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Inflation Dynamics in the Age of Robots: Evidence and Some Theory

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Inflation Dynamics in the Age of Robots: Evidence and Some Theory *

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Abstract

Over the past decade, one of the central questions in macroeconomics has been the missing link observed between inflation and fluctuations in economic activity. We approach this issue with a particular focus on advances in robots, or what are essentially autonomous machines. The contributions of the paper are twofold. First, using a country level balanced panel dataset, we provide significant evidence to show that advances in robots are one factor behind the missing link. Second, we ask a standard New-Keynesian model to rationalize this fact. The distinguishing feature is the introduction of capital which is substituted for human labor, and can therefore be interpreted as the use of robots. Due to this feature and developments in robot, firms can adjust their production by using robots, whose efficiency is getting higher, instead of employing human labor. Hence, the responsiveness of marginal costs to changes in economic activity becomes weakened, and thus, our model supports the empirical fact that advances in robots are one factor behind the missing link.

JEL classification: E12, E22, E31

Keywords: Robot; Labor-substitute capital; Phillips curve; Missing inflation.

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1 Introduction

What is the link between inflation and fluctuations in economic activity? It is a question which is one of the central topics of macroeconomics. A long line of research seeks to find a statistically significant positive link between the two using structural models implied by economic theory, such as the Phillips curve, which plays a central role in monetary policy making.

However, over the past decade, a new challenge has emerged in research on this link. That is, many studies have found that the observed positive link between inflation and economic activity measured by output gap is missing, which suggests that inflation has responded more weakly to economic slack (Bobeica and Jarocinski (2017), Gilchrist and Zakrajsek (2019), Yellen (2017), etc). Inability to explain this missing link is obviously unsettling in the realm of monetary policy. Hence, a rather immediate and central task in monetary policy making is to find the causes of the missing link. We approach this challenge with a particular focus on advances in robots, or what are essentially autonomous machines.

Our focus on robots is motivated by advancements in robot technologies and the proliferation in the use of robots throughout the economy over the same period. We have witnessed the range of tasks in which robots can replace human labor becoming ever wider, including assembling, handling, packaging, painting, welding and so on. Thus, production processes have been converted to the unmanned operation systems conducted by robots, often called robotization. The industrial use of robots has also been remarkable in many advanced economies. The number of world operational robot stocks in 2017 is about three times more than at the end of the 1990s (Figure 1). In particular, industrial use of robots has advanced more rapidly over the past decade. These developments have led to a growing number of studies examining the sensitivity of the real side of the economy to changes in robotization, with a particular focus on employment, economic growth, and productivity (Acemoglu and Restrepo (2017a,b, 2018a,b), Berg et al. (2018), Brynjolfsson et al. (2017), Graetz and Michaels (2018), Gordon (2018), Korinek and Stiglitz (2017), etc).

Along with these recent developments, it seems natural to investigate the responsiveness of the nominal side of the economy to advances in robotization. Specifically, the following questions will be asked: have advances in robotization altered the response of inflation to fluctuations in economic activity? Is robotization among the causes of the weakening link between inflation and fluctuations in overall economic activity? Surprisingly little work has been devoted to examining the extent to which advances in robotization have altered the response of inflation to fluctuations in economic activity. The aim of this paper is to fill this gap. We study the hypothesis that advances in robotization have weakened the response of inflation to fluctuations in economic activity, and thus, robotization plays an important role in accounting for the missing link between the two.

We begin by presenting a systematic empirical relation between advances in robotization and the missing link, using a country level balanced panel dataset including 18 countries from 1998 to 2017. We estimate the Phillips curve to directly examine how the link between inflation and fluctuations in output gap is altered as robotization advances. What we find here is that the Phillips curve becomes flatter as robotization becomes more advanced in the production process. This empirical analysis supports the hypothesis that robotization is one of the sources of the missing link.

After empirically documenting the fact, we move our focus to the underlying economic mechanisms, and present an illustrative framework to explain the empirical finding. Our model is simple (but sufficient) in order to interpret the empirical finding. We focus on examining the role of one particular feature of robots in explaining the missing link: that robots can replace human labor. As is mentioned in recent studies such as Acemoglu and Restrepo (2018a,b), this feature is a striking contrast to other types of capital which are often considered complementary to human labor. To examine the implications of robotization on the missing link, we make a critical addition to the standard New-Keynesian model. Specifically, we introduce two types of capital. One type of capital is complementary to human labor, and the other capital has high elasticity of substitution to labor, which is interpreted as robots, following Berg et al. (2018).

Intuitively, firms can absorb the propagation of structural shocks to marginal costs, using robots (whose production efficiency is increasing), instead of employing human labor. Firms can adjust their production by using robots, helping to absorb the propagation of structural shocks to marginal costs, weakening the link between marginal costs and fluctuations in economic activity. Given the New-Keynesian Phillips curve positively relates inflation to marginal costs, as the economy becomes more dependent on robots, the link between inflation and economic activity goes missing.

Using our model as an illustrative laboratory, we quantify the mechanism presented above and provide some predictions that our model can replicate the weaker the link between inflation and output gap as robotization becomes more advanced. Hence, our model rationalizes the view that robotization would be a factor behind the missing link between inflation and economic activity.

The remainder of this paper is organized as follows. Section 2 presents our empirical finding. Section 3 describes our model to rationalize this finding. Section 4 illustrates parameter values and steady state analysis. Section 5 demonstrates the intuitive mechanisms through which robotization would be a factor behind the missing link between inflation and economic activity. Section 6 gives some concluding remarks.

2 Stylized Fact: Robotization and Inflation

Our hypothesis is that advances in robotization have weakened the response of inflation to fluctuations in economic activity in the short run. We show estimation results on the single-equation Phillips curve with the additional term which is affected by robotization. This exercise reveals that the robotization has a negative impact on the link between inflation and output gap.

2.1 Data Description

We rely on a country level balanced panel dataset that includes information on output gap and inflation.[1] Data are annual and cover the period from 1998 to 2017. We employ output gap as a measure of overall real economic activity. Output gap is picked up from IMF World Economic Outlook (as of October 2018), whereas inflation is calculated by the year-on-year percent change of GDP deflator, which is also from the same IMF World Economic Outlook.

Measure of Robotization We first define the measure of robotization. Following Graetz and Michaels (2018), we use "robot density" as a measure of the extent to which robotization is advanced. Specifically, the robot density for country i at time t is defined as:

$$\text{robot density}_{i,t} = \frac{\text{operational robot stocks}_{i,t}}{\text{thousand of employees}_{i,t}}$$

The number of employees for country i is drawn from IMF World Economic Outlook (as of October 2018). We use data provided by the International Federation of Robotics (IFR) for our data on robot stocks, just like Acemoglu and Restrepo (2017b) and Graetz and Michaels (2018). IFR follows the definition of robots established by International Organization for Standardization (ISO), the internationally recognized standard setting body. ISO defines "industrial robot (ISO8373:2012)" as follows:

Definition. *automatically controlled, reprogrammable, multipurpose manipulator in three or more axes, which can be either fixed in place or mobile for use in industrial automation applications.*

This definition of robot implies that robot capital has a distinguishing feature from other types of capital: robot capital has high elasticity of substitution to labor. Robots are "automatically controlled," meaning they are essentially autonomous machines and do not need human operators when working. This is a distinct feature from standard machines and equipment which usually need the manipulation of human operators. In addition, robots can be programmed for "multiple purposes," just like human labor can deal with various tasks through training and education. These two characteristics indicate that robots can replace human labor.

[1] Countries covered in our dataset are Australia, Austria, Denmark, Finland, France, Germany, Italy, Japan, Korea, Netherlands, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden, U.K., and U.S.

Based on this definition of robots, IFR provides the amount of operational robot stocks by country and year, which are compiled from annual surveys of robot suppliers around the world. Although the initial year of IFR robot statistics is 1993, the data from 1993 to 1997 are relatively limited as the country coverage is sparse. Therefore, we follow the data from 1998 to 2017 to calculate robot density.

2.2 Stylized Fact: Single-Equation Specification

The most straightforward way to formally examine the impact of robotization on inflation dynamics is to estimate the following Phillips curve:[2]

$$\pi_{i,t} = C_1 + \beta_1 x_{i,t} + \beta_2 (x_{i,t} \times rd_{i,t-1}) + \beta_3 rd_{i,t-1}$$

where $\pi_{i,t}$ represents inflation in country i at time t, $x_{i,t}$ refers to output gap, and $rd_{i,t}$ means robot density.[3]

The estimation results are presented in Table 1. Note that we basically include country fixed effects and time dummies in order to control unobserved country specific factors (such as population structures), as well as global common phenomenon (like financial crisis). Firstly, as in consistent with the conventional theoretical prediction, the coefficient of $x_{i,t}$, β_1, is significantly positive in column (1) and (2), meaning that higher positive output gap leads to a higher inflation rate. The crucial term for our analysis is β_2, which is the coefficient of $x_{i,t} \times rd_{i,t-1}$. As in column (4), the coefficient of this interaction term, β_2, is significantly negative, indicating that higher robot density leads to a weaker relation between inflation and output gap. What we stress here is that the link between inflation and output gap becomes weaker as the industrial use of robots becomes more advanced.[4]

Why is this noteworthy? This result sheds new light on our understanding of the economic impact of robotization. What is implied here is that, as the economy is more dependent on robots rather than labor, inflation is more weakly linked to economic activity. This motivates us to examine how robotization would change the evolution of inflation. In particular, after the 2007-2009 Global Financial Crisis, most advanced economies experienced the missing inflation puzzle. That is, inflation has been unexpectedly low in many countries despite their steady recoveries, which has attracted a

[2] Gilchrist and Zakrajsek (2019) estimate the similar form of Phillips curve with a particular attention to trade share, not robot density in our case.

[3] The descriptive statistics of the variables in this estimation are shown in the Appendix Table.

[4] Alternatively, we can use output growth rate instead of output gap, and we can include inflation expectation in the regression equation. In particular, as a robustness check, we include the lagged values of inflation rate as a proxy of inflation expectation in the regression, following Gilchrist and Zakrajsek (2019), Gordon (1982), and Stock and Watson (2009). Even if we control the inflation expectation, the result does not change, suggesting that the link between inflation and output gap becomes weaker as robot density is higher.

great deal of attention worldwide. The result presented here implies that the recent developments in robots plays a substantial role in mitigating the transmission of positive economic activity shocks to the inflation dynamics.

In what follows, we seek to inspect the mechanism at work behind the empirical finding.

3 Model

In this section, we present an illustrative framework to understand the empirical finding. We focus on technological improvements in robots as a source of robotization. We will show a simple mechanism at work through which the degree of robotization leads to a weaker link between inflation and economic activity measured by output gap.

Our model does not have a rich set of nominal and real frictions, along the lines of Christiano et al. (2005) and Smets and Wouters (2007), and thus, comprehensive analysis of a well developed model is beyond the scope of this paper. Instead, we choose to focus our attention on a specific, but (in our opinion) rather important and fascinating feature of robots, among other characteristics: that robots can replace human labor. That is, robots have high elasticity of substitution to human labor. As is mentioned in recent studies such as Acemoglu and Restrepo (2018a,b), this feature is a striking contrast to other types of capital which are often considered as complementary to human labor. We seek to use our model as an illustrative laboratory to examine the role of this feature of robots on inflation dynamics, with particular focus on some of the implications on the missing link between inflation and economic activity.

To this end, we employ a fairly standard New-Keynesian (NK) model with sticky prices, following Justiniano et al. (2011). Departing from the existing literature, we make one critical modification. Specifically, as in Berg et al. (2018) and Lin and Weise (2017), we introduce two types of capital as inputs to the production sector. One type of capital is complementary to human labor, which is common in the literature. Distinguishing from labor-complement capital, the other capital has high elasticity of substitution to labor, which is interpreted as robots. This paper is a first step to study the quantitative implications of this modification, with particular focus on the role technological advances in robots play in the missing link.

The economy is populated by households, final good producers, intermediate goods producers, and a government. Time is discrete. The optimization problems are presented in the subsections below.

3.1 Households

A representative household maximizes a lifetime utility function separable in consumption C_t and hours worked N_t:

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^{t+s} \xi_{t+s} \left\{ \frac{C_{t+s}^{1-\gamma}}{1-\gamma} - \psi \frac{N_{t+s}^{1+\eta}}{1+\eta} \right\} \right] \tag{1}$$

The demand (preference) ξ_t follows an autoregressive process: $\ln \xi_t = \rho_\xi \ln \xi_{t-1} + \epsilon_t^\xi$. ψ means the parameter of labor disutility, and η implies the inverse of the Frisch elasticity of labor supply.

The household's budget constraint is:

$$C_t + \frac{B_{t+1}}{P_t} + I_t^K + I_t^Z + S_K \left(\frac{I_t^K}{K_t} \right) K_t + S_Z \left(\frac{I_t^Z}{Z_t} \right) Z_t + T_t = w_t N_t + r_t^K K_t + r_t^Z Z_t + i_{t-1} \frac{B_t}{P_t} + \frac{Div_t}{P_t} \tag{2}$$

where P_t is the price level and w_t means real wages. Div_t represents dividends from firm and T_t is the lump-sum tax. B_t implies government bonds. i_t is the nominal interest rate. K_t is labor-complement capital and Z_t refers to labor-substitute capital. $S_K\left(\frac{I_t^K}{K_t}\right)$ and $S_Z\left(\frac{I_t^Z}{Z_t}\right)$ are investment adjustment costs which are specified as follows:

$$S_K \left(\frac{I_t^K}{K_t} \right) = \frac{\tau_K}{2} \left(\frac{I_t^K}{K_t} - \delta_K \right)^2 \tag{3}$$

$$S_Z \left(\frac{I_t^Z}{Z_t} \right) = \frac{\tau_Z}{2} \left(\frac{I_t^Z}{Z_t} - \delta_Z \right)^2 \tag{4}$$

Then, the capital accumulation follows:

$$K_{t+1} = (1 - \delta_K) K_t + \mu_K I_t^K \tag{5}$$

$$Z_{t+1} = (1 - \delta_Z) Z_t + \mu_Z I_t^Z \tag{6}$$

μ_K and μ_Z represent the investment specific technologies for labor-complement capital and labor-substitute capital respectively. Investment specific technology describes the investment efficiency with which the final goods are transformed into capital stock. Therefore, it should be highlighted that μ_Z is closely linked to the degree of robotization.

3.2 Firms

Final Good Producers The final good producers are perfectly competitive. At every period t, they produce final output Y_t, based on a continuum of intermediate goods $Y_t(i)$, $i \in [0, 1]$. They use the following production technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{7}$$

where the parameter ε_p is the elasticity of substitution between the differentiated intermediate goods input.

The final good producers take input prices $P_t(j)$ and output prices P_t as given. The profit maximization and the zero profit condition gives us:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon_p} di\right]^{\frac{1}{1-\varepsilon_p}} \tag{8}$$

The demand function for intermediate good i is:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_p} Y_t \tag{9}$$

Intermediate Goods Producers A monopolist produces the intermediate good i, according to the nested-CES (Constant Elasticity of Substitution) production function, as in Berg et al. (2018) and Lin and Weise (2017):

$$Y_t = A_t \left[\theta_K K_t^{\frac{\alpha-1}{\alpha}} + (1-\theta_K) L_t^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \tag{10}$$

where A_t represents exogenous technological progress and follows a stationary AR(1) process. α is the elasticity of substitution between labor-complement capital K_t and labor services L_t. θ_K means a CES distribution parameter for labor-complement capital K_t, which affects the marginal production efficiency of labor-complement capital K_t. The labor services L_t are the composite of labor N_t and labor-substitute capital Z_t, and are defined as follows:

$$L_t = \left[\theta_Z Z_t^{\frac{\phi-1}{\phi}} + (1-\theta_Z) N_t^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{11}$$

where ϕ represents the elasticity of substitution between labor-substitute capital Z_t and labor N_t. θ_Z is a CES distribution parameter for labor-substitute capital Z_t, which has an impact on the marginal production efficiency of labor-substitute capital Z_t. When $\theta_Z = 0$, the marginal product of labor-substitute capital is zero, and even if there exists labor-substitute capital in the economy, it does not contribute to the production at all. In this sense, θ_Z also plays a key role in duplicating the degree of robotization.

As in Calvo (1983), a fraction of ϕ_p of intermediate goods producers cannot choose the price optimally every period, but they can adjust according to the indexation rule:

$$P_{t+s}(i) = \Pi_{t-1,t+s-1}^{\zeta_p} P_t(i) \tag{12}$$

where $\Pi_{t-1,t+s-1} = \frac{P_{t+s-1}}{P_{t-1}}$ is gross inflation, and ζ_p is the parameter on inflation indexation. The remaining fraction $(1-\phi_p)$ of firms choose the price $P_t(i)$ optimally, and they maximize the present discounted value of future profits:

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} (\beta\phi_p)^s \frac{\lambda_{t+s}}{\lambda_t} \left[\left(\frac{\Pi_{t-1,t+s-1}^{\zeta_p} P_t(j)}{P_{t+s}} - \frac{MC_{t+s}}{P_{t+s}} \right) \left(\frac{\Pi_{t-1,t+s-1}^{\zeta_p} P_t(j)}{P_{t+s}} \right)^{-\varepsilon_p} Y_{t+s} \right] \right] \tag{13}$$

subject to the demand function and cost minimization. In this objective, λ_t represents the marginal utility of nominal income for the representative household that owns the firm, while MC_t is the nominal marginal cost.

3.3 Fiscal Policy and Monetary Policy

Fiscal Policy The government spending evolves according to

$$G_t = \omega_t^g Y_t \tag{14}$$

where $\omega_t^g = (1-\rho_g)\bar{\omega}^g + \rho_g \omega_{t-1}^g + \epsilon_t^g$.

To simplify our analysis, we assume that government spending is financed by lump-sum taxes $T_t = G_t$. The exact timing of these taxes is irrelevant to the model outcome, because we assume that the government has access to lump-sum taxes and pursues Ricardian fiscal policy.

Monetary Policy The monetary authority sets the nominal interest rate following a feedback rule of the form

$$\frac{i_t}{i} = \left(\frac{i_{t-1}}{i} \right)^{\rho_i} \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_Y} \right]^{(1-\rho_i)} \epsilon_t^i \tag{15}$$

where i is the steady state of gross nominal interest rate. Interest rates respond to the deviations of inflation from its steady state π, as well as the output growth rate. The monetary policy rule is also perturbed by a monetary policy shock ϵ_t^i.

3.4 Market Clearing

The aggregate resource constraint is given by:

$$Y_t = C_t + I_t^K + I_t^Z + \frac{\tau_K}{2}\left(\frac{I_t^K}{K_t} - \delta_K\right)^2 K_t + \frac{\tau_Z}{2}\left(\frac{I_t^Z}{Z_t} - \delta_Z\right)^2 Z_t + G_t \tag{16}$$

We can compute the non-stochastic steady state of the model and log-linearly approximate the model around this steady state. Then, we solve the resulting linear system of equilibrium conditions to obtain its state space representation. The details of our equilibrium conditions can be found in Appendix 1.

3.5 Robotization in our Model

Among others, we focus on technological improvements in robots as a source of robotization. Variations in two parameters can be interpreted as changes in the degree of robotization, and we will examine quantitative predictions of our model when changing the values of these parameters.

The first is changes in μ_Z. Remember that μ_Z represents changes in the efficiency with which the final good can be transformed into labor-substitute capital ready for intermediate goods production. Hence, higher μ_Z represents increases in the marginal efficiency of investment in labor-substitute capital, which enables firms to accumulate labor-substitute capital more efficiently and with lower costs. Therefore, firms have more incentives to employ labor-substitute capital when producing intermediate goods as μ_Z becomes higher, which leads to advances in robotization.

The second source is changes in θ_Z. Note that θ_Z is a CES distribution parameter in the production of intermediate goods. All other things being equal, higher θ_Z implies higher production efficiency of labor-substitute capital.[5] Hence, since labor-substitute capital can replace human labor, when producing intermediate goods, as μ_Z becomes higher, firms have more incentives to employ labor-substitute capital instead of employing human labor, which leads to advances in robotization.

4 Parameter Values and Steady State Analysis

4.1 Parameter Values

The parameter values used in this paper are shown in Table 2. Each period is a quarter. Most values are fairly in line with the estimates of Smets and Wouters (2007).

A first set of less-controversial parameters are the parameters governing preferences. We set the relative risk aversion at $\gamma = 1$, implying that we use a standard separable utility function with logarithmic consumption. The inverse of the Frisch elasticity of labor supply η is set to 1.83 and the

[5] Alternatively, following a task-based framework developed by Acemoglu and Restrepo (2018a), a rise in θ_Z can be interpreted as an increase in the range of tasks robots can perform.

discount factor is at $\beta = 0.99$.

Regarding the final good production function, the elasticity of substitution between intermediate goods is $\varepsilon_p = 10$. For the production function of intermediate goods, the elasticity of substitution between labor-complement capital K_t and labor services L_t is $\alpha = 0.5$. The elasticity of substitution between labor-substitute capital Z_t and labor N_t is set to $\phi = 2.5$. This implies that labor-substitute capital has higher elasticity of substitution to labor than labor-complement capital.[6]

In addition, regarding our baseline values, we set $\mu_Z = 1.0$. θ_Z and θ_K are set at 0.04 and 0.35 respectively. θ_Z and θ_K are linked to the income share of labor-substitute capital and labor-complement capital. Although there is no perfectly corresponding data for the income share of labor-substitute capital, the income share of labor-substitute capital is derived from ICT capital income share, following Berg et al. (2018).

In what follows, we discuss parameters describing the extent to which robotization has advanced.

Different Values of μ_Z and θ_Z We will examine several experiments designed to illustrate how the link between inflation and overall economic activity changes, depending on changes in the extent to which robotization has advanced. More specifically, we examine some quantitative predictions on the missing link when changing values of μ_Z and θ_Z as follows.

- Several different values of μ_Z

  Our baseline is $\mu_Z = 1.0$, as is presented above. In addition to this, we consider two scenarios. First, we set $\mu_Z = 0$, keeping other parameter values at baseline values, which represents an economy without labor-substitute capital. That is, the model corresponds to a standard NK model. Second, we seek to study the macroeconomic consequences when robotization is more advanced. Then, we investigate what would happen if μ_Z increases from 1.0 to 2.0. This scenario corresponds to an economy with a higher level of robotization.

- Several different values of θ_Z

  Our baseline is $\theta_Z = 0.04$, as is described above. In addition to this, we consider two scenarios. First, we set $\theta_Z = 0$, keeping other parameter values at baseline values, which represents an economy without labor-substitute capital. That is, the model corresponds to a standard NK model. Second, we seek to study the macroeconomic consequences when robotization is more

[6] There is a growing amount of literature on estimate of the substitution elasticity between capital and labor. For example, Chirinko (2008) documents the U.S. elasticity of substitution is approximately $0.4 - 0.6$. Chirinko and Mallick (2017) report that the elasticity of substitution is estimated to be around 0.4 from the U.S. industry data. Furthermore, Karabarbounis and Neiman (2014) claim that the substitution elasticity is approximately 1.25 from the cross-country data. However, all of these estimations do not consider the heterogeneous properties between different types of capitals, such as buildings, structures, equipment and software. Here, in order to consider the different degree of the substitution elasticity, we set $\alpha = 0.5 < 1$ and $\phi = 2.5 > 1$, so that labor-complement capital K_t and labor services L_t are complement, while labor-substitute capital Z_t and labor N_t are substitute. This parameterization is in accordance with the baseline specificiation of Berg et al. (2018).

advanced. Then, we investigate what would happen if θ_Z increases from 0.04 to 0.08. This scenario corresponds to an economy with a higher level of robotization.

4.2 The Impact of Robotization at Steady State

Before studying the role of robotization on the dynamics of economic outcomes, we look at steady state values for each scenario. The results are documented in Table 3-1 and Table 3-2. There are three key points worth noting.

First, as μ_Z or θ_Z becomes higher, labor-substitute capital (Z) increases, but labor (N) decreases.[7] Higher μ_Z implies that firms can use labor-substitute capital with lower costs. Then, the decline in the user cost of labor-substitute capital induces firms to accumulate more labor-substitute capital (Z) instead of human labor (N), leading to the higher robot density ($\frac{Z}{N}$). Turning to θ_Z, a rise in θ_Z means that firms can produce intermediate goods more efficiently by using labor-substitute capital (Z), instead of employing human labor (N). Hence, firms accumulate more labor-substitute capital (Z) and decrease human labor (N).

Second, output (Y) increases as μ_Z or θ_Z becomes higher. In either case of higher μ_Z or higher θ_Z, the marginal efficiency of labor-substitute capital improves, which increases the output level. This change also results in the enhanced labor productivity ($\frac{Y}{N}$). This is because the output (Y) becomes higher, although labor input (N) is lower. Meanwhile, the labor share in national incomes ($\frac{wN}{Y}$) gets lower as robotization become more advanced, due to the higher output (Y) and lower labor (N).

Finally, we would like to stress the steady state change in real wage (w). Two factors affect the level of real wages (w). The first factor is the labor replacement effect: with the introduction of labor-substitute capital, the demand for labor declines, which always reduces real wages. However, there is another factor, which is the productivity gain effect. As the degree of robotization gets higher, labor productivity increases, which pushes real wages upward. Due to these opposite sign effects, the impact of μ_Z or θ_Z on real wages (w) depends on the degree of robotization. From $\mu_Z = 0$ to $\mu_Z = 1.0$, or from $\theta_Z = 0$ to $\theta_Z = 0.04$, real wages go down, because the labor replacement effect is larger than the productivity gain effect. Nevertheless, the further robotization (from $\mu_Z = 1.0$ to $\mu_Z = 2.0$, or from $\theta_Z = 0.04$ to $\theta_Z = 0.08$) pushes real wages to increase, because the productivity gain effect outperforms the labor replacement effect. In fact, these observations are consistent with the theoretical finding by Acemoglu and Restrepo (2018a), although the model specification between Acemoglu and Restrepo (2018a) and ours is different. They claim that whether automation raises real wages or not depends on the stage of automation. They document that the automation which is mediocrely productive decreases real wages, while the automation which is drastically productive increases real wages.

[7] Endogenous variables without time subscript means their steady state values.

5 Discussion

The calibrated model presented above is our illustrative laboratory to study the consequences of robotization on the missing link between inflation and fluctuations in economic activity.

5.1 Intuition: Analytical Example

To start with, we briefly discuss the intuition on the mechanism at work through which changes in robotization (μ_Z and θ_Z) affect the links between inflation and output gap.

As in graduate level textbooks, the basic building block is the following (linearized) NKPC that relates inflation π_t to anticipated future inflation $E_t[\pi_{t+1}]$ and real marginal cost $\tilde{mc}_t$:[8]

$$\pi_t = \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p(1+\beta\phi_p\zeta_p)}\tilde{mc}_t + \frac{\beta}{1+\beta\phi_p\zeta_p}E_t[\pi_{t+1}] + \frac{\zeta_p}{1+\beta\phi_p\zeta_p}\pi_{t-1}. \tag{17}$$

Note that inflation π_t depends positively on real marginal cost $\tilde{mc}_t$. (17) is structurally the same as the standard NKPC common in the textbook.[9]

What is important here is that as robotization is more advanced (increases in μ_Z and θ_Z), the responsiveness of real marginal cost $\tilde{mc}_t$ to structural shocks becomes weaker. This weakness leads to a decline in the responsiveness of inflation to economic slack.

To see this mechanism, we explicitly derive the link between marginal costs $\tilde{mc}_t$ and fluctuations in output gap (a measure of economic activity). As is discussed in Gali and Gertler (1999), we know that the relation between marginal cost and output gap is approximately proportional in the standard sticky price framework without variable capital. However, it is also well known that this relation is no longer proportionate in a model with variable capital, which implies that the relation between marginal costs $\tilde{mc}_t$ and output gap $(Y_t - Y_t^n)$ is given by

$$\tilde{mc}_t = X_1(\tilde{Y}_t - \tilde{Y}_t^n) + X_{2,t} \tag{18}$$

where Y_t^n denotes natural output, and thus, $\tilde{Y}_t - \tilde{Y}_t^n$ is output gap.[10] X_1 relates output gap to marginal cost. $X_{2,t}$ represents a time-varying wedge altering the relation between marginal cost and fluctuations in output gap (a measure of overall economic activity). In particular, it is worth noting that the wedge $X_{2,t}$ is explicitly described as

$$X_{2,t} = \frac{w_t - w_t^n}{w}, \tag{19}$$

[8] $\tilde{x}_t$ denotes the percent deviation from the steady state for variable x_t.
[9] The detailed derivations are shown in Appendix 2.
[10] We show the detailed derivation in Appendix 3.

where w_t^n represents real wage achieved in the absence of sticky price.

What is distinguishing in our model is that the responsiveness of the wedge $X_{2,t}$ to structural shocks depends on the degree of robotization (changes in μ_Z and θ_Z). This is a key to understanding how robotization alters the link between inflation and output gap. In other words, as robotization becomes advanced (increases in μ_Z and θ_Z), the extent to which the wedge $X_{2,t}$ responds to structural shocks is weakened. This is because the responsiveness of real wage w_t to structural shocks declines, due to labor-substitute capital.

The intuition is as follows. In this economy, human labor can be replaced with labor-substitute capital. Higher μ_Z or θ_Z implies increases in the investment efficiency or production efficiency of labor-substitute capital Z_t, and thus, firms have more incentives to employ labor-substitute capital Z_t to produce goods, instead of human labor N_t. That is, while the productivity of human labor N_t is unchanged, the efficiency of labor-substitute capital gets higher, which leads firms to employ labor-substitute capital Z_t, replacing human labor N_t. Then, when a shock hits the economy, firms try to adjust their production by using labor-substitute capital. This adjustment helps absorb the propagation of the shock to wage w_t, which leads to a decline in the responsiveness of $X_{2,t}$ to the shock. Hence, the extent to which marginal costs respond to structural shocks is further weakened as μ_Z or θ_Z becomes higher.

Therefore, as is suggested in (17), the link between inflation and fluctuations in economic activity becomes clearly weaker. This mechanism is key to understanding the impact of advances in labor-substitute capital on the evolution of inflation. In what follows, the numerical results of this model are carefully investigated.

5.2 Impulse Response Functions

We compute the impulse responses of inflation and real marginal cost to four structural shocks for each value of μ_Z and θ_Z: TFP shocks, preference shocks, fiscal policy shocks, and monetary policy shocks. All shocks are normalized to increase output. This analysis allows us to quantitatively study how the propagation of the structural shocks to inflation depends on the degree of robotization. That is, the results illustrate how changes in values of μ_Z and θ_Z affect the impulse response of inflation to structural shocks in our model.

impulse response functions for different values of μ_Z

Figure 2-1 and 2-2 report the impulse response functions of inflation and real marginal costs for $\mu_Z = 0$, 1.0 and 2.0, keeping other parameter values fixed. Red dashed, blue solid, and green dotted lines correspond to $\mu_Z = 0$, 1.0 and 2.0 respectively. The vertical axis in each graph refers to the percent deviation of each variable from its steady state. The unit of the horizontal axis is a quarter. The vertical differences of the impulse response functions in Figure 2-1 and Figure 2-2 represent the evolution of inflation changes, depending on the extent to which robotization is advanced.

We would like to stress two points here. First, the responses of inflation to each structural shock in the case of $\mu_Z = 0$ are common in the literature. Remember that the model corresponds to a standard NK model when we set $\mu_Z = 0$. For example, inflation decreases when a positive TFP shock hits the economy, but inflation increases in the case of a positive preference shock and monetary policy loosening.

Second, and more importantly, Figure 2-1 shows that the responses of inflation to each structural shock become weaker as μ_Z increases. As is shown in Figure 2-2, this is because the responses of real marginal costs get weaker, due to the wedges mentioned above.[11]

IMPULSE RESPONSE FUNCTIONS FOR DIFFERENT VALUES OF θ_Z Similarly, Figure 3-1 and Figure 3-2 show the impulse response functions of inflation and real marginal costs to each structural shock when we change θ_Z. Red dashed, blue solid, and green dotted lines correspond to $\theta_Z = 0$, 0.04 and 0.08 respectively. Note that an increase in θ_Z implies firms can produce goods more efficiently using labor-substitute capital. As in the case of μ_Z, we can verify that the impulse response functions of inflation to each structural shock become lower as θ_Z becomes larger, which implies the results quantify the implications. This is because marginal costs respond weakly to structural shocks, as is reported in Figure 3-2, due to the wedges.

In summary, we know that our model can replicate responses of inflation to shocks becoming weaker as robotization is more advanced through the mechanism presented above.

5.3 Robotization Behind the Missing Link

We now know that robotization would lower the propagation of structural shocks to inflation. In this subsection, we show that our model can rationalize the fact that the link between inflation and fluctuations in output gap becomes weaker as robotization becomes more advanced, due to the mechanism presented above.

To this end, we seek to estimate the NKPC (17), using an artificially generated time series from our model. This exercise allows us to test whether the link between inflation and output gap becomes weaker as robotization becomes more advanced, as is consistent with the empirical finding.

Using Monte Carlo methods, the model is simulated repeatedly to obtain a large number of artificially generated time series samples, based on the volatility in Table 2. We simulate 1000 samples for each structural shock. Each sample has 100 periods. All simulations start at steady

[11] In the case of monetary policy shock, the initial responses of inflation and marginal cost become more pronounced with higher robotization, though the subsequent responses are attenuated. The strong initial response is caused by the fact that the robotization leads to more capital in the economy, which is sensitive to interest rate shock. As Ruper and Šustek (2019) point out, the impact of monetary policy shock on inflation gets stronger with the introduction of investment and capital.

state, and follow the exact same sequence of 100 random shocks in each sample. After simulating the economy, we estimate the following Phillips curves using simulated data.

$$\pi_t = C_1 + \beta_1 \tilde{mc}_t + \beta_2 \pi_{t+1} + \beta_3 \pi_{t-1} \tag{20}$$

$$\pi_t = C_2 + \beta_4 x_t + \beta_5 \pi_{t+1} + \beta_6 \pi_{t-1} \tag{21}$$

where π_t indicates inflation rate, $\tilde{mc}_t$ is the marginal cost, and x_t represents output gap. Note that we define the output gap as a deviation from natural output, which is simulated using the model without nominal frictions.

Our interest is in the slope of the Phillips curve against output gap, β_4 in (21). We will compare it to the slope of the Phillips curve against marginal cost, β_1 in (20). We seek to test whether β_4 becomes smaller, which means the Phillips curve gets flatter against output gap, as μ_Z or θ_Z becomes larger (that is, the robotization becomes more advanced in order to produce goods). On the other hand, β_1, the slope of the Phillips curve against marginal cost in (20), is almost unchanged.

The averages and standard errors of estimation results are reported in Table 4-1 and Table 4-2. As expected, the coefficient of output gap, β_4, decreases for all shocks when either μ_Z or θ_Z increases, while the coefficient of marginal cost, β_1, is relatively unchanged. That is, the Phillips curve becomes flatter as the economy is more dependent on robots to produce goods, which implies robotization would be one of the possible explanations behind the missing link between inflation and economic activity.[12]

6 Conclusion

This paper has investigated whether robotization is the cause of the recent missing link between inflation and a measure of economic activity. The contributions of the paper are twofold. First, we provide empirical evidence that the link between inflation and output gap goes missing, as robotization is advanced. Second, we propose an illustrative model to understand the empirical finding. The distinguishing feature is the introduction of capital which is substitute to human labor. Using this model as an illustrative laboratory, we rationalize the view that robotization would be one factor behind the missing link.

However, our model is an illustrative one and we focus on just one feature of robots. As is discussed in many studies, modelling robots is controversial. Furthermore, our model does not have a rich set of nominal and real frictions, along the lines of Christiano et al. (2005) and Smets and

[12]Here, we use the output gap which is defined as a deviation from natural output. As a robustness check, we also use the output gap which is calculated as the detrended output using Hodrick-Prescott filter. However, the results and implications do not change.

Wouters (2007). Adding to these features is an important issue remaining for future analysis.

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Table 1: Slope of the Phillips curve

	(1)	(2)	(3)	(4)
Dependent variable	$inflation_{i,t}$	$inflation_{i,t}$	$inflation_{i,t}$	$inflation_{i,t}$
Explanatory variables				
$output\ \ gap_{i,t}$ β_1	0.111	0.112	0.267	0.111
	(0.063)	(0.063)	(0.042)	(0.062)
$output\ \ gap_{i,t} \times rd_{i,t-1}$ β_2	–	–	-0.157	-0.166
	–	–	(0.055)	(0.056)
$rd_{i,t-1}$ β_3	–	-0.012	-0.327	-0.038
	–	(0.162)	(0.133)	(0.161)
Fixed effect	yes	yes	yes	yes
Time dummy	yes	yes	no	yes
Sample size	360	360	360	360
$adj - R^2$	0.282	0.280	0.280	0.297

Notes: The number of periods is 20, from 1998 to 2017. The figures in parentheses are standard deviations.
Sources: World Robotics 2018 (International Federation of Robotics); IMF.

Table 2: Parameter values

Parameter		Value
Household		
Discount factor	β	0.99
Relative risk aversion	γ	1.0
Labor disutility	ψ	2.0
Inverse of the Frisch elasticity of labor supply	η	1.83
Firm		
Investment adjustment cost of labor-complement capital	τ_K	5.0
Investment adjustment cost of labor-substitute capital	τ_Z	5.0
Elasticity of substitution between labor-complement capital and labor services	α	0.5
Elasticity of substitution between labor-substitute capital and labor	ϕ	2.5
CES distribution parameter for labor-complement capital	θ_K	0.35
CES distribution parameter for labor-substitute capital	θ_Z	0.04
Investment specific technology of labor-complement capital	μ_K	1.0
Investment specific technology of labor-substitute capital	μ_Z	1.0
Depreciation rate of labor-complement capital	δ_K	0.025
Depreciation rate of labor-substitute capital	δ_Z	0.025
Probability of no price revision	ϕ_p	0.66
Elasticity of substitution between intermediate goods	ε_p	10
Price indexation	ζ_p	0.24
Fiscal policy		
Lagged government spending	ρ_g	0.97
Steady state government spending	ω^g	0.20
Monetary policy		
Laggged interest rate	ρ_i	0.81
Change in inflation	ϕ_π	2.04
Steady state inflation	π	1.0
Change in output	ϕ_Y	0.08
Persistence of shock		
TFP shock	ρ_A	0.9
Preference shock	ρ_ξ	0.9
Standard deviations of shocks		
TFP shock	ϵ^A	0.45
Preference shock	ϵ^ξ	0.1
Fiscal policy shock	ϵ^g	0.53
Monetary policy shock	ϵ^i	0.24

Table 3-1: Steady state values over μ_Z

		μ_Z		
		0	1.0	2.0
Output	Y	1.10	1.12	1.34
Consumption	C	0.80	0.78	0.85
Investment in labor-substitute capital	I^Z	n.a.	0.03	0.12
Investment in labor-complement capital	I^K	0.08	0.08	0.10
Labor-substitute capital	Z	n.a.	1.39	9.26
Labor-complement capital	K	3.29	3.36	4.01
Labor	N	0.81	0.80	0.80
Labor-substitute capital price	r^Z	n.a.	0.04	0.02
Labor-complement capital price	r^K	0.04	0.04	0.04
Wage	w	1.08	1.05	1.13
Inflation rate	π	1.0	1.0	1.0
Robot density	$\frac{Z}{N}$	n.a.	1.72	11.60
Labor productivity	$\frac{Y}{N}$	1.36	1.40	1.68
Labor share	$\frac{wN}{Y}$	0.80	0.75	0.67

Table 3-2: Steady state values over θ_Z

		θ_Z		
		0	0.04	0.08
Output	Y	1.10	1.12	1.71
Consumption	C	0.80	0.78	0.94
Invetsment in labor-substitute capital	I^Z	n.a.	0.03	0.30
Investment in labor-complement capital	I^K	0.08	0.08	0.13
Labor-substitute capital	Z	n.a.	1.39	11.98
Labor-complement capital	K	3.29	3.36	5.13
Labor	N	0.81	0.80	0.78
Labor-substitute capital price	r^Z	n.a.	0.04	0.04
Labor-complement capital price	r^K	0.04	0.04	0.04
Wage	w	1.08	1.05	1.20
Inflation rate	π	1.0	1.0	1.0
Robot density	$\frac{Z}{N}$	n.a.	1.72	15.31
Labor productivity	$\frac{Y}{N}$	1.36	1.40	2.19
Labor share	$\frac{wN}{Y}$	0.80	0.75	0.55

Table 4-1: Slope of Phillips curve (marginal cost representation)

		TFP shock	Preference shock	Fiscal policy shock	Monetary policy shock
NK model without labor-substitute capital		0.248	0.267	0.228	0.243
		(0.011)	(0.007)	(0.029)	(0.001)
NK model with labor-substitute capital	$\mu_Z = 1.0$	0.235	0.256	0.217	0.235
-benchmark case-	$\theta_Z = 0.04$	(0.011)	(0.007)	(0.027)	(0.001)
	$\mu_Z = 2.0$	0.212	0.258	0.230	0.222
NK model with labor-substitute capital	$\theta_Z = 0.04$	(0.016)	(0.009)	(0.016)	(0.005)
-advanced case-	$\mu_Z = 1.0$	0.242	0.241	0.238	0.211
	$\theta_Z = 0.08$	(0.015)	(0.008)	(0.023)	(0.004)

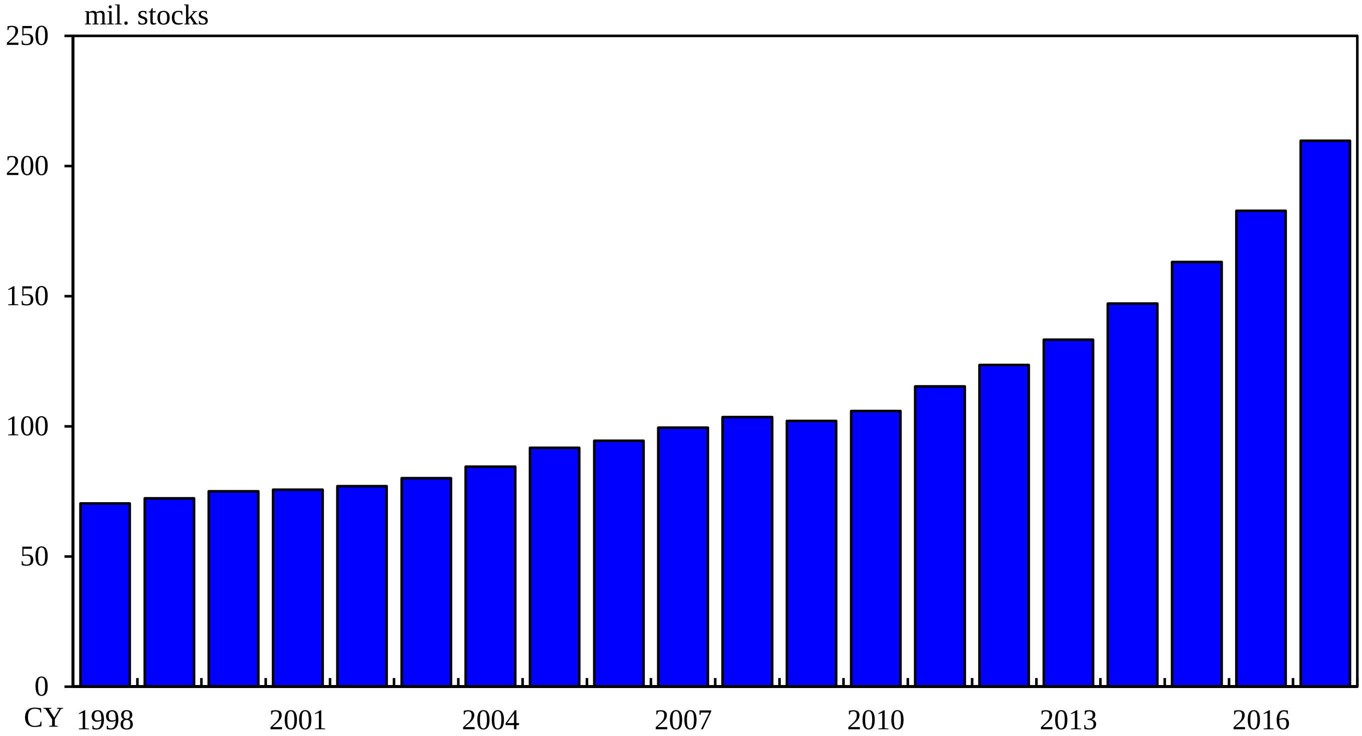
Notes. Using Monte Carlo methods, the model is simulated repeatedly to obtain 1000 artificially generated time series samples, each of which has 100 periods. The numbers in parentheses represent the standard errors of estimated coefficients using simulated data.

Table 4-2: Slope of Phillips curve (output gap representation)

		TFP shock	Preference shock	Fiscal policy shock	Monetary policy shock
NK model without labor-substitute capital		0.714	0.737	0.750	0.655
		(0.028)	(0.010)	(0.036)	(0.001)
NK model with labor-substitute capital	$\mu_Z = 1.0$	0.692	0.702	0.742	0.643
-benchmark case-	$\theta_Z = 0.04$	(0.027)	(0.009)	(0.033)	(0.002)
	$\mu_Z = 2.0$	0.583	0.678	0.715	0.590
NK model with labor-substitute capital	$\theta_Z = 0.04$	(0.037)	(0.008)	(0.029)	(0.024)
-advanced case-	$\mu_Z = 1.0$	0.609	0.642	0.685	0.565
	$\theta_Z = 0.08$	(0.041)	(0.007)	(0.028)	(0.014)

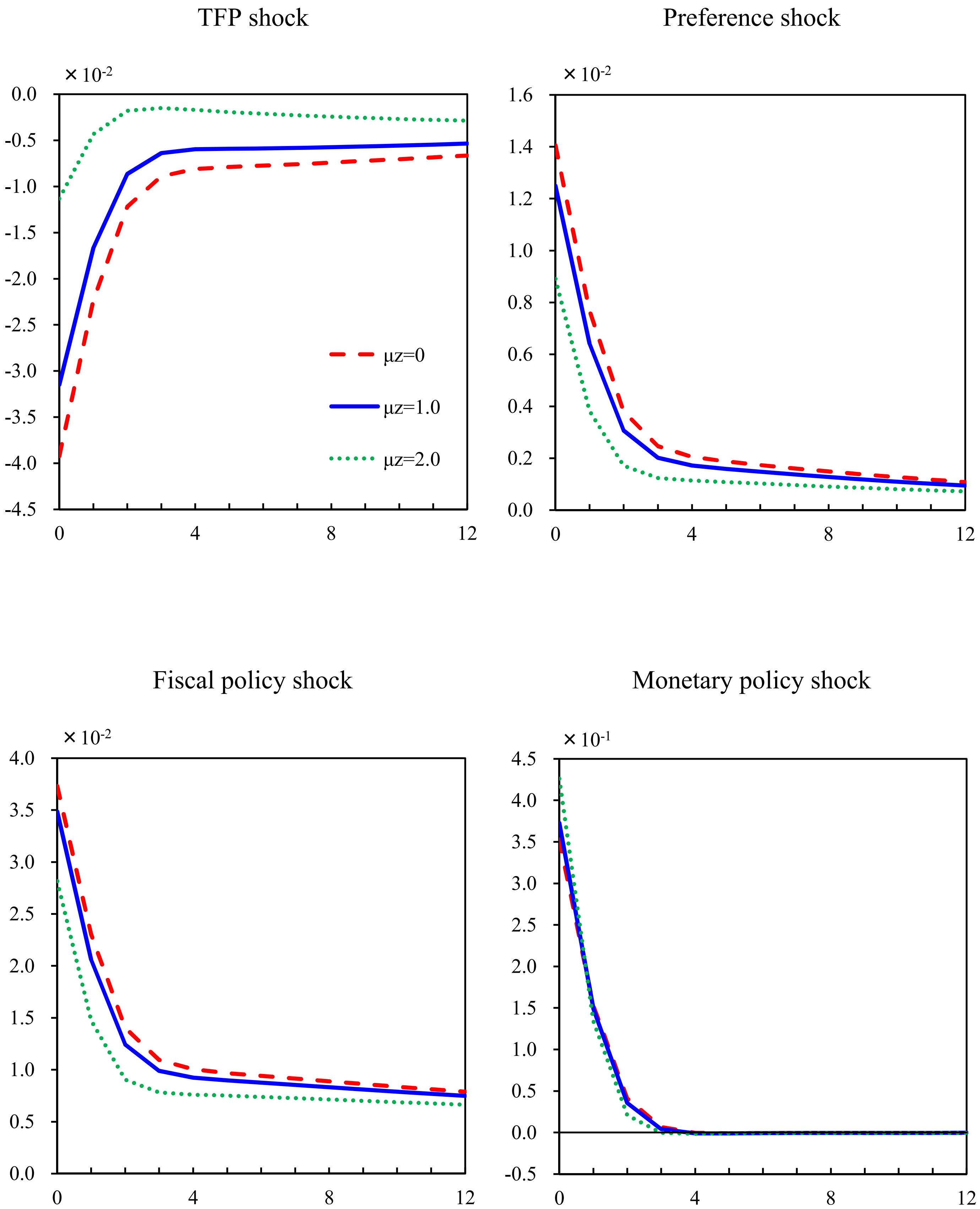
Notes. Using Monte Carlo methods, the model is simulated repeatedly to obtain 1000 artificially generated time series samples, each of which has 100 periods. The numbers in parentheses represent the standard errors of estimated coefficients using simulated data.

Figure 1: World operational robot stock



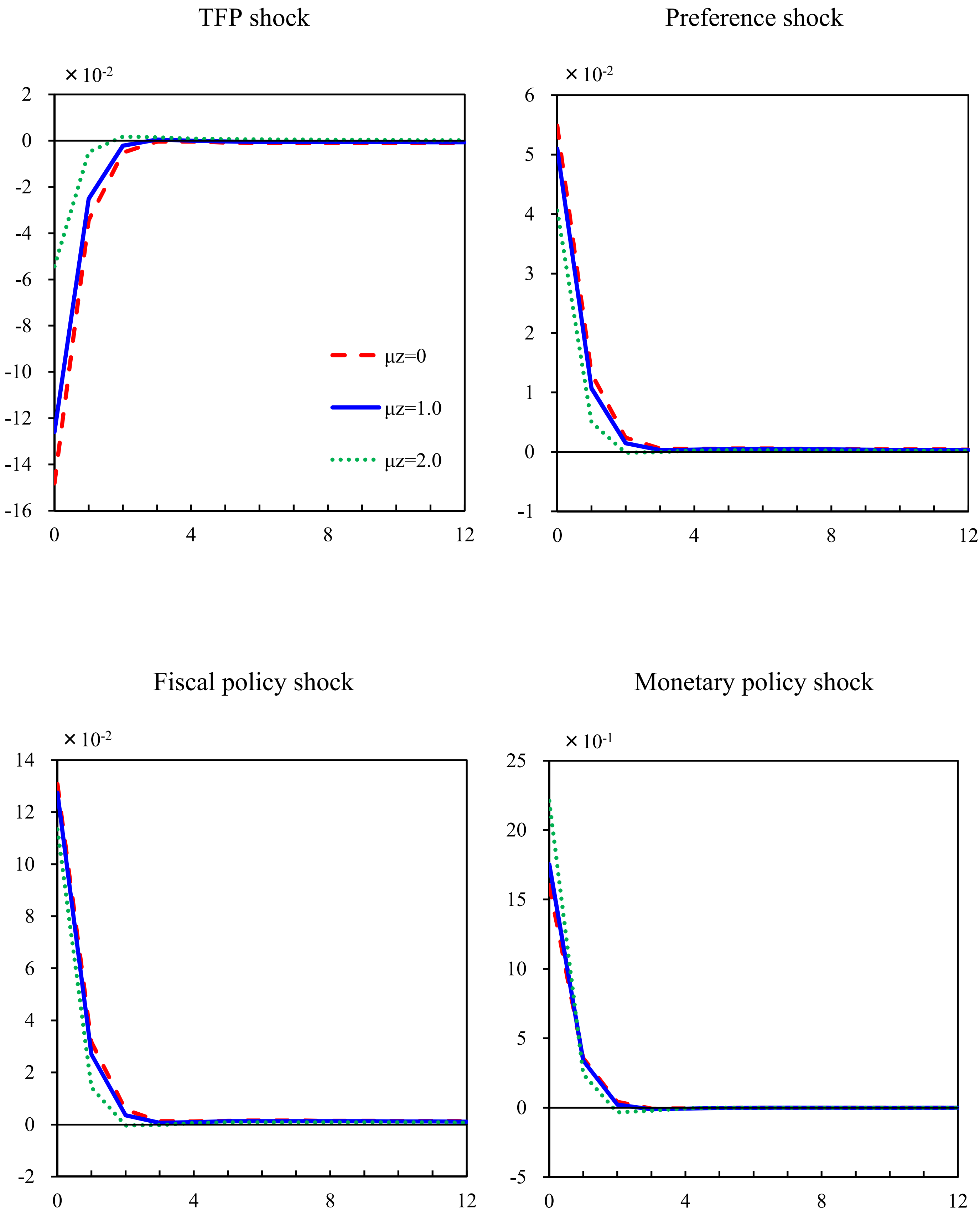
Source: World Robotics 2018 (International Federation of Robotics).

Figure 2-1: IRFs of inflation over μ_Z



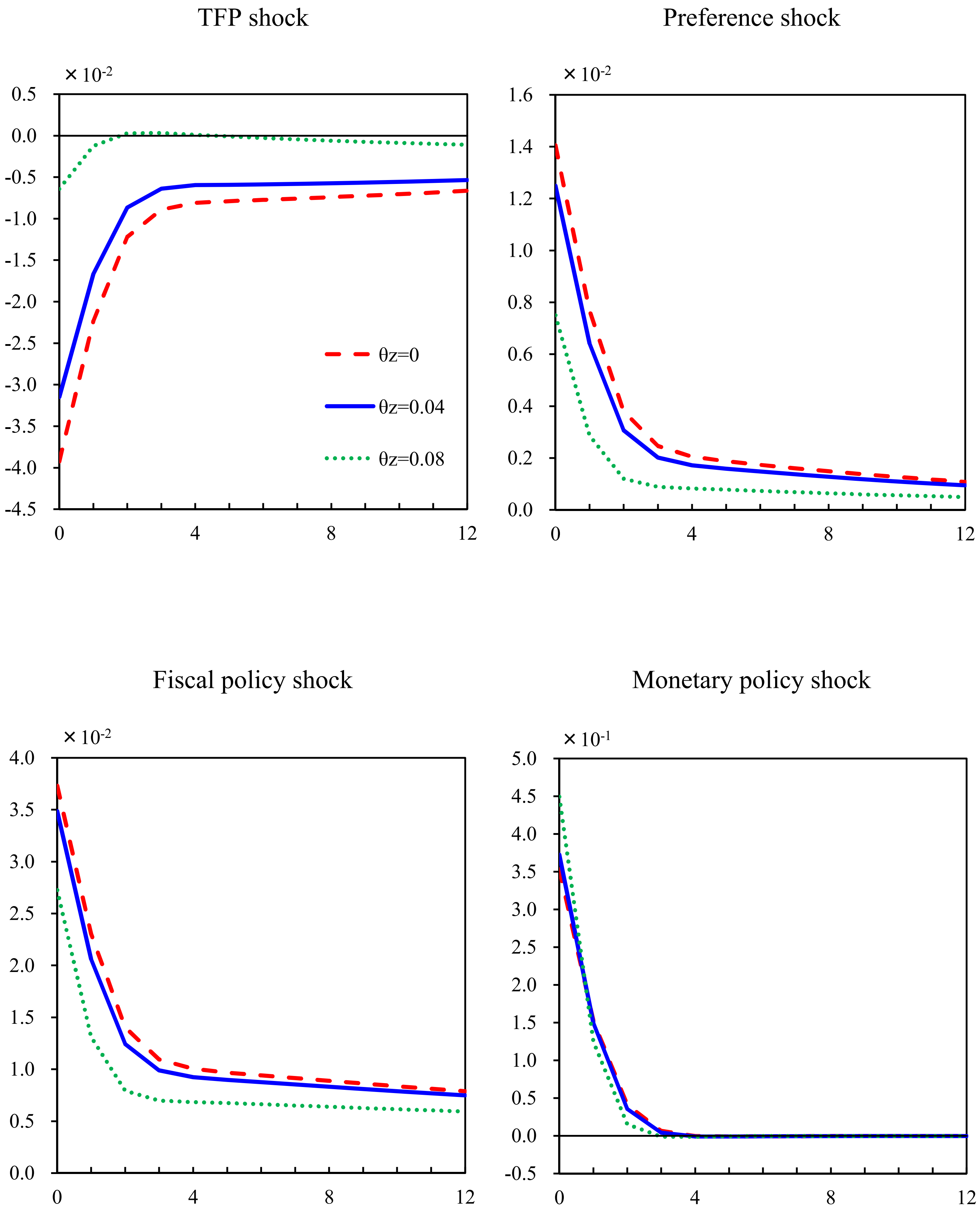
Note: The horizontal axis represents number of quarters after impulse, while the vertical axis represents the percent deviation from the steady state of each variable.

Figure 2-2: IRFs of marginal cost over μ_Z



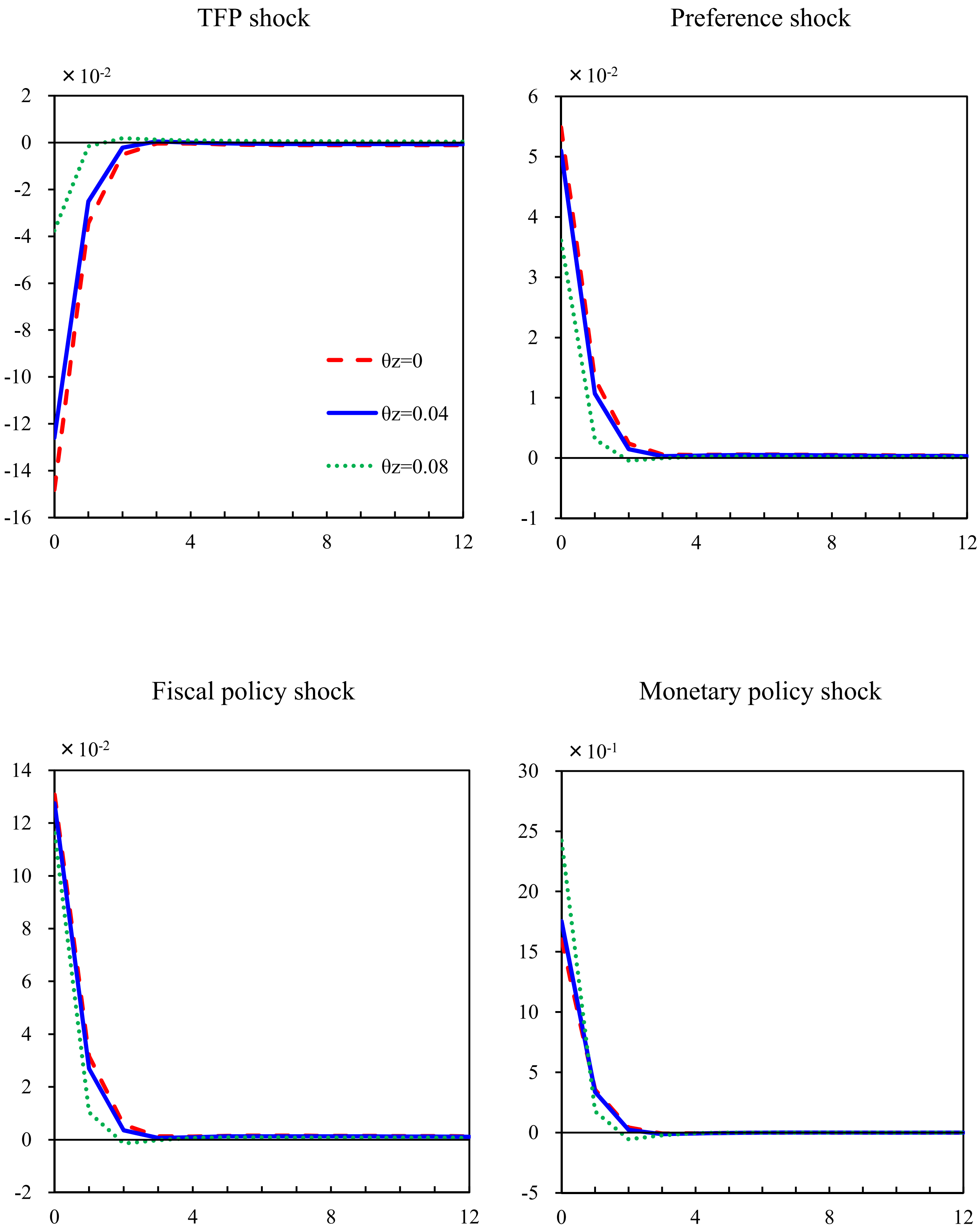
Note: The horizontal axis represents number of quarters after impulse, while the vertical axis represents the percent deviation from the steady state of each variable.

Figure 3-1: IRFs of inflation over θ_Z



Note: The horizontal axis represents number of quarters after impulse, while the vertical axis represents the percent deviation from the steady state of each variable.

Figure 3-2: IRFs of marginal cost over θ_Z



Note: The horizontal axis represents number of quarters after impulse, while the vertical axis represents the percent deviation from the steady state of each variable.

Appendix Table: Descriptive statistics

	Inflation	Output gap	Robot density
Number of observations	360	360	360
Average	1.931	-0.197	1.655
Standard deviation	2.007	2.231	1.555
Median	1.722	-0.137	1.276
Maximum	15.434	7.623	10.219
Minimum	-5.204	-7.761	0.172
Kurtosis	7.582	1.290	5.249
Skewness	1.616	-0.064	2.049

Sources: World Robotics 2018 (International Federation of Robotics); IMF.

Appendix 1: Details of Equilibrium Conditions

1 Full Set of Equilibirum Equations

Household

$$\lambda_t = \xi_t C_t^{-\gamma} \tag{1}$$

$$\lambda_t = \beta E\left[\lambda_{t+1}\frac{i_t}{\pi_{t+1}}\right] \tag{2}$$

$$\mu_K q_t^K = 1 + \tau\left(\frac{I_t^K}{K_t} - \delta_K\right) \tag{3}$$

$$\mu_Z q_t^Z = 1 + \tau\left(\frac{I_t^Z}{Z_t} - \delta_Z\right) \tag{4}$$

$$\lambda_t q_t^K = \beta E\left[\lambda_{t+1}\left\{q_{t+1}^K\left(1-\delta_K\right) + r_{t+1}^K + \frac{\tau}{2}\left(\frac{I_{t+1}^K}{K_{t+1}} - \delta_K\right)\left(\frac{I_{t+1}^K}{K_{t+1}} + \delta_K\right)\right\}\right] \tag{5}$$

$$\lambda_t q_t^Z = \beta E\left[\lambda_{t+1}\left\{q_{t+1}^Z\left(1-\delta_Z\right) + r_{t+1}^Z + \frac{\tau}{2}\left(\frac{I_{t+1}^Z}{Z_{t+1}} - \delta_Z\right)\left(\frac{I_{t+1}^Z}{Z_{t+1}} + \delta_Z\right)\right\}\right] \tag{6}$$

$$K_{t+1} = \left(1-\delta_K\right)K_t + \mu_K I_t^K \tag{7}$$

$$Z_{t+1} = \left(1-\delta_Z\right)Z_t + \mu_Z I_t^Z \tag{8}$$

$$\lambda_t w_t = \psi \xi_t N_t^{\eta} \tag{9}$$

Firm

$$Y_t = \frac{\left[\theta_K\left(\frac{K_t}{L_t}\right)^{\frac{\alpha-1}{\alpha}} + \left(1-\theta_K\right)\right]^{\frac{\alpha}{\alpha-1}}\left[\theta_Z\left(\frac{Z_t}{N_t}\right)^{\frac{\phi-1}{\phi}} + \left(1-\theta_Z\right)\right]^{\frac{\phi}{\phi-1}} N_t}{\vartheta_t^p} \tag{10}$$

$$\vartheta_t^p = \pi_t^{\varepsilon_p}\left\{\left(1-\phi_p\right)\left(\pi_t^{\#}\right)^{-\varepsilon_p} + \phi_p \pi_{t-1}^{-\zeta_p\varepsilon_p}\vartheta_{t-1}^p\right\} \tag{11}$$

$$\pi_t^{1-\varepsilon_p} = \left(1-\phi_p\right)\left(\pi_t^{\#}\right)^{1-\varepsilon_p} + \phi_p \pi_{t-1}^{\zeta_p(1-\varepsilon_p)} \tag{12}$$

$$\pi_t^{\#} = \frac{\varepsilon_p}{\varepsilon_p - 1}\pi_t \frac{x_{1,t}}{x_{2,t}} \tag{13}$$

$$x_{1,t} = \lambda_t mc_t Y_t + \beta\phi_p \pi_t^{-\zeta_p\varepsilon_p} E_t\left[\pi_{t+1}^{\varepsilon_p} x_{1,t+1}\right] \tag{14}$$

$$x_{2,t} = \lambda_t Y_t + \beta\phi_p \pi_t^{\zeta_p(1-\varepsilon_p)} E_t\left[\pi_{t+1}^{\varepsilon_p - 1} x_{2,t+1}\right] \tag{15}$$

$$r_t^K = mc_t \theta_K\left(\frac{Y_t}{K_t}\right)^{\frac{1}{\alpha}} \tag{16}$$

$$r_t^Z = mc_t\left(1-\theta_K\right)\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\alpha}}\theta_Z\left(\frac{L_t}{Z_t}\right)^{\frac{1}{\phi}} \tag{17}$$

$$w_t = mc_t\left(1-\theta_K\right)\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\alpha}}\left(1-\theta_Z\right)\left(\frac{L_t}{N_t}\right)^{\frac{1}{\phi}} \tag{18}$$

$$L_t = \left[\theta_Z Z_t^{\frac{\phi-1}{\phi}} + \left(1-\theta_Z\right)N_t^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{19}$$

Economywide (Resource Constraint)

$$Y_t = C_t + I_t^K + I_t^Z + \frac{\tau}{2}\left(\frac{I_t^K}{K_t} - \delta_K\right)^2 K_t + \frac{\tau}{2}\left(\frac{I_t^Z}{Z_t} - \delta_Z\right)^2 Z_t + G_t \tag{20}$$

Shocks

Fiscal Policy shock

$$G_t = \omega_t^g Y_t \tag{21}$$

$$\omega_t^g = (1-\rho_g)\,\bar{\omega}^g + \rho_g \omega_{t-1}^g + \epsilon_t^g \tag{22}$$

Monetary Policy shock

$$\frac{i_t}{i} = \left(\frac{i_{t-1}}{i}\right)^{\rho_i}\left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_Y}\right]^{1-\rho_i}\epsilon_t^i \tag{23}$$

Preference shock

$$\ln \xi_t = \rho_\xi \ln \xi_{t-1} + \epsilon_t^\xi \tag{24}$$

2 List of Endogenous Variables (24 variables)

$$\left\{C_t,\, N_t,\, \xi_t,\, I_t^K,\, I_t^Z,\, w_t,\, r_t^K,\, r_t^Z,\, K_{t+1},\, Z_{t+1},\, \lambda_t,\, q_t^K,\, q_t^Z,\, Y_t,\, L_t,\, mc_t,\, \vartheta_t^P,\, \pi_t,\, \pi_t^{\#}, x_{1,t},\, x_{2,t},\, G_t,\, \omega_t^g,\, i_t\right\}$$

3 List of Exogenous Variables (3 variables)

$$\left\{\epsilon_t^\xi,\, \epsilon_t^g,\, \epsilon_t^i\right\}$$

4 List of Parameters (24 parameters)

$$\{\beta,\, \gamma,\, \psi,\, \eta,\, \rho_\xi,\, \tau,\, \delta_K,\, \delta_Z,\, \mu_K,\, \mu_Z,\, \varepsilon_p,\, \zeta_p,\, \phi_p,\, \alpha,\, \phi,\, \theta_K,\, \theta_Z,\, \rho_g,\, \rho_i,\, i,\, \phi_\pi,\, \phi_Y,\, \bar{\omega}^g,\, \pi^*\}$$

Appendix 2: Induction of New Keynesian Phillips curve (NKPC)

1 Profit maximization problem of firms

$$\max E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\frac{\lambda_{t+s}}{\lambda_t}\left\{\left(\frac{\Pi_{t-1,t+s-1}^{\zeta_p}P_t(i)}{P_{t+s}}\right)\left(\frac{\Pi_{t-1,t+s-1}^{\zeta_p}P_t(i)}{P_{t+s}}\right)^{-\epsilon_p}Y_{t+s}-\frac{MC_{t+s}}{P_{t+s}}\left(\frac{\Pi_{t-1,t+s-1}^{\zeta_p}P_t(i)}{P_{t+s}}\right)^{-\epsilon_p}Y_{t+s}\right\}\right]$$

$$\Leftrightarrow \max E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\frac{\lambda_{t+s}}{\lambda_t}\left\{\Pi_{t-1,t+s-1}^{\zeta_p}P_t(i)\left(\frac{\Pi_{t-1,t+s-1}^{\zeta_p}P_t(i)}{P_{t+s}}\right)^{-\epsilon_p}Y_{t+s}-MC_{t+s}\left(\frac{\Pi_{t-1,t+s-1}^{\zeta_p}P_t(i)}{P_{t+s}}\right)^{-\epsilon_p}Y_{t+s}\right\}\right]$$

FOC wrt $P_t(i)$;

$$(1-\epsilon_p)E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\frac{\lambda_{t+s}}{\lambda_t}\Pi_{t-1,t+s-1}^{\zeta_p(1-\epsilon_p)}P_t(i)^{-\epsilon_p}P_{t+s}^{\epsilon_p}Y_{t+s}\right]$$

$$+\epsilon_pE_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\frac{\lambda_{t+s}}{\lambda_t}MC_{t+s}\Pi_{t-1,t+s-1}^{-\zeta_p\epsilon_p}P_t(i)^{-\epsilon_p-1}P_{t+s}^{\epsilon_p}Y_{t+s}\right]=0$$

$$\Leftrightarrow(\epsilon_p-1)E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\frac{\lambda_{t+s}}{\lambda_t}\Pi_{t-1,t+s-1}^{\zeta_p(1-\epsilon_p)}P_t(i)^{-\epsilon_p}P_{t+s}^{\epsilon_p}Y_{t+s}\right]$$

$$=\epsilon_pE_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\frac{\lambda_{t+s}}{\lambda_t}MC_{t+s}\Pi_{t-1,t+s-1}^{-\zeta_p\epsilon_p}P_t(i)^{-\epsilon_p-1}P_{t+s}^{\epsilon_p}Y_{t+s}\right]$$

$$\Leftrightarrow P_t(i)=\frac{\epsilon_p}{\epsilon_p-1}\frac{E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\lambda_{t+s}mc_{t+s}P_{t+s}Y_{t+s}(i)\right]}{E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\lambda_{t+s}\Pi_{t-1,t+s-1}^{\zeta_p}Y_{t+s}(i)\right]}$$

$$\Leftrightarrow P_t(i)E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\lambda_{t+s}\Pi_{t-1,t+s-1}^{\zeta_p}Y_{t+s}(i)\right]=\frac{\epsilon_p}{\epsilon_p-1}E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\lambda_{t+s}mc_{t+s}P_{t+s}Y_{t+s}(i)\right]$$

Using Uhlig's log-linearization

$$(LHS)=E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\lambda_{t+s}\Pi_{t-1,t+s-1}^{\zeta_p}P_t(i)Y_{t+s}(i)\right]$$

$$\approx E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\bar{\lambda}\bar{\Pi}^{\zeta_p}\bar{P}(i)\bar{Y}(i)\left(1+\tilde{\lambda}_{t+s}+\zeta_p\tilde{\Pi}_{t-1,t+s-1}+\tilde{P}_t(i)+\tilde{Y}_{t+s}(i)\right)\right]$$

$$=\frac{\bar{\lambda}\bar{\Pi}^{\zeta_p}\bar{P}(i)\bar{Y}(i)}{1-\beta\phi_p}\left(1+\tilde{P}_t(i)\right)+\bar{\lambda}\bar{\Pi}^{\zeta_p}\bar{P}(i)\bar{Y}(i)E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\left(\tilde{\lambda}_{t+s}+\zeta_p\tilde{\Pi}_{t-1,t+s-1}+\tilde{Y}_{t+s}(i)\right)\right]$$

$$(RHS)=\frac{\epsilon_p}{\epsilon_p-1}E_t\left[\sum_{s=0}^{\infty}(\beta\phi_p)^s\lambda_{t+s}mc_{t+s}P_{t+s}Y_{t+s}(i)\right]$$

$$\approx \frac{\epsilon_p}{\epsilon_p - 1} E_t \left[\sum_{s=0}^{\infty} (\beta\phi_p)^s \bar{\lambda} \bar{mc} \bar{P} \bar{Y}(i) \left(1 + \tilde{\lambda}_{t+s} + \tilde{mc}_{t+s} + \tilde{P}_{t+s} + \tilde{Y}_{t+s}(i) \right) \right]$$

Because $\bar{mc} = \frac{\epsilon_p}{\epsilon_p - 1}$ at the steady state:

$$(RHS) = E_t \left[\sum_{s=0}^{\infty} (\beta\phi_p)^s \bar{\lambda} \bar{P} \bar{Y}(i) \left(1 + \tilde{\lambda}_{t+s} + \tilde{mc}_{t+s} + \tilde{P}_{t+s} + \tilde{Y}_{t+s}(i) \right) \right]$$

$$= \bar{\lambda} \bar{P} \bar{Y}(i) E_t \left[\sum_{s=0}^{\infty} (\beta\phi_p)^s \left(1 + \tilde{\lambda}_{t+s} + \tilde{mc}_{t+s} + \tilde{P}_{t+s} + \tilde{Y}_{t+s}(i) \right) \right]$$

At the steady state:

$$\bar{P} = \bar{P}(i)$$

$$\bar{\Pi} = 1$$

so, $\bar{\lambda}\bar{\Pi}^{\zeta_p}\bar{P}(i)\bar{Y}(i)$ on the (LHS) and $\bar{\lambda}\bar{P}\bar{Y}(i)$ on the (RHS) are cancelled out.
Therefore:

$$\frac{1}{1-\beta\phi_p} \left(1 + \tilde{P}_t(i) \right) + E_t \left[\sum_{s=0}^{\infty} (\beta\phi_p)^s \left(\tilde{\lambda}_{t+s} + \zeta_p \tilde{\Pi}_{t-1,t+s-1} + \tilde{Y}_{t+s}(i) \right) \right]$$

$$= E_t \left[\sum_{s=0}^{\infty} (\beta\phi_p)^s \left(1 + \tilde{\lambda}_{t+s} + \tilde{mc}_{t+s} + \tilde{P}_{t+s} + \tilde{Y}_{t+s}(i) \right) \right]$$

$$\Leftrightarrow \tilde{P}_t(i) = (1-\beta\phi_p) E_t \left[\sum_{s=0}^{\infty} (\beta\phi_p)^s \left(\tilde{mc}_{t+s} + \tilde{P}_{t+s} - \zeta_p \tilde{\Pi}_{t-1,t+s-1} \right) \right] \tag{1}$$

2 Inflation Equation (New Keynesian Phillips Curve)

Beginning with the final goods pricing rule:

$$P_t^{1-\epsilon_p} = (1-\phi_p) P_t^{\#}(i)^{1-\epsilon_p} + \phi_p \pi_{t-1}^{\zeta_p(1-\epsilon_p)} P_{t-1}^{1-\epsilon_p}$$

Using Uhlig's log-linearization:

$$\bar{P}^{1-\epsilon_p} \left(1 + (1-\epsilon_p)\tilde{P}_t \right) \approx (1-\phi_p) \bar{P\#}^{1-\epsilon_p} \left(1 + (1-\epsilon_p) \tilde{P\#}_t \right) + \phi_p \bar{\pi}^{\zeta_p(1-\epsilon_p)} \bar{P}^{1-\epsilon_p} \left(1 + \zeta_p (1-\epsilon_p) \tilde{\pi}_{t-1} + (1-\epsilon_p) \tilde{P}_{t-1} \right)$$

$$\Leftrightarrow \tilde{P}_t = (1-\phi_p) \tilde{P\#}_t + \phi_p \left(\zeta_p \tilde{\pi}_{t-1} + \tilde{P}_{t-1} \right)$$

$$\Leftrightarrow \tilde{P}_t = \phi_p \tilde{P}_{t-1} + (1-\phi_p)(1-\beta\phi_p) E_t \left[\sum_{s=0}^{\infty} (\beta\phi_p)^s \left(\tilde{mc}_{t+s} + \tilde{P}_{t+s} - \zeta_p \tilde{\Pi}_{t-1,t+s-1} \right) \right] + \phi_p \zeta_p \tilde{\pi}_{t-1}$$

Using the lag operator L:

$$\Leftrightarrow \left(1 - \beta\phi_p L^{-1} \right) \tilde{P}_t = \left(1 - \beta\phi_p L^{-1} \right) \phi_p \tilde{P}_{t-1}$$

$$+\left(1-\beta\phi_p L^{-1}\right)\left(1-\phi_p\right)\left(1-\beta\phi_p\right)E_t\left[\sum_{s=0}^{\infty}\left(\beta\phi_p\right)^s\left(\tilde{mc}_{t+s}+\tilde{P}_{t+s}-\zeta_p\tilde{\Pi}_{t-1,t+s-1}\right)\right]+\left(1-\beta\phi_p L^{-1}\right)\phi_p\zeta_p\tilde{\pi}_{t-1}$$

$$\Leftrightarrow \tilde{P}_t-\beta\phi_p E_t\left[\tilde{P}_{t+1}\right]=\phi_p\tilde{P}_{t-1}+\left(1-\phi_p\right)\left(1-\beta\phi_p\right)E_t\left[\sum_{s=0}^{\infty}\left(\beta\phi_p\right)^s\left(\tilde{mc}_{t+s}+\tilde{P}_{t+s}-\zeta_p\tilde{\Pi}_{t-1,t+s-1}\right)\right]+\phi_p\zeta_p\tilde{\pi}_{t-1}$$

$$-\beta\phi_p\phi_p\tilde{P}_t-\beta\phi_p\left(1-\phi_p\right)\left(1-\beta\phi_p\right)E_t\left[\sum_{s=0}^{\infty}\left(\beta\phi_p\right)^s\left(\tilde{mc}_{t+1+s}+\tilde{P}_{t+1+s}-\zeta_p\tilde{\Pi}_{t,t+s}\right)\right]-\beta\phi_p\phi_p\zeta_p\tilde{\pi}_t$$

$$\Leftrightarrow \tilde{P}_t-\beta\phi_p E_t\left[\tilde{P}_{t+1}\right]=\phi_p\tilde{P}_{t-1}-\beta\phi_p\phi_p\tilde{P}_t+\left(1-\phi_p\right)\left(1-\beta\phi_p\right)\left(\tilde{mc}_t+\tilde{P}_t-\zeta_p\tilde{\Pi}_{t-1,t-1}\right)+\phi_p\zeta_p\tilde{\pi}_{t-1}-\beta\phi_p\phi_p\zeta_p\tilde{\pi}_t$$

$$\Leftrightarrow \tilde{P}_t-\beta\phi_p E_t\left[\tilde{P}_{t+1}\right]=\phi_p\tilde{P}_{t-1}-\beta\phi_p\phi_p\tilde{P}_t+\left(1-\phi_p\right)\left(1-\beta\phi_p\right)\left(\tilde{mc}_t+\tilde{P}_t\right)+\phi_p\zeta_p\tilde{\pi}_{t-1}-\beta\phi_p\phi_p\zeta_p\tilde{\pi}_t$$

$$\Leftrightarrow \tilde{\pi}_t=\frac{\beta}{1+\beta\phi_p\zeta_p}E_t\left[\tilde{\pi}_{t+1}\right]+\frac{\zeta_p}{1+\beta\phi_p\zeta_p}\tilde{\pi}_{t-1}+\frac{\left(1-\phi_p\right)\left(1-\beta\phi_p\right)}{\phi_p\left(1+\beta\phi_p\zeta_p\right)}\tilde{mc}_t \tag{2}$$

Appendix 3: Marginal Cost and Output Gap

Based on the model specification in the paper, we derive the marginal cost which is represented by output gap.

From the firms' cost minimization problem,

$$\min r_t^K K_t + r_t^Z Z_t + w_t N_t \tag{1}$$

subject to

$$Y_t = A_t \left[\theta_K K_t^{\frac{\alpha-1}{\alpha}} + (1-\theta_K) L_t^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}} \tag{2}$$

$$L_t = \left[\theta_Z Z_t^{\frac{\phi-1}{\phi}} + (1-\theta_Z) N_t^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{3}$$

First order condition with respect to N_t is

$$mc_t = \frac{w_t}{(1-\theta_K)(1-\theta_Z)} A_t^{\frac{1-\alpha}{\alpha}} \left(\frac{L_t}{Y_t} \right)^{\frac{1}{\alpha}} \left(\frac{N_t}{L_t} \right)^{\frac{1}{\phi}} \tag{4}$$

Log-linearizing this equation gives

$$\frac{dmc_t}{mc} = \frac{1-\alpha}{\alpha} A_t + \frac{dw_t}{w} - \frac{1}{\alpha}\frac{dY_t}{Y} + \frac{\phi-\alpha}{\alpha\phi}\frac{dL_t}{L} + \frac{1}{\phi}\frac{dN_t}{N} \tag{5}$$

By log-linearizing (2), (3), and substituting them into (5):

$$\frac{dmc_t}{mc} = \left[\frac{1}{\phi}\frac{1}{1-\theta_K}\frac{1}{1-\theta_Z}\left(\frac{L}{N}\right)^{\frac{\phi-1}{\phi}}\left(\frac{AL}{Y}\right)^{\frac{1-\alpha}{\alpha}} - \frac{1}{\alpha} + \frac{\phi-\alpha}{\alpha\phi}\frac{1}{1-\theta_K}\left(\frac{AL}{Y}\right)^{\frac{1-\alpha}{\alpha}} \right] \frac{Y_t - Y_t^n}{Y} + \frac{w_t - w_t^n}{w} \tag{6}$$

In the representation of the paper:

$$X_1 = \frac{1}{\phi}\frac{1}{1-\theta_K}\frac{1}{1-\theta_Z}\left(\frac{L}{N}\right)^{\frac{\phi-1}{\phi}}\left(\frac{AL}{Y}\right)^{\frac{1-\alpha}{\alpha}} - \frac{1}{\alpha} + \frac{\phi-\alpha}{\alpha\phi}\frac{1}{1-\theta_K}\left(\frac{AL}{Y}\right)^{\frac{1-\alpha}{\alpha}}$$

$$X_{2,t} = \frac{w_t - w_t^n}{w}$$