Bank of Japan Working Paper Series

Natural Rate of Interest in Japan — Measuring its size and identifying drivers based on a DSGE model —

Yosuke Okazaki*
yousuke.okazaki@boj.or.jp

Nao Sudo**
nao.sudou@boj.or.jp

No.18-E-6
March 2018

Bank of Japan
2-1-1 Nihonbashi-Hongokucho, Chuo-ku, Tokyo 103-0021, Japan

* Monetary Affairs Department
** Monetary Affairs Department

Natural Rate of Interest in Japan

-- Measuring its size and identifying drivers based on a DSGE model --[*]

Yosuke Okazaki[†] and Nao Sudo[‡]

March 2018

Abstract

In this paper, we explore the level and determinants of the natural rate of interest in Japan. To this end, we construct a DSGE model that is specifically designed to address potential drivers of the natural rate that are considered important in previous studies, and estimate the model using Japan's data from 1980 to 2017. Our findings are summarized in the following three points. First, the natural rate has shown a secular decline over time, from 400 basis points in the 1980s, to 30 basis points in the last five years. The decline has been mostly attributed to changes in neutral technology. Changes in investment-specific technology, working-age population, and demand factors have also contributed to the decline, but the quantitative impacts have been small. Second, a secular decline and the quantitative importance of neutral technology are also seen when considering the expected future natural rates over a long horizon, indicating that changes in the natural rate have been perceived as persistent rather than temporal changes over the course of history. Third, in the banking crisis starting in the 1990s, financial factors stood out as an important driver that depressed the natural rate. Their contribution holds second place, after changes in neutral technology, when comparing potential drivers by the size of their contribution to variations in the natural rate. Our results suggest the need to monitor the financial intermediation function, as well as the path of neutral technology when analyzing developments in the natural rate.

JEL classification: E32; E43; E44; E52

Keywords: Natural Rate of Interest; Monetary Policy Implementation; DSGE Model

---

[*] The authors would like to thank Kosuke Aoki, Hibiki Ichiue, Takushi Kurozumi, Teppei Nagano, Jouchi Nakajima, Kenji Nishizaki, Masashi Saito, Toshiaki Watanabe, Toshinao Yoshiba, and seminar participants at the Bank of Japan, for their useful comments. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

[†] Monetary Affairs Department, Bank of Japan (E-mail: yousuke.okazaki@boj.or.jp)

[‡] Monetary Affairs Department, Bank of Japan (E-mail: nao.sudou@boj.or.jp)

1 Introduction

The natural rate of interest (hereafter the natural rate) is the real interest rate at which economic activity and prices neither accelerate nor decelerate. Close monitoring of developments in the natural rate is essential for monetary policy implementation, since, with almost no exceptions, monetary easing is attained by driving the real interest rate below the natural rate. There are, however, some difficulties associated with monitoring the natural rate in practice. The natural rate is not observable and needs to be estimated, while the rate is considered to vary over time, reflecting changes in the economic environment that affect the savings and investment decisions of firms and households.

Broadly speaking, there are two approaches to estimating the natural rate. In the first approach, the natural rate is distilled as the trend component of the actual real interest rate, using the Hodrick-Prescott filter, a band-pass filter, or other more sophisticated time series methodologies. The estimation process often exploits the time series of the real interest rate alone, and the theoretical relationship with other variables, such as prices and the output gap, is not taken into account. In the second approach, the natural rate is estimated as the hypothetical real interest rate that is consistent with a theoretical requirement such as that stipulated above, using a structural economic model that exploits the data of variables other than interest rates. One widely used framework in this category is the approach developed by Laubach and Williams (2003, hereafter LW).[1] Another extensively used framework in this approach is a dynamic stochastic general equilibrium (hereafter DSGE) model.[2] One advantage of using DSGE models is that, along with the theoretical consistency of the models, it uncovers the nature of the underlying drivers of the natural rate and helps form clear structural interpretations of their dynamics.[3]

In this paper, we estimate the time path and drivers of the natural rate in Japan from 1980 to 2017, using a New Keynesian DSGE model. Following convention, we define the

---

[1] See Fries et al. (2016), Pescatori and Turunen (2016), Lewis and Vazquez-Grande (2017), and Hakkio and Smith (2017) for estimates based on the approach of LW (2003). See also Holston et al. (2017), who apply the LW methodology to the U.S., Canada, the euro area, and the U.K.

[2] Work based on DSGE models includes, for example, Edge et al. (2008), Justiniano and Primiceri (2010), Barsky et al. (2014), Curdia et al. (2015), Del Negro et al. (2015, 2017), Hristov (2016), and Gerali and Neri (2017).

[3] In addition to the two approaches listed here, some studies, such as Ikeda and Saito (2014), and Sudo and Takizuka (2018), use a calibrated general equilibrium model and compute the real interest rate by feeding into the model the time path of key exogenous variables such as demographic factors and TFP. See also Carvalho et al. (2016) and Gagnon et al. (2016) for related analyses.

natural rate as the ex-ante real short-term interest rate that would prevail if all prices and wages are flexible. Most studies on the natural rate in developed countries agree that the rate has fallen over the years, particularly in the wake of the recent global financial crisis. By contrast, there is less agreement as to what has been the dominant driver of the natural rate over the course of history. We have therefore been careful in our choice of analytical framework so that our analysis not only pins down the level of the natural rate but also serves as a horse racing of potential drivers. That is to say, we focus on selected drivers that have already been proposed and have so far attracted considerable attention in the literature. They are neutral technology, investment-specific technology, financial factors, demographic factors, and demand factors. We design the settings of our DSGE model and our estimation strategy so that the quantitative impacts of these drivers are explicitly addressed in the model and the relevant information on these drivers is appropriately used in the estimation.[4]

In addition to the natural rate, similar to Justiniano and Primiceri (2010), we compute the sequence of expected future natural rates over a long horizon, which we refer to as the expected natural rates, and study how they respond to changes in each driver and how they have evolved over time. This is because this measure summarizes how long a change in the natural rate taking place in the current period will last and serves as a metric for comparing the relative importance of drivers.[5]

We find that the natural rate in Japan shows a decline from about 400 basis points in the 1980s, to 30 basis points in the last five years. The decline was most pronounced in the first half of the 1990s and became moderate in subsequent years. As of 2017, the estimated level of the rate is about 100 basis points. The estimated declining rate accords well with the estimate of the natural rate in Japan based on alternative methodologies such as that of LW (2003) and Imakubo et al. (2015), or estimates based on other DSGE models (Iiboshi et al. [2017]). In our estimate, more than half of the decline is attributed to

[4] Model selection is essential when estimating the level and determinants of the natural rate. Hristov (2016) estimates the natural rate in the euro area using two different DSGE models, a variant of Smets and Wouters (2007), and the same model but equipped with financial frictions. He documents that the size of the estimated decline in the natural rate during the global financial crisis was about 400 basis points in the former, and about 200 basis points in the latter; while the dominant driver of the natural rate was shocks to the discount factor in the former, and shocks to the rate of return on capital in the latter.

[5] As examined in Justiniano and Primiceri (2010), it also serves to gauge the degree of monetary policy implementation. The importance of looking at the expected natural rates over a long horizon is warranted in particular when the prevailing nominal short-term interest rate is floored and alternative policy measures targeting expectations, such as forward guidance, are adopted in monetary policy implementation.

changes in neutral technology. Changes in the working-age population, investment-specific technology, and demand factors have also contributed to the decline, but their quantitative impacts have been limited. Financial factors have driven the natural rate cyclically over the sample period. They dampened the natural rate about 100 basis points during the banking crisis, and enhanced the rate in the years after 2000. When compared by size of contribution to variations in the natural rate, shocks to financial factors hold second place among the five drivers, after the contribution of shocks to neutral technology.

Similar to the natural rate, the expected natural rate also exhibits a decline over the sample period, suggesting that the natural rate decline over time has been perceived as persistent rather than as temporal changes by agents in the economy. This in part reflects the fact that the bulk of variations in the natural rate have been driven by changes in neutral technology whose effects on the natural rate are long-lived. The decline has taken place more gradually, closely tracking the time path of the potential growth rate, particularly in recent years.

Our study is related to studies that estimate the natural rate in various economies, in particular those that use DSGE frameworks. These analyses include Barsky et al. (2014) and Del Negro et al. (2015, 2017) for the U.S., Hristov (2016) for the euro area, Goldby et al. (2015) for the U.K., and Iiboshi et al. (2017) for Japan. Our paper is also related to works on the secular stagnation hypothesis, including Eggertsson et al. (2017), Rachel and Smith (2015), Sajedi and Thwaites (2016), and Summers (2014). Since the remark made by Summers (2013), growing attention has been paid to the question whether the natural rate has merely been declining thus far, or will continue to decline, and if so, why. The five drivers focused on in this paper are all included in the lists in existing studies of potential causes of the decline. Lastly, the current paper is related to studies that estimate a DSGE model using Japan's data, including Sugo and Ueda (2008), Kaihatsu and Kurozumi (2014), Hirakata et al. (2016), and Muto, Sudo, and Yoneyama (2016, hereafter MSY). Our study is close to Hirakata et al. (2016) and MSY (2016) in terms of model settings, in particular those associated with financial frictions.

The contribution of our paper is as follows. First, it provides the time path and drivers of the natural rate over the years in Japan, using a New Keynesian model. To the best of our knowledge, this paper is currently the only paper that estimates the natural rate in

Japan using a medium-scale DSGE model with a financial friction.[6] Second, it focuses on a broad set of drivers considered essential in previous studies, and assesses their quantitative implications for the current natural rate and future expected natural rates, using a model specifically designed for this purpose. We give a chance not only to temporary shocks such as demand shocks but also to long-lived shocks such as growth rate shocks to technology to account for both current and future natural rates, and compare their roles. In particular, our paper differs from existing studies in quantifying the impact of key drivers on expected future natural rates over a long horizon.

The remainder of this paper is organized as follows. Section 2 summarizes the potential drivers of the natural rate discussed in existing studies. Section 3 provides an overview of our model and the estimation strategy. Section 4 reports the estimation results. Section 5 is devoted to the validation check and sensitivity analysis. Section 6 concludes.

2 Potential Drivers of the Natural Rate

In this section, we describe the potential drivers of the natural rate. There are already a number of studies on this topic, in particular in relation to the secular stagnation hypothesis advocated recently by Summers (2013). Here, for the convenience of the analysis below, we classify the drivers into five categories and describe how each driver affects the natural rate. We also briefly discuss the implications for Japan's economy and how we design our model and estimation settings so as to address and quantify these drivers.

Neutral technology

According to the text book growth model, changes in neutral technology are the key source of macroeconomic fluctuations, including those of real interest rates. While neutral technology itself is not directly observable, in developed countries, there is evidence that neutral technology has recently grown at a slower pace than before, as documented for example in Cette et al. (2016) and Eichengreen et al. (2017). Theoretically, stagnation in neutral technology reduces the return on capital, making investment less profitable.

[6] The difference between the estimates by Iiboshi et al. (2017) and our estimates is that, while both are based on a DSGE model, they keep their model framework simple and estimate the model explicitly addressing the non-linearity associated with a zero lower bound of the policy rate. By contrast, while our paper incorporates fairly rich ingredients into the model, such as financial frictions and endogenous changes in the measured TFP, it disregards the non-linearity addressed in their paper.

Consequently, to the extent that a lower return on capital discourages investment, real interest rates should fall. Somewhat surprisingly, however, in most existing estimates based on a DSGE model, a change in neutral technology does not manifest itself as the key driver of the natural rate.[7]

In Japan, as documented in a pioneering paper by Hayashi and Prescott (2002), there was a kink around the early 1990s in the growth rate of measured TFP, the widely agreed proxy of neutral technology in the literature. The growth rate of the measured TFP was on average 1.84% in the 1980s, dropping to 0.49% in the 1990s and 0.46% in the 2000s and beyond, suggesting that changes in neutral technology growth rates may have contributed negatively to developments in the natural rate over the last forty years.

To see if this interpretation holds true, we employ three ingredients in our analysis: First, we incorporate a stochastic trend to neutral technology, similar to previous studies, including Gerali and Neri (2017). Second, we use the time series of measured TFP, the Solow residual, in estimating our model. As discussed in Basu et al. (2006), however, the measured TFP can be driven not only by changes in neutral technology but also by non-technological factors, such as demand shocks, if the actual production process involves the use of intermediate inputs or endogenous changes in capacity utilization of production inputs. As the third element, therefore, we explicitly incorporate these elements when modeling production function of goods producers.

Financial factors

Existing estimates mostly agree that the natural rate in developed countries discontinuously plummeted at the outset of the global financial crisis in 2007.[8] It is therefore natural to see financial factors as an important potential driver of the natural rate. When financial intermediation malfunctions, the amount of borrowing by firms and households should fall, and (risk-adjusted) real interest rates may fall, too. For example, according to an estimate by Goldby et al. (2015), about 400 basis points decline in the natural rate in the U.K. during the period of the global financial crisis was attributed to risk premium shock, which in their interpretation represents credit supply shocks.

[7] One possible reason is because some studies assume that the trend component of neutral technology evolves deterministically and focus only on the effects of changes in the stationary component of neutral technology.

[8] See, for example, Holston et al. (2017) for estimates based on the methodology of LW (2003), and Barsky et al. (2014) and Del Negro et al. (2015) for estimates based on a DSGE model.

In Japan, there have been two crises in the last forty years; the banking crisis that started with an asset price collapse in the early 1990s, and the global financial crisis. The first crisis damaged the balance sheets of financial intermediaries (hereafter FIs) and goods producers, and banks' credit cost rate climbed up to 3.7% at its peak, leading to disruptions to financial intermediation and the collapse of large financial institutions in 1997 and beyond.[9] According to the dating by Reinhart and Rogoff (2011), this banking crisis persisted from 1992 to 2001. The latter crisis was not accompanied by a financial turmoil in Japan. It did, however, trigger a slump in the economy. GDP fell by 8.29% from the peak of the business cycle in 2008:1Q, to the trough in 2009:1Q.[10]

In order to address financial factors, we follow Hirakata, Sudo, and Ueda (2011, 2017, hereafter HSU) and incorporate what they call a chained credit contract into the model. In this framework, both non-financial firms and FIs are credit constrained, raising external funds through credit contracts similar to those adopted in Bernanke, Gertler, and Gilchrist (1999, hereafter BGG).[11] FIs lend to non-financial firms what they borrow from households and their own net worth, and the non-financial firms invest in capital goods, using what they borrow from FIs and their own net worth. Following HSU (2011, 2017), we incorporate shocks to the balance sheets of the two sectors, and these are referred to as financial shocks in the paper. They are shocks to the net worth accumulation of these sectors, and affect macroeconomic variables primarily by changing the terms of the credit contracts. With these shocks, we intend to capture changes in financial intermediation that arise not from macroeconomic conditions, but from financial factors.[12] In addition, when estimating the model, we employ the net worth series for FIs and non-financial firms using the Flow of Funds data, so as to accurately estimate the time path of shocks to the net worth over

[9] See Hoshi and Kashyap (2010) for the sequence of events during this crisis period. See also Bayoumi (2001), for example, for the quantitative impact on economic activity of disruption to financial intermediation during this period.

[10] Here, we follow the dating of Japanese business cycle released by the Cabinet Office. The recession that has coincided with the global financial crisis corresponds to the 14th business cycle based on the dating.

[11] The financial accelerator framework of BGG (1999) is commonly used in existing studies that estimate the natural rate with a DSGE model with a financial friction. See, for example, Del Negro et al. (2017) and Hristov (2016). Another type of financial friction used in the literature is the one used in Eggertsson et al. (2017). In their model, households are subject to borrowing constraint, and an exogenous change in the degree of the constraint reduces the natural rate even to a level below zero.

[12] Existing studies, such as Gilchrist and Leahy (2002) and Nolan and Thoenissen (2009), also consider shocks that are similar in nature to our net worth shocks. Their interpretations of these shocks include "asset bubble and burst of asset bubble," "irrational exuberance," or "innovation in the efficiency of credit contracts."

time.

Demographic landscapes

Population aging and real interest rate declines have gone hand-in-hand over the last few decades in developed countries. Based on this observation, there are a growing number of studies that explore the relationship between real interest rates and demographic factors using a calibrated life-cycle model. These include Carvalho et al. (2016), Gagnon et al. (2016), and Sudo and Takizuka (2018). These studies commonly show that population aging, arising from a declining fertility rate, increasing longevity, or both, depresses real interest rates, since a lower fertility rate is translated into lower labor inputs, and increased longevity increases the amount of capital inputs by giving households added incentive to save.

As documented in Sudo and Takizuka (2018), among the G7 countries, population aging is most pronounced in Japan. In their estimate, from 1980 to 2017, the growth rate of the working-age population has declined by 1.86%, from 0.84% to -1.03%, and life-expectancy has risen by 7.2 years, from 77.2 to 84.4 years in Japan; whereas in the U.S., for example, the corresponding numbers are 1.27% and 3.7 years.

Existing estimates using a DSGE model do not address demographic factors, in part because these factors are considered as primarily affecting the low-frequency components of macroeconomic variables, and in part because some demographic changes are predictable. In the analysis below, we incorporate one element of demographic factors into the model, following Burriel et al. (2010), and study its effect on the natural rate. Namely, we assume that the working-age population evolves stochastically by incorporating shocks to the growth rate of the working-age population, allowing macroeconomic variables, including the natural rate, to react to these shocks. In estimating the model, we use the actual working-age population growth rates to extract the demographic shocks.

Investment-specific technology

Since the pioneering work by Greenwood, Hercowitz and Krusell (1997) documenting the evidence of advances in investment-specific technology over the years in the U.S., investment-specific technology has been considered an important driving force of macroeconomic variables.[13] An improved investment-specific technology makes the price of investment goods cheaper. On the one hand, this implies that a smaller amount of final goods is needed to make the same size of investment, which in turn depresses real interest rates. On the other hand, if the technological improvement induces a larger investment, it should rather boost real interest rates. Whether the former effect dominates the latter depends on whether capital and labor inputs are complements or substitutes in goods production. Sajedi and Thwaites (2016) show, using an overlapping generations (hereafter OG) model where the two production inputs are complementary, that improvements in investment-specific technology reduce the relative price of capital goods, and depress real interest rates.

Changes in investment-specific technology are often measured through changes in the relative price of investment goods. In Japan, based on the investment deflator divided by the GDP deflator, the relative price grew at an average rate of -0.64% from the 1980s to 2017. The growth rate was -1.18% on average during the two decades ending in 2000, and was 0.20% in the rest of the sample period, suggesting that investment-specific technology advanced markedly in the 1980s and 1990s, and slowed down in the subsequent years. We explicitly incorporate into the model investment-specific technology that grows stochastically, following the specification of Fisher (2006), so that changes in the technology affect macroeconomic variables, including the natural rate, and use the time series of the relative price of investment goods as an observable when estimating the model. Following convention, however, we maintain the assumption that the price elasticity of capital and labor inputs is unity in the model.

Demand factors

Changes in household demand, in particular shocks to discount factor, have been considered important drivers of the natural rate. While there is no agreement as to why demand structure changes shape, some existing studies point out that a compositional change in households with different characteristics, in terms of age or wealth, may alter the demand structure of the household sector as a whole. Demand factors are regarded as the key driver of natural rates in some studies. For example, Iiboshi et al. (2017) document that

[13] See, for example, the early work by Fisher (2006) that evaluates the importance of investment-specific technology shocks using a growth model.

the bulk of variations in the natural rate from the 1980s to the present in Japan have been brought about by changes in discount factor. To address demand factors, we incorporate two types of demand shocks: shocks to discount factor and to external demand. Because these demand shocks themselves are not observable, we do not employ specific variables for extracting demand shocks in our estimation procedure.

3 Model Description and Estimation Procedure

3.1 Model Overview

Most of the settings in our model are borrowed from existing studies including HSU (2011) and MSY (2016). In this section, we therefore provide a model overview together with the selected key elements of the model, including how the working-age population changes and affects households' decisions, how monetary policy is implemented, the nature of fundamental shocks, and the definition of the natural rate in the model. We describe the full model structure in Appendix A. See also Figure 1, where the outline of the model is graphically depicted.

Compared to the standard New Keynesian model, such as that in Smets and Wouters (2007), our model includes the following additional elements.

1. Stochastic trends in neutral technology and investment-specific technology.
2. Stochastic trends in the working-age population, following Burriel et al. (2010).
3. Credit contracts between households and FIs, following HSU (2011) and MSY (2016).
4. Credit contracts between FIs and entrepreneurs, or equivalently non-financial firms, following HSU (2011) and MSY (2016).
5. Intermediate inputs and endogenous capacity utilization of capital stock, following Basu (1995) and MSY (2016).
6. Anticipated shocks to short-term nominal interest rates in the future, following Laseen and Svensson (2011) and Del Negro et al. (2017).

The first to fifth elements are incorporated so as to correctly address the potential impacts of the five drivers of the natural rate. The last element typically serves to separately quantify the impact of forward guidance in existing studies. As discussed in Section 5, however, this specification is also intended to serve in accurately estimating the natural rate in this paper.

3.1.1 Changes in Working-age Population

In our model, there is a continuum of households, and each household is composed of H_t identical workers. Each household decides how much each worker works, consumes, and saves so as to maximize the sum of the expected life-time utility of each household member. While existing studies based on a New Keynesian DSGE model often assume that the number of workers H_t grows in a deterministically manner or is unchanged over time, some portion of changes in its growth rate in a period t are in fact unpredictable in a period $t-1$ in the actual economy. We therefore assume, following Burriel et al. (2010), that the working-age population grows stochastically, subject to the law of motion described below:

$$\ln H_t = \ln H_{t-1} + \epsilon_{H,t},$$

where $\epsilon_{H,t}$ is a shock to the growth rate of workers.

Upon the arrival of a shock to the working-age population growth rate, each household changes its decisions about hours worked, consumption, and saving, which in turn affects allocations and prices including the natural rate. For example, denoting the aggregate labor input, hours worked by each worker at a household h by L_t and $l_t(h)$, other things being equal, a positive shock to the growth rate mechanically increases the size of the aggregate labor inputs, since the following equality holds.

$$L_t = H_t \times l_t(h).$$

If the usages of the other production inputs are unchanged, this change makes capital inputs relatively scarce, boosting the real interest rates in the economy.

It is notable that this setting implicitly assumes that the entire change in working-age population growth is unknown. We choose this setting because how expectation about future working-age population growth is formed in practice is not obvious. Our strategy is therefore to give the largest chance to shocks to the working-age population growth to explain the natural rate by simply assuming there is no anticipated component in the growth rate. In later section, we relax this assumption and study how the results are altered when different specifications about the growth rates are assumed.

3.1.2 Monetary Policy

Monetary policy implementation in our model is standard, except that it includes anticipated nominal interest rate shocks. As studied in Laseen and Svensson (2011), these shocks can work in a manner different from unanticipated shocks. We incorporate them because our sample period covers a period when short-term nominal interest rates were around zero, and forward guidance was used as part of monetary policy implementation.

The central bank adjusts the policy rate according to the following Taylor rule with a time-varying target rate of inflation and the exogenous component of the policy rule as described below:

$$R_{n,t} = R_{n,t-1}^{\rho} \left[\overline{R}^{*} \overline{\pi} \left(\frac{\pi_t}{\bar{\pi}_t} \right)^{\varphi_\pi} \right]^{1-\rho} \exp\left(\eta_t\right), \tag{1}$$

where

$$\eta_t = \epsilon_{R_n,t} + \varepsilon_{R_{n,1},t-1} + \varepsilon_{R_{n,2},t-2} + ... + \varepsilon_{R_{n,S},t-S}. \tag{2}$$

Here, $R_{n,t}$ is the nominal interest rate, π_t is the inflation, $\rho \in (0,1)$ is the interest rate smoothing parameter of the monetary policy rule, $\varphi_\pi > 1$ is the policy weight attached to the inflation rate, $\overline{R}^{*}$ is the steady state natural rate, $\overline{\pi}$ is the steady state inflation rate, and $\bar{\pi}_t$ is the target rate of inflation that varies according to the following equation;

$$\ln \bar{\pi}_t = (1 - \rho_\pi) \ln \overline{\pi} + \rho_\pi \ln \bar{\pi}_{t-1} + \epsilon_{\pi,t}.$$

Here, $\rho_\pi \in (0,1)$ is the autoregressive coefficient and $\epsilon_{\pi,t}$ is a shock to the target rate of inflation. Note that η_t is a shock to the monetary policy rule, and is decomposed into the unanticipated component and anticipated component. The unanticipated component $\epsilon_{R_n,t}$ is an i.i.d. shock, and anticipated policy shocks $\varepsilon_{R_{n,s},t-s}$, $s = 1, 2, ..., S$ are known to agents at period $t - s$ in advance, but each of the shocks materializes in the policy rule with a lag of s quarters.

3.1.3 Fundamental Shocks

The model consists of fourteen fundamental shocks and S number of anticipated monetary policy shocks. For our purposes, they are categorized into the following six groups.

1. Shocks to neutral technology: these are shocks that directly affect the neutral production technology of the gross output $Y_{g,t}$, and there are both long-lived and short-lived shocks, denoted as $\epsilon_{Z_a,t}$ and $\epsilon_{A_a,t}$, respectively.

2. Shocks to investment-specific technology: these are shocks that affect capital goods production technology exclusively. Similar to neutral technology shocks, there are both long-lived and short-lived shocks, denoted as $\epsilon_{Z_d,t}$ and $\epsilon_{A_d,t}$, respectively.

3. Shocks to the net worth of FIs and entrepreneurs: these are shocks that affect the accumulation of retained earnings and therefore net worth in the two sectors $N_{F,t}$ and $N_{E,t}$. Positive net worth shocks enhance the balance sheet conditions of these sectors, and negative shocks work in the opposite direction. These shocks are denoted as $\epsilon_{N_F,t}$ and $\epsilon_{N_E,t}$, respectively.

4. Shocks to the working-age population growth rate: these are shocks that change the working-age population growth rate. Because they affect the growth rate, their impact on the level of the working-age population lasts permanently. They are denoted as $\epsilon_{H,t}$.

5. Shocks to demand factors: these are shocks to discount factor and shocks to external demand, denoted as $\epsilon_{d,t}$ and $\epsilon_{G,t}$, respectively.

6. Other shocks: these are shocks that are not categorized above, including shocks to the investment adjustment cost $Z_{I,t}$, the price markup $\theta_{P_Y,t}$, the wage markup $\theta_{W,t}$, the target rate of inflation $\bar{\pi}_t$, and both unanticipated and anticipated monetary policy shocks $\epsilon_{R_n,t}$ and $\varepsilon_{R_{n,s},t-s}$, $s = 1, 2, ..., S$.

3.1.4 Definition of the Natural Rate

The natural rate

We define the natural rate R_t^* as the ex-ante real short-term interest rate that would prevail in a counterfactual economy, which we call the flexible-price economy. The flexible-price economy is exactly the same as the actual economy described in Appendix A, except that both wages W_t and prices P_t are perfectly flexible and there are no markup shocks.[14] In other words, in this flexible-price economy, the parameters associated with adjusting nominal prices are zero, $\kappa_w = \kappa_p = 0$ and markup shocks are zero $\epsilon_{W,t} = \epsilon_{P_Y,t} = 0$ for all t. In what follows, we denote as X_t^* the flexible-price economy counterpart of a variable X_t in the actual economy.

As already discussed in existing studies, such as Justiniano and Primiceri (2010) and Barsky et al. (2014), the difference between the prevailing ex-ante real interest rate R_t and the natural rate R_t^* can be used as a metric when measuring the degree of monetary easing. Denoting the deviation of a variable X_t (or X_t^*) from the steady state by $\tilde{X}_t$ (or $\tilde{X}_t^*$), the Euler equation of the households in our model can be arranged to the following expression.[15]

$$\tilde{c}_t - \tilde{c}_t^* = -\sum_{s=0}^{\infty} \mathrm{E}_t \left(\tilde{R}_{t+s} - \tilde{R}_{t+s}^* \right). \tag{3}$$

Here E_t is the expectation operator. Note that the gap between the actual economy and that in the flexible-price economy is closed when the sum of the sequence of the expected real interest rate $\tilde{R}_{t+s}$ from the current period to the infinite future coincides with that of the natural rate $\tilde{R}_{t+s}^*$ over the same period.[16]

The natural rate at the steady state

The natural rate varies in response to various shocks listed above other than monetary policy shocks and markup shocks. At the steady state, however, the natural rate is determined by the households' discount factor β, and the growth rates of neutral technology and investment-specific technology. Denoting the value of a variable X_t at the non-stochastic steady state as X_{ss}, from the Euler equation, we have:

$$R_{ss} = R_{ss}^* = \frac{1}{\beta} g_{Z_a,ss}^{\frac{1}{(1-\gamma)(\alpha+\alpha_E+\alpha_F)}} g_{Z_d,ss}^{\frac{(1-\alpha-\alpha_E-\alpha_F)}{\alpha+\alpha_E+\alpha_F}} = \frac{1}{\beta}\bar{\gamma}, \tag{4}$$

[14] Similar to the settings in Barsky et al. (2014) and Gerali and Neri (2017), the steady state economy we consider suffers from distortion arising from the presence of markups in the goods producers and households' labor inputs, and that of financial friction. The estimated natural rate does not, therefore, necessarily coincide with the efficient rate.

[15] Here, we consider a hypothetical case in which the degree of internal habit persistence in consumption preferences (θ_h) is zero, for the illustrative purpose.

[16] Note that as in Barsky et al. (2014), we assume that $\tilde{c}_{t+s} - \tilde{c}_{t+s}^*$ goes to zero as s approaches infinity.

where

$$\bar{\gamma} \equiv g_{Z_a,ss}^{\frac{1}{(1-\gamma)(\alpha+\alpha_E+\alpha_F)}} g_{Z_d,ss}^{\frac{(1-\alpha-\alpha_E-\alpha_F)}{\alpha+\alpha_E+\alpha_F}}.$$

Expected natural rate

The equation (3) indicates that from the viewpoint of closing the gap in period t, not only is the level of the current natural rate $\tilde{R}_t^*$ relevant, but equally so is the sequence of the expected natural rates in period $t+1$ and beyond $\sum_{s=1}^{\infty} \mathrm{E}_t \left[\tilde{R}_{t+s}^*\right]$. From this perspective, Justiniano and Primiceri (2010) estimate the right-hand side of the equation, referring to it as the long-term real interest rate gap, for the U.S., and explore its policy implications. Along the same lines, we study the sum of the sequence of the expected natural rate from the current period to a period T quarters ahead, referring it to as the expected natural rate and denote it as $\tilde{R}_{T,t+s}^*$. Thus, we have;

$$\tilde{R}_{T,t+s}^* \equiv T^{-1} \sum_{s=0}^{T-1} \mathrm{E}_t \left[\tilde{R}_{t+s}^*\right].$$

Note that this measure summarizes households' expectation of the sequence of the future natural rates that is formed conditional on the economic state at period t, and considered as a relevant measure to assess the size of shocks that affect the economy through the Euler equation.[17,18]

3.2 Overview of Estimation Strategy

Our estimation methodology follows the standard approach, similar to that used in existing studies such as Smets and Wouters (2007). Because our choice of observables deviates from standard practice, however, in this section we describe the list of variables used in the estimation. The rest of the estimation procedure is provided in Appendix B. See also Table 1 and 2 for the values of calibrated and estimated parameters, respectively.

[17] When projecting the sequence of the future natural rates, we assume that agents at the period foresee no innovations at all periods in the years ahead.

[18] Note that the expected natural rate in T quarters ahead $\mathrm{E}_t\left[R_{t+T}^*\right]$ converges to the steady state value of the natural rate $\bar{\gamma}\beta^{-1}$ as T approaches infinity. This is because our model is absent from the term premium and because none of the fundamental shock delivers a permanent change to the natural rate. We therefore compute the expected natural rate over a finite number of quarters. Given the estimated size of the persistence of fundamental shocks, expected natural rates at periods beyond $T = 40$ are already close to the steady state value.

Data

We use the time series of 23 variables from 1980:2Q to 2017:2Q, and show the data series used for the estimation in Figure 2.[19] The data includes 9 aggregate variables, two net worth series taken from balance sheet data of the FIs and goods-producing sectors, and 12 variables about the expected future policy rate: (1) the real GDP Y_t, (2) real investment I_t, (3) GDP deflator P_t, (4) the deflator of investment goods $P_t Z_{d,t}^{-1} A_{d,t}^{-1}$, (5) the nominal wage per unit of labor input W_t, (6) per capita working hours L_t, (7) the short-term nominal interest rate $R_{n,t}$, (8) the working-age population H_t, (9) the measured TFP (computed as the Solow residual) λ_t, (10) real net worth of the FI sector $N_{F,t} P_t^{-1}$, (11) real net worth of entrepreneurs in the goods-producing sector $N_{E,t} P_t^{-1}$, and (12) to (23) the expected future short-term nominal interest rate $\mathrm{E}_t \left[R_{n,t+s} \right]$ for $s = 1, .., 12$.

The data source of the aggregate variables is mostly the System of National Accounts (hereafter SNA) released by the Cabinet Office of Japan. Series (5) is constructed from the compensation of employees based on the SNA, divided by series (6), where series (6) is obtained from the number of employees based on the Labour Force Survey, multiplied by hours-worked per employee based on the Monthly Labour Survey and divided by series (8). Series (7) is the uncollateralized overnight call rate. Because this series is available only from 1985:3Q and beyond, it is extended backward before 1985:3Q using the collateralized overnight call rate. Series (8) is the size of the population aged 15 to 64 years old, as reported in the Labor Force Survey. The construction methodology of series (9) is similar to that used in Hayashi and Prescott (2002).[20]

Series (10) and (11), the two net worth series, are constructed from the outstanding of shares issued by depository corporations and non-financial corporations, respectively. They are taken from the Flow of Funds Accounts. In the Flow of Funds Accounts, the reported series of outstanding of shares are those evaluated not at market value, but at

[19] All of the series other than series (7) and (12) to (23) are displayed on a year-on-year basis in Figure 2. Note, however, that we use a quarter-on-quarter change rather than a year-on-year change of these variables in our estimation. We use the level series only for series (7) and series (12) to (23) in our estimation. Because of the data limitation associated with the overnight index swap data, the data runs only from 2009:3Q to 2017:2Q for the series (12) to (23).

[20] As in Hayashi and Prescott (2002), our measured TFP series is computed from the logarithm of output growth less the weighted average of the logarithm of labor input and capital input growth. There are, however, two differences between our series and theirs: (*i*) the output series that is used for constructing our series is the GDP series, while the output series used for constructing their series is GNP less government capital consumption; (*ii*) households' residential and foreign assets are not included in our capital stock series, while these two components are included by Hayashi and Prescott (2002).

book value before 1995:4Q for depository corporations, and before 1994:4Q for non-financial corporations. We therefore extend each series evaluated at market value backward using the quarterly growth rates of the market capitalization of banks and of non-financial firms. Also note that, given that these variables are in nature stock prices, we include measurement errors as well following existing studies such as Barsky et al. (2014).

Series (12) to (23) are constructed from the overnight index swap (OIS) rates. We use the spot rates of OIS with a maturity of 3 months, 6 months, 1 year, 2 years, 3 years, and 4 years, imputing the spot rates for periods that fall in the intervals by linearly interpolating the raw data, and derive the expected short-term nominal interest rates for $s = 1, 2, ..., S$, as the forward rates using these rates.[21,22]

In estimating the model, we take the first difference and demean all of the series to obtain the stationary series and remove the deterministic trend, respectively except for the series of current and future policy rates (7) and (12) to (23). To convert the nominal series into the quantity series, we employ the GDP deflator. We also divide all of the quantity series by the series (9) H_t to obtain the series on a per-capita basis.

4 Estimation Results

4.1 Estimated Level of the Natural Rate

Estimated time path of the natural rate

Figure 3 shows the time path of the natural rate R_t^* from 1980:2Q to 2017:2Q, estimated by our DSGE model. For the purpose of comparison, it also shows three measures of the natural rate estimated with different methodologies, the natural rate estimated following LW (2003) and Imakubo et al. (2015), and the potential growth rate released by the Re-

[21] In setting the maximum length of the horizon S, we estimate two identical models that only differ in terms of the value of S, $S = 4$ and 12, and choose as our baseline the model with $S = 12$ based on the comparison of forecast errors. To do this, for the common observables other than expected future short-term nominal interest rate, namely the series (1) to (11), we compute one quarter ahead forecast error from 2009:3Q to 2017:1Q, following Nakajima and Watanabe (2017). We choose 2009:3Q because this is the period when all the actual data of the expected short-term nominal interest rates are available. On average across observables, the model with $S = 12$ outperforms the model with $S = 4$.

[22] While it is technically feasible to extract the data of the expected nominal short-term interest rate beyond $S = 12$, there are some concerns that these data may be susceptible to changes in market liquidity as well as anticipated monetary policy shocks. See, for example, Bank of Japan (2007), about the size of OIS transactions in different tenors.

search and Statistics Department of the Bank of Japan.[23,24] All of the measures, including our DSGE estimate, agree that the rates were far above zero before the early 1990s, ranging from around 250 to greater than 500 basis points depending on measures, fell at a rapid pace to a level around zero in the first half of the 1990s, and stayed at around this low rate in subsequent years. Some estimates deliver rates below zero around the periods of the banking crisis and the global financial crisis. In recent years, in particular since 2010, all measures other than that of Imakubo et al. (2015) have shown a pick-up, recording around 100 basis points in their latest figures.

Comparison with existing studies

Our results are in line with the estimate of the natural rate by Iiboshi et al. (2017) in several dimensions. Most importantly, both estimates show a secular decline over time. In addition, they agree about the timing of when the rates turned to negative values; the early 2000s and around the period of the global financial crisis. They, however, disagree in quantitative aspects. In Iiboshi et al. (2017), the decline in the latter phase reached about 800 basis points, whereas the decline during the same phase was less than 50 basis points in our model.

Our results accord with existing estimates of the natural rate based on a DSGE model in other developed countries, such as those in Barsky et al. (2014) and Gerali and Neri (2017), in that they indicate a secular decline over the last few decades.[25] It is also notable, however, that while these estimates show a prominent decline in the period of the global financial crisis, in our estimate, the largest decline took place in the early 1990s.

4.2 Determinants of the Natural Rate

4.2.1 Impulse Response Functions to Fundamental Shocks

What factors are key to developments in the natural rate? To answer this, we first examine how the key variables respond to each of the following nine fundamental shocks;

[23] See Kawamoto et al. (2017) for the estimation methodology.

[24] As discussed in Fujiwara et al. (2016), the potential growth rate can be used as a measure of the natural rate, when the standard representative agent model is considered. For example, as the equation (4) suggests, on a per capita basis, the potential growth rate at the non-stochastic steady state in our model is given by βR^*_{ss}.

[25] Summers (2016) provides several estimates of the natural rate in the U.S. based on different methodologies, and points out that, by and large, these rates commonly show a secular decline over the years.

- (1) and (2) : non-stationary and stationary shocks to neutral technology $\epsilon_{Z_a,t}$ and $\epsilon_{A_a,t}$,
- (3) and (4) : net worth shocks in the FI sector and goods-producing sector $\epsilon_{N_F,t}$ and $\epsilon_{N_E,t}$,
- (5) : shocks to the growth rate of the working-age population $\epsilon_{H,t}$,
- (6) and (7) : non-stationary and stationary shocks to investment-specific technology $\epsilon_{Z_d,t}$ and $\epsilon_{A_d,t}$,
- (8) : shocks to discount factor $\epsilon_{d,t}$, and
- (9) : shocks to external demand $\epsilon_{G,t}$.

The set of key variables we study here includes the natural rate R_t^*, the ex-ante real interest rate R_t, the expected value of the two rates R_t^* and R_t over a 10-year horizon, namely the expected natural rate and the expected ex-ante real interest rate, which we denote as $R_{40,t}^*$ and $R_{40,t}$, respectively, investment I_t, and net worth $N_{F,t}$ and $N_{E,t}$.[26]Also note that in figures that show the impulse responses, the size of the respective fundamental shocks is one standard deviation, and the sign of a shock is adjusted so that the shock delivers a decline in the natural rate at the impact period.

Responses to shocks to neutral technology

Figure 4 shows the impulse responses of the natural rate R_t^* and other variables to a non-stationary and stationary shock to neutral technology $Z_{a,t}A_{a,t}$. The first column shows the response of technology growth. In the second column, we show the response of the natural rate R_t^* and the ex-ante real interest rate R_t. In the third column, we show the response of the expected natural rate $R_{40,t}^*$ and the expected ex-ante real interest rate over a 10-year horizon $R_{40,t}$. In the fourth and fifth columns, we show the response of investment I_t and net worth $N_{F,t}$ and $N_{E,t}$.[27]

[26] We compute the expected natural rate and expected ex-ante real interest rate over a 10-year horizon at period t by $\left\{40^{-1}\sum_{s=0}^{39}\mathrm{E}_t\left[\tilde{R}_{t+s}^*\right]\right\}$ and $\left\{40^{-1}\sum_{s=0}^{39}\mathrm{E}_t\left[\tilde{R}_{t+s}\right]\right\}$, using the conditional projection of the rates over 40 quarters ahead at period t.

[27] More precisely, while we show the response of investment in both the actual economy and the flexible-price economy in the fourth column, we only show the response of FIs' net worth and entrepreneurs' net worth in the actual economy. This treatment is only for the purpose of saving space. In fact, the response of net worth to a shock studied in Figures 4 to 7 is barely altered across the two economies.

A slowdown in neutral technology growth $g_{Z_a,t}$ leads to a decline in the natural and ex-ante real interest rates, R_t^* and R_t. After the shock, output quickly starts to grow at a slower pace, reflecting the permanent change in the technology. Consumption also declines, albeit gradually, due to households' consumption smoothing motives. The declines in the real interest rate can be understood as mirroring this dynamic of consumption. A decline in the expected natural and ex-ante real interest rates over a 10-year horizon, $R_{40,t}^*$ and $R_{40,t}$, indicate that the effect of neutral technology shocks on the natural rate is long-lived. In both the actual and flexible-price economies, investment I_t and I_t^* fall, since the return on capital $R_{E,t}$ and $R_{E,t}^*$ fall due to lower capital productivity. The balance sheets of FIs and entrepreneurs are also damaged, as a result of the economic downturns induced by the change in neutral technology.

A stationary shock to neutral technology $\epsilon_{A_a,t}$ affects the natural rate in the opposite direction to what a non-stationary shock brings about. That is to say, it is a positive, namely a favorable stationary shock, rather than a negative stationary shock that reduces the natural rates R_t^*. As neutral technology rises only temporarily, households increase spending the most at periods around the impact period, and less so in subsequent quarters, so as to smooth their consumption. Interest rates fall accordingly reflecting the dynamics of consumption.[28] Because higher technology boosts output, larger investment is launched, improving the balance sheet conditions of the two sectors.

Responses to shocks to balance sheets: financial factors

Figure 5 shows the impulse responses of the natural rates R_t^* and other key variables to a negative shock to the balance sheets of the FIs and the entrepreneurs, $\epsilon_{N_F,t}$ and $\epsilon_{N_E,t}$.[29] Net worth shocks are translated to the aggregate economy in the following three steps. First, these shocks directly impair the balance sheets of the two sectors, and increase the default probabilities of entities that borrow external funds through IF or FE credit contracts. Such an increase in default probabilities results in a wider external finance

[28] The feature that the response of real interest rates (or equivalently, real wages) to a technology shock changes depending on the degree of the persistence of technology shocks has already been established in earlier literature on real business cycles. For example, King and Rebelo (1999) document in Figures 9 and 10 of their paper that the response of the real interest rate to a favorable technology shock flips its sign from positive to negative, when the nature of the technology shock changes from a long-lived to a short-lived one.

[29] These shocks are described as an innovation to equations (16) and (17) in Appendix A, respectively.

premium $R_{E,t}R_t^{-1}$ (or $R^*_{E,t}R_t^{*-1}$) borne by the ultimate borrowers, namely entrepreneurs.[30] Second, the higher external finance premium $R_{E,t}R_t^{-1}$ (or $R^*_{E,t}R_t^{*-1}$) reduces demand for external funds and investment I_t (or I_t^*), dampening economic activity. Note also that the economic downturn endogenously damages the balance sheets of the two sectors through equations (16) and (17). Third, these damaged balance sheets cause a second-round effect, similar to the first step, further increasing the default probabilities of borrowers and the external finance premium. As a result, aggregate demand is weakened, likewise further depressing consumption C_t (and C_t^*) and interest rates.

The amplifications to the aggregate economy are greater when net worth shocks strike the FI sector compared with the case when the same size of net worth shocks strike the goods-producing sector. This asymmetry is seen in the size of the decline in net worth in the two sectors at the impact period, namely in (i) and (ii) of panels (1) and (2) of Figure 5. In order for the natural rate R_t^* to fall by about 20 basis points at the impact period, the net worth of the goods-producing sector $N_{E,t}$ needs to fall by 4%, while that of the FI sector $N_{F,t}$ needs to fall by only 0.8%. This asymmetric response can also be seen in the response of investment to the two net worth shocks.[31]

Responses to shocks to the working-age population growth rate

The upper panel of Figure 6 shows the impulse responses of the variables to a negative shock to the working-age population growth rate $\epsilon_{H,t}$. This shock is a non-stationary shock that lowers the size of labor inputs for good. As indicated in equation (4), the natural rate

[30] In the model, the relationship between the external finance premium and net worth of the two sectors can be seen by arranging the investors' participation constraint (13) as follow.

$$\frac{R_{E,t+1}}{R_t} \geq \frac{[Q_t K_t - N_{E,t} - N_{F,t}]}{Q_t K_t} \times \Phi_{F,t+1}^{-1} \times \Phi_{E,t+1}^{-1}.$$

Here, the first term is the inverse of the leverage of investors' lending to the FIs, and this term decreases with net worth of the two sectors $N_{E,t}$ and $N_{F,t}$. The second and third terms represent the inverse of the share of profits of the IF contracts that goes to the investors and that of the FE contracts that goes to the FIs, respectively. Similar to the first term, they are functions of net worth. As theoretically shown in HSU (2017), when the size of net worth becomes smaller, the external premium $R_{E,t+1}R_t^{-1}$ rises through the three terms. This is because a decline in borrowers' net worth increases the default probabilities of borrowers, and lenders in credit contracts require higher ex-ante premiums from borrowers, so as to cover the expected increasing monitoring costs associated with defaulting borrowers.

[31] This observation is consistent with findings in previous studies that compare the aggregate impacts of the two net worth shocks using a model with chained credit contracts similar to ours. For example, HSU (2011, 2017) use a similar model calibrated to the U.S., and show that an impairment of net worth in the FI sector leads to a larger output decline than does that in the goods-producing sector, even if the size of impairments is the same.

R_t^* at the steady state is independent of changes in the working-age population growth rate $g_{H,t}$.[32] In the short-run, however, the natural rate varies with changes in the population growth rate, as indicated in its negative response to a decline in population growth. In the impact period, due to a induced decline in labor inputs L_t^*, the return on capital $R_{E,t}^*$ declines, reflecting the scarcity of labor inputs relative to capital inputs, which in turn depresses the natural rate R_t^*.[33] In the long-run, the effect on the natural rate R_t^* of changes in the working-age population growth rate gradually diminishes, as capital inputs K_t^* are decumulated to a level consistent with the reduced size of labor inputs L_t^*. The natural rate R_t^* then converges to the original steady state rate that is pinned down only by the two technological growth rates and the steady state discount factor.

Responses to shocks to investment-specific technology

The middle and lower panels of Figure 6 show the impulse responses of the variables to a non-stationary and stationary positive shock to investment-specific technology $\epsilon_{Z_d,t}$ and $\epsilon_{A_d,t}$. The non-stationary shock delivers an asymmetric response in the natural rates R_t^* between the short-term and the long-term. In the short-term, as discussed in Sajedi and Thwaits (2016), an improved investment-specific technology $Z_{d,t}$ leads to lower demand for savings, in terms of final goods, depressing interest rates R_t^* and R_t. On the other hand, in the long-term, a permanent increase in the productivity of producing capital inputs is translated into greater output, and households gradually increase consumption C_t(and C_t^*), boosting real interest rates.[34] In contrast to a non-stationary shock $\epsilon_{Z_d,t}$, a stationary shock to investment-specific technology $\epsilon_{A_d,t}$ delivers the short-term dynamics described above exclusively. The natural rate R_t^* therefore drops at the impact and rapidly converges to the steady state in the following quarters. Consequently, its impact on the expected

[32] This is not the case in an OG model such as that used in Sudo and Takizuka (2018), or a model that assumes a different assumption about households' utility function. See Blanchard and Fischer (1989) for a related discussion.

[33] This result is consistent with the findings documented in Sudo and Takizuka (2018). They use an OG model calibrated to Japan's economy and show that the decline in the growth rate of working-age population that has taken place in Japan has exerted downward pressure on real interest rates from the 1960s to the present.

[34] Though not shown in the paper for the purpose of saving space, consumption declines below the steady state level in a few quarters after the shock, and starts to increase, exceeding the steady state level, in subsequent years. A similar consumption response is obtained in Fisher (2006) in which the theoretical implications of non-stationary investment-specific shocks are studied.

natural rate and expected ex-ante real interest rate $R^*_{40,t}$ and $R_{40,t}$ is quantitatively small.[35]

Responses to shocks to demand factors

Figure 7 shows the impulse responses of the variables to a negative shock to discount factor $\epsilon_{d,t}$ and external demand $\epsilon_{G,t}$. In the wake of the former shock, households discount consumption in the future less and save more today, leading to a fall in the real interest rates R^*_t and R_t. Larger savings are translated into greater investment I^*_t and I_t, boosting the economy. The balance sheets of the two sectors improve, reflecting the economic expansion. A decline in external demand also results in a fall in real interest rates. Its impact on the aggregate economic condition is, however, ambiguous. Though both demand shocks are stationary shocks, the estimated autoregressive parameters ρ_d and ρ_G shown in Table 2 indicate that their impacts are fairly long-lived. Consequently, the expected natural rate and expected ex-ante real interest rate $R^*_{40,t}$ and $R_{40,t}$ fall, too.

4.2.2 Historical Decomposition of the Natural Rate

Assessing the contribution of drivers

Figure 8 shows the historical decomposition of the natural rate R^*_t into the contributions of the five drivers. The quantitatively largest driver that has shaped the secular decline in the natural rate has been shocks to neutral technology $Z_{a,t}A_{a,t}$, which is the sum of the contribution of shocks $\epsilon_{Z_a,t}$ and $\epsilon_{A_a,t}$. Throughout the 1980s, neutral technology raised the natural rate R^*_t on average by about 160 basis points. This positive effect diminished gradually and started depressing the rate in the early 2000s. In the last five years, changes in neutral technology have been exerting downward pressure of, on average, about 60 basis points. The secular decline has been reinforced by shocks to the working-age population growth rate $\epsilon_{H,t}$; by shocks to investment-specific technology $Z_{d,t}A_{d,t}$, which is the sum of the contribution of shocks $\epsilon_{Z_d,t}$ and $\epsilon_{A_d,t}$; and by shocks to demand factors, which is the sum of shocks to discount factor and external demand, $\epsilon_{d,t}$ and $\epsilon_{G,t}$. As summarized in Table 3(1), their quantitative impact in the decline of the natural rate R^*_t has been much less than that of shocks to neutral technology, which has been responsible for more than

[35] While both non-stationary and stationary investment-specific shocks are expansionary to the aggregate economy, net worth of the two sectors fall in response to these shocks. This is because favorable investment-specific shocks make the cost of producing capital goods from the final goods cheaper and the demand for the final goods fall at the impact periods. As a result, Tobin's Q falls, dampening the value of net worth accumulated in the two sectors. See HSU (2013) for the related discussion.

half of the decline from the 1980s to the present.

Financial factors, namely the sum of the contribution of net worth shocks to the two sectors $\epsilon_{N_F,t}$ and $\epsilon_{N_E,t}$ have driven the natural rate R_t^* cyclically. They boosted the natural rate in the late 1980s and in the years after the early 2000s. During the period of the banking crisis, which covers from 1992 to 2001 according to Reinhart and Rogoff (2011), they persistently depressed the natural rate R_t^* by about 100 basis points. Table 3(2) shows the decomposition of the natural rate variations from 1980 to 2017 into five components, each of which is variations of the natural rate due to variations in the five drivers.[36] According to this metric, the importance of financial factors holds second place, exceeding the contributions of shocks to the working-age population growth, investment-specific technology, and demand factors.

Comparing the banking crisis and the global financial crisis

Figure 8 is also informative about the nature of the two crises that struck Japan's economy during our sample period; the banking crisis of the 1990s, and the global financial crisis beginning in 2007. In the former crisis, financial factors depressed the natural rate R_t^* massively, indicating that disruptions to financial intermediation significantly undermined economic activity, separately from other possible causes. By contrast, in the latter crisis, financial factors were silent. This result is consistent with the views expressed in existing studies that compare these two crises. Fukuda et al. (2010) study empirically the relationship between the default probabilities of listed firms and their balance sheets information or those of their main banks during the two crisis period, and conclude that the credit crunch took place only in the former crisis and not in the latter. Along similar lines, Nakaso (2017) points out that, compared with the period of the banking crisis, prevailing financial imbalances in Japan were not sizable at the time of the global financial crisis, helping Japan's economy recover quickly to its pre-crisis state.

Comparison of our results with existing works

Our results contrast sharply with existing estimates that underscore the importance of non-technology shocks, such as shocks to discount factor. For example, Iiboshi et al.

[36] When we compute these figures of decomposition, we first compute the sum of squares for the contribution of each of the five factors and that accounted for by other factors to variations in the natural rate over the sample period. We then divide the sum of squares for each factor by the sum of the factors.

(2017) document that almost all of the variations in the natural rate in Japan from the 1980s to 2016 were caused by shocks to discount factor. In our estimate, the contributions of demand factors are small, even if the contributions of discount factor shocks and external demand shocks are combined.[37,38] Our results accord with estimates that underscore financial factors as an important driver of the natural rate in the crisis period. In the estimate of the natural rate in the U.K. by Goldby et al. (2015), risk premium shocks are the key driver of the natural rate decline during the global financial crisis, and the effects of these shocks remained substantial even in 2015. Similarly, the estimated contribution of financial factors in our model substantially hampered the natural rate at the outset of the banking crisis in the early 1990s, and continued depressing the rate for an extended period of time.

4.3 Expected Natural Rate over Long Horizons

Figure 9 shows the time path and determinants of the expected natural rate over a 10-year horizon $R^*_{40,t}$, as well as the potential growth rate released by the Research and Statistics Department of the Bank of Japan. Similar to the natural rate R^*_t, the expected natural rate exhibits a secular decline since the 1980s until 2011 when it started to ascend gradually. In contrast to the natural rate R^*_t, the expected natural rate exhibits less deviations and declines more slowly. Also, it did not fall below zero. Even during the period of the banking crisis, the rate did not plummet, in contrast with the natural rate R^*_t. In the last twenty years, the expected natural rate $R^*_{40,t}$ tracks the time path of the potential growth rate rather than the path of the natural rate R^*_t, suggesting possibility that the two series commonly capture the same long-run components.[39]

[37] There are some DSGE estimates that stress the quantitative importance of technology or productivity shocks. Gerali and Neri (2017) show that the bulk of variations in the natural rate in the U.S. from the 1970s to the present has been attributed to shocks to labour-augmented technology and marginal investment efficiency. Also, Del Negro et al. (2015) show that a decline in productivity has played an important role in depressing the U.S. natural rate in the years since 2008.

[38] In one sense, our estimate of the determinants of the natural rate is consistent with estimates based on the methodology of LW (2003) in stressing the importance of long-lived factors in the dynamics of the natural rate. For example, Holston et al. (2017) estimate the natural rates for Canada, the euro area, the U.K., and the U.S. from the 1960s or 1970s up to 2015, and show that, in all jurisdictions other than the euro area, a large portion of the movement in the natural rate and its secular decline over time are nearly entirely accounted for by a change in the trend growth in output, rather than short-run fluctuations.

[39] One other notable observation from Figure 9 is that investment-specific technology shocks have been depressing the expected natural rate since the mid-2000s. While not explicitly shown in the figure, based on our estimate, this is because a series of negative shocks has been taking place to the growth rate of investment-specific technology. This result is consistent with the view documented in Bank of Japan

Table 4 shows the decomposition of the decline in the expected natural rate over 10 years by drivers at the top, and the decomposition of the variations of the expected natural rate with different horizons by drivers at the bottom.[40] For the latter, we compute the expected natural rate over T quarters $\left\{T^{-1}\sum_{s=0}^{T-1}\mathrm{E}_t\left[R^*_{t+s}\right]\right\}$ for $T=4,\ 20,$ and 40 quarters. Changes in neutral technology maintain the dominant impact both in terms of their contribution to the secular decline and to the variations in the expected natural rate. Roughly speaking, around half of the decline and variations are attributed to these changes. The relative significance of other drivers changes somewhat. For example, the relative significance of financial factors slightly declines when a longer horizon is considered. For example, when the expected natural rate over a 10-year horizon $R^*_{40,t}$ is considered, their relative importance falls to the fourth-largest contributing factor, mirroring the fact that the effects of net worth shocks are relatively short-lived.

5 Validity Checks and Sensitivity Analysis

How reliable and robust are the estimates of the level and determinants of the natural rate obtained so far? To answer this, we conduct two sets of analyses in this section. First, we examine whether our estimates of important drivers of the natural rate appear consistent with the external data series that are considered to contain relevant information on these drivers. Since these data are not employed in our estimation exercise, checking the consistency with those data serves as a validation check of our estimates. Second, we study if our estimates are altered sizably when an alternative estimation methodology is employed. In particular, we focus on the methodologies associated with anticipated shocks to the monetary policy rule and population shocks.

5.1 Validity Check using External Data

We conduct a validity check for the two largest drivers of the natural rate R^*_t, changes in neutral technology and financial factors. The external data we use are the measured TFP

(2017) that stresses that the TFP growth rates in the IT-producing sector and IT-using sector have been decelerating since the middle of 2000s.

[40]The analysis in this section is analogous, in spirit, to what is conducted in Gerali and Neri (2017). They estimate the contributions of fundamental shocks to the natural rate in the U.S. and the euro area for various frequencies.

series released from various institutions for the former factor and the financial position index released by the Bank of Japan for the latter factor.

5.1.1 Neutral Technology

We compare the estimated time path of neutral technology $Z_{a,t}A_{a,t}$ with four external series of measured TFP, each of which is computed by a different institution; (i) Japan Productivity Center (JPC), (ii) the OECD, (iii) the Research and Statistics Department of the Bank of Japan, and (iv) the Cabinet Office of Japan.[41] Though these measures are different from each other in terms of estimation methodology, they are considered as containing the common information about the dynamics of neutral technology.[42]

Figure 10 compares the estimated neutral technology series with series (i) and (ii) in the upper panel, and with series (iii) and (iv) in the lower panel. Note that because the latter two series are computed and released as the low-frequency component of the Solow residual, we compare them with the low-frequency component of the estimated neutral technology series $Z_{a,t}A_{a,t}$.[43,44] In both high- and low-frequency components, our estimated neutral technology captures the key feature of the dynamics that is commonly seen in these alternative measures. Namely, all of the series grew rapidly in the 1980s, slowed down in the early 1990s, and continued growing at a low rate in subsequent years. All series also agree that the slowdown did not take place at once. For example, in the lower panel, the three series all exhibit two humps after the early 1990s, one around the early 2000s, and the other around the early 2010s, just after the global financial crisis.

---

[41] Regarding the construction methodology of each series, see Organisation for Economic Co-operation and Development (OECD) (2001) for the series (ii), Kawamoto et al. (2017) for the series (iii), and Yoshida (2017) for the series (iv).

[42] Similar to our measured TFP, the series (i), (iii) and (iv) are essentially the Solow residuals that are all constructed from the logarithm of output less that of labor and capital inputs. The measurements of these production inputs are, however, not the same. For example, when computing the capacity utilization ratio of the capital stock in the manufacturing sector, series (i) uses the unadjusted raw data released by the government, while series (iii) uses the series in which the downward bias is adjusted, and series (iv) uses the series constructed based on the data of industrial production and working hours.

[43] Note that in Figure 10, we show the growth rate of the measured TFP for the four external measures and the growth rate of the term $(Z_{a,t}A_{a,t})^{\frac{1}{1-\gamma}}$ instead of $(Z_{a,t}A_{a,t})$, for neutral technology estimated in our model. We make this adjustment for the purpose of comparison. As shown in the production function (22), in our model, because firms use intermediate goods as inputs, a change in neutral technology in the production function has an amplification effect on the volume of value-added produced through an increase in the use of intermediate inputs. Such effects are absent in a model where the intermediate inputs are not explicitly incorporated. We adjust for this amplification effect by the factor $(1-\gamma)^{-1}$.

[44] The low-frequency component is extracted using the Hodrick-Prescott filter with $\lambda = 1,600$.

5.1.2 Financial Factors

We also compare the estimated contribution of financial factors to the natural rate with a diffusion index, *TANKAN,* released by the Bank of Japan, which shows the financial position of non-financial firms. This index is calculated by subtracting "tight" from "easy," and larger (lower) values indicate that firms are less (more) financially constrained. Figure 11 shows the time path of this index and the contribution of financial factors.

The movements of the two series are closely aligned over the sample period, indicating that our estimate of net worth shocks successfully captures developments in financial conditions. Similar to the contribution of financial factors, the financial position index exhibits cyclical dynamics over time. The index was "easy" in the latter half of the 1980s, and turned to "tight" immediately after the bubble burst in the early 1990s. It remained "tight" for an extended period, covering the rest of the 1990s and the early 2000s, until it again became "easy" in the 2000s. As of 2017, the index records its highest level in 25 years, indicating that the financial position of firms is the easiest in the same period of time. The asymmetric nature of the two crises discussed in Section 4.2.2 is also evident in the index. In the period of the banking crisis of the 1990s, the index remained "tight" for almost 10 consecutive years. By contrast, during the period of the global financial crisis, the index turned "tight" for only two years, and recovered quickly in the following years.

5.2 Sensitivity Analysis using Alternative Estimation Methodologies

5.2.1 Anticipated Shocks to the Monetary Policy Rule

About half of our sample period overlaps with the period when the Bank of Japan has implemented a low interest rate policy, and forward guidance has played a certain role in monetary policy implementation.[45] As described above, we therefore incorporate news shocks about the future policy rate $R_{n,t}$ into the model, namely $\varepsilon_{R_{n,s},t-s}$ for $s = 1, ...12$, and estimate the time path of these shocks, exploiting information about expected short-term nominal interest rates $\mathrm{E}_t \left[R_{n,t+s}\right]$ for $s = 1, ...12$, from the OIS data. One benefit of

[45] For example, the governor of the Bank of Japan, Haruhiko Kuroda, stated in a speech in November 2017, "in the world of practice, the Bank of Japan has been a pioneer of such forward guidance. When it introduced the zero interest rate policy in 1999, the Bank explicitly committed to continuing with the zero interest rate policy "until deflationary concern is dispelled." I regard this as the first application of forward guidance in practice worldwide. Moreover, in the period of QE from 2001 to 2006, the Bank committed to continuing with QE "until the annual rate of change in the CPI registers zero percent or above in a stable manner.""

using this setting is that it allows us to evaluate separately the impact of monetary policy shocks that occur at the current period and those that are predicted to occur in the future. For our purposes, it is also important in evaluating the current level of the natural rate R_t^*.[46]

To see this, consider a simple case where the economy is at the steady state at period $t-1$, and at period t, a shock to the natural rate ϵ_0 and a news shock $\varepsilon_{R_{n,1},t-1}$ that materializes at $t+1$ strike the economy. For the sake of simplicity, we assume that all the parameter values are known, and the only unknown is the size of an adverse shock ϵ_0 that drives the natural rate. Assume further that the left hand side of equation (3), say $\tilde{c}_t - \tilde{c}_t^* = \overline{c}$, and the sequence of the expected future real interest rates are both observed. Because the future real interest rates are expressed as the linear combination of the component driven by the endogenous response of the monetary policy rule and that driven by news shocks, which we denote by $\omega_{R,m}\epsilon_0$ and $\omega_{N_1,m}\varepsilon_{R_{n,1},t}$ for $m = 1, ...\infty$, respectively,[47] the size of the shock to the natural rate ϵ_0 is expressed as follows.

$$
\begin{aligned}
\overline{c} &= \mathrm{E}_t \left[\sum_{m=0}^{\infty} \left(-\tilde{R}_{t+m} + \omega_m \epsilon_0 \right) \right] \\
&= \mathrm{E}_t \left[\sum_{s=0}^{\infty} \left(-\left(\omega_{R,m}\epsilon_0 + \omega_{N_1,m}\varepsilon_{R_{n,1},t} \right) + \omega_m \epsilon_0 \right) \right], \text{ and} \\
\epsilon_0 &= \frac{\overline{c} + \mathrm{E}_t \left[\sum_{m=0}^{\infty} \tilde{R}_{t+l} \right]}{\sum_{m=0}^{\infty} \omega_m} = \frac{\overline{c} + \mathrm{E}_t \left[\sum_{m=0}^{\infty} \omega_{N_1,m}\varepsilon_{R_{n,1},t} \right]}{\sum_{m=0}^{\infty} \left(\omega_m - \omega_{R,m} \right)}. \qquad (5)
\end{aligned}
$$

Here ω_m is the mth quarter impulse response of the natural rate $\tilde{R}_t^*$ to a one unit shock to the current natural rate and the function of the model's parameters.

Consider an alternative case, where news shock $\varepsilon_{R_{n,1},t}$ is mistakenly disregarded, and the sequence of the real interest rate $\tilde{R}_{t+s}$ is not used to estimate the shock size. In this case, the estimated shock size $\hat{\epsilon}_0$ is derived from the following equation.

---

[46] See also Aoki and Ueno (2012) that show an alternative way to extract the current level of the natural rate using the forward nominal interest rates and other macroeconomic variables such as inflation. Their methodology differs from us, among other things, in the sense that they explicitly consider a possibility that nonzero fundamental shocks may occur in quarters ahead.

[47] Here, $\omega_{R,m}$ and $\omega_{N_1,m}$ are the coefficient determined by model parameters, that represent the response of expected ex-ante real interest rate at period m to a shock to the natural rate at t, ϵ_0, and to a news shock that is anticipated at period t and materializes at period $t+1$, $\varepsilon_{R_{n,1},t}$, respectively.

$$\hat{\epsilon}_0 = \frac{\overline{c}}{\sum_{m=0}^{\infty} (\omega_m - \omega_{R,m})}. \tag{6}$$

The difference between the two values ϵ_0 and $\hat{\epsilon}_0$ is the bias that arises from not exploiting the information on expected future interest rates.

How do incorporating anticipated shocks quantitatively alter the estimated time path of the natural rate? To see this, we estimate the natural rate of the alternative model where anticipated shocks are absent. We also estimate the other alternative model where S is set to 4. Figure 12 shows the estimation results of these alternative models. Each estimation differs in terms of the number of news shocks incorporated in the model, and of the expected short-term nominal interest rate incorporated in the estimation as an observable. The upper and lower panels of the figure show the time path of the natural rate R_t^* and the expected natural rate $R_{40,t}^*$. The long-run developments in both series appear little altered by the choice of the length of horizon S. All series exhibit a common secular decline and a modest pick-up in the last few years. It is noticeable, however, that for both the natural rate and expected natural rate, the estimated rates in the baseline model with $S = 12$ tend to evolve below the estimates based on other two alternative settings over the sample period. Such a tendency is more pronounced when the expected natural rate is considered. This observation suggests a possibility that omitting the anticipated shocks may cause the underestimation of the severity of adverse shocks hitting the economy, and indicates that some caution is called for when estimating the natural rate when the sample period covers a period when forward guidance is implemented.[48]

Table 5 summarizes the contribution of the five drivers to the dynamics of the natural rate and expected natural rate, under these alternative settings. The results are qualitatively unchanged from the baseline. Namely, in all of the specifications, a change in neutral technology manifests itself as the dominant contributing factor. The significance of the contribution of financial factors is also obtained as in the case of the baseline, though their estimated contribution is more susceptible to the choice of specification.

[48] In addition to the comparison of the estimated natural rate, we compare the performance of the model without anticipated shocks and the baseline model based on the one quarter ahead forecast that is the same as that described in footnote 21. The baseline model outperforms the alternative model for most of the common observables. In particular, the forecast error of the short-term nominal interest rate $R_{n,t}$ is about the nine times larger in the latter model.

5.2.2 Demographic Factors

Our baseline setting assumes that the growth rate of the working-age population follows a stochastic trend, and its entire variations are unanticipated. In practice, however, households are to some extent knowledgeable about the prospects for the demographic landscape in the years ahead. We therefore relax the baseline assumption, and study how changes in the predictability of population dynamics alter the estimates of the natural rate and its drivers.

We carry out two alternative estimations together with the baseline estimation. The first exercise assumes that the working-age population H_t grows following the law of motion stated below:

$$\ln H_t = \ln H_{t-1} + v_{H,t},\ v_{H,t} \equiv \rho_H v_{H,t-1} + \epsilon_{H,t}. \tag{7}$$

Here, $\rho_H \in (0,1)$ is the autoregressive coefficient. The second exercise assumes that all of the changes in the working-age population are known to households in advance.

The upper panel of Figure 13 shows the estimated time path of population shocks $\epsilon_{H,t}$ under the three specifications. Under the baseline setting, shocks $\epsilon_{H,t}$ coincide with the data, varying from -20 to 20 basis points, contributing to the secular decline in the natural rate by 70 basis points. In the first alternative specification, realized shocks $\epsilon_{H,t}$ and their contribution become small compared with the case of the baseline. In the second alternative specification, the dynamics of the working-age population are entirely deterministic and these shocks and their contribution are zero.

The middle and lower panel of the figure show the time path of the natural rate R_t^* and expected natural rate $R_{40,t}^*$. While the estimated size and contribution of shocks differ noticeably across settings, the broad picture of the dynamics is unaltered. Table 6 summarizes the contribution of drivers under these alternative settings. The baseline results are again maintained. That is to say, working-age population is not likely the key driver of the natural rate, at least not in our sample period.[49]

[49] Based on an OG model calibrated to Japan's economy, Sudo and Takizuka (2018) show that about 270 out of the 640 basis point decline in the real interest rate from the 1960s to 2015 in Japan was brought about by changes in demographic landscape. Our result is not necessarily inconsistent with their finding. There are three reasons. First, the number reported in their paper also captures the effects of demographic changes that took place before the 1980s that are outside the scope of this paper. Second, some portion of demographic effects may be attributed to demand factors in our model. Related to this, it is notable that, based on Table 3, the contribution of demand shocks ranks only second in the variations of the natural

6 Concluding Remarks

In this paper, we use a DSGE model to compute the level and drivers of the natural rate from 1980 to 2017 in Japan. In particular, we study the quantitative impact of five potential drivers of the natural rate that have been considered essential in existing studies; neutral technology, investment-specific technology, financial factors, demographic landscapes, and demand factors. We design the settings of our DSGE model and choose observables used for our estimation so that our empirical exercise can serve as a "horse racing" across these drivers.

Based on our estimates, the natural rate in Japan has shown a downward trend over time. The rates were on average about 400 basis points in the 1980s, falling at a rapid pace in the early to mid-1990s, and remaining about 30 basis points over the last five years. Changes in neutral technology have played the key role in dampening the natural rate. About half of the decline has been attributed to changes in neutral technology. The other half of the decline has been attributed to changes in working-age population, investment-specific technology, and demand factors. The secular decline and the quantitative importance of neutral technology are confirmed for developments in the expected future natural rates. Changes in financial factors, which are, in our model, represented by shocks to the balance sheets of FIs and non-financial firms, had an important influence on the natural rate cyclically. During the period of the banking crisis in the 1990s, they exerted sizable downward pressure on the natural rate.

Two caveats regarding our analysis are noteworthy. First, while our analysis covers a wide range of potential drivers of the natural rate, in particular those considered the key elements in existing studies, there are drivers which remain unexplored. These include the effects arising from asset, income and consumption inequality across households, or those arising from global factors such as increasing demand for safe assets of developing countries.[50] Second, it is notable that our measure of the expected natural rate is constructed

---

rate, even when all of the demand shocks are considered to be driven by demographic factors. Because of the predictability of demographic changes, however, it is possible that those changes that actually occurred during the 1980s and beyond had already impacted households' economic decisions before the 1980s. In this case, some portions of demographic effects do not show up as demand shocks taking place in 1980 and beyond.

[50] Aside from these factors, Del Negro et al. (2017) estimate how changes in preferences for safe and liquid assets alter the natural rate in the U.S., based on a DSGE framework. Following their paper closely, we estimate the impact of changes in preferences for liquid assets on the natural rate, by extending our baseline model. We replace the utility function in our baseline model with that used in Anzoategui et al.

from expectations of short-term real interest rates exclusively, and does not include the premium components that appear in the actual data of long-term interest rates. Elaborating the current framework, and painting a more comprehensive picture of the underlying drivers of the natural rate, from both a short- and long-term perspective, are left as a future research agenda.

(2016) and estimate the natural rate and its drivers, using the data of the yield of publicly-offered local government bonds as well as data used in the baseline estimation. In contrast to the finding by Del Negro et al. (2017) for the U.S., however, we find that our findings based on the baseline estimation are little changed, and their impact on natural rate dynamics has been small in Japan.

References

[1] Anzoategui, D., D. Comin, M. Gertler, J. Martinez (2016) "Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence," NBER Working Papers 22005, National Bureau of Economic Research, Inc.

[2] Aoki, K., Y. Ueno (2012) "Identification of Structural Shocks under the Zero Lower Bound on Nominal Interest Rates," Bank of Japan Working Paper Series, No.12-E-8.

[3] Bank of Japan (2007) "Survey on Yen-OIS Market in May 2007."

[4] Bank of Japan (2017) "Outlook for Economic Activity and Prices April 2017."

[5] Barsky, R., A. Justiniano, L. Melosi (2014) "The Natural Rate of Interest and Its Usefulness for Monetary Policy," *American Economic Review*, 104, 37–43.

[6] Basu, S. (1995) "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," *American Economic Review,* 85, 512–531.

[7] Basu, S., J. G. Fernald, M. S. Kimball (2006) "Are Technology Improvements Contractionary?" *American Economic Review*, 96, 1418–1448.

[8] Bayoumi, T. (2001) "The Morning after: Explaining the Slowdown in Japanese Growth in the 1990s," *Journal of International Economics,* 53, 241–259.

[9] Bernanke, B., M. Gertler, S. Gilchrist (1999) "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, 1, 1341–1393.

[10] Blanchard, O. J., S. Fischer (1989) "Lectures on Macroeconomics," MIT press.

[11] Burriel, P., J. Fernandez-Villaverde, J. F. Rubio-Ramirez (2010) "MEDEA: A DSGE Model for the Spanish Economy," *SERIEs*, 1, 175–243.

[12] Carvalho, C., A. Ferrero, F. Nechio (2016) "Demographics and Real Interest Rates: Inspecting the Mechanism," *European Economic Review*, 88, 208–226.

[13] Cette, G., J. Fernald, B. Mojon (2016) "The Pre-Great Recession Slowdown in Productivity," *European Economic Review*, 88, 3–20.

[14] Christiano, L., R. Motto, M. Rostagno (2010) "Financial Factors in Economic Fluctuations," ECB Working Paper Series, No. 1192.

[15] Christiano, L., R. Motto, M. Rostagno (2014) "Risk Shocks," *American Economic Review*, 104, 27–65.

[16] Curdia, V., A. Ferrero, G. C. Ng, A. Tambalotti (2015) "Has U.S. Monetary Policy Tracked the Efficient Interest Rate?" *Journal of Monetary Economics*, 70, 72–83.

[17] Del Negro, M., D. Giannone, M. P. Giannoni, A. Tambalotti (2017) "Safety, Liquidity, and the Natural Rate of Interest," Federal Reserve Bank of New York, Staff Report, No. 812.

[18] Del Negro, M., M. P. Giannoni, M. Cocci, S. Shahanaghi, M. Smith (2015) "Why Are Interest Rates So Low?" Federal Reserve Bank of New York, Liberty Street Economics (NY Fed Blog), May 20, 2015.

[19] Edge, R. M., M. T. Kiley, J. Laforte (2008) "Natural Rate Measures in an Estimated DSGE Model of the U.S. Economy," *Journal of Economic Dynamics and Control*, 32, 2512–2535.

[20] Eggertsson, G., N. Mehrotra, J. Robbins (2017) "A Model of Secular Stagnation: Theory and Quantitative Evaluation," NBER Working Papers 23093, National Bureau of Economic Research, Inc.

[21] Eichengreen, B., D. Park, K. Shin (2017) "The Global Productivity Slump: Common and Country-specific Factors," *Asian Economic Papers*, MIT Press, 16, 1–41.

[22] Fisher, J. D. M. (2006) "The Dynamic Effects of Neutral and Investment-specific Technology Shocks," *Journal of Political Economy*, 114, 413–451.

[23] Fries, S., J. Mesonnier, S. Mouabbi, J. Renne (2016) "National Natural Rates of Interest and the Single Monetary Policy in the Euro Area," Banque de France, Working Paper, No. 611.

[24] Fujiwara, S., Y. Iwasaki, I. Muto, K. Nishizaki, N. Sudo (2016) "Developments in the Natural Rate of Interest in Japan," Bank of Japan Review Series, 2016-E-12.

[25] Fukuda, S., M. Kasuya, K. Akashi (2010) "Futatsu no Kinyu Kiki to Waga Kuni no Kigyo Hatan," Bank of Japan Working Paper Series, No.10-J-16 (in Japanese).

[26] Gagnon, E., B. K. Johannsen, D. Lopez-Salido (2016) "Understanding the New Normal: The Role of Demographics," Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 2016-080.

[27] Gerali. A., S. Neri (2017) "Natural Rates across the Atlantic," Bank of Italy, Temi di discussione (Economic working papers) 1140.

[28] Gilchrist, S., J. Leahy (2002) "Monetary Policy and Asset Prices," *Journal of Monetary Economics,* 49, 75–97.

[29] Goldby, M., L. Laureys, K. Reinold (2015) "An Estimate of the UK's Natural Rate of Interest," Bank of England, Bank Underground, August 11, 2015.

[30] Greenwood, J., Z. Hercowitz, P. Krusell (1997) "Long-run Implications of Investment-specific Technological Change," *American Economic Review*, 87, 342–362.

[31] Hakkio, C. S., A. L. Smith (2017) "Bond Premiums and the Natural Real Rate of Interest," Federal Reserve Bank of Kansas City, *KCFED Economic Review*, 102, 5–40.

[32] Hayashi, F., E. Prescott (2002) "The 1990s in Japan: A Lost Decade," *Review of Economic Dynamics*, 5, 206–235.

[33] Hirakata, N., N. Sudo, I. Takei, K. Ueda (2016) "Japan's Financial Crises and Lost Decades," *Japan and the World Economy,* 40, 31–46.

[34] Hirakata, N., N. Sudo, K. Ueda (2011) "Do Banking Shocks Matter for the U.S. Economy?" *Journal of Economic Dynamics and Control*, 35, 2042–2063.

[35] Hirakata, N., N. Sudo, K. Ueda (2013) "Capital Injection, Monetary Policy and Financial Accelerators," *International Journal of Central Banking,* 9, 101–145.

[36] Hirakata, N., N. Sudo, K. Ueda (2017) "Chained Credit Contracts and Financial Accelerators," *Economic Inquiry*, 55, 565–579.

[37] Holston, K., T. Laubach, J. C. Williams (2017) "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics*, 108, 59–75.

[38] Hoshi, T., A. Kashyap (2010) "Will the U.S. Bank Recapitalization Succeed? Eight Lessons from Japan," *Journal of Financial Economics*, 97, 398–417.

[39] Hristov, A. (2016) "Measuring the Natural Rate of Interest in the Eurozone: A DSGE Perspective," CESifo Forum, Ifo Institute - Leibniz Institute for Economic Research at the University of Munich, 17, 86–91.

[40] Iiboshi, H., M. Shintani, K. Ueda (2017) "Estimating the Nonlinear New Keynesian Model with the Zero Lower Bound for Japan," mimeo.

[41] Ikeda, D., M. Saito (2014) "The Effects of Demographic Changes on the Real Interest Rate in Japan," *Japan and the World Economy*, 32, 37–48.

[42] Imakubo, K., H. Kojima, J. Nakajima (2015) "The Natural Yield Curve: Its Concept and Measurement," Bank of Japan Working Paper Series, No.15-E-5.

[43] Justiniano, A., G. Primiceri (2010) "Measuring the Equilibrium Real Interest Rate," Federal Reserve Bank of Chicago, Chicago FED Economic Perspectives, 2010-1Q.

[44] Kaihatsu, S., T. Kurozumi (2014) "What Caused Japan's Great Stagnation in the 1990s? Evidence from an Estimated DSGE Model," *Journal of the Japanese and International Economies,* 34, 217–235.

[45] Kawamoto, T., T. Ozaki, N. Kato, K. Maehashi (2017) "Methodology for Estimating Output Gap and Potential Growth Rate: An Update," Bank of Japan Research Papers 17-05-31.

[46] King, R. G., S. T. Rebelo (1999) "Resuscitating Real Business Cycles," *Handbook of Macroeconomics*, 1, 927–1007.

[47] Kuroda, H. (2017) "Quantitative and Qualitative Monetary Easing and Economic Theory," Speech at the University of Zurich in Switzerland, November 13, 2017.

[48] Laseen, S., L. E. O. Svensson (2011) "Anticipated Alternative Policy Rate Paths in Policy Simulations," *International Journal of Central Banking,* 7, 1–35.

[49] Laubach, T., J. C. Williams (2003) "Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, 85, 1063–1070.

[50] Lewis, K. F., F. Vazquez-Grande (2017) "Measuring the Natural Rate of Interest: Alternative Specifications," Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series No. 2017-059.

[51] Muto, I., N. Sudo, S. Yoneyama (2016) "Productivity Slowdown in Japan's Lost Decades: How Much of It Can Be Attributed to Damaged Balance Sheets?" Bank of Japan Working Paper Series, No.16-E-3.

[52] Nakajima, J., T. Watanabe (2017) "Econometric Analysis of Japanese Exports Using a Time-Varying Parameter Vector Autoregressive Model," *Keizai Kenkyu* 68, 237–249 (in Japanese).

[53] Nakaso, H. (2017) "New Frontier of Macroprudential Policy: Addressing Financial Institutions' Low Profitability and Intensified Competition," Speech at the Kin'yu Konwa Kai (Financial Discussion Meeting) Hosted by the Jiji Press, November 29, 2017.

[54] Nolan, C., C. Thoenissen (2009) "Financial Shocks and the US Business Cycle," *Journal of Monetary Economics,* 56, 596–604.

[55] Organisation for Economic Co-operation and Development (OECD) (2001) "Measuring Productivity – OECD Manual: Measurement of Aggregate and Industry-level Productivity Growth." OECD Publishing.

[56] Pescatori, A., J. Turunen (2016) "Lower for Longer: Neutral Rate in the U.S.," *IMF Economic Review*, 64, 708–731.

[57] Rachel, L., T. Smith (2015) "Secular Drivers of the Global Real Interest Rate" Bank of England Working Paper, No. 571.

[58] Reinhart, C. M., K. S. Rogoff (2011) "From Financial Crash to Debt Crisis," *American Economic Review*, 101, 1676–1706.

[59] Sajedi, R., G. Thwaites (2016) "Why Are Real Interest Rates So Low? The Role of the Relative Price of Investment Goods," *IMF Economic Review,* 64, 635–659.

[60] Smets, F., R. Wouters (2007) "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

[61] Sudo, N., Y. Takizuka (2018) "Population Aging and the Real Interest Rate in the Last and Next 50 Years – A Tale Told by an Overlapping Generations Model –," Bank of Japan Working Paper Series, No.18-E-1.

[62] Sugo, T., K. Ueda (2008) "Estimating a Dynamic Stochastic General Equilibrium Model for Japan," *Journal of the Japanese and International Economies*, 22, 476–502.

[63] Summers, L. H. (2013) Remarks at the IMF Fourteenth Annual Research Conference in Honor of Stanley Fischer, Washington, DC.

[64] Summers, L. H. (2014) "U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound," *Business Economics*, 49, 65–73.

[65] Summers, L. H. (2016) "Secular Stagnation and Monetary Policy," Federal Reserve Bank of St. Louis, *Review*, 98, 93–110.

[66] Yoshida, M. (2017) "A New Methodology for Estimating GDP Gap and Potential Output in Japan," Cabinet Office, Economic and Fiscal Policy Discussion Paper Series, DP 17-3 (in Japanese).

A Model

The economy consists of four sectors: the household sector, the FI sector, the goods-producing sector, and the government sector.

- Household sector: The household sector consists of a continuum of households and investors. Each household is composed of H_t identical workers and supplies labor inputs to the goods-producing sector, earns wages, makes deposits to the investors, and receives repayments in return from deposits. The investors collect deposits from the households at risk-free rates and lend them to the FI sector by making credit contracts with the FI sector, which we call IF contracts.

- FI sector: The FIs raise external funds from the investors through the IF contracts and lend these funds, as well as their net worth, to the entrepreneurs in the goods-producing sector by making credit contracts, which we call FE contracts.

- The goods-producing sector consists of the entrepreneurs, the capital goods producers, and the goods producers. The entrepreneurs raise external funds from the FIs, purchase capital goods from the capital goods producers, and lend them to the goods producers for the rental price. The capital goods producer purchases final goods from the goods producers and converts them into capital goods. The goods producers produce goods from labor inputs, capital inputs, and intermediate goods.

- The government sector consists of the government, which collects taxes from households and spends it on government purchases, and the central bank, which adjusts the nominal interest rate so as to stabilize the inflation rate.

A.1 Credit Contracts

We consider two types of credit contracts. The first type of contract is a deposit contract and is made between households' agents, which we call investors, and FIs. We call these contracts IF contracts. The second type of contract is made between the FIs and entrepreneurs in the goods-producing sector, and we refer to these contracts as FE contracts.

FIs behave monopolistically in the two contracts and choose the borrowing rates so as to maximize their profits.

A.1.1 FE contracts

Setting

In period t, each type i FI offers a loan contract to an infinite number of group j_i entrepreneurs.[51] An entrepreneur in group j_i owns net worth $N_{E,j_i,t}$ and purchases capital of $Q_t K_{j_i,t}$, where Q_t is the price of capital and $K_{j_i,t}$ is the quantity of capital purchased by a group j_i entrepreneur. If the net worth $N_{E,j_i,t}$ is smaller than the amount of the capital purchase $Q_t K_{j_i,t}$, the entrepreneur raises the rest of the fund $Q_t K_{j_i,t} - N_{E,j_i,t}$ by making a credit contract with the type i FI.[52] In period $t+1$, a group j_i entrepreneur receives return of $R_{E,t+1}\omega_{E,j_i,t+1}$ from holding the capital $K_{j_i,t}$, where $R_{E,t+1}$ is the aggregate return on capital and $\omega_{E,j_i,t+1}$ is an idiosyncratic productivity shock that is specific to the group j_i entrepreneurs.[53] There are informational asymmetries, and a type i FI cannot observe the realization of its borrower's idiosyncratic shock $\omega_{E,j_i,t+1}$, unless it pays the monitoring cost. As in the conventional costly state verification problem, a type i FI specifies:

- the amount of debt that a group j_i entrepreneur borrows from a type i FI, $Q_t K_{j_i,t} - N_{E,j_i,t}$, and
- the cut-off value of idiosyncratic productivity shock $\omega_{E,j_i,t+1}$, which we denote by $\overline{\omega}_{E,j_i,t+1}$, such that a group j_i entrepreneur repays its debt if $\omega_{E,j_i,t+1} \geq \overline{\omega}_{E,j_i,t+1}$ and declares a default if otherwise.

Entrepreneur's participation constraint

[51] We assume that the size of the monitoring cost associated with the credit contracts between a type i FI and group j_{i^*} entrepreneurs for $i \neq i^*$ is so high that group j_{i^*} entrepreneurs do not choose to raise funds from a type i FI. By the same assumption, a direct credit contract between the investors and the entrepreneurs is left out from our analysis.

[52] As in BGG (1999), we assume below that net worth does not accumulate infinitely and that the entrepreneurs always raise external funds at the equilibrium. The same argument applies to FIs in the IF contracts.

[53] Following BGG (1999), the idiosyncratic productivity shock is a unit mean, lognormal random variable distributed independently over time and across entrepreneurs. We express its density function by $f_E(\bullet)$ and its cumulative distribution function by $F_E(\bullet)$.

A group j_i entrepreneur joins an FE contract only when the return from joining the contract is at least equal to the opportunity cost. In the FE contract, if the entrepreneur does not default, ex-post, it receives

$$\left(\omega_{E,j_i,t+1} - \overline{\omega}_{E,j_i,t+1}\right) R_{E,t+1} Q_t K_{j_i,t}.$$

The entrepreneurial loan rate $r_{E,j_i,t+1}$ is therefore given by

$$r_{E,j_i,t+1} \equiv \frac{\overline{\omega}_{E,j_i,t+1} R_{E,t+1} Q_t K_{j_i,t}}{Q_t K_{j_i,t} - N_{E,j_i,t}}. \tag{8}$$

Instead of participating in the contract, a group j_i entrepreneur can purchase capital using its own net worth $N_{E,j_i,t}$ and receive the return from holding the capital. In this case, ex-ante, the entrepreneur expects to receive the earning $\mathrm{E}_t\left[\omega_{E,j_i,t+1} R_{E,t+1} N_{E,j_i,t}\right]$, which is equal to $\mathrm{E}_t\left[R_{E,t+1} N_{E,j_i,t}\right]$, and ex-post it receives the earning $\omega_{E,j_i,t+1} R_{E,t+1} N_{E,j_i,t}$. The FE contract is agreed by a group j_i entrepreneur therefore only when the following inequality holds:

$$\begin{aligned} \mathrm{E}_t\left[\left(\int_{\overline{\omega}_{E,j_i,t+1}}^{\infty} \left(\omega_E - \overline{\omega}_{E,j_i,t+1}\right) dF_E\left(\omega_E\right)\right) R_{E,t+1} Q_t K_{j_i,t}\right] \\ \geq \mathrm{E}_t\left[\omega_{E,j_i,t+1} R_{E,t+1} N_{E,j_i,t}\right] \text{ for } \forall j_i. \end{aligned} \tag{9}$$

Note that E_t is the expectation operator.

FIs' earning from FE contracts

The earning of a type i FI from FE contracts is repayments from non-defaulting entrepreneurs minus the monitoring cost paid to assess defaulting entrepreneurs' assets. The expected earnings of a type i FI from FE contracts with group j_i entrepreneurs is thus described as follows

$$\mathrm{E}_t\left[\Phi_{E,i,t+1} R_{E,t+1} Q_t K_{j_i,t}\right],$$

where

$$\Phi_{E,i,t+1} \equiv \int_{\overline{\omega}_{E,j_i,t+1}}^{\infty} \overline{\omega}_{E,j_i,t+1} dF_E(\omega_E) + \int_0^{\overline{\omega}_{E,j_i,t+1}} \omega_E dF_E(\omega_E) - \mu_E \int_0^{\overline{\omega}_{E,j_i,t+1}} \omega_E dF_E(\omega_E). \tag{10}$$

$\Phi_{E,i,t+1}$ in the equation (10) has three terms. The first term stands for the repayment made by the non-defaulting entrepreneurs, the second term stands for realized returns of the defaulting entrepreneurs, and the third term stands for the monitoring cost that the FI pays. The total monitoring cost paid by the FI is given by the third term multiplied by $R_{E,t+1}Q_tK_{j_i,t}$, and the parameter μ_E governs the size of the monitoring cost.

It is also notable that because of constant returns to scale in production and monitoring technology, a type i FI makes contracts with an infinite number of group j_i entrepreneurs with the same size of cut-off value $\overline{\omega}_{E,j_i,t+1}$. In the discussion below, therefore, we drop the subscript j_i.

A.1.2 IF contracts

Setting

An IF contract is made between an investor and a continuum of the FIs. As explained above, in period t, each type i FI, holding the net worth $N_{F,i,t}$, makes loans to group j_i entrepreneurs at an amount of $Q_tK_{i,t} - N_{E,i,t}$, where $K_{i,t}$ is the total amount of capital purchased by group j_i entrepreneurs, and $N_{E,i,t}$ is the total amount of net worth held by group j_i entrepreneurs. An FI i's net worth is smaller than its loans to the entrepreneurs and it raises the external funds $Q_tK_{i,t} - N_{E,i,t} - N_{F,i,t}$ from the investor. After receiving earnings from the FE contracts, an FI is hit by an idiosyncratic productivity shock $\omega_{F,i,t+1}$ that represents technological differences across FIs regarding, for example, management of credit and liquidity risk or loan securitization. Consequently, ex-post, the FI's revenue from the FE contracts after the realization of the idiosyncratic productivity shock is given by[54]

$$\omega_{F,i,t+1}\Phi_{E,i,t+1}R_{E,t+1}Q_tK_{i,t}.$$

[54] We assume that the FI's idiosyncratic productivity shock is a unit mean, lognormal random variable distributed independently over time and across type i FIs. Its density function and its cumulative distribution function are given by $f_F(\bullet)$ and $F_F(\bullet)$, respectively.

There are informational asymmetries between the investor and the FI. The investor can observe the realization of the idiosyncratic shock only if it pays the monitoring cost. Under these circumstances, as with FE contracts, the IF contract specifies:

- the amount of debt that a type i FI borrows from the investor, $Q_t K_{i,t} - N_{E,i,t} - N_{F,i,t}$, and
- the cut-off value of idiosyncratic shock $\omega_{F,i,t+1}$, which we denote by $\overline{\omega}_{F,i,t+1}$, such that the FI repays its debt if $\omega_{F,i,t+1} \geq \overline{\omega}_{F,i,t+1}$ and declares a default if otherwise.

As a result of the IF contracts, a portion of the FIs $\int_{\overline{\omega}_{F,i,t+1}}^{\infty} dF_F(\omega_F)$ do not default, while the remainder default. Ex-post, a default FI i receives nothing and a non-default FI i receives the earnings shown below:

$$\left(\omega_{F,i,t+1} - \overline{\omega}_{F,i,t+1}\right) \Phi_{E,i,t+1} R_{E,t+1} Q_t K_{i,t}. \tag{11}$$

The loan rate that is paid by a non-default FI i to an investor is therefore given by

$$r_{F,i,t+1} \equiv \frac{\overline{\omega}_{F,i,t+1} \Phi_{E,i,t+1} R_{E,t+1} Q_t K_{i,t}}{Q_t K_{i,t} - N_{E,i,t} - N_{F,i,t}}. \tag{12}$$

Investor's participation constraint

An investor participates in an IF contract only when the IF contract is more advantageous. Denoting the risk-free rate in the economy by R_t, an investor's net receipt from the IF contracts must at least equal the return from a risk-free investment. That is for $\forall i$,

$$\Phi_{F,i,t+1} \Phi_{E,i,t+1} R_{E,t+1} Q_t K_{i,t} \geq R_t \left[Q_t K_{i,t} - N_{E,i,t} - N_{F,i,t}\right], \tag{13}$$

where

$$\Phi_{F,i,t+1} \equiv \int_{\overline{\omega}_{F,i,t+1}}^{\infty} \overline{\omega}_{F,i,t+1} dF_F(\omega_F) + \int_0^{\overline{\omega}_{F,i,t+1}} \omega_F dF_F(\omega_F) - \mu_F \int_0^{\overline{\omega}_{F,i,t+1}} \omega_F dF_F(\omega_F). \tag{14}$$

$\Phi_{F,i,t+1}$ has a similar structure to $\Phi_{E,i,t+1}$, as shown in the equation (10). In particular, it is notable that the third term of $\Phi_{F,i,t+1}$ multiplied by the term $\Phi_{E,i,t+1} R_{E,t+1} Q_t K_{i,t}$

shows the total amount of monitoring cost paid by an investor. These costs are used to monitor the outputs of defaulting FIs rather than those of defaulting entrepreneurs.[55,56]

A.1.3 Optimal credit contracts chosen by FIs

At the end of period t, given its own net worth $N_{F,i,t}$ and entrepreneurial net worth $N_{E,i,t}$, a type i FI chooses the terms of the IF and FE contracts so as to maximize its expected profit at the end of the period $t+1$. The terms consist of the amount of loans $Q_tK_{i,s}-N_{E,i,s}$ and borrowings $Q_tK_{i,t}-N_{E,i,t}-N_{F,i,t}$, and the cut-off values $\overline{\omega}_{F,i,t+1}$ and $\{\overline{\omega}_{E,j_i,t+1}\}_{j_i=1}^{\infty}$. As shown in equation (11), the FI's expected profit is given by the FI's revenue minus repayment to investors:

$$\mathrm{E}_t\left[\left(\int_{\overline{\omega}_{F,i,t+1}}^{\infty}\left(\omega_F-\overline{\omega}_{F,i,t+1}\right)dF_F\left(\omega_F\right)\right)\Phi_{E,i,t+1}R_{E,t+1}Q_tK_{i,t}\right] \tag{15}$$

The FI maximizes the term (15) subject to the investor's participation constraint (13) and entrepreneurial participation constraint (9) for all of the group j_i entrepreneurs. As discussed in HSU (2011, 2013), because of constant returns to scale in production and monitoring technology, the expected profit of a type i FI is the same as that of other types of FIs. In what follows, therefore, we drop the subscript i as well.

A.1.4 Dynamic behavior of net worth

The main source of net worth accumulation for the FIs and the entrepreneurs is the earnings from the credit contracts discussed above. In addition, there are two other sources of earnings. First, the FIs and entrepreneurs inelastically supply a unit of labor to the goods producers and receive in return labor income that is depicted by $W_{F,t}$ and $W_{E,t}$, respectively.[57] Second, the net worth accumulation is affected by exogenous disturbances $\varepsilon_{N_{F,t+1}}$ and $\varepsilon_{N_{E,t+1}}$. These shocks are i.i.d. and orthogonal to the earnings from the credit

[55] The two terms $\Phi_{F,i,t}$ and $\Phi_{E,i,t}$ are interpreted as the net share of profits going to the lender in the IF and FE contracts respectively.

[56] It is important to note that, as in BGG (1999), we assume that both FE and IF contracts are contingent on aggregate states and the participation constraints (9) and (13) hold with equality state by state. See for example footnote 16 of Christiano, Motto, and Rostagno (2014, hereafter CMR) for a related discussion. Regarding the IF contracts, we further assume that investors face perfect competition, and at the equilibrium, their earnings from the IF contracts are equal to the amount of repayment to households in every state of the economy.

[57] See BGG (1999) for the reason for introducing inelastic labor supply from the FIs and the entrepreneurs.

contracts. The aggregate net worths of the FIs and the entrepreneurs then evolve according to the equations below:

$$N_{F,t+1} = \gamma_F V_{F,t+1} + \frac{W_{F,t}}{P_t} + \varepsilon_{N_{F,t+1}}, \text{ and} \tag{16}$$

$$N_{E,t+1} = \gamma_E V_{E,t+1} + \frac{W_{E,t}}{P_t} + \varepsilon_{N_E,t+1}, \tag{17}$$

with

$$\begin{aligned} V_{F,t+1} &\equiv (1 - \Gamma_F(\overline{\omega}_{F,t+1}))\,\Phi_E(\overline{\omega}_{E,t+1})\,R_{E,t+1}Q_tK_t, \text{ and} \\ V_{E,t+1} &\equiv (1 - \Gamma_E(\overline{\omega}_{E,t+1}))\,R_{E,t+1}Q_tK_t, \end{aligned}$$

where

$$\begin{aligned} \Gamma_F(\overline{\omega}_{F,t+1}) &\equiv \int_{\overline{\omega}_{F,t+1}}^{\infty} \overline{\omega}_{F,t+1} dF_F(\omega_F) + \int_0^{\overline{\omega}_{F,t+1}} \omega_F dF_F(\omega_F), \text{ and} \\ \Gamma_E(\overline{\omega}_{E,t+1}) &\equiv \int_{\overline{\omega}_{E,t+1}}^{\infty} \overline{\omega}_{E,t+1} dF_E(\omega_E) + \int_0^{\overline{\omega}_{E,t+1}} \omega_E dF_E(\omega_E). \end{aligned}$$ [58]

Here, P_t denotes the nominal price of consumption goods. Note that we assume that FIs and entrepreneurs survive into the next period with a probability γ_F and γ_E, and those who are in business in period t and fail to survive into period $t+1$ consume $(1-\gamma_E)\,V_{E,t+1}$ and $(1-\gamma_F)\,V_{F,t+1}$ and exit from the economy.

A.2 Households

As described in the main text, there is a continuum of households indexed by $h \in [0, 1]$, and each household is composed of H_t identical workers. Denoting total consumption and labor inputs and the variables divided by the household's size as $C_t(h)$, $L(h)$, $c(h)$, and $l(h)$, respectively, the household's expected utility $U_t(h)$ is given by the following equation.

$$U_t(h) \equiv \mathrm{E}_t \left[\sum_{q=0}^{\infty} \beta^q d_{t+q} H_{t+q} \left\{ \ln(c_{t+q}(h) - \theta_h c_{t+q-1}(h)) - \varphi \frac{l_{t+q}(h)^{1+v}}{1+v} \right\} \right], \tag{18}$$

where $\beta \in (0,1)$ is the discount factor, $\theta_h \in (0,1)$ is the degree of internal habit persistence in consumption preferences, $v > 0$ is the inverse of the Frisch labor-supply elasticity, and φ is the weighting assigned to leisure. The variable d_t is a discount factor shock.

The budget constraint for household h is given by

$$c_t(h) H_t + s_t(h) H_t \leq \left[\begin{array}{l} \frac{W_t(h) l_t(h) H_t}{P_t} - \frac{\kappa_w}{2} \left(\frac{W_t(h)}{W_{t-1}(h)} - \bar{\pi}\bar{\gamma} \right)^2 \frac{W_t L_t}{P_t} \\ + R_{t-1} s_{t-1}(h) H_{t-1} + \frac{\Omega_t(h) - \tau_t(h)}{P_t}, \end{array} \right], \tag{19}$$

where $s_t(h)$ is the real saving per a household member, R_t is the real interest rate on deposit, $\Omega_t(h)$ is the nominal profit paid to the household, and τ_t is the lump-sum nominal tax taken by the government. $W_t(h)$ is the nominal wage set by a household h and W_t is the aggregate index of the nominal wage. The second term in the right hand side of the equation is the nominal cost the household needs to pay when it adjusts nominal wage $W_t(h)$, and κ_w, $\overline{\pi}$, and $\overline{\gamma}$ is the parameter associated with the adjustment cost, $\bar{\pi}$ is the steady state inflation rate, and $\bar{\gamma}$ is steady state growth rate of the output on a per capita basis and defined in the equation (4).[59]

A household h has monopolistic power in its differentiated labor input $L_t(h)$. The demand of the differentiated labor is given by

$$L_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\theta_{W,t}} L_t, \tag{20}$$

where L_t is the aggregate index of labor inputs that is defined as

$$L_t = \left[\int_0^1 L_t(h)^{(\theta_{W,t}-1)/\theta_{W,t}} dh \right]^{\theta_{W,t}/(\theta_{W,t}-1)},$$

where $\theta_{W,t} \in (1,\infty)$ is the time-varying elasticity of labor demand for differentiated labor input with respect to wages.

A.3 Goods producers

The goods-producing sector comprises a continuum of firms, each producing differentiated products, indexed by $l \in [0,1]$, as $\{Y_{g,t}(l)\}_{l\in[0,1]}$, and the aggregators that purchase the differentiated goods in a perfectly competitive market and construct the composite of the

[59] The functional form of the nominal wage adjustment costs implies that the costs are zero at the steady state where households adjust their wages by the rate equal to $\overline{\pi\gamma}$.

goods $Y_{g,t}$.

Production function

The aggregators produce the composite of the differentiated goods $Y_{g,t}$ by the technology described below.

$$Y_{g,t} = \left[\int_0^1 Y_{g,t}\left(l\right)^{(\theta_{P_Y,t}-1)/\theta_{P_Y,t}} \, dl \right]^{\theta_{P_Y,t}/(\theta_{P_Y,t}-1)},$$

where $\theta_{P_Y,t} \in (1, \infty)$ denotes the elasticity of substitution between differentiated products. Note also that the demand for each of the differentiated products produced by firm l is derived from the optimization behavior of the aggregator and is represented by

$$Y_{g,t}\left(l\right) = \left[\frac{P_t\left(l\right)}{P_t} \right]^{-\theta_{P_Y,t}} Y_{g,t}, \tag{21}$$

where $\{P_t\left(l\right)\}_{l \in [0,1]}$ is the nominal price of the differentiated products. These prices are related to the nominal price of the final goods by

$$P_t = \left[\int_0^1 P_t\left(l\right)^{1-\theta_{P_Y,t}} \, dl \right]^{\frac{1}{1-\theta_{P_Y,t}}}.$$

The production function of a differentiated firm l is given by

$$Y_{g,t}\left(l\right) = Z_{a,t} A_{a,t} \Psi_t\left(l\right)^{\gamma} \left[L_t\left(l\right)^{\alpha}\right]^{1-\gamma} \left[\left(K_{t-1}\left(l\right) U_t\left(l\right)\right)^{1-\alpha-\alpha_E-\alpha_{FI}} \right]^{1-\gamma} - F_t \tag{22}$$

Here, $Z_{a,t}$ and $A_{a,t}$ are a non-stationary and stationary component of neutral technology, respectively, and $L_t\left(l\right)$, $K_{t-1}\left(l\right)$, and $U_t\left(l\right)$ are labor inputs, capital stock, and capacity utilization rate of the capital stock in firm l.[60]

[60] We assume that the capacity utilization rate U_t is determined by entrepreneurs, and a firm l determines only the product $K_{t-1}\left(l\right) U_t\left(l\right)$. We also assume that entrepreneurs need to pay the real cost of

$$\frac{\kappa_U \left(U_t^{\Upsilon_U+1} - 1 \right)}{\Upsilon_U + 1},$$

in choosing the capacity utilization rate of capital U_t. Here κ_U, Υ_U are parameters. The real net return on capital K_{t-1} received by the entrepreneurs can then be expressed by the following equation.

$$R_{E,t} = \frac{\frac{U_t \tilde{R}_{E,t}}{P_t} - \frac{\kappa_U \left(U_t^{\Upsilon_U+1} - 1 \right)}{\Upsilon_U + 1} + (1-\delta) Q_t}{Q_{t-1}}.$$

where $\tilde{R}_{E,t}$ is the nominal gross return to capital inputs, $K_{t-1}(l) U_t(l)$.

Parameters γ and α are the cost share of intermediate inputs and labor inputs, respectively, and F_t is a fixed cost which is exogenous to firms.[61] Note that a firm l is a price-takers in the input markets, and its cost-minimization problem yields the following marginal cost function $MC_t(l)$:

$$MC_t(l) = \frac{\bar{\phi} P_t^{\gamma}}{A_{a,t} Z_{a,t}} \left[W_t^{\alpha} W_{E,t}^{\alpha_E} W_{F,t}^{\alpha_{FI}} \tilde{R}_{E,t}^{1-\alpha-\alpha_E-\alpha_{FI}} \right]^{1-\gamma}, \tag{23}$$

where $\bar{\phi}$ is a constant.

Price setting

A differentiated firm l is a monopolistic competitor in the product market, and it confronts Rotemberg-type nominal rigidity. Its optimization problem is formalized as choosing its product price $P_t(l)$ in reference to the demand given by (21) and the price adjustment costs as described below:

$$\max_{P_t(l)} \mathrm{E}_t \left[\sum_{q=0}^{\infty} \beta^{t+q} \frac{\Lambda_{t+q}}{\Lambda_t} \frac{\Pi_{t+q}(l)}{P_{t+q}} \right] \tag{24}$$

$$\begin{aligned} s.t.\ \Pi_{t+q}(l) \;=\;& P_{t+q}(l) Y_{g,t+q}(l) - MC_{t+q}(l) \left(Y_{g,t+q}(l) + F_{t+q} \right) \\ & -\frac{\kappa_p}{2} \left(\frac{P_{t+q}(l)}{P_{t+q-1}(l)} - \bar{\pi} \right)^2 P_{t+q} Y_{g,t+q} \end{aligned} \tag{25}$$

where Λ_{t+q} is the Lagrange multiplier associated with budget constraint (19) in period $t+q$, and κ_p is the parameter that governs the price adjustment costs.

How the composite is used

The composite $Y_{g,t}$ serves either as final goods, such as consumption goods and investment goods, as intermediate production inputs, or as goods that are used for financial

[61] Following CMR (2010, 2014), the size of the fixed cost F_t is set so that the profits from operating in the goods-producing sector are zero at the steady state. Following CMR (2010, 2014), we further assume that the fixed cost F_t exogenously grows at the same growth rate as does the non-stationary component of $Y_{g,t}(l)$, that is $Z_{a,t}^{\frac{1}{(\alpha+\alpha_E+\alpha_F)(1-\gamma)}}$, and that firms stop producing goods if the fixed cost exceeds the first term of equation (22).

intermediation activity, namely monitoring costs. The allocation of the gross output is given by

$$
\begin{aligned}
Y_{g,t} &= \overbrace{C_t + \frac{I_t}{Z_{d,t}A_{d,t}} + G_t +}^{\text{Final Output}} \overbrace{\int_0^1 \Psi_t(l)\, dl}^{\text{Intermediate Inputs}} + \overbrace{\frac{\kappa_U \left(U_t^{\gamma_U+1} - 1\right)}{\Upsilon_U + 1} K_{t-1}}^{\text{Inputs for Capacity Utilization}} \\
&+ \overbrace{\left[\mu_E \left(\int_0^{\overline{\omega}_{E,t}} \omega_E dF_E(\omega_E)\right) + \mu_F \left(\int_0^{\overline{\omega}_{F,t}} \omega_F dF_F(\omega_F)\right)\right] R_{E,t} Q_{t-1} K_{t-1}}^{\text{Monitoring Costs}} \\
&+ \overbrace{(1-\gamma_F) V_{F,t} + (1-\gamma_E) V_{E,t}}^{\text{Consumption by Exiting FIs and Entrepreneurs}} . \qquad (26)
\end{aligned}
$$

where I_t is aggregate investment, $Z_{d,t}$ is a non-stationary component of investment-specific technology, $A_{d,t}$ is a stationary component of investment-specific technology, and G_t is government expenditure.

A.4 Capital goods producer

Capital goods producers purchase final goods $I_t/(Z_{d,t}A_{d,t})$, convert them to capital goods K_t with the technology $F_{I,t}$, and sell the goods to the entrepreneurs at price Q_t. Their optimization problem is to maximize the profit shown below:

$$
\max_{i_t} \mathrm{E}_t \left[\sum_{q=0}^{\infty} \beta^{t+q} \frac{\Lambda_{t+q}}{\Lambda_t} \left[Q_{t+q} \left(K_{t+q} - (1-\delta) K_{t+q-1}\right) - \frac{I_{t+q}}{Z_{d,t}A_{d,t}} \right] \right].
$$

Note that the capital depreciates by the rate δ in each period and evolves as follows:

$$
K_t = (1 - F_I(I_t, I_{t-1})) I_t + (1-\delta) K_{t-1}, \qquad (27)
$$

where F_I is defined as follows:

$$
F_I(I_{t+q}, I_{t+q-1}, Z_{I,t+q}) \equiv \frac{\kappa_I}{2} \left(\frac{I_{t+q}}{I_{t+q-1}} Z_{I,t+q} - \bar{\pi}\bar{\gamma} g_{Z_d,ss} \right)^2 .
$$

Here, κ_I and $Z_{I,t+q}$ constitute the investment adjustment cost, and $g_{Z_d,ss}$ is the steady state growth rate of investment-specific technology.

A.5 Defining aggregate variables

As with CMR (2010), the real GDP Y_t in the model is given as follows:

$$Y_t = C_t + \frac{I_t}{Z_{d,t} A_{d,t}} + G_t, \tag{28}$$

The CPI π_t is defined by

$$\pi_t = \frac{P_t}{P_{t-1}}. \tag{29}$$

The real interest rate R_t is given by the Fisher equation that connects the nominal interest rate $R_{n,t}$ and the expected inflation $\mathrm{E}_t\left[\pi_{t+1}\right]$:

$$R_t = \frac{R_{n,t}}{\mathrm{E}_t\left[\pi_{t+1}\right]}.$$

Also, for the purposes of the estimation exercise below, we define the Solow residual as:

$$\lambda_t = \frac{Y_t}{\left(L_t\right)^{\psi_L}\left(K_{t-1}\right)^{1-\psi_L}}, \tag{30}$$

where ψ_L is the steady state labor share of income.

A.6 Government sector

The government collects a per capita lump-sum tax τ_t from households to finance government purchase $P_t G_t$ whose amount is exogenously given. We assume that a balanced budget is maintained in each period t as follows:

$$P_t G_t = \tau_t$$

The central bank adjusts the policy rate according to the Taylor rule that is already described in the equation (1) in the main text.

A.7 Fundamental shocks

As already described, there are six groups of shocks. The law of motion for each fundamental shock is given as follows.

$$\begin{aligned}
\ln Z_{\zeta,t} &= \ln g_{Z_\zeta,t} + \ln Z_{\zeta,t-1} + u_{Z_\zeta,t},\ u_{Z_\zeta,t} = \rho_{Z_\zeta} u_{Z_\zeta,t-1} + \epsilon_{Z_\zeta,t},\ \text{for } \zeta = a \text{ and } d \\
\ln A_{\zeta,t} &= \rho_{A_\zeta} \ln A_{\zeta,t-1} + \epsilon_{A_\zeta,t},\ \text{for } \zeta = a \text{ and } d \\
\varepsilon_{N_\zeta,t} &= \rho_{N_\zeta} \varepsilon_{N_\zeta,t-1} + \epsilon_{N_\zeta,t},\ \text{for } \zeta = F \text{ and } E, \\
\ln H_t &= \ln H_{t-1} + \epsilon_{H,t}. \\
\ln G_t &= \rho_G \ln G_{t-1} + \epsilon_{G,t}, \\
\ln Z_{I,t} &= \rho_{Z_I} \ln Z_{I,t-1} + \epsilon_{Z_I,t}, \\
\ln d_t &= \rho_d \ln d_{t-1} + \epsilon_{d,t}, \\
\ln \theta_{P_Y,t} &= \rho_{P_Y} \ln \theta_{P_Y,t-1} + \epsilon_{P_Y,t}, \\
\ln \theta_{W,t} &= \rho_W \ln \theta_{W,t-1} + \epsilon_{W,t}, \\
\ln \bar{\pi}_t &= (1-\rho_\pi) \ln \bar{\pi} + \rho_\pi \ln \bar{\pi}_{t-1} + \epsilon_{\pi,t},
\end{aligned}$$

where ρ_{Z_a}, ρ_{Z_d}, ρ_{A_d}, ρ_{A_a}, ρ_{N_F}, ρ_{N_E}, ρ_G, ρ_{Z_I}, ρ_d, ρ_{P_Y}, ρ_W, and $\rho_\pi \in (0,1)$ are the autoregressive root of the corresponding shocks, and $\epsilon_{Z_a,t}$, $\epsilon_{Z_d,t}$, $\epsilon_{A_d,t}$, $\epsilon_{A_a,t}$, $\epsilon_{N_F,t}$, $\epsilon_{N_E,t}$, $\epsilon_{H,t}$ $\epsilon_{G,t}$, $\epsilon_{Z_I,t}$, $\epsilon_{d,t}$, $\epsilon_{P_Y,t}$, $\epsilon_{W,t}$, $\epsilon_{\pi,t}$, $\epsilon_{R_n,t}$, and $\varepsilon_{s,t-s}$ are the exogenous i.i.d. shocks that are normally distributed with mean zero.

A.8 Equilibrium

An equilibrium consists of a set of prices, $\{P_t,\ W_t,\ W_{E,t},\ W_{F,t},\ R_{E,t},\ R_t,\ \{R_{t+s}\}_{s=1}^{S},\ Q_t,\ r_{E,t},\ r_{F,t}\}_{t=0}^{\infty}$, and the allocations $\{Y_t,\ C_t,\ I_t,\ Y_{g,t},\ Y_{g,t}(l),\ \Psi_t(l),\ L_t(l),\ K_t(l),\ U_t(l)\ \}_{t=0}^{\infty}$, for all $l \in [0,1]$, for given government policy $\{G_t,\ \tau_t,\ R_{n,t},\ \{R_{n,t+s}\}_{s=1}^{S}\}_{t=0}^{\infty}$, realization of exogenous variables $\{\epsilon_{Z_a,t},\ \epsilon_{Z_d,t},\ \epsilon_{A_d,t},\ \epsilon_{A_a,t},\ \epsilon_{N_F,t},\ \epsilon_{N_E,t},\ \epsilon_{H,t},\ \epsilon_{G,t},\ \epsilon_{Z_I,t},\ \epsilon_{d,t},\ \epsilon_{P_Y,t},\ \epsilon_{W,t},\ \epsilon_{\pi,t},\ \epsilon_{R_n,t},\ \{\varepsilon_{s,t-s}\}_{s=1}^{S}\}_{t=0}^{\infty}$, and initial conditions $\{N_F(s^{-1})\}$, $\{N_E(s^{-1})\}\}$ such that for all t, the following conditions are satisfied.

(i) each household h maximizes its utility given prices;

(ii) each FI i maximizes its profits given prices and its net worth;

(iii) each entrepreneur j_i in the goods-producing sector maximizes its profits given prices and its net worth;

(iv) each goods producer l in the goods-producing sector maximizes its profits given prices;

(v) each capital goods producer in the goods-producing sector maximizes its profits given prices;

(vi) the government budget constraint holds;

(vii) the central bank sets the policy rate following the Taylor rule; and

($viii$) markets clear.

B Estimation

B.1 Estimation strategy

We first detrend the model variables by dividing them by the stochastic trend. We trend the real variables other than the capital stock K_t, such as output Y_t and the net worth N_t, with the term expressed by the function of the steady-state growth rate of the two technologies and that of the working-age population, $Z_{a,t}^{\frac{1}{(1-\gamma)(\alpha+\alpha_E+\alpha_F)}} Z_{d,t}^{\frac{(1-\alpha-\alpha_E-\alpha_F)}{\alpha+\alpha_E+\alpha_F}} H_t$, and detrend the capital stock K_t, with the term $Z_{a,t}^{\frac{1}{(1-\gamma)(\alpha+\alpha_E+\alpha_F)}} Z_{d,t}^{\frac{1}{\alpha+\alpha_E+\alpha_F}} H_t$. We then conduct a Bayesian estimation following existing studies such as CMR (2014). We first write the model's equilibrium conditions in a state-space representation and derive the likelihood function of the system of equilibrium conditions using the Kalman filter. We then combine the likelihood function with the priors for the parameters to obtain the posterior density function numerically. In this process, we use the random walk Metropolis-Hastings algorithm.

B.2 Calibration, Prior Distribution, and Posterior Distribution

Calibrated parameters

Some parameter values are calibrated following existing studies. These include the discount factor β, the elasticity of substitution between differentiated products θ_{P_Y}, the elasticity of substitution between differentiated labor inputs θ_W, the depreciation rate of the capital stock δ, the share of the intermediate input, labor input, entrepreneurial labor input and the FI labor input in goods production γ, α, α_E and α_F, and the utility weight on leisure φ. Values for γ and α are constructed using the historical average of intermediate goods usage divided by gross output, both of which are reported in an input-output table, and the compensation of employees divided by GDP in SNA, respectively. In addition, we set κ_U so that the utilization rate of capital stock is unity at the steady state. See Table 1 for the values of these parameters.

Estimated parameters

We estimate the remaining parameters. See Table 2 for the values of these parameters. The type, mean, and standard deviation of the prior distribution are mostly taken from existing studies such as Edge et al. (2008). They are given in the first to the third columns. In estimating the six parameters that are related to the IF and FE contracts, that include two parameters that govern monitoring costs μ_F and μ_E, variance of idiosyncratic shocks to borrowers σ_F and σ_E, and survival rates γ_F and γ_E, we follow HSU (2011) and set the prior mean of these parameters so that they satisfy the six equilibrium conditions stated below at the steady state: (1) the annualized spread between the FIs' borrowing rate and the risk-free rate $r_F - R$ is 56 bps; (2) the ratio of net worth held by FIs to aggregate capital stock $N_F/(QK)$ is 0.1; (3) the ratio of net worth held by the entrepreneurs in the goods-producing sector to aggregate capital stock $N_E/(QK)$ is 0.6; (4) the annualized failure rate of the FIs is 1%; (5) the annualized failure rate of the entrepreneurs in the goods-producing sector is 1%; and (6) the annualized spread between the FI loan rate and the FI borrowing rate $r_E - r_F$, equals 442 bps. Except for conditions (4) and (5), the conditions above are chosen so that they are consistent with the historical average of Japanese data.[62] We borrow condition (5) from BGG (1999) and assume that the same condition holds in the FI sector as well. This can be seen in condition (4).

Posterior distribution

To calculate the posterior distribution and to evaluate the marginal likelihood of the model, we employ the Metropolis-Hastings algorithm. To do this, we create a sample of 400,000 draws, disregarding the initial 200,000 draws. Estimated posterior distributions of parameters are also shown in Table 2. The last three columns of the table display the posterior mean and the confidence intervals for the estimated parameters.

[62] We take the numbers for conditions (2) and (3) from the Flow of Funds Accounts. We use the long-term prime lending rate and the deposit rate adopted by the Bank of Japan to obtain conditions (1) and (6), respectively.

Table 1: Calibrated Parameters

Values of calibrated parameters		
α	Labor share (household)	0.6
α_E	Labor share (entrepreneur)	0.02
α_{FI}	Labor share (FI)	0.02
γ	Share of intermediate goods	0.583
κ_U	Scaling parameter of capital utilization adjustment cost	0.05
φ	Weight on labor disutility	0.2
β	Households' discount factor (quarterly)	0.998
δ	Capital depreciation rate (quarterly)	0.028
θ_{P_Y}	Elasticity of substitution between differentiated products at the steady state	7
θ_W	Elasticity of substitution between differentiated labor inputs at the steady state	7

Table 2: Estimated Parameters

Values of estimated parameters (prior and posterior distributions)

	Prior distribution			Posterior distribution		
	Distribution	Mean	S.D.	Mean	5th percentiles	95th percentiles
Elasticity of labor supply	gamma	0.8	0.075	0.887	0.763	1.011
Capital stock adjustment cost	gamma	2	0.25	2.275	1.889	2.649
Price adjustment cost	gamma	12	1	6.950	6.690	7.298
Nominal wage adjustment cost	gamma	2.5	0.5	0.887	0.579	1.184
Policy weight on inflation in Taylor rule	normal	2.75	0.05	2.867	2.784	2.946
Monetary policy smoothing	beta	0.5	0.01	0.514	0.498	0.531
Inverse elasticity of capital utilization rate	gamma	5	1	5.888	4.886	7.139
Riskiness of idiosyncratic productivities (FIs)	gamma	0.104	0.002	0.102	0.099	0.106
Riskiness of idiosyncratic productivities (entrepreneurs)	gamma	0.309	0.002	0.309	0.306	0.313
Monitoring cost (IF contract)	gamma	0.539	0.01	0.535	0.519	0.552
Monitoring cost (FE contract)	gamma	0.02	0.01	0.038	0.017	0.060
Survival rates (FIs)	beta	0.86	0.001	0.861	0.859	0.862
Survival rates (entrepreneurs)	beta	0.96	0.001	0.959	0.958	0.961
Steady state value of technology (neutral)	gamma	1.001	0.001	1.000	1.000	1.001
Steady state value of technology (investment-specific)	gamma	1.002	0.001	1.000	1.000	1.001
Steady state value of target inflation rate	normal	1.002	0.001	1.001	1.000	1.002
Degree of internal habit persistence	beta	0.6	0.15	0.136	0.052	0.219
Non-stationary technology shock (neutral) AR	beta	0.5	0.15	0.145	0.052	0.228
Stationary technology shock (neutral) AR	beta	0.5	0.15	0.867	0.818	0.917
Non-stationary technology shock (investment-specific) AR	beta	0.5	0.15	0.517	0.365	0.676
Stationary technology shock (investment-specific) AR	beta	0.5	0.15	0.745	0.631	0.856
Net worth shock (FIs) AR	beta	0.5	0.15	0.194	0.075	0.311
Net worth shock (entrepreneurs) AR	beta	0.5	0.15	0.750	0.612	0.875
External demand shock AR	beta	0.5	0.15	0.961	0.939	0.987
Investment adjustment shock AR	beta	0.5	0.15	0.217	0.104	0.322
Price markup shock AR	beta	0.5	0.15	0.802	0.728	0.873
Nominal wage markup shock AR	beta	0.5	0.15	0.810	0.697	0.927
Discount factor shock AR	beta	0.5	0.15	0.765	0.638	0.908
Target inflation rate shock AR	beta	0.5	0.15	0.450	0.394	0.505
Non-stationary technology shock (neutral) SD	invg	0.01	5	0.002	0.002	0.003
Stationary technology shock (neutral) SD	invg	0.05	5	0.006	0.006	0.006
Non-stationary technology shock (investment-specific) SD	invg	0.01	5	0.002	0.002	0.003
Stationary technology shock (investment-specific) SD	invg	0.05	5	0.006	0.006	0.006
Monetary policy shock SD	invg	0.01	5	0.002	0.002	0.002
Net worth shock (FIs) SD	invg	0.02	5	0.003	0.003	0.004
Net worth shock (entrepreneurs) SD	invg	0.02	5	0.005	0.003	0.006
External demand shock SD	invg	0.01	5	0.071	0.058	0.084
Investment adjustment shock SD	invg	0.01	5	0.031	0.026	0.036
Price markup shock SD	invg	0.01	5	0.029	0.025	0.032
Nominal wage markup shock SD	invg	0.01	5	0.094	0.067	0.120
Discount factor shock SD	invg	0.015	5	0.005	0.003	0.006
Target inflation rate shock SD	invg	0.01	5	0.004	0.003	0.004
1Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
2Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
3Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
4Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
5Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
6Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
7Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
8Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
9Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
10Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
11Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001
12Q ahead anticipated monetary policy shock SD	invg	0.01	5	0.001	0.001	0.001

Table 3: Determinants of the Natural Rate

(1) Decomposition of the natural rate decline

	(i) 1980s (%)	(ii) 2013-2017 (%)	Changes (i) − (ii) (% points)
Natural rate	4.15	0.31	3.84
Neutral technology	1.58	-0.57	2.16
Financial factors	-0.49	0.92	-1.40
Working-age population	0.29	-0.40	0.69
Investment-specific technology	0.14	-0.34	0.48
Demand factors	0.54	-0.04	0.58

(2) Variance decomposition of the natural rate

	(%)
Neutral technology	58.1
Financial factors	24.0
Working-age population	3.8
Investment-specific technology	4.3
Demand factors	7.9
Others	1.9

Notes: 1. In (2), we first compute the sum of squares for the contribution of each factors to variations in the natural rate over the sample period. We then divide the sum of squares for each factor by the sum of the factors.
2. Contribution of neutral technology is the sum of the contribution of both non-stationary and stationary shocks.
3. Contribution of financial factors is the sum of the contribution of net worth shocks to the FIs and that to entrepreneurs.
4. Contribution of investment-specific technology is the sum of the contribution of both non-stationary and stationary shocks.
5. Contribution of demand factors is the sum of the contribution of shocks to the external demand and discount factor.

Table 4: Determinants of the Expected Natural Rate

(1) Decomposition of the expected natural rate decline

	(i) 1980s (%)	(ii) 2013-2017 (%)	Changes (i) − (ii) (% points)
Expected natural rate	2.56	0.49	2.07
Neutral technology	0.77	-0.11	0.87
Financial factors	-0.07	0.19	-0.26
Working-age population	0.13	-0.18	0.32
Investment-specific technology	0.22	-0.22	0.44
Demand factors	0.26	0.02	0.23

(2) Variance decomposition of the natural rate and the expected natural rate

	Natural rate (%)	Expected natural rate		
		1 year (%)	5 year (%)	10 year (%)
Neutral technology	58.1	52.4	54.7	57.1
Financial factors	24.0	27.8	15.9	10.1
Working-age population	3.8	4.9	6.3	6.2
Investment-specific technology	4.3	5.0	11.4	12.5
Demand factors	7.9	8.5	11.4	14.0
Others	1.9	1.4	0.2	0.1

Notes: 1. In (1), the expected natural rate is the average of expected natural rates from the current period to 10 years ahead.
2. In (2), we first compute the sum of squares for the contribution of each factors to variations in the natural rate and expected natural rates over 1 year, 5 year, and 10 year horizons, respectively, over the sample period. We then divide the sum of squares for each factor by the sum of the factors.
3. Contribution of neutral technology is the sum of the contribution of both non-stationary and stationary shocks.
4. Contribution of financial factors is the sum of the contribution of net worth shocks to the FIs and that to entrepreneurs.
5. Contribution of investment-specific technology is the sum of the contribution of both non-stationary and stationary shocks.
6. Contribution of demand factors is the sum of the contribution of shocks to the external demand and discount factor.

Table 5: Robustness Check (1) : Anticipated Shocks

(1) Decomposition of the natural rate and expected natural rate declines

	Changes from 1980s to 2013-2017 (% points)
Natural rate	3.54 ～ 3.82 [3.84]
Neutral technology	1.71 ～ 1.97 [2.16]
Financial factors	-1.44 ～ -1.08 [-1.40]
Working-age population	0.70 [0.69]
Investment-specific technology	0.49 ～ 0.50 [0.48]
Demand factors	0.72 ～ 0.78 [0.58]
Expected natural rate	2.14 ～ 2.22 [2.07]
Neutral technology	0.82 ～ 1.02 [0.87]
Financial factors	-0.24 ～ -0.17 [-0.26]
Working-age population	0.31 ～ 0.32 [0.32]
Investment-specific technology	0.40 ～ 0.45 [0.44]
Demand factors	0.20 ～ 0.25 [0.23]

(2) Variance decomposition of the natural rate and expected natural rate

	Natural rate (%)	Expected natural rate (%)
Neutral technology	47.9 ～ 56.5 [58.1]	61.1 ～ 62.6 [57.1]
Financial factors	12.1 ～ 26.9 [24.0]	2.8 ～ 6.5 [10.1]
Working-age population	3.7 ～ 4.7 [3.8]	5.2 ～ 5.7 [6.2]
Investment-specific technology	3.9 ～ 4.6 [4.3]	10.6 ～ 11.1 [12.5]
Demand factors	13.2 [7.9]	14.9 ～ 18.3 [14.0]
Others	3.0 ～ 10.6 [1.9]	0.2 ～ 1.0 [0.1]

Notes: 1. Figures indicate the range of estimates by various specifications, and those in brackets are values under the baseline model.
2. In (1), the expected natural rate is the average of expected natural rates from the current period to 10 years ahead.
3. In (2), we first compute the sum of squares for the contribution of each factors to variations in the natural rate and expected natural rates over 10 year horizons, over the sample period. We then divide the sum of squares for each factor by the sum of the factors.
4. Contribution of neutral technology is the sum of the contribution of both non-stationary and stationary shocks.
5. Contribution of financial factors is the sum of the contribution of net worth shocks to the FIs and that to entrepreneurs.
6. Contribution of investment-specific technology is the sum of the contribution of both non-stationary and stationary shocks.
7. Contribution of demand factors is the sum of the contribution of shocks to the external demand and discount factor.

Table 6: Robustness Check (2) : Working-age Population

(1) Decomposition of the natural rate and expected natural rate declines

	Changes from 1980s to 2013-2017 (% points)
Natural rate	3.66 ～ 4.08 [3.84]
Neutral technology	2.21 ～ 2.24 [2.16]
Financial factors	-1.41 ～ -0.70 [-1.40]
Working-age population	0.00 ～ 0.05 [0.69]
Investment-specific technology	0.48 [0.48]
Demand factors	0.52 ～ 1.00 [0.58]
Expected natural rate	2.04 ～ 2.09 [2.07]
Neutral technology	0.87 ～ 1.02 [0.87]
Financial factors	-0.27 ～ -0.17 [-0.26]
Working-age population	0.00 ～ 0.15 [0.32]
Investment-specific technology	0.43 ～ 0.44 [0.44]
Demand factors	0.22 ～ 0.39 [0.23]

(2) Variance decomposition of the natural rate and expected natural rate

	Natural rate (%)	Expected natural rate (%)
Neutral technology	61.0 ～ 69.1 [58.1]	60.2 ～ 73.3 [57.1]
Financial factors	12.3 ～ 23.9 [24.0]	5.4 ～ 11.5 [10.1]
Working-age population	0.0 ～ 0.3 [3.8]	0.0 ～ 1.2 [6.2]
Investment-specific technology	1.1 ～ 3.8 [4.3]	10.5 ～ 12.0 [12.5]
Demand factors	1.4 ～ 9.5 [7.9]	10.8 ～ 15.0 [14.0]
Others	1.5 ～ 16.1 [1.9]	0.1 ～ 0.1 [0.1]

Notes: 1. Figures indicate the range of estimates by various specifications, and those in brackets are values under the baseline model.
2. In (1), the expected natural rate is the average of expected natural rates from the current period to 10 years ahead.
3. In (2), we first compute the sum of squares for the contribution of each factors to variations in the natural rate and expected natural rates over 10 year horizons, over the sample period. We then divide the sum of squares for each factor by the sum of the factors.
4. Contribution of neutral technology is the sum of the contribution of both non-stationary and stationary shocks.
5. Contribution of financial factors is the sum of the contribution of net worth shocks to the FIs and that to entrepreneurs.
6. Contribution of investment-specific technology is the sum of the contribution of both non-stationary and stationary shocks.
7. Contribution of demand factors is the sum of the contribution of shocks to the external demand and discount factor.

Figure 1: Outline of the Model

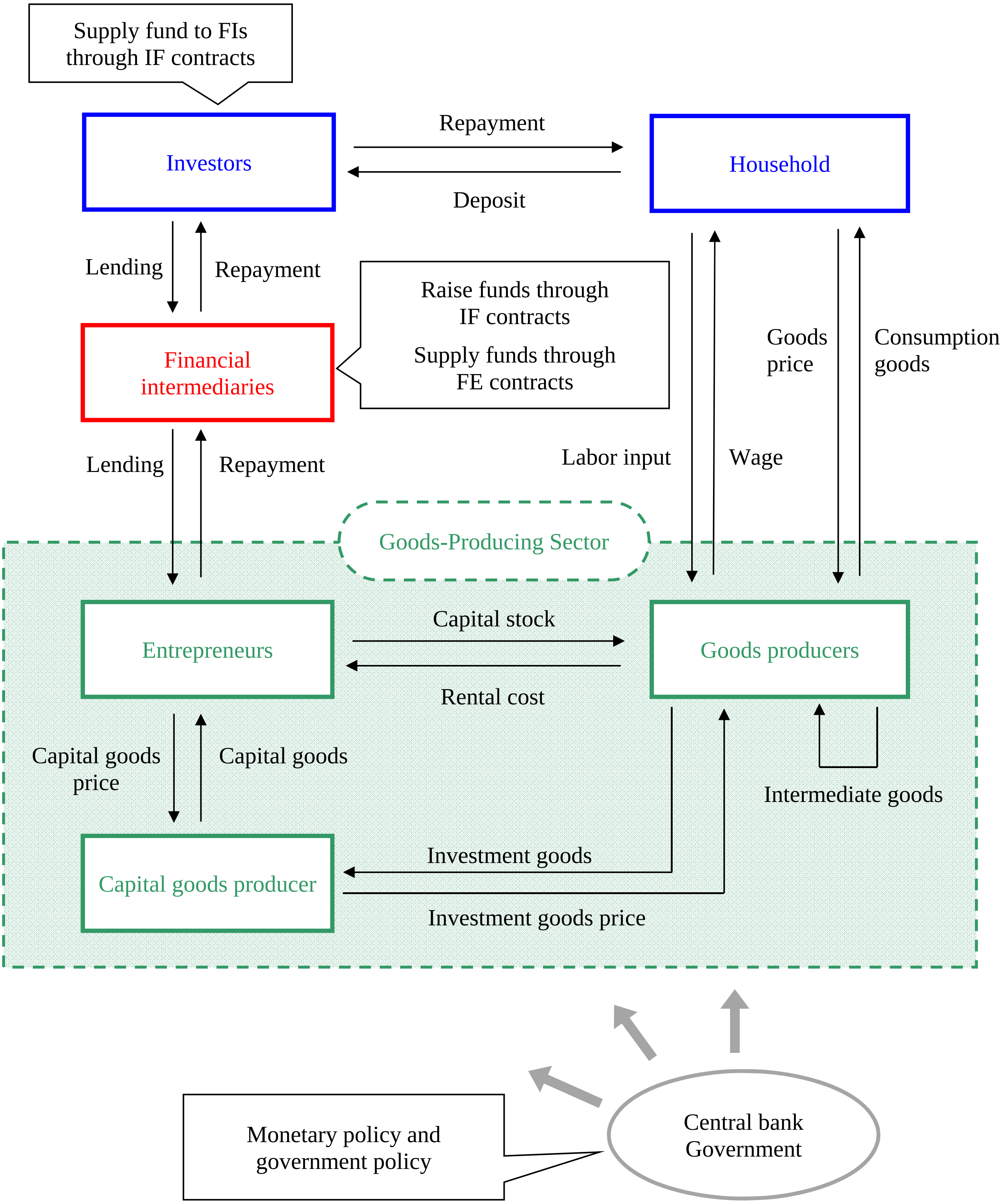
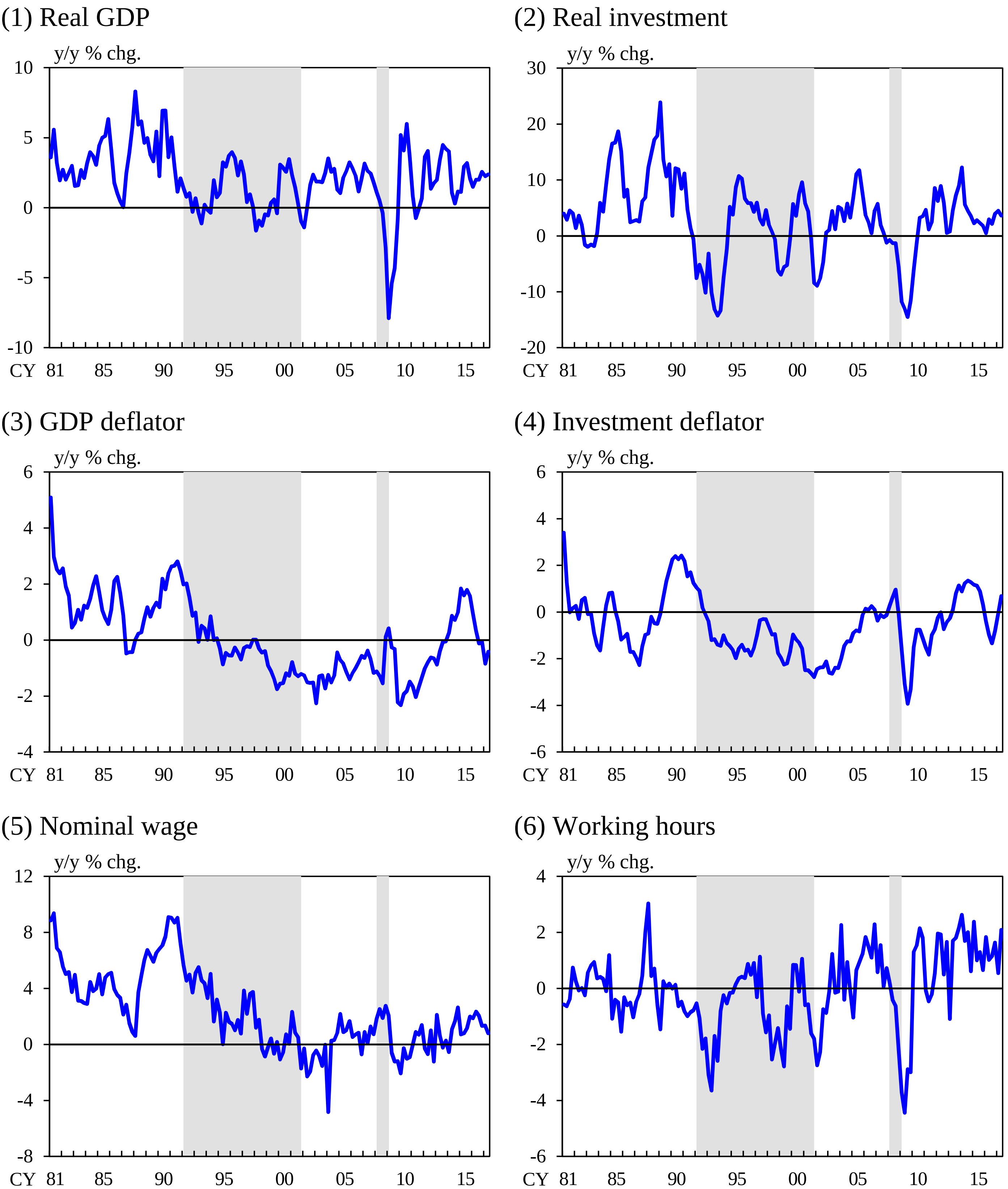


Figure 2: Data Used for Estimation (1)

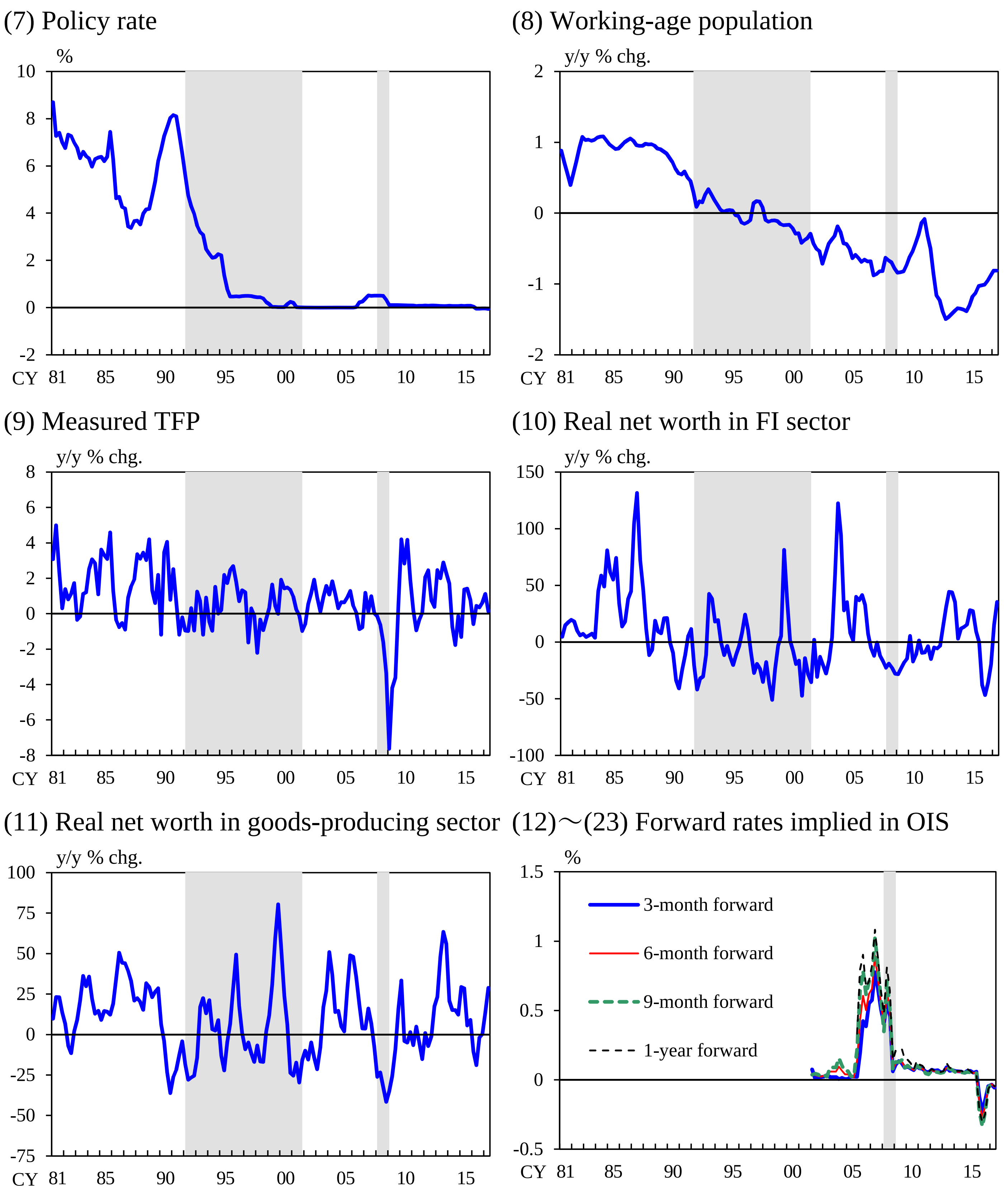


Notes: 1. Series (1), (2), and (6) are on a per capita basis. For all variables, we use a quarter-on-quarter % change of the variable rather than a year-on-year % change.

2. The shaded areas show the period of banking crisis based on the dating by Reinhart and Rogoff (2011) and the recession in the 14th business cycle in Japan based on the dating by the Cabinet Office.

Sources: Cabinet Office, "National Accounts"; Ministry of Health, Labour and Welfare, "Monthly Labour Survey"; Ministry of Internal Affairs and Communications, "Labour Force Survey"; Reinhart and Rogoff (2011).

Figure 2: Data Used for Estimation (2)

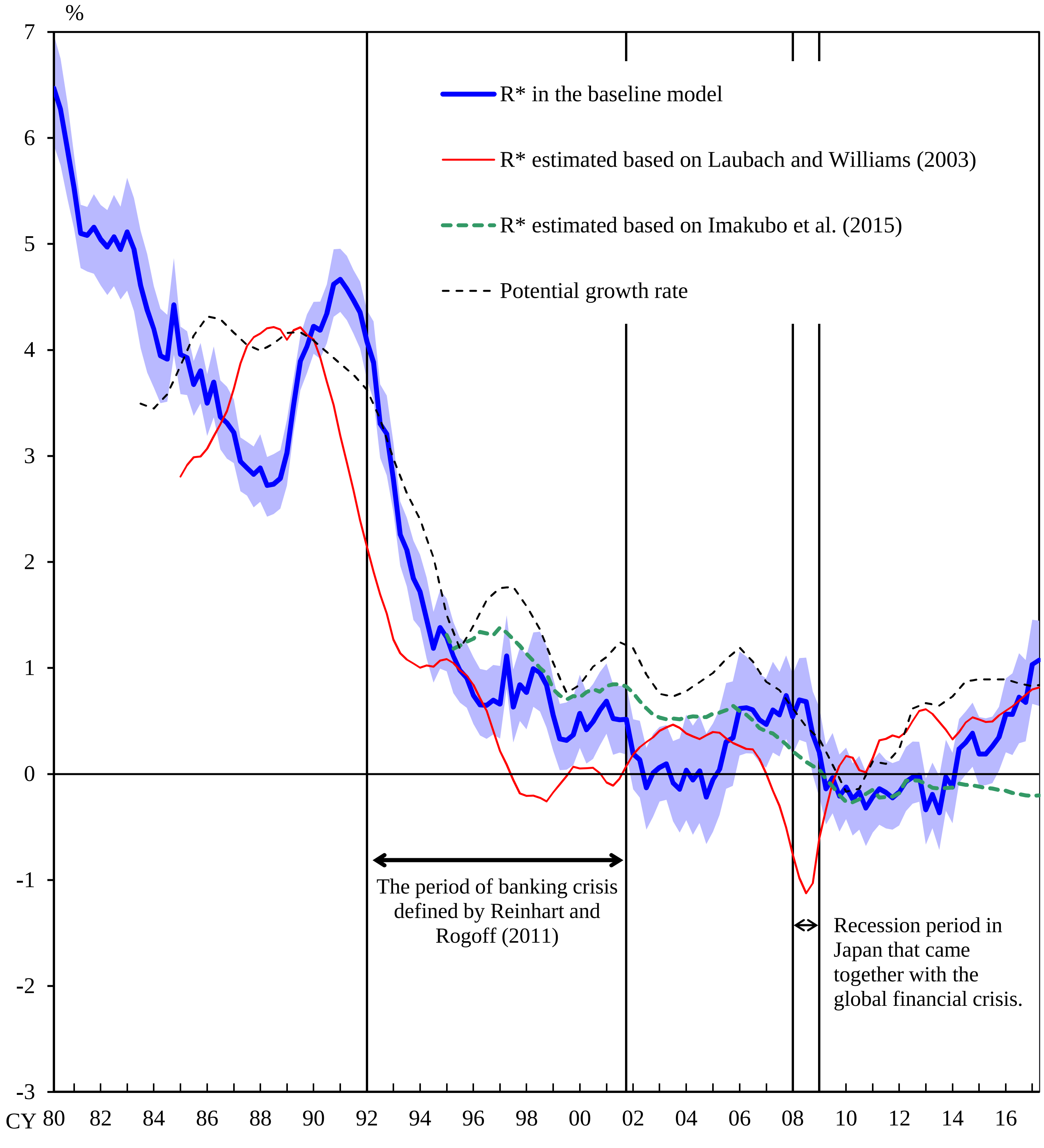


Notes: 1. Series (10) and (11) are on a per capita basis. For all of the series other than series (7) and (12) ～(23), we use a quarter-on-quarter % change of the variable rather than a year-on-year % change. We use the level series for series (7) and (12)～(23) in our estimation. For series (12)～(23), we show selected four among the twelve forward rates to save space.

2. The shaded areas show the period of banking crisis based on the dating by Reinhart and Rogoff (2011) and the recession in the 14th business cycle in Japan based on the dating by the Cabinet Office.

Sources: Bank of Japan, "Flow of Funds Accounts" "Call Rates, Uncollateralized Overnight" "Call Rates, Collateralized Overnight, Average"; Cabinet Office, "National Accounts"; Japan Exchange Group, Inc., "Market Capitalization"; Ministry of Internal Affairs and Communications, "Labour Force Survey"; Reinhart and Rogoff (2011); Bloomberg.

Figure 3: Estimated Level of Natural Rate



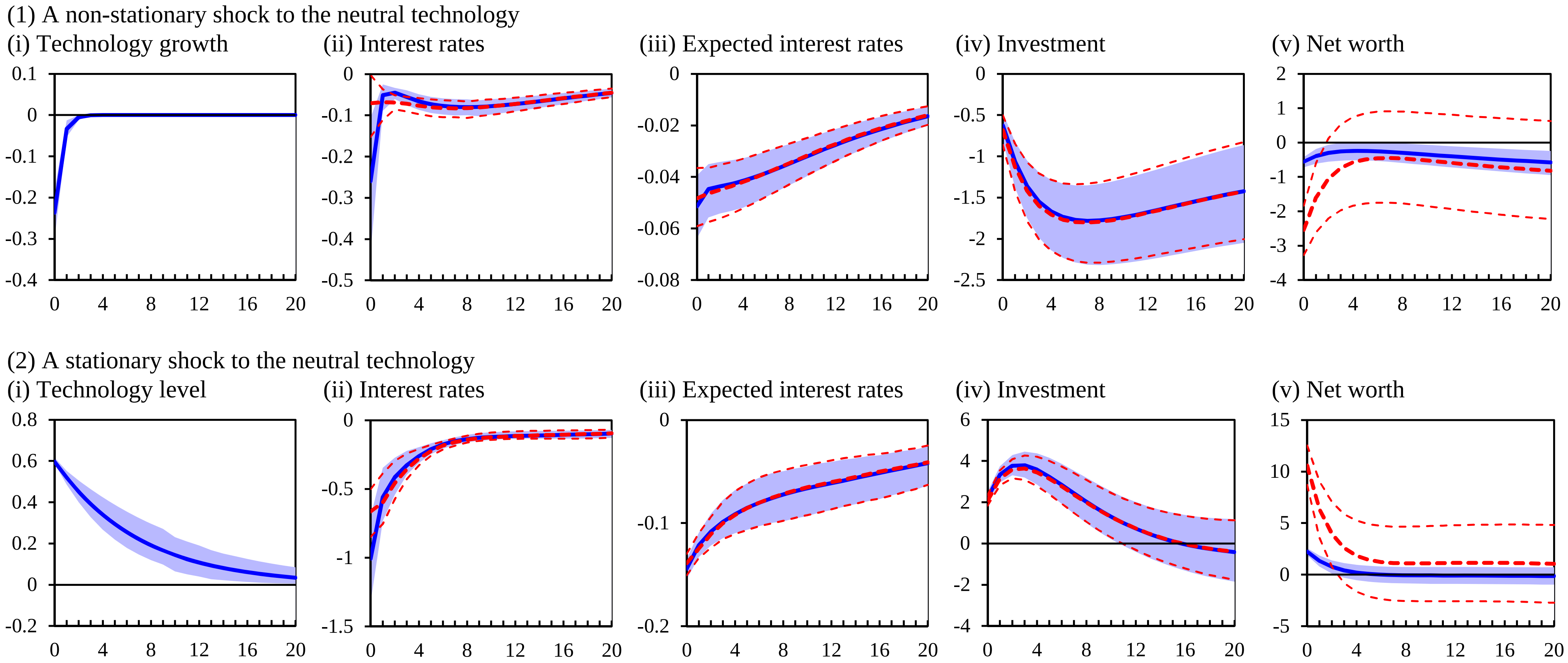
Notes: 1. Figures for the natural rate in the baseline model are the moving average of the point estimate, and the shaded areas are the 90% confidence intervals of the estimate.

2. Figures for the natural rate estimated based on Imakubo et al. (2015) are those for 1-year yield.

3. Figures for the potential growth rate are those released by the Research and Statistics Department of the Bank of Japan.

Sources: Bank of Japan; Bloomberg; Cabinet Office; Consensus Economics Inc., "Consensus Forecasts"; Imakubo et al. (2015); Laubach and Williams (2003); Reinhart and Rogoff (2011).

Figure 4: Impulse Response Functions : Neutral Technology



Notes: 1. Series shown in (ii), (iii) and (1)(i) are the deviation from the non-stochastic steady state, on an annual basis. Others are the percentage deviation from the non-stochastic steady state.

2. The horizontal axis denotes quarters after the shock.

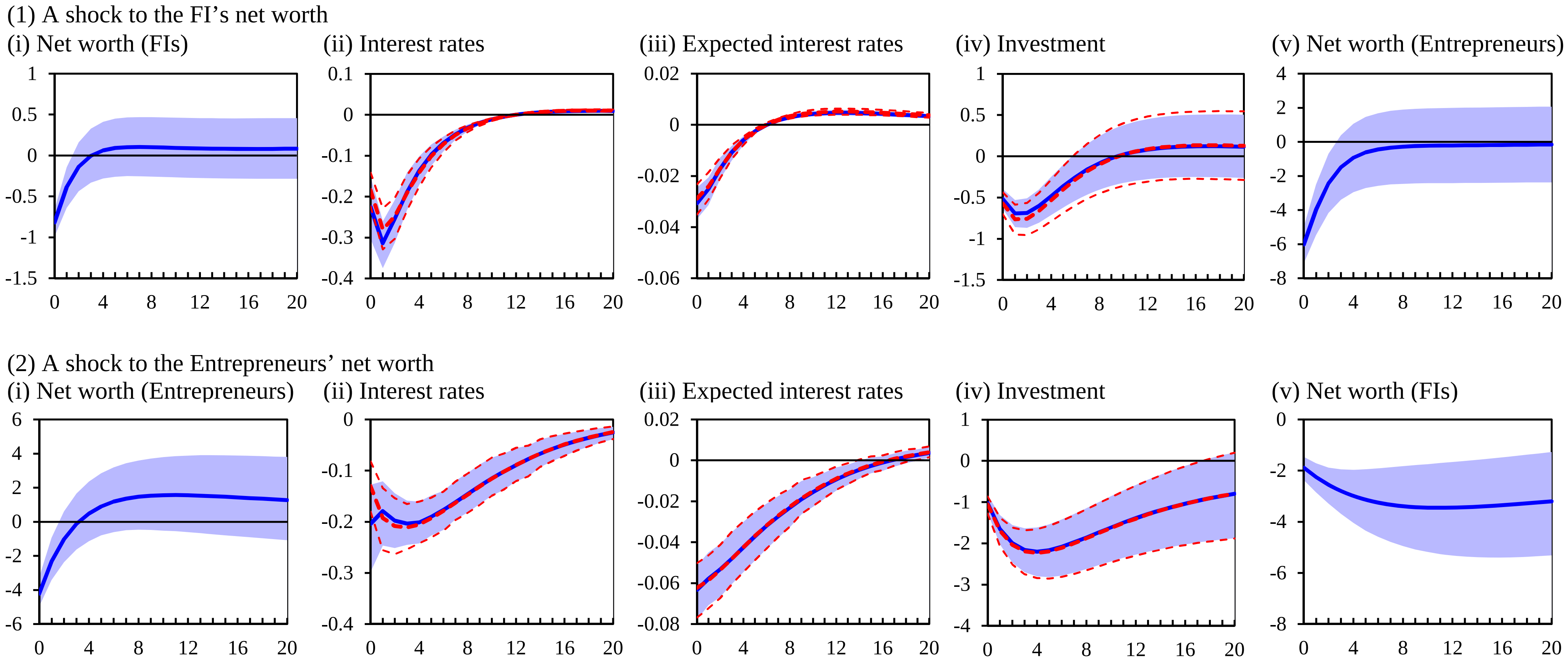
3. In (i), the solid line represents the point estimate of the impulse response, and the shaded areas represent the 90% confidence intervals of the estimate.

4. In (ii) and (iii), the solid blue line represents the point estimate of the impulse response of the natural rate and the expected natural rate over a 40-quarter horizon, respectively. The shaded areas represent the 90% confidence intervals of the estimates. The dotted thick red line represents the point estimate of the impulse response of the ex-ante real interest rate and expected real interest rates, respectively. The dotted thin red lines represent the 90% confidence intervals of the estimates.

5. In (iv), the solid blue line and the dotted thick red line represent the point estimate of the impulse response of investment in the flexible-price economy and the actual economy, respectively. The shaded area and the dotted thin red lines represent the 90% confidence intervals of the estimates.

6. In (v), the solid blue line represents the point estimate of the impulse response of FI's net worth, and the shaded areas represent the 90% confidence intervals of the estimate. The dotted thick red line represents the impulse response of the entrepreneurial net worth, and the dotted thin red lines represent the 90% confidence intervals of the estimate.

Figure 5: Impulse Response Functions : Financial Factors



Notes: 1. Series shown in (ii) and (iii) are the deviation from the non-stochastic steady state, on an annual basis. Others are the percentage deviation from the non-stochastic steady state.

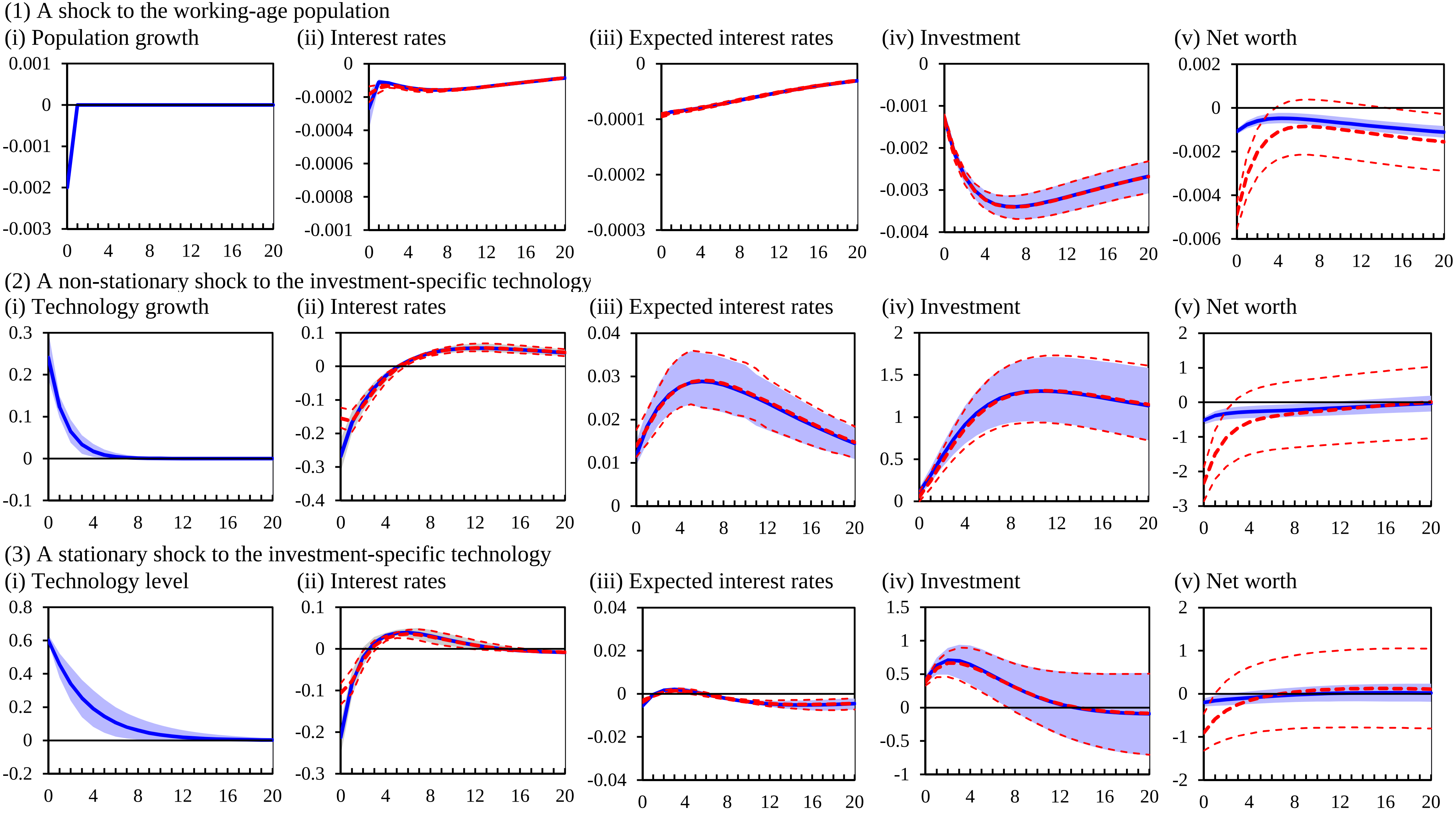
2. The horizontal axis denotes quarters after the shock.

3. In (i) and (v), the solid line represents the point estimate of the impulse response, and the shaded areas represent the 90% confidence intervals of the estimate.

4. In (ii) and (iii), the solid blue line represents the point estimate of the impulse response of the natural rate and the expected natural rate over a 40-quarter horizon, respectively. The shaded areas represent the 90% confidence intervals of the estimates. The dotted thick red line represents the point estimate of the impulse response of the ex-ante real interest rate and expected real interest rates, respectively. The dotted thin red lines represent the 90% confidence intervals of the estimates.

5. In (iv), the solid blue line and the dotted thick red line represent the point estimate of the impulse response of investment in the flexible-price economy and the actual economy, respectively. The shaded area and the dotted thin red lines represent the 90% confidence intervals of the estimates.

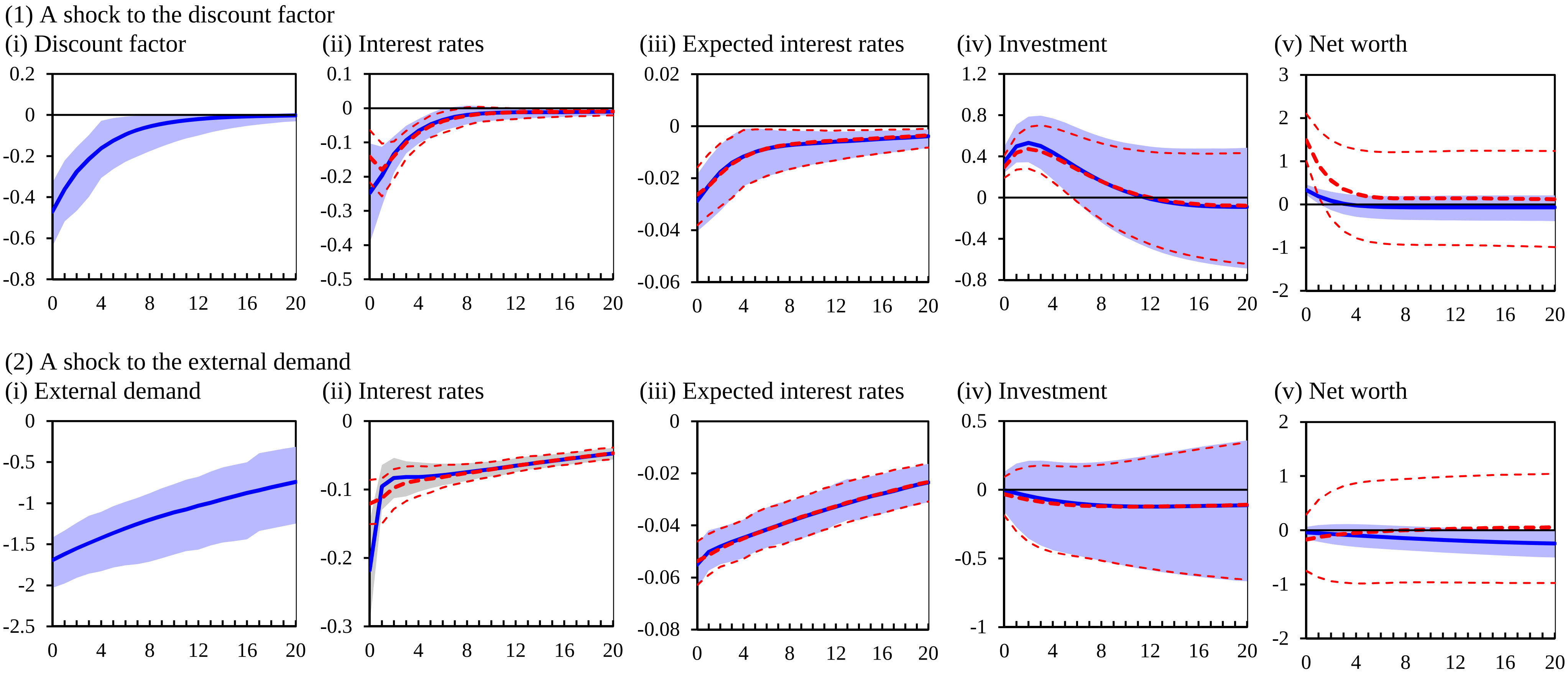
Figure 6: Impulse Response Functions : Working-age Population and Investment-specific Technology



Notes: 1. Series shown in (ii), (iii), (1)(i) and (2)(i) are the deviation from the non-stochastic steady state, on an annual basis. Others are the percentage deviation from the non-stochastic steady state.

2. See footnotes to Figure 4 for the explanation of each line and shaded area.

Figure 7: Impulse Response Functions : Demand Factors



Notes: 1. Series shown in (ii), and (iii) are the deviation from the non-stochastic steady state, on an annual basis. Others are the percentage deviation from the non-stochastic steady state.

2. The horizontal axis denotes quarters after the shock.

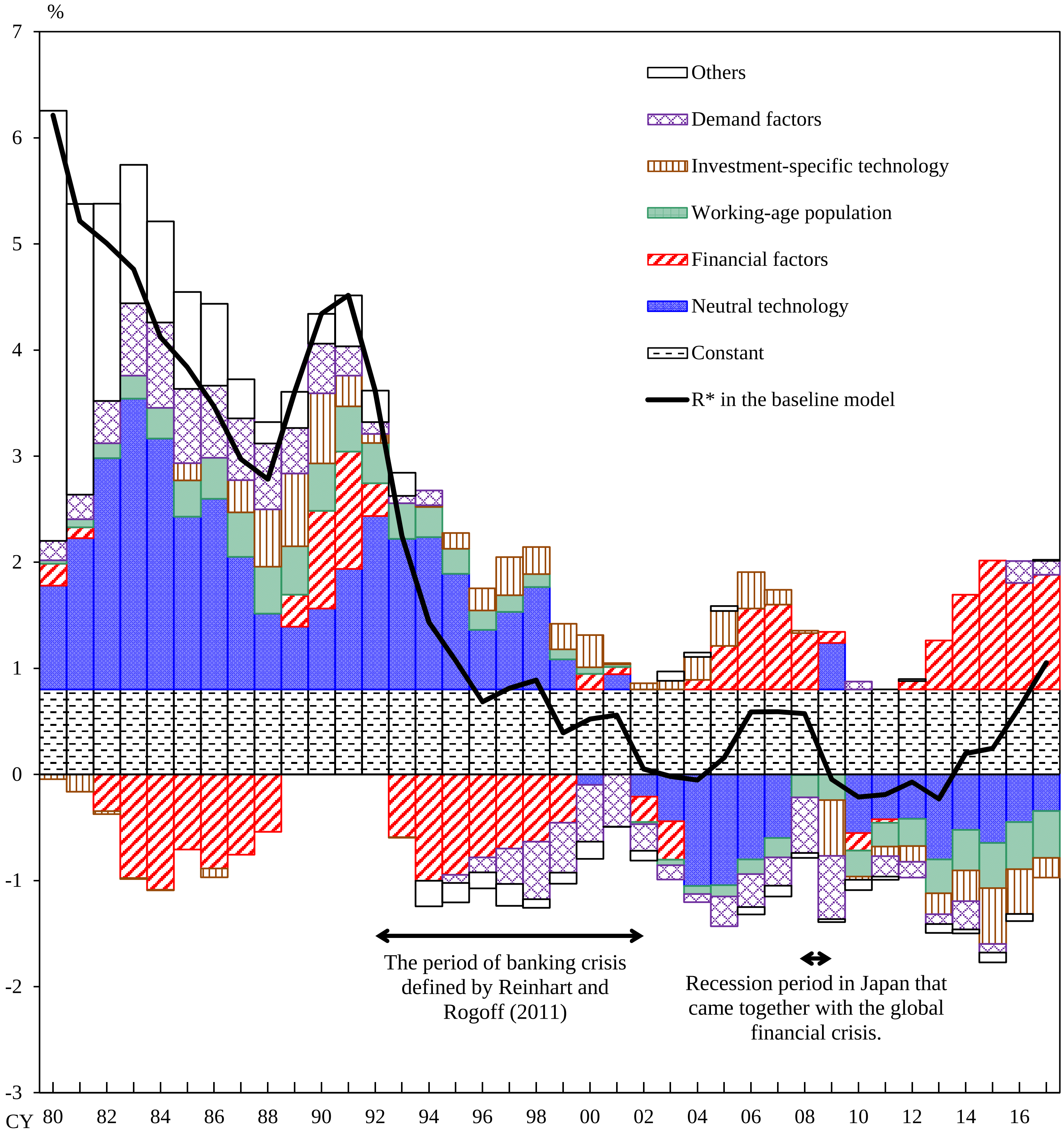
3. In (i), the solid line represents the point estimate of the impulse response, and the shaded areas represent the 90% confidence intervals of the estimate.

4. In (ii) and (iii), the solid blue line represents the point estimate of the impulse response of the natural rate and the expected natural rate over a 40-quarter horizon, respectively. The shaded areas represent the 90% confidence intervals of the estimates. The dotted thick red line represents the point estimate of the impulse response of the ex-ante real interest rate and expected real interest rates, respectively. The dotted thin red lines represent the 90% confidence intervals of the estimates.

5. In (iv), the solid blue line and the dotted thick red line represent the point estimate of the impulse response of investment in the flexible-price economy and the actual economy, respectively. The shaded area and the dotted thin red lines represent the 90% confidence intervals of the estimates.

6. In (v), the solid blue line represents the point estimate of the impulse response of FI's net worth, and the shaded areas represent the 90% confidence intervals of the estimate. The dotted thick red line represents the impulse response of the entrepreneurial net worth, and the dotted thin red lines represent the 90% confidence intervals of the estimate.

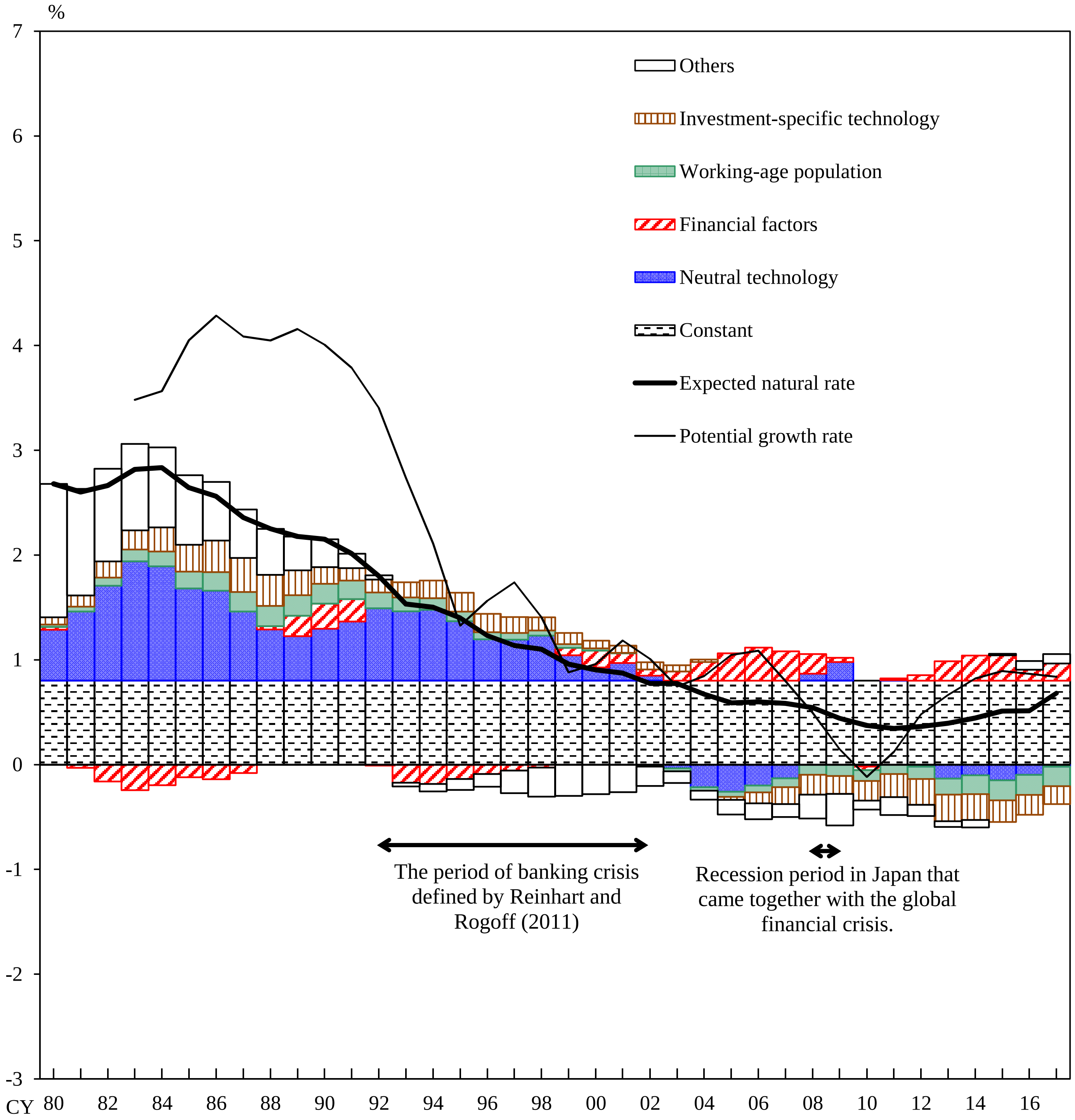
Figure 8: Historical Decomposition of the Natural Rate



Notes: 1. Figures for the natural rate in the baseline model are the average of four quarters.
2. Contribution of neutral technology is the sum of the contribution of both non-stationary and stationary shocks.
3. Contribution of financial factors is the sum of the contribution of net worth shocks to the FIs and that to entrepreneurs.
4. Contribution of investment-specific technology is the sum of the contribution of both non-stationary and stationary shocks.
5. Contribution of demand factors is the sum of the contribution of shocks to the external demand and discount factor.
6. Contribution of constant is the inverse of the subjective discount factor.

Sources: Cabinet Office; Reinhart and Rogoff (2011).

Figure 9: Historical Decomposition of the Expected Natural Rate

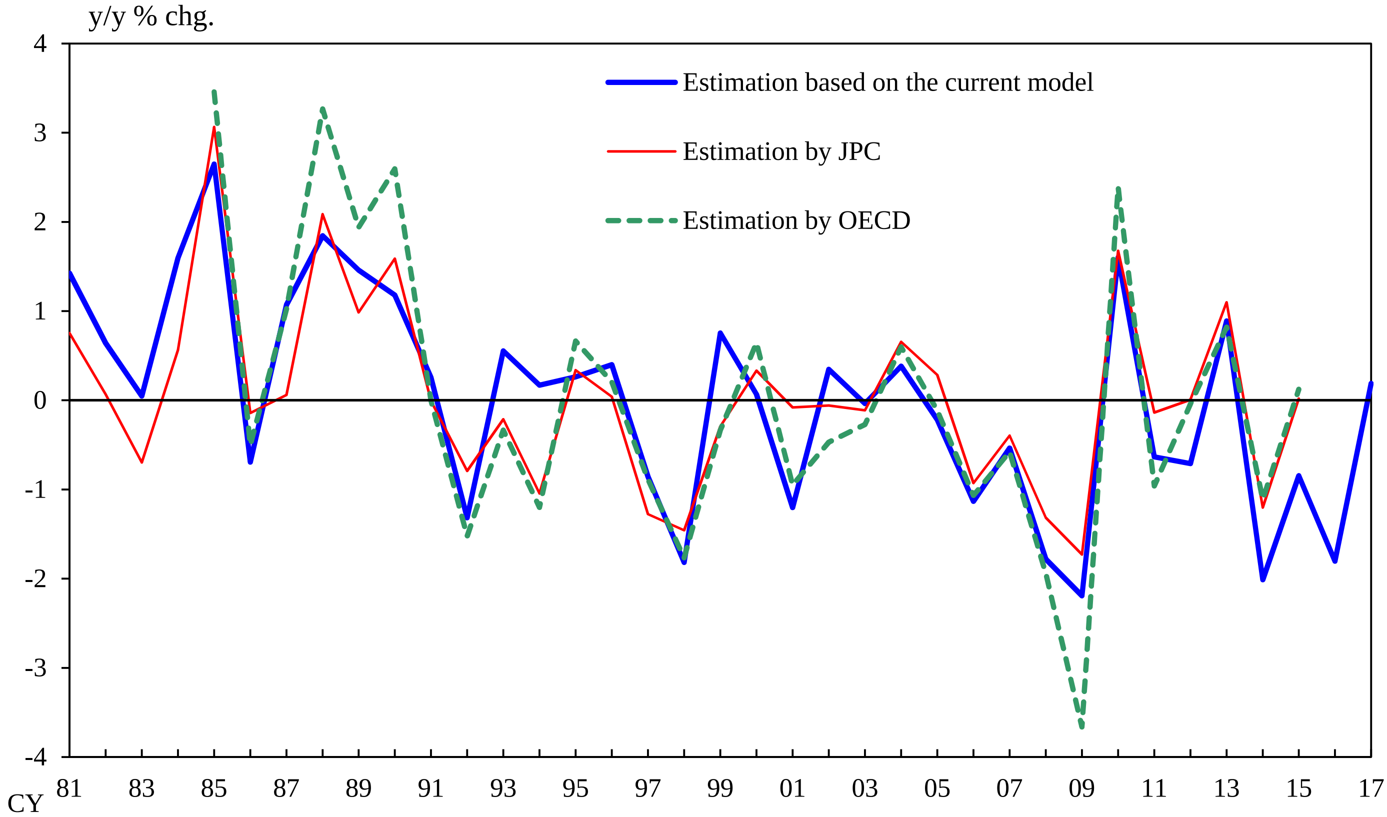


Notes: 1. Figures for the expected natural rate are the average of expected natural rates from the current period to 10 years ahead.
2. Contribution of neutral technology is the sum of the contribution of both non-stationary and stationary shocks.
3. Contribution of financial factors is the sum of the contribution of net worth shocks to the FIs and that to entrepreneurs.
4. Contribution of investment-specific technology is the sum of the contribution of both non-stationary and stationary shocks.
5. Contribution of constant is the inverse of the subjective discount factor.
6. Figures for the potential growth rate are those released by the Research and Statistics Department of the Bank of Japan.

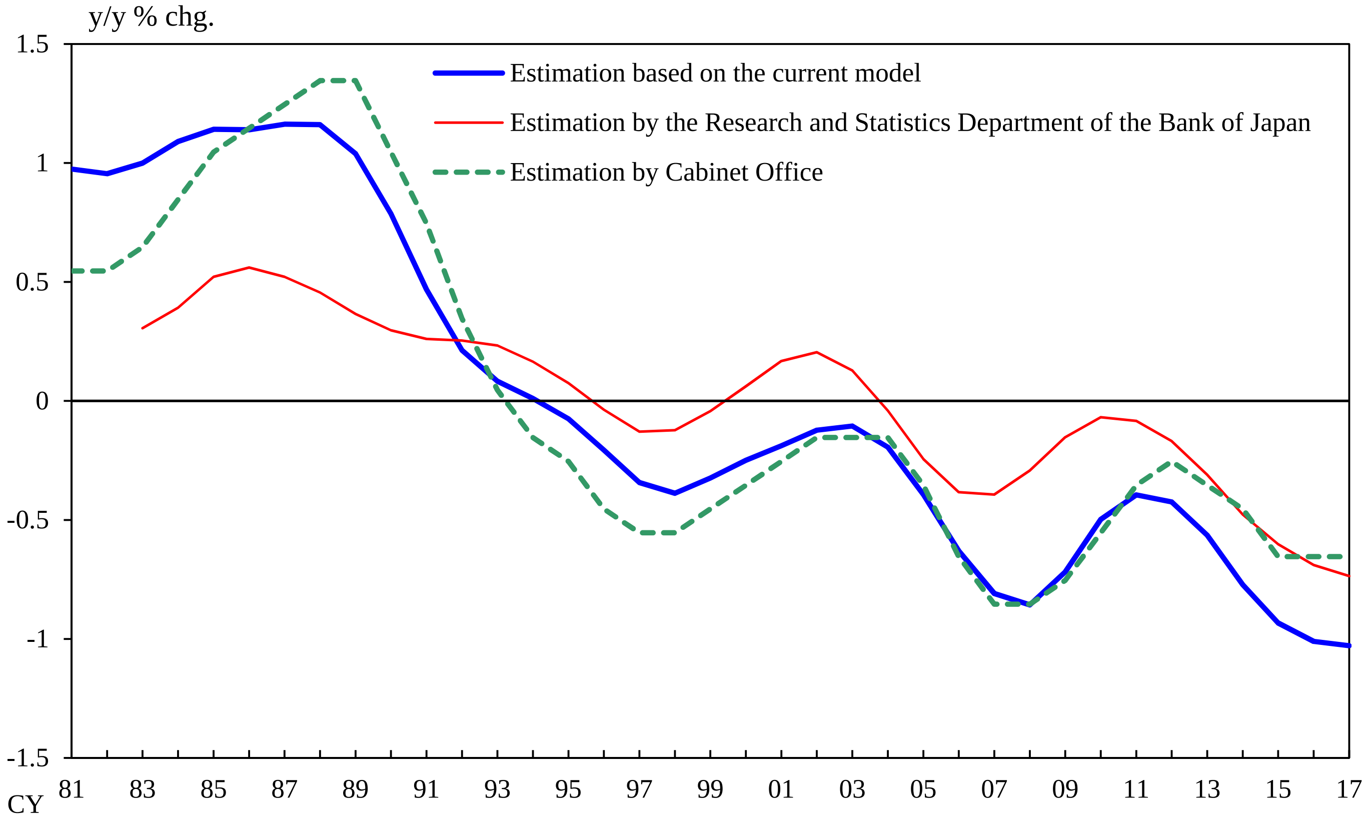
Source: Bank of Japan; Cabinet Office; Reinhart and Rogoff (2011).

Figure 10: Assessment using External Data : Neutral Technology

(1) High frequency movements



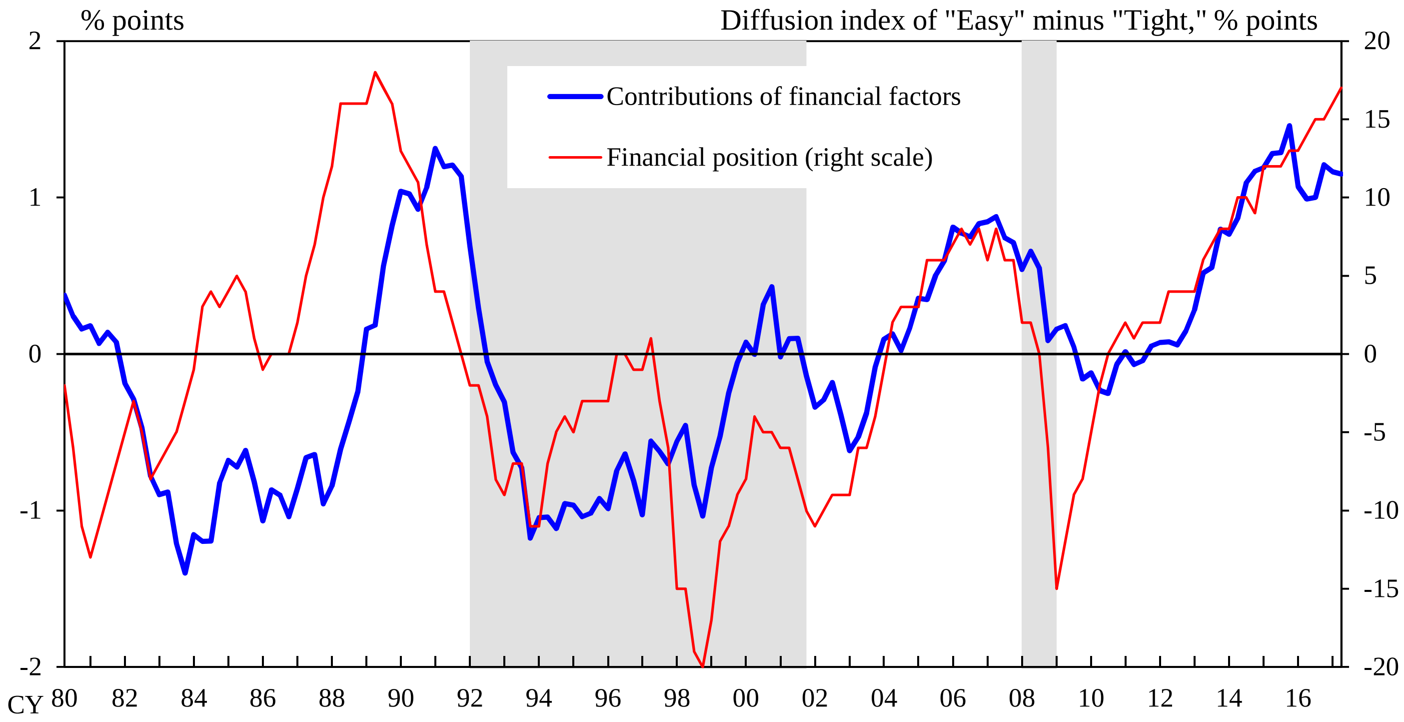
(2) Low frequency movements



Note: All of the series are demeaned.

Sources: Bank of Japan; Cabinet Office; Japan Productivity Center (JPC), "Productivity Statistics"; Organization for Economic Co-operation and Development (OECD).

Figure 11: Assessment using External Data : Financial Factors



Notes: 1. The shaded areas show the period of banking crisis based on the dating by Reinhart and Rogoff (2011) and the recession in the 14th business cycle in Japan based on the dating by the Cabinet Office.

2. Figures for the contributions of financial factors are the contributions of net worth shocks to the two sectors to the dynamics of the natural rate.

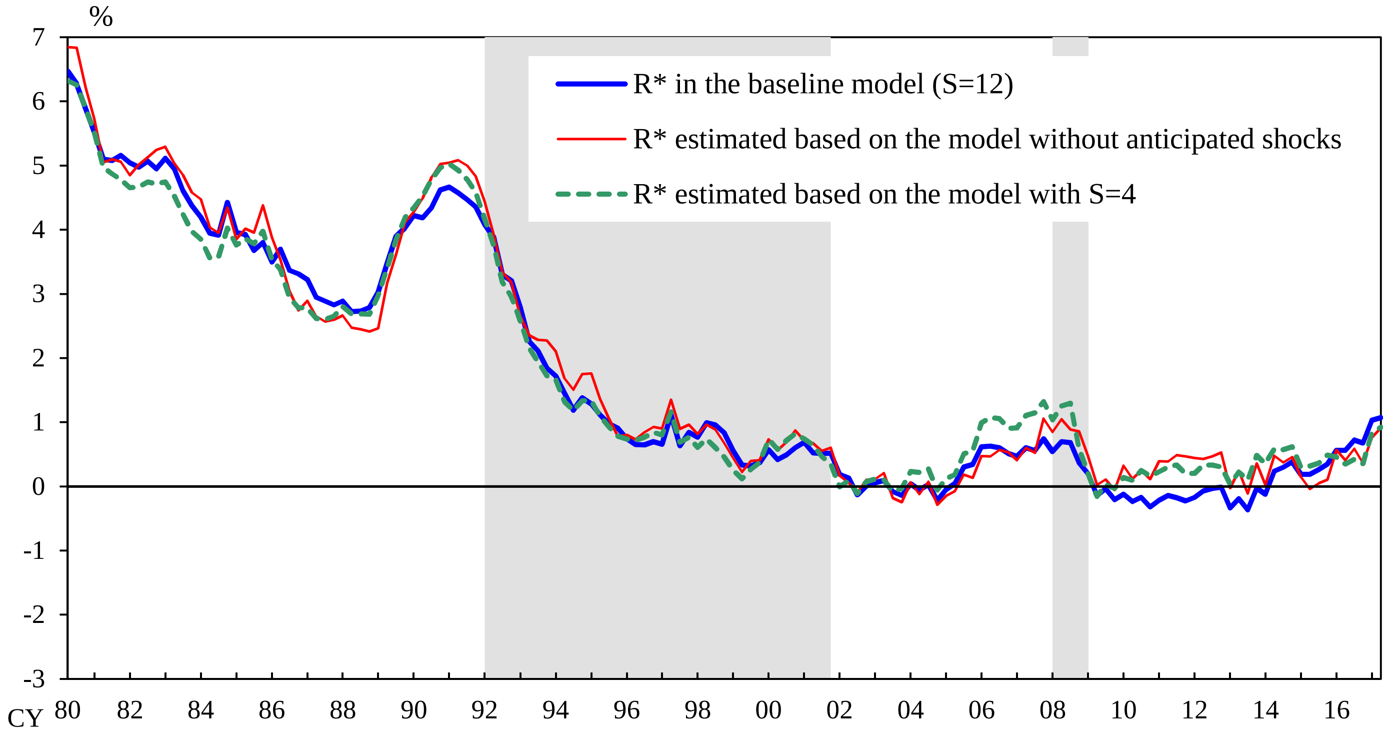
3. Figures for financial position are based on the *Tankan* . All enterprises, all industries.

4. Financial position is calculated by subtracting "tight" from "easy", and larger (lower) values indicate that firms are less (more) financially constrained.

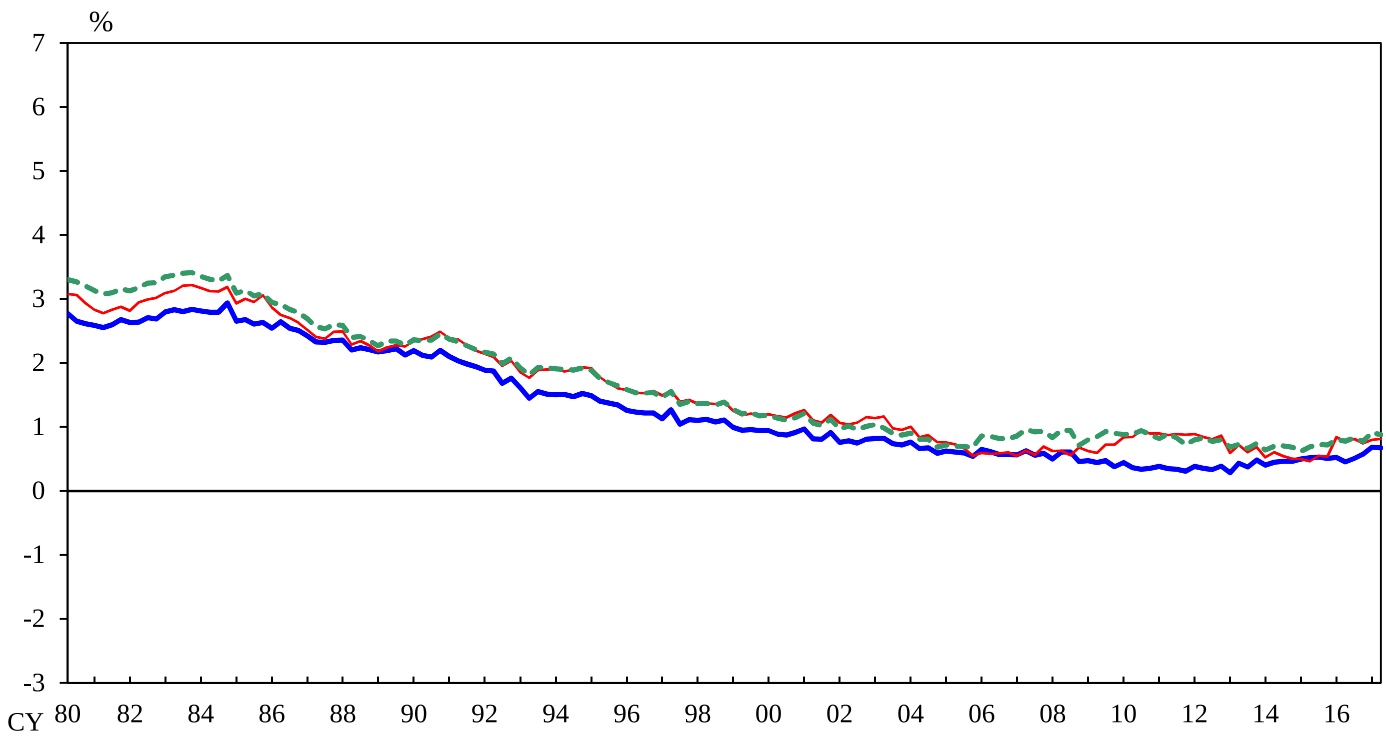
Sources: Bank of Japan; Cabinet Office; Reinhart and Rogoff (2011).

Figure 12: Robustness Check (1) : Anticipated Shocks

(1) Time path of the natural rate estimated by various specifications



(2) Time path of the expected natural rate estimated by various specifications

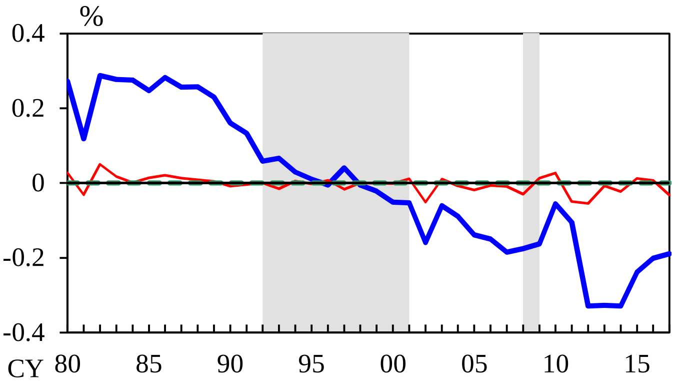


Note: The shaded areas show the period of banking crisis based on the dating by Reinhart and Rogoff (2011) and the recession in the 14th business cycle in Japan based on the dating by the Cabinet Office.

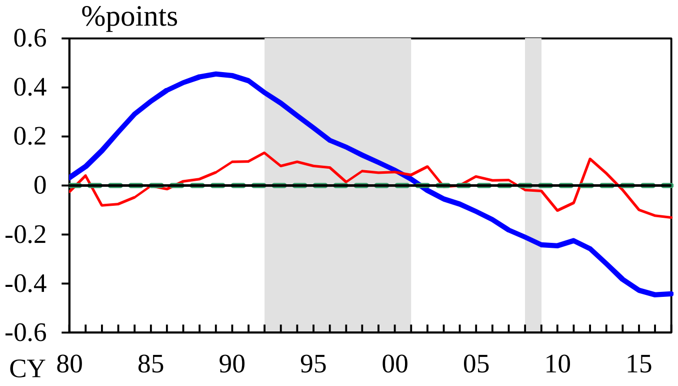
Sources: Cabinet Office; Reinhart and Rogoff (2011).

Figure 13: Robustness Check (2) : Working-age Population

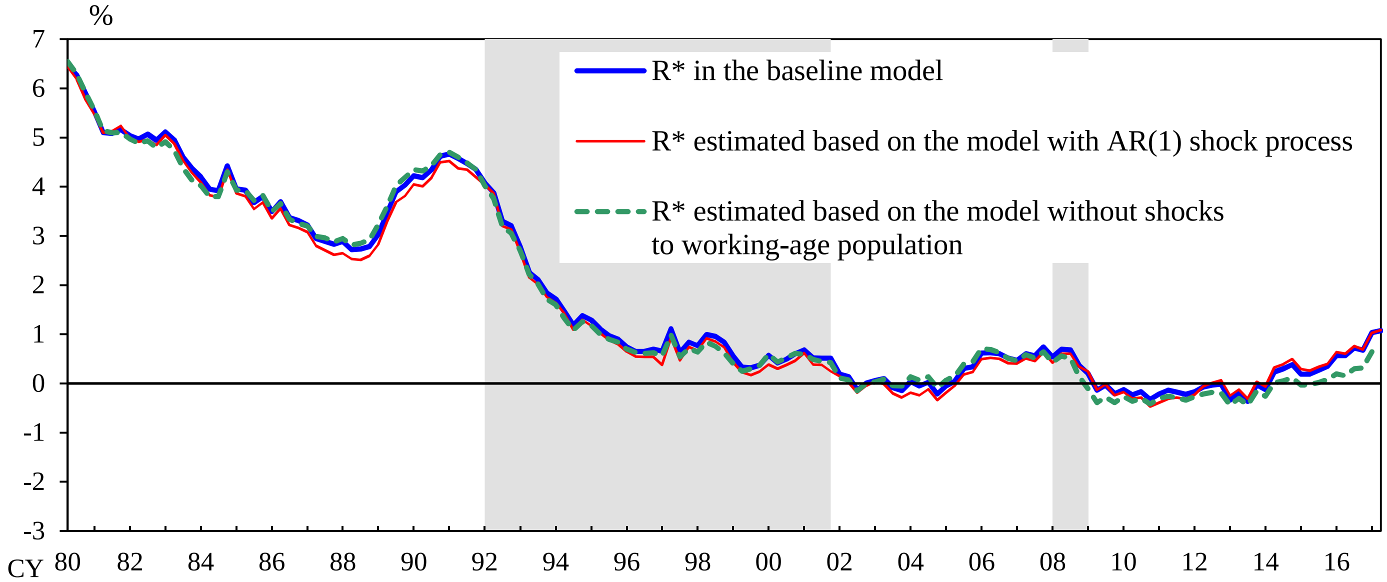
(1) Estimated shocks to working-age population



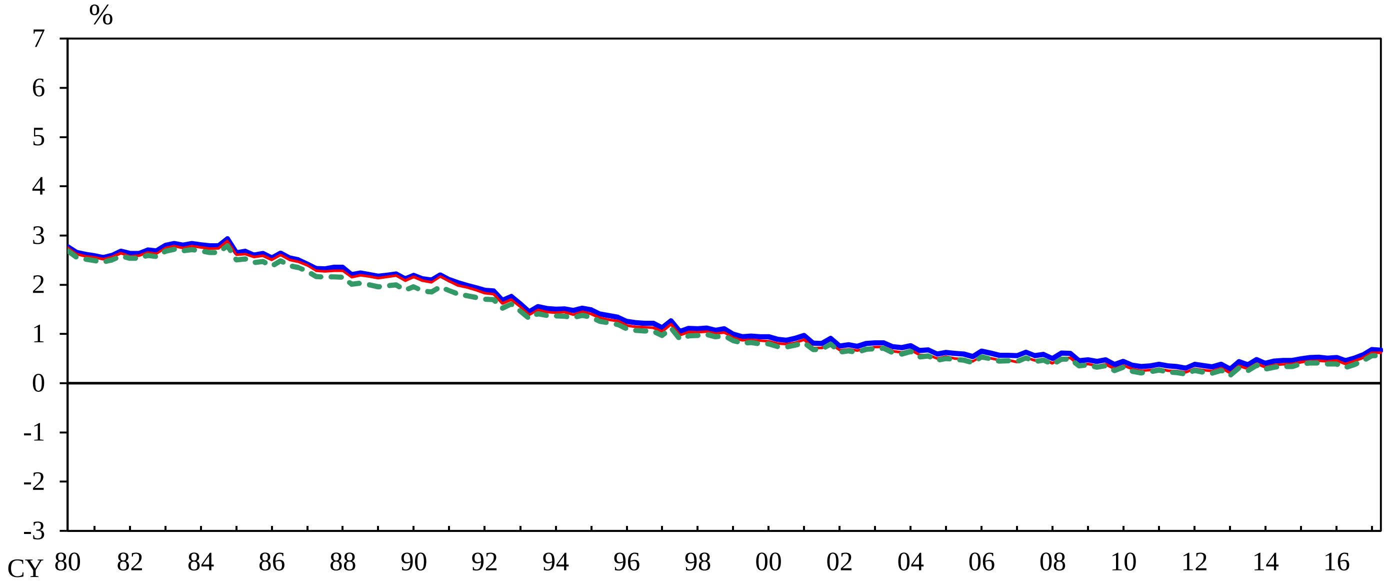
(2) Estimated contribution of shocks to working-age population to the dynamics of the natural rate



(3) Time path of the natural rate estimated by various specifications



(4) Time path of the expected natural rate estimated by various specifications



Notes: 1. The shaded areas show the period of banking crisis based on the dating by Reinhart and Rogoff (2011) and the recession in the 14th business cycle in Japan based on the dating by the Cabinet Office.
2. Figures for (1) and (2) are the average of four quarters.

Sources: Cabinet Office; Reinhart and Rogoff (2011).