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On Liquidity Shocks and Asset Prices

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Abstract

In models of financial frictions, stock market booms tend to follow adverse liquidity shocks. This finding is clearly at odds with the data. We demonstrate that this counterfactual result is specific to real business cycle models with exogenous growth. Once we allow for both endogenous productivity and growth, this puzzling price dynamics easily disappear. Intuitively, the gloomy economic-growth outlook following the adverse liquidity shocks generates a predictable and negative long-run component in dividend growth, leading to the collapse of equity prices.

1 Introduction

The severity and widespread impact of the Great Recession has propelled substantial research to better understand the interconnection between the financial sector and the aggregate economy. One strand of this literature emphasizes the role of financial shocks affecting how easily investors obtain funds for investment. Yet these models of financial frictions (Jermann and Quadrini (2012) and Kiyotaki and Moore (Forthcoming)) have been often criticized because of their counterfactual stock market dynamics (Shi (2015)). More precisely, these models tend to predict a stock market boom following an adverse financial shock, clearly at odds with the data. To complicate matters, the counterfactual asset price response is robust to a wide range of specifications such as nominal rigidities, habit formation in consumption, and adjustment costs in capital or investment. This prediction is troublesome not only because it is counterfactual but also because it arises precisely at the heart of the transmission mechanism they feature. It is therefore crucial to correct these models; otherwise, the reliability of the analysis based on them will be questioned.

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In this paper, we argue that this stock market anomaly is a consequence of the disconnect between productivity and the financial shock. To support our claim, we construct an endogenous productivity model with cash strapped investors. Specifically, we introduce a learning-by-doing mechanism in an otherwise standard liquidity model, allowing for endogenous growth. In this framework, an adverse liquidity disturbance has a long-lasting negative effect on the level of macroeconomic variables. This prediction is consistent with recent empirical work documenting that recessions associated with financial crises tend to be long and painful (Cerra and Saxena (2008) and Jorda, Schularick, and Taylor (2014)). Because of the adverse long-run predictable component in dividend growth associated with the collapse in liquidity, the stock market plunges following the unfavorable shock as long as households have a large enough intertemporal elasticity of substitution. The insight is similar to the one emphasized in the long-run risk literature in finance (Bansal and Yaron (2004)).

2 Model

Our model is based on Shi (2015). The economy is populated by a continuum of households with measure one. Each household has a unit measure of members. At the beginning of the period, all members of the household are identical. During the period, members are separated from each other, and each member receives a shock that determines her role in the period. A member will be an entrepreneur with probability $\pi \in [0, 1]$; otherwise, she will be a worker in the period. These shocks are i.i.d. among members and across time.

A period is divided into three stages: household's decision making, production, and investment. In the household's decision stage, all members of the household are together and share their assets that are s_t units of equity claims. An equity is the ownership of a unit of capital. Aggregate shocks to exogenous state variables are realized. Because all the members of the household are identical in this stage, the head of the household evenly divides the assets among the members. The head of the household also gives contingency plans to each member as follows. If one becomes an entrepreneur, she invests i_t, consumes c_t^e, and makes necessary trades in the stock market so that she holds s_{t+1}^e units of equity claims at the end of the period. In contrast, if the member becomes a worker, she supplies l_t units of labor, consumes c_t^w, and makes necessary trades in the stock market so that he or she holds s_{t+1}^w units of equity claims at the end of the period. After receiving these instructions, the members go to the market. They remain separated from each other for the rest of the period.

At the beginning of the production stage, each member receives the shock whose realization determines whether the individual is an entrepreneur or a worker. Competitive firms produce final consumption goods y_t using labor services l_t^D and capital services k_t^D with the production technology

$$y_t = A_t \left(k_t^D\right)^{\alpha} \left(l_t^D\right)^{1-\alpha}.$$

Here, α is the capital share, the superscript D indicates demand, and A_t is the technology level, which both households and firms take as given. Their first order conditions are standard;

$$\alpha \frac{y_t}{k_t^D} = r_t \quad \text{and} \quad (1-\alpha) \frac{y_t}{l_t^D} = w_t,$$

where r_t denotes the rental price and w_t denotes the wage rate. After production, workers receive wage income, equity holders receive compensation for capital, and a fraction δ of capital depreciates.

Finally, the third stage in the period is the investment stage, where entrepreneurs seek finance and undertake investment projects. An entrepreneur can transform i_t units of consumption goods into i_t units of new capital. Consumption takes place in this stage too. After these events, the period ends. The members of the household get together, their identities are reset, and the new period begins.

The instructions given to the members of the household have to satisfy a number of constraints. First, the budget constraints have to be satisfied;

$$c_t^e + i_t = r_t s_t + q_t \left((1-\delta) s_t + i_t - s_{t+1}^e\right) \tag{1}$$

and

$$c_t^w = r_t s_t + q_t \left((1-\delta) s_t - s_{t+1}^w\right) + w_t l_t \tag{2}$$

where q_t denotes the equity price. Equation (1) is the constraint entrepreneurs face and equation (2) is the constraint workers face. Because the members of the household share their assets at the end of the period, the following identity holds;

$$s_{t+1} = \pi s_{t+1}^e + (1-\pi) s_{t+1}^w. \tag{3}$$

Crucial in the Kiyotaki-Moore model, there are frictions in the equity market. First, an entrepreneur can issue at most θi_t of equity against the new capital. In addition, she can sell at most a fraction ϕ_t of existing capital in the market. Effectively, these constraints introduce a lower bound to the entrepreneur's capital holdings:

$$s_{t+1}^e \geq (1-\theta) i_t + (1-\phi_t)(1-\delta) s_t. \tag{4}$$

ϕ_t is an exogenous random variable we call a liquidity shock. We assume that it follows

$$\log\left(\frac{\phi_t}{\phi}\right) = \rho_\phi \log\left(\frac{\phi_{t-1}}{\phi}\right) + \varepsilon_t \tag{5}$$

where ε_t is an i.i.d. shock. A similar constraint applies to workers, i.e., $s_{t+1}^w \geq (1-\phi_t)(1-\delta)s_t$, but we omit it because it does not bind. Specifically, workers are net buyers of equities in the equilibrium we are interested in, meaning that $s_{t+1}^w > (1-\delta)s_t$ always holds. There are non-negativity constraints for i_t, l_t, c_t^e, c_t^w, and s_{t+1}^w, but we omit them too because they do not bind either.

The head of the household chooses instructions to its members to maximize the value function defined as

$$v(s_t; K_t, \phi_t) = \max_{c,i,l,s} \left\{ \pi \frac{(c_t^e)^{1-\rho} - 1}{1-\rho} + (1-\pi) \frac{\left[c_t^w (1-l_t)^\eta\right]^{1-\rho} - 1}{1-\rho} + \beta \mathbb{E}_t \left[v\left(s_{t+1}; K_{t+1}, \phi_{t+1}\right)\right] \right\} \tag{6}$$

subject to (1), (2), (3), and (4).

Rewriting the constraints is helpful to understand the household's problem. First, multiplying π and $(1-\pi)$ to (1) and (2) respectively, adding them up, and then substituting (3), we obtain

$$\pi c_t^e + (1-\pi) c_t^w + q_t s_{t+1} = r_t s_t + q_t (1-\delta) s_t + (1-\pi) w_t l_t + \pi (q_t - 1) i_t. \tag{7}$$

This is the standard budget constraint at the household level except for the very last term $\pi(q_t - 1) i_t$. The term is zero if q_t is equal to one. If q_t exceeds one, however, i_t relaxes the budget constraint because the household makes money out of investment in this case. But there is a limit on this activity because funding is limited. Specifically, substituting (1) into (4), we find the upper bound on i_t;

$$i_t (1 - \theta q_t) \leq r_t s_t + \phi_t q_t (1-\delta) s_t - c_t^e. \tag{8}$$

The left-hand side is the minimum cost entrepreneurs have to self-finance to conduct investment i_t. They are smaller than i_t because entrepreneurs can issue θi_t of equity against the new capital. The right-hand side is the maximum liquidity available to entrepreneurs after consumption. Hence, (8) is the feasibility constraint for investment.

We can draw a few important implications from (7) and (8). First, if the price of capital q_t is equal to one, the optimal investment level is not uniquely pinned down. Instead, any level of i_t satisfying (8) is optimal. This is because i_t disappears from the budget constraint if q_t is equal to one, and hence, its choice becomes irrelevant to the household's optimization but only s_{t+1} is. Intuitively, investment and equity purchase are perfect substitutes because the price of capital is identical to the marginal costs of creating new one. In this case, the inequality constraint (8) does not bind in general.

However, if the price of capital exceeds one, the inequality constraint (8) must be binding at the optimum. If not, the household can increase i_t without violating (8), which loosens the budget constraint, allowing the household to increase utility by increasing c_t^e and c_t^w for example. Intuitively, if the household is able to make money out of investment, they utilize this opportunity up to the limit.

Finally, the capital price q_t is strictly less than the inverse of θ in the equilibrium. If not, the inequality constraint (8) holds for an arbitrarily large positive i_t because $(1-\theta q_t) \leq 0$ holds. Because $q_t > 1$ also holds, the household can relax the budget constraint (6) freely, resulting in the violation of the market clearing condition.

As in the related literature, we will restrict our attention to the case in which the equilibrium price of capital always exceeds one. Because the inequality constraint (8) always binds in this case, we can combine (8) and (7), obtaining

$$\pi c_t^e + (1-\pi) c_t^w + q_t s_{t+1} = r_t s_t + (1-\pi) w_t l_t + q_t (1-\delta) s_t + \pi \underbrace{\frac{q_t - 1}{1-\theta q_t}}_{\text{liquidity services}} \left(\underbrace{r_t s_t + \phi_t q_t (1-\delta) s_t - c_t^e}_{\text{liquidity after consumption}} \right). \tag{9}$$

The last term is crucial. It is a product of three terms; the fraction of entrepreneurs π, the liquidity held by entrepreneurs after consumption, and the fraction $(q_t - 1)/(1-\theta q_t)$, which Shi (2015) calls liquidity services. The liquidity services are positive because $1 < q_t < 1/\theta$ holds. They measure how much profits entrepreneurs can make using a unit of liquidity. Remember that they can convert a unit of liquidity to $1/(1-\theta q_t)$ units of capital by investment with leverage, each of which is worth q_t in the market.

The optimization problem becomes simpler; the household chooses c_t^e, c_t^w, l_t, and s_{t+1} to maximize the utility function (6) subject to (9). The optimality condition for labor supply is

$$\eta \frac{c_t^w}{1-l_t} = w_t.$$

The left-hand side is the marginal rate of substitution of leisure for consumption. If it is equal to the real wage, workers do not have incentive to readjust time allocation locally.

The optimality condition for the intra-household consumption allocation is

$$\underbrace{\left(c_t^e\right)^{-\rho}}_{\text{marginal utility from } c_t^e} = \left(1 + \underbrace{\frac{q_t - 1}{1-\theta q_t}}_{\text{liquidity services}} \right) \underbrace{\left(c_t^w\right)^{-\rho} (1-l_t)^{\eta(1-\rho)}}_{\text{marginal utility from } c_t^w}.$$

Interestingly, there is a wedge between entrepreneur's marginal utility and worker's. Specifically, the entrepreneur's marginal utility is larger. The reason is clearly seen from the budget constraint (9). The entrepreneur's consumption not only increases the left-hand side but also decreases the right-hand side, while the worker's consumption only increases the left-hand side. In this sense, the entrepreneur's consumption is more expensive than the worker's from the household's point of view. This is because the entrepreneurs can make money using liquidity. Hence, their consumption involves the opportunity costs for giving up this profit.

The Euler equation for investment is

$$q_t = \mathbb{E}_t \left[\underbrace{\beta \left(\frac{c_{t+1}^w}{c_t^w} \right)^{-\rho} \left(\frac{1 - l_{t+1}}{1 - l_t} \right)^{\eta(1-\rho)}}_{\text{stochastic discount factor}} \left(r_{t+1} + (1-\delta) q_{t+1} + \pi \frac{q_{t+1} - 1}{1 - \theta q_{t+1}} \left(\underbrace{r_{t+1} + q_{t+1} \phi_{t+1} (1-\delta)}_{\text{liquidity provision}} \right) \right) \right].$$

This equation determines the equity price. We see the worker's marginal utilities in the stochastic discount factor because they are marginal investors. The parenthesis in the right-hand side summarizes the benefits of holding equity, among which the last term is for providing liquidity to entrepreneurs.

2.1 Equilibrium

The competitive equilibrium is defined in a standard way. Market clearing conditions for final goods, factor services, and equity are

$$\pi c_t^e + (1-\pi) c_t^w + \pi i_t = A_t \left(k_t^D \right)^{\alpha} \left(l_t^D \right)^{1-\alpha},$$

$$l_t^D = (1-\pi) l_t,$$

$$k_t^D = K_t,$$

and

$$K_t = s_t$$

for all t. Capital accumulation rule is $K_{t+1} = (1-\delta) K_t + \pi i_t$.

2.2 Sources of Growth

We compare results under two alternative assumptions regarding the source of growth. In one, we assume that the technology level A_t grows at a constant rate:

$$A_t = A_0 \left(\gamma^{1-\alpha} \right)^t,$$

where A_0 is a scale parameter and γ denotes the (gross) growth rate of the economy along the balanced growth path. This is a standard assumption in the business cycle literature (King, Plosser, and Rebelo (1988)). In the second case, we assume that the level of A_t is endogenous:

$$A_t = A_0 (K_t)^{1-\alpha}.$$

We interpret this assumption following Arrow (1962), Sheshinski (1967), and Romer (1986); namely, knowledge is not only a by-product of investment but also a public good that anyone can access at zero cost.[1] Under this assumption, the production function is written as

$$Y_t = A_0 K_t \left[(1-\pi)\, l_t\right]^{1-\alpha}.$$

The long-run tendency for capital to experience diminishing returns is eliminated. As such, the economy can grow in the long run.

3 Calibration

Because the model has no closed-form solution, we solve it numerically. Table 1 reports our calibration. A period is a quarter of a year. We set the discount factor at $\beta = 0.994$, which is standard in the literature. We set the fraction of investors at $\pi = 0.06$, the capital share at $\alpha = 0.36$, and the persistence in liquidity shock at $\rho_\phi = 0.9$, which are standard choices in the literature (Kiyotaki and Moore (Forthcoming) and Shi (2015)). The relative risk aversion parameter is set at $\rho = 1/1.85$ so that the implied intertemporal elasticity of substitution (the inverse of ρ) is both consistent with the value used in the macro-finance literature (Kung and Schmid (2015)) and within the credible set estimated by Schorfheide, Song, and Yaron (2018). We set in the exogenous growth model the growth rate along the balanced growth path at $\gamma = 1.004$ and normalize the scale parameter by $A_0 = 1$. We calibrate the scale parameter A_0 in the endogenous growth model so that the implied growth rate along the balanced growth path is 1.004. Following Kiyotaki and Moore (Forthcoming), we assume that the fraction of new equity θ is equal to the steady state resalability ϕ. We calibrate ϕ, the curvature in leisure in the utility function η, and the capital depreciation rate δ using the following targets: the aggregate hours of work in the steady state (0.25), the ratio of capital to annual output in steady state (3.32), and the ratio of annual investment to capital in steady state (0.076).

4 Results

The solution method is standard. Many variables in our model are non-stationary, but their ratios to the trend term are stationary along the balanced growth path. After finding the steady state values of these ratios, we linearize the system of equations characterizing the equilibrium around them, then computing impulse response functions (Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016)). The appendix provides a detailed discussion.

[1] Our specification follows chapter 4.3 in Barro and Sala-i-Martin (1999). We use the learning-by-doing structure for simplicity. Similar results would follow from more involved endogenous productivity models such as love-for-varieties or Schumpeterian.

Table 1: Parameters and Calibration Targets

	Value			
Parameter	END	EXO	LOG	Calibration Target
β; discount factor	0.994	0.994	0.994	Exogenously chosen
ρ; relative risk aversion	1/1.85	1/1.85	1	Exogenously chosen
π; fraction of entrepreneurs	0.06	0.06	0.06	Annual fraction of investing firms = 0.24
η; curvature in leisure utility	2.41	2.41	2.45	Hours of work = 0.25
α; capital share	0.36	0.36	0.36	Labor income share $(1-\alpha) = 0.64$
δ; capital depreciation rate	0.015	0.015	0.015	Annual investment/capital = 0.076
A_0; scale parameter	0.183	1	0.182	Normalization; steady-state growth = 1.004
γ; steady-state growth	-	1.004	-	Steady-state growth = 1.004
ϕ; steady-state resalability	0.10	0.10	0.16	Capital stock/annual output = 3.32
ρ_ϕ; persistence in resalability	0.9	0.9	0.9	Exogenously chosen
θ; fraction of new equity	0.10	0.10	0.16	Set equal to ϕ

Figure 1 reports impulse response functions to an unanticipated drop in liquidity. The top panel shows the evolution of the shock; ϕ_t is in the steady state in period 0, but drops unexpectedly in period 1 by 10% relative to its steady state value. No other shocks hit the economy thereafter. Liquidity recovers to the steady state over time, almost fully returning to its pre-shock level in 25 quarters. This is the input to the model in the following exercise.

The other four panels plot the impulse response functions of output y_t, aggregate consumption $\pi c_t^e + (1-\pi) c_t^w$, investment i_t, and labor l_t. The solid blue lines show the responses in the endogenous growth model, and the red dashed lines show responses in the exogenous growth model. We plot responses in levels; for example, the second panel plots the percentage deviation of the output level after the shock from the output level without the shock.[2]

The negative liquidity shock is generally contractionary in the short run, reducing output, investment, and labor. The only exception is consumption, which rises initially.[3] These results are robust to the source of growth; the impulse response functions in period 1 are almost identical in both the endogenous and the exogenous growth models.

However, the responses are similar only in the short run. In the exogenous growth model, all variables in the figure come back to zero, meaning that the liquidity shock generates short-run fluctuations around the trend but the trend itself is unaffected. Yet in the endogenous growth model, the impulse response functions of output, consumption, and investment do not come back to zero, but shift down to a lower level by about 2 percentage points. These persistently adverse

[2] Let y_t^* denote the level of output in period t in the benchmark scenario in which ϕ_t evolves as plotted in the top panel of Figure 1. Let $\tilde{y}_t$ denote the level of the output in period t in the alternative scenario in which no shock hits the economy and therefore ϕ_t is always in the steady state value. We plot $100 \times \log(y_t^*/\tilde{y}_t)$ in the second panel of Figure 1.

[3] A few ways to overturn this prediction are known in the literature such as nominal wage rigidities (Ajello (2016)), insight from the news shock literature (Guerron-Quintana and Jinnai (Forthcoming)), and investment adjustment costs (Shi (2015)).

effects have an important consequence for asset prices.

Figure 2 displays the asset price dynamics. We plot impulse response functions of both the equity price q_t (solid red line) and the aggregate stock market value defined as $Stock_t = q_t K_{t+1}$ (solid blue line), both in the benchmark endogenous growth model (top panel) and the otherwise identical exogenous growth model (middle panel). In the endogenous growth model, a negative liquidity shock decreases both the equity price and the aggregate stock market value. In contrast, the same variables move in the opposite direction in the exogenous growth model.

Loosely speaking, a bad liquidity shock in our endogenous growth model induces business-cycle and long-run fluctuations. In the short run, "liquid" capital becomes scarce, so that its price must rise to restore equilibrium (recall that households value capital in part because of its liquidity services). However, the adverse liquidity shock, via the learning-by-doing channel, compresses the economy and lowers dividends in the long run, thus reducing the demand for capital. This force drives equity prices down. In our calibration, a large intertemporal elasticity of substitution tilts the emphasis to the long-run feature, which leads to the drop in asset prices and the stock market value.

To formalize this intuition, let's take a closer look at the asset pricing equation. As shown in the appendix, the following equation holds both in the endogenous and exogenous growth models:

$$Stock_t = \mathbb{E}_t \left[\underbrace{\beta \left(\frac{c^w_{t+1}}{c^w_t} \right)^{-\rho} \left(\frac{1 - l_{t+1}}{1 - l_t} \right)^{\eta(1-\rho)}}_{\text{stochastic discount factor}} \left(\underbrace{\alpha y_{t+1} + \pi \frac{q_{t+1} - 1}{1 - \theta q_{t+1}} c^e_{t+1} - \pi i_{t+1}}_{\text{dividends}} + Stock_{t+1} \right) \right]. \tag{10}$$

This equation says that the stock market value is the present-discounted value of the future dividend stream, which includes the benefit of relaxing entrepreneur's resource constraint. Taking a log-linear approximation in the endogenous growth model, we find

$$\begin{aligned} \underbrace{\log \left(\frac{Stock_t}{\gamma^t Stock_0} \right)}_{\text{IRF}} \quad = \quad & \underbrace{\log \left(\frac{K_t / K_0}{\gamma^t} \right)}_{\text{realized change in trend}} \\ & \underbrace{+ \left(1 - \beta\gamma^{1-\rho}\right) \sum_{j=1}^{\infty} \left(\beta\gamma^{1-\rho}\right)^{j-1} \mathbb{E}_t \left[(1-\rho) \log \left(\frac{K_{t+j}/K_t}{\gamma^j} \right) \right]}_{\text{expected change in trend}} \\ & \underbrace{+ \sum_{j=1}^{\infty} \left(\beta\gamma^{1-\rho}\right)^{j-1} \mathbb{E}_t \left[cycle_{t+j} \right]}_{\text{cyclical fluctuations around trend}}, \end{aligned} \tag{11}$$

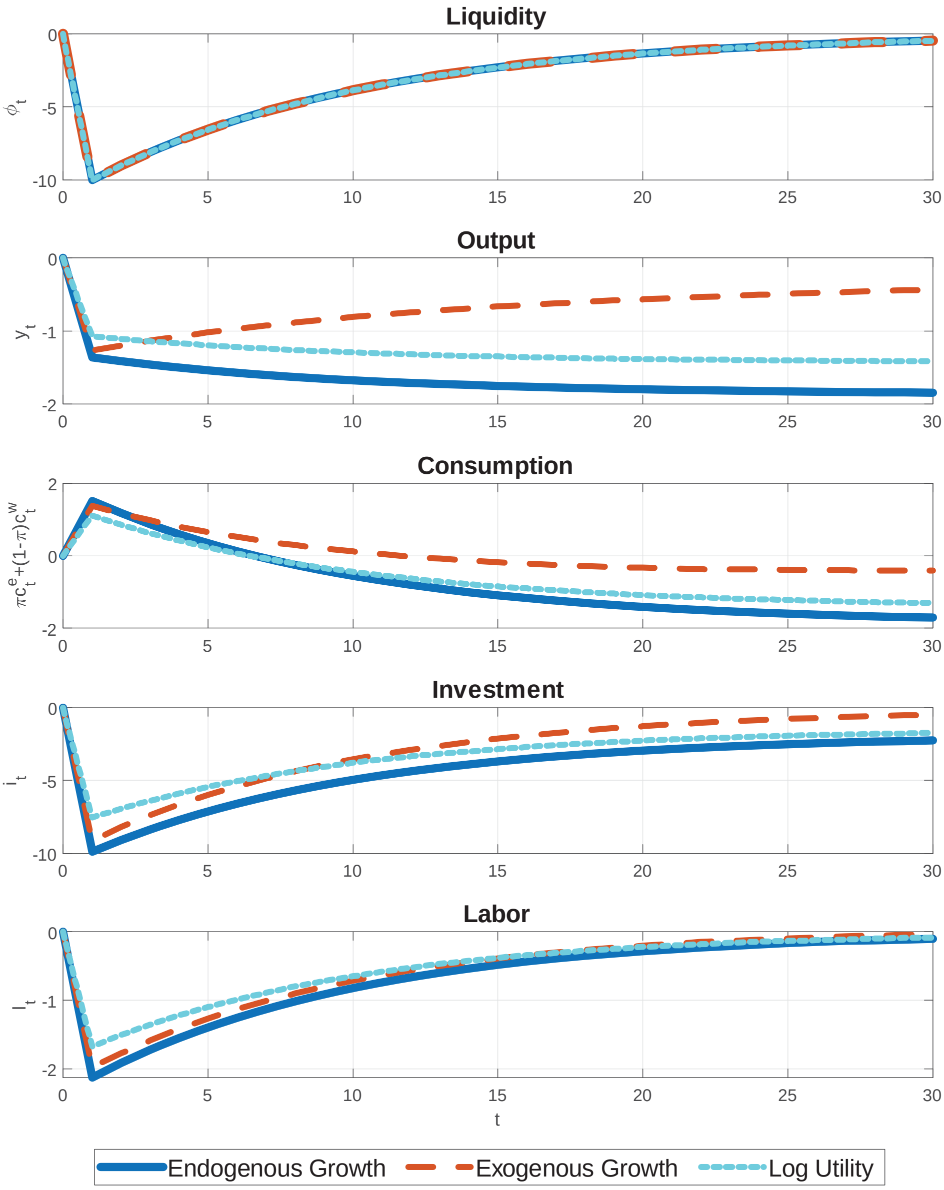

Figure 1: Impulse Response Functions to a Liquidity Shock

where $cycle_{t+j}$ is defined as

$$
\begin{aligned}
cycle_{t+j} \quad = \quad & -\rho \log\left(\frac{\hat{c}^w_{t+j}}{\hat{c}^w_{t+j-1}}\right) - \eta\left(1-\rho\right)\frac{l}{1-l}\log\left(\frac{l_{t+j}}{l_{t+j-1}}\right) \qquad (12)\\
& + \left(1-\beta\gamma^{1-\rho}\right)\frac{\alpha\hat{y}}{\alpha\hat{y}+\pi\frac{q-1}{1-\theta q}\hat{c}^e - \pi\hat{\imath}}\log\left(\frac{\hat{y}_{t+j}}{\hat{y}}\right)\\
& + \left(1-\beta\gamma^{1-\rho}\right)\frac{\pi\frac{q-1}{1-\theta q}\hat{c}^e}{\alpha\hat{y}+\pi\frac{q-1}{1-\theta q}\hat{c}^e - \pi\hat{\imath}}\left[\log\left(\frac{\hat{c}^e_{t+j}}{\hat{c}^e}\right) + \left(\frac{q}{q-1}+\frac{\theta q}{1-\theta q}\right)\log\left(\frac{q_{t+j}}{q}\right)\right]\\
& - \left(1-\beta\gamma^{1-\rho}\right)\frac{\pi\hat{\imath}}{\alpha\hat{y}+\pi\frac{q-1}{1-\theta q}\hat{c}^e - \pi\hat{\imath}}\log\left(\frac{\hat{\imath}_{t+j}}{\hat{\imath}}\right).
\end{aligned}
$$

The left-hand side of equation (11) is the impulse response function of the stock market value in period t. The right-hand side decomposes it into three distinct contributions: realized and expected changes in the endogenous trend term, and the contribution of cyclical fluctuations around the trend. Equation (12) shows the components of the cyclical term. In this equation, variables with a hat indicate that they are divided by the endogenous trend term; for instance, $\hat{y}_t$ denotes $\hat{y}_t \equiv y_t/K_t$. Variables without time subscript denote their steady state values.

Each of these contributions is plotted in Figure 2 in colored bars. In the endogenous growth model, the contributions of the cyclical term to asset prices are positive, but the trend contributions push the stock market into negative territory. In contrast, the exogenous growth model lacks these counterforces. Consequently, the aggregate stock market value is entirely driven by the cyclical term, resulting in a stock market appreciation in the short run. This is a robust prediction in liquidity models with exogenous growth as discussed at large in Shi (2015).[4]

The forces reducing asset prices are fluctuations in the endogenous trend component. Expectations play a key role in the short run, particularly so in period 1. According to equation (11), the stock market response in period 1 is affected by expectations about the future trend as follows:

$$
(1-\rho)\times\left(1-\beta\gamma^{1-\rho}\right)\sum_{j=1}^{\infty}\left(\beta\gamma^{1-\rho}\right)^{j-1}\mathbb{E}_1\left[\log\left(\frac{K_{j+1}/K_1}{\gamma^j}\right)\right]. \qquad (13)
$$

The terms in the weighted sum are the log deviations of the future capital level (K_{j+1}) from the one realized along the balanced growth path ($\gamma^j K_1$). Because the adverse financial shock hurts investment, this term is negative. Since we calibrate the inverse of the intertemporal elasticity of substitution (IES) at $\rho = 1.85^{-1}$, the whole expression is negative, which pushes down the asset price in period 1 (the argument readily extends to $t > 1$).

For the previous argument to go through, the IES ($= 1/\rho$) must be bigger than 1. Although

[4] Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) show that the interaction of a zero lower bound (ZLB) on interest rates and nominal rigidities restores the asset price dynamics. However, the puzzle stands still outside the ZLB or in real models.

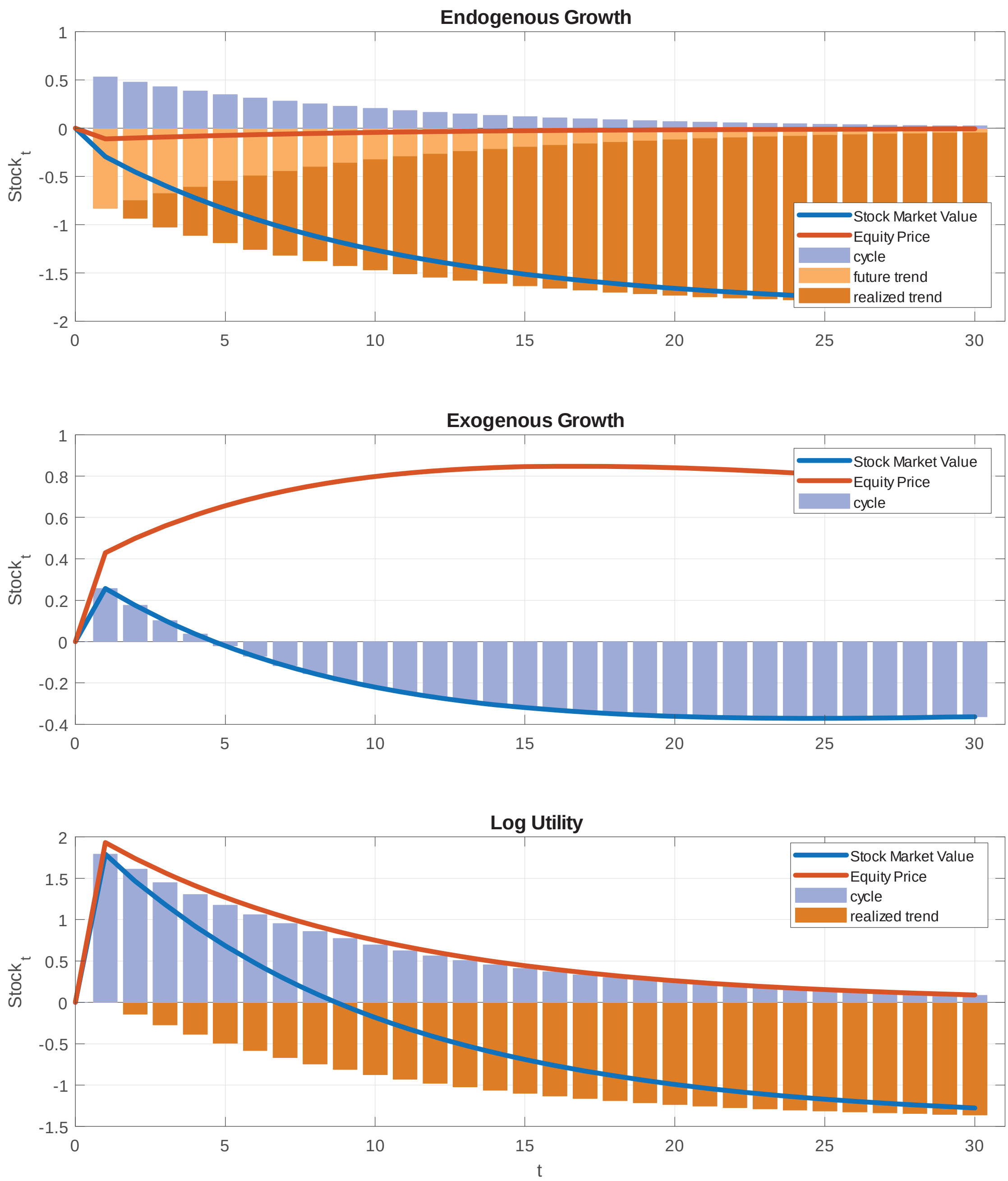

Figure 2: Asset Price Responses to an Adverse Liquidity Shock

this follows mechanically from the future trend element, the role of the IES in equation (13) hides some subtleties brought about by the production structure in our model. To see this, note that a liquidity contraction affects both the dividend and the stochastic discount factor elements of the stock market value. Let's consider dividends, which are the first three terms in the parenthesis in the right-hand side of equation (10). Their impulse response function (not depicted) is similar to that of output plotted in Figure 1. Hence, dividends sink persistently after a negative liquidity shock. If the stochastic discount factor were constant, the stock market value would decrease with this channel alone.

However, the stochastic discount factor is endogenous in general equilibrium. In our model, it is

$$SDF_{t,t+j} \equiv \beta^j \left(\frac{c^w_{t+j}}{c^w_t}\right)^{-\rho} \left(\frac{1-l_{t+j}}{1-l_t}\right)^{\eta(1-\rho)}$$

for $j \geq 1$, which has intertemporal substitutions both in consumption and leisure. Because of growth, the consumption ratio is more important. The impulse response function of the worker's consumption (not depicted) is similar to that of aggregate consumption plotted in Figure 1. As a result, it monotonically decreases after period 1, satisfying

$$\underbrace{\log\left(\frac{c^w_t}{\gamma^t c^w_0}\right)}_{\text{IRF in period } t} > \underbrace{\log\left(\frac{c^w_{t+1}}{\gamma^{t+1} c^w_0}\right)}_{\text{IRF in period } t+1}.$$

Rearranging terms, we find

$$\frac{c^w_{t+1}}{c^w_t} < \gamma$$

for $t \geq 1$. This inequality implies that the worker's consumption growth after a negative liquidity shock is smaller than the growth rate along the balanced growth path (γ). The stochastic discount factor therefore increases, which means that the household discounts future dividends less heavily. This change alone puts upward pressure on the stock market valuation.

Importantly, a higher IES weakens the discount factor channel. This can be seen by noting that a higher IES makes households more willing to substitute future consumption for present consumption if prices change. As a result, the equilibrium price does not move much when an exogenous shock moves the equilibrium allocation. This is why the stochastic discount factor, which is essentially the inverse of the shadow price of future consumption goods, becomes less sensitive to a change in the consumption profile, leading to the weakening of the discount factor channel.

In summary, there are two competing forces driving the dynamics of asset prices, one from the dividends growth and the other from the stochastic discount factor, the latter of which is sensitive to the value of ρ. To highlight this point, the bottom panel of Figure 2 plots the asset price responses in the endogenous growth model with log utility ($\rho = 1$).[5] Unlike the benchmark model, the aggregate stock market value increases in the short run. Moreover, the contribution to asset prices from the expected trend change vanishes, and the contribution from the cyclical terms rises (the blue bars in Figure 2). This is because with a low IES, the discount factor channel operates strongly, putting upward pressure on the asset prices.

The importance of the intertemporal elasticity of substitution for asset price dynamics is familiar in the finance field. Most closely related to our work, the long-run risk literature (Bansal and Yaron (2004)) emphasizes that the IES has to be large to obtain a reasonable stock market response to a change in growth prospects. While the mechanism is similar, we provide a concrete interpretation of long-run risk in a production economy. That is, the long-run risk is an endogenous outcome caused by financial shocks. Our findings might be of interest in the literature searching for a structural origin of the long-run risk.[6]

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[5]Some of the parameters are recalibrated to match the same empirical targets. We show them in Table 1.

[6]Previous work includes Croce (2014), Jinnai (2015), Kaltenbrunner and Lochstoer (2010), and Kung and Schmid (2015). None of them however formally associates the long-run risk with financial shocks.

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5 Appendix (Not for Publication)

5.1 Model Summary

The equilibrium is summarized by the following equations;

$$(c_t^e)^{-\rho} = \left(1 + \frac{q_t - 1}{1 - \theta q_t}\right) (c_t^w)^{-\rho} (1 - l_t)^{\eta(1-\rho)},$$

$$\eta \frac{c_t^w}{1 - l_t} = (1 - \alpha) \frac{y_t}{(1 - \pi) l_t},$$

$$q_t = \mathbb{E}_t \left[\beta \left(\frac{c_{t+1}^w}{c_t^w} \right)^{-\rho} \left(\frac{1 - l_{t+1}}{1 - l_t} \right)^{\eta(1-\rho)} \left(\alpha \frac{y_{t+1}}{K_{t+1}} + (1 - \delta) q_{t+1} + \pi \frac{q_{t+1} - 1}{1 - \theta q_{t+1}} \left(\alpha \frac{y_{t+1}}{K_{t+1}} + q_{t+1} \phi_{t+1} (1 - \delta) \right) \right) \right],$$

$$c_t^e + (1 - \theta q_t) i_t = \alpha y_t + \phi_t q_t (1 - \delta) K_t,$$

$$y_t = A_t (K_t)^\alpha [(1 - \pi) l_t]^{1-\alpha},$$

$$\pi c_t^e + (1 - \pi) c_t^w + \pi i_t = y_t,$$

and

$$K_{t+1} = (1 - \delta) K_t + \pi i_t.$$

5.2 Exogenous Growth Model

We assume $A_t = A_0 (\gamma^{1-\alpha})^t$. The variables are detrended as follows.

$$(\hat{c}_t^e)^{-\rho} = \left(1 + \frac{q_t - 1}{1 - \theta q_t}\right) (\hat{c}_t^w)^{-\rho} (1 - l_t)^{\eta(1-\rho)},$$

$$\eta \frac{\hat{c}_t^w}{1 - l_t} = (1 - \alpha) \frac{\hat{y}_t}{(1 - \pi) l_t},$$

$$q_t = \mathbb{E}_t \left[\beta \left(\frac{\hat{c}_{t+1}^w}{\hat{c}_t^w} \gamma \right)^{-\rho} \left(\frac{1 - l_{t+1}}{1 - l_t} \right)^{\eta(1-\rho)} \left(\alpha \frac{\hat{y}_{t+1}}{\hat{K}_{t+1}} + (1 - \delta) q_{t+1} + \pi \frac{q_{t+1} - 1}{1 - \theta q_{t+1}} \left(\alpha \frac{\hat{y}_{t+1}}{\hat{K}_{t+1}} + q_{t+1} \phi_{t+1} (1 - \delta) \right) \right) \right],$$

$$\hat{c}_t^e + (1 - \theta q_t) \hat{i}_t = \alpha \hat{y}_t + \phi_t q_t (1 - \delta) \hat{K}_t,$$

$$\hat{y}_t = A_0 \left(\hat{K}_t\right)^\alpha [(1 - \pi) l_t]^{1-\alpha},$$

$$\pi \hat{c}_t^e + (1 - \pi) \hat{c}_t^w + \pi \hat{i}_t = \hat{y}_t,$$

and

$$\gamma \hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \pi \hat{i}_t.$$

where variables with hat denote the original variables divided by γ^t, for example, $\hat{c}_t^e = c_t^e / \gamma^t$.

The following parameters are exogenously chosen: $\beta = 0.994$, $\pi = 0.06$, $\alpha = 0.36$, $\gamma = 1.004$, and $A_0 = 1$. The ratio of annual investment to capital in the steady state is set at $4\pi\hat{\imath}/\hat{K} = 0.076$. Depreciation rate of capital δ is found as

$$\delta = \underbrace{1 + \frac{1}{4}\frac{4\pi i}{K} - \gamma}_{\text{known}}.$$

Aggregate hours of work in the steady state is set at $(1-\pi)\,l = 0.25$. Hours of work per worker in the steady state is found as

$$l = \underbrace{\frac{1}{1-\pi}\,(1-\pi)\,l}_{\text{known}}.$$

The ratio of capital to annual output is set at $\hat{K}/\,(4\hat{y}) = 3.32$. Steady state output is backed out from

$$\hat{y} = \underbrace{\left(\frac{\hat{K}}{\hat{y}}\right)^{\frac{\alpha}{1-\alpha}}(1-\pi)\,l}_{\text{known}}.$$

Other steady state values are backed out as follows.

$$\hat{K} = \underbrace{(4\hat{y})\,\frac{\hat{K}}{4\hat{y}}}_{\text{known}}.$$

$$\hat{\imath} = \underbrace{\frac{\hat{K}}{4\pi}\frac{4\pi\hat{\imath}}{\hat{K}}}_{\text{known}}.$$

The following equations are solved for $\hat{c}^e$, $\hat{c}^w$, q, η, and $\phi = \theta$;

$$(\hat{c}^e)^{-\rho} = \left(1 + \frac{q-1}{1-\phi q}\right)(\hat{c}^w)^{-\rho}\,(1-l)^{\eta(1-\rho)},$$

$$\eta\frac{\hat{c}^w}{1-l} = (1-\alpha)\,\frac{\hat{y}}{(1-\pi)\,l},$$

$$q = \beta\,(\gamma)^{-\rho}\left(\alpha\frac{\hat{y}}{\hat{K}} + (1-\delta)\,q + \pi\frac{q-1}{1-\phi q}\left(\alpha\frac{\hat{y}}{\hat{K}} + q\phi\,(1-\delta)\right)\right),$$

$$\hat{c}^e + (1-\phi q)\,\hat{\imath} = \alpha\hat{y} + \phi q\,(1-\delta)\,\hat{K},$$

and

$$\pi\hat{c}^e + (1-\pi)\,\hat{c}^w + \pi\hat{\imath} = \hat{y}.$$

5.3 Endogenous growth model

We assume $A_t = A_0 (K_t)^{1-\alpha}$. The variables are detrended as follows;

$$(\hat{c}_t^e)^{-\rho} = \left(1 + \frac{q_t - 1}{1 - \theta q_t}\right) (\hat{c}_t^w)^{-\rho} (1 - l_t)^{\eta(1-\rho)},$$

$$\eta \frac{\hat{c}_t^w}{1 - l_t} = (1 - \alpha) \frac{\hat{y}_t}{(1 - \pi) l_t},$$

$$q_t = \mathbb{E}_t \left[\beta \left(\frac{\hat{c}_{t+1}^w}{\hat{c}_t^w} \gamma_t \right)^{-\rho} \left(\frac{1 - l_{t+1}}{1 - l_t} \right)^{\eta(1-\rho)} \left(\alpha \hat{y}_{t+1} + (1 - \delta) q_{t+1} + \pi \frac{q_{t+1} - 1}{1 - \theta q_{t+1}} \left(\alpha \hat{y}_{t+1} + q_{t+1} \phi_{t+1} (1 - \delta) \right) \right) \right],$$

$$\hat{c}_t^e + (1 - \theta q_t) \hat{i}_t = \alpha \hat{y}_t + \phi_t q_t (1 - \delta),$$

$$\hat{y}_t = A_0 \left[(1 - \pi) l_t \right]^{1-\alpha},$$

$$\pi \hat{c}_t^e + (1 - \pi) \hat{c}_t^w + \pi \hat{i}_t = \hat{y}_t,$$

and

$$\gamma_t = 1 - \delta + \pi \hat{i}_t.$$

where $\gamma_t = K_{t+1}/K_t$ and hat variables denote the original variables divided by K_t, for example, $\hat{c}_t^w = c_t^w / K_t$.

The following parameters are exogenously chosen: $\beta = 0.994$, $\pi = 0.06$, $\alpha = 0.36$, and $\gamma = 1.004$. The ratio of annual investment to capital in the steady state is set at $4\pi i_t / K_t|_{s.s.} = 4\pi \hat{\imath} = 0.076$. Hence

$$\hat{\imath} = \underbrace{\frac{0.076}{4\pi}}_{\text{known}}.$$

δ is found as

$$\delta = \underbrace{1 + \pi \hat{\imath} - g}_{\text{known}}.$$

Aggregate hours of work in the steady state is set at $(1 - \pi) l = 0.25$. Steady state individual hours of work in the steady state is backed out from

$$l = \underbrace{\frac{1}{1 - \pi} (1 - \pi) l}_{\text{known}}.$$

The ratio of capital to annual output is set at $K_t / (4 y_t)|_{s.s.} = 1 / (4 \hat{y}_t)|_{s.s.} = 3.32$. Hence

$$\hat{y} = \left(\frac{1}{4}\right) \left(\frac{1}{3.32}\right).$$

$$A_0 = \underbrace{\hat{y}\left[\left(1-\pi\right)l\right]^{\alpha-1}}_{\text{known}}.$$

The following equations are solved for $\hat{c}^e$, $\hat{c}^w$, q, η, and $\phi = \theta$;

$$\left(\hat{c}^e\right)^{-\rho} = \left(1 + \frac{q-1}{1-\phi q}\right)\left(\hat{c}^w\right)^{-\rho}\left(1-l\right)^{\eta(1-\rho)},$$

$$\eta\frac{\hat{c}^w}{1-l} = \left(1-\alpha\right)\frac{\hat{y}}{\left(1-\pi\right)l},$$

$$q = \beta\left(\gamma\right)^{-\rho}\left(\alpha\hat{y} + \left(1-\delta\right)q + \pi\frac{q-1}{1-\phi q}\left(\alpha\hat{y} + q\phi\left(1-\delta\right)\right)\right),$$

$$\hat{c}^e + \left(1-\phi q\right)\hat{\imath} = \alpha\hat{y} + \phi q\left(1-\delta\right),$$

and

$$\pi\hat{c}^e + \left(1-\pi\right)\hat{c}^w + \pi\hat{\imath} = \hat{y}.$$

5.4 Asset pricing equation

The Euler equation is

$$q_t = \mathbb{E}_t\left[\beta\left(\frac{c^w_{t+1}}{c^w_t}\right)^{-\rho}\left(\frac{1-l_{t+1}}{1-l_t}\right)^{\eta(1-\rho)}\left(\alpha\frac{y_{t+1}}{K_{t+1}} + \left(1-\delta\right)q_{t+1} + \pi\frac{q_{t+1}-1}{1-\theta q_{t+1}}\left(\alpha\frac{y_{t+1}}{K_{t+1}} + q_{t+1}\phi_{t+1}\left(1-\delta\right)\right)\right)\right].$$

Multiply K_{t+1} to both sides, we find

$$Stock_t = E_t\left[\begin{array}{c}\beta\left(\frac{c^w_{t+1}}{c^w_t}\right)^{-\rho}\left(\frac{1-l_{t+1}}{1-l_t}\right)^{\eta(1-\rho)} \\ \left(\alpha y_{t+1} + \left(1-\delta\right)q_{t+1}K_{t+1} + \pi\frac{q_{t+1}-1}{1-\theta q_{t+1}}\left(\alpha y_{t+1} + q_{t+1}\phi_{t+1}\left(1-\delta\right)K_{t+1}\right)\right)\end{array}\right].$$

Because $c^e_t + \left(1-\theta q_t\right)i_t = \alpha y_t + \phi_t q_t\left(1-\delta\right)K_t$ holds in equilibrium, substituting and rearranging we obtain

$$Stock_t = E_t\left[\beta\left(\frac{c^w_{t+1}}{c^w_t}\right)^{-\rho}\left(\frac{1-l_{t+1}}{1-l_t}\right)^{\eta(1-\rho)}\left(\alpha y_{t+1} + \pi\frac{q_{t+1}-1}{1-\theta q_{t+1}}c^e_{t+1} - \pi i_{t+1} + Stock_{t+1}\right)\right].$$