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Bank of Japan Working Paper Series

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The Labor Share, Capital-Labor Substitution, and Factor Augmenting Technologies*

Naohisa Hirakata† and Yasutaka Koike‡

November 2018

Abstract

In this paper, we analyze the dynamics of the labor share in the United States and Japan using a dynamic stochastic general equilibrium (DSGE) model. For this purpose, we develop a model employing a constant elasticity of substitution (CES) production function with capital- and labor-augmenting technologies and investment specific technology. Our findings are as follows. First, comparing two different specifications of our model - one with a CES production function and one with a Cobb-Douglas production function - using marginal data densities indicates that the former provides a better fit for both the U.S. and Japanese data. Second, our estimates suggest that the elasticity of substitution is larger than one in the United States but less than one in Japan. Third, while capital-augmenting technology shocks have contributed to the decline of the labor share in the United States, they have exerted upward pressure on the labor share in Japan. The difference in the effects of capital-augmenting technology shocks on the labor share is due to the difference in the elasticity of substitution in the United States and Japan. Finally, the estimated models for the United States and Japan successfully replicate the observed relationship between the labor share and inflation.

Keywords: Labor Share, Elasticity of Capital-Labor Substitution, Inflation

JEL Classification: E31, E32

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*We thank Kosuke Aoki, Parantap Basu, Cristiano Cantore, Andrea Ferrero, Hibiki Ichiue, Deb dulal Mallick, Ryo Kato, Takushi Kurozumi, Tatsuyoshi Okimoto, Toshitaka Sekine, Nao Sudo, Tomohiro Sugo, Kenichi Ueda, Kozo Ueda, Francesco Zanetti, and participants of the Summer Workshop on Economic Theory at Hokkaido University and of a workshop at the Asian Development Bank Institute for valuable comments. Any remaining errors are the sole responsibility of the authors. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

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1 Introduction

The past few decades have witnessed a large decline in the labor share of gross value added in countries around the world (see, e.g., Elsby et al., 2013, and Karabarbounis and Neiman, 2014). A variety of reasons have been proposed to explain the declining labor share. The following three factors have been proposed as factors that have contributed to declining the labor share: changes in the relative price of capital led by investment-specific technology (IST), factor-augmenting technological changes – that is, labor and capital-augmenting technological changes – and increases in market concentration.

Karabarbounis and Neiman (2014) suggest that the decline in the relative price of investment goods explains roughly half of the decline in the labor share. Meanwhile, Piketty and Zucman (2014) argue that the declining labor share is the result of increased capital accumulation. The findings of these papers are based on estimates suggesting that the elasticity of substitution of capital and labor is greater than unity. However, some previous studies report that the elasticity is less than unity.\(^1\) When the elasticity is less than unity, the decline of the labor share cannot be explained by the decline in the relative price of investment goods.

Koh et al. (2016) argue that it is changes in capital-augmenting technology that are responsible for the labor share decline. They demonstrate that capital-augmenting technical progress can be interpreted as a form of intellectual property products (IPP) capital deepening. They then show that this IPP capital deepening leads to a decline in the labor share and that the elasticity of substitution between capital and labor is larger than one.

A number of studies suggest that increases in corporate profits related to increases in goods market concentration are another potential reason for the declining labor share.\(^2\) Barkai (2017), for instance, focusing on the United States, finds a negative industry-level

\(^1\)See, for example, Cantore et al. (2015), Oberfield and Raval (2014), and Chirinko (2008) and Antràs (2004).

\(^2\)Some studies argue that monopsony in labor markets may be a factor that has contributed to the decline in the labor share. See, for example, Azar et al. (2017), Dube et al. (2018), and Naidu et al. (2018).
relationship between changes in the labor share and changes in market concentration. He also presents evidence at the aggregate level that profits appear to have risen as a share of GDP and that the pure capital share of income (defined as the value of the capital stock times the required rate of return on capital over GDP) has fallen. Autor et al. (2017), using U.S. firm level data, show that market concentration tends to rise as industries become increasingly dominated by superstar firms with high profits and a low share of labor in firm value-added and sales. As a result, the aggregate labor share tends to fall. Meanwhile, De Loecker and Eeckhout (2017) find that while average markups were fairly constant between 1960 and 1980, there has been a sharp increase since 1980. However, in a more recent study, Traina (2018) reports that firm market power has either remained flat or declined.

In this paper, we examine what factors have contributed to changes in the labor share in the United States and Japan using a dynamic stochastic general equilibrium (DSGE) model. However, before we present our model, let us take a closer look at the developments that we aim to explain. Figure 1 shows developments in the labor share in the United States and Japan, while Figure 2 presents developments in the relative price of investment goods. As can be seen, the labor share in the United States was relatively stable until the early 2000s and only started to decline at the start of the millennium. On the other hand, the relative price of investment goods steadily declined until the early 2000s and since then has moved more or less sideways. These developments suggest that, contrary to Karabarbounis and Neiman’s (2014) assertions, the decline in the labor share in the United States may not be explained by the relative price of investment goods. Similarly, for Japan, the figures show that throughout the period depicted, the labor share has fluctuated around its mean – i.e., no persistent decrease or increase is observed – while developments in the relative price of investment goods are not very different from those in the United States, so that again there does not appear to an obvious link. Given that the relative price of investment goods does not appear to be a major factor explaining the decline in the labor share, the aim of our analysis employing the DSGE model is to examine what other factors – namely, factor augmenting-technological change and changes in markups – in addition to changes in the relative price of investment goods
have played a role.

Since standard New Keynesian models employ a Cobb-Douglas production function, the labor share fluctuates only due to markup changes associated with nominal rigidities. Therefore, standard New Keynesian models cannot address two important questions regarding developments in the labor share. The first is closely related to the results obtained by Karabarbounis and Neiman (2014) and concerns whether changes in factor prices including the relative price of investment goods have played a role in changes in the labor share changes. The second is whether factor-augmenting technological changes have affected capital-labor substitution. To address these questions, we build a model employing a constant elasticity of substitution (CES) production function in a standard New Keynesian models. However, standard New Keynesian models such as the model by Justiniano et al. (2011) assume that IST is non-stationary. When IST is assumed to be non-stationary, a DSGE model with a CES production function does not have a balanced growth path (BGP). In this case, it is not possible to estimate the model using Bayesian techniques. We therefore develop a model that has a BGP when (i) the elasticity of substitution is non-unity, (ii) labor-augmenting technological shocks are non-stationary, and (iii) IST shocks are non-stationary. In order to ensure that our model has a BGP and is stationary, following Uzawa (1961), we introduce non-stationary capital-augmenting technological change which is co-integrated with non-stationary IST change into a dynamic New Keynesian model.\(^3\)

We start by examining the implications of employing a CES production function for the goodness of fit of the models. We therefore estimate models with a Cobb-Douglas production function and models with a CES production function for the United States and Japan and compare the log marginal data densities. The results show that the models with the CES production function better explain the data for both the U.S. and Japanese economies.

Next, we examine the estimation results for the United States and Japan. The

\(^3\)Cantore et al. (2015) also estimate by Bayesian methods a standard medium-sized DSGE model with a CES production function, but they do not assume IST change, which Karabarbounis and Neiman (2014) suggested was the main factor underlying the decline in the labor share.
estimation results of our model using data for the United States are as follows. First, the elasticity of substitution between capital and labor, at 1.47 (at the mean), is greater than unity, which is consistent with the estimate by Karabarbounis and Neiman (2014), who obtained an estimate of around 1.25, but is different from estimates obtained in other studies (e.g., Cantore et al., 2015, and Antràs, 2004), which are less than unity. Second, since the estimated elasticity of substitution is greater than unity, the impulse responses of the labor share to IST shocks and capital-augmenting technology shocks in our model are negative. Third, the decomposition results suggest that about 80 percent of the decline in the labor share in the United States since the early 2000s is explained by positive capital-augmenting technology shocks. On the other hand, IST shocks have had little impact on the U.S. labor share, which contrasts with the results obtained by Karabarbounis and Neiman (2014). However, since Karabarbounis and Neiman’s (2014) analytical framework is based on comparative statics, their results do not reflect the fact that while the labor share in the United State has fallen notably since the early-2000s, the decline of the relative price of investment goods stopped during this period.

The estimation results for the Japanese economy differ from those for the U.S. economy. First, the elasticity of substitution between capital and labor, at 0.20 (at the mean), is less than unity. Second, since the estimated elasticity of substitution is less than unity, the impulse responses of the labor share to IST shocks and capital-augmenting technology shocks are positive, which is contrary to the results for the United States. On the other hand, the response of the labor share to labor-augmenting technology shocks is negative. The reason why the impulse responses are positive in the former case and negative in the latter is that the elasticity of substitution is less than unity. Third, the decomposition analysis shows that, in stark contrast with the United States, in Japan positive capital-augmenting technology shocks and negative labor-augmenting technology shocks put upward pressure on the labor share. This suggests that labor-augmenting technology has stagnated in the post-bubble period since the early 1990s, and this has affected the dynamics of the labor share.

Furthermore, the different production functions have interesting implications for inflation. Figure 3 presents scatterplots of inflation and labor share observations for the
United States and Japan. While the observations for the United States indicate a positive correlation, for Japan a weak negative or no correlation is observed. We ask if the estimated models for the United States and Japan can explain this difference in the link between inflation and the labor share. In the New Keynesian Phillips curve, the real marginal cost (or the inverse of the markup ratio) plays a crucial role in determining inflation dynamics. When the Cobb-Douglas production function is employed, the labor share coincides with the real marginal cost. Many empirical studies on the New Keynesian Phillips curve, such as Sbordone (2002), use this relationship. However, when the CES production function is employed, the real marginal cost generally does not coincide with the labor share. In fact, scatterplots of inflation and labor share observations for the United States and Japan indicate that while the observations for the United States suggest a positive correlation between inflation and the labor share, for Japan a weak negative or no correlation is observed. In order to investigate whether the estimated models for the United States and Japan can explain this difference in the relationship between inflation and the labor share, we conduct stochastic simulation exercises based on the models. The stochastic simulation based on the estimated model for the U.S. economy replicates the positive correlation between inflation and the labor share observed in the U.S. economy. Specifically, labor-augmenting technology shocks, capital-augmenting technology shocks, and IST shocks all contribute to generating the positive correlation between inflation and the labor share. On the other hand, the result of the stochastic simulation based on the estimated model for the Japanese economy replicates the weak correlation between inflation and the labor share, which is consistent with the observed data.

In addition to the studies already mentioned, our paper is also related to the studies by Acemoglu and Restrepo (2018) and Grossman et al. (2017). Developing a model to examine how machines replace human labor and why this might lead to lower employment and wages, Acemoglu and Restrepo (2018) show that automation of tasks previously performed by labor can lead to a permanent reduction in the labor share. Meanwhile, Grossman et al. (2017), extending a standard neoclassical growth model to incorporate endogenous human capital accumulation, demonstrate that in a neoclassical
growth setting with a certain form of capital-skill complementarity a slowdown in productivity growth leads to a deceleration of human capital accumulation and a long-run decline in the labor share.

The remainder of this paper is organized as follows. Section 2 presents our model of the economy. Section 3 describes the estimation method and the data used for the analysis, while Section 4 presents the estimation results. Section 5 then discusses the relationship between the labor share and inflation. Finally, Section 6 concludes.

2 The Model

Our model is broadly based on DSGE models used in recent business cycle studies such as Justiniano et al. (2011) and Hirose and Kurozumi (2012). The main difference from previous studies such as these is that our model employs a CES production function. When we employ a CES rather than a Cobb-Douglas production function and assume IST is non-stationary, the model requires a restriction regarding the steady state growth rate of technological progress to guarantee a balanced growth path. Apart from this difference, our model is essentially the same as the models employed in previous studies such as Justiniano et al. (2011) and Hirose and Kurozumi (2012).

2.1 Firms

2.1.1 Capital Service Firms

In a perfectly competitive environment, capital service firms purchase investment goods $I_t$ at price $P_i^t$ and transform them into capital $K_t$ subject to the prevailing transformation technology. Capital service firms own capital stock $K_t$ and rent utilization adjusted capital service $u_tK_{t-1}$ to intermediate goods firms at real price $R^k_t$. Capital service firms maximize the expected discounted value of future profits,

$$
\max_{K_t, I_t, u_t} E_t \sum_{j=0}^{\infty} \beta^j \frac{A_{t+j}}{A_t} \left( R_{t+j}^k u_{t+j} K_{t+j-1} - \frac{P_i^{t+j}}{P_{t+j}} I_{t+j} \right),
$$

(1)
subject to

\[ K_t = (1 - \delta(u_t)) K_{t-1} + \exp(z^i_t) \left\{ 1 - S \left( \frac{I_t}{I_{t-1}} \frac{1}{z_{ss} \psi_{ss}} \right) \right\} I_t, \] (2)

where \( \Lambda_t \) is households’ marginal utility of consumption, \( \beta \) is the subjective discount factor, and \( \beta \Lambda_{t+j}/\Lambda_t \) is a stochastic discount factor. Further, \( z^i_t \) stands for shocks to the marginal efficiency of investment (MEI), which represent an exogenous disturbance to the process by which investment goods are transformed into capital to be used by intermediate goods firms. The parameters \( z_{ss} \) and \( \psi_{ss} \) stand for the growth rates of labor-augmenting technology and IST in the steady state, respectively. Next, \( S(\bullet) \) stands for investment adjustment costs. We assume that \( S = S' = 0 \) in steady state, \( x = 1 \), and \( S'' > 0 \). Meanwhile, \( \delta(\bullet) \) is the depreciation function, whose properties are \( \delta' > 0 \) and \( \delta'' > 0 \). We specify the adjustment cost function of investment as follows:

\[ S(x) = \frac{\zeta(x - 1)^2}{2}. \] (3)

The first order conditions with respect to \( I_t, u_t, K_t \) are as follows:

\[
\begin{align*}
\frac{P^i_t}{P_t} &= Q_t \exp(z^i_t) \left\{ 1 - S \left( \frac{I_t}{I_{t-1}} \frac{1}{z_{ss} \psi_{ss}} \right) - S' \left( \frac{I_t}{I_{t-1}} \frac{1}{z_{ss} \psi_{ss}} \right) \frac{I_t}{I_{t-1}} \frac{1}{z_{ss} \psi_{ss}} \right\} \\
&\quad + E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} \exp(z^i_t) \left[ S' \left( \frac{I_{t+1}}{I_t} \frac{1}{z_{ss} \psi_{ss}} \right) \left( \frac{I_{t+1}}{I_t} \frac{1}{z_{ss} \psi_{ss}} \right) \frac{1}{z_{ss} \psi_{ss}} \right], \quad \text{(4)}
\end{align*}
\]

\[
\begin{align*}
Q_t &= E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ P_{t+1}^u u_{t+1} + Q_{t+1} (1 - \delta(u_{t+1})) \right\}, \quad \text{and} \\
R^k_t &= Q_t \delta'(u_t). \quad \text{(5)}
\end{align*}
\]

2.1.2 Investment Goods Firms

Perfectly competitive firms purchase \( Y_t^I \) units of final goods to produce investment goods \( I_t \). \( \Psi_t \) stands for the efficiency with which investment goods are produced. Therefore, the marginal cost of investment good production is \( P_t/\Psi_t \). Investment goods firms’ profit
maximization is given by

$$\pi_t^I = P_t^i I_t - P_t Y_t^I = P_t^i I_t - P_t \frac{I_t}{\Psi_t} = \left\{ P_t^i - \frac{P_t}{\Psi_t} \right\} I_t. \quad (7)$$

Since investment goods are produced competitively, their price $P_t^i$ is equal to the marginal cost of production:

$$P_t^i = \frac{P_t}{\Psi_t}. \quad (8)$$

### 2.1.3 Consumption Goods Producers

Final goods producers produce final consumption goods $Y_t$ from intermediate inputs $Y_t(f)$. Final goods producers maximize profits

$$P_t Y_t - \int_0^1 Y_t(f) P_t(f) df \quad (9)$$

subject to the technology for final goods production given by

$$Y_t = \left[ \int_0^1 Y_t(f) \frac{\phi^{P-1}}{\phi_t} df \right] \phi_t \frac{\phi^{P-1}}{\phi_t - 1}. \quad (10)$$

Cost minimization implies that the final goods are $Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\theta_t} Y_t$. Final consumption goods are produced competitively, so that their price is equal to the marginal cost of production:

$$P_t = \left[ \int_0^1 P_t(f)^{1-\theta_t} df \right]^{1-\theta_t}. \quad (11)$$

The market clearing condition for final consumption goods $Y_t$ is equal to the sum of consumption, investment, and other exogenous demand. That is,

$$Y_t = C_t + \frac{I_t}{\Psi_t} + gZ_t \exp(z_t^g), \quad (12)$$

where $g$ stands for the steady state ratio of exogenous demand to output. $z_t^g$ represents exogenous demand shocks and is governed by an autoregressive stationary process.
2.1.4 Intermediate Goods Firms

Producers of intermediate goods $f$ use a production technology characterized by constant returns to scale in capital and labor services, $u_t K_{t-1} (f)$ and $l_t (f)$, to produce output $Y_t (f)$ sold to final goods producers. Specifically, we assume that each intermediate good $Y_t (f)$ is produced based on a CES production function:

$$
Y_t (f) = \left[ \alpha_n (Z_t l_t (f))^\frac{\sigma - 1}{\sigma} + \alpha_k (\Omega_t u_t K_{t-1} (f))^\frac{\sigma - 1}{\sigma} \right]^\frac{\sigma}{\sigma - 1},
$$

(13)

where $\sigma$ denotes the elasticity of substitution between capital and labor service inputs in production, $\alpha_n$ and $\alpha_k$ are distribution parameters, and $Z_t$ and $\Omega_t$ denote labor-augmenting and capital-augmenting technology, respectively. The level of labor-augmenting technology is non-stationary and assumed to follow the stochastic process

$$
\ln Z_t = \ln z_{ss} + \ln Z_{t-1} + z_t^z.
$$

(14)

$z_{ss}$ represents the gross trend rate of labor-augmenting technological change and $z_t^z$ stands for shocks to the rate of change and follows a stationary process.

Each producer’s labor input $l_t (f)$ is an aggregate of differentiated labor services with substitution elasticity $\theta^w_t > 1$ and is given by

$$
l_t (f) = \left[ \int_0^1 l_t (f, h)^{\theta^w_t - 1} \frac{dh}{\theta^w_t} \right]^{\frac{\theta^w_t}{\theta^w_t - 1}}.
$$

(15)

The corresponding aggregate wage is given by

$$
W_t = \left[ \int_0^1 W_t (h)^{1 - \theta^w_t} dh \right]^{-\frac{1}{1 - \theta^w_t}}.
$$

(16)

Producers choose $u_t K_{t-1} (f)$ and $l_t (f)$ to minimize costs given the real capital service price $R^k_t$ and real wage $W_t$. Combining the cost-minimizing conditions with respect to capital and labor services shows that real marginal cost $mc_t$ is identical among interme-
diatic goods firms and is given by

\[ mc_t = \left[ \alpha_n^\sigma \left( \frac{W_t}{Z_t} \right)^{1-\sigma} + \alpha_k^\sigma \left( \frac{R^k_t}{\Omega_t} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \]  \hspace{1cm} (17)

Combining the first order conditions for the optimal choice of capital and labor services inputs yields

\[ \frac{l_t}{u_t K_{t-1}} = \left( \frac{W_t}{R^k_t} \right)^{-\sigma} \left( \frac{Z_t}{\Omega_t} \right)^{\sigma-1} \left( \frac{\alpha_n}{\alpha_k} \right)^\sigma, \]  \hspace{1cm} (18)

where \( l_t = \int_0^1 l_t(f) df \) and \( K_t = \int_0^1 K_t(f) df \). The above equations are obtained using the market clearing conditions with respect to capital and labor services. In addition, using this equation to aggregate the production function over intermediate goods firms, we obtain the following aggregate CES production function:

\[ Y_t d_t = \left[ \alpha_n (Z_t l_t)^{\frac{\sigma-1}{\sigma}} + \alpha_k (\Omega_t u_t K_{t-1})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \]  \hspace{1cm} (19)

where \( d_t \) is the price dispersion of intermediate goods, which is defined as \( \int_0^1 (P_t(f)/P_t)^{-\theta_p} df \).

Given consumption goods producers’ demand, intermediate goods firms set the prices of their products on a staggered basis à la Calvo (1983). Each period, a fraction \( 1 - \xi^p \in [0, 1] \) of intermediate goods prices are reoptimized, while the remaining fraction \( \xi^p \) is set by indexation to a weighted average of past and steady-state inflation rates, \( \Pi_{t-1}^p \Pi_{ss}^{1-\gamma_p} \). \( \gamma_p \in [0, 1] \) is the weight of price indexation to past inflation. Thus, all firms solve the same following problem:

\[ \max_{P_t(f)} E_t \sum_{j=0}^{\infty} \xi^p_j \left( \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \right) \left[ \frac{P_t^p(f)}{P_{t+j}} \prod_{k=1}^{j} \left( \Pi_{t+k-1}^p \Pi_{ss}^{1-\gamma_p} \right) - mc_{t+j} \right] Y_{t+j|t}(f), \]  \hspace{1cm} (20)

subject to

\[ Y_{t+j|t}(f) = Y_{t+j} \left[ \frac{P_t^p(f)}{P_{t+j}} \prod_{k=1}^{j} \left( \Pi_{t+k-1}^p \Pi_{ss}^{1-\gamma_p} \right) \right]^{-\theta_p}, \]  \hspace{1cm} (21)

where \( \beta \Lambda_{t+j}/\Lambda_t \) denotes the stochastic discount factor between periods \( t \) and \( t+j \). The first order condition for the reoptimized price \( P_t^p \) is given by
The consumption goods price equation (11) can be expressed as

\[ 1 = \left( 1 - \xi_p \right) \left\{ \left( \frac{p^o_t}{P_t} \right)^{-\frac{1}{\beta'}} + \sum_{j=1}^{\infty} \left( \xi_p \right)^j \left[ \frac{P_t^o \prod_{k=1}^{j} \left\{ \left( \frac{\Pi_{t+k-1}}{\Pi_{ss}} \right)^{\gamma_p} \frac{\Pi_{ss}}{\Pi_{t+k-1}} \right\} }{P_t} \right]^{-\frac{1}{\beta'}} \right\} , \tag{23} \]

where the price markup \( \lambda_t^p \) is defined as \( \lambda_t^p \equiv 1 / (\theta_t^p - 1) = \lambda_t^p \exp (z_t^p) \). \( z_t^p \) is the price markup shock.

### 2.2 Households

Each household \( h \in [0, 1] \) derives utility from purchasing consumption goods \( C_t (h) \) and disutility from providing differentiated labor supply \( l_t (h) \) to entrepreneurs under monopolistic conditions. Households’ preferences are represented by the utility function

\[ E_t \sum_{t=0}^{\infty} \beta^t \exp \left( z_t^b \right) \left\{ \ln (C_t (h) - bC_{t-1} (h)) - \frac{l_t (h)^{1+\eta}}{1+\eta} \right\} , \tag{24} \]

where \( b \in [0, 1] \) is the degree of habit formation, \( \eta > 0 \) denotes the inverse of the labor supply elasticity, and \( z_t^b \) represents preference shocks. Households’ budget constraint is given by

\[ P_t C_t (h) + B_t (h) = r_t^n B_{t-1} (h) + P_t W_t (h) l_t (h) + T_t (h) , \tag{25} \]

where \( r_t^n \) is the gross nominal interest rate, \( B_t (h) \) represents households’ nominal bond holdings, and \( T_t (h) \) represents lump-sum public transfers and profits received from firms.

In the presence of complete insurance markets, all households purchase the same levels of consumption goods and one-period riskless bonds. Hence, the first order conditions
for utility maximization with respect to consumption and bond-holdings are given by

\[
\Lambda_t = \exp\left(z^b_t\right)(C_t - bC_{t-1})^{-1} + \beta b \exp\left(z^b_{t+1}\right)(C_{t+1} - bC_{t})^{-1}, \quad \text{and} \quad (26)
\]
\[
1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{r^u}{\pi_{t+1}}. \quad (27)
\]

Under monopolistic competition, the demand for labor service \( h \) is given by

\[
l_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\theta^w_t} l_t, \quad (28)
\]

where

\[
l_t = \left[ \int_0^1 l_t(h)^{(\theta^w_t-1)/\theta^w_t} dh \right]^{\theta^w_t/(\theta^w_t-1)}. \quad (29)
\]

(29) is an aggregate of differentiated labor supply. \( \theta^w_t > 1 \) denotes the degree of differentiation of the labor services provided by households. Each household \( h \) has monopoly power and households set nominal wages on a staggered basis à la Calvo (1983). For each period, a fraction \( 1 - \xi^w \in [0, 1] \) of wages is reoptimized, while the remaining fraction \( \xi^w \) is set by indexation to both the gross trend rate of labor-augmenting technology, \( z_{ss} \), and the weighted average of past and steady-state inflation rates \( \Pi_{t-1}^\gamma \Pi_{ss}^{1-\gamma} \), where \( \gamma^w \) is the weight of wage indexation of past inflation relative to steady-state inflation. Specifically, the reoptimized wages are set by solving the following problem:

\[
\max_{W_t(h)} E_t \sum_{j=0}^{\infty} (\beta \xi^w)^j \left[ \Lambda_{t+j} l_{t+j}(h) \frac{P_t W_t(h)}{P_{t+j}} \prod_{k=1}^{j} \left( z_{ss} \Pi_{t+j-1}^\gamma \Pi_{ss}^{1-\gamma} \right) \right]
\]

\[
- \exp(z^b_t(l_t(h))^{1+\eta} l_t(1+\eta) \right). \quad (29)
\]

(30)
The first order condition for the reoptimized wage $W_t^o$ is given by

$$E_t \sum_{j=0}^{\infty} \left( (\beta \xi_w)^j \left( \frac{\Pi_{t+k}}{\Pi_{t+k-1}} \right) I_{t+j} \left[ \frac{(z_{ss})^j W_t^o}{W_t} \prod_{k=1}^{j} \left\{ \left( \frac{\Pi_{t+k-1}}{\Pi_{t+k}} \right)^{\gamma_w} \frac{\Pi_{t+k}}{\Pi_{t+k}} \right\} \right] \right)^{-1+\lambda_{t+j}^w} \times \left( I_{t+j} \left[ \frac{(z_{ss})^j W_t^o}{W_t} \prod_{k=1}^{j} \left\{ \left( \frac{\Pi_{t+k-1}}{\Pi_{t+k}} \right)^{\gamma_w} \frac{\Pi_{t+k}}{\Pi_{t+k}} \right\} \right] \right)^{\lambda_{t+j}^w} = 0. \quad (31)$$

The wage markup $\lambda_t^w$ is defined as $\lambda_t^w \equiv 1/(\theta_t^w - 1) = \bar{\lambda}_w \exp(z_t^w)$, where $z_t^w$ represents wage markup shocks. The aggregate wage equation (16) can be expressed as

$$1 = (1 - \xi_w) \left\{ \left( \frac{W_t^o}{W_t} \right)^{-\frac{1}{\lambda_t^w}} + \sum_{j=1}^{\infty} (\xi_w)^j \left[ \frac{(z_{ss})^j W_t^o}{W_t} \prod_{k=1}^{j} \left\{ \left( \frac{\Pi_{t+k-1}}{\Pi_{t+k}} \right)^{\gamma_w} \frac{\Pi_{t+k}}{\Pi_{t+k}} \right\} \right]^{-\frac{1}{\lambda_t^w}} \right\}. \quad (32)$$

### 2.3 The Central Bank

The central bank conducts monetary policy based on the following Taylor rule:

$$\ln r_t^n = \phi_t \ln r_{t-1}^n + (1 - \phi_t) \left\{ \ln r_{ss}^n + \phi_n \left( \frac{1}{4} \sum_{j=0}^{3} \ln \frac{\pi_{t-j}}{\pi_{ss}} \right) + \phi_y \frac{Y_t}{Y_t^*} \right\}, \quad (33)$$

where $Y_t^*$ is the level of output that would prevail under flexible prices and wages.

### 2.4 Conditions for a Balanced Growth Path

We introduce three types of technology, namely, labor-augmenting technology $Z_t$, capital-augmenting technology $\Omega_t$, and investment-specific technology $\Psi_t$. In our model, we assume that each of the variables representing a particular type of technology follows a stochastic trend. We define that

$$\ln \psi_t \equiv \ln(\Psi_t) - \ln(\Psi_{t-1}), \quad (34)$$

$$\ln \omega_t \equiv \ln(\Omega_t) - \ln(\Omega_{t-1}). \quad (35)$$
Following Grossman et al. (2018), the total growth rate of capital-augmenting technology $\gamma_t$ is defined as the sum of the growth rates of capital-augmenting technology $\omega_t$ and investment-specific technology $\psi_t$. That is,

$$\ln \gamma_t \equiv \ln \omega_t + \ln \psi_t. \quad (36)$$

Uzawa (1961) proposed the condition that a BGP can exist only if $\sigma = 1$ or $\ln \gamma_{ss} = 0$. $\ln \gamma_{ss}$ is the steady state growth rate of the total rate of capital-augmenting technological change.\(^4\) When $\sigma = 1$, the production function of the model is the Cobb-Douglas production function. As we discuss later, since this case is not supported by the data for both the United States and Japan, we introduce the restriction $\ln \gamma_{ss} = 0$. As a result of this restriction, $\ln \gamma_{ss} = 0$, the trend growth of capital-augmenting technology is offset by the trend growth of IST in the steady state.\(^5\)

In order for a BGP to exist in our model, we introduce co-integrated relationships between the non-stationary variables, i.e., between capital-augmenting technology $\Omega_t$ and investment-specific technology $\Psi_t$. To specify the stochastic processes of $\Omega_t$ and $\Psi_t$, we define $\chi_t$ as follows:

$$\exp (\chi_t) \equiv \Omega_t \Psi_t. \quad (37)$$

We assume that $\chi_t$ is stationary and constant at the steady state. The first difference of $\chi_t$ in (37) is given by

$$\chi_t - \chi_{t-1} = \ln \omega_t + \ln \psi_t. \quad (38)$$

The above equation (38) coincides with the definition of the total rate of capital-augmenting technological change $\ln \gamma_t$ provided by (36). As stated earlier, for a BGP to exist, $\ln \gamma_t$ needs to equal zero in the steady state. Finally, we specify the process of capital-augmenting technological change and the process of investment-specific technological change.

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\(^4\)We formulate the condition following Corollary 1 in Grossman et al. (2018).

\(^5\)More intuitively, this restriction prevents capital from growing at a rate that is too high or too low.
change in the following error correction form:

\[
\ln \psi_t = (1 - \rho_\psi) \ln \psi_{ss} + \rho_\psi \ln \psi_{t-1} - \kappa \chi_{t-1} + \epsilon_\psi^t, \text{ and}
\]

\[
\ln \omega_t = (1 - \rho_\omega) \ln \omega_{ss} + \rho_\omega \ln \omega_{t-1} - \kappa \omega \chi_{t-1} + \epsilon_\omega^t, \text{ (40)}
\]

where \( \epsilon_\psi^t \) and \( \epsilon_\omega^t \) are disturbance terms that are normally distributed with mean zero and standard deviation \( \sigma_\psi \) and \( \sigma_\omega \), respectively. To ensure the existence of a BGP, we introduce the following restriction:

\[
\ln \psi_{ss} + \ln \omega_{ss} = 0. \text{ (41)}
\]

The above equation (41) states that the total rate of capital-augmenting technological change \( \ln \gamma_t \) is zero in the steady state, which is equivalent to the condition that \( \chi_t \) is stationary. In the steady state, when the growth rate of IST \( \ln \psi_{ss} \) is negative, the growth rate of capital-augmenting technology \( \ln \omega_{ss} \) is positive. Thus, equation (41) means that technological change is directed toward the factor which is scarce in the steady state.\(^6\)

In addition, we assume that \( \epsilon_\psi^t \) and \( \epsilon_\omega^t \) are governed by stationary stochastic processes.

### 2.5 Income Shares

In our model, nominal output of final consumption goods is paid as wage for labor services, the rental of capital services, and profits:

\[
P_t Y_t = P_t W_t l_t + P_t R_t^k u_t K_{t-1} + P_t Y_t \left(1 - \frac{1}{\mu_t}\right), \text{ (42)}
\]

where \( P_t Y_t (1 - 1/\mu_t) \) is the sum of intermediate goods firms’ profits. \( \mu_t \) denotes the markup, which can be expressed as

\[
\mu_t = \frac{P_t Y_t}{P_t W_t l_t + P_t R_t^k u_t K_{t-1}}. \text{ (43)}
\]

\(^6\)Acemoglu (2002) develops a model in which firms choose to direct technological change toward the relatively abundant factor when the elasticity of substitution is larger than one.
In the steady state, the markup $\mu_t$ in (43) is equal to $\theta_{ss}^p/(\theta_{ss}^p - 1)$. Since our model assumes that prices are sticky, markup $\mu_t$ changes in response not only to changes in firms’ market power $\theta_{ss}^p$ but also to various shocks, such as technology shocks, demand shocks, and so on. Accordingly, the labor share $s_{L,t}$, the capital share $s_{K,t}$, and the profit share $s_{\pi,t}$ are defined as follows:

$$s_{L,t} \equiv \frac{W_t l_t}{\mu_t (W_t l_t + R^k_t u_t K_t)};$$

$$s_{K,t} \equiv \frac{R^k_t u_t K_{t-1}}{\mu_t (W_t l_t + R^k_t u_t K_{t-1})},$$

$$s_{\pi,t} \equiv \left(1 - \frac{1}{\mu_t}\right),$$

where $s_{L,t} + s_{K,t} + s_{\pi,t} = 1$. Other things being equal, an increase in the markup $\mu_t$ reduces the labor share.

### 2.6 The Labor Share, Technologies, and Factor Prices

Let us consider the determinants of the labor share of income $s_{L,t}$ defined by (44). Intermediate goods firms produce intermediate goods based on the CES production function given by (13). Using the first order conditions for cost minimization by intermediate goods firms, the capital-labor ratio can be expressed as a function of capital- and labor-augmenting technologies, $Z_t$ and $\Omega_t$, and factor prices, $W_t$ and $R^k_t$, for a given value of the elasticity of substitution $\sigma$, as shown in (18). Equation (18) states that, given factor prices ($R^k_t$ and $W_t$), the sign of the response of the capital-labor ratio $l_t/u_t K_{t-1}$ to factor-augmenting technologies $Z_t$ and $\Omega_t$ depends on whether the elasticity of substitution $\sigma$ is greater than one or not. When $\sigma > 1$, an increase in $Z_t$ raises the ratio of labor to capital. Thus, when $\sigma > 1$, an increase in $Z_t$ leads to a rise in the labor share. The opposite is the case when $\Omega_t$ increases.

Next, using equation (18), we discuss the effect of IST $\Psi_t$ on the labor share. Although $\Psi_t$ does not appear in the equation, $\Psi_t$ has an effect on the rental rate of capital $R^k_t$, which appears in the equation for the capital-labor ratio (18). The relationship between
the rental rate of capital $R^k_t$ and investment-specific technology $\Psi_t$ is given by

$$E_t \beta \frac{A_{t+1}}{A_t} R^k_{t+1} u_{t+1} = \frac{1}{\Psi_t} - E_t \beta \frac{A_{t+1}}{A_t} [1 - \delta (u_{t+1})] \frac{1}{\Psi_{t+1}}.$$  \hfill (47)

Equation (47) states that a rise in $\Psi_t$ and $\Psi_{t+1}$ leads to a decline in the rental rate of capital $R^k_t$. The decline in the rental rate of capital $R^k_t$ lowers the ratio $l_t / u_t K_{t-1}$. The extent of the decline depends on the size of the elasticity of substitution $\sigma$, as shown in (18). As for the effect of changes in relative factor prices on the labor share, when $\sigma > 1$, the labor share decreases in response to a decline in $R^k_t$ relative to $W_t$. On the other hand, when $\sigma < 1$, the labor share increases in response to a decline in $R^k_t$ relative to $W_t$.

### 3 Estimation Strategy and Data

We use Bayesian methods to characterize the posterior distribution of the structural parameters of the model. The posterior distribution combines the likelihood function with prior information. Before conducting Bayesian estimations, we set some of the parameters to the values observed for the United States and Japan, which are shown in Table 1. The prior distributions for the two countries are shown in Tables 3 and 4, respectively. To calculate the posterior distributions and to evaluate the marginal likelihood of the models, the Metropolis-Hastings algorithm is employed.\(^7\)

\(^7\)For the subsequent analysis, two chains of 1,000,000 draws were generated and the first half of these draws was discarded. The scale factor for the jumping distribution in the Metropolis-Hastings algorithm was adjusted so that an acceptance rate of 25 percent was obtained. The Brooks and Gelman (1998) shrink factor was used to check the convergence of the parameters. All estimations are conducted using Dynare.
3.1 Detrending and Normalization

To use the Bayesian likelihood approach, the equilibrium conditions of the model are rewritten by detrending variables with regard to \( Z_t \) and \( \Psi_t \):

\[
y_t = \frac{Y_t}{Z_t}, \quad c_t = \frac{C_t}{Z_t}, \quad w_t = \frac{W_t}{Z_t}, \quad \lambda_t = \Lambda_t Z_t, \quad i_t = \frac{I_t}{Z_t \Psi_t}, \quad k_t = \frac{K_t}{Z_t \Psi_t}, \quad r_t = R_t \Psi_t, \quad \text{and} \quad q_t = Q_t \Psi_t.
\]

In standard New Keynesian models employing a Cobb-Douglas production function, such as the ones by Justiniano et al. (2011) and Hirose and Kurozumi (2012), the variables are detrended by the composite technology level consisting of labor-augmenting technology and investment specific technology. In contrast, in our model, the variables are detrended only by labor-augmenting technology \( Z_t \), since we introduce the restriction that the total growth rate of capital-augmenting technology is zero in the steady state.

To distinguish the elasticity of substitution between labor and capital from capital-augmented technological changes, we employ a normalized CES function following the re-parameterization procedure proposed by Cantore et al. (2014). The CES production function in deviation form is

\[
Y_t^{d_t} = y_{ss} \left[ (1 - \alpha) \left( \frac{Z_t}{l_{ss}} \right)^{\frac{\sigma - 1}{\sigma}} + \alpha \left( \frac{u_t K_t}{k_{ss} \psi_{ss}} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}, \quad (48)
\]

where \( \alpha \) is obtained by the following re-parameterization of \( \alpha_k \) and \( \alpha_n \):

\[
\alpha = \frac{\alpha_k}{\frac{\alpha_k}{\frac{l_{ss} z_{ss} \psi_{ss}}{k_{ss}}}^{\frac{\sigma - 1}{\sigma}} + \alpha_k}.
\]

3.2 Data and Observation Equations

The observation period for our estimation is from 1985Q1 to 2017Q3 for the U.S. economy and from 1988Q1 to 2017Q3 for the Japanese economy. In the Bayesian estimation, we use the real GDP growth rate, \( \Delta \ln GDP_t \), the consumption growth rate, \( \Delta \ln Consumption_t \), the investment growth rate, \( \Delta \ln Investment_t \), wage inflation, \( \Delta \ln Wage_t \), the inflation rate, \( Inflation_t \), the policy rate, \( PolicyRate_t \), the labor share,
\(LS_t\), and the rate of change in the relative price of investment goods, \(\Delta \ln RPI_t\), as observables.\(^8\) The observation equations are as follow:

\[
\Delta \ln GDP_t = \ln z_t^z + \ln \frac{y_t}{y_{t-1}}, \quad (49)
\]

\[
\Delta \ln Consumption_t = \ln z_t^z + \ln \frac{c_t}{c_{t-1}}, \quad (50)
\]

\[
\Delta \ln Investment_t = \ln z_t^z + \ln \frac{\psi_t}{\psi_{t-1}} + \ln \frac{i_t}{i_{t-1}}, \quad (51)
\]

\[
\Delta \ln Wage_t = \ln z_t^z + \ln \frac{w_t}{w_{t-1}}, \quad (52)
\]

\[
Inflation_t = \pi_{ss} + \pi_t, \quad (53)
\]

\[
PolicyRate_t = r^n_{ss} + r^n_t, \quad (54)
\]

\[
\Delta \ln LS_t = \Delta \ln s_{L,t}, \text{ and} \quad (55)
\]

\[
\Delta \ln RPI_t = \Delta \ln \left( \frac{P_t^I}{P_t} \right) = -\ln \psi_t. \quad (56)
\]

### 3.3 Shock Processes

The structural shocks \(z_t^x, x \in \{z, b, p, w, i, r, g\}\) are all governed by a stationary first order autoregressive process,

\[
z_t^x = \rho_x z_{t-1}^x + \varepsilon_t^x,
\]

where \(\rho_x \in (0, 1)\) is the autoregressive coefficient and \(\varepsilon_t^x\) is a disturbance term that is normally distributed with mean zero and standard deviation \(\sigma_x\). In addition, there are two innovation terms, \(\varepsilon_t^\psi\) and \(\varepsilon_t^\mu\), which appeared in (39) and (40).

### 4 Estimation Results

#### 4.1 Comparison of Models

We start by examining whether our model provides a better fit to the data than the model using the Cobb-Douglas production function. To this end, we estimate the models with the Cobb-Douglas production function to see which model better explains the data for

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\(^8\)For details of the data used for the estimation, see the Appendix.
the U.S. and Japanese economies.

The results of these estimations are shown in Table 2. The values of the marginal data densities for the model with the Cobb-Douglas production function are lower than those for the benchmark model with a CES production function in the case of both the U.S. and Japanese economies. This implies that the models with the CES production function are more successful in explaining the actual data for the two economies. Given equal prior odds, the posterior odds ratios of the models with the CES production function are almost one in the case of both the U.S. and Japanese economies.

Tables 3 and 4 show the estimation results of the models employing a CES production function. The estimates of the elasticity of substitution between capital and labor $\sigma$ are larger than one, 1.47 at the mean, for the United States and less than one, 0.20 at the mean, for the Japanese economy. As explained below, the difference in the elasticities has interesting implications for the dynamics of the labor share.

### 4.2 Impulse Responses

Figure 4 shows the impulse responses of the labor share to various shocks. The response of the labor share to IST shock $\varepsilon_t^V$ and capital-augmenting technology shock $\varepsilon_t^T$ is negative in the case of the United States and positive in the case of Japan. The difference in the responses is caused by the difference in the estimates of the elasticity of substitution between capital and labor, $\sigma$, which, as seen in equation (18), determines the size and direction of the response. The negative response of the labor share to both types of shock in the United States is due to the fact that the elasticity of substitution is larger than one, while the opposite is the case for Japan.

The intuition of the above is as follows. When $\sigma > 1$, an increase in capital-augmenting technology raises the demand for capital more than the demand for labor, since the marginal product of capital increases more than the marginal product of labor. That is, in an environment in which it is easy for firms to substitute labor for capital, i.e., $\sigma > 1$, firms will use more capital when it becomes possible for them to use capital more

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9Cantore et al. (2015) also find that a model with a CES production function explains the U.S. data better than a model with a Cobb-Douglas production function.
efficiently, that is, capital-augmenting technology increases. When $\sigma < 1$, it is not easy to substitute capital for labor, and firms will not use more capital when the efficiency of capital increases through capital-augmenting technological change.

In the case of Japan, since the estimate of the elasticity of substitution $\sigma$ is less than one, the response of the labor share to labor augmenting technological shock $\varepsilon^z_t$ is negative. On the other hand, the response of the labor share in the United States to labor-augmenting technological shock $\varepsilon^z_t$ is initially negative (until around the 7th quarter) but positive after around the 10th quarter. The negative response over the shorter horizon is due to price and wage stickiness. The positive response over the longer horizon is consistent with equation (18).\(^{10}\)

4.3 Historical Decomposition

Next, to examine the reasons why the labor shares in the United States and Japan have followed quite different trajectories over the past few decades, we decompose changes in the labor share in the two countries into the contribution of various shocks. The results of this decomposition are shown in Figure 5. Due to the different estimates of the elasticity of substitution and impulse responses for the two countries, there are stark differences in the decomposition results.

Let us start with the decomposition results for the United States shown in Figure 5(a). The figure indicates that the decline in the labor share in the United States since the early 2000s is mainly explained by positive capital-augmenting technology shocks $\varepsilon^z_t$. Capital-augmenting technological changes can be interpreted as technological changes in IT-using industries. Jorgenson et al. (2011) report that for the period 2000-2007, innovation substantially increased in IT-using industries. This may have led to the labor share decline in the United States and explain why $\sigma > 1$.

On the other hand, IST shocks $\varepsilon^\psi_t$ played only a very small role. This result differs from that of Karabarbounis and Neiman (2014), who show that the decline in the relative price of investment goods explains roughly half of the observed decline in the labor share.

\(^{10}\)For details on the effect of the relative price of investment goods or investment-specific technology on the labor share, see Karabarbounis and Neiman (2014).
The reason for the difference is as follows. Figure 2 shows that the relative price of investment goods in the United States decreased until the early-2000s but since then has largely moved sideways. On the other hand, the labor share in the United States has fallen notably since the early-2000s. Thus, the labor share decline in the United States since the early-2000s is not explained by IST shocks $\varepsilon_t^\psi$ related to the relative price of investment goods. Since the analytical framework of Karabarbounis and Neiman (2014) is based on comparative statics, their results do not reflect the falling labor share and the sideways movements of the relative price of investment goods observed since the early-2000s. Specifically, they estimate equations derived from firms’ factor demand functions using international cross-sectional data. Therefore, their results do not reflect changes in the labor share and the relative price of investment goods over time. Meanwhile, price markup shocks $\varepsilon_t^p$ and wage markup shocks $\varepsilon_t^w$ have also generally put downward pressure on the labor share since the mid-2000s/early 2010s, but they are not a major factor.\(^{11}\)

The results for Japan are shown in Figure 5(b). In stark contrast to the United States, positive capital-augmenting technology shocks $\varepsilon_t^\omega$ and negative labor-augmenting technology shocks have put upward pressure on the labor share.\(^{12}\) This is because the elasticity of substitution between capital and labor is less than one. In addition, markup shocks $\varepsilon_t^p$ have also made a substantial contribution to changes in the labor share, which contrasts with the United States.

### 4.4 The Elasticity of Substitution and the Correlation between the Labor Share and Output Growth

In this subsection, we provide an explanation of the estimation results for the elasticity of substitution – that is, why $\sigma < 1$ for Japan and $\sigma > 1$ for the United States. To

\(^{11}\) As for the effect of markups on the labor share, see Barkai (2017), De Loecker and Eeckhout (2017), and Traina (2018).

\(^{12}\) The positive capital-augmenting technology shocks, as in the United States, likely reflect technology growth in IT-using industries. For details on the pickup in technology growth in IT-using industries in Japan since 2000, see Fueki and Kawamoto (2009).
understand why the estimated elasticities for the two countries differ, it is helpful to look at scatterplots of the labor share and output growth, which are provided in Figures 6(a) and (c). While a negative correlation is observed for Japan, no correlation is observed for the United States.\textsuperscript{13}

According to the impulse response shown in Figure 4, when $\sigma < 1$ as in the case of Japan, an increase in labor-augmenting technology – one of the main drivers of output fluctuations – increases output and decreases the labor share, which suggests the estimated model for Japan can replicate the negative correlation between output growth and the labor share. In fact, as shown in Figure 6(d), the stochastic simulation with all shocks generated simultaneously replicates the negative correlation between output growth and the labor share observed in the Japanese data shown in Figure 6(c).

On the other hand, for the United States, the elasticity of substitution is larger than one, because there is no such negative correlation between output growth and the labor share. In fact, Figure 6(b) shows the result of the stochastic simulation with all shocks generated simultaneously based on the estimated model parameters for the United States shown in Table 3. The result demonstrates that the stochastic simulation replicates the absence of a correlation between output growth and the labor share, which reflects the fact that the elasticity of substitution is larger than one. Furthermore, the simulation also implies that the estimation result that the elasticity of substitution is larger than one is obtained due to the absence of a correlation between output growth and the labor share.

4.5 Importance of the Elasticity of Substitution

To examine the effect of the elasticity of substitution on labor share dynamics in the United States and Japan, we decompose changes in the labor share using the model with the Cobb-Douglas production function (Figure 7) and compare the results with those

\textsuperscript{13}The observation period for the data used in Figure 5(a) is from 1985Q1 to 2017Q3. This observation period is the same as that for the data used in the estimation. No consensus has been reached as to whether the U.S. labor share is procyclical or countercyclical, with studies arriving at conflicting results (see, e.g., Rotemberg and Woodford, 1999, and Nekarda and Ramey, 2013).
for the model with the CES production function (Figure 5). For both the United States and Japan, we find that while shocks to capital- and labor-augmenting technologies, $\epsilon_t^w$ and $\delta_t^z$, are the main determinants of changes in the labor share in the estimations using the model with the CES production function, in the model with the Cobb-Douglas production function they are not. Rather, markup shocks $\epsilon_t^p$ and wage markup shocks $\epsilon_t^w$ are the dominant factors. These results imply that capital-labor substitution in response to technology shocks plays an important role in explaining changes in the labor share. That is, the models with a Cobb-Douglas production function potentially overestimate the contribution of markups and underestimate the contribution of technology shocks to changes in the labor share.

5 Inflation and the Labor Share

The difference in the estimates of the elasticity of substitution in our estimation results has interesting implications for inflation. As shown in Figure 3, which provides scatterplots of inflation and labor share observations for both countries, while the observations for the United States suggest a positive correlation between them, for Japan a weak negative or no correlation is observed. In this section, we examine the relationship between inflation and the labor share using the estimation results presented above.

5.1 Marginal Cost and the Labor Share

In the New Keynesian Phillips curve shown in (22) and (23), the real marginal cost (or the inverse of the markup ratio) plays a crucial role in determining inflation dynamics. When the Cobb-Douglas production function is employed, the labor share coincides with the real marginal cost.$^{14}$ Put differently, the labor share fluctuates only as a result of markup changes associated with nominal price rigidities. Specifically, the labor share in

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$^{14}$For a discussion of the link between the labor share and inflation, see Sbordone (2002).
the Cobb-Douglas case is expressed as follows:

$$s_{L,t} \propto \frac{MC_t}{P_t}. \quad (57)$$

On the other hand, when the CES production function is employed, the real marginal cost generally does not coincide with the labor share. This is because changes in the labor share are caused not only by changes in markups but also by capital-labor substitution. In the CES case, the relationship between the labor share and the real marginal cost is expressed as follows:

$$s_{L,t} \propto \frac{MC_t}{P_t} \left( \frac{Y_t}{l_t} \right)^{\frac{1-\sigma}{\sigma}} Z_t^{\frac{\sigma-1}{\sigma}}. \quad (58)$$

As shown in Figure 4, equation (58) implies that the sign of the response of the labor share to shocks depends on whether the elasticity of substitution $\sigma$ is larger or smaller than unity.

### 5.2 Stochastic Simulations

In order to investigate whether the estimated models for the United States and Japan can explain the difference in the relationship between inflation and the labor share found in Figure 3, we conduct stochastic simulation exercises based on the estimated model parameters shown in Tables 3 and 4.

Figure 8 shows the results for the United States. The stochastic simulation with all shocks generated simultaneously replicates the positive correlation between inflation and the labor share observed in Figure 3. Specifically, Figure 8 shows that labor-augmenting technology shocks, capital-augmenting technology shocks, and IST shocks all contribute to generating the positive correlation between inflation and the labor share. For example, as illustrated by the impulse responses of the labor share and inflation shown in Figure 4(c) and Figure 10(c), positive capital-augmenting technology shocks decrease both the labor share and inflation. As a result of the responses, the positive correlation is replicated as shown in Figure 8(c).

Figure 9 shows the result for Japan. The result of the stochastic simulation with all
shocks generated simultaneously replicates the weak correlation between inflation and the labor share, which is consistent with the observed data in Figure 3. Taking a closer look at Figure 9 shows that it is labor-augmenting technology shocks and capital-augmenting technology shocks that are responsible for the negative correlation between inflation and the labor share. Figure 4(b) and Figure 10(b) show the impulse responses of the labor share and inflation to positive labor-augmenting technology shocks. These responses indicate that a positive labor-augmenting technology shock decreases the labor share but increases inflation, which leads to the negative correlation shown in Figure 9(d).

6 Conclusion

In this paper, we examined the dynamics of the labor share in the United States and Japan using a dynamic stochastic general equilibrium (DSGE) model. For this purpose, we developed a model with a balanced growth path (BGP) that incorporates a CES production function and non-stationary technological progress. To ensure our model has a BGP and is stationary, we introduce the condition that non-stationary capital augmenting technological change is co-integrated with non-stationary IST change into a standard dynamic New Keynesian model. Through Bayesian estimation of our models using data for the United States and Japan, we compared the fit of the model with the CES production function and the Cobb-Douglass production function with the data. Next, we obtained estimates of the elasticity of substitution between capital and labor for the United States and Japan and identified shocks and decomposed changes in the labor share into the contribution of various shocks.

Our findings are as follows. First, comparing the models with the CES production function and the Cobb-Douglas production function using marginal data densities indicates that the former provided a better fit for both the U.S. and Japanese data. Second, we found that the elasticity of substitution between capital and labor in the United States is greater than unity, while that in Japan is less than unity. Third, due to the difference in the elasticity of substitution between the United States and Japan, the signs of the impulse responses of the labor share to technology shocks also differ between the
two countries. The historical decomposition to examine this difference in more detail indicated that whereas in the United States capital- and labor-augmenting technology shocks put downward pressure on the labor share, in Japan they put upward pressure on the labor share.

Moreover, the difference in the elasticity of substitution has interesting implications for inflation. We examine the relationship between inflation and the labor share using the estimated models. In the New Keynesian Phillips curve, the real marginal cost plays a crucial role in determining inflation dynamics. When the Cobb-Douglas production function is employed, the labor share coincides with the real marginal cost. On the other hand, when the CES production function is employed, the real marginal cost generally does not coincide with the labor share. Observations of inflation and the labor share for the United States and Japan indicate that while for the United States there appears to be a positive correlation between inflation and the labor share, for Japan a weak negative or no correlation is observed. The stochastic simulations based on the estimated models replicated the observed correlation between inflation and the labor share in the U.S. and Japanese economies.

References


29


30


A Data

A.1 Data used for estimating the model for the U.S. economy

Calculation of data used as observables

\[ GDP_t = \frac{\text{Real GDP}}{\text{POP}}, \]

\[ Consumption_t = \frac{\text{Real personal consumption expenditures}}{\text{POP}}, \]

\[ Investment_t = \frac{\text{Real investment}}{\text{POP}}, \]

\[ Wage_t = \frac{\text{Nominal wage}}{\text{Consumption deflator}}, \]

\[ RPI_t = \frac{\text{Investment deflator}}{\text{Consumption deflator}}, \]

\[ Inflation_t = \ln(\text{Consumption deflator}) - \ln(\text{Consumption deflator}(-1)), \]

\[ PolicyRate_t = \text{Policy rate}, \text{ and} \]

\[ LS_t = \text{Labor share} \]

Data sources

Real GDP U.S. Bureau of Economic Analysis, Real Gross Domestic Product [GDPC1], retrieved from FRED, Federal Reserve Bank of St. Louis.

Real personal consumption expenditures U.S. Bureau of Economic Analysis, Real Personal Consumption Expenditures [PCECC96], retrieved from FRED, Federal Reserve Bank of St. Louis.

Real investment U.S. Bureau of Economic Analysis, Real Gross Private Domestic Investment [GPDIC1], retrieved from FRED, Federal Reserve Bank of St. Louis.

Nominal wage Nominal hourly compensation, [PRS85006103], sector: nonfarm business, seasonally adjusted, index, 1992 = 100, BLS.

Consumption deflator U.S. Bureau of Economic Analysis, Personal consumption expenditures (implicit price deflator) [DPCERD3Q086SBEA], retrieved from FRED, Federal Reserve Bank of St. Louis.
**Investment deflator** U.S. Bureau of Economic Analysis, Gross private domestic investment (implicit price deflator) [A006RD3Q086SBEA], retrieved from FRED, Federal Reserve Bank of St. Louis.


### A.2 Data used for estimating the model for the Japanese economy

**Calculation of data used as observables**

\[ GDP_t = \frac{\text{Real GDP}}{\text{POP}} \]

\[ Consumption_t = \frac{\text{Real personal consumption expenditures}}{\text{POP}} \]

\[ Investment_t = \frac{\text{Real investment}}{\text{POP}} \]

\[ Wage_t = \frac{\text{Nominal wage}}{\text{Consumption deflator}} \]

\[ RPI_t = \frac{\text{Investment deflator}}{\text{Consumption deflator}} \]

\[ Inflation_t = \ln(\text{CPI}) - \ln(\text{CPI}(-1)) \]

\[ PolicyRate_t = \text{Policy rate, and} \]

\[ LS_t = \text{Compensation of employees/Nominal GDP} \]

**Data sources**

**Real GDP** Cabinet Office, “National Accounts.”

**Real consumption** Cabinet Office, “National Accounts.”

**Real investment** Cabinet Office, “National Accounts.”

CPI  Ministry of Internal Affairs and Communications, “Consumer Price Index (excluding Fresh Food).”

Investment deflator  Cabinet Office, “National Accounts.”

Consumption deflator  Cabinet Office, “National Accounts.”

Policy rate  Bank of Japan, “Collateralized Overnight Call Rate.”

Nominal GDP  Cabinet Office, “National Accounts.”

Compensation of employees  Cabinet Office, “National Accounts.”

Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\pi_{ss}$</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.38</td>
<td>0.36</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>0.015</td>
</tr>
</tbody>
</table>
Table 2: Model Comparison: Log Marginal Data Densities

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
<td>-980.2</td>
<td>-935.2</td>
</tr>
<tr>
<td>Cobb-Douglas</td>
<td>-992.2</td>
<td>-947.2</td>
</tr>
</tbody>
</table>
Table 3: Estimated Parameters for the United States

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr. Mean S.D.</td>
<td>Mean 10th percentile 90th percentile</td>
</tr>
<tr>
<td>$b$</td>
<td>Beta 0.5 0.1</td>
<td>0.837 0.796 0.878</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Gamma 5 1</td>
<td>5.119 3.681 6.007</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Normal 1 0.2</td>
<td>1.475 1.285 1.663</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>Beta 0.66 0.1</td>
<td>0.841 0.814 0.869</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>Beta 0.66 0.1</td>
<td>0.442 0.416 0.470</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>Beta 0.2 0.1</td>
<td>0.128 0.031 0.213</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Beta 0.2 0.1</td>
<td>0.268 0.072 0.453</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>Beta 0.8 0.1</td>
<td>0.812 0.776 0.855</td>
</tr>
<tr>
<td>$\phi_n$</td>
<td>Gamma 1.7 0.1</td>
<td>1.754 1.598 1.912</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Gamma 0.125 0.05</td>
<td>0.152 0.083 0.206</td>
</tr>
<tr>
<td>$z_{ss}$</td>
<td>Gamma 0.5 0.05</td>
<td>0.387 0.327 0.440</td>
</tr>
<tr>
<td>$\tau_{ss}$</td>
<td>Gamma 0.8 0.05</td>
<td>0.899 0.821 0.993</td>
</tr>
<tr>
<td>$\psi_{ss}$</td>
<td>Gamma 0.3 0.05</td>
<td>0.266 0.200 0.328</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>Beta 0.5 0.2</td>
<td>0.686 0.605 0.756</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>Beta 0.5 0.2</td>
<td>0.799 0.704 0.889</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Beta 0.5 0.2</td>
<td>0.260 0.075 0.417</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Beta 0.5 0.2</td>
<td>0.805 0.747 0.870</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>Beta 0.5 0.2</td>
<td>0.682 0.602 0.764</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>Beta 0.5 0.2</td>
<td>0.964 0.945 0.983</td>
</tr>
<tr>
<td>$\rho^i$</td>
<td>Beta 0.8 0.2</td>
<td>0.323 0.190 0.453</td>
</tr>
<tr>
<td>$\rho^o$</td>
<td>Beta 0.8 0.2</td>
<td>0.588 0.336 0.812</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Beta 0.05 0.05</td>
<td>0.015 0.002 0.026</td>
</tr>
<tr>
<td>$\kappa^i$</td>
<td>Beta 0.05 0.05</td>
<td>0.004 0.000 0.010</td>
</tr>
<tr>
<td>$\sigma(\varepsilon^z)$</td>
<td>Inv. Gamma 0.05 Inf.</td>
<td>0.010 0.009 0.011</td>
</tr>
<tr>
<td>$\sigma(\varepsilon^b)$</td>
<td>Inv. Gamma 0.1 Inf.</td>
<td>0.036 0.028 0.043</td>
</tr>
<tr>
<td>$\sigma(\varepsilon^w)$</td>
<td>Inv. Gamma 0.5 Inf.</td>
<td>0.800 0.800 0.800</td>
</tr>
<tr>
<td>$\sigma(\varepsilon^p)$</td>
<td>Inv. Gamma 0.5 Inf.</td>
<td>0.823 0.569 1.130</td>
</tr>
<tr>
<td>$\sigma(\varepsilon^i)$</td>
<td>Inv. Gamma 0.05 Inf.</td>
<td>0.066 0.048 0.083</td>
</tr>
<tr>
<td>$\sigma(\varepsilon^r)$</td>
<td>Inv. Gamma 0.001 Inf.</td>
<td>0.001 0.001 0.001</td>
</tr>
<tr>
<td>$\sigma(\varepsilon^y)$</td>
<td>Inv. Gamma 0.05 Inf.</td>
<td>0.055 0.050 0.059</td>
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<tr>
<td>$\sigma(\varepsilon^o)$</td>
<td>Inv. Gamma 0.01 Inf.</td>
<td>0.004 0.004 0.004</td>
</tr>
<tr>
<td>$\sigma(\varepsilon^r)$</td>
<td>Inv. Gamma 0.01 Inf.</td>
<td>0.007 0.004 0.011</td>
</tr>
</tbody>
</table>

Note: For the posterior distribution, two chains of 1,000,000 draws were created using the Metropolis-Hastings algorithm, and the first half of these was discarded.
Table 4: Estimated Parameters for Japan

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr.</td>
<td>Mean</td>
</tr>
<tr>
<td>(b)</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>(\xi)</td>
<td>Gamma</td>
<td>5</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Normal</td>
<td>1</td>
</tr>
<tr>
<td>(\gamma_p)</td>
<td>Beta</td>
<td>0.66</td>
</tr>
<tr>
<td>(\gamma_w)</td>
<td>Beta</td>
<td>0.66</td>
</tr>
<tr>
<td>(\xi_p)</td>
<td>Beta</td>
<td>0.2</td>
</tr>
<tr>
<td>(\xi_w)</td>
<td>Beta</td>
<td>0.2</td>
</tr>
<tr>
<td>(\phi_r)</td>
<td>Beta</td>
<td>0.8</td>
</tr>
<tr>
<td>(\phi_{\pi})</td>
<td>Gamma</td>
<td>1.7</td>
</tr>
<tr>
<td>(\phi_y)</td>
<td>Gamma</td>
<td>0.125</td>
</tr>
<tr>
<td>(z_{ss})</td>
<td>Gamma</td>
<td>0.5</td>
</tr>
<tr>
<td>(r_{ss}^n)</td>
<td>Gamma</td>
<td>0.8</td>
</tr>
<tr>
<td>(\psi_{ss})</td>
<td>Gamma</td>
<td>0.3</td>
</tr>
<tr>
<td>(\rho_w)</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>(\rho_b)</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>(\rho_p)</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>(\rho_i)</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>(\rho_r)</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>(\rho_g)</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>(\rho_{\psi})</td>
<td>Beta</td>
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</tr>
<tr>
<td>(\rho_{\omega})</td>
<td>Beta</td>
<td>0.8</td>
</tr>
<tr>
<td>(\kappa_{\psi})</td>
<td>Beta</td>
<td>0.05</td>
</tr>
<tr>
<td>(\kappa_{\omega})</td>
<td>Beta</td>
<td>0.05</td>
</tr>
<tr>
<td>(\sigma(\varepsilon_i^z))</td>
<td>Inv. Gamma</td>
<td>0.05</td>
</tr>
<tr>
<td>(\sigma(\varepsilon_b^z))</td>
<td>Inv. Gamma</td>
<td>0.1</td>
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<tr>
<td>(\sigma(\varepsilon_w^z))</td>
<td>Inv. Gamma</td>
<td>0.5</td>
</tr>
<tr>
<td>(\sigma(\varepsilon_i^b))</td>
<td>Inv. Gamma</td>
<td>0.5</td>
</tr>
<tr>
<td>(\sigma(\varepsilon_i^r))</td>
<td>Inv. Gamma</td>
<td>0.05</td>
</tr>
<tr>
<td>(\sigma(\varepsilon_{ss}^r))</td>
<td>Inv. Gamma</td>
<td>0.001</td>
</tr>
<tr>
<td>(\sigma(\varepsilon_i^{s\omega}))</td>
<td>Inv. Gamma</td>
<td>0.05</td>
</tr>
<tr>
<td>(\sigma(\varepsilon_i^{s\psi}))</td>
<td>Inv. Gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma(\varepsilon_i^{s\omega}))</td>
<td>Inv. Gamma</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Note: For the posterior distribution, two chains of 1,000,000 draws were created using the Metropolis-Hastings algorithm, and the first half of these was discarded.
Figure 1: Labor Share in the United States and Japan

(a) United States

(b) Japan
Figure 2: Relative Price of Investment Goods
Figure 3: Inflation and Labor Share in the United States and Japan

(a) United States

(b) Japan
Figure 4: Impulse Responses of Labor Share: United States and Japan

(a) Responses to Markup Shocks

(b) Responses to Labor Augmenting Technology Shocks

(c) Responses to Capital Augmenting Technology Shocks

(d) Responses to Investment Specific Technology Shocks
Notes:
1. The figures show the historical decomposition of the United States and Japan based on the posterior mean estimates of parameters and the Kalman smoothed mean estimates of shocks.
2. "Other" includes preference shocks, external demand shocks, monetary policy shocks, and the initial value.
Figure 6: Output Growth and the Labor Share

United States

Correlation = -0.43

Japan

Correlation = -0.43
Notes:
1. The figures show the historical decomposition of the United States and Japan based on the posterior mean estimates of parameters and the Kalman smoothed mean estimates of shocks.
2. "Other" includes preference shocks, external demand shocks, monetary policy shocks, and the initial value.
Figure 8: Stochastic Simulation: United States

(a) All Shocks

(b) Markup Shocks

(c) Capital-Augmenting Technology Shocks

(d) Labor-Augmenting Technology Shocks

(e) IST Shocks
Figure 9: Stochastic Simulation: Japan

(a) All Shocks

(b) Markup Shocks

(c) Capital-Augmenting Technology Shocks

(d) Labor-Augmenting Technology Shocks

(e) IST Shocks
Figure 10: Impulse Responses of Inflation: United States and Japan

(a) Responses to Markup Shocks

(b) Responses to Labor Augmenting Technology Shocks

(c) Responses to Capital Augmenting Technology Shocks

(d) Responses to Investment Specific Technology Shocks

Percentage points, deviation from steady state

-0.05
-0.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Period

Percentage points, deviation from steady state

-0.10
-0.08
-0.06
-0.04
-0.02
0.00
0.02
0.04
0.06

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Period

Percentage points, deviation from steady state

0.01
0.00
-0.01
-0.02

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Period

Percentage points, deviation from steady state

0.01
0.00
-0.01
-0.02

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Period

United States ($\sigma>1$)

Japan ($\sigma<1$)