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Abstract

We construct an endogenous growth model with public capital and endogenous labor supply and examine quantitatively the welfare effects of fiscal consolidation on the Japanese economy. We consider two modes of fiscal consolidation: the adjustment to a new lower target of the debt-to-GDP and deficit-to-GDP ratios. We find that the debt and deficit reduction rules based only on government consumption and investment expenditure cuts improve households' welfare. This improvement in households' welfare becomes large as the speed of fiscal consolidation rises. Further, reductions in the target debt-to-GDP or deficit-to-GDP ratio generate larger welfare gains. We also discuss the welfare effects of fiscal consolidation with tax increases and transfer payment decreases.

JEL classification: E62, H54, H60

Keywords: Fiscal consolidation, Endogenous growth, Welfare

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1 Introduction

Japanese government debt has climbed to an unprecedented level in recent years. The gross debt-to-GDP ratio in Japan reached 219.3% in 2015, a figure much higher than that in some EU member states. For example, in 2015, the gross debt-to-GDP ratios in Greece, Italy, and Portugal were 183.9%, 159.6%, and 150.7%, respectively. Other EU member states and the United States also suffer from high debt-to-GDP ratios.

To address these challenges, countries aim to implement fiscal consolidation. For example, the Maastricht Treaty enforces EU member states to keep their government deficit-to-GDP ratios below 3% and their debt-to-GDP ratios below 60%. Moreover, the reduction rule is also introduced: the reformed Stability and Growth Pact states that EU members whose debt-to-GDP ratio is above the 60% threshold must reduce their ratios to 60% at an average rate of one-20th per year. In the United States, the 2013 Budget Resolution passed by the US House of Representatives[1] states that the government must cut its expenditure to reduce the budget deficit from 9% of GDP to the pre-financial crisis level by 2022. Likewise, the Japanese government aimed to reduce its primary deficit to achieve a primary surplus by 2020 but abandoned this target.[2] Thus, there are two ways of implementing fiscal consolidation: one is to target the budget deficit and the other is to target government debt. Our study focuses on these rules and compares their effects on the Japanese economy and its welfare. Furthermore, we pay attention to the speed of fiscal consolidation.

To conduct our analyses, we construct an endogenous growth model with public capital and endogenous labor supply. The engine of growth in our model is productive public capital as in Futagami et al. (1993) and Turnovsky (1997).[3] In the model presented herein, public investment is financed not only by taxes on capital income, labor income, and households' consumption but also by issuing government bonds. When implementing fiscal consolidation, we consider the following two policy rules: (i) the government adjusts the public debt-to-output ratio gradually to the target and (ii) the government adjusts the government budget deficit-to-output ratio gradually to the target. In this study, we refer to the former (latter) policy rule as the *debt (deficit) policy rule.*

[1] House Concurrent Resolution, 112th Congress, 2nd Session, House Report No. 112, March 2012.

[2] Doi et al. (2011) study the sustainability of budget deficits of Japan and conclude that the Japanese government must raise tax rates and reduce expenditure. In addition, Arai and Ueda (2013) show that the Japanese government needs to achieve a primary surplus in the long run to ensure fiscal sustainability.

[3] Papageorgiou (2012) considers a similar issue in the Greek economy, using a growth model with productive public capital; however, the author's is not an endogenous growth model.

To implement fiscal consolidation, the government must create fiscal space. Of the various approaches to do this, we focus on the government reducing its expenditure (consumption and investment) under the debt and deficit policy rules. In the short run, when the government reduces its investment expenditure to pursue fiscal consolidation, economic growth slows. However, in the long run, a decline in the interest payments of the government can create sufficient fiscal space to raise public investment, which can stimulate economic growth and improve welfare. Because the IMF (2014) stresses the importance of investment in productive public capital, it is important to study the welfare effects of expenditure cuts under both the debt and the deficit policy rules.

To create fiscal space, the government can also adopt alternative policies such as tax increases and transfer payment reductions to households. If the government chooses tax increases, the distortionary effects can diminish welfare. If the government chooses transfer payment decreases, the budget constraint of households is tightened, which can lower welfare. However, although both these policies have negative welfare effects, the government might not have to reduce its consumption and investment expenditure to pursue fiscal consolidation. Thus, we consider the welfare effects of these two policies.

By using the above framework, we calibrate the model with Japanese data and find the following results:

(i) Under the debt and deficit policy rules, fiscal consolidation based only on government consumption and investment expenditure cuts improves welfare. In addition, the welfare gains become large as the speed of fiscal consolidation rises. Regarding the target, lowering the target debt-to-GDP or deficit-to-GDP ratio generates larger welfare gains. Comparing these two fiscal consolidation policies, the welfare gains under the debt policy rule are larger than those under the deficit policy rule.

In contrast to fiscal consolidation based only on government consumption and investment expenditure cuts, we obtain the following results:

(ii) When fiscal consolidation is pursued by conducting capital or labor income tax increases, both the debt and the deficit policy rules generate smaller welfare gains. In particular, capital income tax increases have sufficiently large negative welfare effects.

(iii) When fiscal consolidation is pursued by conducting consumption tax increases, the debt policy rule generates smaller welfare gains. On the other hand, the deficit policy rule with sufficiently small (large) consumption tax increases generates larger (smaller) welfare gains. However, even in this case, the welfare gains under the debt policy rule are larger than those under the deficit policy rule.

(iv) When fiscal consolidation is pursued by conducting transfer payment reductions, both the debt and the deficit policy rules generate larger welfare gains.

(v) If we incorporate government consumption utility and households' preference for government consumption is sufficiently high, fiscal consolidation pursued by conducting labor income or consumption tax increases can generate larger welfare gains.

The remainder of this paper is organized as follows. Section 2 describes the relevant literature. Section 3 establishes the model. Section 4 investigates the debt policy rule based only on expenditure cuts. Section 5 examines the deficit policy rule based only on expenditure cuts. Section 6 analyzes the welfare effects with tax increases. Section 7 investigates the welfare effects with transfer payment reductions. Section 8 discusses some important issues. Section 9 concludes.

2 Related literature

Previous studies of fiscal sustainability and fiscal consolidation examine two kinds of policy rules. One is based on a rule that the government controls the primary surplus or budget deficit depending on some economic variables. The other is based on a rule that the government controls some policy instruments (e.g., tax rates) to attain the target government debt. The latter studies take account of the rule adopted by EU states or the United States. By contrast, the studies in the first group do not take any government debt targets into account.

The first group of studies includes the following. Greiner (2007, 2012) and Kamiguchi and Tamai (2012) examine the sustainability of government debt by using endogenous growth models with a representative infinitely lived agent and public capital (public spending). All these studies consider the rule that controls the following manner:

$$\frac{PS_t}{Y_t} = \mu + \kappa z_t, \quad \mu \in \mathbb{R}, \ \kappa > 0,$$

where Y_t, PS_t, and z_t stand for GDP, the primary surplus, and the debt-to-GDP ratio, respectively. This rule states that the ratio of the primary surplus to GDP is a positive linear function of the debt-to-GDP ratio. Greiner (2007, 2012) and Kamiguchi and Tamai (2012) show that an economy cannot keep its fiscal sustainability when κ is sufficiently small.

Bräuninger (2005) considers a similar issue based on an endogenous growth model of AK technology with overlapping generations. Yakita (2008a) also considers the same issue based on an endogenous growth model with overlapping generations and public capital. In their models, the issuance of government bonds is tied to a certain ratio to GDP (budget deficit ratio). The following represents this rule:

$$B_{t+1} - B_t = \bar{d} Y_t, \quad \bar{d} > 0.$$

Bräuninger (2005) shows that when the budget deficit ratio is under a certain threshold, fiscal deficits are sustainable. Yakita (2008a) shows that the government deficit is sustainable below the threshold of the initial government debt for an initial level of public capital. However, none of these studies considers any target government debt or deficit.

We next explain the second group. Coenen et al. (2008) use a New Keynesian model that has two regions (the United States and the EU). The government in their model has a target debt-to-GDP ratio in line with the Maastricht Treaty. It then adjusts its policy instruments (e.g., the ratio of government expenditure or tax rates) to attain the target; that is,

$$i_t - i^* = \phi \left(z_t - \bar{z} \right),$$

where i_t represents the policy instrument used and i^* stands for a steady state of their system. $\bar{z}$ stands for the target debt-to-GDP ratio. ϕ is a positive (negative) constant when the instrument is a tax rate (government expenditure). This rule states that when the debt-to-GDP ratio is above the target, the government raises tax rates or decreases the ratio of government expenditure to GDP. Coenen et al. (2008) show that this fiscal consolidation rule has short-run negative effects and long-run positive effects.

Forni et al. (2010) also use a New Keynesian model that has two regions and incorporate a similar rule to Coenen et al. (2008) as follows:

$$\frac{i_t}{i_{t-1}} = \left(\frac{z_t}{\bar{z}} \right)^{\phi_1} \left(\frac{z_t}{z_{t-1}} \right)^{\phi_2} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_3},$$

where ϕ_i $(i = 1, 2, 3)$ is a positive (negative) constant when the instrument is a tax rate (government expenditure). In contrast to Coenen et al. (2008), their policy rule includes that a change of GDP from the last period to the present period and a change of debt from the last period to the present period affect the adjustment of the policy instruments. Moreover, the government adjusts its policy instruments against their values in the last period. They consider a specific case in which the policy rule makes debt reach the target in about 10 years. They then examine the welfare effects of reductions in the debt-to-GDP ratio and show that when the government reduces the target debt-to-GDP ratio, simultaneous decreases in public expenditure and tax rates have significantly positive effects on welfare.

Cogan et al. (2013) use a similar model to Coenen et al. (2008). They consider the following rule:

$$\iota_t = \phi\left(z_t - \bar{z}\right), \quad \phi > 0,$$

where ι_t stands for the ratio of lump sum tax to GDP. Their analyses are based on the 2013 Budget Resolution in the United States and they show that this mode of fiscal consolidation raises GDP not only in the long run but also in the short run.

Hansen and İmrohoroğlu (2016) use a standard neoclassical growth model and consider that the government starts to reduce transfer payment to households or increase tax revenues when the net debt-to-GDP ratio hits the threshold value (250%). Then, the government tries to reduce the ratio to 60%. They show that fiscal adjustment then becomes significantly large.

However, although the above-mentioned studies introduce targets, they do not consider the speed at which fiscal consolidation is adjusted. By contrast, Maebayashi et al. (2017) take account of the speed of adjustment of fiscal consolidation in the following manner:[4]

$$\dot{z}_t = -\phi(z_t - \bar{z}), \quad \phi > 0.$$

They find that fiscal consolidation based on this rule improves welfare and that the improvement in welfare increases as the speed of fiscal consolidation rises. Moreover, they discuss the optimal target and show that the optimal debt-to-GDP ratio is lower than the EU target.[5]

[4]Futagami et al. (2008) consider a similar rule to this; however, the variables in their model are denominated by private capital.

[5]Morimoto et al. (2017) examine the stability of a similar model to Maebayashi et al. (2017) in a small open economy framework and show that the economy can be unstable.

3 Model

3.1 Firms

There is a large number of identical firms, whose size is normalized to one. Firm j produces a single final good by using the production technology given by $Y_{j,t} = AK_{j,t}^{\alpha}(h_t L_{j,t})^{1-\alpha}$ $(0 < \alpha < 1)$, where $Y_{j,t}$ is the output level, $K_{j,t}$ and $L_{j,t}$ are private capital and labor inputs, respectively, and h_t is labor productivity. The profit maximization in competitive markets gives the following necessary conditions: $R_t = \alpha A(K_{j,t}/L_{j,t})^{\alpha-1} h_t^{1-\alpha}$ and $w_t = (1-\alpha)A(K_{j,t}/L_{j,t})^{\alpha} h_t^{1-\alpha}$, where R_t and w_t represent the rental price of capital and wage rate, respectively.

Following Kalaitzidakis and Kalyvitis (2004), Yakita (2008b), and Maebayashi et al. (2017), we assume that aggregate private capital, $K_t = \sum_j K_{j,t}$, and public capital, $K_{g,t}$, positively affect labor productivity and specify $h_t = K_t^{1-\epsilon} K_{g,t}^{\epsilon}$ $(0 < \epsilon < 1)$. Let us denote the aggregate labor input as $L_t = \sum_j L_{j,t}$. In the equilibrium, $K_{j,t}/L_{j,t} = K_t/L_t$ holds. Therefore, the aggregate output and factor prices can be written as follows:

$$Y_t = Ak_{g,t}^{\beta} L_t^{1-\alpha} K_t, \tag{1a}$$

$$R_t = \alpha A k_{g,t}^{\beta} L_t^{1-\alpha}, \tag{1b}$$

$$w_t = (1-\alpha) A k_{g,t}^{\beta} L_t^{-\alpha} K_t, \tag{1c}$$

where $k_{g,t} \equiv K_{g,t}/K_t$ and $\beta \equiv \epsilon(1-\alpha)$.

3.2 Households

We consider a representative household that has an infinite planning horizon and perfect foresight. The size of the population is normalized to one. The objective of the representative household is

$$U_0 = \int_0^{\infty} e^{-\rho t} \left(\log C_t - \psi \frac{n_t^{1+\chi}}{1+\chi} \right) dt, \tag{2}$$

where $\rho > 0$ is the subjective discount rate, C_t is consumption, and n_t is the time devoted to labor supply. χ denotes the inverse intertemporal elasticity of substitution for labor. We assume that each household has one unit of time endowment, and hence leisure time becomes

$1 - n_t$. The budget constraint of the household becomes

$$\dot{K}_t + \dot{B}_t = (1-\tau_k)(R_t - \delta_k)K_t + (1-\tau_b)r_t B_t + (1-\tau_n)w_t n_t - (1+\tau_c)C_t + TR_t,$$

where δ_k is the depreciation of private capital, r_t is the interest rate, B_t is outstanding government debt, and TR_t is transfers from the government. τ_k, τ_b, τ_n, and τ_c represent the capital income tax rate, bond income tax rate, labor income tax rate, and consumption tax rate. Here, we assume that τ_k, τ_b, τ_n, and τ_c are constant over time. Let us define $W_t \equiv K_t + B_t$. The no-arbitrage condition in the asset market is

$$(1-\tau_k)(R_t - \delta_k) = (1-\tau_b)r_t. \tag{3}$$

By using (3), the budget constraint of the household can be rewritten as follows:

$$\dot{W}_t = (1-\tau_k)(R_t - \delta_k)W_t + (1-\tau_n)w_t n_t - (1+\tau_c)C_t + TR_t. \tag{4}$$

From the maximization problem of the household, we obtain

$$\frac{1}{(1+\tau_c)C_t} = \frac{\psi n_t^{\chi}}{(1-\tau_n)w_t}, \tag{5a}$$

$$\frac{\dot{C}_t}{C_t} = (1-\tau_k)(R_t - \delta_k) - \rho, \tag{5b}$$

and the usual transversality condition, $\lim_{t\to\infty} C_t^{-1} W_t e^{-\rho t} = 0$.

3.3 Government

The budget constraint of the government is given by

$$\dot{B}_t = r_t B_t + G_t + TR_t - \left[\tau_k(R_t - \delta_k)K_t + \tau_b r_t B_t + \tau_n w_t n_t + \tau_c C_t\right], \tag{6}$$

where G_t is the sum of government consumption and government investment. The government must satisfy the NPG condition, $\lim_{T\to\infty} B_T e^{-\int_t^T (1-\tau_b) r_v dv} \leq 0$. It allocates a constant proportion, $\theta \in (0,1)$, of G_t to investment in public capital, $I_{g,t}$. Thus, the evolution of public capital is given by

$$\dot{K}_{g,t} = I_{g,t} - \delta_g K_{g,t} = \theta G_t - \delta_g K_{g,t}, \tag{7}$$

where δ_g is the depreciation of public capital. In the following analyses, we assume that government consumption, $(1-\theta)G_t$, does not affect any economic agents.[6] Regarding transfer payment, we assume that the government spends a constant proportion, $\xi \in (0,1)$, of households' labor income, $w_t n_t$, on transfer payment. That is, the following holds:

$$TR_t = \xi w_t n_t. \tag{8}$$

In this study, the government budget deficit is defined as

$$D_t \equiv r_t B_t + G_t + TR_t - \left[\tau_k (R_t - \delta_k) K_t + \tau_b r_t B_t + \tau_n w_t n_t + \tau_c C_t\right]. \tag{9}$$

By using (6) and (9), we obtain $\dot{B}_t = D_t$.

4 Debt policy rule

We first assume that the government gradually adjusts the public debt-to-output ratio to the target and adopts the following rule:

$$\dot{z}_t = -\phi(z_t - \bar{z}), \tag{10}$$

where $z_t \equiv B_t/Y_t$. $\bar{z}$ is the target of z_t and $\phi > 0$ is the adjustment coefficient of the rule. In this study, we refer to the rule (10) as the *debt policy rule*. In Section 5, we consider the alternative policy rule: the government gradually adjusts the government budget deficit-to-output ratio to the target.

4.1 Dynamic system

We consider the dynamic system of the economy. Because the population size is normalized to one, the labor market-clearing condition becomes $L_t = n_t$. The market-clearing condition for the final good is as follows:

$$Y_t = C_t + I_t + G_t, \tag{11}$$

[6] In Section 8.2, we modify this setting to allow government consumption to positively affect households' utility.

where I_t is investment in private capital. Note that the evolution of private capital is given by $\dot{K}_t = I_t - \delta_k K_t$. From (1a) and (11), the growth rate of K_t becomes

$$\frac{\dot{K}_t}{K_t} = Ak_{g,t}^{\beta} n_t^{1-\alpha} - c_t - g_t - \delta_k, \tag{12}$$

where $c_t \equiv C_t/K_t$ and $g_t \equiv G_t/K_t$. By using (7) and (12), we obtain

$$\dot{k}_{g,t} = \theta g_t - \delta_g k_{g,t} - \left(Ak_{g,t}^{\beta} n_t^{1-\alpha} - c_t - g_t - \delta_k \right) k_{g,t}. \tag{13}$$

From (1c) and (5a), the relationship between consumption and labor supply is given by

$$(1+\tau_c)c_t = \frac{(1-\tau_n)(1-\alpha)Ak_{g,t}^{\beta}}{\psi n_t^{\chi+\alpha}}. \tag{14}$$

By using (1b), (5b), (12), (13), and (14), we obtain

$$\begin{aligned}(\chi+\alpha)\frac{\dot{n}_t}{n_t} = &-(1-\tau_k)\left(\alpha Ak_{g,t}^{\beta} n_t^{1-\alpha} - \delta_k\right) + \rho \\ &+ (1-\beta)\left(Ak_{g,t}^{\beta} n_t^{1-\alpha} - c_t - g_t - \delta_k\right) + \beta\left(\theta\frac{g_t}{k_{g,t}} - \delta_g\right).\end{aligned} \tag{15}$$

Because the definition of z_t yields $\dot{z}_t = \frac{\dot{B}_t}{Y_t} - z_t \frac{\dot{Y}_t}{Y_t}$, (1a) and (10) imply

$$-\phi(z_t - \bar{z}) = \frac{\dot{B}_t}{Y_t} - z_t\left[\beta\frac{\dot{k}_{g,t}}{k_{g,t}} + (1-\alpha)\frac{\dot{n}_t}{n_t} + \frac{\dot{K}_t}{K_t}\right]. \tag{16}$$

Here, from (1a), (1b), (1c), (3), (6), and (8), we obtain

$$\begin{aligned}\frac{\dot{B}_t}{Y_t} = &(1-\tau_k)\left(\alpha Ak_{g,t}^{\beta} n_t^{1-\alpha} - \delta_k\right) z_t + \frac{g_t}{Ak_{g,t}^{\beta} n_t^{1-\alpha}} \\ &- \left[\tau_k\left(\alpha - \frac{\delta_k}{Ak_{g,t}^{\beta} n_t^{1-\alpha}}\right) + (\tau_n - \xi)(1-\alpha) + \frac{\tau_c c_t}{Ak_{g,t}^{\beta} n_t^{1-\alpha}}\right].\end{aligned} \tag{17}$$

Substituting (12), (13), (14), (15), and (17) into (16) and solving for g_t leads to

$$g_t = \frac{\Psi_1(k_{g,t}, n_t) - \Psi_2(k_{g,t}, n_t, z_t) - \phi(z_t - \bar{z})Ak_{g,t}^{\beta} n_t^{1-\alpha} + \Psi_3(k_{g,t}, n_t, z_t)}{1 + \frac{\chi+1}{\chi+\alpha}\left[(1-\beta)k_{g,t} - \beta\theta\right] z_t A k_{g,t}^{\beta-1} n_t^{1-\alpha}}, \tag{18}$$

where

$$\Psi_1(k_{g,t}, n_t) \equiv \tau_k \left(\alpha A k_{g,t}^{\beta} n_t^{1-\alpha} - \delta_k\right) + (\tau_n - \xi)(1-\alpha) A k_{g,t}^{\beta} n_t^{1-\alpha} + \tau_c \frac{(1-\tau_n)(1-\alpha) A k_{g,t}^{\beta}}{(1+\tau_c)\psi n_t^{\chi+\alpha}},$$

$$\Psi_2(k_{g,t}, n_t, z_t) \equiv (1-\tau_k) \left(\alpha A k_{g,t}^{\beta} n_t^{1-\alpha} - \delta_k\right) A k_{g,t}^{\beta} n_t^{1-\alpha} z_t,$$

$$\Psi_3(k_{g,t}, n_t, z_t) \equiv \frac{(\chi+1)(1-\beta)}{\chi+\alpha} A k_{g,t}^{\beta} n_t^{1-\alpha} z_t \left[A k_{g,t}^{\beta} n_t^{1-\alpha} - \frac{(1-\tau_n)(1-\alpha) A k_{g,t}^{\beta}}{(1+\tau_c)\psi n_t^{\chi+\alpha}} - \delta_k\right]$$
$$- \frac{1-\alpha}{\chi+\alpha} A k_{g,t}^{\beta} n_t^{1-\alpha} z_t \left[(1-\tau_k)\left(\alpha A k_{g,t}^{\beta} n_t^{1-\alpha} - \delta_k\right) - \rho\right] - \frac{(\chi+1)\beta}{\chi+\alpha} \delta_g A k_{g,t}^{\beta} n_t^{1-\alpha} z_t.$$

The numerator on the right-hand side of (18) is composed of the following parts. $\Psi_1(k_{g,t}, n_t)$ represents tax revenue minus transfer payment. $\Psi_2(k_{g,t}, n_t, z_t)$ represents the interest payments on public debt. $-\phi(z_t - \bar{z}) A k_{g,t}^{\beta} n_t^{1-\alpha}$ represents the government expenditure cuts undertaken to decrease public debt. If ϕ is larger, these government expenditure cuts become larger in the short run. $\Psi_3(k_{g,t}, n_t, z_t)$ is the term related to $\dot{Y}_t/Y_t$.

In the debt policy rule, (10), (13), (14), (15), and (18) characterize the dynamic system with respect to z_t, $k_{g,t}$, and n_t.

4.2 Steady state

Next, we consider the steady state of the economy. In the steady state, z_t, $k_{g,t}$, and n_t become constant over time; that is, $z_t = \bar{z}$, $k_{g,t} = k_g^*$, and $n_t = n^*$ hold. The asterisks represent the variables in the steady state. In addition, the steady-state growth rates must satisfy $\gamma^* \equiv \left(\frac{\dot{K}_t}{K_t}\right)^* = \left(\frac{\dot{C}_t}{C_t}\right)^* = \left(\frac{\dot{B}_t}{B_t}\right)^* = \left(\frac{\dot{K}_{g,t}}{K_{g,t}}\right)^*$. By using (1b), (5b), (7), and (12), we obtain

$$\gamma^* = \theta \frac{g^*}{k_g^*} - \delta_g, \tag{19a}$$

$$= A(k_g^*)^{\beta}(n^*)^{1-\alpha} - c^* - g^* - \delta_k, \tag{19b}$$

$$= (1-\tau_k)\left[\alpha A (k_g^*)^{\beta} (n^*)^{1-\alpha} - \delta_k\right] - \rho. \tag{19c}$$

(18) leads to

$$g^* = \frac{\Psi_1(k_g^*, n^*) - \Psi_2(k_g^*, n^*, \bar{z}) + \Psi_3(k_g^*, n^*, \bar{z})}{1 + \frac{\chi+1}{\chi+\alpha}\left[(1-\beta)k_g^* - \beta\theta\right] \bar{z} A (k_g^*)^{\beta-1} (n^*)^{1-\alpha}}. \tag{20}$$

(14), (19a), and (19b) yield

$$\left(1+\frac{\theta}{k_g^*}\right)g^* - \delta_g = A(k_g^*)^{\beta}(n^*)^{1-\alpha} - \frac{(1-\tau_n)(1-\alpha)A(k_g^*)^{\beta}}{(1+\tau_c)\psi(n^*)^{\chi+\alpha}} - \delta_k. \tag{21}$$

By using (14), (19b), and (19c), we obtain

$$A(k_g^*)^{\beta}(n^*)^{1-\alpha} - \frac{(1-\tau_n)(1-\alpha)A(k_g^*)^{\beta}}{(1+\tau_c)\psi(n^*)^{\chi+\alpha}} - g^* - \delta_k = (1-\tau_k)\left[\alpha A(k_g^*)^{\beta}(n^*)^{1-\alpha} - \delta_k\right] - \rho. \tag{22}$$

By substituting (20) into (21) and (22) and solving these equations, we can obtain the steady-state values k_g^* and n^*. The rest of the steady-state values (c^*, γ^*, and g^*) are determined by (14), (19a), and (20). From these results, we can confirm that the value of ϕ does not affect the steady-state values. However, the steady-state values depend on the value of $\bar{z}$.

4.3 Calibration

In this subsection, we explain the calibration method. As shown by Chetty et al. (2012), the Frisch elasticity of labor supply is 0.5. Because the Frisch elasticity of labor supply in this study is $1/\chi$, we adopt $\chi = 2$. Following Hansen and İmrohoroğlu (2016), the capital income share is set to 0.3783; that is, $\alpha = 0.3783$. We employ $\tau_k = 0.5189$ and $\tau_n = 0.3324$, which are the latest values estimated in Gunji and Miyazaki (2011) for 2007.[7] In addition, we adopt the same value of τ_b as in Hansen and İmrohoroğlu (2016); that is, $\tau_b = 0.2$. The tax rate on consumption in Japan in 2017 was 8%. Therefore, we set $\tau_c = 0.08$. Regarding the depreciation rate of private capital, we adopt $\delta_k = 0.0721$, which is set to be intermediate between the values of Fujiwara et al. (2005), Sugo and Ueda (2008), Nutahara (2015), and Hansen and İmrohoroğlu (2016).[8] The depreciation rate of public capital is set to $\delta_g = 0.0448$ following Kato (2002).

According to the OECD Economic Outlook for 1980–2015 in Japan, the average ratio of the sum of government consumption and government investment to GDP is 0.2148 and the average ratio of government investment to GDP is 0.04985. From these values, the ratio of government investment to the sum of government consumption and government investment is 0.2320. Thus, we set $\theta = 0.2320$. Regarding the elasticity of output with respect to public capital β, we adopt $\beta = 0.160$, which is the value estimated by Miyagawa et al. (2013). This yields $\epsilon = 0.2574$.

[7] We use the tax rate on labor incomes with social security premiums estimated by Gunji and Miyazaki (2011).

[8] In Fujiwara et al. (2005), Sugo and Ueda (2008), and Nutahara (2015), δ_k is set to 0.06. On the contrary, in Hansen and İmrohoroğlu (2016), δ_k is set to 0.0842.

From (1a), (1c), and (8), we obtain $TR_t/Y_t = \xi(1-\alpha)$. In this study, transfer payment, TR_t, are assumed to be social security benefits, and the average ratio of social security benefits to GDP in 1980–2015 is 0.09136. To satisfy this, we set $\xi = 0.1469$.

To calibrate the remaining parameters A, ρ, and ψ, we use the following values on the Japanese economy from the OECD Economic Outlook: (i) the gross public debt-to-GDP ratio in 2015 was 2.1927, (ii) the average growth rate in 1980–2015 was 0.02062, (iii) the average labor supply in 1980–2015 was 0.1875,[9] and (iv) the average ratio of the sum of government consumption and government investment to GDP was 0.2148. To satisfy these values in the steady state, we obtain the following calibration values: $A = 1.5376$, $\rho = 0.02842$, and $\psi = 100.01$.[10] Table 1 summarizes the parameter values.

[Table 1]

Given these parameter values, we calculate the steady-state values of the private consumption-to-GDP ratio and government budget deficit-to-GDP ratio. Table 2 compares these calculations with the Japanese data and shows that these values match well. Moreover, the value of $\rho = 0.02842$ matches well with the conventional value in the macroeconomic literature. In the following analyses, we use this steady state as a starting point.

Table 2 further shows that the steady-state values of the total tax revenue-to-GDP ratio and government interest payments-to-GDP ratio deviate from the Japanese data. In our model, from (3), the real interest rate is determined by the rate of return as private capital, which implies a higher real interest rate than the actual data. Thus, the steady-state value of the government interest payments-to-GDP ratio is about 10 percentage points higher than the Japanese data. To satisfy the government budget constraint, the steady-state value of the total tax revenue-to-GDP ratio is also about 12 percentage points higher than the Japanese data. As mentioned in footnote 11 of Maebayashi et al. (2017), "huge models that incorporate the capital market in the open economy and the exchange rates and other fiscal and monetary policies in each of the countries might be necessary." Indeed, these factors are important when discussing the

[9] In this study, we calculate labor supply in the following way:

$$(\text{labor supply}) = \frac{(\text{the number of employed persons})}{(\text{total population})} \times \frac{(\text{total hours worked in a year})}{14 \times 30 \times 12}$$

We assume that the discretionary hours available per day are 14 hours and that one month is 30 days. From the OECD Economic Outlook, the average ratio of the number of employed persons to the total population was 0.4980 and the average total hours worked in a year was 1897.305 hours in 1980–2015. By using these values, we can calculate the average value of labor supply.

[10] From conditions (i)–(iv), we first calculate the value of k_g^*. Then, we calculate the values of A, ρ, and ψ.

welfare effects of fiscal consolidation. However, huge models may obscure our main arguments. Therefore, in this study, we do not consider these factors.

[Table 2]

4.4 Transitional dynamics

To investigate the transitional dynamics, we assume that the economy is initially in the steady state as in the previous subsection. That is, the government sets $\bar{z} = 2.1927$ before the policy change. At time 0, the government reduces $\bar{z}$ unexpectedly and gradually adjusts z_t to the new target of $\bar{z}$ following the rule (10). The economy exhibits transitional dynamics and converges to the new steady state. We examine these transitional dynamics by using the relaxation algorithm developed by Trimborn et al. (2008).[11] As mentioned in the Introduction, because the Japanese government does not have a public debt target, the new target of $\bar{z}$ is set to 0.6 following the Stability and Growth Pact and Maastricht Treaty. In addition, we vary the values of $\phi \in \{0.01, 0.05, 0.1\}$. Figure 1 presents the results.

[Figure 1]

$I_{g,t}/Y_t$ initially falls and thereafter monotonically increases to the new steady-state level. At time 0, to reduce z_t, the government must improve the government deficit, which leads to a reduction in government investment. When z_t decreases, the interest payments of the government decline, which implies that the government can raise its expenditure. In the long run, government investment is higher than that at the initial steady-state level.

Because households anticipate that the debt reduction raises the long-run growth rate, they break into their savings and increase their consumption at time 0, C_0.[12] Whether the growth rate of K_t initially rises or falls is determined by the initial jump in government expenditure and private consumption. Because the initial decline in government expenditure is sufficiently large, resources are reallocated to investment in K_t. Thus, the growth rate of K_t initially rises. After the initial jump, the growth rate of K_t falls and then rises, converging to the new steady-state level, which is higher than the initial steady-state level.

[11] The MATLAB programs for the relaxation algorithm are available for free download at `http://www.relaxation.uni-siegen.de`.

[12] This is the "non-Keynesian effect" (e.g., Giavazzi and Pagano 1990; Alesina and Ardagna 1998; Perotti 1999).

From (1b) and (5b), the growth rate of C_t is given by

$$\frac{\dot{C}_t}{C_t} = (1 - \tau_k)\left(\alpha A k_{g,t}^{\beta} n_t^{1-\alpha} - \delta_k\right) - \rho. \tag{23}$$

$k_{g,t}$ first falls and then rises, converging to the new steady-state level, which is higher than the initial steady-state level. Furthermore, n_t initially falls and thereafter increases to the new steady-state level, which is higher than the initial steady-state level. From these results, the growth rate of C_t initially falls and thereafter increases to the new steady-state level, which is higher than the initial steady-state level.

In the short and long run, the effect of the growth rate of K_t is sufficiently large. After the initial jump, C_t/K_t decreases to the new steady-state level. From (14), because the effect of C_t/K_t is sufficiently large, the transitional path of n_t begins to run in the opposite direction; that is, n_t initially falls and thereafter increases to the new steady-state level, which is higher than the initial steady-state level.

At the end of this subsection, we mention the effects of ϕ. As in Section 4.2, ϕ does not affect the steady-state values; that is, the long-run effects do not depend on ϕ. However, if ϕ is larger, the short-run government expenditure cuts become larger (see (18)). This enlarges the short-run effects of the other variables.

4.5 Welfare effects based only on expenditure cuts

Next, we examine the welfare effects under the debt policy rule. We define γ_t^C as the growth rate of C_t. Because $C_t = C_0 \exp\left(\int_0^t \gamma_s^C ds\right)$ and $c_t = C_t/K_t$ hold, we obtain

$$\log C_t = \log c_0 + \log K_0 + \int_0^t \gamma_s^C ds. \tag{24}$$

Without any loss of generality, we set $K_0 = 1$. From (2) and (24), we obtain

$$U_0 = \frac{1}{\rho}\log c_0 + \int_0^{\infty} e^{-\rho t}\left(\int_0^t \gamma_s^C ds - \psi\frac{n_t^{1+\chi}}{1+\chi}\right) dt. \tag{25}$$

The welfare level can be calculated by c_0 and the transitional paths of γ_t^C and n_t. Here, U_{ini}^* is defined as the welfare level at which the economy remains in the initial steady state. By using

(25), U^*_{ini} is given by

$$U^*_{ini} = \frac{1}{\rho}\left[\log c^*_{ini} - \psi\frac{(n^*_{ini})^{1+\chi}}{1+\chi}\right] + \frac{1}{\rho^2}\gamma^*_{ini},$$

where c^*_{ini}, n^*_{ini}, and γ^*_{ini} represent the initial steady-state values of c_t, n_t, and the growth rate. We define U_0^{**} as the welfare level after the policy change. In Appendix A, we present the calculation procedure for U_0^{**} under the relaxation algorithm developed by Trimborn et al. (2008). The welfare gains (losses) of the policy change are measured by $\Delta U_0 \equiv (U_0^{**} - U^*_{ini})/|U^*_{ini}|$. Table 3 shows the results of our welfare analysis under the same scenario as in Section 4.4.

[Table 3]

From (25) and $K_0 = 1$, the welfare level, U_0^{**}, depends on C_0 and the transitional paths of γ_t^C and n_t. As shown in Figure 1, the welfare effects of the policy change are as follows. c_0 positively affects U_0^{**}. In the short (long) run, γ_t^C negatively (positively) impacts on U_0^{**}. On the contrary, in the short (long) run, n_t positively (negatively) impacts on U_0^{**}. As shown in Table 3, the positive welfare effects are sufficiently large; that is, a reduction in $\bar{z}$ to 60% improves welfare. Table 3 shows that the welfare gains increase as ϕ rises. Because ϕ does not affect the steady-state values, the short-run positive welfare effects are crucial to the welfare level.

5 Deficit policy rule

Thus far, we have examined the debt policy rule. In this section, we assume that the government gradually adjusts the government budget deficit-to-output ratio to the target and adopts the following rule:

$$\dot{d}_t = -\phi(d_t - \bar{d}), \tag{26}$$

where $d_t \equiv D_t/Y_t$ and $\bar{d}$ is the target of d_t. In this study, we refer to the rule (26) as the *deficit policy rule*.[13]

[13] As mentioned in the Introduction, the Japanese government aimed to achieve a primary surplus by 2020. Based on this policy objective, it might be better to choose the primary deficit (surplus) as a target variable. However, if the government chooses this, the economy cannot keep its fiscal sustainability (see Greiner, 2007, 2012; Kamiguchi and Tamai, 2012).

5.1 Dynamic system

Even under the deficit policy rule, (13), (14), and (15) hold. Then, we consider the government sector. (1a), (12), and $\dot{B}_t = D_t$ yield

$$\dot{b}_t = d_t A k_{g,t}^{\beta} n_t^{1-\alpha} - \left(A k_{g,t}^{\beta} n_t^{1-\alpha} - c_t - g_t - \delta_k \right) b_t, \tag{27}$$

where $b_t \equiv B_t/K_t$. Substituting (1b), (1c), (3), and (8) into (9) and solving for g_t leads to

$$\begin{aligned} g_t = d_t A k_{g,t}^{\beta} n_t^{1-\alpha} - \left(\alpha A k_{g,t}^{\beta} n_t^{1-\alpha} - \delta_k \right) b_t \\ + \tau_k \left(\alpha A k_{g,t}^{\beta} n_t^{1-\alpha} - \delta_k \right)(1 + b_t) + (\tau_n - \xi)(1-\alpha) A k_{g,t}^{\beta} n_t^{1-\alpha} + \tau_c c_t. \end{aligned} \tag{28}$$

g_t is determined by the following components. The first term represents government borrowing. The second term represents the interest payment on government debt. The rest represent tax revenue minus transfer payment. In contrast to the debt policy rule, g_t is not directly affected by the value of ϕ. As shown in the analysis below, the short-run effects of the deficit policy rule are smaller than those of the debt policy rule when ϕ is large.

In the deficit policy rule, the government adjusts the level of d_t according to the rule (26). However, at time 0, the government can freely choose a combination of g_0 and d_0 to satisfy the budget constraint (28). This leads to equilibrium indeterminacy. Therefore, to avoid this problem, we impose the following assumption.

Assumption 1. *Under the deficit policy rule* (26)*, d_t is a predetermined variable.*

As a result, (13), (14), (15), (26), (27), and (28) characterize the dynamic system with respect to d_t, $k_{g,t}$, b_t, and n_t in the deficit policy rule.

5.2 Steady state

In the steady state, d_t, $k_{g,t}$, b_t, and n_t become constant over time; that is, $d_t = \bar{d}$, $k_{g,t} = k_g^*$, $b_t = b^*$, and $n_t = n^*$ hold. In addition, the steady-state growth rates must satisfy $\gamma^* \equiv \left(\frac{\dot{K}_t}{K_t} \right)^* =$

$\left(\frac{\dot{C}_t}{C_t}\right)^* = \left(\frac{\dot{B}_t}{B_t}\right)^* = \left(\frac{\dot{K}_{g,t}}{K_{g,t}}\right)^*$. By using (1b), (5b), (7), (12), and $\dot{B}_t = D_t$, we obtain

$$\gamma^* = \theta\frac{g^*}{k_g^*} - \delta_g, \tag{29a}$$

$$= A(k_g^*)^\beta(n^*)^{1-\alpha} - c^* - g^* - \delta_k, \tag{29b}$$

$$= \bar{d}\frac{A(k_g^*)^\beta(n^*)^{1-\alpha}}{b^*}, \tag{29c}$$

$$= (1-\tau_k)\left[\alpha A(k_g^*)^\beta(n^*)^{1-\alpha} - \delta_k\right] - \rho. \tag{29d}$$

(29c) and (29d) lead to

$$b^* = \frac{\bar{d}A(k_g^*)^\beta(n^*)^{1-\alpha}}{(1-\tau_k)\left[\alpha A(k_g^*)^\beta(n^*)^{1-\alpha} - \delta_k\right] - \rho}. \tag{30}$$

From (14), (28), and (30), we obtain

$$g^* = [\bar{d} + (1-\alpha)(\tau_n - \xi)]A(k_g^*)^\beta(n^*)^{1-\alpha} + \frac{\tau_c}{1+\tau_c}\frac{(1-\tau_n)(1-\alpha)A(k_g^*)^\beta}{\psi(n^*)^{\chi+\alpha}}$$
$$+ \left[\alpha A(k_g^*)^\beta(n^*)^{1-\alpha} - \delta_k\right]\left\{\tau_k - \frac{(1-\tau_k)\bar{d}A(k_g^*)^\beta(n^*)^{1-\alpha}}{(1-\tau_k)\left[\alpha A(k_g^*)^\beta(n^*)^{1-\alpha} - \delta_k\right] - \rho}\right\}. \tag{31}$$

(14), (29a), and (29b) yield

$$\left(1 + \frac{\theta}{k_g^*}\right)g^* - \delta_g = A(k_g^*)^\beta(n^*)^{1-\alpha} - \frac{(1-\tau_n)(1-\alpha)A(k_g^*)^\beta}{(1+\tau_c)\psi(n^*)^{\chi+\alpha}} - \delta_k. \tag{32}$$

By using (14), (29b), and (29d), we obtain

$$A(k_g^*)^\beta(n^*)^{1-\alpha} - \frac{(1-\tau_n)(1-\alpha)A(k_g^*)^\beta}{(1+\tau_c)\psi(n^*)^{\chi+\alpha}} - g^* - \delta_k = (1-\tau_k)\left[\alpha A(k_g^*)^\beta(n^*)^{1-\alpha} - \delta_k\right] - \rho. \tag{33}$$

By substituting (31) into (32) and (33) and solving these equations, we can obtain the steady-state values k_g^* and n^*. The remaining steady-state values (c^*, γ^*, b^*, and g^*) are determined by (14), (29a), (30), and (31). Similarly to the debt policy rule, the value of ϕ does not affect the steady-state values and the steady-state values depend on the value of $\bar{d}$.

5.3 Parameter values

We employ the same parameter values as in Table 2: $(\alpha, A, \epsilon, \chi, \rho, \psi, \theta, \xi, \tau_k, \tau_b, \tau_n, \tau_c, \delta_k, \delta_g) =$ (0.3783, 1.5376, 0.2574, 2, 0.02824, 100.01, 0.2320, 0.1469, 0.5189, 0.2, 0.3324, 0.08, 0.0721, 0.0448). Given these parameter values, we set $\bar{d}$ to 0.04521 initially. Under this value, the initial steady state of the deficit policy rule coincides with that of the debt policy rule described in Section 4.3. In the following analyses, we use this steady state as a starting point.

5.4 Transitional dynamics

To investigate the transitional dynamics, we conduct a similar analysis to in Section 4.4. We assume that the economy is initially in the steady state. That is, the government sets $\bar{d}$ to 0.04521 initially. At time 0, the government reduces $\bar{d}$ unexpectedly and gradually adjusts d_t to the new target of $\bar{d}$ following the rule (26). The new target of $\bar{d}$ is set to 0.01467. Under this new target, the new steady state of the deficit policy rule coincides with that of the debt policy rule described in Sections 4.4 and 4.5. In addition, we vary the values of $\phi \in \{0.01, 0.05, 0.1\}$. Figure 2 presents the results.

[Figure 2]

From Assumption 1 and (28), government expenditure at time 0 depends only on the initial jump in n_t. Hence, the initial jump in $I_{g,t}/Y_t$ is sufficiently small. Then, to reduce d_t according to the rule (26), the government must decrease its expenditure. Thus, $I_{g,t}/Y_t$ first falls. Because a decrease in B_t/Y_t reduces the interest payment of the government, the government can raise its expenditure; that is, $I_{g,t}/Y_t$ increases to the new steady-state level, which is higher than the initial steady-state level.

Because households anticipate that the deficit reduction raises the long-run growth rate, they break into their savings and increase their consumption at time 0, C_0. From (14), n_t initially falls. As a result of the resource reallocation, the growth rate of K_t initially declines. After the policy change, the growth rate of K_t converges to the new steady-state level, which is higher than the initial steady-state level.

As in (23), the growth rate of C_t is determined by the values of $k_{g,t}$ and n_t. $k_{g,t}$ first falls and then rises, converging to the new steady-state level, which is higher than the initial steady-state level. Furthermore, n_t initially falls and thereafter increases to the new steady-state level, which is higher than the initial steady-state level. From these results, the growth rate of C_t

initially falls and thereafter increases to the new steady-state level, which is higher than the initial steady-state level.

In the short (long) run, the growth rate of K_t is lower (higher) than that of C_t. Therefore, C_t/K_t first rises and then falls, converging to the new steady-state level, which is lower than the initial steady-state level. From (14), because the effect of C_t/K_t is sufficiently large, the transitional path of n_t begins to run in the opposite direction; that is, n_t first falls and thereafter increases to the new steady-state level, which is higher than the initial steady-state level.

Similarly to the debt policy rule in Section 4.4, the long-run effects do not depend on ϕ, while the short-run effects become large as ϕ increases. However, compared with the debt policy rule, the deficit policy rule has the following two properties. First, the initial changes are modest because g_t is not directly affected by the value of ϕ (see (28)). Second, it takes a long time to converge to the new steady state. As in the analyses below, these properties mitigate the welfare gains under the deficit policy rule.

5.5 Welfare effects based only on expenditure cuts

In this subsection, we investigate the welfare effects under the deficit policy rule based only on government consumption and investment expenditure cuts. By using the same method as in Section 4.5, we calculate the welfare gains (losses) of the policy change, $\Delta U_0 = (U_0^{**} - U_{ini}^*)/|U_{ini}^*|$. Table 4 shows the results of our welfare analysis under the same scenario as in Section 5.4.

[Table 4]

From (25), the welfare level, U_0^{**}, depends on c_0 and the transitional paths of γ_t^C and n_t. As shown in Figure 2, the welfare effects of the policy change are as follows. c_0 positively affects U_0^{**}. In the short (long) run, γ_t^C negatively (positively) impacts on U_0^{**}. On the contrary, in the short (long) run, n_t positively (negatively) impacts on U_0^{**}. As shown in Table 4, the positive welfare effects exceed the negative welfare effects; that is, reductions in $\bar{d}$ improve welfare. Table 4 further shows that the welfare gains increase as ϕ rises.

The welfare effects under the deficit policy rule are similar to those under the debt policy rule. However, the welfare gains under the deficit policy rule are sufficiently small compared with those under the debt policy rule. From the discussion in the last paragraph of Section 5.4, because the initial changes are modest, the positive welfare effects of c_0 and n_t are sufficiently small. Moreover, it takes a long time to attain higher γ_t^C; that is, the long-run positive welfare effect of γ_t^C is sufficiently small.

6 Welfare effects with tax increases

In the analyses of Sections 4 and 5, the government must decrease its expenditure to pursue fiscal consolidation. As mentioned in the Introduction, we then investigate the welfare effects with tax increases. Under the debt (deficit) policy rule, we consider the following scenario. The economy is initially in the steady state described in Section 4.3 (5.3). At time 0, the government reduces $\bar{z}$ ($\bar{d}$) and raises only one tax rate unexpectedly. After this policy change, the tax rate remains unchanged. Note that if tax increases are not sufficient to pursue fiscal consolidation, the government reduces its expenditure. The new target of $\bar{z}$ ($\bar{d}$) is set to 0.6 (0.01467) and the value of ϕ is set to 0.05.

6.1 Debt policy rule

Figure 3 shows the results of the transitional paths under the debt policy rule with tax increases. The left panels of Figure 3 correspond to the case in which the government increases only the capital income tax rate. Here, $\Delta\tau$ denotes increases in the rates of each tax. Moreover, the center (right) panels of Figure 3 correspond to the case in which the government increases only the tax rate on labor income (consumption).

[Figure 3]

Compared with the case of $\Delta\tau = 0$, for all taxes, increases in the tax rate mitigate the initial decline in $I_{g,t}/Y_t$ and raise $I_{g,t}/Y_t$ during the transitional paths. When τ_k rises, the growth rates of K_t and C_t decrease compared with the case of $\Delta\tau = 0$. An increase in τ_k diminishes households' savings. On the contrary, the effects of increases in τ_n or τ_c on the growth rates of K_t and C_t are sufficiently small. Regarding the initial jump in private consumption, the effect of increases in τ_k on C_0 is modest. However, when τ_n or τ_c rises, the increases in C_0 become small. For all taxes, the effects of tax increases on n_t are sufficiently small.

Table 5 represents the welfare effects under the debt policy rule with tax increases. In all cases, the welfare gains of reductions in $\bar{z}$ based only on government consumption and investment expenditure cuts are larger than those with tax increases. Furthermore, Table 5 shows that sufficiently low ϕ and sufficiently large τ_k increases lead to welfare losses. From

Figure 3, when τ_k increases, the growth rate of C_t falls. This has a sufficiently large negative welfare effect. when τ_n or τ_c increases, the rises in C_0 become small, which has a negative welfare effect.

[Table 5]

6.2 Deficit policy rule

In this subsection, we examine the welfare effects under the deficit policy rule with tax increases. Figure 4 presents the results of the transitional paths. Compared with the case of $\Delta\tau = 0$, increases in τ_n or τ_c raise $I_{g,t}/Y_t$ in the short and long run, whereas increases in τ_k raise the short-run values of $I_{g,t}/Y_t$ but do not seriously affect the long-run values of $I_{g,t}/Y_t$. When τ_k rises, the growth rates of K_t and C_t decrease compared with the case of $\Delta\tau = 0$. On the contrary, the effects of increases in τ_n or τ_c on the growth rates of K_t and C_t are sufficiently small. Regarding the initial jump in private consumption, the effect of an increase in τ_k on C_0 is sufficiently small. However, when τ_n or τ_c increases, C_0 decreases. For all taxes, the effects of tax increases on n_t are sufficiently small.

[Figure 4]

Table 6 represents the welfare effects under the deficit policy rule with tax increases. It shows that the welfare gains of reductions in $\bar{d}$ based only on government consumption and investment expenditure cuts are larger than those with τ_k or τ_n increases. Furthermore, Table 6 shows that when τ_k increases, welfare declines and that sufficiently low ϕ and sufficiently large τ_n increases lead to welfare losses. From Figure 4, when τ_k increases, the growth rate of C_t falls compared with the case of $\Delta\tau = 0$. This has a sufficiently large negative welfare effect. When τ_n increases, C_0 decreases, which has a sufficiently large negative welfare effect. By contrast, Table 6 shows that sufficiently small (large) τ_c increases generate larger (smaller) welfare gains. From Figure 4, when τ_c increases, the growth rate of C_t rises compared with the case of $\Delta\tau = 0$ (this is a positive welfare effect) and C_0 decreases (this is a negative welfare effect). Therefore, when τ_c increases are sufficiently small (large), the positive welfare effect exceeds (falls short of) the negative welfare effect. Although sufficiently small τ_c increases can generate larger welfare gains, these welfare gains are sufficiently small compared with the welfare gains under the debt policy rule.

[Table 6]

7 Welfare effects with transfer payment reductions

Similarly to the analyses of Section 6, we now consider the welfare effects with transfer payment reductions. Under the debt (deficit) policy rule, we consider the following scenario. The economy is initially in the steady state described in Section 4.3 (5.3). At time 0, the government reduces $\bar{z}$ ($\bar{d}$) and ξ unexpectedly. After this policy change, the value of ξ remains unchanged. Note that if transfer payment reductions are not sufficient to pursue fiscal consolidation, the government reduces its expenditure. The new target of $\bar{z}$ ($\bar{d}$) is set to 0.6 (0.01467) and the value of ϕ is set to 0.05.

7.1 Debt policy rule

Figure 5 shows the results of the transitional paths under the debt policy rule with transfer payment decreases. Here, $\Delta\xi$ denotes changes in ξ. Although reductions in transfer payment create fiscal space for the government, they tighten the budget constraint of households. Therefore, compared with the case of $\Delta\xi = 0$, a decrease in ξ raises $I_{g,t}/Y_t$ and reduces C_t/K_t in the short and long run. The transitional path of n_t begins to run in the opposite direction of C_t/K_t; that is, n_t increases during transitional paths. Further, a higher $I_{g,t}/Y_t$ implies higher growth rates of K_t and C_t.

[Figure 5]

Table 7 represents the welfare effects under the debt policy rule with transfer payment decreases. It shows that the welfare gains of reductions in $\bar{z}$ with transfer payment decreases are larger than those based only on government consumption and investment expenditure cuts. From Figure 5, compared with the case of $\Delta\xi = 0$, reductions in ξ have negative welfare effects by reducing C_0 and increasing n_t and have positive welfare effects by increasing the growth rate of C_t. From Table 7, the positive welfare effects exceed the negative welfare effects.

[Table 7]

7.2 Deficit policy rule

In this subsection, we examine the welfare effects under the deficit policy rule with transfer payment decreases. Figure 6 presents the results of the transitional paths. The effects of reductions in transfer payment are similar to those under the debt policy rule. Compared with the case of $\Delta\xi = 0$, reductions in transfer payment raise $I_{g,t}/Y_t$, n_t, and the growth rates of K_t and C_t and reduce C_t/K_t during the transitional paths.

[Figure 6]

Table 8 represents the welfare effects under the deficit policy rule with transfer payment decreases. Similarly to the debt policy rule, the welfare gains of reductions in $\bar{d}$ with transfer payment decreases are larger than those based only on government consumption and investment expenditure cuts. From Figure 6, compared with the case of $\Delta\xi = 0$, reductions in ξ have negative welfare effects by reducing C_0 and increasing n_t and have positive welfare effects by increasing the growth rate of C_t. From Table 8, the positive welfare effects exceed the negative welfare effects.

[Table 8]

8 Discussion

8.1 Changes in new targets

The results presented thus far show that the welfare gains under the debt policy rule are larger than those under the deficit policy rule. This result may depend on the new target of $\bar{z}$ or $\bar{d}$. Therefore, in this subsection, we investigate the welfare effects of changes in $\bar{z}$ and $\bar{d}$.

First, we consider the debt policy rule. The parameter values are the same as in Section 4.3 and the scenario is the same as in Section 4.4. We set ϕ to 0.05 and vary the new targets of $\bar{z}$: 0.2, 0.6, 1, and 1.4. Figure 7 presents the transitional paths and Table 9 shows the results of our welfare analysis. As shown in Figure 7, a lower value of $\bar{z}$ implies the following results: (i) the increase in C_0 is larger, (ii) the short-run growth rate of C_t is lower and the long-run growth rate of C_t is higher, and (iii) the short-run value of n_t is lower and the long-run value of n_t is higher. From Table 9, for lower values of $\bar{z}$, the welfare gains are larger; that is, the positive welfare effects are larger.

[Figure 7 and Table 9]

Next, we consider the deficit policy rule. The parameter values are the same as in Section 5.3 and the scenario is the same as in Section 5.4. We set ϕ to 0.05 and vary the new targets of $\bar{d}$: 0.01, 0.02, 0.03, and 0.04. Figure 8 presents the transitional paths and Table 10 shows the results of our welfare analysis. From Figure 8, a lower value of $\bar{d}$ implies the following results: (i) the increase in C_0 is larger, (ii) the short-run growth rate of C_t is lower and the long-run growth rate of C_t is higher, and (iii) the short-run value of n_t is lower and the long-run value of n_t is higher. From Table 10, for lower values of $\bar{d}$, the welfare gains are larger; that is, the positive welfare effects are larger. However, as shown in Tables 9 and 10, the welfare gains under the deficit policy rule are sufficiently small compared with those under the debt policy rule even if we change the new targets of $\bar{z}$ and $\bar{d}$.

[Figure 8 and Table 10]

8.2 Government consumption into utility

In this subsection, we incorporate government consumption into utility. Following Maebayashi et al. (2017), we modify the utility function (2) as follows:

$$U_0 = \int_0^\infty e^{-\rho t} \left(\log C_t - \psi \frac{n_t^{1+\chi}}{1+\chi} + \eta \log C_{g,t} \right) dt, \tag{34}$$

where $C_{g,t}$ is the government consumption expenditure; that is, $C_{g,t} = (1-\theta)G_t$ holds. η represents households' preference for government consumption. As mentioned by Maebayashi et al. (2017), to avoid equilibrium indeterminacy, we employ a separable utility function for C_t and $C_{g,t}$. Under this modification, the results of the transitional dynamics remain unchanged. Hence, when we consider the debt (deficit) policy rule, we employ the same parameter values as in Section 4.3 (5.3). The welfare gains or losses of the policy change are measured by $\Delta U_0 \equiv (U_0^{***} - U_{ini}^*)/|U_{ini}^*|$, where U_0^{***} is defined as the welfare level after the policy change.[14]

By using the same scenario as in Section 4.5, Table 11 presents the results of our welfare analysis under the debt policy rule based only on government consumption and investment expenditure cuts. In Table 11, we set the new target of $\bar{z}$ to 0.6, vary the values of $\phi \in \{0.01, 0.05, 0.1\}$, and vary the values of $\eta \in \{0, 0.01, 0.05, 0.1\}$. As shown in Table 11, the main results presented in Section 4.5 remain unchanged; that is, reductions in $\bar{z}$ improve welfare and the welfare gains increase as ϕ rises.

[Table 11]

[14] In Appendix A, we present the calculation procedure for U_0^{***} under the relaxation algorithm developed by Trimborn et al. (2008).

By using the same scenario as in Section 5.5, Table 12 presents the results of our welfare analysis under the deficit policy rule based only on government consumption and investment expenditure cuts. In Table 12, we set the new target of $\bar{d}$ to 0.01467, vary the values of $\phi \in \{0.01, 0.05, 0.1\}$, and vary the values of $\eta \in \{0, 0.01, 0.05, 0.1\}$. Table 12 shows that the main results in Section 5.5 remain unchanged. That is, reductions in $\bar{d}$ improve welfare and the welfare gains increase as ϕ rises. From Tables 11 and 12, even for the utility function (34), the welfare gains under the debt policy rule are sufficiently large compared with those under the deficit policy rule.

[Table 12]

Tables 11 and 12 show that an increase in η reduces the welfare gains under the debt and deficit policy rules. From $I_{g,t} = \theta G_t$, $C_{g,t} = (1-\theta)G_t$, and the results of Figures 1 and 2, when fiscal consolidation is implemented, $C_{g,t}$ declines in the short run, which diminishes welfare. A higher η implies that this negative welfare effect becomes larger. As shown in Figures 3–6, tax increases or reductions in transfer payment mitigate the short-run decline in $C_{g,t}$ under the debt policy rule and raise $C_{g,t}$ under the deficit policy rule. Therefore, fiscal consolidation with tax increases or transfer payment reductions might generate larger welfare gains as η increases. To discuss this, we examine the welfare effects under the debt and deficit policy rules with either tax increases or transfer payment decreases, or both. Under the debt (deficit) policy rule, we consider the following scenario. The economy is initially in the steady state described in Section 4.3 (5.3). At time 0, the government reduces $\bar{z}$ ($\bar{d}$) and ξ and raises only one tax rate unexpectedly. After this policy change, the tax rate and value of ξ remain unchanged. The new target of $\bar{z}$ ($\bar{d}$) is set to 0.6 (0.01467) and the value of ϕ is set to 0.05. Table 13 (14) presents the case of $\eta = 0$ under the debt (deficit) policy rule. When $\eta = 0$, the welfare gains under the debt policy rule with transfer payment reductions are the largest.

[Tables 13 and 14]

We then consider a sufficiently low value of η. Table 15 (16) presents the case of $\eta = 0.01$ under the debt (deficit) policy rule. Similarly to the case of $\eta = 0$, the welfare gains under the debt policy rule with transfer payment reductions are the largest.

[Tables 15 and 16]

At the end of this subsection, we consider a sufficiently high value of η. Table 17 (18) presents the case of $\eta = 0.1$ under the debt (deficit) policy rule. In this case, not only fiscal consolidation with reductions in transfer payment but also fiscal consolidation with increases in labor income or consumption taxes generates larger welfare gains. By comparison, fiscal consolidation with increases in consumption tax generates larger welfare gains than that with increases in labor income tax. In summary, the welfare gains under the debt policy rule with both consumption tax increases and transfer payment reductions are the largest.

[Tables 17 and 18]

9 Conclusion

In this study, we constructed an endogenous growth model with public capital and endogenous labor supply and investigated quantitatively the welfare effects of fiscal consolidation by using Japanese data. We found that fiscal consolidation under the debt and deficit policy rules based only on government consumption and investment expenditure cuts improves welfare. A higher speed of fiscal consolidation implies a larger welfare improvement. Lowering the target debt-to-GDP or deficit-to-GDP ratio generates larger welfare gains. Further, the welfare gains under the debt policy rule are larger than those under the deficit policy rule. When we incorporate government consumption utility, the results of our welfare analyses can be summarized as follows. If households' preference for government consumption is sufficiently low, fiscal consolidation under the debt policy rule with transfer payment reductions is appropriate for improving welfare. On the contrary, if households' preference for government consumption is sufficiently high, fiscal consolidation under the debt policy rule with both consumption tax increases and transfer payment reductions is appropriate for improving welfare.

There are several interesting directions for future research. First, as mentioned in Section 4.3, our calibration result shows that the steady-state values of the total tax revenue-to-GDP ratio and government interest payments-to-GDP ratio do not match the Japanese data. Because these factors may be important in our welfare analysis, future research should address this point. Second, Japan is facing a declining birthrate and aging population. Therefore, incorporating intergenerational conflicts could provide interesting insights into fiscal consolidation. Third, we assume that the ratio of government investment to the sum of government consumption and government investment is constant. Owing to population aging, it may be difficult for the Japanese government to cut its consumption expenditure. Hence, investigating the welfare effects of fiscal consolidation with changes in the ratio of government investment to the sum of government consumption and government investment would be an important direction for future research.

Appendix

A Calculation of U_0^{**} and U_0^{***}

Following Maebayashi et al. (2017), we calculate the value of U_0^{**}. Let us define $U_t \equiv \int_t^\infty e^{-\rho(s-t)} \left(\log C_s - \psi \frac{n_s^{1+\chi}}{1+\chi} \right) ds$ and $X_t \equiv U_t - \frac{1}{\rho} \log K_t$. By differentiating X_t with respect to t and using (12) and $c_t = C_t/K_t$, we obtain

$$\dot{X}_t = \rho X_t - \log c_t + \psi \frac{n_t^{1+\chi}}{1+\chi} - \frac{1}{\rho} \left(A k_{g,t}^{\beta} n_t^{1-\alpha} - c_t - g_t - \delta_k \right).$$

In the steady state, X_t becomes

$$X^* = \frac{1}{\rho} \left[\log c^* - \psi \frac{(n^*)^{1+\chi}}{1+\chi} \right] + \frac{1}{\rho^2} \left[A (k_g^*)^{\beta} (n^*)^{1-\alpha} - c^* - g^* - \delta_k \right].$$

Thus, by using the relaxation algorithm, we can calculate the dynamic path of X_t. As in Section 4.5, we set $K_0 = 1$, and hence, $X_0 = U_0$ holds. From these results, we obtain the value of U_0^{**}.

Next, we explain the calculation method under the utility function (34). Let us define $X_{g,t}$ as $X_{g,t} \equiv \int_t^\infty e^{-\rho(s-t)} \eta \log C_{g,s} ds - \frac{\eta}{\rho} \log K_t$. By using $C_{g,t} = (1-\theta) g_t K_t$, we obtain

$$\dot{X}_{g,t} = \rho X_{g,t} - \eta \log g_t - \frac{\eta}{\rho} \left(A k_{g,t}^{\beta} n_t^{1-\alpha} - c_t - g_t - \delta_k \right) - \eta \log(1-\theta).$$

In the steady state, $X_{g,t}$ becomes

$$X_g^* = \frac{\eta}{\rho} \log g^* + \frac{\eta}{\rho^2} \left[A (k_g^*)^{\beta} (n^*)^{1-\alpha} - c^* - g^* - \delta_k \right] + \frac{\eta}{\rho} \log(1-\theta).$$

Thus, by using the relaxation algorithm, we can calculate the dynamic path of $X_{g,t}$. Because we set $K_0 = 1$, we obtain the value of $U_0^{***} = X_0 + X_{g,0}$.

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Parameter	Value	Source
α	0.3783	Hansen and İmrohoroğlu (2016)
A	1.5376	Calibrated
ϵ	0.2574	Calibrated
χ	2	Chetty et al. (2012)
ρ	0.02824	Calibrated
ψ	100.01	Calibrated
θ	0.2320	Data average
ξ	0.1469	Calibrated
τ_k	0.5189	Gunji and Miyazaki (2011)
τ_b	0.2	Hansen and İmrohoroğlu (2016)
τ_n	0.3324	Gunji and Miyazaki (2011)
τ_c	0.08	Data
δ_k	0.0721	Set
δ_g	0.0448	Kato (2002)

Table 1: Parameter values.

Description	Data	Solution
Private consumption-to-GDP	0.5461	0.5832
Government budget deficit-to-GDP	0.04208	0.04521
Total tax revenue-to-GDP	0.2699	0.3949
Government interest payments-to-GDP	0.0287	0.1339

Table 2: Data and solutions.

Source: OECD Economic Outlook.
Note: The data averages of the private consumption-to-GDP ratio, government budget deficit-to-GDP ratio, total tax revenue-to-GDP ratio, and government interest payments-to-GDP ratio are for 1980–2015.

	$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.1$
ΔU_0	1.811%	7.244%	11.941%

Table 3: Welfare gains (losses) based only on expenditure cuts: The debt policy rule.

	$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.1$
ΔU_0	0.092%	0.447%	0.814%

Table 4: Welfare gains (losses) based only on expenditure cuts: The deficit policy rule.

		$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.1$
ΔU_0	no tax change	1.811%	7.244%	11.941%
	$\Delta\tau_k = 0.01$	-1.482%	3.935%	8.625%
	$\Delta\tau_k = 0.02$	-4.813%	0.589%	5.272%
	$\Delta\tau_k = 0.03$	-8.182%	-2.795%	1.881%
	$\Delta\tau_n = 0.01$	1.426%	6.903%	11.632%
	$\Delta\tau_n = 0.02$	0.995%	6.518%	11.278%
	$\Delta\tau_n = 0.03$	0.518%	6.089%	10.882%
	$\Delta\tau_c = 0.01$	1.799%	7.226%	11.923%
	$\Delta\tau_c = 0.02$	1.758%	7.181%	11.878%
	$\Delta\tau_c = 0.03$	1.689%	7.108%	11.806%

Table 5: Welfare gains (losses) with tax increases: The debt policy rule.

		$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.1$
ΔU_0	no tax change	0.092%	0.447%	0.814%
	$\Delta\tau_k = 0.01$	-3.420%	-3.060%	-2.691%
	$\Delta\tau_k = 0.02$	-6.972%	-6.607%	-6.236%
	$\Delta\tau_k = 0.03$	-10.564%	-10.195%	-9.821%
	$\Delta\tau_n = 0.01$	-0.290%	0.069%	0.439%
	$\Delta\tau_n = 0.02$	-0.718%	-0.356%	0.019%
	$\Delta\tau_n = 0.03$	-1.194%	-0.827%	-0.447%
	$\Delta\tau_c = 0.01$	0.118%	0.469%	0.834%
	$\Delta\tau_c = 0.02$	0.112%	0.460%	0.823%
	$\Delta\tau_c = 0.03$	0.078%	0.424%	0.784%

Table 6: Welfare gains (losses) with tax increases: The deficit policy rule.

		$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.1$
ΔU_0	$\Delta\xi = 0$	1.811%	7.244%	11.941%
	$\Delta\xi = -0.01$	2.564%	8.023%	12.740%
	$\Delta\xi = -0.02$	3.293%	8.780%	13.515%
	$\Delta\xi = -0.03$	3.998%	9.514%	14.269%

Table 7: Welfare gains (losses) with transfer payment decreases: The debt policy rule.

		$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.1$
ΔU_0	$\Delta\xi = 0$	0.092%	0.447%	0.814%
	$\Delta\xi = -0.01$	0.909%	1.265%	1.634%
	$\Delta\xi = -0.02$	1.703%	2.059%	2.431%
	$\Delta\xi = -0.03$	2.474%	2.832%	3.206%

Table 8: Welfare gains (losses) with transfer payment decreases: The deficit policy rule.

	$\bar{z} = 0.2$	$\bar{z} = 0.6$	$\bar{z} = 1$	$\bar{z} = 1.4$
ΔU_0	8.872%	7.244%	5.534%	3.746%

Table 9: Welfare gains (losses) based only on expenditure cuts under each value of $\bar{z}$: The debt policy rule.

	$\bar{d} = 0.01$	$\bar{d} = 0.02$	$\bar{d} = 0.03$	$\bar{d} = 0.04$
ΔU_0	0.502%	0.379%	0.238%	0.076%

Table 10: Welfare gains (losses) based only on expenditure cuts under each value of $\bar{d}$: The deficit policy rule.

		$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.1$
ΔU_0	$\eta = 0$	1.811%	7.244%	11.941%
	$\eta = 0.01$	1.796%	7.169%	11.802%
	$\eta = 0.05$	1.742%	6.900%	11.301%
	$\eta = 0.1$	1.685%	6.621%	10.781%

Table 11: Welfare gains (losses) based only on expenditure cuts under government consumption utility: The debt policy rule.

		$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.1$
ΔU_0	$\eta = 0$	0.0922%	0.447%	0.814%
	$\eta = 0.01$	0.0910%	0.441%	0.803%
	$\eta = 0.05$	0.0866%	0.418%	0.766%
	$\eta = 0.1$	0.0820%	0.394%	0.727%

Table 12: Welfare gains (losses) based only on expenditure cuts under government consumption utility: The deficit policy rule.

		$\Delta\xi = 0$	$\Delta\xi = -0.01$	$\Delta\xi = -0.02$	$\Delta\xi = -0.03$
ΔU_0	no tax change	7.244%	8.023%	8.780%	9.514%
	$\Delta\tau_k = 0.01$	3.935%	4.672%	5.386%	6.078%
	$\Delta\tau_k = 0.02$	0.589%	1.283%	1.955%	2.605%
	$\Delta\tau_k = 0.03$	-2.795%	-2.143%	-1.513%	-0.904%
	$\Delta\tau_n = 0.01$	6.903%	7.659%	8.392%	9.104%
	$\Delta\tau_n = 0.02$	6.518%	7.251%	7.962%	8.653%
	$\Delta\tau_n = 0.03$	6.089%	6.800%	7.490%	8.159%
	$\Delta\tau_c = 0.01$	7.226%	7.976%	8.704%	9.411%
	$\Delta\tau_c = 0.02$	7.181%	7.902%	8.603%	9.284%
	$\Delta\tau_c = 0.03$	7.108%	7.804%	8.479%	9.135%

Table 13: Welfare gains (losses) with tax increases and transfer payment decreases under government consumption utility: The debt policy rule. ($\eta = 0$)

		$\Delta\xi = 0$	$\Delta\xi = -0.01$	$\Delta\xi = -0.02$	$\Delta\xi = -0.03$
ΔU_0	no tax change	0.447%	1.265%	2.059%	2.832%
	$\Delta\tau_k = 0.01$	-3.060%	-2.288%	-1.538%	-0.809%
	$\Delta\tau_k = 0.02$	-6.607%	-5.880%	-5.174%	-4.489%
	$\Delta\tau_k = 0.03$	-10.195%	-9.511%	-8.850%	-8.209%
	$\Delta\tau_n = 0.01$	0.069%	0.862%	1.634%	2.385%
	$\Delta\tau_n = 0.02$	-0.356%	0.415%	1.164%	1.894%
	$\Delta\tau_n = 0.03$	-0.827%	-0.078%	0.650%	1.359%
	$\Delta\tau_c = 0.01$	0.469%	1.256%	2.021%	2.765%
	$\Delta\tau_c = 0.02$	0.460%	1.218%	1.956%	2.673%
	$\Delta\tau_c = 0.03$	0.424%	1.154%	1.865%	2.557%

Table 14: Welfare gains (losses) with tax increases and transfer payment decreases under government consumption utility: The deficit policy rule. ($\eta = 0$)

		$\Delta\xi = 0$	$\Delta\xi = -0.01$	$\Delta\xi = -0.02$	$\Delta\xi = -0.03$
ΔU_0	no tax change	7.169%	7.990%	8.788%	9.563%
	$\Delta\tau_k = 0.01$	3.916%	4.695%	5.450%	6.184%
	$\Delta\tau_k = 0.02$	0.626%	1.363%	2.076%	2.768%
	$\Delta\tau_k = 0.03$	-2.701%	-2.007%	-1.335%	-0.684%
	$\Delta\tau_n = 0.01$	6.877%	7.674%	8.448%	9.200%
	$\Delta\tau_n = 0.02$	6.540%	7.314%	8.065%	8.796%
	$\Delta\tau_n = 0.03$	6.159%	6.910%	7.640%	8.349%
	$\Delta\tau_c = 0.01$	7.195%	7.986%	8.754%	9.501%
	$\Delta\tau_c = 0.02$	7.191%	7.953%	8.693%	9.413%
	$\Delta\tau_c = 0.03$	7.159%	7.894%	8.609%	9.303%

Table 15: Welfare gains (losses) with tax increases and transfer payment decreases under government consumption utility: The debt policy rule. ($\eta = 0.01$)

		$\Delta\xi = 0$	$\Delta\xi = -0.01$	$\Delta\xi = -0.02$	$\Delta\xi = -0.03$
ΔU_0	no tax change	0.441%	1.299%	2.134%	2.946%
	$\Delta\tau_k = 0.01$	-3.009%	-2.196%	-1.405%	-0.637%
	$\Delta\tau_k = 0.02$	-6.498%	-5.729%	-4.983%	-4.258%
	$\Delta\tau_k = 0.03$	-10.027%	-9.303%	-8.600%	-7.919%
	$\Delta\tau_n = 0.01$	0.110%	0.944%	1.756%	2.546%
	$\Delta\tau_n = 0.02$	-0.266%	0.544%	1.333%	2.101%
	$\Delta\tau_n = 0.03$	-0.689%	0.099%	0.866%	1.614%
	$\Delta\tau_c = 0.01$	0.505%	1.332%	2.136%	2.920%
	$\Delta\tau_c = 0.02$	0.538%	1.335%	2.111%	2.867%
	$\Delta\tau_c = 0.03$	0.542%	1.310%	2.059%	2.789%

Table 16: Welfare gains (losses) with tax increases and transfer payment decreases under government consumption utility: The deficit policy rule. ($\eta = 0.01$)

		$\Delta\xi = 0$	$\Delta\xi = -0.01$	$\Delta\xi = -0.02$	$\Delta\xi = -0.03$
ΔU_0	no tax change	6.621%	7.749%	8.849%	9.922%
	$\Delta\tau_k = 0.01$	3.778%	4.865%	5.924%	6.957%
	$\Delta\tau_k = 0.02$	0.901%	1.946%	2.965%	3.957%
	$\Delta\tau_k = 0.03$	-2.011%	-1.007%	-0.029%	0.924%
	$\Delta\tau_n = 0.01$	6.684%	7.784%	8.857%	9.904%
	$\Delta\tau_n = 0.02$	6.700%	7.773%	8.820%	9.843%
	$\Delta\tau_n = 0.03$	6.670%	7.717%	8.740%	9.739%
	$\Delta\tau_c = 0.01$	6.962%	8.053%	9.118%	10.157%
	$\Delta\tau_c = 0.02$	7.266%	8.322%	9.352%	10.360%
	$\Delta\tau_c = 0.03$	7.534%	8.557%	9.556%	10.533%

Table 17: Welfare gains (losses) with tax increases and transfer payment decreases under government consumption utility: The debt policy rule. ($\eta = 0.1$)

		$\Delta\xi = 0$	$\Delta\xi = -0.01$	$\Delta\xi = -0.02$	$\Delta\xi = -0.03$
ΔU_0	no tax change	0.394%	1.551%	2.680%	3.783%
	$\Delta\tau_k = 0.01$	-2.635%	-1.522%	-0.435%	0.625%
	$\Delta\tau_k = 0.02$	-5.700%	-4.630%	-3.587%	-2.568%
	$\Delta\tau_k = 0.03$	-8.803%	-7.775%	-6.774%	-5.797%
	$\Delta\tau_n = 0.01$	0.416%	1.545%	2.647%	3.725%
	$\Delta\tau_n = 0.02$	0.390%	1.492%	2.569%	3.622%
	$\Delta\tau_n = 0.03$	0.317%	1.393%	2.445%	3.476%
	$\Delta\tau_c = 0.01$	0.769%	1.888%	2.981%	4.049%
	$\Delta\tau_c = 0.02$	1.105%	2.188%	3.247%	4.283%
	$\Delta\tau_c = 0.03$	1.404%	2.453%	3.480%	4.485%

Table 18: Welfare gains (losses) with tax increases and transfer payment decreases under government consumption utility: The deficit policy rule. ($\eta = 0.1$)

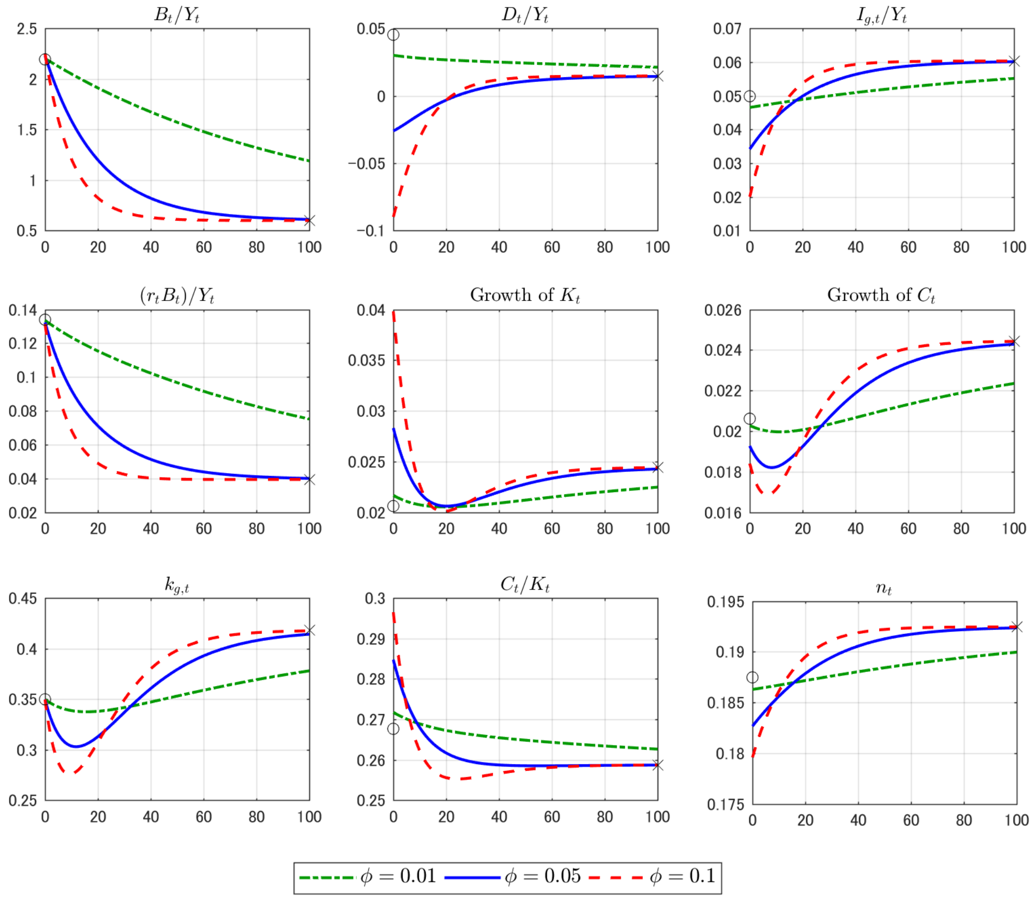

Figure 1: Transition dynamics based only on expenditure cuts: The debt policy rule.

Note: The circle at the left end represents the initial steady-state value and the cross at the right end represents the new steady-state value.

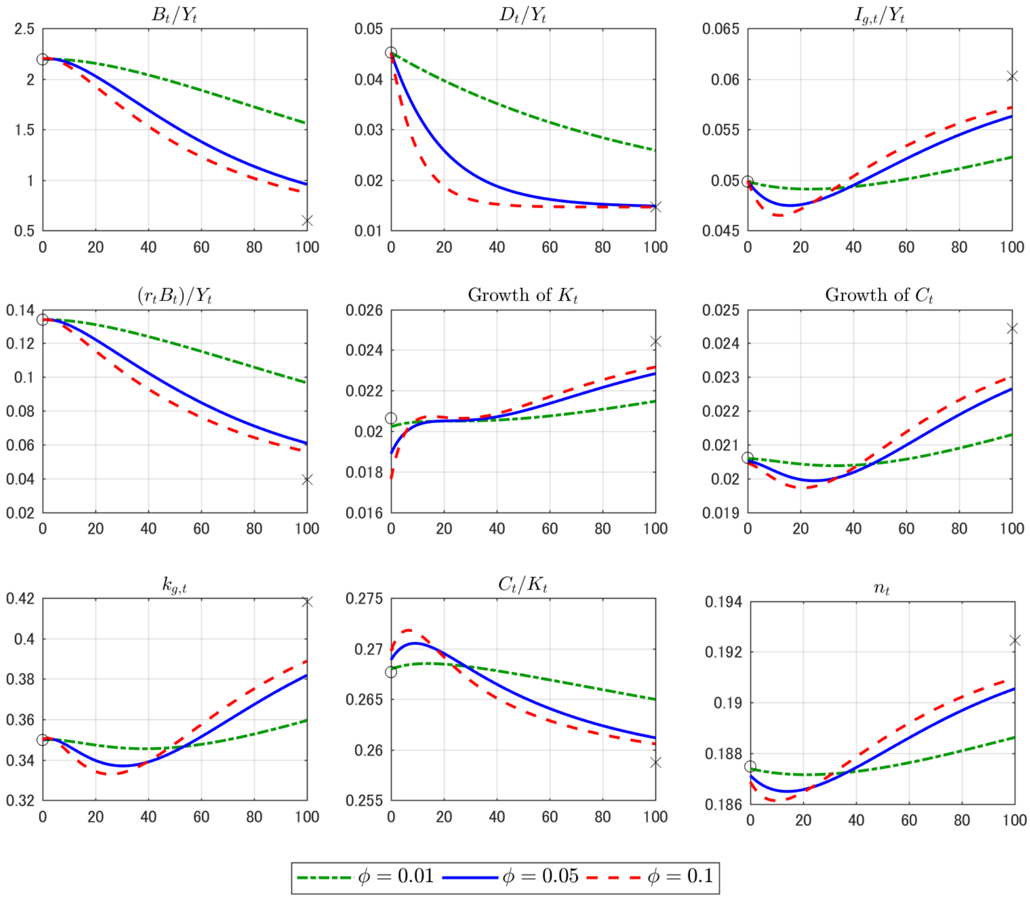

Figure 2: Transition dynamics based only on expenditure cuts: The deficit policy rule.

Note: The circle at the left end represents the initial steady-state value and the cross at the right end represents the new steady-state value.

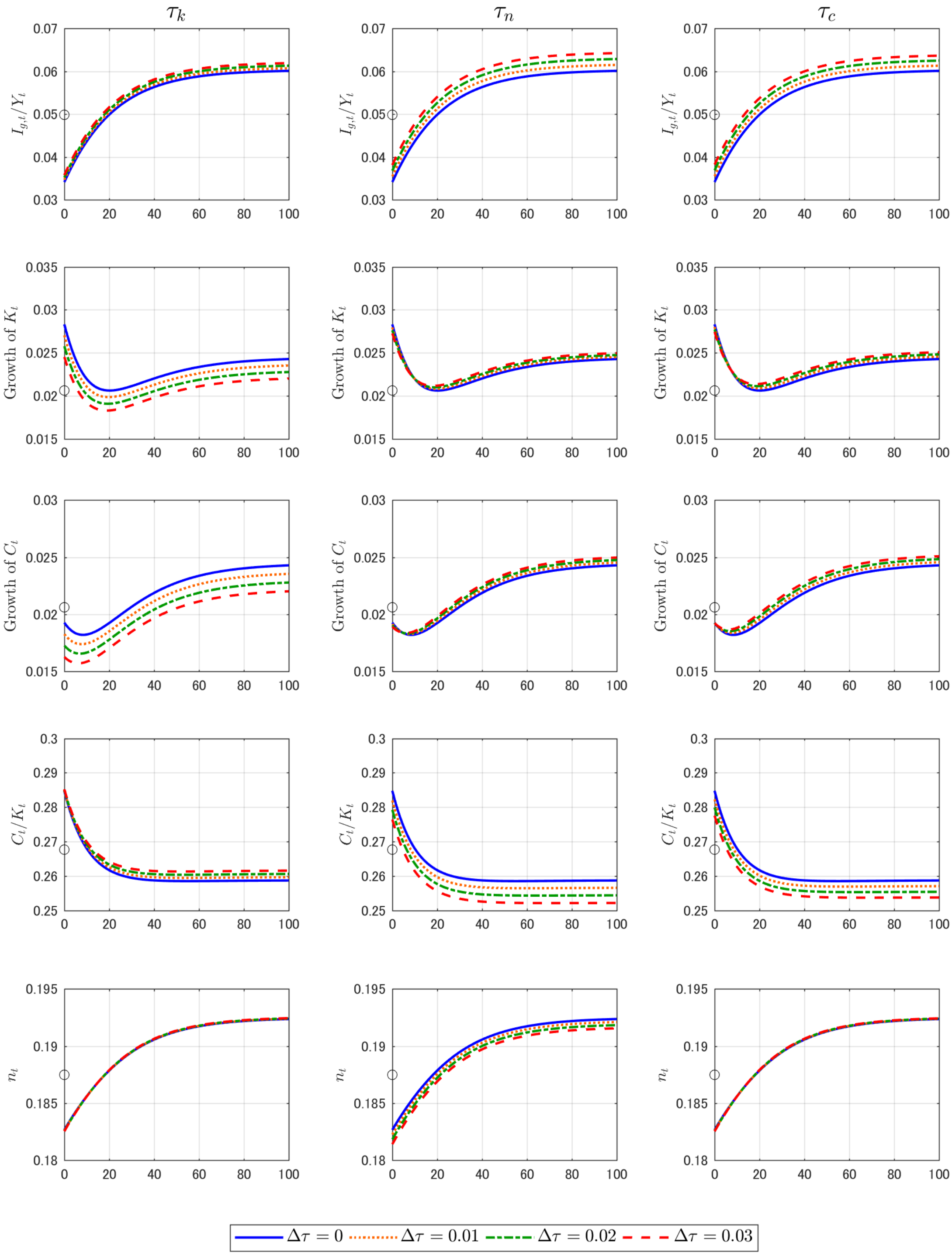

Figure 3: Transition dynamics with tax increases: The debt policy rule.

Note: The circle at the left end represents the initial steady-state value.

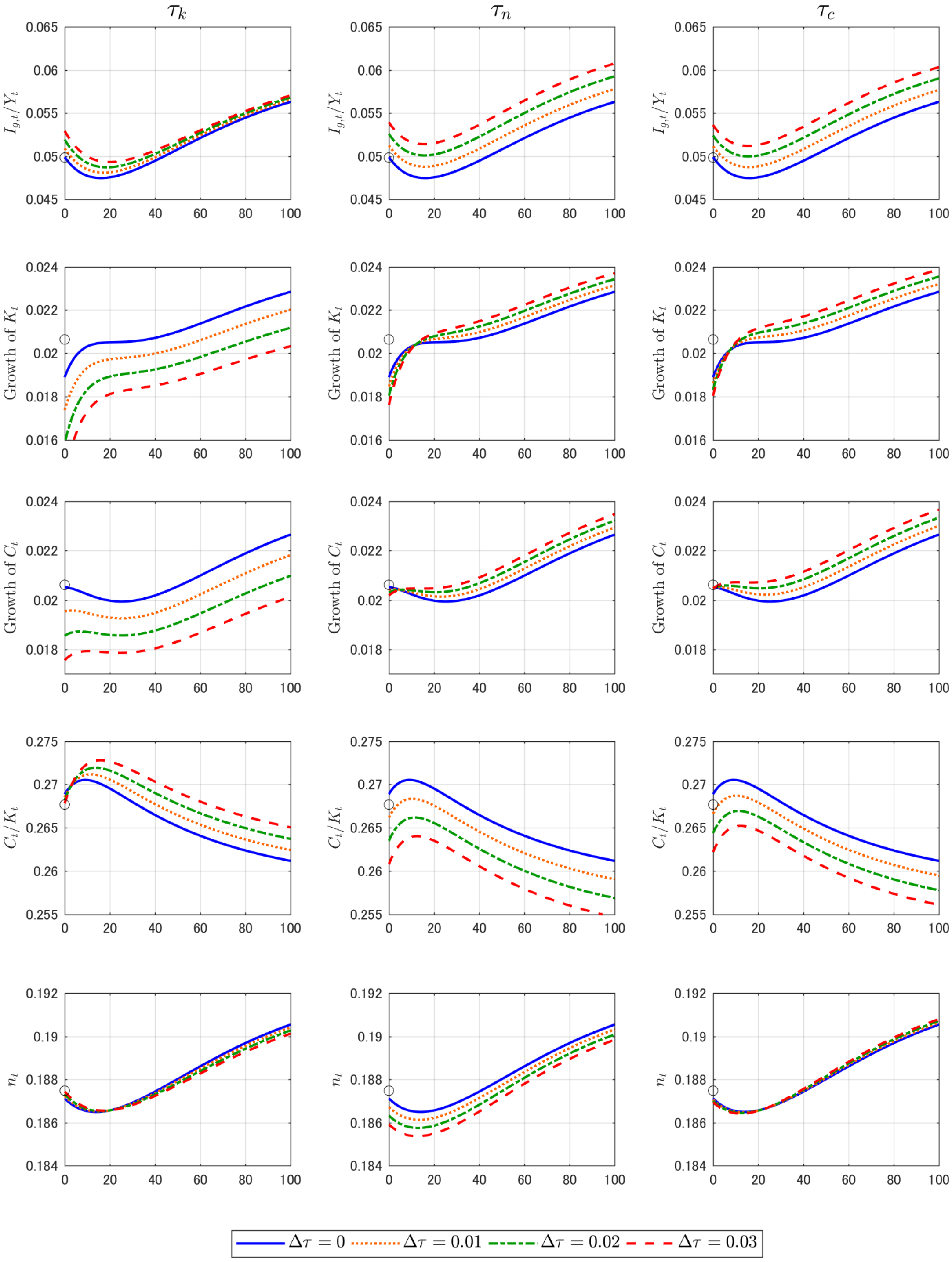

Figure 4: Transition dynamics with tax increases: The deficit policy rule.

Note: The circle at the left end represents the initial steady-state value.

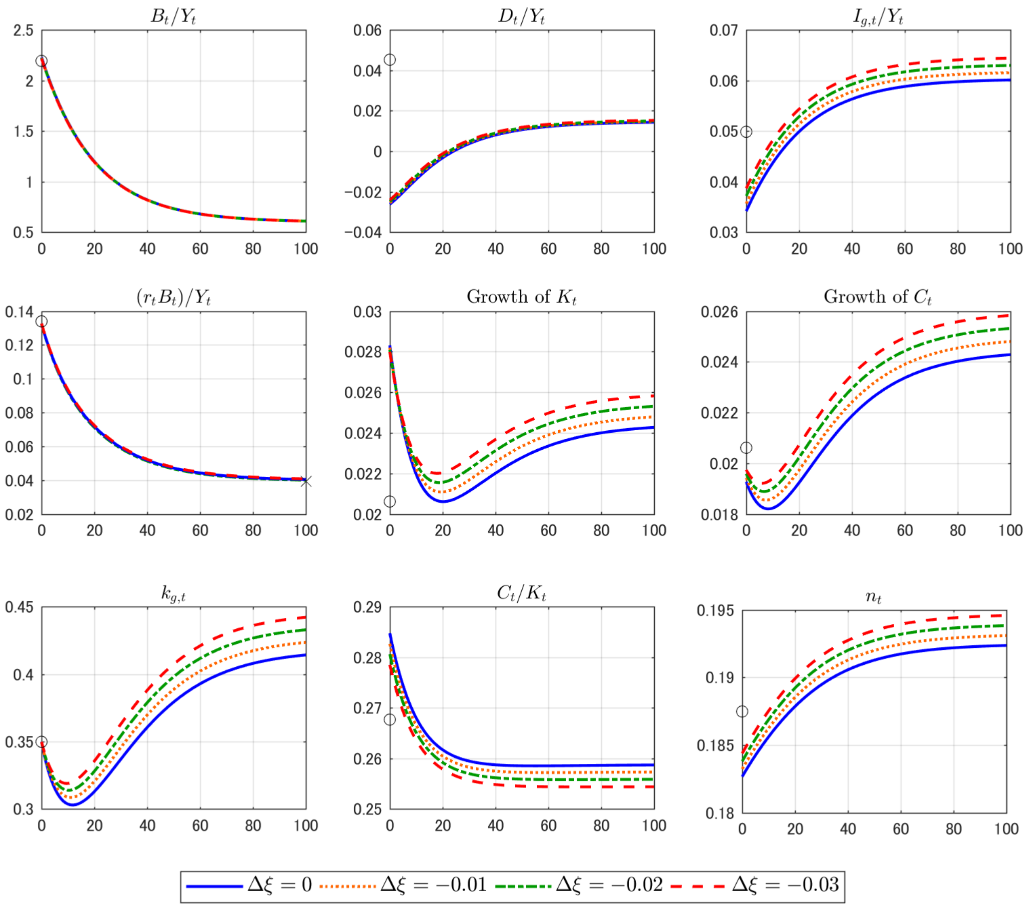

Figure 5: Transition dynamics with transfer payment decreases: The debt policy rule.

Note: The circle at the left end represents the initial steady-state value.

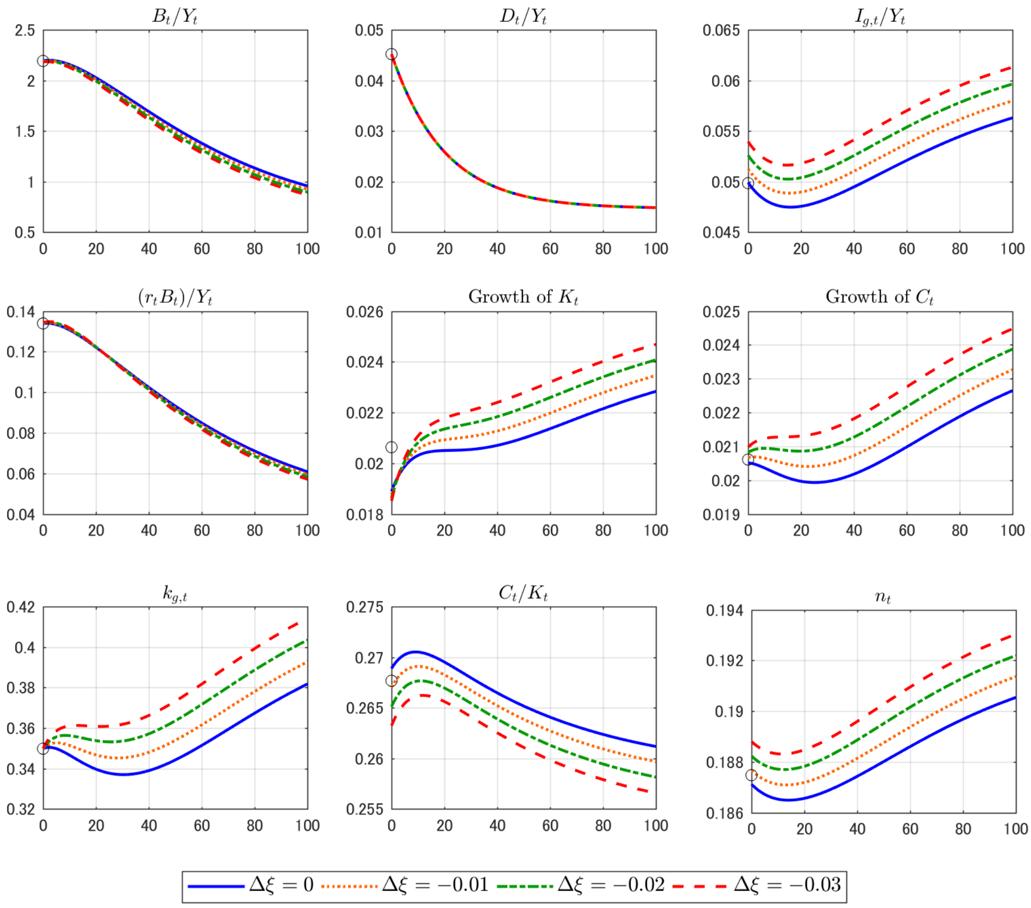

Figure 6: Transition dynamics with transfer payment decreases: The deficit policy rule.

Note: The circle at the left end represents the initial steady-state value.

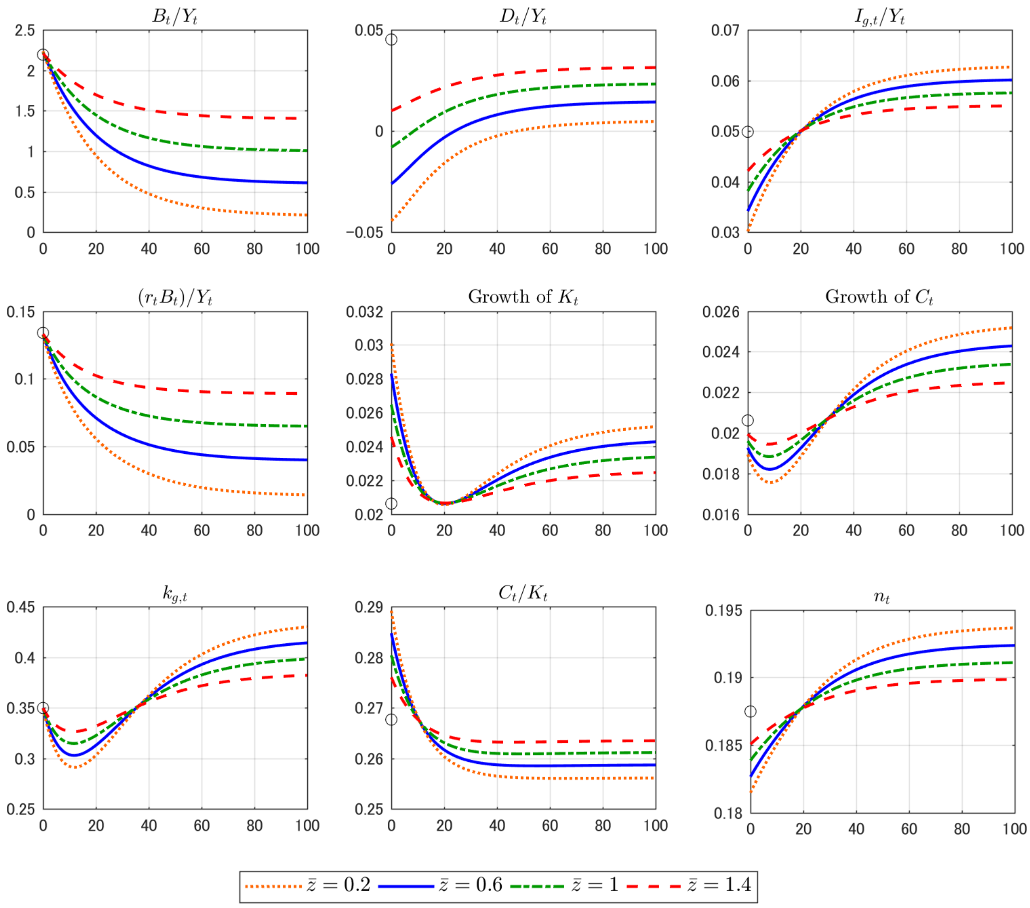

Figure 7: Transition dynamics of changes in $\bar{z}$.

Note: The circle at the left end represents the initial steady-state value.

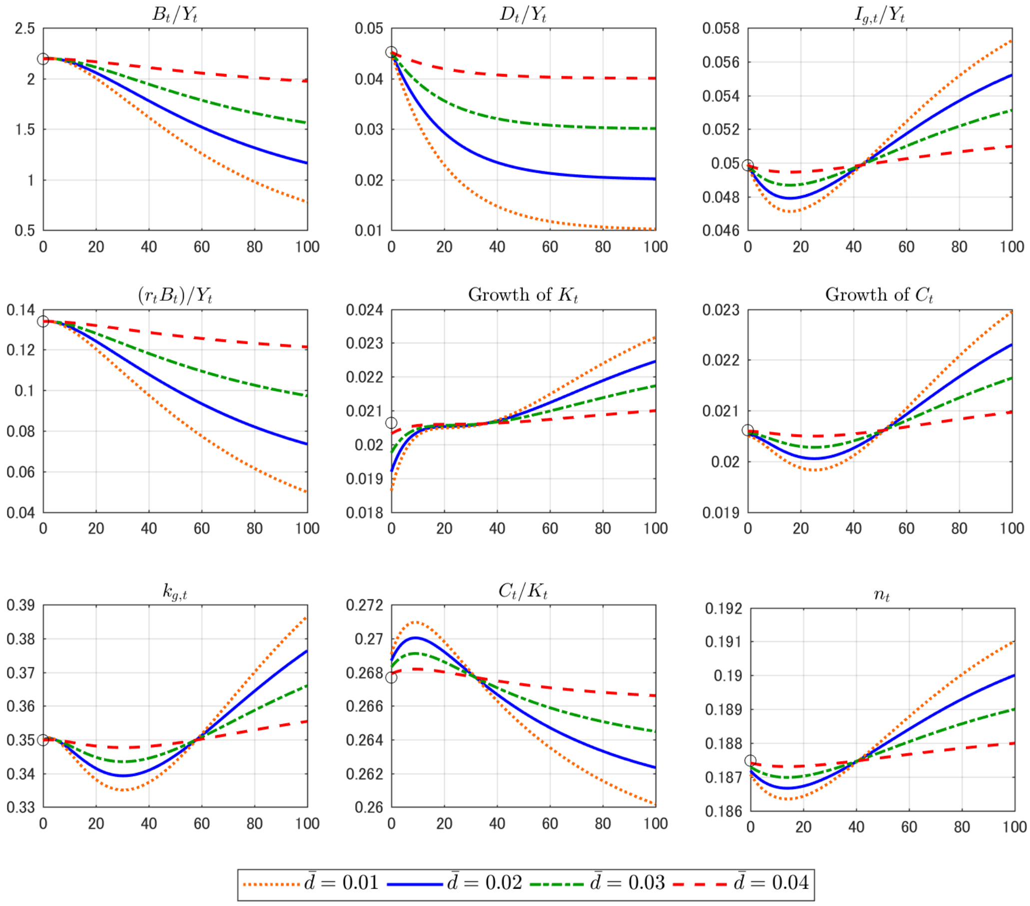

Figure 8: Transition dynamics of changes in $\bar{d}$.

Note: The circle at the left end represents the initial steady-state value.