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# The Quarterly Japanese Economic Model (Q-JEM): 2019 version

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**Abstract**

In this paper, we introduce the updated version of the Quarterly Japanese Economic Model (Q-JEM), which was first developed by Ichiue et al. (2009) and updated by Fukunaga et al. (2011). Q-JEM is a large-scale semi-structural model of the Japanese economy, which is designed to incorporate greater disaggregation of expenditure components and detailed financial market information. Compared to Dynamic Stochastic General Equilibrium (DSGE) models, Q-JEM puts more emphasis on fitting data, while relaxing some theoretical discipline. To improve public access to the model, we share the replication files of the simulations conducted in the paper.

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# 1. Introduction

In this paper, we introduce the updated version of the Quarterly Japanese Economic Model (Q-JEM), which was originally developed by Ichiue et al. (2009) and subsequently updated by Fukunaga et al. (2011).[1]

Q-JEM is a large-scale semi-structural model of the Japanese economy.[2] Some central banks conduct economic projection and simulations with semi-structural models, which differ in important features from Dynamic Stochastic General Equilibrium (DSGE) models.[3] Perhaps the most well-known operative semi-structural model is FRB/US developed by Federal Reserve Board (Brayton and Tinsley, 1996; Reifschneider and Williams, 2000; Brayton et al., 2014; Laforte, 2018).[4] In a similar vein, the Bank of Canada employs Large Empirical and Semi-structural model (LENS) developed by Gervais and Gosselin (2014). The Reserve Bank of Australia utilizes Macroeconomic Relationships for Targeting Inflation (MARTIN, Cusbert and Kendall, 2018). Other semi-structural models operated by central banks and policy institutions include the New Multi-Country Model (NMCM) and ECB Multi-Country model (ECB-MC) developed by the European Central Bank (Dieppe et al., 2012; Dieppe et al., 2018), the Monetary Model of Singapore (MMS) developed by the Monetary Authority of Singapore (Monetary Authority of Singapore, 2014), and the Flexible System of Global Models (FSGM) developed by the International Monetary Fund (Andrle et al., 2015).

At the same time, New Keynesian DSGE models have been widely used in central banks. Many of the DSGE models operated by central banks are a variant of the benchmark medium-scale New Keynesian DSGE model (i.e., Christiano et al., 2005; Smets and Wouters, 2007). These include the Estimated Dynamic Optimization (EDO) model developed by the Federal Reserve Board, which augments greater disaggregation of domestic spending to the standard medium-scale New Keynesian model (Edge et al., 2010; Chung et al., 2010), the FRBNY DSGE Model developed by Federal Reserve Bank of New York, a standard medium-scale New Keynesian model with financial frictions (Del Negro et al. 2013; Del

[1] See Kan et al. (2016) and Bank of Japan (2011, 2015, 2016) for simulation exercises conducted with Q-JEM.
[2] Financial Macroeconometric Model (FMM) is another semi-structural model operated by the Bank of Japan (Kitamura et al., 2014). FMM is developed for the purpose of stress testing, and explicitly models detailed variables of financial sector.
[3] See Lindé et al. (2016), for example, for models in use at central banks.
[4] The FRB/US program is available on the Federal Reserve Board's website.

Negro et al. 2015). Likewise, the European Central Bank employs several DSGE models including the New Area-Wide Model (NAWM) I/II (Christoffel et al., 2008; Coenen et al., 2018). The Bank of England utilizes the Central Organising Model for Projection Analysis and Scenario Simulation (COMPASS), a medium-scale open economy New Keynesian DSGE model (Burgess et al., 2013). Other operative DSGE models at central banks include Terms-of-Trade Economic Model (ToTEM) developed by Bank of Canada (Dorich et al., 2013), and the Riksbank Aggregate Macromodel for Studies of the Economy of Sweden (RAMSES) developed by Sveriges Riksbank (Adolfson et al., 2011).[5]

Semi-structural models and DSGE models have their own advantages and disadvantages. The apparent advantage of a semi-structural model is its flexibility. This enables a model to describe the detailed modeling of GDP expenditure components, financial markets and determinants of inflation, incorporating various transmission channels of policies and exogenous shocks. At the same time, semi-structural models are not necessarilly micro-founded, which makes the models subject to Lucas Critique and limits some structural analysis. On the other hand, because of their theoretical consistency, DSGE models provide richer structural insights, and are more suitable for welfare analysis. However, because DSGE models are more restricted theoretically, many important variables in forecasting and simulations, such as detailed expenditure components or financial market variables, are not easily augmented.

As Tenreyro (2018) points out, economists often resort to different models to analyze different problems. In this regard, Blanchard (2018) argues that models can have different degrees of theoretical purity and simplicity. He argues that, for DSGE models, where theoretical purity is central, closely fitting the data is less important than clarity of structure. On the other hand, models used for policy purposes (e.g., models by central banks or international organizations) must fit the data more closely, and this is likely to require more flexible, less micro-founded, lag structures. Understanding these features, economists and policy makers should select a suitable model for their particular given purpose.[6]

The update of the model from its previous version (Fukunaga et al., 2011) is summarized in the following points. First, in the new version of the model, we change how inflation

[5] DSGE models developed by Bank of Japan include Fueki et al. (2016).

[6] Blanchard (2018) also argues that *ad hoc* macromodels like a variation of IS-LM to Mundell-Fleming model still play an important role in relation to DSGE models. On the other hand, Christiano et al. (2018) argue that DSGE models will remain central to how macroeconomists think about economy and policies as there is no credible alternative.

expectation is modeled, introducing the Trend Inflation Projection System (TIPS), a satellite model developed by Takahashi (2016). In the new version of the model, Phillips Curve augments survey-based inflation expectation, which is modeled with this satellite model. Second, we modify some expenditure component functions, especially investment and import. The investment demand function is modeled based on Tobin's Q theory. The import demand function exploits information of the international Input-Output table to take into account the difference of import-intensity for each expenditure component. In addition to these changes, the model size has been reduced substantially from the previous version. This is aimed at reducing the complexity of the model and enhancing transparency, while maintaining the core transmission mechanism of the model.[7]

To improve public access to the model, we share the replication files of the simulations in Section 3. These simulation programs can be run with EViews, in conjunction with Python. The replication files also contain the full model documentation to aid understanding of the model.

The paper is organized as follows. In Section 2 we describe the core blocks of Q-JEM. Section 3 discusses some simulations that utilize Q-JEM and compare its empirical properties with those of a vector autoregression model. Section 4 concludes.

## 2. Structure of Q-JEM

Q-JEM is the large scale semi-structural model of the Japanese economy with a large number of equations. To capture key elements of Japanese economy, the model includes detailed modeling of GDP expenditure components, financial markets and determinants of inflation. The model employs a small open economy structure and foreign blocks (the US and the rest of the world blocks) are exogenous to the Japanese economy. In the following, we give a brief overview of parameterization and the structure of core blocks. The full-system equations are found in the replication files.[8]

To closely fit the data, most of the parameters are estimated, rather than calibrated. Most key variables are modeled using error-correction models. In the DSGE literature, equations are typically jointly estimated (e.g., Smets and Wouters, 2007). However, the large size of

[7] Another important update is the introduction of a new solution method. See Appendix 1 for detail.
[8] For the simplicity of notation, in Section 2, we omit some dummies embedded in the equations. See *qjemdoc.html* in the replication files for the full equations.

Q-JEM makes it difficult to estimate its equations simultaneously. For that reason, parameters are estimated equation-by-equation, employing the least square approach. The exception is the inflation expectation equations, where parameters are estimated jointly in a satellite model with Bayesian approach.

## 2.1 GDP, Output Gap and Natural Interest Rate

***GDP and expenditure components***

From the expenditure approach to GDP, the GDP identity of Q-JEM is as follows:

$$P_t^Y Y_t = P_t^C C_t + P_t^I I_t + P_t^H H_t + P_t^{II} II_t + P_t^{CG} CG_t + P_t^{IG} IG_t + P_t^{EX} EX_t - P_t^{IM} IM_t. \quad (1)$$

In Q-JEM, nominal GDP $P_t^Y Y_t$ is the sum of domestic expenditure components and net exports: nominal private consumption $P_t^C C_t$, nominal private nonresidential investment $P_t^I I_t$, nominal residential investment $P_t^H H_t$, change in nominal inventory $P_t^{II} II_t$, nominal government consumption $P_t^{CG} CG_t$, nominal public investment $P_t^{IG} IG_t$, nominal exports $P_t^{EX} EX_t$ and nominal imports $P_t^{IM} IM_t$. Expenditure variables are driven by real interest rate, exchange rate and other factors. Specifications of these variables are elaborated below.

***Output gap***

Output gap $GAP_t$ is defined as the deviation of real output $Y_t$ from potential output $Y_t^Q$, where potential output is obtained by the production function approach following Kawamoto et al. (2017):

$$GAP_t = 100 \frac{Y_t}{Y_t^Q} - 100. \quad (2)$$

## 2.2 Phillips Curve and Inflation Expectations

Core inflation $\pi_t$, is determined by the Phillips Curve as follows:

$$\pi_t = \beta_0 \left( \frac{\pi_{t-1} + \pi_{t-2}}{2} \right) + (1 - \beta_0) \pi_{t,l}^e + \beta_1 GAP_t. \quad (3)$$

The Phillips Curve in Q-JEM includes output gap $GAP_t$, long-term (6 to 10 years ahead) inflation expectation $\pi_{t,l}^{e}$ observed by inflation survey (Consensus Economics) and lagged inflation terms $\pi_{t-1}$ and $\pi_{t-2}$.[9]

The long-term inflation expectation $\pi_{t,l}^{e}$ is modeled in the satellite state space model, called the Trend Inflation Projection System (TIPS) developed by Takahashi (2016). For inflation expectation in TIPS, we employ survey-based inflation expectation following Roberts (1995) and Brissimis and Magginas (2008).[10] In TIPS, the long-term inflation expectation $\pi_{t,l}^{e}$ obeys the following process similar to Phillips Curve,

$$\pi_{t,l}^{e} = \beta_0 \pi_{t-1,l}^{e} + (1-\beta_0)\pi_t^{Q} + \beta_1 GAP_{t,24}^{e} + \varepsilon_t, \qquad (4)$$

where $\pi_t^{Q}$ denotes unobserved trend inflation and $GAP_{t,24}^{e}$ denotes 24-quarter ahead expected output gap determined by the AR(2) process as defined below. Trend inflation $\pi_t^{Q}$ is specified as a weighted average of lagged permanent component of inflation $\tau_{t-1}$, and inflation target $\pi_t^{*}$:

$$\pi_t^{Q} = \delta_t \tau_{t-1} + (1-\delta_t)\pi_t^{*}. \qquad (5)$$

In the above, inflation target $\pi_t^{*}$ is an exogenous variable. Lagged permanent component of inflation $\tau_{t-1}$ is obtained by Beveridge-Nelson decomposition of four inflation measures: CPI all items less food and energy; CPI all items less fresh food; CPI all items less fresh food and energy; CPI trimmed mean inflation. Time-varying weight $(1-\delta_t)$ can be interpreted as the share of forward looking agent, therefore the credibility of the central bank. $\delta_t$ follows the following AR (1) process:

$$\delta_t = \omega \delta_{t-1} + \kappa D_t + \mu_t, \qquad (6)$$

where $D_t$ is a dummy variable that takes the value of 1 in the first quarter of 2013. In TIPS, parameters are estimated using the Bayesian approach.

9 In Q-JEM, CPI all items less fresh food and energy is used as the core inflation measure.
10 See Coibion et al. (2018) for the survey of Phillips Curve models that utilize survey-based inflation expectation.

## 2.3 Monetary Policy, Interest Rates and Exchange Rates

***Natural interest rate***

In Q-JEM, natural rate of interest $r_t^*$ is specified as a function of output gap $GAP_\mathrm{t}$ and potential growth rate $\mathrm{dlog}(Y_t^Q)$:[11]

$$r_t^* = \beta_0 + \beta_1 GAP_\mathrm{t} + \beta_2 \mathrm{dlog}(Y_t^Q). \qquad (7)$$

Nominal equilibrium policy rate, $nr_t^Q$ is obtained by the sum of trend inflation $\pi_t^Q$ and the cyclically adjusted natural interest rate ($r_t^* - \beta_1 GAP_t$):

$$nr_t^Q = \pi_t^Q + (r_t^* - \beta_1 GAP_t). \qquad (8)$$

***Monetary policy and interest rate expectation***

In the model, the central bank sets the policy rate according to the Taylor rule with interest rate smoothing,

$$nr_t = (1-\alpha_0) nr_{t-1} + \alpha_0 \left[nr_t^Q + 1.5(\pi_t - \pi_t^*) + 0.5 GAP_\mathrm{t}\right]. \qquad (9)$$

In the policy function, the coefficients on inflation gap and output gap are calibrated (1.5 and 0.5, respectively). The smoothing parameter $\alpha_0$ is estimated by least square. $nr_t^Q$ denotes nominal equilibrium policy rate.

Likewise, $i$-quarter ahead expected policy rate at $t$, $nr_{t,i}^e$ obeys the following process.[12]

$$nr_{t,i}^e = (1-\alpha_0) nr_{t,i-1}^e + \alpha_0 \left[nr_{t,i}^Q + 1.5\left(\pi_{t,i}^e - \pi_t^*\right) + 0.5 GAP_{t,i}^e\right], \qquad (10)$$

where $GAP_{t,i}^e$ is the $i$-quarter ahead expected output gap at $t$, and is obtained by following AR(2) process,

[11] Natural interest rate is not directly observable. Thus, we obtain the natural interest rate by state space model à la Clark and Kozicki (2005). Parameters in equation (7) are obtained by regressing this natural interest rate on output gap and potential growth rate. See Appendix 2 for detail.

[12] We assume that the forward nominal equilibrium interest rate is same as nominal equilibrium interest rate at time *t*. That is, $nr_{t,i}^Q = nr_t^Q$.

$$GAP^e_{t,i} = \beta_1 GAP^e_{t,i-1} + \beta_2 GAP^e_{t,i-2}. \tag{11}$$

In the above, $i$-quarter ahead inflation expectation at $t$, $\pi^e_{t,i}$ is obtained by the linear interpolation of actual core inflation $\pi_t$ and long-term inflation expectation $\pi^e_{t,l}$.

Medium-term interest rate (i.e., average yield on 3-5-year Japanese Government Bonds), $nr^m_t$, is determined by cumulative expected short-term interest rate and term premium. Term premium is assumed to follow AR (1) process. The equation reads as,

$$nr^m_t - \overline{nr}^e_{t,16} = \alpha_0 + \alpha_1\left(nr^m_{t-1} - \overline{nr}^e_{t-1,16}\right), \tag{12}$$

where $\overline{nr}^e_{t,16}$ denotes the four-year average expected interest rate defined as,

$$\overline{nr}^e_{t,16} \equiv \frac{\sum_{i=0}^{15} nr^e_{t,i}}{16}. \tag{13}$$

Similarly, long-term interest rate (i.e., 10-year Japanese Government Bond yield), $nr^l_t$, is obtained by summing cumulative expected short-term interest rate and term premium. Long-term premium is assumed to move in proportion to medium-term premium. Thus, the long-term interest rate is defined as,

$$nr^l_t - \overline{nr}^e_{t,40} = \beta_0 + \beta_1(nr^m_t - \overline{nr}^e_{t,16}), \tag{14}$$

where $\overline{nr}^e_{t,40}$ denotes the 10-year average expected interest rate defined as,

$$\overline{nr}^e_{t,40} \equiv \frac{\sum_{i=0}^{39} nr^e_{t,i}}{40}. \tag{15}$$

As shown above, the theoretical basis of long-term and medium-term interest rate equations is standard expectation hypothesis. That is, the long-term and medium-term interest rates equal an average of expected short-term interest rates, plus a risk premium. This means that, the model does not take into account the Bank of Japan's recent unconventional monetary policies that directly affect long-term interest rates, including Quantitative and

Qualitative Easing (QQE) and Yield Curve Control (YCC).[13] In the simulations, the effect of these policies can be assessed by assuming an exogenous path of long-term interest rate.

***Exchange rates***

The long-run level of US dollar to Japanese yen bilateral nominal exchange rate $\mathcal{E}_t^{\$,Q}$ is determined by the combination of US-Japan long-term real interest rate difference and purchasing power parity,

$$\left[E_t \log\left(RER_{t+40}^{\$,Q}\right) - \log\left(\mathcal{E}_t^{\$,Q} P_t^{US,CORE} / P_t^{CORE}\right)\right]/10 \\ = -\left[nr_t^{US,l} - nr_t^{l} - \left(\bar{\pi}_{t,40}^{US,e} - \bar{\pi}_{t,40}^{e}\right)\right]/100. \quad (16)$$

$E_t \log\left(RER_{t+40}^{\$,Q}\right)$ denotes the long-run level of expected US dollar Japanese yen bilateral real exchange rate. This variable is assumed to be constant and estimated. $nr_t^{US,l}$ denotes long-term US nominal interest rate, $\bar{\pi}_{t,40}^{US,e}$ denotes US 10-year average inflation expectation, $P_t^{US,CORE}$ denotes US price level excluding food and energy and $P_t^{CORE}$ denotes domestic price level excluding food, energy and consumption tax.[14]

Nominal bilateral exchange rate slowly adjusts to this long-run level with the following AR (1) model,

$$\log\left(\mathcal{E}_t^{\$} / \mathcal{E}_{t-1}^{\$,Q}\right) = \beta_0 \log\left(\mathcal{E}_{t-1}^{\$} / \mathcal{E}_{t-2}^{\$,Q}\right). \quad (17)$$

Nominal effective exchange rate, $NEER_t$, is assumed to evolve proportionally to the changes in bilateral exchange rate,

$$\text{dlog}(NEER_t) = \alpha_0 + \alpha_1 \text{dlog}\left(\mathcal{E}_t^{\$}\right). \quad (18)$$

[13] In addition, monetary policy equations presented in this paper do not take into account zero-lower-bound (or effective-lower-bound) of nominal interest rates. Zero-lower-bound of nominal interest rate can be augmented to the model by imposing non-negativity constraints on interest rates. See Fukunaga et al. (2011) for the case zero-lower-bound of nominal interest is considered.

[14] Focusing on the empirical fact that uncovered interest parity tends to hold for longer-term interest rate differentials, Katagiri and Takahashi (2017) develop a small open economy DSGE model with limited asset market participation and estimate the model with Bayesian technique.

Real effective exchange rate, $REER_t$, is obtained by deflating nominal effective exchange rate with relative price,

$$REER_t = NEER_t \times \frac{P_t^{XF}}{P_t^{F,CPI}}, \tag{19}$$

where $P_t^{XF}$ denotes domestic consumer price level (all items less fresh food), and $P_t^{F,CPI}$ denotes aggregate foreign price level.

## 2.4 Expenditure Components

In this subsection, we elaborate on the specifications of main expenditure components: private consumption, private non-residential investment, exports and imports.

### 2.4.1 Private Consumption

Based on the permanent income hypothesis, a standard theory of consumption, the equation of long-run equilibrium level of real private consumption $C_t^Q$ is specified as follows:

$$\log\left(\frac{C_t^Q P_t^C}{YW_t - YP_t}\right) = \alpha_0 + \alpha_1 \left[nr_t^l - \bar{\pi}_{t,40}^e - 100 \times \log\left(\frac{Y_t^Q}{Y_{t-4}^Q}\right)\right] + \alpha_2 \log\left(\frac{FA_t}{YW_t - YP_t}\right), \tag{20}$$

where the left hand side represents the long-run equilibrium level of average propensity to consume. Specifically, $P_t^C$ denotes private consumption deflator, $YW_t$ denotes nominal disposable income and $YP_t$ denotes nominal property income. In calculating the long-run equilibrium level of consumption, we exclude $YP_t$ from disposable income as it exhibits volatile dynamics. On the right-hand side, the second term represents the interest rate gap, where $nr_t^l$ denotes long-term interest rate, $\bar{\pi}_{t,40}^e$ denotes 10-year average inflation expectation, $\log(Y_t^Q/Y_{t-4}^Q)$ is the year-on-year potential growth rate. In addition, the third term represents the wealth-to-income ratio, where $FA_t$ denotes financial assets at the end of $t$. In the model, average propensity to consume increases in response to a lower interest rate gap, as the present value of future disposable income increases. Likewise, it increases in response to an increase in asset prices.

Short-run dynamics of real private consumption is solely driven by long-run equilibrium level of consumption,

$$\mathrm{dlog}(C_t) = \beta_0 + \beta_1 \log\left(\frac{C_{t-1}}{C_{t-1}^{Q}}\right), \tag{21}$$

where $\log(C_{t-1}/C_{t-1}^{Q})$ is an error correction term, which is defined as the deviation from the long-run equilibrium level of consumption. In addition, $\beta_1$ determines the speed of convergence to the long-run equilibrium level. Figure 1 displays the long-run equilibrium level of consumption and the fitted value of short-run dynamics.

## 2.4.2 Private Non-residential Investment

Based on Q theory, the long-run equilibrium level of real private non-residential investment is specified as follows:

$$\log\left(\frac{I_t^{Q}\ P_t^{I}}{Y_t\ P_t^{Y}}\right) = \alpha_0 + \alpha_1 \log(Q_t) + \alpha_2 \log(nr_t^{l} - \bar{\pi}_{t,40}^{e} - r_t^{*} + \delta_t), \tag{22}$$

where $I_t^{Q}$ denotes the long-run equilibrium level of real private non-residential investment, $P_t^{I}$ denotes private non-residential investment deflator, $P_t^{Y}$ denotes GDP deflator, $Q_t$ denotes (average) Tobin's Q, $\delta_t$ denotes depreciation rate.

In a standard empirical specification of the Q theory of investment, investment to capital ratio is determined solely by $Q_t$. However, in Q-JEM, investment is modeled in proportion to output as in equation (22), which also utilizes the relationship between the marginal product of capital and capital to output ratio derived from a firm's profit maximization problem.[15] Therefore, in addition to $Q_t$, the real interest rate is included as an explanatory variable in equation (22).[16] By doing so, the model implicitly incorporates the mechanism that marginal

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[15] Capital-to-output is specified as a decreasing function of user cost of capital, which can be obtained from firm's profit maximization problem. Investment-to-capital ratio is specified as an increasing function of Tobin's Q. Combining these together, investment-to-output ratio can be written as a function of user cost of capital and Tobin's Q.

[16] Natural interest rate $r_t^{*}$ in equation (22) is obtained from long-term real interest rate. Thus, the

product of capital decreases with additional capital, even without explicitly modeling production function. As shown in Figure 2, the relative price of investment (i.e., the ratio of investment deflator to GDP deflator) declined until the early 2000s, and since then has moved more or less sideways. In Q-JEM, the relative price is modeled explicitly to allow for this structural change.

We specify the short-run dynamics of investment as follows:

$$\mathrm{dlog}(I_t) = \beta_0 + \beta_1 \log\left(\frac{I_{t-1}}{I_{t-1}^Q}\right) + \beta_2 \sum_{j=0}^{3} \mathrm{dlog}(EX_{t-j}). \tag{23}$$

In the short-run dynamics, the growth of export is used to capture the effect of demand shocks on the short-term fluctuation of investment as well as error correction term. Figure 3 displays the long-run equilibrium and short-run dynamics of private investment.

### 2.4.3 Exports

Real export follows the simple error correction process. The long-run equilibrium level of real export reads as,

$$\log\left(EX_t^Q\right) = \alpha_0 + \alpha_1 \log(Y_t^F) + \alpha_2 \log(REER_t), \tag{24}$$

where $Y_t^F$ denotes foreign real GDP, where Japan's nominal export shares are used for the weights of individual countries. In addition, $REER_t$ denotes the real effective exchange rate, which reflects Japan's competitiveness. The long-run equilibrium level of real export grows in proportion to foreign real GDP, and also fluctuates depending on real effective exchange rates.

Based on this long-run level of real export, the short-run dynamics of real export is determined by

long-term real interest rate gap is calculated as real interest rate less this natural interest rate. See Appendix 2 for detail.

$$\mathrm{dlog}(EX_t) = \beta_0 + \beta_1 \log\left(\frac{EX_{t-1}}{EX^{Q}_{t-1}}\right) + \beta_2 \mathrm{dlog}(EX_{t-1}) + \beta_3 \mathrm{dlog}(Y^F_t) + \beta_4 \mathrm{dlog}(REER_t). \quad (25)$$

Figure 4 displays the implied long-run equilibrium level of real export, as well as fitted value of short-run dynamics.

### 2.4.4 Imports

The modeling of the dynamics of real import follows the modified version of Bussière et al. (2013), where real import $IM_t$ is driven by import-intensity adjusted demand (IAD),

$$\mathrm{dlog}(IM_t) = \beta_0 + \beta_1 \sum_{j=1}^{16} \left(\frac{EX_{t-j}P^{EX}_{t-j} - IM_{t-j}P^{IM}_{t-j}}{DDN_{t-j}}\right) + \beta_2 \mathrm{dlog}(IAD_t) + \beta_3 \,\mathrm{dlog}(IAD_{t-1}) + \beta_4 \sum_{j=0}^{5} \mathrm{dlog}(REER_{t-j}), \quad (26)$$

where the second term of the right-hand side, $\beta_1 \sum_{j=1}^{16} \left(\frac{EX_{t-j}P^{EX}_{t-j} - IM_{t-j}P^{IM}_{t-j}}{DDN_{t-j}}\right)$, is the moving average of nominal net export over nominal domestic demand (i.e., the sum of consumption, investment and government spending) and puts nominal net export on a stable trajectory in long-term. In addition to net export ratio and IAD, the real import demand function includes $REER_t$, the real effective exchange rate, as a proxy for relative import price.[17] Figure 5 reports IAD and fitted value of equation (26).

***Derivation of IAD***

The import content of expenditure differs among expenditure components. Bussière et al. (2013) show that import demand function that takes into account the heterogeneity of import content of each expenditure category traces trade dynamics well. To reflect the heterogeneity of each expenditure category's import-intensity, they exploit information from the input-output table (I-O table) and construct import-intensity adjusted measure of demand (IAD). Following Bussière et al. (2013), we employ IAD, which is expressed as a log-linear

[17] See Bussière et al. (2013) for the theoretical derivation of import demand function that includes IAD.

combination of each expenditure component:

$$\log(IAD_t) = \alpha_0 \log(C_t) + \alpha_1 \log(CG_t) + \alpha_2 \log(GCF_t) + \alpha_3 \log(EX_t), \quad (27)$$

where $GCF_t$ indicates real gross fixed capital formation, and $\alpha_i$ represents total import content of final expenditure category $i$ and is computed from OECD I-O table.[18] Specifically, $\alpha_i$ is computed as,

$$\alpha_i = \frac{\boldsymbol{u}\boldsymbol{\mathcal{M}}_i^{dir} + \boldsymbol{u}\boldsymbol{\mathcal{M}}_i^{ind}}{\boldsymbol{u}\boldsymbol{\mathcal{F}}_i^{d} + \boldsymbol{u}\boldsymbol{\mathcal{F}}_i^{m}}. \quad (28)$$

In this equation, the numerator represents the sum of direct import content of expenditure $i$ and indirect import induced by expenditure $i$, where $\boldsymbol{u}$ is $1 \times S$ vector with all elements equal to 1, $\boldsymbol{\mathcal{M}}_i^{dir}$ is $\mathrm{S} \times 1$ vector that represents direct import content of expenditure category $i$, $\boldsymbol{\mathcal{M}}_i^{ind}$ is $\mathrm{S} \times 1$ vector that represents indirect import induced by expenditure $i$ and S denotes the number of sectors. The denominator represents the sum of total final demand of domestic goods and final demand of imported goods, where $\boldsymbol{\mathcal{F}}_i^{d}$ is the $\mathrm{S} \times 1$ vector of final demand of domestic goods and services with regard to expenditure category $i$ and $\boldsymbol{\mathcal{F}}_i^{m}$ is the $\mathrm{S} \times 1$ vector of final demand of imported goods and services with regard to expenditure category $i$.

## 2.5 External Sectors

Q-JEM is a small open economy model. Two foreign blocks, the US block and the rest of the world (ROW) block, are exogenous to Japanese economy.[19] These blocks comprise of a small set of variables: GDP, output gap, interest rates and inflation. Due to limited space, we only cover the US block in this paper.[20]

***US block***

The US block consists of a small set of variables: GDP, output gap, interest rates and inflation. US output gap $GAP_t^{US}$ is determined by the reduced-form equation, which includes its lagged value, ROW output gap $GAP_t^{ROW}$, and interest rate gap,

[18] To obtain $\alpha_i$, we compute the average of the values measured at 1995, 2000, 2005 and 2011. See Bussière et al. (2013) for further methodological detail.
[19] In the replication files, ROW variables are called non-US variables.
[20] For the structure of the ROW block, see *qjemdoc.html* in the replication files.

$$
\begin{aligned}
GAP_t^{US} = \alpha_0 + \alpha_1 GAP_{t-1}^{US} + \alpha_2(GAP_t^{ROW} - GAP_{t-1}^{ROW}) \\
+ \alpha_3 \sum_{i=0}^{3} (nr_{t-i}^{US,l} - \bar{\pi}_{t-i,40}^{US,e} - r_{t-i}^{US,Q}) + \alpha_4(GAP_{t-1}^{US} - GAP_{t-2}^{US}) \\
+ \alpha_5(GAP_{t-2}^{US} - GAP_{t-3}^{US}),
\end{aligned}
\qquad (29)
$$

where $GAP_t^{US}$ denotes US output gap, $nr_t^{US,l}$ denotes US long-term interest rate, $\bar{\pi}_{t,40}^{US,e}$ denotes 10 year average inflation expectation estimated from an inflation survey report (Consensus Economics), and $r_t^{US,Q}$ denotes US real natural interest rate estimated by Laubach and Williams (2003).[21]

US core inflation (i.e., inflation excluding food and energy) is determined by the Phillips Curve, where parameters are restricted and the survey-based inflation expectation is employed as an indicator of inflation expectation. The Phillips Curve reads as:

$$
\pi_t^{US} = \gamma_0 \pi_{t-1}^{US} + (1-\gamma_0)\pi_{t,1}^{US,e} + \gamma_1 GAP_t^{US}, \qquad (30)
$$

where $\pi_{t,1}^{US,e}$ indicates one-quarter ahead US inflation expectation at $t$.

US monetary policy follows simple Taylor rule with interest rate smoothing,

$$
nr_t^{US} = (1-\alpha_0) nr_{t-1}^{US} + \alpha_0[nr_t^{US,Q} + 1.5(\pi_t^{US} - \pi_t^{US,Q}) + 0.5 GAP_t^{US}]. \qquad (31)
$$

To construct the US nominal natural interest rate $nr_t^{US,Q}$, we add trend inflation $\pi_t^{US,Q}$ observed by the inflation survey to real natural interest rate $r_t^{US,Q}$.

***Aggregate foreign variables***

US variables and ROW variables are aggregated by their share on Japan's exports. For example, foreign GDP $Y_t^F$ is obtained by the following equation:

$$
\mathrm{dlog}(Y_t^F) = w_t^{US}\mathrm{dlog}(Y_t^{US}) + (1-w_t^{US})\mathrm{dlog}(Y_t^{ROW}), \qquad (32)
$$

[21] US GDP level is determined by US output gap and exogenous US potential output level estimated by Congressional Budget Office (CBO).

where $w_t^{US}$ is the US share on Japanese export at time $t$.[22]

# 3. Simulations

## *3.1 Simulation by Q-JEM*

We now illustrate some key properties of Q-JEM by running some simulations. Specifically, we look at impulse response functions of core variables to exogenous shocks. All simulation codes are available in the replication files.

### *3.1.1 A permanent increase in foreign GDP*

We first consider a scenario in which foreign GDP (both US and ROW) increases by one percent permanently (Figure 6). There are several channels through which this shock is transmitted to domestic economy. First, higher foreign demand directly pushes export up through trade channels. There is also depreciation of the Japanese yen as a result of higher foreign interest rates, which further boosts exports and contains imports. Investment is also stimulated as a result of stronger exports. In the meantime, the oil price increases sharply due to stronger foreign demand and offsets the impact on domestic economy through adversely affecting households' consumption spending. Overall, GDP exhibits the initial response of 0.6 percent, and diminishes gradually thereafter, although remains buoyant. Consumer price index (all items less fresh foods) gradually increases, due to the combination of the higher output gap, yen depreciation, and higher oil price.

### *3.1.2 A permanent decline of oil price*

In this scenario, we assess the effect of a 10 percent permanent decrease in oil price (Figure 7). The main finding is that the decrease in oil price works as a positive supply shock, driving output and inflation to opposite directions. Lower oil price transmits to import price, improving terms-of-trade. This boosts real disposable income of households, leading to stronger real consumption. Overall, GDP cumulatively increases by nearly 0.2 percent by the third year. On the inflation front, lower import price passes through to inflation (all items less fresh foods), pushing inflation 0.1 percent down at the second year peak.

[22] ROW variables are calculated as a weighted average of individual countries comprising of ROW, where the countries' shares of Japan's export are used as a weight.

### 3.1.3 A permanent increase of foreign GDP and a permanent decline of oil price

In addition to scenarios assuming one type of exogenous shock, we consider the scenario in which multiple types of exogenous shocks occur simultaneously. Figure 8 depicts the responses of variables to the combination of a permanent increase in foreign GDP and permanent decline of oil price. Compared with the simulation in Section 3.1.1, where oil price also increases due to higher demand, the response of GDP is stronger due to favorable terms-of-trade. On the inflation front, as opposed to the simulation in Section 3.1.1, price initially lowers due to the lower import price. After the first year, the price level gradually increases due to the combination of higher output gap and weaker nominal exchange rate.

### 3.1.4 A permanent depreciation of exchange rate

Next, we consider a scenario where there is exogenous 10 percent depreciation of nominal exchange rate (Figure 9). Exchange rate depreciation works as a positive demand shock. A weaker yen favorably affects exports by improving competitiveness of Japanese firms, and concurrently weakens imports through expenditure switching channels. Investment also modestly increases due to stronger exports. Overall, GDP cumulatively increases by 0.3 percent by the second year. A combination of higher output gap and exchange rate depreciation cumulatively boosts inflation (all items less fresh foods) by 0.2 percent by the second year.

## 3.2 Comparison with Vector Autoregression model

In this subsection, we compare the impulse responses obtained from Q-JEM with those obtained from a vector autoregression (VAR) model. This exercise allows us to check if the impulse responses obtained from Q-JEM are supported by data.

We consider a structural VAR model with an exogenous foreign block à la Cushman and Zha (1997). This set up is consistent with a small open economy, the structure that Q-JEM employs. The three-variable foreign block is defined as $\boldsymbol{X}_t \equiv \left(\log(Y_t^F), \log\left(P_t^{F,CPI}\right), \log\left(P_t^{oil}\right)\right)'$, where $\log(Y_t^F)$ denotes log of real foreign GDP level, $\log\left(P_t^{F,CPI}\right)$ denotes log of foreign CPI level and $\log\left(P_t^{oil}\right)$ denotes log of nominal oil price level denominated in US dollar. The four-variable domestic block is defined as $\boldsymbol{Z}_t \equiv \left(\log(Y_t), \log(P_t^{XF}), nr_t^l, \log(\mathcal{E}_t^{\$})\right)'$, where $\log(Y_t)$ denotes log of real Japanese GDP,

$\log(P_t^{XF})$ denotes CPI (all items less fresh food), and $nr_t^l$ denotes nominal long-term interest rate, and $\log(\mathcal{E}_t^{\$})$ denotes log of US dollar-yen bilateral nominal exchange rate.[23]

The foreign block is exogenous to Japanese economy, and the domestic block is also affected by the foreign block. Such processes can be written as,

$$\begin{pmatrix} \boldsymbol{X}_t \\ \boldsymbol{Z}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_X \\ \boldsymbol{\alpha}_Z \end{pmatrix} + \begin{pmatrix} \boldsymbol{\beta}_{11}(L) & \mathbf{0} \\ \boldsymbol{\beta}_{21}(L) & \boldsymbol{\beta}_{22}(L) \end{pmatrix} \begin{pmatrix} \boldsymbol{X}_t \\ \boldsymbol{Z}_t \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_t^X \\ \boldsymbol{\varepsilon}_t^Z \end{pmatrix}, \tag{33}$$

where $\boldsymbol{\beta}_{11}(L)$, $\boldsymbol{\beta}_{21}(L)$ and $\boldsymbol{\beta}_{22}(L)$ are $2 \times 2$, $4 \times 2$, and $4 \times 4$ matrices of autoregressive coefficients, respectively. $L$ denotes lag operator, and $\boldsymbol{\varepsilon}_t^X$ and $\boldsymbol{\varepsilon}_t^Z$ are innovation vectors of $2 \times 1$ and $4 \times 1$.

We use quarterly data from 1992 Q1 to 2018 Q4. The number of lags is chosen based on Bayesian Information Criterion; the selected number of lag is one. Shocks are identified by a standard Cholesky decomposition where the order of variables is same as the order mentioned earlier.

Based on this VAR model, we generate shocks comparable with Q-JEM simulations (i.e., one percent increase in foreign GDP, ten percent decrease of oil price, and ten percent depreciation of US dollar-yen bilateral nominal exchange rate) and depict impulse responses of variables of interest: Japanese real GDP and CPI. Figures 10, 11, and 12 compare the impulse responses generated by Q-JEM with those obtained with the VAR model. We find that impulse responses generated by Q-JEM are generally consistent with the VAR model.

# 4. Conclusion

In this paper, we describe the updated version of the Quarterly Japanese Economic Model (Q-JEM), the large-scale semi-structural model of Japanese economy. The model is designed to incorporate greater disaggregation of expenditure components and detailed financial market information, while pursuing theoretical coherence. These properties make Q-JEM a useful tool for producing projections and simulations.

To improve public access to the model, we share the replication files of the simulations

[23] The set-up of the domestic block is the standard three-variable set (i.e., real output, price and nominal interest rate) plus nominal exchange rate. For nominal interest rate, we choose long-term interest rate rather than short-term interest rate because of zero-lower-bound.

conducted in the paper. Needless to say, these simulation results are subject to change in the future, as the current version of the model should be updated and reviewed along with the progress in both theory and empirical techniques. Building and maintaining the model to be used in practice at the central bank are a never-ending task. Our Q-JEM is no exception and the one presented in this paper should be taken as a work-in-progress.

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## Appendix 1: Solver for Q-JEM

Most of the codes for Q-JEM are written in EViews. In finding solutions to Q-JEM equations, we deploy some in-house programs in addition to EViews' built-in function, e.g., "***solve***" and "***control***", to cope with their shortcomings. These shortcomings are summarized in the following points:

- "***solve***" finds the solution to a simultaneous equation model. However, "***solve***" is not necessarily a useful tool where multiple pairs of endogenous and exogenous variables need to be switched. This is because we cannot easily switch pairs of endogenous and exogenous variable using "***solve***".
- "***control***" is a function of EViews for obtaining the solution for a model where a single pair of endogenous and exogenous variables are switched. That is, "***control***" cannot deal with simulations where multiple pairs of endogenous and exogenous variables need to be switched.

To deal with this problem, we develop an in-house solver for Q-JEM. With this solver, we can easily compute a simulation with multiple pairs of endogenous and exogenous variables. A key component of this solver is written in Python, and implements Dulmage-Mendelsohn (DM) decomposition algorithm.[24] The Python programs generate a block equation system to facilitate the efficient computation. By solving the blocked model in EViews, the model solution can be obtained stably and quickly. With these in-house programs, users can easily conduct simulation exercises where multiple pairs of endogenous and exogenous variables are switched.[25]

### *DM decomposition algorithm*

The equation system of the model can be expressed as a matrix of which rows and columns correspond to equations and endogenous variables. Let $A$ be the $N \times N$ square matrix expressing the model which includes $N$ equations and $N$ endogenous variables $x_1, \cdots, x_N$:

[24] The algorithm is originally developed by Dulmage and Mendelsohn (1958).
[25] Some parts of the codes are based on codes written by Takanori Maehara, which is available at http://www.prefield.com/index.html. Sugihara and Murota (2009) are also helpful to the programming.

$A = (a_{ij})$ where

$$a_{ij} = \begin{cases} 1 \ \cdots \ \text{if equation } i \text{ includes endogenous variable } x_j \\ \\ 0 \ \cdots \ \text{else.} \end{cases} \quad \text{(A1)}$$

DM decomposition algorithm converts the given matrix to the block triangular matrix such that each block matrix is irreducible, only by permuting rows and columns (Dulmage and Mendelsohn, 1962). Let $B$ be the blocked triangularization of $A$ obtained by DM decomposition:

$$B = P_{row} A P_{col} = \begin{pmatrix} B_1 & & C \\ & \ddots & \\ 0 & & B_N \end{pmatrix} \quad \text{(A2)}$$

where $P_{row}$ and $P_{col}$ are permutation matrices and $N$ is the number of blocks. $B_1, \cdots, B_N$ are block square matrices, which show irreducible partial models. $C$ shows the partially ordered structure of block matrices. The whole solution is obtained by solving each partial model sequentially.

***Small model example: DM decomposition***

Consider the model with three equations, three endogenous variables, and three exogenous variables:

$$\begin{cases} f(x, y;\ \tilde{\varepsilon}_x) = 0 \\ g(x, z;\ \tilde{\varepsilon}_z) = 0 \\ h(x, y;\ \tilde{\varepsilon}_y) = 0 \end{cases} \quad \text{(A3)}$$

where $x$, $y$, and $z$ are endogenous variables and $\tilde{\varepsilon}_x$, $\tilde{\varepsilon}_y$, and $\tilde{\varepsilon}_z$ are exogenous variables.[26] The matrix expressing the model (A3) can be written as:

$$A^* = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}. \quad \text{(A4)}$$

Rows of $A^*$ correspond to the order of equations ($f$, $g$, and $h$). Columns correspond to the

[26] Throughout the appendix, a variable with tilde denotes an exogenous variable.

order of endogenous variables ($x$, $y$, and $z$). The blocked triangularization of $A^*$ is

$$B^* = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \tag{A5}$$

Rows of $\boldsymbol{B}^*$ correspond to the new order of equations ($\boldsymbol{g}$, $\boldsymbol{f}$, and $\boldsymbol{h}$) and columns correspond to endogenous variables ($\boldsymbol{z}$, $\boldsymbol{y}$, and $\boldsymbol{x}$). $\boldsymbol{B}^*$ indicates how the model (A3) are efficiently solved. First, the partial model with two equations $\boldsymbol{f}$, $\boldsymbol{h}$ should be solved for two endogenous variables $\boldsymbol{x}$, $\boldsymbol{y}$, given exogenous variables $\tilde{\boldsymbol{\varepsilon}}_{\boldsymbol{x}}$, $\tilde{\boldsymbol{\varepsilon}}_{\boldsymbol{y}}$. Second, the other partial model with one equation $\boldsymbol{g}$ are solved for an endogenous variable $\boldsymbol{z}$, given exogenous variable $\tilde{\boldsymbol{\varepsilon}}_{\boldsymbol{z}}$ and endogenous variable $\boldsymbol{x}$ which has already been determined in the first step.

***Small model example:*** **switching endogenous variables and exogenous variables.**

In this example, we consider a simulation with model (A3), where multiple pairs of endogenous and exogenous variables are switched. Let $\tilde{x}$ and $\tilde{z}$ be exogenous paths that $x$ and $z$ follow (i.e., $x = \tilde{x}$ and $z = \tilde{z}$). Correspondingly, change $\tilde{\varepsilon}_x$ and $\tilde{\varepsilon}_z$ in (A3) to endogenous variables (i.e., $\tilde{\varepsilon}_x = \varepsilon_x$ and $\tilde{\varepsilon}_z = \varepsilon_z$). The new matrix expressing the simulation model is

$$A^{**} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{A6}$$

Rows of $A^{**}$ correspond to the order of equations ($f$, $g$, and $h$) and columns of $A^{**}$ correspond to the order of endogenous variables ($\varepsilon_x$, $y$, and $\varepsilon_z$). The block triangularization of $A^{**}$, $B^{**}$, is given as

$$B^{**} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{A7}$$

Rows of $B^{**}$ correspond to the new order of equations ($f$, $g$, and $h$) and the columns correspond to endogenous variables ($\varepsilon_x$, $\varepsilon_z$, and $y$). There are three blocks in $B^{**}$ and each block matrix include at most one pair of exogenous and endogenous variables. For example, top left block contains the pair of $\tilde{x}$ and $\varepsilon_x$, and the center block contains the pair of $\tilde{z}$ and $\varepsilon_z$. Since there is no block with multiple pairs of endogenous and exogenous variables after DM decomposition, the model can be solved with EViews built-in functions "***solve***" and "***control***". This is the case with all Q-JEM simulations presented in this paper, and these simulations can be performed with the EViews built-in functions.

***Detail of in-house programs of DM decomposition***

- "***calc_response_to_exogenous_shocks.prg***" is the main program for the simulation exercises.
- "***solve_qjem.prg***" is a subroutine called by the main program "***calc_response_to_exogenous_shocks.prg***". This routine is composed of (1) generating the blocked model by DM decomposition algorithm and (2) solving the model.
    - (1) "***generate_modeltext_blocked.py***" is called by the program "***solve_qjem.prg***". This routine generates the blocked model by DM decomposition algorithm.
        - ✓ "***DulmageMendelsohnDecomposition.py***" and other subroutines included in the same folder implement DM decomposition algorithm.

Finally, the blocked model is recorded as "***qjem_plain_blocked.txt***", which is used in EViews for obtaining the model solution.

- (2) "***solve_blocks.prg***" is called "***solve_qjem.prg***" and finds the solution of the model using "***qjem_plain_blocked.txt***", which is generated by (1) "***generate_modeltext_blocked.py***".
    - ✓ "***mcontrol.prg***" is called "***solve_blocks.prg***" and finds the model solution.

## Appendix 2: Estimation of Natural Interest Rate

Natural interest rate is not directly observable. Thus, we estimate the natural interest rate with a simple state space model à la Clark and Kozicki (2005). The observation equation is given by IS curve,

$$GAP_t = \alpha_0 GAP_{t-1} + \alpha_1 (r_t^l - r_t^*) + w_t, \tag{A8}$$

where $GAP_t$ denotes output gap, $r_t^l$ denotes real long-term interest rate, $r_t^*$ denotes natural interest rate, and $w_t$ denotes measurement error. In the observation equation, we treat $GAP_t$ and $r_t^l$ as observable variables. Natural interest rate, $r_t^*$, which we wish to estimate, is treated as unobservable.

We suppose that the unobserved natural interest rate follows a simple random-walk process. Thus, the transition equation is given by,

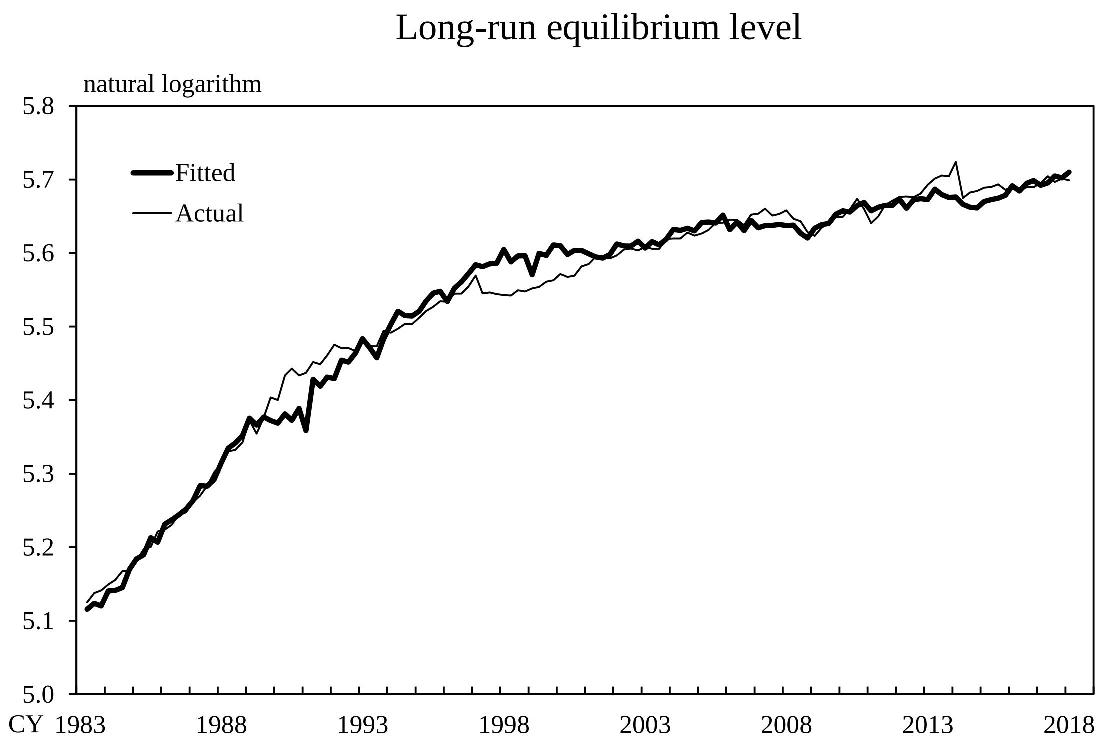
$$r_t^* = r_{t-1}^* + v_t, \tag{A9}$$

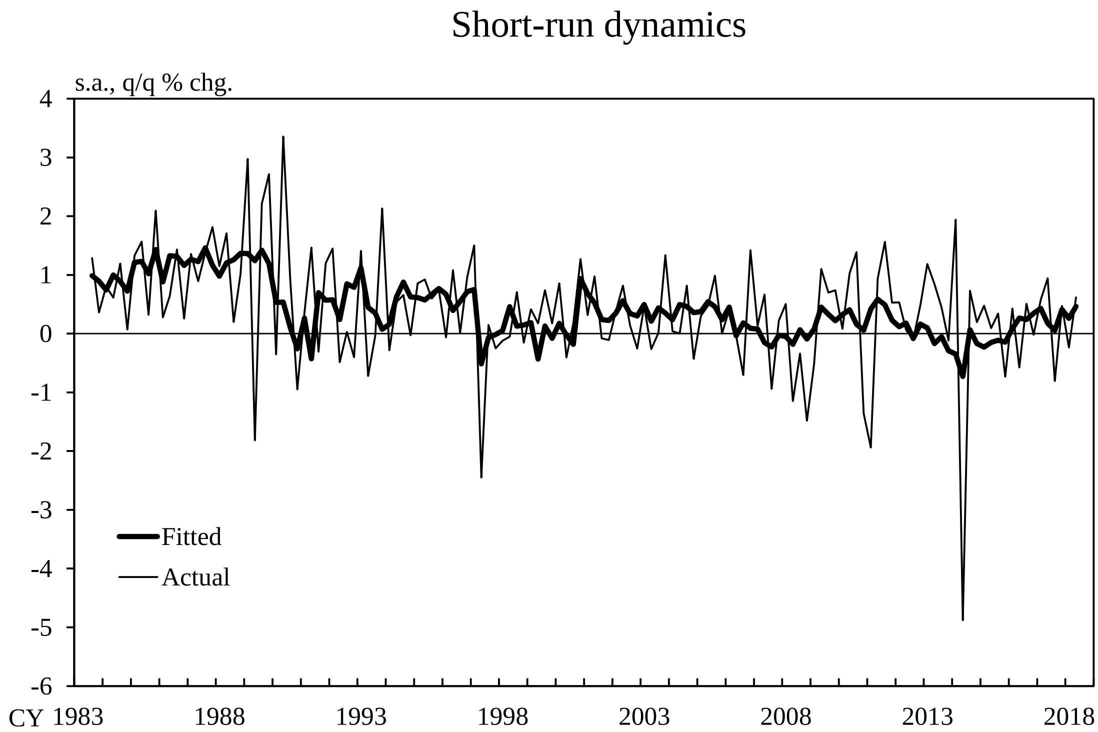
where $v_t$ denotes process noise. Combining equation (A8) and (A9), unobserved $r_t^*$ can be obtained. The parameters are estimated by the maximum likelihood approach.

In this model, we use long-term interest rate rather than short-term interest. In this sense, the estimated natural interest rate $r_t^*$ pertains to long-term interest rate and can be interpreted as an estimate of long-term natural interest rate. This is motivated by the fact that, in many macroeconomic models, as well as in Q-JEM, the long-term interest rate is more important for spending decisions than the short-term interest rate.[27]

[27] With a similar motivation, Roberts (2018) estimates the equilibrium interest rate with US 10-year treasury yield.

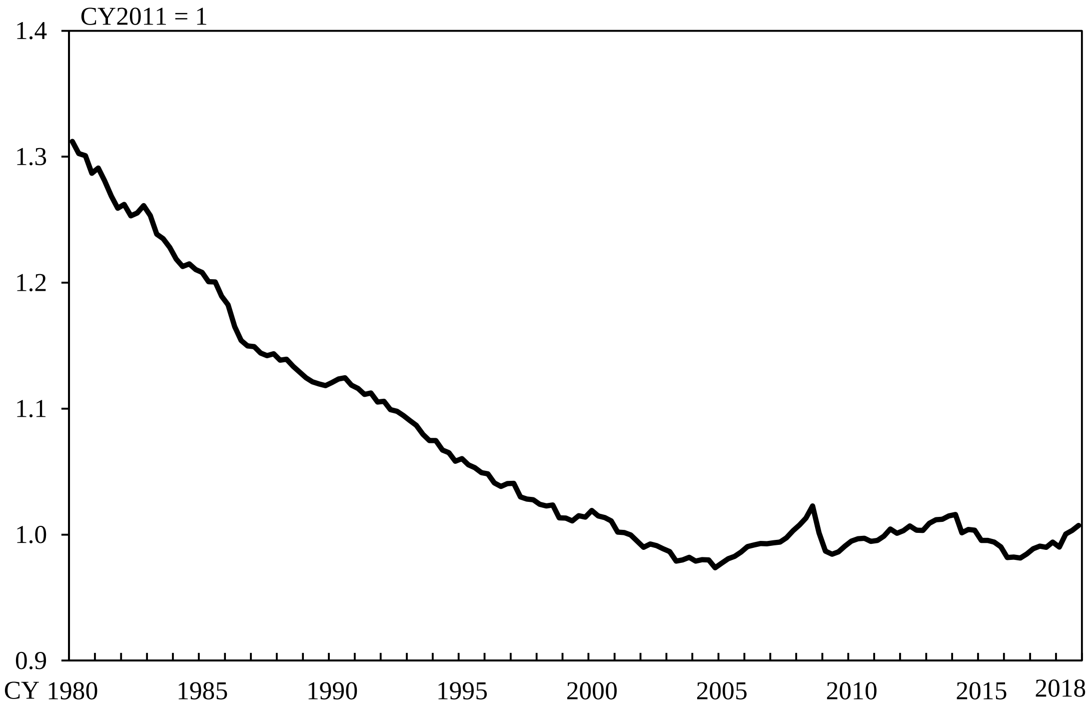
**Figure 1: Long-run equilibrium and short-run dynamics of real private consumption**




*Note*: Real private consumption is at the constant price of 2011 (trillion yen).

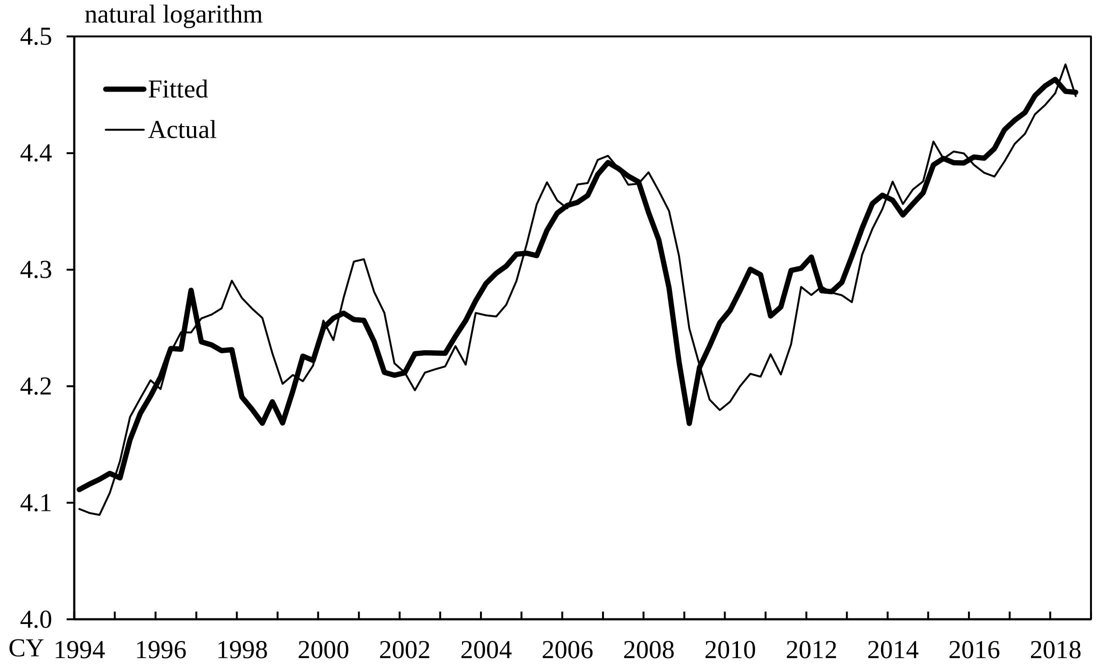
**Figure 2: Relative price of investment goods**



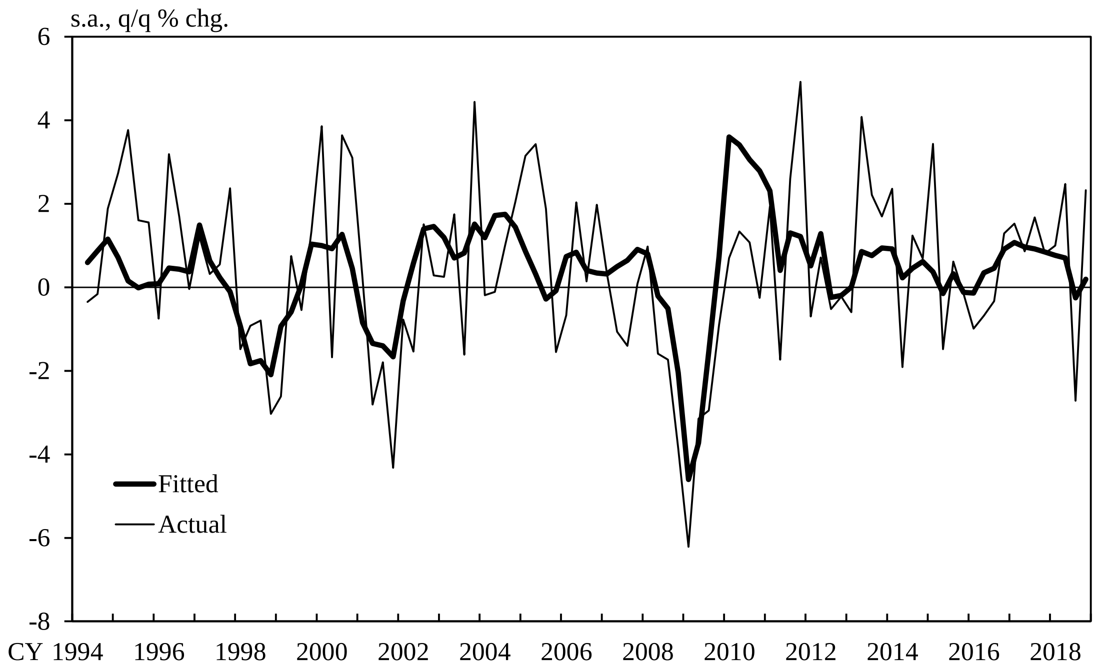

*Note*: The figure shows the ratio of private non-residential investment deflator to GDP deflator.

**Figure 3: Long-run equilibrium and short-run dynamics of real private non-residential investment**

Long-run equilibrium level

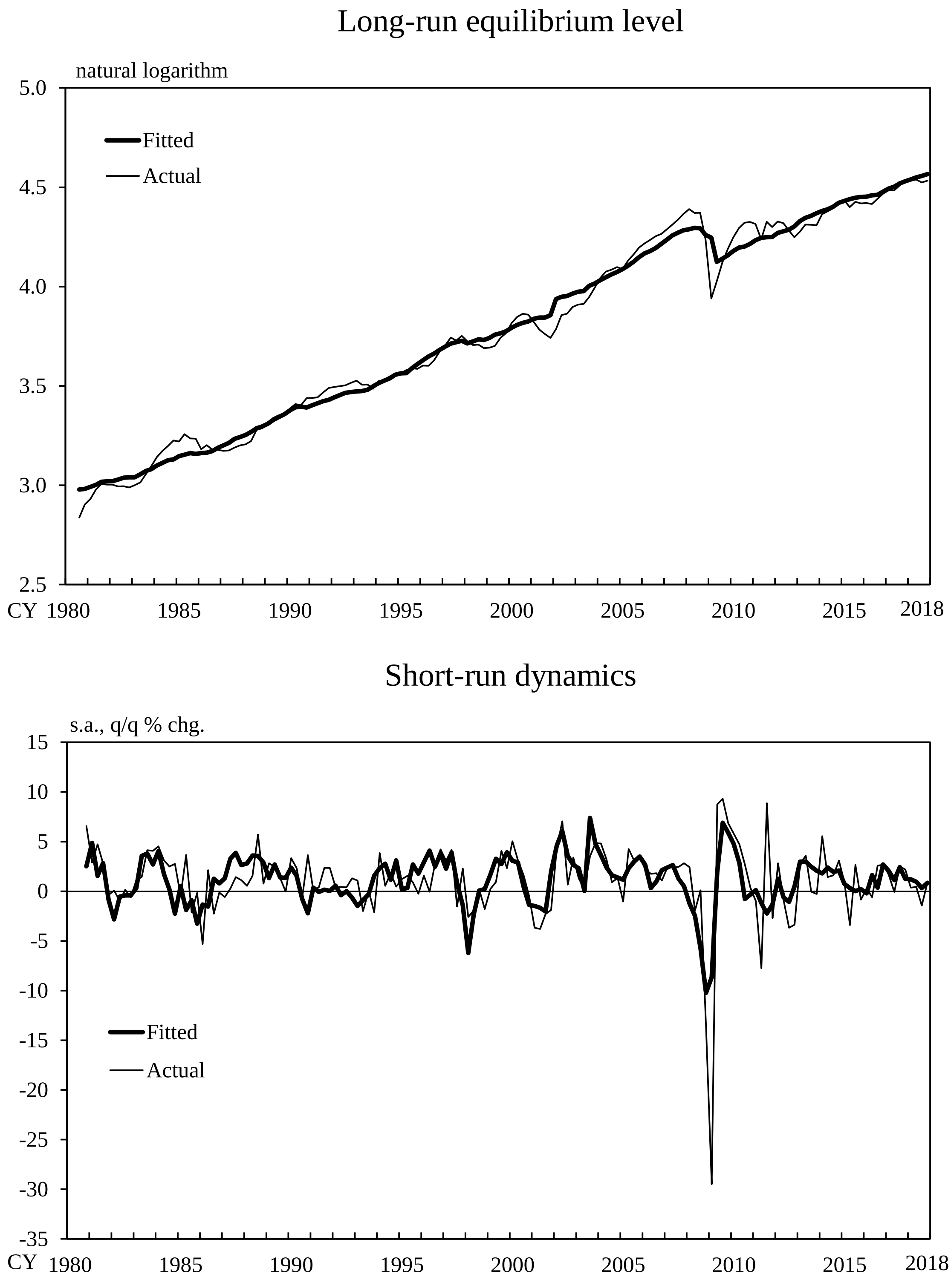



Short-run dynamics



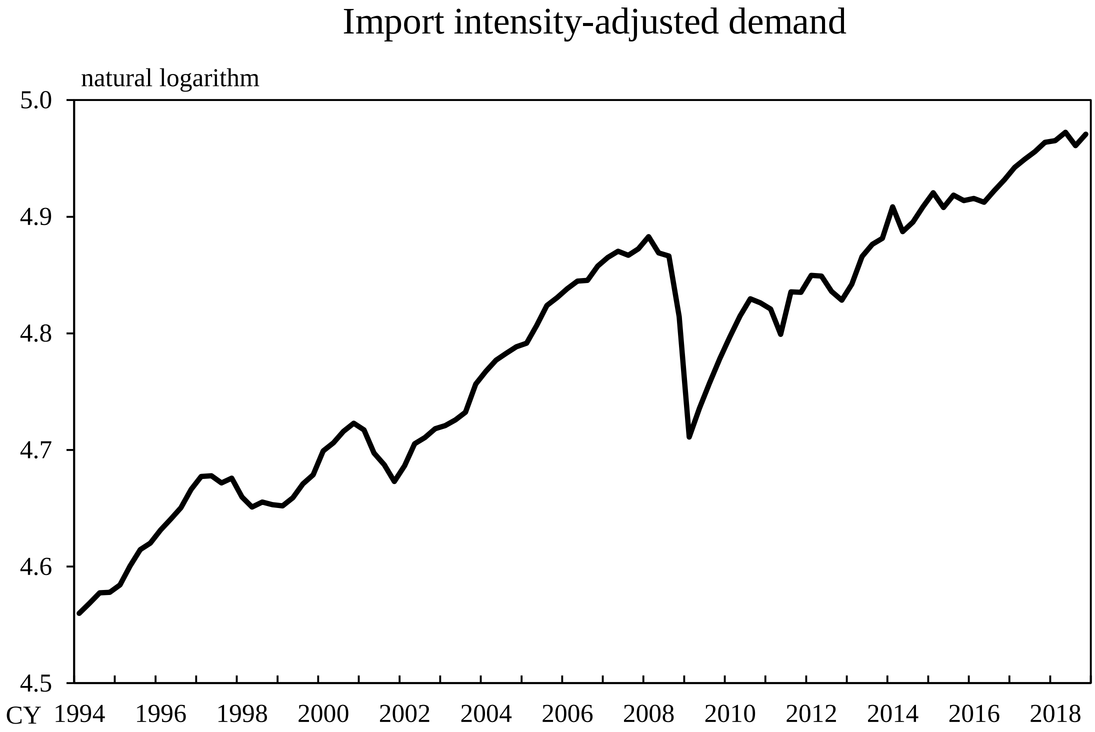

*Note*: Real private non-residential investment is at the constant price of 2011 (trillion yen).

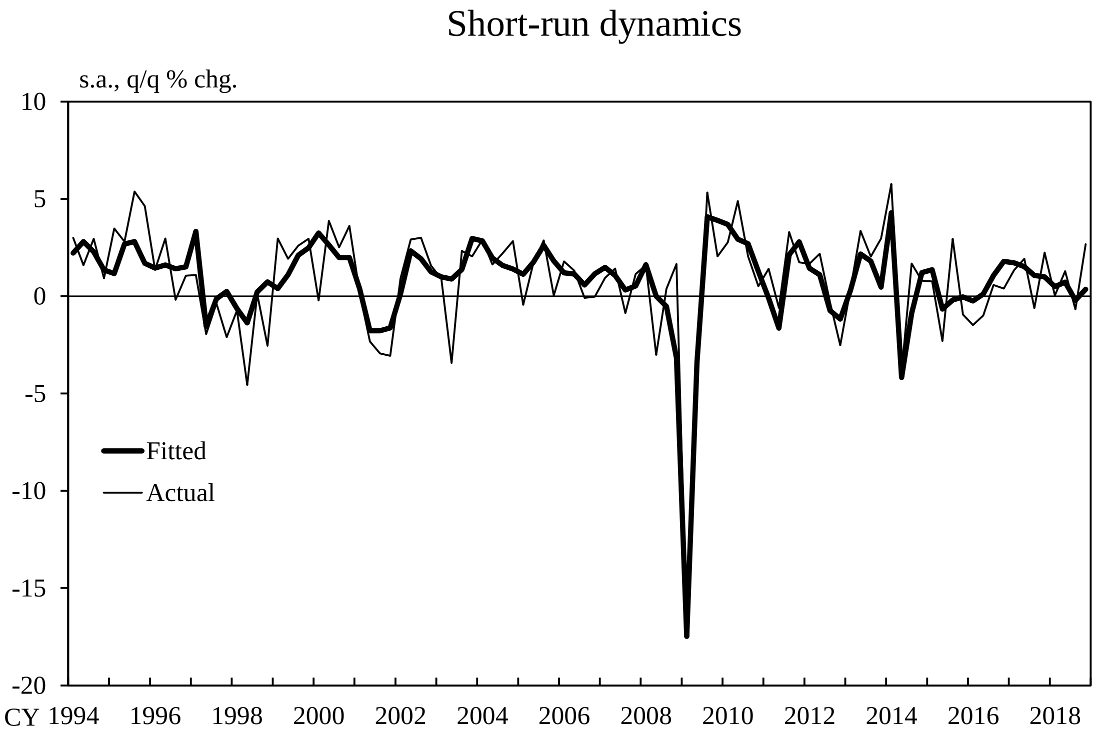
**Figure 4: Long-run equilibrium and short-run dynamics of real export**




*Note*: Real export is at the constant price of 2011 (trillion yen).

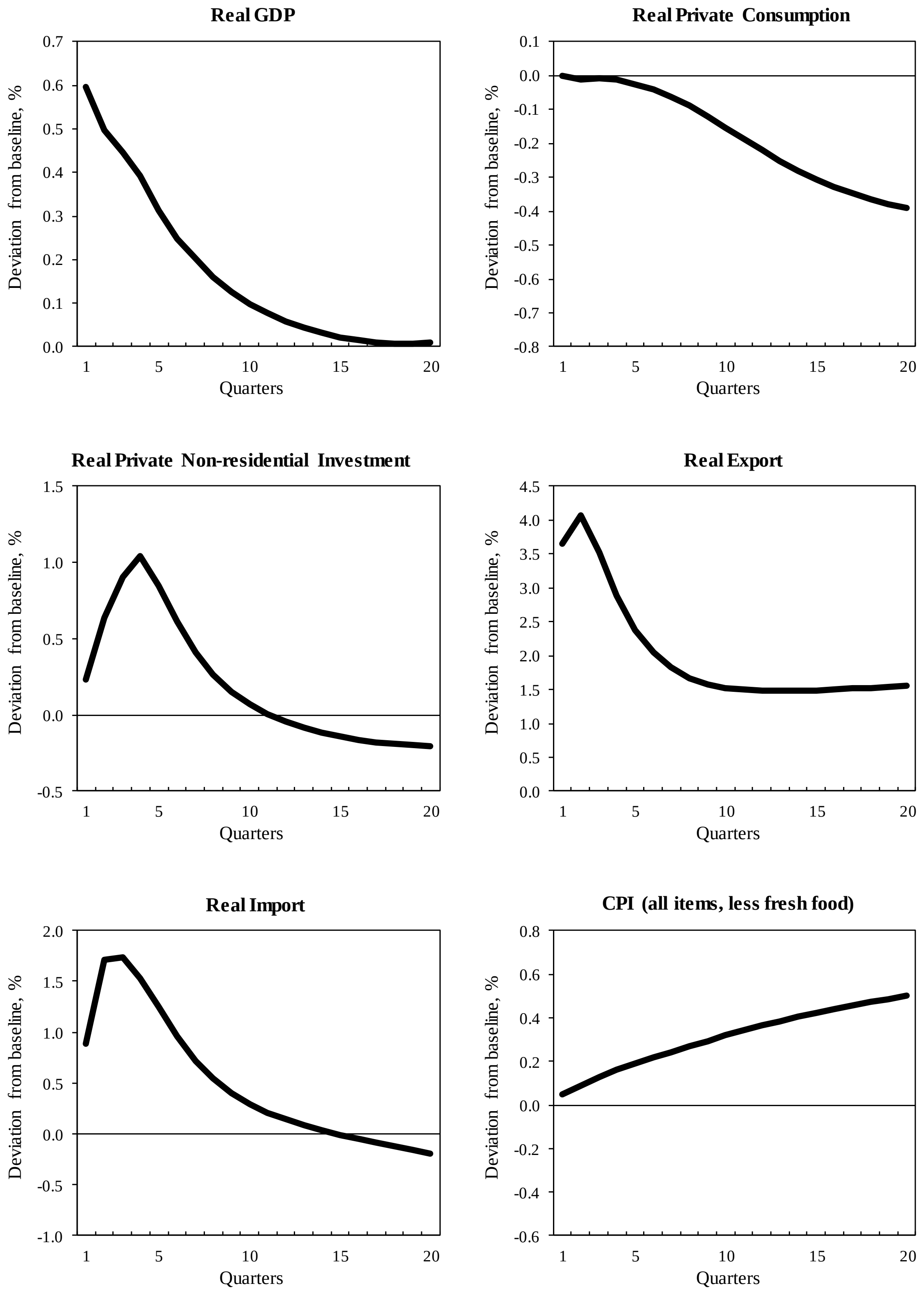
**Figure 5: Import intensity-adjusted demand and short-run dynamics of real import**



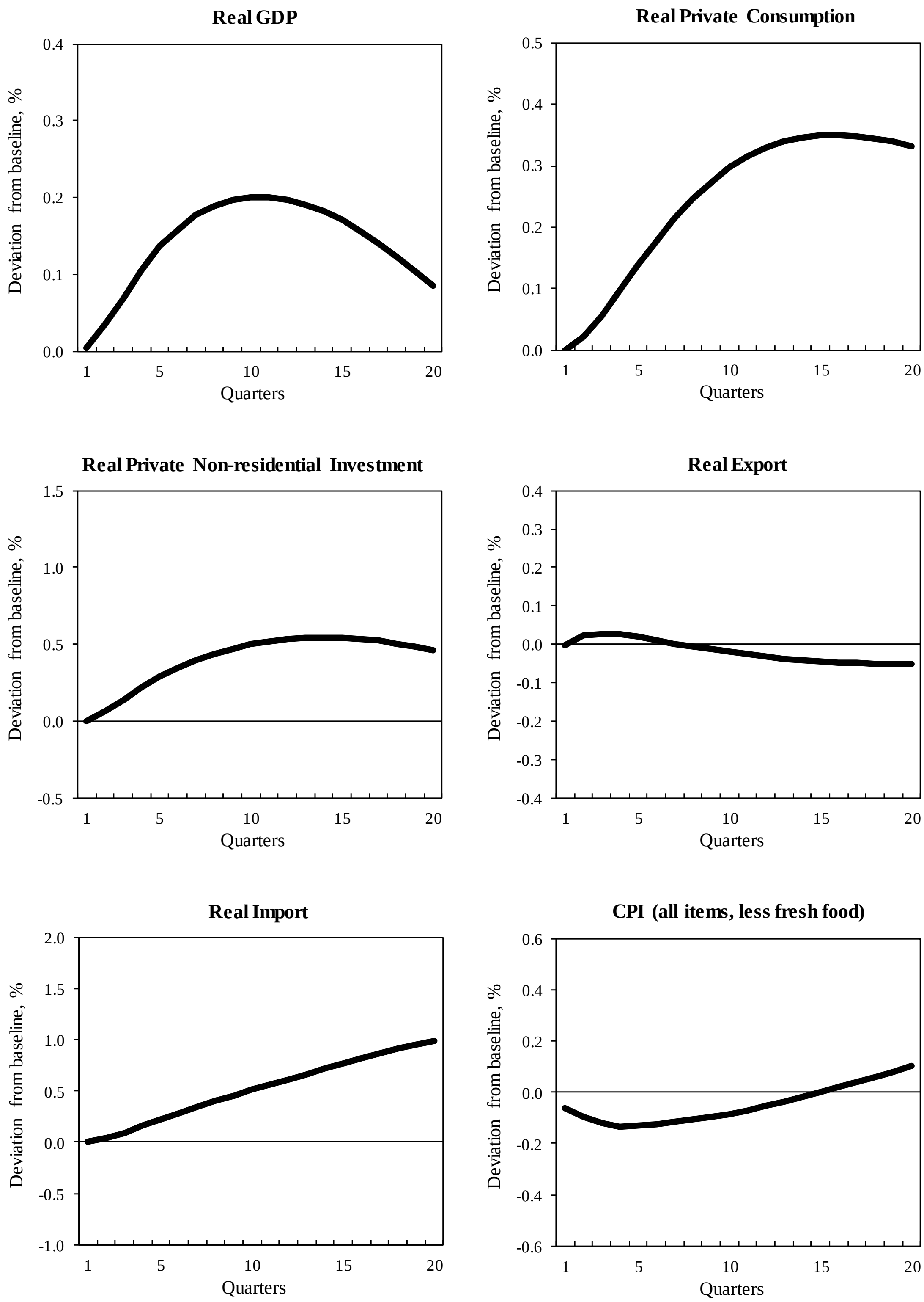




*Note*: Real import and Import intensity adjusted demand is at the constant price of 2011 (trillion yen).

# Figure 6: Responses to one percent permanent increase in foreign GDP



## Figure 7: Responses to 10 percent permanent decrease in oil price



## Figure 8: Responses to one percent permanent increase in foreign GDP and 10 percent permanent decrease in oil price

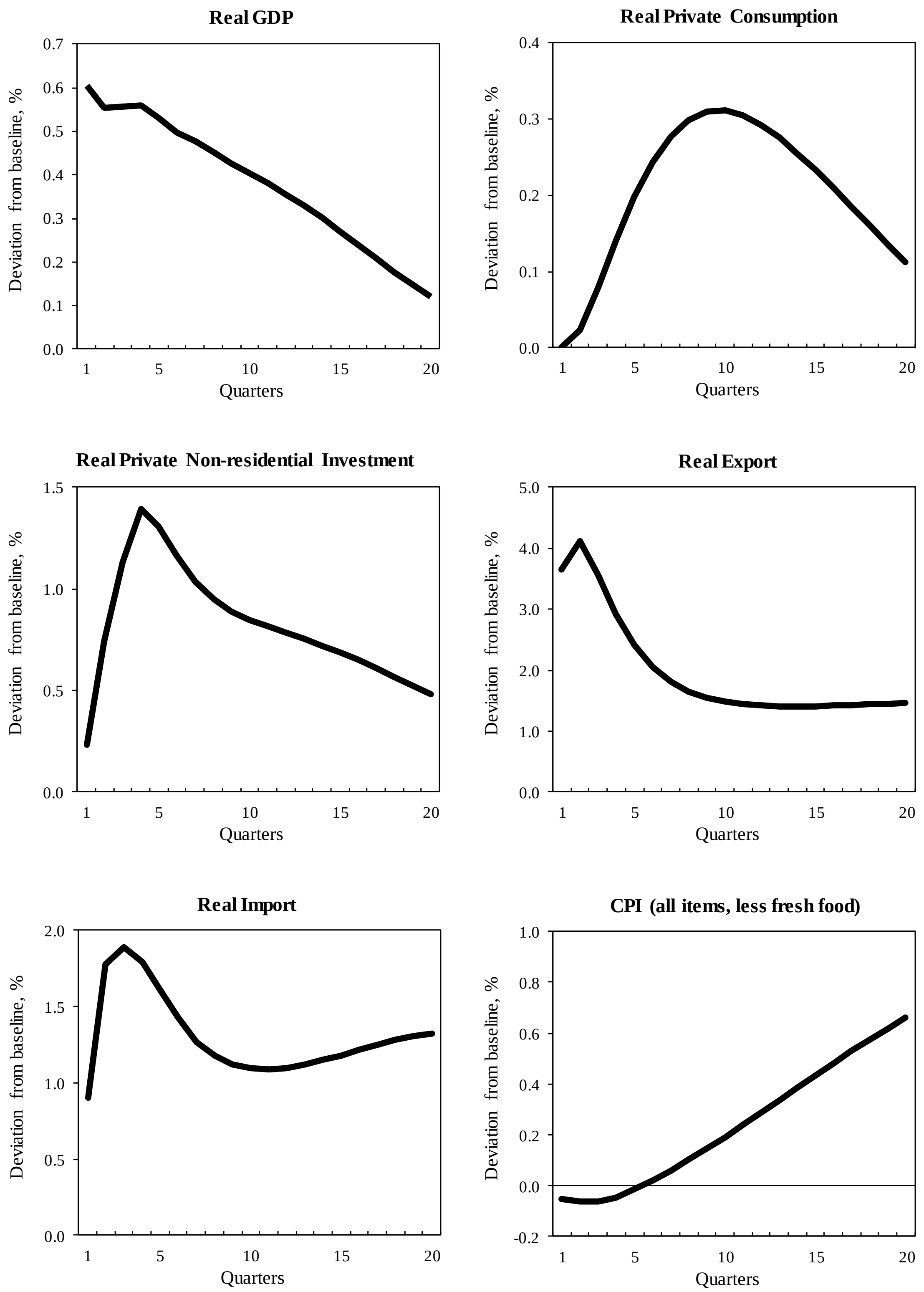
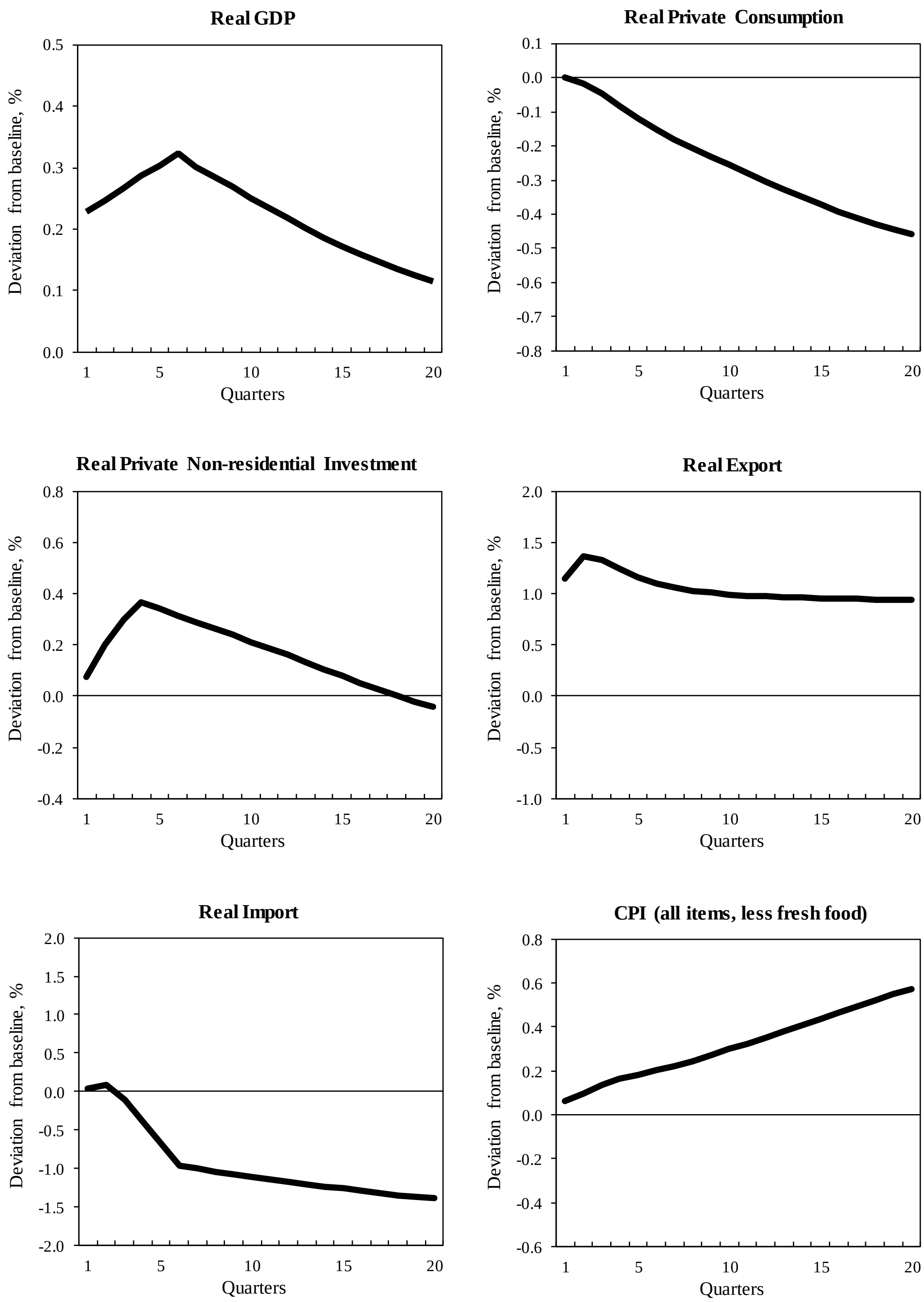
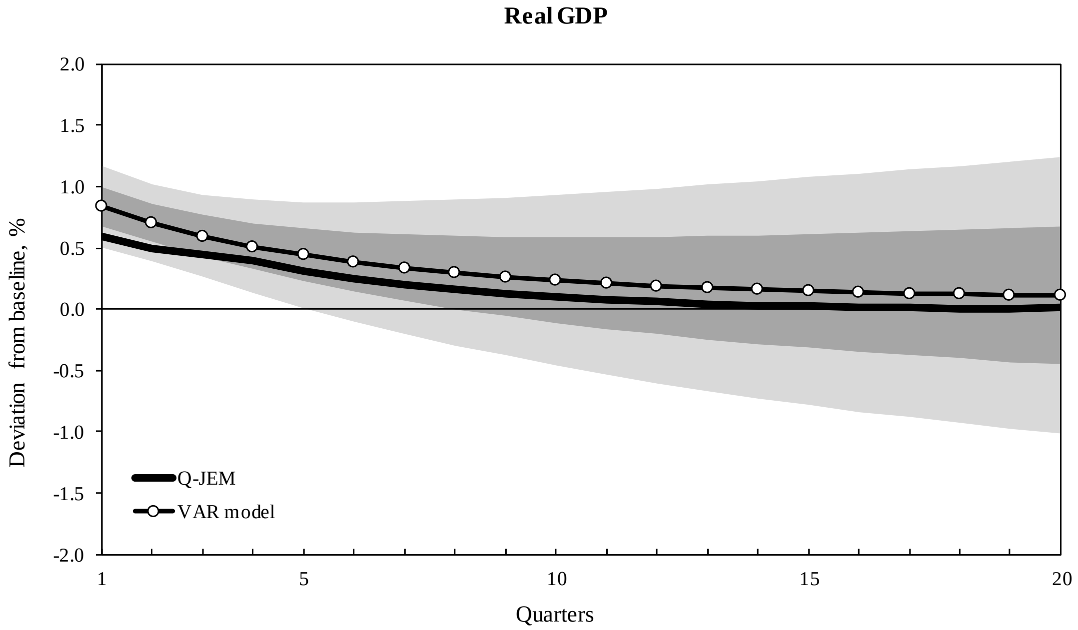
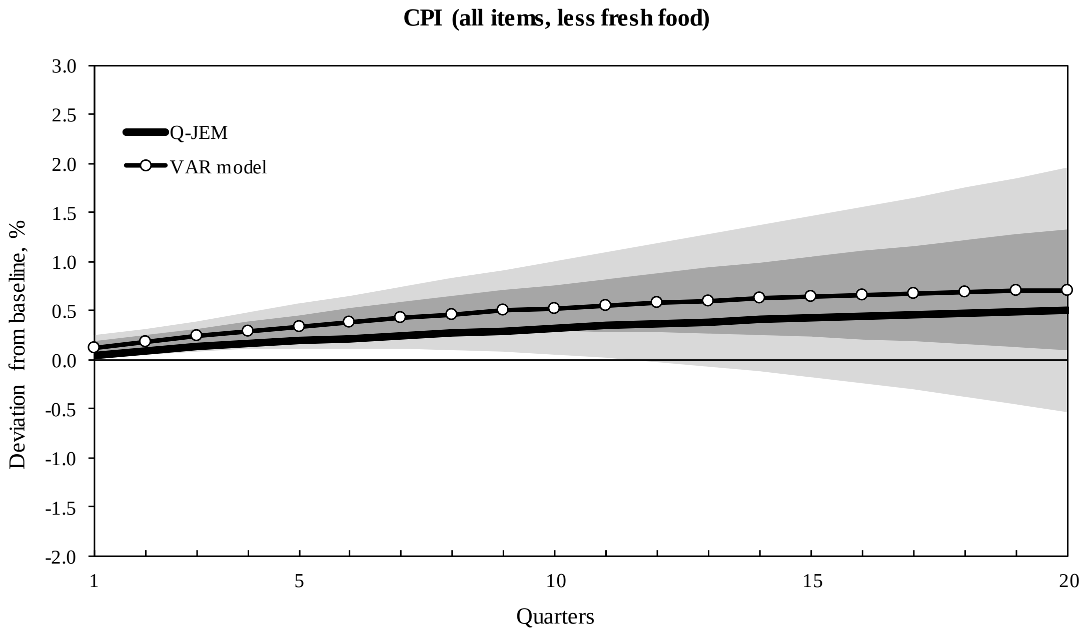


**Figure 9: Responses to 10 percent permanent depreciation in US Dollar-Yen bilateral nominal exchange rate**



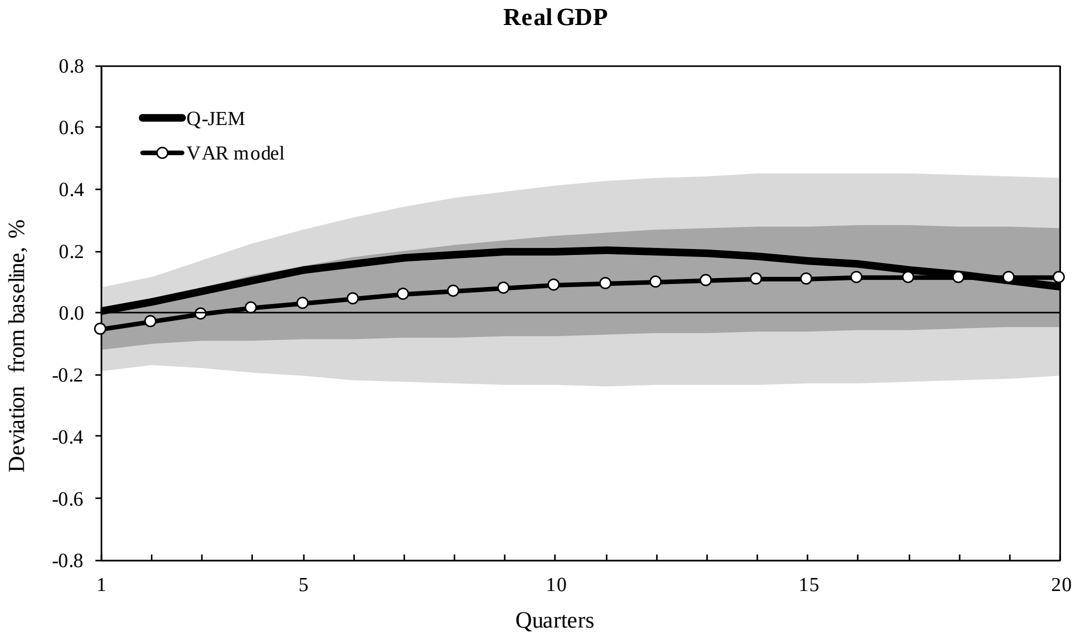
## Figure 10: Responses to a foreign GDP shock: comparison with VAR

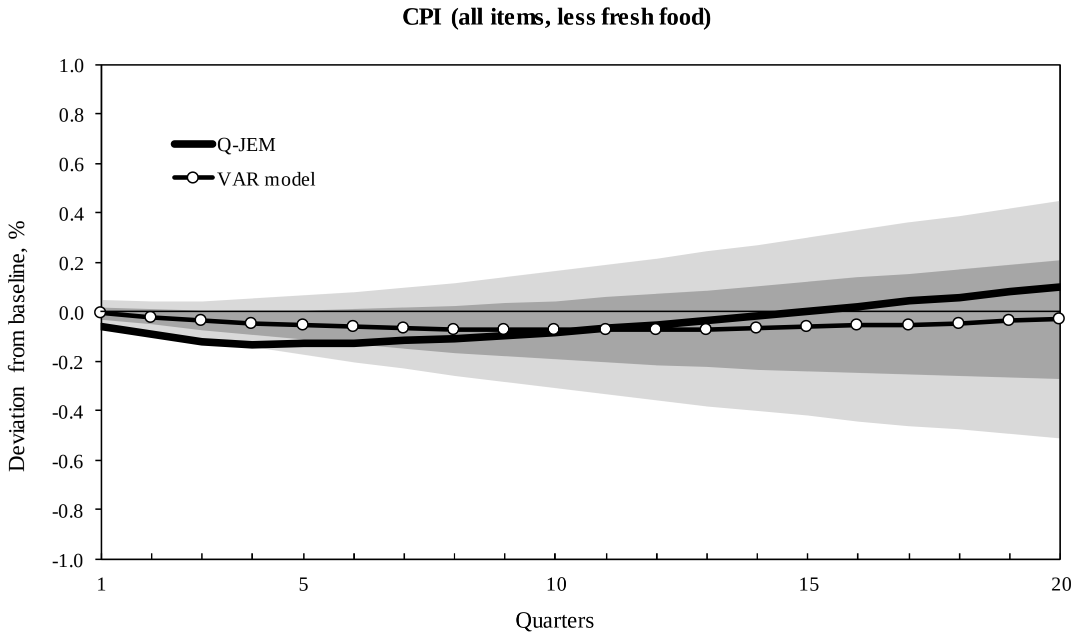



*Note*: Impulse responses of VAR are responses to the shock of a 1 percent increase in foreign GDP. The darker shaded area indicates 1 standard deviation error bands of VAR impulse responses, and the lighter shaded area indicates 2 standard deviation error bands of VAR impulse responses. Impulse responses of Q-JEM are same as Figure 6.

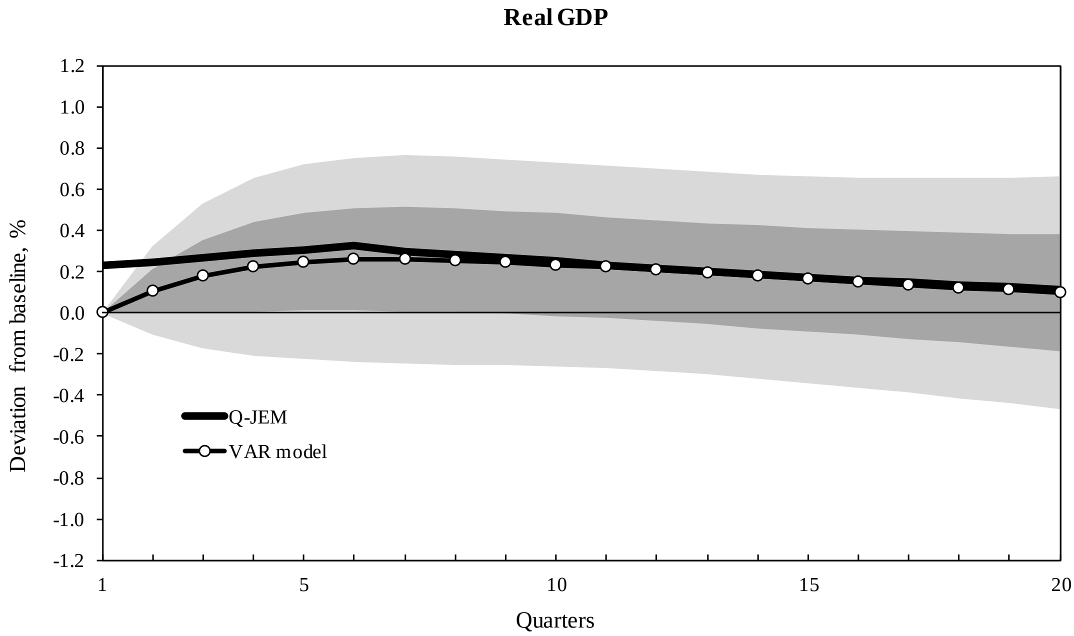
## Figure 11: Responses to an oil price shock: comparison with VAR

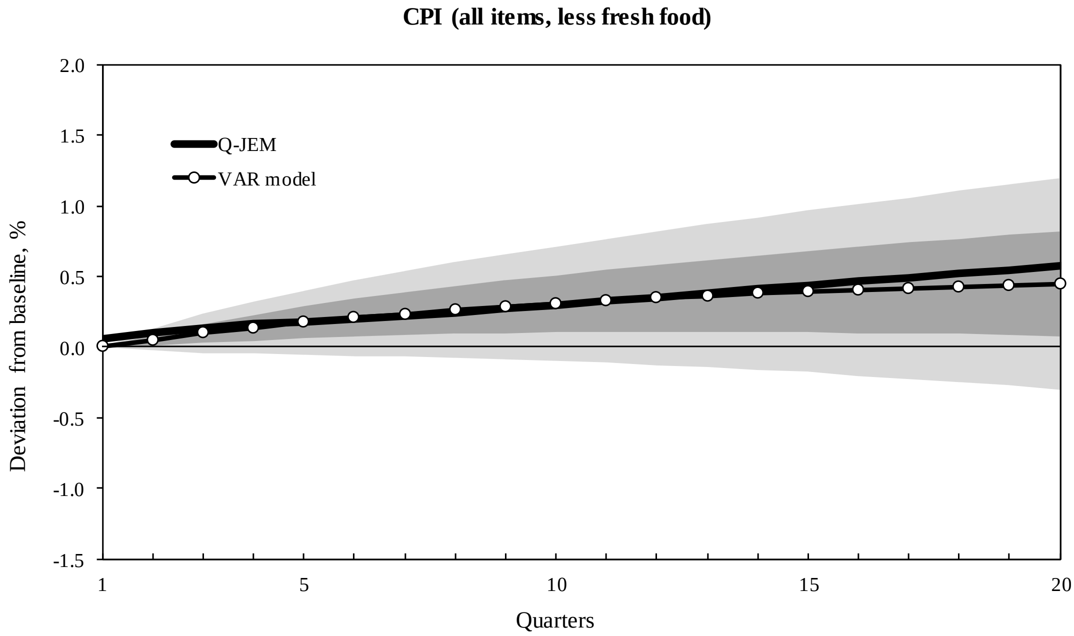



*Note*: Impulse responses of VAR are responses to the shock of a 10 percent decrease in oil price. The darker shaded area indicates 1 standard deviation error bands of VAR impulse responses, and the lighter shaded area indicates 2 standard deviation error bands of VAR impulse responses. Impulse responses of Q-JEM are same as Figure 7.

## Figure 12: Responses to an exchange rate shock: comparison with VAR




*Note*: Impulse responses of VAR are responses to the shock of a 10 percent depreciation of US dollar-yen bilateral nominal exchange rate. The darker shaded area indicates 1 standard deviation error bands of VAR impulse responses, and the lighter shaded area indicates 2 standard deviation error bands of VAR impulse responses. Impulse responses of Q-JEM are same as Figure 9.