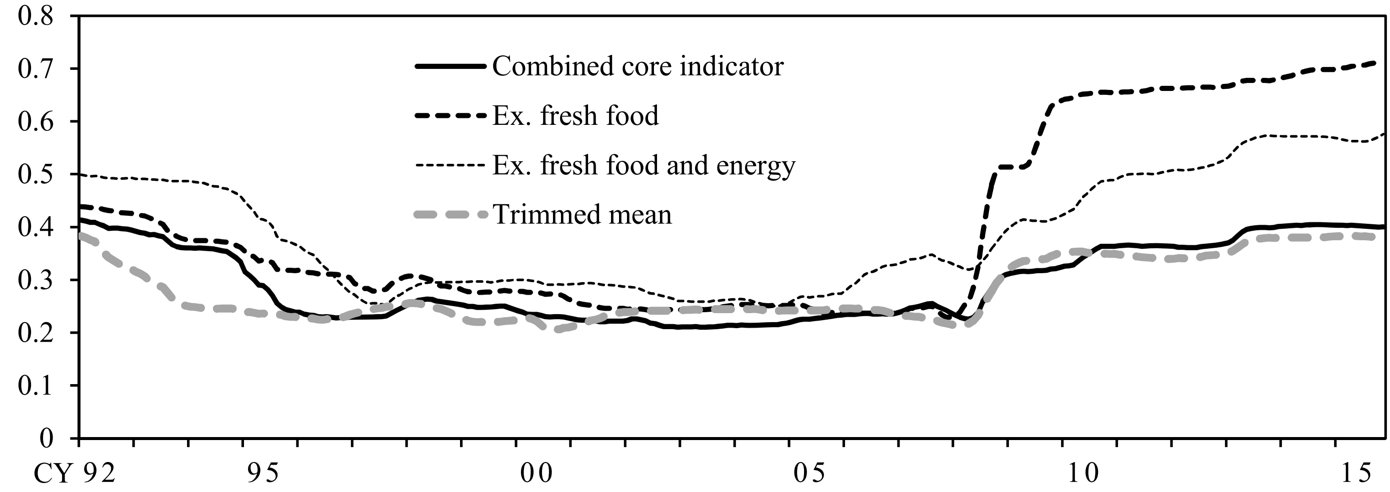


Figure 9

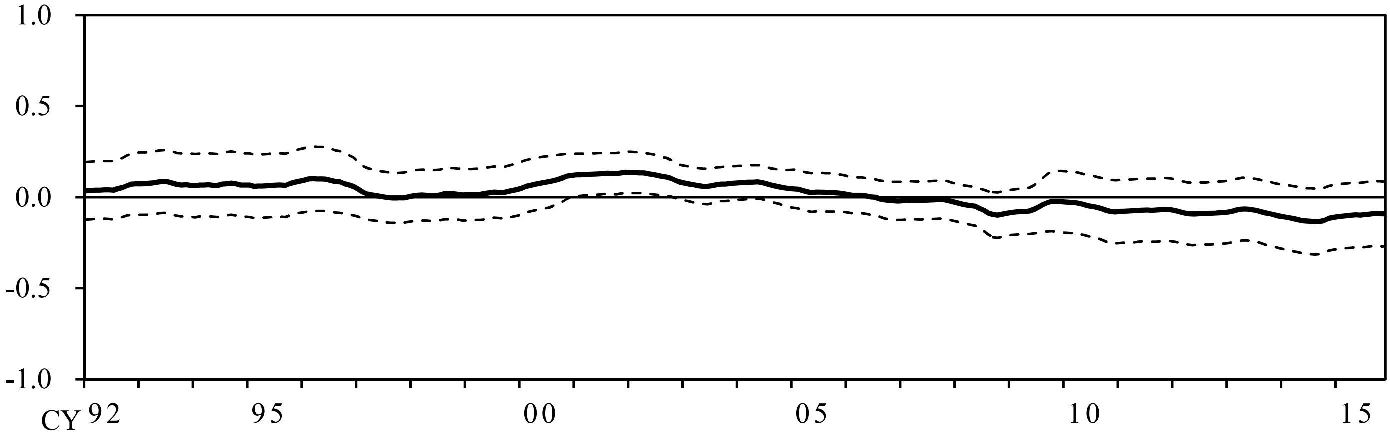
The ability to track the current underlying inflation

(1) Deviations from the trend inflation



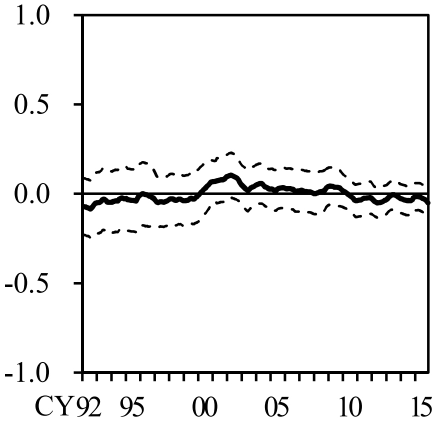
Note: Root mean squared errors from the HP filtered trend are computed repeatedly using subsamples of 120 months ending at each month on the horizontal axis.

(2) Deviation from the overall index

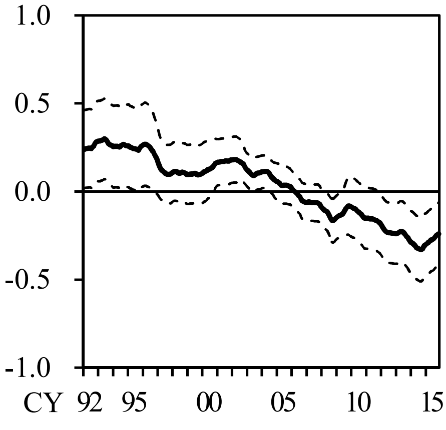


（Reference） Deviation of individual indicators from overall index

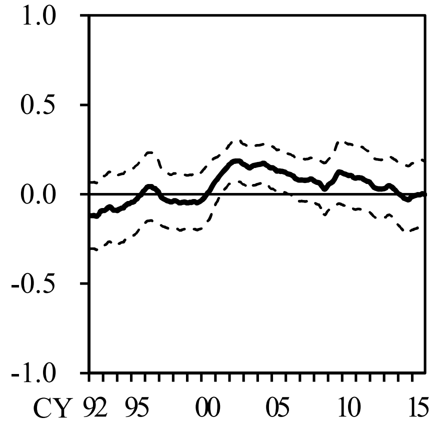
(i)Ex. fresh food



(ii) Ex. fresh food and energy



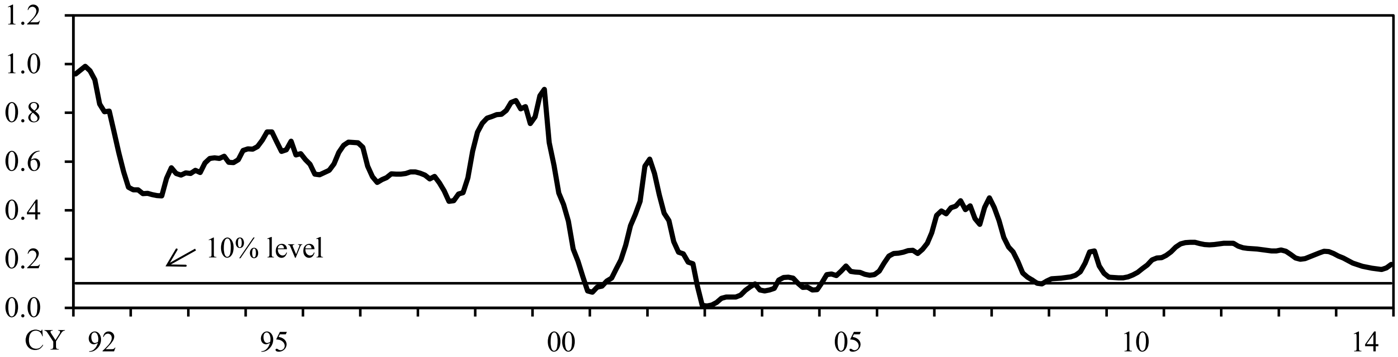
(iii) Trimmed mean



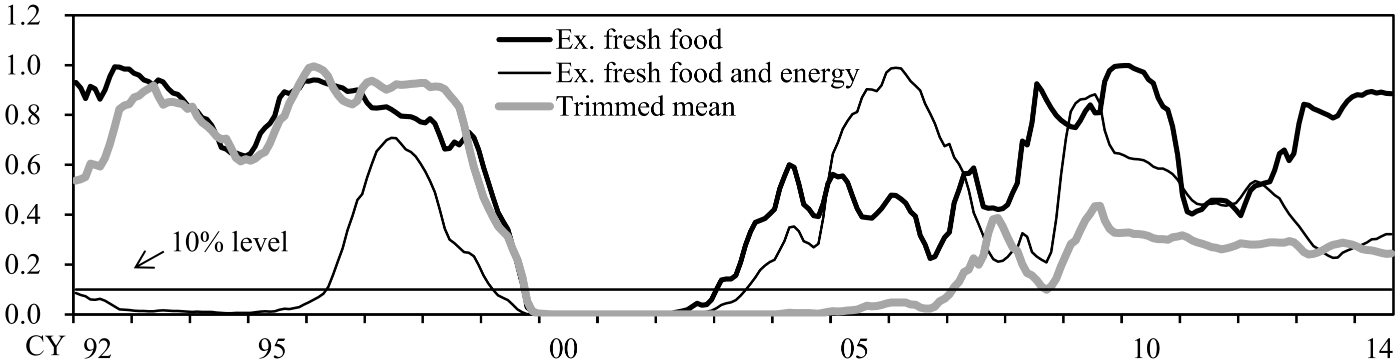
Note: Figures in (2) show estimates of constant terms that are estimated repeatedly using subsamples of 120 months ending at each month on the horizontal axis. The dotted lines show 95% confidence intervals computed from autocorrelation robust standard errors based on Newey-West's method.

Predictive power on future underlying trend

(1) Joint hypothesis test on Equation (12)

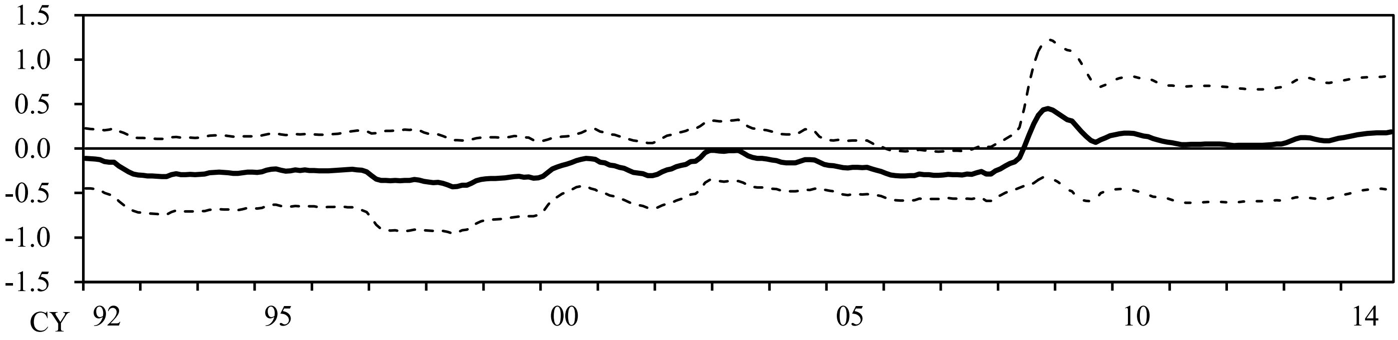


(Reference) Joint hypothesis test on Equation (12) with individual indicators



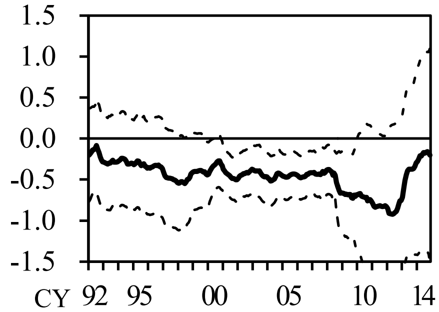
Note: Figures in the charts show p-values for joint hypotheses testing on β_0=0 and β_1=1 in estimation Equation (12) based on the rolling estimation results using subsamples of 120 months ending at each month on the horizontal axis.

(2) Estimates for slope in Equation (13)

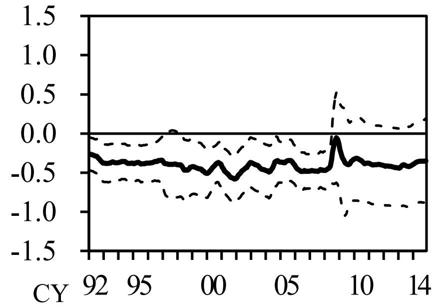


(Reference) Estimates for slope in Equation (13) with individual indicators

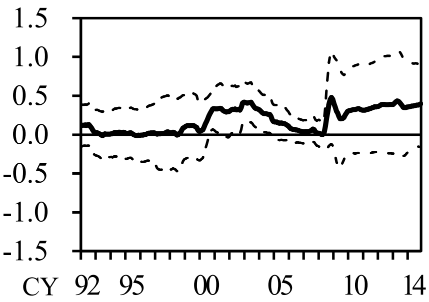
(i) Ex. fresh food



(ii) Ex. fresh food and energy



(iii) Trimmed mean

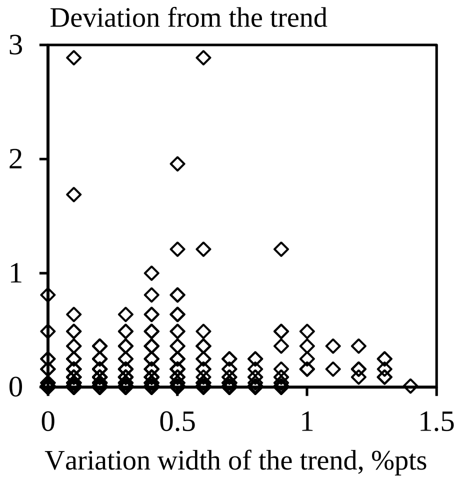


Note: Figures in the charts show estimates for slope in estimation Equation (13) based on the rolling regression results using subsamples of 120 months ending at each month on the horizontal axis. The dotted lines show 95% confidence intervals computed from autocorrelation robust standard errors based on Newey-West's method.

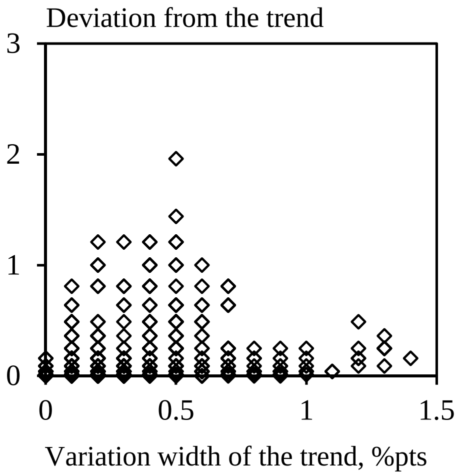
Deviation from the trend and the variation of the trend

(1) Exclusion-based indicators

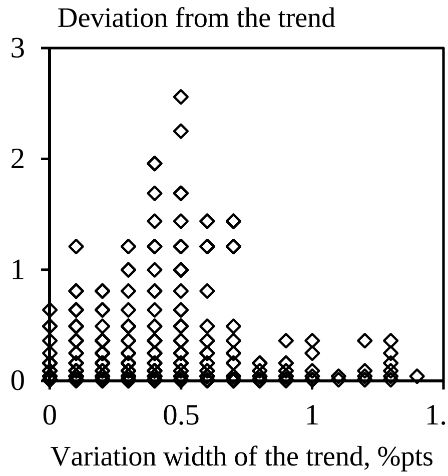
(i) Ex. fresh food



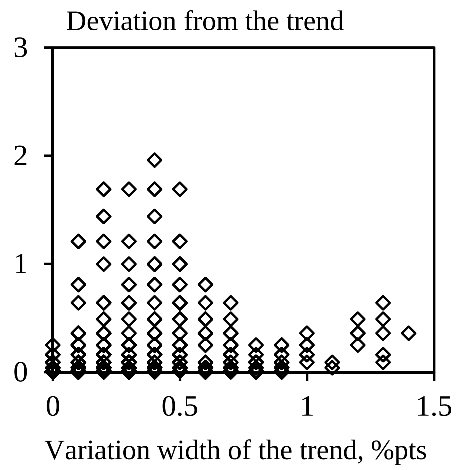
(ii) Ex. fresh food and energy



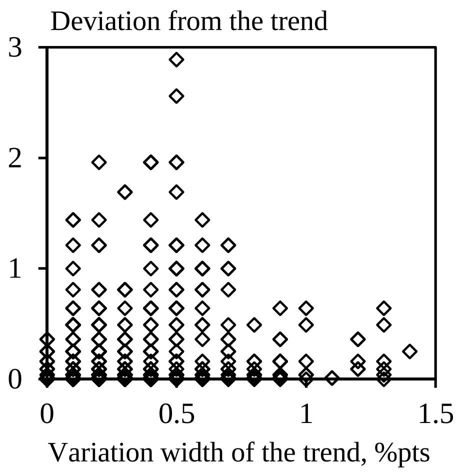
(iii) Ex. food and energy



(iv) Ex. fresh food, energy, and imputed rent

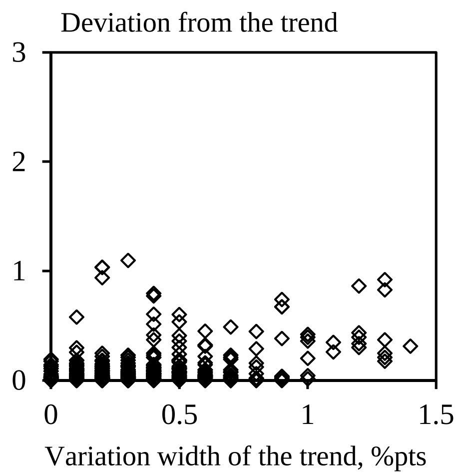


(v) Ex. food, energy, and imputed rent

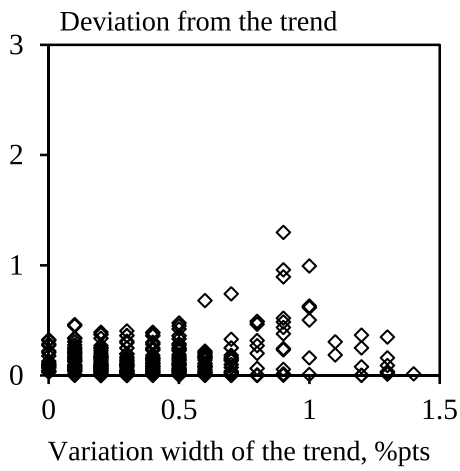


(2) Distribution-based indicators

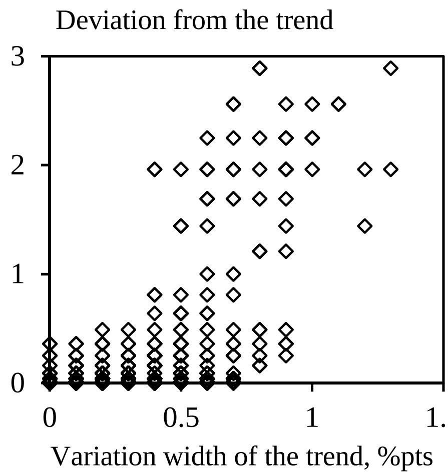
(i) Trimmed mean



(ii) Weighted median



(iii) Mode



Note: "Deviation from the trend" is calculated by root mean squared errors using subsamples of 120 months.
"Variation width of the trend" is the abolute value of the year-on-year difference of the HP filtered trend.